Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.3-Inverse-tangent/150-5.3.4-u-a+b-arctan-c-x-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1301]. This is test number [150].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	98.69 (1284)	1.31 (17)
Mathematica	98.39 (1280)	1.61 (21)
Maple	92.08 (1198)	7.92 (103)
Mupad	57.96 (754)	42.04 (547)
Sympy	44.58 (580)	55.42 (721)
Fricas	43.43 (565)	56.57 (736)
Giac	35.13 (457)	64.87 (844)
Maxima	31.28 (407)	68.72 (894)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

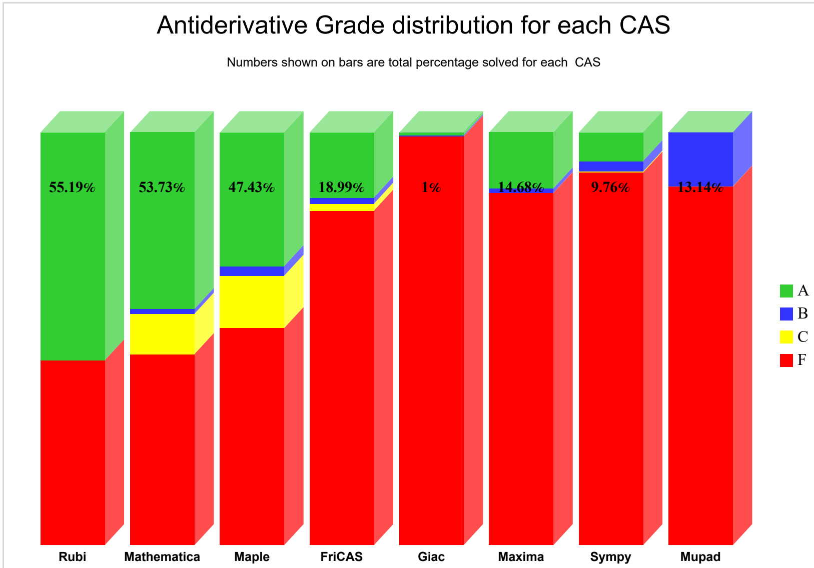
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

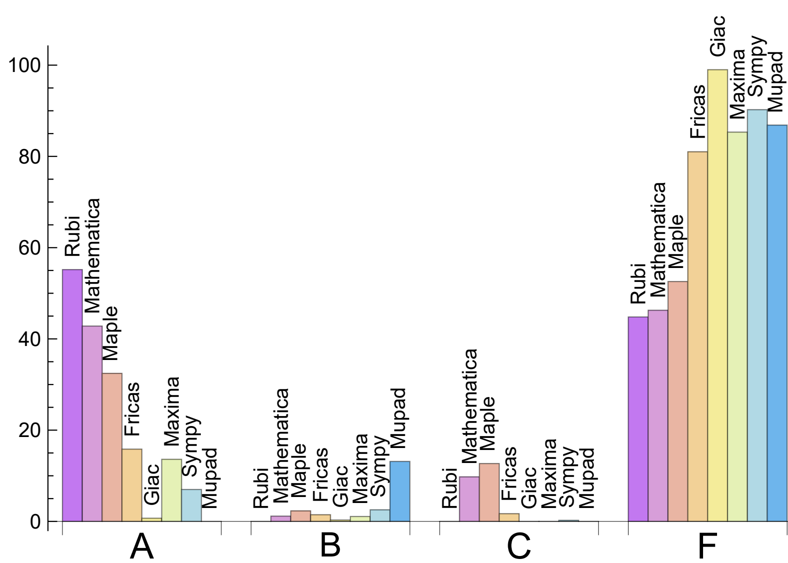
System	% A grade	% B grade	% C grade	% F grade
Rubi	52.652	1.230	0.000	46.118
Mathematica	42.813	1.153	9.762	46.272
Maple	32.437	2.306	12.683	52.575
Fricas	15.834	1.460	1.691	81.015
Maxima	13.605	1.076	0.000	85.319
Sympy	6.995	2.537	0.231	90.238
Giac	0.692	0.307	0.000	99.001
Mupad	0.000	13.144	0.000	86.856

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	17	100.00	0.00	0.00
Mathematica	21	66.67	33.33	0.00
Maple	103	97.09	2.91	0.00
Fricas	736	52.85	0.00	47.15
Maxima	894	42.84	0.67	56.49
Mupad	547	0.00	100.00	0.00
Sympy	721	65.60	32.04	2.36
Giac	844	65.40	12.44	22.16

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.35
Maxima	0.42
Mupad	0.56
Rubi	0.80
Mathematica	2.13
Maple	10.79
Sympy	19.95
Giac	100.84

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	5.14	0.19	3.00	0.13
Mupad	51.98	1.03	24.00	1.00
Sympy	73.86	1.50	29.00	1.20
Maxima	108.24	2.14	73.00	1.09
Fricas	130.55	1.70	50.00	1.14
Rubi	149.95	1.04	65.50	1.00
Mathematica	159.40	1.07	56.00	1.08
Maple	355.33	1.74	26.00	0.92

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

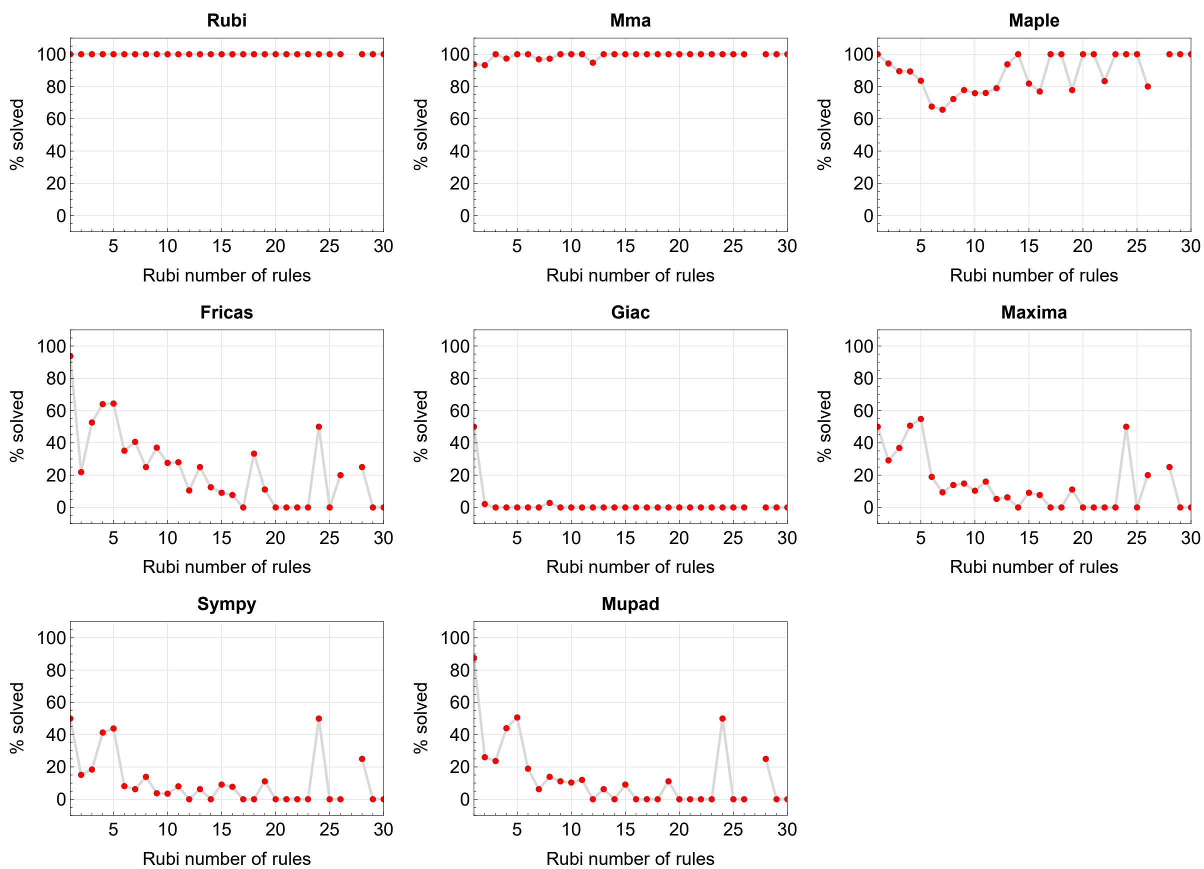


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

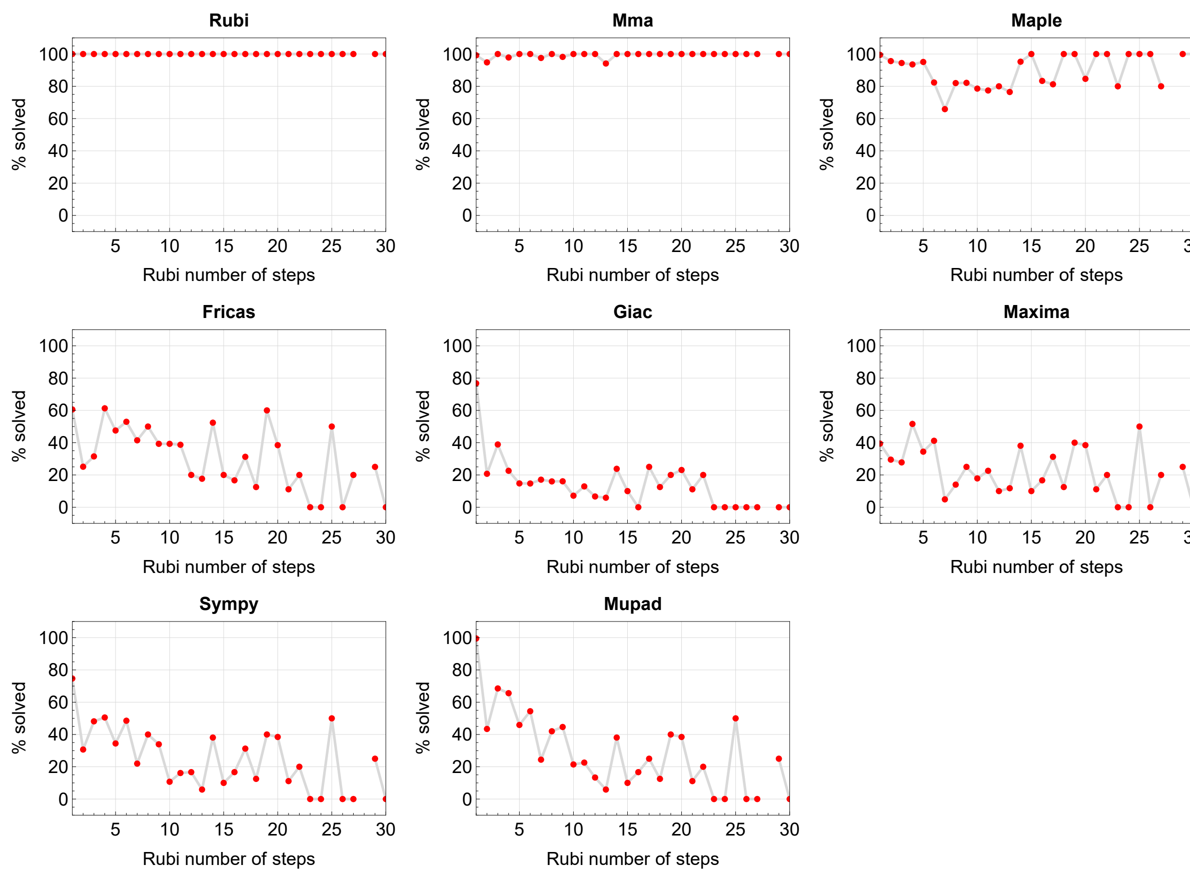


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

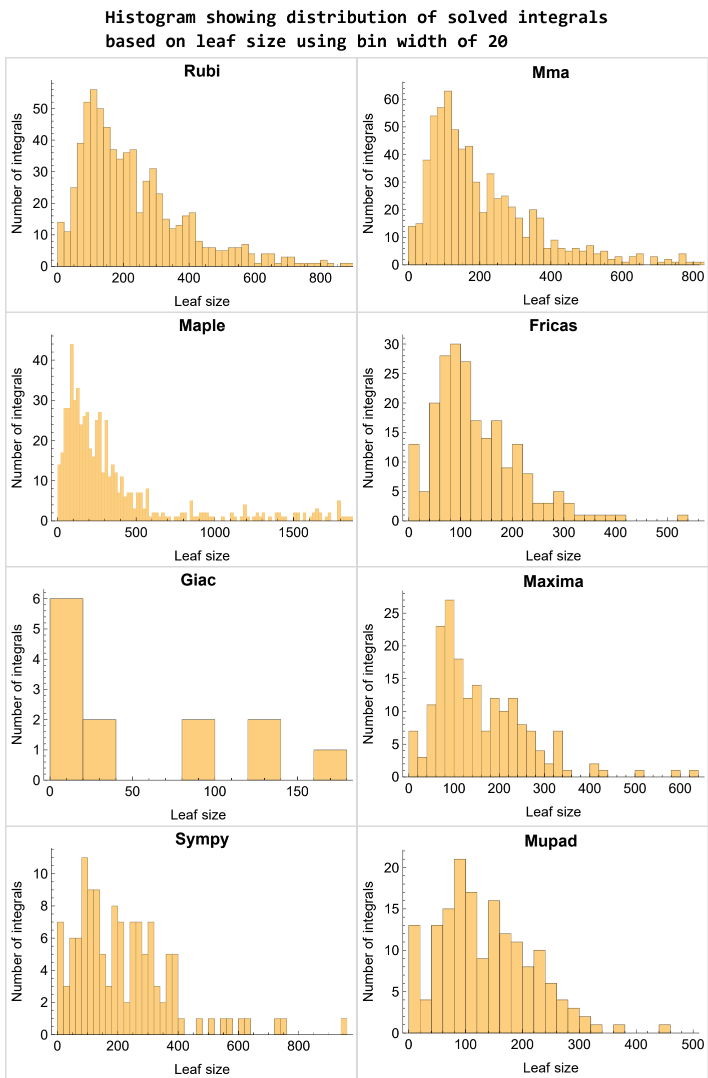


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

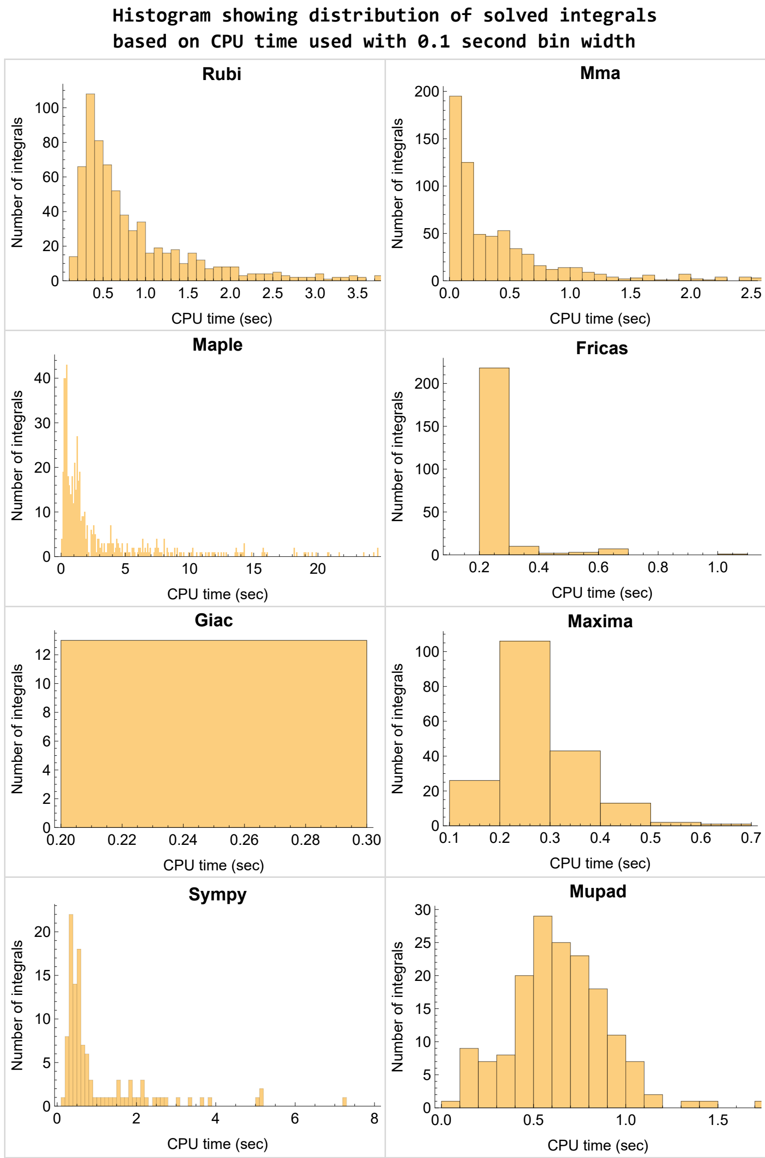


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

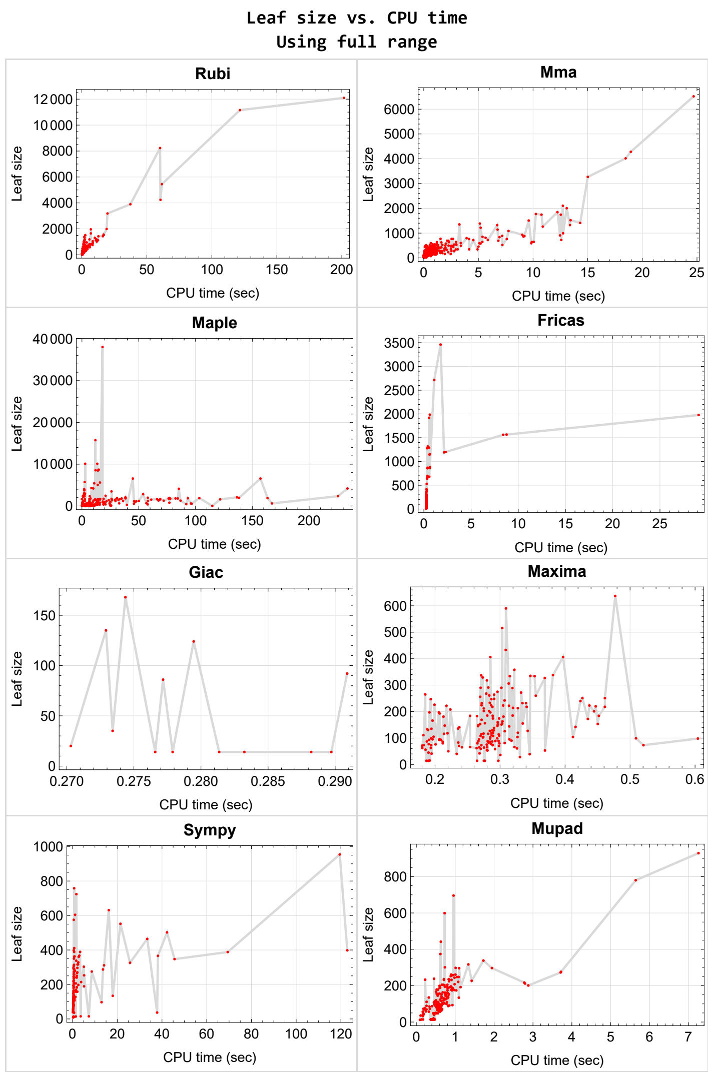


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{133, 148, 251, 252, 253, 254, 255, 256, 257, 355, 356, 357, 358, 359, 360, 361, 362, 458, 459, 460, 461,
462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 485, 486,
487, 488, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 512, 513, 514,
515, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539,
540, 541, 542, 543, 544, 545, 547, 548, 549, 550, 551, 555, 556, 557, 558, 563, 564, 565, 566, 567, 568,
569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 585, 586, 587, 588, 593, 594, 595,
596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616,
617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 631, 632, 633, 634, 639, 640, 641, 642, 644, 645, 646,
647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 672, 673,
674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694,
695, 696, 697, 698, 699, 700, 701, 703, 704, 705, 706, 707, 708, 712, 713, 714, 720, 721, 722, 723, 724,
725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 747,
748, 749, 750, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772,
773, 774, 775, 776, 777, 779, 780, 781, 782, 783, 784, 788, 789, 790, 796, 797, 798, 799, 800, 801, 802,
803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823,
826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850,
851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 862, 863, 864, 865, 869, 870, 871, 877, 878, 879, 880,
881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901,
902, 903, 906, 907, 908, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 929,
930, 931, 935, 936, 937, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958,
959, 960, 961, 964, 965, 966, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985,
987, 988, 989, 990, 994, 995, 996, 997, 998, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012,
1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1029, 1030, 1031,
1032, 1033, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052,
1053, 1054, 1055, 1057, 1058, 1059, 1063, 1064, 1065, 1066, 1067, 1072, 1073, 1074, 1075, 1076, 1077,
1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094,
1095, 1098, 1099, 1100, 1101, 1102, 1107, 1108, 1109, 1110, 1111, 1113, 1174, 1176, 1177, 1178, 1179,
1181, 1184, 1186, 1187, 1188, 1189, 1190, 1191, 1194, 1196, 1197, 1198, 1199, 1200, 1202, 1204, 1205,
1207, 1210, 1213, 1215, 1217, 1222, 1224, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240,
1242, 1244, 1246, 1299}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {1226, 1292, 1294, 1296}

Mathematica {217, 219, 315, 323, 329, 330, 413, 417, 421, 423, 427, 429, 431, 433, 435, 1162, 1170, 1171, 1261, 1263, 1265, 1267}

Maple {72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 262, 264, 270, 272, 278, 280, 283, 285, 287, 289, 291, 295, 297, 303, 305, 364, 366, 368, 370, 372, 374, 376, 378, 380, 382, 384, 386, 387, 389, 390, 392, 393, 395, 396, 400, 401, 403, 408, 409, 411, 510, 511, 516, 518, 519, 1158, 1160, 1161, 1165, 1168, 1169, 1172, 1251, 1253, 1258, 1260, 1269, 1271, 1280, 1291, 1297}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

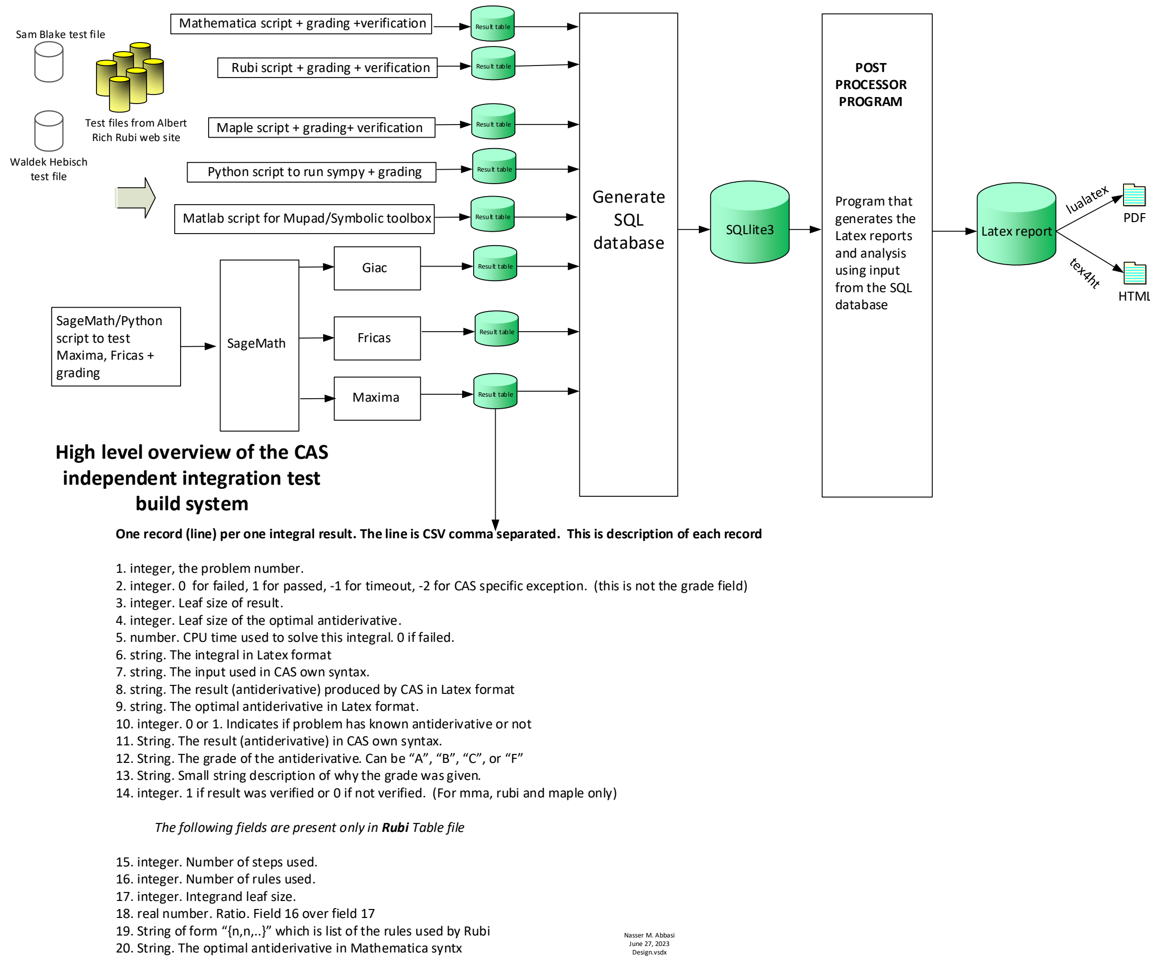
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 308, 309, 310, 311, 312, 313, 314, 317, 318, 319, 320, 321, 322, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 413, 414, 415, 416, 417, 418, 419, 422, 423, 424, 425, 426, 427, 430, 431, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 477, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 546, 552, 554, 560, 561, 562, 581, 582, 589, 590, 591, 592, 622, 628, 629, 630, 635, 636, 637, 638, 643, 660, 661, 668, 669, 670, 671, 702, 709, 710, 711, 715, 716, 717, 718, 719, 745, 746, 751, 752, 753, 754, 778, 785, 786, 787, 791, 792, 793, 794, 795, 824, 825, 831, 832, 833, 834, 859, 866, 867, 868, 872, 873, 874, 875, 876, 904, 905, 909, 910, 911, 912, 928, 932, 933, 934, 938, 939, 940, 941, 942, 962, 963, 967, 968, 969, 970, 986, 991, 992, 993, 999, 1001, 1002, 1027, 1028, 1034, 1035, 1036, 1037, 1056, 1060, 1061, 1062, 1068, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1112, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

B grade { 198, 199, 208, 217, 258, 259, 305, 307, 315, 363, 364, 553, 559, 1000, 1069, 1270 }

C grade { }

F normal fail { 216, 306, 316, 323, 324, 328, 410, 411, 412, 420, 421, 428, 429, 432, 433, 434, 435 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 134, 135, 136, 137, 138, 139, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 422, 424, 425, 426, 427, 428, 430, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 477, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 546, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 622, 628, 629, 630, 635, 636, 637, 638, 643, 660, 661, 668, 669, 670, 671, 702, 709, 711, 719, 752, 778, 786, 825, 859, 866, 868, 905, 912, 928, 933, 967, 970, 986, 991, 993, 1056, 1061, 1112, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1123, 1125, 1126, 1127, 1128, 1129, 1130, 1132, 1134, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1144, 1146, 1148, 1150, 1151, 1152, 1153, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1228, 1229, 1230, 1241, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1265, 1267, 1270, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1292, 1293, 1294, 1295, 1296, 1300 }

B grade { 130, 217, 323, 325, 413, 421, 423, 429, 431, 433, 1261, 1263, 1297, 1298, 1301 }

C grade { 7, 8, 9, 16, 17, 18, 19, 27, 28, 29, 30, 39, 40, 41, 42, 49, 50, 57, 65, 140, 155, 163, 171, 180, 710, 715, 716, 717, 718, 745, 746, 751, 753, 754, 785, 787, 791, 792, 793, 794, 795, 824, 831, 832, 833, 834, 867, 872, 873, 874, 875, 876, 904, 909, 910, 911, 932, 934, 938, 939, 940, 941, 942, 962, 963, 968, 969, 992, 999, 1000, 1001, 1002, 1027, 1028, 1034, 1035, 1036, 1037, 1060, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1120, 1122, 1124, 1131, 1133, 1135, 1143, 1145, 1147, 1149, 1154, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227 }

F normal fail { 141, 142, 143, 144, 145, 146, 1243, 1245, 1268, 1269, 1271, 1272, 1274, 1291 }

F(-1) timedout fail { 147, 509, 515, 1262, 1264, 1266, 1273 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 76, 77, 78, 83, 84, 85, 86, 92, 93, 94, 115, 118, 125, 126, 134, 135, 136, 137, 138, 139, 140, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 201, 203, 204, 206, 209, 211, 212, 213, 214, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 258, 259, 260, 261, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 281, 282, 286, 290, 292, 293, 294, 296, 298, 299, 300, 301, 302, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 353, 354, 363, 365, 367, 369, 371, 373, 375, 377, 379, 381, 383, 385, 388, 391, 394, 397, 398, 399, 402, 404, 405, 406, 407, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 448, 449, 456, 457, 477, 482, 483, 484, 489, 490, 491, 492, 493, 546, 552, 553, 554, 559, 560, 561, 562, 622, 628, 629, 630, 635, 636, 637, 638, 643, 702, 709, 710, 711, 715, 716, 717, 718, 719, 778, 785, 786, 787, 791, 792, 793, 794, 795, 859, 866, 867, 868, 872, 873, 874, 875, 876, 928, 932, 933, 934, 938, 939, 940, 941, 942, 986, 991, 992, 993, 999, 1000, 1001, 1002, 1056, 1060, 1061, 1062, 1068, 1069, 1070, 1071, 1112, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1166, 1167, 1247, 1248, 1249, 1250, 1254, 1255, 1256, 1257, 1259, 1275, 1276, 1277, 1278, 1279, 1286, 1287, 1288, 1289, 1290 }

B grade { 45, 47, 48, 53, 67, 71, 74, 75, 79, 87, 107, 114, 119, 124, 178, 205, 215, 263, 265, 284, 288, 1120, 1162, 1163, 1164, 1170, 1171, 1252, 1264, 1270 }

C grade { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 174, 176, 180, 182, 184, 188, 190, 196, 198, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 248, 249, 250, 262, 264, 270, 272, 278, 280, 283, 285, 287, 289, 291, 295, 297, 303, 305, 341, 342, 349, 350, 351, 352, 364, 366, 368, 370, 372, 374, 376, 378, 380, 382, 384, 386, 387, 389, 390, 392, 393, 395, 396, 400, 401, 403, 408, 409, 411, 446, 447, 452, 453, 454, 455, 510, 511, 516, 517, 518, 519, 581, 582, 589, 590, 591, 592, 660, 661, 668, 669, 670, 671, 1158, 1160, 1161, 1165, 1168, 1169, 1172, 1251, 1253, 1258, 1260, 1269, 1271, 1280, 1291, 1297 }

F normal fail { 334, 340, 348, 438, 439, 444, 445, 450, 451, 745, 746, 751, 752, 753, 754, 824, 825, 831, 832, 833, 834, 904, 905, 909, 910, 911, 912, 962, 963, 967, 968, 969, 970, 1027, 1028, 1034, 1035, 1036, 1037, 1096, 1097, 1103, 1104, 1105, 1106, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1261, 1262, 1263, 1265, 1266, 1267, 1268, 1272, 1273, 1274, 1281, 1282, 1283, 1284, 1285, 1292, 1293, 1294, 1295, 1296, 1298, 1300, 1301 }

F(-1) timedout fail { 102, 598, 616 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 41, 42, 47, 48, 49, 50, 54, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 99, 107, 114, 115, 118, 124, 125, 126, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 349, 350, 351, 352, 391, 397, 398, 399, 404, 405, 406, 407, 446, 447, 452, 453, 454, 455, 477, 546, 622, 643, 702, 778, 859, 928, 986, 1056, 1112, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1142, 1144, 1146, 1148, 1149, 1150, 1159, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1286, 1287, 1288, 1289, 1290 }

B grade { 4, 28, 40, 130, 1166, 1167, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227 }

C grade { 482, 483, 484, 489, 490, 491, 492, 493, 552, 553, 554, 559, 560, 561, 562, 628, 629, 630, 635, 636, 637, 638 }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 51, 52, 53, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 456, 457, 510, 511, 516, 517, 518, 519, 581, 582, 589, 590, 591, 592, 660, 661, 668, 669, 670, 671, 1118, 1120, 1129, 1131, 1133, 1141, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1168, 1169, 1170, 1171, 1172, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1281, 1282, 1283, 1284, 1285, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

F(-1) timedout fail { }

F(-2) exception fail { 687, 688, 689, 691, 692, 693, 695, 696, 697, 699, 700, 701, 703, 704, 705, 706, 708, 709, 710, 711, 712, 714, 715, 716, 717, 718, 719, 720, 722, 723, 724, 726, 727, 728, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 750, 751, 752, 753, 754, 755, 757, 758, 759, 760, 761, 763, 764, 765, 766, 767, 769, 770, 771, 772, 773, 775, 776, 777, 779, 780, 781, 782, 784, 785, 786, 787, 788, 790, 791, 792, 793, 794, 795, 796, 798, 799, 800, 801, 803, 804, 805, 806, 808, 809, 810, 811, 813, 814, 815, 816, 817, 818, 819, 820, 822, 823, 824, 825, 826, 827, 829, 830, 831, 832, 833, 834, 835, 836, 838, 839, 840, 841, 842, 844, 845, 846, 847, 848, 850, 851, 852, 853, 854, 856, 857, 858, 860, 861, 862, 863, 865, 866, 867, 868, 869, 871, 872, 873, 874, 875, 876, 877, 879, 880, 881, 882, 884, 885, 886, 887, 889, 890, 891, 892, 894, 895, 896, 897, 898, 899, 900, 901, 903, 904, 905, 906, 908, 909, 910, 911, 912, 913, 915, 916, 917, 919, 920, 921, 923, 924, 925, 927, 929, 931, 932, 933, 934, 935, 937, 938, 939, 940, 941, 942, 943, 945, 946, 947, 949, 950, 951, 953, 954, 955, 957, 958, 959, 961, 962, 963, 964, 966, 967, 968, 969, 970, 971, 973, 974, 975, 977, 978, 979, 981, 982, 983, 985, 987, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1012, 1013, 1014, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1043, 1044, 1045, 1047, 1048, 1049, 1051, 1052, 1053, 1055, 1057, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1077, 1078, 1079, 1081, 1082, 1083, 1085, 1086, 1087, 1089, 1090, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 17, 18, 19, 20, 21, 22, 24, 25, 29, 30, 31, 35, 36, 37, 42, 58, 59, 60, 61, 62, 66, 114, 115, 118, 125, 126, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 207, 208, 216, 224, 226, 229, 231, 234, 235, 242, 243, 244, 245, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 350, 352, 391, 397, 398, 399, 404, 405, 406, 407, 446, 447, 477, 546, 622, 643, 1114, 1115, 1116, 1117, 1118, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1148, 1149, 1150, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290 }

B grade { 13, 23, 28, 32, 33, 34, 40, 41, 63, 65, 67, 202, 210, 218 }

C grade { }

F normal fail { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 111, 112, 113, 116, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 232, 233, 236, 237, 238, 239, 240, 241, 246, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 291, 295, 296, 297, 303, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 349, 351, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 396, 400, 401, 402, 403, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 628, 629, 630, 635, 636, 637, 638, 660, 661, 668, 669, 670, 671, 1120, 1133, 1147, 1151, 1152, 1153, 1154, 1158, 1160, 1161, 1165, 1168, 1169, 1219, 1221, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1263, 1265, 1267, 1268, 1272, 1274, 1280, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

F(-1) timedout fail { 290, 298, 304, 306, 395, 411 }

F(-2) exception fail { 54, 64, 107, 110, 117, 124, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1155, 1156, 1157, 1159, 1162, 1163, 1164, 1166, 1167, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1220, 1222, 1223, 1224, 1225, 1226, 1227, 1262, 1264, 1266, 1269, 1270, 1271, 1273 }

2.1.6 Giac

A grade { 702, 778, 859, 928, 986, 1056, 1112, 1275, 1276 }

B grade { 177, 1277, 1278, 1279 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 76, 77, 78, 79, 84, 85, 86, 87, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 209, 217, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 241, 243, 244, 245, 246, 247, 248, 249, 250, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 316, 324, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 348, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 379, 380, 381, 382, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 421, 429, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 451, 453, 454, 455, 456, 457, 477, 482, 483, 484, 489, 490, 491, 492, 493, 511, 517, 519, 546, 552, 553, 554, 559, 560, 561, 562, 582, 590, 592, 622, 628, 629, 630, 635, 636, 637, 638, 643, 661, 669, 671, 709, 710, 711, 715, 716, 717, 718, 719, 745, 746, 752, 753, 754, 785, 786, 787, 791, 792, 793, 794, 795, 824, 825, 832, 833, 834, 866, 867, 868, 872, 873, 874, 875, 876, 904, 905, 910, 911, 912, 932, 933, 934, 938, 939, 940, 941, 942, 963, 968, 970, 993, 1002, 1028, 1037, 1062, 1097, 1106, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1175, 1180, 1183, 1185, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1247, 1248, 1249, 1250, 1254, 1255, 1256, 1257, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1297, 1298 }

F(-1) timedout fail { 72, 73, 74, 75, 80, 81, 82, 83, 88, 89, 90, 91, 92, 93, 94, 124, 125, 126, 131, 132, 375, 376, 377, 378, 383, 384, 385, 386, 597, 603, 678, 679, 693, 697, 772, 773, 841, 842, 847, 848, 853, 854, 973, 977, 981, 983, 989, 990, 991, 992, 999, 1000, 1001, 1012, 1016, 1035, 1043, 1047, 1049, 1051, 1053, 1059, 1060, 1061, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1077, 1081, 1085, 1089, 1104, 1181, 1182, 1187, 1188, 1189, 1190, 1191, 1192, 1197, 1198, 1199, 1200, 1251, 1252, 1253, 1258, 1259, 1260, 1291, 1292, 1293, 1294, 1295, 1296, 1299, 1300, 1301 }

F(-2) exception fail { 200, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 232, 240, 242, 253, 254, 255, 307, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 339, 347, 349, 359, 360, 412, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 435, 436, 444, 450, 452, 462, 463, 508, 510, 512, 514, 516, 518, 520, 528, 529, 530, 579, 581, 583, 585, 587, 589, 591, 593, 595, 604, 605, 606, 658, 660, 662, 664, 666, 668, 670, 672, 680, 681, 682, 721, 723, 724, 725, 727, 728, 729, 731, 732, 734, 743, 751, 797, 799, 800, 801, 802, 804, 805, 806, 807, 809, 810, 811, 813, 822, 829, 831, 878, 880, 881, 882, 883, 885, 886, 887, 888, 890, 891, 892, 894, 909, 944, 948, 952, 962, 964, 967, 969, 971, 1007, 1011, 1015, 1025, 1027, 1029, 1031, 1034, 1036, 1038, 1040, 1076, 1080, 1084, 1094, 1096, 1098, 1100, 1103, 1105, 1107, 1109 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 391, 397, 398, 399, 404, 405, 406, 407, 477, 546, 622, 643, 702, 778, 859, 928, 986, 1056, 1112, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1159, 1166, 1167, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1281, 1286, 1287, 1288, 1289, 1290 }

C grade { }

F normal fail { }

F(-1) timedout fail { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 628, 629, 630, 635, 636, 637, 638, 660, 661, 668, 669, 670, 671, 709, 710, 711, 715, 716, 717, 718, 719, 745, 746, 751, 752, 753, 754, 785, 786, 787, 791, 792, 793, 794, 795, 824, 825, 831, 832, 833, 834, 866, 867, 868, 872, 873, 874, 875, 876, 904, 905, 909, 910, 911, 912, 932, 933, 934, 938, 939, 940, 941, 942, 962, 963, 967, 968, 969, 970, 991, 992, 993, 999, 1000, 1001, 1002, 1027, 1028, 1034, 1035, 1036, 1037, 1060, 1061, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1103, 1104, 1105, 1106, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1160, 1161, 1162, 1163, 1164, 1165, 1168, 1169, 1170, 1171, 1172, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1219, 1220, 1221, 1223, 1225, 1226, 1227, 1228, 1229, 1230, 1241, 1243, 1245, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1280, 1282, 1283, 1284, 1285, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1300, 1301 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 8, 9, 10, 11, 12, 18, 19, 20, 21, 30, 31, 32, 42, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 179, 181, 186, 258, 260, 266, 268, 274, 276, 477, 546, 622, 859, 928, 986, 1056, 1114, 1115, 1116, 1117, 1119, 1121, 1122, 1123, 1124, 1125, 1126, 1128, 1130, 1132, 1134, 1135, 1136, 1137, 1138, 1140, 1142, 1144, 1146, 1148, 1150, 1247, 1249, 1254, 1256, 1275, 1276, 1277, 1278, 1279, 1286, 1287, 1288, 1289, 1290 }

B grade { 4, 7, 13, 17, 22, 23, 28, 29, 33, 34, 40, 41, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 177, 183, 189, 191, 192, 194, 197, 199, 1127, 1139, 1149 }

C grade { 1281, 1283, 1285 }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 57, 65, 67, 71, 72, 73, 80, 81, 99, 100, 101, 102, 110, 119, 130, 131, 132, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 185, 190, 193, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 235, 237, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 482, 483, 484, 489, 490, 491, 492, 493, 510, 511, 516, 517, 518, 519, 552, 553, 554, 559, 560, 561, 562, 581, 582, 589, 590, 591, 592, 628, 629, 630, 635, 636, 637, 638, 643, 660, 661, 668, 669, 670, 671, 702, 709, 710, 711, 715, 716, 717, 718, 719, 745, 746, 751, 752, 753, 754, 778, 785, 786, 787, 791, 792, 793, 794, 795, 824, 825, 831, 832, 833, 834, 866, 867, 868, 872, 873, 874, 875, 876, 905, 912, 932, 933, 934, 938, 939, 940, 941, 942, 962, 963, 967, 968, 969, 970, 991, 992, 993, 999, 1000, 1001, 1002, 1027, 1028, 1034, 1035, 1036, 1037, 1060, 1061, 1062, 1068, 1069, 1070, 1071, 1096, 1097, 1112, 1118, 1120, 1129, 1131, 1133, 1141, 1143, 1145, 1147, 1151, 1152, 1153, 1154, 1155, 1156, 1173, 1175, 1180, 1182, 1183, 1185, 1192, 1193, 1195, 1201, 1203, 1206, 1208, 1209, 1211, 1212, 1214, 1216, 1218, 1220, 1226, 1227, 1228, 1229, 1230, 1248, 1250, 1251, 1252, 1253, 1255, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1267, 1280, 1282, 1284, 1291, 1293, 1295 }

F(-1) timedout fail { 26, 27, 35, 36, 37, 38, 39, 51, 55, 56, 58, 59, 60, 64, 68, 69, 70, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 103, 104, 105, 106, 108, 109, 111, 112, 113, 117, 120, 121, 123, 126, 127, 128, 129, 139, 146, 217, 253, 254, 359, 462, 528, 533, 605, 609, 681, 685, 694, 725, 729, 730, 731, 732, 749, 756, 762, 768, 789, 797, 802, 803, 804, 805, 807, 808, 809, 810, 811, 812, 821, 828, 829, 830, 837, 843, 849, 855, 864, 870, 878, 879, 880, 881, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 902, 903, 904, 907, 908, 909, 910, 911, 913, 936, 948, 952, 953, 954, 960, 965, 988, 998, 1011, 1015, 1016, 1017, 1018, 1019, 1024, 1031, 1032, 1033, 1038, 1039, 1040, 1041, 1042, 1046, 1050, 1054, 1058, 1067, 1076, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1092, 1093, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1113, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1217, 1219, 1221, 1223, 1224, 1225, 1232, 1233, 1234, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1266, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1297, 1298, 1300, 1301 }

F(-2) exception fail { 63, 116, 122, 187, 188, 195, 196, 226, 234, 236, 244, 246, 604, 680, 1292, 1294, 1296 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	106	98	99	109	124	184	0	109
N.S.	1	0.91	0.84	0.85	0.93	1.06	1.57	0.00	0.93
time (sec)	N/A	0.275	0.059	1.045	0.299	0.257	1.877	0.000	0.785

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	96	88	91	99	113	167	0	99
N.S.	1	0.91	0.84	0.87	0.94	1.08	1.59	0.00	0.94
time (sec)	N/A	0.276	0.048	0.355	0.296	0.273	1.525	0.000	0.775

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	86	76	82	88	104	158	0	87
N.S.	1	0.95	0.84	0.90	0.97	1.14	1.74	0.00	0.96
time (sec)	N/A	0.254	0.042	0.470	0.281	0.265	1.548	0.000	0.404

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	56	84	70	73	89	128	0	73
N.S.	1	1.06	1.58	1.32	1.38	1.68	2.42	0.00	1.38
time (sec)	N/A	0.205	0.005	0.341	0.271	0.251	1.291	0.000	0.348

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	100	0	0	0	0	63
N.S.	1	1.00	1.00	1.32	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.245	0.006	0.280	0.000	0.000	0.000	0.000	0.644

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	109	0	0	0	0	93
N.S.	1	1.00	0.97	1.42	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.267	0.041	0.240	0.000	0.000	0.000	0.000	0.927

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	60	88	84	75	99	182	0	79
N.S.	1	0.92	1.35	1.29	1.15	1.52	2.80	0.00	1.22
time (sec)	N/A	0.234	0.044	0.674	0.280	0.265	1.896	0.000	0.584

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	95	94	92	87	109	197	0	176
N.S.	1	0.90	0.89	0.87	0.82	1.03	1.86	0.00	1.66
time (sec)	N/A	0.278	0.040	0.477	0.288	0.252	2.601	0.000	0.978

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	109	99	101	102	119	214	0	116
N.S.	1	0.88	0.80	0.81	0.82	0.96	1.73	0.00	0.94
time (sec)	N/A	0.283	0.043	0.692	0.274	0.250	3.847	0.000	0.644

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	138	124	131	185	172	270	0	152
N.S.	1	0.83	0.75	0.79	1.11	1.04	1.63	0.00	0.92
time (sec)	N/A	0.389	0.090	1.511	0.280	0.260	2.135	0.000	0.841

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	130	116	123	174	160	250	0	140
N.S.	1	0.86	0.76	0.81	1.14	1.05	1.64	0.00	0.92
time (sec)	N/A	0.379	0.088	1.190	0.289	0.258	1.923	0.000	0.816

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	118	101	114	155	148	240	0	125
N.S.	1	0.87	0.74	0.84	1.14	1.09	1.76	0.00	0.92
time (sec)	N/A	0.362	0.065	1.606	0.303	0.271	1.839	0.000	0.755

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	70	57	103	138	127	206	0	109
N.S.	1	0.84	0.69	1.24	1.66	1.53	2.48	0.00	1.31
time (sec)	N/A	0.223	0.030	0.780	0.291	0.257	1.513	0.000	0.452

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	103	134	142	0	0	0	131
N.S.	1	1.00	0.80	1.04	1.10	0.00	0.00	0.00	1.02
time (sec)	N/A	0.310	0.109	0.812	0.416	0.000	0.000	0.000	0.801

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	79	112	0	0	0	0	141
N.S.	1	1.00	0.89	1.26	0.00	0.00	0.00	0.00	1.58
time (sec)	N/A	0.300	0.114	0.792	0.000	0.000	0.000	0.000	0.646

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	139	157	0	0	0	0	161
N.S.	1	1.00	0.91	1.03	0.00	0.00	0.00	0.00	1.06
time (sec)	N/A	0.343	0.072	1.011	0.000	0.000	0.000	0.000	0.803

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	80	114	114	144	144	253	0	120
N.S.	1	0.92	1.31	1.31	1.66	1.66	2.91	0.00	1.38
time (sec)	N/A	0.261	0.073	0.800	0.314	0.262	5.171	0.000	0.699

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	136	152	125	152	155	275	0	142
N.S.	1	0.84	0.94	0.78	0.94	0.96	1.71	0.00	0.88
time (sec)	N/A	0.387	0.057	1.351	0.297	0.247	8.593	0.000	0.756

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	142	124	133	183	167	287	0	244
N.S.	1	0.83	0.73	0.78	1.07	0.98	1.68	0.00	1.43
time (sec)	N/A	0.387	0.065	0.963	0.265	0.264	13.613	0.000	0.981

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	169	154	162	261	202	328	0	186
N.S.	1	0.82	0.75	0.79	1.27	0.99	1.60	0.00	0.91
time (sec)	N/A	0.432	0.099	1.968	0.283	0.241	2.573	0.000	0.935

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	161	146	154	242	190	316	0	174
N.S.	1	0.84	0.76	0.81	1.27	0.99	1.65	0.00	0.91
time (sec)	N/A	0.417	0.089	1.411	0.307	0.269	2.479	0.000	0.917

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	136	132	145	222	177	296	0	160
N.S.	1	0.87	0.84	0.92	1.41	1.13	1.89	0.00	1.02
time (sec)	N/A	0.304	0.079	1.105	0.314	0.264	2.158	0.000	0.787

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	96	77	124	197	161	267	0	147
N.S.	1	0.96	0.77	1.24	1.97	1.61	2.67	0.00	1.47
time (sec)	N/A	0.238	0.031	0.883	0.285	0.250	2.023	0.000	0.755

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	139	165	184	0	0	0	196
N.S.	1	1.00	0.82	0.97	1.08	0.00	0.00	0.00	1.15
time (sec)	N/A	0.358	0.153	1.202	0.452	0.000	0.000	0.000	0.889

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	150	159	201	0	0	0	195
N.S.	1	1.00	0.93	0.98	1.24	0.00	0.00	0.00	1.20
time (sec)	N/A	0.359	0.144	1.110	0.445	0.000	0.000	0.000	0.784

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	164	173	0	0	0	0	205
N.S.	1	1.00	0.91	0.96	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.365	0.160	1.388	0.000	0.000	0.000	0.000	0.797

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	170	183	0	0	0	0	221
N.S.	1	1.00	0.90	0.97	0.00	0.00	0.00	0.00	1.17
time (sec)	N/A	0.394	0.082	1.282	0.000	0.000	0.000	0.000	1.077

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	86	165	147	202	174	311	0	154
N.S.	1	0.83	1.60	1.43	1.96	1.69	3.02	0.00	1.50
time (sec)	N/A	0.269	0.096	1.319	0.275	0.254	14.105	0.000	0.747

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	121	185	153	224	185	326	0	174
N.S.	1	0.81	1.23	1.02	1.49	1.23	2.17	0.00	1.16
time (sec)	N/A	0.310	0.073	0.953	0.272	0.255	25.705	0.000	1.027

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	179	188	164	248	198	347	0	192
N.S.	1	0.84	0.88	0.77	1.16	0.93	1.62	0.00	0.90
time (sec)	N/A	0.436	0.096	1.562	0.291	0.257	45.622	0.000	1.138

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	198	290	190	337	230	389	0	217
N.S.	1	0.83	1.22	0.80	1.42	0.97	1.63	0.00	0.91
time (sec)	N/A	0.473	0.096	2.457	0.271	0.263	3.382	0.000	2.776

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	169	276	182	318	218	367	0	205
N.S.	1	0.88	1.43	0.94	1.65	1.13	1.90	0.00	1.06
time (sec)	N/A	0.388	0.080	1.381	0.280	0.248	3.016	0.000	0.687

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	153	264	173	290	206	360	0	191
N.S.	1	0.86	1.48	0.97	1.63	1.16	2.02	0.00	1.07
time (sec)	N/A	0.313	0.066	1.644	0.302	0.255	2.751	0.000	0.852

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	115	77	145	264	188	316	0	175
N.S.	1	0.92	0.62	1.16	2.11	1.50	2.53	0.00	1.40
time (sec)	N/A	0.263	0.022	1.172	0.291	0.271	2.214	0.000	0.801

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	174	193	220	0	0	0	248
N.S.	1	1.00	0.86	0.95	1.08	0.00	0.00	0.00	1.22
time (sec)	N/A	0.408	0.120	1.420	0.447	0.000	0.000	0.000	1.047

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	181	189	240	0	0	0	253
N.S.	1	1.00	0.95	0.99	1.26	0.00	0.00	0.00	1.33
time (sec)	N/A	0.408	0.130	1.287	0.424	0.000	0.000	0.000	0.885

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	163	178	251	0	0	0	258
N.S.	1	1.00	0.94	1.03	1.45	0.00	0.00	0.00	1.49
time (sec)	N/A	0.383	0.126	1.850	0.427	0.000	0.000	0.000	0.941

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	193	195	0	0	0	0	261
N.S.	1	1.00	0.96	0.97	0.00	0.00	0.00	0.00	1.30
time (sec)	N/A	0.424	0.128	1.757	0.000	0.000	0.000	0.000	0.893

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	227	214	0	0	0	0	298
N.S.	1	1.00	1.00	0.94	0.00	0.00	0.00	0.00	1.31
time (sec)	N/A	0.430	0.091	1.896	0.000	0.000	0.000	0.000	1.011

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	191	175	275	202	366	0	186
N.S.	1	0.87	1.63	1.50	2.35	1.73	3.13	0.00	1.59
time (sec)	N/A	0.286	0.125	1.418	0.296	0.266	38.177	0.000	0.805

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	135	235	184	290	216	388	0	208
N.S.	1	0.80	1.40	1.10	1.73	1.29	2.31	0.00	1.24
time (sec)	N/A	0.321	0.094	1.792	0.270	0.270	69.408	0.000	0.984

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	206	293	192	329	228	398	0	317
N.S.	1	0.85	1.21	0.79	1.35	0.94	1.64	0.00	1.30
time (sec)	N/A	0.473	0.075	1.245	0.273	0.255	122.873	0.000	1.337

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	215	166	312	0	0	0	0	0
N.S.	1	1.10	0.85	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.022	0.534	2.036	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	155	132	267	0	0	0	0	0
N.S.	1	0.99	0.85	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.668	0.184	0.777	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	103	108	214	0	0	0	0	0
N.S.	1	0.94	0.98	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.409	0.254	1.072	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	104	0	0	0	0	0
N.S.	1	1.00	1.02	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	0.013	1.005	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	102	148	0	43	0	0	0
N.S.	1	1.00	1.89	2.74	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.260	0.043	0.854	0.000	0.256	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	98	149	200	0	98	0	0	0
N.S.	1	0.98	1.49	2.00	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.485	0.084	0.855	0.000	0.282	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	138	178	251	0	130	0	0	0
N.S.	1	0.86	1.11	1.56	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.726	0.135	1.027	0.000	0.245	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	194	254	278	0	155	0	0	0
N.S.	1	0.98	1.29	1.41	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	1.096	0.100	1.029	0.000	0.255	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	186	317	0	0	0	0	0
N.S.	1	1.00	0.92	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.898	0.937	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	153	271	0	0	0	0	0
N.S.	1	1.00	0.92	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	0.695	0.733	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	128	252	0	0	0	0	0
N.S.	1	1.00	1.05	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.065	0.924	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	66	42	68	0	50	116	0	0
N.S.	1	0.96	0.61	0.99	0.00	0.72	1.68	0.00	0.00
time (sec)	N/A	0.232	0.030	0.730	0.000	0.249	0.885	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	128	207	0	127	0	0	0
N.S.	1	1.00	0.85	1.38	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.374	0.123	0.813	0.000	0.262	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	165	257	0	182	0	0	0
N.S.	1	1.00	0.85	1.32	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.443	0.222	0.883	0.000	0.257	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	222	286	0	221	0	0	0
N.S.	1	1.00	0.91	1.17	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.477	0.245	1.052	0.000	0.255	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	235	364	358	0	0	0	0
N.S.	1	1.00	0.92	1.42	1.40	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	0.876	0.820	0.322	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	216	321	334	0	0	0	0
N.S.	1	1.00	0.96	1.43	1.48	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.716	0.901	0.315	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	187	299	291	0	0	0	0
N.S.	1	1.00	1.06	1.70	1.65	0.00	0.00	0.00	0.00
time (sec)	N/A	0.411	0.132	1.021	0.282	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	86	63	109	70	85	189	0	0
N.S.	1	0.98	0.72	1.24	0.80	0.97	2.15	0.00	0.00
time (sec)	N/A	0.258	0.076	1.201	0.237	0.244	5.146	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	89	55	82	65	75	158	0	0
N.S.	1	0.97	0.60	0.89	0.71	0.82	1.72	0.00	0.00
time (sec)	N/A	0.242	0.035	0.702	0.241	0.251	2.156	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	162	261	406	201	0	0	0
N.S.	1	1.00	0.83	1.34	2.08	1.03	0.00	0.00	0.00
time (sec)	N/A	0.433	0.187	1.411	0.285	0.267	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	227	303	0	263	0	0	0
N.S.	1	1.00	0.91	1.21	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.499	0.253	1.718	0.000	0.267	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	285	345	590	311	0	0	0
N.S.	1	1.00	0.93	1.13	1.93	1.02	0.00	0.00	0.00
time (sec)	N/A	0.552	0.411	1.103	0.309	0.258	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	79	83	93	168	0	0
N.S.	1	1.00	0.73	0.79	0.83	0.93	1.68	0.00	0.00
time (sec)	N/A	0.248	0.042	1.280	0.236	0.246	1.394	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	88	104	126	21	0	0	0
N.S.	1	1.00	1.80	2.12	2.57	0.43	0.00	0.00	0.00
time (sec)	N/A	0.260	0.025	1.084	0.307	0.241	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	285	375	0	0	0	0	0
N.S.	1	1.00	0.99	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.769	0.649	1.481	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	241	352	0	0	0	0	0
N.S.	1	1.00	0.95	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	0.495	1.425	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	208	316	0	0	0	0	0
N.S.	1	1.00	0.99	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.550	0.569	0.697	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	135	151	278	0	0	0	0	0
N.S.	1	1.04	1.16	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.314	1.300	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	216	216	272	3480	0	0	0	0	0
N.S.	1	1.00	1.26	16.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.611	0.296	26.059	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	228	228	289	5609	0	0	0	0	0
N.S.	1	1.00	1.27	24.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	0.322	15.894	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	190	357	0	0	0	0	0
N.S.	1	1.00	1.19	2.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.506	0.237	2.459	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	240	408	0	0	0	0	0
N.S.	1	1.00	1.07	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.608	0.408	3.141	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	342	443	0	0	0	0	0
N.S.	1	1.00	0.92	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.069	0.985	2.094	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	306	420	0	0	0	0	0
N.S.	1	1.00	0.92	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.968	0.970	2.516	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	257	384	0	0	0	0	0
N.S.	1	1.00	0.88	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.790	0.997	1.841	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	188	205	354	0	0	0	0	0
N.S.	1	0.98	1.07	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	0.745	1.199	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	300	300	360	1317	0	0	0	0	0
N.S.	1	1.00	1.20	4.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.761	0.543	29.467	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	317	317	378	5048	0	0	0	0	0
N.S.	1	1.00	1.19	15.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.809	0.680	13.937	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	337	337	388	1381	0	0	0	0	0
N.S.	1	1.00	1.15	4.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.833	0.666	26.520	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	251	253	445	0	0	0	0	0
N.S.	1	0.94	0.95	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	0.590	3.887	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	408	511	0	0	0	0	0
N.S.	1	1.00	0.93	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.443	1.514	2.928	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	369	488	0	0	0	0	0
N.S.	1	1.00	0.92	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.316	1.128	2.845	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	325	452	0	0	0	0	0
N.S.	1	1.00	1.06	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.729	2.092	1.741	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	215	267	412	0	0	0	0	0
N.S.	1	0.95	1.18	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	1.648	1.525	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	385	385	465	1409	0	0	0	0	0
N.S.	1	1.00	1.21	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.949	0.821	23.595	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	402	402	512	1437	0	0	0	0	0
N.S.	1	1.00	1.27	3.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.924	0.553	32.124	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	416	416	500	1523	0	0	0	0	0
N.S.	1	1.00	1.20	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.957	0.952	29.441	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	429	429	595	1655	0	0	0	0	0
N.S.	1	1.00	1.39	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.097	0.644	33.324	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	276	322	501	0	0	0	0	0
N.S.	1	0.94	1.10	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.534	0.692	4.457	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	362	363	541	0	0	0	0	0
N.S.	1	0.94	0.95	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.624	1.077	5.196	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	483	401	564	0	0	0	0	0
N.S.	1	0.94	0.78	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.800	1.365	4.671	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	356	469	421	1105	0	0	0	0	0
N.S.	1	1.32	1.18	3.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.831	1.032	47.766	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	303	330	994	0	0	0	0	0
N.S.	1	1.09	1.19	3.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.454	0.625	25.006	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	192	212	239	2330	0	0	0	0	0
N.S.	1	1.10	1.24	12.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.268	0.730	6.846	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	100	95	851	0	0	0	0	0
N.S.	1	1.02	0.97	8.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.032	24.671	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	88	95	173	1449	0	136	0	0	0
N.S.	1	1.08	1.97	16.47	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.469	0.510	12.258	0.000	0.251	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	186	195	268	8468	0	0	0	0	0
N.S.	1	1.05	1.44	45.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.275	0.880	14.048	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	273	276	373	1873	0	0	0	0	0
N.S.	1	1.01	1.37	6.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.541	1.226	30.951	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	365	418	540	0	0	0	0	0	0
N.S.	1	1.15	1.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.698	0.998	180.000	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	433	433	502	1199	0	0	0	0	0
N.S.	1	1.00	1.16	2.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.033	1.971	36.240	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	364	364	429	1104	0	0	0	0	0
N.S.	1	1.00	1.18	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.828	1.613	21.600	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	292	292	362	4182	0	0	0	0	0
N.S.	1	1.00	1.24	14.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.700	1.182	9.999	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	216	216	300	857	0	0	0	0	0
N.S.	1	1.00	1.39	3.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	0.956	6.031	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	131	72	237	0	103	303	0	0
N.S.	1	1.07	0.59	1.94	0.00	0.84	2.48	0.00	0.00
time (sec)	N/A	0.321	0.651	1.942	0.000	0.249	5.053	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	221	221	299	1670	0	0	0	0	0
N.S.	1	1.00	1.35	7.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.743	0.814	6.572	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	306	306	406	8556	0	0	0	0	0
N.S.	1	1.00	1.33	27.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.907	1.986	11.963	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	403	403	489	1967	0	0	0	0	0
N.S.	1	1.00	1.21	4.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.121	2.593	38.536	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	462	462	578	1215	0	0	0	0	0
N.S.	1	1.00	1.25	2.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.022	2.016	29.517	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	383	383	507	4306	0	0	0	0	0
N.S.	1	1.00	1.32	11.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.848	1.254	8.346	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	304	304	431	960	0	0	0	0	0
N.S.	1	1.00	1.42	3.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.711	1.067	6.790	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	117	315	141	165	502	0	0
N.S.	1	1.00	0.66	1.77	0.79	0.93	2.82	0.00	0.00
time (sec)	N/A	0.401	0.428	2.334	0.237	0.243	42.260	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	184	110	277	136	158	464	0	0
N.S.	1	1.02	0.61	1.54	0.76	0.88	2.58	0.00	0.00
time (sec)	N/A	0.400	0.400	2.382	0.235	0.257	33.404	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	299	435	1783	0	0	0	0	0
N.S.	1	1.00	1.45	5.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.908	1.286	7.606	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	391	391	548	8688	0	0	0	0	0
N.S.	1	1.00	1.40	22.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.113	2.959	14.806	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	155	297	184	206	552	0	0
N.S.	1	1.00	0.75	1.43	0.89	1.00	2.67	0.00	0.00
time (sec)	N/A	0.418	0.352	2.414	0.254	0.250	21.425	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	85	82	193	0	0	0	0	0
N.S.	1	1.12	1.08	2.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	0.233	5.999	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	382	380	693	1513	0	0	0	0	0
N.S.	1	0.99	1.81	3.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.878	2.138	61.006	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	298	307	528	1384	0	0	0	0	0
N.S.	1	1.03	1.77	4.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	1.701	13.643	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	220	232	367	3777	0	0	0	0	0
N.S.	1	1.05	1.67	17.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.523	0.542	2.044	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	141	133	1629	0	0	0	0	0
N.S.	1	1.01	0.96	11.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.622	0.062	2.544	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	186	121	337	0	175	631	0	0
N.S.	1	1.02	0.66	1.85	0.00	0.96	3.47	0.00	0.00
time (sec)	N/A	0.413	0.774	0.824	0.000	0.248	16.189	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	276	183	425	233	265	954	0	0
N.S.	1	1.02	0.68	1.57	0.86	0.98	3.52	0.00	0.00
time (sec)	N/A	0.573	0.581	1.077	0.284	0.248	119.497	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	358	269	518	327	359	0	0	0
N.S.	1	0.99	0.75	1.44	0.91	1.00	0.00	0.00	0.00
time (sec)	N/A	0.788	0.680	1.255	0.369	0.272	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	410	451	541	1359	0	0	0	0	0
N.S.	1	1.10	1.32	3.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.740	1.030	14.228	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	290	393	3018	0	0	0	0	0
N.S.	1	1.05	1.42	10.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.616	0.977	1.888	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	141	133	1629	0	0	0	0	0
N.S.	1	1.01	0.96	11.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.606	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	128	136	268	2795	0	246	0	0	0
N.S.	1	1.06	2.09	21.84	0.00	1.92	0.00	0.00	0.00
time (sec)	N/A	0.642	0.606	2.539	0.000	0.276	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	263	281	439	10105	0	0	0	0	0
N.S.	1	1.07	1.67	38.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.858	1.339	2.853	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	414	421	635	2450	0	0	0	0	0
N.S.	1	1.02	1.53	5.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.819	2.296	20.812	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	43	34	3	23
N.S.	1	1.00	1.09	0.91	1.00	1.95	1.55	0.14	1.05
time (sec)	N/A	0.200	2.905	0.230	0.287	0.244	4.751	30.310	0.524

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	484	377	0	0	0	0	0
N.S.	1	1.00	1.63	1.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	2.493	0.368	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	404	284	0	0	0	0	0
N.S.	1	1.00	1.70	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	1.290	0.338	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	329	224	0	0	0	0	0
N.S.	1	1.00	1.84	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	1.175	0.263	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	138	156	0	0	0	0	0
N.S.	1	1.00	1.00	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.017	0.208	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	169	235	0	0	0	0	0
N.S.	1	1.00	0.93	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.083	0.259	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	223	306	0	0	0	0	0
N.S.	1	1.00	0.96	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	0.120	0.286	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	298	357	0	0	0	0	0
N.S.	1	1.00	1.02	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.495	0.136	0.373	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	598	598	0	2062	0	0	0	0	0
N.S.	1	1.00	0.00	3.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.966	0.000	38.265	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	430	430	0	1710	0	0	0	0	0
N.S.	1	1.00	0.00	3.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.697	0.000	26.048	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	323	323	0	15752	0	0	0	0	0
N.S.	1	1.00	0.00	48.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	0.000	11.855	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	223	223	0	1199	0	0	0	0	0
N.S.	1	1.00	0.00	5.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.000	1.247	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	369	369	0	2337	0	0	0	0	0
N.S.	1	1.00	0.00	6.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.654	0.000	2.355	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	473	473	0	38040	0	0	0	0	0
N.S.	1	1.00	0.00	80.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.848	0.000	18.135	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	C	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	591	591	0	2804	0	0	0	0	0
N.S.	1	1.00	0.00	4.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.058	0.000	53.889	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	25	15	3	20
N.S.	1	1.00	1.11	1.00	1.11	1.39	0.83	0.17	1.11
time (sec)	N/A	0.188	0.564	0.328	0.251	0.244	1.305	54.059	0.571

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	102	69	60	64	57	65	0	57
N.S.	1	1.48	1.00	0.87	0.93	0.83	0.94	0.00	0.83
time (sec)	N/A	0.317	0.009	0.229	0.275	0.251	0.319	0.000	0.322

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	98	66	63	63	62	61	0	58
N.S.	1	1.48	1.00	0.95	0.95	0.94	0.92	0.00	0.88
time (sec)	N/A	0.320	0.027	0.176	0.194	0.252	0.286	0.000	0.246

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	58	51	50	44	54	0	48
N.S.	1	1.10	1.38	1.21	1.19	1.05	1.29	0.00	1.14
time (sec)	N/A	0.211	0.010	0.166	0.220	0.251	0.263	0.000	0.556

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	66	50	46	45	47	48	0	46
N.S.	1	1.32	1.00	0.92	0.90	0.94	0.96	0.00	0.92
time (sec)	N/A	0.238	0.013	0.095	0.190	0.254	0.228	0.000	0.181

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	69	62	74	66	0	0	0	57
N.S.	1	1.11	1.00	1.19	1.06	0.00	0.00	0.00	0.92
time (sec)	N/A	0.323	0.009	0.141	0.322	0.000	0.000	0.000	0.611

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	64	40	41	40	45	41	0	42
N.S.	1	1.60	1.00	1.02	1.00	1.12	1.02	0.00	1.05
time (sec)	N/A	0.303	0.005	0.125	0.233	0.248	0.266	0.000	0.184

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	67	74	95	95	0	0	0	71
N.S.	1	0.96	1.06	1.36	1.36	0.00	0.00	0.00	1.01
time (sec)	N/A	0.328	0.009	0.189	0.314	0.000	0.000	0.000	0.663

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	86	58	56	56	57	61	0	57
N.S.	1	1.37	0.92	0.89	0.89	0.90	0.97	0.00	0.90
time (sec)	N/A	0.304	0.021	0.133	0.185	0.238	0.316	0.000	0.165

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	88	98	91	104	0	89
N.S.	1	1.00	1.00	0.79	0.88	0.82	0.94	0.00	0.80
time (sec)	N/A	0.322	0.045	0.355	0.285	0.237	0.430	0.000	0.491

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	93	95	94	105	0	81
N.S.	1	1.00	1.00	0.88	0.90	0.89	0.99	0.00	0.76
time (sec)	N/A	0.341	0.055	0.281	0.196	0.244	0.374	0.000	0.619

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	65	98	82	62	77	92	0	71
N.S.	1	1.07	1.61	1.34	1.02	1.26	1.51	0.00	1.16
time (sec)	N/A	0.230	0.033	0.250	0.180	0.240	0.324	0.000	0.626

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	115	65	76	77	79	88	0	69
N.S.	1	0.98	0.56	0.65	0.66	0.68	0.75	0.00	0.59
time (sec)	N/A	0.343	0.025	0.216	0.190	0.247	0.294	0.000	0.202

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	119	104	0	0	0	105
N.S.	1	1.00	1.00	1.20	1.05	0.00	0.00	0.00	1.06
time (sec)	N/A	0.291	0.031	0.324	0.327	0.000	0.000	0.000	0.689

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	62	73	71	75	82	0	76
N.S.	1	1.00	0.77	0.90	0.88	0.93	1.01	0.00	0.94
time (sec)	N/A	0.288	0.047	0.221	0.180	0.248	0.361	0.000	0.218

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	120	116	120	0	0	0	110
N.S.	1	1.00	1.33	1.29	1.33	0.00	0.00	0.00	1.22
time (sec)	N/A	0.289	0.036	0.485	0.327	0.000	0.000	0.000	0.565

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	68	75	76	80	87	0	78
N.S.	1	1.00	0.80	0.88	0.89	0.94	1.02	0.00	0.92
time (sec)	N/A	0.297	0.039	0.280	0.201	0.271	0.354	0.000	0.535

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	112	120	113	138	0	111
N.S.	1	1.00	1.00	0.79	0.85	0.80	0.98	0.00	0.79
time (sec)	N/A	0.365	0.048	0.363	0.289	0.258	0.576	0.000	0.498

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	136	116	118	116	138	0	108
N.S.	1	1.00	1.00	0.85	0.87	0.85	1.01	0.00	0.79
time (sec)	N/A	0.406	0.066	0.330	0.219	0.248	0.506	0.000	0.511

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	128	104	73	99	124	0	100
N.S.	1	1.01	1.73	1.41	0.99	1.34	1.68	0.00	1.35
time (sec)	N/A	0.243	0.040	0.309	0.194	0.254	0.423	0.000	0.465

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	158	83	98	99	101	117	0	89
N.S.	1	0.98	0.52	0.61	0.61	0.63	0.73	0.00	0.55
time (sec)	N/A	0.459	0.032	0.280	0.194	0.245	0.391	0.000	0.261

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	143	127	0	0	0	156
N.S.	1	1.00	1.00	1.08	0.96	0.00	0.00	0.00	1.18
time (sec)	N/A	0.338	0.031	0.471	0.343	0.000	0.000	0.000	0.766

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	78	95	93	97	110	0	85
N.S.	1	1.00	0.72	0.88	0.86	0.90	1.02	0.00	0.79
time (sec)	N/A	0.338	0.041	0.313	0.191	0.240	0.475	0.000	0.636

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	154	148	155	0	0	0	152
N.S.	1	1.00	1.12	1.07	1.12	0.00	0.00	0.00	1.10
time (sec)	N/A	0.326	0.034	0.809	0.335	0.000	0.000	0.000	0.646

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	83	99	96	100	117	0	97
N.S.	1	1.00	0.72	0.85	0.83	0.86	1.01	0.00	0.84
time (sec)	N/A	0.342	0.038	0.307	0.207	0.252	0.501	0.000	0.606

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	98	56	58	74	54	70	0	73
N.S.	1	1.22	0.70	0.72	0.92	0.68	0.88	0.00	0.91
time (sec)	N/A	0.599	0.077	0.212	0.288	0.259	0.364	0.000	0.182

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	116	120	152	0	0	0	0	0
N.S.	1	1.03	1.06	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	0.033	0.262	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	48	49	38	54	37	42	0	33
N.S.	1	0.98	1.00	0.78	1.10	0.76	0.86	0.00	0.67
time (sec)	N/A	0.338	0.056	0.227	0.280	0.242	0.269	0.000	0.166

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	77	119	0	0	0	0	0
N.S.	1	1.03	1.07	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.006	0.277	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	36	35	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	2.25	2.19	0.88
time (sec)	N/A	0.170	0.004	0.225	0.297	0.248	0.697	0.273	0.429

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	67	103	134	0	0	0	0	0
N.S.	1	1.05	1.61	2.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.019	0.280	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	51	52	46	53	43	42	0	48
N.S.	1	0.98	1.00	0.88	1.02	0.83	0.81	0.00	0.92
time (sec)	N/A	0.344	0.009	0.281	0.281	0.245	0.407	0.000	0.521

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	104	111	213	0	0	0	0	0
N.S.	1	0.92	0.98	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	0.080	0.392	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	102	88	81	90	71	76	0	78
N.S.	1	1.16	1.00	0.92	1.02	0.81	0.86	0.00	0.89
time (sec)	N/A	0.617	0.021	0.303	0.318	0.241	0.535	0.000	0.547

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	262	90	190	0	0	0	0	0
N.S.	1	1.67	0.57	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.600	0.222	0.432	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	113	79	86	114	81	223	0	94
N.S.	1	1.18	0.82	0.90	1.19	0.84	2.32	0.00	0.98
time (sec)	N/A	0.747	0.069	0.304	0.298	0.242	0.547	0.000	0.519

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	142	77	172	0	0	0	0	0
N.S.	1	1.07	0.58	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	0.121	0.450	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	55	83	49	0	0	48
N.S.	1	1.00	0.73	0.86	1.30	0.77	0.00	0.00	0.75
time (sec)	N/A	0.293	0.057	0.286	0.282	0.238	0.000	0.000	0.432

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	67	39	41	59	40	82	0	40
N.S.	1	1.08	0.63	0.66	0.95	0.65	1.32	0.00	0.65
time (sec)	N/A	0.219	0.040	0.264	0.270	0.242	0.354	0.000	0.179

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	56	78	46	0	0	48
N.S.	1	1.00	0.72	0.92	1.28	0.75	0.00	0.00	0.79
time (sec)	N/A	0.201	0.031	0.297	0.286	0.238	0.000	0.000	0.495

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	132	72	225	0	0	0	0	0
N.S.	1	1.13	0.62	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.155	0.383	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	108	94	95	119	97	274	0	91
N.S.	1	1.11	0.97	0.98	1.23	1.00	2.82	0.00	0.94
time (sec)	N/A	0.606	0.080	0.367	0.287	0.244	0.645	0.000	0.557

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	233	93	253	0	0	0	0	0
N.S.	1	1.49	0.60	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.428	0.368	0.646	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	207	124	121	160	127	362	0	123
N.S.	1	1.52	0.91	0.89	1.18	0.93	2.66	0.00	0.90
time (sec)	N/A	1.520	0.105	0.371	0.296	0.243	0.929	0.000	0.611

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	94	58	63	108	69	209	0	62
N.S.	1	1.09	0.67	0.73	1.26	0.80	2.43	0.00	0.72
time (sec)	N/A	0.254	0.142	0.371	0.270	0.247	0.596	0.000	0.567

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	110	64	81	129	83	0	0	80
N.S.	1	0.99	0.58	0.73	1.16	0.75	0.00	0.00	0.72
time (sec)	N/A	0.312	0.062	0.405	0.290	0.246	0.000	0.000	0.519

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	94	55	63	86	69	209	0	103
N.S.	1	1.12	0.65	0.75	1.02	0.82	2.49	0.00	1.23
time (sec)	N/A	0.232	0.057	0.366	0.284	0.245	0.585	0.000	0.546

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	104	68	93	129	85	0	0	85
N.S.	1	0.99	0.65	0.89	1.23	0.81	0.00	0.00	0.81
time (sec)	N/A	0.285	0.040	0.446	0.288	0.239	0.000	0.000	0.552

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	216	90	256	0	0	0	0	0
N.S.	1	1.36	0.57	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.934	0.199	0.544	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	208	118	136	181	149	604	0	133
N.S.	1	1.46	0.83	0.96	1.27	1.05	4.25	0.00	0.94
time (sec)	N/A	1.095	0.094	0.405	0.305	0.249	1.148	0.000	0.649

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	446	111	292	0	0	0	0	0
N.S.	1	2.18	0.54	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.522	0.552	1.059	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	412	142	161	223	179	724	0	163
N.S.	1	2.25	0.78	0.88	1.22	0.98	3.96	0.00	0.89
time (sec)	N/A	2.995	0.141	0.617	0.305	0.246	1.717	0.000	0.677

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	299	105	176	127	94	0	0	0
N.S.	1	1.87	0.66	1.10	0.79	0.59	0.00	0.00	0.00
time (sec)	N/A	0.763	0.139	0.516	0.284	0.268	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	284	278	199	0	0	0	0	0
N.S.	1	0.95	0.93	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.750	2.532	0.456	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	91	86	156	260	77	0	0	0
N.S.	1	1.06	1.00	1.81	3.02	0.90	0.00	0.00	0.00
time (sec)	N/A	0.245	0.100	0.396	0.355	0.274	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	189	141	178	0	0	0	0	0
N.S.	1	0.77	0.58	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	0.581	0.416	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	175	164	151	0	0	0	0	0
N.S.	1	0.76	0.72	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	0.247	0.489	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	203	163	930	0	0	0	0	0
N.S.	1	0.84	0.67	3.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.705	0.488	0.596	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	231	165	169	0	0	0	0	0
N.S.	1	0.96	0.69	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.690	1.197	0.447	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	85	105	153	73	84	0	0	0
N.S.	1	1.01	1.25	1.82	0.87	1.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.128	0.458	0.289	0.259	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	783	119	199	214	118	0	0	0
N.S.	1	3.61	0.55	0.92	0.99	0.54	0.00	0.00	0.00
time (sec)	N/A	2.308	0.154	0.458	0.320	0.261	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	693	576	221	0	0	0	0	0
N.S.	1	1.94	1.61	0.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.363	5.195	0.462	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	116	101	179	406	98	0	0	0
N.S.	1	1.06	0.93	1.64	3.72	0.90	0.00	0.00	0.00
time (sec)	N/A	0.265	0.163	0.410	0.397	0.268	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	239	351	201	0	0	0	0	0
N.S.	1	0.80	1.18	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.524	2.484	0.409	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	274	233	185	0	0	0	0	0
N.S.	1	0.98	0.83	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.938	0.292	0.490	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	400	218	263	0	0	0	0	0
N.S.	1	1.33	0.73	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.520	1.066	0.529	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	414	301	211	0	0	0	0	0
N.S.	1	1.36	0.99	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.560	1.874	0.504	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	296	263	2335	0	0	0	0	0
N.S.	1	0.95	0.85	7.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.166	0.568	0.791	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	0	129	225	338	154	0	0	0
N.S.	1	0.00	0.45	0.78	1.17	0.53	0.00	0.00	0.00
time (sec)	N/A	0.000	0.210	5.165	0.381	0.288	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	418	1650	907	245	0	0	0	0	0
N.S.	1	3.95	2.17	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.860	12.467	2.936	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	141	111	205	637	130	0	0	0
N.S.	1	1.05	0.83	1.53	4.75	0.97	0.00	0.00	0.00
time (sec)	N/A	0.283	0.202	1.626	0.477	0.269	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	348	289	643	225	0	0	0	0	0
N.S.	1	0.83	1.85	0.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.634	5.361	0.991	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	398	283	198	0	0	0	0	0
N.S.	1	1.21	0.86	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.499	0.336	1.208	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	647	491	265	0	0	0	0	0
N.S.	1	1.82	1.38	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.959	3.395	1.171	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	696	376	256	0	0	0	0	0
N.S.	1	1.91	1.03	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.850	1.906	1.926	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	704	313	270	0	0	0	0	0
N.S.	1	1.89	0.84	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.095	1.012	3.332	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	166	91	165	89	80	0	0	0
N.S.	1	1.38	0.76	1.38	0.74	0.67	0.00	0.00	0.00
time (sec)	N/A	0.439	0.118	0.429	0.293	0.271	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	200	158	184	0	0	0	0	0
N.S.	1	0.80	0.63	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.450	0.619	0.455	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	100	61	64	0	0	0
N.S.	1	1.00	1.02	1.69	1.03	1.08	0.00	0.00	0.00
time (sec)	N/A	0.236	0.068	0.399	0.326	0.262	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	140	90	150	0	0	0	0	0
N.S.	1	0.73	0.47	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.102	0.386	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	124	100	139	0	0	0	0	0
N.S.	1	0.70	0.56	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.232	0.427	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	62	125	36	68	0	0	0
N.S.	1	1.00	1.11	2.23	0.64	1.21	0.00	0.00	0.00
time (sec)	N/A	0.262	0.092	0.411	0.300	0.264	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	184	165	175	0	0	0	0	0
N.S.	1	0.76	0.68	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	0.757	0.449	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	150	110	163	81	89	0	0	0
N.S.	1	1.27	0.93	1.38	0.69	0.75	0.00	0.00	0.00
time (sec)	N/A	0.465	0.117	0.487	0.307	0.278	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	121	107	218	0	102	0	0	0
N.S.	1	1.13	1.00	2.04	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.427	0.124	0.409	0.000	0.273	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	196	155	247	0	0	0	0	0
N.S.	1	0.78	0.62	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.451	0.305	0.379	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	100	28	43	0	0	0
N.S.	1	1.00	0.86	2.04	0.57	0.88	0.00	0.00	0.00
time (sec)	N/A	0.223	0.062	0.306	0.331	0.255	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	38	98	41	40	0	0	0
N.S.	1	1.00	0.84	2.18	0.91	0.89	0.00	0.00	0.00
time (sec)	N/A	0.192	0.048	0.308	0.195	0.242	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	182	141	232	0	0	0	0	0
N.S.	1	0.79	0.62	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.264	0.423	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	111	122	219	0	104	0	0	0
N.S.	1	1.08	1.18	2.13	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.546	0.165	0.437	0.000	0.267	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	376	258	273	0	0	0	0	0
N.S.	1	1.25	0.86	0.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.650	1.231	0.457	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	271	143	273	0	129	0	0	0
N.S.	1	1.64	0.87	1.65	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	1.376	0.273	0.461	0.000	0.279	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	252	131	327	0	140	0	0	0
N.S.	1	1.48	0.77	1.92	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	1.063	0.192	0.825	0.000	0.282	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	290	177	389	0	0	0	0	0
N.S.	1	0.94	0.57	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.044	0.405	0.884	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	118	65	244	65	74	0	0	0
N.S.	1	1.05	0.58	2.18	0.58	0.66	0.00	0.00	0.00
time (sec)	N/A	0.382	0.096	0.655	0.280	0.284	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	81	57	240	93	67	0	0	0
N.S.	1	1.05	0.74	3.12	1.21	0.87	0.00	0.00	0.00
time (sec)	N/A	0.291	0.070	0.607	0.206	0.292	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	81	51	244	66	64	0	0	0
N.S.	1	1.03	0.65	3.09	0.84	0.81	0.00	0.00	0.00
time (sec)	N/A	0.238	0.070	0.401	0.254	0.279	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	103	63	240	86	72	0	0	0
N.S.	1	1.02	0.62	2.38	0.85	0.71	0.00	0.00	0.00
time (sec)	N/A	0.293	0.057	0.505	0.212	0.255	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	273	168	370	0	0	0	0	0
N.S.	1	0.98	0.60	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.081	0.372	0.462	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	224	151	325	0	142	0	0	0
N.S.	1	1.42	0.96	2.06	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	1.095	0.199	0.467	0.000	0.285	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	234	600	0	0	0	0	0
N.S.	1	1.00	0.87	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.462	0.249	167.296	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	175	376	0	0	0	0	0
N.S.	1	1.00	0.87	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.098	57.861	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	111	222	0	0	0	0	0
N.S.	1	1.00	0.90	1.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.090	12.436	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	3	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	0.15	1.10
time (sec)	N/A	0.206	0.931	0.641	0.400	0.269	1.249	58.948	0.393

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	37	29	3	22
N.S.	1	1.00	1.10	1.00	1.10	1.85	1.45	0.15	1.10
time (sec)	N/A	0.209	0.597	0.943	0.410	0.265	4.439	73.957	0.416

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	47	0	0	22
N.S.	1	1.00	1.09	0.91	1.00	2.14	0.00	0.00	1.00
time (sec)	N/A	0.240	1.089	0.743	0.520	0.269	0.000	0.000	0.560

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	0	0	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.231	0.597	0.544	0.417	0.273	0.000	0.000	0.478

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	22	0	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	1.00	0.00	1.00
time (sec)	N/A	0.429	0.118	0.460	0.327	0.259	18.262	0.000	0.415

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.217	0.692	0.423	0.327	0.255	10.291	0.390	0.504

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	49	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	2.23	1.00	1.00	1.00
time (sec)	N/A	0.232	0.688	0.447	0.368	0.278	24.382	0.390	0.610

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	290	89	108	116	97	121	0	102
N.S.	1	2.34	0.72	0.87	0.94	0.78	0.98	0.00	0.82
time (sec)	N/A	1.919	0.064	0.328	0.282	0.272	0.494	0.000	0.662

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	318	104	223	0	0	0	0	0
N.S.	1	2.04	0.67	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.490	0.575	0.724	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	64	87	87	74	94	0	83
N.S.	1	1.01	0.67	0.91	0.91	0.77	0.98	0.00	0.86
time (sec)	N/A	0.330	0.047	0.250	0.187	0.248	0.365	0.000	0.583

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	134	82	200	0	0	0	0	0
N.S.	1	1.05	0.64	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	0.067	0.486	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	209	183	1055	0	0	0	0	0
N.S.	1	1.24	1.08	6.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.186	0.124	18.393	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	164	94	238	0	0	0	0	0
N.S.	1	1.45	0.83	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.002	0.153	0.277	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	196	209	220	1134	0	0	0	0	0
N.S.	1	1.07	1.12	5.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.236	0.123	45.982	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	199	103	271	0	0	0	0	0
N.S.	1	1.47	0.76	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.927	0.291	0.754	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	110	150	169	148	185	0	145
N.S.	1	1.00	0.58	0.79	0.88	0.77	0.97	0.00	0.76
time (sec)	N/A	0.916	0.051	1.166	0.272	0.246	0.582	0.000	0.548

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	133	267	0	0	0	0	0
N.S.	1	1.00	0.59	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.908	1.106	2.314	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	144	84	132	111	123	158	0	135
N.S.	1	0.94	0.55	0.86	0.73	0.80	1.03	0.00	0.88
time (sec)	N/A	0.455	0.053	0.776	0.182	0.246	0.429	0.000	0.321

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	208	112	244	0	0	0	0	0
N.S.	1	1.01	0.55	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.743	0.591	1.851	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	235	235	218	1185	0	0	0	0	0
N.S.	1	1.00	0.93	5.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.689	0.281	50.115	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	167	285	0	0	0	0	0
N.S.	1	1.00	0.81	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.603	0.349	1.184	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	207	207	226	1184	0	0	0	0	0
N.S.	1	1.00	1.09	5.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.634	0.239	58.062	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	189	290	0	0	0	0	0
N.S.	1	1.00	0.88	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.625	0.362	1.470	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	126	188	202	181	241	0	178
N.S.	1	1.00	0.52	0.78	0.84	0.75	1.00	0.00	0.74
time (sec)	N/A	1.281	0.065	1.216	0.283	0.253	0.819	0.000	0.558

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	157	306	0	0	0	0	0
N.S.	1	1.00	0.57	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.202	1.926	3.808	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	187	100	168	133	156	207	0	156
N.S.	1	0.94	0.50	0.84	0.66	0.78	1.04	0.00	0.78
time (sec)	N/A	0.583	0.046	1.066	0.188	0.246	0.580	0.000	0.524

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	290	137	280	0	0	0	0	0
N.S.	1	1.08	0.51	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.068	1.009	3.126	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	287	287	252	1405	0	0	0	0	0
N.S.	1	1.00	0.88	4.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.934	0.463	68.148	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	202	321	0	0	0	0	0
N.S.	1	1.00	0.80	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.834	0.605	1.014	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	299	333	1318	0	0	0	0	0
N.S.	1	1.00	1.11	4.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.821	0.322	67.322	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	221	322	0	0	0	0	0
N.S.	1	1.00	0.88	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	0.524	1.431	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	243	90	224	0	0	0	0	0
N.S.	1	1.46	0.54	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.340	0.269	0.584	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	175	123	844	0	0	0	0	0
N.S.	1	1.04	0.73	4.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.154	0.120	19.895	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	104	69	187	0	0	0	0	0
N.S.	1	1.06	0.70	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.627	0.184	0.525	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	102	106	92	756	0	0	0	0	0
N.S.	1	1.04	0.90	7.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.507	0.027	6.347	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	0	0	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.00	0.88
time (sec)	N/A	0.177	0.006	0.232	0.265	0.234	0.000	0.000	0.173

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	91	107	83	1578	0	0	0	0	0
N.S.	1	1.18	0.91	17.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	0.131	11.532	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	95	73	252	0	0	0	0	0
N.S.	1	1.03	0.79	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.617	0.193	0.503	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	178	173	143	1786	0	0	0	0	0
N.S.	1	0.97	0.80	10.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.215	0.395	35.014	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	215	120	293	0	0	0	0	0
N.S.	1	1.30	0.72	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.195	0.399	0.840	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	192	197	117	855	0	0	0	0	0
N.S.	1	1.03	0.61	4.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.100	0.178	9.502	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	110	68	75	151	69	0	0	96
N.S.	1	1.04	0.64	0.71	1.42	0.65	0.00	0.00	0.91
time (sec)	N/A	0.348	0.175	0.503	0.302	0.235	0.000	0.000	0.464

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	87	47	58	104	48	0	0	50
N.S.	1	0.96	0.52	0.64	1.14	0.53	0.00	0.00	0.55
time (sec)	N/A	0.298	0.055	0.380	0.274	0.244	0.000	0.000	0.486

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	106	65	75	146	67	0	0	101
N.S.	1	1.06	0.65	0.75	1.46	0.67	0.00	0.00	1.01
time (sec)	N/A	0.304	0.098	0.468	0.296	0.243	0.000	0.000	0.571

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	170	195	119	1677	0	0	0	0	0
N.S.	1	1.15	0.70	9.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.054	0.174	16.086	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	197	109	313	0	0	0	0	0
N.S.	1	1.11	0.62	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.221	0.394	0.758	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	365	159	4110	0	0	0	0	0
N.S.	1	1.46	0.64	16.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.987	1.041	85.266	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	409	166	351	0	0	0	0	0
N.S.	1	1.69	0.69	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.444	0.396	1.159	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	145	74	93	185	87	0	0	85
N.S.	1	1.04	0.53	0.66	1.32	0.62	0.00	0.00	0.61
time (sec)	N/A	0.554	0.102	0.628	0.319	0.242	0.000	0.000	0.620

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	312	95	123	232	114	0	0	150
N.S.	1	1.72	0.52	0.68	1.28	0.63	0.00	0.00	0.83
time (sec)	N/A	0.879	0.110	0.618	0.334	0.236	0.000	0.000	0.555

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	133	71	93	163	87	0	0	85
N.S.	1	0.96	0.51	0.67	1.18	0.63	0.00	0.00	0.62
time (sec)	N/A	0.393	0.068	0.584	0.304	0.258	0.000	0.000	0.569

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	220	98	123	232	113	0	0	157
N.S.	1	1.30	0.58	0.73	1.37	0.67	0.00	0.00	0.93
time (sec)	N/A	0.508	0.054	0.668	0.340	0.248	0.000	0.000	0.595

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	236	327	156	1722	0	0	0	0	0
N.S.	1	1.39	0.66	7.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.938	0.218	19.061	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	401	139	370	0	0	0	0	0
N.S.	1	1.60	0.56	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.437	0.341	0.853	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	322	689	227	1970	0	0	0	0	0
N.S.	1	2.14	0.70	6.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.624	0.834	57.977	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	0	189	409	0	0	0	0	0
N.S.	1	0.00	0.60	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.710	1.793	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	1189	360	235	0	0	0	0	0
N.S.	1	3.09	0.94	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.682	1.050	1.392	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	706	267	302	0	0	0	0	0
N.S.	1	1.62	0.61	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.471	1.088	1.395	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	227	260	198	0	0	0	0	0
N.S.	1	0.81	0.93	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.536	0.555	1.280	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	228	201	268	0	0	0	0	0
N.S.	1	0.67	0.59	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.836	0.346	1.290	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	305	250	337	0	0	0	0	0
N.S.	1	0.69	0.57	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.652	0.322	1.668	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	300	265	309	0	0	0	0	0
N.S.	1	0.66	0.58	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.906	0.752	1.573	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	354	222	255	0	0	0	0	0
N.S.	1	1.08	0.68	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.874	1.619	1.497	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	267	239	195	0	0	0	0	0
N.S.	1	0.97	0.87	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.872	1.475	1.378	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	476	3185	797	271	0	0	0	0	0
N.S.	1	6.69	1.67	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	18.376	3.919	1.357	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	0	527	338	0	0	0	0	0
N.S.	1	0.00	0.99	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	2.990	1.434	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	277	601	237	0	0	0	0	0
N.S.	1	0.83	1.80	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.683	3.383	1.220	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	344	439	304	0	0	0	0	0
N.S.	1	0.79	1.00	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.241	0.835	1.279	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	540	496	365	0	0	0	0	0
N.S.	1	1.02	0.94	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.242	2.268	1.573	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	536	376	356	0	0	0	0	0
N.S.	1	0.96	0.68	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.106	0.929	1.515	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	567	667	455	412	0	0	0	0	0
N.S.	1	1.18	0.80	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.754	2.482	1.554	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	575	453	343	0	0	0	0	0
N.S.	1	0.99	0.78	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.293	5.006	1.631	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	578	0	1320	309	0	0	0	0	0
N.S.	1	0.00	2.28	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.722	10.110	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	0	759	376	0	0	0	0	0
N.S.	1	0.00	1.19	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	4.151	6.043	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	327	1087	275	0	0	0	0	0
N.S.	1	0.84	2.81	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.814	7.753	3.836	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	516	485	771	342	0	0	0	0	0
N.S.	1	0.94	1.49	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.730	1.533	2.495	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	825	889	404	0	0	0	0	0
N.S.	1	1.36	1.47	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.721	7.175	3.019	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	655	0	626	399	0	0	0	0	0
N.S.	1	0.00	0.96	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.608	2.932	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	661	1215	761	454	0	0	0	0	0
N.S.	1	1.84	1.15	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	10.824	7.595	3.091	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	675	1119	644	401	0	0	0	0	0
N.S.	1	1.66	0.95	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	10.472	2.945	4.437	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	420	279	206	0	0	0	0	0
N.S.	1	1.33	0.89	0.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.337	0.635	1.255	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	242	175	274	0	0	0	0	0
N.S.	1	0.70	0.51	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.255	0.415	1.211	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	172	126	180	0	0	0	0	0
N.S.	1	0.78	0.57	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	0.265	1.228	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	143	140	0	0	0	0	0	0
N.S.	1	0.56	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	126	145	198	0	0	0	0	0
N.S.	1	0.56	0.64	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.631	0.237	1.270	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	155	128	171	0	0	0	0	0
N.S.	1	0.75	0.62	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.487	1.308	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	221	231	261	0	0	0	0	0
N.S.	1	0.67	0.70	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.302	1.062	1.477	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	382	228	206	0	0	0	0	0
N.S.	1	1.23	0.73	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.465	2.236	1.473	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	264	209	294	0	0	0	0	0
N.S.	1	0.87	0.69	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.900	0.749	1.201	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	224	228	0	0	0	0	0	0
N.S.	1	0.64	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.165	0.401	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	79	50	116	73	51	0	0	0
N.S.	1	1.01	0.64	1.49	0.94	0.65	0.00	0.00	0.00
time (sec)	N/A	0.330	0.082	1.044	0.521	0.268	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	49	114	53	51	0	0	0
N.S.	1	1.00	0.68	1.58	0.74	0.71	0.00	0.00	0.00
time (sec)	N/A	0.228	0.062	0.796	0.369	0.243	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	214	204	307	0	0	0	0	0
N.S.	1	0.69	0.66	0.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.379	0.312	1.357	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	237	226	279	0	0	0	0	0
N.S.	1	0.81	0.77	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.959	0.924	1.073	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	445	371	376	0	0	0	0	0
N.S.	1	1.05	0.88	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.262	1.931	1.088	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	629	270	318	0	0	0	0	0
N.S.	1	1.58	0.68	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.855	2.783	1.265	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	482	229	454	0	0	0	0	0
N.S.	1	1.20	0.57	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.374	1.195	2.346	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	444	391	239	0	0	0	0	0	0
N.S.	1	0.88	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.510	0.562	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	205	81	276	0	92	0	0	0
N.S.	1	1.19	0.47	1.60	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.685	0.163	1.871	0.000	0.264	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	154	80	272	117	88	0	0	0
N.S.	1	1.11	0.58	1.96	0.84	0.63	0.00	0.00	0.00
time (sec)	N/A	0.569	0.108	1.610	0.275	0.254	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	141	71	276	0	82	0	0	0
N.S.	1	1.03	0.52	2.01	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.436	0.089	0.861	0.000	0.259	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	185	86	272	111	93	0	0	0
N.S.	1	1.18	0.55	1.73	0.71	0.59	0.00	0.00	0.00
time (sec)	N/A	0.396	0.074	1.424	0.287	0.260	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	365	246	461	0	0	0	0	0
N.S.	1	0.94	0.63	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.609	0.477	1.306	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	432	296	579	0	0	0	0	0
N.S.	1	1.13	0.78	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.961	1.422	1.221	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	841	37	53	3	24
N.S.	1	1.00	1.09	1.00	38.23	1.68	2.41	0.14	1.09
time (sec)	N/A	0.217	1.432	1.122	9.323	0.262	14.903	110.247	0.485

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	386	22	31	3	22
N.S.	1	1.00	1.10	1.00	19.30	1.10	1.55	0.15	1.10
time (sec)	N/A	0.193	0.718	0.918	4.671	0.255	6.427	107.228	0.466

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	0.14	1.09
time (sec)	N/A	0.221	0.923	0.603	0.490	0.250	1.646	99.654	0.397

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	39	31	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.77	1.41	0.14	1.09
time (sec)	N/A	0.225	0.620	2.138	0.506	0.270	3.089	116.296	0.441

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	0	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.260	1.102	1.456	0.563	0.265	0.000	0.000	0.516

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.00	1.00
time (sec)	N/A	0.254	0.159	1.115	0.411	0.254	29.927	0.000	0.433

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.12	1.00
time (sec)	N/A	0.252	0.642	0.994	0.408	0.245	18.638	41.207	0.503

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.12	1.00
time (sec)	N/A	0.264	0.799	1.225	0.431	0.248	20.645	3.960	0.613

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	711	135	272	0	0	0	0	0
N.S.	1	3.25	0.62	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.438	0.547	1.844	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	503	171	900	0	0	0	0	0
N.S.	1	2.38	0.81	4.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.733	0.492	46.493	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	164	101	237	0	0	0	0	0
N.S.	1	1.02	0.63	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.653	0.083	2.583	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	172	188	144	860	0	0	0	0	0
N.S.	1	1.09	0.84	5.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	0.061	12.824	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	339	264	460	0	0	0	0	0
N.S.	1	1.23	0.96	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.106	0.147	19.538	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	169	236	188	1653	0	0	0	0	0
N.S.	1	1.40	1.11	9.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.606	0.149	14.206	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	328	299	525	0	0	0	0	0
N.S.	1	1.06	0.96	1.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.056	0.301	45.813	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	189	305	179	1807	0	0	0	0	0
N.S.	1	1.61	0.95	9.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.004	0.421	38.365	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	165	331	0	0	0	0	0
N.S.	1	1.00	0.53	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.227	1.030	3.498	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	321	321	233	1256	0	0	0	0	0
N.S.	1	1.00	0.73	3.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.784	0.960	72.002	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	233	131	296	0	0	0	0	0
N.S.	1	0.96	0.54	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.904	0.662	3.733	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	289	312	195	953	0	0	0	0	0
N.S.	1	1.08	0.67	3.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.286	0.477	14.286	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	302	566	0	0	0	0	0
N.S.	1	1.00	0.82	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.128	0.425	24.325	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	284	284	246	1793	0	0	0	0	0
N.S.	1	1.00	0.87	6.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.920	0.325	76.248	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	302	622	0	0	0	0	0
N.S.	1	1.00	0.76	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.004	0.407	32.788	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	311	311	294	1883	0	0	0	0	0
N.S.	1	1.00	0.95	6.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.969	0.475	103.362	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	191	383	0	0	0	0	0
N.S.	1	1.00	0.50	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.479	1.697	4.952	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	389	389	281	1576	0	0	0	0	0
N.S.	1	1.00	0.72	4.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.842	1.604	121.678	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	157	346	0	0	0	0	0
N.S.	1	1.00	0.51	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.219	1.102	5.191	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	388	473	243	1267	0	0	0	0	0
N.S.	1	1.22	0.63	3.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.219	1.002	86.792	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	350	664	0	0	0	0	0
N.S.	1	1.00	0.78	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.800	0.823	56.759	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	354	354	298	1894	0	0	0	0	0
N.S.	1	1.00	0.84	5.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.434	0.605	163.404	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	464	763	0	0	0	0	0
N.S.	1	1.00	0.92	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.434	0.573	77.002	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	336	336	318	1948	0	0	0	0	0
N.S.	1	1.00	0.95	5.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.296	0.660	138.346	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	332	154	888	0	0	0	0	0
N.S.	1	1.53	0.71	4.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.448	0.254	18.163	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	269	162	259	0	0	0	0	0
N.S.	1	1.03	0.62	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.692	0.243	39.600	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	130	136	93	785	0	0	0	0	0
N.S.	1	1.05	0.72	6.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.830	0.206	14.139	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	138	142	119	789	0	0	0	0	0
N.S.	1	1.03	0.86	5.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.643	0.026	15.794	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	0	0	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.00	0.88
time (sec)	N/A	0.180	0.004	0.665	0.277	0.239	0.000	0.000	0.135

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	143	100	1640	0	0	0	0	0
N.S.	1	1.15	0.81	13.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.664	0.133	68.934	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	122	135	108	1609	0	0	0	0	0
N.S.	1	1.11	0.89	13.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.868	0.158	66.129	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	256	189	441	0	0	0	0	0
N.S.	1	0.98	0.72	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.648	0.402	91.860	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	321	153	1831	0	0	0	0	0
N.S.	1	1.41	0.67	8.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.308	0.600	77.326	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	270	281	156	936	0	0	0	0	0
N.S.	1	1.04	0.58	3.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.610	0.166	24.988	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	136	74	88	218	76	0	0	119
N.S.	1	1.01	0.55	0.65	1.61	0.56	0.00	0.00	0.88
time (sec)	N/A	0.437	0.071	1.154	0.341	0.239	0.000	0.000	0.511

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	135	68	75	174	69	0	0	114
N.S.	1	1.02	0.51	0.56	1.31	0.52	0.00	0.00	0.86
time (sec)	N/A	0.422	0.056	0.945	0.299	0.240	0.000	0.000	0.475

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	131	71	88	213	73	0	0	119
N.S.	1	1.02	0.55	0.68	1.65	0.57	0.00	0.00	0.92
time (sec)	N/A	0.393	0.041	1.059	0.327	0.244	0.000	0.000	0.502

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	240	279	156	1787	0	0	0	0	0
N.S.	1	1.16	0.65	7.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.545	0.165	84.450	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	262	157	1731	0	0	0	0	0
N.S.	1	1.12	0.67	7.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.780	0.308	77.723	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	532	243	531	0	0	0	0	0
N.S.	1	1.42	0.65	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.177	0.551	96.556	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	332	580	195	4175	0	0	0	0	0
N.S.	1	1.75	0.59	12.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	6.077	1.511	233.768	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	261	105	123	289	117	0	0	205
N.S.	1	1.23	0.50	0.58	1.36	0.55	0.00	0.00	0.97
time (sec)	N/A	0.721	0.205	1.392	0.319	0.234	0.000	0.000	0.657

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	410	111	145	334	130	0	0	188
N.S.	1	1.73	0.47	0.61	1.41	0.55	0.00	0.00	0.79
time (sec)	N/A	1.487	0.077	1.299	0.353	0.248	0.000	0.000	0.600

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	237	103	123	272	117	0	0	189
N.S.	1	1.14	0.50	0.59	1.31	0.56	0.00	0.00	0.91
time (sec)	N/A	0.617	0.078	1.347	0.332	0.251	0.000	0.000	0.611

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	284	114	153	335	132	0	0	199
N.S.	1	1.26	0.51	0.68	1.49	0.59	0.00	0.00	0.88
time (sec)	N/A	0.934	0.115	1.397	0.347	0.255	0.000	0.000	0.596

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	332	515	208	1841	0	0	0	0	0
N.S.	1	1.55	0.63	5.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.195	0.238	93.257	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	332	539	232	1799	0	0	0	0	0
N.S.	1	1.62	0.70	5.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.123	0.430	82.408	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	0	295	579	0	0	0	0	0
N.S.	1	0.00	0.62	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.811	95.914	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	C	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD
size	432	0	302	2062	0	0	0	0	0
N.S.	1	0.00	0.70	4.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.155	136.483	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	0	262	417	0	0	0	0	0
N.S.	1	0.00	0.50	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.015	4.347	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	747	1306	1844	460	0	0	0	0	0
N.S.	1	1.75	2.47	0.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	9.631	12.232	4.585	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	264	206	370	0	0	0	0	0
N.S.	1	0.71	0.55	0.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.038	0.612	3.947	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	626	400	258	422	0	0	0	0	0
N.S.	1	0.64	0.41	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.362	0.636	4.336	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	600	357	366	454	0	0	0	0	0
N.S.	1	0.60	0.61	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.712	0.556	5.573	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	622	360	768	466	0	0	0	0	0
N.S.	1	0.58	1.23	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.544	2.666	5.532	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	602	560	345	404	0	0	0	0	0
N.S.	1	0.93	0.57	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.458	4.155	4.326	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	387	341	315	0	0	0	0	0
N.S.	1	1.07	0.94	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.167	2.933	3.849	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	652	0	538	469	0	0	0	0	0
N.S.	1	0.00	0.83	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	2.780	4.035	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	A	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	882	0	4015	514	0	0	0	0	0
N.S.	1	0.00	4.55	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	18.474	4.096	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	477	382	441	421	0	0	0	0	0
N.S.	1	0.80	0.92	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.444	2.702	3.632	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	760	652	2105	466	0	0	0	0	0
N.S.	1	0.86	2.77	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.304	12.727	3.747	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	726	629	555	511	0	0	0	0	0
N.S.	1	0.87	0.76	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.837	1.215	4.733	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	901	768	1387	602	0	0	0	0	0
N.S.	1	0.85	1.54	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.792	5.128	5.080	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	919	925	691	592	0	0	0	0	0
N.S.	1	1.01	0.75	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	8.374	6.825	5.247	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	788	755	1508	557	0	0	0	0	0
N.S.	1	0.96	1.91	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	7.700	9.605	4.726	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	798	0	850	525	0	0	0	0	0
N.S.	1	0.00	1.07	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.372	19.268	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	A	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1019	0	6517	566	0	0	0	0	0
N.S.	1	0.00	6.40	0.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	24.694	12.628	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	523	718	477	0	0	0	0	0
N.S.	1	0.93	1.28	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.954	4.535	8.267	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	870	954	4281	518	0	0	0	0	0
N.S.	1	1.10	4.92	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.642	18.945	6.195	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	845	0	723	562	0	0	0	0	0
N.S.	1	0.00	0.86	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	5.863	7.490	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1027	0	3267	655	0	0	0	0	0
N.S.	1	0.00	3.18	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	15.009	7.823	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1043	0	934	660	0	0	0	0	0
N.S.	1	0.00	0.90	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	9.018	7.424	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1061	0	1771	694	0	0	0	0	0
N.S.	1	0.00	1.67	0.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	10.259	7.499	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	460	220	382	0	0	0	0	0
N.S.	1	1.13	0.54	0.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.207	0.785	3.972	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	415	812	428	0	0	0	0	0
N.S.	1	0.66	1.30	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.904	5.657	3.890	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	283	172	168	0	0	0	0	0	0
N.S.	1	0.61	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.728	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	368	201	190	0	0	0	0	0	0
N.S.	1	0.55	0.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.696	0.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	178	208	261	0	0	0	0	0
N.S.	1	0.54	0.64	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.773	0.279	4.419	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	157	174	230	0	0	0	0	0
N.S.	1	0.60	0.67	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.847	0.414	4.268	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	375	345	410	0	0	0	0	0
N.S.	1	0.63	0.58	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.866	3.111	4.351	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	418	343	324	0	0	0	0	0
N.S.	1	1.06	0.87	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.399	4.919	4.206	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	291	308	0	0	0	0	0	0
N.S.	1	0.72	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.554	0.764	0.000	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	495	311	639	0	0	0	0	0	0
N.S.	1	0.63	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.450	1.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	106	61	134	98	62	0	0	0
N.S.	1	0.99	0.57	1.25	0.92	0.58	0.00	0.00	0.00
time (sec)	N/A	0.361	0.115	2.627	0.604	0.249	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	101	56	132	99	58	0	0	0
N.S.	1	1.01	0.56	1.32	0.99	0.58	0.00	0.00	0.00
time (sec)	N/A	0.307	0.076	2.665	0.509	0.261	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	293	295	388	0	0	0	0	0
N.S.	1	0.66	0.67	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.661	0.422	3.828	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	268	301	356	0	0	0	0	0
N.S.	1	0.71	0.80	0.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.623	1.302	3.633	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	534	609	367	0	0	0	0	0	0
N.S.	1	1.14	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.189	1.929	0.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	622	563	691	0	0	0	0	0	0
N.S.	1	0.91	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.657	2.253	0.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	305	104	312	0	113	0	0	0
N.S.	1	1.29	0.44	1.32	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	1.114	0.151	5.302	0.000	0.274	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	239	95	308	0	106	0	0	0
N.S.	1	1.20	0.48	1.55	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.847	0.118	4.792	0.000	0.267	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	220	91	312	0	103	0	0	0
N.S.	1	1.11	0.46	1.57	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.528	0.100	2.458	0.000	0.252	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	274	104	308	0	111	0	0	0
N.S.	1	1.27	0.48	1.43	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.816	0.088	4.166	0.000	0.254	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	523	347	560	0	0	0	0	0
N.S.	1	0.95	0.63	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.372	0.679	4.061	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	552	399	528	0	0	0	0	0
N.S.	1	1.12	0.81	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.991	1.968	3.768	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	952	37	53	3	24
N.S.	1	1.00	1.09	1.00	43.27	1.68	2.41	0.14	1.09
time (sec)	N/A	0.213	1.523	3.824	13.156	0.258	28.468	149.213	0.468

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	435	22	31	3	22
N.S.	1	1.00	1.10	1.00	21.75	1.10	1.55	0.15	1.10
time (sec)	N/A	0.192	0.736	3.265	6.271	0.241	12.472	145.872	0.464

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	0.14	1.09
time (sec)	N/A	0.221	0.950	1.847	0.586	0.250	3.133	136.658	0.401

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	39	31	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.77	1.41	0.14	1.09
time (sec)	N/A	0.221	0.645	4.655	0.607	0.244	4.312	159.301	0.425

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	0	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.259	1.155	4.275	0.692	0.249	0.000	0.000	0.527

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.00	1.00
time (sec)	N/A	0.246	0.162	3.660	0.464	0.236	71.719	0.000	0.458

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.12	1.00
time (sec)	N/A	0.252	0.668	3.778	0.443	0.249	49.953	59.756	0.507

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.12	1.00
time (sec)	N/A	0.268	0.812	3.869	0.478	0.261	59.521	4.246	0.603

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	21	22	3	20
N.S.	1	1.00	1.11	1.00	1.11	1.17	1.22	0.17	1.11
time (sec)	N/A	0.184	0.388	13.419	0.293	0.269	0.690	30.612	0.461

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	19	19	22	3	19
N.S.	1	1.00	1.12	1.00	1.12	1.12	1.29	0.18	1.12
time (sec)	N/A	0.174	0.350	13.406	0.299	0.250	0.614	28.823	0.443

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	22	3	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.10	0.15	1.10
time (sec)	N/A	0.191	0.568	8.867	0.271	0.230	0.922	30.286	0.438

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	36	39	3	22
N.S.	1	1.00	1.10	1.00	1.10	1.80	1.95	0.15	1.10
time (sec)	N/A	0.196	0.477	13.138	0.299	0.246	1.340	36.116	0.414

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	34	39	3	21
N.S.	1	1.00	1.11	1.00	1.11	1.79	2.05	0.16	1.11
time (sec)	N/A	0.181	0.375	28.274	0.289	0.245	0.982	33.645	0.418

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	37	39	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.68	1.77	0.14	1.09
time (sec)	N/A	0.205	0.666	43.575	0.293	0.241	2.187	35.609	0.424

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	47	54	3	22
N.S.	1	1.00	1.10	1.00	1.10	2.35	2.70	0.15	1.10
time (sec)	N/A	0.195	0.501	71.722	0.290	0.240	1.453	40.445	0.442

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	45	54	3	21
N.S.	1	1.00	1.11	1.00	1.11	2.37	2.84	0.16	1.11
time (sec)	N/A	0.181	0.418	59.869	0.294	0.248	1.348	39.037	0.450

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	48	54	3	24
N.S.	1	1.00	1.09	1.00	1.09	2.18	2.45	0.14	1.09
time (sec)	N/A	0.205	0.682	160.841	0.306	0.243	2.649	42.117	0.426

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	0.14	1.09
time (sec)	N/A	0.217	1.253	0.342	0.250	0.236	0.555	19.569	0.394

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	3	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	0.15	1.10
time (sec)	N/A	0.200	0.716	0.401	0.243	0.249	0.564	14.974	0.386

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	15	12	8	0	12
N.S.	1	1.00	1.00	1.08	1.25	1.00	0.67	0.00	1.00
time (sec)	N/A	0.176	0.109	0.883	0.192	0.240	0.266	0.000	0.094

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	23	22	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.05	1.00	0.14	1.09
time (sec)	N/A	0.221	0.148	0.323	0.256	0.245	0.814	15.166	0.379

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	25	24	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.14	1.09	0.14	1.09
time (sec)	N/A	0.222	0.219	0.288	0.272	0.254	0.742	20.443	0.396

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	39	37	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.77	1.68	0.14	1.09
time (sec)	N/A	0.219	3.253	9.681	0.275	0.238	0.788	49.150	0.409

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	39	37	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.77	1.68	0.14	1.09
time (sec)	N/A	0.219	1.724	2.194	0.285	0.240	0.787	47.612	0.406

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	28	25	24	0	74	0	0	0
N.S.	1	0.85	0.76	0.73	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.297	0.088	2.992	0.000	0.249	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	67	0	0	0
N.S.	1	1.00	1.00	0.94	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	0.291	0.055	2.696	0.000	0.240	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	28	23	22	0	70	0	0	0
N.S.	1	0.85	0.70	0.67	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.267	0.036	3.169	0.000	0.245	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	38	37	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.73	1.68	0.14	1.09
time (sec)	N/A	0.215	0.598	3.414	0.286	0.238	1.142	36.890	0.396

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	40	39	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.82	1.77	0.14	1.09
time (sec)	N/A	0.222	1.020	4.403	0.273	0.230	1.254	38.578	0.393

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	50	51	3	24
N.S.	1	1.00	1.09	1.00	1.09	2.27	2.32	0.14	1.09
time (sec)	N/A	0.219	6.442	8.089	0.301	0.245	1.043	72.357	0.432

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	50	51	3	24
N.S.	1	1.00	1.09	1.00	1.09	2.27	2.32	0.14	1.09
time (sec)	N/A	0.216	6.092	8.674	0.282	0.248	1.185	73.615	0.415

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	39	34	35	0	174	0	0	0
N.S.	1	0.78	0.68	0.70	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.321	0.107	3.513	0.000	0.252	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	30	27	26	0	171	0	0	0
N.S.	1	0.86	0.77	0.74	0.00	4.89	0.00	0.00	0.00
time (sec)	N/A	0.303	0.115	2.886	0.000	0.255	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	28	25	24	0	120	0	0	0
N.S.	1	0.85	0.76	0.73	0.00	3.64	0.00	0.00	0.00
time (sec)	N/A	0.304	0.073	3.217	0.000	0.250	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	30	27	26	0	171	0	0	0
N.S.	1	0.86	0.77	0.74	0.00	4.89	0.00	0.00	0.00
time (sec)	N/A	0.284	0.086	2.844	0.000	0.243	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	39	34	33	0	174	0	0	0
N.S.	1	0.78	0.68	0.66	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.285	0.034	3.376	0.000	0.261	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	49	51	3	24
N.S.	1	1.00	1.09	1.00	1.09	2.23	2.32	0.14	1.09
time (sec)	N/A	0.214	0.746	4.447	0.254	0.259	1.624	49.839	0.403

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	51	53	3	24
N.S.	1	1.00	1.09	1.00	1.09	2.32	2.41	0.14	1.09
time (sec)	N/A	0.223	1.125	4.130	0.256	0.234	1.733	53.322	0.419

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	20	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.91	0.14	1.00
time (sec)	N/A	0.222	1.957	5.965	0.322	0.242	0.734	33.371	0.358

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	19	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.90	0.14	1.00
time (sec)	N/A	0.193	0.220	5.789	0.339	0.241	0.546	32.143	0.341

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	20	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.83	0.12	1.00
time (sec)	N/A	0.254	1.290	14.348	0.349	0.243	0.899	34.765	0.353

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	33	20	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.50	0.91	0.14	1.00
time (sec)	N/A	0.233	2.084	19.100	0.345	0.236	4.740	47.165	0.411

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	19	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.90	0.14	1.00
time (sec)	N/A	0.198	0.277	18.430	0.345	0.240	2.966	43.796	0.410

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	20	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.83	0.12	1.00
time (sec)	N/A	0.256	1.438	25.957	0.359	0.233	6.343	46.636	0.397

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	48	20	3	22
N.S.	1	1.00	1.09	0.91	1.00	2.18	0.91	0.14	1.00
time (sec)	N/A	0.229	2.086	16.833	0.397	0.245	15.589	56.898	0.401

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	46	19	3	21
N.S.	1	1.00	1.10	0.90	1.00	2.19	0.90	0.14	1.00
time (sec)	N/A	0.195	0.356	15.849	0.394	0.238	11.124	54.155	0.415

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	49	20	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.04	0.83	0.12	1.00
time (sec)	N/A	0.264	1.596	98.329	0.391	0.251	15.946	56.622	0.405

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	20	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.91	0.14	1.00
time (sec)	N/A	0.223	0.976	1.816	0.322	0.252	0.726	35.821	0.413

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	20	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	0.14	1.00
time (sec)	N/A	0.195	0.202	3.820	0.324	0.251	0.698	33.858	0.412

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.92	0.12	1.00
time (sec)	N/A	0.255	0.722	4.524	0.339	0.248	1.158	34.328	0.430

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	22	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.92	0.00	1.00
time (sec)	N/A	0.265	5.074	1.691	0.345	0.249	1.904	0.000	0.417

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	0	22	24	51	22	3	24
N.S.	1	1.00	0.00	0.92	1.00	2.12	0.92	0.12	1.00
time (sec)	N/A	0.267	0.000	1.643	0.360	0.241	1.984	30.789	0.423

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	40	82	0	0	0	0	0
N.S.	1	1.00	1.03	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	0.185	3.624	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	39	39	40	136	0	0	0	0	0
N.S.	1	1.00	1.03	3.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.342	0.047	3.952	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	50	22	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.08	0.92	0.00	1.00
time (sec)	N/A	0.262	1.350	2.322	0.352	0.249	3.832	0.000	0.429

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	52	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.17	1.00	0.12	1.00
time (sec)	N/A	0.268	1.449	2.332	0.345	0.255	4.435	32.343	0.459

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	22	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	0.92	0.00	1.00
time (sec)	N/A	0.268	6.130	6.646	0.356	0.245	5.470	0.000	0.440

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	0	22	24	62	22	3	24
N.S.	1	1.00	0.00	0.92	1.00	2.58	0.92	0.12	1.00
time (sec)	N/A	0.264	0.000	12.283	0.354	0.238	5.594	75.963	0.422

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	55	55	125	0	0	0	0	0
N.S.	1	0.63	0.63	1.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	0.281	8.078	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	55	53	84	0	0	0	0	0
N.S.	1	0.63	0.61	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.507	0.116	8.891	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	55	51	125	0	0	0	0	0
N.S.	1	0.63	0.59	1.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.137	3.382	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	55	53	179	0	0	0	0	0
N.S.	1	0.63	0.61	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.405	0.102	6.214	0.000	0.000	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	61	22	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.54	0.92	0.00	1.00
time (sec)	N/A	0.274	1.809	2.658	0.355	0.252	9.810	0.000	0.458

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	63	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.62	1.00	0.12	1.00
time (sec)	N/A	0.274	2.164	2.394	0.356	0.257	14.557	61.450	0.439

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	48	66	3	24
N.S.	1	1.00	1.09	1.00	1.09	2.18	3.00	0.14	1.09
time (sec)	N/A	0.220	0.539	4.617	0.394	0.246	12.007	134.381	0.402

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	37	48	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.68	2.18	0.14	1.09
time (sec)	N/A	0.216	0.658	3.980	0.336	0.263	5.203	114.767	0.418

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	27	3	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.35	0.15	1.10
time (sec)	N/A	0.194	0.394	3.806	0.301	0.236	2.620	91.292	0.421

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	0.14	1.09
time (sec)	N/A	0.227	0.463	2.032	0.251	0.246	1.611	62.050	0.402

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	39	37	3	24
N.S.	1	1.00	1.09	1.00	1.09	1.77	1.68	0.14	1.09
time (sec)	N/A	0.226	0.377	5.894	0.265	0.245	3.904	111.847	0.409

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	50	51	3	24
N.S.	1	1.00	1.09	1.00	1.09	2.27	2.32	0.14	1.09
time (sec)	N/A	0.225	0.395	3.872	0.268	0.236	9.711	157.124	0.394

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	49	0	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.04	0.00	0.00	1.00
time (sec)	N/A	0.262	1.289	6.779	0.471	0.231	0.000	0.000	0.514

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	22	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	0.00	1.00
time (sec)	N/A	0.267	0.691	4.982	0.407	0.265	120.092	0.000	0.450

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	22	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	0.00	1.00
time (sec)	N/A	0.248	0.171	4.522	0.343	0.245	6.342	0.000	0.436

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	0.12	1.00
time (sec)	N/A	0.255	0.606	4.395	0.349	0.238	8.081	4.880	0.582

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.92	0.12	1.00
time (sec)	N/A	0.264	0.729	4.230	0.368	0.243	39.301	6.773	0.601

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	0	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	0.00	0.12	1.00
time (sec)	N/A	0.267	0.774	6.388	0.372	0.243	0.000	12.317	0.642

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	67	21	26	3	20
N.S.	1	1.00	1.11	1.00	3.72	1.17	1.44	0.17	1.11
time (sec)	N/A	0.177	0.693	10.666	0.301	0.248	0.771	57.991	0.482

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	60	19	26	3	19
N.S.	1	1.00	1.12	1.00	3.53	1.12	1.53	0.18	1.12
time (sec)	N/A	0.167	0.490	19.673	0.265	0.250	0.535	55.511	0.461

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	74	22	26	3	22
N.S.	1	1.00	1.10	1.00	3.70	1.10	1.30	0.15	1.10
time (sec)	N/A	0.188	0.863	29.711	0.335	0.229	1.009	57.923	0.446

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	101	36	44	3	22
N.S.	1	1.00	1.10	1.00	5.05	1.80	2.20	0.15	1.10
time (sec)	N/A	0.192	0.846	22.591	0.360	0.242	1.199	66.739	0.424

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	92	34	44	3	21
N.S.	1	1.00	1.11	1.00	4.84	1.79	2.32	0.16	1.11
time (sec)	N/A	0.176	1.102	56.341	0.276	0.227	0.973	64.020	0.417

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	108	37	44	3	24
N.S.	1	1.00	1.09	1.00	4.91	1.68	2.00	0.14	1.09
time (sec)	N/A	0.204	1.043	135.176	0.328	0.247	2.000	64.501	0.416

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	123	47	61	3	22
N.S.	1	1.00	1.10	1.00	6.15	2.35	3.05	0.15	1.10
time (sec)	N/A	0.188	0.856	95.803	0.362	0.234	1.501	70.286	0.459

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	114	45	61	3	21
N.S.	1	1.00	1.11	1.00	6.00	2.37	3.21	0.16	1.11
time (sec)	N/A	0.175	0.709	75.081	0.317	0.257	1.342	68.065	0.425

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	130	48	61	3	24
N.S.	1	1.00	1.09	1.00	5.91	2.18	2.77	0.14	1.09
time (sec)	N/A	0.201	1.105	135.071	0.381	0.242	2.670	70.331	0.445

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	36	24	26	3	24
N.S.	1	1.00	1.09	1.00	1.64	1.09	1.18	0.14	1.09
time (sec)	N/A	0.266	0.967	21.056	0.251	0.229	0.645	43.083	0.431

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	34	24	26	3	24
N.S.	1	1.00	1.09	1.00	1.55	1.09	1.18	0.14	1.09
time (sec)	N/A	0.258	0.536	0.356	0.245	0.244	0.577	42.683	0.397

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	30	22	24	3	22
N.S.	1	1.00	1.10	1.00	1.50	1.10	1.20	0.15	1.10
time (sec)	N/A	0.234	0.385	0.952	0.235	0.232	0.617	33.976	0.378

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	10	0	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.71	0.00	1.00
time (sec)	N/A	0.176	0.005	1.283	0.190	0.236	0.314	0.000	0.368

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	37	23	26	3	24
N.S.	1	1.00	1.09	1.00	1.68	1.05	1.18	0.14	1.09
time (sec)	N/A	0.276	0.395	1.047	0.237	0.227	0.825	33.955	0.404

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	40	25	27	3	24
N.S.	1	1.00	1.09	1.00	1.82	1.14	1.23	0.14	1.09
time (sec)	N/A	0.274	0.597	0.421	0.238	0.233	0.799	42.817	0.425

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	40	25	27	3	24
N.S.	1	1.00	1.09	1.00	1.82	1.14	1.23	0.14	1.09
time (sec)	N/A	0.275	0.885	27.566	0.234	0.242	0.850	43.576	0.411

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	40	25	27	3	24
N.S.	1	1.00	1.09	1.00	1.82	1.14	1.23	0.14	1.09
time (sec)	N/A	0.279	1.128	24.217	0.261	0.234	0.935	42.946	0.403

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	103	39	42	3	24
N.S.	1	1.00	1.09	1.00	4.68	1.77	1.91	0.14	1.09
time (sec)	N/A	1.138	3.397	8.637	0.337	0.246	0.884	120.615	0.458

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	37	0	123	0	0	0
N.S.	1	1.00	0.93	0.86	0.00	2.86	0.00	0.00	0.00
time (sec)	N/A	0.420	0.197	6.986	0.000	0.247	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	84	36	38	0	115	0	0	0
N.S.	1	2.05	0.88	0.93	0.00	2.80	0.00	0.00	0.00
time (sec)	N/A	0.782	0.081	8.093	0.000	0.239	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	37	0	112	0	0	0
N.S.	1	1.00	0.83	0.90	0.00	2.73	0.00	0.00	0.00
time (sec)	N/A	0.388	0.119	7.726	0.000	0.251	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	102	38	42	3	24
N.S.	1	1.00	1.09	1.00	4.64	1.73	1.91	0.14	1.09
time (sec)	N/A	1.177	1.275	12.529	0.284	0.242	1.233	84.219	0.407

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	107	40	44	3	24
N.S.	1	1.00	1.09	1.00	4.86	1.82	2.00	0.14	1.09
time (sec)	N/A	0.817	2.593	6.001	0.317	0.247	1.145	88.953	0.421

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	106	40	44	3	24
N.S.	1	1.00	1.09	1.00	4.82	1.82	2.00	0.14	1.09
time (sec)	N/A	1.626	2.825	27.779	0.316	0.244	1.188	89.280	0.455

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	107	40	44	3	24
N.S.	1	1.00	1.09	1.00	4.86	1.82	2.00	0.14	1.09
time (sec)	N/A	1.284	2.584	21.467	0.302	0.233	1.418	89.122	0.425

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	177	83	60	0	292	0	0	0
N.S.	1	2.06	0.97	0.70	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	1.535	0.135	9.345	0.000	0.258	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	102	59	37	0	196	0	0	0
N.S.	1	1.52	0.88	0.55	0.00	2.93	0.00	0.00	0.00
time (sec)	N/A	0.708	0.161	8.045	0.000	0.257	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	95	75	60	0	286	0	0	0
N.S.	1	1.56	1.23	0.98	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.811	0.079	8.665	0.000	0.268	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	57	45	59	0	287	0	0	0
N.S.	1	0.98	0.78	1.02	0.00	4.95	0.00	0.00	0.00
time (sec)	N/A	0.378	0.109	8.247	0.000	0.269	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	135	49	58	3	24
N.S.	1	1.00	1.09	1.00	6.14	2.23	2.64	0.14	1.09
time (sec)	N/A	2.169	1.317	8.642	0.293	0.244	1.977	122.031	0.411

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	140	51	60	3	24
N.S.	1	1.00	1.09	1.00	6.36	2.32	2.73	0.14	1.09
time (sec)	N/A	1.427	1.949	10.731	0.306	0.251	1.824	128.096	0.440

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	139	51	60	3	24
N.S.	1	1.00	1.09	1.00	6.32	2.32	2.73	0.14	1.09
time (sec)	N/A	3.806	2.705	20.690	0.307	0.250	1.755	127.819	0.424

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	140	51	60	3	24
N.S.	1	1.00	1.09	1.00	6.36	2.32	2.73	0.14	1.09
time (sec)	N/A	2.829	3.092	41.624	0.296	0.239	2.197	129.816	0.434

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	1.00	0.14	1.00
time (sec)	N/A	0.224	1.671	26.457	0.336	0.247	0.731	61.049	0.393

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	20	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	0.14	1.00
time (sec)	N/A	0.192	0.398	19.908	0.322	0.246	0.726	59.374	0.361

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	0.12	1.00
time (sec)	N/A	0.248	3.581	60.362	0.371	0.238	1.200	62.056	0.413

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	33	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.50	1.00	0.14	1.00
time (sec)	N/A	0.230	3.734	34.729	0.364	0.234	5.605	76.913	0.447

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	20	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	0.14	1.00
time (sec)	N/A	0.192	0.862	16.615	0.383	0.231	3.553	73.316	0.401

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	0.12	1.00
time (sec)	N/A	0.263	4.108	122.019	0.363	0.230	7.009	77.604	0.432

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	48	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	2.18	1.00	0.14	1.00
time (sec)	N/A	0.228	2.229	62.138	0.412	0.246	29.093	89.074	0.452

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	46	20	3	21
N.S.	1	1.00	1.10	0.90	1.00	2.19	0.95	0.14	1.00
time (sec)	N/A	0.192	0.800	37.740	0.411	0.236	14.734	85.362	0.414

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	49	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.04	0.92	0.12	1.00
time (sec)	N/A	0.257	2.420	118.501	0.403	0.245	20.700	90.581	0.459

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	1.00	0.14	1.00
time (sec)	N/A	0.222	1.252	3.162	0.357	0.251	1.180	69.463	0.455

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	22	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.05	0.14	1.00
time (sec)	N/A	0.194	0.711	6.669	0.362	0.239	1.175	66.129	0.422

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	1.00	0.12	1.00
time (sec)	N/A	0.412	1.380	7.428	0.370	0.244	1.915	65.521	0.435

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.00	1.00
time (sec)	N/A	1.126	9.956	3.334	0.367	0.244	2.817	0.000	0.504

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.12	1.00
time (sec)	N/A	0.949	6.649	2.500	0.372	0.242	2.891	60.962	0.482

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	126	0	0	0	0	0
N.S.	1	1.00	0.80	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	0.126	6.161	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	53	128	0	0	0	0	0
N.S.	1	1.00	0.77	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.510	0.143	5.790	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	50	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.08	1.00	0.00	1.00
time (sec)	N/A	1.374	2.214	3.723	0.382	0.244	5.387	0.000	0.480

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	52	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.17	1.08	0.12	1.00
time (sec)	N/A	1.269	3.902	3.884	0.372	0.236	6.056	62.372	0.493

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	52	26	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.17	1.08	0.00	1.00
time (sec)	N/A	2.145	4.292	20.165	0.391	0.235	8.458	0.000	0.484

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	52	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.17	1.08	0.12	1.00
time (sec)	N/A	2.098	4.877	22.201	0.387	0.233	13.788	65.033	0.491

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	1.00	0.00	1.00
time (sec)	N/A	3.004	11.869	18.231	0.407	0.246	7.146	0.000	0.526

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	1.00	0.12	1.00
time (sec)	N/A	2.167	11.225	17.006	0.374	0.256	6.932	167.803	0.493

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	88	82	309	0	0	0	0	0
N.S.	1	0.75	0.69	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.668	0.213	15.630	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	167	99	309	0	0	0	0	0
N.S.	1	1.18	0.70	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.151	0.338	13.746	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	141	95	301	0	0	0	0	0
N.S.	1	1.22	0.82	2.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.507	0.194	7.331	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	85	61	305	0	0	0	0	0
N.S.	1	0.74	0.53	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.558	0.167	11.792	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	61	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.54	1.00	0.00	1.00
time (sec)	N/A	4.531	2.294	4.321	0.382	0.256	14.955	0.000	0.485

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	63	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.62	1.08	0.12	1.00
time (sec)	N/A	2.343	5.059	4.351	0.370	0.250	18.201	123.323	0.492

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	63	26	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.62	1.08	0.00	1.00
time (sec)	N/A	8.457	6.266	21.625	0.371	0.243	24.580	0.000	0.491

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	63	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.62	1.08	0.12	1.00
time (sec)	N/A	4.690	7.730	33.277	0.405	0.242	31.365	126.876	0.502

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	30	32	28	371	125	126	0	30
N.S.	1	1.00	1.07	0.93	12.37	4.17	4.20	0.00	1.00
time (sec)	N/A	0.259	55.093	5.522	1.582	0.247	108.054	0.000	0.498

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	0	177	48	73	3	24
N.S.	1	1.00	1.09	0.00	8.05	2.18	3.32	0.14	1.09
time (sec)	N/A	0.215	0.593	180.000	0.655	0.252	14.038	174.266	0.420

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	144	37	53	3	24
N.S.	1	1.00	1.09	1.00	6.55	1.68	2.41	0.14	1.09
time (sec)	N/A	0.210	0.658	7.016	0.605	0.239	7.015	154.010	0.399

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	95	22	31	3	22
N.S.	1	1.00	1.10	1.00	4.75	1.10	1.55	0.15	1.10
time (sec)	N/A	0.190	0.405	5.815	0.558	0.254	3.891	133.175	0.445

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	40	24	26	3	24
N.S.	1	1.00	1.09	1.00	1.82	1.09	1.18	0.14	1.09
time (sec)	N/A	0.281	0.540	3.214	0.277	0.245	3.409	112.913	0.435

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	111	39	42	3	24
N.S.	1	1.00	1.09	1.00	5.05	1.77	1.91	0.14	1.09
time (sec)	N/A	0.222	0.466	7.836	0.400	0.255	10.115	230.714	0.445

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	144	50	58	0	24
N.S.	1	1.00	1.09	1.00	6.55	2.27	2.64	0.00	1.09
time (sec)	N/A	0.223	0.484	6.251	0.408	0.240	32.185	0.000	0.458

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	49	0	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.04	0.00	0.00	1.00
time (sec)	N/A	0.261	1.757	10.589	0.451	0.246	0.000	0.000	0.543

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	0	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.262	0.946	10.802	0.402	0.242	0.000	0.000	0.496

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.00	1.00
time (sec)	N/A	0.245	0.251	6.670	0.341	0.242	6.631	0.000	0.467

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.12	1.00
time (sec)	N/A	0.252	0.681	6.639	0.343	0.245	16.989	7.308	0.622

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.12	1.00
time (sec)	N/A	0.263	0.821	6.552	0.370	0.256	67.926	15.122	0.660

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	0	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	0.00	0.12	1.00
time (sec)	N/A	0.264	0.867	14.998	0.391	0.257	0.000	30.337	0.653

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	110	21	26	3	20
N.S.	1	1.00	1.11	1.00	6.11	1.17	1.44	0.17	1.11
time (sec)	N/A	0.183	1.003	31.546	0.353	0.246	0.820	77.717	0.505

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	17	97	19	26	3	19
N.S.	1	1.00	1.12	1.00	5.71	1.12	1.53	0.18	1.12
time (sec)	N/A	0.168	0.998	49.151	0.356	0.264	0.677	74.778	0.485

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	114	22	26	3	22
N.S.	1	1.00	1.10	1.00	5.70	1.10	1.30	0.15	1.10
time (sec)	N/A	0.192	1.327	78.303	0.364	0.240	1.062	79.195	0.446

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	164	36	44	3	22
N.S.	1	1.00	1.10	1.00	8.20	1.80	2.20	0.15	1.10
time (sec)	N/A	0.193	0.842	145.086	0.394	0.240	1.291	87.314	0.444

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	148	34	44	3	21
N.S.	1	1.00	1.11	1.00	7.79	1.79	2.32	0.16	1.11
time (sec)	N/A	0.180	0.518	127.721	0.370	0.242	1.115	86.593	0.429

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	168	37	44	3	24
N.S.	1	1.00	1.09	1.00	7.64	1.68	2.00	0.14	1.09
time (sec)	N/A	0.205	1.042	182.058	0.421	0.237	2.211	88.919	0.425

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	0	196	47	61	3	22
N.S.	1	1.00	1.10	0.00	9.80	2.35	3.05	0.15	1.10
time (sec)	N/A	0.190	0.922	0.000	0.391	0.245	1.620	97.507	0.477

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	181	45	61	3	21
N.S.	1	1.00	1.11	1.00	9.53	2.37	3.21	0.16	1.11
time (sec)	N/A	0.179	1.014	154.424	0.412	0.238	1.466	94.744	0.441

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	201	48	61	3	24
N.S.	1	1.00	1.09	1.00	9.14	2.18	2.77	0.14	1.09
time (sec)	N/A	0.205	1.074	177.360	0.438	0.236	2.844	97.831	0.415

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	65	24	26	3	24
N.S.	1	1.00	1.09	1.00	2.95	1.09	1.18	0.14	1.09
time (sec)	N/A	0.266	0.765	31.523	0.290	0.248	0.666	62.667	0.445

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	62	24	26	3	24
N.S.	1	1.00	1.09	1.00	2.82	1.09	1.18	0.14	1.09
time (sec)	N/A	0.258	0.516	6.822	0.263	0.241	0.693	61.923	0.436

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	55	22	24	3	22
N.S.	1	1.00	1.10	1.00	2.75	1.10	1.20	0.15	1.10
time (sec)	N/A	0.236	0.551	8.965	0.249	0.233	0.719	48.248	0.403

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	0	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.00	0.88
time (sec)	N/A	0.170	0.005	1.418	0.187	0.236	0.445	0.000	0.375

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	59	23	26	3	24
N.S.	1	1.00	1.09	1.00	2.68	1.05	1.18	0.14	1.09
time (sec)	N/A	0.259	0.346	1.362	0.285	0.242	1.018	48.067	0.389

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	69	25	27	3	24
N.S.	1	1.00	1.09	1.00	3.14	1.14	1.23	0.14	1.09
time (sec)	N/A	0.262	0.958	10.285	0.298	0.243	0.932	60.782	0.410

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	70	25	27	3	24
N.S.	1	1.00	1.09	1.00	3.18	1.14	1.23	0.14	1.09
time (sec)	N/A	0.270	1.226	79.315	0.291	0.242	0.951	62.513	0.420

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	71	25	27	3	24
N.S.	1	1.00	1.09	1.00	3.23	1.14	1.23	0.14	1.09
time (sec)	N/A	0.267	2.120	49.905	0.270	0.232	1.138	61.614	0.412

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	137	39	42	3	24
N.S.	1	1.00	1.09	1.00	6.23	1.77	1.91	0.14	1.09
time (sec)	N/A	0.796	6.677	15.901	0.339	0.246	1.092	173.507	0.456

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	113	51	52	0	132	0	0	0
N.S.	1	1.59	0.72	0.73	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.930	0.111	9.066	0.000	0.258	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	51	0	135	0	0	0
N.S.	1	1.00	0.86	0.63	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.475	0.067	8.095	0.000	0.259	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	109	58	52	0	122	0	0	0
N.S.	1	1.68	0.89	0.80	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.910	0.069	8.880	0.000	0.248	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	141	38	42	3	24
N.S.	1	1.00	1.09	1.00	6.41	1.73	1.91	0.14	1.09
time (sec)	N/A	0.836	1.433	9.631	0.367	0.246	1.529	121.357	0.411

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	142	40	44	3	24
N.S.	1	1.00	1.09	1.00	6.45	1.82	2.00	0.14	1.09
time (sec)	N/A	1.410	2.256	9.612	0.346	0.241	1.371	128.167	0.440

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	142	40	44	3	24
N.S.	1	1.00	1.09	1.00	6.45	1.82	2.00	0.14	1.09
time (sec)	N/A	1.263	1.706	81.822	0.364	0.249	1.513	125.058	0.443

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	142	40	44	3	24
N.S.	1	1.00	1.09	1.00	6.45	1.82	2.00	0.14	1.09
time (sec)	N/A	1.900	5.705	32.501	0.379	0.238	1.830	127.474	0.457

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	273	72	88	0	328	0	0	0
N.S.	1	1.54	0.41	0.50	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	1.870	0.198	10.283	0.000	0.247	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	233	60	52	0	215	0	0	0
N.S.	1	1.94	0.50	0.43	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	1.869	0.122	10.582	0.000	0.245	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	194	98	88	0	318	0	0	0
N.S.	1	1.72	0.87	0.78	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	1.245	0.148	10.935	0.000	0.248	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	120	89	89	0	297	0	0	0
N.S.	1	1.48	1.10	1.10	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	0.930	0.140	11.202	0.000	0.257	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	174	49	58	3	24
N.S.	1	1.00	1.09	1.00	7.91	2.23	2.64	0.14	1.09
time (sec)	N/A	2.484	2.133	5.662	0.360	0.235	2.066	180.296	0.422

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	175	51	60	3	24
N.S.	1	1.00	1.09	1.00	7.95	2.32	2.73	0.14	1.09
time (sec)	N/A	2.597	3.122	7.221	0.411	0.241	2.026	187.199	0.446

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	174	51	60	3	24
N.S.	1	1.00	1.09	1.00	7.91	2.32	2.73	0.14	1.09
time (sec)	N/A	4.030	3.917	84.938	0.403	0.237	2.094	184.689	0.444

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	176	51	60	3	24
N.S.	1	1.00	1.09	1.00	8.00	2.32	2.73	0.14	1.09
time (sec)	N/A	4.573	8.253	89.063	0.377	0.245	2.264	185.348	0.455

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	0	0	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.00	0.00	0.88
time (sec)	N/A	0.208	0.224	114.779	0.275	0.228	0.000	0.000	0.475

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	1.00	0.14	1.00
time (sec)	N/A	0.216	1.958	33.536	0.327	0.245	1.220	78.898	0.444

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	20	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	0.14	1.00
time (sec)	N/A	0.187	1.058	32.546	0.350	0.244	1.117	77.582	0.396

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	0.12	1.00
time (sec)	N/A	0.241	3.523	86.745	0.370	0.241	1.670	85.333	0.438

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	33	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.50	1.00	0.14	1.00
time (sec)	N/A	0.226	6.086	50.575	0.384	0.237	6.781	97.468	0.466

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	20	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	0.95	0.14	1.00
time (sec)	N/A	0.190	1.082	66.786	0.359	0.245	4.080	94.055	0.445

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.92	0.12	1.00
time (sec)	N/A	0.255	4.096	85.804	0.367	0.238	9.125	100.816	0.472

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	48	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	2.18	1.00	0.14	1.00
time (sec)	N/A	0.225	2.485	108.876	0.399	0.235	25.980	111.692	0.458

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	46	20	3	21
N.S.	1	1.00	1.10	0.90	1.00	2.19	0.95	0.14	1.00
time (sec)	N/A	0.190	1.160	93.414	0.432	0.238	14.623	108.486	0.441

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	49	22	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.04	0.92	0.12	1.00
time (sec)	N/A	0.248	2.783	177.810	0.442	0.240	19.158	112.270	0.496

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	22	3	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	1.00	0.14	1.00
time (sec)	N/A	0.221	1.503	3.796	0.363	0.246	1.560	90.535	0.475

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	21	21	22	3	21
N.S.	1	1.00	1.10	0.90	1.00	1.00	1.05	0.14	1.00
time (sec)	N/A	0.190	0.657	7.098	0.332	0.237	1.652	86.547	0.453

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	1.00	0.12	1.00
time (sec)	N/A	0.392	4.106	8.912	0.363	0.248	2.887	88.758	0.468

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	37	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.54	1.08	0.12	1.00
time (sec)	N/A	0.249	3.732	8.111	0.342	0.244	4.144	95.279	0.485

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	37	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.54	1.08	0.12	1.00
time (sec)	N/A	0.246	5.531	30.574	0.348	0.240	6.175	94.409	0.500

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.00	1.00
time (sec)	N/A	1.374	6.405	3.608	0.398	0.240	3.834	0.000	0.550

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.12	1.00
time (sec)	N/A	1.100	5.524	2.975	0.364	0.241	4.106	79.785	0.478

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	63	156	0	0	0	0	0
N.S.	1	1.04	0.61	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	0.192	9.250	0.000	0.000	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	105	65	151	0	0	0	0	0
N.S.	1	1.04	0.64	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.621	0.079	9.039	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	50	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.08	1.00	0.00	1.00
time (sec)	N/A	1.660	2.633	4.637	0.370	0.242	7.015	0.000	0.516

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	52	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.17	1.08	0.12	1.00
time (sec)	N/A	1.518	2.849	4.590	0.371	0.254	8.744	82.384	0.517

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	52	26	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.17	1.08	0.00	1.00
time (sec)	N/A	2.598	3.436	36.725	0.396	0.236	14.865	0.000	0.517

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	52	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.17	1.08	0.12	1.00
time (sec)	N/A	2.436	5.959	31.135	0.367	0.253	22.060	82.111	0.539

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	1.00	0.00	1.00
time (sec)	N/A	3.632	7.941	19.579	0.391	0.248	12.723	0.000	0.564

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	1.00	0.12	1.00
time (sec)	N/A	4.869	7.213	18.793	0.401	0.243	12.692	227.272	0.539

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	208	114	359	0	0	0	0	0
N.S.	1	1.16	0.63	1.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.366	0.416	24.648	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	295	119	375	0	0	0	0	0
N.S.	1	1.41	0.57	1.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.089	0.228	20.787	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	294	118	371	0	0	0	0	0
N.S.	1	1.68	0.67	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.054	0.352	11.434	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	177	102	346	0	0	0	0	0
N.S.	1	1.22	0.70	2.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.687	0.190	18.909	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	61	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.54	1.00	0.00	1.00
time (sec)	N/A	5.163	4.172	14.301	0.411	0.258	21.101	0.000	0.540

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	63	26	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.62	1.08	0.12	1.00
time (sec)	N/A	5.038	3.880	13.777	0.390	0.243	29.542	176.072	0.545

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	399	48	73	3	24
N.S.	1	1.00	1.09	1.00	18.14	2.18	3.32	0.14	1.09
time (sec)	N/A	0.215	0.599	10.069	1.236	0.260	17.044	218.438	0.426

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	333	37	53	3	24
N.S.	1	1.00	1.09	1.00	15.14	1.68	2.41	0.14	1.09
time (sec)	N/A	0.215	0.687	9.837	1.145	0.252	8.871	193.405	0.404

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	221	22	31	3	22
N.S.	1	1.00	1.10	1.00	11.05	1.10	1.55	0.15	1.10
time (sec)	N/A	0.193	0.445	10.630	0.999	0.244	6.271	167.102	0.422

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	91	24	26	3	24
N.S.	1	1.00	1.09	1.00	4.14	1.09	1.18	0.14	1.09
time (sec)	N/A	0.281	0.758	4.562	0.389	0.244	6.523	155.561	0.441

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	191	39	42	0	24
N.S.	1	1.00	1.09	1.00	8.68	1.77	1.91	0.00	1.09
time (sec)	N/A	0.219	0.668	11.893	0.686	0.242	18.769	0.000	0.436

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	224	50	58	0	24
N.S.	1	1.00	1.09	1.00	10.18	2.27	2.64	0.00	1.09
time (sec)	N/A	0.225	0.692	7.978	0.709	0.249	52.442	0.000	0.439

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	49	0	0	24
N.S.	1	1.00	1.08	0.92	1.00	2.04	0.00	0.00	1.00
time (sec)	N/A	0.257	2.837	11.556	0.482	0.260	0.000	0.000	0.544

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	0	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.257	1.483	13.616	0.440	0.270	0.000	0.000	0.482

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	0	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.00	1.00
time (sec)	N/A	0.251	0.440	12.389	0.376	0.250	24.522	0.000	0.430

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	24	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	1.00	1.00	0.12	1.00
time (sec)	N/A	0.258	0.913	13.127	0.401	0.255	37.247	9.659	0.593

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	51	24	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.12	1.00	0.12	1.00
time (sec)	N/A	0.270	1.017	12.153	0.395	0.238	122.712	20.726	0.622

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	62	0	3	24
N.S.	1	1.00	1.08	0.92	1.00	2.58	0.00	0.12	1.00
time (sec)	N/A	0.264	1.083	15.069	0.437	0.251	0.000	41.715	0.655

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	22	34	3	22
N.S.	1	1.00	1.09	0.91	0.00	1.00	1.55	0.14	1.00
time (sec)	N/A	0.190	1.064	9.166	0.000	0.256	34.653	52.959	0.574

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	0	29	3	20
N.S.	1	1.00	1.10	0.90	0.00	0.00	1.45	0.15	1.00
time (sec)	N/A	0.250	1.245	4.316	0.000	0.000	1.398	52.399	0.495

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	0	0	27	3	19
N.S.	1	1.00	1.11	0.89	0.00	0.00	1.42	0.16	1.00
time (sec)	N/A	0.165	1.904	3.532	0.000	0.000	1.076	52.073	0.530

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.14	1.00
time (sec)	N/A	0.188	1.056	5.128	0.000	0.000	2.072	233.461	0.494

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	58	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	2.42	0.12	1.00
time (sec)	N/A	0.209	0.756	9.716	0.000	0.269	120.173	53.200	0.506

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	49	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	2.23	0.14	1.00
time (sec)	N/A	0.270	0.951	4.099	0.000	0.000	3.147	52.393	0.437

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	48	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	2.29	0.14	1.00
time (sec)	N/A	0.174	1.081	3.507	0.000	0.000	2.339	52.873	0.454

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.00	1.00
time (sec)	N/A	0.205	0.903	4.954	0.000	0.000	3.628	0.000	0.462

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	48	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.00	0.00	0.12	1.00
time (sec)	N/A	0.208	0.551	10.559	0.000	0.247	0.000	52.165	0.492

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	68	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	3.09	0.14	1.00
time (sec)	N/A	0.278	1.035	4.616	0.000	0.000	6.431	56.693	0.446

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	66	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	3.14	0.14	1.00
time (sec)	N/A	0.174	1.130	3.638	0.000	0.000	4.301	53.335	0.465

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	0.203	0.936	5.171	0.000	0.000	5.657	0.000	0.463

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	24	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.00	0.92	0.12	1.00
time (sec)	N/A	0.212	0.682	6.451	0.000	0.244	4.930	51.736	0.480

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.92	0.12	1.00
time (sec)	N/A	0.425	2.930	3.479	0.000	0.000	0.875	51.882	0.425

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.92	0.12	1.00
time (sec)	N/A	0.335	0.770	3.932	0.000	0.000	0.622	52.618	0.425

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	20	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.91	0.14	1.00
time (sec)	N/A	0.238	0.824	3.739	0.000	0.000	0.465	48.230	0.426

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	0	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.78	0.78
time (sec)	N/A	0.168	0.007	3.865	0.000	0.246	0.000	0.277	0.427

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	19	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.79	0.12	1.00
time (sec)	N/A	0.300	0.333	3.865	0.000	0.000	0.961	24.105	0.466

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.339	0.776	4.250	0.000	0.000	1.136	29.696	0.466

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.488	1.911	3.585	0.000	0.000	1.410	28.661	0.455

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.548	5.333	4.870	0.000	0.000	2.096	28.700	0.463

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	39	32	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.62	1.33	0.12	1.00
time (sec)	N/A	0.207	1.537	13.775	0.000	0.277	29.841	51.841	0.526

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	32	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.33	0.12	1.00
time (sec)	N/A	0.209	3.119	12.627	0.000	0.000	1.129	61.126	0.480

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	66	60	0	0	0	0	0
N.S.	1	1.00	0.82	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	0.335	6.348	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	74	136	45	0	0	0	0	0
N.S.	1	0.94	1.72	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	0.245	5.679	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	68	57	0	0	0	0	0
N.S.	1	1.00	0.88	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.302	6.529	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.21	0.12	1.00
time (sec)	N/A	0.214	0.873	4.711	0.000	0.000	1.592	30.438	0.495

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	50	41	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.08	1.71	0.12	1.00
time (sec)	N/A	0.219	1.577	9.549	0.000	0.278	114.913	52.264	0.538

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	41	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.71	0.12	1.00
time (sec)	N/A	0.214	4.145	12.707	0.000	0.000	2.196	69.474	0.465

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	116	181	102	0	0	0	0	0
N.S.	1	0.83	1.30	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	0.463	6.960	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	111	230	93	0	0	0	0	0
N.S.	1	0.94	1.95	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	0.563	6.398	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	72	141	66	0	0	0	0	0
N.S.	1	0.87	1.70	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	0.371	5.651	0.000	0.000	0.000	0.000	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	111	230	94	0	0	0	0	0
N.S.	1	0.94	1.95	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	0.493	6.375	0.000	0.000	0.000	0.000	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	116	103	102	0	0	0	0	0
N.S.	1	0.83	0.74	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.479	6.403	0.000	0.000	0.000	0.000	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	37	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.54	0.12	1.00
time (sec)	N/A	0.209	1.254	4.442	0.000	0.000	3.256	34.004	0.529

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	1.00	0.00	0.92
time (sec)	N/A	0.246	1.437	12.767	0.000	0.250	63.431	0.000	0.351

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.252	4.656	14.003	0.000	0.000	5.902	77.992	0.343

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.313	3.668	9.270	0.000	0.000	2.875	0.000	0.360

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.00	0.91
time (sec)	N/A	0.191	0.230	12.940	0.000	0.000	1.477	0.000	0.348

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.262	1.447	16.436	0.000	0.255	0.000	0.000	0.344

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.264	4.684	16.084	0.000	0.000	129.201	73.314	0.349

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.325	4.845	13.464	0.000	0.000	60.579	0.000	0.337

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.00	0.91
time (sec)	N/A	0.191	1.848	13.081	0.000	0.000	32.726	0.000	0.345

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	49	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	1.88	0.00	0.00	0.92
time (sec)	N/A	0.256	1.747	22.654	0.000	0.259	0.000	0.000	0.337

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.267	4.068	14.263	0.000	0.000	0.000	73.540	0.332

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.322	4.351	14.536	0.000	0.000	0.000	0.000	0.334

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.190	0.365	13.439	0.000	0.000	0.000	0.000	0.363

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	1.00	0.12	0.92
time (sec)	N/A	0.247	1.348	16.205	0.000	0.263	32.723	51.716	0.353

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.711	4.889	14.576	0.000	0.000	7.968	0.000	0.335

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.438	3.505	13.914	0.000	0.000	4.081	182.161	0.336

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.317	0.945	7.342	0.000	0.000	1.938	191.315	0.334

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.189	0.184	7.868	0.000	0.000	0.856	169.363	0.353

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.249	1.609	14.517	0.000	0.000	1.864	168.353	0.345

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.398	2.361	14.135	0.000	0.000	4.247	184.655	0.334

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.506	8.012	14.829	0.000	0.000	8.014	185.548	0.355

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.766	16.032	14.486	0.000	0.000	19.105	186.459	0.334

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	51	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	1.96	1.00	0.12	0.92
time (sec)	N/A	0.259	1.511	12.403	0.000	0.260	102.355	52.401	0.340

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.271	7.729	7.350	0.000	0.000	9.944	0.000	0.352

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.263	4.198	7.040	0.000	0.000	5.246	67.521	0.332

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	121	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	89	117	0	0	0	0	0	0
N.S.	1	0.98	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.438	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.256	2.648	11.481	0.000	0.000	8.703	35.841	0.344

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.263	4.987	11.353	0.000	0.000	18.055	40.770	0.346

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	62	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	2.38	0.00	0.12	0.92
time (sec)	N/A	0.254	2.381	33.796	0.000	0.268	0.000	51.916	0.333

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.256	7.186	30.049	0.000	0.000	50.886	83.620	0.344

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	143	324	0	0	0	0	0	0
N.S.	1	0.67	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.585	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	135	133	0	0	0	0	0	0
N.S.	1	0.83	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.732	0.388	0.000	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	135	167	0	0	0	0	0	0
N.S.	1	0.83	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.576	0.361	0.000	0.000	0.000	0.000	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	145	252	0	0	0	0	0	0
N.S.	1	0.68	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	0.547	0.000	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.257	3.789	14.428	0.000	0.000	61.562	45.009	0.359

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	22	0	3	22
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.00	0.14	1.00
time (sec)	N/A	0.194	0.908	8.177	0.000	0.251	0.000	51.818	0.619

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	31	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.41	0.14	1.00
time (sec)	N/A	0.194	5.259	4.261	0.000	0.000	8.873	78.967	0.516

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	0	29	3	20
N.S.	1	1.00	1.10	0.90	0.00	0.00	1.45	0.15	1.00
time (sec)	N/A	0.255	0.871	3.771	0.000	0.000	6.413	78.990	0.506

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	0	0	27	3	19
N.S.	1	1.00	1.11	0.89	0.00	0.00	1.42	0.16	1.00
time (sec)	N/A	0.242	2.420	3.594	0.000	0.000	3.853	77.829	0.553

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.14	1.00
time (sec)	N/A	0.190	1.238	4.792	0.000	0.000	4.956	194.848	0.505

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.14	1.00
time (sec)	N/A	0.192	0.874	5.700	0.000	0.000	4.432	192.307	0.529

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	0.00	0.12	1.00
time (sec)	N/A	0.215	0.759	10.661	0.000	0.259	0.000	51.635	0.556

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	51	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.12	0.12	1.00
time (sec)	N/A	0.212	3.697	4.945	0.000	0.000	18.017	78.596	0.465

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	49	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	2.23	0.14	1.00
time (sec)	N/A	0.277	0.918	4.806	0.000	0.000	13.869	78.300	0.453

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	48	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	2.29	0.14	1.00
time (sec)	N/A	0.496	1.378	3.572	0.000	0.000	9.108	78.310	0.450

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.12	1.00
time (sec)	N/A	0.205	1.251	5.474	0.000	0.000	9.748	260.897	0.490

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.12	1.00
time (sec)	N/A	0.210	1.180	5.161	0.000	0.000	7.597	253.696	0.475

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	48	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.00	0.00	0.12	1.00
time (sec)	N/A	0.211	0.557	9.622	0.000	0.261	0.000	52.281	0.517

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	70	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.92	0.12	1.00
time (sec)	N/A	0.214	3.560	4.066	0.000	0.000	37.408	77.701	0.479

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	68	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	3.09	0.14	1.00
time (sec)	N/A	0.275	0.950	4.866	0.000	0.000	29.828	77.003	0.448

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	66	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	3.14	0.14	1.00
time (sec)	N/A	0.739	1.405	3.599	0.000	0.000	18.509	76.854	0.451

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	0.203	1.208	5.001	0.000	0.000	17.762	0.000	0.467

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	0.207	1.430	4.932	0.000	0.000	15.143	0.000	0.467

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	24	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.00	0.92	0.12	1.00
time (sec)	N/A	0.225	0.649	5.833	0.000	0.253	71.781	52.023	0.529

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.92	0.12	1.00
time (sec)	N/A	0.430	3.383	3.618	0.000	0.000	2.701	78.813	0.453

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.92	0.12	1.00
time (sec)	N/A	0.331	0.748	3.617	0.000	0.000	1.765	78.431	0.446

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	20	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.91	0.14	1.00
time (sec)	N/A	0.233	0.799	4.200	0.000	0.000	1.417	74.303	0.438

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	0	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.00	0.78	0.78
time (sec)	N/A	0.175	0.005	4.584	0.000	0.247	0.000	0.281	0.456

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	19	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.79	0.12	1.00
time (sec)	N/A	0.298	0.322	4.918	0.000	0.000	1.214	72.274	0.472

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.341	0.611	3.724	0.000	0.000	1.877	73.134	0.470

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.500	1.646	3.363	0.000	0.000	2.606	73.642	0.488

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.556	4.223	4.246	0.000	0.000	3.633	72.604	0.484

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	39	32	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.62	1.33	0.12	1.00
time (sec)	N/A	0.211	1.476	15.977	0.000	0.253	92.104	51.936	0.559

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	32	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.33	0.12	1.00
time (sec)	N/A	0.211	3.308	14.748	0.000	0.000	3.424	87.403	0.466

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	187	75	0	0	0	0	0
N.S.	1	1.00	1.47	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	0.480	7.453	0.000	0.000	0.000	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	108	75	64	0	0	0	0	0
N.S.	1	0.99	0.69	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	0.101	6.783	0.000	0.000	0.000	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	122	186	75	0	0	0	0	0
N.S.	1	0.98	1.50	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	0.438	7.272	0.000	0.000	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.21	0.12	1.00
time (sec)	N/A	0.224	0.887	4.587	0.000	0.000	2.414	82.786	0.526

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	50	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.08	0.00	0.12	1.00
time (sec)	N/A	0.227	1.423	13.468	0.000	0.257	0.000	57.187	0.516

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	41	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.71	0.12	1.00
time (sec)	N/A	0.221	5.180	14.539	0.000	0.000	9.480	93.596	0.482

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	274	355	132	0	0	0	0	0
N.S.	1	1.19	1.54	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.447	0.746	8.570	0.000	0.000	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	149	350	124	0	0	0	0	0
N.S.	1	0.89	2.08	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.499	0.256	7.556	0.000	0.000	0.000	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	91	353	81	0	0	0	0	0
N.S.	1	0.84	3.27	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.654	6.714	0.000	0.000	0.000	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	149	347	124	0	0	0	0	0
N.S.	1	0.89	2.07	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	0.236	7.315	0.000	0.000	0.000	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	258	355	132	0	0	0	0	0
N.S.	1	1.18	1.62	0.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.167	0.712	7.533	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	37	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.54	0.12	1.00
time (sec)	N/A	0.221	1.591	4.707	0.000	0.000	6.113	87.543	0.520

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.256	1.349	16.533	0.000	0.267	0.000	0.000	0.348

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.260	3.914	13.836	0.000	0.000	92.778	88.924	0.360

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.317	6.228	6.793	0.000	0.000	53.441	0.000	0.336

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.00	0.91
time (sec)	N/A	0.350	0.244	13.706	0.000	0.000	26.595	0.000	0.340

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.00	0.92
time (sec)	N/A	0.251	4.144	13.757	0.000	0.000	18.995	0.000	0.338

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.261	1.495	20.825	0.000	0.274	0.000	0.000	0.348

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.263	4.645	15.967	0.000	0.000	0.000	89.429	0.339

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.324	3.344	15.253	0.000	0.000	0.000	0.000	0.336

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.557	1.632	15.772	0.000	0.000	0.000	0.000	0.358

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.00	0.92
time (sec)	N/A	0.256	3.276	16.240	0.000	0.000	141.534	0.000	0.349

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	49	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	1.88	0.00	0.00	0.92
time (sec)	N/A	0.253	1.741	21.239	0.000	0.260	0.000	0.000	0.337

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.258	3.776	14.992	0.000	0.000	0.000	89.051	0.334

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.321	6.937	13.801	0.000	0.000	0.000	0.000	0.324

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.815	0.391	13.620	0.000	0.000	0.000	0.000	0.355

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.250	3.360	15.102	0.000	0.000	0.000	0.000	0.341

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.12	0.92
time (sec)	N/A	0.244	1.468	17.365	0.000	0.258	0.000	52.384	0.331

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.913	3.952	14.954	0.000	0.000	128.350	0.000	0.354

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.556	3.209	13.642	0.000	0.000	62.975	151.578	0.334

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.306	1.002	7.183	0.000	0.000	36.950	151.317	0.339

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.186	0.189	9.730	0.000	0.000	12.614	143.927	0.340

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.242	2.300	13.964	0.000	0.000	13.697	145.722	0.354

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.385	2.355	15.606	0.000	0.000	18.158	147.543	0.340

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.733	7.357	15.275	0.000	0.000	33.397	149.976	0.344

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	1.100	18.291	15.665	0.000	0.000	58.848	147.842	0.337

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	51	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	1.96	0.00	0.12	0.92
time (sec)	N/A	0.260	1.683	12.276	0.000	0.271	0.000	52.336	0.354

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.257	6.823	8.767	0.000	0.000	154.485	0.000	0.334

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.260	4.656	7.557	0.000	0.000	76.893	85.165	0.344

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	123	128	0	0	0	0	0	0
N.S.	1	0.95	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	0.160	0.000	0.000	0.000	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	86	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.539	0.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.264	3.777	12.046	0.000	0.000	34.595	80.860	0.339

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.264	6.745	10.740	0.000	0.000	57.036	79.229	0.340

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	62	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	2.38	0.00	0.12	0.92
time (sec)	N/A	0.257	2.344	24.108	0.000	0.258	0.000	52.794	0.362

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.256	10.581	27.358	0.000	0.000	0.000	0.000	0.342

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.255	7.135	20.161	0.000	0.000	0.000	96.241	0.350

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	306	272	0	0	0	0	0	0
N.S.	1	1.16	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.378	0.952	0.000	0.000	0.000	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	179	338	0	0	0	0	0	0
N.S.	1	0.72	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	0.456	0.000	0.000	0.000	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	181	261	0	0	0	0	0	0
N.S.	1	0.73	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.601	0.926	0.000	0.000	0.000	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	297	345	0	0	0	0	0	0
N.S.	1	1.18	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.115	0.465	0.000	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.265	4.680	6.063	0.000	0.000	109.900	90.944	0.344

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.267	7.355	6.541	0.000	0.000	176.904	87.683	0.339

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	22	0	3	22
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.00	0.14	1.00
time (sec)	N/A	0.197	0.905	9.056	0.000	0.262	0.000	52.887	0.668

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	31	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.41	0.14	1.00
time (sec)	N/A	0.193	3.983	3.711	0.000	0.000	41.865	108.416	0.536

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	0	29	3	20
N.S.	1	1.00	1.10	0.90	0.00	0.00	1.45	0.15	1.00
time (sec)	N/A	0.342	1.188	1.933	0.000	0.000	26.532	111.290	0.517

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	0	0	27	3	19
N.S.	1	1.00	1.11	0.89	0.00	0.00	1.42	0.16	1.00
time (sec)	N/A	0.246	2.436	1.875	0.000	0.000	21.657	108.650	0.564

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	0	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.00	1.00
time (sec)	N/A	0.190	1.929	2.745	0.000	0.000	16.428	0.000	0.550

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	0	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.00	1.00
time (sec)	N/A	0.193	0.857	2.848	0.000	0.000	19.434	0.000	0.550

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	0.00	0.12	1.00
time (sec)	N/A	0.212	0.775	8.549	0.000	0.257	0.000	53.604	0.561

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	51	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.12	0.12	1.00
time (sec)	N/A	0.211	2.804	4.283	0.000	0.000	80.169	112.394	0.473

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	49	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	2.23	0.14	1.00
time (sec)	N/A	0.611	0.883	1.980	0.000	0.000	51.417	109.500	0.452

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	48	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	2.29	0.14	1.00
time (sec)	N/A	0.490	1.488	2.074	0.000	0.000	40.918	111.914	0.451

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.00	1.00
time (sec)	N/A	0.205	1.245	2.471	0.000	0.000	29.777	0.000	0.473

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.00	1.00
time (sec)	N/A	0.209	1.543	2.535	0.000	0.000	31.241	0.000	0.501

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	48	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.00	0.00	0.12	1.00
time (sec)	N/A	0.212	0.588	10.503	0.000	0.250	0.000	51.063	0.547

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	70	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.92	0.12	1.00
time (sec)	N/A	0.206	2.766	4.445	0.000	0.000	149.491	108.162	0.480

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	68	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	3.09	0.14	1.00
time (sec)	N/A	0.883	0.955	2.187	0.000	0.000	96.844	104.838	0.462

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	66	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	3.14	0.14	1.00
time (sec)	N/A	0.724	1.517	2.368	0.000	0.000	85.955	108.742	0.478

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	0.205	1.055	2.487	0.000	0.000	57.568	0.000	0.506

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	0.207	1.501	2.870	0.000	0.000	61.462	0.000	0.485

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	24	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.00	0.00	0.12	1.00
time (sec)	N/A	0.213	0.642	5.470	0.000	0.247	0.000	52.147	0.528

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.92	0.12	1.00
time (sec)	N/A	0.424	3.689	2.205	0.000	0.000	9.675	108.096	0.462

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	22	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.92	0.12	1.00
time (sec)	N/A	0.326	0.873	1.621	0.000	0.000	7.670	109.589	0.446

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	20	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.91	0.14	1.00
time (sec)	N/A	0.233	0.810	1.151	0.000	0.000	5.064	107.694	0.467

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	15	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.83	0.78	0.78
time (sec)	N/A	0.170	0.004	2.537	0.000	0.245	7.275	0.278	0.442

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	19	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.79	0.12	1.00
time (sec)	N/A	0.294	0.319	1.742	0.000	0.000	3.675	103.475	0.489

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.340	0.600	1.579	0.000	0.000	5.196	107.729	0.494

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.500	1.829	1.657	0.000	0.000	5.408	107.946	0.501

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	20	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	0.83	0.12	1.00
time (sec)	N/A	0.559	3.404	4.455	0.000	0.000	7.183	109.257	0.491

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	39	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.62	0.00	0.12	1.00
time (sec)	N/A	0.209	1.423	9.633	0.000	0.257	0.000	52.150	0.544

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	32	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.33	0.12	1.00
time (sec)	N/A	0.214	3.592	7.579	0.000	0.000	15.147	118.133	0.483

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	161	111	93	0	0	0	0	0
N.S.	1	1.03	0.71	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.761	0.363	25.514	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	153	234	82	0	0	0	0	0
N.S.	1	0.98	1.50	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	0.164	25.740	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	156	108	93	0	0	0	0	0
N.S.	1	1.03	0.72	0.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.754	0.311	25.052	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.21	0.12	1.00
time (sec)	N/A	0.217	1.074	3.253	0.000	0.000	5.277	116.899	0.519

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	50	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.08	0.00	0.12	1.00
time (sec)	N/A	0.223	1.398	7.799	0.000	0.268	0.000	54.135	0.538

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	41	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.71	0.12	1.00
time (sec)	N/A	0.221	6.621	8.124	0.000	0.000	31.006	128.672	0.503

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	346	287	168	0	0	0	0	0
N.S.	1	1.12	0.93	0.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.717	0.599	2.763	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	300	359	157	0	0	0	0	0
N.S.	1	1.17	1.40	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.619	0.604	2.562	0.000	0.000	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	110	185	96	0	0	0	0	0
N.S.	1	0.83	1.39	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.452	1.477	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	286	359	157	0	0	0	0	0
N.S.	1	1.13	1.41	0.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.336	0.563	1.819	0.000	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	330	287	168	0	0	0	0	0
N.S.	1	1.11	0.97	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.610	0.613	2.097	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	37	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.54	0.12	1.00
time (sec)	N/A	0.217	2.040	3.766	0.000	0.000	10.942	125.499	0.530

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.259	1.264	11.497	0.000	0.256	0.000	0.000	0.352

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.253	3.880	5.571	0.000	0.000	0.000	123.403	0.340

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.493	4.579	3.505	0.000	0.000	0.000	0.000	0.329

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.347	0.245	3.582	0.000	0.000	0.000	0.000	0.361

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.00	0.92
time (sec)	N/A	0.249	3.300	3.889	0.000	0.000	168.429	0.000	0.340

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.259	1.440	9.154	0.000	0.252	0.000	0.000	0.344

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.263	4.285	4.161	0.000	0.000	0.000	122.427	0.343

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.725	3.008	2.926	0.000	0.000	0.000	0.000	0.333

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.562	1.745	2.869	0.000	0.000	0.000	0.000	0.334

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.256	3.116	3.763	0.000	0.000	0.000	0.000	0.362

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	49	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	1.88	0.00	0.00	0.92
time (sec)	N/A	0.255	1.701	12.808	0.000	0.257	0.000	0.000	0.351

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.259	3.634	5.083	0.000	0.000	0.000	124.483	0.335

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.999	5.435	3.204	0.000	0.000	0.000	0.000	0.344

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	0	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.818	0.408	3.665	0.000	0.000	0.000	0.000	0.347

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.254	3.351	4.730	0.000	0.000	0.000	0.000	0.339

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.12	0.92
time (sec)	N/A	0.252	1.365	13.726	0.000	0.256	0.000	53.841	0.352

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	1.129	4.019	6.365	0.000	0.000	0.000	0.000	0.348

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.566	3.038	4.425	0.000	0.000	0.000	227.558	0.335

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.308	1.006	4.000	0.000	0.000	0.000	224.849	0.337

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.186	0.184	5.217	0.000	0.000	120.182	213.370	0.339

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.243	2.222	5.536	0.000	0.000	141.358	215.084	0.354

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.394	2.561	5.742	0.000	0.000	171.522	223.397	0.362

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.755	8.137	5.900	0.000	0.000	168.999	224.385	0.344

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	1.484	17.762	5.589	0.000	0.000	177.145	227.335	0.343

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	51	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	1.96	0.00	0.12	0.92
time (sec)	N/A	0.260	1.673	6.594	0.000	0.255	0.000	52.532	0.367

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.261	4.634	5.112	0.000	0.000	0.000	122.195	0.370

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	163	139	0	0	0	0	0	0
N.S.	1	1.01	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	152	97	0	0	0	0	0	0
N.S.	1	0.98	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.631	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.262	3.621	4.796	0.000	0.000	164.288	116.350	0.339

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	62	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	2.38	0.00	0.12	0.92
time (sec)	N/A	0.260	2.231	11.215	0.000	0.263	0.000	55.411	0.356

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.257	7.251	15.820	0.000	0.000	0.000	134.804	0.354

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	350	390	370	0	0	0	0	0	0
N.S.	1	1.11	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.517	0.474	0.000	0.000	0.000	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	344	287	0	0	0	0	0	0
N.S.	1	1.17	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.712	0.959	0.000	0.000	0.000	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	337	356	0	0	0	0	0	0
N.S.	1	1.15	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.332	0.432	0.000	0.000	0.000	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	337	370	176	0	0	0	0	0	0
N.S.	1	1.10	0.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.317	0.211	0.000	0.000	0.000	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.260	5.859	3.101	0.000	0.000	0.000	129.920	0.340

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	22	34	3	22
N.S.	1	1.00	1.09	0.91	0.00	1.00	1.55	0.14	1.00
time (sec)	N/A	0.194	1.014	4.520	0.000	0.250	13.747	64.261	0.693

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	0	29	3	20
N.S.	1	1.00	1.10	0.90	0.00	0.00	1.45	0.15	1.00
time (sec)	N/A	0.184	0.531	1.586	0.000	0.000	1.169	83.681	0.499

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	0	0	29	3	19
N.S.	1	1.00	1.11	0.89	0.00	0.00	1.53	0.16	1.00
time (sec)	N/A	0.170	0.077	1.186	0.000	0.000	0.755	75.741	0.522

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	29	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.32	0.14	1.00
time (sec)	N/A	0.188	1.064	1.353	0.000	0.000	1.290	78.073	0.534

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	58	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	2.42	0.12	1.00
time (sec)	N/A	0.214	0.742	5.321	0.000	0.247	51.809	63.974	0.563

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	49	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	2.23	0.14	1.00
time (sec)	N/A	0.200	0.631	1.222	0.000	0.000	2.321	91.910	0.489

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	49	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	2.33	0.14	1.00
time (sec)	N/A	0.184	0.395	1.069	0.000	0.000	1.708	85.710	0.463

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	0.207	1.050	1.540	0.000	0.000	2.975	88.938	0.470

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	48	80	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.00	3.33	0.12	1.00
time (sec)	N/A	0.218	0.519	5.385	0.000	0.249	160.005	65.116	0.557

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	68	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	3.09	0.14	1.00
time (sec)	N/A	0.195	0.675	1.436	0.000	0.000	4.182	102.266	0.484

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	68	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	3.24	0.14	1.00
time (sec)	N/A	0.178	0.418	1.178	0.000	0.000	3.143	97.826	0.483

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	68	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.83	0.12	1.00
time (sec)	N/A	0.203	1.120	1.533	0.000	0.000	4.128	97.801	0.511

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	24	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.00	1.21	0.12	1.00
time (sec)	N/A	0.219	0.594	4.301	0.000	0.254	12.119	65.275	0.624

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.14	1.00
time (sec)	N/A	0.237	0.729	0.864	0.000	0.000	0.929	47.733	0.437

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	14	12	14	14
N.S.	1	1.00	1.00	0.94	0.00	0.88	0.75	0.88	0.88
time (sec)	N/A	0.173	0.005	1.002	0.000	0.234	0.698	0.288	0.405

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.21	0.12	1.00
time (sec)	N/A	0.215	0.217	0.570	0.000	0.000	1.209	47.637	0.471

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	39	48	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.62	2.00	0.12	1.00
time (sec)	N/A	0.220	1.480	6.687	0.000	0.261	69.825	106.930	0.669

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.12	1.00
time (sec)	N/A	0.216	3.130	5.458	0.000	0.000	1.800	134.140	0.515

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	41	122	38	0	0	0	0	0
N.S.	1	0.87	2.60	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.338	1.750	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	0	0	0	0	0
N.S.	1	1.00	1.00	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.070	1.343	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	41	122	38	0	0	0	0	0
N.S.	1	0.87	2.60	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.279	0.916	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.12	1.00
time (sec)	N/A	0.212	0.728	2.166	0.000	0.000	2.443	110.084	0.542

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	50	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.08	0.00	0.12	1.00
time (sec)	N/A	0.222	1.516	6.608	0.000	0.258	0.000	155.898	0.678

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	65	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.71	0.12	1.00
time (sec)	N/A	0.216	3.709	4.990	0.000	0.000	4.473	190.730	0.524

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	78	230	59	0	0	0	0	0
N.S.	1	0.88	2.58	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	0.496	1.439	0.000	0.000	0.000	0.000	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	131	47	0	0	0	0	0
N.S.	1	0.93	1.85	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.151	1.404	0.000	0.000	0.000	0.000	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	53	229	47	0	0	0	0	0
N.S.	1	0.91	3.95	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.427	1.056	0.000	0.000	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	133	46	0	0	0	0	0
N.S.	1	0.93	1.87	0.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.119	1.230	0.000	0.000	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	78	229	59	0	0	0	0	0
N.S.	1	0.88	2.57	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	0.491	1.006	0.000	0.000	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	65	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.71	0.12	1.00
time (sec)	N/A	0.214	0.955	1.886	0.000	0.000	5.511	158.514	0.535

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	1.00	0.00	0.92
time (sec)	N/A	0.250	1.141	7.631	0.000	0.275	24.686	0.000	0.354

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.220	1.995	2.906	0.000	0.000	1.755	135.756	0.346

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.188	0.235	2.397	0.000	0.000	1.107	126.851	0.369

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.245	4.424	2.654	0.000	0.000	2.090	129.141	0.350

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.257	1.219	6.792	0.000	0.306	0.000	0.000	0.356

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.225	3.218	2.461	0.000	0.000	47.010	168.768	0.348

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.189	0.318	2.443	0.000	0.000	23.363	161.231	0.358

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.248	5.066	2.787	0.000	0.000	21.246	163.413	0.350

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	49	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	1.88	0.00	0.00	0.92
time (sec)	N/A	0.251	1.456	7.094	0.000	0.264	0.000	0.000	0.385

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.225	2.490	2.960	0.000	0.000	0.000	199.360	0.347

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.13	0.91
time (sec)	N/A	0.192	0.333	2.634	0.000	0.000	0.000	191.904	0.352

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.253	4.196	2.681	0.000	0.000	153.800	195.851	0.348

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	1.00	0.12	0.92
time (sec)	N/A	0.248	1.162	7.540	0.000	0.279	31.093	64.145	0.364

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.219	1.144	2.934	0.000	0.000	1.899	133.444	0.358

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	24	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	1.04	0.13	0.91
time (sec)	N/A	0.186	0.192	3.188	0.000	0.000	2.018	133.772	0.344

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.238	1.394	3.293	0.000	0.000	4.532	131.634	0.359

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	51	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	1.96	0.00	0.12	0.92
time (sec)	N/A	0.254	1.407	6.564	0.000	0.277	0.000	36.978	0.354

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.258	12.231	3.947	0.000	0.000	8.831	102.426	0.351

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	97	0	0	0	0	0	0
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	100	0	0	0	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.110	0.000	0.000	0.000	0.000	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.253	1.875	2.720	0.000	0.000	25.669	0.000	0.358

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	62	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	2.38	0.00	0.12	0.92
time (sec)	N/A	0.257	2.010	10.389	0.000	0.273	0.000	71.762	0.364

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.256	5.067	13.657	0.000	0.000	61.957	195.645	0.367

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	99	95	0	0	0	0	0	0
N.S.	1	0.76	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	99	159	0	0	0	0	0	0
N.S.	1	0.76	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.507	0.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	99	156	0	0	0	0	0	0
N.S.	1	0.76	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	0.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	99	93	0	0	0	0	0	0
N.S.	1	0.76	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.260	2.557	3.116	0.000	0.000	133.299	0.000	0.349

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	22	34	3	22
N.S.	1	1.00	1.09	0.91	0.00	1.00	1.55	0.14	1.00
time (sec)	N/A	0.195	0.945	4.820	0.000	0.247	33.824	62.976	0.872

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	0	29	0	20
N.S.	1	1.00	1.10	0.90	0.00	0.00	1.45	0.00	1.00
time (sec)	N/A	0.181	0.817	1.425	0.000	0.000	1.566	0.000	0.620

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	0	0	29	3	19
N.S.	1	1.00	1.11	0.89	0.00	0.00	1.53	0.16	1.00
time (sec)	N/A	0.171	0.727	1.129	0.000	0.000	1.348	169.688	0.581

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	29	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.32	0.14	1.00
time (sec)	N/A	0.190	1.438	1.375	0.000	0.000	2.452	234.078	0.652

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	58	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	2.42	0.12	1.00
time (sec)	N/A	0.213	0.762	5.956	0.000	0.264	61.406	63.636	0.696

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	49	0	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	2.23	0.00	1.00
time (sec)	N/A	0.195	0.917	1.404	0.000	0.000	2.456	0.000	0.571

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	49	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	2.33	0.14	1.00
time (sec)	N/A	0.177	0.813	1.214	0.000	0.000	2.070	209.715	0.521

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	0.205	1.543	1.444	0.000	0.000	3.181	279.869	0.543

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	48	80	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.00	3.33	0.12	1.00
time (sec)	N/A	0.208	0.554	6.629	0.000	0.268	176.373	65.189	0.651

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	68	0	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	3.09	0.00	1.00
time (sec)	N/A	0.192	0.980	1.689	0.000	0.000	4.914	0.000	0.529

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	68	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	3.24	0.14	1.00
time (sec)	N/A	0.178	0.912	1.443	0.000	0.000	3.729	252.599	0.481

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	68	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.83	0.00	1.00
time (sec)	N/A	0.203	1.667	1.509	0.000	0.000	5.202	0.000	0.549

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	24	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.00	1.21	0.12	1.00
time (sec)	N/A	0.276	0.611	4.125	0.000	0.272	56.982	63.837	0.634

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.14	1.00
time (sec)	N/A	0.234	0.671	0.786	0.000	0.000	1.451	123.750	0.458

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	14	14	14	14
N.S.	1	1.00	1.00	0.94	0.00	0.88	0.88	0.88	0.88
time (sec)	N/A	0.175	0.006	0.937	0.000	0.284	1.454	0.283	0.372

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.21	0.12	1.00
time (sec)	N/A	0.267	0.165	1.225	0.000	0.000	2.030	125.815	0.440

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	39	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.62	0.00	0.12	1.00
time (sec)	N/A	0.212	1.228	5.885	0.000	0.264	0.000	141.277	0.664

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.00	1.00
time (sec)	N/A	0.405	3.535	9.916	0.000	0.000	3.420	0.000	0.598

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.00	1.00
time (sec)	N/A	0.733	6.013	5.506	0.000	0.000	3.832	0.000	0.546

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	0	0	0	0	0
N.S.	1	1.00	1.00	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	0.345	1.701	0.000	0.000	0.000	0.000	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	139	158	47	0	0	0	0	0
N.S.	1	1.01	1.14	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.165	1.740	0.000	0.000	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	52	47	0	0	0	0	0
N.S.	1	1.00	0.91	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.276	2.063	0.000	0.000	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.12	1.00
time (sec)	N/A	0.708	2.323	1.817	0.000	0.000	4.572	140.363	0.582

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	0.414	3.435	3.329	0.000	0.000	7.744	148.206	0.591

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	0.430	6.916	2.593	0.000	0.000	10.360	148.479	0.582

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	0.425	6.874	4.518	0.000	0.000	13.373	147.735	0.587

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	50	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.08	0.00	0.12	1.00
time (sec)	N/A	0.229	1.150	7.731	0.000	0.269	0.000	210.411	0.712

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	164	148	86	0	0	0	0	0
N.S.	1	1.71	1.54	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	0.300	2.020	0.000	0.000	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	165	112	53	0	0	0	0	0
N.S.	1	2.46	1.67	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.392	1.751	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	162	156	84	0	0	0	0	0
N.S.	1	1.74	1.68	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.881	0.221	1.407	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	95	144	85	0	0	0	0	0
N.S.	1	1.01	1.53	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	0.374	1.530	0.000	0.000	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	65	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.71	0.12	1.00
time (sec)	N/A	0.762	3.125	1.967	0.000	0.000	13.707	204.942	0.602

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.12	1.00
time (sec)	N/A	0.417	4.778	2.904	0.000	0.000	20.381	211.028	0.594

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.12	1.00
time (sec)	N/A	0.429	7.373	3.102	0.000	0.000	26.363	211.441	0.624

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.12	1.00
time (sec)	N/A	0.427	7.166	3.316	0.000	0.000	33.799	210.168	0.611

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	1.00	0.00	0.92
time (sec)	N/A	0.259	1.117	10.224	0.000	0.268	69.013	0.000	0.351

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.230	2.324	3.409	0.000	0.000	3.963	267.358	0.358

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.196	0.372	2.463	0.000	0.000	5.678	140.578	0.344

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.248	5.498	2.806	0.000	0.000	9.007	145.312	0.349

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.262	1.278	8.119	0.000	0.261	0.000	0.000	0.351

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.231	6.830	2.793	0.000	0.000	62.347	0.000	0.343

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.195	1.231	2.772	0.000	0.000	33.953	187.642	0.367

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.260	11.116	2.689	0.000	0.000	52.241	188.364	0.364

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	49	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	1.88	0.00	0.00	0.92
time (sec)	N/A	0.261	1.547	7.834	0.000	0.267	0.000	0.000	0.351

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.230	3.580	2.994	0.000	0.000	0.000	0.000	0.346

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.13	0.91
time (sec)	N/A	0.195	1.024	2.711	0.000	0.000	0.000	226.748	0.351

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.257	7.241	2.735	0.000	0.000	0.000	240.006	0.360

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.12	0.92
time (sec)	N/A	0.251	1.294	7.007	0.000	0.267	0.000	64.036	0.353

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	3	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.220	1.008	3.066	0.000	0.000	8.353	161.681	0.344

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	24	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	1.04	0.13	0.91
time (sec)	N/A	0.191	0.578	3.527	0.000	0.000	8.457	206.070	0.352

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.398	3.884	3.183	0.000	0.000	23.382	209.343	0.366

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	27	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.04	0.12	0.92
time (sec)	N/A	0.247	7.973	2.778	0.000	0.000	41.894	99.663	0.374

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	51	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	1.96	0.00	0.12	0.92
time (sec)	N/A	0.259	1.566	6.884	0.000	0.267	0.000	89.679	0.376

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.550	6.169	4.562	0.000	0.000	32.937	0.000	0.348

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	1.344	3.658	3.990	0.000	0.000	32.649	209.267	0.365

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	116	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.537	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	107	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.574	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	1.283	6.626	3.343	0.000	0.000	76.767	0.000	0.378

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	27	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.04	0.12	0.92
time (sec)	N/A	0.559	11.599	5.163	0.000	0.000	126.594	84.256	0.360

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.561	14.534	4.753	0.000	0.000	0.000	0.000	0.356

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.555	17.029	6.296	0.000	0.000	0.000	85.453	0.350

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	62	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	2.38	0.00	0.12	0.92
time (sec)	N/A	0.260	1.804	8.956	0.000	0.277	0.000	160.477	0.368

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	134	182	0	0	0	0	0	0
N.S.	1	0.84	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	0.431	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	281	234	241	0	0	0	0	0	0
N.S.	1	0.83	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.103	0.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	232	299	0	0	0	0	0	0
N.S.	1	0.83	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.534	0.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	157	131	158	0	0	0	0	0	0
N.S.	1	0.83	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.576	0.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	1.230	6.843	2.737	0.000	0.000	0.000	0.000	0.371

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.540	15.349	3.195	0.000	0.000	0.000	161.379	0.359

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.549	22.274	4.959	0.000	0.000	0.000	0.000	0.382

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.544	19.886	5.491	0.000	0.000	0.000	163.887	0.353

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	22	0	3	22
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.00	0.14	1.00
time (sec)	N/A	0.191	0.982	4.522	0.000	0.266	0.000	11.189	0.866

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	0	29	0	20
N.S.	1	1.00	1.10	0.90	0.00	0.00	1.45	0.00	1.00
time (sec)	N/A	0.184	1.497	1.604	0.000	0.000	5.463	0.000	0.672

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	0	0	29	3	19
N.S.	1	1.00	1.11	0.89	0.00	0.00	1.53	0.16	1.00
time (sec)	N/A	0.168	1.027	1.012	0.000	0.000	4.987	195.216	0.712

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	29	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.32	0.14	1.00
time (sec)	N/A	0.187	3.421	1.081	0.000	0.000	7.047	294.374	0.720

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	0.00	0.12	1.00
time (sec)	N/A	0.209	0.809	5.012	0.000	0.261	0.000	11.312	0.665

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	49	0	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	2.23	0.00	1.00
time (sec)	N/A	0.193	1.223	1.184	0.000	0.000	5.120	0.000	0.545

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	49	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	2.33	0.14	1.00
time (sec)	N/A	0.180	0.811	0.969	0.000	0.000	5.308	231.348	0.537

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.00	1.00
time (sec)	N/A	0.205	1.961	1.199	0.000	0.000	7.356	0.000	0.584

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	48	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	2.00	0.00	0.12	1.00
time (sec)	N/A	0.210	0.572	6.176	0.000	0.261	0.000	11.302	0.667

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	68	0	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	3.09	0.00	1.00
time (sec)	N/A	0.193	1.344	1.411	0.000	0.000	8.351	0.000	0.576

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	0	68	3	21
N.S.	1	1.00	1.10	0.90	0.00	0.00	3.24	0.14	1.00
time (sec)	N/A	0.179	0.846	1.496	0.000	0.000	7.165	285.306	0.521

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	68	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.83	0.00	1.00
time (sec)	N/A	0.204	2.318	1.485	0.000	0.000	8.488	0.000	0.630

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	24	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.00	0.00	0.12	1.00
time (sec)	N/A	0.277	0.663	3.899	0.000	0.261	0.000	11.324	0.641

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	27	3	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	1.23	0.14	1.00
time (sec)	N/A	0.233	0.909	1.166	0.000	0.000	3.585	184.653	0.426

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	14	15	14	14
N.S.	1	1.00	1.00	0.83	0.00	0.78	0.83	0.78	0.78
time (sec)	N/A	0.169	0.007	0.987	0.000	0.252	3.631	0.290	0.380

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	29	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	1.21	0.12	1.00
time (sec)	N/A	0.265	0.553	0.712	0.000	0.000	4.671	183.133	0.464

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	39	0	3	24
N.S.	1	1.00	1.08	0.92	0.00	1.62	0.00	0.12	1.00
time (sec)	N/A	0.217	1.271	6.023	0.000	0.255	0.000	221.155	0.666

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.00	1.00
time (sec)	N/A	1.610	6.738	4.282	0.000	0.000	11.065	0.000	0.618

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	173	162	62	0	0	0	0	0
N.S.	1	0.96	0.90	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.718	0.520	1.379	0.000	0.000	0.000	0.000	0.000

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	59	0	0	0	0	0
N.S.	1	1.00	0.87	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.106	1.280	0.000	0.000	0.000	0.000	0.000

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	168	170	62	0	0	0	0	0
N.S.	1	0.97	0.98	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.511	0.941	0.000	0.000	0.000	0.000	0.000

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	48	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.00	0.12	1.00
time (sec)	N/A	1.595	3.290	1.763	0.000	0.000	11.869	221.818	0.641

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	1.448	5.916	0.207	0.000	0.000	16.817	227.734	0.650

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	0.771	5.520	0.108	0.000	0.000	22.684	226.728	0.648

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	49	3	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.04	0.12	1.00
time (sec)	N/A	0.771	11.853	0.135	0.000	0.000	29.010	225.314	0.657

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	50	0	0	24
N.S.	1	1.00	1.08	0.92	0.00	2.08	0.00	0.00	1.00
time (sec)	N/A	0.222	1.169	0.378	0.000	0.279	0.000	0.000	0.698

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	297	227	112	0	0	0	0	0
N.S.	1	1.86	1.42	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.133	0.376	0.150	0.000	0.000	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	359	259	68	0	0	0	0	0
N.S.	1	2.78	2.01	0.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.107	0.801	0.197	0.000	0.000	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	292	220	110	0	0	0	0	0
N.S.	1	1.88	1.42	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.061	0.338	0.207	0.000	0.000	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	191	186	113	0	0	0	0	0
N.S.	1	1.53	1.49	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.993	0.688	0.183	0.000	0.000	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	65	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.71	0.00	1.00
time (sec)	N/A	1.281	4.515	0.096	0.000	0.000	28.972	0.000	0.671

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	1.488	6.486	0.102	0.000	0.000	40.831	0.000	0.677

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	0.766	6.028	0.112	0.000	0.000	52.333	0.000	0.704

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	0	66	0	24
N.S.	1	1.00	1.08	0.92	0.00	0.00	2.75	0.00	1.00
time (sec)	N/A	0.780	13.283	0.116	0.000	0.000	64.240	0.000	0.654

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.246	1.164	0.460	0.000	0.263	0.000	0.000	0.358

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.220	3.304	0.230	0.000	0.000	48.956	0.000	0.347

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	22	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.96	0.13	0.91
time (sec)	N/A	0.191	2.034	0.184	0.000	0.000	46.833	164.754	0.359

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	24	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.92	0.12	0.92
time (sec)	N/A	0.244	10.822	0.220	0.000	0.000	61.015	165.700	0.358

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.00	0.92
time (sec)	N/A	0.257	1.352	0.473	0.000	0.258	0.000	0.000	0.358

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.226	3.481	0.282	0.000	0.000	0.000	0.000	0.350

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.13	0.91
time (sec)	N/A	0.192	2.097	0.230	0.000	0.000	0.000	208.943	0.363

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.256	11.484	0.266	0.000	0.000	0.000	210.613	0.355

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	49	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	1.88	0.00	0.00	0.92
time (sec)	N/A	0.253	1.602	0.641	0.000	0.251	0.000	0.000	0.374

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.229	4.207	0.477	0.000	0.000	0.000	0.000	0.347

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	0	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	0.00	0.13	0.91
time (sec)	N/A	0.191	2.314	0.331	0.000	0.000	0.000	255.048	0.349

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.253	8.551	0.347	0.000	0.000	0.000	254.813	0.355

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	24	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.92	0.00	0.12	0.92
time (sec)	N/A	0.256	1.344	0.326	0.000	0.256	0.000	15.191	0.359

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	24	0	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	0.224	2.111	0.202	0.000	0.000	60.652	0.000	0.363

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	19	0	0	24	3	21
N.S.	1	1.00	1.09	0.83	0.00	0.00	1.04	0.13	0.91
time (sec)	N/A	0.191	0.786	0.150	0.000	0.000	61.180	263.100	0.383

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	0.396	4.720	0.189	0.000	0.000	126.300	299.052	0.373

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	0.245	11.331	0.221	0.000	0.000	0.000	261.895	0.369

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	51	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	1.96	0.00	0.12	0.92
time (sec)	N/A	0.258	1.676	0.228	0.000	0.259	0.000	133.749	0.392

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.00	0.92
time (sec)	N/A	2.973	7.778	0.166	0.000	0.000	162.346	0.000	0.390

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	26	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	1.00	0.12	0.92
time (sec)	N/A	1.872	6.673	0.149	0.000	0.000	159.327	296.319	0.353

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	133	124	0	0	0	0	0	0
N.S.	1	1.03	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.730	0.180	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	131	120	0	0	0	0	0	0
N.S.	1	1.04	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.669	0.128	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	2.587	6.674	0.163	0.000	0.000	0.000	0.000	0.353

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	2.972	9.860	0.181	0.000	0.000	0.000	214.478	0.366

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	1.127	14.283	0.217	0.000	0.000	0.000	0.000	0.350

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	1.143	33.776	0.224	0.000	0.000	0.000	218.806	0.346

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	62	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	2.38	0.00	0.12	0.92
time (sec)	N/A	0.260	1.789	0.536	0.000	0.265	0.000	241.727	0.371

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	190	275	255	0	0	0	0	0	0
N.S.	1	1.45	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.386	0.783	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	414	311	0	0	0	0	0	0
N.S.	1	1.85	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.982	0.656	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	411	261	0	0	0	0	0	0
N.S.	1	1.85	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.037	0.769	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	268	300	0	0	0	0	0	0
N.S.	1	1.46	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.730	0.437	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	2.170	9.927	0.183	0.000	0.000	0.000	0.000	0.363

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	2.709	9.373	0.197	0.000	0.000	0.000	254.935	0.358

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	0	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	1.117	16.584	0.222	0.000	0.000	0.000	0.000	0.358

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	22	0	0	0	3	24
N.S.	1	1.00	1.08	0.85	0.00	0.00	0.00	0.12	0.92
time (sec)	N/A	1.121	40.005	0.230	0.000	0.000	0.000	257.280	0.358

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	0	22	19	3	22
N.S.	1	1.00	1.10	1.00	0.00	1.10	0.95	0.15	1.10
time (sec)	N/A	0.264	0.878	0.095	0.000	0.250	0.604	54.735	0.448

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	0	21	0	20	20
N.S.	1	1.00	1.00	1.05	0.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.186	0.005	0.338	0.000	0.249	0.000	0.270	0.443

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	30	0	3	30
N.S.	1	1.00	1.07	1.00	1.07	1.07	0.00	0.11	1.07
time (sec)	N/A	0.245	0.653	1.216	0.645	0.258	0.000	80.871	0.695

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	87	127	108	108	110	138	0	105
N.S.	1	0.81	1.19	1.01	1.01	1.03	1.29	0.00	0.98
time (sec)	N/A	0.284	0.008	0.183	0.289	0.246	0.420	0.000	0.716

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	93	119	104	105	107	128	0	101
N.S.	1	0.99	1.27	1.11	1.12	1.14	1.36	0.00	1.07
time (sec)	N/A	0.299	0.021	0.175	0.198	0.257	0.370	0.000	0.701

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	83	103	87	88	89	114	0	85
N.S.	1	1.01	1.26	1.06	1.07	1.09	1.39	0.00	1.04
time (sec)	N/A	0.261	0.007	0.164	0.267	0.253	0.360	0.000	0.337

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	67	85	76	80	82	94	0	75
N.S.	1	0.99	1.25	1.12	1.18	1.21	1.38	0.00	1.10
time (sec)	N/A	0.251	0.008	0.065	0.212	0.240	0.315	0.000	0.609

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	83	120	104	0	0	0	88
N.S.	1	1.00	1.08	1.56	1.35	0.00	0.00	0.00	1.14
time (sec)	N/A	0.265	0.001	0.142	0.412	0.000	0.000	0.000	0.819

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	61	73	74	73	74	80	0	69
N.S.	1	1.07	1.28	1.30	1.28	1.30	1.40	0.00	1.21
time (sec)	N/A	0.272	0.007	0.090	0.183	0.246	0.328	0.000	0.261

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	86	129	0	0	0	0	91
N.S.	1	1.00	1.12	1.68	0.00	0.00	0.00	0.00	1.18
time (sec)	N/A	0.270	0.008	0.177	0.000	0.000	0.000	0.000	0.849

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	79	98	97	93	85	116	0	92
N.S.	1	0.95	1.18	1.17	1.12	1.02	1.40	0.00	1.11
time (sec)	N/A	0.284	0.027	0.106	0.209	0.243	0.365	0.000	0.653

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	97	96	80	75	99	0	162
N.S.	1	0.88	1.18	1.17	0.98	0.91	1.21	0.00	1.98
time (sec)	N/A	0.249	0.008	0.138	0.299	0.239	0.306	0.000	0.735

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	105	131	119	116	111	153	0	111
N.S.	1	0.95	1.19	1.08	1.05	1.01	1.39	0.00	1.01
time (sec)	N/A	0.307	0.033	0.113	0.214	0.248	0.502	0.000	0.263

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	86	97	109	103	98	122	0	130
N.S.	1	0.82	0.92	1.04	0.98	0.93	1.16	0.00	1.24
time (sec)	N/A	0.263	0.007	0.128	0.285	0.272	0.371	0.000	0.706

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	182	173	197	184	201	260	0	374
N.S.	1	0.98	0.94	1.06	0.99	1.09	1.41	0.00	2.02
time (sec)	N/A	0.419	0.077	0.413	0.289	0.255	0.534	0.000	0.618

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	157	162	181	181	186	245	0	191
N.S.	1	0.98	1.01	1.12	1.12	1.16	1.52	0.00	1.19
time (sec)	N/A	0.428	0.089	0.332	0.213	0.276	0.475	0.000	0.907

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	140	186	156	166	219	0	167
N.S.	1	1.00	1.22	1.62	1.36	1.44	1.90	0.00	1.45
time (sec)	N/A	0.302	0.059	0.309	0.277	0.249	0.412	0.000	0.507

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	122	130	137	147	150	194	0	150
N.S.	1	0.98	1.05	1.10	1.19	1.21	1.56	0.00	1.21
time (sec)	N/A	0.354	0.050	0.197	0.215	0.252	0.393	0.000	0.754

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	123	175	172	0	0	0	157
N.S.	1	1.00	0.90	1.28	1.26	0.00	0.00	0.00	1.15
time (sec)	N/A	0.341	0.078	0.271	0.435	0.000	0.000	0.000	0.818

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	111	114	134	130	140	165	0	135
N.S.	1	1.02	1.05	1.23	1.19	1.28	1.51	0.00	1.24
time (sec)	N/A	0.367	0.080	0.220	0.189	0.245	0.451	0.000	0.818

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	118	178	153	0	0	0	157
N.S.	1	1.00	0.92	1.39	1.20	0.00	0.00	0.00	1.23
time (sec)	N/A	0.333	0.085	0.378	0.450	0.000	0.000	0.000	0.783

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	112	119	139	135	139	180	0	142
N.S.	1	0.97	1.03	1.21	1.17	1.21	1.57	0.00	1.23
time (sec)	N/A	0.377	0.085	0.221	0.194	0.249	0.439	0.000	0.777

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	130	194	0	0	0	0	177
N.S.	1	1.00	0.94	1.40	0.00	0.00	0.00	0.00	1.27
time (sec)	N/A	0.344	0.073	0.372	0.000	0.000	0.000	0.000	0.899

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	143	149	167	166	160	235	0	179
N.S.	1	0.95	0.99	1.11	1.11	1.07	1.57	0.00	1.19
time (sec)	N/A	0.406	0.096	0.278	0.196	0.250	0.546	0.000	0.523

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	113	112	167	145	145	192	0	256
N.S.	1	1.02	1.01	1.50	1.31	1.31	1.73	0.00	2.31
time (sec)	N/A	0.337	0.067	0.264	0.266	0.267	0.420	0.000	0.911

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	178	177	202	197	194	289	0	232
N.S.	1	0.96	0.95	1.09	1.06	1.04	1.55	0.00	1.25
time (sec)	N/A	0.448	0.145	0.262	0.206	0.273	0.722	0.000	0.708

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	303	244	301	268	304	411	0	599
N.S.	1	1.26	1.02	1.25	1.12	1.27	1.71	0.00	2.50
time (sec)	N/A	0.589	0.152	0.316	0.279	0.268	0.758	0.000	0.727

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	236	252	272	265	277	389	0	296
N.S.	1	0.99	1.05	1.14	1.11	1.16	1.63	0.00	1.24
time (sec)	N/A	0.630	0.161	0.323	0.185	0.271	0.721	0.000	1.102

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	157	217	264	232	258	350	0	442
N.S.	1	0.99	1.37	1.67	1.47	1.63	2.22	0.00	2.80
time (sec)	N/A	0.369	0.082	0.515	0.272	0.259	0.609	0.000	0.630

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	185	192	210	222	229	306	0	238
N.S.	1	0.98	1.02	1.12	1.18	1.22	1.63	0.00	1.27
time (sec)	N/A	0.518	0.067	0.265	0.218	0.263	0.495	0.000	0.455

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	190	257	251	0	0	0	232
N.S.	1	1.00	0.83	1.13	1.10	0.00	0.00	0.00	1.02
time (sec)	N/A	0.430	0.108	0.337	0.461	0.000	0.000	0.000	0.905

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	161	169	193	197	206	258	0	236
N.S.	1	1.01	1.06	1.21	1.23	1.29	1.61	0.00	1.48
time (sec)	N/A	0.540	0.088	0.267	0.191	0.285	0.623	0.000	0.710

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	170	240	223	0	0	0	224
N.S.	1	1.00	0.85	1.20	1.12	0.00	0.00	0.00	1.12
time (sec)	N/A	0.393	0.111	0.498	0.437	0.000	0.000	0.000	0.841

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	152	166	195	193	205	272	0	203
N.S.	1	0.96	1.05	1.23	1.22	1.30	1.72	0.00	1.28
time (sec)	N/A	0.517	0.106	0.294	0.207	0.274	0.615	0.000	0.721

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	169	241	218	0	0	0	234
N.S.	1	1.00	0.84	1.20	1.09	0.00	0.00	0.00	1.17
time (sec)	N/A	0.396	0.176	0.530	0.461	0.000	0.000	0.000	0.834

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	171	184	208	208	214	289	0	194
N.S.	1	0.97	1.04	1.18	1.18	1.21	1.63	0.00	1.10
time (sec)	N/A	0.557	0.118	0.269	0.224	0.272	0.589	0.000	0.733

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	175	258	0	0	0	0	261
N.S.	1	1.00	0.77	1.13	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.428	0.107	0.517	0.000	0.000	0.000	0.000	1.005

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	216	230	247	247	243	362	0	236
N.S.	1	0.96	1.03	1.10	1.10	1.08	1.62	0.00	1.05
time (sec)	N/A	0.602	0.122	0.272	0.193	0.281	0.751	0.000	0.835

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	150	154	249	218	228	309	0	301
N.S.	1	0.99	1.01	1.64	1.43	1.50	2.03	0.00	1.98
time (sec)	N/A	0.376	0.125	0.349	0.288	0.263	0.564	0.000	0.733

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	212	245	226	237	314	0	233
N.S.	1	1.00	0.87	1.00	0.93	0.97	1.29	0.00	0.95
time (sec)	N/A	0.606	0.071	0.352	0.199	0.257	0.658	0.000	0.227

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	359	475	503	0	0	0	0	0
N.S.	1	0.99	1.32	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.764	0.275	0.411	0.000	0.000	0.000	0.000	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	441	394	0	0	0	0	0
N.S.	1	1.00	1.42	1.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	0.098	0.269	0.000	0.000	0.000	0.000	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	484	435	0	0	0	0	0
N.S.	1	1.00	1.37	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	0.149	0.298	0.000	0.000	0.000	0.000	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	399	514	571	0	0	0	0	0
N.S.	1	0.98	1.26	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.836	0.228	0.408	0.000	0.000	0.000	0.000	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	550	766	526	0	0	0	0	0
N.S.	1	0.99	1.38	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.978	2.549	0.944	0.000	0.000	0.000	0.000	0.000

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	510	461	400	0	0	0	0	0
N.S.	1	0.99	0.89	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	0.171	0.516	0.000	0.000	0.000	0.000	0.000

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	561	558	467	537	0	0	0	0	0
N.S.	1	0.99	0.83	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.927	0.543	0.891	0.000	0.000	0.000	0.000	0.000

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	403	403	522	716	0	0	0	0	0
N.S.	1	1.00	1.30	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.707	7.195	0.441	0.000	0.000	0.000	0.000	0.000

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	88	98	99	0	234	0	0	696
N.S.	1	0.97	1.08	1.09	0.00	2.57	0.00	0.00	7.65
time (sec)	N/A	0.246	0.126	0.507	0.000	0.277	0.000	0.000	0.957

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	443	443	590	808	0	0	0	0	0
N.S.	1	1.00	1.33	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	5.088	0.484	0.000	0.000	0.000	0.000	0.000

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	489	489	643	851	0	0	0	0	0
N.S.	1	1.00	1.31	1.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.795	9.889	0.611	0.000	0.000	0.000	0.000	0.000

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1335	1335	877	2305	0	0	0	0	0
N.S.	1	1.00	0.66	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.063	9.209	1.858	0.000	0.000	0.000	0.000	0.000

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	819	806	861	2173	0	0	0	0	0
N.S.	1	0.98	1.05	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.170	9.118	1.164	0.000	0.000	0.000	0.000	0.000

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1382	1382	992	2568	0	0	0	0	0
N.S.	1	1.00	0.72	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.886	12.748	1.484	0.000	0.000	0.000	0.000	0.000

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	532	532	589	817	0	0	0	0	0
N.S.	1	1.00	1.11	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.884	9.840	0.657	0.000	0.000	0.000	0.000	0.000

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	151	158	211	0	697	0	0	273
N.S.	1	1.16	1.22	1.62	0.00	5.36	0.00	0.00	2.10
time (sec)	N/A	0.329	2.462	1.122	0.000	0.374	0.000	0.000	3.713

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	150	131	157	0	637	0	0	201
N.S.	1	1.15	1.00	1.20	0.00	4.86	0.00	0.00	1.53
time (sec)	N/A	0.306	0.799	0.704	0.000	0.351	0.000	0.000	2.881

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	574	574	645	903	0	0	0	0	0
N.S.	1	1.00	1.12	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.869	10.059	0.596	0.000	0.000	0.000	0.000	0.000

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	629	629	723	951	0	0	0	0	0
N.S.	1	1.00	1.15	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.958	12.586	1.003	0.000	0.000	0.000	0.000	0.000

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	966	966	1744	3774	0	0	0	0	0
N.S.	1	1.00	1.81	3.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.119	12.537	2.095	0.000	0.000	0.000	0.000	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	893	875	1745	4007	0	0	0	0	0
N.S.	1	0.98	1.95	4.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.324	10.773	1.992	0.000	0.000	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1518	1518	2005	5730	0	0	0	0	0
N.S.	1	1.00	1.32	3.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.712	13.082	2.391	0.000	0.000	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	233	391	0	0	1200	0	0	0
N.S.	1	1.04	1.75	0.00	0.00	5.38	0.00	0.00	0.00
time (sec)	N/A	0.503	0.410	0.000	0.000	2.310	0.000	0.000	0.000

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	0.13	1.00
time (sec)	N/A	0.403	14.025	0.192	0.000	0.265	23.805	95.511	0.698

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	148	279	0	0	879	0	0	0
N.S.	1	1.06	1.99	0.00	0.00	6.28	0.00	0.00	0.00
time (sec)	N/A	0.343	0.424	0.000	0.000	0.630	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	3	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	0.15	1.00
time (sec)	N/A	0.186	5.945	0.186	0.000	0.263	7.153	91.783	0.646

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	0.13	1.00
time (sec)	N/A	0.379	10.410	0.189	0.000	0.247	5.549	283.976	0.852

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	0.13	1.00
time (sec)	N/A	0.364	10.446	0.182	0.000	0.278	2.818	281.837	1.088

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	0.13	1.00
time (sec)	N/A	0.363	11.962	0.212	0.000	0.251	3.777	288.606	1.002

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	142	288	0	0	858	0	0	0
N.S.	1	1.04	2.10	0.00	0.00	6.26	0.00	0.00	0.00
time (sec)	N/A	0.377	0.486	0.000	0.000	0.390	0.000	0.000	0.000

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	0.00	1.00
time (sec)	N/A	0.395	14.134	0.243	0.000	0.267	8.932	0.000	1.096

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	226	413	0	0	1156	0	0	0
N.S.	1	1.01	1.84	0.00	0.00	5.16	0.00	0.00	0.00
time (sec)	N/A	0.494	0.465	0.000	0.000	0.632	0.000	0.000	0.000

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	279	294	418	0	0	1566	0	0	0
N.S.	1	1.05	1.50	0.00	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.605	0.535	0.000	0.000	8.758	0.000	0.000	0.000

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.96	0.13	1.00
time (sec)	N/A	0.442	13.877	0.338	0.000	0.275	63.745	95.980	0.810

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	197	313	0	0	1192	0	0	0
N.S.	1	1.09	1.73	0.00	0.00	6.59	0.00	0.00	0.00
time (sec)	N/A	0.424	0.358	0.000	0.000	2.135	0.000	0.000	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	19	3	20
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.95	0.15	1.00
time (sec)	N/A	0.187	6.383	0.506	0.000	0.254	30.621	93.244	0.676

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	0	23
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	0.00	1.00
time (sec)	N/A	0.405	10.596	0.520	0.000	0.264	22.240	0.000	0.955

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	0.00	1.00
time (sec)	N/A	0.403	11.621	0.320	0.000	0.256	13.825	0.000	1.366

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	0.00	1.00
time (sec)	N/A	0.406	12.436	0.421	0.000	0.264	10.203	0.000	1.143

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	0.00	1.00
time (sec)	N/A	0.405	35.170	0.395	0.000	0.268	9.477	0.000	1.092

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	0.00	1.00
time (sec)	N/A	0.400	13.742	0.398	0.000	0.273	15.590	0.000	1.255

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	186	334	0	0	1145	0	0	0
N.S.	1	1.04	1.88	0.00	0.00	6.43	0.00	0.00	0.00
time (sec)	N/A	0.432	0.443	0.000	0.000	0.602	0.000	0.000	0.000

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	345	365	470	0	0	1978	0	0	0
N.S.	1	1.06	1.36	0.00	0.00	5.73	0.00	0.00	0.00
time (sec)	N/A	0.696	0.683	0.000	0.000	29.096	0.000	0.000	0.000

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	67	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	2.91	0.96	0.13	1.00
time (sec)	N/A	0.481	14.765	0.470	0.000	0.257	176.222	98.120	0.786

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	257	353	0	0	1562	0	0	0
N.S.	1	1.10	1.52	0.00	0.00	6.70	0.00	0.00	0.00
time (sec)	N/A	0.517	0.431	0.000	0.000	8.400	0.000	0.000	0.000

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	61	19	3	20
N.S.	1	1.00	1.10	0.90	0.00	3.05	0.95	0.15	1.00
time (sec)	N/A	0.193	6.895	0.617	0.000	0.251	77.099	94.142	0.711

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	64	20	0	23
N.S.	1	1.00	1.09	0.91	0.00	2.78	0.87	0.00	1.00
time (sec)	N/A	0.424	10.803	0.621	0.000	0.283	40.393	0.000	0.990

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	64	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	2.78	0.96	0.00	1.00
time (sec)	N/A	0.432	12.129	0.589	0.000	0.294	59.851	0.000	1.589

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	64	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	2.78	0.96	0.00	1.00
time (sec)	N/A	0.428	12.449	0.504	0.000	0.262	40.467	0.000	1.266

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	64	22	0	23
N.S.	1	1.00	1.09	0.91	0.00	2.78	0.96	0.00	1.00
time (sec)	N/A	0.434	11.685	0.524	0.000	0.280	42.003	0.000	1.349

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	181	377	0	0	882	0	0	0
N.S.	1	1.03	2.14	0.00	0.00	5.01	0.00	0.00	0.00
time (sec)	N/A	0.430	0.432	0.000	0.000	0.635	0.000	0.000	0.000

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	0.13	1.00
time (sec)	N/A	0.372	12.186	0.184	0.000	0.266	8.833	86.050	0.962

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	104	251	0	0	647	0	0	0
N.S.	1	1.01	2.44	0.00	0.00	6.28	0.00	0.00	0.00
time (sec)	N/A	0.288	0.354	0.000	0.000	0.329	0.000	0.000	0.000

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	3	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	0.15	1.00
time (sec)	N/A	0.181	2.882	0.188	0.000	0.256	2.396	52.414	0.704

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	0.13	1.00
time (sec)	N/A	0.361	4.670	0.198	0.000	0.252	2.920	53.421	1.079

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	108	247	0	0	660	0	0	0
N.S.	1	1.08	2.47	0.00	0.00	6.60	0.00	0.00	0.00
time (sec)	N/A	0.333	0.335	0.000	0.000	0.342	0.000	0.000	0.000

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	0.13	1.00
time (sec)	N/A	0.374	12.264	0.153	0.000	0.253	4.550	57.043	1.087

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	181	372	0	0	868	0	0	0
N.S.	1	1.01	2.08	0.00	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.421	0.395	0.000	0.000	0.389	0.000	0.000	0.000

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	135	321	0	0	1291	0	0	0
N.S.	1	0.99	2.34	0.00	0.00	9.42	0.00	0.00	0.00
time (sec)	N/A	0.359	0.528	0.000	0.000	0.552	0.000	0.000	0.000

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	0.13	1.00
time (sec)	N/A	0.388	23.461	0.508	0.000	0.268	35.224	136.836	0.795

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	210	0	0	379	0	0	0
N.S.	1	1.00	2.96	0.00	0.00	5.34	0.00	0.00	0.00
time (sec)	N/A	0.245	0.332	0.000	0.000	0.316	0.000	0.000	0.000

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	202	0	0	388	0	0	0
N.S.	1	1.00	2.89	0.00	0.00	5.54	0.00	0.00	0.00
time (sec)	N/A	0.246	0.211	0.000	0.000	0.320	0.000	0.000	0.000

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	0.13	1.00
time (sec)	N/A	0.385	10.372	0.411	0.000	0.256	32.726	42.176	1.152

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	143	306	0	0	1317	0	0	0
N.S.	1	1.06	2.27	0.00	0.00	9.76	0.00	0.00	0.00
time (sec)	N/A	0.373	0.482	0.000	0.000	0.425	0.000	0.000	0.000

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	22	3	23
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.96	0.13	1.00
time (sec)	N/A	0.415	14.324	0.379	0.000	0.259	53.824	43.670	1.276

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	245	405	0	0	1920	0	0	0
N.S.	1	0.98	1.63	0.00	0.00	7.71	0.00	0.00	0.00
time (sec)	N/A	1.054	0.564	0.000	0.000	0.554	0.000	0.000	0.000

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	3	23
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	0.13	1.00
time (sec)	N/A	0.432	17.784	0.587	0.000	0.264	0.000	168.206	0.985

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	138	326	0	0	863	0	0	0
N.S.	1	0.97	2.28	0.00	0.00	6.03	0.00	0.00	0.00
time (sec)	N/A	0.366	0.488	0.000	0.000	0.600	0.000	0.000	0.000

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	113	252	0	0	676	0	0	0
N.S.	1	1.04	2.31	0.00	0.00	6.20	0.00	0.00	0.00
time (sec)	N/A	0.339	0.771	0.000	0.000	0.417	0.000	0.000	0.000

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	259	0	0	679	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	6.17	0.00	0.00	0.00
time (sec)	N/A	0.280	0.566	0.000	0.000	0.572	0.000	0.000	0.000

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	145	317	0	0	864	0	0	0
N.S.	1	1.01	2.20	0.00	0.00	6.00	0.00	0.00	0.00
time (sec)	N/A	0.327	0.424	0.000	0.000	0.630	0.000	0.000	0.000

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	20	3	23
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.87	0.13	1.00
time (sec)	N/A	0.411	14.521	0.500	0.000	0.268	65.716	54.183	1.176

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	263	418	0	0	2714	0	0	0
N.S.	1	0.96	1.53	0.00	0.00	9.91	0.00	0.00	0.00
time (sec)	N/A	1.101	0.810	0.000	0.000	1.096	0.000	0.000	0.000

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	3	23
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	0.13	1.00
time (sec)	N/A	0.437	18.037	0.440	0.000	0.260	0.000	55.519	1.260

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	423	398	510	0	0	3460	0	0	0
N.S.	1	0.94	1.21	0.00	0.00	8.18	0.00	0.00	0.00
time (sec)	N/A	1.289	1.307	0.000	0.000	1.772	0.000	0.000	0.000

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	201	345	0	0	1280	0	0	0
N.S.	1	0.97	1.66	0.00	0.00	6.15	0.00	0.00	0.00
time (sec)	N/A	0.962	0.566	0.000	0.000	0.353	0.000	0.000	0.000

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	291	450	0	0	1986	0	0	0
N.S.	1	0.99	1.54	0.00	0.00	6.78	0.00	0.00	0.00
time (sec)	N/A	1.163	0.915	0.000	0.000	0.634	0.000	0.000	0.000

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	378	373	264	0	0	0	0	0	0
N.S.	1	0.99	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.817	0.400	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	230	225	193	0	0	0	0	0	0
N.S.	1	0.98	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.519	0.148	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	120	119	0	0	0	0	0	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	19	3	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.90	0.14	1.10
time (sec)	N/A	0.335	2.280	0.573	0.519	0.262	160.774	154.143	0.700

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	34	0	3	23
N.S.	1	1.00	1.10	1.00	1.10	1.62	0.00	0.14	1.10
time (sec)	N/A	0.330	3.992	1.636	0.507	0.267	0.000	175.282	0.703

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	64	0	23	23
N.S.	1	1.00	1.09	0.91	1.00	2.78	0.00	1.00	1.00
time (sec)	N/A	0.408	3.168	0.692	0.670	0.274	0.000	4.328	0.814

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	40	0	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.74	0.00	1.00	1.00
time (sec)	N/A	0.398	0.096	0.562	0.516	0.268	0.000	2.874	0.776

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.00
time (sec)	N/A	0.376	0.086	0.282	0.347	0.260	42.091	1.447	0.771

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	23	22	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.96	1.00	1.00
time (sec)	N/A	0.382	3.065	0.276	0.384	0.272	22.593	0.470	0.867

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	43	0	23	23
N.S.	1	1.00	1.09	0.91	1.00	1.87	0.00	1.00	1.00
time (sec)	N/A	0.392	4.119	0.568	0.386	0.294	0.000	0.474	0.965

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	23	54	0	23	23
N.S.	1	1.00	1.09	0.91	1.00	2.35	0.00	1.00	1.00
time (sec)	N/A	0.393	5.757	0.583	0.398	0.303	0.000	0.491	0.938

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	0	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10
time (sec)	N/A	0.342	2.525	0.740	0.725	0.284	0.000	2.946	1.204

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.16
time (sec)	N/A	0.385	2.520	0.566	0.590	0.263	0.000	3.312	0.891

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	125	166	0	0	0	0	0	0
N.S.	1	0.97	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.475	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.16
time (sec)	N/A	0.400	2.869	0.632	0.606	0.278	0.000	3.395	0.865

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	285	270	0	0	0	0	0	0	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.16
time (sec)	N/A	0.390	2.968	0.607	0.605	0.300	0.000	3.320	0.818

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	466	432	0	0	0	0	0	0	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.527	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	27	27	0	27	29
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.16
time (sec)	N/A	0.392	2.499	0.862	0.607	0.285	0.000	3.329	0.866

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	240	302	306	289	398	0	338
N.S.	1	1.00	0.89	1.11	1.13	1.07	1.47	0.00	1.25
time (sec)	N/A	0.796	0.133	0.339	0.313	0.296	0.518	0.000	1.725

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	287	406	0	0	0	0	0
N.S.	1	1.00	0.89	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.796	0.686	0.696	0.000	0.000	0.000	0.000	0.000

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	179	248	247	218	296	0	248
N.S.	1	1.00	0.90	1.25	1.24	1.10	1.49	0.00	1.25
time (sec)	N/A	0.578	0.103	0.335	0.287	0.276	0.400	0.000	1.088

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	208	321	0	0	0	0	0
N.S.	1	1.00	0.90	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	0.306	0.539	0.000	0.000	0.000	0.000	0.000

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	217	263	1262	0	0	0	0	0
N.S.	1	1.00	1.21	5.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.637	0.249	6.619	0.000	0.000	0.000	0.000	0.000

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	204	356	0	0	0	0	0
N.S.	1	1.00	1.19	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.513	0.221	0.677	0.000	0.000	0.000	0.000	0.000

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	220	220	273	1289	0	0	0	0	0
N.S.	1	1.00	1.24	5.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.634	0.253	10.206	0.000	0.000	0.000	0.000	0.000

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	502	502	414	552	516	530	758	0	929
N.S.	1	1.00	0.82	1.10	1.03	1.06	1.51	0.00	1.85
time (sec)	N/A	1.302	0.229	0.471	0.303	0.301	0.726	0.000	7.258

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	580	513	634	0	0	0	0	0
N.S.	1	1.00	0.88	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.269	1.181	0.872	0.000	0.000	0.000	0.000	0.000

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	317	492	433	416	575	0	780
N.S.	1	1.00	0.83	1.29	1.14	1.09	1.51	0.00	2.05
time (sec)	N/A	0.945	0.173	0.434	0.309	0.280	0.544	0.000	5.643

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	391	493	0	0	0	0	0
N.S.	1	1.00	0.88	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.924	0.693	0.615	0.000	0.000	0.000	0.000	0.000

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	355	355	389	1524	0	0	0	0	0
N.S.	1	1.00	1.10	4.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.915	0.470	13.602	0.000	0.000	0.000	0.000	0.000

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	349	524	0	0	0	0	0
N.S.	1	1.00	1.02	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.816	0.545	0.717	0.000	0.000	0.000	0.000	0.000

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	320	320	367	1500	0	0	0	0	0
N.S.	1	1.00	1.15	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.806	0.406	15.753	0.000	0.000	0.000	0.000	0.000

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	590	579	1520	0	0	0	0	0	0
N.S.	1	0.98	2.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.227	13.435	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	554	568	0	0	0	0	0	0	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.542	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	492	492	1322	0	0	0	0	0	0
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.541	13.376	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	0	2344	0	0	0	0	0
N.S.	1	1.00	0.00	5.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.512	0.000	225.409	0.000	0.000	0.000	0.000	0.000

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	637	637	1264	0	0	0	0	0	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.921	10.890	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	553	564	0	0	0	0	0	0	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.421	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	745	724	1412	0	0	0	0	0	0
N.S.	1	0.97	1.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.730	14.315	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	943	943	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.862	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	1033	1033	0	6565	0	0	0	0	0
N.S.	1	1.00	0.00	6.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.901	0.000	157.166	0.000	0.000	0.000	0.000	0.000

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	996	836	1175	0	0	0	0	0
N.S.	1	2.18	1.83	2.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.457	6.928	2.576	0.000	0.000	0.000	0.000	0.000

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	1039	1039	0	6565	0	0	0	0	0
N.S.	1	1.00	0.00	6.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.534	0.000	44.793	0.000	0.000	0.000	0.000	0.000

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1087	1087	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1141	1141	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1181	1181	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.256	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	126	79	91	80	72	107	168	82
N.S.	1	1.14	0.71	0.82	0.72	0.65	0.96	1.51	0.74
time (sec)	N/A	0.571	0.020	2.105	0.265	0.256	0.873	0.274	0.525

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	71	62	49	83	124	69
N.S.	1	1.00	0.64	0.81	0.70	0.56	0.94	1.41	0.78
time (sec)	N/A	0.293	0.019	1.629	0.264	0.269	0.566	0.279	0.577

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	97	64	68	65	55	78	135	65
N.S.	1	1.18	0.78	0.83	0.79	0.67	0.95	1.65	0.79
time (sec)	N/A	0.475	0.018	1.659	0.268	0.267	0.360	0.273	0.532

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	38	48	39	33	56	86	48
N.S.	1	1.00	0.78	0.98	0.80	0.67	1.14	1.76	0.98
time (sec)	N/A	0.227	0.014	1.247	0.346	0.261	0.237	0.277	0.495

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	48	38	37	42	33	39	92	39
N.S.	1	1.26	1.00	0.97	1.11	0.87	1.03	2.42	1.03
time (sec)	N/A	0.533	0.008	1.194	0.269	0.253	0.165	0.291	0.454

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	189	165	305	2965	0	0	0	0	0
N.S.	1	0.87	1.61	15.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.660	0.235	2.671	0.000	0.000	0.000	0.000	0.000

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	44	41	0	58	0	37	0	36
N.S.	1	1.07	1.00	0.00	1.41	0.00	0.90	0.00	0.88
time (sec)	N/A	0.404	0.009	0.000	0.292	0.000	37.835	0.000	0.111

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	76	49	0	70	0	0	0	0
N.S.	1	1.10	0.71	0.00	1.01	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.023	0.000	0.298	0.000	0.000	0.000	0.000

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	100	81	0	95	0	97	0	0
N.S.	1	1.23	1.00	0.00	1.17	0.00	1.20	0.00	0.00
time (sec)	N/A	0.650	0.012	0.000	0.268	0.000	12.986	0.000	0.000

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	105	98	0	89	0	0	0	0
N.S.	1	1.03	0.96	0.00	0.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	0.029	0.000	0.298	0.000	0.000	0.000	0.000

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	157	114	0	115	0	134	0	0
N.S.	1	1.38	1.00	0.00	1.01	0.00	1.18	0.00	0.00
time (sec)	N/A	0.969	0.018	0.000	0.271	0.000	17.974	0.000	0.000

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	279	214	289	256	220	338	0	276
N.S.	1	1.00	0.77	1.04	0.92	0.79	1.22	0.00	0.99
time (sec)	N/A	0.859	0.127	2.691	0.269	0.257	1.628	0.000	3.721

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	218	164	243	224	178	279	0	297
N.S.	1	0.99	0.74	1.10	1.01	0.81	1.26	0.00	1.34
time (sec)	N/A	0.456	0.103	1.670	0.286	0.270	1.078	0.000	1.944

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	217	171	217	212	169	258	0	212
N.S.	1	1.02	0.80	1.02	1.00	0.79	1.21	0.00	1.00
time (sec)	N/A	0.725	0.090	1.816	0.275	0.271	0.759	0.000	2.793

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	105	168	149	116	202	0	227
N.S.	1	1.00	0.77	1.23	1.09	0.85	1.47	0.00	1.66
time (sec)	N/A	0.327	0.067	1.244	0.267	0.266	0.528	0.000	1.424

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	112	138	137	153	105	148	0	134
N.S.	1	1.12	1.38	1.37	1.53	1.05	1.48	0.00	1.34
time (sec)	N/A	0.732	0.023	1.145	0.269	0.261	0.318	0.000	1.093

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	282	241	0	5420	0	0	0	0	0
N.S.	1	0.85	0.00	19.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.172	0.000	10.835	0.000	0.000	0.000	0.000	0.000

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	82	111	0	0	0	0	0	0
N.S.	1	0.82	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	147	189	0	0	0	0	0	0
N.S.	1	0.95	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	189	175	181	0	0	0	0	0	0
N.S.	1	0.93	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.329	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	216	260	0	0	0	0	0	0
N.S.	1	0.96	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	248	290	278	0	0	0	0	0	0
N.S.	1	1.17	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.239	0.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	562	554	1140	10102	0	0	0	0	0
N.S.	1	0.99	2.03	17.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.894	6.752	13.535	0.000	0.000	0.000	0.000	0.000

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	656	653	1352	0	0	0	0	0	0
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.682	3.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	43	37	22	0	26
N.S.	1	1.00	1.08	1.00	1.79	1.54	0.92	0.00	1.08
time (sec)	N/A	0.780	0.169	0.712	0.574	0.267	123.045	0.000	1.034

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	672	649	552	0	0	0	0	0	0
N.S.	1	0.97	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.292	0.806	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	528	544	1217	0	0	0	0	0	0
N.S.	1	1.03	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.936	5.202	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [199] had the largest ratio of [1.39999999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	0.91	21	0.190
2	A	4	4	0.91	21	0.190
3	A	4	4	0.95	19	0.211
4	A	5	5	1.06	18	0.278
5	A	2	2	1.00	21	0.095
6	A	2	2	1.00	21	0.095
7	A	4	4	0.92	21	0.190
8	A	4	4	0.90	21	0.190
9	A	4	4	0.88	21	0.190
10	A	4	4	0.83	23	0.174
11	A	4	4	0.86	23	0.174
12	A	4	4	0.87	21	0.190
13	A	5	5	0.84	20	0.250
14	A	2	2	1.00	23	0.087
15	A	2	2	1.00	23	0.087
16	A	2	2	1.00	23	0.087
17	A	4	4	0.92	23	0.174
18	A	4	4	0.84	23	0.174
19	A	4	4	0.83	23	0.174
20	A	4	4	0.82	23	0.174
21	A	4	4	0.84	23	0.174
22	A	4	4	0.87	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
23	A	5	5	0.96	20	0.250
24	A	2	2	1.00	23	0.087
25	A	2	2	1.00	23	0.087
26	A	2	2	1.00	23	0.087
27	A	2	2	1.00	23	0.087
28	A	4	4	0.83	23	0.174
29	A	4	4	0.81	23	0.174
30	A	4	4	0.84	23	0.174
31	A	4	4	0.83	23	0.174
32	A	4	4	0.88	23	0.174
33	A	4	4	0.86	21	0.190
34	A	5	5	0.92	20	0.250
35	A	2	2	1.00	23	0.087
36	A	2	2	1.00	23	0.087
37	A	2	2	1.00	23	0.087
38	A	2	2	1.00	23	0.087
39	A	2	2	1.00	23	0.087
40	A	4	4	0.87	23	0.174
41	A	4	4	0.80	23	0.174
42	A	4	4	0.85	23	0.174
43	A	16	15	1.10	23	0.652
44	A	11	10	0.99	23	0.435
45	A	7	6	0.94	21	0.286
46	A	4	3	1.00	20	0.150
47	A	2	2	1.00	23	0.087
48	A	10	9	0.98	23	0.391
49	A	14	13	0.86	23	0.565
50	A	19	18	0.98	23	0.783
51	A	2	2	1.00	23	0.087
52	A	2	2	1.00	23	0.087
53	A	2	2	1.00	21	0.095
54	A	5	5	0.96	20	0.250
55	A	2	2	1.00	23	0.087
56	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	2	2	1.00	23	0.087
58	A	2	2	1.00	23	0.087
59	A	2	2	1.00	23	0.087
60	A	2	2	1.00	23	0.087
61	A	4	4	0.98	21	0.190
62	A	5	5	0.97	20	0.250
63	A	2	2	1.00	23	0.087
64	A	2	2	1.00	23	0.087
65	A	2	2	1.00	23	0.087
66	A	4	4	1.00	19	0.211
67	A	3	3	1.00	20	0.150
68	A	2	2	1.00	23	0.087
69	A	2	2	1.00	23	0.087
70	A	2	2	1.00	21	0.095
71	A	2	2	1.04	20	0.100
72	A	2	2	1.00	23	0.087
73	A	2	2	1.00	23	0.087
74	A	2	2	1.00	23	0.087
75	A	2	2	1.00	23	0.087
76	A	2	2	1.00	25	0.080
77	A	2	2	1.00	25	0.080
78	A	2	2	1.00	23	0.087
79	A	2	2	0.98	22	0.091
80	A	2	2	1.00	25	0.080
81	A	2	2	1.00	25	0.080
82	A	2	2	1.00	25	0.080
83	A	2	2	0.94	25	0.080
84	A	2	2	1.00	25	0.080
85	A	2	2	1.00	25	0.080
86	A	2	2	1.00	23	0.087
87	A	2	2	0.95	22	0.091
88	A	2	2	1.00	25	0.080
89	A	2	2	1.00	25	0.080
90	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	2	2	1.00	25	0.080
92	A	2	2	0.94	25	0.080
93	A	2	2	0.94	25	0.080
94	A	2	2	0.94	25	0.080
95	A	21	20	1.32	25	0.800
96	A	16	15	1.09	25	0.600
97	A	11	10	1.10	23	0.435
98	A	3	3	1.02	22	0.136
99	A	3	3	1.08	25	0.120
100	A	9	9	1.05	25	0.360
101	A	19	18	1.01	25	0.720
102	A	23	22	1.15	25	0.880
103	A	2	2	1.00	25	0.080
104	A	2	2	1.00	25	0.080
105	A	2	2	1.00	25	0.080
106	A	2	2	1.00	23	0.087
107	A	2	2	1.07	22	0.091
108	A	2	2	1.00	25	0.080
109	A	2	2	1.00	25	0.080
110	A	2	2	1.00	25	0.080
111	A	2	2	1.00	25	0.080
112	A	2	2	1.00	25	0.080
113	A	2	2	1.00	25	0.080
114	A	2	2	1.00	23	0.087
115	A	2	2	1.02	22	0.091
116	A	2	2	1.00	25	0.080
117	A	2	2	1.00	25	0.080
118	A	2	2	1.00	21	0.095
119	A	4	4	1.12	22	0.182
120	A	2	2	0.99	22	0.091
121	A	2	2	1.03	22	0.091
122	A	2	2	1.05	20	0.100
123	A	4	4	1.01	22	0.182
124	A	2	2	1.02	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	2	2	1.02	22	0.091
126	A	2	2	0.99	22	0.091
127	A	17	16	1.10	25	0.640
128	A	9	9	1.05	23	0.391
129	A	4	4	1.01	22	0.182
130	A	4	4	1.06	25	0.160
131	A	10	10	1.07	25	0.400
132	A	16	16	1.02	25	0.640
133	N/A	1	0	1.00	22	0.000
134	A	2	2	1.00	19	0.105
135	A	2	2	1.00	19	0.105
136	A	2	2	1.00	17	0.118
137	A	5	4	1.00	16	0.250
138	A	2	2	1.00	19	0.105
139	A	2	2	1.00	19	0.105
140	A	2	2	1.00	19	0.105
141	A	2	2	1.00	21	0.095
142	A	2	2	1.00	21	0.095
143	A	2	2	1.00	19	0.105
144	A	1	1	1.00	18	0.056
145	A	2	2	1.00	21	0.095
146	A	2	2	1.00	21	0.095
147	A	2	2	1.00	21	0.095
148	N/A	1	0	1.00	18	0.000
149	A	4	4	1.48	18	0.222
150	A	6	5	1.48	18	0.278
151	A	2	2	1.10	16	0.125
152	A	3	3	1.32	15	0.200
153	A	6	6	1.11	18	0.333
154	A	9	8	1.60	18	0.444
155	A	6	6	0.96	18	0.333
156	A	9	8	1.37	18	0.444
157	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	2	2	1.00	20	0.100
159	A	3	3	1.07	18	0.167
160	A	5	5	0.98	17	0.294
161	A	2	2	1.00	20	0.100
162	A	2	2	1.00	20	0.100
163	A	2	2	1.00	20	0.100
164	A	2	2	1.00	20	0.100
165	A	2	2	1.00	20	0.100
166	A	2	2	1.00	20	0.100
167	A	3	3	1.01	18	0.167
168	A	6	6	0.98	17	0.353
169	A	2	2	1.00	20	0.100
170	A	2	2	1.00	20	0.100
171	A	2	2	1.00	20	0.100
172	A	2	2	1.00	20	0.100
173	A	11	10	1.22	20	0.500
174	A	10	9	1.03	20	0.450
175	A	5	5	0.98	20	0.250
176	A	5	4	1.03	18	0.222
177	A	1	1	1.00	17	0.059
178	A	3	3	1.05	20	0.150
179	A	9	8	0.98	20	0.400
180	A	8	8	0.92	20	0.400
181	A	14	13	1.16	20	0.650
182	A	19	18	1.67	20	0.900
183	A	8	8	1.18	20	0.400
184	A	10	9	1.07	20	0.450
185	A	3	3	1.00	20	0.150
186	A	3	3	1.08	18	0.167
187	A	3	3	1.00	17	0.176
188	A	8	8	1.13	20	0.400
189	A	12	11	1.11	20	0.550
190	A	16	16	1.49	20	0.800
191	A	25	24	1.52	20	1.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	5	5	1.09	20	0.250
193	A	4	4	0.99	20	0.200
194	A	4	4	1.12	18	0.222
195	A	4	4	0.99	17	0.235
196	A	13	13	1.36	20	0.650
197	A	16	15	1.46	20	0.750
198	B	21	21	2.18	20	1.050
199	B	29	28	2.25	20	1.400
200	A	13	12	1.87	22	0.545
201	A	9	8	0.95	22	0.364
202	A	5	4	1.06	20	0.200
203	A	3	3	0.77	19	0.158
204	A	6	5	0.76	22	0.227
205	A	8	7	0.84	22	0.318
206	A	6	6	0.96	22	0.273
207	A	6	5	1.01	22	0.227
208	B	27	26	3.61	22	1.182
209	A	17	16	1.94	22	0.727
210	A	6	5	1.06	20	0.250
211	A	4	4	0.80	19	0.211
212	A	11	10	0.98	22	0.455
213	A	12	11	1.33	22	0.500
214	A	12	11	1.36	22	0.500
215	A	14	13	0.95	22	0.591
216	F	0	0	N/A	0.000	N/A
217	B	25	24	3.95	22	1.091
218	A	7	6	1.05	20	0.300
219	A	5	5	0.83	19	0.263
220	A	17	16	1.21	22	0.727
221	A	17	16	1.82	22	0.727
222	A	17	16	1.91	22	0.727
223	A	18	17	1.89	22	0.773
224	A	8	7	1.38	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
225	A	4	4	0.80	22	0.182
226	A	4	3	1.00	20	0.150
227	A	2	2	0.73	19	0.105
228	A	2	2	0.70	22	0.091
229	A	5	4	1.00	22	0.182
230	A	4	4	0.76	22	0.182
231	A	10	9	1.27	22	0.409
232	A	6	5	1.13	22	0.227
233	A	3	3	0.78	22	0.136
234	A	2	2	1.00	20	0.100
235	A	1	1	1.00	19	0.053
236	A	5	5	0.79	22	0.227
237	A	7	6	1.08	22	0.273
238	A	10	10	1.25	22	0.455
239	A	17	16	1.64	22	0.727
240	A	10	9	1.48	22	0.409
241	A	9	8	0.94	22	0.364
242	A	3	3	1.05	22	0.136
243	A	5	4	1.05	22	0.182
244	A	3	3	1.03	20	0.150
245	A	2	2	1.02	19	0.105
246	A	9	9	0.98	22	0.409
247	A	10	9	1.42	22	0.409
248	A	2	2	1.00	20	0.100
249	A	2	2	1.00	20	0.100
250	A	3	3	1.00	18	0.167
251	N/A	1	0	1.00	20	0.000
252	N/A	1	0	1.00	20	0.000
253	N/A	1	0	1.00	22	0.000
254	N/A	1	0	1.00	22	0.000
255	N/A	4	0	1.00	22	0.000
256	N/A	1	0	1.00	22	0.000
257	N/A	1	0	1.00	22	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
258	B	20	19	2.34	20	0.950
259	B	17	16	2.04	20	0.800
260	A	5	5	1.01	18	0.278
261	A	8	7	1.05	17	0.412
262	A	10	10	1.24	20	0.500
263	A	11	10	1.45	20	0.500
264	A	14	13	1.07	20	0.650
265	A	9	9	1.47	20	0.450
266	A	2	2	1.00	22	0.091
267	A	2	2	1.00	22	0.091
268	A	6	6	0.94	20	0.300
269	A	11	10	1.01	19	0.526
270	A	2	2	1.00	22	0.091
271	A	2	2	1.00	22	0.091
272	A	2	2	1.00	22	0.091
273	A	2	2	1.00	22	0.091
274	A	2	2	1.00	22	0.091
275	A	2	2	1.00	22	0.091
276	A	7	7	0.94	20	0.350
277	A	14	13	1.08	19	0.684
278	A	2	2	1.00	22	0.091
279	A	2	2	1.00	22	0.091
280	A	2	2	1.00	22	0.091
281	A	2	2	1.00	22	0.091
282	A	14	13	1.46	22	0.591
283	A	11	11	1.04	22	0.500
284	A	9	8	1.06	22	0.364
285	A	4	4	1.04	20	0.200
286	A	1	1	1.00	19	0.053
287	A	4	4	1.18	22	0.182
288	A	7	7	1.03	22	0.318
289	A	15	14	0.97	22	0.636
290	A	11	11	1.30	22	0.500
291	A	9	9	1.03	22	0.409

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
292	A	5	5	1.04	22	0.227
293	A	4	4	0.96	20	0.200
294	A	5	5	1.06	19	0.263
295	A	9	9	1.15	22	0.409
296	A	12	12	1.11	22	0.545
297	A	22	21	1.46	22	0.955
298	A	23	23	1.69	22	1.045
299	A	5	5	1.04	22	0.227
300	A	11	11	1.72	22	0.500
301	A	5	5	0.96	20	0.250
302	A	9	9	1.30	19	0.474
303	A	14	14	1.39	22	0.636
304	A	21	21	1.60	22	0.955
305	B	27	26	2.14	22	1.182
306	F	0	0	N/A	0.000	N/A
307	B	20	19	3.09	24	0.792
308	A	26	25	1.62	24	1.042
309	A	4	4	0.81	22	0.182
310	A	11	10	0.67	21	0.476
311	A	12	11	0.69	24	0.458
312	A	12	11	0.66	24	0.458
313	A	20	19	1.08	24	0.792
314	A	7	7	0.97	24	0.292
315	B	31	30	6.69	24	1.250
316	F	0	0	N/A	0.000	N/A
317	A	5	5	0.83	22	0.227
318	A	15	14	0.79	21	0.667
319	A	17	16	1.02	24	0.667
320	A	22	21	0.96	24	0.875
321	A	24	23	1.18	24	0.958
322	A	20	19	0.99	24	0.792
323	F	0	0	N/A	0.000	N/A
324	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
325	A	6	6	0.84	22	0.273
326	A	20	19	0.94	21	0.905
327	A	23	22	1.36	24	0.917
328	F	0	0	N/A	0.000	N/A
329	A	29	28	1.84	24	1.167
330	A	30	29	1.66	24	1.208
331	A	8	8	1.33	24	0.333
332	A	12	11	0.70	24	0.458
333	A	3	3	0.78	22	0.136
334	A	8	7	0.56	21	0.333
335	A	8	7	0.56	24	0.292
336	A	3	3	0.75	24	0.125
337	A	13	12	0.67	24	0.500
338	A	8	8	1.23	24	0.333
339	A	5	5	0.87	24	0.208
340	A	11	10	0.64	24	0.417
341	A	2	2	1.01	22	0.091
342	A	2	2	1.00	21	0.095
343	A	11	10	0.69	24	0.417
344	A	6	6	0.81	24	0.250
345	A	23	22	1.05	24	0.917
346	A	15	15	1.58	24	0.625
347	A	13	12	1.20	24	0.500
348	A	16	15	0.88	24	0.625
349	A	7	6	1.19	24	0.250
350	A	4	4	1.11	24	0.167
351	A	3	3	1.03	22	0.136
352	A	5	5	1.18	21	0.238
353	A	15	14	0.94	24	0.583
354	A	12	12	1.13	24	0.500
355	N/A	1	0	1.00	22	0.000
356	N/A	1	0	1.00	20	0.000
357	N/A	1	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
358	N/A	1	0	1.00	22	0.000
359	N/A	1	0	1.00	24	0.000
360	N/A	1	0	1.00	24	0.000
361	N/A	1	0	1.00	24	0.000
362	N/A	1	0	1.00	24	0.000
363	B	23	22	3.25	20	1.100
364	B	21	20	2.38	20	1.000
365	A	10	9	1.02	18	0.500
366	A	7	7	1.09	17	0.412
367	A	15	14	1.23	20	0.700
368	A	10	10	1.40	20	0.500
369	A	13	13	1.06	20	0.650
370	A	16	15	1.61	20	0.750
371	A	2	2	1.00	22	0.091
372	A	2	2	1.00	22	0.091
373	A	12	11	0.96	20	0.550
374	A	12	12	1.08	19	0.632
375	A	2	2	1.00	22	0.091
376	A	2	2	1.00	22	0.091
377	A	2	2	1.00	22	0.091
378	A	2	2	1.00	22	0.091
379	A	2	2	1.00	22	0.091
380	A	2	2	1.00	22	0.091
381	A	15	14	1.00	20	0.700
382	A	17	17	1.22	19	0.895
383	A	2	2	1.00	22	0.091
384	A	2	2	1.00	22	0.091
385	A	2	2	1.00	22	0.091
386	A	2	2	1.00	22	0.091
387	A	15	15	1.53	22	0.682
388	A	14	13	1.03	22	0.591
389	A	8	8	1.05	22	0.364
390	A	5	5	1.03	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
391	A	1	1	1.00	19	0.053
392	A	5	5	1.15	22	0.227
393	A	8	8	1.11	22	0.364
394	A	12	12	0.98	22	0.545
395	A	18	17	1.41	22	0.773
396	A	12	12	1.04	22	0.545
397	A	5	5	1.01	22	0.227
398	A	6	6	1.02	20	0.300
399	A	5	5	1.02	19	0.263
400	A	12	12	1.16	22	0.545
401	A	13	13	1.12	22	0.591
402	A	21	21	1.42	22	0.955
403	A	29	28	1.75	22	1.273
404	A	10	10	1.23	22	0.455
405	A	11	11	1.73	22	0.500
406	A	10	10	1.14	20	0.500
407	A	9	9	1.26	19	0.474
408	A	22	22	1.55	22	1.000
409	A	22	22	1.62	22	1.000
410	F	0	0	N/A	0.000	N/A
411	F	0	0	N/A	0.000	N/A
412	F	0	0	N/A	0.000	N/A
413	A	27	26	1.75	24	1.083
414	A	12	11	0.71	22	0.500
415	A	11	10	0.64	21	0.476
416	A	18	17	0.60	24	0.708
417	A	18	17	0.58	24	0.708
418	A	18	17	0.93	24	0.708
419	A	21	20	1.07	24	0.833
420	F	0	0	N/A	0.000	N/A
421	F	0	0	N/A	0.000	N/A
422	A	16	15	0.80	22	0.682
423	A	13	12	0.86	21	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
424	A	29	28	0.87	24	1.167
425	A	26	25	0.85	24	1.042
426	A	27	26	1.01	24	1.083
427	A	31	30	0.96	24	1.250
428	F	0	0	N/A	0.000	N/A
429	F	0	0	N/A	0.000	N/A
430	A	21	20	0.93	22	0.909
431	A	16	15	1.10	21	0.714
432	F	0	0	N/A	0.000	N/A
433	F	0	0	N/A	0.000	N/A
434	F	0	0	N/A	0.000	N/A
435	F	0	0	N/A	0.000	N/A
436	A	20	19	1.13	24	0.792
437	A	13	12	0.66	24	0.500
438	A	9	8	0.61	22	0.364
439	A	9	8	0.55	21	0.381
440	A	9	8	0.54	24	0.333
441	A	9	8	0.60	24	0.333
442	A	12	11	0.63	24	0.458
443	A	21	20	1.06	24	0.833
444	A	12	11	0.72	24	0.458
445	A	12	11	0.63	24	0.458
446	A	3	3	0.99	22	0.136
447	A	2	2	1.01	21	0.095
448	A	13	12	0.66	24	0.500
449	A	12	11	0.71	24	0.458
450	A	20	19	1.14	24	0.792
451	A	20	19	0.91	24	0.792
452	A	7	7	1.29	24	0.292
453	A	8	7	1.20	24	0.292
454	A	6	6	1.11	22	0.273
455	A	5	5	1.27	21	0.238
456	A	20	19	0.95	24	0.792

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
457	A	18	17	1.12	24	0.708
458	N/A	1	0	1.00	22	0.000
459	N/A	1	0	1.00	20	0.000
460	N/A	1	0	1.00	22	0.000
461	N/A	1	0	1.00	22	0.000
462	N/A	1	0	1.00	24	0.000
463	N/A	1	0	1.00	24	0.000
464	N/A	1	0	1.00	24	0.000
465	N/A	1	0	1.00	24	0.000
466	N/A	1	0	1.00	18	0.000
467	N/A	1	0	1.00	17	0.000
468	N/A	1	0	1.00	20	0.000
469	N/A	1	0	1.00	20	0.000
470	N/A	1	0	1.00	19	0.000
471	N/A	1	0	1.00	22	0.000
472	N/A	1	0	1.00	20	0.000
473	N/A	1	0	1.00	19	0.000
474	N/A	1	0	1.00	22	0.000
475	N/A	1	0	1.00	22	0.000
476	N/A	1	0	1.00	20	0.000
477	A	1	1	1.00	19	0.053
478	N/A	1	0	1.00	22	0.000
479	N/A	1	0	1.00	22	0.000
480	N/A	1	0	1.00	22	0.000
481	N/A	1	0	1.00	22	0.000
482	A	5	4	0.85	22	0.182
483	A	6	5	1.00	20	0.250
484	A	5	4	0.85	19	0.211
485	N/A	1	0	1.00	22	0.000
486	N/A	1	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
487	N/A	1	0	1.00	22	0.000
488	N/A	1	0	1.00	22	0.000
489	A	5	4	0.78	22	0.182
490	A	4	3	0.86	22	0.136
491	A	4	3	0.85	22	0.136
492	A	4	3	0.86	20	0.150
493	A	5	4	0.78	19	0.211
494	N/A	1	0	1.00	22	0.000
495	N/A	1	0	1.00	22	0.000
496	N/A	1	0	1.00	22	0.000
497	N/A	1	0	1.00	21	0.000
498	N/A	1	0	1.00	24	0.000
499	N/A	1	0	1.00	22	0.000
500	N/A	1	0	1.00	21	0.000
501	N/A	1	0	1.00	24	0.000
502	N/A	1	0	1.00	22	0.000
503	N/A	1	0	1.00	21	0.000
504	N/A	1	0	1.00	24	0.000
505	N/A	1	0	1.00	22	0.000
506	N/A	1	0	1.00	21	0.000
507	N/A	1	0	1.00	24	0.000
508	N/A	1	0	1.00	24	0.000
509	N/A	1	0	1.00	24	0.000
510	A	5	4	1.00	22	0.182
511	A	5	4	1.00	21	0.190
512	N/A	1	0	1.00	24	0.000
513	N/A	1	0	1.00	24	0.000
514	N/A	1	0	1.00	24	0.000
515	N/A	1	0	1.00	24	0.000
516	A	6	5	0.63	24	0.208
517	A	5	4	0.63	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
518	A	5	4	0.63	22	0.182
519	A	6	5	0.63	21	0.238
520	N/A	1	0	1.00	24	0.000
521	N/A	1	0	1.00	24	0.000
522	N/A	1	0	1.00	22	0.000
523	N/A	1	0	1.00	22	0.000
524	N/A	1	0	1.00	20	0.000
525	N/A	1	0	1.00	22	0.000
526	N/A	1	0	1.00	22	0.000
527	N/A	1	0	1.00	22	0.000
528	N/A	1	0	1.00	24	0.000
529	N/A	1	0	1.00	24	0.000
530	N/A	1	0	1.00	24	0.000
531	N/A	1	0	1.00	24	0.000
532	N/A	1	0	1.00	24	0.000
533	N/A	1	0	1.00	24	0.000
534	N/A	1	0	1.00	18	0.000
535	N/A	1	0	1.00	17	0.000
536	N/A	1	0	1.00	20	0.000
537	N/A	1	0	1.00	20	0.000
538	N/A	1	0	1.00	19	0.000
539	N/A	1	0	1.00	22	0.000
540	N/A	1	0	1.00	20	0.000
541	N/A	1	0	1.00	19	0.000
542	N/A	1	0	1.00	22	0.000
543	N/A	2	0	1.00	22	0.000
544	N/A	2	0	1.00	22	0.000
545	N/A	2	0	1.00	20	0.000
546	A	1	1	1.00	19	0.053
547	N/A	2	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
548	N/A	2	0	1.00	22	0.000
549	N/A	2	0	1.00	22	0.000
550	N/A	2	0	1.00	22	0.000
551	N/A	14	0	1.00	22	0.000
552	A	8	7	1.00	22	0.318
553	B	11	10	2.05	20	0.500
554	A	8	7	1.00	19	0.368
555	N/A	14	0	1.00	22	0.000
556	N/A	11	0	1.00	22	0.000
557	N/A	17	0	1.00	22	0.000
558	N/A	14	0	1.00	22	0.000
559	B	14	13	2.06	22	0.591
560	A	10	9	1.52	22	0.409
561	A	10	9	1.56	20	0.450
562	A	6	5	0.98	19	0.263
563	N/A	17	0	1.00	22	0.000
564	N/A	14	0	1.00	22	0.000
565	N/A	20	0	1.00	22	0.000
566	N/A	17	0	1.00	22	0.000
567	N/A	1	0	1.00	22	0.000
568	N/A	1	0	1.00	21	0.000
569	N/A	1	0	1.00	24	0.000
570	N/A	1	0	1.00	22	0.000
571	N/A	1	0	1.00	21	0.000
572	N/A	1	0	1.00	24	0.000
573	N/A	1	0	1.00	22	0.000
574	N/A	1	0	1.00	21	0.000
575	N/A	1	0	1.00	24	0.000
576	N/A	1	0	1.00	22	0.000
577	N/A	1	0	1.00	21	0.000
578	N/A	2	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
579	N/A	8	0	1.00	24	0.000
580	N/A	8	0	1.00	24	0.000
581	A	6	5	1.00	22	0.227
582	A	6	5	1.00	21	0.238
583	N/A	8	0	1.00	24	0.000
584	N/A	8	0	1.00	24	0.000
585	N/A	9	0	1.00	24	0.000
586	N/A	9	0	1.00	24	0.000
587	N/A	14	0	1.00	24	0.000
588	N/A	11	0	1.00	24	0.000
589	A	6	5	0.75	24	0.208
590	A	9	8	1.18	24	0.333
591	A	11	10	1.22	22	0.455
592	A	6	5	0.74	21	0.238
593	N/A	19	0	1.00	24	0.000
594	N/A	12	0	1.00	24	0.000
595	N/A	20	0	1.00	24	0.000
596	N/A	13	0	1.00	24	0.000
597	N/A	1	0	1.00	30	0.000
598	N/A	1	0	1.00	22	0.000
599	N/A	1	0	1.00	22	0.000
600	N/A	1	0	1.00	20	0.000
601	N/A	2	0	1.00	22	0.000
602	N/A	1	0	1.00	22	0.000
603	N/A	1	0	1.00	22	0.000
604	N/A	1	0	1.00	24	0.000
605	N/A	1	0	1.00	24	0.000
606	N/A	1	0	1.00	24	0.000
607	N/A	1	0	1.00	24	0.000
608	N/A	1	0	1.00	24	0.000
609	N/A	1	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
610	N/A	1	0	1.00	18	0.000
611	N/A	1	0	1.00	17	0.000
612	N/A	1	0	1.00	20	0.000
613	N/A	1	0	1.00	20	0.000
614	N/A	1	0	1.00	19	0.000
615	N/A	1	0	1.00	22	0.000
616	N/A	1	0	1.00	20	0.000
617	N/A	1	0	1.00	19	0.000
618	N/A	1	0	1.00	22	0.000
619	N/A	2	0	1.00	22	0.000
620	N/A	2	0	1.00	22	0.000
621	N/A	2	0	1.00	20	0.000
622	A	1	1	1.00	19	0.053
623	N/A	2	0	1.00	22	0.000
624	N/A	2	0	1.00	22	0.000
625	N/A	2	0	1.00	22	0.000
626	N/A	2	0	1.00	22	0.000
627	N/A	11	0	1.00	22	0.000
628	A	12	11	1.59	22	0.500
629	A	8	7	1.00	20	0.350
630	A	12	11	1.68	19	0.579
631	N/A	11	0	1.00	22	0.000
632	N/A	15	0	1.00	22	0.000
633	N/A	14	0	1.00	22	0.000
634	N/A	18	0	1.00	22	0.000
635	A	14	13	1.54	22	0.591
636	A	15	14	1.94	22	0.636
637	A	12	11	1.72	20	0.550
638	A	11	10	1.48	19	0.526
639	N/A	17	0	1.00	22	0.000
640	N/A	19	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
641	N/A	20	0	1.00	22	0.000
642	N/A	22	0	1.00	22	0.000
643	A	1	1	1.00	38	0.026
644	N/A	1	0	1.00	22	0.000
645	N/A	1	0	1.00	21	0.000
646	N/A	1	0	1.00	24	0.000
647	N/A	1	0	1.00	22	0.000
648	N/A	1	0	1.00	21	0.000
649	N/A	1	0	1.00	24	0.000
650	N/A	1	0	1.00	22	0.000
651	N/A	1	0	1.00	21	0.000
652	N/A	1	0	1.00	24	0.000
653	N/A	1	0	1.00	22	0.000
654	N/A	1	0	1.00	21	0.000
655	N/A	2	0	1.00	24	0.000
656	N/A	1	0	1.00	24	0.000
657	N/A	1	0	1.00	24	0.000
658	N/A	9	0	1.00	24	0.000
659	N/A	9	0	1.00	24	0.000
660	A	7	6	1.04	22	0.273
661	A	7	6	1.04	21	0.286
662	N/A	9	0	1.00	24	0.000
663	N/A	9	0	1.00	24	0.000
664	N/A	10	0	1.00	24	0.000
665	N/A	10	0	1.00	24	0.000
666	N/A	14	0	1.00	24	0.000
667	N/A	20	0	1.00	24	0.000
668	A	10	9	1.16	24	0.375
669	A	18	17	1.41	24	0.708
670	A	11	10	1.68	22	0.455
671	A	12	11	1.22	21	0.524

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
672	N/A	16	0	1.00	24	0.000
673	N/A	21	0	1.00	24	0.000
674	N/A	1	0	1.00	22	0.000
675	N/A	1	0	1.00	22	0.000
676	N/A	1	0	1.00	20	0.000
677	N/A	2	0	1.00	22	0.000
678	N/A	1	0	1.00	22	0.000
679	N/A	1	0	1.00	22	0.000
680	N/A	1	0	1.00	24	0.000
681	N/A	1	0	1.00	24	0.000
682	N/A	1	0	1.00	24	0.000
683	N/A	1	0	1.00	24	0.000
684	N/A	1	0	1.00	24	0.000
685	N/A	1	0	1.00	24	0.000
686	N/A	1	0	1.00	22	0.000
687	N/A	3	0	1.00	20	0.000
688	N/A	1	0	1.00	19	0.000
689	N/A	1	0	1.00	22	0.000
690	N/A	1	0	1.00	24	0.000
691	N/A	3	0	1.00	22	0.000
692	N/A	1	0	1.00	21	0.000
693	N/A	1	0	1.00	24	0.000
694	N/A	1	0	1.00	24	0.000
695	N/A	3	0	1.00	22	0.000
696	N/A	1	0	1.00	21	0.000
697	N/A	1	0	1.00	24	0.000
698	N/A	1	0	1.00	24	0.000
699	N/A	5	0	1.00	24	0.000
700	N/A	4	0	1.00	24	0.000
701	N/A	2	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
702	A	1	1	1.00	21	0.048
703	N/A	2	0	1.00	24	0.000
704	N/A	4	0	1.00	24	0.000
705	N/A	5	0	1.00	24	0.000
706	N/A	6	0	1.00	24	0.000
707	N/A	1	0	1.00	24	0.000
708	N/A	1	0	1.00	24	0.000
709	A	9	8	1.00	24	0.333
710	A	7	6	0.94	22	0.273
711	A	9	8	1.00	21	0.381
712	N/A	1	0	1.00	24	0.000
713	N/A	1	0	1.00	24	0.000
714	N/A	1	0	1.00	24	0.000
715	A	5	4	0.83	24	0.167
716	A	7	6	0.94	24	0.250
717	A	4	3	0.87	24	0.125
718	A	7	6	0.94	22	0.273
719	A	5	4	0.83	21	0.190
720	N/A	1	0	1.00	24	0.000
721	N/A	1	0	1.00	26	0.000
722	N/A	1	0	1.00	26	0.000
723	N/A	2	0	1.00	24	0.000
724	N/A	1	0	1.00	23	0.000
725	N/A	1	0	1.00	26	0.000
726	N/A	1	0	1.00	26	0.000
727	N/A	2	0	1.00	24	0.000
728	N/A	1	0	1.00	23	0.000
729	N/A	1	0	1.00	26	0.000
730	N/A	1	0	1.00	26	0.000
731	N/A	2	0	1.00	24	0.000
732	N/A	1	0	1.00	23	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
733	N/A	1	0	1.00	26	0.000
734	N/A	3	0	1.00	26	0.000
735	N/A	2	0	1.00	26	0.000
736	N/A	2	0	1.00	24	0.000
737	N/A	1	0	1.00	23	0.000
738	N/A	1	0	1.00	26	0.000
739	N/A	2	0	1.00	26	0.000
740	N/A	2	0	1.00	26	0.000
741	N/A	3	0	1.00	26	0.000
742	N/A	1	0	1.00	26	0.000
743	N/A	1	0	1.00	26	0.000
744	N/A	1	0	1.00	26	0.000
745	A	7	6	1.00	24	0.250
746	A	9	8	0.98	23	0.348
747	N/A	1	0	1.00	26	0.000
748	N/A	1	0	1.00	26	0.000
749	N/A	1	0	1.00	26	0.000
750	N/A	1	0	1.00	26	0.000
751	A	6	5	0.67	26	0.192
752	A	7	6	0.83	26	0.231
753	A	7	6	0.83	24	0.250
754	A	6	5	0.68	23	0.217
755	N/A	1	0	1.00	26	0.000
756	N/A	1	0	1.00	22	0.000
757	N/A	1	0	1.00	22	0.000
758	N/A	3	0	1.00	20	0.000
759	N/A	2	0	1.00	19	0.000
760	N/A	1	0	1.00	22	0.000
761	N/A	1	0	1.00	22	0.000
762	N/A	1	0	1.00	24	0.000
763	N/A	1	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
764	N/A	3	0	1.00	22	0.000
765	N/A	5	0	1.00	21	0.000
766	N/A	1	0	1.00	24	0.000
767	N/A	1	0	1.00	24	0.000
768	N/A	1	0	1.00	24	0.000
769	N/A	1	0	1.00	24	0.000
770	N/A	3	0	1.00	22	0.000
771	N/A	6	0	1.00	21	0.000
772	N/A	1	0	1.00	24	0.000
773	N/A	1	0	1.00	24	0.000
774	N/A	1	0	1.00	24	0.000
775	N/A	5	0	1.00	24	0.000
776	N/A	4	0	1.00	24	0.000
777	N/A	2	0	1.00	22	0.000
778	A	1	1	1.00	21	0.048
779	N/A	2	0	1.00	24	0.000
780	N/A	4	0	1.00	24	0.000
781	N/A	5	0	1.00	24	0.000
782	N/A	6	0	1.00	24	0.000
783	N/A	1	0	1.00	24	0.000
784	N/A	1	0	1.00	24	0.000
785	A	8	7	1.00	24	0.292
786	A	10	9	0.99	22	0.409
787	A	8	7	0.98	21	0.333
788	N/A	1	0	1.00	24	0.000
789	N/A	1	0	1.00	24	0.000
790	N/A	1	0	1.00	24	0.000
791	A	13	12	1.19	24	0.500
792	A	7	6	0.89	24	0.250
793	A	4	3	0.84	24	0.125
794	A	7	6	0.89	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
795	A	13	12	1.18	21	0.571
796	N/A	1	0	1.00	24	0.000
797	N/A	1	0	1.00	26	0.000
798	N/A	1	0	1.00	26	0.000
799	N/A	2	0	1.00	24	0.000
800	N/A	2	0	1.00	23	0.000
801	N/A	1	0	1.00	26	0.000
802	N/A	1	0	1.00	26	0.000
803	N/A	1	0	1.00	26	0.000
804	N/A	2	0	1.00	24	0.000
805	N/A	3	0	1.00	23	0.000
806	N/A	1	0	1.00	26	0.000
807	N/A	1	0	1.00	26	0.000
808	N/A	1	0	1.00	26	0.000
809	N/A	2	0	1.00	24	0.000
810	N/A	4	0	1.00	23	0.000
811	N/A	1	0	1.00	26	0.000
812	N/A	1	0	1.00	26	0.000
813	N/A	4	0	1.00	26	0.000
814	N/A	3	0	1.00	26	0.000
815	N/A	2	0	1.00	24	0.000
816	N/A	1	0	1.00	23	0.000
817	N/A	1	0	1.00	26	0.000
818	N/A	2	0	1.00	26	0.000
819	N/A	3	0	1.00	26	0.000
820	N/A	4	0	1.00	26	0.000
821	N/A	1	0	1.00	26	0.000
822	N/A	1	0	1.00	26	0.000
823	N/A	1	0	1.00	26	0.000
824	A	10	9	0.95	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
825	A	7	6	1.00	23	0.261
826	N/A	1	0	1.00	26	0.000
827	N/A	1	0	1.00	26	0.000
828	N/A	1	0	1.00	26	0.000
829	N/A	1	0	1.00	26	0.000
830	N/A	1	0	1.00	26	0.000
831	A	16	15	1.16	26	0.577
832	A	7	6	0.72	26	0.231
833	A	7	6	0.73	24	0.250
834	A	10	9	1.18	23	0.391
835	N/A	1	0	1.00	26	0.000
836	N/A	1	0	1.00	26	0.000
837	N/A	1	0	1.00	22	0.000
838	N/A	1	0	1.00	22	0.000
839	N/A	4	0	1.00	20	0.000
840	N/A	2	0	1.00	19	0.000
841	N/A	1	0	1.00	22	0.000
842	N/A	1	0	1.00	22	0.000
843	N/A	1	0	1.00	24	0.000
844	N/A	1	0	1.00	24	0.000
845	N/A	6	0	1.00	22	0.000
846	N/A	5	0	1.00	21	0.000
847	N/A	1	0	1.00	24	0.000
848	N/A	1	0	1.00	24	0.000
849	N/A	1	0	1.00	24	0.000
850	N/A	1	0	1.00	24	0.000
851	N/A	7	0	1.00	22	0.000
852	N/A	6	0	1.00	21	0.000
853	N/A	1	0	1.00	24	0.000
854	N/A	1	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
855	N/A	1	0	1.00	24	0.000
856	N/A	5	0	1.00	24	0.000
857	N/A	4	0	1.00	24	0.000
858	N/A	2	0	1.00	22	0.000
859	A	1	1	1.00	21	0.048
860	N/A	2	0	1.00	24	0.000
861	N/A	4	0	1.00	24	0.000
862	N/A	5	0	1.00	24	0.000
863	N/A	6	0	1.00	24	0.000
864	N/A	1	0	1.00	24	0.000
865	N/A	1	0	1.00	24	0.000
866	A	11	10	1.03	24	0.417
867	A	9	8	0.98	22	0.364
868	A	11	10	1.03	21	0.476
869	N/A	1	0	1.00	24	0.000
870	N/A	1	0	1.00	24	0.000
871	N/A	1	0	1.00	24	0.000
872	A	15	14	1.12	24	0.583
873	A	14	13	1.17	24	0.542
874	A	4	3	0.83	24	0.125
875	A	14	13	1.13	22	0.591
876	A	16	15	1.11	21	0.714
877	N/A	1	0	1.00	24	0.000
878	N/A	1	0	1.00	26	0.000
879	N/A	1	0	1.00	26	0.000
880	N/A	3	0	1.00	24	0.000
881	N/A	2	0	1.00	23	0.000
882	N/A	1	0	1.00	26	0.000
883	N/A	1	0	1.00	26	0.000
884	N/A	1	0	1.00	26	0.000
885	N/A	4	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
886	N/A	3	0	1.00	23	0.000
887	N/A	1	0	1.00	26	0.000
888	N/A	1	0	1.00	26	0.000
889	N/A	1	0	1.00	26	0.000
890	N/A	5	0	1.00	24	0.000
891	N/A	4	0	1.00	23	0.000
892	N/A	1	0	1.00	26	0.000
893	N/A	1	0	1.00	26	0.000
894	N/A	5	0	1.00	26	0.000
895	N/A	3	0	1.00	26	0.000
896	N/A	2	0	1.00	24	0.000
897	N/A	1	0	1.00	23	0.000
898	N/A	1	0	1.00	26	0.000
899	N/A	2	0	1.00	26	0.000
900	N/A	3	0	1.00	26	0.000
901	N/A	5	0	1.00	26	0.000
902	N/A	1	0	1.00	26	0.000
903	N/A	1	0	1.00	26	0.000
904	A	8	7	1.01	24	0.292
905	A	10	9	0.98	23	0.391
906	N/A	1	0	1.00	26	0.000
907	N/A	1	0	1.00	26	0.000
908	N/A	1	0	1.00	26	0.000
909	A	14	13	1.11	26	0.500
910	A	17	16	1.17	26	0.615
911	A	11	10	1.15	24	0.417
912	A	13	12	1.10	23	0.522
913	N/A	1	0	1.00	26	0.000
914	N/A	1	0	1.00	22	0.000
915	N/A	1	0	1.00	20	0.000
916	N/A	1	0	1.00	19	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
917	N/A	1	0	1.00	22	0.000
918	N/A	1	0	1.00	24	0.000
919	N/A	1	0	1.00	22	0.000
920	N/A	1	0	1.00	21	0.000
921	N/A	1	0	1.00	24	0.000
922	N/A	1	0	1.00	24	0.000
923	N/A	1	0	1.00	22	0.000
924	N/A	1	0	1.00	21	0.000
925	N/A	1	0	1.00	24	0.000
926	N/A	1	0	1.00	24	0.000
927	N/A	2	0	1.00	22	0.000
928	A	1	1	1.00	21	0.048
929	N/A	1	0	1.00	24	0.000
930	N/A	1	0	1.00	24	0.000
931	N/A	1	0	1.00	24	0.000
932	A	5	4	0.87	24	0.167
933	A	7	6	1.00	22	0.273
934	A	5	4	0.87	21	0.190
935	N/A	1	0	1.00	24	0.000
936	N/A	1	0	1.00	24	0.000
937	N/A	1	0	1.00	24	0.000
938	A	5	4	0.88	24	0.167
939	A	4	3	0.93	24	0.125
940	A	4	3	0.91	24	0.125
941	A	4	3	0.93	22	0.136
942	A	5	4	0.88	21	0.190
943	N/A	1	0	1.00	24	0.000
944	N/A	1	0	1.00	26	0.000
945	N/A	1	0	1.00	24	0.000
946	N/A	1	0	1.00	23	0.000
947	N/A	1	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
948	N/A	1	0	1.00	26	0.000
949	N/A	1	0	1.00	24	0.000
950	N/A	1	0	1.00	23	0.000
951	N/A	1	0	1.00	26	0.000
952	N/A	1	0	1.00	26	0.000
953	N/A	1	0	1.00	24	0.000
954	N/A	1	0	1.00	23	0.000
955	N/A	1	0	1.00	26	0.000
956	N/A	1	0	1.00	26	0.000
957	N/A	1	0	1.00	24	0.000
958	N/A	1	0	1.00	23	0.000
959	N/A	1	0	1.00	26	0.000
960	N/A	1	0	1.00	26	0.000
961	N/A	1	0	1.00	26	0.000
962	A	6	5	1.00	24	0.208
963	A	6	5	1.00	23	0.217
964	N/A	1	0	1.00	26	0.000
965	N/A	1	0	1.00	26	0.000
966	N/A	1	0	1.00	26	0.000
967	A	6	5	0.76	26	0.192
968	A	5	4	0.76	26	0.154
969	A	5	4	0.76	24	0.167
970	A	6	5	0.76	23	0.217
971	N/A	1	0	1.00	26	0.000
972	N/A	1	0	1.00	22	0.000
973	N/A	1	0	1.00	20	0.000
974	N/A	1	0	1.00	19	0.000
975	N/A	1	0	1.00	22	0.000
976	N/A	1	0	1.00	24	0.000
977	N/A	1	0	1.00	22	0.000
978	N/A	1	0	1.00	21	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
979	N/A	1	0	1.00	24	0.000
980	N/A	1	0	1.00	24	0.000
981	N/A	1	0	1.00	22	0.000
982	N/A	1	0	1.00	21	0.000
983	N/A	1	0	1.00	24	0.000
984	N/A	2	0	1.00	24	0.000
985	N/A	2	0	1.00	22	0.000
986	A	1	1	1.00	21	0.048
987	N/A	2	0	1.00	24	0.000
988	N/A	1	0	1.00	24	0.000
989	N/A	3	0	1.00	24	0.000
990	N/A	8	0	1.00	24	0.000
991	A	9	8	1.00	24	0.333
992	A	8	7	1.01	22	0.318
993	A	9	8	1.00	21	0.381
994	N/A	8	0	1.00	24	0.000
995	N/A	3	0	1.00	24	0.000
996	N/A	3	0	1.00	24	0.000
997	N/A	3	0	1.00	24	0.000
998	N/A	1	0	1.00	24	0.000
999	A	9	8	1.71	24	0.333
1000	B	6	5	2.46	24	0.208
1001	A	10	9	1.74	22	0.409
1002	A	6	5	1.01	21	0.238
1003	N/A	8	0	1.00	24	0.000
1004	N/A	3	0	1.00	24	0.000
1005	N/A	3	0	1.00	24	0.000
1006	N/A	3	0	1.00	24	0.000
1007	N/A	1	0	1.00	26	0.000
1008	N/A	1	0	1.00	24	0.000
1009	N/A	1	0	1.00	23	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1010	N/A	1	0	1.00	26	0.000
1011	N/A	1	0	1.00	26	0.000
1012	N/A	1	0	1.00	24	0.000
1013	N/A	1	0	1.00	23	0.000
1014	N/A	1	0	1.00	26	0.000
1015	N/A	1	0	1.00	26	0.000
1016	N/A	1	0	1.00	24	0.000
1017	N/A	1	0	1.00	23	0.000
1018	N/A	1	0	1.00	26	0.000
1019	N/A	1	0	1.00	26	0.000
1020	N/A	1	0	1.00	24	0.000
1021	N/A	1	0	1.00	23	0.000
1022	N/A	2	0	1.00	26	0.000
1023	N/A	1	0	1.00	26	0.000
1024	N/A	1	0	1.00	26	0.000
1025	N/A	2	0	1.00	26	0.000
1026	N/A	8	0	1.00	26	0.000
1027	A	7	6	1.00	24	0.250
1028	A	7	6	1.00	23	0.261
1029	N/A	8	0	1.00	26	0.000
1030	N/A	2	0	1.00	26	0.000
1031	N/A	2	0	1.00	26	0.000
1032	N/A	2	0	1.00	26	0.000
1033	N/A	1	0	1.00	26	0.000
1034	A	6	5	0.84	26	0.192
1035	A	9	8	0.83	26	0.308
1036	A	11	10	0.83	24	0.417
1037	A	6	5	0.83	23	0.217
1038	N/A	8	0	1.00	26	0.000
1039	N/A	2	0	1.00	26	0.000
1040	N/A	2	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1041	N/A	2	0	1.00	26	0.000
1042	N/A	1	0	1.00	22	0.000
1043	N/A	1	0	1.00	20	0.000
1044	N/A	1	0	1.00	19	0.000
1045	N/A	1	0	1.00	22	0.000
1046	N/A	1	0	1.00	24	0.000
1047	N/A	1	0	1.00	22	0.000
1048	N/A	1	0	1.00	21	0.000
1049	N/A	1	0	1.00	24	0.000
1050	N/A	1	0	1.00	24	0.000
1051	N/A	1	0	1.00	22	0.000
1052	N/A	1	0	1.00	21	0.000
1053	N/A	1	0	1.00	24	0.000
1054	N/A	2	0	1.00	24	0.000
1055	N/A	2	0	1.00	22	0.000
1056	A	1	1	1.00	21	0.048
1057	N/A	2	0	1.00	24	0.000
1058	N/A	1	0	1.00	24	0.000
1059	N/A	12	0	1.00	24	0.000
1060	A	9	8	0.96	24	0.333
1061	A	9	8	1.00	22	0.364
1062	A	9	8	0.97	21	0.381
1063	N/A	12	0	1.00	24	0.000
1064	N/A	9	0	1.00	24	0.000
1065	N/A	4	0	1.00	24	0.000
1066	N/A	4	0	1.00	24	0.000
1067	N/A	1	0	1.00	24	0.000
1068	A	8	7	1.86	24	0.292
1069	B	14	13	2.78	24	0.542
1070	A	8	7	1.88	22	0.318
1071	A	11	10	1.53	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1072	N/A	9	0	1.00	24	0.000
1073	N/A	9	0	1.00	24	0.000
1074	N/A	4	0	1.00	24	0.000
1075	N/A	4	0	1.00	24	0.000
1076	N/A	1	0	1.00	26	0.000
1077	N/A	1	0	1.00	24	0.000
1078	N/A	1	0	1.00	23	0.000
1079	N/A	1	0	1.00	26	0.000
1080	N/A	1	0	1.00	26	0.000
1081	N/A	1	0	1.00	24	0.000
1082	N/A	1	0	1.00	23	0.000
1083	N/A	1	0	1.00	26	0.000
1084	N/A	1	0	1.00	26	0.000
1085	N/A	1	0	1.00	24	0.000
1086	N/A	1	0	1.00	23	0.000
1087	N/A	1	0	1.00	26	0.000
1088	N/A	1	0	1.00	26	0.000
1089	N/A	1	0	1.00	24	0.000
1090	N/A	1	0	1.00	23	0.000
1091	N/A	2	0	1.00	26	0.000
1092	N/A	1	0	1.00	26	0.000
1093	N/A	1	0	1.00	26	0.000
1094	N/A	9	0	1.00	26	0.000
1095	N/A	10	0	1.00	26	0.000
1096	A	8	7	1.03	24	0.292
1097	A	8	7	1.04	23	0.304
1098	N/A	10	0	1.00	26	0.000
1099	N/A	9	0	1.00	26	0.000
1100	N/A	3	0	1.00	26	0.000
1101	N/A	3	0	1.00	26	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1102	N/A	1	0	1.00	26	0.000
1103	A	10	9	1.45	26	0.346
1104	A	13	12	1.85	26	0.462
1105	A	11	10	1.85	24	0.417
1106	A	12	11	1.46	23	0.478
1107	N/A	9	0	1.00	26	0.000
1108	N/A	9	0	1.00	26	0.000
1109	N/A	3	0	1.00	26	0.000
1110	N/A	3	0	1.00	26	0.000
1111	N/A	2	0	1.00	20	0.000
1112	A	1	1	1.00	19	0.053
1113	N/A	1	0	1.00	28	0.000
1114	A	5	5	0.81	19	0.263
1115	A	6	5	0.99	19	0.263
1116	A	3	3	1.01	17	0.176
1117	A	6	5	0.99	16	0.312
1118	A	2	2	1.00	19	0.105
1119	A	6	5	1.07	19	0.263
1120	A	2	2	1.00	19	0.105
1121	A	6	5	0.95	19	0.263
1122	A	5	5	0.88	19	0.263
1123	A	6	5	0.95	19	0.263
1124	A	6	6	0.82	19	0.316
1125	A	4	4	0.98	21	0.190
1126	A	6	5	0.98	21	0.238
1127	A	3	3	1.00	19	0.158
1128	A	6	5	0.98	18	0.278
1129	A	2	2	1.00	21	0.095
1130	A	6	5	1.02	21	0.238
1131	A	2	2	1.00	21	0.095
1132	A	6	5	0.97	21	0.238
1133	A	2	2	1.00	21	0.095
1134	A	6	5	0.95	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1135	A	4	4	1.02	21	0.190
1136	A	6	5	0.96	21	0.238
1137	A	9	9	1.26	21	0.429
1138	A	6	5	0.99	21	0.238
1139	A	3	3	0.99	19	0.158
1140	A	6	5	0.98	18	0.278
1141	A	2	2	1.00	21	0.095
1142	A	6	5	1.01	21	0.238
1143	A	2	2	1.00	21	0.095
1144	A	6	5	0.96	21	0.238
1145	A	2	2	1.00	21	0.095
1146	A	6	5	0.97	21	0.238
1147	A	2	2	1.00	21	0.095
1148	A	6	5	0.96	21	0.238
1149	A	4	4	0.99	21	0.190
1150	A	6	5	1.00	14	0.357
1151	A	6	6	0.99	21	0.286
1152	A	2	2	1.00	19	0.105
1153	A	2	2	1.00	21	0.095
1154	A	6	6	0.98	21	0.286
1155	A	7	7	0.99	21	0.333
1156	A	5	5	0.99	18	0.278
1157	A	12	11	0.99	21	0.524
1158	A	2	2	1.00	21	0.095
1159	A	4	4	0.97	19	0.211
1160	A	2	2	1.00	21	0.095
1161	A	2	2	1.00	21	0.095
1162	A	2	2	1.00	21	0.095
1163	A	4	4	0.98	18	0.222
1164	A	2	2	1.00	21	0.095
1165	A	2	2	1.00	21	0.095
1166	A	6	6	1.16	21	0.286
1167	A	5	5	1.15	19	0.263
1168	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1169	A	2	2	1.00	21	0.095
1170	A	2	2	1.00	21	0.095
1171	A	4	4	0.98	18	0.222
1172	A	2	2	1.00	21	0.095
1173	A	10	9	1.04	23	0.391
1174	N/A	7	0	1.00	23	0.000
1175	A	8	7	1.06	21	0.333
1176	N/A	1	0	1.00	20	0.000
1177	N/A	7	0	1.00	23	0.000
1178	N/A	6	0	1.00	23	0.000
1179	N/A	7	0	1.00	23	0.000
1180	A	9	8	1.04	23	0.348
1181	N/A	8	0	1.00	23	0.000
1182	A	11	10	1.01	23	0.435
1183	A	12	11	1.05	23	0.478
1184	N/A	8	0	1.00	23	0.000
1185	A	9	8	1.09	21	0.381
1186	N/A	1	0	1.00	20	0.000
1187	N/A	8	0	1.00	23	0.000
1188	N/A	7	0	1.00	23	0.000
1189	N/A	8	0	1.00	23	0.000
1190	N/A	7	0	1.00	23	0.000
1191	N/A	8	0	1.00	23	0.000
1192	A	11	10	1.04	23	0.435
1193	A	13	12	1.06	23	0.522
1194	N/A	9	0	1.00	23	0.000
1195	A	11	10	1.10	21	0.476
1196	N/A	1	0	1.00	20	0.000
1197	N/A	9	0	1.00	23	0.000
1198	N/A	8	0	1.00	23	0.000
1199	N/A	9	0	1.00	23	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1200	N/A	8	0	1.00	23	0.000
1201	A	9	8	1.03	23	0.348
1202	N/A	6	0	1.00	23	0.000
1203	A	7	6	1.01	21	0.286
1204	N/A	1	0	1.00	20	0.000
1205	N/A	6	0	1.00	23	0.000
1206	A	8	7	1.08	23	0.304
1207	N/A	7	0	1.00	23	0.000
1208	A	9	8	1.01	23	0.348
1209	A	8	7	0.99	23	0.304
1210	N/A	6	0	1.00	23	0.000
1211	A	4	3	1.00	21	0.143
1212	A	6	5	1.00	20	0.250
1213	N/A	7	0	1.00	23	0.000
1214	A	8	7	1.06	23	0.304
1215	N/A	8	0	1.00	23	0.000
1216	A	4	4	0.98	23	0.174
1217	N/A	7	0	1.00	23	0.000
1218	A	7	6	0.97	23	0.261
1219	A	7	6	1.04	23	0.261
1220	A	5	4	1.00	21	0.190
1221	A	7	6	1.01	20	0.300
1222	N/A	8	0	1.00	23	0.000
1223	A	4	4	0.96	23	0.174
1224	N/A	9	0	1.00	23	0.000
1225	A	4	4	0.94	23	0.174
1226	A	8	7	0.97	16	0.438
1227	A	6	5	0.99	16	0.312
1228	A	3	3	0.99	21	0.143
1229	A	3	3	0.98	21	0.143
1230	A	3	3	0.98	19	0.158
1231	N/A	3	0	1.00	21	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1232	N/A	3	0	1.00	21	0.000
1233	N/A	4	0	1.00	23	0.000
1234	N/A	4	0	1.00	23	0.000
1235	N/A	4	0	1.00	23	0.000
1236	N/A	4	0	1.00	23	0.000
1237	N/A	4	0	1.00	23	0.000
1238	N/A	4	0	1.00	23	0.000
1239	N/A	4	0	1.00	21	0.000
1240	N/A	4	0	1.00	25	0.000
1241	A	6	5	0.97	25	0.200
1242	N/A	4	0	1.00	25	0.000
1243	A	4	4	0.95	25	0.160
1244	N/A	4	0	1.00	25	0.000
1245	A	4	4	0.93	25	0.160
1246	N/A	4	0	1.00	25	0.000
1247	A	2	2	1.00	21	0.095
1248	A	2	2	1.00	21	0.095
1249	A	2	2	1.00	19	0.105
1250	A	2	2	1.00	18	0.111
1251	A	2	2	1.00	21	0.095
1252	A	2	2	1.00	21	0.095
1253	A	2	2	1.00	21	0.095
1254	A	2	2	1.00	23	0.087
1255	A	2	2	1.00	23	0.087
1256	A	2	2	1.00	21	0.095
1257	A	2	2	1.00	20	0.100
1258	A	2	2	1.00	23	0.087
1259	A	2	2	1.00	23	0.087
1260	A	2	2	1.00	23	0.087
1261	A	7	7	0.98	23	0.304
1262	A	9	8	1.03	23	0.348
1263	A	2	2	1.00	21	0.095

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1264	A	2	2	1.00	20	0.100
1265	A	2	2	1.00	23	0.087
1266	A	7	7	1.02	23	0.304
1267	A	12	11	0.97	23	0.478
1268	A	2	2	1.00	23	0.087
1269	A	2	2	1.00	23	0.087
1270	B	3	3	2.18	21	0.143
1271	A	2	2	1.00	20	0.100
1272	A	2	2	1.00	23	0.087
1273	A	2	2	1.00	23	0.087
1274	A	2	2	1.00	23	0.087
1275	A	2	2	1.14	12	0.167
1276	A	2	2	1.00	12	0.167
1277	A	2	2	1.18	12	0.167
1278	A	2	2	1.00	10	0.200
1279	A	9	8	1.26	9	0.889
1280	A	8	7	0.87	12	0.583
1281	A	6	5	1.07	12	0.417
1282	A	2	2	1.10	12	0.167
1283	A	12	11	1.23	12	0.917
1284	A	2	2	1.03	12	0.167
1285	A	17	16	1.38	12	1.333
1286	A	2	2	1.00	26	0.077
1287	A	2	2	0.99	26	0.077
1288	A	2	2	1.02	26	0.077
1289	A	2	2	1.00	24	0.083
1290	A	8	7	1.12	23	0.304
1291	A	13	12	0.85	26	0.462
1292	A	9	8	0.82	26	0.308
1293	A	2	2	0.95	26	0.077
1294	A	17	16	0.93	26	0.615
1295	A	2	2	0.96	26	0.077
1296	A	27	26	1.17	26	1.000
1297	A	2	2	0.99	22	0.091

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1298	A	13	12	1.00	21	0.571
1299	N/A	9	0	1.00	24	0.000
1300	A	10	9	0.97	24	0.375
1301	A	2	2	1.03	24	0.083

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int x^3(d+icdx)(a+b\arctan(cx))\,dx$

3.1.1 Optimal result

Integrand size = 21, antiderivative size = 117

$$\begin{aligned}\int x^3(d+icdx)(a+b\arctan(cx))\,dx &= \frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4\\ &\quad -\frac{bd\arctan(cx)}{4c^4}+\frac{1}{4}dx^4(a+b\arctan(cx))\\ &\quad +\frac{1}{5}icdx^5(a+b\arctan(cx))-\frac{ibd\log(1+c^2x^2)}{10c^4}\end{aligned}$$

output
```
1/4*b*d*x/c^3+1/10*I*b*d*x^2/c^2-1/12*b*d*x^3/c-1/20*I*b*d*x^4-1/4*b*d*arc
tan(c*x)/c^4+1/4*d*x^4*(a+b*arctan(c*x))+1/5*I*c*d*x^5*(a+b*arctan(c*x))-1
/10*I*b*d*ln(c^2*x^2+1)/c^4
```

3.1.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int x^3(d+icdx)(a+b\arctan(cx))\,dx\\ &=\frac{d(3ac^4x^4(5+4icx)+bcx(15+6icx-5c^2x^2-3ic^3x^3)+3b(-5+5c^4x^4+4ic^5x^5)\arctan(cx)-6ib\log(1}{60c^4}\end{aligned}$$

input
```
Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*(3*a*c^4*x^4*(5 + (4*I)*c*x) + b*c*x*(15 + (6*I)*c*x - 5*c^2*x^2 - (3*I
)*c^3*x^3) + 3*b*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5)*ArcTan[c*x] - (6*I)*b*Lo
g[1 + c^2*x^2]))/(60*c^4)
```

3.1.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \textbf{5407}$$

$$-bc\int \frac{dx^4(4icx+5)}{20\left(c^2x^2+1\right)}dx+\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{1}{20}bcd\int \frac{x^4(4icx+5)}{c^2x^2+1}dx+\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))$$

$$\Big\downarrow \textbf{523}$$

$$-\frac{1}{20}bcd\int \left(\frac{4ix^3}{c}+\frac{5x^2}{c^2}-\frac{4ix}{c^3}+\frac{4icx+5}{c^4\left(c^2x^2+1\right)}-\frac{5}{c^4}\right)dx+\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))-\frac{1}{20}bcd\left(\frac{5\arctan(cx)}{c^5}-\frac{5x}{c^4}-\frac{2ix^2}{c^3}+\frac{5x^3}{3c^2}+\frac{2i\log\left(c^2x^2+1\right)}{c^5}+\frac{ix^4}{c}\right)$$

input
```
Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcTan[c*x]))/4 + (I/5)*c*d*x^5*(a + b*ArcTan[c*x]) - (b*c*d
*((-5*x)/c^4 - ((2*I)*x^2)/c^3 + (5*x^3)/(3*c^2) + (I*x^4)/c + (5*ArcTan[c
*x])/c^5 + ((2*I)*Log[1 + c^2*x^2])/c^5))/20
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.1.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result
parts	$ad\left(\frac{1}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{bd\left(\frac{i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{cx}{4}-\frac{ic^4x^4}{20}-\frac{c^3x^3}{12}+\frac{ic^2x^2}{10}-\frac{i\ln\left(c^2x^2+1\right)}{10}-\frac{\arctan(cx)}{4}\right)}{c^4}$
derivativedivides	$\frac{ad\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+bd\left(\frac{i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{cx}{4}-\frac{ic^4x^4}{20}-\frac{c^3x^3}{12}+\frac{ic^2x^2}{10}-\frac{i\ln\left(c^2x^2+1\right)}{10}-\frac{\arctan(cx)}{4}\right)}{c^4}$
default	$\frac{ad\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+bd\left(\frac{i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{cx}{4}-\frac{ic^4x^4}{20}-\frac{c^3x^3}{12}+\frac{ic^2x^2}{10}-\frac{i\ln\left(c^2x^2+1\right)}{10}-\frac{\arctan(cx)}{4}\right)}{c^4}$
parallelrisch	$-\frac{-12ic^5bd\arctan(cx)x^5-12ix^5a\,c^5d+3ix^4b\,c^4d-15x^4\arctan(cx)b\,c^4d-15a\,c^4d\,x^4+5c^3x^3db-6ix^2b\,c^2d+6ibd\ln(c^2x^2}{60c^4}$
risch	$\frac{db\left(4x^5c-5ix^4\right)\ln(icx+1)}{40}-\frac{dcb\,x^5\ln(-icx+1)}{10}+\frac{idca\,x^5}{5}+\frac{ad\,x^4}{4}+\frac{id\,x^4b\ln(-icx+1)}{8}-\frac{ibd\,x^4}{20}-\frac{bd\,x^3}{12c}+$

input
```
int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/5*I*c*x^5+1/4*x^4)+b*d/c^4*(1/5*I*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*a
rctan(c*x)+1/4*c*x-1/20*I*c^4*x^4-1/12*c^3*x^3+1/10*I*c^2*x^2-1/10*I*ln(c^
2*x^2+1)-1/4*arctan(c*x))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{24i\,ac^5dx^5+6\,(5\,a-i\,b)c^4dx^4-10\,bc^3dx^3+12i\,bc^2dx^2+30\,bcdx-27i\,bd\log\left(\frac{cx+i}{c}\right)+3i\,bd\log\left(\frac{cx-i}{c}\right)-}{120\,c^4}$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*I*a*c^5*d*x^5 + 6*(5*a - I*b)*c^4*d*x^4 - 10*b*c^3*d*x^3 + 12*I*
b*c^2*d*x^2 + 30*b*c*d*x - 27*I*b*d*log((c*x + I)/c) + 3*I*b*d*log((c*x -
I)/c) - 3*(4*b*c^5*d*x^5 - 5*I*b*c^4*d*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

3.1.6 Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.57

$$\begin{aligned}\int x^3(d+icdx)(a+b\arctan(cx))\,dx &= \frac{iacdx^5}{5}-\frac{bdx^3}{12c}+\frac{ibdx^2}{10c^2}+\frac{bdx}{4c^3}\\ &+\frac{bd\left(\frac{i\log(25bcdx-25ibd)}{40}-\frac{11i\log(25bcdx+25ibd)}{60}\right)}{c^4}\\ &+x^4\left(\frac{ad}{4}-\frac{ibd}{20}\right)+\left(\frac{bcdx^5}{10}-\frac{ibdx^4}{8}\right)\log(icx+1)\\ &+\frac{(-12bc^5dx^5+15ibc^4dx^4-5ibd)\log(-icx+1)}{120c^4}\end{aligned}$$

input
```
integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**5/5 - b*d*x**3/(12*c) + I*b*d*x**2/(10*c**2) + b*d*x/(4*c**3) +
 b*d*(I*log(25*b*c*d*x - 25*I*b*d)/40 - 11*I*log(25*b*c*d*x + 25*I*b*d)/60
)/c**4 + x**4*(a*d/4 - I*b*d/20) + (b*c*d*x**5/10 - I*b*d*x**4/8)*log(I*c*
x + 1) + (-12*b*c**5*d*x**5 + 15*I*b*c**4*d*x**4 - 5*I*b*d)*log(-I*c*x + 1
)/(120*c**4)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{1}{5}i\,acdx^5+\frac{1}{4}\,adx^4+\frac{1}{20}i\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bcd$$
$$+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*I*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 -
2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d + 1/12*(3*x^4*arctan(c*x) - c*
((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d
```

3.1.8 Giac [F]

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx=\int(i\,cdx+d)(b\arctan\left(cx\right)+a)x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$
$$=-\frac{\frac{d\,\left(15\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,6\mathrm{i}\right)}{60}-\frac{b\,c\,d\,x}{4}+\frac{b\,c^3\,d\,x^3}{12}-\frac{b\,c^2\,d\,x^2\,1\mathrm{i}}{10}}{c^4}$$
$$+\frac{d\,\left(15\,a\,x^4+15\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,3\mathrm{i}\right)}{60}+\frac{c\,d\,\left(a\,x^5\,12\mathrm{i}+b\,x^5\,\mathrm{atan}(c\,x)\,12\mathrm{i}\right)}{60}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(15*a*x^4 - b*x^4*3i + 15*b*x^4*atan(c*x)))/60 - ((d*(15*b*atan(c*x) +
b*log(c^2*x^2 + 1)*6i))/60 - (b*c*d*x)/4 - (b*c^2*d*x^2*1i)/10 + (b*c^3*d*
x^3)/12)/c^4 + (c*d*(a*x^5*12i + b*x^5*atan(c*x)*12i))/60
```

3.2 $\int x^2(d+icdx)(a+b\arctan(cx))\,dx$

3.2.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx = \frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3-\frac{ibd\arctan(cx)}{4c^3} + \frac{1}{3}dx^3(a+b\arctan(cx)) + \frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{bd\log(1+c^2x^2)}{6c^3}$$

output
```
1/4*I*b*d*x/c^2-1/6*b*d*x^2/c-1/12*I*b*d*x^3-1/4*I*b*d*arctan(c*x)/c^3+1/3
*d*x^3*(a+b*arctan(c*x))+1/4*I*c*d*x^4*(a+b*arctan(c*x))+1/6*b*d*ln(c^2*x^
2+1)/c^3
```

3.2.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx = \frac{d(ac^3x^3(4+3icx)+bcx(3i-2cx-ic^2x^2)+b(-3i+4c^3x^3+3ic^4x^4)\arctan(cx)+2b\log(1+c^2x^2))}{12c^3}$$

input
```
Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*(a*c^3*x^3*(4 + (3*I)*c*x) + b*c*x*(3*I - 2*c*x - I*c^2*x^2) + b*(-3*I
+ 4*c^3*x^3 + (3*I)*c^4*x^4)*ArcTan[c*x] + 2*b*Log[1 + c^2*x^2]))/(12*c^3)
```

3.2.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{dx^3(3icx+4)}{12\,(c^2x^2+1)}dx+\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bcd\int \frac{x^3(3icx+4)}{c^2x^2+1}dx+\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{523}$$

$$-\frac{1}{12}bcd\int \left(\frac{3ix^2}{c}+\frac{4x}{c^2}+\frac{3i-4cx}{c^3\,(c^2x^2+1)}-\frac{3i}{c^3}\right)dx+\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))-\frac{1}{12}bcd\left(\frac{3i\arctan(cx)}{c^4}-\frac{3ix}{c^3}+\frac{2x^2}{c^2}-\frac{2\log\left(c^2x^2+1\right)}{c^4}+\frac{ix^3}{c}\right)$$

input
```
Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcTan[c*x]))/3 + (I/4)*c*d*x^4*(a + b*ArcTan[c*x]) - (b*c*d
*(((-3*I)*x)/c^3 + (2*x^2)/c^2 + (I*x^3)/c + ((3*I)*ArcTan[c*x])/c^4 - (2*
Log[1 + c^2*x^2])/c^4))/12
```

3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.2.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

method	result
parts	$ad\left(\frac{1}{4}ic\,x^4+\frac{1}{3}x^3\right)+\frac{bd\left(\frac{i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{4}-\frac{ic^3x^3}{12}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}-\frac{i\arctan(cx)}{4}\right)}{c^3}$
derivativedivides	$\frac{ad\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+bd\left(\frac{i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{4}-\frac{ic^3x^3}{12}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}-\frac{i\arctan(cx)}{4}\right)}{c^3}$
default	$\frac{ad\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+bd\left(\frac{i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{4}-\frac{ic^3x^3}{12}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}-\frac{i\arctan(cx)}{4}\right)}{c^3}$
parallelrisch	$\frac{3ix^4\arctan(cx)b\,c^4d+3ix^4a\,c^4d-ix^3b\,c^3d+4x^3\arctan(cx)bd\,c^3+4a\,c^3d\,x^3-2b\,c^2d\,x^2+3ibdxc-3ibd\arctan(cx)+2bd\ln(}{12c^3}$
risch	$\frac{db\left(3c\,x^4-4ix^3\right)\ln(icx+1)}{24}+\frac{iacd\,x^4}{4}-\frac{dc\,x^4b\ln(-icx+1)}{8}+\frac{idb\,x^3\ln(-icx+1)}{6}-\frac{ibd\,x^3}{12}+\frac{x^3da}{3}-\frac{bd\,x^2}{6c}+$

input
```
int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/4*I*c*x^4+1/3*x^3)+b*d/c^3*(1/4*I*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*a
rctan(c*x)+1/4*I*c*x-1/12*I*c^3*x^3-1/6*c^2*x^2+1/6*ln(c^2*x^2+1)-1/4*I*ar
ctan(c*x))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx$$
$$= \frac{6i\,ac^4dx^4+2\,(4\,a-i\,b)c^3dx^3-4\,bc^2dx^2+6i\,bcdx+7\,bd\log\left(\frac{cx+i}{c}\right)+bd\log\left(\frac{cx-i}{c}\right)-(3\,bc^4dx^4-4i\,bc^3dx^3}{24\,c^3}$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/24*(6*I*a*c^4*d*x^4 + 2*(4*a - I*b)*c^3*d*x^3 - 4*b*c^2*d*x^2 + 6*I*b*c*
d*x + 7*b*d*log((c*x + I)/c) + b*d*log((c*x - I)/c) - (3*b*c^4*d*x^4 - 4*I
*b*c^3*d*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

3.2.6 Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.59

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx = \frac{iacdx^4}{4}-\frac{bdx^2}{6c}+\frac{ibdx}{4c^2} + \frac{bd\left(\frac{\log(11bcdx-11ibd)}{24}+\frac{9\log(11bcdx+11ibd)}{40}\right)}{c^3} + x^3\left(\frac{ad}{3}-\frac{ibd}{12}\right)+\left(\frac{bcdx^4}{8}-\frac{ibdx^3}{6}\right)\log(icx+1) + \frac{(-15bc^4dx^4+20ibc^3dx^3+8bd)\log(-icx+1)}{120c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**4/4 - b*d*x**2/(6*c) + I*b*d*x/(4*c**2) + b*d*(log(11*b*c*d*x -
 11*I*b*d)/24 + 9*log(11*b*c*d*x + 11*I*b*d)/40)/c**3 + x**3*(a*d/3 - I*b*
d/12) + (b*c*d*x**4/8 - I*b*d*x**3/6)*log(I*c*x + 1) + (-15*b*c**4*d*x**4
+ 20*I*b*c**3*d*x**3 + 8*b*d)*log(-I*c*x + 1)/(120*c**3)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{1}{4}i\,acdx^4+\frac{1}{3}\,adx^3+\frac{1}{12}i\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd$$
$$+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/4*I*a*c*d*x^4 + 1/3*a*d*x^3 + 1/12*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 -
3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2
 - log(c^2*x^2 + 1)/c^4))*b*d
```

3.2.8 Giac [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx=\int(i\,cdx+d)(b\arctan\left(cx\right)+a)x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx=-\frac{\frac{d\,\left(-2\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}\left(c\,x\right)\,3{}\mathrm{i}\right)}{12}+\frac{b\,c^2\,d\,x^2}{6}-\frac{b\,c\,d\,x\,1{}\mathrm{i}}{4}}{c^3}$$
$$+\frac{d\,\left(4\,a\,x^3+4\,b\,x^3\,\mathrm{atan}\left(c\,x\right)-b\,x^3\,1{}\mathrm{i}\right)}{12}$$
$$+\frac{c\,d\,\left(a\,x^4\,3{}\mathrm{i}+b\,x^4\,\mathrm{atan}\left(c\,x\right)\,3{}\mathrm{i}\right)}{12}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(4*a*x^3 - b*x^3*1i + 4*b*x^3*atan(c*x)))/12 - ((d*(b*atan(c*x)*3i - 2*
b*log(c^2*x^2 + 1)))/12 - (b*c*d*x*1i)/4 + (b*c^2*d*x^2)/6)/c^3 + (c*d*(a*
x^4*3i + b*x^4*atan(c*x)*3i))/12
```

3.2. $\int x^2(d+icdx)(a+b\arctan(cx))\,dx$

3.3 $\int x(d+icdx)(a+b\arctan(cx))\,dx$

3.3.1 Optimal result

Integrand size = 19, antiderivative size = 91

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = -\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{bd\arctan(cx)}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx)) + \frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{ibd\log(1+c^2x^2)}{6c^2}$$

output
```
-1/2*b*d*x/c-1/6*I*b*d*x^2+1/2*b*d*arctan(c*x)/c^2+1/2*d*x^2*(a+b*arctan(c
*x))+1/3*I*c*d*x^3*(a+b*arctan(c*x))+1/6*I*b*d*ln(c^2*x^2+1)/c^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \frac{d(cx(b(-3-icx)+acx(3+2icx))+b(3+3c^2x^2+2ic^3x^3)\arctan(cx)+ib\log(1+c^2x^2))}{6c^2}$$

input
```
Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*(c*x*(b*(-3 - I*c*x) + a*c*x*(3 + (2*I)*c*x)) + b*(3 + 3*c^2*x^2 + (2*I
)*c^3*x^3)*ArcTan[c*x] + I*b*Log[1 + c^2*x^2]))/(6*c^2)
```

3.3.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5407$$

$$-bc\int \frac{dx^2(2icx+3)}{6\left(c^2x^2+1\right)}dx+\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{6}bcd\int \frac{x^2(2icx+3)}{c^2x^2+1}dx+\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))$$

$$\Big\downarrow \ 523$$

$$-\frac{1}{6}bcd\int \left(\frac{2ix}{c}+\frac{i(3i-2cx)}{c^2\left(c^2x^2+1\right)}+\frac{3}{c^2}\right)dx+\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{6}bcd\left(-\frac{3\arctan(cx)}{c^3}+\frac{3x}{c^2}-\frac{i\log\left(c^2x^2+1\right)}{c^3}+\frac{ix^2}{c}\right)$$

input
```
Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^2*(a + b*ArcTan[c*x]))/2 + (I/3)*c*d*x^3*(a + b*ArcTan[c*x]) - (b*c*d
*((3*x)/c^2 + (I*x^2)/c - (3*ArcTan[c*x])/c^3 - (I*Log[1 + c^2*x^2])/c^3))
/6
```

3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.3.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

method	result
parts	$ad\left(\frac{1}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{bd\left(\frac{i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{ic^2x^2}{6}-\frac{cx}{2}+\frac{i\ln\left(c^2x^2+1\right)}{6}+\frac{\arctan(cx)}{2}\right)}{c^2}$
derivativedivides	$\frac{ad\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+bd\left(\frac{i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{ic^2x^2}{6}-\frac{cx}{2}+\frac{i\ln\left(c^2x^2+1\right)}{6}+\frac{\arctan(cx)}{2}\right)}{c^2}$
default	$\frac{ad\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+bd\left(\frac{i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{ic^2x^2}{6}-\frac{cx}{2}+\frac{i\ln\left(c^2x^2+1\right)}{6}+\frac{\arctan(cx)}{2}\right)}{c^2}$
parallelrisch	$\frac{2ix^3\arctan(cx)b\,c^3d+2ia\,c^3d\,x^3-ix^2b\,c^2d+3x^2\arctan(cx)b\,c^2d+3a\,c^2d\,x^2+ibd\ln(c^2x^2+1)-3bcdx+3bd\arctan(cx)}{6c^2}$
risch	$\frac{db(2c\,x^3-3ix^2)\ln(icx+1)}{12}-\frac{dcb\,x^3\ln(-icx+1)}{6}+\frac{iacd\,x^3}{3}+\frac{x^2da}{2}+\frac{id\,x^2b\ln(-icx+1)}{4}-\frac{ibd\,x^2}{6}-\frac{bdx}{2c}+\frac{bd}{}$

input
```
int(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d*(1/3*I*x^3*c+1/2*x^2)+b*d/c^2*(1/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*a
rctan(c*x)-1/6*I*c^2*x^2-1/2*c*x+1/6*I*ln(c^2*x^2+1)+1/2*arctan(c*x))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int x(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{4i\,ac^3dx^3+2\,(3\,a-i\,b)c^2dx^2-6\,bcdx+5i\,bd\log\left(\frac{cx+i}{c}\right)-i\,bd\log\left(\frac{cx-i}{c}\right)-\left(2\,bc^3dx^3-3i\,bc^2dx^2\right)\log\left(-\frac{c}{c}\right.}{12\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12*(4*I*a*c^3*d*x^3 + 2*(3*a - I*b)*c^2*d*x^2 - 6*b*c*d*x + 5*I*b*d*log(
(c*x + I)/c) - I*b*d*log((c*x - I)/c) - (2*b*c^3*d*x^3 - 3*I*b*c^2*d*x^2)*
log(-(c*x + I)/(c*x - I)))/c^2
```

3.3.6 Sympy [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.74

$$\int x(d+icdx)(a+b\arctan(cx))\,dx=\frac{iacdx^3}{3}-\frac{bdx}{2c}+\frac{bd\left(-\frac{i\log\left(9bcdx-9ibd\right)}{12}+\frac{7i\log\left(9bcdx+9ibd\right)}{24}\right)}{c^2}+x^2\left(\frac{ad}{2}-\frac{ibd}{6}\right)+\left(\frac{bcdx^3}{6}-\frac{ibdx^2}{4}\right)\log\left(icx+1\right)+\frac{\left(-4bc^3dx^3+6ibc^2dx^2+3ibd\right)\log\left(-icx+1\right)}{24c^2}$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**3/3 - b*d*x/(2*c) + b*d*(-I*log(9*b*c*d*x - 9*I*b*d)/12 + 7*I*l
og(9*b*c*d*x + 9*I*b*d)/24)/c**2 + x**2*(a*d/2 - I*b*d/6) + (b*c*d*x**3/6
- I*b*d*x**2/4)*log(I*c*x + 1) + (-4*b*c**3*d*x**3 + 6*I*b*c**2*d*x**2 + 3
*I*b*d)*log(-I*c*x + 1)/(24*c**2)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \frac{1}{3}i\,acdx^3 + \frac{1}{6}i\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd + \frac{1}{2}\,adx^2 + \frac{1}{2}\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)bd$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/3*I*a*c*d*x^3 + 1/6*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*b*c*d + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)
/c^3))*b*d
```

3.3.8 Giac [F]

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \int (i\,cdx+d)(b\arctan\left(cx\right)+a)x\,dx$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \frac{d\,\left(3\,a\,x^2+3\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,1\mathrm{i}\right)}{6} + \frac{\frac{d\,\left(3\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}\right)}{6}-\frac{b\,c\,d\,x}{2}}{c^2} + \frac{c\,d\,\left(a\,x^3\,2\mathrm{i}+b\,x^3\,\mathrm{atan}(c\,x)\,2\mathrm{i}\right)}{6}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(3*a*x^2 - b*x^2*1i + 3*b*x^2*atan(c*x)))/6 + ((d*(3*b*atan(c*x) + b*lo
g(c^2*x^2 + 1)*1i))/6 - (b*c*d*x)/2)/c^2 + (c*d*(a*x^3*2i + b*x^3*atan(c*x
)*2i))/6
```

3.3. $\int x(d + icdx)(a + b\arctan(cx))\,dx$

3.4 $\int (d+icdx)(a+b\arctan(cx))\,dx$

3.4.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int (d+icdx)(a+b\arctan(cx))\,dx=-\frac{1}{2}ibdx-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}-\frac{bd\log(i+cx)}{c}$$

output
```
-1/2*I*b*d*x-1/2*I*d*(1+I*c*x)^2*(a+b*arctan(c*x))/c-b*d*ln(c*x+I)/c
```

3.4.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int (d+icdx)(a+b\arctan(cx))\,dx=adx-\frac{1}{2}ibdx+\frac{1}{2}iacdx^2+\frac{ibd\arctan(cx)}{2c}+bdx\arctan(cx)+\frac{1}{2}ibcdx^2\arctan(cx)-\frac{bd\log(1+c^2x^2)}{2c}$$

input
```
Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
a*d*x - (I/2)*b*d*x + (I/2)*a*c*d*x^2 + ((I/2)*b*d*ArcTan[c*x])/c + b*d*x*
ArcTan[c*x] + (I/2)*b*c*d*x^2*ArcTan[c*x] - (b*d*Log[1 + c^2*x^2])/(2*c)
```

3.4.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{ib\int \frac{d^2(icx+1)^2}{c^2x^2+1}dx}{2d}-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}ibd\int \frac{(icx+1)^2}{c^2x^2+1}dx-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{2}ibd\int \frac{icx+1}{1-icx}dx-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}ibd\int \left(\frac{2i}{cx+i}-1\right)dx-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ibd\left(-x+\frac{2i\log(cx+i)}{c}\right)-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

input
```
Int[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/2*I)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/c + (I/2)*b*d*(-x + ((2*I)*
Log[I + c*x])/c)
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

method	result
parts	$-iad\left(-\frac{1}{2}c\,x^2+ix\right)+\frac{bd\left(\frac{i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)+\frac{i\left(-cx+i\ln\left(c^2x^2+1\right)+\arctan(cx)\right)}{2}\right)}{c}$
derivativedivides	$\frac{-iad\left(-\frac{1}{2}c^2x^2+icx\right)+bd\left(\frac{i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)+\frac{i\left(-cx+i\ln\left(c^2x^2+1\right)+\arctan(cx)\right)}{2}\right)}{c}$
default	$\frac{-iad\left(-\frac{1}{2}c^2x^2+icx\right)+bd\left(\frac{i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)+\frac{i\left(-cx+i\ln\left(c^2x^2+1\right)+\arctan(cx)\right)}{2}\right)}{c}$
parallelrisch	$\frac{ibd\arctan(cx)x^2c^2+ia\,c^2d\,x^2-ibdxc+2bcdx\arctan(cx)+ibd\arctan(cx)+2acdx-bd\ln\left(c^2x^2+1\right)}{2c}$
risch	$\frac{bd\left(c\,x^2-2ix\right)\ln(icx+1)}{4}+\frac{iacd\,x^2}{2}-\frac{dc\,x^2b\ln(-icx+1)}{4}+\frac{ibdx\ln(-icx+1)}{2}-\frac{ibdx}{2}+\frac{idb\arctan(cx)}{2c}+adx$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-I*a*d*(-1/2*c*x^2+I*x)+b*d/c*(1/2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)+1
/2*I*(-c*x+I*ln(c^2*x^2+1)+arctan(c*x)))
```

3.4.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(41) = 82.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int (d+icdx)(a+b\arctan(cx))\,dx = \frac{2i\,ac^2dx^2+2\,(2\,a-i\,b)cdx-3\,bd\log\left(\frac{cx+i}{c}\right)-bd\log\left(\frac{cx-i}{c}\right)-(bc^2dx^2-2i\,bcdx)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,c}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/4*(2*I*a*c^2*d*x^2 + 2*(2*a - I*b)*c*d*x - 3*b*d*log((c*x + I)/c) - b*d*
log((c*x - I)/c) - (b*c^2*d*x^2 - 2*I*b*c*d*x)*log(-(c*x + I)/(c*x - I)))/
c
```

3.4.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(44) = 88.

Time = 1.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.42

$$\begin{aligned}\int (d+icdx)(a+b\arctan(cx))\,dx &= \frac{iacdx^2}{2}+\frac{bd\left(-\frac{\log(bcdx-ibd)}{4}-\frac{5\log(bcdx+ibd)}{12}\right)}{c}\\&\quad+x\left(ad-\frac{ibd}{2}\right)+\left(\frac{bcdx^2}{4}-\frac{ibdx}{2}\right)\log(icx+1)\\&\quad+\frac{(-3bc^2dx^2+6ibcdx-4bd)\log(-icx+1)}{12c}\end{aligned}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**2/2 + b*d*(-log(b*c*d*x - I*b*d)/4 - 5*log(b*c*d*x + I*b*d)/12)
/c + x*(a*d - I*b*d/2) + (b*c*d*x**2/4 - I*b*d*x/2)*log(I*c*x + 1) + (-3*b
*c**2*d*x**2 + 6*I*b*c*d*x - 4*b*d)*log(-I*c*x + 1)/(12*c)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int (d+icdx)(a+b\arctan(cx))\,dx = \frac{1}{2}i\,acdx^2 + \frac{1}{2}i\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bcd + adx + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))bd}{2\,c}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/2*I*a*c*d*x^2 + 1/2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*
c*d + a*d*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

3.4.8 Giac [F]

$$\int (d+icdx)(a+b\arctan(cx))\,dx = \int (i\,cdx+d)(b\arctan(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int (d+icdx)(a+b\arctan(cx))\,dx = \frac{d\,(2\,a\,x+2\,b\,x\,\mathrm{atan}(c\,x)-b\,x\,1\mathrm{i})}{2} + \frac{c\,d\,(a\,x^2\,1\mathrm{i}+b\,x^2\,\mathrm{atan}(c\,x)\ 1\mathrm{i})}{2} + \frac{d\,(-b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}(c\,x)\ 1\mathrm{i})}{2\,c}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(2*a*x - b*x*1i + 2*b*x*atan(c*x)))/2 + (c*d*(a*x^2*1i + b*x^2*atan(c*x
)*1i))/2 + (d*(b*atan(c*x)*1i - b*log(c^2*x^2 + 1)))/(2*c)
```

3.5 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx$

3.5.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = iacdx + ibcdx\arctan(cx) + ad\log(x) - \frac{1}{2}ibd\log\left(1+c^2x^2\right) + \frac{1}{2}ibd\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

output
```
I*a*c*d*x+I*b*c*d*x*arctan(c*x)+a*d*ln(x)-1/2*I*b*d*ln(c^2*x^2+1)+1/2*I*b*
d*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = iacdx + ibcdx\arctan(cx) + ad\log(x) - \frac{1}{2}ibd\log\left(1+c^2x^2\right) + \frac{1}{2}ibd\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2
] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

3.5.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{d(a+b\arctan(cx))}{x}+icd(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$iacdx+ad\log(x)+ibcdx\arctan(cx)-\frac{1}{2}ibd\log\left(c^2x^2+1\right)+\frac{1}{2}ibd\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2
] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.5.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

method	result
parts	$ad(icx+\ln(x))+bd\Big(i\arctan(cx)\,cx+\arctan(cx)\ln(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-}{2}$
derivativedivides	$ad(icx+\ln(cx))+bd\Big(i\arctan(cx)\,cx+\arctan(cx)\ln(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(}{2}$
default	$ad(icx+\ln(cx))+bd\Big(i\arctan(cx)\,cx+\arctan(cx)\ln(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(}{2}$
risch	$\frac{\ln(icx+1)bcdx}{2}-\frac{i\ln(icx+1)bd}{2}+\frac{i\,\mathrm{dilog}(icx+1)bd}{2}+ibd-\frac{\ln(-icx+1)bcdx}{2}+\ln(-icx)\,ad+iacdx-$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*d*(I*c*x+ln(x))+b*d*(I*arctan(c*x)*c*x+arctan(c*x)*ln(c*x)+1/2*I*ln(c*x)
*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-
I*c*x)-1/2*I*ln(c^2*x^2+1))
```

3.5.5 Fricas [F]

$$\int\frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx=\int\frac{(i\,cdx+d)(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x
- I)))/x, x)
```

3.5.6 Sympy [F]

$$\int\frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx=id\left(\int ac\,dx+\int\left(-\frac{ia}{x}\right)dx+\int bc\,\mathrm{atan}(cx)\,dx+\int\left(-\frac{ib\,\mathrm{atan}(cx)}{x}\right)dx\right)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x,x)
```

output
```
I*d*(Integral(a*c, x) + Integral(-I*a/x, x) + Integral(b*c*atan(c*x), x) +
 Integral(-I*b*atan(c*x)/x, x))
```

3.5.7 Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx=\int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
I*a*c*d*x + 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d + b*d*integra
te(arctan(c*x)/x, x) + a*d*log(x)
```

3.5.8 Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx=\int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx=&-\frac{b\,d\,\left(\ln{\left(c^2\,x^2+1\right)}\ 1\mathrm{i}-c\,x\,\mathrm{atan}(c\,x)\ 2\mathrm{i}\right)}{2}\\&+a\,d\,\left(\ln{(x)}+c\,x\,1\mathrm{i}\right)\\&-\frac{b\,d\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)\ 1\mathrm{i}}{2}\end{aligned}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x,x)
```

output
```
a*d*(log(x) + c*x*1i) - (b*d*(log(c^2*x^2 + 1)*1i - c*x*atan(c*x)*2i))/2 -
 (b*d*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

3.5. $\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx$

3.6 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx$

3.6.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = &-\frac{d(a+b\arctan(cx))}{x}+iacd\log(x)\\ &+bcd\log(x)-\frac{1}{2}bcd\log\left(1+c^2x^2\right)\\ &-\frac{1}{2}bcd\,\mathrm{PolyLog}(2,-icx)+\frac{1}{2}bcd\,\mathrm{PolyLog}(2,icx)\end{aligned}$$

output
```
-d*(a+b*arctan(c*x))/x+I*a*c*d*ln(x)+b*c*d*ln(x)-1/2*b*c*d*ln(c^2*x^2+1)-1
/2*b*c*d*polylog(2,-I*c*x)+1/2*b*c*d*polylog(2,I*c*x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx\\ &=\frac{d(-2a-2b\arctan(cx)+2iacx\log(x)+2bcx\log(x)-bcx\log\left(1+c^2x^2\right)-bcx\,\mathrm{PolyLog}(2,-icx)+bcx\,\mathrm{Po}}{2x}\end{aligned}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
(d*(-2*a - 2*b*ArcTan[c*x] + (2*I)*a*c*x*Log[x] + 2*b*c*x*Log[x] - b*c*x*L
og[1 + c^2*x^2] - b*c*x*PolyLog[2, (-I)*c*x] + b*c*x*PolyLog[2, I*c*x]))/(
2*x)
```

3.6.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{d(a+b\arctan(cx))}{x^2}+\frac{icd(a+b\arctan(cx))}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{d(a+b\arctan(cx))}{x}+iacd\log(x)-\frac{1}{2}bcd\log\left(c^2x^2+1\right)-\frac{1}{2}bcd\operatorname{PolyLog}(2,-icx)+\frac{1}{2}bcd\operatorname{PolyLog}(2,icx)+bcd\log(x)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcTan[c*x]))/x) + I*a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[
1 + c^2*x^2])/2 - (b*c*d*PolyLog[2, (-I)*c*x])/2 + (b*c*d*PolyLog[2, I*c*x
])/2
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.6.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

method	result
parts	$ad\left(ic\ln\left(x\right)-\frac{1}{x}\right)+bdc\left(i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\arctan(cx)}{cx}-\frac{\ln(cx)\ln(icx+1)}{2}+\frac{\ln(cx)\ln(-icx+1)}{2}\right.$
derivativedivides	$c\left(ad\left(i\ln\left(cx\right)-\frac{1}{cx}\right)+bd\left(i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\arctan(cx)}{cx}-\frac{\ln(cx)\ln(icx+1)}{2}+\frac{\ln(cx)\ln(-icx}{2}\right.\right.$
default	$c\left(ad\left(i\ln\left(cx\right)-\frac{1}{cx}\right)+bd\left(i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\arctan(cx)}{cx}-\frac{\ln(cx)\ln(icx+1)}{2}+\frac{\ln(cx)\ln(-icx}{2}\right.\right.$
risch	$-\frac{bcd\,\mathrm{dilog}(icx+1)}{2}+\frac{bcd\ln(icx)}{2}-\frac{bcd\ln(icx+1)}{2}+\frac{ibd\ln(icx+1)}{2x}+icd\ln\left(-icx\right)a-\frac{ad}{x}+\frac{cd\,\mathrm{dilog}(-ic}{2}$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*d*(I*c*ln(x)-1/x)+b*d*c*(I*arctan(c*x)*ln(c*x)-1/c/x*arctan(c*x)-1/2*ln(
c*x)*ln(1+I*c*x)+1/2*ln(c*x)*ln(1-I*c*x)-1/2*dilog(1+I*c*x)+1/2*dilog(1-I*
c*x)-1/2*ln(c^2*x^2+1)+ln(c*x))
```

3.6.5 Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan\left(cx\right)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x
- I)))/x^2, x)
```

3.6.6 Sympy [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = id\left(\int\left(-\frac{ia}{x^2}\right)dx+\int\frac{ac}{x}\,dx + \int\left(-\frac{ib\operatorname{atan}\left(cx\right)}{x^2}\right)dx+\int\frac{bc\operatorname{atan}\left(cx\right)}{x}\,dx\right)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**2,x)
```

output
```
I*d*(Integral(-I*a/x**2, x) + Integral(a*c/x, x) + Integral(-I*b*atan(c*x)
/x**2, x) + Integral(b*c*atan(c*x)/x, x))
```

3.6.7 Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
I*b*c*d*integrate(arctan(c*x)/x, x) + I*a*c*d*log(x) - 1/2*(c*(log(c^2*x^2
 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d - a*d/x
```

3.6.8 Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx$$
$$= \begin{cases} -\frac{a\,d}{x} & \text{if } c=0 \\ \frac{b\,d\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}+\frac{b\,c\,d\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))}{2}+\frac{a\,d\,(-1+c\,x\,\ln(x)\,1\mathrm{i})}{x}-\frac{b\,d\,\mathrm{atan}(c\,x)}{x} & \text{if } c\neq 0 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^2,x)
```

output
```
piecewise(c == 0, -(a*d)/x, c ~= 0, (b*d*(c^2*log(x) - (c^2*log(c^2*x^2 +
1))/2))/c + (b*c*d*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)))/2 + (a*d*(c*
x*log(x)*1i - 1))/x - (b*d*atan(c*x))/x)
```

3.6. $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx$

3.7 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx$

3.7.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{bcd}{2x}-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2} + ibc^2d\log(x)-ibc^2d\log(i+cx)$$

output
```
-1/2*b*c*d/x-1/2*d*(1+I*c*x)^2*(a+b*arctan(c*x))/x^2+I*b*c^2*d*ln(x)-I*b*c
^2*d*ln(c*x+I)
```

3.7.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{d(a+b\arctan(cx))}{2x^2}-\frac{icd(a+b\arctan(cx))}{x} - \frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x} + \frac{1}{2}ibc^2d\left(2\log(x)-\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcTan[c*x]))/x^2 - (I*c*d*(a + b*ArcTan[c*x]))/x - (b*c*d*
Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x) + (I/2)*b*c^2*d*(2*Log[
x] - Log[1 + c^2*x^2])
```

3.7.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d(i-cx)}{2x^2(cx+i)}dx-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}bcd\int \frac{i-cx}{x^2(cx+i)}dx-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}bcd\int \left(-\frac{2ic^2}{cx+i}+\frac{2ic}{x}+\frac{1}{x^2}\right)dx-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}bcd\left(2ic\log(x)-2ic\log(cx+i)-\frac{1}{x}\right)-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/x^2 + (b*c*d*(-x^(-1) + (2*I)*c
*Log[x] - (2*I)*c*Log[I + c*x]))/2
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.7.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

method	result
parts	$ad\left(-\frac{1}{2x^2}-\frac{ic}{x}\right)+bd\,c^2\left(-\frac{i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}+i\ln(cx)-\frac{1}{2cx}-\frac{i\ln(c^2x^2+1)}{2}-\frac{\arctan(cx)}{2}\right.$
derivativedivides	$c^2\left(ad\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+bd\left(-\frac{i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}+i\ln(cx)-\frac{1}{2cx}-\frac{i\ln(c^2x^2+1)}{2}-\frac{\arctan(}{2}\right.\right.$
default	$c^2\left(ad\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+bd\left(-\frac{i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}+i\ln(cx)-\frac{1}{2cx}-\frac{i\ln(c^2x^2+1)}{2}-\frac{\arctan(}{2}\right.\right.$
parallelrisch	$-\frac{i\ln(c^2x^2+1)x^2b\,c^2d-2ic^2bd\ln(x)x^2+x^2\arctan(cx)b\,c^2d+2ix\arctan(cx)bcd-a\,c^2d\,x^2+2iacdx+bcdx+bd\arctan(cx)}{2x^2}$
risch	$-\frac{(2bcdx-ibd)\ln(icx+1)}{4x^2}+\frac{id(-3b\,c^2\ln(-7cx-7i)x^2+4b\,c^2\ln(-35cx)x^2-b\,c^2\ln(5cx-5i)x^2-4cxa-2ibcx\ln(-icx+}{4x^2}$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*d*(-1/2/x^2-I*c/x)+b*d*c^2*(-I*arctan(c*x)/c/x-1/2/c^2/x^2*arctan(c*x)+I
*ln(c*x)-1/2/c/x-1/2*I*ln(c^2*x^2+1)-1/2*arctan(c*x))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{4i\,bc^2dx^2\log(x)-3i\,bc^2dx^2\log\left(\frac{cx+i}{c}\right)-i\,bc^2dx^2\log\left(\frac{cx-i}{c}\right)-2\,(2i\,a+b)cdx-2\,ad+(2\,bcdx-i\,bd)\log\left(\right.}{4\,x^2}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/4*(4*I*b*c^2*d*x^2*log(x) - 3*I*b*c^2*d*x^2*log((c*x + I)/c) - I*b*c^2*d
*x^2*log((c*x - I)/c) - 2*(2*I*a + b)*c*d*x - 2*a*d + (2*b*c*d*x - I*b*d)*
log(-(c*x + I)/(c*x - I)))/x^2
```

3.7.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(58) = 116.

Time = 1.90 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = {} & ibc^2d\log\left(35b^2c^5d^2x\right) \\ & - \frac{ibc^2d\log\left(35b^2c^5d^2x-35ib^2c^4d^2\right)}{4} \\ & - \frac{3ibc^2d\log\left(35b^2c^5d^2x+35ib^2c^4d^2\right)}{4} \\ & + \frac{-ad+x(-2iacd-bcd)}{2x^2} \\ & + \frac{(-2bcdx+ibd)\log(icx+1)}{4x^2} \\ & + \frac{(2bcdx-ibd)\log(-icx+1)}{4x^2}\end{aligned}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**3,x)
```

output
```
I*b*c**2*d*log(35*b**2*c**5*d**2*x) - I*b*c**2*d*log(35*b**2*c**5*d**2*x -
 35*I*b**2*c**4*d**2)/4 - 3*I*b*c**2*d*log(35*b**2*c**5*d**2*x + 35*I*b**2
*c**4*d**2)/4 + (-a*d + x*(-2*I*a*c*d - b*c*d))/(2*x**2) + (-2*b*c*d*x + I
*b*d)*log(I*c*x + 1)/(4*x**2) + (2*b*c*d*x - I*b*d)*log(-I*c*x + 1)/(4*x**
2)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bcd - \frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bd - \frac{i\,acd}{x}-\frac{ad}{2\,x^2}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d - 1/2*((c
*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - I*a*c*d/x - 1/2*a*d/x^2
```

3.7.8 Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx$$
$$= -\frac{\frac{d\,(a+b\,\mathrm{atan}(c\,x))}{2}+\frac{d\,x\,(a\,c\,2\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,2\mathrm{i})}{2}}{x^2}$$
$$-\frac{d\,\left(b\,c^2\,\mathrm{atan}(c\,x)+b\,c^2\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}-b\,c^2\,\ln\left(x\right)\,2\mathrm{i}\right)}{2}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^3,x)
```

output
```
- ((d*(a + b*atan(c*x)))/2 + (d*x*(a*c*2i + b*c + b*c*atan(c*x)*2i))/2)/x^
2 - (d*(b*c^2*atan(c*x) + b*c^2*log(c^2*x^2 + 1)*1i - b*c^2*log(x)*2i))/2
```

3.8 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx$

3.8.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd}{6x^2}-\frac{ibc^2d}{2x}-\frac{d(a+b\arctan(cx))}{3x^3} - \frac{icd(a+b\arctan(cx))}{2x^2}-\frac{1}{3}bc^3d\log(x) - \frac{1}{12}bc^3d\log(i-cx)+\frac{5}{12}bc^3d\log(i+cx)$$

output
```
-1/6*b*c*d/x^2-1/2*I*b*c^2*d/x-1/3*d*(a+b*arctan(c*x))/x^3-1/2*I*c*d*(a+b*
arctan(c*x))/x^2-1/3*b*c^3*d*ln(x)-1/12*b*c^3*d*ln(I-c*x)+5/12*b*c^3*d*ln(
c*x+I)
```

3.8.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = -\frac{d\left(2a+3iacx+bcx+b(2+3icx)\arctan(cx)+3ibc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+2bc^3x^3\right.}{6x^3}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(d*(2*a + (3*I)*a*c*x + b*c*x + b*(2 + (3*I)*c*x)*ArcTan[c*x] + (3*I)
*b*c^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 2*b*c^3*x^3*Log[x
] - b*c^3*x^3*Log[1 + c^2*x^2]))/x^3
```

3.8.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \ 5407$$

$$-bc\int -\frac{d(3icx+2)}{6x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6}bcd\int \frac{3icx+2}{x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \ 523$$

$$\frac{1}{6}bcd\int \left(-\frac{c^3}{2(cx-i)}+\frac{5c^3}{2(cx+i)}-\frac{2c^2}{x}+\frac{3ic}{x^2}+\frac{2}{x^3}\right)dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}+\frac{1}{6}bcd\left(-2c^2\log(x)-\frac{1}{2}c^2\log(-cx+i)+\frac{5}{2}c^2\log(cx+i)-\frac{3ic}{x}-\frac{1}{x^2}\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcTan[c*x]))/x^3 - ((I/2)*c*d*(a + b*ArcTan[c*x]))/x^2 + (
b*c*d*(-x^(-2) - ((3*I)*c)/x - 2*c^2*Log[x] - (c^2*Log[I - c*x])/2 + (5*c^
2*Log[I + c*x])/2))/6
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.8.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
parts	$ad\left(-\frac{ic}{2x^2}-\frac{1}{3x^3}\right)+bd\,c^3\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{2c^2x^2}-\frac{i}{2cx}-\frac{1}{6c^2x^2}-\frac{\ln(cx)}{3}+\frac{\ln(c^2x^2+1)}{6}-\frac{i\,\mathrm{arc}}{}\right.$
derivativedivides	$c^3\left(ad\left(-\frac{1}{3c^3x^3}-\frac{i}{2c^2x^2}\right)+bd\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{2c^2x^2}-\frac{i}{2cx}-\frac{1}{6c^2x^2}-\frac{\ln(cx)}{3}+\frac{\ln(c^2x^2+1)}{6}-\right.\right.$
default	$c^3\left(ad\left(-\frac{1}{3c^3x^3}-\frac{i}{2c^2x^2}\right)+bd\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{2c^2x^2}-\frac{i}{2cx}-\frac{1}{6c^2x^2}-\frac{\ln(cx)}{3}+\frac{\ln(c^2x^2+1)}{6}-\right.\right.$
parallelrisch	$-\frac{3ix^3\arctan(cx)b\,c^3d-3ia\,c^3d\,x^3-b\,c^3d\ln(c^2x^2+1)x^3+2b\,c^3d\ln(x)x^3-c^3x^3db+3ix^2b\,c^2d+3ix\arctan(cx)bcd+3iacd}{6x^3}$
risch	$-\frac{(3bcdx-2ibd)\ln(icx+1)}{12x^3}+\frac{d\left(5b\,c^3\ln(-cx-i)x^3-b\,c^3\ln(cx-i)x^3-4b\,c^3\ln(-x)x^3-6ib\,c^2x^2-6ixac+3bcx\ln(-icx+\right.}{12x^3}$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*d*(-1/2*I*c/x^2-1/3/x^3)+b*d*c^3*(-1/3*arctan(c*x)/c^3/x^3-1/2*I*arctan(
c*x)/c^2/x^2-1/2*I/c/x-1/6/c^2/x^2-1/3*ln(c*x)+1/6*ln(c^2*x^2+1)-1/2*I*arc
tan(c*x))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = -\frac{4\,bc^3dx^3\log\left(x\right)-5\,bc^3dx^3\log\left(\frac{cx+i}{c}\right)+bc^3dx^3\log\left(\frac{cx-i}{c}\right)+6i\,bc^2dx^2+2\,(3i\,a+b)cdx+4\,ad-(3\,bcdx}{12\,x^3}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/12*(4*b*c^3*d*x^3*log(x) - 5*b*c^3*d*x^3*log((c*x + I)/c) + b*c^3*d*x^3
*log((c*x - I)/c) + 6*I*b*c^2*d*x^2 + 2*(3*I*a + b)*c*d*x + 4*a*d - (3*b*c
*d*x - 2*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^3
```

3.8.6 Sympy [A] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.86

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = & -\frac{bc^3d\log\left(27b^2c^7d^2x\right)}{3} \\ & -\frac{bc^3d\log\left(27b^2c^7d^2x-27ib^2c^6d^2\right)}{12} \\ & +\frac{5bc^3d\log\left(27b^2c^7d^2x+27ib^2c^6d^2\right)}{12} \\ & +\frac{(-3bcdx+2ibd)\log\left(icx+1\right)}{12x^3} \\ & +\frac{(3bcdx-2ibd)\log\left(-icx+1\right)}{12x^3} \\ & +\frac{-2ad-3ibc^2dx^2+x(-3iacd-bcd)}{6x^3}\end{aligned}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**4,x)
```

output
```
-b*c**3*d*log(27*b**2*c**7*d**2*x)/3 - b*c**3*d*log(27*b**2*c**7*d**2*x -
27*I*b**2*c**6*d**2)/12 + 5*b*c**3*d*log(27*b**2*c**7*d**2*x + 27*I*b**2*c
**6*d**2)/12 + (-3*b*c*d*x + 2*I*b*d)*log(I*c*x + 1)/(12*x**3) + (3*b*c*d*
x - 2*I*b*d)*log(-I*c*x + 1)/(12*x**3) + (-2*a*d - 3*I*b*c**2*d*x**2 + x*(
-3*I*a*c*d - b*c*d))/(6*x**3)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\begin{aligned}
&\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx \\
&= -\frac{1}{2}i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bcd \\
&\quad +\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd-\frac{i\,acd}{2\,x^2}-\frac{ad}{3\,x^3}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d + 1/6*((c^2*log(c
^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d - 1/2*I*a*c
*d/x^2 - 1/3*a*d/x^3
```

3.8.8 Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx=\int \frac{(i\,cdx+d)(b\arctan\left(cx\right)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.66

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = \frac{b\,c^3\,d\,\ln\left(c^2\,x^2+1\right)}{6}$$
$$-\frac{\frac{a\,d}{3}-x^5\left(\frac{b\,c^5\,d}{6}+\frac{a\,c^5\,d\,1\mathrm{i}}{2}\right)+\frac{b\,d\,\mathrm{atan}(c\,x)}{3}+\frac{c\,d\,x\,(b+a\,3\mathrm{i})}{6}+\frac{c^2\,d\,x^2\,(2\,a+b\,3\mathrm{i})}{6}+\frac{b\,c^4\,d\,x^4\,1\mathrm{i}}{2}+\frac{b\,c^2\,d\,x^2\,\mathrm{atan}(c\,x)}{3}+\frac{b\,c^3\,d\,x^3\,\mathrm{ata}}{2}}{c^2\,x^5+x^3}$$
$$-\frac{b\,c^3\,d\,\ln(x)}{3}-\frac{b\,d\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,\left(c^2\right)^{3/2}\,1\mathrm{i}}{2}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^4,x)
```

output
```
(b*c^3*d*log(c^2*x^2 + 1))/6 - (b*d*atan((c^2*x)/(c^2)^(1/2))*(c^2)^(3/2)*
1i)/2 - ((a*d)/3 - x^5*((a*c^5*d*1i)/2 + (b*c^5*d)/6) + (b*d*atan(c*x))/3
+ (c*d*x*(a*3i + b))/6 + (c^2*d*x^2*(2*a + b*3i))/6 + (b*c^4*d*x^4*1i)/2 +
 (b*c^2*d*x^2*atan(c*x))/3 + (b*c^3*d*x^3*atan(c*x)*1i)/2 + (b*c*d*x*atan(
c*x)*1i)/2)/(x^3 + c^2*x^5) - (b*c^3*d*log(x))/3
```

3.9 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx$

3.9.1 Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd}{12x^3}-\frac{ibc^2d}{6x^2}+\frac{bc^3d}{4x}-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}-\frac{1}{3}ibc^4d\log(x)+\frac{1}{24}ibc^4d\log(i-cx)+\frac{7}{24}ibc^4d\log(i+cx)$$

output
```
-1/12*b*c*d/x^3-1/6*I*b*c^2*d/x^2+1/4*b*c^3*d/x-1/4*d*(a+b*arctan(c*x))/x^
4-1/3*I*c*d*(a+b*arctan(c*x))/x^3-1/3*I*b*c^4*d*ln(x)+1/24*I*b*c^4*d*ln(I-
c*x)+7/24*I*b*c^4*d*ln(c*x+I)
```

3.9.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx = -\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}-\frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}-\frac{1}{6}ibc^2d\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d*(a + b*ArcTan[c*x]))/x^4 - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 - (
b*c*d*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3) - (I/6)*b*c^2
*d*(x^(-2) + 2*c^2*Log[x] - c^2*Log[1 + c^2*x^2])
```

3.9.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}dx$$
$$\Big\downarrow \ 5407$$
$$-bc\int -\frac{d(4icx+3)}{12x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}$$
$$\Big\downarrow \ 27$$
$$\frac{1}{12}bcd\int \frac{4icx+3}{x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}$$
$$\Big\downarrow \ 523$$
$$\frac{1}{12}bcd\int \left(\frac{ic^4}{2(cx-i)}+\frac{7ic^4}{2(cx+i)}-\frac{4ic^3}{x}-\frac{3c^2}{x^2}+\frac{4ic}{x^3}+\frac{3}{x^4}\right)dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}$$
$$\Big\downarrow \ 2009$$
$$-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}+\frac{1}{12}bcd\left(-4ic^3\log(x)+\frac{1}{2}ic^3\log(-cx+i)+\frac{7}{2}ic^3\log(cx+i)+\frac{3c^2}{x}-\frac{2ic}{x^2}-\frac{1}{x^3}\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d*(a + b*ArcTan[c*x]))/x^4 - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 + (
b*c*d*(-x^(-3) - ((2*I)*c)/x^2 + (3*c^2)/x - (4*I)*c^3*Log[x] + (I/2)*c^3*
Log[I - c*x] + ((7*I)/2)*c^3*Log[I + c*x]))/12
```

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.9.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

method	result
parts	$ad\left(-\frac{1}{4x^4}-\frac{ic}{3x^3}\right)+bd\,c^4\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{3c^3x^3}-\frac{i}{6c^2x^2}-\frac{i\ln(cx)}{3}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{i\ln(c^2x\cdots}{6}\right.$
derivativedivides	$c^4\left(ad\left(-\frac{1}{4c^4x^4}-\frac{i}{3c^3x^3}\right)+bd\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{3c^3x^3}-\frac{i}{6c^2x^2}-\frac{i\ln(cx)}{3}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{i\,l\cdots}{}\right.\right.$
default	$c^4\left(ad\left(-\frac{1}{4c^4x^4}-\frac{i}{3c^3x^3}\right)+bd\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{3c^3x^3}-\frac{i}{6c^2x^2}-\frac{i\ln(cx)}{3}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{i\,l\cdots}{}\right.\right.$
parallelrisch	$\frac{2i\ln(c^2x^2+1)x^4b\,c^4d-4i\ln(x)x^4b\,c^4d+2ix^4b\,c^4d+3x^4\arctan(cx)b\,c^4d+3c^3x^3db-2ix^2b\,c^2d-4ix\arctan(cx)bcd-4iacd\cdots}{12x^4}$
risch	$-\frac{(4bcdx-3ibd)\ln(icx+1)}{24x^4}+\frac{id\left(7b\,c^4\ln(-15cx-15i)x^4+b\,c^4\ln(9cx-9i)x^4-8b\,c^4\ln(-45cx)x^4-6ib\,c^3x^3-4b\,c^2x^2-8c\cdots\right.}{24x^4}$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*d*(-1/4/x^4-1/3*I*c/x^3)+b*d*c^4*(-1/4*arctan(c*x)/c^4/x^4-1/3*I*arctan(
c*x)/c^3/x^3-1/6*I/c^2/x^2-1/3*I*ln(c*x)-1/12/c^3/x^3+1/4/c/x+1/6*I*ln(c^2
*x^2+1)+1/4*arctan(c*x))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{-8i\,bc^4dx^4\log{(x)} + 7i\,bc^4dx^4\log{\left(\frac{cx+i}{c}\right)} + i\,bc^4dx^4\log{\left(\frac{cx-i}{c}\right)} + 6\,bc^3dx^3 - 4i\,bc^2dx^2 - 2\,(4i\,a+b)cdx - 6}{24\,x^4}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/24*(-8*I*b*c^4*d*x^4*log(x) + 7*I*b*c^4*d*x^4*log((c*x + I)/c) + I*b*c^4
*d*x^4*log((c*x - I)/c) + 6*b*c^3*d*x^3 - 4*I*b*c^2*d*x^2 - 2*(4*I*a + b)*
c*d*x - 6*a*d + (4*b*c*d*x - 3*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^4
```

3.9.6 Sympy [A] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.73

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx = -\frac{ibc^4d\log{(135b^2c^9d^2x)}}{3} + \frac{ibc^4d\log{(135b^2c^9d^2x - 135ib^2c^8d^2)}}{24} + \frac{7ibc^4d\log{(135b^2c^9d^2x + 135ib^2c^8d^2)}}{24} + \frac{(-4bcdx + 3ibd)\log{(icx+1)}}{24x^4} + \frac{(4bcdx - 3ibd)\log{(-icx+1)}}{24x^4} + \frac{-3ad + 3bc^3dx^3 - 2ibc^2dx^2 + x(-4iacd - bcd)}{12x^4}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**5,x)
```

output
```
-I*b*c**4*d*log(135*b**2*c**9*d**2*x)/3 + I*b*c**4*d*log(135*b**2*c**9*d**
2*x - 135*I*b**2*c**8*d**2)/24 + 7*I*b*c**4*d*log(135*b**2*c**9*d**2*x + 1
35*I*b**2*c**8*d**2)/24 + (-4*b*c*d*x + 3*I*b*d)*log(I*c*x + 1)/(24*x**4)
+ (4*b*c*d*x - 3*I*b*d)*log(-I*c*x + 1)/(24*x**4) + (-3*a*d + 3*b*c**3*d*x
**3 - 2*I*b*c**2*d*x**2 + x*(-4*I*a*c*d - b*c*d))/(12*x**4)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx \\
&= \frac{1}{6}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bcd \\
&\quad+\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bd-\frac{i\,acd}{3\,x^3}-\frac{ad}{4\,x^4}
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3
)*b*c*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x
)/x^4)*b*d - 1/3*I*a*c*d/x^3 - 1/4*a*d/x^4
```

3.9.8 Giac [F]

$$
\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(i\,cdx+d)(b\arctan\left(cx\right)+a)}{x^5}\,dx
$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{d\left(\frac{3\,b\,c^7\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)}{(c^2)^{3/2}}+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,2\mathrm{i}-b\,c^4\,\ln\left(x\right)\,4\mathrm{i}\right)}{12} - \frac{\frac{d\,(3\,a+3\,b\,\mathrm{atan}(c\,x))}{12}+\frac{d\,x\,(a\,c\,4\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,4\mathrm{i})}{12}-\frac{b\,c^3\,d\,x^3}{4}+\frac{b\,c^2\,d\,x^2\,1\mathrm{i}}{6}}{x^4}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^5,x)
```

output
```
(d*(b*c^4*log(c^2*x^2 + 1)*2i - b*c^4*log(x)*4i + (3*b*c^7*atan((c^2*x)/(c
^2)^(1/2)))/(c^2)^(3/2)))/12 - ((d*(3*a + 3*b*atan(c*x)))/12 + (d*x*(a*c*4
i + b*c + b*c*atan(c*x)*4i))/12 + (b*c^2*d*x^2*1i)/6 - (b*c^3*d*x^3)/4)/x^
4
```

3.10 $\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$

3.10.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx &= \frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5\\&\quad-\frac{5bd^2\arctan(cx)}{12c^4}+\frac{1}{4}d^2x^4(a+b\arctan(cx))\\&\quad+\frac{2}{5}icd^2x^5(a+b\arctan(cx))\\&\quad-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))-\frac{ibd^2\log(1+c^2x^2)}{5c^4}\end{aligned}$$

output
```
5/12*b*d^2*x/c^3+1/5*I*b*d^2*x^2/c^2-5/36*b*d^2*x^3/c-1/10*I*b*d^2*x^4+1/3
0*b*c*d^2*x^5-5/12*b*d^2*arctan(c*x)/c^4+1/4*d^2*x^4*(a+b*arctan(c*x))+2/5
*I*c*d^2*x^5*(a+b*arctan(c*x))-1/6*c^2*d^2*x^6*(a+b*arctan(c*x))-1/5*I*b*d
^2*ln(c^2*x^2+1)/c^4
```

3.10.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$=\frac{d^2(3ac^4x^4(15+24icx-10c^2x^2)+bcx(75+36icx-25c^2x^2-18ic^3x^3+6c^4x^4)+3b(-25+15c^4x^4+24i}{180c^4}$$

input
```
Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*(3*a*c^4*x^4*(15 + (24*I)*c*x - 10*c^2*x^2) + b*c*x*(75 + (36*I)*c*x
- 25*c^2*x^2 - (18*I)*c^3*x^3 + 6*c^4*x^4) + 3*b*(-25 + 15*c^4*x^4 + (24*I
)*c^5*x^5 - 10*c^6*x^6)*ArcTan[c*x] - (36*I)*b*Log[1 + c^2*x^2]))/(180*c^4
)
```

3.10.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^2x^4\left(-10c^2x^2+24icx+15\right)}{60\left(c^2x^2+1\right)}dx-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{60}bcd^2\int \frac{x^4\left(-10c^2x^2+24icx+15\right)}{c^2x^2+1}dx-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{60}bcd^2\int \left(-10x^4+\frac{24ix^3}{c}+\frac{25x^2}{c^2}-\frac{24ix}{c^3}+\frac{24icx+25}{c^4\left(c^2x^2+1\right)}-\frac{25}{c^4}\right)dx-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))-\frac{1}{60}bcd^2\left(\frac{25\arctan(cx)}{c^5}-\frac{25x}{c^4}-\frac{12ix^2}{c^3}+\frac{25x^3}{3c^2}+\frac{12i\log\left(c^2x^2+1\right)}{c^5}+\frac{6ix^4}{c}-2x^5\right)$$

input
```
Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcTan[c*x]))/4 + ((2*I)/5)*c*d^2*x^5*(a + b*ArcTan[c*x])
- (c^2*d^2*x^6*(a + b*ArcTan[c*x]))/6 - (b*c*d^2*((-25*x)/c^4 - ((12*I)*x^
2)/c^3 + (25*x^3)/(3*c^2) + ((6*I)*x^4)/c - 2*x^5 + (25*ArcTan[c*x])/c^5 +
 ((12*I)*Log[1 + c^2*x^2])/c^5))/60
```

3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.10.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79

method	result
parts	$a\,d^2\left(-\frac{1}{6}c^2x^6+\frac{2}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{b\,d^2\left(-\frac{\arctan(cx)c^6x^6}{6}+\frac{2i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{5cx}{12}+\frac{c^5x^5}{30}-\frac{ic^4x^4}{10}\right.}{c^4}$
derivativedivides	$\frac{a\,d^2\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b\,d^2\left(-\frac{\arctan(cx)c^6x^6}{6}+\frac{2i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{5cx}{12}+\frac{c^5x^5}{30}-\frac{ic^4x^4}{10}-\frac{5c^3x^3}{36}\right.}{c^4}$
default	$\frac{a\,d^2\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b\,d^2\left(-\frac{\arctan(cx)c^6x^6}{6}+\frac{2i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{5cx}{12}+\frac{c^5x^5}{30}-\frac{ic^4x^4}{10}-\frac{5c^3x^3}{36}\right.}{c^4}$
parallelrisch	$-\frac{30x^6\arctan(cx)b\,c^6d^2-72ic^5b\,d^2\arctan(cx)x^5+30a\,c^6d^2x^6-72ix^5a\,c^5d^2-6b\,c^5d^2x^5+18ix^4b\,c^4d^2-45x^4\arctan(cx)b}{180c^4}$
risch	$\frac{id^2b(10c^2x^6-24ic\,x^5-15x^4)\ln(icx+1)}{120}-\frac{a\,c^2d^2x^6}{6}-\frac{id^2c^2x^6b\ln(-icx+1)}{12}-\frac{d^2cb\,x^5\ln(-icx+1)}{5}+\frac{bc\,d^2x^5}{30}+$

input
```
int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-1/6*c^2*x^6+2/5*I*c*x^5+1/4*x^4)+b*d^2/c^4*(-1/6*arctan(c*x)*c^6*x
^6+2/5*I*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*arctan(c*x)+5/12*c*x+1/30*c^5*x^5
-1/10*I*c^4*x^4-5/36*c^3*x^3+1/5*I*c^2*x^2-1/5*I*ln(c^2*x^2+1)-5/12*arctan
(c*x))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx =$$
$$-\frac{60\,ac^6d^2x^6+12\,(-12i\,a-b)c^5d^2x^5-18\,(5\,a-2i\,b)c^4d^2x^4+50\,bc^3d^2x^3-72i\,bc^2d^2x^2-150\,bcd^2x+14}{360\,c^4}$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/360*(60*a*c^6*d^2*x^6 + 12*(-12*I*a - b)*c^5*d^2*x^5 - 18*(5*a - 2*I*b)
*c^4*d^2*x^4 + 50*b*c^3*d^2*x^3 - 72*I*b*c^2*d^2*x^2 - 150*b*c*d^2*x + 147
*I*b*d^2*log((c*x + I)/c) - 3*I*b*d^2*log((c*x - I)/c) + 3*(10*I*b*c^6*d^2
*x^6 + 24*b*c^5*d^2*x^5 - 15*I*b*c^4*d^2*x^4)*log(-(c*x + I)/(c*x - I)))/c
^4
```

3.10.6 Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.63

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{ac^2d^2x^6}{6} - \frac{5bd^2x^3}{36c} + \frac{ibd^2x^2}{5c^2} + \frac{5bd^2x}{12c^3} - \frac{bd^2\left(-\frac{i\log\left(291bcd^2x-291ibd^2\right)}{120} + \frac{71i\log\left(291bcd^2x+291ibd^2\right)}{210}\right)}{c^4} - x^5\left(-\frac{2iacd^2}{5} - \frac{bcd^2}{30}\right) - x^4\left(-\frac{ad^2}{4} + \frac{ibd^2}{10}\right) + \left(\frac{ibc^2d^2x^6}{12} + \frac{bcd^2x^5}{5} - \frac{ibd^2x^4}{8}\right)\log\left(icx+1\right) + \frac{\left(-70ibc^6d^2x^6 - 168bc^5d^2x^5 + 105ibc^4d^2x^4 - 59ibd^2\right)\log\left(-icx+1\right)}{840c^4}$$

input
```
integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**6/6 - 5*b*d**2*x**3/(36*c) + I*b*d**2*x**2/(5*c**2) + 5*b*
d**2*x/(12*c**3) - b*d**2*(-I*log(291*b*c*d**2*x - 291*I*b*d**2)/120 + 71*
I*log(291*b*c*d**2*x + 291*I*b*d**2)/210)/c**4 - x**5*(-2*I*a*c*d**2/5 - b
*c*d**2/30) - x**4*(-a*d**2/4 + I*b*d**2/10) + (I*b*c**2*d**2*x**6/12 + b*
c*d**2*x**5/5 - I*b*d**2*x**4/8)*log(I*c*x + 1) + (-70*I*b*c**6*d**2*x**6
- 168*b*c**5*d**2*x**5 + 105*I*b*c**4*d**2*x**4 - 59*I*b*d**2)*log(-I*c*x
+ 1)/(840*c**4)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{1}{6}\,ac^2d^2x^6 + \frac{2}{5}i\,acd^2x^5 + \frac{1}{4}\,ad^2x^4 - \frac{1}{90}\left(15\,x^6\arctan\left(cx\right) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bc^2d^2 + \frac{1}{10}i\left(4\,x^5\arctan\left(cx\right) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bcd^2 + \frac{1}{12}\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd^2$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/6*a*c^2*d^2*x^6 + 2/5*I*a*c*d^2*x^5 + 1/4*a*d^2*x^4 - 1/90*(15*x^6*arct
an(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c
^2*d^2 + 1/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*
x^2 + 1)/c^6))*b*c*d^2 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4
+ 3*arctan(c*x)/c^5))*b*d^2
```

3.10.8 Giac [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^2(b\arctan(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx \\
&= -\frac{\frac{d^2\,\left(75\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,36\mathrm{i}\right)}{180}+\frac{5\,b\,c^3\,d^2\,x^3}{36}-\frac{5\,b\,c\,d^2\,x}{12}-\frac{b\,c^2\,d^2\,x^2\,1\mathrm{i}}{5}}{c^4} \\
&\quad+\frac{d^2\,\left(45\,a\,x^4+45\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,18\mathrm{i}\right)}{180}-\frac{c^2\,d^2\,\left(30\,a\,x^6+30\,b\,x^6\,\mathrm{atan}(c\,x)\right)}{180} \\
&\quad+\frac{c\,d^2\,\left(a\,x^5\,72\mathrm{i}+6\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,72\mathrm{i}\right)}{180}
\end{aligned}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
(d^2*(45*a*x^4 - b*x^4*18i + 45*b*x^4*atan(c*x)))/180 - ((d^2*(75*b*atan(c
*x) + b*log(c^2*x^2 + 1)*36i))/180 - (b*c^2*d^2*x^2*1i)/5 + (5*b*c^3*d^2*x
^3)/36 - (5*b*c*d^2*x)/12)/c^4 - (c^2*d^2*(30*a*x^6 + 30*b*x^6*atan(c*x)))
/180 + (c*d^2*(a*x^5*72i + 6*b*x^5 + b*x^5*atan(c*x)*72i))/180
```

3.11 $\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$

3.11.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned}\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx &= \frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4\\&\quad-\frac{ibd^2\arctan(cx)}{2c^3}+\frac{1}{3}d^2x^3(a+b\arctan(cx))\\&\quad+\frac{1}{2}icd^2x^4(a+b\arctan(cx))\\&\quad-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{4bd^2\log(1+c^2x^2)}{15c^3}\end{aligned}$$

output
```
1/2*I*b*d^2*x/c^2-4/15*b*d^2*x^2/c-1/6*I*b*d^2*x^3+1/20*b*c*d^2*x^4-1/2*I*
b*d^2*arctan(c*x)/c^3+1/3*d^2*x^3*(a+b*arctan(c*x))+1/2*I*c*d^2*x^4*(a+b*a
rctan(c*x))-1/5*c^2*d^2*x^5*(a+b*arctan(c*x))+4/15*b*d^2*ln(c^2*x^2+1)/c^3
```

3.11.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx\\&=\frac{d^2(2ac^3x^3(10+15icx-6c^2x^2)+bcx(30i-16cx-10ic^2x^2+3c^3x^3)+2b(-15i+10c^3x^3+15ic^4x^4-6c^5}{60c^3}\end{aligned}$$

input
```
Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*(2*a*c^3*x^3*(10 + (15*I)*c*x - 6*c^2*x^2) + b*c*x*(30*I - 16*c*x - (
10*I)*c^2*x^2 + 3*c^3*x^3) + 2*b*(-15*I + 10*c^3*x^3 + (15*I)*c^4*x^4 - 6*
c^5*x^5)*ArcTan[c*x] + 16*b*Log[1 + c^2*x^2]))/(60*c^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^2x^3\left(-6c^2x^2+15icx+10\right)}{30\left(c^2x^2+1\right)}dx-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{30}bcd^2\int \frac{x^3\left(-6c^2x^2+15icx+10\right)}{c^2x^2+1}dx-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{30}bcd^2\int \left(-6x^3+\frac{15ix^2}{c}+\frac{16x}{c^2}+\frac{15i-16cx}{c^3\left(c^2x^2+1\right)}-\frac{15i}{c^3}\right)dx-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))-\frac{1}{30}bcd^2\left(\frac{15i\arctan(cx)}{c^4}-\frac{15ix}{c^3}+\frac{8x^2}{c^2}-\frac{8\log\left(c^2x^2+1\right)}{c^4}+\frac{5ix^3}{c}-\frac{3x^4}{2}\right)$$

input
```
Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcTan[c*x]))/3 + (I/2)*c*d^2*x^4*(a + b*ArcTan[c*x]) - (c
^2*d^2*x^5*(a + b*ArcTan[c*x]))/5 - (b*c*d^2*(((-15*I)*x)/c^3 + (8*x^2)/c^
2 + ((5*I)*x^3)/c - (3*x^4)/2 + ((15*I)*ArcTan[c*x])/c^4 - (8*Log[1 + c^2*
x^2])/c^4))/30
```

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.11.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

method	result
parts	$a\,d^2\left(-\frac{1}{5}c^2x^5+\frac{1}{2}ic\,x^4+\frac{1}{3}x^3\right)+\frac{b\,d^2\left(-\frac{c^5x^5\arctan(cx)}{5}+\frac{i\arctan(cx)c^4x^4}{2}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{2}+\frac{c^4x^4}{20}-\frac{ic^3x^3}{6}\right.}{c^3}$
derivativedivides	$\frac{a\,d^2\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+b\,d^2\left(-\frac{c^5x^5\arctan(cx)}{5}+\frac{i\arctan(cx)c^4x^4}{2}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{2}+\frac{c^4x^4}{20}-\frac{ic^3x^3}{6}-\frac{4c^2x^2}{15}+\right.}{c^3}$
default	$\frac{a\,d^2\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+b\,d^2\left(-\frac{c^5x^5\arctan(cx)}{5}+\frac{i\arctan(cx)c^4x^4}{2}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{2}+\frac{c^4x^4}{20}-\frac{ic^3x^3}{6}-\frac{4c^2x^2}{15}+\right.}{c^3}$
parallelrisch	$\frac{-12c^5b\,d^2\arctan(cx)x^5+30ix^4\arctan(cx)b\,c^4d^2-12a\,c^5d^2x^5+30ix^4a\,c^4d^2+3b\,c^4d^2x^4-10ix^3b\,c^3d^2+20x^3\arctan(cx)b}{60c^3}$
risch	$\frac{id^2b\left(6c^2x^5-15ic\,x^4-10x^3\right)\ln(icx+1)}{60}-\frac{id^2c^2b\,x^5\ln(-icx+1)}{10}-\frac{a\,c^2d^2x^5}{5}+\frac{iac\,d^2x^4}{2}-\frac{d^2c\,x^4b\ln(-icx+1)}{4}+$

input
```
int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-1/5*c^2*x^5+1/2*I*x^4*c+1/3*x^3)+b*d^2/c^3*(-1/5*c^5*x^5*arctan(c*
x)+1/2*I*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*arctan(c*x)+1/2*I*c*x+1/20*c^4*x^
4-1/6*I*c^3*x^3-4/15*c^2*x^2+4/15*ln(c^2*x^2+1)-1/2*I*arctan(c*x))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx =$$
$$-\frac{12\,ac^5d^2x^5+3\,(-10i\,a-b)c^4d^2x^4-10\,(2\,a-i\,b)c^3d^2x^3+16\,bc^2d^2x^2-30i\,bcd^2x-31\,bd^2\log\left(\frac{cx+i}{c}\right)-}{60\,c^3}$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/60*(12*a*c^5*d^2*x^5 + 3*(-10*I*a - b)*c^4*d^2*x^4 - 10*(2*a - I*b)*c^3
*d^2*x^3 + 16*b*c^2*d^2*x^2 - 30*I*b*c*d^2*x - 31*b*d^2*log((c*x + I)/c) -
 b*d^2*log((c*x - I)/c) - (-6*I*b*c^5*d^2*x^5 - 15*b*c^4*d^2*x^4 + 10*I*b*
c^3*d^2*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

3.11.6 Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.64

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{ac^2d^2x^5}{5} - \frac{4bd^2x^2}{15c} + \frac{ibd^2x}{2c^2} - \frac{bd^2\left(-\frac{\log(47bcd^2x-47ibd^2)}{60} - \frac{49\log(47bcd^2x+47ibd^2)}{120}\right)}{c^3}$$
$$-x^4\left(-\frac{iacd^2}{2} - \frac{bcd^2}{20}\right) - x^3\left(-\frac{ad^2}{3} + \frac{ibd^2}{6}\right) + \left(\frac{ibc^2d^2x^5}{10} + \frac{bcd^2x^4}{4} - \frac{ibd^2x^3}{6}\right)\log(icx+1)$$
$$+ \frac{(-12ibc^5d^2x^5 - 30bc^4d^2x^4 + 20ibc^3d^2x^3 + 13bd^2)\log(-icx+1)}{120c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**5/5 - 4*b*d**2*x**2/(15*c) + I*b*d**2*x/(2*c**2) - b*d**2*
(-log(47*b*c*d**2*x - 47*I*b*d**2)/60 - 49*log(47*b*c*d**2*x + 47*I*b*d**2
)/120)/c**3 - x**4*(-I*a*c*d**2/2 - b*c*d**2/20) - x**3*(-a*d**2/3 + I*b*d
**2/6) + (I*b*c**2*d**2*x**5/10 + b*c*d**2*x**4/4 - I*b*d**2*x**3/6)*log(I
*c*x + 1) + (-12*I*b*c**5*d**2*x**5 - 30*b*c**4*d**2*x**4 + 20*I*b*c**3*d*
*2*x**3 + 13*b*d**2)*log(-I*c*x + 1)/(120*c**3)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{1}{5}ac^2d^2x^5 + \frac{1}{2}i\,acd^2x^4$$
$$-\frac{1}{20}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4-2x^2}{c^4} + \frac{2\log(c^2x^2+1)}{c^6}\right)\right)bc^2d^2$$
$$+\frac{1}{3}ad^2x^3 + \frac{1}{6}i\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bcd^2$$
$$+\frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^2$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/5*a*c^2*d^2*x^5 + 1/2*I*a*c*d^2*x^4 - 1/20*(4*x^5*arctan(c*x) - c*((c^2
*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1
/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c
*d^2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2
```

3.11.8 Giac [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx=-\frac{\frac{d^2\,\left(-16\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}\left(c\,x\right)\,30{}\mathrm{i}\right)}{60}+\frac{4\,b\,c^2\,d^2\,x^2}{15}-\frac{b\,c\,d^2\,x\,1{}\mathrm{i}}{2}}{c^3}+\frac{d^2\,\left(20\,a\,x^3+20\,b\,x^3\,\mathrm{atan}\left(c\,x\right)-b\,x^3\,10{}\mathrm{i}\right)}{60}-\frac{c^2\,d^2\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atan}\left(c\,x\right)\right)}{60}+\frac{c\,d^2\,\left(a\,x^4\,30{}\mathrm{i}+3\,b\,x^4+b\,x^4\,\mathrm{atan}\left(c\,x\right)\,30{}\mathrm{i}\right)}{60}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
(d^2*(20*a*x^3 - b*x^3*10i + 20*b*x^3*atan(c*x)))/60 - ((d^2*(b*atan(c*x)*
30i - 16*b*log(c^2*x^2 + 1)))/60 + (4*b*c^2*d^2*x^2)/15 - (b*c*d^2*x*1i)/2
)/c^3 - (c^2*d^2*(12*a*x^5 + 12*b*x^5*atan(c*x)))/60 + (c*d^2*(a*x^4*30i +
 3*b*x^4 + b*x^4*atan(c*x)*30i))/60
```

3.12 $\int x(d+icdx)^2(a+b\arctan(cx))\,dx$

3.12.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\begin{aligned}\int x(d+icdx)^2(a+b\arctan(cx))\,dx =& -\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{3bd^2\arctan(cx)}{4c^2}\\ &+\frac{1}{2}d^2x^2(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))\\ &-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{ibd^2\log(1+c^2x^2)}{3c^2}\end{aligned}$$

output
```
-3/4*b*d^2*x/c-1/3*I*b*d^2*x^2+1/12*b*c*d^2*x^3+3/4*b*d^2*arctan(c*x)/c^2+
1/2*d^2*x^2*(a+b*arctan(c*x))+2/3*I*c*d^2*x^3*(a+b*arctan(c*x))-1/4*c^2*d^
2*x^4*(a+b*arctan(c*x))+1/3*I*b*d^2*ln(c^2*x^2+1)/c^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$=\frac{d^2(cx(acx(6+8icx-3c^2x^2)+b(-9-4icx+c^2x^2))+b(9+6c^2x^2+8ic^3x^3-3c^4x^4)\arctan(cx)+4ib\,\mathrm{lo}}{12c^2}$$

input
```
Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*(c*x*(a*c*x*(6 + (8*I)*c*x - 3*c^2*x^2) + b*(-9 - (4*I)*c*x + c^2*x^2
)) + b*(9 + 6*c^2*x^2 + (8*I)*c^3*x^3 - 3*c^4*x^4)*ArcTan[c*x] + (4*I)*b*L
og[1 + c^2*x^2]))/(12*c^2)
```

3.12.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^2x^2\left(-3c^2x^2+8icx+6\right)}{12\left(c^2x^2+1\right)}dx-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bcd^2\int \frac{x^2\left(-3c^2x^2+8icx+6\right)}{c^2x^2+1}dx-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{12}bcd^2\int \left(-3x^2+\frac{8ix}{c}+\frac{i(9i-8cx)}{c^2\left(c^2x^2+1\right)}+\frac{9}{c^2}\right)dx-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))-\frac{1}{12}bcd^2\left(-\frac{9\arctan(cx)}{c^3}+\frac{9x}{c^2}-\frac{4i\log\left(c^2x^2+1\right)}{c^3}+\frac{4ix^2}{c}-x^3\right)$$

input
```
Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^2*(a + b*ArcTan[c*x]))/2 + ((2*I)/3)*c*d^2*x^3*(a + b*ArcTan[c*x])
- (c^2*d^2*x^4*(a + b*ArcTan[c*x]))/4 - (b*c*d^2*((9*x)/c^2 + ((4*I)*x^2)/
c - x^3 - (9*ArcTan[c*x])/c^3 - ((4*I)*Log[1 + c^2*x^2])/c^3))/12
```

3.12.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.12.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

method	result
parts	$a\,d^2\left(-\frac{1}{4}c^2x^4+\frac{2}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{b\,d^2\left(-\frac{c^4x^4\arctan(cx)}{4}+\frac{2i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{3cx}{4}+\frac{c^3x^3}{12}-\frac{ic^2x}{3}\right.}{c^2}$
derivativedivides	$\frac{a\,d^2\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+b\,d^2\left(-\frac{c^4x^4\arctan(cx)}{4}+\frac{2i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{3cx}{4}+\frac{c^3x^3}{12}-\frac{ic^2x^2}{3}+\frac{i\ln(c^2}{3}\right.}{c^2}$
default	$\frac{a\,d^2\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+b\,d^2\left(-\frac{c^4x^4\arctan(cx)}{4}+\frac{2i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{3cx}{4}+\frac{c^3x^3}{12}-\frac{ic^2x^2}{3}+\frac{i\ln(c^2}{3}\right.}{c^2}$
parallelrisch	$\frac{-3x^4\arctan(cx)b\,c^4d^2+8ix^3\arctan(cx)b\,c^3d^2-3a\,c^4d^2x^4+8ix^3a\,c^3d^2+b\,c^3d^2x^3-4ix^2b\,c^2d^2+6x^2\arctan(cx)b\,c^2d^2+6a}{12c^2}$
risch	$\frac{id^2b\left(3c^2x^4-8ic\,x^3-6x^2\right)\ln(icx+1)}{24}-\frac{x^4d^2c^2a}{4}-\frac{id^2c^2x^4b\ln(-icx+1)}{8}-\frac{d^2cb\,x^3\ln(-icx+1)}{3}+\frac{x^3d^2cb}{12}+\frac{2iac}{}$

input
```
int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-1/4*c^2*x^4+2/3*I*c*x^3+1/2*x^2)+b*d^2/c^2*(-1/4*c^4*x^4*arctan(c*
x)+2/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-3/4*c*x+1/12*c^3*x^3-
1/3*I*c^2*x^2+1/3*I*ln(c^2*x^2+1)+3/4*arctan(c*x))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx =$$
$$-\frac{6\,ac^4d^2x^4+2\,(-8i\,a-b)c^3d^2x^3-4\,(3\,a-2i\,b)c^2d^2x^2+18\,bcd^2x-17i\,bd^2\log\left(\frac{cx+i}{c}\right)+i\,bd^2\log\left(\frac{cx-i}{c}\right)}{24\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/24*(6*a*c^4*d^2*x^4 + 2*(-8*I*a - b)*c^3*d^2*x^3 - 4*(3*a - 2*I*b)*c^2*
d^2*x^2 + 18*b*c*d^2*x - 17*I*b*d^2*log((c*x + I)/c) + I*b*d^2*log((c*x -
I)/c) - (-3*I*b*c^4*d^2*x^4 - 8*b*c^3*d^2*x^3 + 6*I*b*c^2*d^2*x^2)*log(-(c
*x + I)/(c*x - I)))/c^2
```

3.12.6 Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.76

$$\begin{aligned}
&\int x(d+icdx)^2(a+b\arctan(cx))\,dx \\
&= -\frac{ac^2d^2x^4}{4} - \frac{3bd^2x}{4c} - \frac{bd^2\left(\frac{i\log\left(67bcd^2x-67ibd^2\right)}{24} - \frac{31i\log\left(67bcd^2x+67ibd^2\right)}{60}\right)}{c^2} \\
&\quad - x^3\left(-\frac{2iacd^2}{3} - \frac{bcd^2}{12}\right) - x^2\left(-\frac{ad^2}{2} + \frac{ibd^2}{3}\right) \\
&\quad + \left(\frac{ibc^2d^2x^4}{8} + \frac{bcd^2x^3}{3} - \frac{ibd^2x^2}{4}\right)\log\left(icx+1\right) \\
&\quad + \frac{\left(-15ibc^4d^2x^4 - 40bc^3d^2x^3 + 30ibc^2d^2x^2 + 23ibd^2\right)\log\left(-icx+1\right)}{120c^2}
\end{aligned}$$

input
```
integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**4/4 - 3*b*d**2*x/(4*c) - b*d**2*(I*log(67*b*c*d**2*x - 67*
I*b*d**2)/24 - 31*I*log(67*b*c*d**2*x + 67*I*b*d**2)/60)/c**2 - x**3*(-2*I
*a*c*d**2/3 - b*c*d**2/12) - x**2*(-a*d**2/2 + I*b*d**2/3) + (I*b*c**2*d**
2*x**4/8 + b*c*d**2*x**3/3 - I*b*d**2*x**2/4)*log(I*c*x + 1) + (-15*I*b*c*
*4*d**2*x**4 - 40*b*c**3*d**2*x**3 + 30*I*b*c**2*d**2*x**2 + 23*I*b*d**2)*
log(-I*c*x + 1)/(120*c**2)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14

$$\begin{aligned}
&\int x(d+icdx)^2(a+b\arctan(cx))\,dx \\
&= -\frac{1}{4}\,ac^2d^2x^4 + \frac{2}{3}i\,acd^2x^3 - \frac{1}{12}\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bc^2d^2 \\
&\quad + \frac{1}{3}i\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd^2 \\
&\quad + \frac{1}{2}\,ad^2x^2 + \frac{1}{2}\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^2
\end{aligned}$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/4*a*c^2*d^2*x^4 + 2/3*I*a*c*d^2*x^3 - 1/12*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^2 + 1/3*I*(2*x^3*arctan(c*x)
 - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^2 + 1/2*a*d^2*x^2 + 1/2*(x^2*
arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^2
```

3.12.8 Giac [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx=\int(i\,cdx+d)^2(b\arctan{(cx)}+a)x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx=\frac{\frac{d^2\,\left(9\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,4\mathrm{i}\right)}{12}-\frac{3\,b\,c\,d^2\,x}{4}}{c^2}+\frac{d^2\,\left(6\,a\,x^2+6\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,4\mathrm{i}\right)}{12}-\frac{c^2\,d^2\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atan}(c\,x)\right)}{12}+\frac{c\,d^2\,\left(a\,x^3\,8\mathrm{i}+b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,8\mathrm{i}\right)}{12}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
((d^2*(9*b*atan(c*x) + b*log(c^2*x^2 + 1)*4i))/12 - (3*b*c*d^2*x)/4)/c^2 +
 (d^2*(6*a*x^2 - b*x^2*4i + 6*b*x^2*atan(c*x)))/12 - (c^2*d^2*(3*a*x^4 + 3
*b*x^4*atan(c*x)))/12 + (c*d^2*(a*x^3*8i + b*x^3 + b*x^3*atan(c*x)*8i))/12
```

3.13 $\int (d+icdx)^2(a+b\arctan(cx))\,dx$

3.13.1 Optimal result

Integrand size = 20, antiderivative size = 83

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{2}{3}ibd^2x-\frac{bd^2(1+icx)^2}{6c}-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}-\frac{4bd^2\log(1-icx)}{3c}$$

output
```
-2/3*I*b*d^2*x-1/6*b*d^2*(1+I*c*x)^2/c-1/3*I*d^2*(1+I*c*x)^3*(a+b*arctan(c
*x))/c-4/3*b*d^2*ln(1-I*c*x)/c
```

3.13.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = \frac{1}{3}d^2\left(\frac{1}{2}bx(-6i+cx)-\frac{(-i+cx)^3(a+b\arctan(cx))}{c}-\frac{4b\log(i+cx)}{c}\right)$$

input
```
Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*((b*x*(-6*I + c*x))/2 - ((-I + c*x)^3*(a + b*ArcTan[c*x]))/c - (4*b*L
og[I + c*x])/c))/3
```

3.13.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{ib\int \frac{d^3(icx+1)^3}{c^2x^2+1}dx}{3d}-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}ibd^2\int \frac{(icx+1)^3}{c^2x^2+1}dx-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{3}ibd^2\int \frac{(icx+1)^2}{1-icx}dx-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}ibd^2\int \left(-icx+\frac{4}{1-icx}-3\right)dx-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}ibd^2\left(-\frac{1}{2}icx^2+\frac{4i\log(cx+i)}{c}-3x\right)-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

input
```
Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/3*I)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x]))/c + (I/3)*b*d^2*(-3*x - (
I/2)*c*x^2 + ((4*I)*Log[I + c*x])/c)
```

3.13.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{ia\,d^2(icx+1)^3}{3}+b\,d^2\left(-\frac{c^3x^3\arctan(cx)}{3}+i\arctan(cx)c^2x^2+cx\arctan(cx)-\frac{i\arctan(cx)}{3}+\frac{i\left(-3cx-\frac{ic^2x^2}{2}+2i\ln\left(c^2x^2+1\right)+\right.}{3}\right.}{c}$
default	$\frac{-\frac{ia\,d^2(icx+1)^3}{3}+b\,d^2\left(-\frac{c^3x^3\arctan(cx)}{3}+i\arctan(cx)c^2x^2+cx\arctan(cx)-\frac{i\arctan(cx)}{3}+\frac{i\left(-3cx-\frac{ic^2x^2}{2}+2i\ln\left(c^2x^2+1\right)+\right.}{3}\right.}{c}$
parts	$-\frac{ia\,d^2(icx+1)^3}{3c}-\frac{b\,d^2c^2x^3\arctan(cx)}{3}+ib\,d^2c\arctan(cx)\,x^2+b\arctan(cx)\,x\,d^2+\frac{id^2b\arctan(cx)}{c}$
parallelrisch	$\frac{-2x^3\arctan(cx)b\,d^2c^3+6ib\,d^2\arctan(cx)x^2c^2-2a\,c^3d^2x^3+6ix^2a\,c^2d^2+b\,c^2d^2x^2-6ib\,d^2xc+6b\arctan(cx)d^2cx+6ib\,d^2\,a}{6c}$
risch	$\frac{id^2(cx-i)^3b\ln(icx+1)}{6c}-\frac{id^2c^2b\,x^3\ln(-icx+1)}{6}-\frac{x^3d^2c^2a}{3}+iac\,d^2x^2-\frac{d^2c\,x^2b\ln(-icx+1)}{2}+\frac{ib\,d^2x\ln(-icx}{2}$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/3*I*a*d^2*(1+I*c*x)^3+b*d^2*(-1/3*c^3*x^3*arctan(c*x)+I*arctan(c*x
)*c^2*x^2+c*x*arctan(c*x)-1/3*I*arctan(c*x)+1/3*I*(-3*c*x-1/2*I*c^2*x^2+2*
I*ln(c^2*x^2+1)+4*arctan(c*x))))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{2\,ac^3d^2x^3-(6i\,a+b)c^2d^2x^2-6\,(a-i\,b)cd^2x+7\,bd^2\log\left(\frac{cx+i}{c}\right)+bd^2\log\left(\frac{cx-i}{c}\right)-(-i\,bc^3d^2x^3-3\,bc^2d}{6\,c}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/6*(2*a*c^3*d^2*x^3 - (6*I*a + b)*c^2*d^2*x^2 - 6*(a - I*b)*c*d^2*x + 7*
b*d^2*log((c*x + I)/c) + b*d^2*log((c*x - I)/c) - (-I*b*c^3*d^2*x^3 - 3*b*
c^2*d^2*x^2 + 3*I*b*c*d^2*x)*log(-(c*x + I)/(c*x - I)))/c
```

3.13.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(73) = 146.

Time = 1.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.48

$$\begin{aligned}\int (d+icdx)^2(a+b\arctan(cx))\,dx &= -\frac{ac^2d^2x^3}{3}-\frac{bd^2\left(\frac{\log(13bcd^2x-13ibd^2)}{6}+\frac{17\log(13bcd^2x+13ibd^2)}{24}\right)}{c}-x^2\left(-iacd^2-\frac{bcd^2}{6}\right)\\&\quad-x\left(-ad^2+ibd^2\right)+\left(\frac{ibc^2d^2x^3}{6}+\frac{bcd^2x^2}{2}-\frac{ibd^2x}{2}\right)\log(icx+1)\\&\quad+\frac{(-4ibc^3d^2x^3-12bc^2d^2x^2+12ibcd^2x-11bd^2)\log(-icx+1)}{24c}\end{aligned}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**3/3 - b*d**2*(log(13*b*c*d**2*x - 13*I*b*d**2)/6 + 17*log(
13*b*c*d**2*x + 13*I*b*d**2)/24)/c - x**2*(-I*a*c*d**2 - b*c*d**2/6) - x*(
-a*d**2 + I*b*d**2) + (I*b*c**2*d**2*x**3/6 + b*c*d**2*x**2/2 - I*b*d**2*x
/2)*log(I*c*x + 1) + (-4*I*b*c**3*d**2*x**3 - 12*b*c**2*d**2*x**2 + 12*I*b
*c*d**2*x - 11*b*d**2)*log(-I*c*x + 1)/(24*c)
```

3.13.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\begin{aligned}\int (d+icdx)^2(a+b\arctan(cx))\,dx = &-\frac{1}{3}\,ac^2d^2x^3\\&-\frac{1}{6}\left(2\,x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bc^2d^2\\&+i\,acd^2x^2\\&+i\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bcd^2\\&+ad^2x+\frac{(2\,cx\arctan(cx)-\log(c^2x^2+1))bd^2}{2\,c}\end{aligned}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/3*a*c^2*d^2*x^3 - 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1
)/c^4))*b*c^2*d^2 + I*a*c*d^2*x^2 + I*(x^2*arctan(c*x) - c*(x/c^2 - arctan
(c*x)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))
*b*d^2/c
```

3.13.8 Giac [F]

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^2(b\arctan(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = \frac{d^2\,(6\,a\,x+6\,b\,x\,\mathrm{atan}(c\,x)-b\,x\,6\mathrm{i})}{6} - \frac{c^2\,d^2\,(2\,a\,x^3+2\,b\,x^3\,\mathrm{atan}(c\,x))}{6} + \frac{d^2\,(-4\,b\,\ln(c^2\,x^2+1)+b\,\mathrm{atan}(c\,x)\;6\mathrm{i})}{6\,c} + \frac{c\,d^2\,(a\,x^2\,6\mathrm{i}+b\,x^2+b\,x^2\,\mathrm{atan}(c\,x)\;6\mathrm{i})}{6}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
(d^2*(6*a*x - b*x*6i + 6*b*x*atan(c*x)))/6 - (c^2*d^2*(2*a*x^3 + 2*b*x^3*a
tan(c*x)))/6 + (d^2*(b*atan(c*x)*6i - 4*b*log(c^2*x^2 + 1)))/(6*c) + (c*d^
2*(a*x^2*6i + b*x^2 + b*x^2*atan(c*x)*6i))/6
```

3.14 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx$

3.14.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx &= 2iacd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\arctan(cx)\\ &\quad+2ibcd^2x\arctan(cx)-\frac{1}{2}c^2d^2x^2(a+b\arctan(cx))\\ &\quad+ad^2\log(x)-ibd^2\log\left(1+c^2x^2\right)\\ &\quad+\frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
2*I*a*c*d^2*x+1/2*b*c*d^2*x-1/2*b*d^2*arctan(c*x)+2*I*b*c*d^2*x*arctan(c*x
)-1/2*c^2*d^2*x^2*(a+b*arctan(c*x))+a*d^2*ln(x)-I*b*d^2*ln(c^2*x^2+1)+1/2*
I*b*d^2*polylog(2,-I*c*x)-1/2*I*b*d^2*polylog(2,I*c*x)
```

3.14.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx &= -\frac{1}{2}d^2\left(-4iacx-bcx+ac^2x^2+b\arctan(cx)\right.\\ &\quad-4ibcx\arctan(cx)+bc^2x^2\arctan(cx)-2a\log(x)\\ &\quad+2ib\log\left(1+c^2x^2\right)-ib\operatorname{PolyLog}(2,-icx)\\ &\quad\left.+ib\operatorname{PolyLog}(2,icx)\right)\end{aligned}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/2*(d^2*((-4*I)*a*c*x - b*c*x + a*c^2*x^2 + b*ArcTan[c*x] - (4*I)*b*c*x*
ArcTan[c*x] + b*c^2*x^2*ArcTan[c*x] - 2*a*Log[x] + (2*I)*b*Log[1 + c^2*x^2
] - I*b*PolyLog[2, (-I)*c*x] + I*b*PolyLog[2, I*c*x]))
```

3.14.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2x(a+b\arctan(cx))+2icd^2(a+b\arctan(cx))+\frac{d^2(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}c^2d^2x^2(a+b\arctan(cx))+2iacd^2x+ad^2\log(x)-\frac{1}{2}bd^2\arctan(cx)+2ibcd^2x\arctan(cx)-ibd^2\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx)+\frac{1}{2}bcd^2x$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(2*I)*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTan[c*x])/2 + (2*I)*b*c*d^2*x*
ArcTan[c*x] - (c^2*d^2*x^2*(a + b*ArcTan[c*x]))/2 + a*d^2*Log[x] - I*b*d^2
*Log[1 + c^2*x^2] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyLog
[2, I*c*x]
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.14.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

method	result
parts	$a\,d^2\left(-\frac{c^2x^2}{2}+2icx+\ln\left(x\right)\right)+b\,d^2\left(-\frac{c^2x^2\arctan(cx)}{2}+2i\arctan\left(cx\right)cx+\arctan\left(cx\right)\ln\left(\right.$
derivativedivides	$a\,d^2\left(-\frac{c^2x^2}{2}+2icx+\ln\left(cx\right)\right)+b\,d^2\left(-\frac{c^2x^2\arctan(cx)}{2}+2i\arctan\left(cx\right)cx+\arctan\left(cx\right)\ln$
default	$a\,d^2\left(-\frac{c^2x^2}{2}+2icx+\ln\left(cx\right)\right)+b\,d^2\left(-\frac{c^2x^2\arctan(cx)}{2}+2i\arctan\left(cx\right)cx+\arctan\left(cx\right)\ln$
risch	$\frac{ib\,d^2\operatorname{dilog}(icx+1)}{2}-d^2bcx\ln\left(-icx+1\right)-\frac{id^2b\ln(-icx+1)c^2x^2}{4}-\frac{id^2b\operatorname{dilog}(-icx+1)}{2}+\frac{bc\,d^2x}{2}+\frac{ib\,d^2}{}$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-1/2*c^2*x^2+2*I*c*x+ln(x))+b*d^2*(-1/2*c^2*x^2*arctan(c*x)+2*I*arc
tan(c*x)*c*x+arctan(c*x)*ln(c*x)+1/2*c*x-I*ln(c^2*x^2+1)-1/2*arctan(c*x)+1
/2*I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/
2*I*dilog(1-I*c*x))
```

3.14.5 Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x
^2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x)
```

3.14.6 Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = -d^2\left(\int\left(-\frac{a}{x}\right)dx + \int(-2iac)\,dx + \int ac^2x\,dx + \int\left(-\frac{b\operatorname{atan}(cx)}{x}\right)dx + \int(-2ibc\operatorname{atan}(cx))\,dx + \int bc^2x\operatorname{atan}(cx)\,dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x,x)
```

output
```
-d**2*(Integral(-a/x, x) + Integral(-2*I*a*c, x) + Integral(a*c**2*x, x) +
 Integral(-b*atan(c*x)/x, x) + Integral(-2*I*b*c*atan(c*x), x) + Integral(
b*c**2*x*atan(c*x), x))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = -\frac{1}{2}ac^2d^2x^2 + 2i\,acd^2x + \frac{1}{2}bcd^2x - \frac{1}{4}\pi bd^2\log\left(c^2x^2+1\right) + bd^2\arctan(cx)\log(cx) + i\left(2\,cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bd^2 - \frac{1}{2}i\,bd^2\mathrm{Li}_2(i\,cx+1) + \frac{1}{2}i\,bd^2\mathrm{Li}_2(-i\,cx+1) + ad^2\log(x) - \frac{1}{2}\left(bc^2d^2x^2 + bd^2\right)\arctan(cx)$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
-1/2*a*c^2*d^2*x^2 + 2*I*a*c*d^2*x + 1/2*b*c*d^2*x - 1/4*pi*b*d^2*log(c^2*
x^2 + 1) + b*d^2*arctan(c*x)*log(c*x) + I*(2*c*x*arctan(c*x) - log(c^2*x^2
 + 1))*b*d^2 - 1/2*I*b*d^2*dilog(I*c*x + 1) + 1/2*I*b*d^2*dilog(-I*c*x + 1
) + a*d^2*log(x) - 1/2*(b*c^2*d^2*x^2 + b*d^2)*arctan(c*x)
```

3.14.8 Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx$$
$$=\begin{cases} a\,d^2\,\ln{(x)} \\ \frac{b\,c\,d^2\,x}{2}+\frac{a\,d^2\,(2\,\ln(x)-c^2\,x^2+c\,x\,4\mathrm{i})}{2}-\frac{b\,d^2\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,d^2\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-b\,d^2\,\ln{(c^2\,x^2+1)}\,1\mathrm{i}-b\,c^2\,d^2\,\mathrm{atan}(c \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x,x)
```

output
```
piecewise(c == 0, a*d^2*log(x), c ~= 0, - b*d^2*log(c^2*x^2 + 1)*1i + (a*d
^2*(2*log(x) + c*x*4i - c^2*x^2))/2 - (b*d^2*dilog(- c*x*1i + 1)*1i)/2 + (
b*d^2*dilog(c*x*1i + 1)*1i)/2 + (b*c*d^2*x)/2 - b*c^2*d^2*atan(c*x)*(1/(2*
c^2) + x^2/2) + b*c*d^2*x*atan(c*x)*2i)
```

3.15 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx$

3.15.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = -ac^2d^2x - bc^2d^2x\arctan(cx) - \frac{d^2(a+b\arctan(cx))}{x} + 2iacd^2\log(x) + bcd^2\log(x) - bcd^2\operatorname{PolyLog}(2,-icx) + bcd^2\operatorname{PolyLog}(2,icx)$$

output
```
-a*c^2*d^2*x-b*c^2*d^2*x*arctan(c*x)-d^2*(a+b*arctan(c*x))/x+2*I*a*c*d^2*l
n(x)+b*c*d^2*ln(x)-b*c*d^2*polylog(2,-I*c*x)+b*c*d^2*polylog(2,I*c*x)
```

3.15.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = -\frac{d^2(a+ac^2x^2+b\arctan(cx)+bc^2x^2\arctan(cx)-2iacx\log(x)-bcx\log(cx)+bcx\operatorname{PolyLog}(2,-icx)-}{x}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d^2*(a + a*c^2*x^2 + b*ArcTan[c*x] + b*c^2*x^2*ArcTan[c*x] - (2*I)*a*c*
x*Log[x] - b*c*x*Log[c*x] + b*c*x*PolyLog[2, (-I)*c*x] - b*c*x*PolyLog[2,
I*c*x]))/x)
```

3.15.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-c^2d^2(a+b\arctan(cx))+\frac{d^2(a+b\arctan(cx))}{x^2}+\frac{2icd^2(a+b\arctan(cx))}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{d^2(a+b\arctan(cx))}{x}-ac^2d^2x+2iacd^2\log(x)-bc^2d^2x\arctan(cx)-bcd^2\operatorname{PolyLog}(2,-icx)+bcd^2\operatorname{PolyLog}(2,icx)+bcd^2\log(x)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-(a*c^2*d^2*x) - b*c^2*d^2*x*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/x + (
2*I)*a*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, (-I)*c*x] + b*c*
d^2*PolyLog[2, I*c*x]
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.15.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

method	result
parts	$a\,d^2\left(-c^2x+2ic\ln\left(x\right)-\frac{1}{x}\right)+b\,d^2c\Big(-cx\arctan\left(cx\right)+2i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\arctan(cx)}{cx}$
derivativedivides	$c\Big(a\,d^2\left(-cx+2i\ln\left(cx\right)-\frac{1}{cx}\right)+b\,d^2\Big(-cx\arctan\left(cx\right)+2i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\arctan(c}{cx}$
default	$c\Big(a\,d^2\left(-cx+2i\ln\left(cx\right)-\frac{1}{cx}\right)+b\,d^2\Big(-cx\arctan\left(cx\right)+2i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\arctan(c}{cx}$
risch	$\frac{ib\,d^2\ln(icx+1)}{2x}-bc\,d^2-\frac{id^2b\ln(-icx+1)}{2x}+c\,d^2b\,\mathrm{dilog}\left(-icx+1\right)+\frac{c\,d^2b\ln(-icx)}{2}+2ic\,d^2a\ln\left(-\right.$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-c^2*x+2*I*c*ln(x)-1/x)+b*d^2*c*(-c*x*arctan(c*x)+2*I*arctan(c*x)*l
n(c*x)-1/c/x*arctan(c*x)-ln(c*x)*ln(1+I*c*x)+ln(c*x)*ln(1-I*c*x)-dilog(1+I
*c*x)+dilog(1-I*c*x)+ln(c*x))
```

3.15.5 Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan\left(cx\right)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x
^2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

3.15.6 Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = -d^2\left(\int ac^2\,dx+\int\left(-\frac{a}{x^2}\right)dx+\int bc^2\operatorname{atan}\left(cx\right)dx + \int\left(-\frac{b\operatorname{atan}\left(cx\right)}{x^2}\right)dx+\int\left(-\frac{2iac}{x}\right)dx + \int\left(-\frac{2ibc\operatorname{atan}\left(cx\right)}{x}\right)dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**2,x)
```

output
```
-d**2*(Integral(a*c**2, x) + Integral(-a/x**2, x) + Integral(b*c**2*atan(c
*x), x) + Integral(-b*atan(c*x)/x**2, x) + Integral(-2*I*a*c/x, x) + Integ
ral(-2*I*b*c*atan(c*x)/x, x))
```

3.15.7 Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-a*c^2*d^2*x - 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^2 + 2*I*b*
c*d^2*integrate(arctan(c*x)/x, x) + 2*I*a*c*d^2*log(x) - 1/2*(c*(log(c^2*x
^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^2 - a*d^2/x
```

3.15.8 Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx$$
$$= \begin{cases} -\frac{a\,d^2}{x} \\ \frac{b\,d^2\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}+b\,c\,d^2\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)+\frac{b\,c\,d^2\,\ln(c^2\,x^2+1)}{2}-\frac{a\,d^2\,(c^2\,x^2+1-c\,x\,\ln(x)}{x} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^2,x)
```

output
```
piecewise(c == 0, -(a*d^2)/x, c ~= 0, (b*d^2*(c^2*log(x) - (c^2*log(c^2*x^
2 + 1))/2))/c + b*c*d^2*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) + (b*c*d
^2*log(c^2*x^2 + 1))/2 - (a*d^2*(c^2*x^2 - c*x*log(x)*2i + 1))/x - (b*d^2*
atan(c*x))/x - b*c^2*d^2*x*atan(c*x))
```

3.16 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx$

3.16.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = -\frac{bcd^2}{2x}-\frac{1}{2}bc^2d^2\arctan(cx)-\frac{d^2(a+b\arctan(cx))}{2x^2} -\frac{2icd^2(a+b\arctan(cx))}{x}-ac^2d^2\log(x) +2ibc^2d^2\log(x)-ibc^2d^2\log\left(1+c^2x^2\right) -\frac{1}{2}ibc^2d^2\,\mathrm{PolyLog}(2,-icx)+\frac{1}{2}ibc^2d^2\,\mathrm{PolyLog}(2,icx)$$

output
```
-1/2*b*c*d^2/x-1/2*b*c^2*d^2*arctan(c*x)-1/2*d^2*(a+b*arctan(c*x))/x^2-2*I
*c*d^2*(a+b*arctan(c*x))/x-a*c^2*d^2*ln(x)+2*I*b*c^2*d^2*ln(x)-I*b*c^2*d^2
*ln(c^2*x^2+1)-1/2*I*b*c^2*d^2*polylog(2,-I*c*x)+1/2*I*b*c^2*d^2*polylog(2
,I*c*x)
```

3.16.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = -\frac{d^2\left(a+4iacx+b\arctan(cx)+4ibcx\arctan(cx)+bcx\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+2ac^2x^2\,\mathrm{l}\cdots\right)}{2x^2\cdots}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + (4*I)*a*c*x + b*ArcTan[c*x] + (4*I)*b*c*x*ArcTan[c*x] + b*c
*x*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 2*a*c^2*x^2*Log[x] - (4*I
)*b*c^2*x^2*Log[x] + (2*I)*b*c^2*x^2*Log[1 + c^2*x^2] + I*b*c^2*x^2*PolyLo
g[2, (-I)*c*x] - I*b*c^2*x^2*PolyLog[2, I*c*x]))/x^2
```

3.16.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{c^2d^2(a+b\arctan(cx))}{x}+\frac{d^2(a+b\arctan(cx))}{x^3}+\frac{2icd^2(a+b\arctan(cx))}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{2x^2}-\frac{2icd^2(a+b\arctan(cx))}{x}-ac^2d^2\log(x)-\frac{1}{2}bc^2d^2\arctan(cx)-\frac{1}{2}ibc^2d^2\operatorname{PolyLog}(2,-icx)+\frac{1}{2}ibc^2d^2\operatorname{PolyLog}(2,icx)-ibc^2d^2\log\left(c^2x^2+1\right)+2ibc^2d^2\log(x)-\frac{bcd^2}{2x}$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^2)/x - (b*c^2*d^2*ArcTan[c*x])/2 - (d^2*(a + b*ArcTan[c*x]))/(
2*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x]))/x - a*c^2*d^2*Log[x] + (2*I)*b*
c^2*d^2*Log[x] - I*b*c^2*d^2*Log[1 + c^2*x^2] - (I/2)*b*c^2*d^2*PolyLog[2,
 (-I)*c*x] + (I/2)*b*c^2*d^2*PolyLog[2, I*c*x]
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.16.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
parts	$a\,d^2\left(-\frac{1}{2x^2}-c^2\ln\left(x\right)-\frac{2ic}{x}\right)+b\,d^2c^2\Big(-\arctan\left(cx\right)\ln\left(cx\right)-\frac{2i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-$
derivativedivides	$c^2\Big(a\,d^2\left(-\ln\left(cx\right)-\frac{2i}{cx}-\frac{1}{2c^2x^2}\right)+b\,d^2\Big(-\arctan\left(cx\right)\ln\left(cx\right)-\frac{2i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-$
default	$c^2\Big(a\,d^2\left(-\ln\left(cx\right)-\frac{2i}{cx}-\frac{1}{2c^2x^2}\right)+b\,d^2\Big(-\arctan\left(cx\right)\ln\left(cx\right)-\frac{2i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-$
risch	$-\frac{ic^2b\,d^2\,\mathrm{dilog}(icx+1)}{2}+\frac{3ic^2b\,d^2\ln(icx)}{4}-\frac{3ic^2b\,d^2\ln(icx+1)}{4}-\frac{5b\,c^2d^2\arctan(cx)}{4}+\frac{c\,d^2b\ln(-icx+1)}{x}-$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-1/2/x^2-c^2*ln(x)-2*I*c/x)+b*d^2*c^2*(-arctan(c*x)*ln(c*x)-2*I*arc
tan(c*x)/c/x-1/2/c^2/x^2*arctan(c*x)-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*
x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)-1/2/c/x+2*I*ln(c*
x)-I*ln(c^2*x^2+1)-1/2*arctan(c*x))
```

3.16.5 Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x
^2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

3.16.6 Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = -d^2\left(\int\left(-\frac{a}{x^3}\right)dx + \int\frac{ac^2}{x}\,dx + \int\left(-\frac{b\,\mathrm{atan}\,(cx)}{x^3}\right)dx + \int\left(-\frac{2iac}{x^2}\right)dx + \int\frac{bc^2\,\mathrm{atan}\,(cx)}{x}\,dx + \int\left(-\frac{2ibc\,\mathrm{atan}\,(cx)}{x^2}\right)dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**3,x)
```

output
```
-d**2*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*atan(c*x
)/x**3, x) + Integral(-2*I*a*c/x**2, x) + Integral(b*c**2*atan(c*x)/x, x)
+ Integral(-2*I*b*c*atan(c*x)/x**2, x))
```

3.16.7 Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-b*c^2*d^2*integrate(arctan(c*x)/x, x) - a*c^2*d^2*log(x) - I*(c*(log(c^2*
x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^2 - 1/2*((c*arctan(c*x) + 1/
x)*c + arctan(c*x)/x^2)*b*d^2 - 2*I*a*c*d^2/x - 1/2*a*d^2/x^2
```

3.16.8 Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.06

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx$$
$$=\begin{cases} -\frac{a\,d^2}{2\,x^2} \\ b\,d^2\left(c^2\,\ln{(x)}-\frac{c^2\,\ln(c^2\,x^2+1)}{2}\right)\,2\mathrm{i}+\frac{b\,c^2\,d^2\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-\frac{b\,c^2\,d^2\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-\frac{b\,d^2\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c}-\frac{a\,d^2\,(2\,c^2\,x}{} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^3,x)
```

output
```
piecewise(c == 0, -(a*d^2)/(2*x^2), c ~= 0, b*d^2*(c^2*log(x) - (c^2*log(c
^2*x^2 + 1))/2)*2i + (b*c^2*d^2*dilog(- c*x*1i + 1)*1i)/2 - (b*c^2*d^2*dil
og(c*x*1i + 1)*1i)/2 - (b*d^2*(c^3*atan(c*x) + c^2/x))/(2*c) - (a*d^2*(c*x
*4i + 2*c^2*x^2*log(x) + 1))/(2*x^2) - (b*d^2*atan(c*x))/(2*x^2) - (b*c*d^
2*atan(c*x)*2i)/x)
```

3.17 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx$

3.17.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd^2}{6x^2}-\frac{ibc^2d^2}{x}-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3} -\frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}bc^3d^2\log(i+cx)$$

output
```
-1/6*b*c*d^2/x^2-I*b*c^2*d^2/x-1/3*d^2*(1+I*c*x)^3*(a+b*arctan(c*x))/x^3-4
/3*b*c^3*d^2*ln(x)+4/3*b*c^3*d^2*ln(c*x+I)
```

3.17.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{d^2\left(2a+6iacx+bcx-6ac^2x^2+2b(1+3icx-3c^2x^2)\arctan(cx)+6ibc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1\right.\right.}{6x^3}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(d^2*(2*a + (6*I)*a*c*x + b*c*x - 6*a*c^2*x^2 + 2*b*(1 + (3*I)*c*x -
3*c^2*x^2)*ArcTan[c*x] + (6*I)*b*c^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -
(c^2*x^2)] + 8*b*c^3*x^3*Log[x] - 4*b*c^3*x^3*Log[1 + c^2*x^2]))/x^3
```

3.17.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{id^2(i-cx)^2}{3x^3(cx+i)}dx-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}ibcd^2\int \frac{(i-cx)^2}{x^3(cx+i)}dx-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{3}ibcd^2\int \left(\frac{4ic^3}{cx+i}-\frac{4ic^2}{x}-\frac{3c}{x^2}+\frac{i}{x^3}\right)dx-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}-\frac{1}{3}ibcd^2\left(-4ic^2\log(x)+4ic^2\log(cx+i)+\frac{3c}{x}-\frac{i}{2x^2}\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x]))/x^3 - (I/3)*b*c*d^2*((-1/2*I)
/x^2 + (3*c)/x - (4*I)*c^2*Log[x] + (4*I)*c^2*Log[I + c*x])
```

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.17.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

method	result
parts	$a\,d^2\left(-\frac{ic}{x^2}+\frac{c^2}{x}-\frac{1}{3x^3}\right)+b\,d^2c^3\Big(-\frac{\arctan(cx)}{3c^3x^3}+\frac{\arctan(cx)}{cx}-\frac{i\arctan(cx)}{c^2x^2}-\frac{i}{cx}-\frac{1}{6c^2x^2}-\frac{4\ln(cx)}{3}$
derivativedivides	$c^3\Big(a\,d^2\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{i}{c^2x^2}\right)+b\,d^2\Big(-\frac{\arctan(cx)}{3c^3x^3}+\frac{\arctan(cx)}{cx}-\frac{i\arctan(cx)}{c^2x^2}-\frac{i}{cx}-\frac{1}{6c^2x^2}-4$
default	$c^3\Big(a\,d^2\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{i}{c^2x^2}\right)+b\,d^2\Big(-\frac{\arctan(cx)}{3c^3x^3}+\frac{\arctan(cx)}{cx}-\frac{i\arctan(cx)}{c^2x^2}-\frac{i}{cx}-\frac{1}{6c^2x^2}-4$
risch	$-\frac{ib\,d^2\left(3c^2x^2-3icx-1\right)\ln(icx+1)}{6x^3}+\frac{d^2\left(7b\,c^3\ln(-cx-i)x^3+b\,c^3\ln(cx-i)x^3-8b\,c^3\ln(-x)x^3+3ib\,x^2\ln(-icx+1)c^2-6\right.}{6x^3}$
parallelrisch	$-\frac{6ix^3\arctan(cx)b\,c^3d^2-6ix^3a\,c^3d^2-4b\,c^3d^2\ln\left(c^2x^2+1\right)x^3+8b\,c^3d^2\ln(x)x^3-b\,c^3d^2x^3+6ix^2b\,c^2d^2-6x^2\arctan(cx)b\,c}{6x^3}$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-I*c/x^2+c^2/x-1/3/x^3)+b*d^2*c^3*(-1/3*arctan(c*x)/c^3/x^3+1/c/x*a
rctan(c*x)-I*arctan(c*x)/c^2/x^2-I/c/x-1/6/c^2/x^2-4/3*ln(c*x)+2/3*ln(c^2*
x^2+1)-I*arctan(c*x))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.66

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{8\,bc^3d^2x^3\log\left(x\right)-7\,bc^3d^2x^3\log\left(\frac{cx+i}{c}\right)-bc^3d^2x^3\log\left(\frac{cx-i}{c}\right)-6\,(a-i\,b)c^2d^2x^2-(-6i\,a-b)cd^2x+2\,a}{6\,x^3}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(8*b*c^3*d^2*x^3*log(x) - 7*b*c^3*d^2*x^3*log((c*x + I)/c) - b*c^3*d^
2*x^3*log((c*x - I)/c) - 6*(a - I*b)*c^2*d^2*x^2 - (-6*I*a - b)*c*d^2*x +
2*a*d^2 - (3*I*b*c^2*d^2*x^2 + 3*b*c*d^2*x - I*b*d^2)*log(-(c*x + I)/(c*x
- I)))/x^3
```

3.17.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(82) = 164$.

Time = 5.17 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.91

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = &-\frac{4bc^3d^2\log\left(135b^2c^7d^4x\right)}{3}\\ &+\frac{bc^3d^2\log\left(135b^2c^7d^4x-135ib^2c^6d^4\right)}{6}\\ &+\frac{7bc^3d^2\log\left(135b^2c^7d^4x+135ib^2c^6d^4\right)}{6}\\ &-\frac{2ad^2+x^2(-6ac^2d^2+6ibc^2d^2)+x(6iacd^2+bcd^2)}{6x^3}\\ &+\frac{(-3ibc^2d^2x^2-3bcd^2x+ibd^2)\log\left(icx+1\right)}{6x^3}\\ &+\frac{(3ibc^2d^2x^2+3bcd^2x-ibd^2)\log\left(-icx+1\right)}{6x^3}\end{aligned}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**4,x)
```

output
```
-4*b*c**3*d**2*log(135*b**2*c**7*d**4*x)/3 + b*c**3*d**2*log(135*b**2*c**7
*d**4*x - 135*I*b**2*c**6*d**4)/6 + 7*b*c**3*d**2*log(135*b**2*c**7*d**4*x
 + 135*I*b**2*c**6*d**4)/6 - (2*a*d**2 + x**2*(-6*a*c**2*d**2 + 6*I*b*c**2
*d**2) + x*(6*I*a*c*d**2 + b*c*d**2))/(6*x**3) + (-3*I*b*c**2*d**2*x**2 -
3*b*c*d**2*x + I*b*d**2)*log(I*c*x + 1)/(6*x**3) + (3*I*b*c**2*d**2*x**2 +
 3*b*c*d**2*x - I*b*d**2)*log(-I*c*x + 1)/(6*x**3)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx &= \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^2d^2 \\ &\quad - i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bcd^2 \\ &\quad + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd^2 \\ &\quad + \frac{ac^2d^2}{x}-\frac{i\,acd^2}{x^2}-\frac{ad^2}{3\,x^3}\end{aligned}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^2 - I*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^2 + 1/6*((c^2*log(c^2*x^2 +
1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^2 + a*c^2*d^2/x - I*
a*c*d^2/x^2 - 1/3*a*d^2/x^3
```

3.17.8 Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan\left(cx\right)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx$$
$$= -\frac{d^2\,\left(8\,b\,c^3\,\ln\left(x\right)-4\,b\,c^3\,\ln\left(c^2\,x^2+1\right)+b\,c^3\,\mathrm{atan}\left(c\,x\right)\,6{}\mathrm{i}\right)}{6}$$
$$-\frac{\frac{d^2\,\left(2\,a+2\,b\,\mathrm{atan}\left(c\,x\right)\right)}{6}+\frac{d^2\,x\,\left(a\,c\,6{}\mathrm{i}+b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,6{}\mathrm{i}\right)}{6}-\frac{d^2\,x^2\,\left(6\,a\,c^2+6\,b\,c^2\,\mathrm{atan}\left(c\,x\right)-b\,c^2\,6{}\mathrm{i}\right)}{6}}{x^3}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^4,x)
```

output
```
- (d^2*(b*c^3*atan(c*x)*6i - 4*b*c^3*log(c^2*x^2 + 1) + 8*b*c^3*log(x)))/6
 - ((d^2*(2*a + 2*b*atan(c*x)))/6 + (d^2*x*(a*c*6i + b*c + b*c*atan(c*x)*6
i))/6 - (d^2*x^2*(6*a*c^2 - b*c^2*6i + 6*b*c^2*atan(c*x)))/6)/x^3
```

3.18 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$

3.18.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd^2}{12x^3}-\frac{ibc^2d^2}{3x^2}+\frac{3bc^3d^2}{4x} - \frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3} + \frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{2}{3}ibc^4d^2\log(x) - \frac{1}{24}ibc^4d^2\log(i-cx)+\frac{17}{24}ibc^4d^2\log(i+cx)$$

output
```
-1/12*b*c*d^2/x^3-1/3*I*b*c^2*d^2/x^2+3/4*b*c^3*d^2/x-1/4*d^2*(a+b*arctan(
c*x))/x^4-2/3*I*c*d^2*(a+b*arctan(c*x))/x^3+1/2*c^2*d^2*(a+b*arctan(c*x))/
x^2-2/3*I*b*c^4*d^2*ln(x)-1/24*I*b*c^4*d^2*ln(I-c*x)+17/24*I*b*c^4*d^2*ln(
c*x+I)
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = \frac{d^2\left(-3a-8iacx+6ac^2x^2-4ibc^2x^2-3b\arctan(cx)-8ibcx\arctan(cx)+6bc^2x^2\arctan(cx)-bcx\,\mathrm{Hyper}\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
(d^2*(-3*a - (8*I)*a*c*x + 6*a*c^2*x^2 - (4*I)*b*c^2*x^2 - 3*b*ArcTan[c*x]
 - (8*I)*b*c*x*ArcTan[c*x] + 6*b*c^2*x^2*ArcTan[c*x] - b*c*x*Hypergeometri
c2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 6*b*c^3*x^3*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^2)] - (8*I)*b*c^4*x^4*Log[x] + (4*I)*b*c^4*x^4*Log[1 + c^2*x^2
]))/(12*x^4)
```

3.18.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^2\left(-6c^2x^2+8icx+3\right)}{12x^4\left(c^2x^2+1\right)}dx+\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bcd^2\int \frac{-6c^2x^2+8icx+3}{x^4\left(c^2x^2+1\right)}dx+\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{12}bcd^2\int \left(-\frac{ic^4}{2(cx-i)}+\frac{17ic^4}{2(cx+i)}-\frac{8ic^3}{x}-\frac{9c^2}{x^2}+\frac{8ic}{x^3}+\frac{3}{x^4}\right)dx+\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}+\frac{1}{12}bcd^2\left(-8ic^3\log(x)-\frac{1}{2}ic^3\log(-cx+i)+\frac{17}{2}ic^3\log(cx+i)+\frac{9c^2}{x}-\frac{4ic}{x^2}-\frac{1}{x^3}\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d^2*(a + b*ArcTan[c*x]))/x^4 - (((2*I)/3)*c*d^2*(a + b*ArcTan[c*x]))
/x^3 + (c^2*d^2*(a + b*ArcTan[c*x]))/(2*x^2) + (b*c*d^2*(-x^(-3) - ((4*I)*
c)/x^2 + (9*c^2)/x - (8*I)*c^3*Log[x] - (I/2)*c^3*Log[I - c*x] + ((17*I)/2
)*c^3*Log[I + c*x]))/12
```

3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.18.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

method	result
parts	$a\,d^2\left(\frac{c^2}{2x^2}-\frac{1}{4x^4}-\frac{2ic}{3x^3}\right)+b\,d^2c^4\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{2i\arctan(cx)}{3c^3x^3}+\frac{\arctan(cx)}{2c^2x^2}-\frac{i}{3c^2x^2}-\frac{2i\ln(cx)}{3}-\right.$
derivativedivides	$c^4\left(a\,d^2\left(-\frac{1}{4c^4x^4}-\frac{2i}{3c^3x^3}+\frac{1}{2c^2x^2}\right)+b\,d^2\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{2i\arctan(cx)}{3c^3x^3}+\frac{\arctan(cx)}{2c^2x^2}-\frac{i}{3c^2x^2}-2i\,l\right.\right.$
default	$c^4\left(a\,d^2\left(-\frac{1}{4c^4x^4}-\frac{2i}{3c^3x^3}+\frac{1}{2c^2x^2}\right)+b\,d^2\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{2i\arctan(cx)}{3c^3x^3}+\frac{\arctan(cx)}{2c^2x^2}-\frac{i}{3c^2x^2}-2i\,l\right.\right.$
risch	$-\frac{ib\,d^2(6c^2x^2-8icx-3)\ln(icx+1)}{24x^4}+\frac{id^2(17b\,c^4\ln(-99cx-99i)x^4-b\,c^4\ln(45cx-45i)x^4-16b\,c^4\ln(-165cx)x^4+6x^2b\,l}{}$
parallelrisch	$\frac{4ic^4b\,d^2\ln(c^2x^2+1)x^4-8ic^4b\,d^2\ln(x)x^4+4ix^4b\,c^4d^2+9x^4\arctan(cx)b\,c^4d^2-6a\,c^4d^2x^4+9b\,c^3d^2x^3-4ix^2b\,c^2d^2+6x^2\,\mathrm{arc}}{12x^4}$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(1/2*c^2/x^2-1/4/x^4-2/3*I*c/x^3)+b*d^2*c^4*(-1/4*arctan(c*x)/c^4/x^
4-2/3*I*arctan(c*x)/c^3/x^3+1/2/c^2/x^2*arctan(c*x)-1/3*I/c^2/x^2-2/3*I*ln
(c*x)-1/12/c^3/x^3+3/4/c/x+1/3*I*ln(c^2*x^2+1)+3/4*arctan(c*x))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$$
$$=\frac{-16i\,bc^4d^2x^4\log(x)+17i\,bc^4d^2x^4\log\left(\frac{cx+i}{c}\right)-i\,bc^4d^2x^4\log\left(\frac{cx-i}{c}\right)+18\,bc^3d^2x^3+4\,(3\,a-2i\,b)c^2d^2x^2-2}{24\,x^4}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/24*(-16*I*b*c^4*d^2*x^4*log(x) + 17*I*b*c^4*d^2*x^4*log((c*x + I)/c) - I
*b*c^4*d^2*x^4*log((c*x - I)/c) + 18*b*c^3*d^2*x^3 + 4*(3*a - 2*I*b)*c^2*d
^2*x^2 - 2*(8*I*a + b)*c*d^2*x - 6*a*d^2 + (6*I*b*c^2*d^2*x^2 + 8*b*c*d^2*
x - 3*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^4
```

3.18.6 Sympy [A] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.71

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$$
$$\begin{aligned}
&= -\frac{2ibc^4d^2\log\left(1485b^2c^9d^4x\right)}{3} - \frac{ibc^4d^2\log\left(1485b^2c^9d^4x - 1485ib^2c^8d^4\right)}{24} \\
&\quad + \frac{17ibc^4d^2\log\left(1485b^2c^9d^4x + 1485ib^2c^8d^4\right)}{24} \\
&\quad + \frac{(-6ibc^2d^2x^2 - 8bcd^2x + 3ibd^2)\log\left(icx+1\right)}{24x^4} \\
&\quad + \frac{(6ibc^2d^2x^2 + 8bcd^2x - 3ibd^2)\log\left(-icx+1\right)}{24x^4} \\
&\quad - \frac{3ad^2 - 9bc^3d^2x^3 + x^2(-6ac^2d^2 + 4ibc^2d^2) + x(8iacd^2 + bcd^2)}{12x^4}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**5,x)
```

output
```
-2*I*b*c**4*d**2*log(1485*b**2*c**9*d**4*x)/3 - I*b*c**4*d**2*log(1485*b**
2*c**9*d**4*x - 1485*I*b**2*c**8*d**4)/24 + 17*I*b*c**4*d**2*log(1485*b**2
*c**9*d**4*x + 1485*I*b**2*c**8*d**4)/24 + (-6*I*b*c**2*d**2*x**2 - 8*b*c*
d**2*x + 3*I*b*d**2)*log(I*c*x + 1)/(24*x**4) + (6*I*b*c**2*d**2*x**2 + 8*
b*c*d**2*x - 3*I*b*d**2)*log(-I*c*x + 1)/(24*x**4) - (3*a*d**2 - 9*b*c**3*
d**2*x**3 + x**2*(-6*a*c**2*d**2 + 4*I*b*c**2*d**2) + x*(8*I*a*c*d**2 + b*
c*d**2))/(12*x**4)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$$
$$\begin{aligned}
&= \frac{1}{2}\left(\left(c\arctan\left(cx\right) + \frac{1}{x}\right)c + \frac{\arctan\left(cx\right)}{x^2}\right)bc^2d^2 \\
&\quad + \frac{1}{3}i\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan\left(cx\right)}{x^3}\right)bcd^2 \\
&\quad + \frac{1}{12}\left(\left(3c^3\arctan\left(cx\right) + \frac{3c^2x^2-1}{x^3}\right)c - \frac{3\arctan\left(cx\right)}{x^4}\right)bd^2 + \frac{ac^2d^2}{2x^2} - \frac{2i\,acd^2}{3x^3} - \frac{ad^2}{4x^4}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^2 + 1/3*I*((c^2*lo
g(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^2 + 1/
12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^2
 + 1/2*a*c^2*d^2/x^2 - 2/3*I*a*c*d^2/x^3 - 1/4*a*d^2/x^4
```

3.18.8 Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.88

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{d^2\left(9\,b\,c^3\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,4\mathrm{i}-b\,c^4\,\ln\left(x\right)\,8\mathrm{i}\right)}{12} - \frac{\frac{d^2\,(3\,a+3\,b\,\mathrm{atan}(c\,x))}{12}+\frac{d^2\,x\,(a\,c\,8\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,8\mathrm{i})}{12}-\frac{d^2\,x^2\,(6\,a\,c^2+6\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,4\mathrm{i})}{12}-\frac{3\,b\,c^3\,d^2\,x^3}{4}}{x^4}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^5,x)
```

output
```
(d^2*(b*c^4*log(c^2*x^2 + 1)*4i - b*c^4*log(x)*8i + 9*b*c^3*atan(x*(c^2)^(
1/2))*(c^2)^(1/2)))/12 - ((d^2*(3*a + 3*b*atan(c*x)))/12 + (d^2*x*(a*c*8i
+ b*c + b*c*atan(c*x)*8i))/12 - (d^2*x^2*(6*a*c^2 - b*c^2*4i + 6*b*c^2*ata
n(c*x)))/12 - (3*b*c^3*d^2*x^3)/4)/x^4
```

3.19 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$

3.19.1 Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^2}{20x^4}-\frac{ibc^2d^2}{6x^3}+\frac{4bc^3d^2}{15x^2}+\frac{ibc^4d^2}{2x} -\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4} +\frac{c^2d^2(a+b\arctan(cx))}{3x^3}+\frac{8}{15}bc^5d^2\log(x) -\frac{1}{60}bc^5d^2\log(i-cx)-\frac{31}{60}bc^5d^2\log(i+cx)$$

output
```
-1/20*b*c*d^2/x^4-1/6*I*b*c^2*d^2/x^3+4/15*b*c^3*d^2/x^2+1/2*I*b*c^4*d^2/x
-1/5*d^2*(a+b*arctan(c*x))/x^5-1/2*I*c*d^2*(a+b*arctan(c*x))/x^4+1/3*c^2*d
^2*(a+b*arctan(c*x))/x^3+8/15*b*c^5*d^2*ln(x)-1/60*b*c^5*d^2*ln(I-c*x)-31/
60*b*c^5*d^2*ln(c*x+I)
```

3.19.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = \frac{d^2\left(-12a-30iacx-3bcx+20ac^2x^2+16bc^3x^3+2b(-6-15icx+10c^2x^2)\arctan(cx)-10ibc^2x^2\,\mathrm{Hyperg}\right.}{60x^5}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(d^2*(-12*a - (30*I)*a*c*x - 3*b*c*x + 20*a*c^2*x^2 + 16*b*c^3*x^3 + 2*b*(
-6 - (15*I)*c*x + 10*c^2*x^2)*ArcTan[c*x] - (10*I)*b*c^2*x^2*Hypergeometri
c2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 32*b*c^5*x^5*Log[x] - 16*b*c^5*x^5*Log[1
 + c^2*x^2]))/(60*x^5)
```

3.19.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \; 5407$$

$$-bc\int -\frac{d^2\left(-10c^2x^2+15icx+6\right)}{30x^5\left(c^2x^2+1\right)}dx+\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}bcd^2\int \frac{-10c^2x^2+15icx+6}{x^5\left(c^2x^2+1\right)}dx+\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}$$

$$\Big\downarrow \; 2333$$

$$\frac{1}{30}bcd^2\int \left(-\frac{c^5}{2(cx-i)}-\frac{31c^5}{2(cx+i)}+\frac{16c^4}{x}-\frac{15ic^3}{x^2}-\frac{16c^2}{x^3}+\frac{15ic}{x^4}+\frac{6}{x^5}\right)dx+\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}+\frac{1}{30}bcd^2\left(16c^4\log(x)-\frac{1}{2}c^4\log(-cx+i)-\frac{31}{2}c^4\log(cx+i)+\frac{15ic^3}{x}+\frac{8c^2}{x^2}-\frac{5ic}{x^3}-\frac{3}{2x^4}\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^2*(a + b*ArcTan[c*x]))/x^5 - ((I/2)*c*d^2*(a + b*ArcTan[c*x]))/x^4
 + (c^2*d^2*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*d^2*(-3/(2*x^4) - ((5*I)*c
)/x^3 + (8*c^2)/x^2 + ((15*I)*c^3)/x + 16*c^4*Log[x] - (c^4*Log[I - c*x])/
2 - (31*c^4*Log[I + c*x])/2))/30
```

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.19.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

method	result
parts	$a\,d^2\left(-\frac{ic}{2x^4}-\frac{1}{5x^5}+\frac{c^2}{3x^3}\right)+b\,d^2c^5\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{i\arctan(cx)}{2c^4x^4}+\frac{\arctan(cx)}{3c^3x^3}-\frac{i}{6c^3x^3}+\frac{i}{2cx}-\frac{1}{20c}\right.$
derivativedivides	$c^5\left(a\,d^2\left(-\frac{1}{5c^5x^5}-\frac{i}{2c^4x^4}+\frac{1}{3c^3x^3}\right)+b\,d^2\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{i\arctan(cx)}{2c^4x^4}+\frac{\arctan(cx)}{3c^3x^3}-\frac{i}{6c^3x^3}+\frac{i}{2cx}\right.\right.$
default	$c^5\left(a\,d^2\left(-\frac{1}{5c^5x^5}-\frac{i}{2c^4x^4}+\frac{1}{3c^3x^3}\right)+b\,d^2\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{i\arctan(cx)}{2c^4x^4}+\frac{\arctan(cx)}{3c^3x^3}-\frac{i}{6c^3x^3}+\frac{i}{2cx}\right.\right.$
risch	$-\frac{ib\,d^2(10c^2x^2-15icx-6)\ln(icx+1)}{60x^5}-\frac{d^2(31b\,c^5\ln(-cx-i)x^5+b\,c^5\ln(cx-i)x^5-32b\,c^5\ln(-x)x^5-30ib\,c^4x^4-10ib\,x^2}{}$
parallelrisch	$\frac{30ic^5b\,d^2\arctan(cx)x^5-16\ln(c^2x^2+1)x^5b\,c^5d^2+32\ln(x)x^5b\,c^5d^2-16b\,c^5d^2x^5+30ix^4b\,c^4d^2+16b\,c^3d^2x^3-10ix^2b\,c^2d^2+}{60x^5}$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(-1/2*I*c/x^4-1/5/x^5+1/3*c^2/x^3)+b*d^2*c^5*(-1/5/c^5/x^5*arctan(c*
x)-1/2*I*arctan(c*x)/c^4/x^4+1/3*arctan(c*x)/c^3/x^3-1/6*I/c^3/x^3+1/2*I/c
/x-1/20/c^4/x^4+4/15/c^2/x^2+8/15*ln(c*x)-4/15*ln(c^2*x^2+1)+1/2*I*arctan(
c*x))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int\frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$
$$=\frac{32\,bc^5d^2x^5\log(x)-31\,bc^5d^2x^5\log\left(\frac{cx+i}{c}\right)-bc^5d^2x^5\log\left(\frac{cx-i}{c}\right)+30i\,bc^4d^2x^4+16\,bc^3d^2x^3+10\,(2\,a-i\,b)c^2}{60\,x^5}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/60*(32*b*c^5*d^2*x^5*log(x) - 31*b*c^5*d^2*x^5*log((c*x + I)/c) - b*c^5*
d^2*x^5*log((c*x - I)/c) + 30*I*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a
 - I*b)*c^2*d^2*x^2 - 3*(10*I*a + b)*c*d^2*x - 12*a*d^2 + (10*I*b*c^2*d^2*
x^2 + 15*b*c*d^2*x - 6*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^5
```

3.19.6 Sympy [A] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.68

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{8bc^5d^2\log(10395b^2c^{11}d^4x)}{15} - \frac{bc^5d^2\log(10395b^2c^{11}d^4x - 10395ib^2c^{10}d^4)}{60}$$
$$- \frac{31bc^5d^2\log(10395b^2c^{11}d^4x + 10395ib^2c^{10}d^4)}{60}$$
$$+ \frac{(-10ibc^2d^2x^2 - 15bcd^2x + 6ibd^2)\log(icx+1)}{60x^5}$$
$$+ \frac{(10ibc^2d^2x^2 + 15bcd^2x - 6ibd^2)\log(-icx+1)}{60x^5}$$
$$- \frac{12ad^2 - 30ibc^4d^2x^4 - 16bc^3d^2x^3 + x^2(-20ac^2d^2 + 10ibc^2d^2) + x(30iacd^2 + 3bcd^2)}{60x^5}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**6,x)
```

output
```
8*b*c**5*d**2*log(10395*b**2*c**11*d**4*x)/15 - b*c**5*d**2*log(10395*b**2
*c**11*d**4*x - 10395*I*b**2*c**10*d**4)/60 - 31*b*c**5*d**2*log(10395*b**
2*c**11*d**4*x + 10395*I*b**2*c**10*d**4)/60 + (-10*I*b*c**2*d**2*x**2 - 1
5*b*c*d**2*x + 6*I*b*d**2)*log(I*c*x + 1)/(60*x**5) + (10*I*b*c**2*d**2*x*
*2 + 15*b*c*d**2*x - 6*I*b*d**2)*log(-I*c*x + 1)/(60*x**5) - (12*a*d**2 -
30*I*b*c**4*d**2*x**4 - 16*b*c**3*d**2*x**3 + x**2*(-20*a*c**2*d**2 + 10*I
*b*c**2*d**2) + x*(30*I*a*c*d**2 + 3*b*c*d**2))/(60*x**5)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.07

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$
$$= -\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bc^2d^2$$
$$+ \frac{1}{6}i\left(\left(3c^3\arctan(cx) + \frac{3c^2x^2-1}{x^3}\right)c - \frac{3\arctan(cx)}{x^4}\right)bcd^2$$
$$- \frac{1}{20}\left(\left(2c^4\log\left(c^2x^2+1\right) - 2c^4\log\left(x^2\right) - \frac{2c^2x^2-1}{x^4}\right)c + \frac{4\arctan(cx)}{x^5}\right)bd^2$$
$$+ \frac{ac^2d^2}{3x^3} - \frac{i\,acd^2}{2x^4} - \frac{ad^2}{5x^5}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)
*b*c^2*d^2 + 1/6*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan
(c*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c
^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2 + 1/3*a*c^2*d^2/x^3 - 1/2*I*
a*c*d^2/x^4 - 1/5*a*d^2/x^5
```

3.19.8 Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.43

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = \frac{8\,b\,c^5\,d^2\,\ln{(x)}}{15} - \frac{4\,b\,c^5\,d^2\,\ln{(c^2\,x^2+1)}}{15} - \frac{\frac{a\,d^2}{5} + \frac{b\,d^2\,\mathrm{atan}(c\,x)}{5} - \frac{4\,b\,c^5\,d^2\,x^5}{15} - \frac{b\,c^6\,d^2\,x^6\,1\mathrm{i}}{2} - \frac{c^4\,d^2\,x^4\,(a+b\,1\mathrm{i})}{3} + \frac{c\,d^2\,x\,(b+a\,10\mathrm{i})}{20} - \frac{c^2\,d^2\,x^2\,(4\,a-b\,5\mathrm{i})}{30} + \frac{c^3\,d^2\,x^3\,(-13\,b+a\,3}{60}}{c^2\,x^7+x^5} + \frac{b\,c^8\,d^2\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,1\mathrm{i}}{2\,(c^2)^{3/2}}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^6,x)
```

output
```
(8*b*c^5*d^2*log(x))/15 - (4*b*c^5*d^2*log(c^2*x^2 + 1))/15 - ((a*d^2)/5 +
 (b*d^2*atan(c*x))/5 - (4*b*c^5*d^2*x^5)/15 - (b*c^6*d^2*x^6*1i)/2 - (c^4*
d^2*x^4*(a + b*1i))/3 + (c*d^2*x*(a*10i + b))/20 - (c^2*d^2*x^2*(4*a - b*5
i))/30 + (c^3*d^2*x^3*(a*30i - 13*b))/60 - (2*b*c^2*d^2*x^2*atan(c*x))/15
+ (b*c^3*d^2*x^3*atan(c*x)*1i)/2 - (b*c^4*d^2*x^4*atan(c*x))/3 + (b*c*d^2*
x*atan(c*x)*1i)/2)/(x^5 + c^2*x^7) + (b*c^8*d^2*atan((c^2*x)/(c^2)^(1/2))*
1i)/(2*(c^2)^(3/2))
```

3.19. $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$

3.20 $\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$

3.20.1 Optimal result

Integrand size = 23, antiderivative size = 205

$$\begin{aligned}\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx = {} & \frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4 \\ & +\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6-\frac{3bd^3\arctan(cx)}{4c^4} \\ & +\frac{1}{4}d^3x^4(a+b\arctan(cx)) \\ & +\frac{3}{5}icd^3x^5(a+b\arctan(cx)) \\ & -\frac{1}{2}c^2d^3x^6(a+b\arctan(cx)) \\ & -\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{13ibd^3\log(1+c^2x^2)}{35c^4}\end{aligned}$$

output
```
3/4*b*d^3*x/c^3+13/35*I*b*d^3*x^2/c^2-1/4*b*d^3*x^3/c-13/70*I*b*d^3*x^4+1/
10*b*c*d^3*x^5+1/42*I*b*c^2*d^3*x^6-3/4*b*d^3*arctan(c*x)/c^4+1/4*d^3*x^4*
(a+b*arctan(c*x))+3/5*I*c*d^3*x^5*(a+b*arctan(c*x))-1/2*c^2*d^3*x^6*(a+b*a
rctan(c*x))-1/7*I*c^3*d^3*x^7*(a+b*arctan(c*x))-13/35*I*b*d^3*ln(c^2*x^2+1
)/c^4
```

3.20.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{d^3(3ac^4x^4(35+84icx-70c^2x^2-20ic^3x^3)+bcx(315+156icx-105c^2x^2-78ic^3x^3+42c^4x^4+10ic^5x^5)}{420c^4}$$

input
```
Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(3*a*c^4*x^4*(35 + (84*I)*c*x - 70*c^2*x^2 - (20*I)*c^3*x^3) + b*c*x*
(315 + (156*I)*c*x - 105*c^2*x^2 - (78*I)*c^3*x^3 + 42*c^4*x^4 + (10*I)*c^
5*x^5) + 3*b*(-105 + 35*c^4*x^4 + (84*I)*c^5*x^5 - 70*c^6*x^6 - (20*I)*c^7
*x^7)*ArcTan[c*x] - (156*I)*b*Log[1 + c^2*x^2]))/(420*c^4)
```

3.20.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^3x^4(-20ic^3x^3-70c^2x^2+84icx+35)}{140(c^2x^2+1)}dx-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{140}bcd^3\int \frac{x^4(-20ic^3x^3-70c^2x^2+84icx+35)}{c^2x^2+1}dx-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{140}bcd^3\int\left(-20icx^5-70x^4+\frac{104ix^3}{c}+\frac{105x^2}{c^2}-\frac{104ix}{c^3}+\frac{104icx+105}{c^4(c^2x^2+1)}-\frac{105}{c^4}\right)dx-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))-\frac{1}{140}bcd^3\left(\frac{105\arctan(cx)}{c^5}-\frac{105x}{c^4}-\frac{52ix^2}{c^3}+\frac{35x^3}{c^2}+\frac{52i\log(c^2x^2+1)}{c^5}-\frac{10}{3}icx^6+\frac{26ix^4}{c}-14x^5\right)$$

input
```
Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^4*(a + b*ArcTan[c*x]))/4 + ((3*I)/5)*c*d^3*x^5*(a + b*ArcTan[c*x])
- (c^2*d^3*x^6*(a + b*ArcTan[c*x]))/2 - (I/7)*c^3*d^3*x^7*(a + b*ArcTan[c*
x]) - (b*c*d^3*((-105*x)/c^4 - ((52*I)*x^2)/c^3 + (35*x^3)/c^2 + ((26*I)*x
^4)/c - 14*x^5 - ((10*I)/3)*c*x^6 + (105*ArcTan[c*x])/c^5 + ((52*I)*Log[1
+ c^2*x^2])/c^5))/140
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.20.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

method	result
parts	$a\,d^3\left(-\frac{1}{7}ic^3x^7-\frac{1}{2}c^2x^6+\frac{3}{5}icx^5+\frac{1}{4}x^4\right)+\frac{b\,d^3\left(-\frac{i\arctan(cx)c^7x^7}{7}-\frac{\arctan(cx)c^6x^6}{2}+\frac{3i\arctan(cx)c^5x^5}{5}+\frac{c^4}{}\right.}{}$
derivativedivides	$\frac{a\,d^3\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b\,d^3\left(-\frac{i\arctan(cx)c^7x^7}{7}-\frac{\arctan(cx)c^6x^6}{2}+\frac{3i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\right.}{c^4}$
default	$\frac{a\,d^3\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b\,d^3\left(-\frac{i\arctan(cx)c^7x^7}{7}-\frac{\arctan(cx)c^6x^6}{2}+\frac{3i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\right.}{c^4}$
parallelrisch	$-\frac{-252ix^5ac^5d^3-156ix^2bc^2d^3-252ic^5bd^3\arctan(cx)x^5+210bc^6d^3\arctan(cx)x^6+60ic^7bd^3\arctan(cx)x^7+210ac^6d^3}{}$
risch	$-\frac{d^3b\left(20c^3x^7-70ic^2x^6-84x^5c+35ix^4\right)\ln(icx+1)}{280}+\frac{d^3c^3bx^7\ln(-icx+1)}{14}-\frac{13ibd^3x^4}{70}-\frac{d^3c^2ax^6}{2}+\frac{ibc^2d^3x^6}{42}$

input
```
int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(-1/7*I*c^3*x^7-1/2*c^2*x^6+3/5*I*c*x^5+1/4*x^4)+b*d^3/c^4*(-1/7*I*a
rctan(c*x)*c^7*x^7-1/2*arctan(c*x)*c^6*x^6+3/5*I*arctan(c*x)*c^5*x^5+1/4*c
^4*x^4*arctan(c*x)+3/4*c*x+1/42*I*c^6*x^6+1/10*c^5*x^5-13/70*I*c^4*x^4-1/4
*c^3*x^3+13/35*I*c^2*x^2-13/35*I*ln(c^2*x^2+1)-3/4*arctan(c*x))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.99

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{-120i\,ac^7d^3x^7-20\,(21\,a-i\,b)c^6d^3x^6-84\,(-6i\,a-b)c^5d^3x^5+6\,(35\,a-26i\,b)c^4d^3x^4-210\,bc^3d^3x^3+31}{}$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/840*(-120*I*a*c^7*d^3*x^7 - 20*(21*a - I*b)*c^6*d^3*x^6 - 84*(-6*I*a - b
)*c^5*d^3*x^5 + 6*(35*a - 26*I*b)*c^4*d^3*x^4 - 210*b*c^3*d^3*x^3 + 312*I*
b*c^2*d^3*x^2 + 630*b*c*d^3*x - 627*I*b*d^3*log((c*x + I)/c) + 3*I*b*d^3*l
og((c*x - I)/c) + 3*(20*b*c^7*d^3*x^7 - 70*I*b*c^6*d^3*x^6 - 84*b*c^5*d^3*
x^5 + 35*I*b*c^4*d^3*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

3.20.6 Sympy [A] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.60

$$
\begin{aligned}
&\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{iac^3d^3x^7}{7} - \frac{bd^3x^3}{4c} + \frac{13ibd^3x^2}{35c^2} + \frac{3bd^3x}{4c^3} \\
&\quad - \frac{bd^3\left(-\frac{i\log(353bcd^3x-353ibd^3)}{280} + \frac{351i\log(353bcd^3x+353ibd^3)}{560}\right)}{c^4} \\
&\quad - x^6\left(\frac{ac^2d^3}{2} - \frac{ibc^2d^3}{42}\right) - x^5\left(-\frac{3iacd^3}{5} - \frac{bcd^3}{10}\right) - x^4\left(-\frac{ad^3}{4} + \frac{13ibd^3}{70}\right) \\
&\quad + \left(-\frac{bc^3d^3x^7}{14} + \frac{ibc^2d^3x^6}{4} + \frac{3bcd^3x^5}{10} - \frac{ibd^3x^4}{8}\right)\log(icx+1) \\
&\quad + \frac{(40bc^7d^3x^7 - 140ibc^6d^3x^6 - 168bc^5d^3x^5 + 70ibc^4d^3x^4 - 67ibd^3)\log(-icx+1)}{560c^4}
\end{aligned}
$$

input
```
integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**7/7 - b*d**3*x**3/(4*c) + 13*I*b*d**3*x**2/(35*c**2) + 3
*b*d**3*x/(4*c**3) - b*d**3*(-I*log(353*b*c*d**3*x - 353*I*b*d**3)/280 + 3
51*I*log(353*b*c*d**3*x + 353*I*b*d**3)/560)/c**4 - x**6*(a*c**2*d**3/2 -
I*b*c**2*d**3/42) - x**5*(-3*I*a*c*d**3/5 - b*c*d**3/10) - x**4*(-a*d**3/4
 + 13*I*b*d**3/70) + (-b*c**3*d**3*x**7/14 + I*b*c**2*d**3*x**6/4 + 3*b*c*
d**3*x**5/10 - I*b*d**3*x**4/8)*log(I*c*x + 1) + (40*b*c**7*d**3*x**7 - 14
0*I*b*c**6*d**3*x**6 - 168*b*c**5*d**3*x**5 + 70*I*b*c**4*d**3*x**4 - 67*I
*b*d**3)*log(-I*c*x + 1)/(560*c**4)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.27

$$
\begin{aligned}
&\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{1}{7}i\,ac^3d^3x^7 - \frac{1}{2}\,ac^2d^3x^6 + \frac{3}{5}i\,acd^3x^5 \\
&\quad - \frac{1}{84}i\left(12\,x^7\arctan{(cx)} - c\left(\frac{2\,c^4x^6 - 3\,c^2x^4 + 6\,x^2}{c^6} - \frac{6\,\log{(c^2x^2+1)}}{c^8}\right)\right)bc^3d^3 \\
&\quad + \frac{1}{4}\,ad^3x^4 - \frac{1}{30}\left(15\,x^6\arctan{(cx)} - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan{(cx)}}{c^7}\right)\right)bc^2d^3 \\
&\quad + \frac{3}{20}i\left(4\,x^5\arctan{(cx)} - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log{(c^2x^2+1)}}{c^6}\right)\right)bcd^3 \\
&\quad + \frac{1}{12}\left(3\,x^4\arctan{(cx)} - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan{(cx)}}{c^5}\right)\right)bd^3
\end{aligned}
$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/7*I*a*c^3*d^3*x^7 - 1/2*a*c^2*d^3*x^6 + 3/5*I*a*c*d^3*x^5 - 1/84*I*(12*
x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 +
 1)/c^8))*b*c^3*d^3 + 1/4*a*d^3*x^4 - 1/30*(15*x^6*arctan(c*x) - c*((3*c^4
*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^3 + 3/20*I*(4*
x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*
d^3 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5
))*b*d^3
```

3.20.8 Giac [F]

$$
\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^3(b\arctan{(cx)}+a)x^3\,dx
$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.20.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.91

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{\frac{d^3\left(315\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,156\mathrm{i}\right)}{420}+\frac{b\,c^3\,d^3\,x^3}{4}-\frac{3\,b\,c\,d^3\,x}{4}-\frac{b\,c^2\,d^3\,x^2\,13\mathrm{i}}{35}}{c^4}$$
$$+\frac{d^3\left(105\,a\,x^4+105\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,78\mathrm{i}\right)}{420}-\frac{c^3\,d^3\left(a\,x^7\,60\mathrm{i}+b\,x^7\,\mathrm{atan}(c\,x)\,60\mathrm{i}\right)}{420}$$
$$+\frac{c\,d^3\left(a\,x^5\,252\mathrm{i}+42\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,252\mathrm{i}\right)}{420}$$
$$-\frac{c^2\,d^3\left(210\,a\,x^6+210\,b\,x^6\,\mathrm{atan}(c\,x)-b\,x^6\,10\mathrm{i}\right)}{420}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
(d^3*(105*a*x^4 - b*x^4*78i + 105*b*x^4*atan(c*x)))/420 - ((d^3*(315*b*ata
n(c*x) + b*log(c^2*x^2 + 1)*156i))/420 - (b*c^2*d^3*x^2*13i)/35 + (b*c^3*d
^3*x^3)/4 - (3*b*c*d^3*x)/4)/c^4 - (c^3*d^3*(a*x^7*60i + b*x^7*atan(c*x)*6
0i))/420 + (c*d^3*(a*x^5*252i + 42*b*x^5 + b*x^5*atan(c*x)*252i))/420 - (c
^2*d^3*(210*a*x^6 - b*x^6*10i + 210*b*x^6*atan(c*x)))/420
```

3.21 $\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$

3.21.1 Optimal result

Integrand size = 23, antiderivative size = 191

$$\begin{aligned}\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx &= \frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4\\&\quad+\frac{1}{30}ibc^2d^3x^5-\frac{11ibd^3\arctan(cx)}{12c^3}\\&\quad+\frac{1}{3}d^3x^3(a+b\arctan(cx))\\&\quad+\frac{3}{4}icd^3x^4(a+b\arctan(cx))\\&\quad-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))\\&\quad-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))+\frac{7bd^3\log(1+c^2x^2)}{15c^3}\end{aligned}$$

output
```
11/12*I*b*d^3*x/c^2-7/15*b*d^3*x^2/c-11/36*I*b*d^3*x^3+3/20*b*c*d^3*x^4+1/
30*I*b*c^2*d^3*x^5-11/12*I*b*d^3*arctan(c*x)/c^3+1/3*d^3*x^3*(a+b*arctan(c
*x))+3/4*I*c*d^3*x^4*(a+b*arctan(c*x))-3/5*c^2*d^3*x^5*(a+b*arctan(c*x))-1
/6*I*c^3*d^3*x^6*(a+b*arctan(c*x))+7/15*b*d^3*ln(c^2*x^2+1)/c^3
```

3.21.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.76

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{d^3(3ac^3x^3(20+45icx-36c^2x^2-10ic^3x^3)+bcx(165i-84cx-55ic^2x^2+27c^3x^3+6ic^4x^4)+3b(-55i+}{180c^3}$$

input
```
Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(3*a*c^3*x^3*(20 + (45*I)*c*x - 36*c^2*x^2 - (10*I)*c^3*x^3) + b*c*x*
(165*I - 84*c*x - (55*I)*c^2*x^2 + 27*c^3*x^3 + (6*I)*c^4*x^4) + 3*b*(-55*
I + 20*c^3*x^3 + (45*I)*c^4*x^4 - 36*c^5*x^5 - (10*I)*c^6*x^6)*ArcTan[c*x]
 + 84*b*Log[1 + c^2*x^2]))/(180*c^3)
```

3.21.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5407$$

$$-bc\int \frac{d^3x^3\left(-10ic^3x^3-36c^2x^2+45icx+20\right)}{60\left(c^2x^2+1\right)}dx-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{60}bcd^3\int \frac{x^3\left(-10ic^3x^3-36c^2x^2+45icx+20\right)}{c^2x^2+1}dx-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \ 2333$$

$$-\frac{1}{60}bcd^3\int\left(-10icx^4-36x^3+\frac{55ix^2}{c}+\frac{56x}{c^2}+\frac{55i-56cx}{c^3\left(c^2x^2+1\right)}-\frac{55i}{c^3}\right)dx-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))-\frac{1}{60}bcd^3\left(\frac{55i\arctan(cx)}{c^4}-\frac{55ix}{c^3}+\frac{28x^2}{c^2}-\frac{28\log\left(c^2x^2+1\right)}{c^4}-2icx^5+\frac{55ix^3}{3c}-9x^4\right)$$

input
```
Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^3*(a + b*ArcTan[c*x]))/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[c*x])
- (3*c^2*d^3*x^5*(a + b*ArcTan[c*x]))/5 - (I/6)*c^3*d^3*x^6*(a + b*ArcTan[
c*x]) - (b*c*d^3*(((-55*I)*x)/c^3 + (28*x^2)/c^2 + (((55*I)/3)*x^3)/c - 9*
x^4 - (2*I)*c*x^5 + ((55*I)*ArcTan[c*x])/c^4 - (28*Log[1 + c^2*x^2])/c^4))
/60
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.21.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81

method	result
parts	$a\,d^{3}\left(-\frac{1}{6}i c^{3} x^{6}-\frac{3}{5}c^{2} x^{5}+\frac{3}{4}i c\,x^{4}+\frac{1}{3}x^{3}\right)+\frac{b\,d^{3}\left(-\frac{i \arctan\left(c x\right) c^{6} x^{6}}{6}-\frac{3 c^{5} x^{5} \arctan\left(c x\right)}{5}+\frac{3 i \arctan\left(c x\right) c^{4} x^{4}}{4}+\frac{c^{3}}{}\right.}{}$
derivativedivides	$\frac{a\,d^{3}\left(-\frac{1}{6}i c^{6} x^{6}-\frac{3}{5}c^{5} x^{5}+\frac{3}{4}i c^{4} x^{4}+\frac{1}{3}c^{3} x^{3}\right)+b\,d^{3}\left(-\frac{i \arctan\left(c x\right) c^{6} x^{6}}{6}-\frac{3 c^{5} x^{5} \arctan\left(c x\right)}{5}+\frac{3 i \arctan\left(c x\right) c^{4} x^{4}}{4}+\frac{c^{3} x^{3} \arctan\left(c x\right)}{3}+\right.}{c^{3}}$
default	$\frac{a\,d^{3}\left(-\frac{1}{6}i c^{6} x^{6}-\frac{3}{5}c^{5} x^{5}+\frac{3}{4}i c^{4} x^{4}+\frac{1}{3}c^{3} x^{3}\right)+b\,d^{3}\left(-\frac{i \arctan\left(c x\right) c^{6} x^{6}}{6}-\frac{3 c^{5} x^{5} \arctan\left(c x\right)}{5}+\frac{3 i \arctan\left(c x\right) c^{4} x^{4}}{4}+\frac{c^{3} x^{3} \arctan\left(c x\right)}{3}+\right.}{c^{3}}$
parallelrisch	$-\frac{-165 i b\,d^{3} x c+55 i x^{3} b\,c^{3} d^{3}+165 i b\,d^{3} \arctan\left(c x\right)+108 c^{5} b\,d^{3} \arctan\left(c x\right) x^{5}-6 i x^{5} b\,c^{5} d^{3}+108 a\,c^{5} d^{3} x^{5}-135 i x^{4} a\,c^{4} d^{3}-27}{}$
risch	$-\frac{d^{3} b\left(10 c^{3} x^{6}-36 i c^{2} x^{5}-45 c\,x^{4}+20 i x^{3}\right) \ln\left(i c x+1\right)}{120}+\frac{11 i b\,d^{3} x}{12 c^{2}}+\frac{d^{3} c^{3} x^{6} b \ln\left(-i c x+1\right)}{12}-\frac{11 i b\,d^{3} \arctan\left(c x\right)}{12 c^{3}}-11 i$

input
```
int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(-1/6*I*c^3*x^6-3/5*c^2*x^5+3/4*I*c*x^4+1/3*x^3)+b*d^3/c^3*(-1/6*I*a
rctan(c*x)*c^6*x^6-3/5*c^5*x^5*arctan(c*x)+3/4*I*arctan(c*x)*c^4*x^4+1/3*c
^3*x^3*arctan(c*x)+11/12*I*c*x+1/30*I*c^5*x^5+3/20*c^4*x^4-11/36*I*c^3*x^3
-7/15*c^2*x^2+7/15*ln(c^2*x^2+1)-11/12*I*arctan(c*x))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.99

$$\int x^{2}(d+icdx)^{3}(a+b\arctan(cx))\,dx$$
$$=\frac{-60i\,ac^{6}d^{3}x^{6}-12\,(18\,a-i\,b)c^{5}d^{3}x^{5}-54\,(-5i\,a-b)c^{4}d^{3}x^{4}+10\,(12\,a-11i\,b)c^{3}d^{3}x^{3}-168\,bc^{2}d^{3}x^{2}+33}{}$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/360*(-60*I*a*c^6*d^3*x^6 - 12*(18*a - I*b)*c^5*d^3*x^5 - 54*(-5*I*a - b)
*c^4*d^3*x^4 + 10*(12*a - 11*I*b)*c^3*d^3*x^3 - 168*b*c^2*d^3*x^2 + 330*I*
b*c*d^3*x + 333*b*d^3*log((c*x + I)/c) + 3*b*d^3*log((c*x - I)/c) + 3*(10*
b*c^6*d^3*x^6 - 36*I*b*c^5*d^3*x^5 - 45*b*c^4*d^3*x^4 + 20*I*b*c^3*d^3*x^3
)*log(-(c*x + I)/(c*x - I)))/c^3
```

3.21.6 Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.65

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{iac^3d^3x^6}{6} - \frac{7bd^3x^2}{15c} + \frac{11ibd^3x}{12c^2} - \frac{bd^3\left(-\frac{\log\left(310bcd^3x-310ibd^3\right)}{120} - \frac{209\log\left(310bcd^3x+310ibd^3\right)}{280}\right)}{c^3}$$
$$- x^5\cdot\left(\frac{3ac^2d^3}{5} - \frac{ibc^2d^3}{30}\right) - x^4\left(-\frac{3iacd^3}{4} - \frac{3bcd^3}{20}\right) - x^3\left(-\frac{ad^3}{3} + \frac{11ibd^3}{36}\right)$$
$$+ \left(-\frac{bc^3d^3x^6}{12} + \frac{3ibc^2d^3x^5}{10} + \frac{3bcd^3x^4}{8} - \frac{ibd^3x^3}{6}\right)\log\left(icx+1\right)$$
$$+ \frac{\left(70bc^6d^3x^6 - 252ibc^5d^3x^5 - 315bc^4d^3x^4 + 140ibc^3d^3x^3 + 150bd^3\right)\log\left(-icx+1\right)}{840c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**6/6 - 7*b*d**3*x**2/(15*c) + 11*I*b*d**3*x/(12*c**2) - b
*d**3*(-log(310*b*c*d**3*x - 310*I*b*d**3)/120 - 209*log(310*b*c*d**3*x +
310*I*b*d**3)/280)/c**3 - x**5*(3*a*c**2*d**3/5 - I*b*c**2*d**3/30) - x**4
*(-3*I*a*c*d**3/4 - 3*b*c*d**3/20) - x**3*(-a*d**3/3 + 11*I*b*d**3/36) + (
-b*c**3*d**3*x**6/12 + 3*I*b*c**2*d**3*x**5/10 + 3*b*c*d**3*x**4/8 - I*b*d
**3*x**3/6)*log(I*c*x + 1) + (70*b*c**6*d**3*x**6 - 252*I*b*c**5*d**3*x**5
 - 315*b*c**4*d**3*x**4 + 140*I*b*c**3*d**3*x**3 + 150*b*d**3)*log(-I*c*x
+ 1)/(840*c**3)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.27

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{1}{6}i\,ac^3d^3x^6 - \frac{3}{5}\,ac^2d^3x^5 + \frac{3}{4}i\,acd^3x^4$$
$$- \frac{1}{90}i\left(15\,x^6\arctan\left(cx\right) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bc^3d^3$$
$$- \frac{3}{20}\left(4\,x^5\arctan\left(cx\right) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^2d^3$$
$$+ \frac{1}{3}\,ad^3x^3 + \frac{1}{4}i\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd^3$$
$$+ \frac{1}{6}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^3$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/6*I*a*c^3*d^3*x^6 - 3/5*a*c^2*d^3*x^5 + 3/4*I*a*c*d^3*x^4 - 1/90*I*(15*
x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c
^7))*b*c^3*d^3 - 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*lo
g(c^2*x^2 + 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/4*I*(3*x^4*arctan(c*x)
- c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctan
(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3
```

3.21.8 Giac [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^3(b\arctan{(cx)}+a)x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx \\ = -\frac{\frac{d^3\,\left(-84\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}(c\,x)\,165\mathrm{i}\right)}{180}+\frac{7\,b\,c^2\,d^3\,x^2}{15}-\frac{b\,c\,d^3\,x\,11\mathrm{i}}{12}}{c^3} \\ +\frac{d^3\,\left(60\,a\,x^3+60\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,55\mathrm{i}\right)}{180}-\frac{c^3\,d^3\,\left(a\,x^6\,30\mathrm{i}+b\,x^6\,\mathrm{atan}(c\,x)\,30\mathrm{i}\right)}{180} \\ +\frac{c\,d^3\,\left(a\,x^4\,135\mathrm{i}+27\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\,135\mathrm{i}\right)}{180} \\ -\frac{c^2\,d^3\,\left(108\,a\,x^5+108\,b\,x^5\,\mathrm{atan}(c\,x)-b\,x^5\,6\mathrm{i}\right)}{180}\end{aligned}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
(d^3*(60*a*x^3 - b*x^3*55i + 60*b*x^3*atan(c*x)))/180 - ((d^3*(b*atan(c*x)
*165i - 84*b*log(c^2*x^2 + 1)))/180 + (7*b*c^2*d^3*x^2)/15 - (b*c*d^3*x*11
i)/12)/c^3 - (c^3*d^3*(a*x^6*30i + b*x^6*atan(c*x)*30i))/180 + (c*d^3*(a*x
^4*135i + 27*b*x^4 + b*x^4*atan(c*x)*135i))/180 - (c^2*d^3*(108*a*x^5 - b*
x^5*6i + 108*b*x^5*atan(c*x)))/180
```

3.21. $\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$

3.22 $\int x(d+icdx)^3(a+b\arctan(cx))\,dx$

3.22.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{3bd^3x}{5c}-\frac{3ibd^3(i-cx)^2}{20c^2}-\frac{bd^3(i-cx)^3}{20c^2}+\frac{ibd^3(i-cx)^4}{20c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{6ibd^3\log(i+cx)}{5c^2}$$

output
```
-3/5*b*d^3*x/c-3/20*I*b*d^3*(I-c*x)^2/c^2-1/20*b*d^3*(I-c*x)^3/c^2+1/20*I*
b*d^3*(I-c*x)^4/c^2+1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))/c^2-1/5*d^3*(1+I
*c*x)^5*(a+b*arctan(c*x))/c^2+6/5*I*b*d^3*ln(c*x+I)/c^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx = \frac{d^3(cx(b(-25-12icx+5c^2x^2+ic^3x^3)+acx(10+20icx-15c^2x^2-4ic^3x^3))+b(25+10c^2x^2+20ic^3x^3}{20c^2}$$

input
```
Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(c*x*(b*(-25 - (12*I)*c*x + 5*c^2*x^2 + I*c^3*x^3) + a*c*x*(10 + (20*
I)*c*x - 15*c^2*x^2 - (4*I)*c^3*x^3)) + b*(25 + 10*c^2*x^2 + (20*I)*c^3*x^
3 - 15*c^4*x^4 - (4*I)*c^5*x^5)*ArcTan[c*x] + (12*I)*b*Log[1 + c^2*x^2]))/
(20*c^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^3(i-cx)^3(1-4icx)}{20c^2(cx+i)}dx-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^3\int \frac{(i-cx)^3(1-4icx)}{cx+i}dx}{20c}-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}$$

$$\Big\downarrow \text{86}$$

$$\frac{bd^3\int \left(-4i(i-cx)^3+3(cx-i)^2-6i(cx-i)+\frac{24i}{cx+i}-12\right)dx}{20c}-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}+\frac{bd^3\left(\frac{i(-cx+i)^4}{c}-\frac{(-cx+i)^3}{c}-\frac{3i(-cx+i)^2}{c}+\frac{24i\log(cx+i)}{c}-12x\right)}{20c}$$

input
```
Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/(4*c^2) - (d^3*(1 + I*c*x)^5*(a +
b*ArcTan[c*x]))/(5*c^2) + (b*d^3*(-12*x - ((3*I)*(I - c*x)^2)/c - (I - c*x
)^3/c + (I*(I - c*x)^4)/c + ((24*I)*Log[I + c*x])/c))/(20*c)
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.22.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

method	result
parts	$a\,d^3\left(-\frac{1}{5}ic^3x^5-\frac{3}{4}c^2x^4+ic\,x^3+\frac{1}{2}x^2\right)+\frac{b\,d^3\left(-\frac{i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{4}+i\arctan(cx)c^3x^3+\frac{c^2x \cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{a\,d^3\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+b\,d^3\left(-\frac{i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{4}+i\arctan(cx)c^3x^3+\frac{c^2x^2\arctan(cx)}{2}-\frac{5 \cdots}{\cdots}\right.}{c^2}$
default	$\frac{a\,d^3\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+b\,d^3\left(-\frac{i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{4}+i\arctan(cx)c^3x^3+\frac{c^2x^2\arctan(cx)}{2}-\frac{5 \cdots}{\cdots}\right.}{c^2}$
parallelrisch	$\frac{-4ic^5b\,d^3\arctan(cx)x^5-4ix^5a\,c^5d^3+ix^4b\,c^4d^3-15x^4\arctan(cx)b\,c^4d^3+20ix^3\arctan(cx)b\,c^3d^3-15a\,c^4d^3x^4+20ix^3a\,c^3 \cdots}{20c^2}$
risch	$-\frac{d^3b\left(4c^3x^5-15ic^2x^4-20c\,x^3+10ix^2\right)\ln(icx+1)}{40}+\frac{d^3c^3b\,x^5\ln(-icx+1)}{10}-\frac{ia\,c^3d^3x^5}{5}-\frac{3a\,c^2d^3x^4}{4}-\frac{3id^3c^2x^4b \cdots}{\cdots}$

input
```
int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(-1/5*I*c^3*x^5-3/4*c^2*x^4+I*c*x^3+1/2*x^2)+b*d^3/c^2*(-1/5*I*arcta
n(c*x)*c^5*x^5-3/4*c^4*x^4*arctan(c*x)+I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*a
rctan(c*x)-5/4*c*x+1/20*I*c^4*x^4+1/4*c^3*x^3-3/5*I*c^2*x^2+3/5*I*ln(c^2*x
^2+1)+5/4*arctan(c*x))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{-8i\,ac^5d^3x^5-2\,(15\,a-i\,b)c^4d^3x^4-10\,(-4i\,a-b)c^3d^3x^3+4\,(5\,a-6i\,b)c^2d^3x^2-50\,bcd^3x+49i\,bd^3\log\left(\cdots\right.}{40\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/40*(-8*I*a*c^5*d^3*x^5 - 2*(15*a - I*b)*c^4*d^3*x^4 - 10*(-4*I*a - b)*c^
3*d^3*x^3 + 4*(5*a - 6*I*b)*c^2*d^3*x^2 - 50*b*c*d^3*x + 49*I*b*d^3*log((c
*x + I)/c) - I*b*d^3*log((c*x - I)/c) + (4*b*c^5*d^3*x^5 - 15*I*b*c^4*d^3*
x^4 - 20*b*c^3*d^3*x^3 + 10*I*b*c^2*d^3*x^2)*log(-(c*x + I)/(c*x - I)))/c^
2
```

3.22.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(138) = 276$.

Time = 2.16 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.89

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{iac^3d^3x^5}{5} - \frac{5bd^3x}{4c} - \frac{bd^3\left(\frac{i\log\left(19bcd^3x-19ibd^3\right)}{40} - \frac{37i\log\left(19bcd^3x+19ibd^3\right)}{40}\right)}{c^2} - x^4\cdot\left(\frac{3ac^2d^3}{4} - \frac{ibc^2d^3}{20}\right) - x^3\left(-iacd^3 - \frac{bcd^3}{4}\right) - x^2\left(-\frac{ad^3}{2} + \frac{3ibd^3}{5}\right) + \left(-\frac{bc^3d^3x^5}{10} + \frac{3ibc^2d^3x^4}{8} + \frac{bcd^3x^3}{2} - \frac{ibd^3x^2}{4}\right)\log\left(icx+1\right) + \frac{\left(4bc^5d^3x^5 - 15ibc^4d^3x^4 - 20bc^3d^3x^3 + 10ibc^2d^3x^2 + 12ibd^3\right)\log\left(-icx+1\right)}{40c^2}$$

input
```
integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**5/5 - 5*b*d**3*x/(4*c) - b*d**3*(I*log(19*b*c*d**3*x - 1
9*I*b*d**3)/40 - 37*I*log(19*b*c*d**3*x + 19*I*b*d**3)/40)/c**2 - x**4*(3*
a*c**2*d**3/4 - I*b*c**2*d**3/20) - x**3*(-I*a*c*d**3 - b*c*d**3/4) - x**2
*(-a*d**3/2 + 3*I*b*d**3/5) + (-b*c**3*d**3*x**5/10 + 3*I*b*c**2*d**3*x**4
/8 + b*c*d**3*x**3/2 - I*b*d**3*x**2/4)*log(I*c*x + 1) + (4*b*c**5*d**3*x*
*5 - 15*I*b*c**4*d**3*x**4 - 20*b*c**3*d**3*x**3 + 10*I*b*c**2*d**3*x**2 +
 12*I*b*d**3)*log(-I*c*x + 1)/(40*c**2)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.41

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{1}{5}i\,ac^3d^3x^5 - \frac{3}{4}\,ac^2d^3x^4$$
$$-\frac{1}{20}i\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^3d^3$$
$$+i\,acd^3x^3 - \frac{1}{4}\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bc^2d^3$$
$$+\frac{1}{2}i\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bcd^3$$
$$+\frac{1}{2}\,ad^3x^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^3$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/5*I*a*c^3*d^3*x^5 - 3/4*a*c^2*d^3*x^4 - 1/20*I*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^3*d^3 + I*a*c*d^3*x^3
 - 1/4*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b
*c^2*d^3 + 1/2*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
b*c*d^3 + 1/2*a*d^3*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^
3))*b*d^3
```

3.22.8 Giac [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^3(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx = \frac{\frac{d^3\,(25\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,12\mathrm{i})}{20}-\frac{5\,b\,c\,d^3\,x}{4}}{c^2} + \frac{d^3\,(10\,a\,x^2+10\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,12\mathrm{i})}{20} - \frac{c^3\,d^3\,(a\,x^5\,4\mathrm{i}+b\,x^5\,\mathrm{atan}(c\,x)\,4\mathrm{i})}{20} + \frac{c\,d^3\,(a\,x^3\,20\mathrm{i}+5\,b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,20\mathrm{i})}{20} - \frac{c^2\,d^3\,(15\,a\,x^4+15\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,1\mathrm{i})}{20}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
((d^3*(25*b*atan(c*x) + b*log(c^2*x^2 + 1)*12i))/20 - (5*b*c*d^3*x)/4)/c^2
 + (d^3*(10*a*x^2 - b*x^2*12i + 10*b*x^2*atan(c*x)))/20 - (c^3*d^3*(a*x^5*
4i + b*x^5*atan(c*x)*4i))/20 + (c*d^3*(a*x^3*20i + 5*b*x^3 + b*x^3*atan(c*
x)*20i))/20 - (c^2*d^3*(15*a*x^4 - b*x^4*1i + 15*b*x^4*atan(c*x)))/20
```

3.23 $\int (d+icdx)^3(a+b\arctan(cx))\,dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 100

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx = -ibd^3x-\frac{bd^3(1+icx)^2}{4c}-\frac{bd^3(1+icx)^3}{12c}-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}-\frac{2bd^3\log(1-icx)}{c}$$

output
```
-I*b*d^3*x-1/4*b*d^3*(1+I*c*x)^2/c-1/12*b*d^3*(1+I*c*x)^3/c-1/4*I*d^3*(1+I
*c*x)^4*(a+b*arctan(c*x))/c-2*b*d^3*ln(1-I*c*x)/c
```

3.23.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{i(3(d+icdx)^4(a+b\arctan(cx))-bd^4(4i-21cx-6ic^2x^2+c^3x^3+24i\log(i+cx)))}{12cd}$$

input
```
Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/12*I)*(3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]) - b*d^4*(4*I - 21*c*x -
(6*I)*c^2*x^2 + c^3*x^3 + (24*I)*Log[I + c*x])))/(c*d)
```

3.23.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5387$$

$$\frac{ib\int \frac{d^4(icx+1)^4}{c^2x^2+1}dx}{4d}-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}ibd^3\int \frac{(icx+1)^4}{c^2x^2+1}dx-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \ 456$$

$$\frac{1}{4}ibd^3\int \frac{(icx+1)^3}{1-icx}dx-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \ 49$$

$$\frac{1}{4}ibd^3\int \left(-(icx+1)^2-2(icx+1)+\frac{8}{1-icx}-4\right)dx-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}ibd^3\left(\frac{i(1+icx)^3}{3c}+\frac{i(1+icx)^2}{c}+\frac{8i\log(cx+i)}{c}-4x\right)-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

input
```
Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/4*I)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/c + (I/4)*b*d^3*(-4*x + (
I*(1 + I*c*x)^2)/c + ((I/3)*(1 + I*c*x)^3)/c + ((8*I)*Log[I + c*x])/c)
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{ia\,d^3(icx+1)^4}{4}+b\,d^3\left(-\frac{i\arctan(cx)c^4x^4}{4}-c^3x^3\arctan(cx)+\frac{3i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)-\frac{i\arctan(cx)}{4}+\frac{i\left(-7cx+\frac{c^3x^3}{3}\right.}{}\right.}{c}$
default	$\frac{-\frac{ia\,d^3(icx+1)^4}{4}+b\,d^3\left(-\frac{i\arctan(cx)c^4x^4}{4}-c^3x^3\arctan(cx)+\frac{3i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)-\frac{i\arctan(cx)}{4}+\frac{i\left(-7cx+\frac{c^3x^3}{3}\right.}{}\right.}{c}$
parts	$-\frac{ia\,d^3(icx+1)^4}{4c}+\frac{b\,d^3\left(-\frac{i\arctan(cx)c^4x^4}{4}-c^3x^3\arctan(cx)+\frac{3i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)-\frac{i\arctan(cx)}{4}+\frac{i\left(-7cx+\right.}{}\right.}{c}$
parallelrisch	$-\frac{3ix^4\arctan(cx)b\,c^4d^3+3ix^4a\,c^4d^3-ix^3b\,c^3d^3+12x^3\arctan(cx)b\,d^3c^3-18ib\,d^3\arctan(cx)x^2c^2+12a\,c^3d^3x^3-18ix^2a\,c}{12c}$
risch	$-\frac{d^3(cx-i)^4b\ln(icx+1)}{8c}-\frac{id^3a\,c^3x^4}{4}+\frac{d^3c^3x^4b\ln(-icx+1)}{8}-\frac{id^3c^2b\,x^3\ln(-icx+1)}{2}+\frac{ib\,c^2d^3x^3}{12}-x^3d^3c^2a$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/4*I*a*d^3*(1+I*c*x)^4+b*d^3*(-1/4*I*arctan(c*x)*c^4*x^4-c^3*x^3*ar
ctan(c*x)+3/2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)-1/4*I*arctan(c*x)+1/4*
I*(-7*c*x+1/3*c^3*x^3-2*I*c^2*x^2+4*I*ln(c^2*x^2+1)+8*arctan(c*x))))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx = \frac{-6i\,ac^4d^3x^4-2\,(12\,a-i\,b)c^3d^3x^3-12\,(-3i\,a-b)c^2d^3x^2+6\,(4\,a-7i\,b)cd^3x-45\,bd^3\log\left(\frac{cx+i}{c}\right)-3\,bd^3\,\mathrm{l}}{24\,c}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/24*(-6*I*a*c^4*d^3*x^4 - 2*(12*a - I*b)*c^3*d^3*x^3 - 12*(-3*I*a - b)*c^
2*d^3*x^2 + 6*(4*a - 7*I*b)*c*d^3*x - 45*b*d^3*log((c*x + I)/c) - 3*b*d^3*
log((c*x - I)/c) + 3*(b*c^4*d^3*x^4 - 4*I*b*c^3*d^3*x^3 - 6*b*c^2*d^3*x^2
+ 4*I*b*c*d^3*x)*log(-(c*x + I)/(c*x - I)))/c
```

3.23.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(85) = 170.

Time = 2.02 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.67

$$\begin{aligned}\int (d+icdx)^3(a+b\arctan(cx))\,dx &= -\frac{iac^3d^3x^4}{4} - \frac{bd^3\left(\frac{\log(22bcd^3x-22ibd^3)}{8}+\frac{49\log(22bcd^3x+22ibd^3)}{40}\right)}{c} \\ &\quad - x^3\left(ac^2d^3-\frac{ibc^2d^3}{12}\right) - x^2\left(-\frac{3iacd^3}{2}-\frac{bcd^3}{2}\right) - x\left(-ad^3+\frac{7ibd^3}{4}\right) \\ &\quad + \left(-\frac{bc^3d^3x^4}{8}+\frac{ibc^2d^3x^3}{2}+\frac{3bcd^3x^2}{4}-\frac{ibd^3x}{2}\right)\log(icx+1) \\ &\quad + \frac{(5bc^4d^3x^4-20ibc^3d^3x^3-30bc^2d^3x^2+20ibcd^3x-26bd^3)\log(-icx+1)}{40c}\end{aligned}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**4/4 - b*d**3*(log(22*b*c*d**3*x - 22*I*b*d**3)/8 + 49*lo
g(22*b*c*d**3*x + 22*I*b*d**3)/40)/c - x**3*(a*c**2*d**3 - I*b*c**2*d**3/1
2) - x**2*(-3*I*a*c*d**3/2 - b*c*d**3/2) - x*(-a*d**3 + 7*I*b*d**3/4) + (-
b*c**3*d**3*x**4/8 + I*b*c**2*d**3*x**3/2 + 3*b*c*d**3*x**2/4 - I*b*d**3*x
/2)*log(I*c*x + 1) + (5*b*c**4*d**3*x**4 - 20*I*b*c**3*d**3*x**3 - 30*b*c*
*2*d**3*x**2 + 20*I*b*c*d**3*x - 26*b*d**3)*log(-I*c*x + 1)/(40*c)
```

3.23.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.97

$$\begin{aligned}
&\int (d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{1}{4}i\,ac^3d^3x^4 - ac^2d^3x^3 - \frac{1}{12}i\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bc^3d^3 \\
&\quad - \frac{1}{2}\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bc^2d^3 \\
&\quad + \frac{3}{2}i\,acd^3x^2 + \frac{3}{2}i\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bcd^3 \\
&\quad + ad^3x + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))bd^3}{2\,c}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/4*I*a*c^3*d^3*x^4 - a*c^2*d^3*x^3 - 1/12*I*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^3*d^3 - 1/2*(2*x^3*arctan(c*x) -
 c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + 3/2*I*a*c*d^3*x^2 + 3/2*I
*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(
2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3/c
```

3.23.8 Giac [F]

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\begin{aligned}\int (d+icdx)^3(a+b\arctan(cx))\,dx=&-\frac{d^3\,\left(a\,x\,12\mathrm{i}+21\,b\,x+b\,x\,\mathrm{atan}(c\,x)\,12\mathrm{i}\right)\,1\mathrm{i}}{12}\\&-\frac{c^3\,d^3\,\left(3\,a\,x^4+3\,b\,x^4\,\mathrm{atan}(c\,x)\right)\,1\mathrm{i}}{12}\\&+\frac{d^3\,\left(21\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,12\mathrm{i}\right)\,1\mathrm{i}}{12\,c}\\&+\frac{c\,d^3\,\left(18\,a\,x^2+18\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,6\mathrm{i}\right)\,1\mathrm{i}}{12}\\&+\frac{c^2\,d^3\,\left(a\,x^3\,12\mathrm{i}+b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,12\mathrm{i}\right)\,1\mathrm{i}}{12}\end{aligned}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
(d^3*(21*b*atan(c*x) + b*log(c^2*x^2 + 1)*12i)*1i)/(12*c) - (c^3*d^3*(3*a*
x^4 + 3*b*x^4*atan(c*x))*1i)/12 - (d^3*(a*x*12i + 21*b*x + b*x*atan(c*x)*1
2i)*1i)/12 + (c*d^3*(18*a*x^2 - b*x^2*6i + 18*b*x^2*atan(c*x))*1i)/12 + (c
^2*d^3*(a*x^3*12i + b*x^3 + b*x^3*atan(c*x)*12i)*1i)/12
```

3.24 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx$

3.24.1 Optimal result

Integrand size = 23, antiderivative size = 170

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx &= 3iacd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}ibc^2d^3x^2-\frac{3}{2}bd^3\arctan(cx)\\&\quad+3ibcd^3x\arctan(cx)-\frac{3}{2}c^2d^3x^2(a+b\arctan(cx))\\&\quad-\frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))\\&\quad+ad^3\log(x)-\frac{5}{3}ibd^3\log\left(1+c^2x^2\right)\\&\quad+\frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
3*I*a*c*d^3*x+3/2*b*c*d^3*x+1/6*I*b*c^2*d^3*x^2-3/2*b*d^3*arctan(c*x)+3*I*
b*c*d^3*x*arctan(c*x)-3/2*c^2*d^3*x^2*(a+b*arctan(c*x))-1/3*I*c^3*d^3*x^3*
(a+b*arctan(c*x))+a*d^3*ln(x)-5/3*I*b*d^3*ln(c^2*x^2+1)+1/2*I*b*d^3*polylo
g(2,-I*c*x)-1/2*I*b*d^3*polylog(2,I*c*x)
```

3.24.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.82

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = -\frac{1}{6}id^3\big(-18acx+9ibcx-9iac^2x^2-bc^2x^2+2ac^3x^3 - 9ib\arctan(cx)-18bcx\arctan(cx) - 9ibc^2x^2\arctan(cx)+2bc^3x^3\arctan(cx) + 6ia\log(x)+10b\log\left(1+c^2x^2\right) - 3b\,\mathrm{PolyLog}(2,-icx)+3b\,\mathrm{PolyLog}(2,icx)\big)$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(-1/6*I)*d^3*(-18*a*c*x + (9*I)*b*c*x - (9*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*
c^3*x^3 - (9*I)*b*ArcTan[c*x] - 18*b*c*x*ArcTan[c*x] - (9*I)*b*c^2*x^2*Arc
Tan[c*x] + 2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*Log[x] + 10*b*Log[1 + c^2*x^2
] - 3*b*PolyLog[2, (-I)*c*x] + 3*b*PolyLog[2, I*c*x])
```

3.24.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3x^2(a+b\arctan(cx))-3c^2d^3x(a+b\arctan(cx))+3icd^3(a+b\arctan(cx))+\frac{d^3(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))-\frac{3}{2}c^2d^3x^2(a+b\arctan(cx))+3iacd^3x+ad^3\log(x)- \frac{3}{2}bd^3\arctan(cx)+3ibcd^3x\arctan(cx)+\frac{1}{6}ibc^2d^3x^2-\frac{5}{3}ibd^3\log\left(c^2x^2+1\right)+ \frac{1}{2}ibd^3\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\,\mathrm{PolyLog}(2,icx)+\frac{3}{2}bcd^3x$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(3*I)*a*c*d^3*x + (3*b*c*d^3*x)/2 + (I/6)*b*c^2*d^3*x^2 - (3*b*d^3*ArcTan[
c*x])/2 + (3*I)*b*c*d^3*x*ArcTan[c*x] - (3*c^2*d^3*x^2*(a + b*ArcTan[c*x])
)/2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x]) + a*d^3*Log[x] - ((5*I)/3)*b*d
^3*Log[1 + c^2*x^2] + (I/2)*b*d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyL
og[2, I*c*x]
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.24.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
parts	$a\,d^3\left(-\frac{ic^3x^3}{3}-\frac{3c^2x^2}{2}+3icx+\ln\left(x\right)\right)+b\,d^3\Big(3i\arctan\left(cx\right)cx-\frac{i\arctan(cx)c^3x^3}{3}-\frac{3c^2x^2\operatorname{arct}}{2}$
derivativedivides	$a\,d^3\left(3icx-\frac{ic^3x^3}{3}-\frac{3c^2x^2}{2}+\ln\left(cx\right)\right)+b\,d^3\Big(3i\arctan\left(cx\right)cx-\frac{i\arctan(cx)c^3x^3}{3}-\frac{3c^2x^2\operatorname{arcta}}{2}$
default	$a\,d^3\left(3icx-\frac{ic^3x^3}{3}-\frac{3c^2x^2}{2}+\ln\left(cx\right)\right)+b\,d^3\Big(3i\arctan\left(cx\right)cx-\frac{i\arctan(cx)c^3x^3}{3}-\frac{3c^2x^2\operatorname{arcta}}{2}$
risch	$-\frac{b\,d^3\ln(icx+1)c^3x^3}{6}-\frac{ix^3a\,c^3d^3}{3}+\frac{3b\,d^3\ln(icx+1)cx}{2}+\frac{3ib\,d^3\ln(icx+1)c^2x^2}{4}+\frac{ix^2b\,c^2d^3}{6}+\frac{3bc\,d^3x}{2}+3iac$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(-1/3*I*c^3*x^3-3/2*c^2*x^2+3*I*c*x+ln(x))+b*d^3*(3*I*arctan(c*x)*c*
x-1/3*I*arctan(c*x)*c^3*x^3-3/2*c^2*x^2*arctan(c*x)+arctan(c*x)*ln(c*x)+1/
2*I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2
*I*dilog(1-I*c*x)+3/2*c*x+1/6*I*c^2*x^2-5/3*I*ln(c^2*x^2+1)-3/2*arctan(c*x
))
```

3.24.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x, x)
```

3.24.6 Sympy [F]

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = -id^3\Bigg(&\int(-3ac)\,dx+\int\frac{ia}{x}\,dx+\int ac^3x^2\,dx\\&+\int(-3bc\,\mathrm{atan}(cx))\,dx+\int\left(-3iac^2x\right)dx\\&+\int\frac{ib\,\mathrm{atan}(cx)}{x}\,dx+\int bc^3x^2\,\mathrm{atan}(cx)\,dx\\&+\int\left(-3ibc^2x\,\mathrm{atan}(cx)\right)dx\Bigg)\end{aligned}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x,x)
```

output
```
-I*d**3*(Integral(-3*a*c, x) + Integral(I*a/x, x) + Integral(a*c**3*x**2,
x) + Integral(-3*b*c*atan(c*x), x) + Integral(-3*I*a*c**2*x, x) + Integral
(I*b*atan(c*x)/x, x) + Integral(b*c**3*x**2*atan(c*x), x) + Integral(-3*I*
b*c**2*x*atan(c*x), x))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = -\frac{1}{3}i\,ac^3d^3x^3 - \frac{3}{2}\,ac^2d^3x^2 + \frac{1}{6}i\,bc^2d^3x^2 + 3i\,acd^3x + \frac{3}{2}\,bcd^3x - \frac{1}{12}\,(3\,\pi + 2i)bd^3\log\left(c^2x^2+1\right) + bd^3\arctan\left(cx\right)\log\left(cx\right) + \frac{3}{2}i\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bd^3 - \frac{1}{2}i\,bd^3\mathrm{Li}_2(i\,cx+1) + \frac{1}{2}i\,bd^3\mathrm{Li}_2(-i\,cx+1) + ad^3\log\left(x\right) + \frac{1}{6}\left(-2i\,bc^3d^3x^3 - 9\,bc^2d^3x^2 - 9\,bd^3\right)\arctan\left(cx\right)$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
-1/3*I*a*c^3*d^3*x^3 - 3/2*a*c^2*d^3*x^2 + 1/6*I*b*c^2*d^3*x^2 + 3*I*a*c*d
^3*x + 3/2*b*c*d^3*x - 1/12*(3*pi + 2*I)*b*d^3*log(c^2*x^2 + 1) + b*d^3*ar
ctan(c*x)*log(c*x) + 3/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3 -
1/2*I*b*d^3*dilog(I*c*x + 1) + 1/2*I*b*d^3*dilog(-I*c*x + 1) + a*d^3*log(x
) + 1/6*(-2*I*b*c^3*d^3*x^3 - 9*b*c^2*d^3*x^2 - 9*b*d^3)*arctan(c*x)
```

3.24.8 Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.15

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx$$

$$= \begin{cases} a\,d^3 \ \\ a\,d^3\,\ln(x) - \frac{b\,d^3\,\ln(c^2\,x^2+1)\,3\mathrm{i}}{2} - \frac{b\,d^3\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} + \frac{b\,d^3\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} - \frac{3\,a\,c^2\,d^3\,x^2}{2} - \frac{a\,c^3\,d^3\,x^3\,1\mathrm{i}}{3} + a\,c\,d^3\,x\,3\mathrm{i} + \frac{3}{} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x,x)
```

output
```
piecewise(c == 0, a*d^3*log(x), c ~= 0, - (b*d^3*log(c^2*x^2 + 1)*3i)/2 +
a*d^3*log(x) - (b*d^3*dilog(- c*x*1i + 1)*1i)/2 + (b*d^3*dilog(c*x*1i + 1)
*1i)/2 - (3*a*c^2*d^3*x^2)/2 - (a*c^3*d^3*x^3*1i)/3 + a*c*d^3*x*3i + (3*b*
c*d^3*x)/2 + (b*c^2*d^3*(x^2/2 - log(c^2*x^2 + 1)/(2*c^2))*1i)/3 - 3*b*c^2
*d^3*atan(c*x)*(1/(2*c^2) + x^2/2) - (b*c^3*d^3*x^3*atan(c*x)*1i)/3 + b*c*
d^3*x*atan(c*x)*3i)
```

3.25 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$

3.25.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx &= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-\frac{1}{2}ibcd^3\arctan(cx)\\ &\quad-3bc^2d^3x\arctan(cx)-\frac{d^3(a+b\arctan(cx))}{x}\\ &\quad-\frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))+3iacd^3\log(x)\\ &\quad+bcd^3\log(x)+bcd^3\log\left(1+c^2x^2\right)\\ &\quad-\frac{3}{2}bcd^3\operatorname{PolyLog}(2,-icx)+\frac{3}{2}bcd^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-3*a*c^2*d^3*x+1/2*I*b*c^2*d^3*x-1/2*I*b*c*d^3*arctan(c*x)-3*b*c^2*d^3*x*a
rctan(c*x)-d^3*(a+b*arctan(c*x))/x-1/2*I*c^3*d^3*x^2*(a+b*arctan(c*x))+3*I
*a*c*d^3*ln(x)+b*c*d^3*ln(x)+b*c*d^3*ln(c^2*x^2+1)-3/2*b*c*d^3*polylog(2,-
I*c*x)+3/2*b*c*d^3*polylog(2,I*c*x)
```

3.25.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{d^3(-2a-6ac^2x^2+ibc^2x^2-iac^3x^3-2b\arctan(cx)-ibcx\arctan(cx)-6bc^2x^2\arctan(cx)-ibc^3x^3\arcta}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
(d^3*(-2*a - 6*a*c^2*x^2 + I*b*c^2*x^2 - I*a*c^3*x^3 - 2*b*ArcTan[c*x] - I
*b*c*x*ArcTan[c*x] - 6*b*c^2*x^2*ArcTan[c*x] - I*b*c^3*x^3*ArcTan[c*x] + (
6*I)*a*c*x*Log[x] + 2*b*c*x*Log[c*x] + 2*b*c*x*Log[1 + c^2*x^2] - 3*b*c*x*
PolyLog[2, (-I)*c*x] + 3*b*c*x*PolyLog[2, I*c*x]))/(2*x)
```

3.25.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$\Big\downarrow \ 5411$$
$$\int \left(-ic^3d^3x(a+b\arctan(cx))-3c^2d^3(a+b\arctan(cx))+\frac{d^3(a+b\arctan(cx))}{x^2}+\frac{3icd^3(a+b\arctan(cx))}{x}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))-\frac{d^3(a+b\arctan(cx))}{x}-3ac^2d^3x+3iacd^3\log(x)-3bc^2d^3x\arctan(cx)-\frac{1}{2}ibcd^3\arctan(cx)+bcd^3\log\left(c^2x^2+1\right)+\frac{1}{2}ibc^2d^3x-\frac{3}{2}bcd^3\operatorname{PolyLog}(2,-icx)+\frac{3}{2}bcd^3\operatorname{PolyLog}(2,icx)+bcd^3\log(x)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-3*a*c^2*d^3*x + (I/2)*b*c^2*d^3*x - (I/2)*b*c*d^3*ArcTan[c*x] - 3*b*c^2*d
^3*x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/x - (I/2)*c^3*d^3*x^2*(a + b*
ArcTan[c*x]) + (3*I)*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 + c^2
*x^2] - (3*b*c*d^3*PolyLog[2, (-I)*c*x])/2 + (3*b*c*d^3*PolyLog[2, I*c*x])
/2
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.25.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

method	result
parts	$a\,d^3\left(-\frac{ic^3x^2}{2}-3c^2x+3ic\ln\left(x\right)-\frac{1}{x}\right)+b\,d^3c\Big(-3cx\arctan\left(cx\right)-\frac{i\arctan(cx)c^2x^2}{2}+3i\,\mathrm{arc}$
derivativedivides	$c\Big(a\,d^3\left(-3cx-\frac{ic^2x^2}{2}+3i\ln\left(cx\right)-\frac{1}{cx}\right)+b\,d^3\Big(-3cx\arctan\left(cx\right)-\frac{i\arctan(cx)c^2x^2}{2}+3i\,\mathrm{ar}$
default	$c\Big(a\,d^3\left(-3cx-\frac{ic^2x^2}{2}+3i\ln\left(cx\right)-\frac{1}{cx}\right)+b\,d^3\Big(-3cx\arctan\left(cx\right)-\frac{i\arctan(cx)c^2x^2}{2}+3i\,\mathrm{ar}$
risch	$-\frac{b\,c^3d^3\ln(icx+1)x^2}{4}-\frac{7id^3ca}{2}+\frac{3bc\,d^3\ln(icx+1)}{4}+\frac{ib\,d^3\ln(icx+1)}{2x}-3bc\,d^3-\frac{3bc\,d^3\,\mathrm{dilog}(icx+1)}{2}+\frac{bc\,d^3}{}$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(-1/2*I*c^3*x^2-3*c^2*x+3*I*c*ln(x)-1/x)+b*d^3*c*(-3*c*x*arctan(c*x)
-1/2*I*arctan(c*x)*c^2*x^2+3*I*arctan(c*x)*ln(c*x)-1/c/x*arctan(c*x)-3/2*l
n(c*x)*ln(1+I*c*x)+3/2*ln(c*x)*ln(1-I*c*x)-3/2*dilog(1+I*c*x)+3/2*dilog(1-
I*c*x)+1/2*I*c*x+ln(c*x)+ln(c^2*x^2+1)-1/2*I*arctan(c*x))
```

3.25.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x^2, x)
```

3.25.6 Sympy [F]

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx = -id^3\Bigg(&\int\left(-3iac^2\right)dx+\int\frac{ia}{x^2}\,dx+\int\left(-\frac{3ac}{x}\right)dx\\&+\int ac^3x\,dx+\int\left(-3ibc^2\operatorname{atan}(cx)\right)dx\\&+\int\frac{ib\operatorname{atan}(cx)}{x^2}\,dx+\int\left(-\frac{3bc\operatorname{atan}(cx)}{x}\right)dx\\&+\int bc^3x\operatorname{atan}(cx)\,dx\Bigg)\end{aligned}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**2,x)
```

output
```
-I*d**3*(Integral(-3*I*a*c**2, x) + Integral(I*a/x**2, x) + Integral(-3*a*
c/x, x) + Integral(a*c**3*x, x) + Integral(-3*I*b*c**2*atan(c*x), x) + Int
egral(I*b*atan(c*x)/x**2, x) + Integral(-3*b*c*atan(c*x)/x, x) + Integral(
b*c**3*x*atan(c*x), x))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.24

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= -\frac{1}{2}i\,ac^3d^3x^2 - 3\,ac^2d^3x + \frac{1}{2}i\,bc^2d^3x - \frac{3}{4}i\,\pi bcd^3\log\left(c^2x^2+1\right) + 3i\,bcd^3\arctan(cx)\log(cx) - \frac{3}{2}\left(2\,cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bcd^3 + \frac{3}{2}\,bcd^3\mathrm{Li}_2(i\,cx+1) - \frac{3}{2}\,bcd^3\mathrm{Li}_2(-i\,cx+1) + 3i\,acd^3\log(x) - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd^3 - \frac{ad^3}{x} + \frac{1}{2}\left(-i\,bc^3d^3x^2 - i\,bcd^3\right)\arctan(cx)$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*I*a*c^3*d^3*x^2 - 3*a*c^2*d^3*x + 1/2*I*b*c^2*d^3*x - 3/4*I*pi*b*c*d^
3*log(c^2*x^2 + 1) + 3*I*b*c*d^3*arctan(c*x)*log(c*x) - 3/2*(2*c*x*arctan(
c*x) - log(c^2*x^2 + 1))*b*c*d^3 + 3/2*b*c*d^3*dilog(I*c*x + 1) - 3/2*b*c*
d^3*dilog(-I*c*x + 1) + 3*I*a*c*d^3*log(x) - 1/2*(c*(log(c^2*x^2 + 1) - lo
g(x^2)) + 2*arctan(c*x)/x)*b*d^3 - a*d^3/x + 1/2*(-I*b*c^3*d^3*x^2 - I*b*c
*d^3)*arctan(c*x)
```

3.25.8 Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \begin{cases} & -\frac{a\,d^3}{x} \\ \frac{b\,d^3\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}-\frac{a\,c^3\,d^3\,x^2\,1\mathrm{i}}{2}-\frac{a\,d^3}{x}+\frac{3\,b\,c\,d^3\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))}{2}+\frac{3\,b\,c\,d^3\,\ln(c^2\,x^2+1)}{2}-3\,a\,c^2\,d^3\,x & \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^2,x)
```

output
```
piecewise(c == 0, -(a*d^3)/x, c ~= 0, - (a*d^3)/x - (a*c^3*d^3*x^2*1i)/2 +
 (b*d^3*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2))/c + (3*b*c*d^3*(dilog(- c
*x*1i + 1) - dilog(c*x*1i + 1)))/2 + (3*b*c*d^3*log(c^2*x^2 + 1))/2 - 3*a*
c^2*d^3*x + (b*c^2*d^3*x*1i)/2 + a*c*d^3*log(x)*3i - (b*d^3*atan(c*x))/x -
 3*b*c^2*d^3*x*atan(c*x) - b*c^3*d^3*atan(c*x)*(1/(2*c^2) + x^2/2)*1i)
```

3.26 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx$

3.26.1 Optimal result

Integrand size = 23, antiderivative size = 180

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = & -\frac{bcd^3}{2x} - iac^3d^3x - \frac{1}{2}bc^2d^3\arctan(cx) \\ & - ibc^3d^3x\arctan(cx) - \frac{d^3(a+b\arctan(cx))}{2x^2} \\ & - \frac{3icd^3(a+b\arctan(cx))}{x} - 3ac^2d^3\log(x) \\ & + 3ibc^2d^3\log(x) - ibc^2d^3\log\left(1+c^2x^2\right) \\ & - \frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,-icx) + \frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/2*b*c*d^3/x-I*a*c^3*d^3*x-1/2*b*c^2*d^3*arctan(c*x)-I*b*c^3*d^3*x*arcta
n(c*x)-1/2*d^3*(a+b*arctan(c*x))/x^2-3*I*c*d^3*(a+b*arctan(c*x))/x-3*a*c^2
*d^3*ln(x)+3*I*b*c^2*d^3*ln(x)-I*b*c^2*d^3*ln(c^2*x^2+1)-3/2*I*b*c^2*d^3*p
olylog(2,-I*c*x)+3/2*I*b*c^2*d^3*polylog(2,I*c*x)
```

3.26.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = -\frac{id^3(-ia+6acx-ibcx+2ac^3x^3-ib\arctan(cx)+6bcx\arctan(cx)-ibc^2x^2\arctan(cx)+2bc^3x^3\arctan$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
((-1/2*I)*d^3*((-I)*a + 6*a*c*x - I*b*c*x + 2*a*c^3*x^3 - I*b*ArcTan[c*x]
+ 6*b*c*x*ArcTan[c*x] - I*b*c^2*x^2*ArcTan[c*x] + 2*b*c^3*x^3*ArcTan[c*x]
- (6*I)*a*c^2*x^2*Log[x] - 6*b*c^2*x^2*Log[c*x] + 2*b*c^2*x^2*Log[1 + c^2*
x^2] + 3*b*c^2*x^2*PolyLog[2, (-I)*c*x] - 3*b*c^2*x^2*PolyLog[2, I*c*x]))/
x^2
```

3.26.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3(a+b\arctan(cx))-\frac{3c^2d^3(a+b\arctan(cx))}{x}+\frac{d^3(a+b\arctan(cx))}{x^3}+\frac{3icd^3(a+b\arctan(cx))}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{2x^2}-\frac{3icd^3(a+b\arctan(cx))}{x}-iac^3d^3x-3ac^2d^3\log(x)-ibc^3d^3x\arctan(cx)-\frac{1}{2}bc^2d^3\arctan(cx)-\frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,-icx)+\frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,icx)-ibc^2d^3\log\left(c^2x^2+1\right)+3ibc^2d^3\log(x)-\frac{bcd^3}{2x}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^3)/x - I*a*c^3*d^3*x - (b*c^2*d^3*ArcTan[c*x])/2 - I*b*c^3*d^3
*x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) - ((3*I)*c*d^3*(a + b*A
rcTan[c*x]))/x - 3*a*c^2*d^3*Log[x] + (3*I)*b*c^2*d^3*Log[x] - I*b*c^2*d^3
*Log[1 + c^2*x^2] - ((3*I)/2)*b*c^2*d^3*PolyLog[2, (-I)*c*x] + ((3*I)/2)*b
*c^2*d^3*PolyLog[2, I*c*x]
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.26.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96

method	result
parts	$a\,d^3\left(-ic^3x-\frac{1}{2x^2}-3c^2\ln\left(x\right)-\frac{3ic}{x}\right)+b\,d^3c^2\Big(-i\arctan\left(cx\right)cx-3\arctan\left(cx\right)\ln\left(cx\right)$
derivativedivides	$c^2\Big(a\,d^3\left(-icx-3\ln\left(cx\right)-\frac{3i}{cx}-\frac{1}{2c^2x^2}\right)+b\,d^3\Big(-i\arctan\left(cx\right)cx-3\arctan\left(cx\right)\ln\left(cx\right)$
default	$c^2\Big(a\,d^3\left(-icx-3\ln\left(cx\right)-\frac{3i}{cx}-\frac{1}{2c^2x^2}\right)+b\,d^3\Big(-i\arctan\left(cx\right)cx-3\arctan\left(cx\right)\ln\left(cx\right)$
risch	$-\frac{b\,c^3d^3\ln(icx+1)x}{2}+\frac{3id^3c^2b\,\mathrm{dilog}(-icx+1)}{2}+\frac{7id^3c^2b\ln(-icx)}{4}-\frac{id^3b\ln(-icx+1)}{4x^2}-ic^2b\,d^3-\frac{3bc\,d^3\ln(ic}{2x}$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(-I*c^3*x-1/2/x^2-3*c^2*ln(x)-3*I*c/x)+b*d^3*c^2*(-I*arctan(c*x)*c*x
-3*arctan(c*x)*ln(c*x)-3*I*arctan(c*x)/c/x-1/2/c^2/x^2*arctan(c*x)-3/2*I*l
n(c*x)*ln(1+I*c*x)+3/2*I*ln(c*x)*ln(1-I*c*x)-3/2*I*dilog(1+I*c*x)+3/2*I*di
log(1-I*c*x)-1/2/c/x+3*I*ln(c*x)-I*ln(c^2*x^2+1)-1/2*arctan(c*x))
```

3.26.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x^3, x)
```

3.26.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**3,x)
```

output
```
Timed out
```

3.26.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-I*a*c^3*d^3*x - 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^2*d^3 -
3*b*c^2*d^3*integrate(arctan(c*x)/x, x) - 3*a*c^2*d^3*log(x) - 3/2*I*(c*(l
og(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^3 - 1/2*((c*arctan(c*
x) + 1/x)*c + arctan(c*x)/x^2)*b*d^3 - 3*I*a*c*d^3/x - 1/2*a*d^3/x^2
```

3.26.8 Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.14

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx$$
$$= \begin{cases} & -\frac{a\,d^3}{2\,x^2} \\ b\,d^3\left(c^2\,\ln\left(x\right)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)\,3\mathrm{i}+\frac{b\,c^2\,d^3\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}}{2}+\frac{b\,c^2\,d^3\,\mathrm{Li}_2\left(1-c\,x\,1\mathrm{i}\right)\,3\mathrm{i}}{2}-\frac{b\,c^2\,d^3\,\mathrm{Li}_2\left(1+c\,x\,1\mathrm{i}\right)\,3\mathrm{i}}{2}-\frac{b\,d^3\left(c^3\,\mathrm{atan}\right.}{2\,c} & \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^3,x)
```

output
```
piecewise(c == 0, -(a*d^3)/(2*x^2), c ~= 0, b*d^3*(c^2*log(x) - (c^2*log(c
^2*x^2 + 1))/2)*3i + (b*c^2*d^3*log(c^2*x^2 + 1)*1i)/2 + (b*c^2*d^3*dilog(
- c*x*1i + 1)*3i)/2 - (b*c^2*d^3*dilog(c*x*1i + 1)*3i)/2 - (b*d^3*(c^3*ata
n(c*x) + c^2/x))/(2*c) - (a*d^3*(c*x*6i + c^3*x^3*2i + 6*c^2*x^2*log(x) +
1))/(2*x^2) - (b*d^3*atan(c*x))/(2*x^2) - (b*c*d^3*atan(c*x)*3i)/x - b*c^3
*d^3*x*atan(c*x)*1i)
```

3.27 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$

3.27.1 Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = &-\frac{bcd^3}{6x^2}-\frac{3ibc^2d^3}{2x}-\frac{3}{2}ibc^3d^3\arctan(cx)\\ &-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3icd^3(a+b\arctan(cx))}{2x^2}\\ &+\frac{3c^2d^3(a+b\arctan(cx))}{x}-iac^3d^3\log(x)\\ &-\frac{10}{3}bc^3d^3\log(x)+\frac{5}{3}bc^3d^3\log\left(1+c^2x^2\right)\\ &+\frac{1}{2}bc^3d^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}bc^3d^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/6*b*c*d^3/x^2-3/2*I*b*c^2*d^3/x-3/2*I*b*c^3*d^3*arctan(c*x)-1/3*d^3*(a+
b*arctan(c*x))/x^3-3/2*I*c*d^3*(a+b*arctan(c*x))/x^2+3*c^2*d^3*(a+b*arctan
(c*x))/x-I*a*c^3*d^3*ln(x)-10/3*b*c^3*d^3*ln(x)+5/3*b*c^3*d^3*ln(c^2*x^2+1
)+1/2*b*c^3*d^3*polylog(2,-I*c*x)-1/2*b*c^3*d^3*polylog(2,I*c*x)
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.90

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{d^3\left(-2a-9iacx-bcx+18ac^2x^2-2b\arctan(cx)-9ibcx\arctan(cx)+18bc^2x^2\arctan(cx)-9ibc^2x^2\,\text{Hyp}\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
(d^3*(-2*a - (9*I)*a*c*x - b*c*x + 18*a*c^2*x^2 - 2*b*ArcTan[c*x] - (9*I)*
b*c*x*ArcTan[c*x] + 18*b*c^2*x^2*ArcTan[c*x] - (9*I)*b*c^2*x^2*Hypergeomet
ric2F1[-1/2, 1, 1/2, -(c^2*x^2)] - (6*I)*a*c^3*x^3*Log[x] - 20*b*c^3*x^3*L
og[x] + 10*b*c^3*x^3*Log[1 + c^2*x^2] + 3*b*c^3*x^3*PolyLog[2, (-I)*c*x] -
 3*b*c^3*x^3*PolyLog[2, I*c*x]))/(6*x^3)
```

3.27.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{ic^3d^3(a+b\arctan(cx))}{x}-\frac{3c^2d^3(a+b\arctan(cx))}{x^2}+\frac{d^3(a+b\arctan(cx))}{x^4}+\frac{3icd^3(a+b\arctan(cx))}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3c^2d^3(a+b\arctan(cx))}{x}-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3icd^3(a+b\arctan(cx))}{2x^2}-iac^3d^3\log(x)-\frac{3}{2}ibc^3d^3\arctan(cx)+\frac{1}{2}bc^3d^3\,\text{PolyLog}(2,-icx)-\frac{1}{2}bc^3d^3\,\text{PolyLog}(2,icx)-\frac{10}{3}bc^3d^3\log(x)-\frac{3ibc^2d^3}{2x}+\frac{5}{3}bc^3d^3\log\left(c^2x^2+1\right)-\frac{bcd^3}{6x^2}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(b*c*d^3)/x^2 - (((3*I)/2)*b*c^2*d^3)/x - ((3*I)/2)*b*c^3*d^3*ArcTan[
c*x] - (d^3*(a + b*ArcTan[c*x]))/(3*x^3) - (((3*I)/2)*c*d^3*(a + b*ArcTan[
c*x]))/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x]))/x - I*a*c^3*d^3*Log[x] - (10*
b*c^3*d^3*Log[x])/3 + (5*b*c^3*d^3*Log[1 + c^2*x^2])/3 + (b*c^3*d^3*PolyLo
g[2, (-I)*c*x])/2 - (b*c^3*d^3*PolyLog[2, I*c*x])/2
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.27.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97

method	result
parts	$a\,d^3\left(-\frac{3ic}{2x^2}-ic^3\ln(x)+\frac{3c^2}{x}-\frac{1}{3x^3}\right)+b\,d^3c^3\left(-\frac{\arctan(cx)}{3c^3x^3}-i\arctan(cx)\ln(cx)+\frac{3\arctan}{cx}\right.$
derivativedivides	$c^3\left(a\,d^3\left(-\frac{1}{3c^3x^3}-i\ln(cx)+\frac{3}{cx}-\frac{3i}{2c^2x^2}\right)+b\,d^3\left(-\frac{\arctan(cx)}{3c^3x^3}-i\arctan(cx)\ln(cx)+\frac{3\,\mathrm{arc}}{}\right.\right.$
default	$c^3\left(a\,d^3\left(-\frac{1}{3c^3x^3}-i\ln(cx)+\frac{3}{cx}-\frac{3i}{2c^2x^2}\right)+b\,d^3\left(-\frac{\arctan(cx)}{3c^3x^3}-i\arctan(cx)\ln(cx)+\frac{3\,\mathrm{arc}}{}\right.\right.$
risch	$\frac{b\,d^3c^3\,\mathrm{dilog}(icx+1)}{2}-\frac{11b\,d^3c^3\ln(icx)}{12}+\frac{5\ln(c^2x^2+1)b\,c^3d^3}{3}-ic^3d^3a\ln(-icx)+\frac{ib\,d^3\ln(icx+1)}{6x^3}-\frac{3ib\,c^3}{}$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(-3/2*I*c/x^2-I*c^3*ln(x)+3*c^2/x-1/3/x^3)+b*d^3*c^3*(-1/3*arctan(c*
x)/c^3/x^3-I*arctan(c*x)*ln(c*x)+3/c/x*arctan(c*x)-3/2*I*arctan(c*x)/c^2/x
^2+1/2*ln(c*x)*ln(1+I*c*x)-1/2*ln(c*x)*ln(1-I*c*x)+1/2*dilog(1+I*c*x)-1/2*
dilog(1-I*c*x)-3/2*I/c/x-1/6/c^2/x^2-10/3*ln(c*x)+5/3*ln(c^2*x^2+1)-3/2*I*
arctan(c*x))
```

3.27.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x^4, x)
```

3.27.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**4,x)
```

output
```
Timed out
```

3.27.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
-I*b*c^3*d^3*integrate(arctan(c*x)/x, x) - I*a*c^3*d^3*log(x) + 3/2*(c*(lo
g(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^3 - 3/2*I*((c*arctan
(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^3 + 1/6*((c^2*log(c^2*x^2 + 1) - c
^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 3/2*I*
a*c*d^3/x^2 - 1/3*a*d^3/x^3
```

3.27.8 Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.27.9 Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.17

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$$
$$= \begin{cases} & -\frac{a}{3} \\ \frac{b\,c^3\,d^3\,\ln\left(-\frac{3\,c^6\,x^2}{2}-\frac{3\,c^4}{2}\right)}{6} - \frac{b\,c^3\,d^3\,\ln(x)}{3} - \frac{b\,c^3\,d^3\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))}{2} - 3\,b\,c\,d^3\left(c^2\,\ln(x) - \frac{c^2\,\ln(c^2\,x^2+1)}{2}\right) - \frac{b\,c}{6} & \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^4,x)
```

output
```
piecewise(c == 0, -(a*d^3)/(3*x^3), c ~= 0, - (b*d^3*(c^3*atan(c*x) + c^2/
x)*3i)/2 - (b*c^3*d^3*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)))/2 - (b*c^
3*d^3*log(x))/3 + (b*c^3*d^3*log(- (3*c^4)/2 - (3*c^6*x^2)/2))/6 - 3*b*c*d
^3*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2) - (b*c*d^3)/(6*x^2) - (a*d^3*(c
*x*9i - 18*c^2*x^2 + c^3*x^3*log(x)*6i + 2))/(6*x^3) - (b*d^3*atan(c*x))/(
3*x^3) - (b*c*d^3*atan(c*x)*3i)/(2*x^2) + (3*b*c^2*d^3*atan(c*x))/x)
```

3.28 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$

3.28.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd^3}{12x^3}-\frac{ibc^2d^3}{2x^2}+\frac{7bc^3d^3}{4x} -\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4} -2ibc^4d^3\log(x)+2ibc^4d^3\log(i+cx)$$

output
```
-1/12*b*c*d^3/x^3-1/2*I*b*c^2*d^3/x^2+7/4*b*c^3*d^3/x-1/4*d^3*(1+I*c*x)^4*
(a+b*arctan(c*x))/x^4-2*I*b*c^4*d^3*ln(x)+2*I*b*c^4*d^3*ln(c*x+I)
```

3.28.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx = \frac{d^3\left(-bcx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)-3i\left(-ia+4acx+6iac^2x^2+2bc^2x^2-4ac^3x^3+b(-i+\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
(d^3*(-(b*c*x*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)]) - (3*I)*((-I)*
a + 4*a*c*x + (6*I)*a*c^2*x^2 + 2*b*c^2*x^2 - 4*a*c^3*x^3 + b*(-I + 4*c*x
+ (6*I)*c^2*x^2 - 4*c^3*x^3)*ArcTan[c*x] + (6*I)*b*c^3*x^3*Hypergeometric2
F1[-1/2, 1, 1/2, -(c^2*x^2)] + 8*b*c^4*x^4*Log[x] - 4*b*c^4*x^4*Log[1 + c^
2*x^2])))/(12*x^4)
```

3.28.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}dx$$

$$\Big\downarrow \textbf{5407}$$

$$-bc\int \frac{d^3(i-cx)^3}{4x^4(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{1}{4}bcd^3\int \frac{(i-cx)^3}{x^4(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \textbf{99}$$

$$-\frac{1}{4}bcd^3\int \left(-\frac{8ic^4}{cx+i}+\frac{8ic^3}{x}+\frac{7c^2}{x^2}-\frac{4ic}{x^3}-\frac{1}{x^4}\right)dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}-\frac{1}{4}bcd^3\left(8ic^3\log(x)-8ic^3\log(cx+i)-\frac{7c^2}{x}+\frac{2ic}{x^2}+\frac{1}{3x^3}\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/x^4 - (b*c*d^3*(1/(3*x^3) + (
(2*I)*c)/x^2 - (7*c^2)/x + (8*I)*c^3*Log[x] - (8*I)*c^3*Log[I + c*x]))/4
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.28.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.43

method	result
parts	$a\,d^3\left(\frac{3c^2}{2x^2}-\frac{1}{4x^4}+\frac{ic^3}{x}-\frac{ic}{x^3}\right)+b\,d^3c^4\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{c^3x^3}+\frac{i\arctan(cx)}{cx}+\frac{3\arctan(cx)}{2c^2x^2}-\right.$
derivativedivides	$c^4\left(a\,d^3\left(-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}+\frac{3}{2c^2x^2}\right)+b\,d^3\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{c^3x^3}+\frac{i\arctan(cx)}{cx}+\frac{3\arctan}{2c^2x}\right.\right.$
default	$c^4\left(a\,d^3\left(-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}+\frac{3}{2c^2x^2}\right)+b\,d^3\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{c^3x^3}+\frac{i\arctan(cx)}{cx}+\frac{3\arctan}{2c^2x}\right.\right.$
parallelrisch	$-12ix\arctan(cx)bc\,d^3+12ix^3a\,c^3d^3-6ix^2b\,c^2d^3+21x^4\arctan(cx)b\,c^4d^3-12iac\,d^3x-18a\,c^4d^3x^4+12ix^3\arctan(cx)b\,c^3d$
risch	$\frac{(4b\,c^3d^3x^3-6ix^2b\,c^2d^3-4bc\,d^3x+ib\,d^3)\ln(icx+1)}{8x^4}-\frac{id^3(-3b\,c^4\ln(119cx-119i)x^4-45b\,c^4\ln(-217cx-217i)x^4+48b\,c}{}$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(3/2*c^2/x^2-1/4/x^4+I*c^3/x-I*c/x^3)+b*d^3*c^4*(-1/4*arctan(c*x)/c^
4/x^4-I*arctan(c*x)/c^3/x^3+I*arctan(c*x)/c/x+3/2/c^2/x^2*arctan(c*x)-1/2*
I/c^2/x^2-2*I*ln(c*x)-1/12/c^3/x^3+7/4/c/x+I*ln(c^2*x^2+1)+7/4*arctan(c*x)
)
```

3.28.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(85) = 170$.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.69

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{-48i\,bc^4d^3x^4\log(x)+45i\,bc^4d^3x^4\log\left(\frac{cx+i}{c}\right)+3i\,bc^4d^3x^4\log\left(\frac{cx-i}{c}\right)-6\,(-4i\,a-7\,b)c^3d^3x^3+12\,(3\,a-i\,b}{24\,x^4}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/24*(-48*I*b*c^4*d^3*x^4*log(x) + 45*I*b*c^4*d^3*x^4*log((c*x + I)/c) + 3
*I*b*c^4*d^3*x^4*log((c*x - I)/c) - 6*(-4*I*a - 7*b)*c^3*d^3*x^3 + 12*(3*a
 - I*b)*c^2*d^3*x^2 - 2*(12*I*a + b)*c*d^3*x - 6*a*d^3 - 3*(4*b*c^3*d^3*x^
3 - 6*I*b*c^2*d^3*x^2 - 4*b*c*d^3*x + I*b*d^3)*log(-(c*x + I)/(c*x - I)))/
x^4
```

3.28.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(99) = 198.

Time = 14.11 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.02

$$
\begin{aligned}
&\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx \\
&= -2ibc^4d^3\log\left(3689b^2c^9d^6x\right)+\frac{ibc^4d^3\log\left(3689b^2c^9d^6x-3689ib^2c^8d^6\right)}{8} \\
&\quad+\frac{15ibc^4d^3\log\left(3689b^2c^9d^6x+3689ib^2c^8d^6\right)}{8} \\
&\quad-\frac{3ad^3+x^3(-12iac^3d^3-21bc^3d^3)+x^2(-18ac^2d^3+6ibc^2d^3)+x(12iacd^3+bcd^3)}{12x^4} \\
&\quad+\frac{(-4bc^3d^3x^3+6ibc^2d^3x^2+4bcd^3x-ibd^3)\log\left(-icx+1\right)}{8x^4} \\
&\quad+\frac{(4bc^3d^3x^3-6ibc^2d^3x^2-4bcd^3x+ibd^3)\log\left(icx+1\right)}{8x^4}
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**5,x)
```

output
```
-2*I*b*c**4*d**3*log(3689*b**2*c**9*d**6*x) + I*b*c**4*d**3*log(3689*b**2*
c**9*d**6*x - 3689*I*b**2*c**8*d**6)/8 + 15*I*b*c**4*d**3*log(3689*b**2*c*
*9*d**6*x + 3689*I*b**2*c**8*d**6)/8 - (3*a*d**3 + x**3*(-12*I*a*c**3*d**3
 - 21*b*c**3*d**3) + x**2*(-18*a*c**2*d**3 + 6*I*b*c**2*d**3) + x*(12*I*a*
c*d**3 + b*c*d**3))/(12*x**4) + (-4*b*c**3*d**3*x**3 + 6*I*b*c**2*d**3*x**
2 + 4*b*c*d**3*x - I*b*d**3)*log(-I*c*x + 1)/(8*x**4) + (4*b*c**3*d**3*x**
3 - 6*I*b*c**2*d**3*x**2 - 4*b*c*d**3*x + I*b*d**3)*log(I*c*x + 1)/(8*x**4
)
```

3.28.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.96

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bc^3d^3$$
$$+\frac{3}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^2d^3$$
$$+\frac{1}{2}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bcd^3+\frac{i\,ac^3d^3}{x}$$
$$+\frac{1}{12}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bd^3+\frac{3\,ac^2d^3}{2\,x^2}-\frac{i\,acd^3}{x^3}-\frac{ad^3}{4\,x^4}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^3 + 3/2*
((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^3 + 1/2*I*((c^2*log(c^
2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^3 + I*a*c^
3*d^3/x + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x
)/x^4)*b*d^3 + 3/2*a*c^2*d^3/x^2 - I*a*c*d^3/x^3 - 1/4*a*d^3/x^4
```

3.28.8 Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx =$$
$$-\frac{\frac{d^3\,(3\,a+3\,b\,\mathrm{atan}(c\,x))}{12}+\frac{d^3\,x\,(a\,c\,12\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,12\mathrm{i})}{12}-\frac{d^3\,x^2\,\left(18\,a\,c^2+18\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,6\mathrm{i}\right)}{12}-\frac{d^3\,x^3\,\left(a\,c^3\,12\mathrm{i}+21\,b\,c^3+b\,c^3\,\mathrm{atan}\right.}{12}}{x^4}$$
$$+\frac{d^3\,\left(21\,b\,c^4\,\mathrm{atan}(c\,x)+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,12\mathrm{i}-b\,c^4\,\ln\left(x\right)\,24\mathrm{i}\right)}{12}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^5,x)
```

output
```
(d^3*(21*b*c^4*atan(c*x) + b*c^4*log(c^2*x^2 + 1)*12i - b*c^4*log(x)*24i))
/12 - ((d^3*(3*a + 3*b*atan(c*x)))/12 + (d^3*x*(a*c*12i + b*c + b*c*atan(c
*x)*12i))/12 - (d^3*x^2*(18*a*c^2 - b*c^2*6i + 18*b*c^2*atan(c*x)))/12 - (
d^3*x^3*(a*c^3*12i + 21*b*c^3 + b*c^3*atan(c*x)*12i))/12)/x^4
```

3.29 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$

3.29.1 Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^3}{20x^4}-\frac{ibc^2d^3}{4x^3}+\frac{3bc^3d^3}{5x^2}+\frac{5ibc^4d^3}{4x} - \frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5} + \frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4} + \frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\log(i+cx)$$

output
```
-1/20*b*c*d^3/x^4-1/4*I*b*c^2*d^3/x^3+3/5*b*c^3*d^3/x^2+5/4*I*b*c^4*d^3/x-
1/5*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))/x^5+1/20*I*c*d^3*(1+I*c*x)^4*(a+b*ar
ctan(c*x))/x^4+6/5*b*c^5*d^3*ln(x)-6/5*b*c^5*d^3*ln(c*x+I)
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.23

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$

$$= \frac{d^3(-4a-15iacx-bcx+20ac^2x^2+10iac^3x^3+12bc^3x^3-4b\arctan(cx)-15ibcx\arctan(cx)+20bc^2x^2\,\mathrm{a}}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(d^3*(-4*a - (15*I)*a*c*x - b*c*x + 20*a*c^2*x^2 + (10*I)*a*c^3*x^3 + 12*b
*c^3*x^3 - 4*b*ArcTan[c*x] - (15*I)*b*c*x*ArcTan[c*x] + 20*b*c^2*x^2*ArcTa
n[c*x] + (10*I)*b*c^3*x^3*ArcTan[c*x] - (5*I)*b*c^2*x^2*Hypergeometric2F1[
-3/2, 1, -1/2, -(c^2*x^2)] + (10*I)*b*c^4*x^4*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^2)] + 24*b*c^5*x^5*Log[x] - 12*b*c^5*x^5*Log[1 + c^2*x^2]))/(2
0*x^5)
```

3.29.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^3(icx+1)^3(cx+4i)}{20x^5(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}bcd^3\int \frac{(icx+1)^3(cx+4i)}{x^5(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}$$

$$\Big\downarrow \text{165}$$

$$\frac{1}{20}bcd^3\int \left(-\frac{24c^5}{cx+i}+\frac{24c^4}{x}-\frac{25ic^3}{x^2}-\frac{24c^2}{x^3}+\frac{15ic}{x^4}+\frac{4}{x^5}\right)dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}+\frac{1}{20}bcd^3\left(24c^4\log(x)-24c^4\log(cx+i)+\frac{25ic^3}{x}+\frac{12c^2}{x^2}-\frac{5ic}{x^3}-\frac{1}{x^4}\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/x^5 + ((I/20)*c*d^3*(1 + I*c*
x)^4*(a + b*ArcTan[c*x]))/x^4 + (b*c*d^3*(-x^(-4) - ((5*I)*c)/x^3 + (12*c^
2)/x^2 + ((25*I)*c^3)/x + 24*c^4*Log[x] - 24*c^4*Log[I + c*x]))/20
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.29.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.02

method	result
parts	$a\,d^3\left(\frac{ic^3}{2x^2}-\frac{3ic}{4x^4}-\frac{1}{5x^5}+\frac{c^2}{x^3}\right)+b\,d^3c^5\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{3i\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{c^3x^3}+\frac{i\arctan(cx)}{2c^2x^2}-\right.$
derivativedivides	$c^5\left(a\,d^3\left(-\frac{1}{5c^5x^5}-\frac{3i}{4c^4x^4}+\frac{1}{c^3x^3}+\frac{i}{2c^2x^2}\right)+b\,d^3\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{3i\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{c^3x^3}+\right.\right.$
default	$c^5\left(a\,d^3\left(-\frac{1}{5c^5x^5}-\frac{3i}{4c^4x^4}+\frac{1}{c^3x^3}+\frac{i}{2c^2x^2}\right)+b\,d^3\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{3i\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{c^3x^3}+\right.\right.$
parallelrisch	$-15ix\arctan(cx)bc\,d^3-10ix^5a\,c^5d^3-12b\,c^5d^3\ln(c^2x^2+1)x^5+24b\,c^5d^3\ln(x)x^5-12b\,c^5d^3x^5+10ix^3a\,c^3d^3-5ix^2b\,c^2d^3-$
risch	$\frac{(10b\,c^3d^3x^3-20ix^2b\,c^2d^3-15bc\,d^3x+4ib\,d^3)\ln(icx+1)}{40x^5}-\frac{d^3(49b\,c^5\ln(-cx-i)x^5-b\,c^5\ln(cx-i)x^5-48b\,c^5\ln(-x)x^5-$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(1/2*I*c^3/x^2-3/4*I*c/x^4-1/5/x^5+c^2/x^3)+b*d^3*c^5*(-1/5/c^5/x^5*
arctan(c*x)-3/4*I*arctan(c*x)/c^4/x^4+arctan(c*x)/c^3/x^3+1/2*I*arctan(c*x
)/c^2/x^2-1/4*I/c^3/x^3+5/4*I/c/x-1/20/c^4/x^4+3/5/c^2/x^2+6/5*ln(c*x)-3/5
*ln(c^2*x^2+1)+5/4*I*arctan(c*x))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.23

$$\int\frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$=\frac{48\,bc^5d^3x^5\log(x)-49\,bc^5d^3x^5\log\left(\frac{cx+i}{c}\right)+bc^5d^3x^5\log\left(\frac{cx-i}{c}\right)+50i\,bc^4d^3x^4-4\,(-5i\,a-6\,b)c^3d^3x^3+10\,(}{40}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/40*(48*b*c^5*d^3*x^5*log(x) - 49*b*c^5*d^3*x^5*log((c*x + I)/c) + b*c^5*
d^3*x^5*log((c*x - I)/c) + 50*I*b*c^4*d^3*x^4 - 4*(-5*I*a - 6*b)*c^3*d^3*x
^3 + 10*(4*a - I*b)*c^2*d^3*x^2 - 2*(15*I*a + b)*c*d^3*x - 8*a*d^3 - (10*b
*c^3*d^3*x^3 - 20*I*b*c^2*d^3*x^2 - 15*b*c*d^3*x + 4*I*b*d^3)*log(-(c*x +
I)/(c*x - I)))/x^5
```

3.29.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(144) = 288$.

Time = 25.71 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.17

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{6bc^5d^3\log\left(113975b^2c^{11}d^6x\right)}{5} + \frac{bc^5d^3\log\left(113975b^2c^{11}d^6x - 113975ib^2c^{10}d^6\right)}{40}$$
$$- \frac{49bc^5d^3\log\left(113975b^2c^{11}d^6x + 113975ib^2c^{10}d^6\right)}{40}$$
$$+ \frac{(-10bc^3d^3x^3 + 20ibc^2d^3x^2 + 15bcd^3x - 4ibd^3)\log\left(-icx+1\right)}{40x^5}$$
$$+ \frac{(10bc^3d^3x^3 - 20ibc^2d^3x^2 - 15bcd^3x + 4ibd^3)\log\left(icx+1\right)}{40x^5}$$
$$- \frac{4ad^3 - 25ibc^4d^3x^4 + x^3(-10iac^3d^3 - 12bc^3d^3) + x^2(-20ac^2d^3 + 5ibc^2d^3) + x(15iacd^3 + bcd^3)}{20x^5}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**6,x)
```

output
```
6*b*c**5*d**3*log(113975*b**2*c**11*d**6*x)/5 + b*c**5*d**3*log(113975*b**
2*c**11*d**6*x - 113975*I*b**2*c**10*d**6)/40 - 49*b*c**5*d**3*log(113975*
b**2*c**11*d**6*x + 113975*I*b**2*c**10*d**6)/40 + (-10*b*c**3*d**3*x**3 +
 20*I*b*c**2*d**3*x**2 + 15*b*c*d**3*x - 4*I*b*d**3)*log(-I*c*x + 1)/(40*x
**5) + (10*b*c**3*d**3*x**3 - 20*I*b*c**2*d**3*x**2 - 15*b*c*d**3*x + 4*I*
b*d**3)*log(I*c*x + 1)/(40*x**5) - (4*a*d**3 - 25*I*b*c**4*d**3*x**4 + x**
3*(-10*I*a*c**3*d**3 - 12*b*c**3*d**3) + x**2*(-20*a*c**2*d**3 + 5*I*b*c**
2*d**3) + x*(15*I*a*c*d**3 + b*c*d**3))/(20*x**5)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.49

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{1}{2}i\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^3d^3$$
$$-\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^2d^3$$
$$+\frac{1}{4}i\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bcd^3$$
$$-\frac{1}{20}\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan(cx)}{x^5}\right)bd^3$$
$$+\frac{i\,ac^3d^3}{2x^2}+\frac{ac^2d^3}{x^3}-\frac{3i\,acd^3}{4x^4}-\frac{ad^3}{5x^5}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^3 - 1/2*((c^2*lo
g(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^3 +
1/4*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*
c*d^3 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x
^4)*c + 4*arctan(c*x)/x^5)*b*d^3 + 1/2*I*a*c^3*d^3/x^2 + a*c^2*d^3/x^3 - 3
/4*I*a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

3.29.8 Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.29.9 Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.16

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{d^3\left(24\,b\,c^5\,\ln(x) - 12\,b\,c^5\,\ln(c^2\,x^2+1) + b\,c^4\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}\,25\mathrm{i}\right)}{20} + \frac{-\frac{d^3\,(4\,a+4\,b\,\mathrm{atan}(c\,x))}{20} - \frac{d^3\,x\,(a\,c\,15\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,15\mathrm{i})}{20} + \frac{d^3\,x^3\,\left(a\,c^3\,10\mathrm{i}+12\,b\,c^3+b\,c^3\,\mathrm{atan}(c\,x)\,10\mathrm{i}\right)}{20} + \frac{d^3\,x^2\,\left(20\,a\,c^2+20\,b\,c^2\,\mathrm{atan}(c\,x)\right.}{20}}{x^5}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^6,x)
```

output
```
(d^3*(24*b*c^5*log(x) - 12*b*c^5*log(c^2*x^2 + 1) + b*c^4*atan(x*(c^2)^(1/
2))*(c^2)^(1/2)*25i))/20 + ((d^3*x^3*(a*c^3*10i + 12*b*c^3 + b*c^3*atan(c*
x)*10i))/20 - (d^3*x*(a*c*15i + b*c + b*c*atan(c*x)*15i))/20 - (d^3*(4*a +
 4*b*atan(c*x)))/20 + (d^3*x^2*(20*a*c^2 - b*c^2*5i + 20*b*c^2*atan(c*x)))
/20 + (b*c^4*d^3*x^4*5i)/4)/x^5
```

3.30 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$

3.30.1 Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx =& -\frac{bcd^3}{30x^5}-\frac{3ibc^2d^3}{20x^4}+\frac{11bc^3d^3}{36x^3}+\frac{7ibc^4d^3}{15x^2}\\&-\frac{11bc^5d^3}{12x}-\frac{d^3(a+b\arctan(cx))}{6x^6}\\&-\frac{3icd^3(a+b\arctan(cx))}{5x^5}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}\\&+\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{14}{15}ibc^6d^3\log(x)\\&-\frac{1}{120}ibc^6d^3\log(i-cx)-\frac{37}{40}ibc^6d^3\log(i+cx)\end{aligned}$$

output
```
-1/30*b*c*d^3/x^5-3/20*I*b*c^2*d^3/x^4+11/36*b*c^3*d^3/x^3+7/15*I*b*c^4*d^
3/x^2-11/12*b*c^5*d^3/x-1/6*d^3*(a+b*arctan(c*x))/x^6-3/5*I*c*d^3*(a+b*arc
tan(c*x))/x^5+3/4*c^2*d^3*(a+b*arctan(c*x))/x^4+1/3*I*c^3*d^3*(a+b*arctan(
c*x))/x^3+14/15*I*b*c^6*d^3*ln(x)-1/120*I*b*c^6*d^3*ln(I-c*x)-37/40*I*b*c^
6*d^3*ln(c*x+I)
```

3.30.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.88

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{d^3\left(-2bcx\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)+i\left(10ia-36acx-45iac^2x^2-9bc^2x^2+20ac^3x^3+28\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
(d^3*(-2*b*c*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + I*((10*I)*a
- 36*a*c*x - (45*I)*a*c^2*x^2 - 9*b*c^2*x^2 + 20*a*c^3*x^3 + 28*b*c^4*x^4
+ (10*I)*b*ArcTan[c*x] - 36*b*c*x*ArcTan[c*x] - (45*I)*b*c^2*x^2*ArcTan[c*
x] + 20*b*c^3*x^3*ArcTan[c*x] - (15*I)*b*c^3*x^3*Hypergeometric2F1[-3/2, 1
, -1/2, -(c^2*x^2)] + 56*b*c^6*x^6*Log[x] - 28*b*c^6*x^6*Log[1 + c^2*x^2])
))/(60*x^6)
```

3.30.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$\Big\downarrow \text{5407}$$
$$-bc\int -\frac{d^3\left(-20ic^3x^3-45c^2x^2+36icx+10\right)}{60x^6\left(c^2x^2+1\right)}dx+\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{60}bcd^3\int \frac{-20ic^3x^3-45c^2x^2+36icx+10}{x^6\left(c^2x^2+1\right)}dx+\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{60}bcd^3\int \left(-\frac{ic^6}{2(cx-i)}-\frac{111ic^6}{2(cx+i)}+\frac{56ic^5}{x}+\frac{55c^4}{x^2}-\frac{56ic^3}{x^3}-\frac{55c^2}{x^4}+\frac{36ic}{x^5}+\frac{10}{x^6}\right)dx+\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}+\frac{1}{60}bcd^3\left(56ic^5\log(x)-\frac{1}{2}ic^5\log(-cx+i)-\frac{111}{2}ic^5\log(cx+i)-\frac{55c^4}{x}+\frac{28ic^3}{x^2}+\frac{55c^2}{3x^3}-\frac{9ic}{x^4}-\frac{2}{x^5}\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(d^3*(a + b*ArcTan[c*x]))/x^6 - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*x]))
/x^5 + (3*c^2*d^3*(a + b*ArcTan[c*x]))/(4*x^4) + ((I/3)*c^3*d^3*(a + b*Arc
Tan[c*x]))/x^3 + (b*c*d^3*(-2/x^5 - ((9*I)*c)/x^4 + (55*c^2)/(3*x^3) + ((2
8*I)*c^3)/x^2 - (55*c^4)/x + (56*I)*c^5*Log[x] - (I/2)*c^5*Log[I - c*x] -
((111*I)/2)*c^5*Log[I + c*x]))/60
```

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.30.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.77

method	result
parts	$a\,d^3\left(\frac{3c^2}{4x^4}-\frac{3ic}{5x^5}-\frac{1}{6x^6}+\frac{ic^3}{3x^3}\right)+b\,d^3c^6\left(-\frac{3i\arctan(cx)}{5c^5x^5}+\frac{3\arctan(cx)}{4c^4x^4}+\frac{i\arctan(cx)}{3c^3x^3}-\frac{\arctan(cx)}{6c^6x^6}\right.$
derivativedivides	$c^6\left(a\,d^3\left(-\frac{3i}{5c^5x^5}+\frac{3}{4c^4x^4}+\frac{i}{3c^3x^3}-\frac{1}{6c^6x^6}\right)+b\,d^3\left(-\frac{3i\arctan(cx)}{5c^5x^5}+\frac{3\arctan(cx)}{4c^4x^4}+\frac{i\arctan(cx)}{3c^3x^3}-\right.\right.$
default	$c^6\left(a\,d^3\left(-\frac{3i}{5c^5x^5}+\frac{3}{4c^4x^4}+\frac{i}{3c^3x^3}-\frac{1}{6c^6x^6}\right)+b\,d^3\left(-\frac{3i\arctan(cx)}{5c^5x^5}+\frac{3\arctan(cx)}{4c^4x^4}+\frac{i\arctan(cx)}{3c^3x^3}-\right.\right.$
parallelrisch	$-\frac{108ix\arctan(cx)bc\,d^3+84i\ln(c^2x^2+1)x^6b\,c^6d^3-60ix^3a\,c^3d^3+165b\,c^6d^3\arctan(cx)x^6+165b\,c^5d^3x^5+27ix^2b\,c^2d^3+}{}$
risch	$\frac{(20b\,c^3d^3x^3-45ix^2b\,c^2d^3-36bc\,d^3x+10ib\,d^3)\ln(icx+1)}{120x^6}-\frac{id^3(333b\,c^6\ln(-12265cx-12265i)x^6-336b\,c^6\ln(-25199c}{}$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
a*d^3*(3/4*c^2/x^4-3/5*I*c/x^5-1/6/x^6+1/3*I*c^3/x^3)+b*d^3*c^6*(-3/5*I*ar
ctan(c*x)/c^5/x^5+3/4*arctan(c*x)/c^4/x^4+1/3*I*arctan(c*x)/c^3/x^3-1/6*ar
ctan(c*x)/c^6/x^6+14/15*I*ln(c*x)-3/20*I/c^4/x^4+7/15*I/c^2/x^2-1/30/c^5/x
^5+11/36/c^3/x^3-11/12/c/x-7/15*I*ln(c^2*x^2+1)-11/12*arctan(c*x))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int\frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$=\frac{336i\,bc^6d^3x^6\log(x)-333i\,bc^6d^3x^6\log\left(\frac{cx+i}{c}\right)-3i\,bc^6d^3x^6\log\left(\frac{cx-i}{c}\right)-330\,bc^5d^3x^5+168i\,bc^4d^3x^4-10\,(-}{}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/360*(336*I*b*c^6*d^3*x^6*log(x) - 333*I*b*c^6*d^3*x^6*log((c*x + I)/c) -
 3*I*b*c^6*d^3*x^6*log((c*x - I)/c) - 330*b*c^5*d^3*x^5 + 168*I*b*c^4*d^3*
x^4 - 10*(-12*I*a - 11*b)*c^3*d^3*x^3 + 54*(5*a - I*b)*c^2*d^3*x^2 - 12*(1
8*I*a + b)*c*d^3*x - 60*a*d^3 - 3*(20*b*c^3*d^3*x^3 - 45*I*b*c^2*d^3*x^2 -
 36*b*c*d^3*x + 10*I*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^6
```

3.30.6 Sympy [A] (verification not implemented)

Time = 45.62 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.62

$$
\begin{aligned}
&\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx \\
&= \frac{14ibc^6d^3\log\left(1385945b^2c^{13}d^6x\right)}{15} - \frac{ibc^6d^3\log\left(1385945b^2c^{13}d^6x - 1385945ib^2c^{12}d^6\right)}{120} \\
&\quad - \frac{37ibc^6d^3\log\left(1385945b^2c^{13}d^6x + 1385945ib^2c^{12}d^6\right)}{40} \\
&\quad + \frac{(-20bc^3d^3x^3 + 45ibc^2d^3x^2 + 36bcd^3x - 10ibd^3)\log\left(-icx+1\right)}{120x^6} \\
&\quad + \frac{(20bc^3d^3x^3 - 45ibc^2d^3x^2 - 36bcd^3x + 10ibd^3)\log\left(icx+1\right)}{120x^6} \\
&\quad - \frac{30ad^3 + 165bc^5d^3x^5 - 84ibc^4d^3x^4 + x^3(-60iac^3d^3 - 55bc^3d^3) + x^2(-135ac^2d^3 + 27ibc^2d^3) + x(108iac}{180x^6}
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**7,x)
```

output
```
14*I*b*c**6*d**3*log(1385945*b**2*c**13*d**6*x)/15 - I*b*c**6*d**3*log(138
5945*b**2*c**13*d**6*x - 1385945*I*b**2*c**12*d**6)/120 - 37*I*b*c**6*d**3
*log(1385945*b**2*c**13*d**6*x + 1385945*I*b**2*c**12*d**6)/40 + (-20*b*c*
*3*d**3*x**3 + 45*I*b*c**2*d**3*x**2 + 36*b*c*d**3*x - 10*I*b*d**3)*log(-I
*c*x + 1)/(120*x**6) + (20*b*c**3*d**3*x**3 - 45*I*b*c**2*d**3*x**2 - 36*b
*c*d**3*x + 10*I*b*d**3)*log(I*c*x + 1)/(120*x**6) - (30*a*d**3 + 165*b*c*
*5*d**3*x**5 - 84*I*b*c**4*d**3*x**4 + x**3*(-60*I*a*c**3*d**3 - 55*b*c**3
*d**3) + x**2*(-135*a*c**2*d**3 + 27*I*b*c**2*d**3) + x*(108*I*a*c*d**3 +
6*b*c*d**3))/(180*x**6)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.16

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$= -\frac{1}{6}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^3d^3$$
$$-\frac{1}{4}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bc^2d^3$$
$$-\frac{3}{20}i\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bcd^3$$
$$-\frac{1}{90}\left(\left(15\,c^5\arctan\left(cx\right)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan\left(cx\right)}{x^6}\right)bd^3$$
$$+\frac{i\,ac^3d^3}{3\,x^3}+\frac{3\,ac^2d^3}{4\,x^4}-\frac{3i\,acd^3}{5\,x^5}-\frac{ad^3}{6\,x^6}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^
3)*b*c^3*d^3 - 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan
(c*x)/x^4)*b*c^2*d^3 - 3/20*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) -
(2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c*d^3 - 1/90*((15*c^5*arctan
(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^3 +
1/3*I*a*c^3*d^3/x^3 + 3/4*a*c^2*d^3/x^4 - 3/5*I*a*c*d^3/x^5 - 1/6*a*d^3/x^
6
```

3.30.8 Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)}{x^7}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
sage0*x
```

3.30.9 Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx =$$

$$-\frac{\frac{d^3\,(30\,a+30\,b\,\mathrm{atan}(c\,x))}{180}+\frac{d^3\,x\,(a\,c\,108\mathrm{i}+6\,b\,c+b\,c\,\mathrm{atan}(c\,x)\,108\mathrm{i})}{180}-\frac{d^3\,x^3\,\left(a\,c^3\,60\mathrm{i}+55\,b\,c^3+b\,c^3\,\mathrm{atan}(c\,x)\,60\mathrm{i}\right)}{180}-\frac{d^3\,x^2\,\left(135\,a\,c^2+135\,b\,c^2\right.}{180}}{x^6}$$

$$-\frac{d^3\left(\frac{165\,b\,c^9\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)}{{\left(c^2\right)}^{3/2}}+b\,c^6\,\ln\left(c^2\,x^2+1\right)\,84\mathrm{i}-b\,c^6\,\ln\left(x\right)\,168\mathrm{i}\right)}{180}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^7,x)
```

output
```
- ((d^3*(30*a + 30*b*atan(c*x)))/180 + (d^3*x*(a*c*108i + 6*b*c + b*c*atan
(c*x)*108i))/180 - (d^3*x^3*(a*c^3*60i + 55*b*c^3 + b*c^3*atan(c*x)*60i))/
180 - (d^3*x^2*(135*a*c^2 - b*c^2*27i + 135*b*c^2*atan(c*x)))/180 - (b*c^4
*d^3*x^4*7i)/15 + (11*b*c^5*d^3*x^5)/12)/x^6 - (d^3*(b*c^6*log(c^2*x^2 + 1
)*84i - b*c^6*log(x)*168i + (165*b*c^9*atan((c^2*x)/(c^2)^(1/2)))/(c^2)^(3
/2)))/180
```

3.31 $\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$

3.31.1 Optimal result

Integrand size = 23, antiderivative size = 238

$$\begin{aligned}\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx &= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4\\ &+\frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x^6-\frac{1}{56}bc^3d^4x^7\\ &-\frac{11bd^4\arctan(cx)}{8c^4}+\frac{1}{4}d^4x^4(a+b\arctan(cx))\\ &+\frac{4}{5}icd^4x^5(a+b\arctan(cx))\\ &-c^2d^4x^6(a+b\arctan(cx))\\ &-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))\\ &+\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{24ibd^4\log(1+c^2x^2)}{35c^4}\end{aligned}$$

output
```
11/8*b*d^4*x/c^3+24/35*I*b*d^4*x^2/c^2-11/24*b*d^4*x^3/c-12/35*I*b*d^4*x^4
+9/40*b*c*d^4*x^5+2/21*I*b*c^2*d^4*x^6-1/56*b*c^3*d^4*x^7-11/8*b*d^4*arcta
n(c*x)/c^4+1/4*d^4*x^4*(a+b*arctan(c*x))+4/5*I*c*d^4*x^5*(a+b*arctan(c*x))
-c^2*d^4*x^6*(a+b*arctan(c*x))-4/7*I*c^3*d^4*x^7*(a+b*arctan(c*x))+1/8*c^4
*d^4*x^8*(a+b*arctan(c*x))-24/35*I*b*d^4*ln(c^2*x^2+1)/c^4
```

3.31.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.22

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}+\frac{1}{4}ad^4x^4 - \frac{12}{35}ibd^4x^4+\frac{4}{5}iacd^4x^5+\frac{9}{40}bcd^4x^5 - ac^2d^4x^6+\frac{2}{21}ibc^2d^4x^6-\frac{4}{7}iac^3d^4x^7 - \frac{1}{56}bc^3d^4x^7+\frac{1}{8}ac^4d^4x^8-\frac{11bd^4\arctan(cx)}{8c^4} + \frac{1}{4}bd^4x^4\arctan(cx)+\frac{4}{5}ibcd^4x^5\arctan(cx) - bc^2d^4x^6\arctan(cx)-\frac{4}{7}ibc^3d^4x^7\arctan(cx) + \frac{1}{8}bc^4d^4x^8\arctan(cx)-\frac{24ibd^4\log(1+c^2x^2)}{35c^4}$$

input
```
Integrate[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(11*b*d^4*x)/(8*c^3) + (((24*I)/35)*b*d^4*x^2)/c^2 - (11*b*d^4*x^3)/(24*c)
 + (a*d^4*x^4)/4 - ((12*I)/35)*b*d^4*x^4 + ((4*I)/5)*a*c*d^4*x^5 + (9*b*c*
d^4*x^5)/40 - a*c^2*d^4*x^6 + ((2*I)/21)*b*c^2*d^4*x^6 - ((4*I)/7)*a*c^3*d
^4*x^7 - (b*c^3*d^4*x^7)/56 + (a*c^4*d^4*x^8)/8 - (11*b*d^4*ArcTan[c*x])/(
8*c^4) + (b*d^4*x^4*ArcTan[c*x])/4 + ((4*I)/5)*b*c*d^4*x^5*ArcTan[c*x] - b
*c^2*d^4*x^6*ArcTan[c*x] - ((4*I)/7)*b*c^3*d^4*x^7*ArcTan[c*x] + (b*c^4*d^
4*x^8*ArcTan[c*x])/8 - (((24*I)/35)*b*d^4*Log[1 + c^2*x^2])/c^4
```

3.31.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^4x^4(35c^4x^4-160ic^3x^3-280c^2x^2+224icx+70)}{280\,(c^2x^2+1)}dx+\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{280}bcd^4\int \frac{x^4(35c^4x^4-160ic^3x^3-280c^2x^2+224icx+70)}{c^2x^2+1}dx+\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))$$

$$\Big\downarrow \; 2333$$

$$-\frac{1}{280}bcd^4\int \left(35c^2x^6-160icx^5-315x^4+\frac{384ix^3}{c}+\frac{385x^2}{c^2}-\frac{384ix}{c^3}+\frac{384icx+385}{c^4\,(c^2x^2+1)}-\frac{385}{c^4}\right)dx+\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))-\frac{1}{280}bcd^4\left(\frac{385\arctan(cx)}{c^5}-\frac{385x}{c^4}-\frac{192ix^2}{c^3}+5c^2x^7+\frac{385x^3}{3c^2}+\frac{192i\log\left(c^2x^2+1\right)}{c^5}-\frac{80}{3}icx^6+\frac{96ix^4}{c}-63x^5\right)$$

input
```
Int[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(d^4*x^4*(a + b*ArcTan[c*x]))/4 + ((4*I)/5)*c*d^4*x^5*(a + b*ArcTan[c*x])
- c^2*d^4*x^6*(a + b*ArcTan[c*x]) - ((4*I)/7)*c^3*d^4*x^7*(a + b*ArcTan[c*
x]) + (c^4*d^4*x^8*(a + b*ArcTan[c*x]))/8 - (b*c*d^4*((-385*x)/c^4 - ((192
*I)*x^2)/c^3 + (385*x^3)/(3*c^2) + ((96*I)*x^4)/c - 63*x^5 - ((80*I)/3)*c*
x^6 + 5*c^2*x^7 + (385*ArcTan[c*x])/c^5 + ((192*I)*Log[1 + c^2*x^2])/c^5))
/280
```

3.31. $\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$

3.31.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.31.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.80

method	result
parts	$d^4 a\left(\frac{1}{8}c^4x^8-\frac{4}{7}ic^3x^7-c^2x^6+\frac{4}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{d^4 b\left(\frac{\arctan(cx)c^8x^8}{8}-\frac{4i\arctan(cx)c^7x^7}{7}-\arctan(cx)c^6x^6\right.}{}$
derivativedivides	$\frac{d^4a\left(\frac{1}{8}c^8x^8-\frac{4}{7}ic^7x^7-c^6x^6+\frac{4}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^4b\left(\frac{\arctan(cx)c^8x^8}{8}-\frac{4i\arctan(cx)c^7x^7}{7}-\arctan(cx)c^6x^6+\frac{4i\arctan(cx)c^5x^5}{5}\right.}{c^4}$
default	$\frac{d^4a\left(\frac{1}{8}c^8x^8-\frac{4}{7}ic^7x^7-c^6x^6+\frac{4}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^4b\left(\frac{\arctan(cx)c^8x^8}{8}-\frac{4i\arctan(cx)c^7x^7}{7}-\arctan(cx)c^6x^6+\frac{4i\arctan(cx)c^5x^5}{5}\right.}{c^4}$
parallelrisch	$-\frac{-105b\,c^8d^4\arctan(cx)x^8-80ix^6b\,c^6d^4-105a\,c^8d^4x^8+288ix^4b\,c^4d^4+15b\,c^7d^4x^7+480ic^7b\,d^4\arctan(cx)x^7+840b\,c^6d^4}{}$
risch	$-\frac{id^4c^2x^6b\ln(-icx+1)}{2}+\frac{d^4c^4a\,x^8}{8}-\frac{4id^4c^3a\,x^7}{7}+\frac{2d^4c^3b\,x^7\ln(-icx+1)}{7}-\frac{b\,c^3d^4x^7}{56}-\frac{id^4b(35c^4x^8-160ic^3x^7}{}$

input
```
int(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/8*c^4*x^8-4/7*I*c^3*x^7-c^2*x^6+4/5*I*c*x^5+1/4*x^4)+d^4*b/c^4*(1
/8*arctan(c*x)*c^8*x^8-4/7*I*arctan(c*x)*c^7*x^7-arctan(c*x)*c^6*x^6+4/5*I
*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*arctan(c*x)+11/8*c*x-1/56*c^7*x^7+2/21*I*
c^6*x^6+9/40*c^5*x^5-12/35*I*c^4*x^4-11/24*c^3*x^3+24/35*I*c^2*x^2-24/35*I
*ln(c^2*x^2+1)-11/8*arctan(c*x))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.97

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$=\frac{210\,ac^8d^4x^8-30\,(32i\,a+b)c^7d^4x^7-80\,(21\,a-2i\,b)c^6d^4x^6-42\,(-32i\,a-9\,b)c^5d^4x^5+12\,(35\,a-48i\,b)c}{}$$

input
```
integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(210*a*c^8*d^4*x^8 - 30*(32*I*a + b)*c^7*d^4*x^7 - 80*(21*a - 2*I*b
)*c^6*d^4*x^6 - 42*(-32*I*a - 9*b)*c^5*d^4*x^5 + 12*(35*a - 48*I*b)*c^4*d^
4*x^4 - 770*b*c^3*d^4*x^3 + 1152*I*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 2307*I
*b*d^4*log((c*x + I)/c) + 3*I*b*d^4*log((c*x - I)/c) - 3*(-35*I*b*c^8*d^4*
x^8 - 160*b*c^7*d^4*x^7 + 280*I*b*c^6*d^4*x^6 + 224*b*c^5*d^4*x^5 - 70*I*b
*c^4*d^4*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

3.31.6 Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.63

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx=\frac{ac^4d^4x^8}{8}-\frac{11bd^4x^3}{24c}+\frac{24ibd^4x^2}{35c^2}+\frac{11bd^4x}{8c^3}$$
$$+\frac{bd^4\left(\frac{i\log\left(5893bcd^4x-5893ibd^4\right)}{560}-\frac{1471i\log\left(5893bcd^4x+5893ibd^4\right)}{1260}\right)}{c^4}+x^7\left(-\frac{4iac^3d^4}{7}-\frac{bc^3d^4}{56}\right)$$
$$+x^6\left(-ac^2d^4+\frac{2ibc^2d^4}{21}\right)+x^5\cdot\left(\frac{4iacd^4}{5}+\frac{9bcd^4}{40}\right)+x^4\left(\frac{ad^4}{4}-\frac{12ibd^4}{35}\right)$$
$$+\left(-\frac{ibc^4d^4x^8}{16}-\frac{2bc^3d^4x^7}{7}+\frac{ibc^2d^4x^6}{2}+\frac{2bcd^4x^5}{5}-\frac{ibd^4x^4}{8}\right)\log\left(icx+1\right)$$
$$+\frac{(315ibc^8d^4x^8+1440bc^7d^4x^7-2520ibc^6d^4x^6-2016bc^5d^4x^5+630ibc^4d^4x^4-1037ibd^4)\log\left(-icx+1\right)}{5040c^4}$$

input
```
integrate(x**3*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**8/8 - 11*b*d**4*x**3/(24*c) + 24*I*b*d**4*x**2/(35*c**2) +
11*b*d**4*x/(8*c**3) + b*d**4*(I*log(5893*b*c*d**4*x - 5893*I*b*d**4)/560
- 1471*I*log(5893*b*c*d**4*x + 5893*I*b*d**4)/1260)/c**4 + x**7*(-4*I*a*c*
*3*d**4/7 - b*c**3*d**4/56) + x**6*(-a*c**2*d**4 + 2*I*b*c**2*d**4/21) + x
**5*(4*I*a*c*d**4/5 + 9*b*c*d**4/40) + x**4*(a*d**4/4 - 12*I*b*d**4/35) +
(-I*b*c**4*d**4*x**8/16 - 2*b*c**3*d**4*x**7/7 + I*b*c**2*d**4*x**6/2 + 2*
b*c*d**4*x**5/5 - I*b*d**4*x**4/8)*log(I*c*x + 1) + (315*I*b*c**8*d**4*x**
8 + 1440*b*c**7*d**4*x**7 - 2520*I*b*c**6*d**4*x**6 - 2016*b*c**5*d**4*x**
5 + 630*I*b*c**4*d**4*x**4 - 1037*I*b*d**4)*log(-I*c*x + 1)/(5040*c**4)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.42

$$\begin{aligned}\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx &= \frac{1}{8}\,ac^4d^4x^8 - \frac{4}{7}i\,ac^3d^4x^7 - ac^2d^4x^6 + \frac{4}{5}i\,acd^4x^5 \\ &+ \frac{1}{840}\left(105\,x^8\arctan(cx) - c\left(\frac{15\,c^6x^7 - 21\,c^4x^5 + 35\,c^2x^3 - 105\,x}{c^8} + \frac{105\,\arctan(cx)}{c^9}\right)\right)bc^4d^4 \\ &- \frac{1}{21}i\left(12\,x^7\arctan(cx) - c\left(\frac{2\,c^4x^6 - 3\,c^2x^4 + 6\,x^2}{c^6} - \frac{6\,\log(c^2x^2+1)}{c^8}\right)\right)bc^3d^4 \\ &+ \frac{1}{4}\,ad^4x^4 - \frac{1}{15}\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)bc^2d^4 \\ &+ \frac{1}{5}i\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bcd^4 \\ &+ \frac{1}{12}\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bd^4\end{aligned}$$

input
```
integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*c^4*d^4*x^8 - 4/7*I*a*c^3*d^4*x^7 - a*c^2*d^4*x^6 + 4/5*I*a*c*d^4*x^
5 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3
- 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*c^4*d^4 - 1/21*I*(12*x^7*arctan(c*x
) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*c^
3*d^4 + 1/4*a*d^4*x^4 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x
^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^4 + 1/5*I*(4*x^5*arctan(c*x)
 - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d^4 + 1/12*(3*x
^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^4
```

3.31.8 Giac [F]

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^4(b\arctan(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.31.9 Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx &= \frac{c^4\,d^4\,(105\,a\,x^8+105\,b\,x^8\,\mathrm{atan}(c\,x))}{840}+\frac{d^4\,(210\,a\,x^4+210\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,288\mathrm{i})}{840}\\&\quad-\frac{\frac{d^4\,(1155\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,576\mathrm{i})}{840}+\frac{11\,b\,c^3\,d^4\,x^3}{24}-\frac{11\,b\,c\,d^4\,x}{8}-\frac{b\,c^2\,d^4\,x^2\,24\mathrm{i}}{35}}{c^4}\\&\quad+\frac{c\,d^4\,(a\,x^5\,672\mathrm{i}+189\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,672\mathrm{i})}{840}\\&\quad-\frac{c^3\,d^4\,(a\,x^7\,480\mathrm{i}+15\,b\,x^7+b\,x^7\,\mathrm{atan}(c\,x)\,480\mathrm{i})}{840}\\&\quad-\frac{c^2\,d^4\,(840\,a\,x^6+840\,b\,x^6\,\mathrm{atan}(c\,x)-b\,x^6\,80\mathrm{i})}{840}\end{aligned}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
(d^4*(210*a*x^4 - b*x^4*288i + 210*b*x^4*atan(c*x)))/840 - ((d^4*(1155*b*a
tan(c*x) + b*log(c^2*x^2 + 1)*576i))/840 - (b*c^2*d^4*x^2*24i)/35 + (11*b*
c^3*d^4*x^3)/24 - (11*b*c*d^4*x)/8)/c^4 + (c^4*d^4*(105*a*x^8 + 105*b*x^8*
atan(c*x)))/840 + (c*d^4*(a*x^5*672i + 189*b*x^5 + b*x^5*atan(c*x)*672i))/
840 - (c^3*d^4*(a*x^7*480i + 15*b*x^7 + b*x^7*atan(c*x)*480i))/840 - (c^2*
d^4*(840*a*x^6 - b*x^6*80i + 840*b*x^6*atan(c*x)))/840
```

3.32 $\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$

3.32.1 Optimal result

Integrand size = 23, antiderivative size = 193

$$\begin{aligned}\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx &= \frac{5ibd^4x}{3c^2}-\frac{88bd^4x^2}{105c}-\frac{5}{9}ibd^4x^3\\ &\quad+\frac{47}{140}bcd^4x^4+\frac{2}{15}ibc^2d^4x^5-\frac{1}{42}bc^3d^4x^6\\ &\quad+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}\\ &\quad-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}\\ &\quad+\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}\\ &\quad+\frac{176bd^4\log(i+cx)}{105c^3}\end{aligned}$$

output
```
5/3*I*b*d^4*x/c^2-88/105*b*d^4*x^2/c-5/9*I*b*d^4*x^3+47/140*b*c*d^4*x^4+2/
15*I*b*c^2*d^4*x^5-1/42*b*c^3*d^4*x^6+1/5*I*d^4*(1+I*c*x)^5*(a+b*arctan(c*
x))/c^3-1/3*I*d^4*(1+I*c*x)^6*(a+b*arctan(c*x))/c^3+1/7*I*d^4*(1+I*c*x)^7*
(a+b*arctan(c*x))/c^3+176/105*b*d^4*ln(c*x+I)/c^3
```

3.32.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.43

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{5ibd^4x}{3c^2} - \frac{88bd^4x^2}{105c} + \frac{1}{3}ad^4x^3 - \frac{5}{9}ibd^4x^3 + iacd^4x^4 + \frac{47}{140}bcd^4x^4 - \frac{6}{5}ac^2d^4x^5 + \frac{2}{15}ibc^2d^4x^5 - \frac{2}{3}iac^3d^4x^6 - \frac{1}{42}bc^3d^4x^6 + \frac{1}{7}ac^4d^4x^7 - \frac{5ibd^4\arctan(cx)}{3c^3} + \frac{1}{3}bd^4x^3\arctan(cx) + ibcd^4x^4\arctan(cx) - \frac{6}{5}bc^2d^4x^5\arctan(cx) - \frac{2}{3}ibc^3d^4x^6\arctan(cx) + \frac{1}{7}bc^4d^4x^7\arctan(cx) + \frac{88bd^4\log(1+c^2x^2)}{105c^3}$$

input
```
Integrate[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(((5*I)/3)*b*d^4*x)/c^2 - (88*b*d^4*x^2)/(105*c) + (a*d^4*x^3)/3 - ((5*I)/
9)*b*d^4*x^3 + I*a*c*d^4*x^4 + (47*b*c*d^4*x^4)/140 - (6*a*c^2*d^4*x^5)/5
+ ((2*I)/15)*b*c^2*d^4*x^5 - ((2*I)/3)*a*c^3*d^4*x^6 - (b*c^3*d^4*x^6)/42
+ (a*c^4*d^4*x^7)/7 - (((5*I)/3)*b*d^4*ArcTan[c*x])/c^3 + (b*d^4*x^3*ArcTa
n[c*x])/3 + I*b*c*d^4*x^4*ArcTan[c*x] - (6*b*c^2*d^4*x^5*ArcTan[c*x])/5 -
((2*I)/3)*b*c^3*d^4*x^6*ArcTan[c*x] + (b*c^4*d^4*x^7*ArcTan[c*x])/7 + (88*
b*d^4*Log[1 + c^2*x^2])/(105*c^3)
```

3.32.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5407$$

$$-bc\int -\frac{d^4(i-cx)^4\left(-15c^2x^2-5icx+1\right)}{105c^3(cx+i)}dx+\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{bd^4\int \frac{(i-cx)^4(-15c^2x^2-5icx+1)}{cx+i}dx}{105c^2}+\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}$$

$$\Big\downarrow \textbf{1195}$$

$$\frac{bd^4\int \left(-15c^5x^5+70ic^4x^4+141c^3x^3-175ic^2x^2-176cx+\frac{176}{cx+i}+175i\right)dx}{105c^2}+\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}+\frac{bd^4\left(-\frac{5}{2}c^5x^6+14ic^4x^5+\frac{141c^3x^4}{4}-\frac{175}{3}ic^2x^3-88cx^2+\frac{176\log(cx+i)}{c}+175ix\right)}{105c^2}$$

input
```
Int[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
((I/5)*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/c^3 - ((I/3)*d^4*(1 + I*c*x)
^6*(a + b*ArcTan[c*x]))/c^3 + ((I/7)*d^4*(1 + I*c*x)^7*(a + b*ArcTan[c*x])
)/c^3 + (b*d^4*((175*I)*x - 88*c*x^2 - ((175*I)/3)*c^2*x^3 + (141*c^3*x^4)
/4 + (14*I)*c^4*x^5 - (5*c^5*x^6)/2 + (176*Log[I + c*x])/c))/(105*c^2)
```

3.32. $\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.32.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94

method	result
parts	$d^4 a\left(\frac{1}{7}c^4x^7-\frac{2}{3}ic^3x^6-\frac{6}{5}c^2x^5+ic\,x^4+\frac{1}{3}x^3\right)+\frac{d^4 b\left(\frac{\arctan(cx)c^7x^7}{7}-\frac{2i\arctan(cx)c^6x^6}{3}-\frac{6c^5x^5\arctan(cx)}{5}\right.}{}$
derivativedivides	$\frac{d^4 a\left(\frac{1}{7}c^7x^7-\frac{2}{3}ic^6x^6-\frac{6}{5}c^5x^5+ic^4x^4+\frac{1}{3}c^3x^3\right)+d^4 b\left(\frac{\arctan(cx)c^7x^7}{7}-\frac{2i\arctan(cx)c^6x^6}{3}-\frac{6c^5x^5\arctan(cx)}{5}+i\arctan(cx)c^4x\right.}{c^3}$
default	$\frac{d^4 a\left(\frac{1}{7}c^7x^7-\frac{2}{3}ic^6x^6-\frac{6}{5}c^5x^5+ic^4x^4+\frac{1}{3}c^3x^3\right)+d^4 b\left(\frac{\arctan(cx)c^7x^7}{7}-\frac{2i\arctan(cx)c^6x^6}{3}-\frac{6c^5x^5\arctan(cx)}{5}+i\arctan(cx)c^4x\right.}{c^3}$
parallelrisch	$-\frac{-180c^7b\,d^4\arctan(cx)x^7-2100ib\,d^4xc-180a\,c^7d^4x^7-168ix^5b\,c^5d^4+30b\,c^6d^4x^6-1260ix^4\arctan(cx)b\,c^4d^4+1512c^5b}{}$
risch	$-\frac{5ib\,d^4x^3}{9}-\frac{id^4b\left(15c^4x^7-70ic^3x^6-126c^2x^5+105ic\,x^4+35x^3\right)\ln(icx+1)}{210}+\frac{a\,c^4d^4x^7}{7}+\frac{id^4c^4b\,x^7\ln(-icx+1)}{14}+$

input
```
int(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/7*c^4*x^7-2/3*I*c^3*x^6-6/5*c^2*x^5+I*c*x^4+1/3*x^3)+d^4*b/c^3*(1
/7*arctan(c*x)*c^7*x^7-2/3*I*arctan(c*x)*c^6*x^6-6/5*c^5*x^5*arctan(c*x)+I
*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*arctan(c*x)+5/3*I*c*x-1/42*c^6*x^6+2/15*I
*c^5*x^5+47/140*c^4*x^4-5/9*I*c^3*x^3-88/105*c^2*x^2+88/105*ln(c^2*x^2+1)-
5/3*I*arctan(c*x))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.13

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$=\frac{180\,ac^7d^4x^7-30\,(28i\,a+b)c^6d^4x^6-168\,(9\,a-i\,b)c^5d^4x^5-9\,(-140i\,a-47\,b)c^4d^4x^4+140\,(3\,a-5i\,b)c^3d^4\ldots}{}$$

input
```
integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/1260*(180*a*c^7*d^4*x^7 - 30*(28*I*a + b)*c^6*d^4*x^6 - 168*(9*a - I*b)*
c^5*d^4*x^5 - 9*(-140*I*a - 47*b)*c^4*d^4*x^4 + 140*(3*a - 5*I*b)*c^3*d^4*
x^3 - 1056*b*c^2*d^4*x^2 + 2100*I*b*c*d^4*x + 2106*b*d^4*log((c*x + I)/c)
+ 6*b*d^4*log((c*x - I)/c) - 6*(-15*I*b*c^7*d^4*x^7 - 70*b*c^6*d^4*x^6 + 1
26*I*b*c^5*d^4*x^5 + 105*b*c^4*d^4*x^4 - 35*I*b*c^3*d^4*x^3)*log(-(c*x + I
)/(c*x - I)))/c^3
```

3.32.6 Sympy [A] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.90

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$=\frac{ac^4d^4x^7}{7}-\frac{88bd^4x^2}{105c}+\frac{5ibd^4x}{3c^2}+\frac{bd^4\left(\frac{\log\left(2299bcd^4x-2299ibd^4\right)}{210}+\frac{769\log\left(2299bcd^4x+2299ibd^4\right)}{560}\right)}{c^3}$$
$$+x^6\left(-\frac{2iac^3d^4}{3}-\frac{bc^3d^4}{42}\right)+x^5\left(-\frac{6ac^2d^4}{5}+\frac{2ibc^2d^4}{15}\right)+x^4\left(iacd^4+\frac{47bcd^4}{140}\right)$$
$$+x^3\left(\frac{ad^4}{3}-\frac{5ibd^4}{9}\right)+\left(-\frac{ibc^4d^4x^7}{14}-\frac{bc^3d^4x^6}{3}+\frac{3ibc^2d^4x^5}{5}+\frac{bcd^4x^4}{2}-\frac{ibd^4x^3}{6}\right)\log\left(icx+1\right)$$
$$+\frac{(120ibc^7d^4x^7+560bc^6d^4x^6-1008ibc^5d^4x^5-840bc^4d^4x^4+280ibc^3d^4x^3+501bd^4)\log\left(-icx+1\right)}{1680c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**7/7 - 88*b*d**4*x**2/(105*c) + 5*I*b*d**4*x/(3*c**2) + b*d*
*4*(log(2299*b*c*d**4*x - 2299*I*b*d**4)/210 + 769*log(2299*b*c*d**4*x + 2
299*I*b*d**4)/560)/c**3 + x**6*(-2*I*a*c**3*d**4/3 - b*c**3*d**4/42) + x**
5*(-6*a*c**2*d**4/5 + 2*I*b*c**2*d**4/15) + x**4*(I*a*c*d**4 + 47*b*c*d**4
/140) + x**3*(a*d**4/3 - 5*I*b*d**4/9) + (-I*b*c**4*d**4*x**7/14 - b*c**3*
d**4*x**6/3 + 3*I*b*c**2*d**4*x**5/5 + b*c*d**4*x**4/2 - I*b*d**4*x**3/6)*
log(I*c*x + 1) + (120*I*b*c**7*d**4*x**7 + 560*b*c**6*d**4*x**6 - 1008*I*b
*c**5*d**4*x**5 - 840*b*c**4*d**4*x**4 + 280*I*b*c**3*d**4*x**3 + 501*b*d*
*4)*log(-I*c*x + 1)/(1680*c**3)
```

3.32.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(153) = 306$.

Time = 0.28 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.65

$$
\begin{aligned}
&\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx \\
&= \frac{1}{7}\,ac^4d^4x^7 - \frac{2}{3}i\,ac^3d^4x^6 - \frac{6}{5}\,ac^2d^4x^5 \\
&\quad + \frac{1}{84}\left(12\,x^7\arctan(cx) - c\left(\frac{2\,c^4x^6 - 3\,c^2x^4 + 6\,x^2}{c^6} - \frac{6\,\log(c^2x^2+1)}{c^8}\right)\right)bc^4d^4 \\
&\quad + i\,acd^4x^4 - \frac{2}{45}i\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)bc^3d^4 \\
&\quad - \frac{3}{10}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^2d^4 \\
&\quad + \frac{1}{3}\,ad^4x^3 + \frac{1}{3}i\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bcd^4 \\
&\quad + \frac{1}{6}\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^4
\end{aligned}
$$

input
```
integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^4*d^4*x^7 - 2/3*I*a*c^3*d^4*x^6 - 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7
*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)
/c^8))*b*c^4*d^4 + I*a*c*d^4*x^4 - 2/45*I*(15*x^6*arctan(c*x) - c*((3*c^4*
x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^3*d^4 - 3/10*(4*x^5
*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d
^4 + 1/3*a*d^4*x^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3
*arctan(c*x)/c^5))*b*c*d^4 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2
*x^2 + 1)/c^4))*b*d^4
```

3.32.8 Giac [F]

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx=\int (i\,cdx+d)^4(b\arctan\left(cx\right)+a)x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx\\
&=\frac{c^4\,d^4\,(180\,a\,x^7+180\,b\,x^7\,\mathrm{atan}(c\,x))}{1260}+\frac{d^4\,(420\,a\,x^3+420\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,700\mathrm{i})}{1260}\\
&\quad-\frac{\frac{d^4\,(-1056\,b\,\ln(c^2\,x^2+1)+b\,\mathrm{atan}(c\,x)\,2100\mathrm{i})}{1260}+\frac{88\,b\,c^2\,d^4\,x^2}{105}-\frac{b\,c\,d^4\,x\,5\mathrm{i}}{3}}{c^3}\\
&\quad+\frac{c\,d^4\,(a\,x^4\,1260\mathrm{i}+423\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\;1260\mathrm{i})}{1260}\\
&\quad-\frac{c^3\,d^4\,(a\,x^6\,840\mathrm{i}+30\,b\,x^6+b\,x^6\,\mathrm{atan}(c\,x)\;840\mathrm{i})}{1260}\\
&\quad-\frac{c^2\,d^4\,(1512\,a\,x^5+1512\,b\,x^5\,\mathrm{atan}(c\,x)-b\,x^5\,168\mathrm{i})}{1260}
\end{aligned}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
(d^4*(420*a*x^3 - b*x^3*700i + 420*b*x^3*atan(c*x)))/1260 - ((d^4*(b*atan(
c*x)*2100i - 1056*b*log(c^2*x^2 + 1)))/1260 + (88*b*c^2*d^4*x^2)/105 - (b*
c*d^4*x*5i)/3)/c^3 + (c^4*d^4*(180*a*x^7 + 180*b*x^7*atan(c*x)))/1260 + (c
*d^4*(a*x^4*1260i + 423*b*x^4 + b*x^4*atan(c*x)*1260i))/1260 - (c^3*d^4*(a
*x^6*840i + 30*b*x^6 + b*x^6*atan(c*x)*840i))/1260 - (c^2*d^4*(1512*a*x^5
- b*x^5*168i + 1512*b*x^5*atan(c*x)))/1260
```

3.32. $\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$

3.33 $\int x(d+icdx)^4(a+b\arctan(cx))\,dx$

3.33.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int x(d+icdx)^4(a+b\arctan(cx))\,dx = &-\frac{16bd^4x}{15c}-\frac{4ibd^4(i-cx)^2}{15c^2}-\frac{4bd^4(i-cx)^3}{45c^2}\\&+\frac{ibd^4(i-cx)^4}{30c^2}+\frac{bd^4(i-cx)^5}{30c^2}\\&+\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}\\&-\frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2}\\&+\frac{32ibd^4\log(i+cx)}{15c^2}\end{aligned}$$

output
```
-16/15*b*d^4*x/c-4/15*I*b*d^4*(I-c*x)^2/c^2-4/45*b*d^4*(I-c*x)^3/c^2+1/30*
I*b*d^4*(I-c*x)^4/c^2+1/30*b*d^4*(I-c*x)^5/c^2+1/5*d^4*(1+I*c*x)^5*(a+b*ar
ctan(c*x))/c^2-1/6*d^4*(1+I*c*x)^6*(a+b*arctan(c*x))/c^2+32/15*I*b*d^4*ln(
c*x+I)/c^2
```

3.33.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.48

$$\begin{aligned}\int x(d+icdx)^4(a+b\arctan(cx))\,dx = & -\frac{13bd^4x}{6c}+\frac{1}{2}ad^4x^2-\frac{16}{15}ibd^4x^2+\frac{4}{3}iacd^4x^3 \\ & +\frac{5}{9}bcd^4x^3-\frac{3}{2}ac^2d^4x^4+\frac{1}{5}ibc^2d^4x^4-\frac{4}{5}iac^3d^4x^5 \\ & -\frac{1}{30}bc^3d^4x^5+\frac{1}{6}ac^4d^4x^6+\frac{13bd^4\arctan(cx)}{6c^2} \\ & +\frac{1}{2}bd^4x^2\arctan(cx)+\frac{4}{3}ibcd^4x^3\arctan(cx) \\ & -\frac{3}{2}bc^2d^4x^4\arctan(cx)-\frac{4}{5}ibc^3d^4x^5\arctan(cx) \\ & +\frac{1}{6}bc^4d^4x^6\arctan(cx)+\frac{16ibd^4\log(1+c^2x^2)}{15c^2}\end{aligned}$$

input
```
Integrate[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(-13*b*d^4*x)/(6*c) + (a*d^4*x^2)/2 - ((16*I)/15)*b*d^4*x^2 + ((4*I)/3)*a*
c*d^4*x^3 + (5*b*c*d^4*x^3)/9 - (3*a*c^2*d^4*x^4)/2 + (I/5)*b*c^2*d^4*x^4
- ((4*I)/5)*a*c^3*d^4*x^5 - (b*c^3*d^4*x^5)/30 + (a*c^4*d^4*x^6)/6 + (13*b
*d^4*ArcTan[c*x])/(6*c^2) + (b*d^4*x^2*ArcTan[c*x])/2 + ((4*I)/3)*b*c*d^4*
x^3*ArcTan[c*x] - (3*b*c^2*d^4*x^4*ArcTan[c*x])/2 - ((4*I)/5)*b*c^3*d^4*x^
5*ArcTan[c*x] + (b*c^4*d^4*x^6*ArcTan[c*x])/6 + (((16*I)/15)*b*d^4*Log[1 +
 c^2*x^2])/c^2
```

3.33.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5407$$

$$-bc\int \frac{d^4(i-cx)^4(5cx+i)}{30c^2(cx+i)}dx-\frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2}+\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{bd^4 \int \frac{(i-cx)^4(5cx+i)}{cx+i}dx}{30c} - \frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2} + \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}$$

$$\Big\downarrow \textcolor{blue}{86}$$

$$-\frac{bd^4 \int \left(5(i-cx)^4 - 4i(cx-i)^3 - 8(cx-i)^2 + 16i(cx-i) - \frac{64i}{cx+i} + 32\right)dx}{30c} - \frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2} + \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2} + \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2} - \frac{bd^4\left(-\frac{(-cx+i)^5}{c} - \frac{i(-cx+i)^4}{c} + \frac{8(-cx+i)^3}{3c} + \frac{8i(-cx+i)^2}{c} - \frac{64i\log(cx+i)}{c} + 32x\right)}{30c}$$

input
```
Int[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) - (d^4*(1 + I*c*x)^6*(a +
b*ArcTan[c*x]))/(6*c^2) - (b*d^4*(32*x + ((8*I)*(I - c*x)^2)/c + (8*(I - c
*x)^3)/(3*c) - (I*(I - c*x)^4)/c - (I - c*x)^5/c - ((64*I)*Log[I + c*x])/c
))/(30*c)
```

3.33.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.33.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.97

method	result
parts	$d^4a\left(\frac{1}{6}c^4x^6-\frac{4}{5}ic^3x^5-\frac{3}{2}c^2x^4+\frac{4}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{d^4b\left(\frac{\arctan(cx)c^6x^6}{6}-\frac{4i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(c}{2}\right.}{}$
derivativedivides	$\frac{d^4a\left(\frac{1}{6}c^6x^6-\frac{4}{5}ic^5x^5-\frac{3}{2}c^4x^4+\frac{4}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^4b\left(\frac{\arctan(cx)c^6x^6}{6}-\frac{4i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{2}+\frac{4i\arctan(cx)c^3}{3}\right.}{c^2}$
default	$\frac{d^4a\left(\frac{1}{6}c^6x^6-\frac{4}{5}ic^5x^5-\frac{3}{2}c^4x^4+\frac{4}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^4b\left(\frac{\arctan(cx)c^6x^6}{6}-\frac{4i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{2}+\frac{4i\arctan(cx)c^3}{3}\right.}{c^2}$
parallelrisch	$\frac{15b\,c^6d^4\arctan(cx)x^6+18ix^4b\,c^4d^4+15a\,c^6d^4x^6+96ib\,d^4\ln(c^2x^2+1)-3b\,c^5d^4x^5+120ix^3\arctan(cx)b\,c^3d^4-135x^4\operatorname{arcta}}{}$
risch	$-\frac{id^4b\left(5c^4x^6-24ic^3x^5-45c^2x^4+40ic\,x^3+15x^2\right)\ln(icx+1)}{60}+\frac{a\,c^4d^4x^6}{6}+\frac{ib\,c^2d^4x^4}{5}+\frac{2d^4c^3b\,x^5\ln(-icx+1)}{5}-\frac{b}{}$

input
```
int(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/6*c^4*x^6-4/5*I*c^3*x^5-3/2*c^2*x^4+4/3*I*c*x^3+1/2*x^2)+d^4*b/c^
2*(1/6*arctan(c*x)*c^6*x^6-4/5*I*arctan(c*x)*c^5*x^5-3/2*c^4*x^4*arctan(c*
x)+4/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-13/6*c*x-1/30*c^5*x^5
+1/5*I*c^4*x^4+5/9*c^3*x^3-16/15*I*c^2*x^2+16/15*I*ln(c^2*x^2+1)+13/6*arct
an(c*x))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$=\frac{30\,ac^6d^4x^6-6\,(24i\,a+b)c^5d^4x^5-18\,(15\,a-2i\,b)c^4d^4x^4-20\,(-12i\,a-5\,b)c^3d^4x^3+6\,(15\,a-32i\,b)c^2d^4x^2}{}$$

input
```
integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*d^4*x^6 - 6*(24*I*a + b)*c^5*d^4*x^5 - 18*(15*a - 2*I*b)*c
^4*d^4*x^4 - 20*(-12*I*a - 5*b)*c^3*d^4*x^3 + 6*(15*a - 32*I*b)*c^2*d^4*x^
2 - 390*b*c*d^4*x + 387*I*b*d^4*log((c*x + I)/c) - 3*I*b*d^4*log((c*x - I)
/c) - 3*(-5*I*b*c^6*d^4*x^6 - 24*b*c^5*d^4*x^5 + 45*I*b*c^4*d^4*x^4 + 40*b
*c^3*d^4*x^3 - 15*I*b*c^2*d^4*x^2)*log(-(c*x + I)/(c*x - I)))/c^2
```

3.33.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(156) = 312.

Time = 2.75 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.02

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx=\frac{ac^4d^4x^6}{6}-\frac{13bd^4x}{6c}$$
$$+\frac{bd^4\left(-\frac{i\log\left(709bcd^4x-709ibd^4\right)}{60}+\frac{117i\log\left(709bcd^4x+709ibd^4\right)}{70}\right)}{c^2}+x^5\left(-\frac{4iac^3d^4}{5}-\frac{bc^3d^4}{30}\right)$$
$$+x^4\left(-\frac{3ac^2d^4}{2}+\frac{ibc^2d^4}{5}\right)+x^3\cdot\left(\frac{4iacd^4}{3}+\frac{5bcd^4}{9}\right)+x^2\left(\frac{ad^4}{2}-\frac{16ibd^4}{15}\right)$$
$$+\left(-\frac{ibc^4d^4x^6}{12}-\frac{2bc^3d^4x^5}{5}+\frac{3ibc^2d^4x^4}{4}+\frac{2bcd^4x^3}{3}-\frac{ibd^4x^2}{4}\right)\log\left(icx+1\right)$$
$$+\frac{(35ibc^6d^4x^6+168bc^5d^4x^5-315ibc^4d^4x^4-280bc^3d^4x^3+105ibc^2d^4x^2+201ibd^4)\log\left(-icx+1\right)}{420c^2}$$

input
```
integrate(x*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**6/6 - 13*b*d**4*x/(6*c) + b*d**4*(-I*log(709*b*c*d**4*x - 7
09*I*b*d**4)/60 + 117*I*log(709*b*c*d**4*x + 709*I*b*d**4)/70)/c**2 + x**5
*(-4*I*a*c**3*d**4/5 - b*c**3*d**4/30) + x**4*(-3*a*c**2*d**4/2 + I*b*c**2
*d**4/5) + x**3*(4*I*a*c*d**4/3 + 5*b*c*d**4/9) + x**2*(a*d**4/2 - 16*I*b*
d**4/15) + (-I*b*c**4*d**4*x**6/12 - 2*b*c**3*d**4*x**5/5 + 3*I*b*c**2*d**
4*x**4/4 + 2*b*c*d**4*x**3/3 - I*b*d**4*x**2/4)*log(I*c*x + 1) + (35*I*b*c
**6*d**4*x**6 + 168*b*c**5*d**4*x**5 - 315*I*b*c**4*d**4*x**4 - 280*b*c**3
*d**4*x**3 + 105*I*b*c**2*d**4*x**2 + 201*I*b*d**4)*log(-I*c*x + 1)/(420*c
**2)
```

3.33.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(138) = 276$.

Time = 0.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.63

$$\begin{aligned}
&\int x(d+icdx)^4(a+b\arctan(cx))\,dx \\
&= \frac{1}{6}\,ac^4d^4x^6 - \frac{4}{5}i\,ac^3d^4x^5 - \frac{3}{2}\,ac^2d^4x^4 \\
&\quad + \frac{1}{90}\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)bc^4d^4 \\
&\quad - \frac{1}{5}i\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^3d^4 \\
&\quad + \frac{4}{3}i\,acd^4x^3 - \frac{1}{2}\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bc^2d^4 \\
&\quad + \frac{2}{3}i\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bcd^4 \\
&\quad + \frac{1}{2}\,ad^4x^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^4
\end{aligned}$$

input
```
integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^4*d^4*x^6 - 4/5*I*a*c^3*d^4*x^5 - 3/2*a*c^2*d^4*x^4 + 1/90*(15*x^6
*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)
)*b*c^4*d^4 - 1/5*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(
c^2*x^2 + 1)/c^6))*b*c^3*d^4 + 4/3*I*a*c*d^4*x^3 - 1/2*(3*x^4*arctan(c*x)
- c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^4 + 2/3*I*(2*x^3*ar
ctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 +
1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^4
```

3.33.8 Giac [F]

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^4(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{\frac{d^4\,(195\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,96\mathrm{i})}{90}-\frac{13\,b\,c\,d^4\,x}{6}}{c^2} + \frac{d^4\,(45\,a\,x^2+45\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,96\mathrm{i})}{90} + \frac{c^4\,d^4\,(15\,a\,x^6+15\,b\,x^6\,\mathrm{atan}(c\,x))}{90} + \frac{c\,d^4\,(a\,x^3\,120\mathrm{i}+50\,b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,120\mathrm{i})}{90} - \frac{c^3\,d^4\,(a\,x^5\,72\mathrm{i}+3\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,72\mathrm{i})}{90} - \frac{c^2\,d^4\,(135\,a\,x^4+135\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,18\mathrm{i})}{90}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
((d^4*(195*b*atan(c*x) + b*log(c^2*x^2 + 1)*96i))/90 - (13*b*c*d^4*x)/6)/c
^2 + (d^4*(45*a*x^2 - b*x^2*96i + 45*b*x^2*atan(c*x)))/90 + (c^4*d^4*(15*a
*x^6 + 15*b*x^6*atan(c*x)))/90 + (c*d^4*(a*x^3*120i + 50*b*x^3 + b*x^3*ata
n(c*x)*120i))/90 - (c^3*d^4*(a*x^5*72i + 3*b*x^5 + b*x^5*atan(c*x)*72i))/9
0 - (c^2*d^4*(135*a*x^4 - b*x^4*18i + 135*b*x^4*atan(c*x)))/90
```

3.34 $\int (d+icdx)^4(a+b\arctan(cx))\,dx$

3.34.1 Optimal result

Integrand size = 20, antiderivative size = 125

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx = -\frac{8}{5}ibd^4x - \frac{2bd^4(1+icx)^2}{5c} - \frac{2bd^4(1+icx)^3}{15c} - \frac{bd^4(1+icx)^4}{20c} - \frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c} - \frac{16bd^4\log(1-icx)}{5c}$$

output
```
-8/5*I*b*d^4*x-2/5*b*d^4*(1+I*c*x)^2/c-2/15*b*d^4*(1+I*c*x)^3/c-1/20*b*d^4
*(1+I*c*x)^4/c-1/5*I*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/c-16/5*b*d^4*ln(1-I
*c*x)/c
```

3.34.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx = \frac{d^4(12(-i+cx)^5(a+b\arctan(cx)) - b(35+180icx-66c^2x^2-20ic^3x^3+3c^4x^4+192\log(i+cx)))}{60c}$$

input
```
Integrate[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(d^4*(12*(-I + c*x)^5*(a + b*ArcTan[c*x]) - b*(35 + (180*I)*c*x - 66*c^2*x
^2 - (20*I)*c^3*x^3 + 3*c^4*x^4 + 192*Log[I + c*x])))/(60*c)
```

3.34.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{ib\int \frac{d^5(icx+1)^5}{c^2x^2+1}dx}{5d}-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}ibd^4\int \frac{(icx+1)^5}{c^2x^2+1}dx-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{5}ibd^4\int \frac{(icx+1)^4}{1-icx}dx-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{5}ibd^4\int \left(-(icx+1)^3-2(icx+1)^2-4(icx+1)+\frac{16}{1-icx}-8\right)dx-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}ibd^4\left(\frac{i(1+icx)^4}{4c}+\frac{2i(1+icx)^3}{3c}+\frac{2i(1+icx)^2}{c}+\frac{16i\log(cx+i)}{c}-8x\right)-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

input
```
Int[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/5*I)*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/c + (I/5)*b*d^4*(-8*x + (
(2*I)*(1 + I*c*x)^2)/c + (((2*I)/3)*(1 + I*c*x)^3)/c + ((I/4)*(1 + I*c*x)^
4)/c + ((16*I)*Log[I + c*x])/c)
```

3.34.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.34.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{-\frac{id^4a(icx+1)^5}{5}+d^4b\left(\frac{c^5x^5\arctan(cx)}{5}-i\arctan(cx)c^4x^4-2c^3x^3\arctan(cx)+2i\arctan(cx)c^2x^2+cx\arctan(cx)-\right.}{c}$
default	$\frac{-\frac{id^4a(icx+1)^5}{5}+d^4b\left(\frac{c^5x^5\arctan(cx)}{5}-i\arctan(cx)c^4x^4-2c^3x^3\arctan(cx)+2i\arctan(cx)c^2x^2+cx\arctan(cx)-\right.}{c}$
parts	$-\frac{id^4a(icx+1)^5}{5c}+\frac{d^4b\left(\frac{c^5x^5\arctan(cx)}{5}-i\arctan(cx)c^4x^4-2c^3x^3\arctan(cx)+2i\arctan(cx)c^2x^2+cx\arctan(cx)-\right.}{c}$
parallelrisch	$-\frac{-12c^5b\,d^4\arctan(cx)x^5+180ib\,d^4xc-12a\,c^5d^4x^5+60ix^4\arctan(cx)b\,c^4d^4+3b\,c^4d^4x^4-180ib\,d^4\arctan(cx)+120x^3}{}$
risch	$-id^4c^2b\,x^3\ln\left(-icx+1\right)-id^4a\,c^3x^4+\frac{d^4a\,c^4x^5}{5}+2iac\,d^4x^2+\frac{d^4c^3x^4b\ln(-icx+1)}{2}+$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/5*I*d^4*a*(1+I*c*x)^5+d^4*b*(1/5*c^5*x^5*arctan(c*x)-I*arctan(c*x)
*c^4*x^4-2*c^3*x^3*arctan(c*x)+2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)-1/5
*I*arctan(c*x)+1/5*I*(-15*c*x+1/4*I*c^4*x^4+5/3*c^3*x^3-11/2*I*c^2*x^2+8*I
*ln(c^2*x^2+1)+16*arctan(c*x))))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.50

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{12\,ac^5d^4x^5 - 3\,(20i\,a+b)c^4d^4x^4 - 20\,(6\,a-i\,b)c^3d^4x^3 - 6\,(-20i\,a-11\,b)c^2d^4x^2 + 60\,(a-3i\,b)cd^4x - 18}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/60*(12*a*c^5*d^4*x^5 - 3*(20*I*a + b)*c^4*d^4*x^4 - 20*(6*a - I*b)*c^3*d
^4*x^3 - 6*(-20*I*a - 11*b)*c^2*d^4*x^2 + 60*(a - 3*I*b)*c*d^4*x - 186*b*d
^4*log((c*x + I)/c) - 6*b*d^4*log((c*x - I)/c) - 6*(-I*b*c^5*d^4*x^5 - 5*b
*c^4*d^4*x^4 + 10*I*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 - 5*I*b*c*d^4*x)*log(
-(c*x + I)/(c*x - I)))/c
```

3.34.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(110) = 220$.

Time = 2.21 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.53

$$\int (d + icdx)^4 (a + b \arctan(cx))\, dx$$
$$= \frac{ac^4d^4x^5}{5} + \frac{bd^4\left(-\frac{\log(41bcd^4x - 41ibd^4)}{10} - \frac{43\log(41bcd^4x + 41ibd^4)}{20}\right)}{c} + x^4\left(-iac^3d^4 - \frac{bc^3d^4}{20}\right)$$
$$+ x^3\left(-2ac^2d^4 + \frac{ibc^2d^4}{3}\right) + x^2 \cdot \left(2iacd^4 + \frac{11bcd^4}{10}\right) + x(ad^4 - 3ibd^4)$$
$$+ \left(-\frac{ibc^4d^4x^5}{10} - \frac{bc^3d^4x^4}{2} + ibc^2d^4x^3 + bcd^4x^2 - \frac{ibd^4x}{2}\right)\log(icx + 1)$$
$$+ \frac{(2ibc^5d^4x^5 + 10bc^4d^4x^4 - 20ibc^3d^4x^3 - 20bc^2d^4x^2 + 10ibcd^4x - 19bd^4)\log(-icx + 1)}{20c}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**5/5 + b*d**4*(-log(41*b*c*d**4*x - 41*I*b*d**4)/10 - 43*log
(41*b*c*d**4*x + 41*I*b*d**4)/20)/c + x**4*(-I*a*c**3*d**4 - b*c**3*d**4/2
0) + x**3*(-2*a*c**2*d**4 + I*b*c**2*d**4/3) + x**2*(2*I*a*c*d**4 + 11*b*c
*d**4/10) + x*(a*d**4 - 3*I*b*d**4) + (-I*b*c**4*d**4*x**5/10 - b*c**3*d**
4*x**4/2 + I*b*c**2*d**4*x**3 + b*c*d**4*x**2 - I*b*d**4*x/2)*log(I*c*x +
1) + (2*I*b*c**5*d**4*x**5 + 10*b*c**4*d**4*x**4 - 20*I*b*c**3*d**4*x**3 -
 20*b*c**2*d**4*x**2 + 10*I*b*c*d**4*x - 19*b*d**4)*log(-I*c*x + 1)/(20*c)
```

3.34.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.11

$$
\begin{aligned}
&\int (d+icdx)^4(a+b\arctan(cx))\,dx \\
&= \frac{1}{5}ac^4d^4x^5 - i\,ac^3d^4x^4 + \frac{1}{20}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^4d^4 \\
&\quad - 2\,ac^2d^4x^3 - \frac{1}{3}i\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bc^3d^4 \\
&\quad - \left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bc^2d^4 \\
&\quad + 2i\,acd^4x^2 + 2i\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bcd^4 \\
&\quad + ad^4x + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))bd^4}{2\,c}
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^4*x^5 - I*a*c^3*d^4*x^4 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^
4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^4*d^4 - 2*a*c^2*d^4*x^3 - 1/
3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^
3*d^4 - (2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^4
 + 2*I*a*c*d^4*x^2 + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
*c*d^4 + a*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^4/c
```

3.34.8 Giac [F]

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx = \int (i\,cdx+d)^4(b\arctan(cx)+a)\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx = \frac{d^4\,(60\,a\,x+60\,b\,x\,\mathrm{atan}(c\,x)-b\,x\,180\mathrm{i})}{60} + \frac{c^4\,d^4\,(12\,a\,x^5+12\,b\,x^5\,\mathrm{atan}(c\,x))}{60} + \frac{d^4\,(-96\,b\,\ln(c^2\,x^2+1)+b\,\mathrm{atan}(c\,x)\,180\mathrm{i})}{60\,c} + \frac{c\,d^4\,(a\,x^2\,120\mathrm{i}+66\,b\,x^2+b\,x^2\,\mathrm{atan}(c\,x)\,120\mathrm{i})}{60} - \frac{c^3\,d^4\,(a\,x^4\,60\mathrm{i}+3\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\,60\mathrm{i})}{60} - \frac{c^2\,d^4\,(120\,a\,x^3+120\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,20\mathrm{i})}{60}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
(d^4*(60*a*x - b*x*180i + 60*b*x*atan(c*x)))/60 + (c^4*d^4*(12*a*x^5 + 12*
b*x^5*atan(c*x)))/60 + (d^4*(b*atan(c*x)*180i - 96*b*log(c^2*x^2 + 1)))/(6
0*c) + (c*d^4*(a*x^2*120i + 66*b*x^2 + b*x^2*atan(c*x)*120i))/60 - (c^3*d^
4*(a*x^4*60i + 3*b*x^4 + b*x^4*atan(c*x)*60i))/60 - (c^2*d^4*(120*a*x^3 -
b*x^3*20i + 120*b*x^3*atan(c*x)))/60
```

3.35 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$

3.35.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx &= 4iacd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}ibc^2d^4x^2\\&\quad-\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\arctan(cx)\\&\quad+4ibcd^4x\arctan(cx)-3c^2d^4x^2(a+b\arctan(cx))\\&\quad-\frac{4}{3}ic^3d^4x^3(a+b\arctan(cx))\\&\quad+\frac{1}{4}c^4d^4x^4(a+b\arctan(cx))\\&\quad+ad^4\log(x)-\frac{8}{3}ibd^4\log\left(1+c^2x^2\right)\\&\quad+\frac{1}{2}ibd^4\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
4*I*a*c*d^4*x+13/4*b*c*d^4*x+2/3*I*b*c^2*d^4*x^2-1/12*b*c^3*d^4*x^3-13/4*b
*d^4*arctan(c*x)+4*I*b*c*d^4*x*arctan(c*x)-3*c^2*d^4*x^2*(a+b*arctan(c*x))
-4/3*I*c^3*d^4*x^3*(a+b*arctan(c*x))+1/4*c^4*d^4*x^4*(a+b*arctan(c*x))+a*d
^4*ln(x)-8/3*I*b*d^4*ln(c^2*x^2+1)+1/2*I*b*d^4*polylog(2,-I*c*x)-1/2*I*b*d
^4*polylog(2,I*c*x)
```

3.35.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \frac{1}{12}d^4\big(48iacx+39bcx-36ac^2x^2+8ibc^2x^2-16iac^3x^3 - bc^3x^3+3ac^4x^4-39b\arctan(cx)+48ibcx\arctan(cx) - 36bc^2x^2\arctan(cx)-16ibc^3x^3\arctan(cx) + 3bc^4x^4\arctan(cx)+12a\log(x)-32ib\log\left(1+c^2x^2\right) + 6ib\,\mathrm{PolyLog}(2,-icx)-6ib\,\mathrm{PolyLog}(2,icx)\big)$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(d^4*((48*I)*a*c*x + 39*b*c*x - 36*a*c^2*x^2 + (8*I)*b*c^2*x^2 - (16*I)*a*
c^3*x^3 - b*c^3*x^3 + 3*a*c^4*x^4 - 39*b*ArcTan[c*x] + (48*I)*b*c*x*ArcTan
[c*x] - 36*b*c^2*x^2*ArcTan[c*x] - (16*I)*b*c^3*x^3*ArcTan[c*x] + 3*b*c^4*
x^4*ArcTan[c*x] + 12*a*Log[x] - (32*I)*b*Log[1 + c^2*x^2] + (6*I)*b*PolyLo
g[2, (-I)*c*x] - (6*I)*b*PolyLog[2, I*c*x]))/12
```

3.35.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \Big(c^4d^4x^3(a+b\arctan(cx))-4ic^3d^4x^2(a+b\arctan(cx))-6c^2d^4x(a+b\arctan(cx))+4icd^4(a+b\arctan(cx))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}c^4d^4x^4(a+b\arctan(cx))-\frac{4}{3}ic^3d^4x^3(a+b\arctan(cx))-3c^2d^4x^2(a+b\arctan(cx))+4iacd^4x+ ad^4\log(x)-\frac{13}{4}bd^4\arctan(cx)+4ibcd^4x\arctan(cx)-\frac{1}{12}bc^3d^4x^3+\frac{2}{3}ibc^2d^4x^2- \frac{8}{3}ibd^4\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^4\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^4\operatorname{PolyLog}(2,icx)+\frac{13}{4}bcd^4x$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(4*I)*a*c*d^4*x + (13*b*c*d^4*x)/4 + ((2*I)/3)*b*c^2*d^4*x^2 - (b*c^3*d^4*
x^3)/12 - (13*b*d^4*ArcTan[c*x])/4 + (4*I)*b*c*d^4*x*ArcTan[c*x] - 3*c^2*d
^4*x^2*(a + b*ArcTan[c*x]) - ((4*I)/3)*c^3*d^4*x^3*(a + b*ArcTan[c*x]) + (
c^4*d^4*x^4*(a + b*ArcTan[c*x]))/4 + a*d^4*Log[x] - ((8*I)/3)*b*d^4*Log[1
+ c^2*x^2] + (I/2)*b*d^4*PolyLog[2, (-I)*c*x] - (I/2)*b*d^4*PolyLog[2, I*c
*x]
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.35.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

method	result
parts	$d^4a\left(\frac{c^4x^4}{4}-\frac{4ic^3x^3}{3}-3c^2x^2+4icx+\ln(x)\right)+d^4b\left(4i\arctan(cx)cx+\frac{c^4x^4\arctan(cx)}{4}-\frac{4i}{}\right.$
derivativedivides	$d^4a\left(4icx+\frac{c^4x^4}{4}-\frac{4ic^3x^3}{3}-3c^2x^2+\ln(cx)\right)+d^4b\left(4i\arctan(cx)cx+\frac{c^4x^4\arctan(cx)}{4}-\right.$
default	$d^4a\left(4icx+\frac{c^4x^4}{4}-\frac{4ic^3x^3}{3}-3c^2x^2+\ln(cx)\right)+d^4b\left(4i\arctan(cx)cx+\frac{c^4x^4\arctan(cx)}{4}-\right.$
risch	$-3x^2d^4c^2a+\frac{13bc\,d^4x}{4}-\frac{103d^4a}{12}-\frac{b\,c^3d^4x^3}{12}-\frac{ib\,d^4\ln(icx+1)c^4x^4}{8}+\frac{ib\,d^4\operatorname{dilog}(icx+1)}{2}+\frac{a\,c^4d^4x^4}{4}-$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/4*c^4*x^4-4/3*I*c^3*x^3-3*c^2*x^2+4*I*c*x+ln(x))+d^4*b*(4*I*arcta
n(c*x)*c*x+1/4*c^4*x^4*arctan(c*x)-4/3*I*arctan(c*x)*c^3*x^3-3*c^2*x^2*arc
tan(c*x)+arctan(c*x)*ln(c*x)+1/2*I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-
I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x)+13/4*c*x-1/12*c^3*x^3+2/3
*I*c^2*x^2-8/3*I*ln(c^2*x^2+1)-13/4*arctan(c*x))
```

3.35.5 Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan{(cx)}+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x, x)
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x,x)
```

output
```
Timed out
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.08

$$
\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx \\
&= \frac{1}{4}\,ac^4d^4x^4 - \frac{4}{3}i\,ac^3d^4x^3 - \frac{1}{12}\,bc^3d^4x^3 - 3\,ac^2d^4x^2 + \frac{2}{3}i\,bc^2d^4x^2 + 4i\,acd^4x \\
&\quad + \frac{13}{4}\,bcd^4x - \frac{1}{12}\,(3\,\pi + 8i)bd^4\log\left(c^2x^2+1\right) + bd^4\arctan\left(cx\right)\log\left(cx\right) \\
&\quad + 2i\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bd^4 - \frac{1}{2}i\,bd^4\mathrm{Li}_2(i\,cx+1) + \frac{1}{2}i\,bd^4\mathrm{Li}_2(-i\,cx+1) \\
&\quad + ad^4\log\left(x\right) + \frac{1}{12}\left(3\,bc^4d^4x^4 - 16i\,bc^3d^4x^3 - 36\,bc^2d^4x^2 - 39\,bd^4\right)\arctan\left(cx\right)
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*c^4*d^4*x^4 - 4/3*I*a*c^3*d^4*x^3 - 1/12*b*c^3*d^4*x^3 - 3*a*c^2*d^4
*x^2 + 2/3*I*b*c^2*d^4*x^2 + 4*I*a*c*d^4*x + 13/4*b*c*d^4*x - 1/12*(3*pi +
 8*I)*b*d^4*log(c^2*x^2 + 1) + b*d^4*arctan(c*x)*log(c*x) + 2*I*(2*c*x*arc
tan(c*x) - log(c^2*x^2 + 1))*b*d^4 - 1/2*I*b*d^4*dilog(I*c*x + 1) + 1/2*I*
b*d^4*dilog(-I*c*x + 1) + a*d^4*log(x) + 1/12*(3*b*c^4*d^4*x^4 - 16*I*b*c^
3*d^4*x^3 - 36*b*c^2*d^4*x^2 - 39*b*d^4)*arctan(c*x)
```

3.35.8 Giac [F]

$$
\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \int \frac{\left(i\,cdx+d\right)^4\left(b\arctan\left(cx\right)+a\right)}{x}\,dx
$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.35.9 Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$$

$$= \begin{cases} \\ a\,d^4\,\ln(x) - b\,d^4\,\ln(c^2\,x^2+1)\;2\mathrm{i} - \frac{b\,d^4\,(3\,\mathrm{atan}(c\,x)-3\,c\,x+c^3\,x^3)}{12} - \frac{b\,d^4\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} + \frac{b\,d^4\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} - 3\,a\,c^2 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x,x)
```

output
```
piecewise(c == 0, a*d^4*log(x), c ~= 0, - (b*d^4*(3*atan(c*x) - 3*c*x + c^
3*x^3))/12 - b*d^4*log(c^2*x^2 + 1)*2i + a*d^4*log(x) - (b*d^4*dilog(- c*x
*1i + 1)*1i)/2 + (b*d^4*dilog(c*x*1i + 1)*1i)/2 - 3*a*c^2*d^4*x^2 - (a*c^3
*d^4*x^3*4i)/3 + (a*c^4*d^4*x^4)/4 + a*c*d^4*x*4i + 3*b*c*d^4*x + (b*c^2*d
^4*(x^2/2 - log(c^2*x^2 + 1)/(2*c^2))*4i)/3 - 6*b*c^2*d^4*atan(c*x)*(1/(2*
c^2) + x^2/2) - (b*c^3*d^4*x^3*atan(c*x)*4i)/3 + (b*c^4*d^4*x^4*atan(c*x))
/4 + b*c*d^4*x*atan(c*x)*4i)
```

3.36 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$

3.36.1 Optimal result

Integrand size = 23, antiderivative size = 190

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx &= -6ac^2d^4x+2ibc^2d^4x-\frac{1}{6}bc^3d^4x^2 \\ &\quad -2ibcd^4\arctan(cx)-6bc^2d^4x\arctan(cx) \\ &\quad -\frac{d^4(a+b\arctan(cx))}{x}-2ic^3d^4x^2(a+b\arctan(cx)) \\ &\quad +\frac{1}{3}c^4d^4x^3(a+b\arctan(cx))+4iacd^4\log(x) \\ &\quad +bcd^4\log(x)+\frac{8}{3}bcd^4\log\left(1+c^2x^2\right) \\ &\quad -2bcd^4\operatorname{PolyLog}(2,-icx)+2bcd^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-6*a*c^2*d^4*x+2*I*b*c^2*d^4*x-1/6*b*c^3*d^4*x^2-2*I*b*c*d^4*arctan(c*x)-6
*b*c^2*d^4*x*arctan(c*x)-d^4*(a+b*arctan(c*x))/x-2*I*c^3*d^4*x^2*(a+b*arct
an(c*x))+1/3*c^4*d^4*x^3*(a+b*arctan(c*x))+4*I*a*c*d^4*ln(x)+b*c*d^4*ln(x)
+8/3*b*c*d^4*ln(c^2*x^2+1)-2*b*c*d^4*polylog(2,-I*c*x)+2*b*c*d^4*polylog(2
,I*c*x)
```

3.36.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{d^4(-6a-36ac^2x^2+12ibc^2x^2-12iac^3x^3-bc^3x^3+2ac^4x^4-6b\arctan(cx)-12ibcx\arctan(cx)-36bc^2x}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
(d^4*(-6*a - 36*a*c^2*x^2 + (12*I)*b*c^2*x^2 - (12*I)*a*c^3*x^3 - b*c^3*x^
3 + 2*a*c^4*x^4 - 6*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] - 36*b*c^2*x^
2*ArcTan[c*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 2*b*c^4*x^4*ArcTan[c*x] + (
24*I)*a*c*x*Log[x] + 6*b*c*x*Log[c*x] + 16*b*c*x*Log[1 + c^2*x^2] - 12*b*c
*x*PolyLog[2, (-I)*c*x] + 12*b*c*x*PolyLog[2, I*c*x]))/(6*x)
```

3.36.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(c^4d^4x^2(a+b\arctan(cx))-4ic^3d^4x(a+b\arctan(cx))-6c^2d^4(a+b\arctan(cx))+\frac{d^4(a+b\arctan(cx))}{x^2}+4ic\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}c^4d^4x^3(a+b\arctan(cx))-2ic^3d^4x^2(a+b\arctan(cx))-\frac{d^4(a+b\arctan(cx))}{x}-6ac^2d^4x+4iacd^4\log(x)-6bc^2d^4x\arctan(cx)-2ibcd^4\arctan(cx)-\frac{1}{6}bc^3d^4x^2+\frac{8}{3}bcd^4\log\left(c^2x^2+1\right)+2ibc^2d^4x-2bcd^4\operatorname{PolyLog}(2,-icx)+2bcd^4\operatorname{PolyLog}(2,icx)+bcd^4\log(x)$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-6*a*c^2*d^4*x + (2*I)*b*c^2*d^4*x - (b*c^3*d^4*x^2)/6 - (2*I)*b*c*d^4*Arc
Tan[c*x] - 6*b*c^2*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/x - (2*I)
*c^3*d^4*x^2*(a + b*ArcTan[c*x]) + (c^4*d^4*x^3*(a + b*ArcTan[c*x]))/3 + (
4*I)*a*c*d^4*Log[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 + c^2*x^2])/3 - 2*
b*c*d^4*PolyLog[2, (-I)*c*x] + 2*b*c*d^4*PolyLog[2, I*c*x]
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.36.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

method	result
parts	$d^4 a\left(\frac{c^4 x^3}{3}-2ic^3x^2-6c^2x+4ic\ln\left(x\right)-\frac{1}{x}\right)+d^4 bc\left(-6cx\arctan\left(cx\right)+\frac{c^3x^3\arctan(cx)}{3}-\right.$
derivativedivides	$c\left(d^4 a\left(-6cx+\frac{c^3x^3}{3}-2ic^2x^2-\frac{1}{cx}+4i\ln\left(cx\right)\right)+d^4 b\left(-6cx\arctan\left(cx\right)+\frac{c^3x^3\arctan(cx)}{3}\right.\right.$
default	$c\left(d^4 a\left(-6cx+\frac{c^3x^3}{3}-2ic^2x^2-\frac{1}{cx}+4i\ln\left(cx\right)\right)+d^4 b\left(-6cx\arctan\left(cx\right)+\frac{c^3x^3\arctan(cx)}{3}\right.\right.$
risch	$-\frac{119bc\,d^4}{18}-6a\,c^2d^4x-\frac{b\,c^3d^4x^2}{6}-\frac{id^4b\ln(-icx+1)}{2x}+4id^4ca\ln\left(-icx\right)+\frac{ib\,d^4\ln(icx+1)}{2x}-$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/3*c^4*x^3-2*I*c^3*x^2-6*c^2*x+4*I*c*ln(x)-1/x)+d^4*b*c*(-6*c*x*ar
ctan(c*x)+1/3*c^3*x^3*arctan(c*x)-2*I*arctan(c*x)*c^2*x^2-1/c/x*arctan(c*x
)+4*I*arctan(c*x)*ln(c*x)-2*ln(c*x)*ln(1+I*c*x)+2*ln(c*x)*ln(1-I*c*x)-2*di
log(1+I*c*x)+2*dilog(1-I*c*x)+2*I*c*x-1/6*c^2*x^2+ln(c*x)+8/3*ln(c^2*x^2+1
)-2*I*arctan(c*x))
```

3.36.5 Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

3.36.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**2,x)
```

output
```
Timed out
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.26

$$\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx \\
&= \frac{1}{3}\,ac^4d^4x^3 - 2i\,ac^3d^4x^2 - \frac{1}{6}\,bc^3d^4x^2 - 6\,ac^2d^4x + 2i\,bc^2d^4x \\
&\quad - \frac{1}{6}\,(6i\,\pi-1)bcd^4\log\left(c^2x^2+1\right) + 4i\,bcd^4\arctan(cx)\log(cx) \\
&\quad - 3\left(2\,cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bcd^4 + 2\,bcd^4\mathrm{Li}_2(i\,cx+1) - 2\,bcd^4\mathrm{Li}_2(-i\,cx+1) \\
&\quad + 4i\,acd^4\log(x) - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd^4 \\
&\quad - \frac{ad^4}{x} + \frac{1}{3}\left(bc^4d^4x^3 - 6i\,bc^3d^4x^2 - 6i\,bcd^4\right)\arctan(cx)
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*c^4*d^4*x^3 - 2*I*a*c^3*d^4*x^2 - 1/6*b*c^3*d^4*x^2 - 6*a*c^2*d^4*x
+ 2*I*b*c^2*d^4*x - 1/6*(6*I*pi - 1)*b*c*d^4*log(c^2*x^2 + 1) + 4*I*b*c*d^
4*arctan(c*x)*log(c*x) - 3*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^4
+ 2*b*c*d^4*dilog(I*c*x + 1) - 2*b*c*d^4*dilog(-I*c*x + 1) + 4*I*a*c*d^4*l
og(x) - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^4 - a*
d^4/x + 1/3*(b*c^4*d^4*x^3 - 6*I*b*c^3*d^4*x^2 - 6*I*b*c*d^4)*arctan(c*x)
```

3.36.8 Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.33

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$$
$$= \left\{ \frac{a\,c^4\,d^4\,x^3}{3} - \frac{a\,d^4}{x} + \frac{b\,d^4\left(c^2\,\ln(x) - \frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c} + 2\,b\,c\,d^4\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i}) - \mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right) + 3\,b\,c\,d^4\,\ln\left(c^2\,x^2\right.\right.$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^2,x)
```

output
```
piecewise(c == 0, -(a*d^4)/x, c ~= 0, - (a*d^4)/x - a*c^3*d^4*x^2*2i + (a*
c^4*d^4*x^3)/3 + (b*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2))/c + 2*b*c
*d^4*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) + 3*b*c*d^4*log(c^2*x^2 + 1
) - 6*a*c^2*d^4*x + b*c^2*d^4*x*2i - (b*c^3*d^4*(x^2/2 - log(c^2*x^2 + 1)/
(2*c^2)))/3 + a*c*d^4*log(x)*4i - (b*d^4*atan(c*x))/x - 6*b*c^2*d^4*x*atan
(c*x) - b*c^3*d^4*atan(c*x)*(1/(2*c^2) + x^2/2)*4i + (b*c^4*d^4*x^3*atan(c
*x))/3)
```

3.36. $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$

3.37 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$

3.37.1 Optimal result

Integrand size = 23, antiderivative size = 173

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx = &-\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\arctan(cx)\\ &-\frac{d^4(a+b\arctan(cx))}{2x^2}-\frac{4icd^4(a+b\arctan(cx))}{x}\\ &+\frac{1}{2}c^4d^4x^2(a+b\arctan(cx))-6ac^2d^4\log(x)\\ &+4ibc^2d^4\log(x)-3ibc^2d^4\operatorname{PolyLog}(2,-icx)\\ &+3ibc^2d^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/2*b*c*d^4/x-4*I*a*c^3*d^4*x-1/2*b*c^3*d^4*x-4*I*b*c^3*d^4*x*arctan(c*x)
-1/2*d^4*(a+b*arctan(c*x))/x^2-4*I*c*d^4*(a+b*arctan(c*x))/x+1/2*c^4*d^4*x
^2*(a+b*arctan(c*x))-6*a*c^2*d^4*ln(x)+4*I*b*c^2*d^4*ln(x)-3*I*b*c^2*d^4*p
olylog(2,-I*c*x)+3*I*b*c^2*d^4*polylog(2,I*c*x)
```

3.37.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{d^4(-a-8iacx-bcx-8iac^3x^3-bc^3x^3+ac^4x^4-b\arctan(cx)-8ibcx\arctan(cx)-8ibc^3x^3\arctan(cx)\ \cdots}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
(d^4*(-a - (8*I)*a*c*x - b*c*x - (8*I)*a*c^3*x^3 - b*c^3*x^3 + a*c^4*x^4 -
 b*ArcTan[c*x] - (8*I)*b*c*x*ArcTan[c*x] - (8*I)*b*c^3*x^3*ArcTan[c*x] + b
*c^4*x^4*ArcTan[c*x] - 12*a*c^2*x^2*Log[x] + (8*I)*b*c^2*x^2*Log[c*x] - (6
*I)*b*c^2*x^2*PolyLog[2, (-I)*c*x] + (6*I)*b*c^2*x^2*PolyLog[2, I*c*x]))/(
2*x^2)
```

3.37.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(c^4d^4x(a+b\arctan(cx)) - 4ic^3d^4(a+b\arctan(cx)) - \frac{6c^2d^4(a+b\arctan(cx))}{x} + \frac{d^4(a+b\arctan(cx))}{x^3} + \frac{4icd$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c^4d^4x^2(a+b\arctan(cx)) - \frac{d^4(a+b\arctan(cx))}{2x^2} - \frac{4icd^4(a+b\arctan(cx))}{x} - 4iac^3d^4x - 6ac^2d^4\log(x) - 4ibc^3d^4x\arctan(cx) - \frac{1}{2}bc^3d^4x - 3ibc^2d^4\operatorname{PolyLog}(2,-icx) + 3ibc^2d^4\operatorname{PolyLog}(2,icx) + 4ibc^2d^4\log(x) - \frac{bcd^4}{2x}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^4)/x - (4*I)*a*c^3*d^4*x - (b*c^3*d^4*x)/2 - (4*I)*b*c^3*d^4*x
*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(2*x^2) - ((4*I)*c*d^4*(a + b*Arc
Tan[c*x]))/x + (c^4*d^4*x^2*(a + b*ArcTan[c*x]))/2 - 6*a*c^2*d^4*Log[x] +
(4*I)*b*c^2*d^4*Log[x] - (3*I)*b*c^2*d^4*PolyLog[2, (-I)*c*x] + (3*I)*b*c^
2*d^4*PolyLog[2, I*c*x]
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.37.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03

method	result
parts	$d^4a\left(\frac{c^4x^2}{2}-4ic^3x-\frac{1}{2x^2}-6c^2\ln\left(x\right)-\frac{4ic}{x}\right)+d^4b\,c^2\Big(-4i\arctan\left(cx\right)cx+\frac{c^2x^2\arctan(cx)}{2}$
derivativedivides	$c^2\left(d^4a\left(-4icx+\frac{c^2x^2}{2}-6\ln\left(cx\right)-\frac{4i}{cx}-\frac{1}{2c^2x^2}\right)+d^4b\Big(-4i\arctan\left(cx\right)cx+\frac{c^2x^2\arctan(cx)}{2}\right.$
default	$c^2\left(d^4a\left(-4icx+\frac{c^2x^2}{2}-6\ln\left(cx\right)-\frac{4i}{cx}-\frac{1}{2c^2x^2}\right)+d^4b\Big(-4i\arctan\left(cx\right)cx+\frac{c^2x^2\arctan(cx)}{2}\right.$
risch	$\frac{9a\,c^2d^4}{2}-\frac{bc\,d^4}{2x}-\frac{b\,c^3d^4x}{2}-4ia\,c^3d^4x+\frac{7ib\,d^4c^2\ln(icx)}{4}+3ic^2d^4b\,\mathrm{dilog}\left(-icx+1\right)-\frac{id^4b\ln(-icx}{4x^2}$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/2*c^4*x^2-4*I*c^3*x-1/2/x^2-6*c^2*ln(x)-4*I*c/x)+d^4*b*c^2*(-4*I*
arctan(c*x)*c*x+1/2*c^2*x^2*arctan(c*x)-6*arctan(c*x)*ln(c*x)-4*I*arctan(c
*x)/c/x-1/2/c^2/x^2*arctan(c*x)-1/2*c*x+4*I*ln(c*x)-1/2/c/x-3*I*ln(c*x)*ln
(1+I*c*x)+3*I*ln(c*x)*ln(1-I*c*x)-3*I*dilog(1+I*c*x)+3*I*dilog(1-I*c*x))
```

3.37.5 Fricas [F]

$$\int\frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx=\int\frac{(i\,cdx+d)^4(b\arctan\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

3.37.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**3,x)
```

output
```
Timed out
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.45

$$\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx \\
&= \frac{1}{2}\,ac^4d^4x^2 - 4i\,ac^3d^4x - \frac{1}{2}\,bc^3d^4x + \frac{3}{2}\,\pi bc^2d^4\log\left(c^2x^2+1\right) \\
&\quad - 6\,bc^2d^4\arctan\left(cx\right)\log\left(cx\right) - 2i\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bc^2d^4 \\
&\quad + 3i\,bc^2d^4\mathrm{Li}_2(i\,cx+1) - 3i\,bc^2d^4\mathrm{Li}_2(-i\,cx+1) - 6\,ac^2d^4\log\left(x\right) \\
&\quad - 2i\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\,\arctan\left(cx\right)}{x}\right)bcd^4 \\
&\quad - \frac{1}{2}\left(\left(c\arctan\left(cx\right) + \frac{1}{x}\right)c + \frac{\arctan\left(cx\right)}{x^2}\right)bd^4 \\
&\quad - \frac{4i\,acd^4}{x} - \frac{ad^4}{2\,x^2} + \frac{1}{2}\left(bc^4d^4x^2 + bc^2d^4\right)\arctan\left(cx\right)
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*c^4*d^4*x^2 - 4*I*a*c^3*d^4*x - 1/2*b*c^3*d^4*x + 3/2*pi*b*c^2*d^4*l
og(c^2*x^2 + 1) - 6*b*c^2*d^4*arctan(c*x)*log(c*x) - 2*I*(2*c*x*arctan(c*x
) - log(c^2*x^2 + 1))*b*c^2*d^4 + 3*I*b*c^2*d^4*dilog(I*c*x + 1) - 3*I*b*c
^2*d^4*dilog(-I*c*x + 1) - 6*a*c^2*d^4*log(x) - 2*I*(c*(log(c^2*x^2 + 1) -
 log(x^2)) + 2*arctan(c*x)/x)*b*c*d^4 - 1/2*((c*arctan(c*x) + 1/x)*c + arc
tan(c*x)/x^2)*b*d^4 - 4*I*a*c*d^4/x - 1/2*a*d^4/x^2 + 1/2*(b*c^4*d^4*x^2 +
 b*c^2*d^4)*arctan(c*x)
```

3.37.8 Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.37.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.49

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$$
$$= \begin{cases} \frac{a\,c^4\,d^4\,x^2}{2} - \frac{\frac{a\,d^4}{2}+a\,c\,d^4\,x\,4\mathrm{i}}{x^2} - 6\,a\,c^2\,d^4\,\ln(x) - \frac{b\,d^4\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - \frac{b\,c^3\,d^4\,x}{2} - \frac{b\,d^4\,\mathrm{atan}(c\,x)}{2\,x^2} + b\,c^4\,d^4\,\mathrm{atan}(c\,x)\,\Big(\end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^3,x)
```

output
```
piecewise(c == 0, -(a*d^4)/(2*x^2), c ~= 0, - ((a*d^4)/2 + a*c*d^4*x*4i)/x
^2 + b*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2)*4i + b*c^2*d^4*log(c^2*
x^2 + 1)*2i + (a*c^4*d^4*x^2)/2 - 6*a*c^2*d^4*log(x) + b*c^2*d^4*dilog(- c
*x*1i + 1)*3i - b*c^2*d^4*dilog(c*x*1i + 1)*3i - (b*d^4*(c^3*atan(c*x) + c
^2/x))/(2*c) - a*c^3*d^4*x*4i - (b*c^3*d^4*x)/2 - (b*d^4*atan(c*x))/(2*x^2
) - (b*c*d^4*atan(c*x)*4i)/x - b*c^3*d^4*x*atan(c*x)*4i + b*c^4*d^4*atan(c
*x)*(1/(2*c^2) + x^2/2))
```

3.38 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$

3.38.1 Optimal result

Integrand size = 23, antiderivative size = 201

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = &-\frac{bcd^4}{6x^2}-\frac{2ibc^2d^4}{x}+ac^4d^4x\\ &-2ibc^3d^4\arctan(cx)+bc^4d^4x\arctan(cx)\\ &-\frac{d^4(a+b\arctan(cx))}{3x^3}-\frac{2icd^4(a+b\arctan(cx))}{x^2}\\ &+\frac{6c^2d^4(a+b\arctan(cx))}{x}-4iac^3d^4\log(x)\\ &-\frac{19}{3}bc^3d^4\log(x)+\frac{8}{3}bc^3d^4\log\left(1+c^2x^2\right)\\ &+2bc^3d^4\operatorname{PolyLog}(2,-icx)-2bc^3d^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/6*b*c*d^4/x^2-2*I*b*c^2*d^4/x+a*c^4*d^4*x-2*I*b*c^3*d^4*arctan(c*x)+b*c
^4*d^4*x*arctan(c*x)-1/3*d^4*(a+b*arctan(c*x))/x^3-2*I*c*d^4*(a+b*arctan(c
*x))/x^2+6*c^2*d^4*(a+b*arctan(c*x))/x-4*I*a*c^3*d^4*ln(x)-19/3*b*c^3*d^4*
ln(x)+8/3*b*c^3*d^4*ln(c^2*x^2+1)+2*b*c^3*d^4*polylog(2,-I*c*x)-2*b*c^3*d^
4*polylog(2,I*c*x)
```

3.38.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{d^4(-2a-12iacx-bcx+36ac^2x^2-12ibc^2x^2+6ac^4x^4-2b\arctan(cx)-12ibcx\arctan(cx)+36bc^2x^2\operatorname{ar}}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
(d^4*(-2*a - (12*I)*a*c*x - b*c*x + 36*a*c^2*x^2 - (12*I)*b*c^2*x^2 + 6*a*
c^4*x^4 - 2*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] + 36*b*c^2*x^2*ArcTan
[c*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 6*b*c^4*x^4*ArcTan[c*x] - (24*I)*a*
c^3*x^3*Log[x] - 38*b*c^3*x^3*Log[c*x] + 16*b*c^3*x^3*Log[1 + c^2*x^2] + 1
2*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 12*b*c^3*x^3*PolyLog[2, I*c*x]))/(6*x^3
)
```

3.38.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(c^4d^4(a+b\arctan(cx))-\frac{4ic^3d^4(a+b\arctan(cx))}{x}-\frac{6c^2d^4(a+b\arctan(cx))}{x^2}+\frac{d^4(a+b\arctan(cx))}{x^4}+\frac{4icd^4}{}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{6c^2d^4(a+b\arctan(cx))}{x}-\frac{d^4(a+b\arctan(cx))}{3x^3}-\frac{2icd^4(a+b\arctan(cx))}{x^2}+ac^4d^4x-4iac^3d^4\log(x)+bc^4d^4x\arctan(cx)-2ibc^3d^4\arctan(cx)+2bc^3d^4\operatorname{PolyLog}(2,-icx)-2bc^3d^4\operatorname{PolyLog}(2,icx)-\frac{19}{3}bc^3d^4\log(x)-\frac{2ibc^2d^4}{x}+\frac{8}{3}bc^3d^4\log\left(c^2x^2+1\right)-\frac{bcd^4}{6x^2}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(b*c*d^4)/x^2 - ((2*I)*b*c^2*d^4)/x + a*c^4*d^4*x - (2*I)*b*c^3*d^4*A
rcTan[c*x] + b*c^4*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(3*x^3) -
 ((2*I)*c*d^4*(a + b*ArcTan[c*x]))/x^2 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/x
 - (4*I)*a*c^3*d^4*Log[x] - (19*b*c^3*d^4*Log[x])/3 + (8*b*c^3*d^4*Log[1 +
 c^2*x^2])/3 + 2*b*c^3*d^4*PolyLog[2, (-I)*c*x] - 2*b*c^3*d^4*PolyLog[2, I
*c*x]
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.38.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

method	result
parts	$d^4a\left(c^4x-\frac{2ic}{x^2}-4ic^3\ln(x)+\frac{6c^2}{x}-\frac{1}{3x^3}\right)+d^4b\,c^3\left(cx\arctan(cx)-\frac{\arctan(cx)}{3c^3x^3}-4i\arctan\right.$
derivativedivides	$c^3\left(d^4a\left(cx-\frac{1}{3c^3x^3}-4i\ln(cx)+\frac{6}{cx}-\frac{2i}{c^2x^2}\right)+d^4b\left(cx\arctan(cx)-\frac{\arctan(cx)}{3c^3x^3}-4i\arctan\right.\right.$
default	$c^3\left(d^4a\left(cx-\frac{1}{3c^3x^3}-4i\ln(cx)+\frac{6}{cx}-\frac{2i}{c^2x^2}\right)+d^4b\left(cx\arctan(cx)-\frac{\arctan(cx)}{3c^3x^3}-4i\arctan\right.\right.$
risch	$b\,c^3d^4-\frac{bc\,d^4}{6x^2}+\frac{8b\,c^3d^4\ln(c^2x^2+1)}{3}+\frac{id^4c^4b\ln(-icx+1)x}{2}+ia\,c^3d^4+a\,c^4d^4x-2ib\,c^3d^4\arctan($

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(c^4*x-2*I*c/x^2-4*I*c^3*ln(x)+6*c^2/x-1/3/x^3)+d^4*b*c^3*(c*x*arcta
n(c*x)-1/3*arctan(c*x)/c^3/x^3-4*I*arctan(c*x)*ln(c*x)+6/c/x*arctan(c*x)-2
*I*arctan(c*x)/c^2/x^2+2*ln(c*x)*ln(1+I*c*x)-2*ln(c*x)*ln(1-I*c*x)+2*dilog
(1+I*c*x)-2*dilog(1-I*c*x)-1/6/c^2/x^2-2*I/c/x-19/3*ln(c*x)+8/3*ln(c^2*x^2
+1)-2*I*arctan(c*x))
```

3.38.5 Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**4,x)
```

output
```
Timed out
```

3.38.7 Maxima [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
a*c^4*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^3*d^4 - 4*I*b
*c^3*d^4*integrate(arctan(c*x)/x, x) - 4*I*a*c^3*d^4*log(x) + 3*(c*(log(c^
2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^4 - 2*I*((c*arctan(c*x)
+ 1/x)*c + arctan(c*x)/x^2)*b*c*d^4 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log
(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^4 + 6*a*c^2*d^4/x - 2*I*a*c*d^4/
x^2 - 1/3*a*d^4/x^3
```

3.38.8 Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.30

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$$

$$= \begin{cases} \frac{b\,c^3\,d^4\,\ln\left(-\frac{3\,c^6\,x^2}{2}-\frac{3\,c^4}{2}\right)}{6} - \frac{b\,c^3\,d^4\,\ln(c^2\,x^2+1)}{2} - \frac{b\,c^3\,d^4\,\ln(x)}{3} - 2\,b\,c^3\,d^4\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i}) - \mathrm{Li}_2(1+c\,x\,1\mathrm{i})) - 6\,b\,c\,d^4 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^4,x)
```

output
```
piecewise(c == 0, -(a*d^4)/(3*x^3), c ~= 0, - b*d^4*(c^3*atan(c*x) + c^2/x
)*2i - 2*b*c^3*d^4*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) - (b*c^3*d^4*
log(c^2*x^2 + 1))/2 - (b*c^3*d^4*log(x))/3 + (b*c^3*d^4*log(- (3*c^4)/2 -
(3*c^6*x^2)/2))/6 - 6*b*c*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2) - (b
*c*d^4)/(6*x^2) - (a*d^4*(c*x*6i - 18*c^2*x^2 - 3*c^4*x^4 + c^3*x^3*log(x)
*12i + 1))/(3*x^3) - (b*d^4*atan(c*x))/(3*x^3) - (b*c*d^4*atan(c*x)*2i)/x^
2 + b*c^4*d^4*x*atan(c*x) + (6*b*c^2*d^4*atan(c*x))/x)
```

3.39 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$

3.39.1 Optimal result

Integrand size = 23, antiderivative size = 227

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = &-\frac{bcd^4}{12x^3}-\frac{2ibc^2d^4}{3x^2}+\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\arctan(cx)\\ &-\frac{d^4(a+b\arctan(cx))}{4x^4}-\frac{4icd^4(a+b\arctan(cx))}{3x^3}\\ &+\frac{3c^2d^4(a+b\arctan(cx))}{x^2}+\frac{4ic^3d^4(a+b\arctan(cx))}{x}\\ &+ac^4d^4\log(x)-\frac{16}{3}ibc^4d^4\log(x)+\frac{8}{3}ibc^4d^4\log\left(1\right.\\ &\left.+c^2x^2\right)\\ &+\frac{1}{2}ibc^4d^4\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ibc^4d^4\,\mathrm{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/12*b*c*d^4/x^3-2/3*I*b*c^2*d^4/x^2+13/4*b*c^3*d^4/x+13/4*b*c^4*d^4*arct
an(c*x)-1/4*d^4*(a+b*arctan(c*x))/x^4-4/3*I*c*d^4*(a+b*arctan(c*x))/x^3+3*
c^2*d^4*(a+b*arctan(c*x))/x^2+4*I*c^3*d^4*(a+b*arctan(c*x))/x+a*c^4*d^4*ln
(x)-16/3*I*b*c^4*d^4*ln(x)+8/3*I*b*c^4*d^4*ln(c^2*x^2+1)+1/2*I*b*c^4*d^4*p
olylog(2,-I*c*x)-1/2*I*b*c^4*d^4*polylog(2,I*c*x)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{d^4(-3a-16iacx+36ac^2x^2-8ibc^2x^2+48iac^3x^3-3b\arctan(cx)-16ibcx\arctan(cx)+36bc^2x^2\arctan(}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
(d^4*(-3*a - (16*I)*a*c*x + 36*a*c^2*x^2 - (8*I)*b*c^2*x^2 + (48*I)*a*c^3*
x^3 - 3*b*ArcTan[c*x] - (16*I)*b*c*x*ArcTan[c*x] + 36*b*c^2*x^2*ArcTan[c*x
] + (48*I)*b*c^3*x^3*ArcTan[c*x] - b*c*x*Hypergeometric2F1[-3/2, 1, -1/2,
-(c^2*x^2)] + 36*b*c^3*x^3*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 1
2*a*c^4*x^4*Log[x] - (64*I)*b*c^4*x^4*Log[x] + (32*I)*b*c^4*x^4*Log[1 + c^
2*x^2] + (6*I)*b*c^4*x^4*PolyLog[2, (-I)*c*x] - (6*I)*b*c^4*x^4*PolyLog[2,
 I*c*x]))/(12*x^4)
```

3.39.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{c^4d^4(a+b\arctan(cx))}{x}-\frac{4ic^3d^4(a+b\arctan(cx))}{x^2}-\frac{6c^2d^4(a+b\arctan(cx))}{x^3}+\frac{d^4(a+b\arctan(cx))}{x^5}+\frac{4icd^4}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{4ic^3d^4(a+b\arctan(cx))}{x}+\frac{3c^2d^4(a+b\arctan(cx))}{x^2}-\frac{d^4(a+b\arctan(cx))}{4x^4}-\frac{4icd^4(a+b\arctan(cx))}{3x^3}+ac^4d^4\log(x)+\frac{13}{4}bc^4d^4\arctan(cx)+\frac{1}{2}ibc^4d^4\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibc^4d^4\operatorname{PolyLog}(2,icx)-\frac{16}{3}ibc^4d^4\log(x)+\frac{13bc^3d^4}{4x}-\frac{2ibc^2d^4}{3x^2}+\frac{8}{3}ibc^4d^4\log\left(c^2x^2+1\right)-\frac{bcd^4}{12x^3}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*d^4)/x^3 - (((2*I)/3)*b*c^2*d^4)/x^2 + (13*b*c^3*d^4)/(4*x) + (
13*b*c^4*d^4*ArcTan[c*x])/4 - (d^4*(a + b*ArcTan[c*x]))/(4*x^4) - (((4*I)/
3)*c*d^4*(a + b*ArcTan[c*x]))/x^3 + (3*c^2*d^4*(a + b*ArcTan[c*x]))/x^2 +
((4*I)*c^3*d^4*(a + b*ArcTan[c*x]))/x + a*c^4*d^4*Log[x] - ((16*I)/3)*b*c^
4*d^4*Log[x] + ((8*I)/3)*b*c^4*d^4*Log[1 + c^2*x^2] + (I/2)*b*c^4*d^4*Poly
Log[2, (-I)*c*x] - (I/2)*b*c^4*d^4*PolyLog[2, I*c*x]
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.39.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.94

method	result
parts	$d^4a\left(\frac{3c^2}{x^2}+c^4\ln(x)-\frac{1}{4x^4}+\frac{4ic^3}{x}-\frac{4ic}{3x^3}\right)+d^4b\,c^4\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{4i\arctan(cx)}{3c^3x^3}+\arctan(cx)\right.$
derivativedivides	$c^4\left(d^4a\left(-\frac{1}{4c^4x^4}-\frac{4i}{3c^3x^3}+\ln(cx)+\frac{4i}{cx}+\frac{3}{c^2x^2}\right)+d^4b\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{4i\arctan(cx)}{3c^3x^3}+\arctan\right.\right.$
default	$c^4\left(d^4a\left(-\frac{1}{4c^4x^4}-\frac{4i}{3c^3x^3}+\ln(cx)+\frac{4i}{cx}+\frac{3}{c^2x^2}\right)+d^4b\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{4i\arctan(cx)}{3c^3x^3}+\arctan\right.\right.$
risch	$-\frac{bc\,d^4}{12x^3}+\frac{13b\,c^3d^4}{4x}-\frac{25ib\,d^4c^4\ln(icx)}{24}-\frac{3ib\,d^4c^2\ln(icx+1)}{2x^2}+\frac{8ib\,c^4d^4\ln(c^2x^2+1)}{3}+\frac{3ic^2d^4b\ln(-icx+1)}{2x^2}+$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(3*c^2/x^2+c^4*ln(x)-1/4/x^4+4*I*c^3/x-4/3*I*c/x^3)+d^4*b*c^4*(-1/4*
arctan(c*x)/c^4/x^4-4/3*I*arctan(c*x)/c^3/x^3+arctan(c*x)*ln(c*x)+4*I*arct
an(c*x)/c/x+3/c^2/x^2*arctan(c*x)-2/3*I/c^2/x^2-16/3*I*ln(c*x)-1/12/c^3/x^
3+13/4/c/x+8/3*I*ln(c^2*x^2+1)+13/4*arctan(c*x)+1/2*I*ln(c*x)*ln(1+I*c*x)-
1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x))
```

3.39.5 Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan\left(cx\right)+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5, x)
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**5,x)
```

output
```
Timed out
```

3.39.7 Maxima [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan{(cx)}+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
b*c^4*d^4*integrate(arctan(c*x)/x, x) + a*c^4*d^4*log(x) + 2*I*(c*(log(c^2
*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^4 + 3*((c*arctan(c*x) + 1
/x)*c + arctan(c*x)/x^2)*b*c^2*d^4 + 2/3*I*((c^2*log(c^2*x^2 + 1) - c^2*lo
g(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^4 + 4*I*a*c^3*d^4/x + 1/12*((
3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^4 + 3*
a*c^2*d^4/x^2 - 4/3*I*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

3.39.8 Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan{(cx)}+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.39.9 Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.31

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$$
$$= \begin{cases} \\ 3\,b\,c\,d^4\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)-\frac{b\,d^4\left(\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\mathrm{atan}(c\,x)\right)}{4\,c}-\frac{b\,c^4\,d^4\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,c^4\,d^4\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-b\,c^2\,d^4\,(\end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^5,x)
```

output
```
piecewise(c == 0, -(a*d^4)/(4*x^4), c ~= 0, - (b*d^4*(c^4*log(x) - (c^4*lo
g(- (c^4*(3*c^2*x^2 + 1))/2 - c^4))/2 + c^2/(2*x^2))*4i)/3 - (b*d^4*((c^2/
3 - c^4*x^2)/x^3 - c^5*atan(c*x)))/(4*c) - (b*c^4*d^4*dilog(- c*x*1i + 1)*
1i)/2 + (b*c^4*d^4*dilog(c*x*1i + 1)*1i)/2 - b*c^2*d^4*(c^2*log(x) - (c^2*
log(c^2*x^2 + 1))/2)*4i + (a*d^4*(- c*x*16i + 36*c^2*x^2 + c^3*x^3*48i + 1
2*c^4*x^4*log(x) - 3))/(12*x^4) - (b*d^4*atan(c*x))/(4*x^4) + 3*b*c*d^4*(c
^3*atan(c*x) + c^2/x) - (b*c*d^4*atan(c*x)*4i)/(3*x^3) + (3*b*c^2*d^4*atan
(c*x))/x^2 + (b*c^3*d^4*atan(c*x)*4i)/x)
```

3.39. $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$

3.40 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$

3.40.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^4}{20x^4}-\frac{ibc^2d^4}{3x^3}+\frac{11bc^3d^4}{10x^2}+\frac{3ibc^4d^4}{x} -\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5} +\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(i+cx)$$

output
```
-1/20*b*c*d^4/x^4-1/3*I*b*c^2*d^4/x^3+11/10*b*c^3*d^4/x^2+3*I*b*c^4*d^4/x-
1/5*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^5+16/5*b*c^5*d^4*ln(x)-16/5*b*c^5*
d^4*ln(c*x+I)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.63

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx =$$

$$-\frac{d^4\left(20ibc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+3\left(4a+20iacx+bcx-40ac^2x^2-40iac^3x^3-22\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/60*(d^4*((20*I)*b*c^2*x^2*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)]
+ 3*(4*a + (20*I)*a*c*x + b*c*x - 40*a*c^2*x^2 - (40*I)*a*c^3*x^3 - 22*b*c
^3*x^3 + 20*a*c^4*x^4 + 4*b*(1 + (5*I)*c*x - 10*c^2*x^2 - (10*I)*c^3*x^3 +
 5*c^4*x^4)*ArcTan[c*x] - (40*I)*b*c^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2,
 -(c^2*x^2)] - 64*b*c^5*x^5*Log[x] + 32*b*c^5*x^5*Log[1 + c^2*x^2]))/x^5
```

3.40.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{id^4(i-cx)^4}{5x^5(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}ibcd^4\int \frac{(i-cx)^4}{x^5(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{5}ibcd^4\int \left(\frac{16ic^5}{cx+i}-\frac{16ic^4}{x}-\frac{15c^3}{x^2}+\frac{11ic^2}{x^3}+\frac{5c}{x^4}-\frac{i}{x^5}\right)dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}ibcd^4\left(-16ic^4\log(x)+16ic^4\log(cx+i)+\frac{15c^3}{x}-\frac{11ic^2}{2x^2}-\frac{5c}{3x^3}+\frac{i}{4x^4}\right)-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/x^5 + (I/5)*b*c*d^4*((I/4)/x^
4 - (5*c)/(3*x^3) - (((11*I)/2)*c^2)/x^2 + (15*c^3)/x - (16*I)*c^4*Log[x]
+ (16*I)*c^4*Log[I + c*x])
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.40.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.50

method	result
parts	$d^4 a\left(\frac{2ic^3}{x^2}-\frac{ic}{x^4}-\frac{1}{5x^5}-\frac{c^4}{x}+\frac{2c^2}{x^3}\right)+d^4 b\,c^5\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{i\arctan(cx)}{c^4x^4}+\frac{2\arctan(cx)}{c^3x^3}-\frac{\arctan(cx}{cx}\right.$
derivativedivides	$c^5\left(d^4 a\left(-\frac{1}{5c^5x^5}-\frac{i}{c^4x^4}+\frac{2}{c^3x^3}-\frac{1}{cx}+\frac{2i}{c^2x^2}\right)+d^4 b\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{i\arctan(cx)}{c^4x^4}+\frac{2\arctan(cx)}{c^3x^3}-\right.\right.$
default	$c^5\left(d^4 a\left(-\frac{1}{5c^5x^5}-\frac{i}{c^4x^4}+\frac{2}{c^3x^3}-\frac{1}{cx}+\frac{2i}{c^2x^2}\right)+d^4 b\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{i\arctan(cx)}{c^4x^4}+\frac{2\arctan(cx)}{c^3x^3}-\right.\right.$
parallelrisch	$\frac{180ix^4 b\,c^4 d^4+120ix^3\arctan(cx)b\,c^3 d^4-96b\,c^5 d^4\ln(c^2x^2+1)x^5+192b\,c^5 d^4\ln(x)x^5-66b\,c^5 d^4 x^5+180ic^5 b\,d^4\arctan(cx)x}{}$
risch	$\frac{id^4b(5c^4x^4-10ic^3x^3-10c^2x^2+5icx+1)\ln(icx+1)}{10x^5}-\frac{d^4\left(186b\,c^5\ln(-cx-i)x^5+6b\,c^5\ln(cx-i)x^5-192b\,c^5\ln(-x)x^5-6\right.}{}$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(2*I*c^3/x^2-I*c/x^4-1/5/x^5-c^4/x+2*c^2/x^3)+d^4*b*c^5*(-1/5/c^5/x^
5*arctan(c*x)-I*arctan(c*x)/c^4/x^4+2*arctan(c*x)/c^3/x^3-1/c/x*arctan(c*x
)+2*I*arctan(c*x)/c^2/x^2-1/3*I/c^3/x^3+3*I/c/x-1/20/c^4/x^4+11/10/c^2/x^2
+16/5*ln(c*x)-8/5*ln(c^2*x^2+1)+3*I*arctan(c*x))
```

3.40.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(97) = 194$.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{192\,bc^5d^4x^5\log(x) - 186\,bc^5d^4x^5\log\left(\frac{cx+i}{c}\right) - 6\,bc^5d^4x^5\log\left(\frac{cx-i}{c}\right) - 60\,(a-3i\,b)c^4d^4x^4 - 6\,(-20i\,a - 11\,b}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/60*(192*b*c^5*d^4*x^5*log(x) - 186*b*c^5*d^4*x^5*log((c*x + I)/c) - 6*b*
c^5*d^4*x^5*log((c*x - I)/c) - 60*(a - 3*I*b)*c^4*d^4*x^4 - 6*(-20*I*a - 1
1*b)*c^3*d^4*x^3 + 20*(6*a - I*b)*c^2*d^4*x^2 - 3*(20*I*a + b)*c*d^4*x - 1
2*a*d^4 - 6*(5*I*b*c^4*d^4*x^4 + 10*b*c^3*d^4*x^3 - 10*I*b*c^2*d^4*x^2 - 5
*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5
```

3.40.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(114) = 228$.

Time = 38.18 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.13

$$
\begin{aligned}
\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx &= \frac{16bc^5d^4\log\left(10395b^2c^{11}d^8x\right)}{5} \\
&- \frac{bc^5d^4\log\left(10395b^2c^{11}d^8x - 10395ib^2c^{10}d^8\right)}{10} \\
&- \frac{31bc^5d^4\log\left(10395b^2c^{11}d^8x + 10395ib^2c^{10}d^8\right)}{10} \\
&+ \frac{-12ad^4 + x^4(-60ac^4d^4 + 180ibc^4d^4) + x^3\cdot(120iac^3d^4 + 66bc^3d^4) + x^2\cdot(120ac^2d^4 - 20ibc^2d^4) + x(-6}{60x^5} \\
&+ \frac{(-5ibc^4d^4x^4 - 10bc^3d^4x^3 + 10ibc^2d^4x^2 + 5bcd^4x - ibd^4)\log(-icx+1)}{10x^5} \\
&+ \frac{(5ibc^4d^4x^4 + 10bc^3d^4x^3 - 10ibc^2d^4x^2 - 5bcd^4x + ibd^4)\log(icx+1)}{10x^5}
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**6,x)
```

output
```
16*b*c**5*d**4*log(10395*b**2*c**11*d**8*x)/5 - b*c**5*d**4*log(10395*b**2
*c**11*d**8*x - 10395*I*b**2*c**10*d**8)/10 - 31*b*c**5*d**4*log(10395*b**
2*c**11*d**8*x + 10395*I*b**2*c**10*d**8)/10 + (-12*a*d**4 + x**4*(-60*a*c
**4*d**4 + 180*I*b*c**4*d**4) + x**3*(120*I*a*c**3*d**4 + 66*b*c**3*d**4)
+ x**2*(120*a*c**2*d**4 - 20*I*b*c**2*d**4) + x*(-60*I*a*c*d**4 - 3*b*c*d*
*4))/(60*x**5) + (-5*I*b*c**4*d**4*x**4 - 10*b*c**3*d**4*x**3 + 10*I*b*c**
2*d**4*x**2 + 5*b*c*d**4*x - I*b*d**4)*log(-I*c*x + 1)/(10*x**5) + (5*I*b*
c**4*d**4*x**4 + 10*b*c**3*d**4*x**3 - 10*I*b*c**2*d**4*x**2 - 5*b*c*d**4*
x + I*b*d**4)*log(I*c*x + 1)/(10*x**5)
```

3.40.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.35

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$
$$= -\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^4d^4$$
$$+2i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^3d^4$$
$$-\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^2d^4$$
$$-\frac{ac^4d^4}{x}+\frac{1}{3}i\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bcd^4$$
$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bd^4$$
$$+\frac{2i\,ac^3d^4}{x^2}+\frac{2\,ac^2d^4}{x^3}-\frac{i\,acd^4}{x^4}-\frac{ad^4}{5\,x^5}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^4*d^4 + 2*I*(
(c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^4 - ((c^2*log(c^2*x^2 +
 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^4 - a*c^4*d^4/x
 + 1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)
*b*c*d^4 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1
)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^4 + 2*I*a*c^3*d^4/x^2 + 2*a*c^2*d^4/x^3
- I*a*c*d^4/x^4 - 1/5*a*d^4/x^5
```

3.40.8 Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx=\int \frac{(i\,cdx+d)^4(b\arctan\left(cx\right)+a)}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.59

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{d^4\,(192\,b\,c^5\,\ln(x) - 96\,b\,c^5\,\ln(c^2\,x^2+1) + b\,c^5\,\mathrm{atan}(c\,x)\ 180\mathrm{i})}{60}$$
$$- \frac{\frac{d^4\,(12\,a+12\,b\,\mathrm{atan}(c\,x))}{60} + \frac{d^4\,x\,(a\,c\,60\mathrm{i}+3\,b\,c+b\,c\,\mathrm{atan}(c\,x)\,60\mathrm{i})}{60} - \frac{d^4\,x^2\,(120\,a\,c^2+120\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,20\mathrm{i})}{60} + \frac{d^4\,x^4\,(60\,a\,c^4+60\,b\,c^4\,\mathrm{ata}}{60}}{x^5}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^6,x)
```

output
```
(d^4*(b*c^5*atan(c*x)*180i - 96*b*c^5*log(c^2*x^2 + 1) + 192*b*c^5*log(x))
)/60 - ((d^4*(12*a + 12*b*atan(c*x)))/60 + (d^4*x*(a*c*60i + 3*b*c + b*c*a
tan(c*x)*60i))/60 - (d^4*x^2*(120*a*c^2 - b*c^2*20i + 120*b*c^2*atan(c*x))
)/60 + (d^4*x^4*(60*a*c^4 - b*c^4*180i + 60*b*c^4*atan(c*x)))/60 - (d^4*x^
3*(a*c^3*120i + 66*b*c^3 + b*c^3*atan(c*x)*120i))/60)/x^5
```

3.41 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$

3.41.1 Optimal result

Integrand size = 23, antiderivative size = 168

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd^4}{30x^5}-\frac{ibc^2d^4}{5x^4}+\frac{5bc^3d^4}{9x^3}+\frac{16ibc^4d^4}{15x^2} -\frac{13bc^5d^4}{6x}-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6} +\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5} +\frac{32}{15}ibc^6d^4\log(x)-\frac{32}{15}ibc^6d^4\log(i+cx)$$

output
```
-1/30*b*c*d^4/x^5-1/5*I*b*c^2*d^4/x^4+5/9*b*c^3*d^4/x^3+16/15*I*b*c^4*d^4/
x^2-13/6*b*c^5*d^4/x-1/6*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^6+1/30*I*c*d^
4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^5+32/15*I*b*c^6*d^4*ln(x)-32/15*I*b*c^6*
d^4*ln(c*x+I)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.40

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx = -\frac{d^4\left(5a+24iacx-45ac^2x^2+6ibc^2x^2-40iac^3x^3+15ac^4x^4-32ibc^4x^4+5b\arctan(cx)+24ibcx\arctan\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/30*(d^4*(5*a + (24*I)*a*c*x - 45*a*c^2*x^2 + (6*I)*b*c^2*x^2 - (40*I)*a
*c^3*x^3 + 15*a*c^4*x^4 - (32*I)*b*c^4*x^4 + 5*b*ArcTan[c*x] + (24*I)*b*c*
x*ArcTan[c*x] - 45*b*c^2*x^2*ArcTan[c*x] - (40*I)*b*c^3*x^3*ArcTan[c*x] +
15*b*c^4*x^4*ArcTan[c*x] + b*c*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^
2)] - 15*b*c^3*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 15*b*c^5
*x^5*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] - (64*I)*b*c^6*x^6*Log[x]
 + (32*I)*b*c^6*x^6*Log[1 + c^2*x^2]))/x^6
```

3.41.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^4(i-cx)^4(cx+5i)}{30x^6(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}bcd^4\int \frac{(i-cx)^4(cx+5i)}{x^6(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}$$

$$\Big\downarrow \text{165}$$

$$\frac{1}{30}bcd^4\int \left(-\frac{64ic^6}{cx+i}+\frac{64ic^5}{x}+\frac{65c^4}{x^2}-\frac{64ic^3}{x^3}-\frac{50c^2}{x^4}+\frac{24ic}{x^5}+\frac{5}{x^6}\right)dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}+\frac{1}{30}bcd^4\left(64ic^5\log(x)-64ic^5\log(cx+i)-\frac{65c^4}{x}+\frac{32ic^3}{x^2}+\frac{50c^2}{3x^3}-\frac{6ic}{x^4}-\frac{1}{x^5}\right)$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/x^6 + ((I/30)*c*d^4*(1 + I*c*
x)^5*(a + b*ArcTan[c*x]))/x^5 + (b*c*d^4*(-x^(-5) - ((6*I)*c)/x^4 + (50*c^
2)/(3*x^3) + ((32*I)*c^3)/x^2 - (65*c^4)/x + (64*I)*c^5*Log[x] - (64*I)*c^
5*Log[I + c*x]))/30
```

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.41.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

method	result
parts	$d^4 a\left(-\frac{c^4}{2x^2}+\frac{3c^2}{2x^4}-\frac{4ic}{5x^5}-\frac{1}{6x^6}+\frac{4ic^3}{3x^3}\right)+d^4 b\, c^6\left(-\frac{4i\arctan(cx)}{5c^5x^5}+\frac{3\arctan(cx)}{2c^4x^4}+\frac{4i\arctan(cx)}{3c^3x^3}-\right.$
derivativedivides	$c^6\left(d^4 a\left(-\frac{4i}{5c^5x^5}+\frac{3}{2c^4x^4}+\frac{4i}{3c^3x^3}-\frac{1}{2c^2x^2}-\frac{1}{6c^6x^6}\right)+d^4 b\left(-\frac{4i\arctan(cx)}{5c^5x^5}+\frac{3\arctan(cx)}{2c^4x^4}+\right.\right.$
default	$c^6\left(d^4 a\left(-\frac{4i}{5c^5x^5}+\frac{3}{2c^4x^4}+\frac{4i}{3c^3x^3}-\frac{1}{2c^2x^2}-\frac{1}{6c^6x^6}\right)+d^4 b\left(-\frac{4i\arctan(cx)}{5c^5x^5}+\frac{3\arctan(cx)}{2c^4x^4}+\right.\right.$
parallelrisch	$-\frac{96ic^6bd^4\ln(c^2x^2+1)x^6+96ix^6bc^6d^4-96ix^4bc^4d^4+195bc^6d^4\arctan(cx)x^6-45ac^6d^4x^6+195bc^5d^4x^5-120ix^3\cdots}{\cdots}$
risch	$\frac{id^4b(15c^4x^4-40ic^3x^3-45c^2x^2+24icx+5)\ln(icx+1)}{60x^6}-\frac{id^4(387bc^6\ln(-16705cx-16705i)x^6-3bc^6\ln(8255cx-8255i)\cdots}{\cdots}$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(-1/2/x^2*c^4+3/2*c^2/x^4-4/5*I*c/x^5-1/6/x^6+4/3*I*c^3/x^3)+d^4*b*c
^6*(-4/5*I*arctan(c*x)/c^5/x^5+3/2*arctan(c*x)/c^4/x^4+4/3*I*arctan(c*x)/c
^3/x^3-1/2/c^2/x^2*arctan(c*x)-1/6*arctan(c*x)/c^6/x^6+32/15*I*ln(c*x)-1/5
*I/c^4/x^4+16/15*I/c^2/x^2-1/30/c^5/x^5+5/9/c^3/x^3-13/6/c/x-16/15*I*ln(c^
2*x^2+1)-13/6*arctan(c*x))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.29

$$\int\frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$
$$=\frac{384i\,bc^6d^4x^6\log(x)-387i\,bc^6d^4x^6\log\left(\frac{cx+i}{c}\right)+3i\,bc^6d^4x^6\log\left(\frac{cx-i}{c}\right)-390\,bc^5d^4x^5-6\,(15\,a-32i\,b)c^4d^4x\cdots}{\cdots}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/180*(384*I*b*c^6*d^4*x^6*log(x) - 387*I*b*c^6*d^4*x^6*log((c*x + I)/c) +
 3*I*b*c^6*d^4*x^6*log((c*x - I)/c) - 390*b*c^5*d^4*x^5 - 6*(15*a - 32*I*b
)*c^4*d^4*x^4 - 20*(-12*I*a - 5*b)*c^3*d^4*x^3 + 18*(15*a - 2*I*b)*c^2*d^4
*x^2 - 6*(24*I*a + b)*c*d^4*x - 30*a*d^4 - 3*(15*I*b*c^4*d^4*x^4 + 40*b*c^
3*d^4*x^3 - 45*I*b*c^2*d^4*x^2 - 24*b*c*d^4*x + 5*I*b*d^4)*log(-(c*x + I)/
(c*x - I)))/x^6
```

3.41.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(163) = 326.

Time = 69.41 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.31

$$
\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx \\
&= \frac{32ibc^6d^4\log\left(2121535b^2c^{13}d^8x\right)}{15} + \frac{ibc^6d^4\log\left(2121535b^2c^{13}d^8x - 2121535ib^2c^{12}d^8\right)}{60} \\
&\quad - \frac{43ibc^6d^4\log\left(2121535b^2c^{13}d^8x + 2121535ib^2c^{12}d^8\right)}{20} \\
&\quad + \frac{(-15ibc^4d^4x^4 - 40bc^3d^4x^3 + 45ibc^2d^4x^2 + 24bcd^4x - 5ibd^4)\log(-icx+1)}{60x^6} \\
&\quad + \frac{(15ibc^4d^4x^4 + 40bc^3d^4x^3 - 45ibc^2d^4x^2 - 24bcd^4x + 5ibd^4)\log(icx+1)}{60x^6} \\
&\quad + \frac{-15ad^4 - 195bc^5d^4x^5 + x^4(-45ac^4d^4 + 96ibc^4d^4) + x^3\cdot(120iac^3d^4 + 50bc^3d^4) + x^2\cdot(135ac^2d^4 - 18ib}{90x^6}
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**7,x)
```

output
```
32*I*b*c**6*d**4*log(2121535*b**2*c**13*d**8*x)/15 + I*b*c**6*d**4*log(212
1535*b**2*c**13*d**8*x - 2121535*I*b**2*c**12*d**8)/60 - 43*I*b*c**6*d**4*
log(2121535*b**2*c**13*d**8*x + 2121535*I*b**2*c**12*d**8)/20 + (-15*I*b*c
**4*d**4*x**4 - 40*b*c**3*d**4*x**3 + 45*I*b*c**2*d**4*x**2 + 24*b*c*d**4*
x - 5*I*b*d**4)*log(-I*c*x + 1)/(60*x**6) + (15*I*b*c**4*d**4*x**4 + 40*b*
c**3*d**4*x**3 - 45*I*b*c**2*d**4*x**2 - 24*b*c*d**4*x + 5*I*b*d**4)*log(I
*c*x + 1)/(60*x**6) + (-15*a*d**4 - 195*b*c**5*d**4*x**5 + x**4*(-45*a*c**
4*d**4 + 96*I*b*c**4*d**4) + x**3*(120*I*a*c**3*d**4 + 50*b*c**3*d**4) + x
**2*(135*a*c**2*d**4 - 18*I*b*c**2*d**4) + x*(-72*I*a*c*d**4 - 3*b*c*d**4)
)/(90*x**6)
```

3.41.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(134) = 268.

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.73

$$\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx \\
&= -\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^4d^4 \\
&\quad -\frac{2}{3}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^3d^4 \\
&\quad -\frac{1}{2}\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bc^2d^4 \\
&\quad -\frac{1}{5}i\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan(cx)}{x^5}\right)bcd^4 \\
&\quad -\frac{ac^4d^4}{2x^2}-\frac{1}{90}\left(\left(15c^5\arctan(cx)+\frac{15c^4x^4-5c^2x^2+3}{x^5}\right)c+\frac{15\arctan(cx)}{x^6}\right)bd^4 \\
&\quad +\frac{4i\,ac^3d^4}{3x^3}+\frac{3ac^2d^4}{2x^4}-\frac{4i\,acd^4}{5x^5}-\frac{ad^4}{6x^6}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^4*d^4 - 2/3*I*((c^2*l
og(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^3*d^4 -
 1/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c
^2*d^4 - 1/5*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)
/x^4)*c + 4*arctan(c*x)/x^5)*b*c*d^4 - 1/2*a*c^4*d^4/x^2 - 1/90*((15*c^5*a
rctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d
^4 + 4/3*I*a*c^3*d^4/x^3 + 3/2*a*c^2*d^4/x^4 - 4/5*I*a*c*d^4/x^5 - 1/6*a*d
^4/x^6
```

3.41.8 Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx=\int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^7}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
sage0*x
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.24

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$
$$= -\frac{d^4\left(195\,b\,c^5\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}+b\,c^6\,\ln\left(c^2\,x^2+1\right)\,96\mathrm{i}-b\,c^6\,\ln\left(x\right)\,192\mathrm{i}\right)}{90}$$
$$-\frac{\frac{d^4\,\left(15\,a+15\,b\,\mathrm{atan}(c\,x)\right)}{90}+\frac{d^4\,x\,\left(a\,c\,72\mathrm{i}+3\,b\,c+b\,c\,\mathrm{atan}(c\,x)\,72\mathrm{i}\right)}{90}+\frac{d^4\,x^4\,\left(45\,a\,c^4+45\,b\,c^4\,\mathrm{atan}(c\,x)-b\,c^4\,96\mathrm{i}\right)}{90}-\frac{d^4\,x^2\,\left(135\,a\,c^2+135\,b\,c^2\,\mathrm{ata}\right.}{90}}{x^6}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^7,x)
```

output
```
- (d^4*(b*c^6*log(c^2*x^2 + 1)*96i - b*c^6*log(x)*192i + 195*b*c^5*atan(x*
(c^2)^(1/2))*(c^2)^(1/2)))/90 - ((d^4*(15*a + 15*b*atan(c*x)))/90 + (d^4*x
*(a*c*72i + 3*b*c + b*c*atan(c*x)*72i))/90 + (d^4*x^4*(45*a*c^4 - b*c^4*96
i + 45*b*c^4*atan(c*x)))/90 - (d^4*x^2*(135*a*c^2 - b*c^2*18i + 135*b*c^2*
atan(c*x)))/90 - (d^4*x^3*(a*c^3*120i + 50*b*c^3 + b*c^3*atan(c*x)*120i))/
90 + (13*b*c^5*d^4*x^5)/6)/x^6
```

3.42 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$

3.42.1 Optimal result

Integrand size = 23, antiderivative size = 243

$$
\begin{aligned}
\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx = & -\frac{bcd^4}{42x^6}-\frac{2ibc^2d^4}{15x^5}+\frac{47bc^3d^4}{140x^4} \\
& +\frac{5ibc^4d^4}{9x^3}-\frac{88bc^5d^4}{105x^2}-\frac{5ibc^6d^4}{3x} \\
& -\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6} \\
& +\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4} \\
& -\frac{c^4d^4(a+b\arctan(cx))}{3x^3}-\frac{176}{105}bc^7d^4\log(x) \\
& +\frac{1}{210}bc^7d^4\log(i-cx)+\frac{117}{70}bc^7d^4\log(i+cx)
\end{aligned}
$$

output
```
-1/42*b*c*d^4/x^6-2/15*I*b*c^2*d^4/x^5+47/140*b*c^3*d^4/x^4+5/9*I*b*c^4*d^
4/x^3-88/105*b*c^5*d^4/x^2-5/3*I*b*c^6*d^4/x-1/7*d^4*(a+b*arctan(c*x))/x^7
-2/3*I*c*d^4*(a+b*arctan(c*x))/x^6+6/5*c^2*d^4*(a+b*arctan(c*x))/x^5+I*c^3
*d^4*(a+b*arctan(c*x))/x^4-1/3*c^4*d^4*(a+b*arctan(c*x))/x^3-176/105*b*c^7
*d^4*ln(x)+1/210*b*c^7*d^4*ln(I-c*x)+117/70*b*c^7*d^4*ln(c*x+I)
```

3.42.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx = &-\frac{ad^4}{7x^7}-\frac{2iacd^4}{3x^6}-\frac{bcd^4}{42x^6}+\frac{6ac^2d^4}{5x^5}+\frac{iac^3d^4}{x^4}\\&+\frac{47bc^3d^4}{140x^4}-\frac{ac^4d^4}{3x^3}-\frac{88bc^5d^4}{105x^2}-\frac{bd^4\arctan(cx)}{7x^7}\\&-\frac{2ibcd^4\arctan(cx)}{3x^6}+\frac{6bc^2d^4\arctan(cx)}{5x^5}\\&+\frac{ibc^3d^4\arctan(cx)}{x^4}-\frac{bc^4d^4\arctan(cx)}{3x^3}\\&-\frac{2ibc^2d^4\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{15x^5}\\&+\frac{ibc^4d^4\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{3x^3}\\&-\frac{176}{105}bc^7d^4\log(x)+\frac{88}{105}bc^7d^4\log\left(1+c^2x^2\right)\end{aligned}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(a*d^4)/x^7 - (((2*I)/3)*a*c*d^4)/x^6 - (b*c*d^4)/(42*x^6) + (6*a*c^2
*d^4)/(5*x^5) + (I*a*c^3*d^4)/x^4 + (47*b*c^3*d^4)/(140*x^4) - (a*c^4*d^4)
/(3*x^3) - (88*b*c^5*d^4)/(105*x^2) - (b*d^4*ArcTan[c*x])/(7*x^7) - (((2*I
)/3)*b*c*d^4*ArcTan[c*x])/x^6 + (6*b*c^2*d^4*ArcTan[c*x])/(5*x^5) + (I*b*c
^3*d^4*ArcTan[c*x])/x^4 - (b*c^4*d^4*ArcTan[c*x])/(3*x^3) - (((2*I)/15)*b*
c^2*d^4*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)])/x^5 + ((I/3)*b*c^4*d
^4*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/x^3 - (176*b*c^7*d^4*Log[
x])/105 + (88*b*c^7*d^4*Log[1 + c^2*x^2])/105
```

3.42.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^4(35c^4x^4-105ic^3x^3-126c^2x^2+70icx+15)}{105x^7(c^2x^2+1)}dx-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}bcd^4\int \frac{35c^4x^4-105ic^3x^3-126c^2x^2+70icx+15}{x^7(c^2x^2+1)}dx-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{105}bcd^4\int \left(\frac{c^7}{2(cx-i)}+\frac{351c^7}{2(cx+i)}-\frac{176c^6}{x}+\frac{175ic^5}{x^2}+\frac{176c^4}{x^3}-\frac{175ic^3}{x^4}-\frac{141c^2}{x^5}+\frac{70ic}{x^6}+\frac{15}{x^7}\right)dx-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}+\frac{1}{105}bcd^4\left(-176c^6\log(x)+\frac{1}{2}c^6\log(-cx+i)+\frac{351}{2}c^6\log(cx+i)-\frac{175ic^5}{x}-\frac{88c^4}{x^2}+\frac{175ic^3}{3x^3}+\frac{141c^2}{4x^4}-\frac{14ic}{x^5}-\frac{5}{2x}\right.$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^4*(a + b*ArcTan[c*x]))/x^7 - (((2*I)/3)*c*d^4*(a + b*ArcTan[c*x]))
/x^6 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/(5*x^5) + (I*c^3*d^4*(a + b*ArcTan[
c*x]))/x^4 - (c^4*d^4*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*d^4*(-5/(2*x^6)
- ((14*I)*c)/x^5 + (141*c^2)/(4*x^4) + (((175*I)/3)*c^3)/x^3 - (88*c^4)/x^
2 - ((175*I)*c^5)/x - 176*c^6*Log[x] + (c^6*Log[I - c*x])/2 + (351*c^6*Log
[I + c*x])/2))/105
```

3.42. $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.42.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.79

method	result
parts	$d^4 a\left(-\frac{1}{7x^7}+\frac{ic^3}{x^4}+\frac{6c^2}{5x^5}-\frac{2ic}{3x^6}-\frac{c^4}{3x^3}\right)+d^4 b\,c^7\left(\frac{6\arctan(cx)}{5c^5x^5}+\frac{i\arctan(cx)}{c^4x^4}-\frac{\arctan(cx)}{3c^3x^3}-\frac{\arctan(c}{7c^7x^7}\right.$
derivativedivides	$c^7\left(d^4 a\left(\frac{6}{5c^5x^5}+\frac{i}{c^4x^4}-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}-\frac{2i}{3c^6x^6}\right)+d^4 b\left(\frac{6\arctan(cx)}{5c^5x^5}+\frac{i\arctan(cx)}{c^4x^4}-\frac{\arctan(cx)}{3c^3x^3}-\right.\right.$
default	$c^7\left(d^4 a\left(\frac{6}{5c^5x^5}+\frac{i}{c^4x^4}-\frac{1}{3c^3x^3}-\frac{1}{7c^7x^7}-\frac{2i}{3c^6x^6}\right)+d^4 b\left(\frac{6\arctan(cx)}{5c^5x^5}+\frac{i\arctan(cx)}{c^4x^4}-\frac{\arctan(cx)}{3c^3x^3}-\right.\right.$
parallelrisch	$-\frac{2100ix^6b\,c^6d^4-1056\ln(c^2x^2+1)x^7b\,c^7d^4+2112\ln(x)x^7b\,c^7d^4-1056b\,c^7d^4x^7-700ix^4b\,c^4d^4+1056b\,c^5d^4x^5+2100ic^7b}{}$
risch	$\frac{id^4b(35c^4x^4-105ic^3x^3-126c^2x^2+70icx+15)\ln(icx+1)}{210x^7}+\frac{d^4(2106b\,c^7\ln(-cx-i)x^7+6b\,c^7\ln(cx-i)x^7-2112b\,c^7\ln(-}{}$

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(-1/7/x^7+I*c^3/x^4+6/5*c^2/x^5-2/3*I*c/x^6-1/3/x^3*c^4)+d^4*b*c^7*(
6/5/c^5/x^5*arctan(c*x)+I*arctan(c*x)/c^4/x^4-1/3*arctan(c*x)/c^3/x^3-1/7*
arctan(c*x)/c^7/x^7-2/3*I*arctan(c*x)/c^6/x^6-5/3*I/c/x-2/15*I/c^5/x^5+5/9
*I/c^3/x^3-1/42/c^6/x^6+47/140/c^4/x^4-88/105/c^2/x^2-176/105*ln(c*x)+88/1
05*ln(c^2*x^2+1)-5/3*I*arctan(c*x))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx =$$
$$-\frac{2112\,bc^7d^4x^7\log(x) - 2106\,bc^7d^4x^7\log\left(\frac{cx+i}{c}\right) - 6\,bc^7d^4x^7\log\left(\frac{cx-i}{c}\right) + 2100i\,bc^6d^4x^6 + 1056\,bc^5d^4x^5 + \cdots}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/1260*(2112*b*c^7*d^4*x^7*log(x) - 2106*b*c^7*d^4*x^7*log((c*x + I)/c) -
 6*b*c^7*d^4*x^7*log((c*x - I)/c) + 2100*I*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*
x^5 + 140*(3*a - 5*I*b)*c^4*d^4*x^4 + 9*(-140*I*a - 47*b)*c^3*d^4*x^3 - 16
8*(9*a - I*b)*c^2*d^4*x^2 + 30*(28*I*a + b)*c*d^4*x + 180*a*d^4 + 6*(35*I*
b*c^4*d^4*x^4 + 105*b*c^3*d^4*x^3 - 126*I*b*c^2*d^4*x^2 - 70*b*c*d^4*x + 1
5*I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^7
```

3.42.6 Sympy [A] (verification not implemented)

Time = 122.87 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.64

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$$
$$= -\frac{176bc^7d^4\log(43427825b^2c^{15}d^8x)}{105} + \frac{bc^7d^4\log(43427825b^2c^{15}d^8x - 43427825ib^2c^{14}d^8)}{210}$$
$$+ \frac{117bc^7d^4\log(43427825b^2c^{15}d^8x + 43427825ib^2c^{14}d^8)}{70}$$
$$+ \frac{(-35ibc^4d^4x^4 - 105bc^3d^4x^3 + 126ibc^2d^4x^2 + 70bcd^4x - 15ibd^4)\log(-icx+1)}{210x^7}$$
$$+ \frac{(35ibc^4d^4x^4 + 105bc^3d^4x^3 - 126ibc^2d^4x^2 - 70bcd^4x + 15ibd^4)\log(icx+1)}{210x^7}$$
$$+ \frac{-180ad^4 - 2100ibc^6d^4x^6 - 1056bc^5d^4x^5 + x^4(-420ac^4d^4 + 700ibc^4d^4) + x^3\cdot(1260iac^3d^4 + 423bc^3d^4) - \cdots}{1260x^7}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**8,x)
```

output
```
-176*b*c**7*d**4*log(43427825*b**2*c**15*d**8*x)/105 + b*c**7*d**4*log(434
27825*b**2*c**15*d**8*x - 43427825*I*b**2*c**14*d**8)/210 + 117*b*c**7*d**
4*log(43427825*b**2*c**15*d**8*x + 43427825*I*b**2*c**14*d**8)/70 + (-35*I
*b*c**4*d**4*x**4 - 105*b*c**3*d**4*x**3 + 126*I*b*c**2*d**4*x**2 + 70*b*c
*d**4*x - 15*I*b*d**4)*log(-I*c*x + 1)/(210*x**7) + (35*I*b*c**4*d**4*x**4
 + 105*b*c**3*d**4*x**3 - 126*I*b*c**2*d**4*x**2 - 70*b*c*d**4*x + 15*I*b*
d**4)*log(I*c*x + 1)/(210*x**7) + (-180*a*d**4 - 2100*I*b*c**6*d**4*x**6 -
 1056*b*c**5*d**4*x**5 + x**4*(-420*a*c**4*d**4 + 700*I*b*c**4*d**4) + x**
3*(1260*I*a*c**3*d**4 + 423*b*c**3*d**4) + x**2*(1512*a*c**2*d**4 - 168*I*
b*c**2*d**4) + x*(-840*I*a*c*d**4 - 30*b*c*d**4))/(1260*x**7)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.35

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$$
$$\begin{aligned}
&= \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^4d^4 \\
&\quad -\frac{1}{3}i\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bc^3d^4 \\
&\quad +\frac{3}{10}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bc^2d^4 \\
&\quad -\frac{2}{45}i\left(\left(15\,c^5\arctan\left(cx\right)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan\left(cx\right)}{x^6}\right)bcd^4 \\
&\quad +\frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right)-6\,c^6\log\left(x^2\right)-\frac{6\,c^4x^4-3\,c^2x^2+2}{x^6}\right)c-\frac{12\arctan\left(cx\right)}{x^7}\right)bd^4 \\
&\quad -\frac{ac^4d^4}{3\,x^3}+\frac{i\,ac^3d^4}{x^4}+\frac{6\,ac^2d^4}{5\,x^5}-\frac{2i\,acd^4}{3\,x^6}-\frac{ad^4}{7\,x^7}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*c^4*d^4 - 1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(
c*x)/x^4)*b*c^3*d^4 + 3/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*
c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c^2*d^4 - 2/45*I*((15*c^5*arcta
n(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*c*d^4
 + 1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2
 + 2)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 + I*a*c^3*d^4
/x^4 + 6/5*a*c^2*d^4/x^5 - 2/3*I*a*c*d^4/x^6 - 1/7*a*d^4/x^7
```

3.42.8 Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx=\int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^8}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

output
```
sage0*x
```

3.42.9 Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.30

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx=\frac{88\,b\,c^7\,d^4\,\ln(c^2\,x^2+1)}{105} - \frac{\frac{a\,d^4}{7}+\frac{b\,d^4\,\mathrm{atan}(c\,x)}{7}+\frac{88\,b\,c^7\,d^4\,x^7}{105}+\frac{b\,c^8\,d^4\,x^8\,5\mathrm{i}}{3}+\frac{c\,d^4\,x\,(b+a\,28\mathrm{i})}{42}+\frac{c^6\,d^4\,x^6\,(3\,a+b\,10\mathrm{i})}{9}-\frac{c^4\,d^4\,x^4\,(39\,a+b\,19\mathrm{i})}{45}-\frac{c^2\,d^4\,x^2\,(111}{105}}{} - \frac{176\,b\,c^7\,d^4\,\ln(x)}{105} - \frac{b\,c^{10}\,d^4\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,5\mathrm{i}}{3\,(c^2)^{3/2}}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^8,x)
```

output
```
(88*b*c^7*d^4*log(c^2*x^2 + 1))/105 - ((a*d^4)/7 + (b*d^4*atan(c*x))/7 + (
88*b*c^7*d^4*x^7)/105 + (b*c^8*d^4*x^8*5i)/3 + (c*d^4*x*(a*28i + b))/42 +
(c^6*d^4*x^6*(3*a + b*10i))/9 - (c^4*d^4*x^4*(39*a + b*19i))/45 - (c^2*d^4
*x^2*(111*a - b*14i))/105 - (c^3*d^4*x^3*(a*140i + 131*b))/420 - c^5*d^4*x
^5*(a*1i - (211*b)/420) - (37*b*c^2*d^4*x^2*atan(c*x))/35 - (b*c^3*d^4*x^3
*atan(c*x)*1i)/3 - (13*b*c^4*d^4*x^4*atan(c*x))/15 - b*c^5*d^4*x^5*atan(c*
x)*1i + (b*c^6*d^4*x^6*atan(c*x))/3 + (b*c*d^4*x*atan(c*x)*2i)/3)/(x^7 + c
^2*x^9) - (176*b*c^7*d^4*log(x))/105 - (b*c^10*d^4*atan((c^2*x)/(c^2)^(1/2
))*5i)/(3*(c^2)^(3/2))
```

3.42. $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$

3.43 $\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx$

3.43.1 Optimal result

Integrand size = 23, antiderivative size = 196

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx = {} & \frac{iax}{c^3d}-\frac{bx}{2c^3d}+\frac{ibx^2}{6c^2d}+\frac{b\arctan(cx)}{2c^4d} \\ & +\frac{ibx\arctan(cx)}{c^3d}+\frac{x^2(a+b\arctan(cx))}{2c^2d} \\ & -\frac{ix^3(a+b\arctan(cx))}{3cd}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d} \\ & -\frac{2ib\log(1+c^2x^2)}{3c^4d}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d}\end{aligned}$$

output
```
I*a*x/c^3/d-1/2*b*x/c^3/d+1/6*I*b*x^2/c^2/d+1/2*b*arctan(c*x)/c^4/d+I*b*x*
arctan(c*x)/c^3/d+1/2*x^2*(a+b*arctan(c*x))/c^2/d-1/3*I*x^3*(a+b*arctan(c*
x))/c/d+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d-2/3*I*b*ln(c^2*x^2+1)/c^4/
d+1/2*I*b*polylog(2,1-2/(1+I*c*x))/c^4/d
```

3.43.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx =$$

$$-\frac{i\left(-b-6acx-3ibcx+3iac^2x^2-bc^2x^2+2ac^3x^3+6b\arctan(cx)^2+\arctan(cx)\left(6a+b(3i-6cx+3ic\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-1/6*I)*(-b - 6*a*c*x - (3*I)*b*c*x + (3*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*
c^3*x^3 + 6*b*ArcTan[c*x]^2 + ArcTan[c*x]*(6*a + b*(3*I - 6*c*x + (3*I)*c^
2*x^2 + 2*c^3*x^3) + (6*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (3*I)*a*Log
[1 + c^2*x^2] + 4*b*Log[1 + c^2*x^2] + 3*b*PolyLog[2, -E^((2*I)*ArcTan[c*x
])]))/(c^4*d)
```

3.43.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5401, 27, 5361, 243, 49, 2009, 5401, 5361, 262, 216, 5401, 2009, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}dx$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{d(icx+1)}dx}{c}-\frac{i\int x^2(a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\int x^2(a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int \frac{x^3}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{243}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \frac{x^2}{c^2x^2+1}dx^2\right)}{cd}$$

$$\Big\downarrow \text{49}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2\right)}{cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\int x(a+b\arctan(cx))dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int \frac{x^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{262}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int \frac{1}{c^2x^2+1}dx}{c^2}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{216}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\left(\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))dx}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{2009}$$

3.43. $\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\int\frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right.\right.}{c}\right.}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\text{arcta}\right.\right.}{c}\right.}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{2752}$$

3.43. $\displaystyle\int\frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx$

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\text{arcta}}{c}\right.\right.}{c}\right.$$

$$cd$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-I)*((x^3*(a + b*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4)
)/6))/(c*d) + (I*(((-I)*((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTa
n[c*x]/c^3))/2))/c + (I*(((-I)*(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2
])/(2*c)))/c + (I*((I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (b*PolyL
og[2, 1 - 2/(1 + I*c*x)])/(2*c)))/c))/c))/(c*d)
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

3.43. $\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx$

3.43.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.59

method	result
derivativedivides	$-\frac{5ib\ln\left(c^4x^4+10c^2x^2+9\right)}{48d}-\frac{ib\arctan(cx)c^3x^3}{3d}+\frac{a\,c^2x^2}{2d}-\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{ib\,c^2x^2}{6d}+\frac{ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-$
default	$-\frac{5ib\ln\left(c^4x^4+10c^2x^2+9\right)}{48d}-\frac{ib\arctan(cx)c^3x^3}{3d}+\frac{a\,c^2x^2}{2d}-\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{ib\,c^2x^2}{6d}+\frac{ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-$
risch	$\frac{iax}{c^3d}-\frac{b\left(\frac{1}{3}c^2x^3+\frac{1}{2}ic\,x^2-x\right)\ln(icx+1)}{2c^3d}+\frac{ib\,x^2}{6c^2d}-\frac{5a}{6d\,c^4}+\frac{a\,x^2}{2d\,c^2}-\frac{11ib\ln\left(c^2x^2+1\right)}{24c^4d}+\frac{b\,x^3\ln(-icx+1)}{6dc}+ ib\ln$
parts	$\frac{iax}{c^3d}+\frac{a\,x^2}{2d\,c^2}-\frac{ib\arctan(cx)x^3}{3dc}-\frac{a\ln\left(c^2x^2+1\right)}{2d\,c^4}+\frac{ib\,x^2}{6c^2d}-\frac{5ib\ln\left(c^4x^4+10c^2x^2+9\right)}{48d\,c^4}+\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d\,c^4}+ b$

input
```
int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-5/48*I/d*b*ln(c^4*x^4+10*c^2*x^2+9)-1/3*I/d*b*arctan(c*x)*c^3*x^3+
1/2/d*a*c^2*x^2-1/2/d*a*ln(c^2*x^2+1)+1/2*I/d*b*dilog(-1/2*I*(c*x+I))+1/6*
I/d*b*c^2*x^2+1/2*I/d*b*ln(-1/2*I*(c*x+I))*ln(c*x-I)+1/2/d*b*arctan(c*x)*c
^2*x^2-1/d*b*arctan(c*x)*ln(c*x-I)-11/24*I/d*b*ln(c^2*x^2+1)-1/3*I/d*a*c^3
*x^3-1/4*I/d*b*ln(c*x-I)^2-1/2/d*b*c*x+I/d*a*c*x+2/3*I/d*b-I/d*a*arctan(c*
x)+5/24/d*b*arctan(1/2*c*x)-5/24/d*b*arctan(1/6*c^3*x^3+7/6*c*x)-5/12/d*b*
arctan(1/2*c*x-1/2*I)+I/d*b*arctan(c*x)*c*x+11/12/d*b*arctan(c*x))
```

3.43.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(c*d*x - I*d),
x)
```

3.43.6 Sympy [F]

$$\int \frac{x^3(a + b\arctan(cx))}{d + icdx}\,dx =$$
$$-\frac{i\left(\int \frac{6ib\log(icx+1)}{c^2x^2+1}\,dx + \int \frac{12ac^4x^4}{c^2x^2+1}\,dx + \int \frac{6bcx}{c^2x^2+1}\,dx + \int \frac{bc^3x^3}{c^2x^2+1}\,dx + \int \frac{12iac^3x^3}{c^2x^2+1}\,dx + \int \frac{3ibc^2x^2}{c^2x^2+1}\,dx + \int \left(-\frac{2ibc^4x}{c^2x^2+}\right.\right.}{12c^3d}$$
$$+ \frac{(2bc^3x^3 + 3ibc^2x^2 - 6bcx - 6ib\log(icx+1))\log(-icx+1)}{12c^4d}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
-I*(Integral(6*I*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(12*a*c**4
*x**4/(c**2*x**2 + 1), x) + Integral(6*b*c*x/(c**2*x**2 + 1), x) + Integra
l(b*c**3*x**3/(c**2*x**2 + 1), x) + Integral(12*I*a*c**3*x**3/(c**2*x**2 +
 1), x) + Integral(3*I*b*c**2*x**2/(c**2*x**2 + 1), x) + Integral(-2*I*b*c
**4*x**4/(c**2*x**2 + 1), x) + Integral(-6*b*c*x*log(I*c*x + 1)/(c**2*x**2
 + 1), x) + Integral(6*b*c**3*x**3*log(I*c*x + 1)/(c**2*x**2 + 1), x) + In
tegral(-6*I*b*c**4*x**4*log(I*c*x + 1)/(c**2*x**2 + 1), x))/(12*c**3*d) +
(2*b*c**3*x**3 + 3*I*b*c**2*x**2 - 6*b*c*x - 6*I*b*log(I*c*x + 1))*log(-I*
c*x + 1)/(12*c**4*d)
```

3.43.7 Maxima [F]

$$\int \frac{x^3(a + b\arctan(cx))}{d + icdx}\,dx = \int \frac{(b\arctan(cx) + a)x^3}{i\,cdx + d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*a*(I*(2*c^2*x^3 + 3*I*c*x^2 - 6*x)/(c^3*d) + 6*log(I*c*x + 1)/(c^4*d)
) - 1/72*(432*I*c^8*d*integrate(1/12*x^4*arctan(c*x)/(c^5*d*x^2 + c^3*d),
x) + 216*c^8*d*integrate(1/12*x^4*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x)
 - 432*c^7*d*integrate(1/12*x^3*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) + 216*
I*c^7*d*integrate(1/12*x^3*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 432*
c^5*d*integrate(1/12*x*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) - 216*I*c^5*d*i
ntegrate(1/12*x*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 4*c^3*x^3 - 216
*c^4*d*integrate(1/12*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 3*I*c^2*x
^2 - 30*c*x - 6*(-2*I*c^3*x^3 + 3*c^2*x^2 + 6*I*c*x - 5)*arctan(c*x) + 18*
I*arctan(c*x)^2 - 3*(2*c^3*x^3 + 3*I*c^2*x^2 - 6*c*x + I)*log(c^2*x^2 + 1)
 + 9*I*log(c^2*x^2 + 1)^2 + 18*I*log(12*c^5*d*x^2 + 12*c^3*d))*b/(c^4*d)
```

3.43.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

3.44 $\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx$

3.44.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \frac{ax}{c^2d}+\frac{ibx}{2c^2d}-\frac{ib\arctan(cx)}{2c^3d}+\frac{bx\arctan(cx)}{c^2d} - \frac{ix^2(a+b\arctan(cx))}{2cd}-\frac{i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d} - \frac{b\log\left(1+c^2x^2\right)}{2c^3d}+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}$$

output
```
a*x/c^2/d+1/2*I*b*x/c^2/d-1/2*I*b*arctan(c*x)/c^3/d+b*x*arctan(c*x)/c^2/d-
1/2*I*x^2*(a+b*arctan(c*x))/c/d-I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d-
1/2*b*ln(c^2*x^2+1)/c^3/d+1/2*b*polylog(2,1-2/(1+I*c*x))/c^3/d
```

3.44.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.85

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = -\frac{-2acx-ibcx+iac^2x^2+2b\arctan(cx)^2+i\arctan(cx)\left(-2ia+b+2ibcx+bc^2x^2+2b\log\left(1+e^{2i\arctan}\right.\right.}{2c^3d}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
-1/2*(-2*a*c*x - I*b*c*x + I*a*c^2*x^2 + 2*b*ArcTan[c*x]^2 + I*ArcTan[c*x]
*((-2*I)*a + b + (2*I)*b*c*x + b*c^2*x^2 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x
])]) - I*a*Log[1 + c^2*x^2] + b*Log[1 + c^2*x^2] + b*PolyLog[2, -E^((2*I)*
ArcTan[c*x])])/(c^3*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5401, 27, 5361, 262, 216, 5401, 2009, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}dx$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{d(icx+1)}dx}{c}-\frac{i\int x(a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\int x(a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int \frac{x^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{262}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int \frac{1}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{216}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

3.44. $\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx$

output
```
((-I)*((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2))/(
c*d) + (I*(((-I)*(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)))/c +
 (I*((I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (b*PolyLog[2, 1 - 2/(1
 + I*c*x)])/(2*c)))/c))/(c*d)
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

3.44.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{\frac{acx}{d}-\frac{ia\,c^2x^2}{2d}+\frac{ibcx}{2d}-\frac{a\arctan(cx)}{d}+\frac{b\arctan(cx)cx}{d}+\frac{ib\arctan(cx)\ln(cx-i)}{d}-\frac{ib\arctan\left(\frac{cx}{2}\right)}{8d}+\frac{b\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d}+}{}$
default	$\frac{\frac{acx}{d}-\frac{ia\,c^2x^2}{2d}+\frac{ibcx}{2d}-\frac{a\arctan(cx)}{d}+\frac{b\arctan(cx)cx}{d}+\frac{ib\arctan(cx)\ln(cx-i)}{d}-\frac{ib\arctan\left(\frac{cx}{2}\right)}{8d}+\frac{b\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d}+}{}$
risch	$\frac{b\ln(icx+1)^2}{4c^3d}-\frac{b\left(\frac{1}{2}c\,x^2+ix\right)\ln(icx+1)}{2c^2d}+\frac{ia\ln(c^2x^2+1)}{2c^3d}+\frac{ax}{d\,c^2}-\frac{a\arctan(cx)}{c^3d}+\frac{ibx}{2c^2d}-\frac{ia\,x^2}{2cd}+$
parts	$\frac{ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{8c^3d}+\frac{ax}{d\,c^2}+\frac{ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{4c^3d}-\frac{a\arctan(cx)}{c^3d}+\frac{bx\arctan(cx)}{c^2d}+\frac{ia\ln(c^2x^2+1)}{2c^3d}+\frac{ibx}{2c^2d}$

input
```
int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/d*a*c*x-1/2*I/d*a*c^2*x^2+1/2*I/d*b*c*x-1/d*a*arctan(c*x)+1/d*b*a
rctan(c*x)*c*x+I/d*b*arctan(c*x)*ln(c*x-I)-1/8*I/d*b*arctan(1/2*c*x)+1/2/d
*b*ln(-1/2*I*(c*x+I))*ln(c*x-I)+1/2/d*b*dilog(-1/2*I*(c*x+I))-1/4/d*b*ln(c
*x-I)^2+1/2/d*b+1/4*I/d*b*arctan(1/2*c*x-1/2*I)-1/16/d*b*ln(c^4*x^4+10*c^2
*x^2+9)+1/2*I/d*a*ln(c^2*x^2+1)+1/8*I/d*b*arctan(1/6*c^3*x^3+7/6*c*x)-1/2*
I/d*b*arctan(c*x)*c^2*x^2-3/8/d*b*ln(c^2*x^2+1)-3/4*I/d*b*arctan(c*x))
```

3.44.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(c*d*x - I*d),
x)
```

3.44.6 Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = -\frac{i\left(\int \frac{2b\log(icx+1)}{c^2x^2+1}\,dx + \int \frac{4ac^3x^3}{c^2x^2+1}\,dx + \int \frac{bc^2x^2}{c^2x^2+1}\,dx + \int \frac{4iac^2x^2}{c^2x^2+1}\,dx + \int \left(-\frac{2ibcx}{c^2x^2+1}\right)dx + \int \left(-\frac{ibc^3x^3}{c^2x^2+1}\right)dx + \int\right.}{4c^2d} + \frac{(bc^2x^2+2ibcx-2b\log(icx+1))\log(-icx+1)}{4c^3d}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
-I*(Integral(2*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(4*a*c**3*x*
*3/(c**2*x**2 + 1), x) + Integral(b*c**2*x**2/(c**2*x**2 + 1), x) + Integr
al(4*I*a*c**2*x**2/(c**2*x**2 + 1), x) + Integral(-2*I*b*c*x/(c**2*x**2 +
1), x) + Integral(-I*b*c**3*x**3/(c**2*x**2 + 1), x) + Integral(2*b*c**2*x
**2*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(2*I*b*c*x*log(I*c*x + 1)
/(c**2*x**2 + 1), x) + Integral(-2*I*b*c**3*x**3*log(I*c*x + 1)/(c**2*x**2
 + 1), x))/(4*c**2*d) + (b*c**2*x**2 + 2*I*b*c*x - 2*b*log(I*c*x + 1))*log
(-I*c*x + 1)/(4*c**3*d)
```

3.44.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*a*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/8*(32*I*
c^6*d*integrate(1/8*x^3*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*c^6*d*int
egrate(1/8*x^3*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*c^5*d*integra
te(1/8*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*I*c^5*d*integrate(1/8*
x^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*I*c^4*d*integrate(1/8*x*
arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 16*c^4*d*integrate(1/8*x*log(c^2*x^2
 + 1)/(c^4*d*x^2 + c^2*d), x) + 16*I*c^3*d*integrate(1/8*log(c^2*x^2 + 1)/
(c^4*d*x^2 + c^2*d), x) + c^2*x^2 + 2*I*c*x - 2*(-I*c^2*x^2 + 2*c*x + I)*a
rctan(c*x) + 2*arctan(c*x)^2 - (c^2*x^2 + 2*I*c*x + 1)*log(c^2*x^2 + 1) +
log(c^2*x^2 + 1)^2 + 2*log(8*c^4*d*x^2 + 8*c^2*d))*b/(c^3*d)
```

3.44.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

3.45 $\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx$

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = -\frac{iax}{cd}-\frac{ibx\arctan(cx)}{cd}-\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d}+\frac{ib\log(1+c^2x^2)}{2c^2d}-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d}$$

output
```
-I*a*x/c/d-I*b*x*arctan(c*x)/c/d-(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2/d+1
/2*I*b*ln(c^2*x^2+1)/c^2/d-1/2*I*b*polylog(2,1-2/(1+I*c*x))/c^2/d
```

3.45.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = \frac{-2iacx+2ib\arctan(cx)^2+2i\arctan(cx)\left(a-bcx+ib\log\left(1+e^{2i\arctan(cx)}\right)\right)+a\log(1+c^2x^2)+ib\log(\ldots}{2c^2d}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-2*I)*a*c*x + (2*I)*b*ArcTan[c*x]^2 + (2*I)*ArcTan[c*x]*(a - b*c*x + I*b
*Log[1 + E^((2*I)*ArcTan[c*x])]) + a*Log[1 + c^2*x^2] + I*b*Log[1 + c^2*x^
2] + I*b*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5401, 27, 2009, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx$$

$$\Big\downarrow \ 5401$$

$$\frac{i\int \frac{a+b\arctan(cx)}{d(icx+1)}dx}{c}-\frac{i\int (a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \ 27$$

$$\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{cd}-\frac{i\int (a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \ 2009$$

$$\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

$$\Big\downarrow \ 5379$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

$$\Big\downarrow \ 2849$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}\right)}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

$$\Big\downarrow \ 2752$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}\right)}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-I)*(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)))/(c*d) + (I*((I
*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (b*PolyLog[2, 1 - 2/(1 + I*c*
x)])/(2*c)))/(c*d)
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

3.45.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(100) = 200$.

Time = 1.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{-\frac{iacx}{d}+\frac{a\ln\left(c^{2}x^{2}+1\right)}{2d}+\frac{ia\arctan(cx)}{d}-\frac{ib\arctan(cx)cx}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}-\frac{ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d}-\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d}}{c^{2}}$
default	$\frac{-\frac{iacx}{d}+\frac{a\ln\left(c^{2}x^{2}+1\right)}{2d}+\frac{ia\arctan(cx)}{d}-\frac{ib\arctan(cx)cx}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}-\frac{ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d}-\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d}}{c^{2}}$
risch	$-\frac{ib\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d\,c^{2}}-\frac{bx\ln(icx+1)}{2cd}+\frac{i\ln(-icx+1)b}{2d\,c^{2}}-\frac{b\arctan(cx)}{2d\,c^{2}}+\frac{bx\ln(-icx+1)}{2dc}+\frac{a\ln(c^{2}x^{2}+1)}{2d\,c^{2}}+$
parts	$-\frac{iax}{cd}+\frac{a\ln(c^{2}x^{2}+1)}{2d\,c^{2}}+\frac{ia\arctan(cx)}{d\,c^{2}}-\frac{ibx\arctan(cx)}{cd}+\frac{b\arctan(cx)\ln(cx-i)}{c^{2}d}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2c^{2}d}$

input
```
int(x*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-I/d*a*c*x+1/2/d*a*ln(c^2*x^2+1)+I/d*a*arctan(c*x)-I/d*b*arctan(c*x
)*c*x+1/d*b*arctan(c*x)*ln(c*x-I)-1/2*I/d*b*ln(-1/2*I*(c*x+I))*ln(c*x-I)-1
/2*I/d*b*dilog(-1/2*I*(c*x+I))+1/4*I/d*b*ln(c*x-I)^2+1/8*I/d*b*ln(c^8*x^8+
12*c^6*x^6+30*c^4*x^4+28*c^2*x^2+9)-1/4/d*b*arctan(1/12*c^3*x^3+13/12*c*x)
-1/4/d*b*arctan(1/4*c*x)+1/2/d*b*arctan(1/2*c*x-1/2*I))
```

3.45.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*x*log(-(c*x + I)/(c*x - I)) - 2*I*a*x)/(c*d*x - I*d), x)
```

3.45.6 Sympy [F]

$$\int \frac{x(a + b\arctan(cx))}{d + icdx}\,dx =$$
$$-\frac{i\left(\int\left(-\frac{ib\log(icx+1)}{c^2x^2+1}\right)dx + \int\frac{2ac^2x^2}{c^2x^2+1}\,dx + \int\left(-\frac{bcx}{c^2x^2+1}\right)dx + \int\frac{2iacx}{c^2x^2+1}\,dx + \int\left(-\frac{ibc^2x^2}{c^2x^2+1}\right)dx + \int\frac{2bcx\log(icx}{c^2x^2+1}\right.}{2cd}$$
$$+\frac{(bcx + ib\log(icx+1))\log(-icx+1)}{2c^2d}$$

input
```
integrate(x*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
-I*(Integral(-I*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(2*a*c**2*x
**2/(c**2*x**2 + 1), x) + Integral(-b*c*x/(c**2*x**2 + 1), x) + Integral(2
*I*a*c*x/(c**2*x**2 + 1), x) + Integral(-I*b*c**2*x**2/(c**2*x**2 + 1), x)
 + Integral(2*b*c*x*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-I*b*c**
2*x**2*log(I*c*x + 1)/(c**2*x**2 + 1), x))/(2*c*d) + (b*c*x + I*b*log(I*c*
x + 1))*log(-I*c*x + 1)/(2*c**2*d)
```

3.45.7 Maxima [F]

$$\int \frac{x(a + b\arctan(cx))}{d + icdx}\,dx = \int \frac{(b\arctan(cx) + a)x}{i\,cdx + d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) - 1/8*(8*I*c^4*d*integrate(1/2*x^2
*arctan(c*x)/(c^3*d*x^2 + c*d), x) + 4*c^4*d*integrate(1/2*x^2*log(c^2*x^2
 + 1)/(c^3*d*x^2 + c*d), x) - 16*c^3*d*integrate(1/2*x*arctan(c*x)/(c^3*d*
x^2 + c*d), x) + 8*I*c^3*d*integrate(1/2*x*log(c^2*x^2 + 1)/(c^3*d*x^2 + c
*d), x) + 4*c^2*d*integrate(1/2*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) - 2
*c*x*log(c^2*x^2 + 1) + 4*c*x - 4*(-I*c*x + 1)*arctan(c*x) - 2*I*arctan(c*
x)^2 - I*log(c^2*x^2 + 1)^2 - 2*I*log(2*c^3*d*x^2 + 2*c*d))*b/(c^2*d)
```

3.45.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{(b\arctan{(cx)}+a)x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

3.46 $\int \frac{a+b\arctan(cx)}{d+icdx}\,dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\frac{i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd}$$

output
```
I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/d-1/2*b*polylog(2,1-2/(1+I*c*x))/c/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\frac{2i(a+b\arctan(cx))\log\left(\frac{2d}{d+icdx}\right)-b\,\mathrm{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)}{2cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]
```

output
```
((2*I)*(a + b*ArcTan[c*x])*Log[(2*d)/(d + I*c*d*x)] - b*PolyLog[2, (I + c*
x)/(-I + c*x)])/(2*c*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{d+icdx}dx$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{cd}-\frac{ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{2849}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{cd}-\frac{b\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{cd}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2cd}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*d) - (b*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(2*c*d)
```

3.46.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

3.46.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{-\frac{ia\ln\left(c^2x^2+1\right)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-i\ln(icx+1)\arctan(cx)-\frac{\left(\ln(icx+1)-\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2}+\frac{\mathrm{dilog}\left(\frac{1}{2}+\frac{icx}{2}\right)}{2}+\frac{\ln(icx+1)^2}{4}\right)}{d}}{c}$
default	$\frac{-\frac{ia\ln\left(c^2x^2+1\right)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-i\ln(icx+1)\arctan(cx)-\frac{\left(\ln(icx+1)-\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2}+\frac{\mathrm{dilog}\left(\frac{1}{2}+\frac{icx}{2}\right)}{2}+\frac{\ln(icx+1)^2}{4}\right)}{d}}{c}$
parts	$-\frac{ia\ln(c^2x^2+1)}{2cd}+\frac{a\arctan(cx)}{cd}+\frac{b\left(-i\ln(icx+1)\arctan(cx)-\frac{\left(\ln(icx+1)-\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2}+\frac{\mathrm{dilog}\left(\frac{1}{2}+\frac{icx}{2}\right)}{2}\right)}{dc}$
risch	$-\frac{b\ln(icx+1)^2}{4dc}-\frac{ia\ln(c^2x^2+1)}{2cd}-\frac{b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2cd}+\frac{b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2cd}+\frac{a\arctan(cx)}{cd}-$ b dilo

input
```
int((a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I/d*a*ln(c^2*x^2+1)+1/d*a*arctan(c*x)+1/d*b*(-I*ln(1+I*c*x)*arct
an(c*x)-1/2*(ln(1+I*c*x)-ln(1/2+1/2*I*c*x))*ln(1/2-1/2*I*c*x)+1/2*dilog(1/
2+1/2*I*c*x)+1/4*ln(1+I*c*x)^2))
```

3.46.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\int \frac{b\arctan(cx)+a}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(c*d*x - I*d), x)
```

3.46.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx = \frac{b\log(-icx+1)\log(icx+1)}{2cd} - \frac{i\left(\int \frac{ia}{c^2x^2+1}\,dx + \int \frac{acx}{c^2x^2+1}\,dx + \int \left(-\frac{ibcx\log(icx+1)}{c^2x^2+1}\right)dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
b*log(-I*c*x + 1)*log(I*c*x + 1)/(2*c*d) - I*(Integral(I*a/(c**2*x**2 + 1)
, x) + Integral(a*c*x/(c**2*x**2 + 1), x) + Integral(-I*b*c*x*log(I*c*x +
1)/(c**2*x**2 + 1), x))/d
```

3.46.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx = \int \frac{b\arctan(cx)+a}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/8*(8*I*c^2*d*integrate(x*arctan(c*x)/(c^2*d*x^2 + d), x) + 4*c^2*d*inte
grate(x*log(c^2*x^2 + 1)/(c^2*d*x^2 + d), x) - 4*arctan(c*x)^2 - log(c^2*x
^2 + 1)^2)*b/(c*d) - I*a*log(I*c*d*x + d)/(c*d)
```

3.46.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx = \int \frac{b\arctan(cx)+a}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{d + icdx} \, dx = \int \frac{a + b \operatorname{atan}(c\,x)}{d + c\,d\,x\,1\mathrm{i}} \, dx$$

input
```
int((a + b*atan(c*x))/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))/(d + c*d*x*1i), x)
```

3.47 $\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx$

3.47.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx=\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{ib\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
(a+b*arctan(c*x))*ln(2-2/(1+I*c*x))/d+1/2*I*b*polylog(2,-1+2/(1+I*c*x))/d
```

3.47.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.89

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx=\frac{a\log(x)}{d}+\frac{(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right)}{d}+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d}-\frac{ib\operatorname{PolyLog}(2,icx)}{2d}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{i+cx}{i-cx}\right)}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

output
```
(a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)])/d + ((I/2)*b*Pol
yLog[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d + ((I/2)*b*PolyLog[2,
 -((I + c*x)/(I - c*x))])/d
```

3.47.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx$$

$$\Big\downarrow \text{5403}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d}-\frac{bc\int \frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d}+\frac{ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

output
```
((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + ((I/2)*b*PolyLog[2, -1 +
2/(1 + I*c*x)])/d
```

3.47.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

3.47.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(49) = 98.

Time = 0.85 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

method	result
risch	$\frac{ib\ln(icx+1)^2}{4d}+\frac{ib\,\mathrm{dilog}(icx+1)}{2d}-\frac{a\ln(c^2x^2+1)}{2d}+\frac{\ln(-icx)a}{d}-\frac{i\ln(-icx+1)\ln(\frac{1}{2}+\frac{icx}{2})b}{2d}+\frac{i\ln(\frac{1}{2}-\frac{icx}{2})\ln(\frac{1}{2}}{2d}$
parts	$\frac{a\ln(x)}{d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{b\left(\arctan(cx)\ln(cx)-\arctan(cx)\ln(cx-i)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(}{2}\right.}{}$
derivativedivides	$\frac{a\ln(cx)}{d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{b\left(\arctan(cx)\ln(cx)-\arctan(cx)\ln(cx-i)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln}{}\right.}{}$
default	$\frac{a\ln(cx)}{d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{b\left(\arctan(cx)\ln(cx)-\arctan(cx)\ln(cx-i)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln}{}\right.}{}$

input
```
int((a+b*arctan(c*x))/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/d*b*ln(1+I*c*x)^2+1/2*I/d*b*dilog(1+I*c*x)-1/2/d*a*ln(c^2*x^2+1)+1/d
*ln(-I*c*x)*a-1/2*I/d*ln(1-I*c*x)*ln(1/2+1/2*I*c*x)*b+1/2*I/d*ln(1/2-1/2*I
*c*x)*ln(1/2+1/2*I*c*x)*b-I/d*a*arctan(c*x)+1/2*I/d*b*dilog(1/2-1/2*I*c*x)
-1/2*I/d*dilog(1-I*c*x)*b
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx=\frac{-i\,b\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+2\,a\log\left(x\right)-2\,a\log\left(\frac{cx-i}{c}\right)}{2\,d}$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/2*(-I*b*dilog((c*x + I)/(c*x - I) + 1) + 2*a*log(x) - 2*a*log((c*x - I)/
c))/d
```

3.47.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx = -\frac{i\left(\int \frac{a}{cx^2-ix}\,dx+\int \frac{b\,\mathrm{atan}\,(cx)}{cx^2-ix}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/x/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a/(c*x**2 - I*x), x) + Integral(b*atan(c*x)/(c*x**2 - I*x), x
))/d
```

3.47.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*b*(I*arctan(c*x)^2/d - 2*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x))
 - a*(log(I*c*x + 1)/d - log(x)/d)
```

3.47.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)), x)
```

3.48 $\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx$

3.48.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx=-\frac{a+b\arctan(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log(1+c^2x^2)}{2d}-\frac{ic(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{bc\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
(-a-b*arctan(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(c^2*x^2+1)/d-I*c*(a+b*arctan
(c*x))*ln(2-2/(1+I*c*x))/d+1/2*b*c*polylog(2,-1+2/(1+I*c*x))/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.49

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx=-\frac{a+b\arctan(cx)}{dx}-\frac{iac\log(x)}{d}-\frac{ic(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right)}{d}+\frac{bc(2\log(x)-\log(1+c^2x^2))}{2d}+\frac{bc\,\text{PolyLog}(2,-icx)}{2d}-\frac{bc\,\text{PolyLog}(2,icx)}{2d}+\frac{bc\,\text{PolyLog}\left(2,-\frac{i+cx}{i-cx}\right)}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

output
```
-((a + b*ArcTan[c*x])/(d*x)) - (I*a*c*Log[x])/d - (I*c*(a + b*ArcTan[c*x])
*Log[(2*I)/(I - c*x)])/d + (b*c*(2*Log[x] - Log[1 + c^2*x^2]))/(2*d) + (b*
c*PolyLog[2, (-I)*c*x])/(2*d) - (b*c*PolyLog[2, I*c*x])/(2*d) + (b*c*PolyL
og[2, -((I + c*x)/(I - c*x))])/(2*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5405, 27, 5361, 243, 47, 14, 16, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^2}dx}{d}-ic\int \frac{a+b\arctan(cx)}{dx(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^2}dx}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \textcolor{blue}{16}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \textcolor{blue}{5403}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))-bc\int\frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \textcolor{blue}{2897}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{1}{2}ib\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

output
```
(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2)/d - (I*
c*((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)] + (I/2)*b*PolyLog[2, -1 + 2/
(1 + I*c*x)]))/d
```

3.48.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

3.48. $\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx$

3.48.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(95) = 190$.

Time = 0.86 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.00

method	result
parts	$-\frac{a}{dx}-\frac{iac\ln(x)}{d}+\frac{ica\ln(c^2x^2+1)}{2d}-\frac{ca\arctan(cx)}{d}+\frac{bc\left(-\frac{\arctan(cx)}{cx}-i\arctan(cx)\ln(cx)+i\arctan(cx)\ln(cx-\right.}{}$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ia\ln(cx)}{d}+\frac{ia\ln(c^2x^2+1)}{2d}-\frac{a\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{cx}-i\arctan(cx)\ln(cx)+i\arctan(cx)\ln(cx\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ia\ln(cx)}{d}+\frac{ia\ln(c^2x^2+1)}{2d}-\frac{a\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{cx}-i\arctan(cx)\ln(cx)+i\arctan(cx)\ln(cx\right.}{}\right.$
risch	$\frac{bc\ln(icx+1)^2}{4d}+\frac{bc\,\mathrm{dilog}(icx+1)}{2d}+\frac{bc\ln(icx)}{2d}-\frac{bc\ln(icx+1)}{2d}+\frac{ib\ln(icx+1)}{2dx}-\frac{a}{dx}-\frac{ic\ln(-icx)a}{d}+\frac{ica\ln(c^2}{2d}$

input
```
int((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x-I/d*a*c*ln(x)+1/2*I*c/d*a*ln(c^2*x^2+1)-c/d*a*arctan(c*x)+1/d*b*c*(
-1/c/x*arctan(c*x)-I*arctan(c*x)*ln(c*x)+I*arctan(c*x)*ln(c*x-I)+1/2*ln(c*
x)*ln(1+I*c*x)-1/2*ln(c*x)*ln(1-I*c*x)+1/2*dilog(1+I*c*x)-1/2*dilog(1-I*c*
x)+1/2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*dilog(-1/2*I*(c*x+I))-1/4*ln(c*x-I
)^2-1/2*ln(c^2*x^2+1)+ln(c*x))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx = -\frac{bcx\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+2\,(i\,a-b)cx\log(x)+bcx\log\left(\frac{cx+i}{c}\right)-(2i\,a-b)cx\log\left(\frac{cx-i}{c}\right)+i\,b\log\left(-\frac{cx+i}{cx-i}\right)+2\,a}{2\,dx}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
-1/2*(b*c*x*dilog((c*x + I)/(c*x - I) + 1) + 2*(I*a - b)*c*x*log(x) + b*c*
x*log((c*x + I)/c) - (2*I*a - b)*c*x*log((c*x - I)/c) + I*b*log(-(c*x + I)
/(c*x - I)) + 2*a)/(d*x)
```

3.48.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx=-\frac{i\left(\int \frac{a}{cx^3-ix^2}\,dx+\int \frac{b\,\mathrm{atan}\,(cx)}{cx^3-ix^2}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a/(c*x**3 - I*x**2), x) + Integral(b*atan(c*x)/(c*x**3 - I*x*
*2), x))/d
```

3.48.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
(-I*c*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x) + integrate(arctan(c*x)/
(c^2*d*x^4 + d*x^2), x))*b + a*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d
*x))
```

3.48.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)), x)
```

3.49 $\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx$

3.49.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx = & -\frac{bc}{2dx}-\frac{bc^2\arctan(cx)}{2d}-\frac{a+b\arctan(cx)}{2dx^2} \\ & +\frac{ic(a+b\arctan(cx))}{dx}-\frac{ibc^2\log(x)}{d}+\frac{ibc^2\log(1+c^2x^2)}{2d} \\ & -\frac{c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d} \\ & -\frac{ibc^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

output
```
-1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d+1/2*(-a-b*arctan(c*x))/d/x^2+I*c*(a+b
*arctan(c*x))/d/x-I*b*c^2*ln(x)/d+1/2*I*b*c^2*ln(c^2*x^2+1)/d-c^2*(a+b*arc
tan(c*x))*ln(2-2/(1+I*c*x))/d-1/2*I*b*c^2*polylog(2,-1+2/(1+I*c*x))/d
```

3.49.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx =$$
$$-\frac{\frac{a+b\arctan(cx)}{x^2}-\frac{2ic(a+b\arctan(cx))}{x}+\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x}+2ac^2\log(x)+2c^2(a+b\arctan(cx))\log}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

output
```
-1/2*((a + b*ArcTan[c*x])/x^2 - ((2*I)*c*(a + b*ArcTan[c*x]))/x + (b*c*Hyp
ergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 2*a*c^2*Log[x] + 2*c^2*(a +
b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + I*b*c^2*(2*Log[x] - Log[1 + c^2*x^2]
) + I*b*c^2*PolyLog[2, (-I)*c*x] - I*b*c^2*PolyLog[2, I*c*x] + I*b*c^2*Pol
yLog[2, (I + c*x)/(-I + c*x)])/d
```

3.49.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5405, 27, 5361, 264, 216, 5405, 5361, 243, 47, 14, 16, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^3}dx}{d}-ic\int \frac{a+b\arctan(cx)}{dx^2(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^3}dx}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(\int\frac{a+b\arctan(cx)}{x^2}dx-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \textbf{5361}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+bc\int\frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \textbf{47}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\left(\int\frac{1}{x^2}dx^2-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \textbf{14}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)}{d}$$

$$\Big\downarrow \textbf{5403}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))-bc\int\frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right.\right.\right.}{d}$$

$$\Big\downarrow \textbf{2897}$$

3.49. $\int\frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-$$

$$\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{1}{2}ib\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\log\left(c^2\right.\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2)/d - (I*
c*(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2 - I*c*
((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)] + (I/2)*b*PolyLog[2, -1 + 2/(1
 + I*c*x)])))/d
```

3.49.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

3.49. $\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx$

3.49.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.56

method	result
derivativedivides	$c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{ia}{dcx}-\frac{a\ln(cx)}{d}+\frac{a\ln(c^2x^2+1)}{2d}+\frac{ia\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{i\arctan(cx)}{cx}-\arctan(cx)\ln\right.}{}\right.$
default	$c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{ia}{dcx}-\frac{a\ln(cx)}{d}+\frac{a\ln(c^2x^2+1)}{2d}+\frac{ia\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{i\arctan(cx)}{cx}-\arctan(cx)\ln\right.}{}\right.$
parts	$-\frac{a}{2d\,x^2}+\frac{iac}{dx}-\frac{a\,c^2\ln(x)}{d}+\frac{c^2a\ln(c^2x^2+1)}{2d}+\frac{ic^2a\arctan(cx)}{d}+\frac{b\,c^2\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{i\arctan(cx)}{cx}-\arctan(cx)\ln\right.}{}$
risch	$-\frac{ic^2b\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d}-\frac{ib\ln(-icx+1)}{4d\,x^2}-\frac{ic^2b\ln(-icx)}{4d}+\frac{ic^2b\ln(-icx+1)}{4d}-\frac{3b\,c^2\arctan(cx)}{4d}+\frac{bc\ln(icx+1)}{2dx}-$

input
```
int((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2/d*a/c^2/x^2+I/d*a/c/x-1/d*a*ln(c*x)+1/2/d*a*ln(c^2*x^2+1)+I/d*a*
arctan(c*x)+1/d*b*(-1/2/c^2/x^2*arctan(c*x)+I*arctan(c*x)/c/x-arctan(c*x)*
ln(c*x)+arctan(c*x)*ln(c*x-I)-I*ln(c*x)-1/2/c/x+1/2*I*ln(c^2*x^2+1)-1/2*ar
ctan(c*x)-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(
1+I*c*x)+1/2*I*dilog(1-I*c*x)-1/2*I*(dilog(-1/2*I*(c*x+I))+ln(c*x-I)*ln(-1
/2*I*(c*x+I)))+1/4*I*ln(c*x-I)^2))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx = \frac{2i\,bc^2x^2\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(a+i\,b)c^2x^2\log(x)+i\,bc^2x^2\log\left(\frac{cx+i}{c}\right)+(4\,a+3i\,b)c^2x^2\log\left(\frac{cx-i}{c}\right)-2\,(-2i\,a}{4\,dx^2}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/4*(2*I*b*c^2*x^2*dilog((c*x + I)/(c*x - I) + 1) - 4*(a + I*b)*c^2*x^2*lo
g(x) + I*b*c^2*x^2*log((c*x + I)/c) + (4*a + 3*I*b)*c^2*x^2*log((c*x - I)/
c) - 2*(-2*I*a + b)*c*x - (2*b*c*x + I*b)*log(-(c*x + I)/(c*x - I)) - 2*a)
/(d*x^2)
```

3.49.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx=-\frac{i\left(\int \frac{a}{cx^4-ix^3}\,dx+\int \frac{b\,\mathrm{atan}\,(cx)}{cx^4-ix^3}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a/(c*x**4 - I*x**3), x) + Integral(b*atan(c*x)/(c*x**4 - I*x*
*3), x))/d
```

3.49.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx=\int \frac{b\arctan\,(cx)+a}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a +
(-I*c*integrate(arctan(c*x)/(c^2*d*x^4 + d*x^2), x) + integrate(arctan(c*x
)/(c^2*d*x^5 + d*x^3), x))*b
```

3.49.8 Giac [F]

$$\int \frac{a + b\arctan(cx)}{x^3(d + icdx)}\,dx = \int \frac{b\arctan(cx) + a}{(i\,cdx + d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^3(d + icdx)}\,dx = \int \frac{a + b\,\mathrm{atan}(c\,x)}{x^3\,(d + c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)), x)
```

3.50 $\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$

3.50.1 Optimal result

Integrand size = 23, antiderivative size = 197

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx = &-\frac{bc}{6dx^2}+\frac{ibc^2}{2dx}+\frac{ibc^3\arctan(cx)}{2d}-\frac{a+b\arctan(cx)}{3dx^3}\\ &+\frac{ic(a+b\arctan(cx))}{2dx^2}+\frac{c^2(a+b\arctan(cx))}{dx}-\frac{4bc^3\log(x)}{3d}\\ &+\frac{2bc^3\log(1+c^2x^2)}{3d}+\frac{ic^3(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}\\ &-\frac{bc^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

output
```
-1/6*b*c/d/x^2+1/2*I*b*c^2/d/x+1/2*I*b*c^3*arctan(c*x)/d+1/3*(-a-b*arctan(
c*x))/d/x^3+1/2*I*c*(a+b*arctan(c*x))/d/x^2+c^2*(a+b*arctan(c*x))/d/x-4/3*
b*c^3*ln(x)/d+2/3*b*c^3*ln(c^2*x^2+1)/d+I*c^3*(a+b*arctan(c*x))*ln(2-2/(1+
I*c*x))/d-1/2*b*c^3*polylog(2,-1+2/(1+I*c*x))/d
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.29

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$$
$$= \frac{-2a+3iacx-bcx+6ac^2x^2-2b\arctan(cx)+3ibcx\arctan(cx)+6bc^2x^2\arctan(cx)+3ibc^2x^2\,\text{Hypergeo}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

output
```
(-2*a + (3*I)*a*c*x - b*c*x + 6*a*c^2*x^2 - 2*b*ArcTan[c*x] + (3*I)*b*c*x*
ArcTan[c*x] + 6*b*c^2*x^2*ArcTan[c*x] + (3*I)*b*c^2*x^2*Hypergeometric2F1[
-1/2, 1, 1/2, -(c^2*x^2)] + (6*I)*a*c^3*x^3*Log[x] - 8*b*c^3*x^3*Log[x] +
(6*I)*a*c^3*x^3*Log[(2*I)/(I - c*x)] + (6*I)*b*c^3*x^3*ArcTan[c*x]*Log[(2*
I)/(I - c*x)] + 4*b*c^3*x^3*Log[1 + c^2*x^2] - 3*b*c^3*x^3*PolyLog[2, (-I)
*c*x] + 3*b*c^3*x^3*PolyLog[2, I*c*x] - 3*b*c^3*x^3*PolyLog[2, (I + c*x)/(
-I + c*x)])/(6*d*x^3)
```

3.50.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {5405, 27, 5361, 243, 54, 2009, 5405, 5361, 264, 216, 5405, 5361, 243, 47, 14, 16, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$$
$$\Big\downarrow \; 5405$$
$$\frac{\int \frac{a+b\arctan(cx)}{x^4}dx}{d}-ic\int \frac{a+b\arctan(cx)}{dx^3(icx+1)}dx$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{a+b\arctan(cx)}{x^4}dx}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{3}bc\int \frac{1}{x^3(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{6}bc\int \frac{1}{x^4(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{\frac{1}{6}bc\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(\int \frac{a+b\arctan(cx)}{x^3}dx-ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx+\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\Big\downarrow \text{5405}$$

3.50. $\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$
$$\frac{ic\left(-ic\left(\int\frac{a+b\arctan(cx)}{x^2}dx-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\Big\downarrow \; 5361$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$
$$\frac{ic\left(-ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+bc\int\frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$
$$\frac{ic\left(-ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\Big\downarrow \; 47$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$
$$ic\Big(-ic\Big(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\left(\int\frac{1}{x^2}dx^2-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\Big)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\big(-c\arctan($$
$$d$$

$$\Big\downarrow \; 14$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$
$$ic\Big(-ic\Big(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\Big)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\big(-c\arctan($$
$$d$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$
$$ic\Big(-ic\Big(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\Big)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\big(-c\arctan(c$$
$$d$$

$$\Big\downarrow \; 5403$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$
$$ic\Bigg(-ic\Bigg(-ic\Bigg(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))-bc\int\frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\Bigg)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\big(\log\left(x^2\right)-\log\big(c^2x$$
$$d$$

3.50. $\int\frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$

$$\Big\downarrow \text{2897}$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\text{lo}\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

output
```
(-1/3*(a + b*ArcTan[c*x])/x^3 + (b*c*(-x^(-2) - c^2*Log[x^2] + c^2*Log[1 +
 c^2*x^2]))/6)/d - (I*c*(-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*
ArcTan[c*x]))/2 - I*c*(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 +
 c^2*x^2]))/2 - I*c*((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)] + (I/2)*b*
PolyLog[2, -1 + 2/(1 + I*c*x)]))))/d
```

3.50.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

3.50. $\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

3.50.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.41

method	result
derivativedivides	$c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ia}{2d\,c^2x^2}+\frac{ia\ln(cx)}{d}+\frac{a}{dcx}-\frac{ia\ln(c^2x^2+1)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{3c^3x^3}+\frac{i\arctan(cx)}{2c^2x^2}+\right.}{}\right.$
default	$c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ia}{2d\,c^2x^2}+\frac{ia\ln(cx)}{d}+\frac{a}{dcx}-\frac{ia\ln(c^2x^2+1)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{3c^3x^3}+\frac{i\arctan(cx)}{2c^2x^2}+\right.}{}\right.$
parts	$-\frac{a}{3d\,x^3}+\frac{ica}{2d\,x^2}+\frac{ia\,c^3\ln(x)}{d}+\frac{c^2a}{dx}-\frac{ic^3a\ln(c^2x^2+1)}{2d}+\frac{c^3a\arctan(cx)}{d}+\frac{b\,c^3\left(-\frac{\arctan(cx)}{3c^3x^3}+\frac{i\arctan(cx)}{2c^2x^2}+\right.}{}$
risch	$-\frac{bc}{6d\,x^2}+\frac{11b\,c^3\ln(c^2x^2+1)}{24d}+\frac{ib\,c^2}{2dx}+\frac{ib\ln(icx+1)}{6d\,x^3}+\frac{ic^3\ln(-icx)a}{d}-\frac{c^3\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)b}{2d}+\frac{c^3b\ln\left(\frac{1}{2}\right.}{}$

input
```
int((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3/d*a/c^3/x^3+1/2*I/d*a/c^2/x^2+I/d*a*ln(c*x)+a/d/c/x-1/2*I/d*a*ln
(c^2*x^2+1)+1/d*a*arctan(c*x)+1/d*b*(-1/3*arctan(c*x)/c^3/x^3+1/2*I*arctan
(c*x)/c^2/x^2+1/2*I/c/x+1/c/x*arctan(c*x)+I*arctan(c*x)*ln(c*x)-1/2*ln(c*x
-I)*ln(-1/2*I*(c*x+I))-1/2*dilog(-1/2*I*(c*x+I))+1/4*ln(c*x-I)^2-1/2*ln(c*
x)*ln(1+I*c*x)+1/2*ln(c*x)*ln(1-I*c*x)-1/2*dilog(1+I*c*x)+1/2*dilog(1-I*c*
x)+1/2*I*arctan(c*x)-1/6/c^2/x^2-4/3*ln(c*x)+2/3*ln(c^2*x^2+1)-I*arctan(c*
x)*ln(c*x-I)))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$$
$$= \frac{6\,bc^3x^3\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(-3i\,a+4\,b)c^3x^3\log\left(x\right)+5\,bc^3x^3\log\left(\frac{cx+i}{c}\right)+(-12i\,a+11\,b)c^3x^3\log\left(\frac{cx-i}{c}\right)+6}{12\,dx^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/12*(6*b*c^3*x^3*dilog((c*x + I)/(c*x - I) + 1) - 4*(-3*I*a + 4*b)*c^3*x^
3*log(x) + 5*b*c^3*x^3*log((c*x + I)/c) + (-12*I*a + 11*b)*c^3*x^3*log((c*
x - I)/c) + 6*(2*a + I*b)*c^2*x^2 - 2*(-3*I*a + b)*c*x + (6*I*b*c^2*x^2 -
3*b*c*x - 2*I*b)*log(-(c*x + I)/(c*x - I)) - 4*a)/(d*x^3)
```

3.50.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx = -\frac{i\left(\int \frac{a}{cx^5-ix^4}\,dx+\int \frac{b\,\mathrm{atan}\,(cx)}{cx^5-ix^4}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/x**4/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a/(c*x**5 - I*x**4), x) + Integral(b*atan(c*x)/(c*x**5 - I*x**
*4), x))/d
```

3.50.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*(6*I*c^3*log(I*c*x + 1)/d - 6*I*c^3*log(x)/d - (6*c^2*x^2 + 3*I*c*x -
 2)/(d*x^3))*a + (-I*c*integrate(arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + int
egrate(arctan(c*x)/(c^2*d*x^6 + d*x^4), x))*b
```

3.50.8 Giac [F]

$$\int \frac{a + b \arctan(cx)}{x^4(d + icdx)}\, dx = \int \frac{b \arctan(cx) + a}{(i\, cdx + d)x^4}\, dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x^4(d + icdx)}\, dx = \int \frac{a + b\, \mathrm{atan}(c\, x)}{x^4\, (d + c\, d\, x\, 1\mathrm{i})}\, dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + c*d*x*1i)), x)
```

3.51 $\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx$

3.51.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx = &-\frac{2iax}{c^3d^2}+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(i-cx)}-\frac{b\arctan(cx)}{c^4d^2}\\ &-\frac{2ibx\arctan(cx)}{c^3d^2}-\frac{x^2(a+b\arctan(cx))}{2c^2d^2}\\ &+\frac{i(a+b\arctan(cx))}{c^4d^2(i-cx)}-\frac{3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^2}\\ &+\frac{ib\log(1+c^2x^2)}{c^4d^2}-\frac{3ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^2}\end{aligned}$$

output
```
-2*I*a*x/c^3/d^2+1/2*b*x/c^3/d^2+1/2*b/c^4/d^2/(I-c*x)-b*arctan(c*x)/c^4/d
^2-2*I*b*x*arctan(c*x)/c^3/d^2-1/2*x^2*(a+b*arctan(c*x))/c^2/d^2+I*(a+b*ar
ctan(c*x))/c^4/d^2/(I-c*x)-3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d^2+I*b
*ln(c^2*x^2+1)/c^4/d^2-3/2*I*b*polylog(2,1-2/(1+I*c*x))/c^4/d^2
```

3.51.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx =$$
$$-\frac{8iacx+2ac^2x^2+\frac{4ia}{-i+cx}-12ia\arctan(cx)-6a\log\left(1+c^2x^2\right)+b\big(-2cx-12i\arctan(cx)^2+i\cos(2\operatorname{arc}}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-1/4*((8*I)*a*c*x + 2*a*c^2*x^2 + ((4*I)*a)/(-I + c*x) - (12*I)*a*ArcTan[c
*x] - 6*a*Log[1 + c^2*x^2] + b*(-2*c*x - (12*I)*ArcTan[c*x]^2 + I*Cos[2*Ar
cTan[c*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((2*I)*ArcTan[c*
x])] + 2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan[c*x]] + 6*Log
[1 + E^((2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[c*x]]))
/(c^4*d^2)
```

3.51.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{3(a+b\arctan(cx))}{c^3d^2(cx-i)}-\frac{2i(a+b\arctan(cx))}{c^3d^2}+\frac{i(a+b\arctan(cx))}{c^3d^2(cx-i)^2}-\frac{x(a+b\arctan(cx))}{c^2d^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{c^4d^2(-cx+i)}-\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^2}-\frac{x^2(a+b\arctan(cx))}{2c^2d^2}-\frac{2iax}{c^3d^2}-\frac{b\arctan(cx)}{c^4d^2}-\frac{2ibx\arctan(cx)}{c^3d^2}-\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^4d^2}+\frac{b}{2c^4d^2(-cx+i)}+\frac{bx}{2c^3d^2}+\frac{ib\log\left(c^2x^2+1\right)}{c^4d^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
((-2*I)*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(I - c*x)) - (b*
ArcTan[c*x])/(c^4*d^2) - ((2*I)*b*x*ArcTan[c*x])/(c^3*d^2) - (x^2*(a + b*A
rcTan[c*x]))/(2*c^2*d^2) + (I*(a + b*ArcTan[c*x]))/(c^4*d^2*(I - c*x)) - (
3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^2) + (I*b*Log[1 + c^2*x^2
])/(c^4*d^2) - (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^2)
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.51.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.56

method	result
derivativedivides	$-\frac{2ib\arctan(cx)cx}{d^2}-\frac{a\,c^2x^2}{2d^2}-\frac{ia}{d^2(cx-i)}+\frac{3a\ln\left(c^2x^2+1\right)}{2d^2}+\frac{3ib\ln(cx-i)^2}{4d^2}+\frac{3ia\arctan(cx)}{d^2}-\frac{b\arctan(cx)c^2x^2}{2d^2}+\frac{ib\ln\left(c^4x^4+10c^2x^2 \cdots\right.}{8d^2} \cdots$
default	$-\frac{2ib\arctan(cx)cx}{d^2}-\frac{a\,c^2x^2}{2d^2}-\frac{ia}{d^2(cx-i)}+\frac{3a\ln\left(c^2x^2+1\right)}{2d^2}+\frac{3ib\ln(cx-i)^2}{4d^2}+\frac{3ia\arctan(cx)}{d^2}-\frac{b\arctan(cx)c^2x^2}{2d^2}+\frac{ib\ln\left(c^4x^4+10c^2x^2 \cdots\right.}{8d^2} \cdots$
parts	$-\frac{x^2a}{2d^2c^2}+\frac{ib\ln(c^4x^4+10c^2x^2+9)}{8d^2c^4}-\frac{3ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d^2c^4}+\frac{3a\ln(c^2x^2+1)}{2d^2c^4}-\frac{ib}{2d^2c^4}-\frac{b\arctan(cx)x^2}{2d^2c^2}-\frac{2ibx \cdots}{\cdots}$
risch	$\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(-cx+i)}-\frac{i\ln(-icx+1)b\,x^2}{4d^2c^2}-\frac{3b\arctan(cx)}{2c^4d^2}-\frac{ib\ln(-icx+1)}{4d^2c^4(-icx-1)}-\frac{b\ln(-icx+1)x}{4d^2c^3(-icx-1)}-\frac{3ib\ln\left(\frac{1}{2}+ \cdots\right.}{\cdots}$

input
```
int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-2*I*b/d^2*arctan(c*x)*c*x-1/2*a/d^2*c^2*x^2-I*a/d^2/(c*x-I)+3/2*a/
d^2*ln(c^2*x^2+1)+3/4*I*b/d^2*ln(c*x-I)^2+3*I*a/d^2*arctan(c*x)-1/2*b/d^2*
arctan(c*x)*c^2*x^2+1/8*I*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)+3*b/d^2*arctan(c*
x)*ln(c*x-I)-3/2*I*b/d^2*dilog(-1/2*I*(c*x+I))-1/2*I*b/d^2-2*I*a/d^2*c*x+1
/2*b/d^2*c*x-I*b/d^2*arctan(c*x)/(c*x-I)+3/4*I*b/d^2*ln(c^2*x^2+1)-1/4*b/d
^2*arctan(1/2*c*x)+1/4*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/2*b/d^2*arctan(
1/2*c*x-1/2*I)-1/2*b/d^2/(c*x-I)-3/2*I*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)-
3/2*b*arctan(c*x)/d^2)
```

3.51.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-I*b*x^3*log(-(c*x + I)/(c*x - I)) - 2*a*x^3)/(c^2*d^2*x^2 -
 2*I*c*d^2*x - d^2), x)
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.51.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(2*I/(c^5*d^2*x - I*c^4*d^2) + (c*x^2 + 4*I*x)/(c^3*d^2) - 6*log(c*
x - I)/(c^4*d^2)) + 1/8*(I*c^3*x^3 - 5*c^2*x^2 - 2*c*x*(arctan2(1, c*x) -
3*I) - 12*(-I*c*x - 1)*arctan(c*x)^2 - 3*(-I*c*x - 1)*log(c^2*x^2 + 1)^2 -
 3*(c^5*d^2*x - I*c^4*d^2)*((c*(x/(c^7*d^2*x^2 + c^5*d^2) + arctan(c*x)/(c
^6*d^2)) - 2*arctan(c*x)/(c^7*d^2*x^2 + c^5*d^2))*c + 16*integrate(1/8*log
(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x)) + 3*(-I*c^5*d^2
*x - c^4*d^2)*(c*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/(c^7*d^2*
x^2 + c^5*d^2)) + 32*integrate(1/8*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^
2 + c^3*d^2), x)) + 6*(c^6*d^2*x - I*c^5*d^2)*(c*(x/(c^7*d^2*x^2 + c^5*d^2
) + arctan(c*x)/(c^6*d^2)) - 16*c*integrate(1/8*x^2*log(c^2*x^2 + 1)/(c^7*
d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 2*arctan(c*x)/(c^7*d^2*x^2 + c^5*
d^2)) - 6*(I*c^6*d^2*x + c^5*d^2)*(32*c*integrate(1/8*x^2*arctan(c*x)/(c^7
*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - c^2/(c^9*d^2*x^2 + c^7*d^2) - lo
g(c^2*x^2 + 1)/(c^7*d^2*x^2 + c^5*d^2)) - 2*(c^3*x^3 + 3*I*c^2*x^2 + 4*c*x
 + 2*I)*arctan(c*x) - 16*(c^9*d^2*x - I*c^8*d^2)*integrate(1/8*(2*c*x^5*ar
ctan(c*x) + x^4*log(c^2*x^2 + 1))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2),
 x) - 16*(-I*c^9*d^2*x - c^8*d^2)*integrate(1/8*(c*x^5*log(c^2*x^2 + 1) -
2*x^4*arctan(c*x))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 16*(I*c^8
*d^2*x + c^7*d^2)*integrate(1/8*(2*c*x^4*arctan(c*x) + x^3*log(c^2*x^2 + 1
))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 16*(c^8*d^2*x - I*c^7*...
```

3.51.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

3.52 $\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx$

3.52.1 Optimal result

Integrand size = 23, antiderivative size = 167

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = &-\frac{ax}{c^2d^2}-\frac{ib}{2c^3d^2(i-cx)}+\frac{ib\arctan(cx)}{2c^3d^2}-\frac{bx\arctan(cx)}{c^2d^2}\\&+\frac{a+b\arctan(cx)}{c^3d^2(i-cx)}+\frac{2i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2}\\&+\frac{b\log(1+c^2x^2)}{2c^3d^2}-\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}\end{aligned}$$

output
```
-a*x/c^2/d^2-1/2*I*b/c^3/d^2/(I-c*x)+1/2*I*b*arctan(c*x)/c^3/d^2-b*x*arcta
n(c*x)/c^2/d^2+(a+b*arctan(c*x))/c^3/d^2/(I-c*x)+2*I*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c^3/d^2+1/2*b*ln(c^2*x^2+1)/c^3/d^2-b*polylog(2,1-2/(1+I*c*x)
)/c^3/d^2
```

3.52.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx =$$

$$-\frac{4acx+\frac{4a}{-i+cx}-8a\arctan(cx)+4ia\log(1+c^2x^2)+b\big(-8\arctan(cx)^2+\cos(2\arctan(cx))-2\log(1+}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-1/4*(4*a*c*x + (4*a)/(-I + c*x) - 8*a*ArcTan[c*x] + (4*I)*a*Log[1 + c^2*x
^2] + b*(-8*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*Po
lyLog[2, -E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2
*c*x + I*Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2
*ArcTan[c*x]])))/(c^3*d^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{2i(a+b\arctan(cx))}{c^2d^2(cx-i)}-\frac{a+b\arctan(cx)}{c^2d^2}+\frac{a+b\arctan(cx)}{c^2d^2(cx-i)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a+b\arctan(cx)}{c^3d^2(-cx+i)}+\frac{2i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^3d^2}-\frac{ax}{c^2d^2}+\frac{ib\arctan(cx)}{2c^3d^2}-\frac{bx\arctan(cx)}{c^2d^2}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^3d^2}-\frac{ib}{2c^3d^2(-cx+i)}+\frac{b\log\left(c^2x^2+1\right)}{2c^3d^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-((a*x)/(c^2*d^2)) - ((I/2)*b)/(c^3*d^2*(I - c*x)) + ((I/2)*b*ArcTan[c*x])
/(c^3*d^2) - (b*x*ArcTan[c*x])/(c^2*d^2) + (a + b*ArcTan[c*x])/(c^3*d^2*(I
 - c*x)) + ((2*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + (b*L
og[1 + c^2*x^2])/(2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.52.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.62

method	result
derivativedivides	$-\frac{acx}{d^2}-\frac{a}{d^2(cx-i)}+\frac{2a\arctan(cx)}{d^2}+\frac{ib\arctan\left(\frac{cx}{2}\right)}{8d^2}-\frac{b\arctan(cx)cx}{d^2}-\frac{b\arctan(cx)}{d^2(cx-i)}-\frac{2ib\arctan(cx)\ln(cx-i)}{d^2}-\frac{b\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx}{d^2}$
default	$-\frac{acx}{d^2}-\frac{a}{d^2(cx-i)}+\frac{2a\arctan(cx)}{d^2}+\frac{ib\arctan\left(\frac{cx}{2}\right)}{8d^2}-\frac{b\arctan(cx)cx}{d^2}-\frac{b\arctan(cx)}{d^2(cx-i)}-\frac{2ib\arctan(cx)\ln(cx-i)}{d^2}-\frac{b\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx}{d^2}$
parts	$-\frac{ax}{c^2d^2}+\frac{a}{d^2c^3(-cx+i)}-\frac{2ib\arctan(cx)\ln(cx-i)}{c^3d^2}+\frac{2a\arctan(cx)}{d^2c^3}-\frac{bx\arctan(cx)}{c^2d^2}-\frac{b\arctan(cx)}{c^3d^2(cx-i)}+\frac{3ib\,\mathrm{arcta}}{4c^3}$
risch	$-\frac{b\ln(icx+1)^2}{2c^3d^2}+\left(\frac{ibx}{2c^2d^2}+\frac{ib}{2c^3d^2(cx-i)}\right)\ln(icx+1)+\frac{3ib\arctan(cx)}{4c^3d^2}-\frac{b}{2c^3d^2}-\frac{b\ln(-icx+1)}{4d^2c^3(-icx-1)}-$

input
```
int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a/d^2*c*x-a/d^2/(c*x-I)+2*a/d^2*arctan(c*x)+1/8*I*b/d^2*arctan(1/2
*c*x)-b/d^2*arctan(c*x)*c*x-b/d^2*arctan(c*x)/(c*x-I)-2*I*b/d^2*arctan(c*x
)*ln(c*x-I)-b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)-b/d^2*dilog(-1/2*I*(c*x+I))
+1/2*b/d^2*ln(c*x-I)^2+1/16*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)+3/4*I*b/d^2*arc
tan(c*x)-I*a/d^2*ln(c^2*x^2+1)+1/2*I*b/d^2/(c*x-I)-1/4*I*b/d^2*arctan(1/2*
c*x-1/2*I)+3/8*b/d^2*ln(c^2*x^2+1)-1/8*I*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)
)
```

3.52.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-I*b*x^2*log(-(c*x + I)/(c*x - I)) - 2*a*x^2)/(c^2*d^2*x^2 -
 2*I*c*d^2*x - d^2), x)
```

3.52.6 Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx$$
$$= \frac{(-ibc^2x^2 + 2bcx\log(icx+1) - bcx - 2ib\log(icx+1) - ib)\log(-icx+1)}{2c^4d^2x - 2ic^3d^2}$$
$$- \frac{\int\left(-\frac{b}{c^3x^3-ic^2x^2+cx-i}\right)dx + \int\left(-\frac{2b\log(icx+1)}{c^3x^3-ic^2x^2+cx-i}\right)dx + \int\frac{2ac^3x^3}{c^3x^3-ic^2x^2+cx-i}\,dx + \int\left(-\frac{2bc^2x^2}{c^3x^3-ic^2x^2+cx-i}\right)dx + }{}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
(-I*b*c**2*x**2 + 2*b*c*x*log(I*c*x + 1) - b*c*x - 2*I*b*log(I*c*x + 1) -
I*b)*log(-I*c*x + 1)/(2*c**4*d**2*x - 2*I*c**3*d**2) - (Integral(-b/(c**3*
x**3 - I*c**2*x**2 + c*x - I), x) + Integral(-2*b*log(I*c*x + 1)/(c**3*x**
3 - I*c**2*x**2 + c*x - I), x) + Integral(2*a*c**3*x**3/(c**3*x**3 - I*c**
2*x**2 + c*x - I), x) + Integral(-2*b*c**2*x**2/(c**3*x**3 - I*c**2*x**2 +
 c*x - I), x) + Integral(2*I*a*c**2*x**2/(c**3*x**3 - I*c**2*x**2 + c*x -
I), x) + Integral(-I*b*c**3*x**3/(c**3*x**3 - I*c**2*x**2 + c*x - I), x) +
 Integral(3*b*c**2*x**2*log(I*c*x + 1)/(c**3*x**3 - I*c**2*x**2 + c*x - I)
, x) + Integral(-4*I*b*c*x*log(I*c*x + 1)/(c**3*x**3 - I*c**2*x**2 + c*x -
 I), x) + Integral(-I*b*c**3*x**3*log(I*c*x + 1)/(c**3*x**3 - I*c**2*x**2
+ c*x - I), x))/(2*c**2*d**2)
```

3.52.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-a*(1/(c^4*d^2*x - I*c^3*d^2) + x/(c^2*d^2) + 2*I*log(c*x - I)/(c^3*d^2))
+ 1/4*(2*I*c^2*x^2 + 4*(c*x - I)*arctan(c*x)^2 + (c*x - I)*log(c^2*x^2 + 1
)^2 - (-I*c^4*d^2*x - c^3*d^2)*((c*(x/(c^6*d^2*x^2 + c^4*d^2) + arctan(c*x
)/(c^5*d^2)) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*d^2))*c + 8*integrate(1/4*
log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x)) - (c^4*d^2*x
 - I*c^3*d^2)*(c*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(c^6*d^2*
x^2 + c^4*d^2)) + 16*integrate(1/4*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^
2 + c^2*d^2), x)) + 2*(-I*c^5*d^2*x - c^4*d^2)*(c*(x/(c^6*d^2*x^2 + c^4*d^
2) + arctan(c*x)/(c^5*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^6*
d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*
d^2)) - 2*(c^5*d^2*x - I*c^4*d^2)*(16*c*integrate(1/4*x^2*arctan(c*x)/(c^6
*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - c^2/(c^8*d^2*x^2 + c^6*d^2) - lo
g(c^2*x^2 + 1)/(c^6*d^2*x^2 + c^4*d^2)) + 2*c*x - 2*(c^2*x^2 - I*c*x + 1)*
arctan(c*x) - 4*(c^7*d^2*x - I*c^6*d^2)*integrate(1/4*(2*c*x^4*arctan(c*x)
 + x^3*log(c^2*x^2 + 1))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 4*(
-I*c^7*d^2*x - c^6*d^2)*integrate(1/4*(c*x^4*log(c^2*x^2 + 1) - 2*x^3*arct
an(c*x))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 12*(I*c^6*d^2*x + c
^5*d^2)*integrate(1/4*(2*c*x^3*arctan(c*x) + x^2*log(c^2*x^2 + 1))/(c^6*d^
2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 12*(c^6*d^2*x - I*c^5*d^2)*integrat
e(1/4*(c*x^3*log(c^2*x^2 + 1) - 2*x^2*arctan(c*x))/(c^6*d^2*x^4 + 2*c^4...
```

3.52.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

3.53 $\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx$

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = -\frac{b}{2c^2d^2(i-cx)}+\frac{b\arctan(cx)}{2c^2d^2}-\frac{i(a+b\arctan(cx))}{c^2d^2(i-cx)}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d^2}$$

output
```
-1/2*b/c^2/d^2/(I-c*x)+1/2*b*arctan(c*x)/c^2/d^2-I*(a+b*arctan(c*x))/c^2/d
^2/(I-c*x)+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2/d^2+1/2*I*b*polylog(2,1-2
/(1+I*c*x))/c^2/d^2
```

3.53.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = -\frac{i(a+b\arctan(cx))}{c^2d^2(i-cx)}-\frac{b\left(\frac{1}{c(i-cx)}-\frac{\arctan(cx)}{c}\right)}{2cd^2}+\frac{(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right)}{c^2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{i+cx}{i-cx}\right)}{2c^2d^2}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
((-I)*(a + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) - (b*(1/(c*(I - c*x)) - Arc
Tan[c*x]/c))/(2*c*d^2) + ((a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)])/(c^2*d
^2) + ((I/2)*b*PolyLog[2, -((I + c*x)/(I - c*x))])/(c^2*d^2)
```

3.53.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{a+b\arctan(cx)}{cd^2(cx-i)}-\frac{i(a+b\arctan(cx))}{cd^2(cx-i)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(a+b\arctan(cx))}{c^2d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^2d^2}+\frac{b\arctan(cx)}{2c^2d^2}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^2d^2}-\frac{b}{2c^2d^2(-cx+i)}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-1/2*b/(c^2*d^2*(I - c*x)) + (b*ArcTan[c*x])/(2*c^2*d^2) - (I*(a + b*ArcTa
n[c*x]))/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c
^2*d^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2)
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.53.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(110) = 220$.

Time = 0.92 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.07

method	result
derivativedivides	$\frac{ia}{d^2(cx-i)}-\frac{a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{ia\arctan(cx)}{d^2}+\frac{ib\arctan(cx)}{d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{d^2}+\frac{ib\ln\left(c^4x^4+10c^2x^2+9\right)}{16d^2}-\frac{b\arctan\left(\frac{cx}{2}\right)}{8d^2}+\frac{b\,\text{arcta}}{}$
default	$\frac{ia}{d^2(cx-i)}-\frac{a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{ia\arctan(cx)}{d^2}+\frac{ib\arctan(cx)}{d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{d^2}+\frac{ib\ln\left(c^4x^4+10c^2x^2+9\right)}{16d^2}-\frac{b\arctan\left(\frac{cx}{2}\right)}{8d^2}+\frac{b\,\text{arcta}}{}$
risch	$\frac{ib\ln(icx+1)^2}{4c^2d^2}+\frac{b\ln(icx+1)}{2c^2d^2(cx-i)}+\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2c^2d^2}-\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2c^2d^2}+\frac{ib\,\text{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2c^2d^2}-\frac{ib\,\text{l}}{}$
parts	$-\frac{ia}{d^2c^2(-cx+i)}-\frac{a\ln(c^2x^2+1)}{2c^2d^2}-\frac{ia\arctan(cx)}{c^2d^2}+\frac{ib\arctan(cx)}{c^2d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{c^2d^2}+\frac{ib\ln(c^4x^4+10c^2x^2}{16c^2d^2}$

input
```
int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(I*a/d^2/(c*x-I)-1/2*a/d^2*ln(c^2*x^2+1)-I*a/d^2*arctan(c*x)+I*b/d^2
*arctan(c*x)/(c*x-I)-b/d^2*arctan(c*x)*ln(c*x-I)+1/16*I*b/d^2*ln(c^4*x^4+1
0*c^2*x^2+9)-1/8*b/d^2*arctan(1/2*c*x)+1/8*b/d^2*arctan(1/6*c^3*x^3+7/6*c*
x)+1/4*b/d^2*arctan(1/2*c*x-1/2*I)+1/2*b/d^2/(c*x-I)-1/8*I*b/d^2*ln(c^2*x^
2+1)+1/4*b*arctan(c*x)/d^2+1/2*I*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)+1/2*I*
b/d^2*dilog(-1/2*I*(c*x+I))-1/4*I*b/d^2*ln(c*x-I)^2)
```

3.53.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-I*b*x*log(-(c*x + I)/(c*x - I)) - 2*a*x)/(c^2*d^2*x^2 - 2*I
*c*d^2*x - d^2), x)
```

3.53.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \frac{(-ibcx\log(icx+1)-b\log(icx+1)-b)\log(-icx+1)}{2c^3d^2x-2ic^2d^2} - \frac{\int \frac{ib}{c^3x^3-ic^2x^2+cx-i}\,dx + \int \frac{ib\log(icx+1)}{c^3x^3-ic^2x^2+cx-i}\,dx + \int \frac{2ac^2x^2}{c^3x^3-ic^2x^2+cx-i}\,dx + \int \left(-\frac{bcx}{c^3x^3-ic^2x^2+cx-i}\right)\,dx + \int \frac{2iac}{c^3x^3-ic^2x^2}}{2cd^2}$$

input
```
integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
(-I*b*c*x*log(I*c*x + 1) - b*log(I*c*x + 1) - b)*log(-I*c*x + 1)/(2*c**3*d
**2*x - 2*I*c**2*d**2) - (Integral(I*b/(c**3*x**3 - I*c**2*x**2 + c*x - I)
, x) + Integral(I*b*log(I*c*x + 1)/(c**3*x**3 - I*c**2*x**2 + c*x - I), x)
 + Integral(2*a*c**2*x**2/(c**3*x**3 - I*c**2*x**2 + c*x - I), x) + Integr
al(-b*c*x/(c**3*x**3 - I*c**2*x**2 + c*x - I), x) + Integral(2*I*a*c*x/(c*
*3*x**3 - I*c**2*x**2 + c*x - I), x) + Integral(-b*c*x*log(I*c*x + 1)/(c**
3*x**3 - I*c**2*x**2 + c*x - I), x) + Integral(-2*I*b*c**2*x**2*log(I*c*x
+ 1)/(c**3*x**3 - I*c**2*x**2 + c*x - I), x))/(2*c*d**2)
```

3.53.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
a*(I/(c^3*d^2*x - I*c^2*d^2) - log(c*x - I)/(c^2*d^2)) - 1/8*(4*(I*c*x + 1
)*arctan(c*x)^2 + 4*c*x*arctan2(1, c*x) - (-I*c*x - 1)*log(c^2*x^2 + 1)^2
- (c^3*d^2*x - I*c^2*d^2)*((c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(c*x)/(c^
4*d^2)) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2))*c + 8*integrate(1/4*log(c
^2*x^2 + 1)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x)) - (I*c^3*d^2*x + c^
2*d^2)*(c*(c^2/(c^7*d^2*x^2 + c^5*d^2) + log(c^2*x^2 + 1)/(c^5*d^2*x^2 + c
^3*d^2)) + 16*integrate(1/4*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d
^2), x)) + (c^4*d^2*x - I*c^3*d^2)*(c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(
c*x)/(c^4*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^5*d^2*x^4 + 2*
c^3*d^2*x^2 + c*d^2), x) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2)) + (-I*c^
4*d^2*x - c^3*d^2)*(16*c*integrate(1/4*x^2*arctan(c*x)/(c^5*d^2*x^4 + 2*c^
3*d^2*x^2 + c*d^2), x) - c^2/(c^7*d^2*x^2 + c^5*d^2) - log(c^2*x^2 + 1)/(c
^5*d^2*x^2 + c^3*d^2)) + 16*(c^5*d^2*x - I*c^4*d^2)*integrate(1/4*(2*c*x^3
*arctan(c*x) + x^2*log(c^2*x^2 + 1))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2)
, x) + 16*(-I*c^5*d^2*x - c^4*d^2)*integrate(1/4*(c*x^3*log(c^2*x^2 + 1) -
 2*x^2*arctan(c*x))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 4*I*arctan
(c*x) - 4*I*arctan2(1, c*x) + 2*log(c^2*x^2 + 1))*b/(c^3*d^2*x - I*c^2*d^2
)
```

3.53.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

3.54 $\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx$

3.54.1 Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx=\frac{ib}{2cd^2(i-cx)}-\frac{ib\arctan(cx)}{2cd^2}+\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}$$

output
```
1/2*I*b/c/d^2/(I-c*x)-1/2*I*b*arctan(c*x)/c/d^2+I*(a+b*arctan(c*x))/c/d^2/
(1+I*c*x)
```

3.54.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx=\frac{2a-ib+(b-ibcx)\arctan(cx)}{2cd^2(-i+cx)}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
```

output
```
(2*a - I*b + (b - I*b*c*x)*ArcTan[c*x])/(2*c*d^2*(-I + c*x))
```

3.54.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5387, 27, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \frac{1}{d(icx+1)(c^2x^2+1)}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \frac{1}{(icx+1)(c^2x^2+1)}dx}{d^2}$$

$$\Big\downarrow \text{456}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \frac{1}{(1-icx)(icx+1)^2}dx}{d^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \left(\frac{1}{2(c^2x^2+1)}-\frac{1}{2(cx-i)^2}\right)dx}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\left(\frac{\arctan(cx)}{2c}-\frac{1}{2c(-cx+i)}\right)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
```

output
```
(I*(a + b*ArcTan[c*x]))/(c*d^2*(1 + I*c*x)) - (I*b*(-1/2*1/(c*(I - c*x)) +
 ArcTan[c*x]/(2*c)))/d^2
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.54.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$\frac{\frac{ia}{d^2(icx+1)}+\frac{ib\arctan(cx)}{d^2(icx+1)}-\frac{ib\arctan(cx)}{2d^2}-\frac{ib}{2d^2(cx-i)}}{c}$	68
default	$\frac{\frac{ia}{d^2(icx+1)}+\frac{ib\arctan(cx)}{d^2(icx+1)}-\frac{ib\arctan(cx)}{2d^2}-\frac{ib}{2d^2(cx-i)}}{c}$	68
parts	$\frac{ia}{d^2c(icx+1)}+\frac{ib\arctan(cx)}{c\,d^2(icx+1)}-\frac{ib\arctan(cx)}{2c\,d^2}-\frac{ib}{2c\,d^2(cx-i)}$	76
risch	$-\frac{ib\ln(icx+1)}{2c\,d^2(cx-i)}+\frac{2ib\ln(-icx+1)+\ln(-cx-i)bcx-\ln(cx-i)bcx-i\ln(-cx-i)b+i\ln(cx-i)b-2ib+4a}{4d^2(cx-i)c}$	111

input
```
int((a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(I*a/d^2/(1+I*c*x)+I*b/d^2/(1+I*c*x)*arctan(c*x)-1/2*I*b/d^2*arctan(c*
x)-1/2*I*b/d^2/(c*x-I))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx = \frac{(bcx+i\,b)\log\left(-\frac{cx+i}{cx-i}\right)+4\,a-2i\,b}{4\left(c^2d^2x-i\,cd^2\right)}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/4*((b*c*x + I*b)*log(-(c*x + I)/(c*x - I)) + 4*a - 2*I*b)/(c^2*d^2*x - I
*c*d^2)
```

3.54.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(51) = 102.

Time = 0.88 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx = \frac{ib\log\left(-icx+1\right)}{2c^2d^2x-2icd^2} - \frac{ib\log\left(icx+1\right)}{2c^2d^2x-2icd^2} - \frac{b\left(\frac{\log\left(bx-\frac{ib}{c}\right)}{4} - \frac{\log\left(bx+\frac{ib}{c}\right)}{4}\right)}{cd^2} - \frac{-2a+ib}{2c^2d^2x-2icd^2}$$

input
```
integrate((a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
I*b*log(-I*c*x + 1)/(2*c**2*d**2*x - 2*I*c*d**2) - I*b*log(I*c*x + 1)/(2*c
**2*d**2*x - 2*I*c*d**2) - b*(log(b*x - I*b/c)/4 - log(b*x + I*b/c)/4)/(c*
d**2) - (-2*a + I*b)/(2*c**2*d**2*x - 2*I*c*d**2)
```

3.54.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.54.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^2, x)
```

3.55 $\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx$

3.55.1 Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \frac{b}{2d^2(i-cx)}-\frac{b\arctan(cx)}{2d^2}+\frac{i(a+b\arctan(cx))}{d^2(i-cx)}+\frac{a\log(x)}{d^2}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d^2}-\frac{ib\operatorname{PolyLog}(2,icx)}{2d^2}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}$$

output
```
1/2*b/d^2/(I-c*x)-1/2*b*arctan(c*x)/d^2+I*(a+b*arctan(c*x))/d^2/(I-c*x)+a*
ln(x)/d^2+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d^2+1/2*I*b*polylog(2,-I*c*x)/
d^2-1/2*I*b*polylog(2,I*c*x)/d^2+1/2*I*b*polylog(2,1-2/(1+I*c*x))/d^2
```

3.55.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx$$
$$=\frac{b\left(\frac{1}{i-cx}-\arctan(cx)\right)-\frac{2i(a+b\arctan(cx))}{-i+cx}+2a\log(x)+2(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right)+ib\operatorname{PolyLog}(2,-icx)}{2d^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

output
```
(b*((I - c*x)^(-1) - ArcTan[c*x]) - ((2*I)*(a + b*ArcTan[c*x]))/(-I + c*x)
 + 2*a*Log[x] + 2*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + I*b*PolyLog[2
, (-I)*c*x] - I*b*PolyLog[2, I*c*x] + I*b*PolyLog[2, (I + c*x)/(-I + c*x)]
)/(2*d^2)
```

3.55.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{a+b\arctan(cx)}{d^2x}-\frac{c(a+b\arctan(cx))}{d^2(cx-i)}+\frac{ic(a+b\arctan(cx))}{d^2(cx-i)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^2}+\frac{a\log(x)}{d^2}-\frac{b\arctan(cx)}{2d^2}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d^2}-\frac{ib\,\text{PolyLog}(2,icx)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^2}+\frac{b}{2d^2(-cx+i)}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

output
```
b/(2*d^2*(I - c*x)) - (b*ArcTan[c*x])/(2*d^2) + (I*(a + b*ArcTan[c*x]))/(d
^2*(I - c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
d^2 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2
 + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.55.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38

method	result
parts	$\frac{a\ln(x)}{d^2}+\frac{ia}{d^2(-cx+i)}-\frac{a\ln(c^2x^2+1)}{2d^2}-\frac{ia\arctan(cx)}{d^2}+\frac{b\left(\arctan(cx)\ln(cx)-\frac{i\arctan(cx)}{cx-i}-\arctan(cx)\ln(cx-i)-\right.}{}$
derivativedivides	$\frac{a\ln(cx)}{d^2}-\frac{ia}{d^2(cx-i)}-\frac{a\ln(c^2x^2+1)}{2d^2}-\frac{ia\arctan(cx)}{d^2}+\frac{b\left(\arctan(cx)\ln(cx)-\frac{i\arctan(cx)}{cx-i}-\arctan(cx)\ln(cx-i)+\right.}{}$
default	$\frac{a\ln(cx)}{d^2}-\frac{ia}{d^2(cx-i)}-\frac{a\ln(c^2x^2+1)}{2d^2}-\frac{ia\arctan(cx)}{d^2}+\frac{b\left(\arctan(cx)\ln(cx)-\frac{i\arctan(cx)}{cx-i}-\arctan(cx)\ln(cx-i)+\right.}{}$
risch	$\frac{ib\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d^2}+\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d^2}-\frac{ib\,\mathrm{dilog}(-icx+1)}{2d^2}-\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d^2}-\frac{ib\ln(-icx+1)}{4d^2(-icx-1)}-$

input
```
int((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*ln(x)+I*a/d^2/(-c*x+I)-1/2*a/d^2*ln(c^2*x^2+1)-I*a/d^2*arctan(c*x)+b
/d^2*(arctan(c*x)*ln(c*x)-I*arctan(c*x)/(c*x-I)-arctan(c*x)*ln(c*x-I)+1/2*
I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I
*dilog(1-I*c*x)-1/2*arctan(c*x)-1/2/(c*x-I)+1/2*I*(dilog(-1/2*I*(c*x+I))+l
n(c*x-I)*ln(-1/2*I*(c*x+I)))-1/4*I*ln(c*x-I)^2)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = -\frac{2\,(i\,bcx+b)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(acx-i\,a)\log\left(x\right)-2\,b\log\left(-\frac{cx+i}{cx-i}\right)-(-i\,bcx-b)\log\left(\frac{cx+i}{c}\right)+((4\,a-i}{4\,(cd^2x-i\,d^2)}$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(I*b*c*x + b)*dilog((c*x + I)/(c*x - I) + 1) - 4*(a*c*x - I*a)*log
(x) - 2*b*log(-(c*x + I)/(c*x - I)) - (-I*b*c*x - b)*log((c*x + I)/c) + ((
4*a - I*b)*c*x - 4*I*a - b)*log((c*x - I)/c) + 4*I*a + 2*b)/(c*d^2*x - I*d
^2)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.55.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(i\,cdx+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
(-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x) - in
tegrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x
))*b + a*(-I/(c*d^2*x - I*d^2) - log(c*x - I)/d^2 + log(x)/d^2)
```

3.55.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^2), x)
```

3.56 $\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx$

3.56.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx = & -\frac{ibc}{2d^2(i-cx)}+\frac{ibc\arctan(cx)}{2d^2}-\frac{a+b\arctan(cx)}{d^2x} \\ & +\frac{c(a+b\arctan(cx))}{d^2(i-cx)}-\frac{2iac\log(x)}{d^2} \\ & +\frac{bc\log(x)}{d^2}-\frac{2ic(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^2} \\ & -\frac{bc\log\left(1+c^2x^2\right)}{2d^2}+\frac{bc\operatorname{PolyLog}(2,-icx)}{d^2} \\ & -\frac{bc\operatorname{PolyLog}(2,icx)}{d^2}+\frac{bc\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2}\end{aligned}$$

output
```
-1/2*I*b*c/d^2/(I-c*x)+1/2*I*b*c*arctan(c*x)/d^2+(-a-b*arctan(c*x))/d^2/x+
c*(a+b*arctan(c*x))/d^2/(I-c*x)-2*I*a*c*ln(x)/d^2+b*c*ln(x)/d^2-2*I*c*(a+b
*arctan(c*x))*ln(2/(1+I*c*x))/d^2-1/2*b*c*ln(c^2*x^2+1)/d^2+b*c*polylog(2,
-I*c*x)/d^2-b*c*polylog(2,I*c*x)/d^2+b*c*polylog(2,1-2/(1+I*c*x))/d^2
```

3.56.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx =$$

$$-\,ibc\left(\frac{1}{i-cx}-\arctan(cx)\right)+\frac{2(a+b\arctan(cx))}{x}+\frac{2c(a+b\arctan(cx))}{-i+cx}+4iac\log(x)+4ic(a+b\arctan(cx))\log\left(\frac{2i}{i-c}\right.$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(I*b*c*((I - c*x)^(-1) - ArcTan[c*x]) + (2*(a + b*ArcTan[c*x]))/x + (
2*c*(a + b*ArcTan[c*x]))/(-I + c*x) + (4*I)*a*c*Log[x] + (4*I)*c*(a + b*Ar
cTan[c*x])*Log[(2*I)/(I - c*x)] + b*c*(-2*Log[x] + Log[1 + c^2*x^2]) - 2*b
*c*PolyLog[2, (-I)*c*x] + 2*b*c*PolyLog[2, I*c*x] - 2*b*c*PolyLog[2, (I +
c*x)/(-I + c*x)])/d^2
```

3.56.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{2ic^2(a+b\arctan(cx))}{d^2(cx-i)}+\frac{c^2(a+b\arctan(cx))}{d^2(cx-i)^2}+\frac{a+b\arctan(cx)}{d^2x^2}-\frac{2ic(a+b\arctan(cx))}{d^2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{a+b\arctan(cx)}{d^2x}-\frac{2ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^2}-\frac{2iac\log(x)}{d^2}+\frac{ibc\arctan(cx)}{2d^2}-\frac{bc\log(c^2x^2+1)}{2d^2}+\frac{bc\operatorname{PolyLog}(2,-icx)}{d^2}-\frac{bc\operatorname{PolyLog}(2,icx)}{d^2}+\frac{bc\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{d^2}-\frac{ibc}{2d^2(-cx+i)}+\frac{bc\log(x)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```

output
```
((-1/2*I)*b*c)/(d^2*(I - c*x)) + ((I/2)*b*c*ArcTan[c*x])/d^2 - (a + b*ArcT
an[c*x])/(d^2*x) + (c*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) - ((2*I)*a*c*Lo
g[x])/d^2 + (b*c*Log[x])/d^2 - ((2*I)*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c
*x)])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, (-I)*c*x])/d^
2 - (b*c*PolyLog[2, I*c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.56.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.32

method	result
parts	$-\frac{a}{d^2 x}-\frac{2iac\ln(x)}{d^2}+\frac{ac}{d^2(-cx+i)}-\frac{2ca\arctan(cx)}{d^2}+\frac{ica\ln(c^2x^2+1)}{d^2}+\frac{bc\left(-\frac{\arctan(cx)}{cx}-2i\arctan(cx)\ln(cx)-\right.}{}$
derivativedivides	$c\left(-\frac{a}{d^2cx}-\frac{2ia\ln(cx)}{d^2}-\frac{a}{d^2(cx-i)}-\frac{2a\arctan(cx)}{d^2}+\frac{ia\ln(c^2x^2+1)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{cx}-2i\arctan(cx)\ln(cx)-\right.}{}\right.$
default	$c\left(-\frac{a}{d^2cx}-\frac{2ia\ln(cx)}{d^2}-\frac{a}{d^2(cx-i)}-\frac{2a\arctan(cx)}{d^2}+\frac{ia\ln(c^2x^2+1)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{cx}-2i\arctan(cx)\ln(cx)-\right.}{}\right.$
risch	$-\frac{cb\ln(-icx+1)}{4d^2(-icx-1)}+\frac{bc\ln(c^2x^2+1)}{8d^2}-\frac{cb\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{d^2}+\frac{ic^2b\ln(-icx+1)x}{4d^2(-icx-1)}+\frac{cb\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{d^2}$

input
```
int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d^2/x-2*I*a*c*ln(x)/d^2+a/d^2*c/(-c*x+I)-2/d^2*c*a*arctan(c*x)+I/d^2*c*
a*ln(c^2*x^2+1)+b/d^2*c*(-1/c/x*arctan(c*x)-2*I*arctan(c*x)*ln(c*x)-arctan
(c*x)/(c*x-I)+2*I*arctan(c*x)*ln(c*x-I)-dilog(-I*(c*x+I))-ln(c*x)*ln(-I*(c
*x+I))+(ln(c*x)-ln(-I*c*x))*ln(-I*(-c*x+I))-dilog(-I*c*x)+dilog(-1/2*I*(c*
x+I))+ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2-1/2*ln(c^2*x^2+1)+1/2*I
*arctan(c*x)+ln(c*x)+1/2*I/(c*x-I))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)^2}\,dx = -\frac{2\,(4\,a - i\,b)cx + 4\,(bc^2x^2 - i\,bcx)\mathrm{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) + 4\,((2i\,a - b)c^2x^2 + (2\,a + i\,b)cx)\log\left(x\right) + 2\,(2i\,bcx + \cdots}{4\,(cd^2x^2 - i\,c \cdots}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(4*a - I*b)*c*x + 4*(b*c^2*x^2 - I*b*c*x)*dilog((c*x + I)/(c*x - I
) + 1) + 4*((2*I*a - b)*c^2*x^2 + (2*a + I*b)*c*x)*log(x) + 2*(2*I*b*c*x +
 b)*log(-(c*x + I)/(c*x - I)) + 3*(b*c^2*x^2 - I*b*c*x)*log((c*x + I)/c) -
 ((8*I*a - b)*c^2*x^2 + (8*a + I*b)*c*x)*log((c*x - I)/c) - 4*I*a)/(c*d^2*
x^2 - I*d^2*x)
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.56.7 Maxima [F]

$$\int \frac{a + b \arctan(cx)}{x^2(d + icdx)^2} \, dx = \int \frac{b \arctan(cx) + a}{(i\, cdx + d)^2 x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
(-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) -
integrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2
), x))*b - a*(c/(c*d^2*x - I*d^2) - 2*I*c*log(c*x - I)/d^2 + 2*I*c*log(x)/
d^2 + 1/(d^2*x))
```

3.56.8 Giac [F]

$$\int \frac{a + b \arctan(cx)}{x^2(d + icdx)^2} \, dx = \int \frac{b \arctan(cx) + a}{(i\, cdx + d)^2 x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x^2(d + icdx)^2} \, dx = \int \frac{a + b \operatorname{atan}(c\, x)}{x^2 \, (d + c\, d\, x\, 1\mathrm{i})^2} \, dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^2), x)
```

3.57 $\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx$

3.57.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx &= -\frac{bc}{2d^2x}-\frac{bc^2}{2d^2(i-cx)}-\frac{a+b\arctan(cx)}{2d^2x^2}\\&\quad+\frac{2ic(a+b\arctan(cx))}{d^2x}-\frac{ic^2(a+b\arctan(cx))}{d^2(i-cx)}-\frac{3ac^2\log(x)}{d^2}\\&\quad-\frac{2ibc^2\log(x)}{d^2}-\frac{3c^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}\\&\quad+\frac{ibc^2\log(1+c^2x^2)}{d^2}-\frac{3ibc^2\operatorname{PolyLog}(2,-icx)}{2d^2}\\&\quad+\frac{3ibc^2\operatorname{PolyLog}(2,icx)}{2d^2}-\frac{3ibc^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}\end{aligned}$$

output
```
-1/2*b*c/d^2/x-1/2*b*c^2/d^2/(I-c*x)+1/2*(-a-b*arctan(c*x))/d^2/x^2+2*I*c*
(a+b*arctan(c*x))/d^2/x-I*c^2*(a+b*arctan(c*x))/d^2/(I-c*x)-3*a*c^2*ln(x)/
d^2-2*I*b*c^2*ln(x)/d^2-3*c^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d^2+I*b*c^
2*ln(c^2*x^2+1)/d^2-3/2*I*b*c^2*polylog(2,-I*c*x)/d^2+3/2*I*b*c^2*polylog(
2,I*c*x)/d^2-3/2*I*b*c^2*polylog(2,1-2/(1+I*c*x))/d^2
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx =$$
$$-\frac{-bc^2\left(\frac{1}{-i+cx}+\arctan(cx)\right)+\frac{a+b\arctan(cx)}{x^2}-\frac{4ic(a+b\arctan(cx))}{x}-\frac{2ic^2(a+b\arctan(cx))}{-i+cx}+\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\right.}{x}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(-(b*c^2*((-I + c*x)^(-1) + ArcTan[c*x])) + (a + b*ArcTan[c*x])/x^2 -
 ((4*I)*c*(a + b*ArcTan[c*x]))/x - ((2*I)*c^2*(a + b*ArcTan[c*x]))/(-I + c
*x) + (b*c*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 6*a*c^2*Log[x]
 + 6*c^2*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + (2*I)*b*c^2*(2*Log[x]
- Log[1 + c^2*x^2]) + (3*I)*b*c^2*PolyLog[2, (-I)*c*x] - (3*I)*b*c^2*PolyL
og[2, I*c*x] + (3*I)*b*c^2*PolyLog[2, (I + c*x)/(-I + c*x)])/d^2
```

3.57.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{3c^3(a+b\arctan(cx))}{d^2(cx-i)}-\frac{ic^3(a+b\arctan(cx))}{d^2(cx-i)^2}-\frac{3c^2(a+b\arctan(cx))}{d^2x}+\frac{a+b\arctan(cx)}{d^2x^3}-\frac{2ic(a+b\arctan}{d^2x^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{ic^2(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^2}-\frac{a+b\arctan(cx)}{2d^2x^2}+\frac{2ic(a+b\arctan(cx))}{d^2x}-\frac{3ac^2\log(x)}{d^2}-\frac{3ibc^2\operatorname{PolyLog}(2,-icx)}{2d^2}+\frac{3ibc^2\operatorname{PolyLog}(2,icx)}{2d^2}-\frac{3ibc^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^2}+\frac{ibc^2\log\left(c^2x^2+1\right)}{d^2}-\frac{bc^2}{2d^2(-cx+i)}-\frac{2ibc^2\log(x)}{d^2}-\frac{bc}{2d^2x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(b*c)/(d^2*x) - (b*c^2)/(2*d^2*(I - c*x)) - (a + b*ArcTan[c*x])/(2*d^
2*x^2) + ((2*I)*c*(a + b*ArcTan[c*x]))/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x]
))/(d^2*(I - c*x)) - (3*a*c^2*Log[x])/d^2 - ((2*I)*b*c^2*Log[x])/d^2 - (3*
c^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^2 + (I*b*c^2*Log[1 + c^2*x^2
])/d^2 - (((3*I)/2)*b*c^2*PolyLog[2, (-I)*c*x])/d^2 + (((3*I)/2)*b*c^2*Pol
yLog[2, I*c*x])/d^2 - (((3*I)/2)*b*c^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.57.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.17

method	result
derivativedivides	$c^2\left(-\frac{a}{2d^2c^2x^2}+\frac{2ia}{d^2cx}-\frac{3a\ln(cx)}{d^2}+\frac{ia}{d^2(cx-i)}+\frac{3a\ln(c^2x^2+1)}{2d^2}+\frac{3ia\arctan(cx)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{2i\,\mathrm{arct}}{c}\right.}{}\right.$
default	$c^2\left(-\frac{a}{2d^2c^2x^2}+\frac{2ia}{d^2cx}-\frac{3a\ln(cx)}{d^2}+\frac{ia}{d^2(cx-i)}+\frac{3a\ln(c^2x^2+1)}{2d^2}+\frac{3ia\arctan(cx)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{2i\,\mathrm{arct}}{c}\right.}{}\right.$
parts	$-\frac{a}{2d^2x^2}+\frac{2iac}{x\,d^2}-\frac{3a\,c^2\ln(x)}{d^2}-\frac{ia\,c^2}{d^2(-cx+i)}+\frac{3c^2a\ln(c^2x^2+1)}{2d^2}+\frac{3ic^2a\arctan(cx)}{d^2}+\frac{b\,c^2\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{2i\,\mathrm{ar}}{}\right.}{}$
risch	$-\frac{a}{2d^2x^2}+\frac{b\,c^2\arctan(cx)}{4d^2}-\frac{bc}{2d^2x}+\frac{c^3b\ln(-icx+1)x}{4d^2(-icx-1)}+\frac{ic^2b\ln(-icx+1)}{4d^2(-icx-1)}+\frac{3ic^2b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d^2}-3$

input
```
int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a/d^2/c^2/x^2+2*I*a/d^2/c/x-3*a/d^2*ln(c*x)+I*a/d^2/(c*x-I)+3/2*
a/d^2*ln(c^2*x^2+1)+3*I*a/d^2*arctan(c*x)+b/d^2*(-1/2/c^2/x^2*arctan(c*x)+
2*I*arctan(c*x)/c/x-3*arctan(c*x)*ln(c*x)+I*arctan(c*x)/(c*x-I)+3*arctan(c
*x)*ln(c*x-I)-3/2*I*ln(c*x)*ln(1+I*c*x)+3/2*I*ln(c*x)*ln(1-I*c*x)-3/2*I*di
log(1+I*c*x)+3/2*I*dilog(1-I*c*x)-3/2*I*(dilog(-1/2*I*(c*x+I))+ln(c*x-I)*l
n(-1/2*I*(c*x+I)))+3/4*I*ln(c*x-I)^2+I*ln(c^2*x^2+1)-1/2/c/x-2*I*ln(c*x)+1
/2/(c*x-I)))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int\frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx$$
$$=\frac{12i\,ac^2x^2+2\,(3\,a+i\,b)cx-6\,(-i\,bc^3x^3-bc^2x^2)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,((3\,a+2i\,b)c^3x^3+(-3i\,a+2\,b)c^2x^2)\,\mathrm{l}}{}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/4*(12*I*a*c^2*x^2 + 2*(3*a + I*b)*c*x - 6*(-I*b*c^3*x^3 - b*c^2*x^2)*dil
og((c*x + I)/(c*x - I) + 1) - 4*((3*a + 2*I*b)*c^3*x^3 + (-3*I*a + 2*b)*c^
2*x^2)*log(x) - (6*b*c^2*x^2 - 3*I*b*c*x + b)*log(-(c*x + I)/(c*x - I)) -
4*(-I*b*c^3*x^3 - b*c^2*x^2)*log((c*x + I)/c) + 4*((3*a + I*b)*c^3*x^3 - (
3*I*a - b)*c^2*x^2)*log((c*x - I)/c) + 2*I*a)/(c*d^2*x^3 - I*d^2*x^2)
```

3.57.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx = -\frac{\int \frac{a}{c^2x^5-2icx^4-x^3}\,dx + \int \frac{b\,\mathrm{atan}\,(cx)}{c^2x^5-2icx^4-x^3}\,dx}{d^2}$$

input
```
integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**2,x)
```

output
```
-(Integral(a/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(b*atan(c*x)/(c
**2*x**5 - 2*I*c*x**4 - x**3), x))/d**2
```

3.57.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(i\,cdx+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
(-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
- integrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x
^3), x))*b - 1/2*a*(-2*I*c^2/(c*d^2*x - I*d^2) - 6*c^2*log(c*x - I)/d^2 +
6*c^2*log(x)/d^2 - (4*I*c*x - 1)/(d^2*x^2))
```

3.57.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^2), x)
```

3.58 $\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx$

3.58.1 Optimal result

Integrand size = 23, antiderivative size = 256

$$\begin{aligned}\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx = &-\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{b}{8c^5d^3(i-cx)^2}-\frac{15ib}{8c^5d^3(i-cx)}\\&+\frac{19ib\arctan(cx)}{8c^5d^3}-\frac{3bx\arctan(cx)}{c^4d^3}\\&+\frac{ix^2(a+b\arctan(cx))}{2c^3d^3}-\frac{i(a+b\arctan(cx))}{2c^5d^3(i-cx)^2}\\&+\frac{4(a+b\arctan(cx))}{c^5d^3(i-cx)}+\frac{6i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^5d^3}\\&+\frac{3b\log(1+c^2x^2)}{2c^5d^3}-\frac{3b\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3}\end{aligned}$$

output
```
-3*a*x/c^4/d^3-1/2*I*b*x/c^4/d^3-1/8*b/c^5/d^3/(I-c*x)^2-15/8*I*b/c^5/d^3/
(I-c*x)+19/8*I*b*arctan(c*x)/c^5/d^3-3*b*x*arctan(c*x)/c^4/d^3+1/2*I*x^2*(
a+b*arctan(c*x))/c^3/d^3-1/2*I*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)^2+4*(a+b*
arctan(c*x))/c^5/d^3/(I-c*x)+6*I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5/d^3
+3/2*b*ln(c^2*x^2+1)/c^5/d^3-3*b*polylog(2,1-2/(1+I*c*x))/c^5/d^3
```

3.58.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$
$$= \frac{-96acx+16iac^2x^2-\frac{16ia}{(-i+cx)^2}-\frac{128a}{-i+cx}+192a\arctan(cx)-96ia\log\left(1+c^2x^2\right)+b\big(-16icx+192\arctan($$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(-96*a*c*x + (16*I)*a*c^2*x^2 - ((16*I)*a)/(-I + c*x)^2 - (128*a)/(-I + c*
x) + 192*a*ArcTan[c*x] - (96*I)*a*Log[1 + c^2*x^2] + b*((-16*I)*c*x + 192*
ArcTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + c^
2*x^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*ArcTan[c*x]]
 + (4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcTan[c*x]] +
 Cos[4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)*Sin[2*Arc
Tan[c*x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]))/(32*c^5*d^3)
```

3.58.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{6i(a+b\arctan(cx))}{c^4d^3(cx-i)}+\frac{4(a+b\arctan(cx))}{c^4d^3(cx-i)^2}-\frac{3(a+b\arctan(cx))}{c^4d^3}+\frac{i(a+b\arctan(cx))}{c^4d^3(cx-i)^3}+\frac{ix(a+b\arctan(\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{4(a+b\arctan(cx))}{c^5d^3(-cx+i)}-\frac{i(a+b\arctan(cx))}{2c^5d^3(-cx+i)^2}+\frac{6i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^5d^3}+\frac{ix^2(a+b\arctan(cx))}{2c^3d^3}-\frac{3ax}{c^4d^3}+\frac{19ib\arctan(cx)}{8c^5d^3}-\frac{3bx\arctan(cx)}{c^4d^3}-\frac{3b\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^5d^3}-\frac{15ib}{8c^5d^3(-cx+i)}-\frac{b}{8c^5d^3(-cx+i)^2}-\frac{ibx}{2c^4d^3}+\frac{3b\log\left(c^2x^2+1\right)}{2c^5d^3}$$

input
```
Int[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(-3*a*x)/(c^4*d^3) - ((I/2)*b*x)/(c^4*d^3) - b/(8*c^5*d^3*(I - c*x)^2) - (
((15*I)/8)*b)/(c^5*d^3*(I - c*x)) + (((19*I)/8)*b*ArcTan[c*x])/(c^5*d^3) -
 (3*b*x*ArcTan[c*x])/(c^4*d^3) + ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c^3*d^3)
 - ((I/2)*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)^2) + (4*(a + b*ArcTan[c*
x]))/(c^5*d^3*(I - c*x)) + ((6*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
(c^5*d^3) + (3*b*Log[1 + c^2*x^2])/(2*c^5*d^3) - (3*b*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(c^5*d^3)
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.58.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\overline{-\frac{3acx}{d^3}-\frac{3ia\ln\left(c^2x^2+1\right)}{d^3}-\frac{ib\arctan(cx)}{2d^3(cx-i)^2}-\frac{4a}{d^3(cx-i)}+\frac{6a\arctan(cx)}{d^3}+\frac{43ib\arctan(cx)}{16d^3}-\frac{3b\arctan(cx)cx}{d^3}-\frac{ia}{2d^3(cx-i)^2}+\frac{ib\arctan(c\cdots}{2d^3}}$
default	$\overline{-\frac{3acx}{d^3}-\frac{3ia\ln\left(c^2x^2+1\right)}{d^3}-\frac{ib\arctan(cx)}{2d^3(cx-i)^2}-\frac{4a}{d^3(cx-i)}+\frac{6a\arctan(cx)}{d^3}+\frac{43ib\arctan(cx)}{16d^3}-\frac{3b\arctan(cx)cx}{d^3}-\frac{ia}{2d^3(cx-i)^2}+\frac{ib\arctan(c\cdots}{2d^3}}$
parts	$-\frac{5ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{32c^5d^3}-\frac{3ax}{c^4d^3}+\frac{4a}{d^3c^5(-cx+i)}+\frac{5ib\arctan\left(\frac{cx}{2}\right)}{32c^5d^3}+\frac{6a\arctan(cx)}{c^5d^3}+\frac{43ib\arctan(cx)}{16c^5d^3}-3b\cdots$
risch	$-\frac{3ax}{c^4d^3}+\frac{43b\ln(c^2x^2+1)}{32c^5d^3}-\frac{9b}{8c^5d^3}-\frac{b}{8c^5d^3(-cx+i)^2}+\frac{5b\ln(-icx+1)}{4c^5d^3}-\frac{3b\operatorname{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{c^5d^3}-\frac{b}{8c^5d^3(-icx-1)\cdots}$

input
```
int(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-3*a/d^3*c*x-3*I*a/d^3*ln(c^2*x^2+1)-1/2*I*b/d^3*arctan(c*x)/(c*x-I
)^2-4*a/d^3/(c*x-I)+6*a/d^3*arctan(c*x)+43/16*I*b/d^3*arctan(c*x)-3*b/d^3*
arctan(c*x)*c*x-1/2*I*a/d^3/(c*x-I)^2+1/2*I*b/d^3*arctan(c*x)*c^2*x^2-4*b/
d^3*arctan(c*x)/(c*x-I)-6*I*b/d^3*arctan(c*x)*ln(c*x-I)-1/2*b/d^3-1/2*I*b/
d^3*c*x+5/64*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-5/32*I*b/d^3*arctan(1/6*c^3*x^
3+7/6*c*x)+1/2*I*a/d^3*c^2*x^2+15/8*I*b/d^3/(c*x-I)-5/16*I*b/d^3*arctan(1/
2*c*x-1/2*I)-1/8*b/d^3/(c*x-I)^2+43/32*b/d^3*ln(c^2*x^2+1)+5/32*I*b/d^3*ar
ctan(1/2*c*x)-3*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/2*b/d^3*ln(c*x-I)^2-3
*b/d^3*dilog(-1/2*I*(c*x+I)))
```

3.58.5 Fricas [F]

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(b*x^4*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^4)/(c^3*d^3*x^3 -
 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.40

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$
$$=\frac{8i\,ac^4x^4-8\,(4\,a+i\,b)c^3x^3+(b(5i\,\arctan(1,cx)-16)+88i\,a)c^2x^2+2\,(b(5\,\arctan(1,cx)+19i)-8\,a)c}{}$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/16*(8*I*a*c^4*x^4 - 8*(4*a + I*b)*c^3*x^3 + (b*(5*I*arctan2(1, c*x) - 16
) + 88*I*a)*c^2*x^2 + 2*(b*(5*arctan2(1, c*x) + 19*I) - 8*a)*c*x + 24*(b*c
^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)^2 + 6*(b*c^2*x^2 - 2*I*b*c*x - b)*log(
c^2*x^2 + 1)^2 - 24*(I*b*c^2*x^2 + 2*b*c*x - I*b)*arctan(c*x)*log(1/4*c^2*
x^2 + 1/4) + b*(-5*I*arctan2(1, c*x) + 28) + (8*I*b*c^4*x^4 - 32*b*c^3*x^3
 + (96*a + 131*I*b)*c^2*x^2 - 2*(96*I*a - 35*b)*c*x - 96*a + 13*I*b)*arcta
n(c*x) - 48*(b*c^2*x^2 - 2*I*b*c*x - b)*dilog(1/2*I*c*x + 1/2) - 12*(2*(2*
I*a - b)*c^2*x^2 + 4*(2*a + I*b)*c*x + (b*c^2*x^2 - 2*I*b*c*x - b)*log(1/4
*c^2*x^2 + 1/4) - 4*I*a + 2*b)*log(c^2*x^2 + 1) + 56*I*a)/(c^7*d^3*x^2 - 2
*I*c^6*d^3*x - c^5*d^3)
```

3.58.8 Giac [F]

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{x^4\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^4*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x^4*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.59 $\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx$

3.59.1 Optimal result

Integrand size = 23, antiderivative size = 225

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = {} & \frac{iax}{c^3d^3}+\frac{ib}{8c^4d^3(i-cx)^2}-\frac{11b}{8c^4d^3(i-cx)} \\ & +\frac{11b\arctan(cx)}{8c^4d^3}+\frac{ibx\arctan(cx)}{c^3d^3}-\frac{a+b\arctan(cx)}{2c^4d^3(i-cx)^2} \\ & -\frac{3i(a+b\arctan(cx))}{c^4d^3(i-cx)}+\frac{3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^3} \\ & -\frac{ib\log(1+c^2x^2)}{2c^4d^3}+\frac{3ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^3}\end{aligned}$$

output
```
I*a*x/c^3/d^3+1/8*I*b/c^4/d^3/(I-c*x)^2-11/8*b/c^4/d^3/(I-c*x)+11/8*b*arct
an(c*x)/c^4/d^3+I*b*x*arctan(c*x)/c^3/d^3+1/2*(-a-b*arctan(c*x))/c^4/d^3/(
I-c*x)^2-3*I*(a+b*arctan(c*x))/c^4/d^3/(I-c*x)+3*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c^4/d^3-1/2*I*b*ln(c^2*x^2+1)/c^4/d^3+3/2*I*b*polylog(2,1-2/(1+I*
c*x))/c^4/d^3
```

3.59.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{32iacx - \frac{16a}{(-i+cx)^2} + \frac{96ia}{-i+cx} - 96ia\arctan(cx) - 48a\log\left(1+c^2x^2\right) + ib\big(-96\arctan(cx)^2 + 20\cos(2\arctan}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
((32*I)*a*c*x - (16*a)/(-I + c*x)^2 + ((96*I)*a)/(-I + c*x) - (96*I)*a*Arc
Tan[c*x] - 48*a*Log[1 + c^2*x^2] + I*b*(-96*ArcTan[c*x]^2 + 20*Cos[2*ArcTa
n[c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*PolyLog[2, -E^((2*
I)*ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[c*x]*(8*c*x + (10*
I)*Cos[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Log[1 + E^((2*I)*Arc
Tan[c*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[
c*x]]))/(32*c^4*d^3)
```

3.59.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \ 5411$$

$$\int \left(-\frac{3(a+b\arctan(cx))}{c^3d^3(cx-i)} - \frac{3i(a+b\arctan(cx))}{c^3d^3(cx-i)^2} + \frac{i(a+b\arctan(cx))}{c^3d^3} + \frac{a+b\arctan(cx)}{c^3d^3(cx-i)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3i(a+b\arctan(cx))}{c^4d^3(-cx+i)}-\frac{a+b\arctan(cx)}{2c^4d^3(-cx+i)^2}+\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^3}+\frac{iax}{c^3d^3}+\frac{11b\arctan(cx)}{8c^4d^3}+\frac{ibx\arctan(cx)}{c^3d^3}+\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^4d^3}-\frac{11b}{8c^4d^3(-cx+i)}+\frac{ib}{8c^4d^3(-cx+i)^2}-\frac{ib\log(c^2x^2+1)}{2c^4d^3}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(I*a*x)/(c^3*d^3) + ((I/8)*b)/(c^4*d^3*(I - c*x)^2) - (11*b)/(8*c^4*d^3*(I
 - c*x)) + (11*b*ArcTan[c*x])/(8*c^4*d^3) + (I*b*x*ArcTan[c*x])/(c^3*d^3)
- (a + b*ArcTan[c*x])/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x])
)/(c^4*d^3*(I - c*x)) + (3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^
3) - ((I/2)*b*Log[1 + c^2*x^2])/(c^4*d^3) + (((3*I)/2)*b*PolyLog[2, 1 - 2/
(1 + I*c*x)])/(c^4*d^3)
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.59.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{-\frac{3ib\ln(cx-i)^2}{4d^3}-\frac{a}{2d^3(cx-i)^2}+\frac{ib}{8d^3(cx-i)^2}-\frac{3a\ln\left(c^2x^2+1\right)}{2d^3}+\frac{iacx}{d^3}+\frac{3ib\ln\left(c^4x^4+10c^2x^2+9\right)}{64d^3}-\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{3ia\arctan(cx)}{d^3}-3b}{}$
default	$\frac{-\frac{3ib\ln(cx-i)^2}{4d^3}-\frac{a}{2d^3(cx-i)^2}+\frac{ib}{8d^3(cx-i)^2}-\frac{3a\ln\left(c^2x^2+1\right)}{2d^3}+\frac{iacx}{d^3}+\frac{3ib\ln\left(c^4x^4+10c^2x^2+9\right)}{64d^3}-\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{3ia\arctan(cx)}{d^3}-3b}{}$
parts	$\frac{ib}{8c^4d^3(cx-i)^2}+\frac{iax}{c^3d^3}-\frac{3a\ln(c^2x^2+1)}{2d^3c^4}+\frac{3ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2c^4d^3}-\frac{a}{2d^3c^4(-cx+i)^2}-\frac{3ib\ln(cx-i)^2}{4c^4d^3}+\frac{ibx\,\mathrm{arcta}}{c^3d^3}$
risch	$-\frac{3b}{2c^4d^3(-cx+i)}+\frac{19b\arctan(cx)}{16c^4d^3}+\frac{iax}{c^3d^3}+\frac{ib}{8c^4d^3(-cx+i)^2}-\frac{3ia\arctan(cx)}{d^3c^4}-\frac{ib\ln(-icx+1)}{2d^3c^4}+\frac{3ib\,\mathrm{dilog}(\frac{1}{2}}{2d^3c^4}$

input
```
int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-3/4*I*b/d^3*ln(c*x-I)^2-1/2*a/d^3/(c*x-I)^2+1/8*I*b/d^3/(c*x-I)^2-
3/2*a/d^3*ln(c^2*x^2+1)+I*a/d^3*c*x+3/64*I*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-
1/2*b/d^3*arctan(c*x)/(c*x-I)^2-3*I*a/d^3*arctan(c*x)-3*b/d^3*arctan(c*x)*
ln(c*x-I)+3*I*b/d^3*arctan(c*x)/(c*x-I)-3/32*b/d^3*arctan(1/2*c*x)+3/32*b/
d^3*arctan(1/6*c^3*x^3+7/6*c*x)+3/16*b/d^3*arctan(1/2*c*x-1/2*I)+3*I*a/d^3
/(c*x-I)+3/2*I*b/d^3*ln(-1/2*I*(c*x+I))*ln(c*x-I)+19/16*b*arctan(c*x)/d^3+
11/8*b/d^3/(c*x-I)+3/2*I*b/d^3*dilog(-1/2*I*(c*x+I))-19/32*I*b/d^3*ln(c^2*
x^2+1)+I*b/d^3*arctan(c*x)*c*x)
```

3.59.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(c^3*d^3*x^3 -
 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.48

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{-16i\,ac^3x^3 - 32\,ac^2x^2 - 2\,(16i\,a + 11\,b)cx - 12\,(-i\,bc^2x^2 - 2\,bcx + i\,b)\arctan{(cx)}^2 - 3\,(-i\,bc^2x^2 - 2\,b}{}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*(-16*I*a*c^3*x^3 - 32*a*c^2*x^2 - 2*(16*I*a + 11*b)*c*x - 12*(-I*b*c
^2*x^2 - 2*b*c*x + I*b)*arctan(c*x)^2 - 3*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*l
og(c^2*x^2 + 1)^2 + 12*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)*log(1/4*c^2
*x^2 + 1/4) + (-16*I*b*c^3*x^3 - 3*(-16*I*a + 17*b)*c^2*x^2 + 6*(16*a + I*
b)*c*x - 48*I*a - 21*b)*arctan(c*x) + 3*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan
2(c*x, -1) - 24*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog(1/2*I*c*x + 1/2) + 2*(
4*(3*a + I*b)*c^2*x^2 - 8*(3*I*a - b)*c*x - 3*(I*b*c^2*x^2 + 2*b*c*x - I*b
)*log(1/4*c^2*x^2 + 1/4) - 12*a - 4*I*b)*log(c^2*x^2 + 1) - 40*a + 20*I*b)
/(c^6*d^3*x^2 - 2*I*c^5*d^3*x - c^4*d^3)
```

3.59.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.60 $\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx$

3.60.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx = {} & \frac{b}{8c^3d^3(i-cx)^2}+\frac{7ib}{8c^3d^3(i-cx)}-\frac{7ib\arctan(cx)}{8c^3d^3}\\ & +\frac{i(a+b\arctan(cx))}{2c^3d^3(i-cx)^2}-\frac{2(a+b\arctan(cx))}{c^3d^3(i-cx)}\\ & -\frac{i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^3}+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d^3}\end{aligned}$$

output
```
1/8*b/c^3/d^3/(I-c*x)^2+7/8*I*b/c^3/d^3/(I-c*x)-7/8*I*b*arctan(c*x)/c^3/d^
3+1/2*I*(a+b*arctan(c*x))/c^3/d^3/(I-c*x)^2-2*(a+b*arctan(c*x))/c^3/d^3/(I
-c*x)-I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d^3+1/2*b*polylog(2,1-2/(1+I
*c*x))/c^3/d^3
```

3.60.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx =$$

$$-\frac{i\left(12a-6ib+16iacx+7bcx-8a\log\left(\frac{2i}{i-cx}\right)-16iacx\log\left(\frac{2i}{i-cx}\right)+8ac^2x^2\log\left(\frac{2i}{i-cx}\right)+b\arctan(cx)\left(5\right.\right.}{8c^3d^3(-i+cx)^2}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
((-1/8*I)*(12*a - (6*I)*b + (16*I)*a*c*x + 7*b*c*x - 8*a*Log[(2*I)/(I - c*
x)] - (16*I)*a*c*x*Log[(2*I)/(I - c*x)] + 8*a*c^2*x^2*Log[(2*I)/(I - c*x)]
 + b*ArcTan[c*x]*(5 + (2*I)*c*x + 7*c^2*x^2 + 8*(-I + c*x)^2*Log[(2*I)/(I
- c*x)]) + (4*I)*b*(-I + c*x)^2*PolyLog[2, (I + c*x)/(-I + c*x)]))/(c^3*d^
3*(-I + c*x)^2)
```

3.60.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{i(a+b\arctan(cx))}{c^2d^3(cx-i)}-\frac{2(a+b\arctan(cx))}{c^2d^3(cx-i)^2}-\frac{i(a+b\arctan(cx))}{c^2d^3(cx-i)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(a+b\arctan(cx))}{c^3d^3(-cx+i)}+\frac{i(a+b\arctan(cx))}{2c^3d^3(-cx+i)^2}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^3d^3}-\frac{7ib\arctan(cx)}{8c^3d^3}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^3d^3}+\frac{7ib}{8c^3d^3(-cx+i)}+\frac{b}{8c^3d^3(-cx+i)^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
b/(8*c^3*d^3*(I - c*x)^2) + (((7*I)/8)*b)/(c^3*d^3*(I - c*x)) - (((7*I)/8)
*b*ArcTan[c*x])/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)
^2) - (2*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (I*(a + b*ArcTan[c*x])
*Log[2/(1 + I*c*x)])/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*
d^3)
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.60.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{\frac{ia\ln\left(c^2x^2+1\right)}{2d^3}+\frac{2a}{d^3(cx-i)}+\frac{ib\arctan(cx)}{2d^3(cx-i)^2}-\frac{a\arctan(cx)}{d^3}-\frac{7ib\arctan(cx)}{16d^3}+\frac{2b\arctan(cx)}{d^3(cx-i)}+\frac{ia}{2d^3(cx-i)^2}+\frac{7b\ln\left(c^4x^4+10c^2x^2+9\right)}{64d^3}+}{}$
default	$\frac{\frac{ia\ln\left(c^2x^2+1\right)}{2d^3}+\frac{2a}{d^3(cx-i)}+\frac{ib\arctan(cx)}{2d^3(cx-i)^2}-\frac{a\arctan(cx)}{d^3}-\frac{7ib\arctan(cx)}{16d^3}+\frac{2b\arctan(cx)}{d^3(cx-i)}+\frac{ia}{2d^3(cx-i)^2}+\frac{7b\ln\left(c^4x^4+10c^2x^2+9\right)}{64d^3}+}{}$
parts	$-\frac{2a}{d^3c^3(-cx+i)}-\frac{7ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{32c^3d^3}-\frac{a\arctan(cx)}{c^3d^3}-\frac{7ib}{8c^3d^3(cx-i)}+\frac{2b\arctan(cx)}{c^3d^3(cx-i)}+\frac{7ib\arctan\left(\frac{cx}{2}\right)}{32c^3d^3}-$
risch	$\frac{b\ln(icx+1)^2}{4c^3d^3}+\frac{\left(-\frac{ibx}{c^2}-\frac{3b}{4c^3}\right)\ln(icx+1)}{d^3(cx-i)^2}-\frac{ib\ln(-icx+1)x}{8c^2d^3(-icx-1)^2}-\frac{a\arctan(cx)}{c^3d^3}-\frac{ib\ln(-icx+1)x}{2c^2d^3(-icx-1)}+\frac{ib}{d^3c^3(-cx+i)}$

input
```
int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/2*I*a/d^3*ln(c^2*x^2+1)+2*a/d^3/(c*x-I)+1/2*I*b/d^3*arctan(c*x)/(
c*x-I)^2-a/d^3*arctan(c*x)-7/16*I*b/d^3*arctan(c*x)+2*b/d^3*arctan(c*x)/(c
*x-I)+1/2*I*a/d^3/(c*x-I)^2+7/64*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)+I*b/d^3*ar
ctan(c*x)*ln(c*x-I)-7/32*I*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-7/8*I*b/d^3/(
c*x-I)-7/16*I*b/d^3*arctan(1/2*c*x-1/2*I)+1/8*b/d^3/(c*x-I)^2-7/32*b/d^3*l
n(c^2*x^2+1)+7/32*I*b/d^3*arctan(1/2*c*x)+1/2*b/d^3*ln(c*x-I)*ln(-1/2*I*(c
*x+I))+1/2*b/d^3*dilog(-1/2*I*(c*x+I))-1/4*b/d^3*ln(c*x-I)^2)
```

3.60.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(c^3*d^3*x^3 -
 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.65

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx=$$
$$-\frac{-7i\,bc^2x^2\arctan(1,cx)-2\,(7\,b(\arctan(1,cx)-i)+16\,a)cx+4\,(bc^2x^2-2i\,bcx-b)\arctan(cx)^2+(b$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*(-7*I*b*c^2*x^2*arctan2(1, c*x) - 2*(7*b*(arctan2(1, c*x) - I) + 16*
a)*c*x + 4*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)^2 + (b*c^2*x^2 - 2*I*b*
c*x - b)*log(c^2*x^2 + 1)^2 - 4*(I*b*c^2*x^2 + 2*b*c*x - I*b)*arctan(c*x)*
log(1/4*c^2*x^2 + 1/4) + b*(7*I*arctan2(1, c*x) + 12) + ((16*a + 7*I*b)*c^
2*x^2 - 2*(16*I*a + 9*b)*c*x - 16*a + 17*I*b)*arctan(c*x) - 8*(b*c^2*x^2 -
 2*I*b*c*x - b)*dilog(1/2*I*c*x + 1/2) - 2*(4*I*a*c^2*x^2 + 8*a*c*x + (b*c
^2*x^2 - 2*I*b*c*x - b)*log(1/4*c^2*x^2 + 1/4) - 4*I*a)*log(c^2*x^2 + 1) +
 24*I*a)/(c^5*d^3*x^2 - 2*I*c^4*d^3*x - c^3*d^3)
```

3.60.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.61 $\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{ib}{8c^2d^3(i-cx)^2}+\frac{3b}{8c^2d^3(i-cx)}+\frac{b\arctan(cx)}{8c^2d^3}+\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}$$

output
```
-1/8*I*b/c^2/d^3/(I-c*x)^2+3/8*b/c^2/d^3/(I-c*x)+1/8*b*arctan(c*x)/c^2/d^3
+1/2*x^2*(a+b*arctan(c*x))/d^3/(1+I*c*x)^2
```

3.61.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{b(2i-3cx)+a(-4-8icx)-b(1+2icx+3c^2x^2)\arctan(cx)}{8c^2d^3(-i+cx)^2}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(b*(2*I - 3*c*x) + a*(-4 - (8*I)*c*x) - b*(1 + (2*I)*c*x + 3*c^2*x^2)*ArcT
an[c*x])/(8*c^2*d^3*(-I + c*x)^2)
```

3.61.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}dx$$

$$\Big\downarrow \text{5407}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-bc\int \frac{x^2}{2d^3(i-cx)^3(cx+i)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-\frac{bc\int \frac{x^2}{(i-cx)^3(cx+i)}dx}{2d^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-\frac{bc\int \left(-\frac{3}{4c^2(cx-i)^2}-\frac{i}{2c^2(cx-i)^3}-\frac{1}{4c^2(c^2x^2+1)}\right)dx}{2d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-\frac{bc\left(-\frac{\arctan(cx)}{4c^3}-\frac{3}{4c^3(-cx+i)}+\frac{i}{4c^3(-cx+i)^2}\right)}{2d^3}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x]))/(2*d^3*(1 + I*c*x)^2) - (b*c*((I/4)/(c^3*(I - c*
x)^2) - 3/(4*c^3*(I - c*x)) - ArcTan[c*x]/(4*c^3)))/(2*d^3)
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

3.61.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{ib\arctan(cx)}{d^3(cx-i)}-\frac{3b\arctan(cx)}{8d^3}-\frac{ib}{8d^3(cx-i)^2}-\frac{3b}{8d^3(cx-i)}}{c^2}$
default	$\frac{\frac{a\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{ib\arctan(cx)}{d^3(cx-i)}-\frac{3b\arctan(cx)}{8d^3}-\frac{ib}{8d^3(cx-i)^2}-\frac{3b}{8d^3(cx-i)}}{c^2}$
parts	$\frac{a\left(\frac{i}{c^2(-cx+i)}+\frac{1}{2(-cx+i)^2c^2}\right)}{d^3}-\frac{ib\arctan(cx)}{c^2d^3(cx-i)}+\frac{b\arctan(cx)}{2c^2d^3(cx-i)^2}-\frac{3b\arctan(cx)}{8c^2d^3}-\frac{ib}{8c^2d^3(cx-i)^2}-\frac{3b}{8c^2d^3(cx-i)}$
risch	$-\frac{b(2cx-i)\ln(icx+1)}{4c^2d^3(cx-i)^2}-\frac{i(-3\ln(-cx+i)b\,c^2x^2+3\ln(cx+i)b\,c^2x^2+6i\ln(-cx+i)bcx-6i\ln(cx+i)bcx+8ibcx\ln(-icx+\cdots}{16c^2d^3(cx-i)^2}$

input
```
int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a/d^3*(1/2/(c*x-I)^2-I/(c*x-I))+1/2*b/d^3*arctan(c*x)/(c*x-I)^2-I*b
/d^3*arctan(c*x)/(c*x-I)-3/8*b*arctan(c*x)/d^3-1/8*I*b/d^3/(c*x-I)^2-3/8*b
/d^3/(c*x-I))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{2\,(8i\,a+3\,b)cx-(-3i\,bc^2x^2+2\,bcx-i\,b)\log\left(-\frac{cx+i}{cx-i}\right)+8\,a-4i\,b}{16\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*(8*I*a + 3*b)*c*x - (-3*I*b*c^2*x^2 + 2*b*c*x - I*b)*log(-(c*x +
I)/(c*x - I)) + 8*a - 4*I*b)/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

3.61.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(75) = 150.

Time = 5.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{b\left(\frac{3i\log\left(x-\frac{i}{c}\right)}{16}-\frac{3i\log\left(x+\frac{i}{c}\right)}{16}\right)}{c^2d^3}+\frac{(-2bcx+ib)\log(icx+1)}{4c^4d^3x^2-8ic^3d^3x-4c^2d^3}+\frac{(2bcx-ib)\log(-icx+1)}{4c^4d^3x^2-8ic^3d^3x-4c^2d^3}+\frac{-4a+2ib+x(-8iac-3bc)}{8c^4d^3x^2-16ic^3d^3x-8c^2d^3}$$

input
```
integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
b*(3*I*log(x - I/c)/16 - 3*I*log(x + I/c)/16)/(c**2*d**3) + (-2*b*c*x + I*
b)*log(I*c*x + 1)/(4*c**4*d**3*x**2 - 8*I*c**3*d**3*x - 4*c**2*d**3) + (2*
b*c*x - I*b)*log(-I*c*x + 1)/(4*c**4*d**3*x**2 - 8*I*c**3*d**3*x - 4*c**2*
d**3) + (-4*a + 2*I*b + x*(-8*I*a*c - 3*b*c))/(8*c**4*d**3*x**2 - 16*I*c**
3*d**3*x - 8*c**2*d**3)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{(8i\,a+3\,b)cx+(3\,bc^2x^2+2i\,bcx+b)\arctan{(cx)}+4\,a-2i\,b}{8\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/8*((8*I*a + 3*b)*c*x + (3*b*c^2*x^2 + 2*I*b*c*x + b)*arctan(c*x) + 4*a
- 2*I*b)/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

3.61.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{(b\arctan{(cx)}+a)x}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

3.62 $\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx$

3.62.1 Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx=-\frac{b}{8cd^3(i-cx)^2}+\frac{ib}{8cd^3(i-cx)}-\frac{ib\arctan(cx)}{8cd^3}+\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}$$

output
```
-1/8*b/c/d^3/(I-c*x)^2+1/8*I*b/c/d^3/(I-c*x)-1/8*I*b*arctan(c*x)/c/d^3+1/2
*I*(a+b*arctan(c*x))/c/d^3/(1+I*c*x)^2
```

3.62.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx=-\frac{i(4a+b(-2i+cx)+b(3-2icx+c^2x^2)\arctan(cx))}{8cd^3(-i+cx)^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
```

output
```
((-1/8*I)*(4*a + b*(-2*I + c*x) + b*(3 - (2*I)*c*x + c^2*x^2)*ArcTan[c*x])
)/(c*d^3*(-I + c*x)^2)
```

3.62.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5387, 27, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \frac{1}{d^2(icx+1)^2(c^2x^2+1)}dx}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \frac{1}{(icx+1)^2(c^2x^2+1)}dx}{2d^3}$$

$$\Big\downarrow \text{456}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \frac{1}{(1-icx)(icx+1)^3}dx}{2d^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \left(-\frac{1}{4(cx-i)^2}+\frac{i}{2(cx-i)^3}+\frac{1}{4(c^2x^2+1)}\right)dx}{2d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\left(\frac{\arctan(cx)}{4c}-\frac{1}{4c(-cx+i)}-\frac{i}{4c(-cx+i)^2}\right)}{2d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
```

output
```
((I/2)*(a + b*ArcTan[c*x]))/(c*d^3*(1 + I*c*x)^2) - ((I/2)*b*((-1/4*I)/(c*
(I - c*x)^2) - 1/(4*c*(I - c*x)) + ArcTan[c*x]/(4*c)))/d^3
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.62.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{ia}{2d^3(icx+1)^2}+\frac{ib\arctan(cx)}{2d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8d^3}-\frac{b}{8d^3(cx-i)^2}-\frac{ib}{8d^3(cx-i)}}{c}$
default	$\frac{\frac{ia}{2d^3(icx+1)^2}+\frac{ib\arctan(cx)}{2d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8d^3}-\frac{b}{8d^3(cx-i)^2}-\frac{ib}{8d^3(cx-i)}}{c}$
parts	$\frac{ia}{2d^3(icx+1)^2c}+\frac{ib\arctan(cx)}{2c\,d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8c\,d^3}-\frac{b}{8c\,d^3(cx-i)^2}-\frac{ib}{8c\,d^3(cx-i)}$
risch	$-\frac{b\ln(icx+1)}{4c\,d^3(cx-i)^2}+\frac{4b\ln(-icx+1)-\ln(-cx+i)b\,c^2x^2+\ln(cx+i)b\,c^2x^2+2i\ln(-cx+i)bcx-2i\ln(cx+i)bcx+b\ln(-cx+i)-}{16d^3(cx-i)^2c}$

input
```
int((a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*I*a/d^3/(1+I*c*x)^2+1/2*I*b/d^3/(1+I*c*x)^2*arctan(c*x)-1/8*I*b/d
^3*arctan(c*x)-1/8*b/d^3/(c*x-I)^2-1/8*I*b/d^3/(c*x-I))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \frac{-2i\,bcx+(bc^2x^2-2i\,bcx+3\,b)\log\left(-\frac{cx+i}{cx-i}\right)-8i\,a-4\,b}{16\,(c^3d^3x^2-2i\,c^2d^3x-cd^3)}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
1/16*(-2*I*b*c*x + (b*c^2*x^2 - 2*I*b*c*x + 3*b)*log(-(c*x + I)/(c*x - I))
 - 8*I*a - 4*b)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

3.62.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(70) = 140.

Time = 2.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \frac{b\log(-icx+1)}{4c^3d^3x^2-8ic^2d^3x-4cd^3} - \frac{b\log(icx+1)}{4c^3d^3x^2-8ic^2d^3x-4cd^3} + \frac{b\left(-\frac{\log\left(bx-\frac{ib}{c}\right)}{16}+\frac{\log\left(bx+\frac{ib}{c}\right)}{16}\right)}{cd^3} + \frac{-4ia-ibcx-2b}{8c^3d^3x^2-16ic^2d^3x-8cd^3}$$

input
```
integrate((a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
b*log(-I*c*x + 1)/(4*c**3*d**3*x**2 - 8*I*c**2*d**3*x - 4*c*d**3) - b*log(
I*c*x + 1)/(4*c**3*d**3*x**2 - 8*I*c**2*d**3*x - 4*c*d**3) + b*(-log(b*x -
 I*b/c)/16 + log(b*x + I*b/c)/16)/(c*d**3) + (-4*I*a - I*b*c*x - 2*b)/(8*c
**3*d**3*x**2 - 16*I*c**2*d**3*x - 8*c*d**3)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = -\frac{i\,bcx + (i\,bc^2x^2 + 2\,bcx + 3i\,b)\arctan(cx) + 4i\,a + 2\,b}{8\,(c^3d^3x^2 - 2i\,c^2d^3x - cd^3)}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/8*(I*b*c*x + (I*b*c^2*x^2 + 2*b*c*x + 3*I*b)*arctan(c*x) + 4*I*a + 2*b)
/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

3.62.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^3, x)
```

3.63 $\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx$

3.63.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx &= \frac{ib}{8d^3(i-cx)^2}+\frac{5b}{8d^3(i-cx)}-\frac{5b\arctan(cx)}{8d^3} \\ &\quad-\frac{a+b\arctan(cx)}{2d^3(i-cx)^2}+\frac{i(a+b\arctan(cx))}{d^3(i-cx)}+\frac{a\log(x)}{d^3} \\ &\quad+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d^3} \\ &\quad-\frac{ib\operatorname{PolyLog}(2,icx)}{2d^3}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output
```
1/8*I*b/d^3/(I-c*x)^2+5/8*b/d^3/(I-c*x)-5/8*b*arctan(c*x)/d^3+1/2*(-a-b*ar
ctan(c*x))/d^3/(I-c*x)^2+I*(a+b*arctan(c*x))/d^3/(I-c*x)+a*ln(x)/d^3+(a+b*
arctan(c*x))*ln(2/(1+I*c*x))/d^3+1/2*I*b*polylog(2,-I*c*x)/d^3-1/2*I*b*pol
ylog(2,I*c*x)/d^3+1/2*I*b*polylog(2,1-2/(1+I*c*x))/d^3
```

3.63.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx$$
$$= \frac{\frac{5b}{i-cx}+\frac{ib}{(-i+cx)^2}-5b\arctan(cx)-\frac{4(a+b\arctan(cx))}{(-i+cx)^2}-\frac{8i(a+b\arctan(cx))}{-i+cx}+8a\log(x)+8(a+b\arctan(cx))\log\left(\frac{}{i}\right.}{8d^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```

output
```
((5*b)/(I - c*x) + (I*b)/(-I + c*x)^2 - 5*b*ArcTan[c*x] - (4*(a + b*ArcTan
[c*x]))/(-I + c*x)^2 - ((8*I)*(a + b*ArcTan[c*x]))/(-I + c*x) + 8*a*Log[x]
 + 8*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + (4*I)*b*PolyLog[2, (-I)*c*
x] - (4*I)*b*PolyLog[2, I*c*x] + (4*I)*b*PolyLog[2, (I + c*x)/(-I + c*x)])
/(8*d^3)
```

3.63.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx$$
$$\Big\downarrow \; 5411$$
$$\int \left(\frac{a+b\arctan(cx)}{d^3x}-\frac{c(a+b\arctan(cx))}{d^3(cx-i)}+\frac{ic(a+b\arctan(cx))}{d^3(cx-i)^2}+\frac{c(a+b\arctan(cx))}{d^3(cx-i)^3}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{i(a+b\arctan(cx))}{d^3(-cx+i)}-\frac{a+b\arctan(cx)}{2d^3(-cx+i)^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^3}+\frac{a\log(x)}{d^3}-\frac{5b\arctan(cx)}{8d^3}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d^3}-\frac{ib\,\text{PolyLog}(2,icx)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^3}+\frac{5b}{8d^3(-cx+i)}+\frac{ib}{8d^3(-cx+i)^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```

output
```
((I/8)*b)/(d^3*(I - c*x)^2) + (5*b)/(8*d^3*(I - c*x)) - (5*b*ArcTan[c*x])/
(8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x])
)/(d^3*(I - c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/d^3 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*PolyLog[2, I*c*x])
/d^3 + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.63.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a\ln(cx)}{d^3}-\frac{a}{2d^3(cx-i)^2}-\frac{ia}{d^3(cx-i)}-\frac{a\ln(c^2x^2+1)}{2d^3}-\frac{ia\arctan(cx)}{d^3}+\frac{b\left(\arctan(cx)\ln(cx)-\frac{\arctan(cx)}{2(cx-i)^2}-\frac{i\arctan}{cx-}\right.}{}$
default	$\frac{a\ln(cx)}{d^3}-\frac{a}{2d^3(cx-i)^2}-\frac{ia}{d^3(cx-i)}-\frac{a\ln(c^2x^2+1)}{2d^3}-\frac{ia\arctan(cx)}{d^3}+\frac{b\left(\arctan(cx)\ln(cx)-\frac{\arctan(cx)}{2(cx-i)^2}-\frac{i\arctan}{cx-}\right.}{}$
parts	$\frac{a\ln(x)}{d^3}-\frac{a}{2d^3(-cx+i)^2}+\frac{ia}{d^3(-cx+i)}-\frac{a\ln(c^2x^2+1)}{2d^3}-\frac{ia\arctan(cx)}{d^3}+\frac{b\left(\arctan(cx)\ln(cx)-\frac{\arctan(cx)}{2(cx-i)^2}-\frac{i\,\mathrm{arc}}{c}\right.}{}$
risch	$\frac{ib\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d^3}-\frac{ib\ln(icx+1)}{2d^3(icx+1)}+\frac{ib\ln(-icx+1)c^2x^2}{16d^3(-icx-1)^2}-\frac{ia\arctan(cx)}{d^3}+\frac{i\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)b}{2d^3}-\frac{ib\ln(icx+}{4d^3(icx+1}$

input
```
int((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d^3*ln(c*x)-1/2*a/d^3/(c*x-I)^2-I*a/d^3/(c*x-I)-1/2*a/d^3*ln(c^2*x^2+1)-
I*a/d^3*arctan(c*x)+b/d^3*(arctan(c*x)*ln(c*x)-1/2*arctan(c*x)/(c*x-I)^2-I
*arctan(c*x)/(c*x-I)-arctan(c*x)*ln(c*x-I)-5/8*arctan(c*x)+1/8*I/(c*x-I)^2
-5/8/(c*x-I)-1/2*I*(dilog(-I*(c*x+I))+ln(c*x)*ln(-I*(c*x+I)))+1/2*I*((ln(c
*x)-ln(-I*c*x))*ln(-I*(-c*x+I))-dilog(-I*c*x))+1/2*I*(dilog(-1/2*I*(c*x+I)
)+ln(c*x-I)*ln(-1/2*I*(c*x+I)))-1/4*I*ln(c*x-I)^2)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx = -\frac{2\,(8i\,a+5\,b)cx+8\,(i\,bc^2x^2+2\,bcx-i\,b)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-16\,(ac^2x^2-2i\,acx-a)\log{(x)}-4\,(2\,bcx-3i$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*(8*I*a + 5*b)*c*x + 8*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog((c*x +
I)/(c*x - I) + 1) - 16*(a*c^2*x^2 - 2*I*a*c*x - a)*log(x) - 4*(2*b*c*x - 3
*I*b)*log(-(c*x + I)/(c*x - I)) + 5*(I*b*c^2*x^2 + 2*b*c*x - I*b)*log((c*x
 + I)/c) + ((16*a - 5*I*b)*c^2*x^2 + 2*(-16*I*a - 5*b)*c*x - 16*a + 5*I*b)
*log((c*x - I)/c) + 24*a - 12*I*b)/(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)
```

3.63.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

3.63.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(151) = 302$.

Time = 0.29 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.08

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx$$
$$= \frac{2(-8ia-5b)cx+4(-ibc^2x^2-2bcx+ib)\arctan(cx)^2-(ibc^2x^2+2bcx-ib)\log(c^2x^2+1)^2-4(bc^2}{}$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/16*(2*(-8*I*a - 5*b)*c*x + 4*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*arctan(c*x)^
2 - (I*b*c^2*x^2 + 2*b*c*x - I*b)*log(c^2*x^2 + 1)^2 - 4*(b*c^2*x^2 - 2*I*
b*c*x - b)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + 16*(b*c^2*x^2 - 2*I*b*c*x
- b)*arctan(c*x)*log(c*x) - ((16*I*a + 5*b)*c^2*x^2 + 2*(16*a + 3*I*b)*c*x
 - 16*I*a + 19*b)*arctan(c*x) + 5*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan2(c*x,
 -1) + 8*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*dilog(I*c*x + 1) + 8*(I*b*c^2*x^2
+ 2*b*c*x - I*b)*dilog(1/2*I*c*x + 1/2) + 8*(I*b*c^2*x^2 + 2*b*c*x - I*b)*
dilog(-I*c*x + 1) - 2*(2*(pi*b + 2*a)*c^2*x^2 - 4*(I*pi*b + 2*I*a)*c*x - 2
*pi*b - (I*b*c^2*x^2 + 2*b*c*x - I*b)*log(1/4*c^2*x^2 + 1/4) - 4*a)*log(c^
2*x^2 + 1) + 16*(a*c^2*x^2 - 2*I*a*c*x - a)*log(x) - 24*a + 12*I*b)/(c^2*d
^3*x^2 - 2*I*c*d^3*x - d^3)
```

3.63.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx = \int \frac{b\arctan(cx)+a}{(icdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^3), x)
```

3.64 $\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx$

3.64.1 Optimal result

Integrand size = 23, antiderivative size = 250

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx = {} & \frac{bc}{8d^3(i-cx)^2}-\frac{9ibc}{8d^3(i-cx)}+\frac{9ibc\arctan(cx)}{8d^3}-\frac{a+b\arctan(cx)}{d^3x} \\ & +\frac{ic(a+b\arctan(cx))}{2d^3(i-cx)^2}+\frac{2c(a+b\arctan(cx))}{d^3(i-cx)}-\frac{3iac\log(x)}{d^3} \\ & +\frac{bc\log(x)}{d^3}-\frac{3ic(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^3} \\ & -\frac{bc\log\left(1+c^2x^2\right)}{2d^3}+\frac{3bc\,\mathrm{PolyLog}(2,-icx)}{2d^3} \\ & -\frac{3bc\,\mathrm{PolyLog}(2,icx)}{2d^3}+\frac{3bc\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output
```
1/8*b*c/d^3/(I-c*x)^2-9/8*I*b*c/d^3/(I-c*x)+9/8*I*b*c*arctan(c*x)/d^3+(-a-
b*arctan(c*x))/d^3/x+1/2*I*c*(a+b*arctan(c*x))/d^3/(I-c*x)^2+2*c*(a+b*arct
an(c*x))/d^3/(I-c*x)-3*I*a*c*ln(x)/d^3+b*c*ln(x)/d^3-3*I*c*(a+b*arctan(c*x
))*ln(2/(1+I*c*x))/d^3-1/2*b*c*ln(c^2*x^2+1)/d^3+3/2*b*c*polylog(2,-I*c*x)
/d^3-3/2*b*c*polylog(2,I*c*x)/d^3+3/2*b*c*polylog(2,1-2/(1+I*c*x))/d^3
```

3.64.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx$$
$$= \frac{-8ibc\left(\frac{1}{i-cx}-\arctan(cx)\right)-\frac{8(a+b\arctan(cx))}{x}+\frac{4ic(a+b\arctan(cx))}{(-i+cx)^2}-\frac{16c(a+b\arctan(cx))}{-i+cx}+\frac{bc(2+icx+i(-i+cx)^2\arctan(cx}{(-i+cx)^2}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```

output
```
((-8*I)*b*c*((I - c*x)^(-1) - ArcTan[c*x]) - (8*(a + b*ArcTan[c*x]))/x + (
(4*I)*c*(a + b*ArcTan[c*x]))/(-I + c*x)^2 - (16*c*(a + b*ArcTan[c*x]))/(-I
 + c*x) + (b*c*(2 + I*c*x + I*(-I + c*x)^2*ArcTan[c*x]))/(-I + c*x)^2 - (2
4*I)*a*c*Log[x] - (24*I)*c*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + 4*b*
c*(2*Log[x] - Log[1 + c^2*x^2]) + 12*b*c*PolyLog[2, (-I)*c*x] - 12*b*c*Pol
yLog[2, I*c*x] + 12*b*c*PolyLog[2, (I + c*x)/(-I + c*x)])/(8*d^3)
```

3.64.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{3ic^2(a+b\arctan(cx))}{d^3(cx-i)}+\frac{2c^2(a+b\arctan(cx))}{d^3(cx-i)^2}-\frac{ic^2(a+b\arctan(cx))}{d^3(cx-i)^3}+\frac{a+b\arctan(cx)}{d^3x^2}-\frac{3ic(a+b\operatorname{arcta}}{d^3x}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{2c(a+b\arctan(cx))}{d^3(-cx+i)}+\frac{ic(a+b\arctan(cx))}{2d^3(-cx+i)^2}-\frac{a+b\arctan(cx)}{d^3x}-\frac{3ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^3}-\frac{3iac\log(x)}{d^3}+\frac{9ibc\arctan(cx)}{8d^3}-\frac{bc\log(c^2x^2+1)}{2d^3}+\frac{3bc\operatorname{PolyLog}(2,-icx)}{2d^3}-\frac{3bc\operatorname{PolyLog}(2,icx)}{2d^3}+\frac{3bc\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^3}-\frac{9ibc}{8d^3(-cx+i)}+\frac{bc}{8d^3(-cx+i)^2}+\frac{bc\log(x)}{d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```

output
```
(b*c)/(8*d^3*(I - c*x)^2) - (((9*I)/8)*b*c)/(d^3*(I - c*x)) + (((9*I)/8)*b
*c*ArcTan[c*x])/d^3 - (a + b*ArcTan[c*x])/(d^3*x) + ((I/2)*c*(a + b*ArcTan
[c*x]))/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - ((
3*I)*a*c*Log[x])/d^3 + (b*c*Log[x])/d^3 - ((3*I)*c*(a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/d^3 - (b*c*Log[1 + c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2,
(-I)*c*x])/(2*d^3) - (3*b*c*PolyLog[2, I*c*x])/(2*d^3) + (3*b*c*PolyLog[2,
 1 - 2/(1 + I*c*x)])/(2*d^3)
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.64.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.21

method	result
derivativedivides	$c\left(-\frac{a}{d^3cx}-\frac{3ia\ln(cx)}{d^3}+\frac{ia}{2d^3(cx-i)^2}+\frac{3ia\ln(c^2x^2+1)}{2d^3}-\frac{3a\arctan(cx)}{d^3}-\frac{2a}{d^3(cx-i)}+\frac{b\left(-\frac{\arctan(cx)}{cx}+\frac{}{8(c}\right.}{}\right.$
default	$c\left(-\frac{a}{d^3cx}-\frac{3ia\ln(cx)}{d^3}+\frac{ia}{2d^3(cx-i)^2}+\frac{3ia\ln(c^2x^2+1)}{2d^3}-\frac{3a\arctan(cx)}{d^3}-\frac{2a}{d^3(cx-i)}+\frac{b\left(-\frac{\arctan(cx)}{cx}+\frac{}{8(c}\right.}{}\right.$
parts	$-\frac{a}{d^3x}-\frac{3iac\ln(x)}{d^3}+\frac{iac}{2d^3(-cx+i)^2}+\frac{3ica\ln(c^2x^2+1)}{2d^3}-\frac{3ca\arctan(cx)}{d^3}+\frac{2ac}{d^3(-cx+i)}+\frac{bc\left(-\frac{\arctan(cx)}{cx}+\frac{}{8}\right.}{}$
risch	$-\frac{a}{d^3x}+\frac{9bc\ln(c^2x^2+1)}{32d^3}-\frac{3ca\arctan(cx)}{d^3}+\frac{ic^2b\ln(-icx+1)x}{2d^3(-icx-1)}-\frac{ic^2b\ln(-icx+1)x}{8d^3(-icx-1)^2}+\frac{3c\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right.}{2d^3}$

input
```
int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-a/d^3/c/x-3*I*a/d^3*ln(c*x)+1/2*I*a/d^3/(c*x-I)^2+3/2*I*a/d^3*ln(c^2*x
^2+1)-3*a/d^3*arctan(c*x)-2*a/d^3/(c*x-I)+b/d^3*(-1/c/x*arctan(c*x)+9/8*I/
(c*x-I)+1/2*I*arctan(c*x)/(c*x-I)^2-3*I*arctan(c*x)*ln(c*x)-2*arctan(c*x)/
(c*x-I)-1/2*ln(c^2*x^2+1)+9/8*I*arctan(c*x)+ln(c*x)+3*I*arctan(c*x)*ln(c*x
-I)+1/8/(c*x-I)^2-3/2*dilog(-I*(c*x+I))-3/2*ln(c*x)*ln(-I*(c*x+I))+3/2*(ln
(c*x)-ln(-I*c*x))*ln(-I*(-c*x+I))-3/2*dilog(-I*c*x)+3/2*ln(c*x-I)*ln(-1/2*
I*(c*x+I))+3/2*dilog(-1/2*I*(c*x+I))-3/4*ln(c*x-I)^2))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.05

$$\int\frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx = -\frac{6\,(8\,a-3i\,b)c^2x^2+4\,(-18i\,a-5\,b)cx+24\,(bc^3x^3-2i\,bc^2x^2-bcx)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+16\,((3i\,a-b)c^3x^3-}{}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*(8*a - 3*I*b)*c^2*x^2 + 4*(-18*I*a - 5*b)*c*x + 24*(b*c^3*x^3 - 2
*I*b*c^2*x^2 - b*c*x)*dilog((c*x + I)/(c*x - I) + 1) + 16*((3*I*a - b)*c^3
*x^3 + 2*(3*a + I*b)*c^2*x^2 + (-3*I*a + b)*c*x)*log(x) + 4*(6*I*b*c^2*x^2
 + 9*b*c*x - 2*I*b)*log(-(c*x + I)/(c*x - I)) + 17*(b*c^3*x^3 - 2*I*b*c^2*
x^2 - b*c*x)*log((c*x + I)/c) - ((48*I*a + b)*c^3*x^3 + 2*(48*a - I*b)*c^2
*x^2 + (-48*I*a - b)*c*x)*log((c*x - I)/c) - 16*a)/(c^2*d^3*x^3 - 2*I*c*d^
3*x^2 - d^3*x)
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.64.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^3), x)
```

3.65 $\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx$

3.65.1 Optimal result

Integrand size = 23, antiderivative size = 306

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx &= -\frac{bc}{2d^3x}-\frac{ibc^2}{8d^3(i-cx)^2}-\frac{13bc^2}{8d^3(i-cx)}+\frac{9bc^2\arctan(cx)}{8d^3}\\&\quad-\frac{a+b\arctan(cx)}{2d^3x^2}+\frac{3ic(a+b\arctan(cx))}{d^3x}\\&\quad+\frac{c^2(a+b\arctan(cx))}{2d^3(i-cx)^2}-\frac{3ic^2(a+b\arctan(cx))}{d^3(i-cx)}-\frac{6ac^2\log(x)}{d^3}\\&\quad-\frac{3ibc^2\log(x)}{d^3}-\frac{6c^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}\\&\quad+\frac{3ibc^2\log(1+c^2x^2)}{2d^3}-\frac{3ibc^2\,\mathrm{PolyLog}(2,-icx)}{d^3}\\&\quad+\frac{3ibc^2\,\mathrm{PolyLog}(2,icx)}{d^3}-\frac{3ibc^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^3}\end{aligned}$$

output
```
-1/2*b*c/d^3/x-1/8*I*b*c^2/d^3/(I-c*x)^2-13/8*b*c^2/d^3/(I-c*x)+9/8*b*c^2*
arctan(c*x)/d^3+1/2*(-a-b*arctan(c*x))/d^3/x^2+3*I*c*(a+b*arctan(c*x))/d^3
/x+1/2*c^2*(a+b*arctan(c*x))/d^3/(I-c*x)^2-3*I*c^2*(a+b*arctan(c*x))/d^3/(
I-c*x)-6*a*c^2*ln(x)/d^3-3*I*b*c^2*ln(x)/d^3-6*c^2*(a+b*arctan(c*x))*ln(2/
(1+I*c*x))/d^3+3/2*I*b*c^2*ln(c^2*x^2+1)/d^3-3*I*b*c^2*polylog(2,-I*c*x)/d
^3+3*I*b*c^2*polylog(2,I*c*x)/d^3-3*I*b*c^2*polylog(2,1-2/(1+I*c*x))/d^3
```

3.65.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.93

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx =$$
$$-\frac{12bc^2\left(\frac{1}{i-cx}-\arctan(cx)\right)+\frac{4(a+b\arctan(cx))}{x^2}-\frac{24ic(a+b\arctan(cx))}{x}-\frac{4c^2(a+b\arctan(cx))}{(-i+cx)^2}-\frac{24ic^2(a+b\arctan(cx))}{-i+cx}-\frac{b}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

output
```
-1/8*(12*b*c^2*((I - c*x)^(-1) - ArcTan[c*x]) + (4*(a + b*ArcTan[c*x]))/x^
2 - ((24*I)*c*(a + b*ArcTan[c*x]))/x - (4*c^2*(a + b*ArcTan[c*x]))/(-I + c
*x)^2 - ((24*I)*c^2*(a + b*ArcTan[c*x]))/(-I + c*x) - (b*c^2*(-2*I + c*x +
 (-I + c*x)^2*ArcTan[c*x]))/(-I + c*x)^2 + (4*b*c*Hypergeometric2F1[-1/2,
1, 1/2, -(c^2*x^2)])/x + 48*a*c^2*Log[x] + 48*c^2*(a + b*ArcTan[c*x])*Log[
(2*I)/(I - c*x)] + (12*I)*b*c^2*(2*Log[x] - Log[1 + c^2*x^2]) + (24*I)*b*c
^2*PolyLog[2, (-I)*c*x] - (24*I)*b*c^2*PolyLog[2, I*c*x] + (24*I)*b*c^2*Po
lyLog[2, (I + c*x)/(-I + c*x)])/d^3
```

3.65.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{6c^3(a+b\arctan(cx))}{d^3(cx-i)}-\frac{3ic^3(a+b\arctan(cx))}{d^3(cx-i)^2}-\frac{c^3(a+b\arctan(cx))}{d^3(cx-i)^3}-\frac{6c^2(a+b\arctan(cx))}{d^3x}+\frac{a+b\,\mathrm{arcta}}{d^3x^3}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3ic^2(a+b\arctan(cx))}{d^3(-cx+i)}+\frac{c^2(a+b\arctan(cx))}{2d^3(-cx+i)^2}-\frac{6c^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^3}-\frac{a+b\arctan(cx)}{2d^3x^2}+\frac{3ic(a+b\arctan(cx))}{d^3x}-\frac{6ac^2\log(x)}{d^3}+\frac{9bc^2\arctan(cx)}{8d^3}-\frac{3ibc^2\operatorname{PolyLog}(2,-icx)}{d^3}+\frac{3ibc^2\operatorname{PolyLog}(2,icx)}{d^3}-\frac{3ibc^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{d^3}+\frac{3ibc^2\log\left(c^2x^2+1\right)}{2d^3}-\frac{13bc^2}{8d^3(-cx+i)}-\frac{ibc^2}{8d^3(-cx+i)^2}-\frac{3ibc^2\log(x)}{d^3}-\frac{bc}{2d^3x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

output
```
-1/2*(b*c)/(d^3*x) - ((I/8)*b*c^2)/(d^3*(I - c*x)^2) - (13*b*c^2)/(8*d^3*(
I - c*x)) + (9*b*c^2*ArcTan[c*x])/(8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*x^2
) + ((3*I)*c*(a + b*ArcTan[c*x]))/(d^3*x) + (c^2*(a + b*ArcTan[c*x]))/(2*d
^3*(I - c*x)^2) - ((3*I)*c^2*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - (6*a*c
^2*Log[x])/d^3 - ((3*I)*b*c^2*Log[x])/d^3 - (6*c^2*(a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/d^3 + (((3*I)/2)*b*c^2*Log[1 + c^2*x^2])/d^3 - ((3*I)*b*c
^2*PolyLog[2, (-I)*c*x])/d^3 + ((3*I)*b*c^2*PolyLog[2, I*c*x])/d^3 - ((3*I
)*b*c^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

3.65.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.65.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.13

method	result
derivativedivides	$c^2\left(-\frac{a}{2d^3c^2x^2}+\frac{3ia}{d^3cx}-\frac{6a\ln(cx)}{d^3}+\frac{3ia}{d^3(cx-i)}+\frac{a}{2d^3(cx-i)^2}+\frac{3a\ln(c^2x^2+1)}{d^3}+\frac{6ia\arctan(cx)}{d^3}+\cdots\right.$
default	$c^2\left(-\frac{a}{2d^3c^2x^2}+\frac{3ia}{d^3cx}-\frac{6a\ln(cx)}{d^3}+\frac{3ia}{d^3(cx-i)}+\frac{a}{2d^3(cx-i)^2}+\frac{3a\ln(c^2x^2+1)}{d^3}+\frac{6ia\arctan(cx)}{d^3}+\cdots\right.$
parts	$-\frac{a}{2d^3x^2}+\frac{3ica}{d^3x}-\frac{6a\,c^2\ln(x)}{d^3}-\frac{3ia\,c^2}{d^3(-cx+i)}+\frac{a\,c^2}{2d^3(-cx+i)^2}+\frac{6ic^2a\arctan(cx)}{d^3}+\frac{3c^2a\ln(c^2x^2+1)}{d^3}+\cdots$
risch	$-\frac{bc}{2d^3x}-\frac{15b\,c^2\arctan(cx)}{16d^3}+\frac{3ic^2b\ln(-icx+1)}{4d^3(-icx-1)}-\frac{3ic^2b\ln(-icx+1)}{16d^3(-icx-1)^2}+\frac{3c^3b\ln(-icx+1)x}{4d^3(-icx-1)}-\frac{c^3b\ln(-icx+1)x}{8d^3(-icx-1)^2}\cdots$

input
```
int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a/d^3/c^2/x^2+3*I*a/d^3/c/x-6*a/d^3*ln(c*x)+3*I*a/d^3/(c*x-I)+1/
2*a/d^3/(c*x-I)^2+3*a/d^3*ln(c^2*x^2+1)+6*I*a/d^3*arctan(c*x)+b/d^3*(-1/2/
c^2/x^2*arctan(c*x)+3/2*I*ln(c^2*x^2+1)-6*arctan(c*x)*ln(c*x)+3/2*I*ln(c*x
-I)^2+1/2*arctan(c*x)/(c*x-I)^2+6*arctan(c*x)*ln(c*x-I)-3*I*(dilog(-1/2*I*
(c*x+I))+ln(c*x-I)*ln(-1/2*I*(c*x+I)))+9/8*arctan(c*x)-3*I*((ln(c*x)-ln(-I
*c*x))*ln(-I*(-c*x+I))-dilog(-I*c*x))-1/2/c/x-3*I*ln(c*x)+13/8/(c*x-I)-1/8
*I/(c*x-I)^2+3*I*(dilog(-I*(c*x+I))+ln(c*x)*ln(-I*(c*x+I)))+3*I*arctan(c*x
)/c/x+3*I*arctan(c*x)/(c*x-I)))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.02

$$\int\frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx = -\frac{6(-16i\,a-3\,b)c^3x^3-12\,(12\,a-i\,b)c^2x^2+8\,(4i\,a-b)cx+48\,(-i\,bc^4x^4-2\,bc^3x^3+i\,bc^2x^2)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+\cdots\right.}{\cdots}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*(-16*I*a - 3*b)*c^3*x^3 - 12*(12*a - I*b)*c^2*x^2 + 8*(4*I*a - b)
*c*x + 48*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*dilog((c*x + I)/(c*x
- I) + 1) + 48*((2*a + I*b)*c^4*x^4 + 2*(-2*I*a + b)*c^3*x^3 - (2*a + I*b)
*c^2*x^2)*log(x) + 4*(12*b*c^3*x^3 - 18*I*b*c^2*x^2 - 4*b*c*x - I*b)*log(-
(c*x + I)/(c*x - I)) + 33*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*log((
c*x + I)/c) - 3*((32*a + 5*I*b)*c^4*x^4 - 2*(32*I*a - 5*b)*c^3*x^3 - (32*a
 + 5*I*b)*c^2*x^2)*log((c*x - I)/c) - 8*a)/(c^2*d^3*x^4 - 2*I*c*d^3*x^3 -
d^3*x^2)
```

3.65.6 Sympy [F]

$$\int \frac{a + b\arctan(cx)}{x^3(d + icdx)^3}\,dx = \frac{i\left(\int \frac{a}{c^3x^6 - 3ic^2x^5 - 3cx^4 + ix^3}\,dx + \int \frac{b\,\mathrm{atan}\,(cx)}{c^3x^6 - 3ic^2x^5 - 3cx^4 + ix^3}\,dx\right)}{d^3}$$

input
```
integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**3,x)
```

output
```
I*(Integral(a/(c**3*x**6 - 3*I*c**2*x**5 - 3*c*x**4 + I*x**3), x) + Integr
al(b*atan(c*x)/(c**3*x**6 - 3*I*c**2*x**5 - 3*c*x**4 + I*x**3), x))/d**3
```

3.65.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(253) = 506$.

Time = 0.31 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.93

$$\int \frac{a + b\arctan(cx)}{x^3(d + icdx)^3}\,dx = -\frac{33\,bc^4x^4\arctan\left(1, cx\right) + 6\left(b(-11i\arctan\left(1, cx\right) - 3) - 16i\,a\right)c^3x^3 - 3\left(b(11\arctan\left(1, cx\right) - 4i) + 48\,a\right.}{}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*(33*b*c^4*x^4*arctan2(1, c*x) + 6*(b*(-11*I*arctan2(1, c*x) - 3) - 1
6*I*a)*c^3*x^3 - 3*(b*(11*arctan2(1, c*x) - 4*I) + 48*a)*c^2*x^2 + 8*(4*I*
a - b)*c*x + 24*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*arctan(c*x)^2 +
 6*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*log(c^2*x^2 + 1)^2 - 24*(b*c
^4*x^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + 9
6*(b*c^4*x^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(c*x) + (3*(-32*I
*a + 5*b)*c^4*x^4 - 6*(32*a + 21*I*b)*c^3*x^3 + 3*(32*I*a - 53*b)*c^2*x^2
+ 32*I*b*c*x - 8*b)*arctan(c*x) + 48*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2
*x^2)*dilog(I*c*x + 1) + 48*(I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^2*x^2)*dilo
g(1/2*I*c*x + 1/2) + 48*(I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^2*x^2)*dilog(-I
*c*x + 1) - 12*(2*((pi + I)*b + 2*a)*c^4*x^4 - 4*((I*pi - 1)*b + 2*I*a)*c^
3*x^3 - 2*((pi + I)*b + 2*a)*c^2*x^2 - (I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^
2*x^2)*log(1/4*c^2*x^2 + 1/4))*log(c^2*x^2 + 1) + 48*((2*a + I*b)*c^4*x^4
+ 2*(-2*I*a + b)*c^3*x^3 - (2*a + I*b)*c^2*x^2)*log(x) - 8*a)/(c^2*d^3*x^4
 - 2*I*c*d^3*x^3 - d^3*x^2)
```

3.65.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(i\,cdx+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(d+c\,d\,x\,1\mathrm{i}\right)}^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^3), x)
```

3.66 $\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx$

3.66.1 Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = -\frac{ib}{18c(i-cx)^3} - \frac{b}{24c(i-cx)^2} + \frac{ib}{24c(i-cx)} - \frac{ib\arctan(cx)}{24c} + \frac{i(a+b\arctan(cx))}{3c(1+icx)^3}$$

output
```
-1/18*I*b/c/(I-c*x)^3-1/24*b/c/(I-c*x)^2+1/24*I*b/c/(I-c*x)-1/24*I*b*arcta
n(c*x)/c+1/3*I*(a+b*arctan(c*x))/c/(1+I*c*x)^3
```

3.66.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = \frac{-24a+b(10i-9cx-3ic^2x^2)+3b(-7+3icx-3c^2x^2-ic^3x^3)\arctan(cx)}{72c(-i+cx)^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
```

output
```
(-24*a + b*(10*I - 9*c*x - (3*I)*c^2*x^2) + 3*b*(-7 + (3*I)*c*x - 3*c^2*x^
2 - I*c^3*x^3)*ArcTan[c*x])/(72*c*(-I + c*x)^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5387, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\int \frac{1}{(icx+1)^3(c^2x^2+1)}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\int \frac{1}{(1-icx)(icx+1)^4}dx$$

$$\Big\downarrow \text{54}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\int \left(-\frac{1}{8(cx-i)^2}+\frac{i}{4(cx-i)^3}+\frac{1}{2(cx-i)^4}+\frac{1}{8(c^2x^2+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\left(\frac{\arctan(cx)}{8c}-\frac{1}{8c(-cx+i)}-\frac{i}{8c(-cx+i)^2}+\frac{1}{6c(-cx+i)^3}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
```

output
```
((I/3)*(a + b*ArcTan[c*x]))/(c*(1 + I*c*x)^3) - (I/3)*b*(1/(6*c*(I - c*x)^
3) - (I/8)/(c*(I - c*x)^2) - 1/(8*c*(I - c*x)) + ArcTan[c*x]/(8*c))
```

3.66.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.66.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{ia}{3(icx+1)^3}+\frac{ib\arctan(cx)}{3(icx+1)^3}-\frac{ib\arctan(cx)}{24}-\frac{b}{24(cx-i)^2}+\frac{ib}{18(cx-i)^3}-\frac{ib}{24(cx-i)}}{c}$
default	$\frac{\frac{ia}{3(icx+1)^3}+\frac{ib\arctan(cx)}{3(icx+1)^3}-\frac{ib\arctan(cx)}{24}-\frac{b}{24(cx-i)^2}+\frac{ib}{18(cx-i)^3}-\frac{ib}{24(cx-i)}}{c}$
parts	$\frac{ia}{3(icx+1)^3c}+\frac{ib\arctan(cx)}{3c(icx+1)^3}-\frac{ib\arctan(cx)}{24c}-\frac{b}{24c(cx-i)^2}+\frac{ib}{18c(cx-i)^3}-\frac{ib}{24c(cx-i)}$
risch	$\frac{ib\ln(icx+1)}{6c(cx-i)^3}-\frac{24ib\ln(-icx+1)-3bc^3\ln(-cx-i)x^3+3bc^3\ln(cx-i)x^3+9i\ln(-cx-i)bc^2x^2-9i\ln(cx-i)bc^2x^2+9\ln(-}{144(cx-i)^3c}$

input
```
int((a+b*arctan(c*x))/(1+I*c*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*I*a/(1+I*c*x)^3+1/3*I*b/(1+I*c*x)^3*arctan(c*x)-1/24*I*b*arctan(c
*x)-1/24*b/(c*x-I)^2+1/18*I*b/(c*x-I)^3-1/24*I*b/(c*x-I))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{a + b\arctan(cx)}{(1 + icx)^4}\,dx$$
$$= \frac{-6i\,bc^2x^2 - 18\,bcx + 3\,(bc^3x^3 - 3i\,bc^2x^2 - 3\,bcx - 7i\,b)\log\left(-\frac{cx+i}{cx-i}\right) - 48\,a + 20i\,b}{144\,(c^4x^3 - 3i\,c^3x^2 - 3\,c^2x + i\,c)}$$

input
```
integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="fricas")
```

output
```
1/144*(-6*I*b*c^2*x^2 - 18*b*c*x + 3*(b*c^3*x^3 - 3*I*b*c^2*x^2 - 3*b*c*x
- 7*I*b)*log(-(c*x + I)/(c*x - I)) - 48*a + 20*I*b)/(c^4*x^3 - 3*I*c^3*x^2
 - 3*c^2*x + I*c)
```

3.66.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.

Time = 1.39 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.68

$$\int \frac{a + b\arctan(cx)}{(1 + icx)^4}\,dx = -\frac{ib\log(-icx + 1)}{6c^4x^3 - 18ic^3x^2 - 18c^2x + 6ic} + \frac{ib\log(icx + 1)}{6c^4x^3 - 18ic^3x^2 - 18c^2x + 6ic} + \frac{b\left(-\frac{\log\left(bx - \frac{ib}{c}\right)}{48} + \frac{\log\left(bx + \frac{ib}{c}\right)}{48}\right)}{c} + \frac{-24a - 3ibc^2x^2 - 9bcx + 10ib}{72c^4x^3 - 216ic^3x^2 - 216c^2x + 72ic}$$

input
```
integrate((a+b*atan(c*x))/(1+I*c*x)**4,x)
```

output
```
-I*b*log(-I*c*x + 1)/(6*c**4*x**3 - 18*I*c**3*x**2 - 18*c**2*x + 6*I*c) +
I*b*log(I*c*x + 1)/(6*c**4*x**3 - 18*I*c**3*x**2 - 18*c**2*x + 6*I*c) + b*
(-log(b*x - I*b/c)/48 + log(b*x + I*b/c)/48)/c + (-24*a - 3*I*b*c**2*x**2
- 9*b*c*x + 10*I*b)/(72*c**4*x**3 - 216*I*c**3*x**2 - 216*c**2*x + 72*I*c)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = -\frac{3i\,bc^2x^2+9\,bcx-3\left(-i\,bc^3x^3-3\,bc^2x^2+3i\,bcx-7\,b\right)\arctan(cx)+24\,a-10i\,b}{72\left(c^4x^3-3i\,c^3x^2-3\,c^2x+i\,c\right)}$$

input
```
integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="maxima")
```

output
```
-1/72*(3*I*b*c^2*x^2 + 9*b*c*x - 3*(-I*b*c^3*x^3 - 3*b*c^2*x^2 + 3*I*b*c*x
 - 7*b)*arctan(c*x) + 24*a - 10*I*b)/(c^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*
c)
```

3.66.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cx+1)^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(1+c\,x\,1\mathrm{i})^4}\,dx$$

input
```
int((a + b*atan(c*x))/(c*x*1i + 1)^4,x)
```

output
```
int((a + b*atan(c*x))/(c*x*1i + 1)^4, x)
```

3.67 $\int \frac{\arctan(ax)}{cx+iacx^2} dx$

3.67.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{\arctan(ax)}{cx+iacx^2}\,dx = \frac{\arctan(ax)\log\left(2-\frac{2}{1+iax}\right)}{c}+\frac{i\,\text{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)}{2c}$$

output
```
arctan(a*x)*ln(2-2/(1+I*a*x))/c+1/2*I*polylog(2,-1+2/(1+I*a*x))/c
```

3.67.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int \frac{\arctan(ax)}{cx+iacx^2}\,dx = \frac{\arctan(ax)\log\left(\frac{2i}{i-ax}\right)}{c}+\frac{i\,\text{PolyLog}(2,-iax)}{2c} - \frac{i\,\text{PolyLog}(2,iax)}{2c}+\frac{i\,\text{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2c}$$

input
```
Integrate[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

output
```
(ArcTan[a*x]*Log[(2*I)/(I - a*x)])/c + ((I/2)*PolyLog[2, (-I)*a*x])/c - ((
I/2)*PolyLog[2, I*a*x])/c + ((I/2)*PolyLog[2, -((I + a*x)/(I - a*x))])/c
```

3.67.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2026, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{cx+iacx^2}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{\arctan(ax)}{x(c+iacx)}dx$$
$$\Big\downarrow \text{ 5403}$$
$$\frac{\arctan(ax)\log\left(2-\frac{2}{1+iax}\right)}{c}-\frac{a\int \frac{\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \text{ 2897}$$
$$\frac{\arctan(ax)\log\left(2-\frac{2}{1+iax}\right)}{c}+\frac{i\,\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)}{2c}$$

input
```
Int[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

output
```
(ArcTan[a*x]*Log[2 - 2/(1 + I*a*x)])/c + ((I/2)*PolyLog[2, -1 + 2/(1 + I*a
*x)])/c
```

3.67.3.1 Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

3.67.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 1.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.12

method	result
risch	$\frac{i\ln(iax+1)^2}{4c}+\frac{i\,\mathrm{dilog}(iax+1)}{2c}+\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln\left(\frac{1}{2}+\frac{iax}{2}\right)}{2c}-\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{2c}-\frac{i\,\mathrm{dilog}(-iax+1)}{2c}+$ i di
derivativedivides	$\frac{a\arctan(ax)\ln(ax)}{c}-\frac{a\arctan(ax)\ln(ax-i)}{c}-\frac{a\left(-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\mathrm{dilog}(iax+1)}{2}+\frac{i\,\mathrm{dilog}(-iax+1)}{2}-\right.}{c}$ i(d … / a
default	$\frac{a\arctan(ax)\ln(ax)}{c}-\frac{a\arctan(ax)\ln(ax-i)}{c}-\frac{a\left(-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\mathrm{dilog}(iax+1)}{2}+\frac{i\,\mathrm{dilog}(-iax+1)}{2}-\right.}{c}$ i(d … / a
parts	$-\frac{\arctan(ax)\ln(-ax+i)}{c}+\frac{\arctan(ax)\ln(x)}{c}-\frac{a\left(-\frac{i\ln(x)(\ln(iax+1)-\ln(-iax+1))}{2a}-\frac{i(\mathrm{dilog}(iax+1)-\mathrm{dilog}(-iax+1))}{2a}+\right.}{}$

input
```
int(arctan(a*x)/(c*x+I*a*c*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/c*ln(1+I*a*x)^2+1/2*I/c*dilog(1+I*a*x)+1/2*I/c*ln(1/2-1/2*I*a*x)*ln(
1/2+1/2*I*a*x)-1/2*I/c*ln(1/2+1/2*I*a*x)*ln(1-I*a*x)-1/2*I/c*dilog(1-I*a*x
)+1/2*I/c*dilog(1/2-1/2*I*a*x)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.43

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx = -\frac{i\,\mathrm{Li}_2\left(\frac{ax+i}{ax-i} + 1\right)}{2\,c}$$

input
```
integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="fricas")
```

output
```
-1/2*I*dilog((a*x + I)/(a*x - I) + 1)/c
```

3.67.6 Sympy [F]

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx = -\frac{i\int \frac{\mathrm{atan}\,(ax)}{ax^2 - ix}\,dx}{c}$$

input
```
integrate(atan(a*x)/(c*x+I*a*c*x**2),x)
```

output
```
-I*Integral(atan(a*x)/(a*x**2 - I*x), x)/c
```

3.67.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(40) = 80.

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx$$
$$= \frac{1}{4}a\left(-\frac{i\,\log\left(i\,ax+1\right)^2}{ac} + \frac{2i\left(\log\left(i\,ax+1\right)\log\left(-\frac{1}{2}i\,ax+\frac{1}{2}\right) + \mathrm{Li}_2\left(\frac{1}{2}i\,ax+\frac{1}{2}\right)\right)}{ac} + \frac{2i\left(\log\left(i\,ax+1\right)\log\left(x\right.\right.}{ac}\right.$$
$$\left. - \left(\frac{\log\left(i\,ax+1\right)}{c} - \frac{\log\left(x\right)}{c}\right)\arctan\left(ax\right)\right.$$

input
```
integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="maxima")
```

output
```
1/4*a*(-I*log(I*a*x + 1)^2/(a*c) + 2*I*(log(I*a*x + 1)*log(-1/2*I*a*x + 1/
2) + dilog(1/2*I*a*x + 1/2))/(a*c) + 2*I*(log(I*a*x + 1)*log(x) + dilog(-I
*a*x))/(a*c) - 2*I*(log(-I*a*x + 1)*log(x) + dilog(I*a*x))/(a*c)) - (log(I
*a*x + 1)/c - log(x)/c)*arctan(a*x)
```

3.67.8 Giac [F]

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx = \int \frac{\arctan(ax)}{i\,acx^2 + cx}\,dx$$

input
```
integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx = \int \frac{\operatorname{atan}(a\,x)}{1\mathrm{i}\,a\,c\,x^2 + c\,x}\,dx$$

input
```
int(atan(a*x)/(c*x + a*c*x^2*1i),x)
```

output
```
int(atan(a*x)/(c*x + a*c*x^2*1i), x)
```

3.68 $\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx$

3.68.1 Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx &= \frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{ib^2dx^3}{30c}\\&+\frac{3ib^2d\arctan(cx)}{10c^4}+\frac{b^2dx\arctan(cx)}{2c^3}\\&+\frac{ibdx^2(a+b\arctan(cx))}{5c^2}-\frac{bdx^3(a+b\arctan(cx))}{6c}\\&-\frac{1}{10}ibdx^4(a+b\arctan(cx))\\&-\frac{9d(a+b\arctan(cx))^2}{20c^4}+\frac{1}{4}dx^4(a+b\arctan(cx))^2\\&+\frac{1}{5}icdx^5(a+b\arctan(cx))^2\\&+\frac{2ibd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^4}\\&-\frac{b^2d\log(1+c^2x^2)}{3c^4}-\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}\end{aligned}$$

output
```
1/2*a*b*d*x/c^3-3/10*I*b^2*d*x/c^3+1/12*b^2*d*x^2/c^2+1/30*I*b^2*d*x^3/c+3
/10*I*b^2*d*arctan(c*x)/c^4+1/2*b^2*d*x*arctan(c*x)/c^3+1/5*I*b*d*x^2*(a+b
*arctan(c*x))/c^2-1/6*b*d*x^3*(a+b*arctan(c*x))/c-1/10*I*b*d*x^4*(a+b*arct
an(c*x))-9/20*d*(a+b*arctan(c*x))^2/c^4+1/4*d*x^4*(a+b*arctan(c*x))^2+1/5*
I*c*d*x^5*(a+b*arctan(c*x))^2+2/5*I*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/
c^4-1/3*b^2*d*ln(c^2*x^2+1)/c^4-1/5*b^2*d*polylog(2,1-2/(1+I*c*x))/c^4
```

3.68.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$= \frac{d(18iab+5b^2+30abcx-18ib^2cx+12iabc^2x^2+5b^2c^2x^2-10abc^3x^3+2ib^2c^3x^3+15a^2c^4x^4-6iabc^4x^4+}{}$$

input
```
Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d*((18*I)*a*b + 5*b^2 + 30*a*b*c*x - (18*I)*b^2*c*x + (12*I)*a*b*c^2*x^2
+ 5*b^2*c^2*x^2 - 10*a*b*c^3*x^3 + (2*I)*b^2*c^3*x^3 + 15*a^2*c^4*x^4 - (6
*I)*a*b*c^4*x^4 + (12*I)*a^2*c^5*x^5 + 3*b^2*(-1 + 5*c^4*x^4 + (4*I)*c^5*x
^5)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(9*I + 15*c*x + (6*I)*c^2*x^2 - 5*c
^3*x^3 - (3*I)*c^4*x^4) + 3*a*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5) + (12*I)*b*
Log[1 + E^((2*I)*ArcTan[c*x])]) - (12*I)*a*b*Log[1 + c^2*x^2] - 20*b^2*Log
[1 + c^2*x^2] + 12*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^4)
```

3.68.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \ 5411$$
$$\int \left(dx^3(a+b\arctan(cx))^2+icdx^4(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{9d(a+b\arctan(cx))^2}{20c^4}+\frac{2ibd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^4}+\frac{ibdx^2(a+b\arctan(cx))}{5c^2}+\frac{1}{5}icdx^5(a+b\arctan(cx))^2+\frac{1}{4}dx^4(a+b\arctan(cx))^2-\frac{1}{10}ibdx^4(a+b\arctan(cx))-\frac{bdx^3(a+b\arctan(cx))}{6c}+\frac{abdx}{2c^3}+\frac{3ib^2d\arctan(cx)}{10c^4}+\frac{b^2dx\arctan(cx)}{2c^3}-\frac{b^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^4}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}-\frac{b^2d\log\left(c^2x^2+1\right)}{3c^4}+\frac{ib^2dx^3}{30c}$$

input
```
Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*b*d*x)/(2*c^3) - (((3*I)/10)*b^2*d*x)/c^3 + (b^2*d*x^2)/(12*c^2) + ((I/
30)*b^2*d*x^3)/c + (((3*I)/10)*b^2*d*ArcTan[c*x])/c^4 + (b^2*d*x*ArcTan[c*
x])/(2*c^3) + ((I/5)*b*d*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d*x^3*(a + b*Ar
cTan[c*x]))/(6*c) - (I/10)*b*d*x^4*(a + b*ArcTan[c*x]) - (9*d*(a + b*ArcTa
n[c*x])^2)/(20*c^4) + (d*x^4*(a + b*ArcTan[c*x])^2)/4 + (I/5)*c*d*x^5*(a +
 b*ArcTan[c*x])^2 + (((2*I)/5)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])
/c^4 - (b^2*d*Log[1 + c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 + I*c
*x)])/(5*c^4)
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.68.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.31

method	result
parts	$a^2 d\left(\frac{1}{5} i c\, x^5+\frac{1}{4} x^4\right)+\frac{d\, b^2 \left(-\frac{i \arctan(cx) c^4 x^4}{10}+\frac{c^4 x^4 \arctan(cx)^2}{4}+\frac{cx \arctan(cx)}{2}-\frac{i \arctan(cx) \ln\left(c^2 x^2+1\right)}{5}-\frac{c^3 x^3 \operatorname{arct}}{6}\right.}{}$
derivativedivides	$\frac{a^2 d\left(\frac{1}{5} i c^5 x^5+\frac{1}{4} c^4 x^4\right)+d\, b^2\left(-\frac{i \arctan(cx) c^4 x^4}{10}+\frac{c^4 x^4 \arctan(cx)^2}{4}+\frac{cx \arctan(cx)}{2}-\frac{i \arctan(cx) \ln\left(c^2 x^2+1\right)}{5}-\frac{c^3 x^3 \arctan(cx)}{6}\right.}{}$
default	$\frac{a^2 d\left(\frac{1}{5} i c^5 x^5+\frac{1}{4} c^4 x^4\right)+d\, b^2\left(-\frac{i \arctan(cx) c^4 x^4}{10}+\frac{c^4 x^4 \arctan(cx)^2}{4}+\frac{cx \arctan(cx)}{2}-\frac{i \arctan(cx) \ln\left(c^2 x^2+1\right)}{5}-\frac{c^3 x^3 \arctan(cx)}{6}\right.}{}$
risch	$\frac{b^2 d\, x^2}{12 c^2}-\frac{b^2 d \ln\left(c^2 x^2+1\right)}{3 c^4}+\frac{abdx}{2 c^3}+\frac{d\, x^4 a^2}{4}-\frac{9 d\, a^2}{20 c^4}-\frac{bda \arctan(cx)}{2 c^4}-\frac{dab\, x^3}{6c}+\frac{5 b^2 d}{12 c^4}+\left(\frac{i d\, b^2\left(4 x^5 c-5 i x\right.}{40}\right.$

input
```
int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*(1/5*I*c*x^5+1/4*x^4)+d*b^2/c^4*(-1/10*I*arctan(c*x)*c^4*x^4+1/4*c^4
*x^4*arctan(c*x)^2+1/2*c*x*arctan(c*x)-1/5*I*arctan(c*x)*ln(c^2*x^2+1)-1/6
*c^3*x^3*arctan(c*x)+1/30*I*c^3*x^3+1/5*I*arctan(c*x)*c^2*x^2-1/4*arctan(c
*x)^2+1/10*ln(c*x-I)*ln(c^2*x^2+1)-1/10*dilog(-1/2*I*(c*x+I))-1/10*ln(c*x-
I)*ln(-1/2*I*(c*x+I))-1/20*ln(c*x-I)^2-1/10*ln(c*x+I)*ln(c^2*x^2+1)+1/10*d
ilog(1/2*I*(c*x-I))+1/10*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/20*ln(c*x+I)^2+3/10
*I*arctan(c*x)-3/10*I*c*x+1/12*c^2*x^2-1/3*ln(c^2*x^2+1)+1/5*I*arctan(c*x)
^2*c^5*x^5)+2*a*b*d/c^4*(1/5*I*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*arctan(c*x)
+1/4*c*x-1/20*I*c^4*x^4-1/12*c^3*x^3+1/10*I*c^2*x^2-1/10*I*ln(c^2*x^2+1)-1
/4*arctan(c*x))
```

3.68.5 Fricas [F]

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/80*(-4*I*b^2*c*d*x^5 - 5*b^2*d*x^4)*log(-(c*x + I)/(c*x - I))^2 + integr
al(1/20*(20*I*a^2*c^3*d*x^6 + 20*a^2*c^2*d*x^5 + 20*I*a^2*c*d*x^4 + 20*a^2
*d*x^3 - (20*a*b*c^3*d*x^6 + 4*(-5*I*a*b - b^2)*c^2*d*x^5 + 5*(4*a*b + I*b
^2)*c*d*x^4 - 20*I*a*b*d*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.68.7 Maxima [F]

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan{(cx)}+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*I*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctan(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*I*(
4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*
b*c*d + 1/80*I*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 + 80*integrat
e(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(c^2*x^6 + x
^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x
))*b^2*c*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*
x)/c^5))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arcta
n(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d
```

3.68.8 Giac [F]

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.69 $\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx$

3.69.1 Optimal result

Integrand size = 23, antiderivative size = 255

$$\begin{aligned}\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx &= \frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{ib^2dx^2}{12c}-\frac{b^2d\arctan(cx)}{3c^3}\\ &\quad+\frac{ib^2dx\arctan(cx)}{2c^2}-\frac{bdx^2(a+b\arctan(cx))}{3c}\\ &\quad-\frac{1}{6}ibdx^3(a+b\arctan(cx))\\ &\quad-\frac{7id(a+b\arctan(cx))^2}{12c^3}+\frac{1}{3}dx^3(a+b\arctan(cx))^2\\ &\quad+\frac{1}{4}icdx^4(a+b\arctan(cx))^2\\ &\quad-\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\ &\quad-\frac{ib^2d\log(1+c^2x^2)}{3c^3}-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\end{aligned}$$

output
```
1/2*I*a*b*d*x/c^2+1/3*b^2*d*x/c^2+1/12*I*b^2*d*x^2/c-1/3*b^2*d*arctan(c*x)
/c^3+1/2*I*b^2*d*x*arctan(c*x)/c^2-1/3*b*d*x^2*(a+b*arctan(c*x))/c-1/6*I*b
*d*x^3*(a+b*arctan(c*x))-7/12*I*d*(a+b*arctan(c*x))^2/c^3+1/3*d*x^3*(a+b*a
rctan(c*x))^2+1/4*I*c*d*x^4*(a+b*arctan(c*x))^2-2/3*b*d*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/c^3-1/3*I*b^2*d*ln(c^2*x^2+1)/c^3-1/3*I*b^2*d*polylog(2,1-
2/(1+I*c*x))/c^3
```

3.69.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.95

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{id\left(b^2+6abcx-4ib^2cx+4iabc^2x^2+b^2c^2x^2-4ia^2c^3x^3-2abc^3x^3+3a^2c^4x^4+b^2(1-4ic^3x^3+3c^4x^4)\operatorname{arc}\right.}{}$$

input
```
Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((I/12)*d*(b^2 + 6*a*b*c*x - (4*I)*b^2*c*x + (4*I)*a*b*c^2*x^2 + b^2*c^2*x
^2 - (4*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(1 - (4*I)*c^
3*x^3 + 3*c^4*x^4)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(2*I + 3*c*x + (2*I)
*c^2*x^2 - c^3*x^3) + a*(-3 - (4*I)*c^3*x^3 + 3*c^4*x^4) + (4*I)*b*Log[1 +
 E^((2*I)*ArcTan[c*x])]) - (4*I)*a*b*Log[1 + c^2*x^2] - 4*b^2*Log[1 + c^2*
x^2] + 4*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c^3
```

3.69.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(dx^2(a+b\arctan(cx))^2+icdx^3(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{7id(a+b\arctan(cx))^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{1}{4}icdx^4(a+b\arctan(cx))^2+\frac{1}{3}dx^3(a+b\arctan(cx))^2-\frac{1}{6}ibdx^3(a+b\arctan(cx))-\frac{bdx^2(a+b\arctan(cx))}{3c}+\frac{iabdx}{2c^2}-\frac{b^2d\arctan(cx)}{3c^3}+\frac{ib^2dx\arctan(cx)}{2c^2}-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2dx}{3c^2}-\frac{ib^2d\log(c^2x^2+1)}{3c^3}+\frac{ib^2dx^2}{12c}$$

input
```
Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((I/2)*a*b*d*x)/c^2 + (b^2*d*x)/(3*c^2) + ((I/12)*b^2*d*x^2)/c - (b^2*d*Ar
cTan[c*x])/(3*c^3) + ((I/2)*b^2*d*x*ArcTan[c*x])/c^2 - (b*d*x^2*(a + b*Arc
Tan[c*x]))/(3*c) - (I/6)*b*d*x^3*(a + b*ArcTan[c*x]) - (((7*I)/12)*d*(a +
b*ArcTan[c*x])^2)/c^3 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/4)*c*d*x^4*(a
 + b*ArcTan[c*x])^2 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^
3) - ((I/3)*b^2*d*Log[1 + c^2*x^2])/c^3 - ((I/3)*b^2*d*PolyLog[2, 1 - 2/(1
 + I*c*x)])/c^3
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.69.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.38

method	result
parts	$a^2 d\left(\frac{1}{4}icx^4+\frac{1}{3}x^3\right)+\frac{db^2\left(\frac{i\arctan(cx)^2c^4x^4}{4}+\frac{c^3x^3\arctan(cx)^2}{3}-\frac{i\arctan(cx)c^3x^3}{6}-\frac{i\arctan(cx)^2}{4}+\frac{i\arctan(cx)cx}{2}-\right.}{}$
derivativedivides	$\frac{a^2 d\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+db^2\left(\frac{i\arctan(cx)^2c^4x^4}{4}+\frac{c^3x^3\arctan(cx)^2}{3}-\frac{i\arctan(cx)c^3x^3}{6}-\frac{i\arctan(cx)^2}{4}+\frac{i\arctan(cx)cx}{2}-\right.}{}$
default	$\frac{a^2 d\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+db^2\left(\frac{i\arctan(cx)^2c^4x^4}{4}+\frac{c^3x^3\arctan(cx)^2}{3}-\frac{i\arctan(cx)c^3x^3}{6}-\frac{i\arctan(cx)^2}{4}+\frac{i\arctan(cx)cx}{2}-\right.}{}$
risch	$\frac{b^2dx}{3c^2}-\frac{115b^2d\arctan(cx)}{288c^3}-\frac{abd}{c^3}+\frac{a^2dx^3}{3}+\frac{bda\ln(c^2x^2+1)}{3c^3}-\frac{dabx^2}{3c}+\frac{idab\ln(-icx+1)x^3}{3}-$

input
```
int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*(1/4*I*c*x^4+1/3*x^3)+d*b^2/c^3*(1/4*I*arctan(c*x)^2*c^4*x^4+1/3*c^3
*x^3*arctan(c*x)^2-1/6*I*arctan(c*x)*c^3*x^3-1/4*I*arctan(c*x)^2+1/2*I*arc
tan(c*x)*c*x-1/3*c^2*x^2*arctan(c*x)+1/3*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x
+1/12*I*c^2*x^2-1/3*I*ln(c^2*x^2+1)-1/3*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c^
2*x^2+1)-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^
2)-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*
(c*x-I))-1/2*ln(c*x+I)^2))+2*a*b*d/c^3*(1/4*I*arctan(c*x)*c^4*x^4+1/3*c^3*
x^3*arctan(c*x)+1/4*I*c*x-1/12*I*c^3*x^3-1/6*c^2*x^2+1/6*ln(c^2*x^2+1)-1/4
*I*arctan(c*x))
```

3.69.5 Fricas [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/48*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(-(c*x + I)/(c*x - I))^2 + integr
al(1/12*(12*I*a^2*c^3*d*x^5 + 12*a^2*c^2*d*x^4 + 12*I*a^2*c*d*x^3 + 12*a^2
*d*x^2 - (12*a*b*c^3*d*x^5 + 3*(-4*I*a*b - b^2)*c^2*d*x^4 + 4*(3*a*b + I*b
^2)*c*d*x^3 - 12*I*a*b*d*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*I*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3
 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d + 1/3*(2*x^3*arctan(c*x) - c*(x^
2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d - 1/48*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^
3)*arctan(c*x)^2 - 1/48*(3*b^2*c*d*x^4 - 4*I*b^2*d*x^3)*arctan(c*x)*log(c^
2*x^2 + 1) + 1/192*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + I
*integrate(-1/48*(14*b^2*c^2*d*x^4*arctan(c*x) - 36*(b^2*c^3*d*x^5 + b^2*c
*d*x^3)*arctan(c*x)^2 - 3*(b^2*c^3*d*x^5 + b^2*c*d*x^3)*log(c^2*x^2 + 1)^2
 - (3*b^2*c^3*d*x^5 - 4*b^2*c*d*x^3 - 12*(b^2*c^2*d*x^4 + b^2*d*x^2)*arcta
n(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + integrate(1/48*(36*(b^2*c^2*
d*x^4 + b^2*d*x^2)*arctan(c*x)^2 + 3*(b^2*c^2*d*x^4 + b^2*d*x^2)*log(c^2*x
^2 + 1)^2 + 2*(3*b^2*c^3*d*x^5 - 4*b^2*c*d*x^3)*arctan(c*x) + (7*b^2*c^2*d
*x^4 + 12*(b^2*c^3*d*x^5 + b^2*c*d*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^
2*x^2 + 1), x)
```

3.69.8 Giac [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx = \int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.70 $\int x(d+icdx)(a+b\arctan(cx))^2\,dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 211

$$\begin{aligned}\int x(d+icdx)(a+b\arctan(cx))^2\,dx &= -\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{ib^2d\arctan(cx)}{3c^2}-\frac{b^2dx\arctan(cx)}{c}\\ &\quad-\frac{1}{3}ibdx^2(a+b\arctan(cx))+\frac{5d(a+b\arctan(cx))^2}{6c^2}\\ &\quad+\frac{1}{2}dx^2(a+b\arctan(cx))^2+\frac{1}{3}icdx^3(a+b\arctan(cx))^2\\ &\quad-\frac{2ibd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^2}\\ &\quad+\frac{b^2d\log(1+c^2x^2)}{2c^2}+\frac{b^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}\end{aligned}$$

output
```
-a*b*d*x/c+1/3*I*b^2*d*x/c-1/3*I*b^2*d*arctan(c*x)/c^2-b^2*d*x*arctan(c*x)
/c-1/3*I*b*d*x^2*(a+b*arctan(c*x))+5/6*d*(a+b*arctan(c*x))^2/c^2+1/2*d*x^2
*(a+b*arctan(c*x))^2+1/3*I*c*d*x^3*(a+b*arctan(c*x))^2-2/3*I*b*d*(a+b*arct
an(c*x))*ln(2/(1+I*c*x))/c^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2+1/3*b^2*d*polylog
(2,1-2/(1+I*c*x))/c^2
```

3.70.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$= \frac{d(-6abcx+2ib^2cx+3a^2c^2x^2-2iabc^2x^2+2ia^2c^3x^3+b^2(1+3c^2x^2+2ic^3x^3)\arctan(cx)^2+2b\arctan(cx}{}$$

input
```
Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d*(-6*a*b*c*x + (2*I)*b^2*c*x + 3*a^2*c^2*x^2 - (2*I)*a*b*c^2*x^2 + (2*I)
*a^2*c^3*x^3 + b^2*(1 + 3*c^2*x^2 + (2*I)*c^3*x^3)*ArcTan[c*x]^2 + 2*b*Arc
Tan[c*x]*((-I)*b*(1 - (3*I)*c*x + c^2*x^2) + a*(3 + 3*c^2*x^2 + (2*I)*c^3*
x^3) - (2*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2
] + 3*b^2*Log[1 + c^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(6
*c^2)
```

3.70.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(dx(a+b\arctan(cx))^2+icdx^2(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{5d(a+b\arctan(cx))^2}{6c^2}-\frac{2ibd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^2}+\frac{1}{3}icdx^3(a+b\arctan(cx))^2+\frac{1}{2}dx^2(a+b\arctan(cx))^2-\frac{1}{3}ibdx^2(a+b\arctan(cx))-\frac{abdx}{c}-\frac{ib^2d\arctan(cx)}{3c^2}-\frac{b^2dx\arctan(cx)}{c}+\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^2}+\frac{b^2d\log\left(c^2x^2+1\right)}{2c^2}+\frac{ib^2dx}{3c}$$

input
```
Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
-((a*b*d*x)/c) + ((I/3)*b^2*d*x)/c - ((I/3)*b^2*d*ArcTan[c*x])/c^2 - (b^2*
d*x*ArcTan[c*x])/c - (I/3)*b*d*x^2*(a + b*ArcTan[c*x]) + (5*d*(a + b*ArcTa
n[c*x])^2)/(6*c^2) + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (I/3)*c*d*x^3*(a +
b*ArcTan[c*x])^2 - (((2*I)/3)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
c^2 + (b^2*d*Log[1 + c^2*x^2])/(2*c^2) + (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*
x)])/(3*c^2)
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.70.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.50

method	result
parts	$a^2 d\left(\frac{1}{3} i c\, x^3+\frac{1}{2} x^2\right)+\frac{d\, b^2 \left(\frac{i \arctan(cx) \ln\left(c^2 x^2+1\right)}{3}+\frac{c^2 x^2 \arctan(cx)^2}{2}-\frac{i \arctan(cx) c^2 x^2}{3}+\frac{i \arctan(cx)^2 c^3 x^3}{3}+\right.}{}$
derivativedivides	$\frac{a^2 d\left(\frac{1}{3} i c^3 x^3+\frac{1}{2} c^2 x^2\right)+d\, b^2 \left(\frac{i \arctan(cx) \ln\left(c^2 x^2+1\right)}{3}+\frac{c^2 x^2 \arctan(cx)^2}{2}-\frac{i \arctan(cx) c^2 x^2}{3}+\frac{i \arctan(cx)^2 c^3 x^3}{3}+\frac{\arctan(cx)^2}{2}-\right.}{}$
default	$\frac{a^2 d\left(\frac{1}{3} i c^3 x^3+\frac{1}{2} c^2 x^2\right)+d\, b^2 \left(\frac{i \arctan(cx) \ln\left(c^2 x^2+1\right)}{3}+\frac{c^2 x^2 \arctan(cx)^2}{2}-\frac{i \arctan(cx) c^2 x^2}{3}+\frac{i \arctan(cx)^2 c^3 x^3}{3}+\frac{\arctan(cx)^2}{2}-\right.}{}$
risch	$\frac{a^2 d\, x^2}{2}+\frac{73 b^2 d \ln(c^2 x^2+1)}{144 c^2}-\frac{a b d x}{c}-\frac{b^2 d}{3 c^2}-\frac{i b d\, x^2 a}{3}+\frac{d\, b^2 \ln(-icx+1) x^2}{6}-\frac{i d\, b^2 \ln(-icx+1) x}{2c}+$

input
```
int(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*(1/3*I*x^3*c+1/2*x^2)+d*b^2/c^2*(1/3*I*arctan(c*x)*ln(c^2*x^2+1)+1/2
*c^2*x^2*arctan(c*x)^2-1/3*I*arctan(c*x)*c^2*x^2+1/3*I*arctan(c*x)^2*c^3*x
^3+1/2*arctan(c*x)^2-c*x*arctan(c*x)-1/6*ln(c*x-I)*ln(c^2*x^2+1)+1/6*dilog
(-1/2*I*(c*x+I))+1/6*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/12*ln(c*x-I)^2+1/6*ln(
c*x+I)*ln(c^2*x^2+1)-1/6*dilog(1/2*I*(c*x-I))-1/6*ln(c*x+I)*ln(1/2*I*(c*x-
I))-1/12*ln(c*x+I)^2-1/3*I*arctan(c*x)+1/2*ln(c^2*x^2+1)+1/3*I*c*x)+2*a*b*
d/c^2*(1/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-1/6*I*c^2*x^2-1/2
*c*x+1/6*I*ln(c^2*x^2+1)+1/2*arctan(c*x))
```

3.70.5 Fricas [F]

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/24*(-2*I*b^2*c*d*x^3 - 3*b^2*d*x^2)*log(-(c*x + I)/(c*x - I))^2 + integr
al(1/6*(6*I*a^2*c^3*d*x^4 + 6*a^2*c^2*d*x^3 + 6*I*a^2*c*d*x^2 + 6*a^2*d*x
- (6*a*b*c^3*d*x^4 + 2*(-3*I*a*b - b^2)*c^2*d*x^3 + 3*(2*a*b + I*b^2)*c*d*
x^2 - 6*I*a*b*d*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.70.7 Maxima [F]

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*I*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctan(c*x)^2 + 1/3*I*(2*x^3*arctan(c*x)
 - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d + 1/48*I*(4*x^3*arctan(c*x)
^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/48*(4*c^2*x^4*log(c^2*x^2 + 1
) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*arctan(c*x)^2 + 3*(c^2*x^4 +
x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2*c*d + 1/2*a^2*d*x^2 + (x^2
*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d - 1/2*(2*c*(x/c^2 - arct
an(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d
```

3.70.8 Giac [F]

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\int x\,\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)\,dx$$

input
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.71 $\int (d+icdx)(a+b\arctan(cx))^2\,dx$

3.71.1 Optimal result

Integrand size = 20, antiderivative size = 130

$$\begin{aligned}\int (d+icdx)(a+b\arctan(cx))^2\,dx &= -iabdx-ib^2dx\arctan(cx)\\ &\quad-\frac{id(1+icx)^2(a+b\arctan(cx))^2}{2c}\\ &\quad+\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c}\\ &\quad+\frac{ib^2d\log(1+c^2x^2)}{2c}-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}\end{aligned}$$

output
```
-I*a*b*d*x-I*b^2*d*x*arctan(c*x)-1/2*I*d*(1+I*c*x)^2*(a+b*arctan(c*x))^2/c
+2*b*d*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/c+1/2*I*b^2*d*ln(c^2*x^2+1)/c-I*b
^2*d*polylog(2,1-2/(1-I*c*x))/c
```

3.71.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{id\left(-2ia^2cx-2abcx+a^2c^2x^2+b^2(-i+cx)^2\arctan(cx)^2+2b\arctan(cx)\left(a-2iacx-bcx+ac^2x^2-2ib\right.\right.}{2c}$$

input
```
Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((I/2)*d*((-2*I)*a^2*c*x - 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-I + c*x)^2*ArcT
an[c*x]^2 + 2*b*ArcTan[c*x]*(a - (2*I)*a*c*x - b*c*x + a*c^2*x^2 - (2*I)*b
*Log[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2] + b^2*Log[1
+ c^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
```

3.71.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{ib\int \left(-\frac{2i(i-cx)(a+b\arctan(cx))d^2}{c^2x^2+1}-(a+b\arctan(cx))d^2\right)dx}{d}-\frac{id(1+icx)^2(a+b\arctan(cx))^2}{2c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ib\left(-\frac{2id^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-ad^2x-bd^2x\arctan(cx)+\frac{bd^2\log(c^2x^2+1)}{2c}-\frac{bd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}\right)}{d}-\frac{id(1+icx)^2(a+b\arctan(cx))^2}{2c}$$

input
```
Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/2*I)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x])^2)/c + (I*b*(-(a*d^2*x) - b*
d^2*x*ArcTan[c*x] - ((2*I)*d^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c +
 (b*d^2*Log[1 + c^2*x^2])/(2*c) - (b*d^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/c)
)/d
```

3.71.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.71.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(118) = 236$.

Time = 1.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.14

method	result
parts	$-ia^2d\left(-\frac{1}{2}c\,x^2+ix\right)+\frac{d\,b^2\left(\frac{i\arctan(cx)^2c^2x^2}{2}+\arctan(cx)^2cx+i\left(i\arctan(cx)\ln(c^2x^2+1)+\frac{\arctan(cx)^2}{2}-cx\,\mathrm{ar}\right.\right.}{}$
derivativedivides	$\frac{-ia^2d\left(-\frac{1}{2}c^2x^2+icx\right)+d\,b^2\left(\frac{i\arctan(cx)^2c^2x^2}{2}+\arctan(cx)^2cx+i\left(i\arctan(cx)\ln(c^2x^2+1)+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)\right.\right.}{}$
default	$\frac{-ia^2d\left(-\frac{1}{2}c^2x^2+icx\right)+d\,b^2\left(\frac{i\arctan(cx)^2c^2x^2}{2}+\arctan(cx)^2cx+i\left(i\arctan(cx)\ln(c^2x^2+1)+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)\right.\right.}{}$
risch	$a^2dx-\frac{abd\ln(c^2x^2+1)}{4c}-\frac{3\ln(-icx+1)abd}{2c}+\frac{3ia^2d}{2c}+\frac{ib^2\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)d}{c}+\frac{ia^2cd\,x^2}{2}-\frac{iabd\arctan(cx)}{2c}+$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-I*a^2*d*(-1/2*c*x^2+I*x)+d*b^2/c*(1/2*I*arctan(c*x)^2*c^2*x^2+arctan(c*x)
^2*c*x+I*(I*arctan(c*x)*ln(c^2*x^2+1)+1/2*arctan(c*x)^2-c*x*arctan(c*x)-1/
2*ln(c*x-I)*ln(c^2*x^2+1)+1/2*dilog(-1/2*I*(c*x+I))+1/2*ln(c*x-I)*ln(-1/2*
I*(c*x+I))+1/4*ln(c*x-I)^2+1/2*ln(c*x+I)*ln(c^2*x^2+1)-1/2*dilog(1/2*I*(c*
x-I))-1/2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/4*ln(c*x+I)^2+1/2*ln(c^2*x^2+1)))+
2*a*b*d/c*(1/2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)+1/2*I*(-c*x+I*ln(c^2*
x^2+1)+arctan(c*x)))
```

3.71.5 Fricas [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/8*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/
2*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d*x + 2*a^2*d - (2*a*b*
c^3*d*x^3 - (2*I*a*b + b^2)*c^2*d*x^2 + 2*(a*b + I*b^2)*c*d*x - 2*I*a*b*d)
*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.71.6 Sympy [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{id\left(\int\left(-\frac{4ia^2}{c^2x^2+1}\right)dx+\int\left(-\frac{ib^2\log(icx+1)}{c^2x^2+1}\right)dx+\int\left(-\frac{4ab\log(icx+1)}{c^2x^2+1}\right)dx+\int\frac{4a^2cx}{c^2x^2+1}\,dx+\int\frac{4a^2c^3x^3}{c^2x^2+1}\,dx+\int\frac{2b^2}{c^2x^2}}{}$$
$$+\left(-\frac{ib^2cdx^2}{8}-\frac{b^2dx}{4}\right)\log(icx+1)^2+\frac{(-ib^2c^2dx^2-2b^2cdx-3ib^2d)\log(-icx+1)^2}{8c}$$
$$+\frac{(-2abc^2dx^2+4iabcdx+ib^2c^2dx^2\log(icx+1)+2b^2cdx\log(icx+1)+2b^2cdx-ib^2d\log(icx+1))\log}{4c}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
I*d*(Integral(-4*I*a**2/(c**2*x**2 + 1), x) + Integral(-I*b**2*log(I*c*x +
 1)/(c**2*x**2 + 1), x) + Integral(-4*a*b*log(I*c*x + 1)/(c**2*x**2 + 1),
x) + Integral(4*a**2*c*x/(c**2*x**2 + 1), x) + Integral(4*a**2*c**3*x**3/(
c**2*x**2 + 1), x) + Integral(2*b**2*c*x/(c**2*x**2 + 1), x) + Integral(-4
*I*a**2*c**2*x**2/(c**2*x**2 + 1), x) + Integral(2*I*b**2*c**2*x**2/(c**2*
x**2 + 1), x) + Integral(-6*a*b*c**2*x**2/(c**2*x**2 + 1), x) + Integral(5
*b**2*c*x*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(4*I*a*b*c*x/(c**2*
x**2 + 1), x) + Integral(-2*I*a*b*c**3*x**3/(c**2*x**2 + 1), x) + Integral
(2*I*b**2*c**2*x**2*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-4*a*b*c
**2*x**2*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-4*I*a*b*c*x*log(I*
c*x + 1)/(c**2*x**2 + 1), x) + Integral(-4*I*a*b*c**3*x**3*log(I*c*x + 1)/
(c**2*x**2 + 1), x))/4 + (-I*b**2*c*d*x**2/8 - b**2*d*x/4)*log(I*c*x + 1)*
*2 + (-I*b**2*c**2*d*x**2 - 2*b**2*c*d*x - 3*I*b**2*d)*log(-I*c*x + 1)**2/
(8*c) + (-2*a*b*c**2*d*x**2 + 4*I*a*b*c*d*x + I*b**2*c**2*d*x**2*log(I*c*x
 + 1) + 2*b**2*c*d*x*log(I*c*x + 1) + 2*b**2*c*d*x - I*b**2*d*log(I*c*x +
1))*log(-I*c*x + 1)/(4*c)
```

3.71.7 Maxima [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan{(cx)}+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
 x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1), x) + 1/2*I
*a^2*c*d*x^2 + 12*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1)
, x) + b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) +
 6*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + I*(x^
2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*c*d + 1/4*b^2*d*arctan(c*
x)^3/c + 4*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2
+ 1), x) - 8*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*
d*x + b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*a
rctan(c*x) - log(c^2*x^2 + 1))*a*b*d/c - 1/8*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*
arctan(c*x)^2 - 1/8*(b^2*c*d*x^2 - 2*I*b^2*d*x)*arctan(c*x)*log(c^2*x^2 +
1) + 1/32*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(c^2*x^2 + 1)^2 + I*integrate(-1
/16*(12*b^2*c^2*d*x^2*arctan(c*x) - 12*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(
c*x)^2 - (b^2*c^3*d*x^3 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(b^2*c^3*d*x^3
 - 2*b^2*c*d*x - 2*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/
(c^2*x^2 + 1), x)
```

3.71.8 Giac [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)(b\arctan{(cx)}+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

3.72 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx$

3.72.1 Optimal result

Integrand size = 23, antiderivative size = 216

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx &= -d(a+b\arctan(cx))^2+icdx(a+b\arctan(cx))^2\\
&\quad+2d(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\
&\quad+2ibd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)\\
&\quad-b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\
&\quad-ibd(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\
&\quad+ibd(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\
&\quad-\frac{1}{2}b^2d\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\
&\quad+\frac{1}{2}b^2d\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-d*(a+b*arctan(c*x))^2+I*c*d*x*(a+b*arctan(c*x))^2-2*d*(a+b*arctan(c*x))^2
*arctanh(-1+2/(1+I*c*x))+2*I*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))-b^2*d*p
olylog(2,1-2/(1+I*c*x))-I*b*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I
*b*d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*d*polylog(3,1-2/(
1+I*c*x))+1/2*b^2*d*polylog(3,-1+2/(1+I*c*x))
```

3.72.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.26

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx$$
$$= d\Big(ia^2cx + a^2\log(cx) + iab\big(2cx\arctan(cx) - \log\left(1+c^2x^2\right)\big) + b^2\left(\arctan(cx)\left((1+icx)\arctan(cx) + 2i\log\left(1+e^{2i\arctan(cx)}\right)\right) + \operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right) + iab(\operatorname{PolyLog}(2,-icx) - \operatorname{PolyLog}(2,icx)) + b^2\Big(-\frac{i\pi^3}{24} + \frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\Big)\Big)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
d*(I*a^2*c*x + a^2*Log[c*x] + I*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2])
 + b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*I)*ArcT
an[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + I*a*b*(PolyLog[2, (-I)*
c*x] - PolyLog[2, I*c*x]) + b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3
+ ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^
((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] +
I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*Ar
cTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2))
```

3.72.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx$$
$$\Big\downarrow \; 5411$$

$$\int \left(\frac{d(a+b\arctan(cx))^2}{x}+icd(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2d\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-ibd\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-d(a+b\arctan(cx))^2+icdx(a+b\arctan(cx))^2+2ibd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+b^2(-d)\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
-(d*(a + b*ArcTan[c*x])^2) + I*c*d*x*(a + b*ArcTan[c*x])^2 + 2*d*(a + b*Ar
cTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (2*I)*b*d*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d*(a + b*ArcT
an[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog
[2, -1 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d
*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.72.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.06 (sec) , antiderivative size = 3480, normalized size of antiderivative = 16.11

method	result	size
parts	Expression too large to display	3480
derivativedivides	Expression too large to display	3482
default	Expression too large to display	3482

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*(I*c*x+ln(x))+d*b^2*(-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*poly
log(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2
))+1/2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+arctan(c*x)^2+I*arctan(c*x)*pol
ylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+di
log(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*Pi*cs
gn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*(I*arctan(c*x)
*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c
^2*x^2+1)^(1/2)))-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+1/2*Pi*arcta
n(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*Pi*arctan(c*x)*ln(1-I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))-1/2*Pi*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+a
rctan(c*x)^2*ln(c*x)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arcta
n(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*(2*I*arctan(c*x)*ln(1+(1+I*c*x
)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1/2*
I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*(I*ar
ctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*...
```

3.72.5 Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
 I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x, x
)
```

3.72.6 Sympy [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx = id\left(\int a^2c\,dx + \int \left(-\frac{ia^2}{x}\right)dx + \int b^2c\,\mathrm{atan}^2(cx)\,dx + \int \left(-\frac{ib^2\,\mathrm{atan}^2(cx)}{x}\right)dx + \int 2abc\,\mathrm{atan}(cx)\,dx + \int \left(-\frac{2iab\,\mathrm{atan}(cx)}{x}\right)dx\right)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x,x)
```

output
```
I*d*(Integral(a**2*c, x) + Integral(-I*a**2/x, x) + Integral(b**2*c*atan(c
*x)**2, x) + Integral(-I*b**2*atan(c*x)**2/x, x) + Integral(2*a*b*c*atan(c
*x), x) + Integral(-2*I*a*b*atan(c*x)/x, x))
```

3.72.7 Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/4*I*b^2*c*d*x*arctan(c*x)^2 + 12*I*b^2*c^3*d*integrate(1/16*x^3*arctan(c
*x)^2/(c^2*x^3 + x), x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c
^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2
 + 1)^2/(c^2*x^3 + x), x) + 8*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^
2*x^3 + x), x) + 4*I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^
3 + x), x) - 1/4*b^2*c*d*x*arctan(c*x)*log(c^2*x^2 + 1) - 1/16*I*b^2*c*d*x
*log(c^2*x^2 + 1)^2 + 1/4*I*b^2*d*arctan(c*x)^3 + 12*b^2*c^2*d*integrate(1
/16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*c^2*d*integrate(1/16*x^2
*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1
/16*x^2*arctan(c*x)/(c^2*x^3 + x), x) - 8*I*b^2*c^2*d*integrate(1/16*x^2*a
rctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + I*a^2*c*d*x
 + 4*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x),
x) + I*b^2*c*d*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 1/1
6*b^2*d*log(c^2*x^2 + 1)^2 + I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*
d + 12*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*d*in
tegrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + b^2*d*integr
ate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d*integrate(1/16*ar
ctan(c*x)/(c^2*x^3 + x), x) + a^2*d*log(x)
```

3.72.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
Timed out
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x, x)
```

3.73 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx$

3.73.1 Optimal result

Integrand size = 23, antiderivative size = 228

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = {} & -icd(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{x} \\ & +2icd(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\ & +2bcd(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) \\ & -ib^2cd\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) \\ & +bcd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\ & -bcd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\ & -\frac{1}{2}ib^2cd\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) \\ & +\frac{1}{2}ib^2cd\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-I*c*d*(a+b*arctan(c*x))^2-d*(a+b*arctan(c*x))^2/x-2*I*c*d*(a+b*arctan(c*x
))^2*arctanh(-1+2/(1+I*c*x))+2*b*c*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-I
*b^2*c*d*polylog(2,-1+2/(1-I*c*x))+b*c*d*(a+b*arctan(c*x))*polylog(2,1-2/(
1+I*c*x))-b*c*d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*I*b^2*c*d*
polylog(3,1-2/(1+I*c*x))+1/2*I*b^2*c*d*polylog(3,-1+2/(1+I*c*x))
```

3.73.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.27

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{id\left(ia^2+a^2cx\log(x)+iab(2\arctan(cx)+cx(-2\log(cx)+\log(1+c^2x^2)))+ib^2\left(\arctan(cx)^2-2cx\,\mathrm{arcta}\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(I*d*(I*a^2 + a^2*c*x*Log[x] + I*a*b*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + L
og[1 + c^2*x^2])) + I*b^2*(ArcTan[c*x]^2 - 2*c*x*ArcTan[c*x]*Log[1 - E^((2
*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x]
)])) + I*a*b*c*x*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*c*x*((-
I)*Pi^3 + (16*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan
[c*x])] - 24*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[
c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E
^((2*I)*ArcTan[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog
[3, -E^((2*I)*ArcTan[c*x])]))/24))/x
```

3.73.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^2}+\frac{icd(a+b\arctan(cx))^2}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$2icd\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2+bcd\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-bcd\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-icd(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{x}+2bcd\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-ib^2cd\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-\frac{1}{2}ib^2cd\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}ib^2cd\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-I)*c*d*(a + b*ArcTan[c*x])^2 - (d*(a + b*ArcTan[c*x])^2)/x + (2*I)*c*d*(
a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + 2*b*c*d*(a + b*ArcTan[c*
x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + b*
c*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - b*c*d*(a + b*ArcTa
n[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - (I/2)*b^2*c*d*PolyLog[3, 1 - 2/(1
 + I*c*x)] + (I/2)*b^2*c*d*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.73.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.89 (sec) , antiderivative size = 5609, normalized size of antiderivative = 24.60

method	result	size
parts	Expression too large to display	5609
derivativedivides	Expression too large to display	5613
default	Expression too large to display	5613

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.73.5 Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
 I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x^2,
 x)
```

3.73.6 Sympy [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = id\Bigg(\int\left(-\frac{ia^2}{x^2}\right)dx+\int\frac{a^2c}{x}\,dx + \int\left(-\frac{ib^2\operatorname{atan}^2{(cx)}}{x^2}\right)dx+\int\frac{b^2c\operatorname{atan}^2{(cx)}}{x}\,dx + \int\left(-\frac{2iab\operatorname{atan}{(cx)}}{x^2}\right)dx+\int\frac{2abc\operatorname{atan}{(cx)}}{x}\,dx\Bigg)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**2,x)
```

output
```
I*d*(Integral(-I*a**2/x**2, x) + Integral(a**2*c/x, x) + Integral(-I*b**2*
atan(c*x)**2/x**2, x) + Integral(b**2*c*atan(c*x)**2/x, x) + Integral(-2*I
*a*b*atan(c*x)/x**2, x) + Integral(2*a*b*c*atan(c*x)/x, x))
```

3.73.7 Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
I*a^2*c*d*log(x) - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b
*d - a^2*d/x - 1/96*(24*b^2*d*arctan(c*x)^2 + 24*I*b^2*d*arctan(c*x)*log(c
^2*x^2 + 1) - 6*b^2*d*log(c^2*x^2 + 1)^2 - 24*(b^2*c*d*arctan(c*x)^3 + 16*
b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
 x) + 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x
) - 16*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) +
 16*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
 x) + 32*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*b^2
*d*integrate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*b^2*d*integrate(1/
16*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x - I*(1152*b^2*c^3*d*integrate
(1/16*x^3*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 3072*a*b*c^3*d*integrate(1/1
6*x^3*arctan(c*x)/(c^2*x^4 + x^2), x) + b^2*c*d*log(c^2*x^2 + 1)^3 + 24*b^
2*c*d*arctan(c*x)^2 - 384*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)*log(c^2
*x^2 + 1)/(c^2*x^4 + x^2), x) + 1152*b^2*c*d*integrate(1/16*x*arctan(c*x)^
2/(c^2*x^4 + x^2), x) + 96*b^2*c*d*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^
2*x^4 + x^2), x) + 3072*a*b*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^
2), x) + 384*b^2*c*d*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x)
 - 384*b^2*d*integrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
x))*x)/x
```

3.73.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^2, x)
```

3.74 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx$

3.74.1 Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bcd(a+b\arctan(cx))}{x}+\frac{1}{2}c^2d(a+b\arctan(cx))^2 - \frac{d(a+b\arctan(cx))^2}{2x^2}-\frac{icd(a+b\arctan(cx))^2}{x} + b^2c^2d\log(x)-\frac{1}{2}b^2c^2d\log\left(1+c^2x^2\right) + 2ibc^2d(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) + b^2c^2d\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

output
```
-b*c*d*(a+b*arctan(c*x))/x+1/2*c^2*d*(a+b*arctan(c*x))^2-1/2*d*(a+b*arctan
(c*x))^2/x^2-I*c*d*(a+b*arctan(c*x))^2/x+b^2*c^2*d*ln(x)-1/2*b^2*c^2*d*ln(
c^2*x^2+1)+2*I*b*c^2*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+b^2*c^2*d*polyl
og(2,-1+2/(1-I*c*x))
```

3.74.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.19

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx =$$
$$-\frac{d\Big(a^2+2ia^2cx+2abcx-b^2(-i+cx)^2\arctan(cx)^2+2b\arctan(cx)\left(a+2iacx+bcx+ac^2x^2-2ibc^2x^2\right.}{}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-1/2*(d*(a^2 + (2*I)*a^2*c*x + 2*a*b*c*x - b^2*(-I + c*x)^2*ArcTan[c*x]^2
+ 2*b*ArcTan[c*x]*(a + (2*I)*a*c*x + b*c*x + a*c^2*x^2 - (2*I)*b*c^2*x^2*L
og[1 - E^((2*I)*ArcTan[c*x])]) - (4*I)*a*b*c^2*x^2*Log[c*x] - 2*b^2*c^2*x^
2*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (2*I)*a*b*c^2*x^2*Log[1 + c^2*x^2] - 2*b^
2*c^2*x^2*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^2
```

3.74.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx$$
$$\Big\downarrow \; 5411$$
$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^3}+\frac{icd(a+b\arctan(cx))^2}{x^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}c^2d(a+b\arctan(cx))^2+2ibc^2d\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{d(a+b\arctan(cx))^2}{2x^2}-\frac{icd(a+b\arctan(cx))^2}{x}-\frac{bcd(a+b\arctan(cx))}{x}+b^2c^2d\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-\frac{1}{2}b^2c^2d\log\left(c^2x^2+1\right)+b^2c^2d\log(x)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d*(a + b*ArcTan[c*x]))/x) + (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
 + b*ArcTan[c*x])^2)/(2*x^2) - (I*c*d*(a + b*ArcTan[c*x])^2)/x + b^2*c^2*d
*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 + (2*I)*b*c^2*d*(a + b*ArcTan[c*x
])*Log[2 - 2/(1 - I*c*x)] + b^2*c^2*d*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.74.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(149) = 298$.

Time = 2.46 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.25

method	result
parts	$a^2d\left(-\frac{1}{2x^2}-\frac{ic}{x}\right)+d\,b^2c^2\Bigg(-i\arctan\left(cx\right)\ln\left(c^2x^2+1\right)-\frac{\arctan(cx)^2}{2c^2x^2}+2i\arctan\left(cx\right)\ln\left(c\right.$
derivativedivides	$c^2\Bigg(a^2d\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+d\,b^2\Bigg(-i\arctan\left(cx\right)\ln\left(c^2x^2+1\right)-\frac{\arctan(cx)^2}{2c^2x^2}+2i\arctan\left(cx\right)\ln$
default	$c^2\Bigg(a^2d\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+d\,b^2\Bigg(-i\arctan\left(cx\right)\ln\left(c^2x^2+1\right)-\frac{\arctan(cx)^2}{2c^2x^2}+2i\arctan\left(cx\right)\ln$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*(-1/2/x^2-I*c/x)+d*b^2*c^2*(-I*arctan(c*x)*ln(c^2*x^2+1)-1/2/c^2/x^2
*arctan(c*x)^2+2*I*arctan(c*x)*ln(c*x)-1/c/x*arctan(c*x)-I*arctan(c*x)^2/c
/x-1/2*arctan(c*x)^2+ln(c*x)-1/2*ln(c^2*x^2+1)-ln(c*x)*ln(1+I*c*x)+ln(c*x)
*ln(1-I*c*x)-dilog(1+I*c*x)+dilog(1-I*c*x)+1/2*ln(c*x-I)*ln(c^2*x^2+1)-1/2
*dilog(-1/2*I*(c*x+I))-1/2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/4*ln(c*x-I)^2-1/
2*ln(c*x+I)*ln(c^2*x^2+1)+1/2*dilog(1/2*I*(c*x-I))+1/2*ln(c*x+I)*ln(1/2*I*
(c*x-I))+1/4*ln(c*x+I)^2)+2*a*b*d*c^2*(-I*arctan(c*x)/c/x-1/2/c^2/x^2*arct
an(c*x)+I*ln(c*x)-1/2/c/x-1/2*I*ln(c^2*x^2+1)-1/2*arctan(c*x))
```

3.74.5 Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(8*x^2*integral(1/2*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d
*x + 2*a^2*d - (2*a*b*c^3*d*x^3 + 2*(-I*a*b + b^2)*c^2*d*x^2 + (2*a*b - I*
b^2)*c*d*x - 2*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^5 + x^3), x) + (
2*I*b^2*c*d*x + b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^2
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Timed out
```

3.74.7 Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c*d - ((c*arcta
n(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d + 1/2*((arctan(c*x)^2 - log(c^2*x
^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b^2*d + 1
/16*I*(4*(c*arctan(c*x)^3 + 4*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c
^2*x^4 + x^2), x) - 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 +
x^2), x) + 32*c*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*inte
grate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*integrate(1/16*log(c^2*x^
2 + 1)^2/(c^2*x^4 + x^2), x))*x - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^
2*c*d/x - I*a^2*c*d/x - 1/2*b^2*d*arctan(c*x)^2/x^2 - 1/2*a^2*d/x^2
```

3.74.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^3, x)
```

3.75 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx$

3.75.1 Optimal result

Integrand size = 23, antiderivative size = 224

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = &-\frac{b^2c^2d}{3x}-\frac{1}{3}b^2c^3d\arctan(cx)-\frac{bcd(a+b\arctan(cx))}{3x^2}\\ &-\frac{ibc^2d(a+b\arctan(cx))}{x}-\frac{1}{6}ic^3d(a+b\arctan(cx))^2\\ &-\frac{d(a+b\arctan(cx))^2}{3x^3}-\frac{icd(a+b\arctan(cx))^2}{2x^2}\\ &+ib^2c^3d\log(x)-\frac{1}{2}ib^2c^3d\log\left(1+c^2x^2\right)\\ &-\frac{2}{3}bc^3d(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ &+\frac{1}{3}ib^2c^3d\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d/x-1/3*b^2*c^3*d*arctan(c*x)-1/3*b*c*d*(a+b*arctan(c*x))/x^2
-I*b*c^2*d*(a+b*arctan(c*x))/x-1/6*I*c^3*d*(a+b*arctan(c*x))^2-1/3*d*(a+b*
arctan(c*x))^2/x^3-1/2*I*c*d*(a+b*arctan(c*x))^2/x^2+I*b^2*c^3*d*ln(x)-1/2
*I*b^2*c^3*d*ln(c^2*x^2+1)-2/3*b*c^3*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))
+1/3*I*b^2*c^3*d*polylog(2,-1+2/(1-I*c*x))
```

3.75.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.07

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{d\Big(-2a^2-3ia^2cx-2abcx-6iabc^2x^2-2b^2c^2x^2-ib^2(-2i+3cx+c^3x^3)\arctan(cx)^2-2b\arctan(cx)\,(bc}{}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
(d*(-2*a^2 - (3*I)*a^2*c*x - 2*a*b*c*x - (6*I)*a*b*c^2*x^2 - 2*b^2*c^2*x^2
 - I*b^2*(-2*I + 3*c*x + c^3*x^3)*ArcTan[c*x]^2 - 2*b*ArcTan[c*x]*(b*c*x*(
1 + (3*I)*c*x + c^2*x^2) + a*(2 + (3*I)*c*x + (3*I)*c^3*x^3) + 2*b*c^3*x^3
*Log[1 - E^((2*I)*ArcTan[c*x])]) - 4*a*b*c^3*x^3*Log[c*x] + (6*I)*b^2*c^3*
x^3*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 2*a*b*c^3*x^3*Log[1 + c^2*x^2] + (2*I)*
b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(6*x^3)
```

3.75.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx$$
$$\Big\downarrow \; 5411$$
$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^4}+\frac{icd(a+b\arctan(cx))^2}{x^3}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{1}{6}ic^3d(a+b\arctan(cx))^2-\frac{2}{3}bc^3d\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{ibc^2d(a+b\arctan(cx))}{x}-\frac{d(a+b\arctan(cx))^2}{3x^3}-\frac{icd(a+b\arctan(cx))^2}{2x^2}-\frac{bcd(a+b\arctan(cx))}{3x^2}-\frac{1}{3}b^2c^3d\arctan(cx)+\frac{1}{3}ib^2c^3d\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+ib^2c^3d\log(x)-\frac{b^2c^2d}{3x}-\frac{1}{2}ib^2c^3d\log\left(c^2x^2+1\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
-1/3*(b^2*c^2*d)/x - (b^2*c^3*d*ArcTan[c*x])/3 - (b*c*d*(a + b*ArcTan[c*x]
))/(3*x^2) - (I*b*c^2*d*(a + b*ArcTan[c*x]))/x - (I/6)*c^3*d*(a + b*ArcTan
[c*x])^2 - (d*(a + b*ArcTan[c*x])^2)/(3*x^3) - ((I/2)*c*d*(a + b*ArcTan[c*
x])^2)/x^2 + I*b^2*c^3*d*Log[x] - (I/2)*b^2*c^3*d*Log[1 + c^2*x^2] - (2*b*
c^3*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*d*Poly
Log[2, -1 + 2/(1 - I*c*x)]
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.75.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(198) = 396.

Time = 3.14 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.82

method	result
parts	$a^2 d\left(-\frac{ic}{2x^2}-\frac{1}{3x^3}\right)+d\,b^2c^3\left(-\frac{\arctan(cx)^2}{3c^3x^3}-\frac{i\ln(c^2x^2+1)}{2}+i\ln(cx)-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\arctan(cx)\ln}{3}\right.$
derivativedivides	$c^3\left(a^2 d\left(-\frac{1}{3c^3x^3}-\frac{i}{2c^2x^2}\right)+d\,b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}-\frac{i\ln(c^2x^2+1)}{2}+i\ln(cx)-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\arctan}{}\right.\right.$
default	$c^3\left(a^2 d\left(-\frac{1}{3c^3x^3}-\frac{i}{2c^2x^2}\right)+d\,b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}-\frac{i\ln(c^2x^2+1)}{2}+i\ln(cx)-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\arctan}{}\right.\right.$

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*(-1/2*I*c/x^2-1/3/x^3)+d*b^2*c^3*(-1/3*arctan(c*x)^2/c^3/x^3-1/2*I*l
n(c^2*x^2+1)+I*ln(c*x)-1/3/c^2/x^2*arctan(c*x)-2/3*arctan(c*x)*ln(c*x)+1/3
*arctan(c*x)*ln(c^2*x^2+1)+1/3*I*ln(c*x)*ln(1-I*c*x)-I*arctan(c*x)/c/x-1/3
*I*ln(c*x)*ln(1+I*c*x)-1/3/c/x-1/2*I*arctan(c*x)^2/c^2/x^2-1/3*arctan(c*x)
+1/6*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*
(c*x+I))-1/2*ln(c*x-I)^2)-1/3*I*dilog(1+I*c*x)-1/6*I*(ln(c*x+I)*ln(c^2*x^2
+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2)-1/2*
I*arctan(c*x)^2+1/3*I*dilog(1-I*c*x))+2*a*b*d*c^3*(-1/3*arctan(c*x)/c^3/x^
3-1/2*I*arctan(c*x)/c^2/x^2-1/2*I/c/x-1/6/c^2/x^2-1/3*ln(c*x)+1/6*ln(c^2*x
^2+1)-1/2*I*arctan(c*x))
```

3.75.5 Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/24*(24*x^3*integral(1/6*(6*I*a^2*c^3*d*x^3 + 6*a^2*c^2*d*x^2 + 6*I*a^2*c
*d*x + 6*a^2*d - (6*a*b*c^3*d*x^3 + 3*(-2*I*a*b + b^2)*c^2*d*x^2 + 2*(3*a*
b - I*b^2)*c*d*x - 6*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^6 + x^4),
x) + (3*I*b^2*c*d*x + 2*b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**4,x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d + 1/3*((c^2*log(c^2
*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*d - 1/2*I*a^2
*c*d/x^2 - 1/3*a^2*d/x^3 + 1/96*(96*I*x^3*integrate(1/48*(20*b^2*c^2*d*x^2
*arctan(c*x) + 36*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x)^2 + 3*(b^2*c^3*d
*x^3 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(3*b^2*c^3*d*x^3 - 2*b^2*c*d*x +
6*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^6 + x^4),
x) + 96*x^3*integrate(1/48*(36*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x)^2 + 3*(
b^2*c^2*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 - 4*(3*b^2*c^3*d*x^3 - 2*b^2*c*d
*x)*arctan(c*x) - 2*(5*b^2*c^2*d*x^2 - 6*(b^2*c^3*d*x^3 + b^2*c*d*x)*arcta
n(c*x))*log(c^2*x^2 + 1))/(c^2*x^6 + x^4), x) - 4*(3*I*b^2*c*d*x + 2*b^2*d
)*arctan(c*x)^2 + 4*(3*b^2*c*d*x - 2*I*b^2*d)*arctan(c*x)*log(c^2*x^2 + 1)
 + (3*I*b^2*c*d*x + 2*b^2*d)*log(c^2*x^2 + 1)^2)/x^3
```

3.75.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,(d+c\,d\,x\,1\mathrm{i})}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^4,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^4, x)
```

3.76 $\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx$

3.76.1 Optimal result

Integrand size = 25, antiderivative size = 373

$$\begin{aligned}\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx &= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{ib^2d^2x^3}{15c}\\
&-\frac{1}{60}b^2d^2x^4+\frac{3ib^2d^2\arctan(cx)}{5c^4}\\
&+\frac{5b^2d^2x\arctan(cx)}{6c^3}+\frac{2ibd^2x^2(a+b\arctan(cx))}{5c^2}\\
&-\frac{5bd^2x^3(a+b\arctan(cx))}{18c}\\
&-\frac{1}{5}ibd^2x^4(a+b\arctan(cx))\\
&+\frac{1}{15}bcd^2x^5(a+b\arctan(cx))\\
&-\frac{49d^2(a+b\arctan(cx))^2}{60c^4}\\
&+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2\\
&+\frac{2}{5}icd^2x^5(a+b\arctan(cx))^2\\
&-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))^2\\
&+\frac{4ibd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^4}\\
&-\frac{53b^2d^2\log(1+c^2x^2)}{90c^4}\\
&-\frac{2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}\end{aligned}$$

output
```
5/6*a*b*d^2*x/c^3+2/5*I*b*d^2*x^2*(a+b*arctan(c*x))/c^2+31/180*b^2*d^2*x^2
/c^2+3/5*I*b^2*d^2*arctan(c*x)/c^4-1/60*b^2*d^2*x^4-1/5*I*b*d^2*x^4*(a+b*a
rctan(c*x))+5/6*b^2*d^2*x*arctan(c*x)/c^3+1/15*I*b^2*d^2*x^3/c-5/18*b*d^2*
x^3*(a+b*arctan(c*x))/c+2/5*I*c*d^2*x^5*(a+b*arctan(c*x))^2+1/15*b*c*d^2*x
^5*(a+b*arctan(c*x))-49/60*d^2*(a+b*arctan(c*x))^2/c^4+1/4*d^2*x^4*(a+b*ar
ctan(c*x))^2-3/5*I*b^2*d^2*x/c^3-1/6*c^2*d^2*x^6*(a+b*arctan(c*x))^2+4/5*I
*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4-53/90*b^2*d^2*ln(c^2*x^2+1)/c
^4-2/5*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^4
```

3.76.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.92

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{d^2\left(108iab+34b^2+150abcx-108ib^2cx+72iabc^2x^2+31b^2c^2x^2-50abc^3x^3+12ib^2c^3x^3+45a^2c^4x^4-36\right.}{}$$

input
```
Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^2*((108*I)*a*b + 34*b^2 + 150*a*b*c*x - (108*I)*b^2*c*x + (72*I)*a*b*c^
2*x^2 + 31*b^2*c^2*x^2 - 50*a*b*c^3*x^3 + (12*I)*b^2*c^3*x^3 + 45*a^2*c^4*
x^4 - (36*I)*a*b*c^4*x^4 - 3*b^2*c^4*x^4 + (72*I)*a^2*c^5*x^5 + 12*a*b*c^5
*x^5 - 30*a^2*c^6*x^6 - 3*b^2*(1 - 15*c^4*x^4 - (24*I)*c^5*x^5 + 10*c^6*x^
6)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(54*I + 75*c*x + (36*I)*c^2*x^2 - 25
*c^3*x^3 - (18*I)*c^4*x^4 + 6*c^5*x^5) + a*(-75 + 45*c^4*x^4 + (72*I)*c^5*
x^5 - 30*c^6*x^6) + (72*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (72*I)*a*b*
Log[1 + c^2*x^2] - 106*b^2*Log[1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^((2*I)*
ArcTan[c*x])]))/(180*c^4)
```

3.76.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2x^5(a+b\arctan(cx))^2+2icd^2x^4(a+b\arctan(cx))^2+d^2x^3(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{49d^2(a+b\arctan(cx))^2}{60c^4}+\frac{4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^4}-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))^2+\frac{2ibd^2x^2(a+b\arctan(cx))}{5c^2}+\frac{2}{5}icd^2x^5(a+b\arctan(cx))^2+\frac{1}{15}bcd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2-\frac{1}{5}ibd^2x^4(a+b\arctan(cx))-\frac{5bd^2x^3(a+b\arctan(cx))}{18c}+\frac{5abd^2x}{6c^3}+\frac{3ib^2d^2\arctan(cx)}{5c^4}+\frac{5b^2d^2x\arctan(cx)}{6c^3}-\frac{2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^4}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}-\frac{53b^2d^2\log(c^2x^2+1)}{90c^4}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4$$

input
```
Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(5*a*b*d^2*x)/(6*c^3) - (((3*I)/5)*b^2*d^2*x)/c^3 + (31*b^2*d^2*x^2)/(180*
c^2) + ((I/15)*b^2*d^2*x^3)/c - (b^2*d^2*x^4)/60 + (((3*I)/5)*b^2*d^2*ArcT
an[c*x])/c^4 + (5*b^2*d^2*x*ArcTan[c*x])/(6*c^3) + (((2*I)/5)*b*d^2*x^2*(a
 + b*ArcTan[c*x]))/c^2 - (5*b*d^2*x^3*(a + b*ArcTan[c*x]))/(18*c) - (I/5)*
b*d^2*x^4*(a + b*ArcTan[c*x]) + (b*c*d^2*x^5*(a + b*ArcTan[c*x]))/15 - (49
*d^2*(a + b*ArcTan[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 +
 ((2*I)/5)*c*d^2*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^6*(a + b*ArcTan[c*
x])^2)/6 + (((4*I)/5)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 -
(53*b^2*d^2*Log[1 + c^2*x^2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(5*c^4)
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.76.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.19

method	result
parts	$a^2d^2\left(-\frac{1}{6}c^2x^6+\frac{2}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{b^2d^2\left(-\frac{\arctan(cx)^2c^6x^6}{6}-\frac{i\arctan(cx)c^4x^4}{5}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{5cx\arctan(cx)}{6}\right.}{}$
derivativedivides	$\frac{a^2d^2\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b^2d^2\left(-\frac{\arctan(cx)^2c^6x^6}{6}-\frac{i\arctan(cx)c^4x^4}{5}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{5cx\arctan(cx)}{6}+\frac{c^5x^5\operatorname{arct}}{15}\right.}{}$
default	$\frac{a^2d^2\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b^2d^2\left(-\frac{\arctan(cx)^2c^6x^6}{6}-\frac{i\arctan(cx)c^4x^4}{5}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{5cx\arctan(cx)}{6}+\frac{c^5x^5\operatorname{arct}}{15}\right.}{}$
risch	$\frac{31b^2d^2x^2}{180c^2}-\frac{8713b^2d^2\ln(c^2x^2+1)}{14400c^4}-\frac{b^2d^2x^4}{60}+\frac{5ab\,d^2x}{6c^3}+\frac{77b^2d^2}{90c^4}+\frac{2id^2x^2ab}{5c^2}-\frac{2ib\,d^2a\ln(c^2x^2+1)}{5c^4}+\frac{ib^2d^2}{}$

input
```
int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*(-1/6*c^2*x^6+2/5*I*c*x^5+1/4*x^4)+b^2*d^2/c^4*(-1/6*arctan(c*x)^2
*c^6*x^6-1/5*I*arctan(c*x)*c^4*x^4+1/4*c^4*x^4*arctan(c*x)^2+5/6*c*x*arcta
n(c*x)+1/15*c^5*x^5*arctan(c*x)-2/5*I*arctan(c*x)*ln(c^2*x^2+1)-5/18*c^3*x
^3*arctan(c*x)+1/15*I*c^3*x^3+2/5*I*arctan(c*x)*c^2*x^2-5/12*arctan(c*x)^2
+1/5*ln(c*x-I)*ln(c^2*x^2+1)-1/5*ln(c*x+I)*ln(c^2*x^2+1)-1/5*ln(c*x-I)*ln(
-1/2*I*(c*x+I))-1/10*ln(c*x-I)^2+1/10*ln(c*x+I)^2+1/5*ln(c*x+I)*ln(1/2*I*(
c*x-I))-1/5*dilog(-1/2*I*(c*x+I))+1/5*dilog(1/2*I*(c*x-I))+3/5*I*arctan(c*
x)-1/60*c^4*x^4-3/5*I*c*x+31/180*c^2*x^2-53/90*ln(c^2*x^2+1)+2/5*I*arctan(
c*x)^2*c^5*x^5)+2*a*d^2*b/c^4*(-1/6*arctan(c*x)*c^6*x^6+2/5*I*arctan(c*x)*
c^5*x^5+1/4*c^4*x^4*arctan(c*x)+5/12*c*x+1/30*c^5*x^5-1/10*I*c^4*x^4-5/36*
c^3*x^3+1/5*I*c^2*x^2-1/5*I*ln(c^2*x^2+1)-5/12*arctan(c*x))
```

3.76.5 Fricas [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/240*(10*b^2*c^2*d^2*x^6 - 24*I*b^2*c*d^2*x^5 - 15*b^2*d^2*x^4)*log(-(c*x
 + I)/(c*x - I))^2 + integral(-1/60*(60*a^2*c^4*d^2*x^7 - 120*I*a^2*c^3*d^
2*x^6 - 120*I*a^2*c*d^2*x^4 - 60*a^2*d^2*x^3 - (-60*I*a*b*c^4*d^2*x^7 - 10
*(12*a*b - I*b^2)*c^3*d^2*x^6 + 24*b^2*c^2*d^2*x^5 - 15*(8*a*b + I*b^2)*c*
d^2*x^4 + 60*I*a*b*d^2*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.76.7 Maxima [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)^2(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/6*a^2*c^2*d^2*x^6 + 2/5*I*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctan(c*x)^2
 + 1/4*a^2*d^2*x^4 - 1/45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3
+ 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b*c^2*d^2 + 1/5*I*(4*x^5*arctan(c*x)
- c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^2 + 1/6*(3*x
^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^2 - 1/
12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 +
 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 - 1/120*(5*b^2*c^2*d^2
*x^6 - 12*I*b^2*c*d^2*x^5)*arctan(c*x)^2 + 1/120*(-5*I*b^2*c^2*d^2*x^6 - 1
2*b^2*c*d^2*x^5)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(5*b^2*c^2*d^2*x^6 -
 12*I*b^2*c*d^2*x^5)*log(c^2*x^2 + 1)^2 - integrate(-1/240*(68*b^2*c^3*d^2
*x^6*arctan(c*x) - 180*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*arctan(c*x)^2 -
 15*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*log(c^2*x^2 + 1)^2 - 2*(5*b^2*c^4*
d^2*x^7 - 12*b^2*c^2*d^2*x^5 - 60*(b^2*c^3*d^2*x^6 + b^2*c*d^2*x^4)*arctan
(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + I*integrate(1/120*(180*(b^2*c
^3*d^2*x^6 + b^2*c*d^2*x^4)*arctan(c*x)^2 + 15*(b^2*c^3*d^2*x^6 + b^2*c*d^
2*x^4)*log(c^2*x^2 + 1)^2 + 2*(5*b^2*c^4*d^2*x^7 - 12*b^2*c^2*d^2*x^5)*arc
tan(c*x) + (17*b^2*c^3*d^2*x^6 + 30*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*ar
ctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

3.76.8 Giac [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.77 $\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx$

3.77.1 Optimal result

Integrand size = 25, antiderivative size = 333

$$\begin{aligned}\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx = {} & \frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3 \\ & -\frac{19b^2d^2\arctan(cx)}{30c^3}+\frac{ib^2d^2x\arctan(cx)}{c^2} \\ & -\frac{8bd^2x^2(a+b\arctan(cx))}{15c} \\ & -\frac{1}{3}ibd^2x^3(a+b\arctan(cx)) \\ & +\frac{1}{10}bcd^2x^4(a+b\arctan(cx)) \\ & -\frac{31id^2(a+b\arctan(cx))^2}{30c^3} \\ & +\frac{1}{3}d^2x^3(a+b\arctan(cx))^2 \\ & +\frac{1}{2}icd^2x^4(a+b\arctan(cx))^2 \\ & -\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))^2 \\ & -\frac{16bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^3} \\ & -\frac{2ib^2d^2\log\left(1+c^2x^2\right)}{3c^3} \\ & -\frac{8ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}\end{aligned}$$

output
```
1/6*I*b^2*d^2*x^2/c+19/30*b^2*d^2*x/c^2-1/3*I*b*d^2*x^3*(a+b*arctan(c*x))-
1/30*b^2*d^2*x^3-19/30*b^2*d^2*arctan(c*x)/c^3+1/2*I*c*d^2*x^4*(a+b*arctan
(c*x))^2-8/15*b*d^2*x^2*(a+b*arctan(c*x))/c-31/30*I*d^2*(a+b*arctan(c*x))^
2/c^3+1/10*b*c*d^2*x^4*(a+b*arctan(c*x))+I*b^2*d^2*x*arctan(c*x)/c^2+1/3*d
^2*x^3*(a+b*arctan(c*x))^2-2/3*I*b^2*d^2*ln(c^2*x^2+1)/c^3-1/5*c^2*d^2*x^5
*(a+b*arctan(c*x))^2-16/15*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3-8/1
5*I*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^3+I*a*b*d^2*x/c^2
```

3.77.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.92

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx =$$
$$-\frac{d^2\left(9ab-5ib^2-30iabcx-19b^2cx+16abc^2x^2-5ib^2c^2x^2-10a^2c^3x^3+10iabc^3x^3+b^2c^3x^3-15ia^2c^4x^4\right.}{}$$

input
```
Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-1/30*(d^2*(9*a*b - (5*I)*b^2 - (30*I)*a*b*c*x - 19*b^2*c*x + 16*a*b*c^2*x
^2 - (5*I)*b^2*c^2*x^2 - 10*a^2*c^3*x^3 + (10*I)*a*b*c^3*x^3 + b^2*c^3*x^3
 - (15*I)*a^2*c^4*x^4 - 3*a*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-I + c*x)^3*(
-1 + (3*I)*c*x + 6*c^2*x^2)*ArcTan[c*x]^2 + b*ArcTan[c*x]*(b*(19 - (30*I)*
c*x + 16*c^2*x^2 + (10*I)*c^3*x^3 - 3*c^4*x^4) + 2*a*(15*I - 10*c^3*x^3 -
(15*I)*c^4*x^4 + 6*c^5*x^5) + 32*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 16*a*
b*Log[1 + c^2*x^2] + (20*I)*b^2*Log[1 + c^2*x^2] - (16*I)*b^2*PolyLog[2, -
E^((2*I)*ArcTan[c*x])]))/c^3
```

3.77.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = $\{5411, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2x^4(a+b\arctan(cx))^2+2icd^2x^3(a+b\arctan(cx))^2+d^2x^2(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{31id^2(a+b\arctan(cx))^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{15c^3}-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))^2+\frac{1}{2}icd^2x^4(a+b\arctan(cx))^2+\frac{1}{10}bcd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))^2-\frac{1}{3}ibd^2x^3(a+b\arctan(cx))-\frac{8bd^2x^2(a+b\arctan(cx))}{15c}+\frac{iabd^2x}{c^2}-\frac{19b^2d^2\arctan(cx)}{30c^3}+\frac{ib^2d^2x\arctan(cx)}{c^2}-\frac{8ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{15c^3}+\frac{19b^2d^2x}{30c^2}-\frac{2ib^2d^2\log\left(c^2x^2+1\right)}{3c^3}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3$$

input
```
Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(I*a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + ((I/6)*b^2*d^2*x^2)/c - (b^2
*d^2*x^3)/30 - (19*b^2*d^2*ArcTan[c*x])/(30*c^3) + (I*b^2*d^2*x*ArcTan[c*x
])/c^2 - (8*b*d^2*x^2*(a + b*ArcTan[c*x]))/(15*c) - (I/3)*b*d^2*x^3*(a + b
*ArcTan[c*x]) + (b*c*d^2*x^4*(a + b*ArcTan[c*x]))/10 - (((31*I)/30)*d^2*(a
 + b*ArcTan[c*x])^2)/c^3 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/2)*c*d^2
*x^4*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^5*(a + b*ArcTan[c*x])^2)/5 - (16*b
*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((2*I)/3)*b^2*d^2
*Log[1 + c^2*x^2])/c^3 - (((8*I)/15)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)]
)/c^3
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.77.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.26

method	result
parts	$a^2d^2\left(-\frac{1}{5}c^2x^5+\frac{1}{2}icx^4+\frac{1}{3}x^3\right)+\frac{b^2d^2\left(-\frac{\arctan(cx)^2c^5x^5}{5}-\frac{2i\ln\left(c^2x^2+1\right)}{3}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{4i\left(\ln(cx-i)\ln\left(c\right.\right.}{}\right.}{}$
derivativedivides	$\frac{a^2d^2\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+b^2d^2\left(-\frac{\arctan(cx)^2c^5x^5}{5}-\frac{2i\ln\left(c^2x^2+1\right)}{3}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{4i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\ln\right.}{}\right.}{}$
default	$\frac{a^2d^2\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+b^2d^2\left(-\frac{\arctan(cx)^2c^5x^5}{5}-\frac{2i\ln\left(c^2x^2+1\right)}{3}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{4i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\ln\right.}{}\right.}{}$
risch	$\frac{19b^2d^2x}{30c^2}-\frac{2537b^2d^2\arctan(cx)}{3600c^3}-\frac{b^2d^2x^3}{30}+\frac{a^2d^2x^3}{3}-\frac{a^2c^2d^2x^5}{5}+\frac{abc\,d^2x^4}{10}-\frac{59ab\,d^2}{30c^3}+\frac{iab\,d^2x}{c^2}-\frac{8id^2b^2}{}$

input
```
int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*(-1/5*c^2*x^5+1/2*I*x^4*c+1/3*x^3)+b^2*d^2/c^3*(-1/5*arctan(c*x)^2
*c^5*x^5-2/3*I*ln(c^2*x^2+1)+1/3*c^3*x^3*arctan(c*x)^2+4/15*I*(ln(c*x-I)*l
n(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x
-I)^2)+1/10*c^4*x^4*arctan(c*x)-4/15*I*(ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*
I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)^2)-8/15*c^2*x^2*arcta
n(c*x)+8/15*arctan(c*x)*ln(c^2*x^2+1)-1/2*I*arctan(c*x)^2+19/30*c*x-1/30*c
^3*x^3+1/2*I*arctan(c*x)^2*c^4*x^4-1/3*I*arctan(c*x)*c^3*x^3-19/30*arctan(
c*x)+I*arctan(c*x)*c*x+1/6*I*c^2*x^2)+2*a*d^2*b/c^3*(-1/5*c^5*x^5*arctan(c
*x)+1/2*I*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*arctan(c*x)+1/2*I*c*x+1/20*c^4*x
^4-1/6*I*c^3*x^3-4/15*c^2*x^2+4/15*ln(c^2*x^2+1)-1/2*I*arctan(c*x))
```

3.77.5 Fricas [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/120*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c*d^2*x^4 - 10*b^2*d^2*x^3)*log(-(c*x
+ I)/(c*x - I))^2 + integral(-1/30*(30*a^2*c^4*d^2*x^6 - 60*I*a^2*c^3*d^2*
x^5 - 60*I*a^2*c*d^2*x^3 - 30*a^2*d^2*x^2 - (-30*I*a*b*c^4*d^2*x^6 - 6*(10
*a*b - I*b^2)*c^3*d^2*x^5 + 15*b^2*c^2*d^2*x^4 - 10*(6*a*b + I*b^2)*c*d^2*
x^3 + 30*I*a*b*d^2*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.77.7 Maxima [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)^2(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/5*a^2*c^2*d^2*x^5 + 1/2*I*a^2*c*d^2*x^4 - 1/10*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2
*x^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c
^5))*a*b*c*d^2 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^
4))*a*b*d^2 - 1/120*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c*d^2*x^4 - 10*b^2*d^2*x
^3)*arctan(c*x)^2 + 1/120*(-6*I*b^2*c^2*d^2*x^5 - 15*b^2*c*d^2*x^4 + 10*I*
b^2*d^2*x^3)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(6*b^2*c^2*d^2*x^5 - 15*
I*b^2*c*d^2*x^4 - 10*b^2*d^2*x^3)*log(c^2*x^2 + 1)^2 - integrate(1/240*(18
0*(b^2*c^4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x)^2 + 15*(b^2*c^4*d^2*x^6 - b^
2*d^2*x^2)*log(c^2*x^2 + 1)^2 - 4*(21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x^3)*
arctan(c*x) + 2*(6*b^2*c^4*d^2*x^6 - 25*b^2*c^2*d^2*x^4 - 60*(b^2*c^3*d^2*
x^5 + b^2*c*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + I*
integrate(1/120*(180*(b^2*c^3*d^2*x^5 + b^2*c*d^2*x^3)*arctan(c*x)^2 + 15*
(b^2*c^3*d^2*x^5 + b^2*c*d^2*x^3)*log(c^2*x^2 + 1)^2 + 2*(6*b^2*c^4*d^2*x^
6 - 25*b^2*c^2*d^2*x^4)*arctan(c*x) + (21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x
^3 + 30*(b^2*c^4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^
2*x^2 + 1), x)
```

3.77.8 Giac [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int x^2\,\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^2\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.78 $\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx$

3.78.1 Optimal result

Integrand size = 23, antiderivative size = 293

$$\begin{aligned}\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx = &-\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{1}{12}b^2d^2x^2-\frac{2ib^2d^2\arctan(cx)}{3c^2}\\ &-\frac{3b^2d^2x\arctan(cx)}{2c}-\frac{2}{3}ibd^2x^2(a+b\arctan(cx))\\ &+\frac{1}{6}bcd^2x^3(a+b\arctan(cx))\\ &+\frac{17d^2(a+b\arctan(cx))^2}{12c^2}\\ &+\frac{1}{2}d^2x^2(a+b\arctan(cx))^2\\ &+\frac{2}{3}icd^2x^3(a+b\arctan(cx))^2\\ &-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))^2\\ &-\frac{4ibd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^2}\\ &+\frac{5b^2d^2\log(1+c^2x^2)}{6c^2}\\ &+\frac{2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}\end{aligned}$$

output
```
-3/2*a*b*d^2*x/c+2/3*I*b^2*d^2*x/c-1/12*b^2*d^2*x^2-2/3*I*b^2*d^2*arctan(c
*x)/c^2-3/2*b^2*d^2*x*arctan(c*x)/c-2/3*I*b*d^2*x^2*(a+b*arctan(c*x))+1/6*
b*c*d^2*x^3*(a+b*arctan(c*x))+17/12*d^2*(a+b*arctan(c*x))^2/c^2+1/2*d^2*x^
2*(a+b*arctan(c*x))^2+2/3*I*c*d^2*x^3*(a+b*arctan(c*x))^2-1/4*c^2*d^2*x^4*
(a+b*arctan(c*x))^2-4/3*I*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2+5/6*
b^2*d^2*ln(c^2*x^2+1)/c^2+2/3*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^2
```

3.78.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.88

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx =$$
$$-\frac{d^2\left(b^2+18abcx-8ib^2cx-6a^2c^2x^2+8iabc^2x^2+b^2c^2x^2-8ia^2c^3x^3-2abc^3x^3+3a^2c^4x^4+b^2(-i+cx)\right.}{}$$

input
```
Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-1/12*(d^2*(b^2 + 18*a*b*c*x - (8*I)*b^2*c*x - 6*a^2*c^2*x^2 + (8*I)*a*b*c
^2*x^2 + b^2*c^2*x^2 - (8*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 +
 b^2*(-I + c*x)^3*(I + 3*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(4*I + 9*
c*x + (4*I)*c^2*x^2 - c^3*x^3) + a*(-9 - 6*c^2*x^2 - (8*I)*c^3*x^3 + 3*c^4
*x^4) + (8*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (8*I)*a*b*Log[1 + c^2*x^
2] - 10*b^2*Log[1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/
c^2
```

3.78.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2 d^2 x^3 (a+b\arctan(cx))^2 + 2icd^2 x^2 (a+b\arctan(cx))^2 + d^2 x (a+b\arctan(cx))^2\right) dx$$

$$\big\downarrow \text{2009}$$

$$-\frac{1}{4}c^2 d^2 x^4 (a+b\arctan(cx))^2 + \frac{17d^2 (a+b\arctan(cx))^2}{12c^2} - \frac{4ibd^2 \log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^2} + \frac{2}{3}icd^2 x^3 (a+b\arctan(cx))^2 + \frac{1}{6}bcd^2 x^3 (a+b\arctan(cx)) + \frac{1}{2}d^2 x^2 (a+b\arctan(cx))^2 - \frac{2}{3}ibd^2 x^2 (a+b\arctan(cx)) - \frac{3abd^2 x}{2c} - \frac{2ib^2 d^2 \arctan(cx)}{3c^2} - \frac{3b^2 d^2 x\arctan(cx)}{2c} + \frac{2b^2 d^2 \operatorname{PolyLog}\left(2, 1-\frac{2}{icx+1}\right)}{3c^2} + \frac{5b^2 d^2 \log(c^2x^2+1)}{6c^2} + \frac{2ib^2 d^2 x}{3c} - \frac{1}{12}b^2 d^2 x^2$$

input
```
Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(-3*a*b*d^2*x)/(2*c) + (((2*I)/3)*b^2*d^2*x)/c - (b^2*d^2*x^2)/12 - (((2*I
)/3)*b^2*d^2*ArcTan[c*x])/c^2 - (3*b^2*d^2*x*ArcTan[c*x])/(2*c) - ((2*I)/3
)*b*d^2*x^2*(a + b*ArcTan[c*x]) + (b*c*d^2*x^3*(a + b*ArcTan[c*x]))/6 + (1
7*d^2*(a + b*ArcTan[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2
+ ((2*I)/3)*c*d^2*x^3*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^4*(a + b*ArcTan[c
*x])^2)/4 - (((4*I)/3)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 +
 (5*b^2*d^2*Log[1 + c^2*x^2])/(6*c^2) + (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I
*c*x)])/(3*c^2)
```

3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.78.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.31

method	result
parts	$a^2d^2\left(-\frac{1}{4}c^2x^4+\frac{2}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{b^2d^2\left(-\frac{c^4x^4\arctan(cx)^2}{4}+\frac{2i\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}-\frac{2i\,\mathrm{arc}}{}\right.}{}$
derivativedivides	$\frac{a^2d^2\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+b^2d^2\left(-\frac{c^4x^4\arctan(cx)^2}{4}+\frac{2i\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}-\frac{2i\arctan(cx)c^2x^2}{3}\right.}{}$
default	$\frac{a^2d^2\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+b^2d^2\left(-\frac{c^4x^4\arctan(cx)^2}{4}+\frac{2i\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}-\frac{2i\arctan(cx)c^2x^2}{3}\right.}{}$
risch	$-\frac{b^2d^2x^2}{12}+\frac{503b^2d^2\ln(c^2x^2+1)}{576c^2}-\frac{3ab\,d^2x}{2c}-\frac{3b^2d^2}{4c^2}-\frac{2id^2x^2ab}{3}+\frac{d^2b^2\ln(-icx+1)x^2}{3}+\frac{abc\,d^2x^3}{6}+\frac{a^2d^2x}{2}$

input
```
int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*(-1/4*c^2*x^4+2/3*I*c*x^3+1/2*x^2)+b^2*d^2/c^2*(-1/4*c^4*x^4*arcta
n(c*x)^2+2/3*I*arctan(c*x)*ln(c^2*x^2+1)+1/2*c^2*x^2*arctan(c*x)^2-2/3*I*a
rctan(c*x)*c^2*x^2+1/6*c^3*x^3*arctan(c*x)+2/3*I*arctan(c*x)^2*c^3*x^3+3/4
*arctan(c*x)^2-3/2*c*x*arctan(c*x)-1/3*ln(c*x-I)*ln(c^2*x^2+1)+1/3*ln(c*x+
I)*ln(c^2*x^2+1)+1/3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/6*ln(c*x-I)^2-1/6*ln(c
*x+I)^2-1/3*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/3*dilog(-1/2*I*(c*x+I))-1/3*dilo
g(1/2*I*(c*x-I))-2/3*I*arctan(c*x)-1/12*c^2*x^2+5/6*ln(c^2*x^2+1)+2/3*I*c*
x)+2*a*d^2*b/c^2*(-1/4*c^4*x^4*arctan(c*x)+2/3*I*arctan(c*x)*c^3*x^3+1/2*c
^2*x^2*arctan(c*x)-3/4*c*x+1/12*c^3*x^3-1/3*I*c^2*x^2+1/3*I*ln(c^2*x^2+1)+
3/4*arctan(c*x))
```

3.78.5 Fricas [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^3 - 6*b^2*d^2*x^2)*log(-(c*x + I
)/(c*x - I))^2 + integral(-1/12*(12*a^2*c^4*d^2*x^5 - 24*I*a^2*c^3*d^2*x^4
 - 24*I*a^2*c*d^2*x^2 - 12*a^2*d^2*x - (-12*I*a*b*c^4*d^2*x^5 - 3*(8*a*b -
 I*b^2)*c^3*d^2*x^4 + 8*b^2*c^2*d^2*x^3 - 6*(4*a*b + I*b^2)*c*d^2*x^2 + 12
*I*a*b*d^2*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*c^2*d^2*x^4 + 2/3*I*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctan(c*x)^2
 - 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a
*b*c^2*d^2 + 2/3*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4)
)*a*b*c*d^2 + 1/2*a^2*d^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/
c^3))*a*b*d^2 - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c
*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d^2 - 1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2
*c*d^2*x^3)*arctan(c*x)^2 + 1/48*(-3*I*b^2*c^2*d^2*x^4 - 8*b^2*c*d^2*x^3)*
arctan(c*x)*log(c^2*x^2 + 1) + 1/192*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^
3)*log(c^2*x^2 + 1)^2 - integrate(-1/48*(22*b^2*c^3*d^2*x^4*arctan(c*x) -
36*(b^2*c^4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x)^2 - 3*(b^2*c^4*d^2*x^5
+ b^2*c^2*d^2*x^3)*log(c^2*x^2 + 1)^2 - (3*b^2*c^4*d^2*x^5 - 8*b^2*c^2*d^2
*x^3 - 24*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))
/(c^2*x^2 + 1), x) + I*integrate(1/48*(72*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2
)*arctan(c*x)^2 + 6*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*log(c^2*x^2 + 1)^2 +
 2*(3*b^2*c^4*d^2*x^5 - 8*b^2*c^2*d^2*x^3)*arctan(c*x) + (11*b^2*c^3*d^2*x
^4 + 12*(b^2*c^4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))
/(c^2*x^2 + 1), x)
```

3.78.8 Giac [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

input
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.79 $\int (d+icdx)^2(a+b\arctan(cx))^2\,dx$

3.79.1 Optimal result

Integrand size = 22, antiderivative size = 192

$$\begin{aligned}\int (d+icdx)^2(a+b\arctan(cx))^2\,dx = &-2iabd^2x-\frac{1}{3}b^2d^2x+\frac{b^2d^2\arctan(cx)}{3c}\\ &-2ib^2d^2x\arctan(cx)+\frac{1}{3}bcd^2x^2(a+b\arctan(cx))\\ &-\frac{id^2(1+icx)^3(a+b\arctan(cx))^2}{3c}\\ &+\frac{8bd^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{3c}\\ &+\frac{ib^2d^2\log(1+c^2x^2)}{c}-\frac{4ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{3c}\end{aligned}$$

output
```
-2*I*a*b*d^2*x-1/3*b^2*d^2*x+1/3*b^2*d^2*arctan(c*x)/c-2*I*b^2*d^2*x*arcta
n(c*x)+1/3*b*c*d^2*x^2*(a+b*arctan(c*x))-1/3*I*d^2*(1+I*c*x)^3*(a+b*arctan
(c*x))^2/c+8/3*b*d^2*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/c+I*b^2*d^2*ln(c^2*
x^2+1)/c-4/3*I*b^2*d^2*polylog(2,1-2/(1-I*c*x))/c
```

3.79.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx = -\frac{d^2\left(-3a^2cx+6iabcx+b^2cx-3ia^2c^2x^2-abc^2x^2+a^2c^3x^3+b^2(-i+cx)^3\arctan(cx)^2-b\arctan(cx)\left(\right.\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-1/3*(d^2*(-3*a^2*c*x + (6*I)*a*b*c*x + b^2*c*x - (3*I)*a^2*c^2*x^2 - a*b*
c^2*x^2 + a^2*c^3*x^3 + b^2*(-I + c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*
(1 - (6*I)*c*x + c^2*x^2) + a*(6*I + 6*c*x + (6*I)*c^2*x^2 - 2*c^3*x^3) +
8*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + 4*a*b*Log[1 + c^2*x^2] - (3*I)*b^2*L
og[1 + c^2*x^2] + (4*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
```

3.79.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{2ib\int \left(-icx(a+b\arctan(cx))d^3-\frac{4i(i-cx)(a+b\arctan(cx))d^3}{c^2x^2+1}-3(a+b\arctan(cx))d^3\right)dx}{3d}-\frac{id^2(1+icx)^3(a+b\arctan(cx))^2}{3c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ib\left(-\frac{1}{2}icd^3x^2(a+b\arctan(cx))-\frac{4id^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-3ad^3x-3bd^3x\arctan(cx)-\frac{ibd^3\arctan(cx)}{2c}+\frac{3bd^3}{}\right.}{3d}$$

$$\frac{id^2(1+icx)^3(a+b\arctan(cx))^2}{3c}$$

input
```
Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/3*I)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^2)/c + (((2*I)/3)*b*(-3*a*
d^3*x + (I/2)*b*d^3*x - ((I/2)*b*d^3*ArcTan[c*x])/c - 3*b*d^3*x*ArcTan[c*x
] - (I/2)*c*d^3*x^2*(a + b*ArcTan[c*x]) - ((4*I)*d^3*(a + b*ArcTan[c*x])*L
og[2/(1 - I*c*x)])/c + (3*b*d^3*Log[1 + c^2*x^2])/(2*c) - (2*b*d^3*PolyLog
[2, 1 - 2/(1 - I*c*x)])/c))/d
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.79.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(172) = 344$.

Time = 1.20 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.84

method	result
derivativedivides	$\frac{-\frac{ia^2d^2(icx+1)^3}{3}+b^2d^2\left(-\frac{c^3x^3\arctan(cx)^2}{3}+i\arctan(cx)^2c^2x^2+\arctan(cx)^2cx-\frac{i\arctan(cx)^2}{3}+\frac{2i\left(-3cx\arctan(cx)+\frac{icx}{2}+2\right.}{}\right.}{}$
default	$\frac{-\frac{ia^2d^2(icx+1)^3}{3}+b^2d^2\left(-\frac{c^3x^3\arctan(cx)^2}{3}+i\arctan(cx)^2c^2x^2+\arctan(cx)^2cx-\frac{i\arctan(cx)^2}{3}+\frac{2i\left(-3cx\arctan(cx)+\frac{icx}{2}+2\right.}{}\right.}{}$
parts	$-\frac{ia^2d^2(icx+1)^3}{3c}+\frac{b^2d^2\left(-\frac{c^3x^3\arctan(cx)^2}{3}+i\arctan(cx)^2c^2x^2+\arctan(cx)^2cx-\frac{i\arctan(cx)^2}{3}+\frac{2i\left(-3cx\arctan(cx)+\right.}{}\right.}{}$
risch	$-\frac{b^2d^2x}{3}+\frac{7ab\,d^2}{3c}+x\,d^2a^2+i\ln(-icx+1)\,xab\,d^2+\frac{4ib^2\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)d^2}{3c}-\frac{4ib^2\ln(-icx+1}{3}$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/3*I*a^2*d^2*(1+I*c*x)^3+b^2*d^2*(-1/3*c^3*x^3*arctan(c*x)^2+I*arct
an(c*x)^2*c^2*x^2+arctan(c*x)^2*c*x-1/3*I*arctan(c*x)^2+2/3*I*(-3*c*x*arct
an(c*x)+1/2*I*c*x+2*I*arctan(c*x)*ln(c^2*x^2+1)+2*arctan(c*x)^2-ln(c*x-I)*
ln(c^2*x^2+1)+ln(c*x+I)*ln(c^2*x^2+1)+ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*ln(
c*x-I)^2-1/2*ln(c*x+I)^2-ln(c*x+I)*ln(1/2*I*(c*x-I))+dilog(-1/2*I*(c*x+I))
-dilog(1/2*I*(c*x-I))-1/2*I*arctan(c*x)*c^2*x^2+3/2*ln(c^2*x^2+1)-1/2*I*ar
ctan(c*x)))+2*a*d^2*b*(-1/3*c^3*x^3*arctan(c*x)+I*arctan(c*x)*c^2*x^2+c*x*
arctan(c*x)-1/3*I*arctan(c*x)+1/3*I*(-3*c*x-1/2*I*c^2*x^2+2*I*ln(c^2*x^2+1
)+4*arctan(c*x))))
```

3.79.5 Fricas [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2*x)*log(-(c*x + I)/(c
*x - I))^2 + integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*
a^2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - (6*a*b - I*b^2)*c^3*d^2*
x^3 + 3*b^2*c^2*d^2*x^2 - 3*(2*a*b + I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c
*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.79.7 Maxima [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/3*a^2*c^2*d^2*x^3 - 36*b^2*c^4*d^2*integrate(1/48*x^4*arctan(c*x)^2/(c^
2*x^2 + 1), x) - 3*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*
x^2 + 1), x) - 4*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2
+ 1), x) + 24*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/
(c^2*x^2 + 1), x) + 32*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2
 + 1), x) - 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a
*b*c^2*d^2 + I*a^2*c*d^2*x^2 + 24*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x
^2 + 1)/(c^2*x^2 + 1), x) + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/
c^3))*a*b*c*d^2 + 1/4*b^2*d^2*arctan(c*x)^3/c + 24*b^2*c*d^2*integrate(1/4
8*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 24*b^2*c*d^2*integrat
e(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^2*x + 3*b^2*d^2*integrate(1
/48*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^
2 + 1))*a*b*d^2/c - 1/12*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2*
x)*arctan(c*x)^2 + 1/12*(-I*b^2*c^2*d^2*x^3 - 3*b^2*c*d^2*x^2 + 3*I*b^2*d^
2*x)*arctan(c*x)*log(c^2*x^2 + 1) + 1/48*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*
x^2 - 3*b^2*d^2*x)*log(c^2*x^2 + 1)^2 + I*integrate(1/24*(36*(b^2*c^3*d^2*
x^3 + b^2*c*d^2*x)*arctan(c*x)^2 + 3*(b^2*c^3*d^2*x^3 + b^2*c*d^2*x)*log(c
^2*x^2 + 1)^2 + 4*(b^2*c^4*d^2*x^4 - 6*b^2*c^2*d^2*x^2)*arctan(c*x) + 2*(4
*b^2*c^3*d^2*x^3 - 3*b^2*c*d^2*x + 3*(b^2*c^4*d^2*x^4 - b^2*d^2)*arctan(c*
x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

3.79.8 Giac [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int (d + icdx)^2 (a + b \arctan(cx))^2 \, dx = \int (a + b \operatorname{atan}(c\,x))^2 \, (d + c\,d\,x\,1\mathrm{i})^2 \, dx$$

input
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

3.80 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx$

3.80.1 Optimal result

Integrand size = 25, antiderivative size = 300

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx &= abcd^2x+b^2cd^2x\arctan(cx)-\frac{5}{2}d^2(a+b\arctan(cx))^2\\ &\quad+2icd^2x(a+b\arctan(cx))^2\\ &\quad-\frac{1}{2}c^2d^2x^2(a+b\arctan(cx))^2\\ &\quad+2d^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &\quad+4ibd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)\\ &\quad-\frac{1}{2}b^2d^2\log\left(1+c^2x^2\right)\\ &\quad-2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &\quad-ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &\quad+ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\ &\quad-\frac{1}{2}b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &\quad+\frac{1}{2}b^2d^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
a*b*c*d^2*x+b^2*c*d^2*x*arctan(c*x)-5/2*d^2*(a+b*arctan(c*x))^2+2*I*c*d^2*
x*(a+b*arctan(c*x))^2-1/2*c^2*d^2*x^2*(a+b*arctan(c*x))^2-2*d^2*(a+b*arcta
n(c*x))^2*arctanh(-1+2/(1+I*c*x))+4*I*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))-1/2*b^2*d^2*ln(c^2*x^2+1)-2*b^2*d^2*polylog(2,1-2/(1+I*c*x))-I*b*d^2*(
a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*d^2*(a+b*arctan(c*x))*polylo
g(2,-1+2/(1+I*c*x))-1/2*b^2*d^2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*d^2*polyl
og(3,-1+2/(1+I*c*x))
```

3.80.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.20

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx$$
$$= \frac{1}{2}d^2\Bigg(4ia^2cx - a^2c^2x^2 + 2b^2cx\arctan(cx) - b^2(1+c^2x^2)\arctan(cx)^2 - 2ab(-cx+(1+c^2x^2)\arctan(cx)) + 2a^2\log(cx) + 4iab(2cx\arctan(cx) - \log(1+c^2x^2)) - b^2\log(1+c^2x^2) + 4b^2\left(\arctan(cx)\left((1+icx)\arctan(cx) + 2i\log\left(1+e^{2i\arctan(cx)}\right)\right) + \operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right) + 2iab(\operatorname{PolyLog}(2,-icx) - \operatorname{PolyLog}(2,icx)) + 2b^2\Bigg(-\frac{i\pi^3}{24} + \frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\Bigg)\Bigg)$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
(d^2*((4*I)*a^2*c*x - a^2*c^2*x^2 + 2*b^2*c*x*ArcTan[c*x] - b^2*(1 + c^2*x
^2)*ArcTan[c*x]^2 - 2*a*b*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]) + 2*a^2*Log
[c*x] + (4*I)*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - b^2*Log[1 + c^2
*x^2] + 4*b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*
I)*ArcTan[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*(PolyL
og[2, (-I)*c*x] - PolyLog[2, I*c*x]) + 2*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*A
rcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]
^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*Arc
Tan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3,
 E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2))/2
```

3.80.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2x(a+b\arctan(cx))^2+2icd^2(a+b\arctan(cx))^2+\frac{d^2(a+b\arctan(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2d^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}c^2d^2x^2(a+b\arctan(cx))^2-ibd^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+2icd^2x(a+b\arctan(cx))^2-\frac{5}{2}d^2(a+b\arctan(cx))^2+4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+abcd^2x+b^2cd^2x\arctan(cx)-\frac{1}{2}b^2d^2\log\left(c^2x^2+1\right)-2b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
a*b*c*d^2*x + b^2*c*d^2*x*ArcTan[c*x] - (5*d^2*(a + b*ArcTan[c*x])^2)/2 +
(2*I)*c*d^2*x*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^2*(a + b*ArcTan[c*x])^2)/
2 + 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (4*I)*b*d^2*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*d^2*Log[1 + c^2*x^2])/2 - 2*b
^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d^2*(a + b*ArcTan[c*x])*PolyLog
[2, 1 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1
+ I*c*x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^2*PolyLog[3
, -1 + 2/(1 + I*c*x)])/2
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.80.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 29.47 (sec) , antiderivative size = 1317, normalized size of antiderivative = 4.39

method	result	size
parts	Expression too large to display	1317
derivativedivides	Expression too large to display	1319
default	Expression too large to display	1319

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*(-1/2*c^2*x^2+2*I*c*x+ln(x))+b^2*d^2*(-1/2*polylog(3,-(1+I*c*x)^2/
(c^2*x^2+1))+2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*
x)/(c^2*x^2+1)^(1/2))-1/2*c^2*x^2*arctan(c*x)^2+ln(1+(1+I*c*x)^2/(c^2*x^2+
1))+3/2*arctan(c*x)^2+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+4*
dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+4*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(
1/2))+arctan(c*x)*(c*x-I)-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arct
an(c*x)^2*ln(c*x)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c
*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,(1+I*c*x
)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2
))+1/2*I*Pi*arctan(c*x)^2+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+
I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*
x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2
*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+2*I*arct
an(c*x)^2*c*x+1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^3*arctan(c*x)^2-1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c...
```

3.80.5 Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*
d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*
b*c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x
)
```

3.80.6 Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = -d^2\left(\int\left(-\frac{a^2}{x}\right)dx+\int\left(-2ia^2c\right)dx+\int a^2c^2x\,dx+\int\left(-\frac{b^2\operatorname{atan}^2(cx)}{x}\right)dx+\int\left(-2ib^2c\operatorname{atan}^2(cx)\right)dx+\int\left(-\frac{2ab\operatorname{atan}(cx)}{x}\right)dx+\int b^2c^2x\operatorname{atan}^2(cx)\,dx+\int\left(-4iabc\operatorname{atan}(cx)\right)dx+\int 2abc^2x\operatorname{atan}(cx)\,dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x,x)
```

output
```
-d**2*(Integral(-a**2/x, x) + Integral(-2*I*a**2*c, x) + Integral(a**2*c**
2*x, x) + Integral(-b**2*atan(c*x)**2/x, x) + Integral(-2*I*b**2*c*atan(c*
x)**2, x) + Integral(-2*a*b*atan(c*x)/x, x) + Integral(b**2*c**2*x*atan(c*
x)**2, x) + Integral(-4*I*a*b*c*atan(c*x), x) + Integral(2*a*b*c**2*x*atan
(c*x), x))
```

3.80.7 Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
-12*b^2*c^4*d^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*I*b
^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x
) - b^2*c^4*d^2*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) +
2*I*b^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 32*a*b*c
^4*d^2*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 2*b^2*c^4*d^2*in
tegrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) - 1/2*a^2*c^2*d^2*x^2
+ 12*I*b^2*c^3*d^2*integrate(1/8*x^3*arctan(c*x)^2/(c^2*x^3 + x), x) + 8*b
^2*c^3*d^2*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x),
x) + I*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x)
+ 20*b^2*c^3*d^2*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 5*I*b^
2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 1/2*I*b^2
*d^2*arctan(c*x)^3 - 8*I*b^2*c^2*d^2*integrate(1/8*x^2*arctan(c*x)/(c^2*x^
3 + x), x) + 2*I*a^2*c*d^2*x + 8*b^2*c*d^2*integrate(1/16*x*arctan(c*x)*lo
g(c^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c*d^2*integrate(1/8*x*log(c^2*x^2
 + 1)^2/(c^2*x^3 + x), x) + 1/8*b^2*d^2*log(c^2*x^2 + 1)^2 + 2*I*(2*c*x*ar
ctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2 + 12*b^2*d^2*integrate(1/16*arctan(c
*x)^2/(c^2*x^3 + x), x) - 2*I*b^2*d^2*integrate(1/8*arctan(c*x)*log(c^2*x^
2 + 1)/(c^2*x^3 + x), x) + b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*
x^3 + x), x) + 32*a*b*d^2*integrate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + a
^2*d^2*log(x) - 1/8*(b^2*c^2*d^2*x^2 - 4*I*b^2*c*d^2*x)*arctan(c*x)^2 +...
```

3.80.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
Timed out
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x, x)
```

3.81 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx$

3.81.1 Optimal result

Integrand size = 25, antiderivative size = 317

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx &= -2icd^2(a+b\arctan(cx))^2-\frac{d^2(a+b\arctan(cx))^2}{x}\\
&\quad -c^2d^2x(a+b\arctan(cx))^2\\
&\quad +4icd^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\
&\quad -2bcd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)\\
&\quad +2bcd^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\
&\quad -ib^2cd^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\
&\quad -ib^2cd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\
&\quad +2bcd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\
&\quad -2bcd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\
&\quad -ib^2cd^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\
&\quad +ib^2cd^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-2*I*c*d^2*(a+b*arctan(c*x))^2-d^2*(a+b*arctan(c*x))^2/x-c^2*d^2*x*(a+b*ar
ctan(c*x))^2-4*I*c*d^2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-2*b*c*d
^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))+2*b*c*d^2*(a+b*arctan(c*x))*ln(2-2/(1
-I*c*x))-I*b^2*c*d^2*polylog(2,-1+2/(1-I*c*x))-I*b^2*c*d^2*polylog(2,1-2/(
1+I*c*x))+2*b*c*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))-2*b*c*d^2*(
a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-I*b^2*c*d^2*polylog(3,1-2/(1+I*
c*x))+I*b^2*c*d^2*polylog(3,-1+2/(1+I*c*x))
```

3.81.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.19

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx =$$
$$-\frac{d^2\left(12a^2-b^2c\pi^3x+12a^2c^2x^2+24ab\arctan(cx)+24abc^2x^2\arctan(cx)+12b^2\arctan(cx)^2+12b^2c^2x^2\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
-1/12*(d^2*(12*a^2 - b^2*c*Pi^3*x + 12*a^2*c^2*x^2 + 24*a*b*ArcTan[c*x] +
24*a*b*c^2*x^2*ArcTan[c*x] + 12*b^2*ArcTan[c*x]^2 + 12*b^2*c^2*x^2*ArcTan[
c*x]^2 + 16*b^2*c*x*ArcTan[c*x]^3 - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E
^((-2*I)*ArcTan[c*x])] - 24*b^2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*
x])] + 24*b^2*c*x*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*b^2*
c*x*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*a^2*c*x*Log[c*x]
 - 24*a*b*c*x*Log[c*x] + 24*b^2*c*x*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTa
n[c*x])] + 12*b^2*c*x*(-I + 2*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x
])] + (12*I)*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + 24*a*b*c*x*PolyLo
g[2, (-I)*c*x] - 24*a*b*c*x*PolyLog[2, I*c*x] - (12*I)*b^2*c*x*PolyLog[3,
E^((-2*I)*ArcTan[c*x])] + (12*I)*b^2*c*x*PolyLog[3, -E^((2*I)*ArcTan[c*x])
]))/x
```

3.81.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \ 5411$$
$$\int \left(-c^2d^2(a+b\arctan(cx))^2+\frac{d^2(a+b\arctan(cx))^2}{x^2}+\frac{2icd^2(a+b\arctan(cx))^2}{x}\right)dx$$
$$\Big\downarrow \ 2009$$
$$4icd^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-c^2d^2x(a+b\arctan(cx))^2+2bcd^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-2bcd^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-2icd^2(a+b\arctan(cx))^2-\frac{d^2(a+b\arctan(cx))^2}{x}-2bcd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+2bcd^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-ib^2cd^2\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-ib^2cd^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-ib^2cd^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+ib^2cd^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-2*I)*c*d^2*(a + b*ArcTan[c*x])^2 - (d^2*(a + b*ArcTan[c*x])^2)/x - c^2*d
^2*x*(a + b*ArcTan[c*x])^2 + (4*I)*c*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 -
 2/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] + 2*b*c
*d^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -
1 + 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + 2*b*c*d^2
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcT
an[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - I*b^2*c*d^2*PolyLog[3, 1 - 2/(1
+ I*c*x)] + I*b^2*c*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.81.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.94 (sec) , antiderivative size = 5048, normalized size of antiderivative = 15.92

method	result	size
parts	Expression too large to display	5048
derivativedivides	Expression too large to display	5050
default	Expression too large to display	5050

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.81.5 Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*
d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*
b*c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2,
 x)
```

3.81.6 Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = -d^2\left(\int a^2c^2\,dx + \int\left(-\frac{a^2}{x^2}\right)dx + \int b^2c^2\,\mathrm{atan}^2\,(cx)\,dx + \int\left(-\frac{b^2\,\mathrm{atan}^2\,(cx)}{x^2}\right)dx + \int\left(-\frac{2ia^2c}{x}\right)dx + \int 2abc^2\,\mathrm{atan}\,(cx)\,dx + \int\left(-\frac{2ab\,\mathrm{atan}\,(cx)}{x^2}\right)dx + \int\left(-\frac{2ib^2c\,\mathrm{atan}^2\,(cx)}{x}\right)dx + \int\left(-\frac{4iabc\,\mathrm{atan}\,(cx)}{x}\right)dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**2,x)
```

output
```
-d**2*(Integral(a**2*c**2, x) + Integral(-a**2/x**2, x) + Integral(b**2*c*
*2*atan(c*x)**2, x) + Integral(-b**2*atan(c*x)**2/x**2, x) + Integral(-2*I
*a**2*c/x, x) + Integral(2*a*b*c**2*atan(c*x), x) + Integral(-2*a*b*atan(c
*x)/x**2, x) + Integral(-2*I*b**2*c*atan(c*x)**2/x, x) + Integral(-4*I*a*b
*c*atan(c*x)/x, x))
```

3.81.7 Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-a^2*c^2*d^2*x - (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*c*d^2 + 2*I*a^
2*c*d^2*log(x) - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*d
^2 - a^2*d^2/x + 1/16*(8*b^2*c^2*d^2*x^2 - 2*b^2*c^2*d^2*x*integrate(4*arc
tan(c*x)^2 + log(c^2*x^2 + 1)^2, x) + 4*I*b^2*c^2*d^2*x*integrate(-1/4*(8*
(c^2*x^2 + 1)*c*x*arctan(c*x)^2 - 2*(c^2*x^2 + 1)*c*x*log(c^2*x^2 + 1)^2 +
 8*(c^2*x^2 + 1)*arctan(c*x)*log(c^2*x^2 + 1) - (4*(c^2*x^2 + 1)^(3/2)*arc
tan(c*x)*cos(2*arctan(c*x))*log(c^2*x^2 + 1) + 4*sqrt(c^2*x^2 + 1)*arctan(
c*x)*log(c^2*x^2 + 1) + (4*(c^2*x^2 + 1)^(3/2)*arctan(c*x)^2 - (c^2*x^2 +
1)^(3/2)*log(c^2*x^2 + 1)^2)*sin(2*arctan(c*x)))*sqrt(c^2*x^2 + 1))/((c^2*
x^2 + 1)^3*cos(2*arctan(c*x))^2 + (c^2*x^2 + 1)^3*sin(2*arctan(c*x))^2 + c
^2*x^2 + 2*(c^2*x^2 + 1)^2*cos(2*arctan(c*x)) + 4*(c^2*x^2 + 1)^2 - 4*((c^
2*x^2 + 1)^(3/2)*c*x*sin(2*arctan(c*x)) + (c^2*x^2 + 1)^(3/2)*cos(2*arctan
(c*x)) + sqrt(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1) + 1), x) - 4*b^2*c^2*d^2*x*i
ntegrate(1/4*(8*(c^2*x^2 + 1)*c*x*arctan(c*x)*log(c^2*x^2 + 1) - 8*(c^2*x^
2 + 1)*arctan(c*x)^2 + 2*(c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - (4*(c^2*x^2 +
1)^(3/2)*arctan(c*x)*log(c^2*x^2 + 1)*sin(2*arctan(c*x)) - 4*sqrt(c^2*x^2
+ 1)*arctan(c*x)^2 + sqrt(c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - (4*(c^2*x^2 +
1)^(3/2)*arctan(c*x)^2 - (c^2*x^2 + 1)^(3/2)*log(c^2*x^2 + 1)^2)*cos(2*arc
tan(c*x)))*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^3*cos(2*arctan(c*x))^2 + (c^2
*x^2 + 1)^3*sin(2*arctan(c*x))^2 + c^2*x^2 + 2*(c^2*x^2 + 1)^2*cos(2*ar...
```

3.81.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^2}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^2, x)
```

3.82 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx$

3.82.1 Optimal result

Integrand size = 25, antiderivative size = 337

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = &-\frac{bcd^2(a+b\arctan(cx))}{x}+\frac{3}{2}c^2d^2(a+b\arctan(cx))^2\\ &-\frac{d^2(a+b\arctan(cx))^2}{2x^2}-\frac{2icd^2(a+b\arctan(cx))^2}{x}\\ &-2c^2d^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &+b^2c^2d^2\log(x)-\frac{1}{2}b^2c^2d^2\log\left(1+c^2x^2\right)\\ &+4ibc^2d^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ &+2b^2c^2d^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\ &+ibc^2d^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &-ibc^2d^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1\right.\\ &\quad\left.+\frac{2}{1+icx}\right)+\frac{1}{2}b^2c^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &-\frac{1}{2}b^2c^2d^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-b*c*d^2*(a+b*arctan(c*x))/x+3/2*c^2*d^2*(a+b*arctan(c*x))^2-1/2*d^2*(a+b*
arctan(c*x))^2/x^2-2*I*c*d^2*(a+b*arctan(c*x))^2/x+2*c^2*d^2*(a+b*arctan(c
*x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(c^2*x^
2+1)+4*I*b*c^2*d^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+2*b^2*c^2*d^2*polyl
og(2,-1+2/(1-I*c*x))+I*b*c^2*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x)
)-I*b*c^2*d^2*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+1/2*b^2*c^2*d^2*
polylog(3,1-2/(1+I*c*x))-1/2*b^2*c^2*d^2*polylog(3,-1+2/(1+I*c*x))
```

3.82.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.15

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx =$$
$$-\frac{d^2\Big(a^2+4ia^2cx+2ab(\arctan(cx)+cx(1+cx\arctan(cx)))+2a^2c^2x^2\log(x)+b^2\Big(2cx\arctan(cx)+(1}{}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-1/2*(d^2*(a^2 + (4*I)*a^2*c*x + 2*a*b*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[
c*x])) + 2*a^2*c^2*x^2*Log[x] + b^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*Arc
Tan[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (4*I)*a*b*c*x*(2*Ar
cTan[c*x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])) + (4*I)*b^2*c*x*(ArcTan[
c*x]^2 - 2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[
c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])) + (2*I)*a*b*c^2*x^2*(PolyLog[
2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*c^2*x^2*((-I)*Pi^3 + (16*I)*ArcTa
n[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*
x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2
*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
+ 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c
*x])]))/12))/x^2
```

3.82.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{c^2d^2(a+b\arctan(cx))^2}{x}+\frac{d^2(a+b\arctan(cx))^2}{x^3}+\frac{2icd^2(a+b\arctan(cx))^2}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-2c^2d^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2+ibc^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-ibc^2d^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{3}{2}c^2d^2(a+b\arctan(cx))^2+4ibc^2d^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{d^2(a+b\arctan(cx))^2}{2x^2}-\frac{2icd^2(a+b\arctan(cx))^2}{x}-\frac{bcd^2(a+b\arctan(cx))}{x}+2b^2c^2d^2\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)-\frac{1}{2}b^2c^2d^2\log\left(c^2x^2+1\right)+b^2c^2d^2\log(x)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d^2*(a + b*ArcTan[c*x]))/x) + (3*c^2*d^2*(a + b*ArcTan[c*x])^2)/2 -
 (d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x])^2)
/x - 2*c^2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*
d^2*Log[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (4*I)*b*c^2*d^2*(a + b*Arc
Tan[c*x])*Log[2 - 2/(1 - I*c*x)] + 2*b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 - I*
c*x)] + I*b*c^2*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*
b*c^2*d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (b^2*c^2*d^
2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 + I
*c*x)])/2
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.82.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.52 (sec) , antiderivative size = 1381, normalized size of antiderivative = 4.10

method	result	size
parts	Expression too large to display	1381
derivativedivides	Expression too large to display	1383
default	Expression too large to display	1383

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*(-1/2/x^2-c^2*ln(x)-2*I*c/x)+b^2*d^2*c^2*(-1/2/c^2/x^2*arctan(c*x)
^2+1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-2*polylog(3,(1+I*c*x)/(c^2*x^2+
1)^(1/2))+ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*c*x)/(c^2*x^
2+1)^(1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+3/2*arctan(c*x)^2+arctan(c*x
)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-arctan(c*x)^2*ln(c*x)-arctan(c*x)^2*ln(1
-(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/
2))+4*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*dilog((1+I*c*x)/(c^2*x^2+1)^(
1/2))-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
))*arctan(c*x)^2-I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2*I*arc
tan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*arctan(c*x)*polylog(2,
-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*Pi*arctan(c*x)^2+4*I*arctan(c*x)*ln(1+
(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(
1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1
)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arct
an(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*c...
```

3.82.5 Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*
d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*
b*c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3,
 x)
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Timed out
```

3.82.7 Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-a^2*c^2*d^2*log(x) - 2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)
/x)*a*b*c*d^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^2 - 2*I*
a^2*c*d^2/x - 1/2*a^2*d^2/x^2 + 1/96*(48*I*(b^2*c^2*d^2*arctan(c*x)^3 + 4*
b^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3)
, x) + 2*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3),
 x) - 8*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x)
 + 20*b^2*c^2*d^2*integrate(1/8*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 24*b
^2*c*d^2*integrate(1/8*x*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 2*b^2*c*d^2*i
ntegrate(1/8*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 2*b^2*c*d^2*integr
ate(1/8*x*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 4*b^2*d^2*integrate(1/8*a
rctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x))*x^2 - (1152*b^2*c^4*d^2*i
ntegrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^4*d^2*int
egrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + b^2*c^2*d^2*log(c^2*x^2
+ 1)^3 + 48*b^2*c^2*d^2*arctan(c*x)^2 - 768*b^2*c^3*d^2*integrate(1/16*x^3
*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 3072*a*b*c^2*d^2*integ
rate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 960*b^2*c^2*d^2*integrate(
1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 768*b^2*c*d^2*integrate(1/
16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 384*b^2*c*d^2*inte
grate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) - 1152*b^2*d^2*integrate(1/16
*arctan(c*x)^2/(c^2*x^5 + x^3), x) - 96*b^2*d^2*integrate(1/16*log(c^2*...
```

3.82.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^3, x)
```

3.83 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx$

3.83.1 Optimal result

Integrand size = 25, antiderivative size = 267

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = &-\frac{b^2c^2d^2}{3x}-\frac{1}{3}b^2c^3d^2\arctan(cx)\\ &-\frac{bcd^2(a+b\arctan(cx))}{3x^2}-\frac{2ibc^2d^2(a+b\arctan(cx))}{x}\\ &-\frac{d^2(1+icx)^3(a+b\arctan(cx))^2}{3x^3}\\ &-\frac{8}{3}abc^3d^2\log(x)+2ib^2c^3d^2\log(x)\\ &-\frac{8}{3}bc^3d^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)\\ &-ib^2c^3d^2\log\left(1+c^2x^2\right)-\frac{4}{3}ib^2c^3d^2\operatorname{PolyLog}(2,-icx)\\ &+\frac{4}{3}ib^2c^3d^2\operatorname{PolyLog}(2,icx)\\ &+\frac{4}{3}ib^2c^3d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d^2/x-1/3*b^2*c^3*d^2*arctan(c*x)-1/3*b*c*d^2*(a+b*arctan(c*x
))/x^2-2*I*b*c^2*d^2*(a+b*arctan(c*x))/x-1/3*d^2*(1+I*c*x)^3*(a+b*arctan(c
*x))^2/x^3-8/3*a*b*c^3*d^2*ln(x)+2*I*b^2*c^3*d^2*ln(x)-8/3*b*c^3*d^2*(a+b*
arctan(c*x))*ln(2/(1-I*c*x))-I*b^2*c^3*d^2*ln(c^2*x^2+1)-4/3*I*b^2*c^3*d^2
*polylog(2,-I*c*x)+4/3*I*b^2*c^3*d^2*polylog(2,I*c*x)+4/3*I*b^2*c^3*d^2*po
lylog(2,1-2/(1-I*c*x))
```

3.83.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{d^2\left(-a^2-3ia^2cx-abcx+3a^2c^2x^2-6iabc^2x^2-b^2c^2x^2+b^2(-1-icx)^3\arctan(cx)^2-b\arctan(cx)\left(bcx\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
(d^2*(-a^2 - (3*I)*a^2*c*x - a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a*b*c^2*x^2 -
 b^2*c^2*x^2 + b^2*(-1 - I*c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*c*x*(1
+ (6*I)*c*x + c^2*x^2) + a*(2 + (6*I)*c*x - 6*c^2*x^2 + (6*I)*c^3*x^3) + 8
*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + (6*I
)*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*a*b*c^3*x^3*Log[1 + c^2*x^2
] + (4*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(3*x^3)
```

3.83.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx$$
$$\Big\downarrow \text{5409}$$
$$-2bc\int\left(-\frac{4d^2(a+b\arctan(cx))c^3}{3(cx+i)}+\frac{4d^2(a+b\arctan(cx))c^2}{3x}-\frac{id^2(a+b\arctan(cx))c}{x^2}-\frac{d^2(a+b\arctan(cx))}{3x^3}\right)$$
$$\frac{d^2(1+icx)^3(a+b\arctan(cx))^2}{3x^3}$$
$$\Big\downarrow \text{2009}$$

$$-2bc\left(\frac{4}{3}c^2d^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))+\frac{d^2(a+b\arctan(cx))}{6x^2}+\frac{icd^2(a+b\arctan(cx))}{x}+\frac{4}{3}ac^2d^2\log(x)-\right.$$
$$\frac{d^2(1+icx)^3(a+b\arctan(cx))^2}{3x^3}$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^2)/x^3 - 2*b*c*((b*c*d^2)/(6*x
) + (b*c^2*d^2*ArcTan[c*x])/6 + (d^2*(a + b*ArcTan[c*x]))/(6*x^2) + (I*c*d
^2*(a + b*ArcTan[c*x]))/x + (4*a*c^2*d^2*Log[x])/3 - I*b*c^2*d^2*Log[x] +
(4*c^2*d^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/3 + (I/2)*b*c^2*d^2*Log
[1 + c^2*x^2] + ((2*I)/3)*b*c^2*d^2*PolyLog[2, (-I)*c*x] - ((2*I)/3)*b*c^2
*d^2*PolyLog[2, I*c*x] - ((2*I)/3)*b*c^2*d^2*PolyLog[2, 1 - 2/(1 - I*c*x)]
)
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

3.83.4 Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.67

method	result
parts	$a^2d^2\left(-\frac{ic}{x^2}+\frac{c^2}{x}-\frac{1}{3x^3}\right)+b^2d^2c^3\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)^2}{cx}-i\ln\left(c^2x^2+1\right)+2i\ln\left(cx\right)\right.$ -
derivativedivides	$c^3\left(a^2d^2\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{i}{c^2x^2}\right)+b^2d^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)^2}{cx}-i\ln\left(c^2x^2+1\right)+2i\ln\left(c\right.\right.\right.$
default	$c^3\left(a^2d^2\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{i}{c^2x^2}\right)+b^2d^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)^2}{cx}-i\ln\left(c^2x^2+1\right)+2i\ln\left(c\right.\right.\right.$

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*(-I*c/x^2+c^2/x-1/3/x^3)+b^2*d^2*c^3*(-1/3*arctan(c*x)^2/c^3/x^3+a
rctan(c*x)^2/c/x-I*ln(c^2*x^2+1)+2*I*ln(c*x)-1/3/c^2/x^2*arctan(c*x)-8/3*a
rctan(c*x)*ln(c*x)+4/3*arctan(c*x)*ln(c^2*x^2+1)+4/3*I*ln(c*x)*ln(1-I*c*x)
-2*I*arctan(c*x)/c/x-1/3/c/x-4/3*I*ln(c*x)*ln(1+I*c*x)-I*arctan(c*x)^2/c^2
/x^2-1/3*arctan(c*x)+2/3*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))-
ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)-4/3*I*dilog(1+I*c*x)-2/3*I*(
ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1
/2*ln(c*x+I)^2)-I*arctan(c*x)^2+4/3*I*dilog(1-I*c*x))+2*a*d^2*b*c^3*(-1/3*
arctan(c*x)/c^3/x^3+1/c/x*arctan(c*x)-I*arctan(c*x)/c^2/x^2-I/c/x-1/6/c^2/
x^2-4/3*ln(c*x)+2/3*ln(c^2*x^2+1)-I*arctan(c*x))
```

3.83.5 Fricas [F]

$$\int\frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx=\int\frac{(i\,cdx+d)^2(b\arctan\left(cx\right)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/12*(12*x^3*integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*
a^2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - 3*(2*a*b + I*b^2)*c^3*d^
2*x^3 - 3*b^2*c^2*d^2*x^2 - (6*a*b - I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c
*x + I)/(c*x - I)))/(c^2*x^6 + x^4), x) - (3*b^2*c^2*d^2*x^2 - 3*I*b^2*c*d
^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**4,x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c^2*d^2 - 2*I*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d^2 + 1/3*((c^2*log(c^2*x^2
+ 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*d^2 + a^2*c^2*d^2/
x - I*a^2*c*d^2/x^2 - 1/3*a^2*d^2/x^3 - 1/48*(12*(b^2*c^3*d^2*arctan(c*x)^
3 + 12*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4),
x) - 48*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x
) - 96*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^
6 + x^4), x) + 192*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^6 + x
^4), x) + 64*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^
4), x) - 96*b^2*c*d^2*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x
^6 + x^4), x) - 32*b^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x^6 + x^4),
 x) - 144*b^2*d^2*integrate(1/48*arctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^
2*d^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 + 12*I*(b
^2*c^3*d^2*arctan(c*x)^2 - 24*b^2*c^4*d^2*integrate(1/24*x^4*arctan(c*x)*l
og(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 144*b^2*c^3*d^2*integrate(1/24*x^3*a
rctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^2*c^3*d^2*integrate(1/24*x^3*log(c
^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) + 48*b^2*c^3*d^2*integrate(1/24*x^3*log(
c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 64*b^2*c^2*d^2*integrate(1/24*x^2*arcta
n(c*x)/(c^2*x^6 + x^4), x) - 144*b^2*c*d^2*integrate(1/24*x*arctan(c*x)^2/
(c^2*x^6 + x^4), x) - 12*b^2*c*d^2*integrate(1/24*x*log(c^2*x^2 + 1)^2/...
```

3.83.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^4,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^4, x)
```

3.84 $\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx$

3.84.1 Optimal result

Integrand size = 25, antiderivative size = 438

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx = \frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44ib^2d^3x^3}{315c} -\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib^2cd^3x^5+\frac{122ib^2d^3\arctan(cx)}{105c^4} +\frac{3b^2d^3x\arctan(cx)}{2c^3}+\frac{26ibd^3x^2(a+b\arctan(cx))}{35c^2} -\frac{bd^3x^3(a+b\arctan(cx))}{2c} -\frac{13}{35}ibd^3x^4(a+b\arctan(cx)) +\frac{1}{5}bcd^3x^5(a+b\arctan(cx)) +\frac{1}{21}ibc^2d^3x^6(a+b\arctan(cx)) -\frac{209d^3(a+b\arctan(cx))^2}{140c^4} +\frac{1}{4}d^3x^4(a+b\arctan(cx))^2 +\frac{3}{5}icd^3x^5(a+b\arctan(cx))^2 -\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))^2 -\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))^2 +\frac{52ibd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{35c^4} -\frac{11b^2d^3\log(1+c^2x^2)}{10c^4} -\frac{26b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{35c^4}$$

output
```
3/2*a*b*d^3*x/c^3-1/105*I*b^2*c*d^3*x^5+7/20*b^2*d^3*x^2/c^2+1/21*I*b*c^2*
d^3*x^6*(a+b*arctan(c*x))-1/20*b^2*d^3*x^4+44/315*I*b^2*d^3*x^3/c-13/35*I*
b*d^3*x^4*(a+b*arctan(c*x))+3/2*b^2*d^3*x*arctan(c*x)/c^3-122/105*I*b^2*d^
3*x/c^3-1/2*b*d^3*x^3*(a+b*arctan(c*x))/c+52/35*I*b*d^3*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/c^4+1/5*b*c*d^3*x^5*(a+b*arctan(c*x))+3/5*I*c*d^3*x^5*(a+b
*arctan(c*x))^2-209/140*d^3*(a+b*arctan(c*x))^2/c^4+1/4*d^3*x^4*(a+b*arcta
n(c*x))^2+122/105*I*b^2*d^3*arctan(c*x)/c^4-1/2*c^2*d^3*x^6*(a+b*arctan(c*
x))^2+26/35*I*b*d^3*x^2*(a+b*arctan(c*x))/c^2-1/7*I*c^3*d^3*x^7*(a+b*arcta
n(c*x))^2-11/10*b^2*d^3*ln(c^2*x^2+1)/c^4-26/35*b^2*d^3*polylog(2,1-2/(1+I
*c*x))/c^4
```

3.84.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.93

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^3\left(1464iab+504b^2+1890abcx-1464ib^2cx+936iabc^2x^2+441b^2c^2x^2-630abc^3x^3+176ib^2c^3x^3+315a\right.}{}$$

input
```
Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^3*((1464*I)*a*b + 504*b^2 + 1890*a*b*c*x - (1464*I)*b^2*c*x + (936*I)*a
*b*c^2*x^2 + 441*b^2*c^2*x^2 - 630*a*b*c^3*x^3 + (176*I)*b^2*c^3*x^3 + 315
*a^2*c^4*x^4 - (468*I)*a*b*c^4*x^4 - 63*b^2*c^4*x^4 + (756*I)*a^2*c^5*x^5
+ 252*a*b*c^5*x^5 - (12*I)*b^2*c^5*x^5 - 630*a^2*c^6*x^6 + (60*I)*a*b*c^6*
x^6 - (180*I)*a^2*c^7*x^7 + 9*b^2*(-I + c*x)^4*(-1 + (4*I)*c*x + 10*c^2*x^
2 - (20*I)*c^3*x^3)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*(244*I + 315*c*x +
(156*I)*c^2*x^2 - 105*c^3*x^3 - (78*I)*c^4*x^4 + 42*c^5*x^5 + (10*I)*c^6*x
^6) + 3*a*(-105 + 35*c^4*x^4 + (84*I)*c^5*x^5 - 70*c^6*x^6 - (20*I)*c^7*x^
7) + (312*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (936*I)*a*b*Log[1 + c^2*x
^2] - 1386*b^2*Log[1 + c^2*x^2] + 936*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x]
)]))/(1260*c^4)
```

3.84.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \ 5411$$

$$\int \left(-ic^3d^3x^6(a+b\arctan(cx))^2-3c^2d^3x^5(a+b\arctan(cx))^2+3icd^3x^4(a+b\arctan(cx))^2+d^3x^3(a+b\arctan(c$$

$$\Big\downarrow \ 2009$$

$$-\frac{209d^3(a+b\arctan(cx))^2}{140c^4}+\frac{52ibd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{35c^4}-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))^2-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))^2+\frac{1}{21}ibc^2d^3x^6(a+b\arctan(cx))+\frac{26ibd^3x^2(a+b\arctan(cx))}{35c^2}+\frac{3}{5}icd^3x^5(a+b\arctan(cx))^2+\frac{1}{5}bcd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))^2-\frac{13}{35}ibd^3x^4(a+b\arctan(cx))-\frac{bd^3x^3(a+b\arctan(cx))}{2c}+\frac{3abd^3x}{2c^3}+\frac{122ib^2d^3\arctan(cx)}{105c^4}+\frac{3b^2d^3x\arctan(cx)}{2c^3}-\frac{26b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{35c^4}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}-\frac{11b^2d^3\log(c^2x^2+1)}{10c^4}-\frac{1}{105}ib^2cd^3x^5+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4$$

input
```
Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(3*a*b*d^3*x)/(2*c^3) - (((122*I)/105)*b^2*d^3*x)/c^3 + (7*b^2*d^3*x^2)/(2
0*c^2) + (((44*I)/315)*b^2*d^3*x^3)/c - (b^2*d^3*x^4)/20 - (I/105)*b^2*c*d
^3*x^5 + (((122*I)/105)*b^2*d^3*ArcTan[c*x])/c^4 + (3*b^2*d^3*x*ArcTan[c*x
])/(2*c^3) + (((26*I)/35)*b*d^3*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d^3*x^3*
(a + b*ArcTan[c*x]))/(2*c) - ((13*I)/35)*b*d^3*x^4*(a + b*ArcTan[c*x]) + (
b*c*d^3*x^5*(a + b*ArcTan[c*x]))/5 + (I/21)*b*c^2*d^3*x^6*(a + b*ArcTan[c*
x]) - (209*d^3*(a + b*ArcTan[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTan[c
*x])^2)/4 + ((3*I)/5)*c*d^3*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^3*x^6*(a +
b*ArcTan[c*x])^2)/2 - (I/7)*c^3*d^3*x^7*(a + b*ArcTan[c*x])^2 + (((52*I)/3
5)*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (11*b^2*d^3*Log[1 +
 c^2*x^2])/(10*c^4) - (26*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(35*c^4)
```

3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.84.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.17

method	result
parts	$d^3a^2\left(-\frac{1}{7}ic^3x^7-\frac{1}{2}c^2x^6+\frac{3}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{b^2d^3\left(\frac{3cx\arctan(cx)}{2}-\frac{c^3x^3\arctan(cx)}{2}+\frac{c^5x^5\arctan(cx)}{5}-\frac{11\ln(c\ldots}{\ldots}\right.}{\ldots}$
derivativedivides	$\frac{d^3a^2\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b^2d^3\left(\frac{3cx\arctan(cx)}{2}-\frac{c^3x^3\arctan(cx)}{2}+\frac{c^5x^5\arctan(cx)}{5}-\frac{11\ln(c^2x^2+1)}{10}+\frac{7c^2x\ldots}{20}\right.}{\ldots}$
default	$\frac{d^3a^2\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+b^2d^3\left(\frac{3cx\arctan(cx)}{2}-\frac{c^3x^3\arctan(cx)}{2}+\frac{c^5x^5\arctan(cx)}{5}-\frac{11\ln(c^2x^2+1)}{10}+\frac{7c^2x\ldots}{20}\right.}{\ldots}$
risch	$-\frac{b^2d^3x^4}{20}+\frac{7b^2d^3x^2}{20c^2}-\frac{11b^2d^3\ln(c^2x^2+1)}{10c^4}+\frac{3ab\,d^3x}{2c^3}+\frac{77b^2d^3}{45c^4}+\frac{d^3x^4a^2}{4}-\frac{3d^3ba\arctan(cx)}{2c^4}-\frac{d^3ab\,x^3}{2c}-\ldots$

input
```
int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/7*I*c^3*x^7-1/2*c^2*x^6+3/5*I*c*x^5+1/4*x^4)+b^2*d^3/c^4*(3/2*
c*x*arctan(c*x)-1/2*c^3*x^3*arctan(c*x)+1/5*c^5*x^5*arctan(c*x)-11/10*ln(c
^2*x^2+1)+7/20*c^2*x^2-3/4*arctan(c*x)^2-1/20*c^4*x^4-13/35*dilog(-1/2*I*(
c*x+I))+1/4*c^4*x^4*arctan(c*x)^2+13/35*dilog(1/2*I*(c*x-I))-13/70*ln(c*x-
I)^2+13/70*ln(c*x+I)^2+13/35*ln(c*x-I)*ln(c^2*x^2+1)-13/35*ln(c*x-I)*ln(-1
/2*I*(c*x+I))-13/35*ln(c*x+I)*ln(c^2*x^2+1)+13/35*ln(c*x+I)*ln(1/2*I*(c*x-
I))-1/2*arctan(c*x)^2*c^6*x^6-13/35*I*arctan(c*x)*c^4*x^4+26/35*I*arctan(c
*x)*c^2*x^2+1/21*I*arctan(c*x)*c^6*x^6-1/7*I*arctan(c*x)^2*c^7*x^7+3/5*I*a
rctan(c*x)^2*c^5*x^5+122/105*I*arctan(c*x)-122/105*I*c*x-1/105*I*c^5*x^5+4
4/315*I*c^3*x^3-26/35*I*arctan(c*x)*ln(c^2*x^2+1))+2*a*d^3*b/c^4*(-1/7*I*a
rctan(c*x)*c^7*x^7-1/2*arctan(c*x)*c^6*x^6+3/5*I*arctan(c*x)*c^5*x^5+1/4*c
^4*x^4*arctan(c*x)+3/4*c*x+1/42*I*c^6*x^6+1/10*c^5*x^5-13/70*I*c^4*x^4-1/4
*c^3*x^3+13/35*I*c^2*x^2-13/35*I*ln(c^2*x^2+1)-3/4*arctan(c*x))
```

3.84.5 Fricas [F]

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/560*(20*I*b^2*c^3*d^3*x^7 + 70*b^2*c^2*d^3*x^6 - 84*I*b^2*c*d^3*x^5 - 35
*b^2*d^3*x^4)*log(-(c*x + I)/(c*x - I))^2 + integral(1/140*(-140*I*a^2*c^5
*d^3*x^8 - 420*a^2*c^4*d^3*x^7 + 280*I*a^2*c^3*d^3*x^6 - 280*a^2*c^2*d^3*x
^5 + 420*I*a^2*c*d^3*x^4 + 140*a^2*d^3*x^3 + (140*a*b*c^5*d^3*x^8 - 20*(21
*I*a*b + b^2)*c^4*d^3*x^7 - 70*(4*a*b - I*b^2)*c^3*d^3*x^6 - 28*(10*I*a*b
- 3*b^2)*c^2*d^3*x^5 - 35*(12*a*b + I*b^2)*c*d^3*x^4 + 140*I*a*b*d^3*x^3)*
log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.84.7 Maxima [F]

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/7*I*a^2*c^3*d^3*x^7 - 1/2*a^2*c^2*d^3*x^6 + 3/5*I*a^2*c*d^3*x^5 + 1/4*b
^2*d^3*x^4*arctan(c*x)^2 - 1/42*I*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*
c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*
x^4 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 1
5*arctan(c*x)/c^7))*a*b*c^2*d^3 + 3/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4
- 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^3 + 1/6*(3*x^4*arctan(c*x)
 - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^3 - 1/12*(2*c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)
^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^3 + 1/280*(-10*I*b^2*c^3*d^3*x^7 - 35*
b^2*c^2*d^3*x^6 + 42*I*b^2*c*d^3*x^5)*arctan(c*x)^2 + 1/280*(10*b^2*c^3*d^
3*x^7 - 35*I*b^2*c^2*d^3*x^6 - 42*b^2*c*d^3*x^5)*arctan(c*x)*log(c^2*x^2 +
 1) - 1/1120*(-10*I*b^2*c^3*d^3*x^7 - 35*b^2*c^2*d^3*x^6 + 42*I*b^2*c*d^3*
x^5)*log(c^2*x^2 + 1)^2 - I*integrate(1/560*(420*(b^2*c^5*d^3*x^8 - 2*b^2*
c^3*d^3*x^6 - 3*b^2*c*d^3*x^4)*arctan(c*x)^2 + 35*(b^2*c^5*d^3*x^8 - 2*b^2
*c^3*d^3*x^6 - 3*b^2*c*d^3*x^4)*log(c^2*x^2 + 1)^2 - 12*(15*b^2*c^4*d^3*x^
7 - 14*b^2*c^2*d^3*x^5)*arctan(c*x) + 2*(10*b^2*c^5*d^3*x^8 - 77*b^2*c^3*d
^3*x^6 - 210*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*arctan(c*x))*log(c^2*x^2
+ 1))/(c^2*x^2 + 1), x) - integrate(1/560*(1260*(b^2*c^4*d^3*x^7 + b^2*c^2
*d^3*x^5)*arctan(c*x)^2 + 105*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*log(c^2*
x^2 + 1)^2 + 4*(10*b^2*c^5*d^3*x^8 - 77*b^2*c^3*d^3*x^6)*arctan(c*x) + ...
```

3.84.8 Giac [F]

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.85 $\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx$

3.85.1 Optimal result

Integrand size = 25, antiderivative size = 402

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx = \frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61ib^2d^3x^2}{180c} - \frac{1}{10}b^2d^3x^3-\frac{1}{60}ib^2cd^3x^4-\frac{37b^2d^3\arctan(cx)}{30c^3} + \frac{11ib^2d^3x\arctan(cx)}{6c^2}-\frac{14bd^3x^2(a+b\arctan(cx))}{15c} - \frac{11}{18}ibd^3x^3(a+b\arctan(cx)) + \frac{3}{10}bcd^3x^4(a+b\arctan(cx)) + \frac{1}{15}ibc^2d^3x^5(a+b\arctan(cx)) - \frac{37id^3(a+b\arctan(cx))^2}{20c^3} + \frac{1}{3}d^3x^3(a+b\arctan(cx))^2 + \frac{3}{4}icd^3x^4(a+b\arctan(cx))^2 - \frac{3}{5}c^2d^3x^5(a+b\arctan(cx))^2 - \frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))^2 - \frac{28bd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^3} - \frac{113ib^2d^3\log(1+c^2x^2)}{90c^3} - \frac{14ib^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}$$

output
```
11/6*I*a*b*d^3*x/c^2+37/30*b^2*d^3*x/c^2-11/18*I*b*d^3*x^3*(a+b*arctan(c*x
))-1/10*b^2*d^3*x^3+61/180*I*b^2*d^3*x^2/c-37/30*b^2*d^3*arctan(c*x)/c^3-1
13/90*I*b^2*d^3*ln(c^2*x^2+1)/c^3-14/15*b*d^3*x^2*(a+b*arctan(c*x))/c-1/60
*I*b^2*c*d^3*x^4+3/10*b*c*d^3*x^4*(a+b*arctan(c*x))-14/15*I*b^2*d^3*polylo
g(2,1-2/(1+I*c*x))/c^3-37/20*I*d^3*(a+b*arctan(c*x))^2/c^3+1/3*d^3*x^3*(a+
b*arctan(c*x))^2+1/15*I*b*c^2*d^3*x^5*(a+b*arctan(c*x))-3/5*c^2*d^3*x^5*(a
+b*arctan(c*x))^2+11/6*I*b^2*d^3*x*arctan(c*x)/c^2-28/15*b*d^3*(a+b*arctan
(c*x))*ln(2/(1+I*c*x))/c^3-1/6*I*c^3*d^3*x^6*(a+b*arctan(c*x))^2+3/4*I*c*d
^3*x^4*(a+b*arctan(c*x))^2
```

3.85.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.92

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$= \frac{d^3\left(-162ab+64ib^2+330iabcx+222b^2cx-168abc^2x^2+61ib^2c^2x^2+60a^2c^3x^3-110iabc^3x^3-18b^2c^3x^3\right.}{}$$

input
```
Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^3*(-162*a*b + (64*I)*b^2 + (330*I)*a*b*c*x + 222*b^2*c*x - 168*a*b*c^2*
x^2 + (61*I)*b^2*c^2*x^2 + 60*a^2*c^3*x^3 - (110*I)*a*b*c^3*x^3 - 18*b^2*c
^3*x^3 + (135*I)*a^2*c^4*x^4 + 54*a*b*c^4*x^4 - (3*I)*b^2*c^4*x^4 - 108*a^
2*c^5*x^5 + (12*I)*a*b*c^5*x^5 - (30*I)*a^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(
I + 4*c*x - (10*I)*c^2*x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(-111 + (16
5*I)*c*x - 84*c^2*x^2 - (55*I)*c^3*x^3 + 27*c^4*x^4 + (6*I)*c^5*x^5) + 3*a
*(-55*I + 20*c^3*x^3 + (45*I)*c^4*x^4 - 36*c^5*x^5 - (10*I)*c^6*x^6) - 168
*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + 168*a*b*Log[1 + c^2*x^2] - (226*I)*b^
2*Log[1 + c^2*x^2] + (168*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(180
*c^3)
```

3.85.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \ 5411$$
$$\int \left(-ic^3d^3x^5(a+b\arctan(cx))^2-3c^2d^3x^4(a+b\arctan(cx))^2+3icd^3x^3(a+b\arctan(cx))^2+d^3x^2(a+b\arctan(\right.$$
$$\Big\downarrow \ 2009$$

$$-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))^2-\frac{37id^3(a+b\arctan(cx))^2}{20c^3}-\frac{28bd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{15c^3}-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))^2+\frac{1}{15}ibc^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))^2+\frac{3}{10}bcd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))^2-\frac{11}{18}ibd^3x^3(a+b\arctan(cx))-\frac{14bd^3x^2(a+b\arctan(cx))}{15c}+\frac{11iabd^3x}{6c^2}-\frac{37b^2d^3\arctan(cx)}{30c^3}+\frac{11ib^2d^3x\arctan(cx)}{6c^2}-\frac{14ib^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{15c^3}+\frac{37b^2d^3x}{30c^2}-\frac{113ib^2d^3\log(c^2x^2+1)}{90c^3}-\frac{1}{60}ib^2cd^3x^4+\frac{61ib^2d^3x^2}{180c}-\frac{1}{10}b^2d^3x^3$$

input
```
Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(((11*I)/6)*a*b*d^3*x)/c^2 + (37*b^2*d^3*x)/(30*c^2) + (((61*I)/180)*b^2*d
^3*x^2)/c - (b^2*d^3*x^3)/10 - (I/60)*b^2*c*d^3*x^4 - (37*b^2*d^3*ArcTan[c
*x])/(30*c^3) + (((11*I)/6)*b^2*d^3*x*ArcTan[c*x])/c^2 - (14*b*d^3*x^2*(a
+ b*ArcTan[c*x]))/(15*c) - ((11*I)/18)*b*d^3*x^3*(a + b*ArcTan[c*x]) + (3*
b*c*d^3*x^4*(a + b*ArcTan[c*x]))/10 + (I/15)*b*c^2*d^3*x^5*(a + b*ArcTan[c
*x]) - (((37*I)/20)*d^3*(a + b*ArcTan[c*x])^2)/c^3 + (d^3*x^3*(a + b*ArcTa
n[c*x])^2)/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^5*
(a + b*ArcTan[c*x])^2)/5 - (I/6)*c^3*d^3*x^6*(a + b*ArcTan[c*x])^2 - (28*b
*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((113*I)/90)*b^2*
d^3*Log[1 + c^2*x^2])/c^3 - (((14*I)/15)*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c
*x)])/c^3
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.85.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.21

method	result
parts	$d^3a^2\left(-\frac{1}{6}ic^3x^6-\frac{3}{5}c^2x^5+\frac{3}{4}ic\,x^4+\frac{1}{3}x^3\right)+\frac{b^2d^3\left(-\frac{113i\ln\left(c^2x^2+1\right)}{90}-\frac{3\arctan(cx)^2c^5x^5}{5}+\frac{i\arctan(cx)c^5x^5}{15}+\right.}{}$
derivativedivides	$\frac{d^3a^2\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+b^2d^3\left(-\frac{113i\ln\left(c^2x^2+1\right)}{90}-\frac{3\arctan(cx)^2c^5x^5}{5}+\frac{i\arctan(cx)c^5x^5}{15}+\frac{c^3x^3\arctan(cx}{3}\right.}{}$
default	$\frac{d^3a^2\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+b^2d^3\left(-\frac{113i\ln\left(c^2x^2+1\right)}{90}-\frac{3\arctan(cx)^2c^5x^5}{5}+\frac{i\arctan(cx)c^5x^5}{15}+\frac{c^3x^3\arctan(cx}{3}\right.}{}$
risch	$-\frac{b^2d^3x^3}{10}+\frac{37b^2d^3x}{30c^2}-\frac{37b^2d^3\arctan(cx)}{30c^3}+\frac{3abc\,d^3x^4}{10}-\frac{3a^2c^2d^3x^5}{5}-\frac{337ab\,d^3}{90c^3}+\frac{a^2d^3x^3}{3}+\frac{14ab\,d^3\ln(c^2x}{15c^3}$

input
```
int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/6*I*c^3*x^6-3/5*c^2*x^5+3/4*I*c*x^4+1/3*x^3)+b^2*d^3/c^3*(-113
/90*I*ln(c^2*x^2+1)-3/5*arctan(c*x)^2*c^5*x^5+1/15*I*arctan(c*x)*c^5*x^5+1
/3*c^3*x^3*arctan(c*x)^2-11/18*I*arctan(c*x)*c^3*x^3+11/6*I*arctan(c*x)*c*
x+3/10*c^4*x^4*arctan(c*x)+61/180*I*c^2*x^2-14/15*c^2*x^2*arctan(c*x)+14/1
5*arctan(c*x)*ln(c^2*x^2+1)+7/15*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(
c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)-7/15*I*(ln(c*x+I)*ln
(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*ln(c*x+I)
^2)-11/12*I*arctan(c*x)^2+3/4*I*arctan(c*x)^2*c^4*x^4+37/30*c*x-1/60*I*c^4
*x^4-1/10*c^3*x^3-1/6*I*arctan(c*x)^2*c^6*x^6-37/30*arctan(c*x))+2*a*d^3*b
/c^3*(-1/6*I*arctan(c*x)*c^6*x^6-3/5*c^5*x^5*arctan(c*x)+3/4*I*arctan(c*x)
*c^4*x^4+1/3*c^3*x^3*arctan(c*x)+11/12*I*c*x+1/30*I*c^5*x^5+3/20*c^4*x^4-1
1/36*I*c^3*x^3-7/15*c^2*x^2+7/15*ln(c^2*x^2+1)-11/12*I*arctan(c*x))
```

3.85.5 Fricas [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/240*(10*I*b^2*c^3*d^3*x^6 + 36*b^2*c^2*d^3*x^5 - 45*I*b^2*c*d^3*x^4 - 20
*b^2*d^3*x^3)*log(-(c*x + I)/(c*x - I))^2 + integral(1/60*(-60*I*a^2*c^5*d
^3*x^7 - 180*a^2*c^4*d^3*x^6 + 120*I*a^2*c^3*d^3*x^5 - 120*a^2*c^2*d^3*x^4
 + 180*I*a^2*c*d^3*x^3 + 60*a^2*d^3*x^2 + (60*a*b*c^5*d^3*x^7 - 10*(18*I*a
*b + b^2)*c^4*d^3*x^6 - 12*(10*a*b - 3*I*b^2)*c^3*d^3*x^5 - 15*(8*I*a*b -
3*b^2)*c^2*d^3*x^4 - 20*(9*a*b + I*b^2)*c*d^3*x^3 + 60*I*a*b*d^3*x^2)*log(
-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.85.7 Maxima [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/6*I*a^2*c^3*d^3*x^6 - 3/5*a^2*c^2*d^3*x^5 + 3/4*I*a^2*c*d^3*x^4 - 1/45*
I*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(
c*x)/c^7))*a*b*c^3*d^3 - 3/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^
4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3*x^3 + 1/2*I*(3*x^4*
arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d^3 + 1/3
*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^3 + 1/240*
(-10*I*b^2*c^3*d^3*x^6 - 36*b^2*c^2*d^3*x^5 + 45*I*b^2*c*d^3*x^4 + 20*b^2*
d^3*x^3)*arctan(c*x)^2 + 1/240*(10*b^2*c^3*d^3*x^6 - 36*I*b^2*c^2*d^3*x^5
- 45*b^2*c*d^3*x^4 + 20*I*b^2*d^3*x^3)*arctan(c*x)*log(c^2*x^2 + 1) - 1/96
0*(-10*I*b^2*c^3*d^3*x^6 - 36*b^2*c^2*d^3*x^5 + 45*I*b^2*c*d^3*x^4 + 20*b^
2*d^3*x^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/240*(180*(b^2*c^5*d^3*x^7 -
2*b^2*c^3*d^3*x^5 - 3*b^2*c*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^7 -
 2*b^2*c^3*d^3*x^5 - 3*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - 2*(46*b^2*c^4*d
^3*x^6 - 65*b^2*c^2*d^3*x^4)*arctan(c*x) + (10*b^2*c^5*d^3*x^7 - 81*b^2*c^
3*d^3*x^5 + 20*b^2*c*d^3*x^3 - 60*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 -
 b^2*d^3*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate
(1/240*(180*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*arctan(c
*x)^2 + 15*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*log(c^2*x
^2 + 1)^2 + 2*(10*b^2*c^5*d^3*x^7 - 81*b^2*c^3*d^3*x^5 + 20*b^2*c*d^3*x^3)
*arctan(c*x) + (46*b^2*c^4*d^3*x^6 - 65*b^2*c^2*d^3*x^4 + 60*(b^2*c^5*d...
```

3.85.8 Giac [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.86 $\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx$

3.86.1 Optimal result

Integrand size = 23, antiderivative size = 307

$$\begin{aligned}\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx = & -\frac{5abd^3x}{2c}+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2 \\ & -\frac{1}{30}ib^2cd^3x^3-\frac{13ib^2d^3\arctan(cx)}{10c^2} \\ & -\frac{5b^2d^3x\arctan(cx)}{2c}-\frac{6}{5}ibd^3x^2(a+b\arctan(cx)) \\ & +\frac{1}{2}bcd^3x^3(a+b\arctan(cx)) \\ & +\frac{1}{10}ibc^2d^3x^4(a+b\arctan(cx)) \\ & +\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4c^2} \\ & -\frac{d^3(1+icx)^5(a+b\arctan(cx))^2}{5c^2} \\ & -\frac{12ibd^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{5c^2} \\ & +\frac{3b^2d^3\log(1+c^2x^2)}{2c^2} \\ & -\frac{6b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{5c^2}\end{aligned}$$

output
```
-5/2*a*b*d^3*x/c+13/10*I*b^2*d^3*x/c-1/4*b^2*d^3*x^2-1/30*I*b^2*c*d^3*x^3-
13/10*I*b^2*d^3*arctan(c*x)/c^2-5/2*b^2*d^3*x*arctan(c*x)/c-6/5*I*b*d^3*x^
2*(a+b*arctan(c*x))+1/2*b*c*d^3*x^3*(a+b*arctan(c*x))+1/10*I*b*c^2*d^3*x^4
*(a+b*arctan(c*x))+1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/c^2-1/5*d^3*(1+
I*c*x)^5*(a+b*arctan(c*x))^2/c^2-12/5*I*b*d^3*(a+b*arctan(c*x))*ln(2/(1-I*
c*x))/c^2+3/2*b^2*d^3*ln(c^2*x^2+1)/c^2-6/5*b^2*d^3*polylog(2,1-2/(1-I*c*x
))/c^2
```

3.86.2 Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.06

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^3\left(-18iab-15b^2-150abcx+78ib^2cx+30a^2c^2x^2-72iabc^2x^2-15b^2c^2x^2+60ia^2c^3x^3+30abc^3x^3-2ib\right.}{}$$

input
```
Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^3*((-18*I)*a*b - 15*b^2 - 150*a*b*c*x + (78*I)*b^2*c*x + 30*a^2*c^2*x^2
 - (72*I)*a*b*c^2*x^2 - 15*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 30*a*b*c^3*x
^3 - (2*I)*b^2*c^3*x^3 - 45*a^2*c^4*x^4 + (6*I)*a*b*c^4*x^4 - (12*I)*a^2*c
^5*x^5 + 3*b^2*(1 - (4*I)*c*x)*(-I + c*x)^4*ArcTan[c*x]^2 + 6*b*ArcTan[c*x
]*(b*(-13*I - 25*c*x - (12*I)*c^2*x^2 + 5*c^3*x^3 + I*c^4*x^4) + a*(25 + 1
0*c^2*x^2 + (20*I)*c^3*x^3 - 15*c^4*x^4 - (4*I)*c^5*x^5) - (24*I)*b*Log[1
+ E^((2*I)*ArcTan[c*x])]) + (72*I)*a*b*Log[1 + c^2*x^2] + 90*b^2*Log[1 + c
^2*x^2] - 72*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^2)
```

3.86.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{i(d+icdx)^3(a+b\arctan(cx))^2}{c}-\frac{i(d+icdx)^4(a+b\arctan(cx))^2}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}ibc^2d^3x^4(a+b\arctan(cx))-\frac{d^3(1+icx)^5(a+b\arctan(cx))^2}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4c^2}-\frac{12ibd^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{5c^2}+\frac{1}{2}bcd^3x^3(a+b\arctan(cx))-\frac{6}{5}ibd^3x^2(a+b\arctan(cx))-\frac{5abd^3x}{2c}-\frac{13ib^2d^3\arctan(cx)}{10c^2}-\frac{5b^2d^3x\arctan(cx)}{2c}-\frac{6b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{5c^2}+\frac{3b^2d^3\log(c^2x^2+1)}{2c^2}-\frac{1}{30}ib^2cd^3x^3+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2$$

input
```
Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(-5*a*b*d^3*x)/(2*c) + (((13*I)/10)*b^2*d^3*x)/c - (b^2*d^3*x^2)/4 - (I/30
)*b^2*c*d^3*x^3 - (((13*I)/10)*b^2*d^3*ArcTan[c*x])/c^2 - (5*b^2*d^3*x*Arc
Tan[c*x])/(2*c) - ((6*I)/5)*b*d^3*x^2*(a + b*ArcTan[c*x]) + (b*c*d^3*x^3*(
a + b*ArcTan[c*x]))/2 + (I/10)*b*c^2*d^3*x^4*(a + b*ArcTan[c*x]) + (d^3*(1
 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*Arc
Tan[c*x])^2)/(5*c^2) - (((12*I)/5)*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*
c*x)])/c^2 + (3*b^2*d^3*Log[1 + c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2,
1 - 2/(1 - I*c*x)])/(5*c^2)
```

3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.86.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.47

method	result
parts	$d^3a^2\left(-\frac{1}{5}ic^3x^5-\frac{3}{4}c^2x^4+icx^3+\frac{1}{2}x^2\right)+\frac{b^2d^3\left(\frac{i\arctan(cx)c^4x^4}{10}-\frac{3c^4x^4\arctan(cx)^2}{4}+\frac{6i\arctan(cx)\ln\left(c^2x^2+\right.}{5}\right.}{}$
derivativedivides	$\frac{d^3a^2\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+b^2d^3\left(\frac{i\arctan(cx)c^4x^4}{10}-\frac{3c^4x^4\arctan(cx)^2}{4}+\frac{6i\arctan(cx)\ln\left(c^2x^2+1\right)}{5}+\frac{c^2x^2\arctan}{2}\right.}{}$
default	$\frac{d^3a^2\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+b^2d^3\left(\frac{i\arctan(cx)c^4x^4}{10}-\frac{3c^4x^4\arctan(cx)^2}{4}+\frac{6i\arctan(cx)\ln\left(c^2x^2+1\right)}{5}+\frac{c^2x^2\arctan}{2}\right.}{}$
risch	$-\frac{x^2d^3b^2}{4}+\frac{3b^2d^3\ln(c^2x^2+1)}{2c^2}-\frac{5ab\,d^3x}{2c}-\frac{19b^2d^3}{12c^2}+\frac{3d^3b^2\ln(-icx+1)x^2}{5}+\frac{abc\,d^3x^3}{2}-\frac{6ib\,d^3x^2a}{5}+\frac{a^2d^3x}{2}$

input
```
int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/5*I*c^3*x^5-3/4*c^2*x^4+I*c*x^3+1/2*x^2)+b^2*d^3/c^2*(1/10*I*a
rctan(c*x)*c^4*x^4-3/4*c^4*x^4*arctan(c*x)^2+6/5*I*arctan(c*x)*ln(c^2*x^2+
1)+1/2*c^2*x^2*arctan(c*x)^2-1/30*I*c^3*x^3-6/5*I*arctan(c*x)*c^2*x^2+1/2*
c^3*x^3*arctan(c*x)+I*arctan(c*x)^2*c^3*x^3+5/4*arctan(c*x)^2-5/2*c*x*arct
an(c*x)-3/5*ln(c*x-I)*ln(c^2*x^2+1)+3/5*ln(c*x+I)*ln(c^2*x^2+1)+3/5*ln(c*x
-I)*ln(-1/2*I*(c*x+I))+3/10*ln(c*x-I)^2-3/10*ln(c*x+I)^2-3/5*ln(c*x+I)*ln(
1/2*I*(c*x-I))+3/5*dilog(-1/2*I*(c*x+I))-3/5*dilog(1/2*I*(c*x-I))-13/10*I*
arctan(c*x)+13/10*I*c*x-1/4*c^2*x^2+3/2*ln(c^2*x^2+1)-1/5*I*arctan(c*x)^2*
c^5*x^5)+2*a*d^3*b/c^2*(-1/5*I*arctan(c*x)*c^5*x^5-3/4*c^4*x^4*arctan(c*x)
+I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-5/4*c*x+1/20*I*c^4*x^4+1/4*
c^3*x^3-3/5*I*c^2*x^2+3/5*I*ln(c^2*x^2+1)+5/4*arctan(c*x))
```

3.86.5 Fricas [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/80*(4*I*b^2*c^3*d^3*x^5 + 15*b^2*c^2*d^3*x^4 - 20*I*b^2*c*d^3*x^3 - 10*b
^2*d^3*x^2)*log(-(c*x + I)/(c*x - I))^2 + integral(1/20*(-20*I*a^2*c^5*d^3
*x^6 - 60*a^2*c^4*d^3*x^5 + 40*I*a^2*c^3*d^3*x^4 - 40*a^2*c^2*d^3*x^3 + 60
*I*a^2*c*d^3*x^2 + 20*a^2*d^3*x + (20*a*b*c^5*d^3*x^6 - 4*(15*I*a*b + b^2)
*c^4*d^3*x^5 - 5*(8*a*b - 3*I*b^2)*c^3*d^3*x^4 - 20*(2*I*a*b - b^2)*c^2*d^
3*x^3 - 10*(6*a*b + I*b^2)*c*d^3*x^2 + 20*I*a*b*d^3*x)*log(-(c*x + I)/(c*x
 - I)))/(c^2*x^2 + 1), x)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.86.7 Maxima [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)^3(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/5*I*a^2*c^3*d^3*x^5 - 3/4*a^2*c^2*d^3*x^4 - 1/10*I*(4*x^5*arctan(c*x) -
 c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^3*d^3 + I*a^2*c
*d^3*x^3 + 1/2*b^2*d^3*x^2*arctan(c*x)^2 - 1/2*(3*x^4*arctan(c*x) - c*((c^
2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c^2*d^3 + I*(2*x^3*arctan(c*x)
- c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d^3 + 1/2*a^2*d^3*x^2 + (x^2*a
rctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^3 - 1/2*(2*c*(x/c^2 - arct
an(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d^3
 + 1/80*(-4*I*b^2*c^3*d^3*x^5 - 15*b^2*c^2*d^3*x^4 + 20*I*b^2*c*d^3*x^3)*a
rctan(c*x)^2 + 1/80*(4*b^2*c^3*d^3*x^5 - 15*I*b^2*c^2*d^3*x^4 - 20*b^2*c*d
^3*x^3)*arctan(c*x)*log(c^2*x^2 + 1) - 1/320*(-4*I*b^2*c^3*d^3*x^5 - 15*b^
2*c^2*d^3*x^4 + 20*I*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/80*
(60*(b^2*c^5*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x)^2
+ 5*(b^2*c^5*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*log(c^2*x^2 +
1)^2 - 2*(19*b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3)*arctan(c*x) + (4*b^2*c^
5*d^3*x^6 - 35*b^2*c^3*d^3*x^4 - 60*(b^2*c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*ar
ctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate(1/80*(180*(b^2*
c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^4*d^3*x^5 + b^2*c
^2*d^3*x^3)*log(c^2*x^2 + 1)^2 + 2*(4*b^2*c^5*d^3*x^6 - 35*b^2*c^3*d^3*x^4
)*arctan(c*x) + (19*b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3 + 20*(b^2*c^5*d^3
*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x))*log(c^2*x^2 + ...
```

3.86.8 Giac [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.87 $\int (d+icdx)^3(a+b\arctan(cx))^2\,dx$

3.87.1 Optimal result

Integrand size = 22, antiderivative size = 226

$$\begin{aligned}\int (d+icdx)^3(a+b\arctan(cx))^2\,dx = &-\frac{7}{2}iabd^3x-b^2d^3x-\frac{1}{12}ib^2cd^3x^2+\frac{b^2d^3\arctan(cx)}{c}\\&-\frac{7}{2}ib^2d^3x\arctan(cx)+bcd^3x^2(a+b\arctan(cx))\\&+\frac{1}{6}ibc^2d^3x^3(a+b\arctan(cx))\\&-\frac{id^3(1+icx)^4(a+b\arctan(cx))^2}{4c}\\&+\frac{4bd^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c}\\&+\frac{11ib^2d^3\log(1+c^2x^2)}{6c}\\&-\frac{2ib^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}\end{aligned}$$

output
```
-7/2*I*a*b*d^3*x-b^2*d^3*x-1/12*I*b^2*c*d^3*x^2+b^2*d^3*arctan(c*x)/c-7/2*
I*b^2*d^3*x*arctan(c*x)+b*c*d^3*x^2*(a+b*arctan(c*x))+1/6*I*b*c^2*d^3*x^3*
(a+b*arctan(c*x))-1/4*I*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/c+4*b*d^3*(a+b
*arctan(c*x))*ln(2/(1-I*c*x))/c+11/6*I*b^2*d^3*ln(c^2*x^2+1)/c-2*I*b^2*d^3
*polylog(2,1-2/(1-I*c*x))/c
```

3.87.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.18

$$\int (d + icdx)^3(a + b\arctan(cx))^2\,dx =$$
$$-\frac{id^3\left(b^2 + 12ia^2cx + 42abcx - 12ib^2cx - 18a^2c^2x^2 + 12iabc^2x^2 + b^2c^2x^2 - 12ia^2c^3x^3 - 2abc^3x^3 + 3a^2c^4x\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/12*I)*d^3*(b^2 + (12*I)*a^2*c*x + 42*a*b*c*x - (12*I)*b^2*c*x - 18*a^
2*c^2*x^2 + (12*I)*a*b*c^2*x^2 + b^2*c^2*x^2 - (12*I)*a^2*c^3*x^3 - 2*a*b*
c^3*x^3 + 3*a^2*c^4*x^4 + 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*
x]*(b*(6*I + 21*c*x + (6*I)*c^2*x^2 - c^3*x^3) + 3*a*(-7 + (4*I)*c*x - 6*c
^2*x^2 - (4*I)*c^3*x^3 + c^4*x^4) + (24*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])
]) - (24*I)*a*b*Log[1 + c^2*x^2] - 22*b^2*Log[1 + c^2*x^2] + 24*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])]))/c
```

3.87.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + icdx)^3(a + b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5389}$$
$$\frac{ib\int \left(c^2x^2(a+b\arctan(cx))d^4 - 4icx(a+b\arctan(cx))d^4 - \frac{8i(i-cx)(a+b\arctan(cx))d^4}{c^2x^2+1} - 7(a+b\arctan(cx))d^4\right)dx}{2d} - \frac{id^3(1+icx)^4(a+b\arctan(cx))^2}{4c}$$
$$\Big\downarrow \text{2009}$$

$$\frac{ib\left(\frac{1}{3}c^2d^4x^3(a+b\arctan(cx))-2icd^4x^2(a+b\arctan(cx))-\frac{8id^4\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-7ad^4x-7bd^4x\arctan(\right.}{2d}$$

$$\frac{id^3(1+icx)^4(a+b\arctan(cx))^2}{4c}$$

input
```
Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/4*I)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/c + ((I/2)*b*(-7*a*d^4*
x + (2*I)*b*d^4*x - (b*c*d^4*x^2)/6 - ((2*I)*b*d^4*ArcTan[c*x])/c - 7*b*d^
4*x*ArcTan[c*x] - (2*I)*c*d^4*x^2*(a + b*ArcTan[c*x]) + (c^2*d^4*x^3*(a +
b*ArcTan[c*x]))/3 - ((8*I)*d^4*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c +
 (11*b*d^4*Log[1 + c^2*x^2])/(3*c) - (4*b*d^4*PolyLog[2, 1 - 2/(1 - I*c*x)
])/c))/d
```

3.87.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.87.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(204) = 408$.

Time = 1.52 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.82

method	result
derivativedivides	$-\frac{id^3a^2(icx+1)^4}{4}+b^2d^3\Bigg(-\frac{i\arctan(cx)^2c^4x^4}{4}-c^3x^3\arctan(cx)^2+\frac{3i\arctan(cx)^2c^2x^2}{2}+\arctan(cx)^2cx-\frac{i\arctan(cx)^2}{4}+\frac{i\big(-7c}{}$
default	$-\frac{id^3a^2(icx+1)^4}{4}+b^2d^3\Bigg(-\frac{i\arctan(cx)^2c^4x^4}{4}-c^3x^3\arctan(cx)^2+\frac{3i\arctan(cx)^2c^2x^2}{2}+\arctan(cx)^2cx-\frac{i\arctan(cx)^2}{4}+\frac{i\big(-7c}{}$
parts	$-\frac{id^3a^2(icx+1)^4}{4c}+\frac{b^2d^3\Bigg(-\frac{i\arctan(cx)^2c^4x^4}{4}-c^3x^3\arctan(cx)^2+\frac{3i\arctan(cx)^2c^2x^2}{2}+\arctan(cx)^2cx-\frac{i\arctan(cx)^2}{4}+\frac{i}{}}{}$
risch	$-b^2d^3x-\frac{2b\ln(c^2x^2+1)a\,d^3}{c}+\frac{47b^2d^3\arctan(cx)}{32c}+\frac{14ab\,d^3}{3c}+x\,d^3a^2-\frac{7iab\,d^3x}{2}+\frac{7d^3b^2\ln(-icx+1)x}{4}-$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/4*I*d^3*a^2*(1+I*c*x)^4+b^2*d^3*(-1/4*I*arctan(c*x)^2*c^4*x^4-c^3*
x^3*arctan(c*x)^2+3/2*I*arctan(c*x)^2*c^2*x^2+arctan(c*x)^2*c*x-1/4*I*arct
an(c*x)^2+1/2*I*(-7*c*x*arctan(c*x)+1/3*c^3*x^3*arctan(c*x)+4*I*arctan(c*x
)*ln(c^2*x^2+1)-2*I*arctan(c*x)*c^2*x^2+4*arctan(c*x)^2-2*ln(c*x-I)*ln(c^2
*x^2+1)+2*ln(c*x+I)*ln(c^2*x^2+1)+2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+ln(c*x-I)
^2-ln(c*x+I)^2-2*ln(c*x+I)*ln(1/2*I*(c*x-I))+2*dilog(-1/2*I*(c*x+I))-2*dil
og(1/2*I*(c*x-I))-2*I*arctan(c*x)-1/6*c^2*x^2+11/3*ln(c^2*x^2+1)+2*I*c*x))
+2*a*d^3*b*(-1/4*I*arctan(c*x)*c^4*x^4-c^3*x^3*arctan(c*x)+3/2*I*arctan(c*
x)*c^2*x^2+c*x*arctan(c*x)-1/4*I*arctan(c*x)+1/4*I*(-7*c*x+1/3*c^3*x^3-2*I
*c^2*x^2+4*I*ln(c^2*x^2+1)+8*arctan(c*x))))
```

3.87.5 Fricas [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/16*(I*b^2*c^3*d^3*x^4 + 4*b^2*c^2*d^3*x^3 - 6*I*b^2*c*d^3*x^2 - 4*b^2*d^
3*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12
*a^2*c^4*d^3*x^4 + 8*I*a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^
3*x + 4*a^2*d^3 + (4*a*b*c^5*d^3*x^5 + (-12*I*a*b - b^2)*c^4*d^3*x^4 - 4*(
2*a*b - I*b^2)*c^3*d^3*x^3 - 2*(4*I*a*b - 3*b^2)*c^2*d^3*x^2 - 4*(3*a*b +
I*b^2)*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

3.87.7 Maxima [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*I*a^2*c^3*d^3*x^4 - 4*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)*log(
c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 2*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c
*x)/(c^2*x^2 + 1), x) - a^2*c^2*d^3*x^3 - 36*b^2*c^4*d^3*integrate(1/16*x^
4*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^2*c^4*d^3*integrate(1/16*x^4*log(c
^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 5*b^2*c^4*d^3*integrate(1/16*x^4*log(c^2
*x^2 + 1)/(c^2*x^2 + 1), x) - 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x
)/c^4 + 3*arctan(c*x)/c^5))*a*b*c^3*d^3 + 8*b^2*c^3*d^3*integrate(1/16*x^3
*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 20*b^2*c^3*d^3*integrate
(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1), x) - (2*x^3*arctan(c*x) - c*(x^2/c^2
- log(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 + 3/2*I*a^2*c*d^3*x^2 - 24*b^2*c^2*d^
3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) - 2*b^2*c^2*d^3*integ
rate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 10*b^2*c^2*d^3*integr
ate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*I*(x^2*arctan(c*x) - c
*(x/c^2 - arctan(c*x)/c^3))*a*b*c*d^3 + 1/4*b^2*d^3*arctan(c*x)^3/c + 12*b
^2*c*d^3*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) -
 8*b^2*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^3*x +
b^2*d^3*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arcta
n(c*x) - log(c^2*x^2 + 1))*a*b*d^3/c + 1/16*(-I*b^2*c^3*d^3*x^4 - 4*b^2*c^
2*d^3*x^3 + 6*I*b^2*c*d^3*x^2 + 4*b^2*d^3*x)*arctan(c*x)^2 + 1/16*(b^2*c^3
*d^3*x^4 - 4*I*b^2*c^2*d^3*x^3 - 6*b^2*c*d^3*x^2 + 4*I*b^2*d^3*x)*arcta...
```

3.87.8 Giac [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

3.88 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx$

3.88.1 Optimal result

Integrand size = 25, antiderivative size = 385

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = 3abcd^3x - \frac{1}{3}ib^2cd^3x + \frac{1}{3}ib^2d^3\arctan(cx) + 3b^2cd^3x\arctan(cx) + \frac{1}{3}ibc^2d^3x^2(a+b\arctan(cx)) - \frac{29}{6}d^3(a+b\arctan(cx))^2 + 3icd^3x(a+b\arctan(cx))^2 - \frac{3}{2}c^2d^3x^2(a+b\arctan(cx))^2 - \frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))^2 + 2d^3(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + \frac{20}{3}ibd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) - \frac{3}{2}b^2d^3\log\left(1+c^2x^2\right) - \frac{10}{3}b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - \frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + \frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
3*a*b*c*d^3*x+1/3*I*b^2*d^3*arctan(c*x)-1/3*I*c^3*d^3*x^3*(a+b*arctan(c*x)
)^2+3*b^2*c*d^3*x*arctan(c*x)+1/3*I*b*c^2*d^3*x^2*(a+b*arctan(c*x))-29/6*d
^3*(a+b*arctan(c*x))^2-1/3*I*b^2*c*d^3*x-3/2*c^2*d^3*x^2*(a+b*arctan(c*x))
^2+I*b*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-2*d^3*(a+b*arctan(c
*x))^2*arctanh(-1+2/(1+I*c*x))+3*I*c*d^3*x*(a+b*arctan(c*x))^2-3/2*b^2*d^3
*ln(c^2*x^2+1)-10/3*b^2*d^3*polylog(2,1-2/(1+I*c*x))+20/3*I*b*d^3*(a+b*arc
tan(c*x))*ln(2/(1+I*c*x))-I*b*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x
))-1/2*b^2*d^3*polylog(3,1-2/(1+I*c*x))+1/2*b^2*d^3*polylog(3,-1+2/(1+I*c*
x))
```

3.88.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = -\frac{1}{24}id^3\big(b^2\pi^3 - 72a^2cx + 72iabcx + 8b^2cx - 36ia^2c^2x^2 - 8abc^2x^2 + 8a^2c^3x^3 - 72iab\arctan(cx) - 8b^2\arctan(cx) - 144abcx\arctan(cx) + 72ib^2cx\arctan(cx) - 72iabc^2x^2\arctan(cx) - 8b^2c^2x^2\arctan(cx) + 16abc^3x^3\arctan(cx) + 44ib^2\arctan(cx)^2 - 72b^2cx\arctan(cx)^2 - 36ib^2c^2x^2\arctan(cx)^2 + 8b^2c^3x^3\arctan(cx)^2 - 16b^2\arctan(cx)^3 + 24ib^2\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - 160b^2\arctan(cx)\log\left(1+e^{2i\arctan(cx)}\right) - 24ib^2\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + 24ia^2\log(cx) + 80ab\log\left(1+c^2x^2\right) - 36ib^2\log\left(1+c^2x^2\right) - 24b^2\arctan(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) - 8b^2(-10i+3\arctan(cx))\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) - 24ab\,\mathrm{PolyLog}(2,-icx) + 24ab\,\mathrm{PolyLog}(2,icx) + 12ib^2\,\mathrm{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - 12ib^2\,\mathrm{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\big)$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
(-1/24*I)*d^3*(b^2*Pi^3 - 72*a^2*c*x + (72*I)*a*b*c*x + 8*b^2*c*x - (36*I)
*a^2*c^2*x^2 - 8*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - (72*I)*a*b*ArcTan[c*x] - 8*
b^2*ArcTan[c*x] - 144*a*b*c*x*ArcTan[c*x] + (72*I)*b^2*c*x*ArcTan[c*x] - (
72*I)*a*b*c^2*x^2*ArcTan[c*x] - 8*b^2*c^2*x^2*ArcTan[c*x] + 16*a*b*c^3*x^3
*ArcTan[c*x] + (44*I)*b^2*ArcTan[c*x]^2 - 72*b^2*c*x*ArcTan[c*x]^2 - (36*I
)*b^2*c^2*x^2*ArcTan[c*x]^2 + 8*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*ArcTan[
c*x]^3 + (24*I)*b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 160*b^
2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*b^2*ArcTan[c*x]^2*Lo
g[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*a^2*Log[c*x] + 80*a*b*Log[1 + c^2*x^
2] - (36*I)*b^2*Log[1 + c^2*x^2] - 24*b^2*ArcTan[c*x]*PolyLog[2, E^((-2*I)
*ArcTan[c*x])] - 8*b^2*(-10*I + 3*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan
[c*x])] - 24*a*b*PolyLog[2, (-I)*c*x] + 24*a*b*PolyLog[2, I*c*x] + (12*I)*
b^2*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*PolyLog[3, -E^((2*I)*A
rcTan[c*x])])
```

3.88.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3x^2(a+b\arctan(cx))^2-3c^2d^3x(a+b\arctan(cx))^2+3icd^3(a+b\arctan(cx))^2+\frac{d^3(a+b\arctan(cx))^2}{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$2d^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))^2-\frac{3}{2}c^2d^3x^2(a+b\arctan(cx))^2+\frac{1}{3}ibc^2d^3x^2(a+b\arctan(cx))-ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd^3\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+3icd^3x(a+b\arctan(cx))^2-\frac{29}{6}d^3(a+b\arctan(cx))^2+\frac{20}{3}ibd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+3abcd^3x+\frac{1}{3}ib^2d^3\arctan(cx)+3b^2cd^3x\arctan(cx)-\frac{3}{2}b^2d^3\log\left(c^2x^2+1\right)-\frac{10}{3}b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)-\frac{1}{3}ib^2cd^3x$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
3*a*b*c*d^3*x - (I/3)*b^2*c*d^3*x + (I/3)*b^2*d^3*ArcTan[c*x] + 3*b^2*c*d^
3*x*ArcTan[c*x] + (I/3)*b*c^2*d^3*x^2*(a + b*ArcTan[c*x]) - (29*d^3*(a + b
*ArcTan[c*x])^2)/6 + (3*I)*c*d^3*x*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^2*
(a + b*ArcTan[c*x])^2)/2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x])^2 + 2*d^3
*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + ((20*I)/3)*b*d^3*(a +
b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (3*b^2*d^3*Log[1 + c^2*x^2])/2 - (10*b
^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/3 - I*b*d^3*(a + b*ArcTan[c*x])*Poly
Log[2, 1 - 2/(1 + I*c*x)] + I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/
(1 + I*c*x)] - (b^2*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^3*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/2
```

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.88.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.60 (sec) , antiderivative size = 1409, normalized size of antiderivative = 3.66

method	result	size
parts	Expression too large to display	1409
derivativedivides	Expression too large to display	1411
default	Expression too large to display	1411

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/3*I*c^3*x^3-3/2*c^2*x^2+3*I*c*x+ln(x))+b^2*d^3*(-1/2*polylog(3
,-(1+I*c*x)^2/(c^2*x^2+1))+2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*poly
log(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*c^2*x^2*arctan(c*x)^2+3*ln(1+(1+I*
c*x)^2/(c^2*x^2+1))+11/6*arctan(c*x)^2+I*arctan(c*x)*polylog(2,-(1+I*c*x)^
2/(c^2*x^2+1))+20/3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+20/3*dilog(1-I*
(1+I*c*x)/(c^2*x^2+1)^(1/2))+11/3*arctan(c*x)*(c*x-I)-arctan(c*x)^2*ln((1+
I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(c*x)+arctan(c*x)^2*ln(1-(1+I*c*x)
/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*ar
ctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2
,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*arctan(c*x)^2+1/2*I*Pi*csgn(I/(1+(
1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*I*Pi*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I
*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*I*Pi
*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1
)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^3*arctan(c*x)^2-1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x...
```

3.88.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x, x)
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x,x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
-1/3*I*a^2*c^3*d^3*x^3 - 36*I*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)^2
/(c^2*x^3 + x), x) - 12*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)*log(c^2
*x^2 + 1)/(c^2*x^3 + x), x) - 3*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x
^2 + 1)^2/(c^2*x^3 + x), x) - 96*I*a*b*c^5*d^3*integrate(1/48*x^5*arctan(c
*x)/(c^2*x^3 + x), x) - 8*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)/(c^2*
x^3 + x), x) - 4*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x^2 + 1)/(c^2*x^
3 + x), x) - 108*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^3 + x
), x) + 36*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(
c^2*x^3 + x), x) - 9*b^2*c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^
2*x^3 + x), x) - 288*a*b*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 +
 x), x) + 44*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 + x), x
) - 22*b^2*c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) -
 3/2*a^2*c^2*d^3*x^2 + 72*I*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)^2/(
c^2*x^3 + x), x) + 24*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)*log(c^2*x
^2 + 1)/(c^2*x^3 + x), x) + 6*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2
 + 1)^2/(c^2*x^3 + x), x) - 96*I*a*b*c^3*d^3*integrate(1/48*x^3*arctan(c*x
)/(c^2*x^3 + x), x) + 108*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)/(c^2*
x^3 + x), x) + 54*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*x
^3 + x), x) + 3/4*I*b^2*d^3*arctan(c*x)^3 - 72*b^2*c^2*d^3*integrate(1/48*
x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 24*I*b^2*c^2*d^3*integrate(1/48*x...
```

3.88.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
Timed out
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^3}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x, x)
```

3.89 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx$

3.89.1 Optimal result

Integrand size = 25, antiderivative size = 402

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = iabc^2d^3x+ib^2c^2d^3x\arctan(cx) - \frac{9}{2}icd^3(a+b\arctan(cx))^2 - \frac{d^3(a+b\arctan(cx))^2}{x} - 3c^2d^3x(a+b\arctan(cx))^2 - \frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))^2 + 6icd^3(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) - 6bcd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) - \frac{1}{2}ib^2cd^3\log\left(1+c^2x^2\right) + 2bcd^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - ib^2cd^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) - 3ib^2cd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + 3bcd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - 3bcd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - \frac{3}{2}ib^2cd^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + \frac{3}{2}ib^2cd^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-I*b^2*c*d^3*polylog(2,-1+2/(1-I*c*x))+3/2*I*b^2*c*d^3*polylog(3,-1+2/(1+I
*c*x))-9/2*I*c*d^3*(a+b*arctan(c*x))^2-d^3*(a+b*arctan(c*x))^2/x-3*c^2*d^3
*x*(a+b*arctan(c*x))^2-3*I*b^2*c*d^3*polylog(2,1-2/(1+I*c*x))-1/2*I*c^3*d^
3*x^2*(a+b*arctan(c*x))^2-6*b*c*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))-6*I*
c*d^3*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+2*b*c*d^3*(a+b*arctan(c*
x))*ln(2-2/(1-I*c*x))-3/2*I*b^2*c*d^3*polylog(3,1-2/(1+I*c*x))-1/2*I*b^2*c
*d^3*ln(c^2*x^2+1)+3*b*c*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))-3*
b*c*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+I*a*b*c^2*d^3*x+I*b^2*
c^2*d^3*x*arctan(c*x)
```

3.89.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.27

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{d^3\left(-8a^2+b^2c\pi^3x-24a^2c^2x^2+8iabc^2x^2-4ia^2c^3x^3-16ab\arctan(cx)-8iabcx\arctan(cx)-48abc^2x^2\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(d^3*(-8*a^2 + b^2*c*Pi^3*x - 24*a^2*c^2*x^2 + (8*I)*a*b*c^2*x^2 - (4*I)*a
^2*c^3*x^3 - 16*a*b*ArcTan[c*x] - (8*I)*a*b*c*x*ArcTan[c*x] - 48*a*b*c^2*x
^2*ArcTan[c*x] + (8*I)*b^2*c^2*x^2*ArcTan[c*x] - (8*I)*a*b*c^3*x^3*ArcTan[
c*x] - 8*b^2*ArcTan[c*x]^2 + (12*I)*b^2*c*x*ArcTan[c*x]^2 - 24*b^2*c^2*x^2
*ArcTan[c*x]^2 - (4*I)*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*c*x*ArcTan[c*x]^
3 + (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 16*b^2*
c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] - 48*b^2*c*x*ArcTan[c*x]*Lo
g[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 + E^((2*
I)*ArcTan[c*x])] + (24*I)*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c*x] + 16*a*b*c*
x*Log[1 + c^2*x^2] - (4*I)*b^2*c*x*Log[1 + c^2*x^2] - 24*b^2*c*x*ArcTan[c*
x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 24*b^2*c*x*(-I + ArcTan[c*x])*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - (8*I)*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[
c*x])] - 24*a*b*c*x*PolyLog[2, (-I)*c*x] + 24*a*b*c*x*PolyLog[2, I*c*x] +
(12*I)*b^2*c*x*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*c*x*PolyLog
[3, -E^((2*I)*ArcTan[c*x])]))/(8*x)
```

3.89.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3x(a+b\arctan(cx))^2-3c^2d^3(a+b\arctan(cx))^2+\frac{d^3(a+b\arctan(cx))^2}{x^2}+\frac{3icd^3(a+b\arctan(cx))^2}{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$6icd^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))^2-3c^2d^3x(a+b\arctan(cx))^2+3bcd^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-3bcd^3\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-\frac{9}{2}icd^3(a+b\arctan(cx))^2-\frac{d^3(a+b\arctan(cx))^2}{x}-6bcd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+2bcd^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+iabc^2d^3x+ib^2c^2d^3x\arctan(cx)-\frac{1}{2}ib^2cd^3\log\left(c^2x^2+1\right)-ib^2cd^3\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-3ib^2cd^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{3}{2}ib^2cd^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{3}{2}ib^2cd^3\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
I*a*b*c^2*d^3*x + I*b^2*c^2*d^3*x*ArcTan[c*x] - ((9*I)/2)*c*d^3*(a + b*Arc
Tan[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/x - 3*c^2*d^3*x*(a + b*ArcTan[c*
x])^2 - (I/2)*c^3*d^3*x^2*(a + b*ArcTan[c*x])^2 + (6*I)*c*d^3*(a + b*ArcTa
n[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - 6*b*c*d^3*(a + b*ArcTan[c*x])*Log[2
/(1 + I*c*x)] - (I/2)*b^2*c*d^3*Log[1 + c^2*x^2] + 2*b*c*d^3*(a + b*ArcTan
[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^3*PolyLog[2, -1 + 2/(1 - I*c*x)]
 - (3*I)*b^2*c*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + 3*b*c*d^3*(a + b*ArcTan
[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - 3*b*c*d^3*(a + b*ArcTan[c*x])*PolyL
og[2, -1 + 2/(1 + I*c*x)] - ((3*I)/2)*b^2*c*d^3*PolyLog[3, 1 - 2/(1 + I*c*
x)] + ((3*I)/2)*b^2*c*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

3.89. $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx$

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.89.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 32.12 (sec) , antiderivative size = 1437, normalized size of antiderivative = 3.57

method	result	size
parts	Expression too large to display	1437
derivativedivides	Expression too large to display	1439
default	Expression too large to display	1439

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/2*I*c^3*x^2-3*c^2*x+3*I*c*ln(x)-1/x)+b^2*d^3*c*(-3*arctan(c*x)
^2*c*x-3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-6*arctan(c*x)*ln(
1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))+6*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*arctan(c*x
)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*Pi*arctan(c*x)^2-3/2*Pi*csgn
(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+3/
2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-3/2*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-
1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+3/2*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+3/2*Pi*csgn(I/(1+(1+I*c*x)^
2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2
+1)))^2*arctan(c*x)^2-3/2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+3/2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-3/2*Pi*csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-arctan(c*x)^2/c/x
+2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*arctan(c*x)^2*c^2*x
^2+I*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^...
```

3.89.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**2,x)
```

output
```
Timed out
```

3.89.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-1/2*I*a^2*c^3*d^3*x^2 - 3*a^2*c^2*d^3*x - 3*(2*c*x*arctan(c*x) - log(c^2*
x^2 + 1))*a*b*c*d^3 + 3*I*a^2*c*d^3*log(x) - (c*(log(c^2*x^2 + 1) - log(x^
2)) + 2*arctan(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/96*(12*(-I*b^2*c^3*d^3*x^3
- 6*b^2*c^2*d^3*x^2 - 2*b^2*d^3)*arctan(c*x)^2 + 12*(b^2*c^3*d^3*x^3 - 6*I
*b^2*c^2*d^3*x^2 - 2*I*b^2*d^3)*arctan(c*x)*log(c^2*x^2 + 1) - 3*(-I*b^2*c
^3*d^3*x^3 - 6*b^2*c^2*d^3*x^2 - 2*b^2*d^3)*log(c^2*x^2 + 1)^2 - 2*I*(576*
b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 48*b^2*
c^5*d^3*integrate(1/16*x^5*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 1536*a
*b*c^5*d^3*integrate(1/16*x^5*arctan(c*x)/(c^2*x^4 + x^2), x) + 96*b^2*c^5
*d^3*integrate(1/16*x^5*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 576*b^2*c^4
*d^3*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) -
 1344*b^2*c^4*d^3*integrate(1/16*x^4*arctan(c*x)/(c^2*x^4 + x^2), x) - 115
2*b^2*c^3*d^3*integrate(1/16*x^3*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 3072*
a*b*c^3*d^3*integrate(1/16*x^3*arctan(c*x)/(c^2*x^4 + x^2), x) - b^2*c*d^3
*log(c^2*x^2 + 1)^3 - 12*b^2*c*d^3*arctan(c*x)^2 - 384*b^2*c^2*d^3*integra
te(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 9*b^2*c*d^3
*log(c^2*x^2 + 1)^2 - 1728*b^2*c*d^3*integrate(1/16*x*arctan(c*x)^2/(c^2*x
^4 + x^2), x) - 144*b^2*c*d^3*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^4
 + x^2), x) - 4608*a*b*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2),
 x) - 192*b^2*c*d^3*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), ...
```

3.89.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^2, x)
```

3.90 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx$

3.90.1 Optimal result

Integrand size = 25, antiderivative size = 416

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bcd^3(a+b\arctan(cx))}{x}+\frac{7}{2}c^2d^3(a+b\arctan(cx))^2 - \frac{d^3(a+b\arctan(cx))^2}{2x^2}-\frac{3icd^3(a+b\arctan(cx))^2}{x} - ic^3d^3x(a+b\arctan(cx))^2 - 6c^2d^3(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + b^2c^2d^3\log(x) - 2ibc^2d^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) - \frac{1}{2}b^2c^2d^3\log\left(1+c^2x^2\right) + 6ibc^2d^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) + 3b^2c^2d^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) + b^2c^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + 3ibc^2d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - 3ibc^2d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)+\frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) - \frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-b*c*d^3*(a+b*arctan(c*x))/x+7/2*c^2*d^3*(a+b*arctan(c*x))^2-1/2*d^3*(a+b*
arctan(c*x))^2/x^2-2*I*b*c^2*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))+3*I*b*c
^2*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+6*c^2*d^3*(a+b*arctan(c*
x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d^3*ln(x)+6*I*b*c^2*d^3*(a+b*arctan(
c*x))*ln(2-2/(1-I*c*x))-1/2*b^2*c^2*d^3*ln(c^2*x^2+1)-I*c^3*d^3*x*(a+b*arc
tan(c*x))^2+3*b^2*c^2*d^3*polylog(2,-1+2/(1-I*c*x))+b^2*c^2*d^3*polylog(2,
1-2/(1+I*c*x))-3*I*c*d^3*(a+b*arctan(c*x))^2/x-3*I*b*c^2*d^3*(a+b*arctan(c
*x))*polylog(2,-1+2/(1+I*c*x))+3/2*b^2*c^2*d^3*polylog(3,1-2/(1+I*c*x))-3/
2*b^2*c^2*d^3*polylog(3,-1+2/(1+I*c*x))
```

3.90.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.20

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx$$
$$= \frac{1}{2}d^3\left(-\frac{a^2}{x^2}-\frac{6ia^2c}{x}-2ia^2c^3x-\frac{2ab(\arctan(cx)+cx(1+cx\arctan(cx)))}{x^2}-6a^2c^2\log(x)\right.$$
$$-\frac{b^2\left(2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{x^2}$$
$$-2iabc^2\left(2cx\arctan(cx)-\log\left(1+c^2x^2\right)\right)$$
$$-\frac{6iabc(2\arctan(cx)+cx(-2\log(cx)+\log(1+c^2x^2)))}{x}$$
$$-2ib^2c^2\left(\arctan(cx)\left((-i+cx)\arctan(cx)+2\log\left(1+e^{2i\arctan(cx)}\right)\right)\right.$$
$$\left.-i\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right)$$
$$+\frac{6b^2c\left(\arctan(cx)\left((-i+cx)\arctan(cx)+2icx\log\left(1-e^{2i\arctan(cx)}\right)\right)+cx\,\mathrm{PolyLog}\left(2,e^{2i\arctan(cx)}\right)\right)}{x}$$
$$-6iabc^2(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx))+6b^2c^2\left(\frac{i\pi^3}{24}-\frac{2}{3}i\arctan(cx)^3\right.$$
$$-\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)+\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)$$
$$-i\arctan(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)-i\arctan(cx)\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)$$
$$\left.\left.-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)\right)$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
(d^3*(-(a^2/x^2) - ((6*I)*a^2*c)/x - (2*I)*a^2*c^3*x - (2*a*b*(ArcTan[c*x]
 + c*x*(1 + c*x*ArcTan[c*x])))/x^2 - 6*a^2*c^2*Log[x] - (b^2*(2*c*x*ArcTan
[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2
]]))/x^2 - (2*I)*a*b*c^2*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - ((6*I)*a
*b*c*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])))/x - (2*I)*b^2
*c^2*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x]
)]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (6*b^2*c*(ArcTan[c*x]*((-I +
 c*x)*ArcTan[c*x] + (2*I)*c*x*Log[1 - E^((2*I)*ArcTan[c*x])]) + c*x*PolyLo
g[2, E^((2*I)*ArcTan[c*x])]))/x - (6*I)*a*b*c^2*(PolyLog[2, (-I)*c*x] - Po
lyLog[2, I*c*x]) + 6*b^2*c^2*((I/24)*Pi^3 - ((2*I)/3)*ArcTan[c*x]^3 - ArcT
an[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + ArcTan[c*x]^2*Log[1 + E^((2*I)
*ArcTan[c*x])] - I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - I*ArcT
an[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - PolyLog[3, E^((-2*I)*ArcTan[c
*x])]/2 + PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2)))/2
```

3.90.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(-ic^3d^3(a+b\arctan(cx))^2-\frac{3c^2d^3(a+b\arctan(cx))^2}{x}+\frac{d^3(a+b\arctan(cx))^2}{x^3}+\frac{3icd^3(a+b\arctan(cx))^2}{x^2}\right) d$$

$$\Big\downarrow \; 2009$$

$$-6c^2d^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-ic^3d^3x(a+b\arctan(cx))^2+$$
$$3ibc^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-3ibc^2d^3\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+$$
$$b\arctan(cx))+\frac{7}{2}c^2d^3(a+b\arctan(cx))^2-2ibc^2d^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+$$
$$6ibc^2d^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{d^3(a+b\arctan(cx))^2}{2x^2}-\frac{3icd^3(a+b\arctan(cx))^2}{x}-$$
$$\frac{bcd^3(a+b\arctan(cx))}{x}+3b^2c^2d^3\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+b^2c^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)+$$
$$\frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)-\frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)-\frac{1}{2}b^2c^2d^3\log\left(c^2x^2+1\right)+$$
$$b^2c^2d^3\log(x)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d^3*(a + b*ArcTan[c*x]))/x) + (7*c^2*d^3*(a + b*ArcTan[c*x])^2)/2 -
 (d^3*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((3*I)*c*d^3*(a + b*ArcTan[c*x])^2)
/x - I*c^3*d^3*x*(a + b*ArcTan[c*x])^2 - 6*c^2*d^3*(a + b*ArcTan[c*x])^2*A
rcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^3*Log[x] - (2*I)*b*c^2*d^3*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*c^2*d^3*Log[1 + c^2*x^2])/2 + (6*I)*b
*c^2*d^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] + 3*b^2*c^2*d^3*PolyLo
g[2, -1 + 2/(1 - I*c*x)] + b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + (3*
I)*b*c^2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - (3*I)*b*c
^2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (3*b^2*c^2*d^3
*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 - (3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 +
I*c*x)])/2
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.90.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 29.44 (sec) , antiderivative size = 1523, normalized size of antiderivative = 3.66

method	result	size
derivativedivides	Expression too large to display	1523
default	Expression too large to display	1523
parts	Expression too large to display	1523

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^3*a^2*(-I*c*x-3*ln(c*x)-3*I/c/x-1/2/c^2/x^2)+b^2*d^3*(-1/2/c^2/x^2*
arctan(c*x)^2+3/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-6*polylog(3,(1+I*c*x
)/(c^2*x^2+1)^(1/2))+ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*polylog(3,-(1+I*c
*x)/(c^2*x^2+1)^(1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+3/2*arctan(c*x)^2
-3*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*dilog(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*arctan(c*x)
^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-3*arctan(c*x)^2*ln(c*x)-3*arctan(c*x)^2*l
n(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1
)^(1/2))+6*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*dilog((1+I*c*x)/(c^2*x^2
+1)^(1/2))-1/2*arctan(c*x)*(I*c*x-(c^2*x^2+1)^(1/2)+1)/c/x-1/2*arctan(c*x)
*(I*c*x+(c^2*x^2+1)^(1/2)+1)/c/x-3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1
)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+6*I*arctan(c*x)*polylog(2,(1+I
*c*x)/(c^2*x^2+1)^(1/2))+6*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^
(1/2))-3/2*I*Pi*arctan(c*x)^2-2*I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))-2*I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*arctan(c*x
)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-3/2*I*Pi*csgn(((1+I*c*x
)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+3/2*I*Pi*c
sgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c...
```

3.90.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Timed out
```

3.90.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-I*a^2*c^3*d^3*x - I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*c^2*d^3 -
3*a^2*c^2*d^3*log(x) - 3*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x
)/x)*a*b*c*d^3 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^3 - 3*I
*a^2*c*d^3/x - 1/2*a^2*d^3/x^2 - 1/32*(16*I*(24*b^2*c^5*d^3*integrate(1/16
*x^5*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 2*b^2*c^5*d^3*integrate(1/16*x^5*
log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 8*b^2*c^5*d^3*integrate(1/16*x^5*
log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - b^2*c^2*d^3*arctan(c*x)^3 - 24*b^2*
c^4*d^3*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x
) - 16*b^2*c^4*d^3*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) - 4*
b^2*c^3*d^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 24
*b^2*c^3*d^3*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 16*
b^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3
), x) - 56*b^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x)
- 72*b^2*c*d^3*integrate(1/16*x*arctan(c*x)^2/(c^2*x^5 + x^3), x) - 6*b^2*
c*d^3*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 4*b^2*c*d^
3*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 8*b^2*d^3*integr
ate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x))*x^2 + (128*b^2*
c^5*d^3*integrate(1/16*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x
) + 256*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)/(c^2*x^5 + x^3), x) + 1
152*b^2*c^4*d^3*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + ...
```

3.90.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^3, x)
```

3.91 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx$

3.91.1 Optimal result

Integrand size = 25, antiderivative size = 429

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = &-\frac{b^2c^2d^3}{3x}-\frac{1}{3}b^2c^3d^3\arctan(cx)\\&-\frac{bcd^3(a+b\arctan(cx))}{3x^2}-\frac{3ibc^2d^3(a+b\arctan(cx))}{x}\\&+\frac{11}{6}ic^3d^3(a+b\arctan(cx))^2\\&-\frac{d^3(a+b\arctan(cx))^2}{3x^3}-\frac{3icd^3(a+b\arctan(cx))^2}{2x^2}\\&+\frac{3c^2d^3(a+b\arctan(cx))^2}{x}\\&-2ic^3d^3(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\&+3ib^2c^3d^3\log(x)-\frac{3}{2}ib^2c^3d^3\log\left(1+c^2x^2\right)\\&-\frac{20}{3}bc^3d^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\&+\frac{10}{3}ib^2c^3d^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\&-bc^3d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\&+bc^3d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\&+\frac{1}{2}ib^2c^3d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\&-\frac{1}{2}ib^2c^3d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d^3/x-1/3*b^2*c^3*d^3*arctan(c*x)-1/3*b*c*d^3*(a+b*arctan(c*x
))/x^2-3*I*b*c^2*d^3*(a+b*arctan(c*x))/x+10/3*I*b^2*c^3*d^3*polylog(2,-1+2
/(1-I*c*x))-1/3*d^3*(a+b*arctan(c*x))^2/x^3+11/6*I*c^3*d^3*(a+b*arctan(c*x
))^2+3*c^2*d^3*(a+b*arctan(c*x))^2/x+2*I*c^3*d^3*(a+b*arctan(c*x))^2*arcta
nh(-1+2/(1+I*c*x))-1/2*I*b^2*c^3*d^3*polylog(3,-1+2/(1+I*c*x))-3/2*I*b^2*c
^3*d^3*ln(c^2*x^2+1)-20/3*b*c^3*d^3*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-3/
2*I*c*d^3*(a+b*arctan(c*x))^2/x^2-b*c^3*d^3*(a+b*arctan(c*x))*polylog(2,1-
2/(1+I*c*x))+b*c^3*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+3*I*b^2
*c^3*d^3*ln(x)+1/2*I*b^2*c^3*d^3*polylog(3,1-2/(1+I*c*x))
```

3.91.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.39

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{d^3\left(-8a^2-36ia^2cx-8abcx+72a^2c^2x^2-72iabc^2x^2-8b^2c^2x^2-b^2c^3\pi^3x^3-16ab\arctan(cx)-72iabcx\,\text{a}\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
(d^3*(-8*a^2 - (36*I)*a^2*c*x - 8*a*b*c*x + 72*a^2*c^2*x^2 - (72*I)*a*b*c^
2*x^2 - 8*b^2*c^2*x^2 - b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTan[c*x] - (72*I)*a*b
*c*x*ArcTan[c*x] - 8*b^2*c*x*ArcTan[c*x] + 144*a*b*c^2*x^2*ArcTan[c*x] - (
72*I)*b^2*c^2*x^2*ArcTan[c*x] - (72*I)*a*b*c^3*x^3*ArcTan[c*x] - 8*b^2*c^3
*x^3*ArcTan[c*x] - 8*b^2*ArcTan[c*x]^2 - (36*I)*b^2*c*x*ArcTan[c*x]^2 + 72
*b^2*c^2*x^2*ArcTan[c*x]^2 + (44*I)*b^2*c^3*x^3*ArcTan[c*x]^2 + 16*b^2*c^3
*x^3*ArcTan[c*x]^3 - (24*I)*b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 - E^((-2*I)*Ar
cTan[c*x])] - 160*b^2*c^3*x^3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] +
 (24*I)*b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*
a^2*c^3*x^3*Log[x] - 160*a*b*c^3*x^3*Log[c*x] + (72*I)*b^2*c^3*x^3*Log[(c*
x)/Sqrt[1 + c^2*x^2]] + 80*a*b*c^3*x^3*Log[1 + c^2*x^2] + 24*b^2*c^3*x^3*A
rcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 24*b^2*c^3*x^3*ArcTan[c*x]
*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (80*I)*b^2*c^3*x^3*PolyLog[2, E^((2*
I)*ArcTan[c*x])] + 24*a*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 24*a*b*c^3*x^3*Po
lyLog[2, I*c*x] - (12*I)*b^2*c^3*x^3*PolyLog[3, E^((-2*I)*ArcTan[c*x])] +
(12*I)*b^2*c^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(24*x^3)
```

3.91.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx$$
$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{ic^3d^3(a+b\arctan(cx))^2}{x}-\frac{3c^2d^3(a+b\arctan(cx))^2}{x^2}+\frac{d^3(a+b\arctan(cx))^2}{x^4}+\frac{3icd^3(a+b\arctan(cx))^2}{x^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2ic^3d^3\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-bc^3d^3\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+bc^3d^3\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{11}{6}ic^3d^3(a+b\arctan(cx))^2-\frac{20}{3}bc^3d^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+\frac{3c^2d^3(a+b\arctan(cx))^2}{x}-\frac{3ibc^2d^3(a+b\arctan(cx))}{x}-\frac{d^3(a+b\arctan(cx))^2}{3x^3}-\frac{3icd^3(a+b\arctan(cx))^2}{2x^2}-\frac{bcd^3(a+b\arctan(cx))}{3x^2}-\frac{1}{3}b^2c^3d^3\arctan(cx)+\frac{10}{3}ib^2c^3d^3\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{1}{2}ib^2c^3d^3\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)-\frac{1}{2}ib^2c^3d^3\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)+3ib^2c^3d^3\log(x)-\frac{b^2c^2d^3}{3x}-\frac{3}{2}ib^2c^3d^3\log\left(c^2x^2+1\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
-1/3*(b^2*c^2*d^3)/x - (b^2*c^3*d^3*ArcTan[c*x])/3 - (b*c*d^3*(a + b*ArcTa
n[c*x]))/(3*x^2) - ((3*I)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x + ((11*I)/6)*c^
3*d^3*(a + b*ArcTan[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (((3*I
)/2)*c*d^3*(a + b*ArcTan[c*x])^2)/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/
x - (2*I)*c^3*d^3*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (3*I)
*b^2*c^3*d^3*Log[x] - ((3*I)/2)*b^2*c^3*d^3*Log[1 + c^2*x^2] - (20*b*c^3*d
^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + ((10*I)/3)*b^2*c^3*d^3*
PolyLog[2, -1 + 2/(1 - I*c*x)] - b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 + I*c*x)] + b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 +
I*c*x)] + (I/2)*b^2*c^3*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] - (I/2)*b^2*c^3*
d^3*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

3.91. $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx$

3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.91.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.32 (sec) , antiderivative size = 1655, normalized size of antiderivative = 3.86

method	result	size
derivativedivides	Expression too large to display	1655
default	Expression too large to display	1655
parts	Expression too large to display	1727

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(8/3*b^2*d^3*arctan(c*x)+1/2*b^2*d^3*Pi*arctan(c*x)^2+b^2*d^3*arctan(c
*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3*b^2*d^3*arctan(c*x)^2/c/x+1/2*b^
2*d^3*Pi*arctan(c*x)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
-2*b^2*d^3*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*b^2*d^3*ar
ctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-20/3*b^2*d^3*arctan(c*x)
*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*a*d^3*b*(-1/3*arctan(c*x)/c^3/x^3-I*a
rctan(c*x)*ln(c*x)+3/c/x*arctan(c*x)-3/2*I*arctan(c*x)/c^2/x^2+1/2*ln(c*x)
*ln(1+I*c*x)-1/2*ln(c*x)*ln(1-I*c*x)+1/2*dilog(1+I*c*x)-1/2*dilog(1-I*c*x)
-3/2*I/c/x-1/6/c^2/x^2-10/3*ln(c*x)+5/3*ln(c^2*x^2+1)-3/2*I*arctan(c*x))+1
1/6*I*b^2*d^3*arctan(c*x)^2+1/2*I*b^2*d^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+
1))-2*I*b^2*d^3*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*b^2*d^3*polylo
g(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+20/3*I*b^2*d^3*dilog(1+(1+I*c*x)/(c^2*x^2
+1)^(1/2))-20/3*I*b^2*d^3*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*b^2*d^3*l
n(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*b^2*d^3*ln((1+I*c*x)/(c^2*x^2+1)^(1/2
)-1)+d^3*a^2*(-1/3/c^3/x^3-I*ln(c*x)+3/c/x-3/2*I/c^2/x^2)+1/2*b^2*d^3*Pi*a
rctan(c*x)^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))
+1/2*b^2*d^3*Pi*arctan(c*x)^2*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x
)^2/(c^2*x^2+1)))^3-1/2*b^2*d^3*Pi*arctan(c*x)^2*csgn(((1+I*c*x)^2/(c^2...
```

3.91.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**4,x)
```

output
```
Timed out
```

3.91.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-I*a^2*c^3*d^3*log(x) + 3*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)
/x)*a*b*c^2*d^3 - 3*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d^
3 + 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x
^3)*a*b*d^3 + 3*a^2*c^2*d^3/x - 3/2*I*a^2*c*d^3/x^2 - 1/3*a^2*d^3/x^3 - 1/
96*(24*(3*b^2*c^3*d^3*arctan(c*x)^3 + 48*b^2*c^5*d^3*integrate(1/48*x^5*ar
ctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) + 36*b^2*c^4*d^3*integrate(
1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) - 144*b^2*c^4*d^3*integrat
e(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 96*b^2*c^3*d^3*integrate
(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) + 432*b^2*c^3*d
^3*integrate(1/48*x^3*arctan(c*x)/(c^2*x^6 + x^4), x) + 288*b^2*c^2*d^3*in
tegrate(1/48*x^2*arctan(c*x)^2/(c^2*x^6 + x^4), x) + 24*b^2*c^2*d^3*integr
ate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) + 88*b^2*c^2*d^3*integ
rate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 144*b^2*c*d^3*integra
te(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 32*b^2*c*d^3*
integrate(1/48*x*arctan(c*x)/(c^2*x^6 + x^4), x) - 144*b^2*d^3*integrate(1
/48*arctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^2*d^3*integrate(1/48*log(c^2*
x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 + I*(3456*b^2*c^5*d^3*integrate(1/48*x
^5*arctan(c*x)^2/(c^2*x^6 + x^4), x) + 9216*a*b*c^5*d^3*integrate(1/48*x^5
*arctan(c*x)/(c^2*x^6 + x^4), x) + b^2*c^3*d^3*log(c^2*x^2 + 1)^3 + 72*b^2
*c^3*d^3*arctan(c*x)^2 - 3456*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x...
```

3.91.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^4,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^4, x)
```

3.92 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx$

3.92.1 Optimal result

Integrand size = 25, antiderivative size = 293

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = & -\frac{b^2c^2d^3}{12x^2}-\frac{ib^2c^3d^3}{x}-ib^2c^4d^3\arctan(cx) \\ & -\frac{bcd^3(a+b\arctan(cx))}{6x^3}-\frac{ibc^2d^3(a+b\arctan(cx))}{x^2} \\ & +\frac{7bc^3d^3(a+b\arctan(cx))}{2x} \\ & -\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4x^4} \\ & -4iabc^4d^3\log(x)-\frac{11}{3}b^2c^4d^3\log(x) \\ & -4ibc^4d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right) \\ & +\frac{11}{6}b^2c^4d^3\log\left(1+c^2x^2\right)+2b^2c^4d^3\operatorname{PolyLog}(2,-icx) \\ & -2b^2c^4d^3\operatorname{PolyLog}(2,icx) \\ & -2b^2c^4d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/12*b^2*c^2*d^3/x^2-I*b^2*c^3*d^3/x-I*b^2*c^4*d^3*arctan(c*x)-1/6*b*c*d^
3*(a+b*arctan(c*x))/x^3-I*b*c^2*d^3*(a+b*arctan(c*x))/x^2+7/2*b*c^3*d^3*(a
+b*arctan(c*x))/x-1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/x^4-4*I*a*b*c^4*
d^3*ln(x)-11/3*b^2*c^4*d^3*ln(x)-4*I*b*c^4*d^3*(a+b*arctan(c*x))*ln(2/(1-I
*c*x))+11/6*b^2*c^4*d^3*ln(c^2*x^2+1)+2*b^2*c^4*d^3*polylog(2,-I*c*x)-2*b^
2*c^4*d^3*polylog(2,I*c*x)-2*b^2*c^4*d^3*polylog(2,1-2/(1-I*c*x))
```

3.92.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.10

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx$$
$$= \frac{d^3\left(-3a^2-12ia^2cx-2abcx+18a^2c^2x^2-12iabc^2x^2-b^2c^2x^2+12ia^2c^3x^3+42abc^3x^3-12ib^2c^3x^3-b^2c^4\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```

output
```
(d^3*(-3*a^2 - (12*I)*a^2*c*x - 2*a*b*c*x + 18*a^2*c^2*x^2 - (12*I)*a*b*c^
2*x^2 - b^2*c^2*x^2 + (12*I)*a^2*c^3*x^3 + 42*a*b*c^3*x^3 - (12*I)*b^2*c^3
*x^3 - b^2*c^4*x^4 - 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b
*c*x*(-1 - (6*I)*c*x + 21*c^2*x^2 - (6*I)*c^3*x^3) + 3*a*(-1 - (4*I)*c*x +
 6*c^2*x^2 + (4*I)*c^3*x^3 + 7*c^4*x^4) - (24*I)*b*c^4*x^4*Log[1 - E^((2*I
)*ArcTan[c*x])]) - (48*I)*a*b*c^4*x^4*Log[c*x] - 44*b^2*c^4*x^4*Log[(c*x)/
Sqrt[1 + c^2*x^2]] + (24*I)*a*b*c^4*x^4*Log[1 + c^2*x^2] - 24*b^2*c^4*x^4*
PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(12*x^4)
```

3.92.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx$$
$$\Big\downarrow \; 5409$$
$$-2bc\int \left(-\frac{2id^3(a+b\arctan(cx))c^4}{cx+i}+\frac{2id^3(a+b\arctan(cx))c^3}{x}+\frac{7d^3(a+b\arctan(cx))c^2}{4x^2}-\frac{id^3(a+b\arctan(cx)}{x^3}\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4x^4}$$
$$\Big\downarrow \; 2009$$

$$-2bc\left(2ic^3d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{7c^2d^3(a+b\arctan(cx))}{4x}+\frac{d^3(a+b\arctan(cx))}{12x^3}+\frac{icd^3(a+b\operatorname{arc}}{2x^2}\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4x^4}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```

output
```
-1/4*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/x^4 - 2*b*c*((b*c*d^3)/(24*
x^2) + ((I/2)*b*c^2*d^3)/x + (I/2)*b*c^3*d^3*ArcTan[c*x] + (d^3*(a + b*Arc
Tan[c*x]))/(12*x^3) + ((I/2)*c*d^3*(a + b*ArcTan[c*x]))/x^2 - (7*c^2*d^3*(
a + b*ArcTan[c*x]))/(4*x) + (2*I)*a*c^3*d^3*Log[x] + (11*b*c^3*d^3*Log[x])
/6 + (2*I)*c^3*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)] - (11*b*c^3*d^3*
Log[1 + c^2*x^2])/12 - b*c^3*d^3*PolyLog[2, (-I)*c*x] + b*c^3*d^3*PolyLog[
2, I*c*x] + b*c^3*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

3.92.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.71

method	result
parts	$d^3a^2\left(\frac{3c^2}{2x^2}-\frac{1}{4x^4}+\frac{ic^3}{x}-\frac{ic}{x^3}\right)+b^2d^3c^4\left(\frac{3\arctan(cx)^2}{2c^2x^2}+\frac{11\ln(c^2x^2+1)}{6}+\frac{7\arctan(cx)^2}{4}-\frac{11\ln(cx)}{3}-\right.$
derivativedivides	$c^4\left(d^3a^2\left(-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}+\frac{3}{2c^2x^2}\right)+b^2d^3\left(\frac{3\arctan(cx)^2}{2c^2x^2}+\frac{11\ln(c^2x^2+1)}{6}+\frac{7\arctan(cx)^2}{4}-11\right.\right.$
default	$c^4\left(d^3a^2\left(-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}+\frac{3}{2c^2x^2}\right)+b^2d^3\left(\frac{3\arctan(cx)^2}{2c^2x^2}+\frac{11\ln(c^2x^2+1)}{6}+\frac{7\arctan(cx)^2}{4}-11\right.\right.$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(3/2*c^2/x^2-1/4/x^4+I*c^3/x-I*c/x^3)+b^2*d^3*c^4*(3/2/c^2/x^2*arc
tan(c*x)^2+11/6*ln(c^2*x^2+1)+7/4*arctan(c*x)^2-11/3*ln(c*x)+dilog(-1/2*I*
(c*x+I))-1/12/c^2/x^2-1/4/c^4/x^4*arctan(c*x)^2-dilog(1/2*I*(c*x-I))+7/2/c
/x*arctan(c*x)+2*dilog(1+I*c*x)-2*dilog(1-I*c*x)-I*arctan(c*x)+1/2*ln(c*x-
I)^2-1/2*ln(c*x+I)^2-ln(c*x-I)*ln(c^2*x^2+1)+ln(c*x-I)*ln(-1/2*I*(c*x+I))+
ln(c*x+I)*ln(c^2*x^2+1)-ln(c*x+I)*ln(1/2*I*(c*x-I))+2*ln(c*x)*ln(1+I*c*x)-
2*ln(c*x)*ln(1-I*c*x)-I/c/x-1/6*arctan(c*x)/c^3/x^3-I*arctan(c*x)/c^2/x^2-
4*I*arctan(c*x)*ln(c*x)+2*I*arctan(c*x)*ln(c^2*x^2+1)+I*arctan(c*x)^2/c/x-
I*arctan(c*x)^2/c^3/x^3)+2*a*d^3*b*c^4*(-1/4*arctan(c*x)/c^4/x^4-I*arctan(
c*x)/c^3/x^3+I*arctan(c*x)/c/x+3/2/c^2/x^2*arctan(c*x)-1/2*I/c^2/x^2-2*I*l
n(c*x)-1/12/c^3/x^3+7/4/c/x+I*ln(c^2*x^2+1)+7/4*arctan(c*x))
```

3.92.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/16*(16*x^4*integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12*a^2*c^4*d^3*x^4 + 8*I
*a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x + 4*a^2*d^3 + (4*a
*b*c^5*d^3*x^5 - 4*(3*I*a*b - b^2)*c^4*d^3*x^4 - 2*(4*a*b + 3*I*b^2)*c^3*d
^3*x^3 - 4*(2*I*a*b + b^2)*c^2*d^3*x^2 - (12*a*b - I*b^2)*c*d^3*x + 4*I*a*
b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^7 + x^5), x) + (-4*I*b^2*c^3*d^3*
x^3 - 6*b^2*c^2*d^3*x^2 + 4*I*b^2*c*d^3*x + b^2*d^3)*log(-(c*x + I)/(c*x -
 I))^2)/x^4
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**5,x)
```

output
```
Timed out
```

3.92.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)^2}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c^3*d^3 + 3*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c^2*d^3 + I*((c^2*log(c^2*x^2
+ 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*c*d^3 + I*a^2*c^3*
d^3/x + 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x
^4)*a*b*d^3 + 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c
*x) - (3*c^2*x^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*lo
g(x) + 1)*c^2/x^2)*b^2*d^3 + 3/2*a^2*c^2*d^3/x^2 - I*a^2*c*d^3/x^3 - 1/4*b
^2*d^3*arctan(c*x)^2/x^4 - 1/4*a^2*d^3/x^4 - 1/32*(8*I*(b^2*c^4*d^3*arctan
(c*x)^3 + 4*b^2*c^5*d^3*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x
^4), x) - 16*b^2*c^5*d^3*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^
4), x) - 48*b^2*c^4*d^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c
^2*x^6 + x^4), x) + 80*b^2*c^4*d^3*integrate(1/16*x^3*arctan(c*x)/(c^2*x^6
 + x^4), x) - 96*b^2*c^3*d^3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^6 + x
^4), x) - 8*b^2*c^3*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^6 + x
^4), x) + 40*b^2*c^3*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^
4), x) - 48*b^2*c^2*d^3*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2
*x^6 + x^4), x) - 32*b^2*c^2*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^6 + x
^4), x) - 144*b^2*c*d^3*integrate(1/16*arctan(c*x)^2/(c^2*x^6 + x^4), x) -
 12*b^2*c*d^3*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 -
 8*(b^2*c^4*d^3*arctan(c*x)^2 - 16*b^2*c^5*d^3*integrate(1/16*x^4*arcta...
```

3.92.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

output
```
Timed out
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^5,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^5, x)
```

3.93 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx$

3.93.1 Optimal result

Integrand size = 25, antiderivative size = 384

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = &-\frac{b^2c^2d^3}{30x^3}-\frac{ib^2c^3d^3}{4x^2}+\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\arctan(cx)\\ &-\frac{bcd^3(a+b\arctan(cx))}{10x^4}-\frac{ibc^2d^3(a+b\arctan(cx))}{2x^3}\\ &+\frac{6bc^3d^3(a+b\arctan(cx))}{5x^2}\\ &+\frac{5ibc^4d^3(a+b\arctan(cx))}{2x}\\ &-\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{5x^5}\\ &+\frac{icd^3(1+icx)^4(a+b\arctan(cx))^2}{20x^4}\\ &+\frac{12}{5}abc^5d^3\log(x)-3ib^2c^5d^3\log(x)\\ &+\frac{12}{5}bc^5d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)\\ &+\frac{3}{2}ib^2c^5d^3\log\left(1+c^2x^2\right)+\frac{6}{5}ib^2c^5d^3\operatorname{PolyLog}(2,-icx)\\ &-\frac{6}{5}ib^2c^5d^3\operatorname{PolyLog}(2,icx)\\ &-\frac{6}{5}ib^2c^5d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/30*b^2*c^2*d^3/x^3+1/20*I*c*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/x^4+13/
10*b^2*c^4*d^3/x+13/10*b^2*c^5*d^3*arctan(c*x)-1/10*b*c*d^3*(a+b*arctan(c*
x))/x^4+5/2*I*b*c^4*d^3*(a+b*arctan(c*x))/x+6/5*b*c^3*d^3*(a+b*arctan(c*x)
)/x^2-1/2*I*b*c^2*d^3*(a+b*arctan(c*x))/x^3-1/5*d^3*(1+I*c*x)^4*(a+b*arcta
n(c*x))^2/x^5-6/5*I*b^2*c^5*d^3*polylog(2,1-2/(1-I*c*x))+12/5*a*b*c^5*d^3*
ln(x)-6/5*I*b^2*c^5*d^3*polylog(2,I*c*x)+12/5*b*c^5*d^3*(a+b*arctan(c*x))*
ln(2/(1-I*c*x))+3/2*I*b^2*c^5*d^3*ln(c^2*x^2+1)-1/4*I*b^2*c^3*d^3/x^2+6/5*
I*b^2*c^5*d^3*polylog(2,-I*c*x)-3*I*b^2*c^5*d^3*ln(x)
```

3.93.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx$$
$$= \frac{d^3\Big(-12a^2-45ia^2cx-6abcx+60a^2c^2x^2-30iabc^2x^2-2b^2c^2x^2+30ia^2c^3x^3+72abc^3x^3-15ib^2c^3x^3+1$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```

output
```
(d^3*(-12*a^2 - (45*I)*a^2*c*x - 6*a*b*c*x + 60*a^2*c^2*x^2 - (30*I)*a*b*c
^2*x^2 - 2*b^2*c^2*x^2 + (30*I)*a^2*c^3*x^3 + 72*a*b*c^3*x^3 - (15*I)*b^2*
c^3*x^3 + (150*I)*a*b*c^4*x^4 + 78*b^2*c^4*x^4 - (15*I)*b^2*c^5*x^5 + (3*I
)*b^2*(-I + c*x)^4*(4*I + c*x)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*c*x*(-1
- (5*I)*c*x + 12*c^2*x^2 + (25*I)*c^3*x^3 + 13*c^4*x^4) + a*(-4 - (15*I)*c
*x + 20*c^2*x^2 + (10*I)*c^3*x^3 + (25*I)*c^5*x^5) + 24*b*c^5*x^5*Log[1 -
E^((2*I)*ArcTan[c*x])]) + 144*a*b*c^5*x^5*Log[c*x] - (180*I)*b^2*c^5*x^5*L
og[(c*x)/Sqrt[1 + c^2*x^2]] - 72*a*b*c^5*x^5*Log[1 + c^2*x^2] - (72*I)*b^2
*c^5*x^5*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(60*x^5)
```

3.93.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx$$
$$\Big\downarrow \text{5409}$$
$$-2bc\int \left(\frac{6d^3(a+b\arctan(cx))c^5}{5(cx+i)} - \frac{6d^3(a+b\arctan(cx))c^4}{5x} + \frac{5id^3(a+b\arctan(cx))c^3}{4x^2} + \frac{6d^3(a+b\arctan(cx))c}{5x^3}\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{5x^5} + \frac{icd^3(1+icx)^4(a+b\arctan(cx))^2}{20x^4}$$
$$\Big\downarrow \text{2009}$$
$$-2bc\left(-\frac{6}{5}c^4d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx)) - \frac{5ic^3d^3(a+b\arctan(cx))}{4x} - \frac{3c^2d^3(a+b\arctan(cx))}{5x^2} + \frac{d^3(a+b\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{5x^5} + \frac{icd^3(1+icx)^4(a+b\arctan(cx))^2}{20x^4}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```

output
```
-1/5*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/x^5 + ((I/20)*c*d^3*(1 + I*
c*x)^4*(a + b*ArcTan[c*x])^2)/x^4 - 2*b*c*((b*c*d^3)/(60*x^3) + ((I/8)*b*c
^2*d^3)/x^2 - (13*b*c^3*d^3)/(20*x) - (13*b*c^4*d^3*ArcTan[c*x])/20 + (d^3
*(a + b*ArcTan[c*x]))/(20*x^4) + ((I/4)*c*d^3*(a + b*ArcTan[c*x]))/x^3 - (
3*c^2*d^3*(a + b*ArcTan[c*x]))/(5*x^2) - (((5*I)/4)*c^3*d^3*(a + b*ArcTan[
c*x]))/x - (6*a*c^4*d^3*Log[x])/5 + ((3*I)/2)*b*c^4*d^3*Log[x] - (6*c^4*d^
3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/5 - ((3*I)/4)*b*c^4*d^3*Log[1 +
c^2*x^2] - ((3*I)/5)*b*c^4*d^3*PolyLog[2, (-I)*c*x] + ((3*I)/5)*b*c^4*d^3*
PolyLog[2, I*c*x] + ((3*I)/5)*b*c^4*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

3.93.4 Maple [A] (verified)

Time = 5.20 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.41

method	result
parts	$d^3a^2\left(\frac{ic^3}{2x^2}-\frac{3ic}{4x^4}-\frac{1}{5x^5}+\frac{c^2}{x^3}\right)+b^2d^3c^5\left(-\frac{\arctan(cx)^2}{5c^5x^5}+\frac{3i\ln(c^2x^2+1)}{2}+\frac{\arctan(cx)^2}{c^3x^3}-\frac{i\arctan(cx}{2c^3x^3}\right.$
derivativedivides	$c^5\left(d^3a^2\left(-\frac{1}{5c^5x^5}-\frac{3i}{4c^4x^4}+\frac{1}{c^3x^3}+\frac{i}{2c^2x^2}\right)+b^2d^3\left(-\frac{\arctan(cx)^2}{5c^5x^5}+\frac{3i\ln(c^2x^2+1)}{2}+\frac{\arctan(cx)^2}{c^3x^3}-\right.\right.$
default	$c^5\left(d^3a^2\left(-\frac{1}{5c^5x^5}-\frac{3i}{4c^4x^4}+\frac{1}{c^3x^3}+\frac{i}{2c^2x^2}\right)+b^2d^3\left(-\frac{\arctan(cx)^2}{5c^5x^5}+\frac{3i\ln(c^2x^2+1)}{2}+\frac{\arctan(cx)^2}{c^3x^3}-\right.\right.$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/2*I*c^3/x^2-3/4*I*c/x^4-1/5/x^5+c^2/x^3)+b^2*d^3*c^5*(-1/5*arct
an(c*x)^2/c^5/x^5+3/2*I*ln(c^2*x^2+1)+arctan(c*x)^2/c^3/x^3-1/2*I*arctan(c
*x)/c^3/x^3-3*I*ln(c*x)-6/5*I*ln(c*x)*ln(1-I*c*x)-1/10*arctan(c*x)/c^4/x^4
+6/5/c^2/x^2*arctan(c*x)+12/5*arctan(c*x)*ln(c*x)-6/5*arctan(c*x)*ln(c^2*x
^2+1)+5/2*I*arctan(c*x)/c/x+6/5*I*ln(c*x)*ln(1+I*c*x)-1/4*I/c^2/x^2+1/2*I*
arctan(c*x)^2/c^2/x^2-3/5*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(c*x+I))
-ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)+6/5*I*dilog(1+I*c*x)+3/5*I*
(ln(c*x+I)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))-
1/2*ln(c*x+I)^2)+5/4*I*arctan(c*x)^2-3/4*I*arctan(c*x)^2/c^4/x^4-6/5*I*dil
og(1-I*c*x)-1/30/c^3/x^3+13/10/c/x+13/10*arctan(c*x))+2*a*d^3*b*c^5*(-1/5/
c^5/x^5*arctan(c*x)-3/4*I*arctan(c*x)/c^4/x^4+arctan(c*x)/c^3/x^3+1/2*I*ar
ctan(c*x)/c^2/x^2-1/4*I/c^3/x^3+5/4*I/c/x-1/20/c^4/x^4+3/5/c^2/x^2+6/5*ln(
c*x)-3/5*ln(c^2*x^2+1)+5/4*I*arctan(c*x))
```

3.93.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="fricas")
```

output
```
1/80*(80*x^5*integral(1/20*(-20*I*a^2*c^5*d^3*x^5 - 60*a^2*c^4*d^3*x^4 + 4
0*I*a^2*c^3*d^3*x^3 - 40*a^2*c^2*d^3*x^2 + 60*I*a^2*c*d^3*x + 20*a^2*d^3 +
 (20*a*b*c^5*d^3*x^5 - 10*(6*I*a*b - b^2)*c^4*d^3*x^4 - 20*(2*a*b + I*b^2)
*c^3*d^3*x^3 - 5*(8*I*a*b + 3*b^2)*c^2*d^3*x^2 - 4*(15*a*b - I*b^2)*c*d^3*
x + 20*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^8 + x^6), x) + (-10*I*
b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 + 15*I*b^2*c*d^3*x + 4*b^2*d^3)*log(-
(c*x + I)/(c*x - I))^2)/x^5
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**6,x)
```

output
```
Timed out
```

3.93.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="maxima")
```

output
```
I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c^3*d^3 - ((c^2*log(c^2*
x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*c^2*d^3 + 1/2*
I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b*c*
d^3 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4
)*c + 4*arctan(c*x)/x^5)*a*b*d^3 + 1/2*I*a^2*c^3*d^3/x^2 + a^2*c^2*d^3/x^3
 - 3/4*I*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^5 - 1/320*(320*I*x^5*integrate(1/80
*(60*(b^2*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x)^2 +
 5*(b^2*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*log(c^2*x^2 + 1)^
2 + 2*(30*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2)*arctan(c*x) - (10*b^2*c^5*
d^3*x^5 - 35*b^2*c^3*d^3*x^3 + 4*b^2*c*d^3*x + 20*(3*b^2*c^4*d^3*x^4 + 2*b
^2*c^2*d^3*x^2 - b^2*d^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6),
x) + 320*x^5*integrate(1/80*(60*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b
^2*d^3)*arctan(c*x)^2 + 5*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3
)*log(c^2*x^2 + 1)^2 - 2*(10*b^2*c^5*d^3*x^5 - 35*b^2*c^3*d^3*x^3 + 4*b^2*
c*d^3*x)*arctan(c*x) - (30*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2 - 20*(b^2*
c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x))*log(c^2*x^2
+ 1))/(c^2*x^8 + x^6), x) - 4*(10*I*b^2*c^3*d^3*x^3 + 20*b^2*c^2*d^3*x^2 -
 15*I*b^2*c*d^3*x - 4*b^2*d^3)*arctan(c*x)^2 + 4*(10*b^2*c^3*d^3*x^3 - 20*
I*b^2*c^2*d^3*x^2 - 15*b^2*c*d^3*x + 4*I*b^2*d^3)*arctan(c*x)*log(c^2*x^2
+ 1) + (10*I*b^2*c^3*d^3*x^3 + 20*b^2*c^2*d^3*x^2 - 15*I*b^2*c*d^3*x - ...
```

3.93.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="giac")
```

output
```
Timed out
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^6,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^6, x)
```

3.94 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx$

3.94.1 Optimal result

Integrand size = 25, antiderivative size = 513

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = -\frac{b^2c^2d^3}{60x^4} - \frac{ib^2c^3d^3}{10x^3} + \frac{61b^2c^4d^3}{180x^2} + \frac{37ib^2c^5d^3}{30x} + \frac{37}{30}ib^2c^6d^3\arctan(cx) - \frac{bcd^3(a+b\arctan(cx))}{15x^5} - \frac{3ibc^2d^3(a+b\arctan(cx))}{10x^4} + \frac{11bc^3d^3(a+b\arctan(cx))}{18x^3} + \frac{14ibc^4d^3(a+b\arctan(cx))}{15x^2} - \frac{11bc^5d^3(a+b\arctan(cx))}{6x} - \frac{d^3(a+b\arctan(cx))^2}{6x^6} - \frac{3icd^3(a+b\arctan(cx))^2}{5x^5} + \frac{3c^2d^3(a+b\arctan(cx))^2}{4x^4} + \frac{ic^3d^3(a+b\arctan(cx))^2}{3x^3} + \frac{28}{15}iabc^6d^3\log(x) + \frac{113}{45}b^2c^6d^3\log(x) + \frac{37}{20}ibc^6d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right) + \frac{1}{60}ibc^6d^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) - \frac{113}{90}b^2c^6d^3\log\left(1+c^2x^2\right) - \frac{14}{15}b^2c^6d^3\,\mathrm{PolyLog}(2,-icx) + \frac{14}{15}b^2c^6d^3\,\mathrm{PolyLog}(2,icx) + \frac{37}{40}b^2c^6d^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right) - \frac{1}{120}b^2c^6d^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)$$

output
```
-1/60*b^2*c^2*d^3/x^4+14/15*I*b*c^4*d^3*(a+b*arctan(c*x))/x^2+61/180*b^2*c
^4*d^3/x^2-3/10*I*b*c^2*d^3*(a+b*arctan(c*x))/x^4+37/30*I*b^2*c^5*d^3/x-1/
15*b*c*d^3*(a+b*arctan(c*x))/x^5+37/20*I*b*c^6*d^3*(a+b*arctan(c*x))*ln(2/
(1-I*c*x))+11/18*b*c^3*d^3*(a+b*arctan(c*x))/x^3+1/60*I*b*c^6*d^3*(a+b*arc
tan(c*x))*ln(2/(1+I*c*x))-11/6*b*c^5*d^3*(a+b*arctan(c*x))/x-1/6*d^3*(a+b*
arctan(c*x))^2/x^6+28/15*I*a*b*c^6*d^3*ln(x)+3/4*c^2*d^3*(a+b*arctan(c*x))
^2/x^4+1/3*I*c^3*d^3*(a+b*arctan(c*x))^2/x^3-3/5*I*c*d^3*(a+b*arctan(c*x))
^2/x^5+113/45*b^2*c^6*d^3*ln(x)-1/10*I*b^2*c^3*d^3/x^3+37/30*I*b^2*c^6*d^3
*arctan(c*x)-113/90*b^2*c^6*d^3*ln(c^2*x^2+1)-14/15*b^2*c^6*d^3*polylog(2,
-I*c*x)+14/15*b^2*c^6*d^3*polylog(2,I*c*x)+37/40*b^2*c^6*d^3*polylog(2,1-2
/(1-I*c*x))-1/120*b^2*c^6*d^3*polylog(2,1-2/(1+I*c*x))
```

3.94.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.78

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx$$
$$= \frac{d^3\left(-30a^2 - 108ia^2cx - 12abcx + 135a^2c^2x^2 - 54iabc^2x^2 - 3b^2c^2x^2 + 60ia^2c^3x^3 + 110abc^3x^3 - 18ib^2c^3x^3\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

output
```
(d^3*(-30*a^2 - (108*I)*a^2*c*x - 12*a*b*c*x + 135*a^2*c^2*x^2 - (54*I)*a*
b*c^2*x^2 - 3*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 110*a*b*c^3*x^3 - (18*I)*
b^2*c^3*x^3 + (168*I)*a*b*c^4*x^4 + 61*b^2*c^4*x^4 - 330*a*b*c^5*x^5 + (22
2*I)*b^2*c^5*x^5 + 64*b^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(-10 + (4*I)*c*x +
c^2*x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*c*x*(-6 - (27*I)*c*x + 55*c^2*
x^2 + (84*I)*c^3*x^3 - 165*c^4*x^4 + (111*I)*c^5*x^5) - 3*a*(10 + (36*I)*c
*x - 45*c^2*x^2 - (20*I)*c^3*x^3 + 55*c^6*x^6) + (168*I)*b*c^6*x^6*Log[1 -
 E^((2*I)*ArcTan[c*x])]) + (336*I)*a*b*c^6*x^6*Log[c*x] + 452*b^2*c^6*x^6*
Log[(c*x)/Sqrt[1 + c^2*x^2]] - (168*I)*a*b*c^6*x^6*Log[1 + c^2*x^2] + 168*
b^2*c^6*x^6*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(180*x^6)
```

3.94.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx$$

$$\Big\downarrow \; 5409$$

$$-2bc\int \left(-\frac{id^3(a+b\arctan(cx))c^6}{120(i-cx)}+\frac{37id^3(a+b\arctan(cx))c^6}{40(cx+i)}-\frac{14id^3(a+b\arctan(cx))c^5}{15x}-\frac{11d^3(a+b\arctan\right. \frac{ic^3d^3(a+b\arctan(cx))^2}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))^2}{4x^4}-\frac{d^3(a+b\arctan(cx))^2}{6x^6}-\frac{3icd^3(a+b\arctan(cx))^2}{5x^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{ic^3d^3(a+b\arctan(cx))^2}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))^2}{4x^4}- 2bc\left(-\frac{37}{40}ic^5d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{120}ic^5d^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{11c^4d^3(a+b\arct\right. \frac{d^3(a+b\arctan(cx))^2}{6x^6}-\frac{3icd^3(a+b\arctan(cx))^2}{5x^5}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

output
```
-1/6*(d^3*(a + b*ArcTan[c*x])^2)/x^6 - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*x]
)^2)/x^5 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/(4*x^4) + ((I/3)*c^3*d^3*(a +
 b*ArcTan[c*x])^2)/x^3 - 2*b*c*((b*c*d^3)/(120*x^4) + ((I/20)*b*c^2*d^3)/x
^3 - (61*b*c^3*d^3)/(360*x^2) - (((37*I)/60)*b*c^4*d^3)/x - ((37*I)/60)*b*
c^5*d^3*ArcTan[c*x] + (d^3*(a + b*ArcTan[c*x]))/(30*x^5) + (((3*I)/20)*c*d
^3*(a + b*ArcTan[c*x]))/x^4 - (11*c^2*d^3*(a + b*ArcTan[c*x]))/(36*x^3) -
(((7*I)/15)*c^3*d^3*(a + b*ArcTan[c*x]))/x^2 + (11*c^4*d^3*(a + b*ArcTan[c
*x]))/(12*x) - ((14*I)/15)*a*c^5*d^3*Log[x] - (113*b*c^5*d^3*Log[x])/90 -
((37*I)/40)*c^5*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)] - (I/120)*c^5*d
^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] + (113*b*c^5*d^3*Log[1 + c^2*x^2
])/180 + (7*b*c^5*d^3*PolyLog[2, (-I)*c*x])/15 - (7*b*c^5*d^3*PolyLog[2, I
*c*x])/15 - (37*b*c^5*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])/80 + (b*c^5*d^3*P
olyLog[2, 1 - 2/(1 + I*c*x)])/240)
```

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

3.94.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.10

method	result
parts	$d^3a^2\left(\frac{3c^2}{4x^4}-\frac{3ic}{5x^5}-\frac{1}{6x^6}+\frac{ic^3}{3x^3}\right)+b^2d^3c^6\left(-\frac{\arctan(cx)}{15c^5x^5}-\frac{113\ln(c^2x^2+1)}{90}-\frac{1}{60c^4x^4}-\frac{11\arctan(cx)^2}{12}\right.$
derivativedivides	$c^6\left(d^3a^2\left(-\frac{3i}{5c^5x^5}+\frac{3}{4c^4x^4}+\frac{i}{3c^3x^3}-\frac{1}{6c^6x^6}\right)+b^2d^3\left(-\frac{\arctan(cx)}{15c^5x^5}-\frac{113\ln(c^2x^2+1)}{90}-\frac{1}{60c^4x^4}-\right.\right.$
default	$c^6\left(d^3a^2\left(-\frac{3i}{5c^5x^5}+\frac{3}{4c^4x^4}+\frac{i}{3c^3x^3}-\frac{1}{6c^6x^6}\right)+b^2d^3\left(-\frac{\arctan(cx)}{15c^5x^5}-\frac{113\ln(c^2x^2+1)}{90}-\frac{1}{60c^4x^4}-\right.\right.$

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(3/4*c^2/x^4-3/5*I*c/x^5-1/6/x^6+1/3*I*c^3/x^3)+b^2*d^3*c^6*(-1/15
/c^5/x^5*arctan(c*x)-113/90*ln(c^2*x^2+1)-1/60/c^4/x^4-11/12*arctan(c*x)^2
+113/45*ln(c*x)-7/15*dilog(-1/2*I*(c*x+I))+61/180/c^2/x^2+3/4/c^4/x^4*arct
an(c*x)^2+7/15*dilog(1/2*I*(c*x-I))-11/6/c/x*arctan(c*x)-14/15*dilog(1+I*c
*x)+14/15*dilog(1-I*c*x)-7/30*ln(c*x-I)^2+7/30*ln(c*x+I)^2+7/15*ln(c*x-I)*
ln(c^2*x^2+1)-7/15*ln(c*x-I)*ln(-1/2*I*(c*x+I))-7/15*ln(c*x+I)*ln(c^2*x^2+
1)+7/15*ln(c*x+I)*ln(1/2*I*(c*x-I))-14/15*ln(c*x)*ln(1+I*c*x)+14/15*ln(c*x
)*ln(1-I*c*x)+11/18*arctan(c*x)/c^3/x^3+37/30*I*arctan(c*x)-1/10*I/c^3/x^3
+37/30*I/c/x-14/15*I*arctan(c*x)*ln(c^2*x^2+1)+28/15*I*arctan(c*x)*ln(c*x)
-1/6*arctan(c*x)^2/c^6/x^6+14/15*I*arctan(c*x)/c^2/x^2-3/5*I*arctan(c*x)^2
/c^5/x^5+1/3*I*arctan(c*x)^2/c^3/x^3-3/10*I*arctan(c*x)/c^4/x^4)+2*a*d^3*b
*c^6*(-3/5*I*arctan(c*x)/c^5/x^5+3/4*arctan(c*x)/c^4/x^4+1/3*I*arctan(c*x)
/c^3/x^3-1/6*arctan(c*x)/c^6/x^6+14/15*I*ln(c*x)-3/20*I/c^4/x^4+7/15*I/c^2
/x^2-1/30/c^5/x^5+11/36/c^3/x^3-11/12/c/x-7/15*I*ln(c^2*x^2+1)-11/12*arcta
n(c*x))
```

3.94.5 Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)^2}{x^7}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="fricas")
```

output
```
1/240*(240*x^6*integral(1/60*(-60*I*a^2*c^5*d^3*x^5 - 180*a^2*c^4*d^3*x^4
+ 120*I*a^2*c^3*d^3*x^3 - 120*a^2*c^2*d^3*x^2 + 180*I*a^2*c*d^3*x + 60*a^2
*d^3 + (60*a*b*c^5*d^3*x^5 - 20*(9*I*a*b - b^2)*c^4*d^3*x^4 - 15*(8*a*b +
3*I*b^2)*c^3*d^3*x^3 - 12*(10*I*a*b + 3*b^2)*c^2*d^3*x^2 - 10*(18*a*b - I*
b^2)*c*d^3*x + 60*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^9 + x^7), x
) + (-20*I*b^2*c^3*d^3*x^3 - 45*b^2*c^2*d^3*x^2 + 36*I*b^2*c*d^3*x + 10*b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2)/x^6
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**7,x)
```

output
```
Timed out
```

3.94.7 Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^7}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="maxima")
```

output
```
-1/3*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^
3)*a*b*c^3*d^3 - 1/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arct
an(c*x)/x^4)*a*b*c^2*d^3 - 3/10*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2
) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*a*b*c*d^3 - 1/45*((15*c^5*
arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*a*
b*d^3 - 1/180*(4*(15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c
*arctan(c*x) - (30*c^4*x^4*arctan(c*x)^2 - 46*c^4*x^4*log(c^2*x^2 + 1) + 9
2*c^4*x^4*log(x) + 16*c^2*x^2 - 3)*c^2/x^4)*b^2*d^3 + 1/3*I*a^2*c^3*d^3/x^
3 + 3/4*a^2*c^2*d^3/x^4 - 3/5*I*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctan(c*x)^2/
x^6 - 1/6*a^2*d^3/x^6 - 1/960*(960*I*x^5*integrate(1/240*(180*(b^2*c^5*d^3
*x^4 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c*d^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^
4 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c*d^3)*log(c^2*x^2 + 1)^2 + 2*(65*b^2*c^4*d^
3*x^3 - 36*b^2*c^2*d^3*x)*arctan(c*x) - (20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d
^3*x^2 + 180*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*arctan(c*x))*log(c^2*x^2 +
1))/(c^2*x^8 + x^6), x) + 960*x^5*integrate(1/240*(540*(b^2*c^4*d^3*x^3 +
b^2*c^2*d^3*x)*arctan(c*x)^2 + 45*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*log(c^
2*x^2 + 1)^2 - 2*(20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d^3*x^2)*arctan(c*x) - (
65*b^2*c^4*d^3*x^3 - 36*b^2*c^2*d^3*x - 60*(b^2*c^5*d^3*x^4 - 2*b^2*c^3*d^
3*x^2 - 3*b^2*c*d^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6), x) -
4*(20*I*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x - 36*I*b^2*c*d^3)*arctan(c*x...
```

3.94.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="giac")
```

output
```
Timed out
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^3}{x^7}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^7,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^7, x)
```

3.95 $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

3.95.1 Optimal result

Integrand size = 25, antiderivative size = 356

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = & -\frac{abx}{c^3d}-\frac{ib^2x}{3c^3d}+\frac{ib^2\arctan(cx)}{3c^4d}-\frac{b^2x\arctan(cx)}{c^3d} \\ & +\frac{ibx^2(a+b\arctan(cx))}{3c^2d}-\frac{5(a+b\arctan(cx))^2}{6c^4d} \\ & +\frac{ix(a+b\arctan(cx))^2}{c^3d}+\frac{x^2(a+b\arctan(cx))^2}{2c^2d} \\ & -\frac{ix^3(a+b\arctan(cx))^2}{3cd}+\frac{8ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^4d} \\ & +\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^4d} \\ & +\frac{b^2\log(1+c^2x^2)}{2c^4d}-\frac{4b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^4d} \\ & +\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d} \\ & +\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d}\end{aligned}$$

output
```
-a*b*x/c^3/d-1/3*I*b^2*x/c^3/d+1/3*I*b^2*arctan(c*x)/c^4/d-b^2*x*arctan(c*
x)/c^3/d+1/3*I*b*x^2*(a+b*arctan(c*x))/c^2/d-5/6*(a+b*arctan(c*x))^2/c^4/d
+I*x*(a+b*arctan(c*x))^2/c^3/d+1/2*x^2*(a+b*arctan(c*x))^2/c^2/d-1/3*I*x^3
*(a+b*arctan(c*x))^2/c/d+8/3*I*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d+(
a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d+1/2*b^2*ln(c^2*x^2+1)/c^4/d-4/3*b
^2*polylog(2,1-2/(1+I*c*x))/c^4/d+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I
*c*x))/c^4/d+1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^4/d
```

3.95.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.18

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \frac{ia^2x}{c^3d}+\frac{a^2x^2}{2c^2d}-\frac{ia^2x^3}{3cd}-\frac{ia^2\arctan(cx)}{c^4d}-\frac{a^2\log(1+c^2x^2)}{2c^4d}$$
$$-\frac{iab\Big(-3icx-8cx\arctan(cx)+6\arctan(cx)^2+(1+c^2x^2)\left(-1+3i\arctan(cx)+2cx\arctan(cx)\right)+6i\,a}{3c^4d}$$
$$-\frac{ib^2\Big(2cx-6icx\arctan(cx)-2(1+c^2x^2)\arctan(cx)+8i\arctan(cx)^2-8cx\arctan(cx)^2+3i(1+c^2x^2)}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
(I*a^2*x)/(c^3*d) + (a^2*x^2)/(2*c^2*d) - ((I/3)*a^2*x^3)/(c*d) - (I*a^2*A
rcTan[c*x])/(c^4*d) - (a^2*Log[1 + c^2*x^2])/(2*c^4*d) - ((I/3)*a*b*((-3*I
)*c*x - 8*c*x*ArcTan[c*x] + 6*ArcTan[c*x]^2 + (1 + c^2*x^2)*(-1 + (3*I)*Ar
cTan[c*x] + 2*c*x*ArcTan[c*x]) + (6*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan
[c*x])] - 8*Log[1/Sqrt[1 + c^2*x^2]] + 3*PolyLog[2, -E^((2*I)*ArcTan[c*x])
]))/(c^4*d) - ((I/6)*b^2*(2*c*x - (6*I)*c*x*ArcTan[c*x] - 2*(1 + c^2*x^2)*
ArcTan[c*x] + (8*I)*ArcTan[c*x]^2 - 8*c*x*ArcTan[c*x]^2 + (3*I)*(1 + c^2*x
^2)*ArcTan[c*x]^2 + 2*c*x*(1 + c^2*x^2)*ArcTan[c*x]^2 + 4*ArcTan[c*x]^3 -
16*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (6*I)*ArcTan[c*x]^2*Log[1
+ E^((2*I)*ArcTan[c*x])] - (6*I)*Log[1/Sqrt[1 + c^2*x^2]] + (8*I + 6*ArcTa
n[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*I)*PolyLog[3, -E^((2*I)*Ar
cTan[c*x])]))/(c^4*d)
```

3.95.3 Rubi [A] (verified)

Time = 3.83 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.32, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5401, 27, 5361, 5401, 5361, 5401, 5345, 5379, 5451, 2009, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$$
$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))^2}{d(icx+1)}dx}{c}-\frac{i\int x^2(a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \ 27$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\int x^2(a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \ 5361$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \ 5401$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\int x(a+b\arctan(cx))^2dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \ 5361$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \ 5401$$

$$\frac{i\left(\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))^2dx}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \ 5345$$

3.95. $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\int\frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,\mathrm{arc}\right.}{}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5451}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,\mathrm{arc}\right.}{}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{2009}$$

3.95. $\int\frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,\mathrm{arc}\cdots\right.}{\cdots}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 5361$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,\mathrm{arc}\cdots\right.}{\cdots}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 262$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,\mathrm{arc}\cdots\right.}{\cdots}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 216$$

3.95. $\int\frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,\mathrm{arc}\cdots\right.}{\cdots}\right.}{cd}$$

$$-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 5419$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,\mathrm{arc}\cdots\right.}{\cdots}\right.}{cd}$$

$$-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 5455$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{c}\cdots\right.}{c}\cdots\right.}{cd}$$

$$-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 5379$$

3.95. $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}\right.}{c}\right.\right.}{c}\right.}{c}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+}{}}{c^2}\right.\right.}{cd}$$

$$\Big\downarrow \text{2849}$$

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b}{c}}{c}\right.\right.}{c}\right.}{c}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}}{c^2}\right.\right.}{cd}$$

$$\Big\downarrow \text{2752}$$

3.95. $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan\cdots}{c}\cdots\right.\right.}{c}\cdots}{c}\cdots\right.}{cd}\cdots$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\mathrm{P}\cdots}{\cdots}}{c}}{c^2}\cdots\right.\right.}{cd}\cdots$$

$$\Big\downarrow\ 5529$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a\cdots}{\cdots}\right.\right.}{\cdots}\cdots}{c}\cdots\right.}{\cdots}\cdots$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\mathrm{P}\cdots}{\cdots}}{c}}{c^2}\cdots\right.\right.}{cd}\cdots$$

$$\Big\downarrow\ 7164$$

3.95. $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan}{}\right.\right.}{}}{c}}{}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{P}}{}}{c}}{c^2}\right.\right.}{cd}$$

input

```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output

```
((-I)*((x^3*(a + b*ArcTan[c*x])^2)/3 - (2*b*c*(((x^2*(a + b*ArcTan[c*x]))/
2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*x]
)^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*Poly
Log[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3))/(c*d) + (I*(((-I)*((x^2*(a + b*
ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*A
rcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)))/c + (I*(((-I)*(x*(a + b*Ar
cTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)
])/c)/c)))/c + (I*((I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - (2*I)*
b*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(4*c))))/c))/c))/(c*d)
```

3.95.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

3.95. $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.95. $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

3.95.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 47.77 (sec) , antiderivative size = 1105, normalized size of antiderivative = 3.10

method	result	size
derivativedivides	Expression too large to display	1105
default	Expression too large to display	1105
parts	Expression too large to display	1149

input
```
int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(4/3*I/d*a*b+I/d*a*b*ln(-1/2*I*(c*x+I))*ln(c*x-I)+1/2*a^2/d*c^2*x^2-
1/2*a^2/d*ln(c^2*x^2+1)+I*a^2/d*c*x+b^2/d*(-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2
+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/3*I*arctan(c*x)*(c*x-I)
^2+1/2*c^2*x^2*arctan(c*x)^2-arctan(c*x)^2*ln(c*x-I)+11/6*arctan(c*x)^2+8/
3*I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-ln(1+(1+I*c*x)^2/(c^2*
x^2+1))+8/3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+8/3*dilog(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))+I*Pi*arctan(c*x)^2-I*arctan(c*x)*polylog(2,-(1+I*c*x)^
2/(c^2*x^2+1))+2/3*I*arctan(c*x)*(c*x+I)*(c*x-I)-1/3*arctan(c*x)*(c*x-I)-1
/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan
(c*x)^2-1/3*I*(c*x+I)-1/3*I*arctan(c*x)^2*c^3*x^3-2/3*I*arctan(c*x)^3+8/3*
I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/(1+(1+I*
c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+
1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+
I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2
+1)))*arctan(c*x)^2-1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^
2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+I*arctan(c*x)^2
*c*x+arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+1/2*polylog(3,-(1+I*c*x
)^2/(c^2*x^2+1)))-5/24*I/d*a*b*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I/d*a*b*ln(c*x
-I)^2+1/d*a*b*arctan(c*x)*c^2*x^2-2/d*a*b*arctan(c*x)*ln(c*x-I)-I*a^2/d*ar
ctan(c*x)-2/3*I/d*a*b*arctan(c*x)*c^3*x^3+1/3*I/d*a*b*c^2*x^2-1/d*a*b*c...
```

3.95.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^3*log(-(c*x
+ I)/(c*x - I)) - 4*I*a^2*x^3)/(c*d*x - I*d), x)
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.95.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*a^2*(I*(2*c^2*x^3 + 3*I*c*x^2 - 6*x)/(c^3*d) + 6*log(I*c*x + 1)/(c^4*
d)) - 1/96*(16*I*(216*b^2*c^4*integrate(1/48*x^4*arctan(c*x)^2/(c^5*d*x^2
+ c^3*d), x) + 18*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^5*d*x^2
 + c^3*d), x) + 576*a*b*c^4*integrate(1/48*x^4*arctan(c*x)/(c^5*d*x^2 + c^
3*d), x) + 24*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3
*d), x) + 72*b^2*c^3*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^5*
d*x^2 + c^3*d), x) + 24*b^2*c^3*integrate(1/48*x^3*arctan(c*x)/(c^5*d*x^2
+ c^3*d), x) - 36*b^2*c^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^5*d*x^2 +
 c^3*d), x) + 144*b^2*c*integrate(1/48*x*arctan(c*x)/(c^5*d*x^2 + c^3*d),
x) - 36*b^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^5*d*x^2 + c^3*d), x) - b^
2*arctan(c*x)^3/(c^4*d))*c^4*d - 96*c^4*d*integrate(1/48*(12*(3*b^2*c^2*x^
3 - 2*b^2*x)*arctan(c*x)^2 + 3*(b^2*c^2*x^3 - 2*b^2*x)*log(c^2*x^2 + 1)^2
- 4*(2*b^2*c^3*x^4 - 24*a*b*c^2*x^3 - 3*b^2*c*x^2)*arctan(c*x) - 2*(6*b^2*
c^3*x^4*arctan(c*x) - b^2*c^2*x^3 - 6*b^2*x)*log(c^2*x^2 + 1))/(c^4*d*x^2
+ c^2*d), x) + 24*I*b^2*arctan(c*x)^3 - 3*b^2*log(c^2*x^2 + 1)^3 - 4*(-2*I
*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 6*I*b^2*c*x)*arctan(c*x)^2 + (-2*I*b^2*c^3*
x^3 + 3*b^2*c^2*x^2 + 6*I*b^2*c*x + 6*I*b^2*arctan(c*x))*log(c^2*x^2 + 1)^
2 - 4*(3*b^2*arctan(c*x)^2 + (2*b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 - 6*b^2*c*x)
*arctan(c*x))*log(c^2*x^2 + 1))/(c^4*d)
```

3.95.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

3.95. $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$

3.96 $\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$

3.96.1 Optimal result

Integrand size = 25, antiderivative size = 277

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx &= \frac{iabx}{c^2d}+\frac{ib^2x\arctan(cx)}{c^2d}+\frac{i(a+b\arctan(cx))^2}{2c^3d}\\ &+\frac{x(a+b\arctan(cx))^2}{c^2d}-\frac{ix^2(a+b\arctan(cx))^2}{2cd}\\ &+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d}\\ &-\frac{i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d}\\ &-\frac{ib^2\log(1+c^2x^2)}{2c^3d}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d}\\ &+\frac{b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d}\\ &-\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d}\end{aligned}$$

output
```
I*a*b*x/c^2/d+I*b^2*x*arctan(c*x)/c^2/d+1/2*I*(a+b*arctan(c*x))^2/c^3/d+x*
(a+b*arctan(c*x))^2/c^2/d-1/2*I*x^2*(a+b*arctan(c*x))^2/c/d+2*b*(a+b*arcta
n(c*x))*ln(2/(1+I*c*x))/c^3/d-I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d-
1/2*I*b^2*ln(c^2*x^2+1)/c^3/d+I*b^2*polylog(2,1-2/(1+I*c*x))/c^3/d+b*(a+b*
arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3/d-1/2*I*b^2*polylog(3,1-2/(1+I*c
*x))/c^3/d
```

3.96.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.19

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx =$$
$$-\frac{i\big(6ia^2cx-6abcx+3a^2c^2x^2-6ia^2\arctan(cx)+6ab\arctan(cx)+12iabcx\arctan(cx)-6b^2cx\arctan(c}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-1/6*I)*((6*I)*a^2*c*x - 6*a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a^2*ArcTan[c*
x] + 6*a*b*ArcTan[c*x] + (12*I)*a*b*c*x*ArcTan[c*x] - 6*b^2*c*x*ArcTan[c*x
] + 6*a*b*c^2*x^2*ArcTan[c*x] - (12*I)*a*b*ArcTan[c*x]^2 + 9*b^2*ArcTan[c*
x]^2 + (6*I)*b^2*c*x*ArcTan[c*x]^2 + 3*b^2*c^2*x^2*ArcTan[c*x]^2 - (4*I)*b
^2*ArcTan[c*x]^3 + 12*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (12
*I)*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b^2*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] - 3*a^2*Log[1 + c^2*x^2] - (6*I)*a*b*Log[1 +
 c^2*x^2] + 3*b^2*Log[1 + c^2*x^2] + 6*b*((-I)*a + b - I*b*ArcTan[c*x])*Po
lyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])
]))/(c^3*d)
```

3.96.3 Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5401, 27, 5361, 5401, 5345, 5379, 5451, 2009, 5419, 5455, 5379, 2849, 2752, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$$
$$\Big\downarrow \; 5401$$
$$\frac{i\int \frac{x(a+b\arctan(cx))^2}{d(icx+1)}dx}{c}-\frac{i\int x(a+b\arctan(cx))^2dx}{cd}$$
$$\Big\downarrow \; 27$$

$$\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\int x(a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))^2dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5345}$$

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5451}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int (a+b\arctan(cx))dx}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{2009}$$

3.96. $\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 5419$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \; 5455$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \; 5379$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\cdots\right)}{c^2x^2+1}dx}{c}\cdots\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \; 2849$$

3.96. $\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)\cdots}{c}\right.\right.}{c}\right.\cdots}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \; 2752$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\,\cdots}{c}\right.\right.}{c}\right.\cdots}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \; 5529$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(\cdots\right.\right.}{\cdots}\right.\cdots}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \; 7164$$

3.96. $\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$
\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(-\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\mathrm{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a}{}\right.\right.}{}\right.}{cd}
$$

$$
\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}
$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-I)*((x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*
c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)))/(c*d) +
 (I*(((-I)*(x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])
^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyL
og[2, 1 - 2/(1 + I*c*x)])/c)/c)))/c + (I*((I*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/c - (2*I)*b*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(
1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)])/(4*c))))/c))/(c*d)
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.96. $\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.96.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.01 (sec) , antiderivative size = 994, normalized size of antiderivative = 3.59

method	result	size
derivativedivides	Expression too large to display	994
default	Expression too large to display	994
parts	Expression too large to display	1038

input
```
int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^2/d*c*x+2*I/d*a*b*arctan(c*x)*ln(c*x-I)-I/d*a*b*arctan(c*x)*c^2*x
^2-a^2/d*arctan(c*x)+b^2/d*(arctan(c*x)^2*c*x-1/2*I*arctan(c*x)^2*c^2*x^2-
2*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*dilog(1+I*(1+I*c*x)/(c^2*x^
2+1)^(1/2))+I*arctan(c*x)^2*ln(c*x-I)+I*arctan(c*x)*(c*x-I)+2*arctan(c*x)*
ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^
2+1)^(1/2))+Pi*arctan(c*x)^2+I*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*arctan(
c*x)^2-Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arct
an(c*x)^2-1/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+1/2*Pi*csgn(I/(1+(1+
I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))^2*arctan(c*x)^2-1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+
I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2
+1)))*arctan(c*x)^2-1/2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-2/3*arctan(c
*x)^3-arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-I*arctan(c*x)^2*ln(2
*I*(1+I*c*x)^2/(c^2*x^2+1)))+2/d*a*b*arctan(c*x)*c*x+1/2*I*a^2/d*ln(c^2*x^
2+1)+1/4*I/d*a*b*arctan(1/6*c^3*x^3+7/6*c*x)+1/d*a*b*ln(-1/2*I*(c*x+I))*ln
(c*x-I)+1/d*a*b*dilog(-1/2*I*(c*x+I))-1/2/d*a*b*ln(c*x-I)^2+1/d*a*b-1/4*I/
d*a*b*arctan(1/2*c*x)-1/8/d*a*b*ln(c^4*x^4+10*c^2*x^2+9)+1/2*I/d*a*b*arcta
n(1/2*c*x-1/2*I)-1/2*I*a^2/d*c^2*x^2+I/d*a*b*c*x-3/4/d*a*b*ln(c^2*x^2+1...
```

3.96.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^2x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^2*log(-(c*x
+ I)/(c*x - I)) - 4*I*a^2*x^2)/(c*d*x - I*d), x)
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.96.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*a^2*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/96*(16
*(24*b^2*c^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d*x^2 +
c^2*d), x) + 24*b^2*c^3*integrate(1/16*x^3*arctan(c*x)/(c^4*d*x^2 + c^2*d)
, x) - 72*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x)
 - 6*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x)
 - 192*a*b*c^2*integrate(1/16*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 12
*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) + 48*
b^2*c*integrate(1/16*x*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 12*b^2*integr
ate(1/16*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - b^2*arctan(c*x)^3/(c
^3*d))*c^3*d + 24*b^2*arctan(c*x)^3 + 96*I*c^3*d*integrate(1/16*(4*(3*b^2*
c^2*x^3 - 2*b^2*x)*arctan(c*x)^2 + (b^2*c^2*x^3 - 2*b^2*x)*log(c^2*x^2 + 1
)^2 + 4*(8*a*b*c^2*x^3 + b^2*c*x^2)*arctan(c*x) + 2*(b^2*c^2*x^3 + 2*b^2*c
*x^2*arctan(c*x) + 2*b^2*x)*log(c^2*x^2 + 1))/(c^3*d*x^2 + c*d), x) + 3*I*
b^2*log(c^2*x^2 + 1)^3 - 12*(-I*b^2*c^2*x^2 + 2*b^2*c*x)*arctan(c*x)^2 + 3
*(-I*b^2*c^2*x^2 + 2*b^2*c*x + 2*b^2*arctan(c*x))*log(c^2*x^2 + 1)^2 - 12*
(-I*b^2*arctan(c*x)^2 + (b^2*c^2*x^2 + 2*I*b^2*c*x)*arctan(c*x))*log(c^2*x
^2 + 1))/(c^3*d)
```

3.96.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

3.97 $\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx$

3.97.1 Optimal result

Integrand size = 23, antiderivative size = 192

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = {} & \frac{(a+b\arctan(cx))^2}{c^2d}-\frac{ix(a+b\arctan(cx))^2}{cd} \\ & -\frac{2ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d} \\ & -\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d}+\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d} \\ & -\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d} \\ & -\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d}\end{aligned}$$

output
```
(a+b*arctan(c*x))^2/c^2/d-I*x*(a+b*arctan(c*x))^2/c/d-2*I*b*(a+b*arctan(c*
x))*ln(2/(1+I*c*x))/c^2/d-(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^2/d+b^2*po
lylog(2,1-2/(1+I*c*x))/c^2/d-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x)
)/c^2/d-1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^2/d
```

3.97.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.24

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx =$$
$$-\frac{i\left(6a^2cx-6a^2\arctan(cx)+12abcx\arctan(cx)-12ab\arctan(cx)^2-6ib^2\arctan(cx)^2+6b^2cx\arctan(c\right.}{}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-1/6*I)*(6*a^2*c*x - 6*a^2*ArcTan[c*x] + 12*a*b*c*x*ArcTan[c*x] - 12*a*b
*ArcTan[c*x]^2 - (6*I)*b^2*ArcTan[c*x]^2 + 6*b^2*c*x*ArcTan[c*x]^2 - 4*b^2
*ArcTan[c*x]^3 - (12*I)*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 1
2*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^2*ArcTan[c*x]^2
*Log[1 + E^((2*I)*ArcTan[c*x])] + (3*I)*a^2*Log[1 + c^2*x^2] - 6*a*b*Log[1
 + c^2*x^2] - 6*b*(a + I*b + b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*
x])] - (3*I)*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(c^2*d)
```

3.97.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5401, 27, 5345, 5379, 5455, 5379, 2849, 2752, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx$$
$$\Big\downarrow \text{5401}$$
$$\frac{i\int \frac{(a+b\arctan(cx))^2}{d(icx+1)}dx}{c}-\frac{i\int (a+b\arctan(cx))^2dx}{cd}$$
$$\Big\downarrow \text{27}$$
$$\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\int (a+b\arctan(cx))^2dx}{cd}$$
$$\Big\downarrow \text{5345}$$

$$\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5455}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\int \frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{2752}$$

3.97. $\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)\right)}{cd}$$

$$\Big\downarrow \text{5529}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)\right)}{cd}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)\right)}{cd}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-I)*(x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(
b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2,
 1 - 2/(1 + I*c*x)])/c)/c)))/(c*d) + (I*((I*(a + b*ArcTan[c*x])^2*Log[2/(1
 + I*c*x)])/c - (2*I)*b*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)])/(4*c))))/(c*d)
```

3.97. $\int\frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx$

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.97. $\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx$

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.97.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.85 (sec) , antiderivative size = 2330, normalized size of antiderivative = 12.14

method	result	size
derivativedivides	Expression too large to display	2330
default	Expression too large to display	2330
parts	Expression too large to display	2363

input
```
int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-I*a^2/d*c*x+1/2*a^2/d*ln(c^2*x^2+1)+I*a^2/d*arctan(c*x)+b^2/d*(-1/
2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+
1))-arctan(c*x)^2+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-dilog(
1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-Pi
*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-Pi*arctan(c*x)*ln(1-I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))+Pi*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-I*a
rctan(c*x)^2*c*x-arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+arctan(c*x)
^2*ln(c*x-I)-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2
/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*(I
*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I
*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*
(1+I*c*x)/(c^2*x^2+1)^(1/2)))-1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+
I*c*x)^2/(c^2*x^2+1)))^3*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c
^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+1/4*I*Pi*csgn((1+
I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*(2*I*arctan(c*x)*ln(1+
(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1
)))-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*(I*ar
ctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*...
```

3.97.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{(b\arctan{(cx)}+a)^2x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*x*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x*log(-(c*x + I)
/(c*x - I)) - 4*I*a^2*x)/(c*d*x - I*d), x)
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.97.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^2*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) + 1/96*(-24*I*b^2*c*x*arctan(c*x
)^2 + 24*I*b^2*arctan(c*x)^3 - 3*b^2*log(c^2*x^2 + 1)^3 - 16*I*(72*b^2*c^2
*integrate(1/16*x^2*arctan(c*x)^2/(c^3*d*x^2 + c*d), x) + 6*b^2*c^2*integr
ate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x) + 192*a*b*c^2*integr
ate(1/16*x^2*arctan(c*x)/(c^3*d*x^2 + c*d), x) + 24*b^2*c^2*integrate(1/16
*x^2*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) + 24*b^2*c*integrate(1/16*x*ar
ctan(c*x)*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) - 48*b^2*c*integrate(1/16
*x*arctan(c*x)/(c^3*d*x^2 + c*d), x) + 12*b^2*integrate(1/16*log(c^2*x^2 +
 1)^2/(c^3*d*x^2 + c*d), x) + b^2*arctan(c*x)^3/(c^2*d))*c^2*d + 96*c^2*d*
integrate(1/16*(20*b^2*x*arctan(c*x)^2 + 3*b^2*x*log(c^2*x^2 + 1)^2 - 8*(b
^2*c*x^2 - 4*a*b*x)*arctan(c*x) - 4*(b^2*c*x^2*arctan(c*x) + b^2*x)*log(c^
2*x^2 + 1))/(c^2*d*x^2 + d), x) + 6*(I*b^2*c*x + I*b^2*arctan(c*x))*log(c^
2*x^2 + 1)^2 + 12*(2*b^2*c*x*arctan(c*x) - b^2*arctan(c*x)^2)*log(c^2*x^2
+ 1))/(c^2*d)
```

3.97.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2 x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{x\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

3.98 $\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx$

3.98.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \frac{i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{cd} + \frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd}$$

output
```
I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c/d-b*(a+b*arctan(c*x))*polylog(2,1-
2/(1+I*c*x))/c/d+1/2*I*b^2*polylog(3,1-2/(1+I*c*x))/c/d
```

3.98.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \frac{i\left(2(a+b\arctan(cx))^2\log\left(\frac{2d}{d+icdx}\right)+2ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)+b^2\operatorname{PolyLog}\left(3,\frac{i+cx}{-i+cx}\right)\right)}{2cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

output
```
((I/2)*(2*(a + b*ArcTan[c*x])^2*Log[(2*d)/(d + I*c*d*x)] + (2*I)*b*(a + b*
ArcTan[c*x])*PolyLog[2, (I + c*x)/(-I + c*x)] + b^2*PolyLog[3, (I + c*x)/(
-I + c*x)]))/(c*d)
```

3.98.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx$$

$$\Big\downarrow \ 5379$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{cd}-\frac{2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \ 5529$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{cd}-\frac{2ib\left(\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)}{d}$$

$$\Big\downarrow \ 7164$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{cd}-\frac{2ib\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c*d) - ((2*I)*b*(((-1/2*I)*(
a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/
(1 + I*c*x)])/(4*c)))/d
```

3.98.3.1 Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.98.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.67 (sec) , antiderivative size = 851, normalized size of antiderivative = 8.68

method	result
derivativedivides	$-\frac{i a^2 \ln\left(c^2x^2+1\right)}{2d}+\frac{a^2 \arctan(cx)}{d}+\frac{b^2\left(-i\ln(icx+1)\arctan(cx)^2+2i\left(\frac{\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)}{2}-\frac{i\arctan(cx)^3}{3}+\frac{i\pi\left(\operatorname{csgn}\left(\frac{\cdots}{1+\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{i a^2 \ln\left(c^2x^2+1\right)}{2d}+\frac{a^2 \arctan(cx)}{d}+\frac{b^2\left(-i\ln(icx+1)\arctan(cx)^2+2i\left(\frac{\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)}{2}-\frac{i\arctan(cx)^3}{3}+\frac{i\pi\left(\operatorname{csgn}\left(\frac{\cdots}{1+\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$
parts	$-\frac{i a^2 \ln(c^2x^2+1)}{2dc}+\frac{a^2 \arctan(cx)}{dc}+\frac{b^2\left(-i\ln(icx+1)\arctan(cx)^2+2i\left(\frac{\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)}{2}-\frac{i\pi\left(-\operatorname{csgn}\left(\frac{\cdots}{(c\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$

input
```
int((a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I*a^2/d*ln(c^2*x^2+1)+a^2/d*arctan(c*x)+b^2/d*(-I*ln(1+I*c*x)*ar
ctan(c*x)^2+2*I*(1/2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/3*I*a
rctan(c*x)^3+1/4*I*Pi*(csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^
2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^
2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*
c*x)^2/(c^2*x^2+1)))+csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+
1)))^3-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*
c*x)^2/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(
I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+csgn((1+I*c*x)^2/
(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2-1)*arctan(c*x)^2-1/2*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(
c^2*x^2+1))+1/4*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))))+2/d*a*b*(-I*ln(1+I*c
*x)*arctan(c*x)-1/2*(ln(1+I*c*x)-ln(1/2+1/2*I*c*x))*ln(1/2-1/2*I*c*x)+1/2*
dilog(1/2+1/2*I*c*x)+1/4*ln(1+I*c*x)^2))
```

3.98.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*
x - I)) - 4*I*a^2)/(c*d*x - I*d), x)
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-I*a^2*log(I*c*d*x + d)/(c*d) + 1/96*(24*b^2*arctan(c*x)^3 + 12*I*b^2*arct
an(c*x)^2*log(c^2*x^2 + 1) + 6*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2 + 3*I*b^
2*log(c^2*x^2 + 1)^3 - 8*(48*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^
2 + 1)/(c^2*d*x^2 + d), x) - b^2*arctan(c*x)^3/(c*d) + 12*b^2*integrate(1/
16*log(c^2*x^2 + 1)^2/(c^2*d*x^2 + d), x) - 12*a*b*arctan(c*x)^2/(c*d))*c*
d - 96*I*c*d*integrate(1/16*(20*b^2*c*x*arctan(c*x)^2 + 3*b^2*c*x*log(c^2*
x^2 + 1)^2 + 32*a*b*c*x*arctan(c*x) + 4*b^2*arctan(c*x)*log(c^2*x^2 + 1))/
(c^2*d*x^2 + d), x))/(c*d)
```

3.98.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{d+c\,d\,x\,1{}\mathrm{i}}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i), x)
```

3.99 $\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx$

3.99.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d} + \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/d+I*b*(a+b*arctan(c*x))*polylog(2,-1
+2/(1+I*c*x))/d+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d
```

3.99.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.97

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = -\frac{i\left(b^2\pi^3+24a^2\arctan(cx)+48ab\arctan(cx)^2+24ib^2\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)+48iab\arctan(\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

output
```
((-1/24*I)*(b^2*Pi^3 + 24*a^2*ArcTan[c*x] + 48*a*b*ArcTan[c*x]^2 + (24*I)*
b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (48*I)*a*b*ArcTan[c*x]
*Log[1 - E^((2*I)*ArcTan[c*x])] + (24*I)*a^2*Log[c*x] - (12*I)*a^2*Log[1 +
 c^2*x^2] - 24*b^2*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 24*a*b
*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*b^2*PolyLog[3, E^((-2*I)*ArcTa
n[c*x])]))/d
```

3.99.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5403, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx$$

$$\Big\downarrow \text{5403}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}-\frac{2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{5529}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}-\frac{2bc\left(\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}\right)}{d}$$

$$\Big\downarrow \text{7164}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}-\frac{2bc\left(-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{4c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

output
```
((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)])/d - (2*b*c*(((-1/2*I)*(a +
b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1
 + I*c*x)])/(4*c)))/d
```

3.99.3.1 Defintions of rubi rules used

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.99.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.26 (sec) , antiderivative size = 1449, normalized size of antiderivative = 16.47

method	result	size
parts	Expression too large to display	1449
derivativedivides	Expression too large to display	1451
default	Expression too large to display	1451

input
```
int((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*ln(x)-1/2*a^2/d*ln(c^2*x^2+1)-I*a^2/d*arctan(c*x)+b^2/d*(arctan(c*x)
^2*ln(c*x)-arctan(c*x)^2*ln(c*x-I)+arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x
^2+1))-2/3*I*arctan(c*x)^3-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arc
tan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,(1+
I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(
c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,-(1+I*c
*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*
(csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+
I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*
c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x
^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csg
n((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I*((1+I*c*x)
^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^3-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(((1+I*c*x...
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.55

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx =$$
$$-\frac{b^2\log\left(\frac{2\,cx}{cx-i}\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+2\,b^2\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right)+4i\,ab\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,a^2\log\left(x\right)+4\,a^2\operatorname{lo}}{4\,d}$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
-1/4*(b^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 2*b^2*dilog(-
2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) + 4*I*a*b*dilog((c*x + I)/(
c*x - I) + 1) - 4*a^2*log(x) + 4*a^2*log((c*x - I)/c) - 2*b^2*polylog(3, -
(c*x + I)/(c*x - I)))/d
```

3.99.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^2}{cx^2-ix}\,dx + \int \frac{b^2\,\mathrm{atan}^2\,(cx)}{cx^2-ix}\,dx + \int \frac{2ab\,\mathrm{atan}\,(cx)}{cx^2-ix}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**2/(c*x**2 - I*x), x) + Integral(b**2*atan(c*x)**2/(c*x**2
- I*x), x) + Integral(2*a*b*atan(c*x)/(c*x**2 - I*x), x))/d
```

3.99.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-a^2*(log(I*c*x + 1)/d - log(x)/d) + 1/96*(-24*I*b^2*arctan(c*x)^3 + 12*b^
2*arctan(c*x)^2*log(c^2*x^2 + 1) - 6*I*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2
+ 3*b^2*log(c^2*x^2 + 1)^3 - 2*(384*b^2*c^2*integrate(1/16*x^2*arctan(c*x)
^2/(c^2*d*x^3 + d*x), x) + 192*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*
x^2 + 1)/(c^2*d*x^3 + d*x), x) + b^2*log(c^2*x^2 + 1)^3/d - 576*b^2*integr
ate(1/16*arctan(c*x)^2/(c^2*d*x^3 + d*x), x) - 48*b^2*integrate(1/16*log(c
^2*x^2 + 1)^2/(c^2*d*x^3 + d*x), x) - 1536*a*b*integrate(1/16*arctan(c*x)/
(c^2*d*x^3 + d*x), x))*d - 8*I*(b^2*arctan(c*x)^3/d - 12*b^2*c*integrate(1
/16*x*log(c^2*x^2 + 1)^2/(c^2*d*x^3 + d*x), x) + 12*a*b*arctan(c*x)^2/d +
48*b^2*integrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x))*
d)/d
```

3.99.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)), x)
```

3.100 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx$

3.100.1 Optimal result

Integrand size = 25, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = &-\frac{ic(a+b\arctan(cx))^2}{d}-\frac{(a+b\arctan(cx))^2}{dx}\\&+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d}\\&-\frac{ic(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d}\\&-\frac{ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}\\&+\frac{bc(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d}\\&-\frac{ib^2c\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*b*c*(a+b*arctan(c*x))
*ln(2-2/(1-I*c*x))/d-I*c*(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/d-I*b^2*c*p
olylog(2,-1+2/(1-I*c*x))/d+b*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))
/d-1/2*I*b^2*c*polylog(3,-1+2/(1+I*c*x))/d
```

3.100.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx =$$
$$-\frac{\frac{2a^2}{x}+2a^2c\arctan(cx)+2ia^2c\log(x)-ia^2c\log\left(1+c^2x^2\right)+\frac{2ab\left(2cx\arctan(cx)^2+\arctan(cx)\left(2+2icx\log\left(1-e^{2i\arctan(}\right.\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

output
```
-1/2*((2*a^2)/x + 2*a^2*c*ArcTan[c*x] + (2*I)*a^2*c*Log[x] - I*a^2*c*Log[1
 + c^2*x^2] + (2*a*b*(2*c*x*ArcTan[c*x]^2 + ArcTan[c*x]*(2 + (2*I)*c*x*Log
[1 - E^((2*I)*ArcTan[c*x])]) + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2]) + c*x*
PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x + (2*I)*b^2*c*((-1/24*I)*Pi^3 + ArcT
an[c*x]^2 - (I*ArcTan[c*x]^2)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcT
an[c*x])] + (2*I)*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*
x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x])]
+ PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2))/d
```

3.100.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5405, 27, 5361, 5403, 5459, 5403, 2897, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx$$

$$\Big\downarrow \textbf{5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^2}dx}{d}-ic\int \frac{(a+b\arctan(cx))^2}{dx(icx+1)}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^2}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \; 5361$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \; 5403$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \; 5459$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \; 5403$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \; 2897$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \; 5529$$

3.100. $\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}dx$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}\right)\right)}{d}$$

$$\Big\downarrow \; 7164$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{4c}\right)\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

output
```
(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/
(1 - I*c*x)])/2)))/d - (I*c*((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)]
- 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/c -
 (b*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c))))/d
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.100.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.05 (sec) , antiderivative size = 8468, normalized size of antiderivative = 45.53

method	result	size
parts	Expression too large to display	8468
derivativedivides	Expression too large to display	8470
default	Expression too large to display	8470

input
```
int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.100.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/4*(I*b^2*c*x*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 2*I*b^2*
c*x*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 2*I*b^2*c*x*po
lylog(3, -(c*x + I)/(c*x - I)) + b^2*log(-(c*x + I)/(c*x - I))^2 + 4*d*x*i
ntegral((-I*a^2*c*x + a^2 + ((a*b + I*b^2)*c*x + I*a*b)*log(-(c*x + I)/(c*
x - I)))/(c^2*d*x^4 + d*x^2), x))/(d*x)
```

3.100.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx=-\frac{i\left(\int \frac{a^2}{cx^3-ix^2}\,dx+\int \frac{b^2\operatorname{atan}^2(cx)}{cx^3-ix^2}\,dx+\int \frac{2ab\operatorname{atan}(cx)}{cx^3-ix^2}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**2/(c*x**3 - I*x**2), x) + Integral(b**2*atan(c*x)**2/(c*x*
*3 - I*x**2), x) + Integral(2*a*b*atan(c*x)/(c*x**3 - I*x**2), x))/d
```

3.100.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^2*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x)) - 1/96*(24*b^2*c*x*arc
tan(c*x)^3 + 3*I*b^2*c*x*log(c^2*x^2 + 1)^3 + 24*b^2*arctan(c*x)^2 - 2*I*(
384*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + b^2
*c*log(c^2*x^2 + 1)^3/d + 12*b^2*c*arctan(c*x)^2/d - 576*b^2*c*integrate(1
/16*x*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) - 48*b^2*c*integrate(1/16*x*lo
g(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 1536*a*b*c*integrate(1/16*x*arc
tan(c*x)/(c^2*d*x^4 + d*x^2), x) + 192*b^2*c*integrate(1/16*x*log(c^2*x^2
+ 1)/(c^2*d*x^4 + d*x^2), x) - 192*b^2*integrate(1/16*arctan(c*x)*log(c^2*
x^2 + 1)/(c^2*d*x^4 + d*x^2), x))*d*x - 16*(b^2*c*arctan(c*x)^3/d + 12*b^2
*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 24*b^
2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 24*b^2
*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) +
 48*b^2*c*integrate(1/16*x*arctan(c*x)/(c^2*d*x^4 + d*x^2), x) + 72*b^2*in
tegrate(1/16*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + 6*b^2*integrate(1/16*
log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 192*a*b*integrate(1/16*arctan
(c*x)/(c^2*d*x^4 + d*x^2), x))*d*x + 6*(b^2*c*x*arctan(c*x) - b^2)*log(c^2
*x^2 + 1)^2 + 12*(I*b^2*c*x*arctan(c*x)^2 + 2*I*b^2*arctan(c*x))*log(c^2*x
^2 + 1))/(d*x)
```

3.100.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)), x)
```

3.101 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$

3.101.1 Optimal result

Integrand size = 25, antiderivative size = 273

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx = & -\frac{bc(a+b\arctan(cx))}{dx}-\frac{3c^2(a+b\arctan(cx))^2}{2d} \\ & -\frac{(a+b\arctan(cx))^2}{2dx^2}+\frac{ic(a+b\arctan(cx))^2}{dx}+\frac{b^2c^2\log(x)}{d} \\ & -\frac{b^2c^2\log(1+c^2x^2)}{2d}-\frac{2ibc^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d} \\ & -\frac{c^2(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} \\ & -\frac{b^2c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d} \\ & -\frac{ibc^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d} \\ & -\frac{b^2c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

output
```
-b*c*(a+b*arctan(c*x))/d/x-3/2*c^2*(a+b*arctan(c*x))^2/d-1/2*(a+b*arctan(c
*x))^2/d/x^2+I*c*(a+b*arctan(c*x))^2/d/x+b^2*c^2*ln(x)/d-1/2*b^2*c^2*ln(c^
2*x^2+1)/d-2*I*b*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d-c^2*(a+b*arctan
(c*x))^2*ln(2-2/(1+I*c*x))/d-b^2*c^2*polylog(2,-1+2/(1-I*c*x))/d-I*b*c^2*(
a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d-1/2*b^2*c^2*polylog(3,-1+2/(1
+I*c*x))/d
```

3.101.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.37

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$$
$$= \frac{-\frac{a^2}{x^2}+\frac{2ia^2c}{x}+2ia^2c^2\arctan(cx)-2a^2c^2\log(x)+a^2c^2\log\left(1+c^2x^2\right)+\frac{2iab(2c^2x^2\arctan(cx)^2+\arctan(cx)(i+2cx+ic}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```

output
```
(-(a^2/x^2) + ((2*I)*a^2*c)/x + (2*I)*a^2*c^2*ArcTan[c*x] - 2*a^2*c^2*Log[
x] + a^2*c^2*Log[1 + c^2*x^2] + ((2*I)*a*b*(2*c^2*x^2*ArcTan[c*x]^2 + ArcT
an[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x
])]) + c*x*(I - 2*c*x*Log[c*x] + c*x*Log[1 + c^2*x^2]) + c^2*x^2*PolyLog[2
, E^((2*I)*ArcTan[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/(c*x
) - (3*ArcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2)/(c
*x) - ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*Lo
g[1 - E^((2*I)*ArcTan[c*x])] + Log[c*x] - Log[1 + c^2*x^2]/2 - I*ArcTan[c*
x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x])]
- PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2))/(2*d)
```

3.101.3 Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {5405, 27, 5361, 5405, 5361, 5403, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 2897, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^3}dx}{d}-ic\int \frac{(a+b\arctan(cx))^2}{dx^2(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^3}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \ 5361$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \ 5405$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \ 5361$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)}{d}$$

$$\Big\downarrow \ 5403$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}\ldots}{}\right.$$

$$d$$

$$\Big\downarrow \ 5453$$

$$\frac{bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}\ldots}{}\right.$$

$$d$$

$$\Big\downarrow \ 5361$$

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}\ldots}{}\right.$$

$$d$$

$$\Big\downarrow \ 243$$

3.101. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}dx$

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}}{}\right.}{d}$$

$\downarrow$ 47

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}}{}\right.}{d}$$

$\downarrow$ 14

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}}{}\right.}{d}$$

$\downarrow$ 16

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}}{}\right.}{d}$$

$\downarrow$ 5419

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b\,\mathrm{ar}}{}\right.}{d}$$

$\downarrow$ 5459

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a}{}\right.\right.}{d}$$

3.101. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}dx$

$$\Big\downarrow \text{5403}$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-ic\left(\log\left(2-\frac{2}{1+icx}\right)\right.\right.}{d}$$

$$\Big\downarrow \text{2897}$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a\right.\right.\right.}{d}$$

$$\Big\downarrow \text{5529}$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}\right.\right.\right.}{}$$

$$\Big\downarrow \text{7164}$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-ic\left(\log\left(2-\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(-((a + b*ArcTan[c*x])/x) - (c*(a +
b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2))/d - (I*c
*(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
 I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2
/(1 - I*c*x)])/2)) - I*c*((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)] - 2
*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/c - (b
*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c)))))/d
```

3.101. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$

3.101.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

3.101. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.101. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$

3.101.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.95 (sec) , antiderivative size = 1873, normalized size of antiderivative = 6.86

method	result	size
derivativedivides	Expression too large to display	1873
default	Expression too large to display	1873
parts	Expression too large to display	1877

input
```
int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^2/d/c^2/x^2+I*a^2/d/c/x-a^2/d*ln(c*x)+1/2*a^2/d*ln(c^2*x^2+1)+
I*a^2/d*arctan(c*x)+b^2/d*(-1/2/c^2/x^2*arctan(c*x)^2-2*polylog(3,(1+I*c*x
)/(c^2*x^2+1)^(1/2))+ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*c
*x)/(c^2*x^2+1)^(1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-3/2*arctan(c*x)^2
+arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-arctan(c*x)^2*ln(c*x)-arctan(
c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2
*x^2+1)^(1/2))-2*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*dilog((1+I*c*x)/(c
^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/
(c^2*x^2+1)))*arctan(c*x)^2+2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1
)^(1/2))+2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I
*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*
Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*I
*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2...
```

3.101.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/8*(2*b^2*c^2*x^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 4*b^
2*c^2*x^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 4*b^2*c^
2*x^2*polylog(3, -(c*x + I)/(c*x - I)) + 8*d*x^2*integral(1/2*(-2*I*a^2*c*
x + 2*a^2 + (2*b^2*c^2*x^2 + (2*a*b + I*b^2)*c*x + 2*I*a*b)*log(-(c*x + I)
/(c*x - I)))/(c^2*d*x^5 + d*x^3), x) + (-2*I*b^2*c*x + b^2)*log(-(c*x + I)
/(c*x - I))^2)/(d*x^2)
```

3.101.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^2}{cx^4-ix^3}\,dx + \int \frac{b^2\operatorname{atan}^2(cx)}{cx^4-ix^3}\,dx + \int \frac{2ab\operatorname{atan}(cx)}{cx^4-ix^3}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**2/(c*x**4 - I*x**3), x) + Integral(b**2*atan(c*x)**2/(c*x*
*4 - I*x**3), x) + Integral(2*a*b*atan(c*x)/(c*x**4 - I*x**3), x))/d
```

3.101.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a^2
- 1/96*(-24*I*b^2*c^2*x^2*arctan(c*x)^3 + 3*b^2*c^2*x^2*log(c^2*x^2 + 1)^3
 - 2*(384*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x)
 + b^2*c^2*log(c^2*x^2 + 1)^3/d + 12*b^2*c^2*arctan(c*x)^2/d + 96*b^2*c^2*
integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 192*b^2*c*in
tegrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 192*
b^2*c*integrate(1/16*x*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 576*b^2*integ
rate(1/16*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) + 48*b^2*integrate(1/16*lo
g(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 1536*a*b*integrate(1/16*arctan(
c*x)/(c^2*d*x^5 + d*x^3), x))*d*x^2 + 16*I*(b^2*c^2*arctan(c*x)^3/d + 12*b
^2*c^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) - 24*
b^2*c^3*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 24*b
^2*c^2*integrate(1/16*x^2*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 72*b^2*c*i
ntegrate(1/16*x*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) + 6*b^2*c*integrate(
1/16*x*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 192*a*b*c*integrate(1/
16*x*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) - 12*b^2*c*integrate(1/16*x*log(c
^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 24*b^2*integrate(1/16*arctan(c*x)*lo
g(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x))*d*x^2 + 12*(-2*I*b^2*c*x + b^2)*ar
ctan(c*x)^2 - 3*(2*I*b^2*c^2*x^2*arctan(c*x) - 2*I*b^2*c*x + b^2)*log(c^2*
x^2 + 1)^2 + 12*(b^2*c^2*x^2*arctan(c*x)^2 + (2*b^2*c*x + I*b^2)*arctan...
```

3.101.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^3\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)), x)
```

3.102 $\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$

3.102.1 Optimal result

Integrand size = 25, antiderivative size = 365

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = &-\frac{b^2c^2}{3dx}-\frac{b^2c^3\arctan(cx)}{3d}-\frac{bc(a+b\arctan(cx))}{3dx^2}\\
&+\frac{ibc^2(a+b\arctan(cx))}{dx}+\frac{11ic^3(a+b\arctan(cx))^2}{6d}\\
&-\frac{(a+b\arctan(cx))^2}{3dx^3}+\frac{ic(a+b\arctan(cx))^2}{2dx^2}\\
&+\frac{c^2(a+b\arctan(cx))^2}{dx}-\frac{ib^2c^3\log(x)}{d}+\frac{ib^2c^3\log(1+c^2x^2)}{2d}\\
&-\frac{8bc^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{3d}\\
&+\frac{ic^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d}\\
&+\frac{4ib^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{3d}\\
&-\frac{bc^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d}\\
&+\frac{ib^2c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

output
```
-1/3*b^2*c^2/d/x-1/3*b^2*c^3*arctan(c*x)/d-1/3*b*c*(a+b*arctan(c*x))/d/x^2
+I*b*c^2*(a+b*arctan(c*x))/d/x+11/6*I*c^3*(a+b*arctan(c*x))^2/d-1/3*(a+b*a
rctan(c*x))^2/d/x^3+1/2*I*c*(a+b*arctan(c*x))^2/d/x^2+c^2*(a+b*arctan(c*x)
)^2/d/x-I*b^2*c^3*ln(x)/d+1/2*I*b^2*c^3*ln(c^2*x^2+1)/d-8/3*b*c^3*(a+b*arc
tan(c*x))*ln(2-2/(1-I*c*x))/d+I*c^3*(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/
d+4/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))/d-b*c^3*(a+b*arctan(c*x))*polylo
g(2,-1+2/(1+I*c*x))/d+1/2*I*b^2*c^3*polylog(3,-1+2/(1+I*c*x))/d
```

3.102.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.48

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$$
$$= -\frac{a^2}{3dx^3}+\frac{ia^2c}{2dx^2}+\frac{a^2c^2}{dx}+\frac{a^2c^3\arctan(cx)}{d}+\frac{ia^2c^3\log(x)}{d}-\frac{ia^2c^3\log(1+c^2x^2)}{2d}$$
$$-\frac{2iabc^3\left(-\frac{1}{2cx}-\frac{i(1+c^2x^2)}{6c^2x^2}+\frac{4i\arctan(cx)}{3cx}-\frac{i(1+c^2x^2)\arctan(cx)}{3c^3x^3}-\frac{(1+c^2x^2)\arctan(cx)}{2c^2x^2}+\frac{1}{2}i\arctan(cx)^2-\arctan(c\right.}{d}$$
$$+\frac{b^2c^3\left(\pi^3-\frac{8}{cx}+\frac{24i\arctan(cx)}{cx}-\frac{8(1+c^2x^2)\arctan(cx)}{c^2x^2}+32i\arctan(cx)^2+\frac{32\arctan(cx)^2}{cx}-\frac{8(1+c^2x^2)\arctan(cx)^2}{c^3x^3}+\frac{1}{}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

output
```
-1/3*a^2/(d*x^3) + ((I/2)*a^2*c)/(d*x^2) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcT
an[c*x])/d + (I*a^2*c^3*Log[x])/d - ((I/2)*a^2*c^3*Log[1 + c^2*x^2])/d - (
(2*I)*a*b*c^3*(-1/2*1/(c*x) - ((I/6)*(1 + c^2*x^2))/(c^2*x^2) + (((4*I)/3)
*ArcTan[c*x])/(c*x) - ((I/3)*(1 + c^2*x^2)*ArcTan[c*x])/(c^3*x^3) - ((1 +
c^2*x^2)*ArcTan[c*x])/(2*c^2*x^2) + (I/2)*ArcTan[c*x]^2 - ArcTan[c*x]*Log[
1 - E^((2*I)*ArcTan[c*x])] - ((4*I)/3)*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (I/2
)*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])))/d + (b^2*c^3*(Pi^3
 - 8/(c*x) + ((24*I)*ArcTan[c*x])/(c*x) - (8*(1 + c^2*x^2)*ArcTan[c*x])/(c
^2*x^2) + (32*I)*ArcTan[c*x]^2 + (32*ArcTan[c*x]^2)/(c*x) - (8*(1 + c^2*x^
2)*ArcTan[c*x]^2)/(c^3*x^3) + ((12*I)*(1 + c^2*x^2)*ArcTan[c*x]^2)/(c^2*x^
2) + (24*I)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 64*ArcTan[c*x]
*Log[1 - E^((2*I)*ArcTan[c*x])] - (24*I)*Log[c*x] - (24*I)*Log[1/Sqrt[1 +
c^2*x^2]] - 24*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (32*I)*Pol
yLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*ArcTan[c*x])]
))/(24*d)
```

3.102.3 Rubi [A] (verified)

Time = 3.70 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.15, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {5405, 27, 5361, 5405, 5361, 5405, 5361, 5403, 5453, 5361, 243, 47, 14, 16, 264, 216, 5419, 5459, 5403, 2897, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^4}dx}{d}-ic\int \frac{(a+b\arctan(cx))^2}{dx^3(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^4}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(\int \frac{(a+b\arctan(cx))^2}{x^3}dx-ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-ic\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx\right)-\frac{(a+b\arctan(cx))^2}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx - ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx - \frac{(a+b\arctan(cx))^2}{x}\right) - \frac{(a+b\arctan(cx))^2}{2x^2}\right)}{d}$$

$$\Big\downarrow \; 5403$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx - ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2 - 2bc\int \frac{(a+b\arctan(cx))}{c^2x^2}\right.\right.\right.}{d}$$

$$\Big\downarrow \; 5453$$

$$\frac{\frac{2}{3}bc\left(\int \frac{a+b\arctan(cx)}{x^3}dx - c^2\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right) - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx - c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right) - ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx\right.\right.\right.}{d}$$

$$\Big\downarrow \; 5361$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right) + \frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx - \frac{a+b\arctan(cx)}{2x^2}\right) - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right) + bc\int \frac{1}{x(c^2x^2+1)}dx - \frac{a+b\arctan(cx)}{x}\right) - ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - ic\left(\log\left(2-\frac{2}{1+icx}\right.\right.\right.\right.}{d}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right) + \frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx - \frac{a+b\arctan(cx)}{2x^2}\right) - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right) + \frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2 - \frac{a+b\arctan(cx)}{x}\right) - ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - ic\left(\log\left(2-\right.\right.\right.\right.}{d}$$

$$\Big\downarrow \; 47$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right) + \frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx - \frac{a+b\arctan(cx)}{2x^2}\right) - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right) + \frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2 - c^2\int \frac{1}{c^2x^2+1}dx^2\right) - \frac{a+b\arctan(cx)}{x}\right) - ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - ic\right.\right.}{}$$

$$\Big\downarrow \; 14$$

3.102. $\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$

$$\frac{ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\right.}{}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$

$$\frac{ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\right.}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\left(c^2\left(-\int\frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$

$$\frac{ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\right.}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$

$$\frac{ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\right.}{d}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$

$$\frac{ic\left(-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\right.\right.}{d}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\int\frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)}{d}$$

$$\frac{ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\int\frac{a+b\arctan(cx)}{x(cx+i)}dx\right.\right.\right.}{}$$

3.102. $\int\frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$

$$\Big\downarrow \; 5403$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right.\right.\right.}{ic\left(-ic\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-ic\left(\log\left(2-\right.\right.\right.\right.}$$

$$d$$

$$\Big\downarrow \; 2897$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan}{2b}\right.\right.\right.}{ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\left(-i\log\left(2-\frac{2}{1-ic}\right.\right.\right.\right.\right.}$$

$$d$$

$$\Big\downarrow \; 5529$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan}{2b}\right.\right.\right.}{ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan}{2c}\right.\right.\right.\right.}$$

$$d$$

$$\Big\downarrow \; 7164$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan}{2b}\right.\right.\right.}{ic\left(bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\right.\right.\right.\right.}$$

$$d$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

3.102. $\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$

output
```
(-1/3*(a + b*ArcTan[c*x])^2/x^3 + (2*b*c*(-1/2*(a + b*ArcTan[c*x])/x^2 + (
b*c*(-x^(-1) - c*ArcTan[c*x]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b
 + I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 +
 2/(1 - I*c*x)])/2))))/3)/d - (I*c*(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(
-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2
] - Log[1 + c^2*x^2]))/2) - I*c*(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/
2*I)*(a + b*ArcTan[c*x])^2)/b + I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 -
 I*c*x)] - (b*PolyLog[2, -1 + 2/(1 - I*c*x)])/2)) - I*c*((a + b*ArcTan[c*x
])^2*Log[2 - 2/(1 + I*c*x)] - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog
[2, -1 + 2/(1 + I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c))))))
/d
```

3.102.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.102. $\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.102.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x)
```

output
```
int((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x)
```

3.102.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/24*(-6*I*b^2*c^3*x^3*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 -
12*I*b^2*c^3*x^3*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) + 1
2*I*b^2*c^3*x^3*polylog(3, -(c*x + I)/(c*x - I)) + 24*d*x^3*integral(1/6*(
-6*I*a^2*c*x + 6*a^2 + (-6*I*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 2*(3*a*b + I*b^
2)*c*x + 6*I*a*b)*log(-(c*x + I)/(c*x - I)))/(c^2*d*x^6 + d*x^4), x) - (6*
b^2*c^2*x^2 + 3*I*b^2*c*x - 2*b^2)*log(-(c*x + I)/(c*x - I))^2)/(d*x^3)
```

3.102.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^2}{cx^5-ix^4}\,dx+\int \frac{b^2\operatorname{atan}^2(cx)}{cx^5-ix^4}\,dx+\int \frac{2ab\operatorname{atan}(cx)}{cx^5-ix^4}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**2/x**4/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**2/(c*x**5 - I*x**4), x) + Integral(b**2*atan(c*x)**2/(c*x*
*5 - I*x**4), x) + Integral(2*a*b*atan(c*x)/(c*x**5 - I*x**4), x))/d
```

3.102.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*(6*I*c^3*log(I*c*x + 1)/d - 6*I*c^3*log(x)/d - (6*c^2*x^2 + 3*I*c*x -
 2)/(d*x^3))*a^2 + 1/96*(24*b^2*c^3*x^3*arctan(c*x)^3 + 3*I*b^2*c^3*x^3*lo
g(c^2*x^2 + 1)^3 - 2*I*(1152*b^2*c^5*integrate(1/48*x^5*arctan(c*x)^2/(c^2
*d*x^6 + d*x^4), x) + b^2*c^3*log(c^2*x^2 + 1)^3/d + 12*b^2*c^3*arctan(c*x
)^2/d + 288*b^2*c^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4
), x) + 192*b^2*c^2*integrate(1/48*x^2*arctan(c*x)/(c^2*d*x^6 + d*x^4), x)
 + 1728*b^2*c*integrate(1/48*x*arctan(c*x)^2/(c^2*d*x^6 + d*x^4), x) + 144
*b^2*c*integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*d*x^6 + d*x^4), x) + 4608*
a*b*c*integrate(1/48*x*arctan(c*x)/(c^2*d*x^6 + d*x^4), x) - 192*b^2*c*int
egrate(1/48*x*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x) + 576*b^2*integrate
(1/48*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x))*d*x^3 - 16*(b^
2*c^3*arctan(c*x)^3/d + 36*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(
c^2*d*x^6 + d*x^4), x) - 72*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c
^2*d*x^6 + d*x^4), x) + 72*b^2*c^3*integrate(1/48*x^3*arctan(c*x)/(c^2*d*x
^6 + d*x^4), x) - 12*b^2*c^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*d*x^
6 + d*x^4), x) + 72*b^2*c*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c
^2*d*x^6 + d*x^4), x) - 48*b^2*c*integrate(1/48*x*arctan(c*x)/(c^2*d*x^6 +
 d*x^4), x) - 216*b^2*integrate(1/48*arctan(c*x)^2/(c^2*d*x^6 + d*x^4), x)
 - 18*b^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*d*x^6 + d*x^4), x) - 576*
a*b*integrate(1/48*arctan(c*x)/(c^2*d*x^6 + d*x^4), x))*d*x^3 + 4*(6*b^...
```

3.102.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^4\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^4*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^4*(d + c*d*x*1i)), x)
```

3.103 $\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$

3.103.1 Optimal result

Integrand size = 25, antiderivative size = 433

$$\begin{aligned}\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx &= \frac{2iabx}{c^4d^2}-\frac{b^2x}{3c^4d^2}+\frac{b^2}{2c^5d^2(i-cx)}-\frac{b^2\arctan(cx)}{6c^5d^2}\\
&+\frac{2ib^2x\arctan(cx)}{c^4d^2}+\frac{bx^2(a+b\arctan(cx))}{3c^3d^2}\\
&+\frac{ib(a+b\arctan(cx))}{c^5d^2(i-cx)}+\frac{11i(a+b\arctan(cx))^2}{6c^5d^2}\\
&+\frac{3x(a+b\arctan(cx))^2}{c^4d^2}-\frac{ix^2(a+b\arctan(cx))^2}{c^3d^2}\\
&-\frac{x^3(a+b\arctan(cx))^2}{3c^2d^2}-\frac{(a+b\arctan(cx))^2}{c^5d^2(i-cx)}\\
&+\frac{20b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^5d^2}\\
&-\frac{4i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^5d^2}\\
&-\frac{ib^2\log(1+c^2x^2)}{c^5d^2}+\frac{10ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^5d^2}\\
&+\frac{4b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^2}\\
&-\frac{2ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^2}\end{aligned}$$

output
```
2*I*b^2*x*arctan(c*x)/c^4/d^2-1/3*b^2*x/c^4/d^2+1/2*b^2/c^5/d^2/(I-c*x)-1/
6*b^2*arctan(c*x)/c^5/d^2+11/6*I*(a+b*arctan(c*x))^2/c^5/d^2+1/3*b*x^2*(a+
b*arctan(c*x))/c^3/d^2-4*I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^5/d^2+10/
3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5/d^2+3*x*(a+b*arctan(c*x))^2/c^4/d^2+2
*I*a*b*x/c^4/d^2-1/3*x^3*(a+b*arctan(c*x))^2/c^2/d^2-(a+b*arctan(c*x))^2/c
^5/d^2/(I-c*x)+20/3*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5/d^2-I*b^2*ln(c
^2*x^2+1)/c^5/d^2-2*I*b^2*polylog(3,1-2/(1+I*c*x))/c^5/d^2-I*x^2*(a+b*arct
an(c*x))^2/c^3/d^2+4*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^5/d^2+
I*b*(a+b*arctan(c*x))/c^5/d^2/(I-c*x)
```

3.103.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.16

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = -\frac{-36a^2cx+12ia^2c^2x^2+4a^2c^3x^3-\frac{12a^2}{-i+cx}+48a^2\arctan(cx)-24ia^2\log(1+c^2x^2)+2ab(-2-12icx-}{}$$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/12*(-36*a^2*c*x + (12*I)*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(-I + c
*x) + 48*a^2*ArcTan[c*x] - (24*I)*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2 - (12*I
)*c*x - 2*c^2*x^2 + 48*ArcTan[c*x]^2 - 3*Cos[2*ArcTan[c*x]] + 20*Log[1 + c
^2*x^2] + 24*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 2*ArcTan[c*x]*(6*I - 18*
c*x + (6*I)*c^2*x^2 + 2*c^3*x^3 - (3*I)*Cos[2*ArcTan[c*x]] + (24*I)*Log[1
+ E^((2*I)*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]]) + (3*I)*Sin[2*ArcTan[c*x]
]) + b^2*(4*c*x - 4*ArcTan[c*x] - (24*I)*c*x*ArcTan[c*x] - 4*c^2*x^2*ArcTa
n[c*x] + (52*I)*ArcTan[c*x]^2 - 36*c*x*ArcTan[c*x]^2 + (12*I)*c^2*x^2*ArcT
an[c*x]^2 + 4*c^3*x^3*ArcTan[c*x]^2 + 32*ArcTan[c*x]^3 + (3*I)*Cos[2*ArcTa
n[c*x]] - 6*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]^2*Cos[2*Arc
Tan[c*x]] - 80*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (48*I)*ArcTan[
c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*Log[1 + c^2*x^2] + 8*(5*I +
 6*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (24*I)*PolyLog[3, -E^
((2*I)*ArcTan[c*x])] + 3*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Sin[2*ArcT
an[c*x]] - 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(c^5*d^2)
```

3.103.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{4i(a+b\arctan(cx))^2}{c^4d^2(cx-i)}+\frac{3(a+b\arctan(cx))^2}{c^4d^2}-\frac{(a+b\arctan(cx))^2}{c^4d^2(cx-i)^2}-\frac{2ix(a+b\arctan(cx))^2}{c^3d^2}-\frac{x^2(a+b\arctan(cx))^2}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^5d^2}-\frac{(a+b\arctan(cx))^2}{c^5d^2(-cx+i)}+\frac{11i(a+b\arctan(cx))^2}{6c^5d^2}+\frac{ib(a+b\arctan(cx))}{c^5d^2(-cx+i)}-\frac{4i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^5d^2}+\frac{20b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^5d^2}+\frac{3x(a+b\arctan(cx))^2}{c^4d^2}-\frac{ix^2(a+b\arctan(cx))^2}{c^3d^2}+\frac{bx^2(a+b\arctan(cx))}{3c^3d^2}-\frac{x^3(a+b\arctan(cx))^2}{3c^2d^2}+\frac{2iabx}{c^4d^2}-\frac{b^2\arctan(cx)}{6c^5d^2}+\frac{2ib^2x\arctan(cx)}{c^4d^2}+\frac{10ib^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^5d^2}-\frac{2ib^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^5d^2}+\frac{b^2}{2c^5d^2(-cx+i)}-\frac{b^2x}{3c^4d^2}-\frac{ib^2\log\left(c^2x^2+1\right)}{c^5d^2}$$

input
```
Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
((2*I)*a*b*x)/(c^4*d^2) - (b^2*x)/(3*c^4*d^2) + b^2/(2*c^5*d^2*(I - c*x))
- (b^2*ArcTan[c*x])/(6*c^5*d^2) + ((2*I)*b^2*x*ArcTan[c*x])/(c^4*d^2) + (b
*x^2*(a + b*ArcTan[c*x]))/(3*c^3*d^2) + (I*b*(a + b*ArcTan[c*x]))/(c^5*d^2
*(I - c*x)) + (((11*I)/6)*(a + b*ArcTan[c*x])^2)/(c^5*d^2) + (3*x*(a + b*A
rcTan[c*x])^2)/(c^4*d^2) - (I*x^2*(a + b*ArcTan[c*x])^2)/(c^3*d^2) - (x^3*
(a + b*ArcTan[c*x])^2)/(3*c^2*d^2) - (a + b*ArcTan[c*x])^2/(c^5*d^2*(I - c
*x)) + (20*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^5*d^2) - ((4*I)*
(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^5*d^2) - (I*b^2*Log[1 + c^2*x
^2])/(c^5*d^2) + (((10*I)/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2)
+ (4*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) - ((2*
I)*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^5*d^2)
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.103.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 36.24 (sec) , antiderivative size = 1199, normalized size of antiderivative = 2.77

method	result	size
derivativedivides	Expression too large to display	1199
default	Expression too large to display	1199
parts	Expression too large to display	1254

input
```
int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(2*I*a*b/d^2*c*x+2*a*b/d^2*arctan(c*x)/(c*x-I)+4*a*b/d^2*ln(c*x-I)*l
n(-1/2*I*(c*x+I))-11/12*I*a*b/d^2*arctan(1/2*c*x)+11/6*I*a*b/d^2*arctan(1/
2*c*x-1/2*I)-I*a*b/d^2/(c*x-I)+11/12*I*a*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)
-29/6*I*a*b/d^2*arctan(c*x)+1/3*a*b/d^2*c^2*x^2-I*a^2/d^2*c^2*x^2+2*I*a^2/
d^2*ln(c^2*x^2+1)+3*a^2/d^2*c*x-1/3*a^2/d^2*c^3*x^3-2*a*b/d^2*ln(c*x-I)^2+
4*a*b/d^2*dilog(-1/2*I*(c*x+I))-11/24*a*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)-29/
12*a*b/d^2*ln(c^2*x^2+1)-2*I*a*b/d^2*arctan(c*x)*c^2*x^2+6*a*b/d^2*arctan(
c*x)*c*x-2/3*a*b/d^2*arctan(c*x)*c^3*x^3+8*I*a*b/d^2*arctan(c*x)*ln(c*x-I)
+7/3*a*b/d^2+b^2/d^2*(-1/3*I-1/3*c*x-1/3*c^3*x^3*arctan(c*x)^2-8/3*arctan(
c*x)^3+3*arctan(c*x)^2*c*x-4*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1
))+2/3*arctan(c*x)*(c*x-I)*(c*x+I)-1/3*arctan(c*x)*(c*x-I)^2+20/3*arctan(c
*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+20/3*arctan(c*x)*ln(1-I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))+4*Pi*arctan(c*x)^2-2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+
1))-4*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arcta
n(c*x)^2-2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*
arctan(c*x)^2-2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2
+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+2*Pi*csgn(I/(1+(1+I*c*x)^
2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^
2*arctan(c*x)^2-2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/
(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*...
```

3.103.5 Fricas [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^4*log(-(c*x
+ I)/(c*x - I)) - 4*a^2*x^4)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.103.7 Maxima [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3/(c^6*d^2*x - I*c^5*d^2) - (c^2*x^3 + 3*I*c*x^2 - 9*x)/(c^4*d^2)
 + 12*I*log(c*x - I)/(c^5*d^2)) - 1/48*(48*(b^2*c*x - I*b^2)*arctan(c*x)^3
 - 6*(-I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 + 4*(b^2*c^4*x^4 + 2*I*b^2*c^3*
x^3 - 6*b^2*c^2*x^2 + 9*I*b^2*c*x - 3*b^2)*arctan(c*x)^2 - (b^2*c^4*x^4 +
2*I*b^2*c^3*x^3 - 6*b^2*c^2*x^2 + 9*I*b^2*c*x - 3*b^2 - 12*(b^2*c*x - I*b^
2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(c^6*d^2*x - I*c^5*d^2)*(288*b^2*c^
6*integrate(1/48*x^6*arctan(c*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2)
, x) + 24*b^2*c^6*integrate(1/48*x^6*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c
^6*d^2*x^2 + c^4*d^2), x) + 768*a*b*c^6*integrate(1/48*x^6*arctan(c*x)/(c^
8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 32*b^2*c^6*integrate(1/48*x^6*l
og(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 192*b^2*c^5*
integrate(1/48*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x
^2 + c^4*d^2), x) + 128*b^2*c^5*integrate(1/48*x^5*arctan(c*x)/(c^8*d^2*x^
4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 288*b^2*c^4*integrate(1/48*x^4*arctan(c
*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 24*b^2*c^4*integrate(1
/48*x^4*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 7
68*a*b*c^4*integrate(1/48*x^4*arctan(c*x)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c
^4*d^2), x) - 160*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^8*d^2*x^4
 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 704*b^2*c^3*integrate(1/48*x^3*arctan(c*
x)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 768*b^2*c^2*integrate(...
```

3.103.8 Giac [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{x^4\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.104 $\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$

3.104.1 Optimal result

Integrand size = 25, antiderivative size = 364

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx &= \frac{abx}{c^3d^2}-\frac{ib^2}{2c^4d^2(i-cx)}+\frac{ib^2\arctan(cx)}{2c^4d^2}+\frac{b^2x\arctan(cx)}{c^3d^2}\\&\quad+\frac{b(a+b\arctan(cx))}{c^4d^2(i-cx)}+\frac{(a+b\arctan(cx))^2}{c^4d^2}\\&\quad-\frac{2ix(a+b\arctan(cx))^2}{c^3d^2}-\frac{x^2(a+b\arctan(cx))^2}{2c^2d^2}\\&\quad+\frac{i(a+b\arctan(cx))^2}{c^4d^2(i-cx)}-\frac{4ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^2}\\&\quad-\frac{3(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^4d^2}\\&\quad-\frac{b^2\log\left(1+c^2x^2\right)}{2c^4d^2}+\frac{2b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^2}\\&\quad-\frac{3ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^2}\\&\quad-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^2}\end{aligned}$$

output
```
a*b*x/c^3/d^2-1/2*I*b^2/c^4/d^2/(I-c*x)+1/2*I*b^2*arctan(c*x)/c^4/d^2+b^2*
x*arctan(c*x)/c^3/d^2+b*(a+b*arctan(c*x))/c^4/d^2/(I-c*x)+(a+b*arctan(c*x)
)^2/c^4/d^2-2*I*x*(a+b*arctan(c*x))^2/c^3/d^2-1/2*x^2*(a+b*arctan(c*x))^2/
c^2/d^2+I*(a+b*arctan(c*x))^2/c^4/d^2/(I-c*x)-4*I*b*(a+b*arctan(c*x))*ln(2
/(1+I*c*x))/c^4/d^2-3*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d^2-1/2*b^2*
ln(c^2*x^2+1)/c^4/d^2+2*b^2*polylog(2,1-2/(1+I*c*x))/c^4/d^2-3*I*b*(a+b*ar
ctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^4/d^2-3/2*b^2*polylog(3,1-2/(1+I*c*x
))/c^4/d^2
```

3.104.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.18

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx =$$

$$-\frac{8ia^2cx+2a^2c^2x^2+\frac{4ia^2}{-i+cx}-12ia^2\arctan(cx)-6a^2\log\left(1+c^2x^2\right)+2ab\big(-2cx-12i\arctan(cx)^2+i\,\mathrm{co}}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/4*((8*I)*a^2*c*x + 2*a^2*c^2*x^2 + ((4*I)*a^2)/(-I + c*x) - (12*I)*a^2*
ArcTan[c*x] - 6*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2*c*x - (12*I)*ArcTan[c*x]^
2 + I*Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((
2*I)*ArcTan[c*x])] + 2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan
[c*x]] + 6*Log[1 + E^((2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*
ArcTan[c*x]]) + b^2*(-4*c*x*ArcTan[c*x] + 10*ArcTan[c*x]^2 + (8*I)*c*x*Arc
Tan[c*x]^2 + 2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*ArcTan[c*x]^3 + Cos[2*ArcTan[
c*x]] + (2*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 2*ArcTan[c*x]^2*Cos[2*ArcTa
n[c*x]] + (16*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*ArcTan[c*
x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 2*Log[1 + c^2*x^2] + 4*(2 - (3*I)*Ar
cTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 6*PolyLog[3, -E^((2*I)*Arc
Tan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (2*
I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(c^4*d^2)
```

3.104.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{3(a+b\arctan(cx))^2}{c^3d^2(cx-i)}-\frac{2i(a+b\arctan(cx))^2}{c^3d^2}+\frac{i(a+b\arctan(cx))^2}{c^3d^2(cx-i)^2}-\frac{x(a+b\arctan(cx))^2}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^4d^2}+\frac{b(a+b\arctan(cx))}{c^4d^2(-cx+i)}+\frac{i(a+b\arctan(cx))^2}{c^4d^2(-cx+i)}+\frac{(a+b\arctan(cx))^2}{c^4d^2}-\frac{4ib\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^2}-\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^4d^2}-\frac{2ix(a+b\arctan(cx))^2}{c^3d^2}-\frac{x^2(a+b\arctan(cx))^2}{2c^2d^2}+\frac{abx}{c^3d^2}+\frac{ib^2\arctan(cx)}{2c^4d^2}+\frac{b^2x\arctan(cx)}{c^3d^2}+\frac{2b^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^4d^2}-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^4d^2}-\frac{ib^2}{2c^4d^2(-cx+i)}-\frac{b^2\log(c^2x^2+1)}{2c^4d^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
(a*b*x)/(c^3*d^2) - ((I/2)*b^2)/(c^4*d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*
x])/(c^4*d^2) + (b^2*x*ArcTan[c*x])/(c^3*d^2) + (b*(a + b*ArcTan[c*x]))/(c
^4*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(c^4*d^2) - ((2*I)*x*(a + b*ArcT
an[c*x])^2)/(c^3*d^2) - (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d^2) + (I*(a +
b*ArcTan[c*x])^2)/(c^4*d^2*(I - c*x)) - ((4*I)*b*(a + b*ArcTan[c*x])*Log[2
/(1 + I*c*x)])/(c^4*d^2) - (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c
^4*d^2) - (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(c^4*d^2) - ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^4*d^2) - (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^4*d^2)
```

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.104.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.60 (sec) , antiderivative size = 1104, normalized size of antiderivative = 3.03

method	result	size
derivativedivides	Expression too large to display	1104
default	Expression too large to display	1104
parts	Expression too large to display	1158

input
```
int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(3/2*I*a*b/d^2*ln(c^2*x^2+1)-1/2*a^2/d^2*c^2*x^2-3*I*a*b/d^2*dilog(-
1/2*I*(c*x+I))+3/2*a^2/d^2*ln(c^2*x^2+1)-2*I*a^2/d^2*c*x+b^2/d^2*(3*I*Pi*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1
/2*c^2*x^2*arctan(c*x)^2+I*arctan(c*x)*(c*x+I)/(2*c*x-2*I)+3*arctan(c*x)^2
*ln(c*x-I)-3*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-4*I*arctan(c*x)
*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*Pi*arctan(c*x)^2-3/2*polylog(3,-(
1+I*c*x)^2/(c^2*x^2+1))-4*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*dilog(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arctan(c*x)^2+3*I*arctan(c*x)*polylog(2,
-(1+I*c*x)^2/(c^2*x^2+1))+arctan(c*x)*(c*x-I)+3/2*I*Pi*csgn((1+I*c*x)^2/(c
^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+ln(1+(1+I*c*x)^2/(c
^2*x^2+1))-I*arctan(c*x)^2/(c*x-I)+2*I*arctan(c*x)^3-4*I*arctan(c*x)*ln(1-
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))
)*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^
2+3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2
+3/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-2*I*arctan(c*x)^2*c*x+1/4*(c*x+I)/
(c*x-I))-I*a^2/d^2/(c*x-I)-a*b/d^2*arctan(c*x)*c^2*x^2-2*I*a*b/d^2*arctan(
c*x)/(c*x-I)+6*a*b/d^2*arctan(c*x)*ln(c*x-I)+3/2*I*a*b/d^2*ln(c*x-I)^2+1/4
*I*a*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)-3*I*a*b/d^2*ln(-1/2*I*(c*x+I))*ln(c...
```

3.104.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2x^3}{\left(i\,cdx+d\right)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^3*log(-(c*x
+ I)/(c*x - I)) - 4*a^2*x^3)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(2*I/(c^5*d^2*x - I*c^4*d^2) + (c*x^2 + 4*I*x)/(c^3*d^2) - 6*log(
c*x - I)/(c^4*d^2)) + 1/32*(24*(I*b^2*c*x + b^2)*arctan(c*x)^3 - 3*(b^2*c*
x - I*b^2)*log(c^2*x^2 + 1)^3 - 4*(b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 + 4*b^2*c
*x + 2*I*b^2)*arctan(c*x)^2 + (b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 + 4*b^2*c*x +
 2*I*b^2 + 6*(I*b^2*c*x + b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 - 2*(c^5*d^
2*x - I*c^4*d^2)*(192*b^2*c^5*integrate(1/16*x^5*arctan(c*x)^2/(c^7*d^2*x^
4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 16*b^2*c^5*integrate(1/16*x^5*log(c^2*x
^2 + 1)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 512*a*b*c^5*integr
ate(1/16*x^5*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 32*
b^2*c^5*integrate(1/16*x^5*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 +
 c^3*d^2), x) + 128*b^2*c^4*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1
)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 192*b^2*c^4*integrate(1/16
*x^4*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 6*b^2*c^3*(
c^2/(c^11*d^2*x^2 + c^9*d^2) + log(c^2*x^2 + 1)/(c^9*d^2*x^2 + c^7*d^2)) -
 576*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2
 + c^3*d^2), x) - 112*b^2*c^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^7*d
^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 512*a*b*c^3*integrate(1/16*x^3*arc
tan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 160*b^2*c^3*integra
te(1/16*x^3*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) +
 320*b^2*c^2*integrate(1/16*x^2*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^...
```

3.104.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.105 $\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$

3.105.1 Optimal result

Integrand size = 25, antiderivative size = 292

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = & -\frac{b^2}{2c^3d^2(i-cx)}+\frac{b^2\arctan(cx)}{2c^3d^2}-\frac{ib(a+b\arctan(cx))}{c^3d^2(i-cx)}\\ & -\frac{i(a+b\arctan(cx))^2}{2c^3d^2}-\frac{x(a+b\arctan(cx))^2}{c^2d^2}\\ & +\frac{(a+b\arctan(cx))^2}{c^3d^2(i-cx)}-\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2}\\ & +\frac{2i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d^2}\\ & -\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}\\ & -\frac{2b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}\\ & +\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^3d^2}\end{aligned}$$

output
```
-1/2*b^2/c^3/d^2/(I-c*x)+1/2*b^2*arctan(c*x)/c^3/d^2-I*b*(a+b*arctan(c*x))
/c^3/d^2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^2/c^3/d^2-x*(a+b*arctan(c*x))^2/c
^2/d^2+(a+b*arctan(c*x))^2/c^3/d^2/(I-c*x)-2*b*(a+b*arctan(c*x))*ln(2/(1+I
*c*x))/c^3/d^2+2*I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d^2-I*b^2*polyl
og(2,1-2/(1+I*c*x))/c^3/d^2-2*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))
/c^3/d^2+I*b^2*polylog(3,1-2/(1+I*c*x))/c^3/d^2
```

3.105.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.24

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx =$$
$$-\frac{12a^2cx+\frac{12a^2}{-i+cx}-24a^2\arctan(cx)+12ia^2\log\left(1+c^2x^2\right)+b^2\left(-12i\arctan(cx)^2+12cx\arctan(cx)^2-\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/12*(12*a^2*c*x + (12*a^2)/(-I + c*x) - 24*a^2*ArcTan[c*x] + (12*I)*a^2*
Log[1 + c^2*x^2] + b^2*((-12*I)*ArcTan[c*x]^2 + 12*c*x*ArcTan[c*x]^2 - 16*
ArcTan[c*x]^3 - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan[c*x]*Cos[2*ArcTan[c*x]
] + (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]*Log[1 + E^((2*
I)*ArcTan[c*x])] - (24*I)*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 1
2*(I + 2*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (12*I)*PolyLog[
3, -E^((2*I)*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[
2*ArcTan[c*x]] + 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + 6*a*b*(-8*ArcTan[c*
x]^2 + Cos[2*ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*PolyLog[2, -E^((2*I)*Ar
cTan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2*c*x + I*Cos[2*ArcTan
[c*x]] - (4*I)*Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(c^3
*d^2)
```

3.105.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{2i(a+b\arctan(cx))^2}{c^2d^2(cx-i)}-\frac{(a+b\arctan(cx))^2}{c^2d^2}+\frac{(a+b\arctan(cx))^2}{c^2d^2(cx-i)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2b\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^3d^2}-\frac{ib(a+b\arctan(cx))}{c^3d^2(-cx+i)}+\frac{(a+b\arctan(cx))^2}{c^3d^2(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{2c^3d^2}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^3d^2}+\frac{2i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^3d^2}-\frac{x(a+b\arctan(cx))^2}{c^2d^2}+\frac{b^2\arctan(cx)}{2c^3d^2}-\frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^3d^2}+\frac{ib^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^3d^2}-\frac{b^2}{2c^3d^2(-cx+i)}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/2*b^2/(c^3*d^2*(I - c*x)) + (b^2*ArcTan[c*x])/(2*c^3*d^2) - (I*b*(a + b
*ArcTan[c*x]))/(c^3*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^
2) - (x*(a + b*ArcTan[c*x])^2)/(c^2*d^2) + (a + b*ArcTan[c*x])^2/(c^3*d^2*
(I - c*x)) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + ((2*
I)*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^2) - (I*b^2*PolyLog[2,
 1 - 2/(1 + I*c*x)])/(c^3*d^2) - (2*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(c^3*d^2) + (I*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.105.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.00 (sec) , antiderivative size = 4182, normalized size of antiderivative = 14.32

method	result	size
derivativedivides	Expression too large to display	4182
default	Expression too large to display	4182
parts	Expression too large to display	4230

input
```
int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a^2/d^2*c*x-a^2/d^2/(c*x-I)+2*a^2/d^2*arctan(c*x)+1/4*I*a*b/d^2*ar
ctan(1/2*c*x)+b^2/d^2*(4/3*arctan(c*x)^3-arctan(c*x)^2*c*x-3/2*arctan(c*x)
*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x
^2+1))-1/2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)
*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*Pi*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))+2*Pi*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*Pi*arctan(c*x)^2-Pi*p
olylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3
/2*I*arctan(c*x)^2+2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x
^2+1)))^2*arctan(c*x)^2+Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^3*arctan(c*x)^2+Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)
^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-Pi*csgn(I/(1+(
1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^2*arctan(c*x)^2+Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*
c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1
)))*arctan(c*x)^2+1/2*arctan(c*x)*(c*x+I)/(c*x-I)-arctan(c*x)^2/(c*x-I)-I*
Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+
I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-2*I*Pi*
arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*I*Pi*arctan(c*x)*ln(1+I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))+2*I*Pi*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))-1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^...
```

3.105.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^2*log(-(c*x
+ I)/(c*x - I)) - 4*a^2*x^2)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.105.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-a^2*(1/(c^4*d^2*x - I*c^3*d^2) + x/(c^2*d^2) + 2*I*log(c*x - I)/(c^3*d^2)
) + 1/16*(8*(b^2*c*x - I*b^2)*arctan(c*x)^3 - (-I*b^2*c*x - b^2)*log(c^2*x
^2 + 1)^3 - 4*(b^2*c^2*x^2 - I*b^2*c*x + b^2)*arctan(c*x)^2 + (b^2*c^2*x^2
 - I*b^2*c*x + b^2 + 2*(b^2*c*x - I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 -
 2*(c^4*d^2*x - I*c^3*d^2)*(96*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c
^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 8*b^2*c^4*integrate(1/16*x^4*l
og(c^2*x^2 + 1)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 256*a*b*c^
4*integrate(1/16*x^4*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2),
x) + 32*b^2*c^4*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d
^2*x^2 + c^2*d^2), x) + 64*b^2*c^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*
x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 64*b^2*c^3*integrat
e(1/16*x^3*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 32*b^
2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*
d^2), x) + 24*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^6*d^2*x^4 +
 2*c^4*d^2*x^2 + c^2*d^2), x) - 256*a*b*c^2*integrate(1/16*x^2*arctan(c*x)
/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 64*b^2*c^2*integrate(1/16*x
^2*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - (c*(x/(c
^6*d^2*x^2 + c^4*d^2) + arctan(c*x)/(c^5*d^2)) - 2*arctan(c*x)/(c^6*d^2*x^
2 + c^4*d^2))*b^2*c + 128*b^2*integrate(1/16*arctan(c*x)^2/(c^6*d^2*x^4 +
2*c^4*d^2*x^2 + c^2*d^2), x) + 32*b^2*integrate(1/16*log(c^2*x^2 + 1)^2...
```

3.105.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan{(cx)}+a)^2x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.106 $\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$

3.106.1 Optimal result

Integrand size = 23, antiderivative size = 216

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx &= \frac{ib^2}{2c^2d^2(i-cx)}-\frac{ib^2\arctan(cx)}{2c^2d^2}-\frac{b(a+b\arctan(cx))}{c^2d^2(i-cx)}\\ &+\frac{(a+b\arctan(cx))^2}{2c^2d^2}-\frac{i(a+b\arctan(cx))^2}{c^2d^2(i-cx)}\\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2}\\ &+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2}\\ &+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d^2}\end{aligned}$$

output
```
1/2*I*b^2/c^2/d^2/(I-c*x)-1/2*I*b^2*arctan(c*x)/c^2/d^2-b*(a+b*arctan(c*x)
)/c^2/d^2/(I-c*x)+1/2*(a+b*arctan(c*x))^2/c^2/d^2-I*(a+b*arctan(c*x))^2/c^
2/d^2/(I-c*x)+(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^2/d^2+I*b*(a+b*arctan(
c*x))*polylog(2,1-2/(1+I*c*x))/c^2/d^2+1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^
2/d^2
```

3.106.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.39

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$$
$$= \frac{\frac{12ia^2}{-i+cx} - 12ia^2\arctan(cx) - 6a^2\log\left(1+c^2x^2\right) - 6iab\left(4\arctan(cx)^2 - \cos(2\arctan(cx)) + 2\,\mathrm{PolyLog}\left(2, -\right.\right.}{}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
(((12*I)*a^2)/(-I + c*x) - (12*I)*a^2*ArcTan[c*x] - 6*a^2*Log[1 + c^2*x^2]
 - (6*I)*a*b*(4*ArcTan[c*x]^2 - Cos[2*ArcTan[c*x]] + 2*PolyLog[2, -E^((2*I
)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*(Cos[2*ArcTan[c*x]] - 2*Log[1 + E^((2*
I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + I*Sin[2*ArcTan[c*x]]) + b^2*((-
8*I)*ArcTan[c*x]^3 + 3*Cos[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Cos[2*ArcTan
[c*x]] - 6*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 12*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] - (12*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])
] + 6*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - (3*I)*Sin[2*ArcTan[c*x]] + 6*Ar
cTan[c*x]*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(1
2*c^2*d^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$$
$$\Big\downarrow \; 5411$$
$$\int \left(-\frac{(a+b\arctan(cx))^2}{cd^2(cx-i)} - \frac{i(a+b\arctan(cx))^2}{cd^2(cx-i)^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^2d^2}-\frac{b(a+b\arctan(cx))}{c^2d^2(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{c^2d^2(-cx+i)}+\frac{(a+b\arctan(cx))^2}{2c^2d^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^2d^2}-\frac{ib^2\arctan(cx)}{2c^2d^2}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^2d^2}+\frac{ib^2}{2c^2d^2(-cx+i)}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
((I/2)*b^2)/(c^2*d^2*(I - c*x)) - ((I/2)*b^2*ArcTan[c*x])/(c^2*d^2) - (b*(
a + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(2*c^2*d^2
) - (I*(a + b*ArcTan[c*x])^2)/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])^2
*Log[2/(1 + I*c*x)])/(c^2*d^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(c^2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d^2)
```

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.106.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.03 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.97

method	result
derivativedivides	$\dfrac{-\frac{iab\ln\left(c^2x^2+1\right)}{4d^2}-\frac{a^2\ln\left(c^2x^2+1\right)}{2d^2}-\frac{iab\ln(cx-i)^2}{2d^2}+\frac{b^2\left(\frac{i\arctan(cx)^2}{cx-i}-\arctan(cx)^2\ln(cx-i)+\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)-\frac{2i\,\mathrm{ar}}{}\right.}{}}{}$
default	$\dfrac{-\frac{iab\ln\left(c^2x^2+1\right)}{4d^2}-\frac{a^2\ln\left(c^2x^2+1\right)}{2d^2}-\frac{iab\ln(cx-i)^2}{2d^2}+\frac{b^2\left(\frac{i\arctan(cx)^2}{cx-i}-\arctan(cx)^2\ln(cx-i)+\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)-\frac{2i\,\mathrm{ar}}{}\right.}{}}{}$
parts	$-\frac{ia^2}{d^2c^2(-cx+i)}-\frac{a^2\ln(c^2x^2+1)}{2d^2c^2}-\frac{ia^2\arctan(cx)}{d^2c^2}+\frac{b^2\left(\frac{i\arctan(cx)^2}{cx-i}-\arctan(cx)^2\ln(cx-i)+\arctan(cx)^2\ln\left(\frac{2i(}{c}\right.\right.}{}$

input
```
int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/4*I*a*b/d^2*ln(c^2*x^2+1)-1/2*a^2/d^2*ln(c^2*x^2+1)-1/2*I*a*b/d^
2*ln(c*x-I)^2+b^2/d^2*(I*arctan(c*x)^2/(c*x-I)-arctan(c*x)^2*ln(c*x-I)+arc
tan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2/3*I*arctan(c*x)^3+1/2*I*Pi*cs
gn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x
)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+
1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+
(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x
^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(
c*x)^2-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*ar
ctan(c*x)^2+1/2*arctan(c*x)^2+I*Pi*arctan(c*x)^2-2*I*arctan(c*x)*(c*x+I)/(
4*c*x-4*I)-1/4*(c*x+I)/(c*x-I)-I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x
^2+1))+1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+I*a*b/d^2*dilog(-1/2*I*(c*
x+I))-2*a*b/d^2*arctan(c*x)*ln(c*x-I)+1/8*I*a*b/d^2*ln(c^4*x^4+10*c^2*x^2+
9)-1/4*a*b/d^2*arctan(1/2*c*x)+1/4*a*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/2
*a*b/d^2*arctan(1/2*c*x-1/2*I)+a*b/d^2/(c*x-I)+I*a*b/d^2*ln(-1/2*I*(c*x+I)
)*ln(c*x-I)+1/2*a*b/d^2*arctan(c*x)+I*a^2/d^2/(c*x-I)-I*a^2/d^2*arctan(c*x
)+2*I*a*b/d^2*arctan(c*x)/(c*x-I))
```

3.106.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x*log(-(c*x + I)
/(c*x - I)) - 4*a^2*x)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.106.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
a^2*(I/(c^3*d^2*x - I*c^2*d^2) - log(c*x - I)/(c^2*d^2)) - 1/32*(-8*I*b^2*
arctan(c*x)^2 - 8*(-I*b^2*c*x - b^2)*arctan(c*x)^3 - (b^2*c*x - I*b^2)*log
(c^2*x^2 + 1)^3 - 2*(-I*b^2 + (-I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2
+ 1)^2 - (2*b^2*c^3*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/(c^7*d
^2*x^2 + c^5*d^2)) - 640*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^5*d^2
*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 96*b^2*c^3*integrate(1/16*x^3*log(c^2*
x^2 + 1)^2/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 1024*a*b*c^3*integr
ate(1/16*x^3*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 256*b
^2*c^2*integrate(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^5*d^2*x^4 + 2*c^
3*d^2*x^2 + c*d^2), x) + 256*b^2*c^2*integrate(1/16*x^2*arctan(c*x)/(c^5*d
^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) + 16*(c*(x/(c^5*d^2*x^2 + c^3*d^2) + a
rctan(c*x)/(c^4*d^2)) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2))*a*b*c + 128
*b^2*c*integrate(1/16*x*arctan(c*x)^2/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2
), x) + b^2*c*log(c^2*x^2 + 1)^2/(c^5*d^2*x^2 + c^3*d^2) + 256*b^2*integra
te(1/16*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x))*(c^3*d^2*x
- I*c^2*d^2) - 32*(I*c^3*d^2*x + c^2*d^2)*integrate(-1/8*(32*a*b*c^2*x^2*a
rctan(c*x) - b^2*log(c^2*x^2 + 1)^2 + 4*(2*b^2*c^2*x^2 - b^2)*arctan(c*x)^
2 - 2*(b^2*c^2*x^2 + b^2 + (b^2*c^3*x^3 - b^2*c*x)*arctan(c*x))*log(c^2*x^
2 + 1))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) + 4*(2*b^2*arctan(c*x) -
 (b^2*c*x - I*b^2)*arctan(c*x)^2)*log(c^2*x^2 + 1))/(c^3*d^2*x - I*c^2*...
```

3.106.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

3.107 $\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$

3.107.1 Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{b^2}{2cd^2(i-cx)} - \frac{b^2\arctan(cx)}{2cd^2} + \frac{ib(a+b\arctan(cx))}{cd^2(i-cx)} - \frac{i(a+b\arctan(cx))^2}{2cd^2} + \frac{i(a+b\arctan(cx))^2}{cd^2(1+icx)}$$

output
```
1/2*b^2/c/d^2/(I-c*x)-1/2*b^2*arctan(c*x)/c/d^2+I*b*(a+b*arctan(c*x))/c/d^
2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^2/c/d^2+I*(a+b*arctan(c*x))^2/c/d^2/(1+I
*c*x)
```

3.107.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = -\frac{-2a^2+2iab+b^2+b(2ia+b)(i+cx)\arctan(cx)+b^2(-1+icx)\arctan(cx)^2}{2cd^2(-i+cx)}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

output
```
-1/2*(-2*a^2 + (2*I)*a*b + b^2 + b*((2*I)*a + b)*(I + c*x)*ArcTan[c*x] + b
^2*(-1 + I*c*x)*ArcTan[c*x]^2)/(c*d^2*(-I + c*x))
```

3.107.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^2}{cd^2(1+icx)}-\frac{2ib\int \left(\frac{a+b\arctan(cx)}{2d(c^2x^2+1)}-\frac{a+b\arctan(cx)}{2d(i-cx)^2}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^2}{cd^2(1+icx)}-\frac{2ib\left(\frac{(a+b\arctan(cx))^2}{4bcd}-\frac{a+b\arctan(cx)}{2cd(-cx+i)}-\frac{ib\arctan(cx)}{4cd}+\frac{ib}{4cd(-cx+i)}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

output
```
(I*(a + b*ArcTan[c*x])^2)/(c*d^2*(1 + I*c*x)) - ((2*I)*b*(((I/4)*b)/(c*d*(
I - c*x)) - ((I/4)*b*ArcTan[c*x])/(c*d) - (a + b*ArcTan[c*x])/(2*c*d*(I -
c*x)) + (a + b*ArcTan[c*x])^2/(4*b*c*d)))/d
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.107.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(110) = 220$.

Time = 1.94 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{i a^2}{d^2(icx+1)} + \frac{b^2\left(\frac{i\arctan(cx)^2}{icx+1} - 2i\left(\frac{i\arctan(cx)\ln(cx+i)}{4} - \frac{i\arctan(cx)\ln(cx-i)}{4} + \frac{\arctan(cx)}{2cx-2i} - \frac{\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{8} + \frac{\ln(cx-i)^2}{16}\right.\right.}{d^2}$
default	$\frac{i a^2}{d^2(icx+1)} + \frac{b^2\left(\frac{i\arctan(cx)^2}{icx+1} - 2i\left(\frac{i\arctan(cx)\ln(cx+i)}{4} - \frac{i\arctan(cx)\ln(cx-i)}{4} + \frac{\arctan(cx)}{2cx-2i} - \frac{\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{8} + \frac{\ln(cx-i)^2}{16}\right.\right.}{d^2}$
parts	$\frac{i a^2}{d^2c(icx+1)} + \frac{b^2\left(\frac{i\arctan(cx)^2}{icx+1} - 2i\left(\frac{i\arctan(cx)\ln(cx+i)}{4} - \frac{i\arctan(cx)\ln(cx-i)}{4} + \frac{\arctan(cx)}{2cx-2i} - \frac{\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{8} + \ln(\right.\right.}{d^2c}$
risch	$\frac{i(cx+i)b^2\ln(icx+1)^2}{8d^2(cx-i)c} - \frac{ib(bcx\ln(-icx+1)+ib\ln(-icx+1)-2ib+4a)\ln(icx+1)}{4d^2(cx-i)c} + \frac{8i\ln(-icx+1)ab+4b^2\ln(-icx+1)}{}$

input
```
int((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(I*a^2/d^2/(1+I*c*x)+b^2/d^2*(I/(1+I*c*x)*arctan(c*x)^2-2*I*(1/4*I*arc
tan(c*x)*ln(c*x+I)-1/4*I*arctan(c*x)*ln(c*x-I)+1/2*arctan(c*x)/(c*x-I)-1/8
*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/16*ln(c*x-I)^2-1/4*I*arctan(c*x)-1/4*I/(c*
x-I)-1/8*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(-1/2*I*(-c*x+I))+1/16*ln(c*x+I)
^2))+2*I*a*b/d^2/(1+I*c*x)*arctan(c*x)-I*a*b/d^2*arctan(c*x)-I*a*b/d^2/(c*
x-I))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{(a + b\arctan(cx))^2}{(d + icdx)^2}\,dx$$
$$= \frac{(i\,b^2cx - b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2 + 8\,a^2 - 8i\,ab - 4\,b^2 + 2\left((2\,ab - i\,b^2)cx + 2i\,ab + b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{8\left(c^2d^2x - i\,cd^2\right)}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/8*((I*b^2*c*x - b^2)*log(-(c*x + I)/(c*x - I))^2 + 8*a^2 - 8*I*a*b - 4*b
^2 + 2*((2*a*b - I*b^2)*c*x + 2*I*a*b + b^2)*log(-(c*x + I)/(c*x - I)))/(c
^2*d^2*x - I*c*d^2)
```

3.107.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(94) = 188$.

Time = 5.05 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.48

$$\int \frac{(a + b\arctan(cx))^2}{(d + icdx)^2}\,dx = -\frac{b(2a-ib)\log\left(-\frac{ib(2a-ib)}{c} + x(2ab-ib^2)\right)}{4cd^2} + \frac{b(2a-ib)\log\left(\frac{ib(2a-ib)}{c} + x(2ab-ib^2)\right)}{4cd^2} + \frac{(-2iab-b^2)\log(icx+1)}{2c^2d^2x-2icd^2} + \frac{(ib^2cx-b^2)\log(-icx+1)^2}{8c^2d^2x-8icd^2} + \frac{(ib^2cx-b^2)\log(icx+1)^2}{8c^2d^2x-8icd^2} + \frac{(4iab - ib^2cx\log(icx+1) + b^2\log(icx+1) + 2b^2)\log(-icx+1)}{4c^2d^2x-4icd^2} - \frac{-2a^2+2iab+b^2}{2c^2d^2x-2icd^2}$$

input
```
integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
-b*(2*a - I*b)*log(-I*b*(2*a - I*b)/c + x*(2*a*b - I*b**2))/(4*c*d**2) + b
*(2*a - I*b)*log(I*b*(2*a - I*b)/c + x*(2*a*b - I*b**2))/(4*c*d**2) + (-2*
I*a*b - b**2)*log(I*c*x + 1)/(2*c**2*d**2*x - 2*I*c*d**2) + (I*b**2*c*x -
b**2)*log(-I*c*x + 1)**2/(8*c**2*d**2*x - 8*I*c*d**2) + (I*b**2*c*x - b**2
)*log(I*c*x + 1)**2/(8*c**2*d**2*x - 8*I*c*d**2) + (4*I*a*b - I*b**2*c*x*l
og(I*c*x + 1) + b**2*log(I*c*x + 1) + 2*b**2)*log(-I*c*x + 1)/(4*c**2*d**2
*x - 4*I*c*d**2) - (-2*a**2 + 2*I*a*b + b**2)/(2*c**2*d**2*x - 2*I*c*d**2)
```

3.107.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\arctan(cx))^2}{(d + icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.107.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^2, x)
```

3.108 $\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx$

3.108.1 Optimal result

Integrand size = 25, antiderivative size = 221

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx =& -\frac{ib^2}{2d^2(i-cx)}+\frac{ib^2\arctan(cx)}{2d^2}+\frac{b(a+b\arctan(cx))}{d^2(i-cx)}\\ &-\frac{(a+b\arctan(cx))^2}{2d^2}+\frac{i(a+b\arctan(cx))^2}{d^2(i-cx)}\\ &+\frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2}\\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^2}\\ &+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2}\\ &+\frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}\end{aligned}$$

output
```
-1/2*I*b^2/d^2/(I-c*x)+1/2*I*b^2*arctan(c*x)/d^2+b*(a+b*arctan(c*x))/d^2/(
I-c*x)-1/2*(a+b*arctan(c*x))^2/d^2+I*(a+b*arctan(c*x))^2/d^2/(I-c*x)-2*(a+
b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^2+(a+b*arctan(c*x))^2*ln(2/(1+I
*c*x))/d^2+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2+1/2*b^2*pol
ylog(3,-1+2/(1+I*c*x))/d^2
```

3.108.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx$$
$$= \frac{-\frac{24ia^2}{-i+cx} - 24ia^2\arctan(cx) + 24a^2\log(cx) - 12a^2\log(1+c^2x^2) - 12ab(4i\arctan(cx)^2 + i\cos(2\arctan(c}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

output
```
(((-24*I)*a^2)/(-I + c*x) - (24*I)*a^2*ArcTan[c*x] + 24*a^2*Log[c*x] - 12*
a^2*Log[1 + c^2*x^2] - 12*a*b*((4*I)*ArcTan[c*x]^2 + I*Cos[2*ArcTan[c*x]]
+ (2*I)*PolyLog[2, E^((2*I)*ArcTan[c*x])] - 2*ArcTan[c*x]*(Cos[2*ArcTan[c*
x]] + 2*Log[1 - E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + Sin[2*Arc
Tan[c*x]]) + b^2*((-I)*Pi^3 - 6*Cos[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Co
s[2*ArcTan[c*x]] + 12*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]^2*
Log[1 - E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*
ArcTan[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (6*I)*Sin[2*ArcTan
[c*x]] - 12*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]^2*Sin[2*Ar
cTan[c*x]]))/(24*d^2)
```

3.108.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{(a+b\arctan(cx))^2}{d^2x} - \frac{c(a+b\arctan(cx))^2}{d^2(cx-i)} + \frac{ic(a+b\arctan(cx))^2}{d^2(cx-i)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{ib\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}+\frac{b(a+b\arctan(cx))}{d^2(-cx+i)}+\frac{i(a+b\arctan(cx))^2}{d^2(-cx+i)}-\frac{(a+b\arctan(cx))^2}{2d^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{ib^2\arctan(cx)}{2d^2}+\frac{b^2\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^2}-\frac{ib^2}{2d^2(-cx+i)}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

output
```
((-1/2*I)*b^2)/(d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x])/d^2 + (b*(a + b*A
rcTan[c*x]))/(d^2*(I - c*x)) - (a + b*ArcTan[c*x])^2/(2*d^2) + (I*(a + b*A
rcTan[c*x])^2)/(d^2*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
 + I*c*x)])/d^2 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (I*b*(a
 + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + (b^2*PolyLog[3, -1
 + 2/(1 + I*c*x)])/(2*d^2)
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.108.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.57 (sec) , antiderivative size = 1670, normalized size of antiderivative = 7.56

method	result	size
parts	Expression too large to display	1670
derivativedivides	Expression too large to display	1671
default	Expression too large to display	1671

input
```
int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^2*ln(x)+I*a^2/d^2/(-c*x+I)-1/2*a^2/d^2*ln(c^2*x^2+1)-I*a^2/d^2*arcta
n(c*x)+b^2/d^2*(2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I
*c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)^2-arctan(c*x)^2*ln((1+I*c*x)^2/(c
^2*x^2+1)-1)+arctan(c*x)^2*ln(c*x)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1
)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*p
olylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,-(1+I*c*x)
/(c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x
)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)
))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arcta
n(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-1
/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*ar
ctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^3*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+
(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/
(c^2*x^2+1)))^2*arctan(c*x)^2+arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2...
```

3.108.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 - (b^2*c*x - I*b^2)*log(2*c*x/(c*x
- I))*log(-(c*x + I)/(c*x - I))^2 - 2*(b^2*c*x - I*b^2)*dilog(-2*c*x/(c*x
- I) + 1)*log(-(c*x + I)/(c*x - I)) + 4*(c*d^2*x - I*d^2)*integral(-(a^2*c
*x + I*a^2 - ((-I*a*b - b^2)*c*x + a*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^
2*x^4 - I*c^2*d^2*x^3 + c*d^2*x^2 - I*d^2*x), x) + 2*(b^2*c*x - I*b^2)*pol
ylog(3, -(c*x + I)/(c*x - I)))/(c*d^2*x - I*d^2)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.108.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
a^2*(-I/(c*d^2*x - I*d^2) - log(c*x - I)/d^2 + log(x)/d^2) - 1/32*(8*I*b^2
*arctan(c*x)^2 - 8*(-I*b^2*c*x - b^2)*arctan(c*x)^3 - (b^2*c*x - I*b^2)*lo
g(c^2*x^2 + 1)^3 - 2*(I*b^2 + (-I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2
+ 1)^2 - (6*b^2*c^4*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(c^6*d
^2*x^2 + c^4*d^2)) - 256*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^4*d^2
*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 64*b^2*c^4*integrate(1/16*x^4*log(c^2*
x^2 + 1)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 256*b^2*c^3*integra
te(1/16*x^3*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 16*(c*
(x/(c^4*d^2*x^2 + c^2*d^2) + arctan(c*x)/(c^3*d^2)) - 2*arctan(c*x)/(c^4*d
^2*x^2 + c^2*d^2))*a*b*c^2 - 640*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/
(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 3*b^2*c^2*log(c^2*x^2 + 1)^2/(
c^4*d^2*x^2 + c^2*d^2) - 256*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^
2 + 1)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 256*b^2*c*integrate(1/1
6*x*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 384*b^2*integr
ate(1/16*arctan(c*x)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 32*b^2*
integrate(1/16*log(c^2*x^2 + 1)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x
) + 1024*a*b*integrate(1/16*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2
*x), x))*(c*d^2*x - I*d^2) - 32*(I*c*d^2*x + d^2)*integrate(1/8*(b^2*c^3*x
^3*log(c^2*x^2 + 1)^2 - 32*a*b*c*x*arctan(c*x) + 4*(b^2*c^3*x^3 - 2*b^2*c*
x)*arctan(c*x)^2 - 2*(b^2*c^3*x^3 + b^2*c*x - (b^2*c^2*x^2 - b^2)*arcta...
```

3.108.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^2), x)
```

3.109 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx$

3.109.1 Optimal result

Integrand size = 25, antiderivative size = 306

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = &-\frac{b^2c}{2d^2(i-cx)}+\frac{b^2c\arctan(cx)}{2d^2}\\
&-\frac{ibc(a+b\arctan(cx))}{d^2(i-cx)}-\frac{ic(a+b\arctan(cx))^2}{2d^2}\\
&-\frac{(a+b\arctan(cx))^2}{d^2x}+\frac{c(a+b\arctan(cx))^2}{d^2(i-cx)}\\
&-\frac{4ic(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2}\\
&-\frac{2ic(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^2}\\
&+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^2}\\
&-\frac{ib^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}\\
&+\frac{2bc(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2}\\
&-\frac{ib^2c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{d^2}\end{aligned}$$

output
```
-1/2*b^2*c/d^2/(I-c*x)+1/2*b^2*c*arctan(c*x)/d^2-I*b*c*(a+b*arctan(c*x))/d
^2/(I-c*x)-1/2*I*c*(a+b*arctan(c*x))^2/d^2-(a+b*arctan(c*x))^2/d^2/x+c*(a+
b*arctan(c*x))^2/d^2/(I-c*x)+4*I*c*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c
*x))/d^2-2*I*c*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/d^2+2*b*c*(a+b*arctan(c
*x))*ln(2-2/(1-I*c*x))/d^2-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d^2+2*b*c*(a+
b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2-I*b^2*c*polylog(3,-1+2/(1+I*c
*x))/d^2
```

3.109.2 Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx =$$

$$-\frac{\frac{12a^2}{x}+\frac{12a^2c}{-i+cx}+24a^2c\arctan(cx)+24ia^2c\log(cx)-12ia^2c\log\left(1+c^2x^2\right)+b^2c\Big(\pi^3+12i\arctan(cx)^2+}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```

output
```
-1/12*((12*a^2)/x + (12*a^2*c)/(-I + c*x) + 24*a^2*c*ArcTan[c*x] + (24*I)*
a^2*c*Log[c*x] - (12*I)*a^2*c*Log[1 + c^2*x^2] + b^2*c*(Pi^3 + (12*I)*ArcT
an[c*x]^2 + (12*ArcTan[c*x]^2)/(c*x) - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan
[c*x]*Cos[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + (24*I)
*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]*Log[1 - E^
((2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] +
 (12*I)*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*Ar
cTan[c*x])] - 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]]
+ 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + (6*a*b*(8*c*x*ArcTan[c*x]^2 + 4*c*
x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + c*x*(Cos[2*ArcTan[c*x]] - 4*Log[c*x]
 + 2*Log[1 + c^2*x^2] - I*Sin[2*ArcTan[c*x]]) + 2*ArcTan[c*x]*(2 + I*c*x*C
os[2*ArcTan[c*x]] + (4*I)*c*x*Log[1 - E^((2*I)*ArcTan[c*x])] + c*x*Sin[2*A
rcTan[c*x]])))/x)/d^2
```

3.109.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{2ic^2(a+b\arctan(cx))^2}{d^2(cx-i)}+\frac{c^2(a+b\arctan(cx))^2}{d^2(cx-i)^2}+\frac{(a+b\arctan(cx))^2}{d^2x^2}-\frac{2ic(a+b\arctan(cx))^2}{d^2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4ic\,\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{2bc\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}-\frac{ibc(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{(a+b\arctan(cx))^2}{d^2x}+\frac{c(a+b\arctan(cx))^2}{d^2(-cx+i)}-\frac{ic(a+b\arctan(cx))^2}{2d^2}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}-\frac{2ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{b^2c\arctan(cx)}{2d^2}-\frac{ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^2}-\frac{ib^2c\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{d^2}-\frac{b^2c}{2d^2(-cx+i)}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(b^2*c)/(d^2*(I - c*x)) + (b^2*c*ArcTan[c*x])/(2*d^2) - (I*b*c*(a + b
*ArcTan[c*x]))/(d^2*(I - c*x)) - ((I/2)*c*(a + b*ArcTan[c*x])^2)/d^2 - (a
+ b*ArcTan[c*x])^2/(d^2*x) + (c*(a + b*ArcTan[c*x])^2)/(d^2*(I - c*x)) - (
(4*I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - ((2*I)*c*(
a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*
Log[2 - 2/(1 - I*c*x)])/d^2 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2
 + (2*b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (I*b^2
*c*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^2
```

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.109.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.96 (sec) , antiderivative size = 8556, normalized size of antiderivative = 27.96

method	result	size
parts	Expression too large to display	8556
derivativedivides	Expression too large to display	8557
default	Expression too large to display	8557

input
```
int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.109.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(-I*b^2*c^2*x^2 - b^2*c*x)*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*
x - I))^2 + 4*(-I*b^2*c^2*x^2 - b^2*c*x)*dilog(-2*c*x/(c*x - I) + 1)*log(-
(c*x + I)/(c*x - I)) - (2*b^2*c*x - I*b^2)*log(-(c*x + I)/(c*x - I))^2 - 4
*(c*d^2*x^2 - I*d^2*x)*integral(-(a^2*c*x + I*a^2 - (2*I*b^2*c^2*x^2 + (-I
*a*b + b^2)*c*x + a*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^2*x^5 - I*c^2*d^2
*x^4 + c*d^2*x^3 - I*d^2*x^2), x) + 4*(I*b^2*c^2*x^2 + b^2*c*x)*polylog(3,
 -(c*x + I)/(c*x - I)))/(c*d^2*x^2 - I*d^2*x)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-a^2*(c/(c*d^2*x - I*d^2) - 2*I*c*log(c*x - I)/d^2 + 2*I*c*log(x)/d^2 + 1/
(d^2*x)) - 1/16*(8*(b^2*c^2*x^2 - I*b^2*c*x)*arctan(c*x)^3 - (-I*b^2*c^2*x
^2 - b^2*c*x)*log(c^2*x^2 + 1)^3 + 4*(2*b^2*c*x - I*b^2)*arctan(c*x)^2 - (
2*b^2*c*x - I*b^2 - 2*(b^2*c^2*x^2 - I*b^2*c*x)*arctan(c*x))*log(c^2*x^2 +
 1)^2 - 2*(128*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^4*d^2*x^6 + 2*c
^2*d^2*x^4 + d^2*x^2), x) + 32*b^2*c^4*integrate(1/16*x^4*log(c^2*x^2 + 1)
^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 64*b^2*c^4*integrate(1/16
*x^4*log(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + (c*(x/
(c^4*d^2*x^2 + c^2*d^2) + arctan(c*x)/(c^3*d^2)) - 2*arctan(c*x)/(c^4*d^2*
x^2 + c^2*d^2))*b^2*c^3 + 32*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^4
*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 24*b^2*c^2*integrate(1/16*x^2*lo
g(c^2*x^2 + 1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 256*a*b*c^2
*integrate(1/16*x^2*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x
) - 64*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c^2*d^
2*x^4 + d^2*x^2), x) - 64*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 +
 1)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 64*b^2*c*integrate(1/16*
x*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 96*b^2*integra
te(1/16*arctan(c*x)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 8*b^2*
integrate(1/16*log(c^2*x^2 + 1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2),
 x) + 256*a*b*integrate(1/16*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 +...
```

3.109.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{(i\,cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^2), x)
```

3.110 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx$

3.110.1 Optimal result

Integrand size = 25, antiderivative size = 403

$$
\begin{aligned}
\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx ={}& \frac{ib^2c^2}{2d^2(i-cx)}-\frac{ib^2c^2\arctan(cx)}{2d^2}\\
&-\frac{bc(a+b\arctan(cx))}{d^2x}-\frac{bc^2(a+b\arctan(cx))}{d^2(i-cx)}\\
&-\frac{2c^2(a+b\arctan(cx))^2}{d^2}-\frac{(a+b\arctan(cx))^2}{2d^2x^2}\\
&+\frac{2ic(a+b\arctan(cx))^2}{d^2x}-\frac{ic^2(a+b\arctan(cx))^2}{d^2(i-cx)}\\
&-\frac{6c^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2}\\
&+\frac{b^2c^2\log(x)}{d^2}-\frac{3c^2(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^2}\\
&-\frac{b^2c^2\log(1+c^2x^2)}{2d^2}-\frac{4ibc^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^2}\\
&-\frac{2b^2c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}\\
&-\frac{3ibc^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2}\\
&-\frac{3b^2c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}
\end{aligned}
$$

output
```
-I*c^2*(a+b*arctan(c*x))^2/d^2/(I-c*x)+1/2*I*b^2*c^2/d^2/(I-c*x)-b*c*(a+b*
arctan(c*x))/d^2/x-b*c^2*(a+b*arctan(c*x))/d^2/(I-c*x)-2*c^2*(a+b*arctan(c
*x))^2/d^2-1/2*(a+b*arctan(c*x))^2/d^2/x^2+2*I*c*(a+b*arctan(c*x))^2/d^2/x
-1/2*I*b^2*c^2*arctan(c*x)/d^2+6*c^2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I
*c*x))/d^2+b^2*c^2*ln(x)/d^2-3*c^2*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/d^2
-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2-4*I*b*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x
))/d^2-2*b^2*c^2*polylog(2,-1+2/(1-I*c*x))/d^2-3*I*b*c^2*(a+b*arctan(c*x))
*polylog(2,-1+2/(1+I*c*x))/d^2-3/2*b^2*c^2*polylog(3,-1+2/(1+I*c*x))/d^2
```

3.110.2 Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx$$
$$= \frac{-\frac{4a^2}{x^2}+\frac{16ia^2c}{x}+\frac{8ia^2c^2}{-i+cx}+24ia^2c^2\arctan(cx)-24a^2c^2\log(x)+12a^2c^2\log(1+c^2x^2)-b^2c^2\left(-i\pi^3+\frac{8\arctan}{cx}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```

output
```
((-4*a^2)/x^2 + ((16*I)*a^2*c)/x + ((8*I)*a^2*c^2)/(-I + c*x) + (24*I)*a^2
*c^2*ArcTan[c*x] - 24*a^2*c^2*Log[x] + 12*a^2*c^2*Log[1 + c^2*x^2] - b^2*c
^2*((-I)*Pi^3 + (8*ArcTan[c*x])/(c*x) + 20*ArcTan[c*x]^2 + (4*ArcTan[c*x]^
2)/(c^2*x^2) - ((16*I)*ArcTan[c*x]^2)/(c*x) - 2*Cos[2*ArcTan[c*x]] - (4*I)
*ArcTan[c*x]*Cos[2*ArcTan[c*x]] + 4*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*
ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (32*I)*ArcTan[c*x]*Log[1 -
 E^((2*I)*ArcTan[c*x])] - 8*Log[c*x] + 4*Log[1 + c^2*x^2] + (24*I)*ArcTan[
c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 16*PolyLog[2, E^((2*I)*ArcTan[c*
x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (2*I)*Sin[2*ArcTan[c*x]] -
4*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]])
 + (4*I)*a*b*c^2*((2*I)/(c*x) + 12*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 8*
Log[c*x] + 4*Log[1 + c^2*x^2] + 6*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*Si
n[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(I + I/(c^2*x^2) + 4/(c*x) + I*Cos[2*ArcT
an[c*x]] + (6*I)*Log[1 - E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(8
*d^2)
```

3.110.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx$$

$$\Big\downarrow \ 5411$$

$$\int \left(\frac{3c^3(a+b\arctan(cx))^2}{d^2(cx-i)}-\frac{ic^3(a+b\arctan(cx))^2}{d^2(cx-i)^2}-\frac{3c^2(a+b\arctan(cx))^2}{d^2x}+\frac{(a+b\arctan(cx))^2}{d^2x^3}-\frac{2ic(a+b\ldots}{d\ldots}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{6c^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}-\frac{3ibc^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}-\frac{ic^2(a+b\arctan(cx))^2}{d^2(-cx+i)}-\frac{2c^2(a+b\arctan(cx))^2}{d^2}-\frac{bc^2(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}-\frac{4ibc^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}-\frac{(a+b\arctan(cx))^2}{2d^2x^2}+\frac{2ic(a+b\arctan(cx))^2}{d^2x}-\frac{bc(a+b\arctan(cx))}{d^2x}-\frac{ib^2c^2\arctan(cx)}{2d^2}-\frac{2b^2c^2\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^2}-\frac{3b^2c^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^2}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d^2}+\frac{ib^2c^2}{2d^2(-cx+i)}+\frac{b^2c^2\log(x)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```

output
```
((I/2)*b^2*c^2)/(d^2*(I - c*x)) - ((I/2)*b^2*c^2*ArcTan[c*x])/d^2 - (b*c*(
a + b*ArcTan[c*x]))/(d^2*x) - (b*c^2*(a + b*ArcTan[c*x]))/(d^2*(I - c*x))
- (2*c^2*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d^2*x^2) +
((2*I)*c*(a + b*ArcTan[c*x])^2)/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x])^2)/(d
^2*(I - c*x)) - (6*c^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d
^2 + (b^2*c^2*Log[x])/d^2 - (3*c^2*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)
])/d^2 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) - ((4*I)*b*c^2*(a + b*ArcTan[c
*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - (2*b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x
)])/d^2 - ((3*I)*b*c^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])
/d^2 - (3*b^2*c^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d^2)
```

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.110.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 38.54 (sec) , antiderivative size = 1967, normalized size of antiderivative = 4.88

method	result	size
derivativedivides	Expression too large to display	1967
default	Expression too large to display	1967
parts	Expression too large to display	1975

input
```
int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^2/d^2/c^2/x^2+2*I*a^2/d^2/c/x-3*a^2/d^2*ln(c*x)+I*a^2/d^2/(c*x
-I)+3/2*a^2/d^2*ln(c^2*x^2+1)+3*I*a^2/d^2*arctan(c*x)+b^2/d^2*(-1/2/c^2/x^
2*arctan(c*x)^2-6*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln(1+(1+I*c*x)/(c
^2*x^2+1)^(1/2))-6*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln((1+I*c*x)/(c
^2*x^2+1)^(1/2)-1)-2*arctan(c*x)^2+3*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2
+1)-1)-3*arctan(c*x)^2*ln(c*x)-3*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^
(1/2))-3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*dilog(1+(1+I*c*
x)/(c^2*x^2+1)^(1/2))+4*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)
*(I*c*x-(c^2*x^2+1)^(1/2)+1)/c/x-1/2*arctan(c*x)*(I*c*x+(c^2*x^2+1)^(1/2)+
1)/c/x-3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1
)))*arctan(c*x)^2+6*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6
*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-3/2*
I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arcta
n(c*x)^2+3/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2
+1)))^2*arctan(c*x)^2+3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+3
/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+3/2*I*Pi*csgn(I*((1+I...
```

3.110.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/8*(6*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*log(2*c*x/(c*x - I))*log(-(c*x + I)/(
c*x - I))^2 + 12*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*dilog(-2*c*x/(c*x - I) + 1)
*log(-(c*x + I)/(c*x - I)) + (-6*I*b^2*c^2*x^2 - 3*b^2*c*x - I*b^2)*log(-(
c*x + I)/(c*x - I))^2 + 8*(c*d^2*x^3 - I*d^2*x^2)*integral(-1/2*(2*a^2*c*x
 + 2*I*a^2 - (6*b^2*c^3*x^3 - 3*I*b^2*c^2*x^2 + (-2*I*a*b + b^2)*c*x + 2*a
*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^2*x^6 - I*c^2*d^2*x^5 + c*d^2*x^4 -
I*d^2*x^3), x) - 12*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*polylog(3, -(c*x + I)/(c
*x - I)))/(c*d^2*x^3 - I*d^2*x^2)
```

3.110.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = -\frac{\int \frac{a^2}{c^2x^5-2icx^4-x^3}\,dx + \int \frac{b^2\operatorname{atan}^2(cx)}{c^2x^5-2icx^4-x^3}\,dx + \int \frac{2ab\operatorname{atan}(cx)}{c^2x^5-2icx^4-x^3}\,dx}{d^2}$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x)**2,x)
```

output
```
-(Integral(a**2/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(b**2*atan(c
*x)**2/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(2*a*b*atan(c*x)/(c**
2*x**5 - 2*I*c*x**4 - x**3), x))/d**2
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.110.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^3\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)^2), x)
```

3.111 $\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$

3.111.1 Optimal result

Integrand size = 25, antiderivative size = 462

$$\begin{aligned}\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = & -\frac{iabx}{c^4d^3}+\frac{ib^2}{16c^5d^3(i-cx)^2}-\frac{29b^2}{16c^5d^3(i-cx)} \\ & +\frac{29b^2\arctan(cx)}{16c^5d^3}-\frac{ib^2x\arctan(cx)}{c^4d^3} \\ & -\frac{b(a+b\arctan(cx))}{4c^5d^3(i-cx)^2}-\frac{15ib(a+b\arctan(cx))}{4c^5d^3(i-cx)} \\ & -\frac{5i(a+b\arctan(cx))^2}{8c^5d^3}-\frac{3x(a+b\arctan(cx))^2}{c^4d^3} \\ & +\frac{ix^2(a+b\arctan(cx))^2}{2c^3d^3}-\frac{i(a+b\arctan(cx))^2}{2c^5d^3(i-cx)^2} \\ & +\frac{4(a+b\arctan(cx))^2}{c^5d^3(i-cx)}-\frac{6b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^5d^3} \\ & +\frac{6i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^5d^3} \\ & +\frac{ib^2\log(1+c^2x^2)}{2c^5d^3}-\frac{3ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3} \\ & -\frac{6b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3} \\ & +\frac{3ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^3}\end{aligned}$$

output
```
-15/4*I*b*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)+1/2*I*b^2*ln(c^2*x^2+1)/c^5/d^
3-29/16*b^2/c^5/d^3/(I-c*x)+29/16*b^2*arctan(c*x)/c^5/d^3-I*b^2*x*arctan(c
*x)/c^4/d^3-1/4*b*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)^2-1/2*I*(a+b*arctan(c*
x))^2/c^5/d^3/(I-c*x)^2-3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5/d^3-3*x*(a+b*
arctan(c*x))^2/c^4/d^3+1/2*I*x^2*(a+b*arctan(c*x))^2/c^3/d^3+1/16*I*b^2/c^
5/d^3/(I-c*x)^2+4*(a+b*arctan(c*x))^2/c^5/d^3/(I-c*x)-6*b*(a+b*arctan(c*x)
)*ln(2/(1+I*c*x))/c^5/d^3+3*I*b^2*polylog(3,1-2/(1+I*c*x))/c^5/d^3-5/8*I*(
a+b*arctan(c*x))^2/c^5/d^3+6*I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^5/d^3
-6*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^5/d^3-I*a*b*x/c^4/d^3
```

3.111.2 Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.25

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$
$$= \frac{-48a^2cx+8ia^2c^2x^2-\frac{8ia^2}{(-i+cx)^2}-\frac{64a^2}{-i+cx}+96a^2\arctan(cx)-48ia^2\log\left(1+c^2x^2\right)+ab\left(-16icx+192\,\text{arcta}\right.}{}$$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(-48*a^2*c*x + (8*I)*a^2*c^2*x^2 - ((8*I)*a^2)/(-I + c*x)^2 - (64*a^2)/(-I
 + c*x) + 96*a^2*ArcTan[c*x] - (48*I)*a^2*Log[1 + c^2*x^2] + a*b*((-16*I)*
c*x + 192*ArcTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*
Log[1 + c^2*x^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*Ar
cTan[c*x]] + (4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcT
an[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)
*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]) + (16*
I)*b^2*(-(c*x*ArcTan[c*x]) + 3*ArcTan[c*x]^2 + (3*I)*c*x*ArcTan[c*x]^2 + (
(1 + c^2*x^2)*ArcTan[c*x]^2)/2 - (4*I)*ArcTan[c*x]^3 - (7*(-1 - (2*I)*ArcT
an[c*x] + 2*ArcTan[c*x]^2)*Cos[2*ArcTan[c*x]])/8 - Cos[4*ArcTan[c*x]]/64 -
 (I/16)*ArcTan[c*x]*Cos[4*ArcTan[c*x]] + (ArcTan[c*x]^2*Cos[4*ArcTan[c*x]]
)/8 + (6*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] + Log[1 + c^2*x^2]/2 + (3 - (6*I)*ArcTan[c*x
])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*ArcTan[c*x]
)] - ((7*I)/8)*Sin[2*ArcTan[c*x]] + (7*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/4 +
 ((7*I)/4)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]] + (I/64)*Sin[4*ArcTan[c*x]] -
(ArcTan[c*x]*Sin[4*ArcTan[c*x]])/16 - (I/8)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x
]]))/(16*c^5*d^3)
```

3.111.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(-\frac{6i(a+b\arctan(cx))^2}{c^4d^3(cx-i)}+\frac{4(a+b\arctan(cx))^2}{c^4d^3(cx-i)^2}-\frac{3(a+b\arctan(cx))^2}{c^4d^3}+\frac{i(a+b\arctan(cx))^2}{c^4d^3(cx-i)^3}+\frac{ix(a+b\operatorname{arc}}{c^3d}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{6b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^5d^3}-\frac{15ib(a+b\arctan(cx))}{4c^5d^3(-cx+i)}-\frac{b(a+b\arctan(cx))}{4c^5d^3(-cx+i)^2}+\frac{4(a+b\arctan(cx))^2}{c^5d^3(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{2c^5d^3(-cx+i)^2}-\frac{5i(a+b\arctan(cx))^2}{8c^5d^3}-\frac{6b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^5d^3}+\frac{6i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^5d^3}-\frac{3x(a+b\arctan(cx))^2}{c^4d^3}+\frac{ix^2(a+b\arctan(cx))^2}{2c^3d^3}-\frac{iabx}{c^4d^3}+\frac{29b^2\arctan(cx)}{16c^5d^3}-\frac{ib^2x\arctan(cx)}{c^4d^3}-\frac{3ib^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^5d^3}+\frac{3ib^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^5d^3}-\frac{29b^2}{16c^5d^3(-cx+i)}+\frac{ib^2}{16c^5d^3(-cx+i)^2}+\frac{ib^2\log\left(c^2x^2+1\right)}{2c^5d^3}$$

input
```
Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
((-I)*a*b*x)/(c^4*d^3) + ((I/16)*b^2)/(c^5*d^3*(I - c*x)^2) - (29*b^2)/(16
*c^5*d^3*(I - c*x)) + (29*b^2*ArcTan[c*x])/(16*c^5*d^3) - (I*b^2*x*ArcTan[
c*x])/(c^4*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c^5*d^3*(I - c*x)^2) - (((15*
I)/4)*b*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)) - (((5*I)/8)*(a + b*ArcTa
n[c*x])^2)/(c^5*d^3) - (3*x*(a + b*ArcTan[c*x])^2)/(c^4*d^3) + ((I/2)*x^2*
(a + b*ArcTan[c*x])^2)/(c^3*d^3) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^5*d^3*
(I - c*x)^2) + (4*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c*x)) - (6*b*(a + b
*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^5*d^3) + ((6*I)*(a + b*ArcTan[c*x])^2
*Log[2/(1 + I*c*x)])/(c^5*d^3) + ((I/2)*b^2*Log[1 + c^2*x^2])/(c^5*d^3) -
((3*I)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) - (6*b*(a + b*ArcTan[c
*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) + ((3*I)*b^2*PolyLog[3, 1 -
2/(1 + I*c*x)])/(c^5*d^3)
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.111.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 29.52 (sec) , antiderivative size = 1215, normalized size of antiderivative = 2.63

method	result	size
derivativedivides	Expression too large to display	1215
default	Expression too large to display	1215
parts	Expression too large to display	1279

input
```
int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-6*a*b/d^3*arctan(c*x)*c*x+b^2/d^3*(4*arctan(c*x)^3-3*arctan(c*x)^2
*c*x+6*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-6*arctan(c*x)*ln(1+
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))+1/2*I*arctan(c*x)^2*c^2*x^2-6*Pi*arctan(c*x)^2+6*I*dilog(1+I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))+6*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+3*P
i*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^
2+3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*
c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1
)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x
)^2+3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))
*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+7
/4*arctan(c*x)*(c*x+I)/(c*x-I)-4*arctan(c*x)^2/(c*x-I)-I*ln(1+(1+I*c*x)^2/
(c^2*x^2+1))+3*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+43/8*I*arctan(c*x)^2-
I*arctan(c*x)*(c*x-I)+6*I*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/
2*I*arctan(c*x)^2/(c*x-I)^2-6*I*arctan(c*x)^2*ln(c*x-I)+1/16*(c*x+I)^2*arc
tan(c*x)/(c*x-I)^2-7*I*(c*x+I)/(8*c*x-8*I)-1/64*I*(c*x+I)^2/(c*x-I)^2)-4*a
^2/d^3/(c*x-I)+6*a^2/d^3*arctan(c*x)+1/2*I*a^2/d^3*c^2*x^2-8*a*b/d^3*arcta
n(c*x)/(c*x-I)-6*a*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+5/16*I*a*b/d^3*arcta
n(1/2*c*x)-5/8*I*a*b/d^3*arctan(1/2*c*x-1/2*I)+15/4*I*a*b/d^3/(c*x-I)-5...
```

3.111.5 Fricas [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan{(cx)}+a)^2x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-I*b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^4*log(-(c*x
 + I)/(c*x - I)) + 4*I*a^2*x^4)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.111.7 Maxima [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2 x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/128*(64*I*a^2*c^4*x^4 - 256*a^2*c^3*x^3 - 32*a^2*c^2*x^2*(15*arctan2(1,
c*x) - 22*I) + 64*a^2*c*x*(15*I*arctan2(1, c*x) - 2) + 192*(b^2*c^2*x^2 -
2*I*b^2*c*x - b^2)*arctan(c*x)^3 + 24*(I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*
log(c^2*x^2 + 1)^3 + 32*a^2*(15*arctan2(1, c*x) + 14*I) + 16*(I*b^2*c^4*x^
4 - 4*b^2*c^3*x^3 + 11*I*b^2*c^2*x^2 - 2*b^2*c*x + 7*I*b^2)*arctan(c*x)^2
- 4*(I*b^2*c^4*x^4 - 4*b^2*c^3*x^3 + 11*I*b^2*c^2*x^2 - 2*b^2*c*x + 7*I*b^
2 - 12*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 -
 36*(I*b^2*c^8*d^3*x^2 + 2*b^2*c^7*d^3*x - I*b^2*c^6*d^3)*(((8*c^2*x^2 + 7
)*c^2/(c^16*d^3*x^4 + 2*c^14*d^3*x^2 + c^12*d^3) + 2*(4*c^2*x^2 + 3)*log(c
^2*x^2 + 1)/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3))*c^4 + 2*(2*c^2*x^2
 + 1)*c^2*log(c^2*x^2 + 1)^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) - c
^2*(c^2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*log(c^2*x^2 + 1)/(c
^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3)) - 2048*c^2*integrate(1/64*x^3*arc
tan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) -
2*log(c^2*x^2 + 1)^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 2048*integ
rate(1/64*x*arctan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 +
c^4*d^3), x)) + 12*(-I*b^2*c^10*d^3*x^2 - 2*b^2*c^9*d^3*x + I*b^2*c^8*d^3)
*(((8*c^2*x^2 + 7)*c^2/(c^16*d^3*x^4 + 2*c^14*d^3*x^2 + c^12*d^3) + 2*(4*c
^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3))*c
^2 + 2048*c^2*integrate(1/64*x^5*arctan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^...
```

3.111.8 Giac [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{x^4\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.112 $\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$

3.112.1 Optimal result

Integrand size = 25, antiderivative size = 383

$$
\begin{aligned}
\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = {} & \frac{b^2}{16c^4d^3(i-cx)^2}+\frac{21ib^2}{16c^4d^3(i-cx)}-\frac{21ib^2\arctan(cx)}{16c^4d^3} \\
& +\frac{ib(a+b\arctan(cx))}{4c^4d^3(i-cx)^2}-\frac{11b(a+b\arctan(cx))}{4c^4d^3(i-cx)} \\
& +\frac{3(a+b\arctan(cx))^2}{8c^4d^3}+\frac{ix(a+b\arctan(cx))^2}{c^3d^3} \\
& -\frac{(a+b\arctan(cx))^2}{2c^4d^3(i-cx)^2}-\frac{3i(a+b\arctan(cx))^2}{c^4d^3(i-cx)} \\
& +\frac{2ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^3} \\
& +\frac{3(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^4d^3}-\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^3} \\
& +\frac{3ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^3} \\
& +\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^3}
\end{aligned}
$$

output
```
1/16*b^2/c^4/d^3/(I-c*x)^2+21/16*I*b^2/c^4/d^3/(I-c*x)-21/16*I*b^2*arctan(
c*x)/c^4/d^3+1/4*I*b*(a+b*arctan(c*x))/c^4/d^3/(I-c*x)^2-11/4*b*(a+b*arcta
n(c*x))/c^4/d^3/(I-c*x)+3/8*(a+b*arctan(c*x))^2/c^4/d^3+I*x*(a+b*arctan(c*
x))^2/c^3/d^3-1/2*(a+b*arctan(c*x))^2/c^4/d^3/(I-c*x)^2-3*I*(a+b*arctan(c*
x))^2/c^4/d^3/(I-c*x)+2*I*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d^3+3*(a
+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d^3-b^2*polylog(2,1-2/(1+I*c*x))/c^4
/d^3+3*I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^4/d^3+3/2*b^2*poly
log(3,1-2/(1+I*c*x))/c^4/d^3
```

3.112.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.32

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{64ia^2cx - \frac{32a^2}{(-i+cx)^2} + \frac{192ia^2}{-i+cx} - 192ia^2\arctan(cx) - 96a^2\log\left(1+c^2x^2\right) + 4iab\left(-96\arctan(cx)^2 + 20\cos(2\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
((64*I)*a^2*c*x - (32*a^2)/(-I + c*x)^2 + ((192*I)*a^2)/(-I + c*x) - (192*
I)*a^2*ArcTan[c*x] - 96*a^2*Log[1 + c^2*x^2] + (4*I)*a*b*(-96*ArcTan[c*x]^
2 + 20*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*
PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[
c*x]*(8*c*x + (10*I)*Cos[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Lo
g[1 + E^((2*I)*ArcTan[c*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]])
 + I*Sin[4*ArcTan[c*x]]) + I*b^2*((-64*I)*ArcTan[c*x]^2 + 64*c*x*ArcTan[c*
x]^2 - 128*ArcTan[c*x]^3 - (40*I)*Cos[2*ArcTan[c*x]] + 80*ArcTan[c*x]*Cos[
2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + I*Cos[4*ArcTan[
c*x]] - 4*ArcTan[c*x]*Cos[4*ArcTan[c*x]] - (8*I)*ArcTan[c*x]^2*Cos[4*ArcTa
n[c*x]] + 128*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (192*I)*ArcTan[
c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 64*(I + 3*ArcTan[c*x])*PolyLog[2,
-E^((2*I)*ArcTan[c*x])] - (96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - 40*S
in[2*ArcTan[c*x]] - (80*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + 80*ArcTan[c*x]
^2*Sin[2*ArcTan[c*x]] + Sin[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Sin[4*ArcTa
n[c*x]] - 8*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(64*c^4*d^3)
```

3.112.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{3(a+b\arctan(cx))^2}{c^3d^3(cx-i)}-\frac{3i(a+b\arctan(cx))^2}{c^3d^3(cx-i)^2}+\frac{i(a+b\arctan(cx))^2}{c^3d^3}+\frac{(a+b\arctan(cx))^2}{c^3d^3(cx-i)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^4d^3}-\frac{11b(a+b\arctan(cx))}{4c^4d^3(-cx+i)}+\frac{ib(a+b\arctan(cx))}{4c^4d^3(-cx+i)^2}-\frac{3i(a+b\arctan(cx))^2}{c^4d^3(-cx+i)}-\frac{(a+b\arctan(cx))^2}{2c^4d^3(-cx+i)^2}+\frac{3(a+b\arctan(cx))^2}{8c^4d^3}+\frac{2ib\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^3}+\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^4d^3}+\frac{ix(a+b\arctan(cx))^2}{c^3d^3}-\frac{21ib^2\arctan(cx)}{16c^4d^3}-\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^4d^3}+\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^4d^3}+\frac{21ib^2}{16c^4d^3(-cx+i)}+\frac{b^2}{16c^4d^3(-cx+i)^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
b^2/(16*c^4*d^3*(I - c*x)^2) + (((21*I)/16)*b^2)/(c^4*d^3*(I - c*x)) - (((
21*I)/16)*b^2*ArcTan[c*x])/(c^4*d^3) + ((I/4)*b*(a + b*ArcTan[c*x]))/(c^4*
d^3*(I - c*x)^2) - (11*b*(a + b*ArcTan[c*x]))/(4*c^4*d^3*(I - c*x)) + (3*(
a + b*ArcTan[c*x])^2)/(8*c^4*d^3) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d^3)
- (a + b*ArcTan[c*x])^2/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x
])^2)/(c^4*d^3*(I - c*x)) + ((2*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)
])/(c^4*d^3) + (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^3) - (b
^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + ((3*I)*b*(a + b*ArcTan[c*x])
*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + (3*b^2*PolyLog[3, 1 - 2/(1 + I
*c*x)])/(2*c^4*d^3)
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.112.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.35 (sec) , antiderivative size = 4306, normalized size of antiderivative = 11.24

method	result	size
derivativedivides	Expression too large to display	4306
default	Expression too large to display	4306
parts	Expression too large to display	4365

input
```
int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(3*I*a*b/d^3*dilog(-1/2*I*(c*x+I))-1/2*a^2/d^3/(c*x-I)^2-3/2*I*a*b/d
^3*ln(c*x-I)^2-3/2*a^2/d^3*ln(c^2*x^2+1)+3/32*I*a*b/d^3*ln(c^4*x^4+10*c^2*
x^2+9)+b^2/d^3*(3/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+19/16*polylog(2,-(
1+I*c*x)^2/(c^2*x^2+1))+19/8*arctan(c*x)^2-3*I*arctan(c*x)*polylog(2,-(1+I
*c*x)^2/(c^2*x^2+1))-2*I*arctan(c*x)^3-3/8*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))-3/8*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*Pi*arctan(c*x)*ln(1+I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*Pi*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-3*Pi*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+I*arctan(c*x)^2*c*
x+3*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-3*arctan(c*x)^2*ln(c*x-I
)-5/8*(c*x+I)/(c*x-I)-3/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*Pi*csgn
((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)*ln(1+(
1+I*c*x)^2/(c^2*x^2+1))-3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*Pi*csgn((
1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln(1+(1+
I*c*x)^2/(c^2*x^2+1))-3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*Pi*csgn((
1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)*ln(1+I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csg
n((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln...
```

3.112.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(icdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^3*log(-(c*x
 + I)/(c*x - I)) + 4*I*a^2*x^3)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.112.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/128*(128*I*a^2*c^3*x^3 + 32*a^2*c^2*x^2*(3*I*arctan2(1, c*x) + 8) + 64*a
^2*c*x*(3*arctan2(1, c*x) + 4*I) + 96*(-I*b^2*c^2*x^2 - 2*b^2*c*x + I*b^2)
*arctan(c*x)^3 + 12*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 +
 32*a^2*(-3*I*arctan2(1, c*x) + 10) + 16*(2*I*b^2*c^3*x^3 + 4*b^2*c^2*x^2
+ 4*I*b^2*c*x + 5*b^2)*arctan(c*x)^2 - 4*(2*I*b^2*c^3*x^3 + 4*b^2*c^2*x^2
+ 4*I*b^2*c*x + 5*b^2 - 6*(-I*b^2*c^2*x^2 - 2*b^2*c*x + I*b^2)*arctan(c*x)
)*log(c^2*x^2 + 1)^2 - 18*(b^2*c^7*d^3*x^2 - 2*I*b^2*c^6*d^3*x - b^2*c^5*d
^3)*(((8*c^2*x^2 + 7)*c^2/(c^15*d^3*x^4 + 2*c^13*d^3*x^2 + c^11*d^3) + 2*(
4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3))
*c^4 + 2*(2*c^2*x^2 + 1)*c^2*log(c^2*x^2 + 1)^2/(c^11*d^3*x^4 + 2*c^9*d^3*
x^2 + c^7*d^3) - c^2*(c^2/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3) + 2*lo
g(c^2*x^2 + 1)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3)) - 4096*c^2*integr
ate(1/128*x^3*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 +
 c^3*d^3), x) - 2*log(c^2*x^2 + 1)^2/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^
3) + 4096*integrate(1/128*x*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3
*c^5*d^3*x^2 + c^3*d^3), x)) - 8*(b^2*c^9*d^3*x^2 - 2*I*b^2*c^8*d^3*x - b^
2*c^7*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^15*d^3*x^4 + 2*c^13*d^3*x^2 + c^11*d^3
) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^
9*d^3))*c^2 + 4096*c^2*integrate(1/128*x^5*arctan(c*x)^2/(c^9*d^3*x^6 + 3*
c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 1024*c^2*integrate(1/128*x...
```

3.112.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.113 $\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$

3.113.1 Optimal result

Integrand size = 25, antiderivative size = 304

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = & -\frac{ib^2}{16c^3d^3(i-cx)^2}+\frac{13b^2}{16c^3d^3(i-cx)}-\frac{13b^2\arctan(cx)}{16c^3d^3} \\ & +\frac{b(a+b\arctan(cx))}{4c^3d^3(i-cx)^2}+\frac{7ib(a+b\arctan(cx))}{4c^3d^3(i-cx)} \\ & -\frac{7i(a+b\arctan(cx))^2}{8c^3d^3}+\frac{i(a+b\arctan(cx))^2}{2c^3d^3(i-cx)^2} \\ & -\frac{2(a+b\arctan(cx))^2}{c^3d^3(i-cx)}-\frac{i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d^3} \\ & +\frac{b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^3} \\ & -\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d^3}\end{aligned}$$

output
```
-1/16*I*b^2/c^3/d^3/(I-c*x)^2+13/16*b^2/c^3/d^3/(I-c*x)-13/16*b^2*arctan(c
*x)/c^3/d^3+1/4*b*(a+b*arctan(c*x))/c^3/d^3/(I-c*x)^2+7/4*I*b*(a+b*arctan(
c*x))/c^3/d^3/(I-c*x)-7/8*I*(a+b*arctan(c*x))^2/c^3/d^3+1/2*I*(a+b*arctan(
c*x))^2/c^3/d^3/(I-c*x)^2-2*(a+b*arctan(c*x))^2/c^3/d^3/(I-c*x)-I*(a+b*arc
tan(c*x))^2*ln(2/(1+I*c*x))/c^3/d^3+b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I
*c*x))/c^3/d^3-1/2*I*b^2*polylog(3,1-2/(1+I*c*x))/c^3/d^3
```

3.113.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.42

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$
$$= \frac{\frac{96ia^2}{(-i+cx)^2}+\frac{384a^2}{-i+cx}-192a^2\arctan(cx)+96ia^2\log(1+c^2x^2)-b^2\big(128\arctan(cx)^3+72i\cos(2\arctan(cx))-}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(((96*I)*a^2)/(-I + c*x)^2 + (384*a^2)/(-I + c*x) - 192*a^2*ArcTan[c*x] +
(96*I)*a^2*Log[1 + c^2*x^2] - b^2*(128*ArcTan[c*x]^3 + (72*I)*Cos[2*ArcTan
[c*x]] - 144*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]^2*Cos[2*
ArcTan[c*x]] - (3*I)*Cos[4*ArcTan[c*x]] + 12*ArcTan[c*x]*Cos[4*ArcTan[c*x]
] + (24*I)*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + (192*I)*ArcTan[c*x]^2*Log[1
+ E^((2*I)*ArcTan[c*x])] + 192*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x
])] + (96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] + 72*Sin[2*ArcTan[c*x]] +
(144*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 144*ArcTan[c*x]^2*Sin[2*ArcTan[c*
x]] - 3*Sin[4*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Sin[4*ArcTan[c*x]] + 24*Ar
cTan[c*x]^2*Sin[4*ArcTan[c*x]]) - 12*a*b*(32*ArcTan[c*x]^2 - 12*Cos[2*ArcT
an[c*x]] + Cos[4*ArcTan[c*x]] + 16*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (1
2*I)*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*((-6*I)*Cos
[2*ArcTan[c*x]] + I*Cos[4*ArcTan[c*x]] + (8*I)*Log[1 + E^((2*I)*ArcTan[c*x
])] - 6*Sin[2*ArcTan[c*x]] + Sin[4*ArcTan[c*x]])))/(192*c^3*d^3)
```

3.113.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{i(a+b\arctan(cx))^2}{c^2d^3(cx-i)}-\frac{2(a+b\arctan(cx))^2}{c^2d^3(cx-i)^2}-\frac{i(a+b\arctan(cx))^2}{c^2d^3(cx-i)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^3d^3}+\frac{7ib(a+b\arctan(cx))}{4c^3d^3(-cx+i)}+\frac{b(a+b\arctan(cx))}{4c^3d^3(-cx+i)^2}-\frac{2(a+b\arctan(cx))^2}{c^3d^3(-cx+i)}+\frac{i(a+b\arctan(cx))^2}{2c^3d^3(-cx+i)^2}-\frac{7i(a+b\arctan(cx))^2}{8c^3d^3}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^3d^3}-\frac{13b^2\arctan(cx)}{16c^3d^3}-\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^3d^3}+\frac{13b^2}{16c^3d^3(-cx+i)}-\frac{ib^2}{16c^3d^3(-cx+i)^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
((-1/16*I)*b^2)/(c^3*d^3*(I - c*x)^2) + (13*b^2)/(16*c^3*d^3*(I - c*x)) -
(13*b^2*ArcTan[c*x])/(16*c^3*d^3) + (b*(a + b*ArcTan[c*x]))/(4*c^3*d^3*(I
- c*x)^2) + (((7*I)/4)*b*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (((7*I
)/8)*(a + b*ArcTan[c*x])^2)/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3
*d^3*(I - c*x)^2) - (2*(a + b*ArcTan[c*x])^2)/(c^3*d^3*(I - c*x)) - (I*(a
+ b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^3) + (b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d^3) - ((I/2)*b^2*PolyLog[3, 1 - 2/(1
+ I*c*x)])/(c^3*d^3)
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.113.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.79 (sec) , antiderivative size = 960, normalized size of antiderivative = 3.16

method	result	size
derivativedivides	Expression too large to display	960
default	Expression too large to display	960
parts	Expression too large to display	1015

input
```
int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/2*I*a^2/d^3/(c*x-I)^2+2*a^2/d^3/(c*x-I)-7/4*I*a*b/d^3/(c*x-I)-a^2
/d^3*arctan(c*x)+b^2/d^3*(3*I*(c*x+I)/(8*c*x-8*I)+2*arctan(c*x)^2/(c*x-I)+
I*arctan(c*x)^2*ln(c*x-I)+1/2*I*arctan(c*x)^2/(c*x-I)^2-2/3*arctan(c*x)^3+
1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c
^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+
(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-1/2*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arct
an(c*x)^2-Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*a
rctan(c*x)^2+Pi*arctan(c*x)^2+1/64*I*(c*x+I)^2/(c*x-I)^2-1/16*(c*x+I)^2*ar
ctan(c*x)/(c*x-I)^2-7/8*I*arctan(c*x)^2-arctan(c*x)*polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1))-1/2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3/4*arctan(c*x)*(c
*x+I)/(c*x-I)-I*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1)))-7/8*I*a*b/d
^3*arctan(1/2*c*x-1/2*I)+4*a*b/d^3*arctan(c*x)/(c*x-I)+1/2*I*a^2/d^3*ln(c^
2*x^2+1)+7/32*a*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-7/8*I*a*b/d^3*arctan(c*x)-7
/16*I*a*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)+7/16*I*a*b/d^3*arctan(1/2*c*x)+I
*a*b/d^3*arctan(c*x)/(c*x-I)^2+1/4*a*b/d^3/(c*x-I)^2-7/16*a*b/d^3*ln(c^2*x
^2+1)+2*I*a*b/d^3*arctan(c*x)*ln(c*x-I)+a*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I
))+a*b/d^3*dilog(-1/2*I*(c*x+I))-1/2*a*b/d^3*ln(c*x-I)^2)
```

3.113.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^2*log(-(c*x
 + I)/(c*x - I)) + 4*I*a^2*x^2)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.113.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/128*(144*a^2*c^2*x^2*arctan2(1, c*x) - 32*a^2*c*x*(9*I*arctan2(1, c*x) -
 8) - 32*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x)^3 + 4*(-I*b^2*c^2*x
^2 - 2*b^2*c*x + I*b^2)*log(c^2*x^2 + 1)^3 - 48*a^2*(3*arctan2(1, c*x) + 4
*I) + 16*(4*b^2*c*x - 3*I*b^2)*arctan(c*x)^2 - 4*(4*b^2*c*x - 3*I*b^2 + 2*
(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(I*b
^2*c^6*d^3*x^2 + 2*b^2*c^5*d^3*x - I*b^2*c^4*d^3)*(((8*c^2*x^2 + 7)*c^2/(c
^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 +
 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2
*log(c^2*x^2 + 1)^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) - c^2*(c^2/(c
^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*log(c^2*x^2 + 1)/(c^10*d^3*x^4
 + 2*c^8*d^3*x^2 + c^6*d^3)) - 512*c^2*integrate(1/16*x^3*arctan(c*x)^2/(c
^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) - 2*log(c^2*x^2
+ 1)^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) + 512*integrate(1/16*x*arct
an(c*x)^2/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x)) - 4
*(-I*b^2*c^8*d^3*x^2 - 2*b^2*c^7*d^3*x + I*b^2*c^6*d^3)*(((8*c^2*x^2 + 7)*
c^2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2
*x^2 + 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^2 + 512*c^2*integra
te(1/16*x^5*arctan(c*x)^2/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c
^2*d^3), x) + 128*c^2*integrate(1/16*x^5*log(c^2*x^2 + 1)^2/(c^8*d^3*x^6 +
 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + 2*(2*c^2*x^2 + 1)*log(c...
```

3.113.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(icdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.114 $\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$

3.114.1 Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{b^2}{16c^2d^3(i-cx)^2} - \frac{5ib^2}{16c^2d^3(i-cx)} + \frac{5ib^2\arctan(cx)}{16c^2d^3} - \frac{ib(a+b\arctan(cx))}{4c^2d^3(i-cx)^2} + \frac{3b(a+b\arctan(cx))}{4c^2d^3(i-cx)} + \frac{(a+b\arctan(cx))^2}{8c^2d^3} + \frac{x^2(a+b\arctan(cx))^2}{2d^3(1+icx)^2}$$

output
```
-1/16*b^2/c^2/d^3/(I-c*x)^2-5/16*I*b^2/c^2/d^3/(I-c*x)+5/16*I*b^2*arctan(c
*x)/c^2/d^3-1/4*I*b*(a+b*arctan(c*x))/c^2/d^3/(I-c*x)^2+3/4*b*(a+b*arctan(
c*x))/c^2/d^3/(I-c*x)+1/8*(a+b*arctan(c*x))^2/c^2/d^3+1/2*x^2*(a+b*arctan(
c*x))^2/d^3/(1+I*c*x)^2
```

3.114.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.66

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{4ab(2i-3cx)+b^2(4+5icx)+a^2(-8-16icx)+b(i+cx)(a(4i-12cx)+b(3+5icx))\arctan(cx)-2b^2}{16c^2d^3(-i+cx)^2}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(4*a*b*(2*I - 3*c*x) + b^2*(4 + (5*I)*c*x) + a^2*(-8 - (16*I)*c*x) + b*(I
+ c*x)*(a*(4*I - 12*c*x) + b*(3 + (5*I)*c*x))*ArcTan[c*x] - 2*b^2*(1 + (2*
I)*c*x + 3*c^2*x^2)*ArcTan[c*x]^2)/(16*c^2*d^3*(-I + c*x)^2)
```

3.114.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}dx$$

$$\Big\downarrow \text{5409}$$

$$\frac{x^2(a+b\arctan(cx))^2}{2d^3(1+icx)^2}-$$
$$2bc\int\left(-\frac{a+b\arctan(cx)}{8c^2d^3\left(c^2x^2+1\right)}-\frac{3(a+b\arctan(cx))}{8c^2d^3(i-cx)^2}+\frac{i(a+b\arctan(cx))}{4c^2d^3(i-cx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(a+b\arctan(cx))^2}{2d^3(1+icx)^2}-$$
$$2bc\left(-\frac{(a+b\arctan(cx))^2}{16bc^3d^3}-\frac{3(a+b\arctan(cx))}{8c^3d^3(-cx+i)}+\frac{i(a+b\arctan(cx))}{8c^3d^3(-cx+i)^2}-\frac{5ib\arctan(cx)}{32c^3d^3}+\frac{5ib}{32c^3d^3(-cx+i)}+ 32\right.$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x])^2)/(2*d^3*(1 + I*c*x)^2) - 2*b*c*(b/(32*c^3*d^3*(
I - c*x)^2) + (((5*I)/32)*b)/(c^3*d^3*(I - c*x)) - (((5*I)/32)*b*ArcTan[c*
x])/(c^3*d^3) + ((I/8)*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)^2) - (3*(a
+ b*ArcTan[c*x]))/(8*c^3*d^3*(I - c*x)) - (a + b*ArcTan[c*x])^2/(16*b*c^3*
d^3))
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

3.114.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(156) = 312$.

Time = 2.33 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b^2\left(\frac{\arctan(cx)^2}{2(cx-i)^2}-\frac{i\arctan(cx)^2}{cx-i}-\frac{3i\arctan(cx)\ln(cx+i)}{8}+\frac{3i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}-\frac{3\arctan(c}{4(cx-i)}\right.}{}}{}$
default	$\frac{\frac{a^2\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b^2\left(\frac{\arctan(cx)^2}{2(cx-i)^2}-\frac{i\arctan(cx)^2}{cx-i}-\frac{3i\arctan(cx)\ln(cx+i)}{8}+\frac{3i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}-\frac{3\arctan(c}{4(cx-i)}\right.}{}}{}$
parts	$\frac{a^2\left(\frac{i}{c^2(-cx+i)}+\frac{1}{2(-cx+i)^2c^2}\right)}{d^3}+\frac{b^2\left(\frac{\arctan(cx)^2}{2(cx-i)^2}-\frac{i\arctan(cx)^2}{cx-i}-\frac{3i\arctan(cx)\ln(cx+i)}{8}+\frac{3i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(c}{4(cx-i)^2}\right.}{}$
risch	$\frac{(3b^2c^2x^2+2ib^2cx+b^2)\ln(icx+1)^2}{32c^2d^3(cx-i)^2}-\frac{(2i\ln(-icx+1)b^2cx+b^2\ln(-icx+1)+3b^2\ln(-icx+1)c^2x^2-6ib^2cx+16abcx-8ia}{16c^2d^3(cx-i)^2}$

input
```
int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a^2/d^3*(1/2/(c*x-I)^2-I/(c*x-I))+b^2/d^3*(1/2*arctan(c*x)^2/(c*x-I
)^2-I*arctan(c*x)^2/(c*x-I)-3/8*I*arctan(c*x)*ln(c*x+I)+3/8*I*arctan(c*x)*
ln(c*x-I)-1/4*I*arctan(c*x)/(c*x-I)^2-3/4*arctan(c*x)/(c*x-I)+5/16*I*arcta
n(c*x)+5/16*I/(c*x-I)-1/16/(c*x-I)^2+3/16*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3/3
2*ln(c*x-I)^2+3/16*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(-1/2*I*(-c*x+I))-3/32
*ln(c*x+I)^2)+a*b/d^3*arctan(c*x)/(c*x-I)^2-2*I*a*b/d^3*arctan(c*x)/(c*x-I
)-3/4*a*b/d^3*arctan(c*x)-1/4*I*a*b/d^3/(c*x-I)^2-3/4*a*b/d^3/(c*x-I))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{2\,(16i\,a^2+12\,ab-5i\,b^2)cx-(3\,b^2c^2x^2+2i\,b^2cx+b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+16\,a^2-16i\,ab-8\,b^2-((-12i\,ab}{32\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/32*(2*(16*I*a^2 + 12*a*b - 5*I*b^2)*c*x - (3*b^2*c^2*x^2 + 2*I*b^2*c*x
+ b^2)*log(-(c*x + I)/(c*x - I))^2 + 16*a^2 - 16*I*a*b - 8*b^2 - ((-12*I*a
*b - 5*b^2)*c^2*x^2 + 2*(4*a*b - I*b^2)*c*x - 4*I*a*b - 3*b^2)*log(-(c*x +
 I)/(c*x - I)))/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

3.114.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(156) = 312$.

Time = 42.26 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.82

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{ib(12a-5ib)\log\left(-\frac{ib(12a-5ib)}{c}+x(12ab-5ib^2)\right)}{32c^2d^3} - \frac{ib(12a-5ib)\log\left(\frac{ib(12a-5ib)}{c}+x(12ab-5ib^2)\right)}{32c^2d^3} + \frac{(3b^2c^2x^2+2ib^2cx+b^2)\log(-icx+1)^2}{32c^4d^3x^2-64ic^3d^3x-32c^2d^3} + \frac{(3b^2c^2x^2+2ib^2cx+b^2)\log(icx+1)^2}{32c^4d^3x^2-64ic^3d^3x-32c^2d^3} + \frac{-8a^2+8iab+4b^2+x(-16ia^2c-12abc+5ib^2c)}{16c^4d^3x^2-32ic^3d^3x-16c^2d^3} + \frac{(16abcx-8iab-3b^2c^2x^2\log(icx+1)-2ib^2cx\log(icx+1)-6ib^2cx-b^2\log(icx+1)-4b^2)\log(-ic}{16c^4d^3x^2-32ic^3d^3x-16c^2d^3} + \frac{(-8abcx+4iab+3ib^2cx+2b^2)\log(icx+1)}{8c^4d^3x^2-16ic^3d^3x-8c^2d^3}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
I*b*(12*a - 5*I*b)*log(-I*b*(12*a - 5*I*b)/c + x*(12*a*b - 5*I*b**2))/(32*
c**2*d**3) - I*b*(12*a - 5*I*b)*log(I*b*(12*a - 5*I*b)/c + x*(12*a*b - 5*I
*b**2))/(32*c**2*d**3) + (3*b**2*c**2*x**2 + 2*I*b**2*c*x + b**2)*log(-I*c
*x + 1)**2/(32*c**4*d**3*x**2 - 64*I*c**3*d**3*x - 32*c**2*d**3) + (3*b**2
*c**2*x**2 + 2*I*b**2*c*x + b**2)*log(I*c*x + 1)**2/(32*c**4*d**3*x**2 - 6
4*I*c**3*d**3*x - 32*c**2*d**3) + (-8*a**2 + 8*I*a*b + 4*b**2 + x*(-16*I*a
**2*c - 12*a*b*c + 5*I*b**2*c))/(16*c**4*d**3*x**2 - 32*I*c**3*d**3*x - 16
*c**2*d**3) + (16*a*b*c*x - 8*I*a*b - 3*b**2*c**2*x**2*log(I*c*x + 1) - 2*
I*b**2*c*x*log(I*c*x + 1) - 6*I*b**2*c*x - b**2*log(I*c*x + 1) - 4*b**2)*l
og(-I*c*x + 1)/(16*c**4*d**3*x**2 - 32*I*c**3*d**3*x - 16*c**2*d**3) + (-8
*a*b*c*x + 4*I*a*b + 3*I*b**2*c*x + 2*b**2)*log(I*c*x + 1)/(8*c**4*d**3*x*
*2 - 16*I*c**3*d**3*x - 8*c**2*d**3)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx =$$
$$-\frac{(16i\,a^2+12\,ab-5i\,b^2)cx+2\left(3\,b^2c^2x^2+2i\,b^2cx+b^2\right)\arctan\left(cx\right)^2+8\,a^2-8i\,ab-4\,b^2+\left((12\,ab-5i\,b\right.}{16\left(c^4d^3x^2-2i\,c^3d^3x-c^2d^3\right)}$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*((16*I*a^2 + 12*a*b - 5*I*b^2)*c*x + 2*(3*b^2*c^2*x^2 + 2*I*b^2*c*x
+ b^2)*arctan(c*x)^2 + 8*a^2 - 8*I*a*b - 4*b^2 + ((12*a*b - 5*I*b^2)*c^2*x
^2 - 2*(-4*I*a*b - b^2)*c*x + 4*a*b - 3*I*b^2)*arctan(c*x))/(c^4*d^3*x^2 -
 2*I*c^3*d^3*x - c^2*d^3)
```

3.114.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2x}{\left(i\,cdx+d\right)^3}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{x\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{\left(d+c\,d\,x\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

3.115 $\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$

3.115.1 Optimal result

Integrand size = 22, antiderivative size = 180

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{ib^2}{16cd^3(i-cx)^2}+\frac{3b^2}{16cd^3(i-cx)}-\frac{3b^2\arctan(cx)}{16cd^3} - \frac{b(a+b\arctan(cx))}{4cd^3(i-cx)^2}+\frac{ib(a+b\arctan(cx))}{4cd^3(i-cx)} - \frac{i(a+b\arctan(cx))^2}{8cd^3}+\frac{i(a+b\arctan(cx))^2}{2cd^3(1+icx)^2}$$

output
```
1/16*I*b^2/c/d^3/(I-c*x)^2+3/16*b^2/c/d^3/(I-c*x)-3/16*b^2*arctan(c*x)/c/d
^3-1/4*b*(a+b*arctan(c*x))/c/d^3/(I-c*x)^2+1/4*I*b*(a+b*arctan(c*x))/c/d^3
/(I-c*x)-1/8*I*(a+b*arctan(c*x))^2/c/d^3+1/2*I*(a+b*arctan(c*x))^2/c/d^3/(
1+I*c*x)^2
```

3.115.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{i(8a^2+b^2(-4-3icx)+4ab(-2i+cx)+b(i+cx)(b(-5-3icx)+4a(-3i+cx))\arctan(cx)+2b^2(3}{16cd^3(-i+cx)^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```

output
```
((-1/16*I)*(8*a^2 + b^2*(-4 - (3*I)*c*x) + 4*a*b*(-2*I + c*x) + b*(I + c*x
)*(b*(-5 - (3*I)*c*x) + 4*a*(-3*I + c*x))*ArcTan[c*x] + 2*b^2*(3 - (2*I)*c
*x + c^2*x^2)*ArcTan[c*x]^2))/(c*d^3*(-I + c*x)^2)
```

3.115.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3} \, dx$$

$$\Big\downarrow \ 5389$$

$$\frac{i(a+b\arctan(cx))^2}{2cd^3(1+icx)^2}-\frac{ib\int \left(\frac{a+b\arctan(cx)}{4d^2(c^2x^2+1)}-\frac{a+b\arctan(cx)}{4d^2(i-cx)^2}-\frac{i(a+b\arctan(cx))}{2d^2(i-cx)^3}\right)dx}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{i(a+b\arctan(cx))^2}{2cd^3(1+icx)^2}-\frac{ib\left(\frac{(a+b\arctan(cx))^2}{8bcd^2}-\frac{a+b\arctan(cx)}{4cd^2(-cx+i)}-\frac{i(a+b\arctan(cx))}{4cd^2(-cx+i)^2}-\frac{3ib\arctan(cx)}{16cd^2}+\frac{3ib}{16cd^2(-cx+i)}-\frac{b}{16cd^2(-cx+i)^2}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```

output
```
((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^3*(1 + I*c*x)^2) - (I*b*(-1/16*b/(c*d^2
*(I - c*x)^2) + (((3*I)/16)*b)/(c*d^2*(I - c*x)) - (((3*I)/16)*b*ArcTan[c*
x])/(c*d^2) - ((I/4)*(a + b*ArcTan[c*x]))/(c*d^2*(I - c*x)^2) - (a + b*Arc
Tan[c*x])/(4*c*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(8*b*c*d^2)))/d
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.115.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{ia^2}{2d^3(icx+1)^2}+\frac{b^2\left(\frac{i\arctan(cx)^2}{2(icx+1)^2}-i\left(\frac{i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{\arctan(cx)}{4cx-4i}-\frac{\left(\ln(cx+i)-\ln\left(-\frac{i(c}{}\right.\right.}{}\right.\right.}{}$
default	$\frac{ia^2}{2d^3(icx+1)^2}+\frac{b^2\left(\frac{i\arctan(cx)^2}{2(icx+1)^2}-i\left(\frac{i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{\arctan(cx)}{4cx-4i}-\frac{\left(\ln(cx+i)-\ln\left(-\frac{i(c}{}\right.\right.}{}\right.\right.}{}$
parts	$\frac{ia^2}{2d^3(icx+1)^2c}+\frac{b^2\left(\frac{i\arctan(cx)^2}{2(icx+1)^2}-i\left(\frac{i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{\arctan(cx)}{4cx-4i}-\frac{\left(\ln(cx+i)-\ln\right.}{}\right.\right.}{}$
risch	$\frac{ib^2(c^2x^2-2icx+3)\ln(icx+1)^2}{32d^3(cx-i)^2c}-\frac{(3i\ln(-icx+1)b^2+i\ln(-icx+1)b^2c^2x^2+2\ln(-icx+1)b^2cx+2b^2cx-4ib^2+8ab)\ln(ic}{16d^3(cx-i)^2c}$

input
```
int((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*I*a^2/d^3/(1+I*c*x)^2+b^2/d^3*(1/2*I/(1+I*c*x)^2*arctan(c*x)^2-I*
(1/8*I*arctan(c*x)*ln(c*x+I)-1/8*I*arctan(c*x)*ln(c*x-I)-1/4*I*arctan(c*x)
/(c*x-I)^2+1/4*arctan(c*x)/(c*x-I)-1/16*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(
-1/2*I*(-c*x+I))+1/32*ln(c*x+I)^2-3/16*I*arctan(c*x)-3/16*I/(c*x-I)-1/16/(
c*x-I)^2-1/16*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/32*ln(c*x-I)^2))+I*a*b/d^3/(1
+I*c*x)^2*arctan(c*x)-1/4*I*a*b/d^3*arctan(c*x)-1/4*a*b/d^3/(c*x-I)^2-1/4*
I*a*b/d^3/(c*x-I))
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx =$$
$$-\frac{2\,(4i\,ab+3\,b^2)cx-(i\,b^2c^2x^2+2\,b^2cx+3i\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+16i\,a^2+16\,ab-8i\,b^2-((4\,ab-3i\,b^2)c^2x^2}{32\,(c^3d^3x^2-2i\,c^2d^3x-cd^3)}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/32*(2*(4*I*a*b + 3*b^2)*c*x - (I*b^2*c^2*x^2 + 2*b^2*c*x + 3*I*b^2)*log
(-(c*x + I)/(c*x - I))^2 + 16*I*a^2 + 16*a*b - 8*I*b^2 - ((4*a*b - 3*I*b^2
)*c^2*x^2 - 2*(4*I*a*b + b^2)*c*x + 12*a*b - 5*I*b^2)*log(-(c*x + I)/(c*x
- I)))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

3.115.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(144) = 288.

Time = 33.40 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.58

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{b(4a-3ib)\log\left(-\frac{ib(4a-3ib)}{c}+x(4ab-3ib^2)\right)}{32cd^3}$$
$$+\frac{b(4a-3ib)\log\left(\frac{ib(4a-3ib)}{c}+x(4ab-3ib^2)\right)}{32cd^3}$$
$$+\frac{(-4ab-b^2cx+2ib^2)\log(icx+1)}{8c^3d^3x^2-16ic^2d^3x-8cd^3}+\frac{(ib^2c^2x^2+2b^2cx+3ib^2)\log(-icx+1)^2}{32c^3d^3x^2-64ic^2d^3x-32cd^3}$$
$$+\frac{(ib^2c^2x^2+2b^2cx+3ib^2)\log(icx+1)^2}{32c^3d^3x^2-64ic^2d^3x-32cd^3}+\frac{-8ia^2-8ab+4ib^2+x(-4iabc-3b^2c)}{16c^3d^3x^2-32ic^2d^3x-16cd^3}$$
$$+\frac{(8ab-ib^2c^2x^2\log(icx+1)-2b^2cx\log(icx+1)+2b^2cx-3ib^2\log(icx+1)-4ib^2)\log(-icx+1)}{16c^3d^3x^2-32ic^2d^3x-16cd^3}$$

input
```
integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
-b*(4*a - 3*I*b)*log(-I*b*(4*a - 3*I*b)/c + x*(4*a*b - 3*I*b**2))/(32*c*d*
*3) + b*(4*a - 3*I*b)*log(I*b*(4*a - 3*I*b)/c + x*(4*a*b - 3*I*b**2))/(32*
c*d**3) + (-4*a*b - b**2*c*x + 2*I*b**2)*log(I*c*x + 1)/(8*c**3*d**3*x**2
- 16*I*c**2*d**3*x - 8*c*d**3) + (I*b**2*c**2*x**2 + 2*b**2*c*x + 3*I*b**2
)*log(-I*c*x + 1)**2/(32*c**3*d**3*x**2 - 64*I*c**2*d**3*x - 32*c*d**3) +
(I*b**2*c**2*x**2 + 2*b**2*c*x + 3*I*b**2)*log(I*c*x + 1)**2/(32*c**3*d**3
*x**2 - 64*I*c**2*d**3*x - 32*c*d**3) + (-8*I*a**2 - 8*a*b + 4*I*b**2 + x*
(-4*I*a*b*c - 3*b**2*c))/(16*c**3*d**3*x**2 - 32*I*c**2*d**3*x - 16*c*d**3
) + (8*a*b - I*b**2*c**2*x**2*log(I*c*x + 1) - 2*b**2*c*x*log(I*c*x + 1) +
 2*b**2*c*x - 3*I*b**2*log(I*c*x + 1) - 4*I*b**2)*log(-I*c*x + 1)/(16*c**3
*d**3*x**2 - 32*I*c**2*d**3*x - 16*c*d**3)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx =$$
$$-\frac{(4i\,ab+3\,b^2)cx-2\left(-i\,b^2c^2x^2-2\,b^2cx-3i\,b^2\right)\arctan\left(cx\right)^2+8i\,a^2+8\,ab-4i\,b^2+((4i\,ab+3\,b^2)c^2x^2}{16\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*((4*I*a*b + 3*b^2)*c*x - 2*(-I*b^2*c^2*x^2 - 2*b^2*c*x - 3*I*b^2)*ar
ctan(c*x)^2 + 8*I*a^2 + 8*a*b - 4*I*b^2 + ((4*I*a*b + 3*b^2)*c^2*x^2 + 2*(
4*a*b - I*b^2)*c*x + 12*I*a*b + 5*b^2)*arctan(c*x))/(c^3*d^3*x^2 - 2*I*c^2
*d^3*x - c*d^3)
```

3.115.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(i\,cdx+d\right)^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^3, x)
```

3.116 $\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx$

3.116.1 Optimal result

Integrand size = 25, antiderivative size = 299

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx &= \frac{b^2}{16d^3(i-cx)^2}-\frac{11ib^2}{16d^3(i-cx)} \\ &+\frac{11ib^2\arctan(cx)}{16d^3}+\frac{ib(a+b\arctan(cx))}{4d^3(i-cx)^2} \\ &+\frac{5b(a+b\arctan(cx))}{4d^3(i-cx)}-\frac{5(a+b\arctan(cx))^2}{8d^3} \\ &-\frac{(a+b\arctan(cx))^2}{2d^3(i-cx)^2}+\frac{i(a+b\arctan(cx))^2}{d^3(i-cx)} \\ &+\frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^3} \\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^3} \\ &+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^3} \\ &+\frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output
```
1/16*b^2/d^3/(I-c*x)^2-11/16*I*b^2/d^3/(I-c*x)+11/16*I*b^2*arctan(c*x)/d^3
+1/4*I*b*(a+b*arctan(c*x))/d^3/(I-c*x)^2+5/4*b*(a+b*arctan(c*x))/d^3/(I-c*
x)-5/8*(a+b*arctan(c*x))^2/d^3-1/2*(a+b*arctan(c*x))^2/d^3/(I-c*x)^2+I*(a+
b*arctan(c*x))^2/d^3/(I-c*x)-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))
/d^3+(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/d^3+I*b*(a+b*arctan(c*x))*polylog
(2,-1+2/(1+I*c*x))/d^3+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d^3
```

3.116.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.45

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx$$
$$= \frac{-\frac{96a^2}{(-i+cx)^2} - \frac{192ia^2}{-i+cx} - 192ia^2\arctan(cx) + 192a^2\log(cx) - 96a^2\log(1+c^2x^2) + 12iab\big(-32\arctan(cx)^2 -}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

output
```
((-96*a^2)/(-I + c*x)^2 - ((192*I)*a^2)/(-I + c*x) - (192*I)*a^2*ArcTan[c*
x] + 192*a^2*Log[c*x] - 96*a^2*Log[1 + c^2*x^2] + (12*I)*a*b*(-32*ArcTan[c
*x]^2 - 12*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*PolyLog[2, E^((2*I
)*ArcTan[c*x])] + (12*I)*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]*(6*Cos[2*A
rcTan[c*x]] + Cos[4*ArcTan[c*x]] + 8*Log[1 - E^((2*I)*ArcTan[c*x])] - (6*I
)*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[c*x]]) + b^2
*((-8*I)*Pi^3 - 72*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]*Cos[2*ArcTan[c
*x]] + 144*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] - 3*Cos[4*ArcTan[c*x]] - (12*I
)*ArcTan[c*x]*Cos[4*ArcTan[c*x]] + 24*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + 1
92*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (192*I)*ArcTan[c*x]*Pol
yLog[2, E^((-2*I)*ArcTan[c*x])] + 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] +
(72*I)*Sin[2*ArcTan[c*x]] - 144*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (144*I)*A
rcTan[c*x]^2*Sin[2*ArcTan[c*x]] + (3*I)*Sin[4*ArcTan[c*x]] - 12*ArcTan[c*x
]*Sin[4*ArcTan[c*x]] - (24*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(192*d^3)
```

3.116.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx$$
$$\Big\downarrow \; 5411$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{d^3x} - \frac{c(a+b\arctan(cx))^2}{d^3(cx-i)} + \frac{ic(a+b\arctan(cx))^2}{d^3(cx-i)^2} + \frac{c(a+b\arctan(cx))^2}{d^3(cx-i)^3} \right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3} + \frac{ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^3} + \frac{5b(a+b\arctan(cx))}{4d^3(-cx+i)} + \frac{ib(a+b\arctan(cx))}{4d^3(-cx+i)^2} + \frac{i(a+b\arctan(cx))^2}{d^3(-cx+i)} - \frac{(a+b\arctan(cx))^2}{2d^3(-cx+i)^2} - \frac{5(a+b\arctan(cx))^2}{8d^3} + \frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3} + \frac{11ib^2\arctan(cx)}{16d^3} + \frac{b^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^3} - \frac{11ib^2}{16d^3(-cx+i)} + \frac{b^2}{16d^3(-cx+i)^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

output
```
b^2/(16*d^3*(I - c*x)^2) - (((11*I)/16)*b^2)/(d^3*(I - c*x)) + (((11*I)/16
)*b^2*ArcTan[c*x])/d^3 + ((I/4)*b*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)^2) +
 (5*b*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)) - (5*(a + b*ArcTan[c*x])^2)/(
8*d^3) - (a + b*ArcTan[c*x])^2/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x]
)^2)/(d^3*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)]
)/d^3 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (I*b*(a + b*ArcTa
n[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*PolyLog[3, -1 + 2/(1 +
I*c*x)])/(2*d^3)
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.116.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.61 (sec) , antiderivative size = 1783, normalized size of antiderivative = 5.96

method	result	size
derivativedivides	Expression too large to display	1783
default	Expression too large to display	1783
parts	Expression too large to display	1783

input
```
int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^3*ln(c*x)-1/2*a^2/d^3/(c*x-I)^2-I*a^2/d^3/(c*x-I)-1/2*a^2/d^3*ln(c^2
*x^2+1)-I*a^2/d^3*arctan(c*x)+b^2/d^3*(2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(
1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-5/8*arctan(c*x)^2-arctan(c
*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(c*x)+arctan(c*x)^2*ln
(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(
1/2))-2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*
x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/
(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*I*Pi*csgn(I/(1+
(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arcta
n(c*x)^2+1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2
+1)))^3*arctan(c*x)^2-1/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^
2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+arctan(c*x)^2*ln(2...
```

3.116.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(icdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*log(2*c*x/(c*x - I))*log(-(c*x +
 I)/(c*x - I))^2 + 4*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*dilog(-2*c*x/(c*x -
 I) + 1)*log(-(c*x + I)/(c*x - I)) - (2*I*b^2*c*x + 3*b^2)*log(-(c*x + I)/
(c*x - I))^2 - 8*(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)*integral(1/2*(2*I*a^2*c
*x - 2*a^2 - (2*b^2*c^2*x^2 + (2*a*b - 3*I*b^2)*c*x + 2*I*a*b)*log(-(c*x +
 I)/(c*x - I)))/(c^4*d^3*x^5 - 2*I*c^3*d^3*x^4 - 2*I*c*d^3*x^2 - d^3*x), x
) - 4*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*polylog(3, -(c*x + I)/(c*x - I)))/
(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)
```

3.116.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

3.116.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(icdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/128*(-16*I*a^2*c^2*x^2*arctan2(1, c*x) - 32*a^2*c*x*(arctan2(1, c*x) -
4*I) + 32*(I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*arctan(c*x)^3 - 4*(b^2*c^2*x
^2 - 2*I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 + 16*a^2*(I*arctan2(1, c*x) + 1
2) + 16*(2*I*b^2*c*x + 3*b^2)*arctan(c*x)^2 - 4*(2*I*b^2*c*x + 3*b^2 - 2*(
I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(b^
2*c^4*d^3*x^2 - 2*I*b^2*c^3*d^3*x - b^2*c^2*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^
12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1
)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2*lo
g(c^2*x^2 + 1)^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - c^2*(c^2/(c^10*
d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 2*log(c^2*x^2 + 1)/(c^8*d^3*x^4 + 2*c
^6*d^3*x^2 + c^4*d^3)) - 512*c^2*integrate(1/16*x^3*arctan(c*x)^2/(c^6*d^3
*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x) - 2*log(c^2*x^2 + 1)^2/(c^
6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + 512*integrate(1/16*x*arctan(c*x)^2/
(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)) - 2*(b^2*c^2*d^3*
x^2 - 2*I*b^2*c*d^3*x - b^2*d^3)*(c^4*(c^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 +
 c^6*d^3) + 2*log(c^2*x^2 + 1)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)) -
512*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*
c^2*d^3*x^3 + d^3*x), x) + 2*c^2*log(c^2*x^2 + 1)^2/(c^6*d^3*x^4 + 2*c^4*d
^3*x^2 + c^2*d^3) + 512*integrate(1/16*arctan(c*x)^2/(c^6*d^3*x^7 + 3*c^4*
d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x) + 128*integrate(1/16*log(c^2*x^2 +...
```

3.116.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^3), x)
```

3.117 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx$

3.117.1 Optimal result

Integrand size = 25, antiderivative size = 391

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = &-\frac{ib^2c}{16d^3(i-cx)^2}-\frac{19b^2c}{16d^3(i-cx)}+\frac{19b^2c\arctan(cx)}{16d^3}\\ &+\frac{bc(a+b\arctan(cx))}{4d^3(i-cx)^2}-\frac{9ibc(a+b\arctan(cx))}{4d^3(i-cx)}\\ &+\frac{ic(a+b\arctan(cx))^2}{8d^3}-\frac{(a+b\arctan(cx))^2}{d^3x}\\ &+\frac{ic(a+b\arctan(cx))^2}{2d^3(i-cx)^2}+\frac{2c(a+b\arctan(cx))^2}{d^3(i-cx)}\\ &-\frac{6ic(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^3}\\ &-\frac{3ic(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^3}\\ &+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^3}\\ &-\frac{ib^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^3}\\ &+\frac{3bc(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^3}\\ &-\frac{3ib^2c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output
```
-1/16*I*b^2*c/d^3/(I-c*x)^2-19/16*b^2*c/d^3/(I-c*x)+19/16*b^2*c*arctan(c*x
)/d^3+1/4*b*c*(a+b*arctan(c*x))/d^3/(I-c*x)^2-9/4*I*b*c*(a+b*arctan(c*x))/
d^3/(I-c*x)+1/8*I*c*(a+b*arctan(c*x))^2/d^3-(a+b*arctan(c*x))^2/d^3/x+1/2*
I*c*(a+b*arctan(c*x))^2/d^3/(I-c*x)^2+2*c*(a+b*arctan(c*x))^2/d^3/(I-c*x)+
6*I*c*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3-3*I*c*(a+b*arctan(c*
x))^2*ln(2/(1+I*c*x))/d^3+2*b*c*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^3-I*
b^2*c*polylog(2,-1+2/(1-I*c*x))/d^3+3*b*c*(a+b*arctan(c*x))*polylog(2,-1+2
/(1+I*c*x))/d^3-3/2*I*b^2*c*polylog(3,-1+2/(1+I*c*x))/d^3
```

3.117.2 Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx =$$

$$-\frac{\frac{64a^2}{x}-\frac{32ia^2c}{(-i+cx)^2}+\frac{128a^2c}{-i+cx}+192a^2c\arctan(cx)+192ia^2c\log(x)-96ia^2c\log\left(1+c^2x^2\right)-ib^2c\Big(8i\pi^3-64\ldots}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

output
```
-1/64*((64*a^2)/x - ((32*I)*a^2*c)/(-I + c*x)^2 + (128*a^2*c)/(-I + c*x) +
 192*a^2*c*ArcTan[c*x] + (192*I)*a^2*c*Log[x] - (96*I)*a^2*c*Log[1 + c^2*x
^2] - I*b^2*c*((8*I)*Pi^3 - 64*ArcTan[c*x]^2 + ((64*I)*ArcTan[c*x]^2)/(c*x
) + 40*Cos[2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 80*Arc
Tan[c*x]^2*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Cos
[4*ArcTan[c*x]] - 8*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] - 192*ArcTan[c*x]^2*L
og[1 - E^((-2*I)*ArcTan[c*x])] - (128*I)*ArcTan[c*x]*Log[1 - E^((2*I)*ArcT
an[c*x])] - (192*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 64*Po
lyLog[2, E^((2*I)*ArcTan[c*x])] - 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] -
(40*I)*Sin[2*ArcTan[c*x]] + 80*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (80*I)*Arc
Tan[c*x]^2*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*Sin[4
*ArcTan[c*x]] + (8*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]) + (4*a*b*(96*c*x*A
rcTan[c*x]^2 + 48*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + c*x*(20*Cos[2*Ar
cTan[c*x]] + Cos[4*ArcTan[c*x]] - 32*Log[c*x] + 16*Log[1 + c^2*x^2] - (20*
I)*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) + 4*ArcTan[c*x]*(8 + (10*I)*
c*x*Cos[2*ArcTan[c*x]] + I*c*x*Cos[4*ArcTan[c*x]] + (24*I)*c*x*Log[1 - E^(
(2*I)*ArcTan[c*x])] + 10*c*x*Sin[2*ArcTan[c*x]] + c*x*Sin[4*ArcTan[c*x]]))
)/x)/d^3
```

3.117.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{3ic^2(a+b\arctan(cx))^2}{d^3(cx-i)}+\frac{2c^2(a+b\arctan(cx))^2}{d^3(cx-i)^2}-\frac{ic^2(a+b\arctan(cx))^2}{d^3(cx-i)^3}+\frac{(a+b\arctan(cx))^2}{d^3x^2}-\frac{3ic(a+b\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6ic\,\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}+\frac{3bc\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^3}-\frac{9ibc(a+b\arctan(cx))}{4d^3(-cx+i)}+\frac{bc(a+b\arctan(cx))}{4d^3(-cx+i)^2}-\frac{(a+b\arctan(cx))^2}{d^3x}+\frac{2c(a+b\arctan(cx))^2}{d^3(-cx+i)}+\frac{ic(a+b\arctan(cx))^2}{2d^3(-cx+i)^2}+\frac{ic(a+b\arctan(cx))^2}{8d^3}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}-\frac{3ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}+\frac{19b^2c\arctan(cx)}{16d^3}-\frac{ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^3}-\frac{3ib^2c\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^3}-\frac{19b^2c}{16d^3(-cx+i)}-\frac{ib^2c}{16d^3(-cx+i)^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

output
```
((-1/16*I)*b^2*c)/(d^3*(I - c*x)^2) - (19*b^2*c)/(16*d^3*(I - c*x)) + (19*
b^2*c*ArcTan[c*x])/(16*d^3) + (b*c*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)^2
) - (((9*I)/4)*b*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) + ((I/8)*c*(a + b*
ArcTan[c*x])^2)/d^3 - (a + b*ArcTan[c*x])^2/(d^3*x) + ((I/2)*c*(a + b*ArcT
an[c*x])^2)/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x])^2)/(d^3*(I - c*x)
) - ((6*I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 - ((3*I
)*c*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (2*b*c*(a + b*ArcTan[c
*x])*Log[2 - 2/(1 - I*c*x)])/d^3 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)]
)/d^3 + (3*b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 - (
((3*I)/2)*b^2*c*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.117.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.81 (sec) , antiderivative size = 8688, normalized size of antiderivative = 22.22

method	result	size
derivativedivides	Expression too large to display	8688
default	Expression too large to display	8688
parts	Expression too large to display	8690

input
```
int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.117.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/8*(6*(-I*b^2*c^3*x^3 - 2*b^2*c^2*x^2 + I*b^2*c*x)*log(2*c*x/(c*x - I))*
log(-(c*x + I)/(c*x - I))^2 + 12*(-I*b^2*c^3*x^3 - 2*b^2*c^2*x^2 + I*b^2*c
*x)*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - (6*b^2*c^2*x^2
 - 9*I*b^2*c*x - 2*b^2)*log(-(c*x + I)/(c*x - I))^2 - 8*(c^2*d^3*x^3 - 2*I
*c*d^3*x^2 - d^3*x)*integral(1/2*(2*I*a^2*c*x - 2*a^2 + (6*I*b^2*c^3*x^3 +
 9*b^2*c^2*x^2 - 2*(a*b + I*b^2)*c*x - 2*I*a*b)*log(-(c*x + I)/(c*x - I)))
/(c^4*d^3*x^6 - 2*I*c^3*d^3*x^5 - 2*I*c*d^3*x^3 - d^3*x^2), x) + 12*(I*b^2
*c^3*x^3 + 2*b^2*c^2*x^2 - I*b^2*c*x)*polylog(3, -(c*x + I)/(c*x - I)))/(c
^2*d^3*x^3 - 2*I*c*d^3*x^2 - d^3*x)
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.117.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^3), x)
```

3.118 $\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx$

3.118.1 Optimal result

Integrand size = 21, antiderivative size = 207

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = -\frac{b^2}{54c(i-cx)^3}+\frac{5ib^2}{144c(i-cx)^2}+\frac{11b^2}{144c(i-cx)} - \frac{11b^2\arctan(cx)}{144c}-\frac{ib(a+b\arctan(cx))}{9c(i-cx)^3} - \frac{b(a+b\arctan(cx))}{12c(i-cx)^2}+\frac{ib(a+b\arctan(cx))}{12c(i-cx)} - \frac{i(a+b\arctan(cx))^2}{24c}+\frac{i(a+b\arctan(cx))^2}{3c(1+icx)^3}$$

output
```
-1/54*b^2/c/(I-c*x)^3+5/144*I*b^2/c/(I-c*x)^2+11/144*b^2/c/(I-c*x)-11/144*
b^2*arctan(c*x)/c-1/9*I*b*(a+b*arctan(c*x))/c/(I-c*x)^3-1/12*b*(a+b*arctan
(c*x))/c/(I-c*x)^2+1/12*I*b*(a+b*arctan(c*x))/c/(I-c*x)-1/24*I*(a+b*arctan
(c*x))^2/c+1/3*I*(a+b*arctan(c*x))^2/c/(1+I*c*x)^3
```

3.118.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx =$$
$$-\frac{144a^2+12ab(-10i+9cx+3ic^2x^2)+b^2(-56-81icx+33c^2x^2)+3b(i+cx)\,(12a(-7i+4cx+ic^2x^2)}{432c(-i+cx)^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```

output
```
-1/432*(144*a^2 + 12*a*b*(-10*I + 9*c*x + (3*I)*c^2*x^2) + b^2*(-56 - (81*
I)*c*x + 33*c^2*x^2) + 3*b*(I + c*x)*(12*a*(-7*I + 4*c*x + I*c^2*x^2) + b*
(-29 - (32*I)*c*x + 11*c^2*x^2))*ArcTan[c*x] + 18*b^2*(7 - (3*I)*c*x + 3*c
^2*x^2 + I*c^3*x^3)*ArcTan[c*x]^2)/(c*(-I + c*x)^3)
```

3.118.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx$$
$$\Big\downarrow \text{5389}$$
$$\frac{i(a+b\arctan(cx))^2}{3c(1+icx)^3}-$$
$$\frac{2}{3}ib\int \left(\frac{a+b\arctan(cx)}{8\,(c^2x^2+1)}-\frac{a+b\arctan(cx)}{8(i-cx)^2}-\frac{i(a+b\arctan(cx))}{4(i-cx)^3}+\frac{a+b\arctan(cx)}{2(i-cx)^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{i(a+b\arctan(cx))^2}{3c(1+icx)^3}-$$
$$\frac{2}{3}ib\left(\frac{(a+b\arctan(cx))^2}{16bc}-\frac{a+b\arctan(cx)}{8c(-cx+i)}-\frac{i(a+b\arctan(cx))}{8c(-cx+i)^2}+\frac{a+b\arctan(cx)}{6c(-cx+i)^3}-\frac{11ib\arctan(cx)}{96c}+\frac{1}{96c(-}\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```

output
```
((I/3)*(a + b*ArcTan[c*x])^2)/(c*(1 + I*c*x)^3) - ((2*I)/3)*b*(((-1/36*I)*
b)/(c*(I - c*x)^3) - (5*b)/(96*c*(I - c*x)^2) + (((11*I)/96)*b)/(c*(I - c*
x)) - (((11*I)/96)*b*ArcTan[c*x])/c + (a + b*ArcTan[c*x])/(6*c*(I - c*x)^3
) - ((I/8)*(a + b*ArcTan[c*x]))/(c*(I - c*x)^2) - (a + b*ArcTan[c*x])/(8*c
*(I - c*x)) + (a + b*ArcTan[c*x])^2/(16*b*c))
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.118.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{\frac{ia^2}{3(icx+1)^3}+b^2\left(\frac{i\arctan(cx)^2}{3(icx+1)^3}-\frac{2i\left(\frac{i\arctan(cx)\ln(cx+i)}{16}-\frac{i\arctan(cx)\ln(cx-i)}{16}-\frac{i\arctan(cx)}{8(cx-i)^2}-\frac{\arctan(cx)}{6(cx-i)^3}+\frac{\arctan(cx)}{8cx-8i}-\frac{11i\arcta}{96}\right.}{}\right.}{}$
default	$\frac{\frac{ia^2}{3(icx+1)^3}+b^2\left(\frac{i\arctan(cx)^2}{3(icx+1)^3}-\frac{2i\left(\frac{i\arctan(cx)\ln(cx+i)}{16}-\frac{i\arctan(cx)\ln(cx-i)}{16}-\frac{i\arctan(cx)}{8(cx-i)^2}-\frac{\arctan(cx)}{6(cx-i)^3}+\frac{\arctan(cx)}{8cx-8i}-\frac{11i\arcta}{96}\right.}{}\right.}{}$
parts	$\frac{ia^2}{3(icx+1)^3c}+\frac{b^2\left(\frac{i\arctan(cx)^2}{3(icx+1)^3}-\frac{2i\left(\frac{i\arctan(cx)\ln(cx+i)}{16}-\frac{i\arctan(cx)\ln(cx-i)}{16}-\frac{i\arctan(cx)}{8(cx-i)^2}-\frac{\arctan(cx)}{6(cx-i)^3}+\frac{\arctan(cx)}{8cx-8i}-11i\right.}{}\right.}{}$
risch	$\frac{ib^2(c^3x^3-3ic^2x^2-3cx-7i)\ln(icx+1)^2}{96(cx-i)^3c}+\frac{ib(-3bc^3x^3\ln(-icx+1)+9ibx^2\ln(-icx+1)c^2+6ibc^2x^2+9bcx\ln(-icx+1)-}{144(cx-i)^3c}$

input
```
int((a+b*arctan(c*x))^2/(1+I*c*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*I*a^2/(1+I*c*x)^3+b^2*(1/3*I/(1+I*c*x)^3*arctan(c*x)^2-2/3*I*(1/1
6*I*arctan(c*x)*ln(c*x+I)-1/16*I*arctan(c*x)*ln(c*x-I)-1/8*I*arctan(c*x)/(
c*x-I)^2-1/6*arctan(c*x)/(c*x-I)^3+1/8*arctan(c*x)/(c*x-I)-11/96*I*arctan(
c*x)+1/36*I/(c*x-I)^3-11/96*I/(c*x-I)-5/96/(c*x-I)^2-1/32*ln(c*x-I)*ln(-1/
2*I*(c*x+I))+1/64*ln(c*x-I)^2-1/32*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(-1/2*
I*(-c*x+I))+1/64*ln(c*x+I)^2))+2/3*I*a*b/(1+I*c*x)^3*arctan(c*x)-1/12*I*a*
b*arctan(c*x)-1/12*a*b/(c*x-I)^2+1/9*I*a*b/(c*x-I)^3-1/12*I*a*b/(c*x-I))
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx =$$
$$-\frac{6\,(12i\,ab+11\,b^2)c^2x^2+54\,(4\,ab-3i\,b^2)cx+9\,(-i\,b^2c^3x^3-3\,b^2c^2x^2+3i\,b^2cx-7\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+28}{864\,(c^4x}$$

input
```
integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="fricas")
```

output
```
-1/864*(6*(12*I*a*b + 11*b^2)*c^2*x^2 + 54*(4*a*b - 3*I*b^2)*c*x + 9*(-I*b
^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*I*b^2*c*x - 7*b^2)*log(-(c*x + I)/(c*x - I)
)^2 + 288*a^2 - 240*I*a*b - 112*b^2 - 3*((12*a*b - 11*I*b^2)*c^3*x^3 - 3*(
12*I*a*b + 7*b^2)*c^2*x^2 - 3*(12*a*b + I*b^2)*c*x - 84*I*a*b - 29*b^2)*lo
g(-(c*x + I)/(c*x - I)))/(c^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*c)
```

3.118.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(158) = 316.

Time = 21.42 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.67

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = -\frac{b(12a-11ib)\log\left(-\frac{ib(12a-11ib)}{c}+x(12ab-11ib^2)\right)}{288c}$$
$$+\frac{b(12a-11ib)\log\left(\frac{ib(12a-11ib)}{c}+x(12ab-11ib^2)\right)}{288c}$$
$$+\frac{-144a^2+120iab+56b^2+x^2(-36iabc^2-33b^2c^2)+x(-108abc+81ib^2c)}{432c^4x^3-1296ic^3x^2-1296c^2x+432ic}$$
$$+\frac{(-48iab-3ib^2c^3x^3\log(icx+1)-9b^2c^2x^2\log(icx+1)+6b^2c^2x^2+9ib^2cx\log(icx+1)-18ib^2cx-2}{144c^4x^3-432ic^3x^2-432c^2x+144ic}$$
$$+\frac{(ib^2c^3x^3+3b^2c^2x^2-3ib^2cx+7b^2)\log(-icx+1)^2}{96c^4x^3-288ic^3x^2-288c^2x+96ic}$$
$$+\frac{(ib^2c^3x^3+3b^2c^2x^2-3ib^2cx+7b^2)\log(icx+1)^2}{96c^4x^3-288ic^3x^2-288c^2x+96ic}$$
$$+\frac{(24iab-3b^2c^2x^2+9ib^2cx+10b^2)\log(icx+1)}{72c^4x^3-216ic^3x^2-216c^2x+72ic}$$

input
```
integrate((a+b*atan(c*x))**2/(1+I*c*x)**4,x)
```

output
```
-b*(12*a - 11*I*b)*log(-I*b*(12*a - 11*I*b)/c + x*(12*a*b - 11*I*b**2))/(2
88*c) + b*(12*a - 11*I*b)*log(I*b*(12*a - 11*I*b)/c + x*(12*a*b - 11*I*b**
2))/(288*c) + (-144*a**2 + 120*I*a*b + 56*b**2 + x**2*(-36*I*a*b*c**2 - 33
*b**2*c**2) + x*(-108*a*b*c + 81*I*b**2*c))/(432*c**4*x**3 - 1296*I*c**3*x
**2 - 1296*c**2*x + 432*I*c) + (-48*I*a*b - 3*I*b**2*c**3*x**3*log(I*c*x +
 1) - 9*b**2*c**2*x**2*log(I*c*x + 1) + 6*b**2*c**2*x**2 + 9*I*b**2*c*x*lo
g(I*c*x + 1) - 18*I*b**2*c*x - 21*b**2*log(I*c*x + 1) - 20*b**2)*log(-I*c*
x + 1)/(144*c**4*x**3 - 432*I*c**3*x**2 - 432*c**2*x + 144*I*c) + (I*b**2*
c**3*x**3 + 3*b**2*c**2*x**2 - 3*I*b**2*c*x + 7*b**2)*log(-I*c*x + 1)**2/(
96*c**4*x**3 - 288*I*c**3*x**2 - 288*c**2*x + 96*I*c) + (I*b**2*c**3*x**3
+ 3*b**2*c**2*x**2 - 3*I*b**2*c*x + 7*b**2)*log(I*c*x + 1)**2/(96*c**4*x**
3 - 288*I*c**3*x**2 - 288*c**2*x + 96*I*c) + (24*I*a*b - 3*b**2*c**2*x**2
+ 9*I*b**2*c*x + 10*b**2)*log(I*c*x + 1)/(72*c**4*x**3 - 216*I*c**3*x**2 -
 216*c**2*x + 72*I*c)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx$$
$$= \frac{3\,(-12i\,ab-11\,b^2)c^2x^2-27\,(4\,ab-3i\,b^2)cx+18\,(-i\,b^2c^3x^3-3\,b^2c^2x^2+3i\,b^2cx-7\,b^2)\arctan{(cx)}^2-14}{432\,(c^4x^3}$$

input
```
integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="maxima")
```

output
```
1/432*(3*(-12*I*a*b - 11*b^2)*c^2*x^2 - 27*(4*a*b - 3*I*b^2)*c*x + 18*(-I*
b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*I*b^2*c*x - 7*b^2)*arctan(c*x)^2 - 144*a^2
 + 120*I*a*b + 56*b^2 + 3*((-12*I*a*b - 11*b^2)*c^3*x^3 - 3*(12*a*b - 7*I*
b^2)*c^2*x^2 + 3*(12*I*a*b - b^2)*c*x - 84*a*b + 29*I*b^2)*arctan(c*x))/(c
^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*c)
```

3.118.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{(i\,cx+1)^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(1+c\,x\,1\mathrm{i})^4}\,dx$$

input
```
int((a + b*atan(c*x))^2/(c*x*1i + 1)^4,x)
```

output
```
int((a + b*atan(c*x))^2/(c*x*1i + 1)^4, x)
```

3.119 $\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx$

3.119.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx=\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}+\frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
arctan(a*x)^2*ln(2-2/(1-I*a*x))/c-I*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/
c+1/2*polylog(3,-1+2/(1-I*a*x))/c
```

3.119.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx=\frac{-i\pi^3+16i\arctan(ax)^3+24\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)+24i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax}\right.}{24c}$$

input
```
Integrate[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

output
```
((-I)*Pi^3 + (16*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*Arc
Tan[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*Po
lyLog[3, E^((-2*I)*ArcTan[a*x])])/(24*c)
```

3.119.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2026, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{\arctan(ax)^2}{x(c - iacx)}dx$$

$$\Big\downarrow \text{ 5403}$$

$$\frac{\arctan(ax)^2 \log\left(2 - \frac{2}{1-iax}\right)}{c} - \frac{2a\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 5527}$$

$$\frac{\arctan(ax)^2 \log\left(2 - \frac{2}{1-iax}\right)}{c} - \frac{2a\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a} - \frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\arctan(ax)^2 \log\left(2 - \frac{2}{1-iax}\right)}{c} - \frac{2a\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a} - \frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)}{c}$$

input
```
Int[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

output
```
(ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (2*a*(((I/2)*ArcTan[a*x]*PolyLo
g[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a)))/c
```

3.119.3.1 Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.119.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(70) = 140$.

Time = 6.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.54

method	result
derivativedivides	$\frac{\frac{a\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{2ia\arctan(ax)\,\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+\frac{2a\,\mathrm{polylog}\left(3,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+\frac{a\arctan(ax)^2\ln\left(1+\frac{iax+}{\sqrt{a^2x^2}}\right)}{c}}{a}$
default	$\frac{\frac{a\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{2ia\arctan(ax)\,\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+\frac{2a\,\mathrm{polylog}\left(3,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}+\frac{a\arctan(ax)^2\ln\left(1+\frac{iax+}{\sqrt{a^2x^2}}\right)}{c}}{a}$

input
```
int(arctan(a*x)^2/(c*x-I*a*c*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a/c*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*a/c*arctan(a*
x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a/c*polylog(3,(1+I*a*x)/(a^2*x
^2+1)^(1/2))+a/c*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*a/c*a
rctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a/c*polylog(3,-(1+I*a
*x)/(a^2*x^2+1)^(1/2)))
```

3.119.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx=\int \frac{\arctan\left(ax\right)^2}{-i\,acx^2+cx}\,dx$$

input
```
integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="fricas")
```

output
```
integral(-1/4*I*log(-(a*x + I)/(a*x - I))^2/(a*c*x^2 + I*c*x), x)
```

3.119.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx=\frac{i\int \frac{\operatorname{atan}^2(ax)}{ax^2+ix}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/(c*x-I*a*c*x**2),x)
```

output
```
I*Integral(atan(a*x)**2/(a*x**2 + I*x), x)/c
```

3.119.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx=\int \frac{\arctan\left(ax\right)^2}{-i\,acx^2+cx}\,dx$$

input
```
integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="maxima")
```

output
```
1/96*(8*I*arctan(a*x)^3 - 12*arctan(a*x)^2*log(a^2*x^2 + 1) - 6*I*arctan(a
*x)*log(a^2*x^2 + 1)^2 + log(a^2*x^2 + 1)^3 + 24*I*(arctan(a*x)^3/c + 4*a*
integrate(1/16*x*log(a^2*x^2 + 1)^2/(a^2*c*x^3 + c*x), x) - 16*integrate(1
/16*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*c*x^3 + c*x), x))*c + 96*c*integrate
(1/16*(4*a*x*arctan(a*x)*log(a^2*x^2 + 1) + 12*arctan(a*x)^2 + log(a^2*x^2
 + 1)^2)/(a^2*c*x^3 + c*x), x))/c
```

3.119.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \int \frac{\arctan(ax)^2}{-i\,acx^2 + cx}\,dx$$

input
```
integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{c\,x - a\,c\,x^2\,1\mathrm{i}}\,dx$$

input
```
int(atan(a*x)^2/(c*x - a*c*x^2*1i),x)
```

output
```
int(atan(a*x)^2/(c*x - a*c*x^2*1i), x)
```

3.120 $\int (d+icdx)^3(a+b\arctan(cx))^3\,dx$

3.120.1 Optimal result

Integrand size = 22, antiderivative size = 382

$$
\begin{aligned}
\int (d+icdx)^3(a+b\arctan(cx))^3\,dx = & -3ab^2d^3x+\frac{1}{4}ib^3d^3x-\frac{ib^3d^3\arctan(cx)}{4c} \\
& -3b^3d^3x\arctan(cx)-\frac{1}{4}ib^2cd^3x^2(a+b\arctan(cx)) \\
& +\frac{7bd^3(a+b\arctan(cx))^2}{c} \\
& -\frac{21}{4}ibd^3x(a+b\arctan(cx))^2 \\
& +\frac{3}{2}bcd^3x^2(a+b\arctan(cx))^2 \\
& +\frac{1}{4}ibc^2d^3x^3(a+b\arctan(cx))^2 \\
& -\frac{id^3(1+icx)^4(a+b\arctan(cx))^3}{4c} \\
& +\frac{6bd^3(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c} \\
& -\frac{11ib^2d^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} \\
& +\frac{3b^3d^3\log\left(1+c^2x^2\right)}{2c} \\
& -\frac{6ib^2d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c} \\
& +\frac{11b^3d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c} \\
& +\frac{3b^3d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}
\end{aligned}
$$

output
```
-3*a*b^2*d^3*x+1/4*I*b*c^2*d^3*x^3*(a+b*arctan(c*x))^2-1/4*I*b^2*c*d^3*x^2
*(a+b*arctan(c*x))-3*b^3*d^3*x*arctan(c*x)-21/4*I*b*d^3*x*(a+b*arctan(c*x)
)^2+7*b*d^3*(a+b*arctan(c*x))^2/c-1/4*I*b^3*d^3*arctan(c*x)/c+3/2*b*c*d^3*
x^2*(a+b*arctan(c*x))^2+1/4*I*b^3*d^3*x-6*I*b^2*d^3*(a+b*arctan(c*x))*poly
log(2,1-2/(1-I*c*x))/c+6*b*d^3*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/c-11*I*
b^2*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+3/2*b^3*d^3*ln(c^2*x^2+1)/c-1/
4*I*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^3/c+11/2*b^3*d^3*polylog(2,1-2/(1+I*
c*x))/c+3*b^3*d^3*polylog(3,1-2/(1-I*c*x))/c
```

3.120.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.81

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx =$$

$$-\frac{id^3\left(ab^2+4ia^3cx+21a^2bcx-12iab^2cx-b^3cx-6a^3c^2x^2+6ia^2bc^2x^2+ab^2c^2x^2-4ia^3c^3x^3-a^2bc^3x^3\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```

output
```
((-1/4*I)*d^3*(a*b^2 + (4*I)*a^3*c*x + 21*a^2*b*c*x - (12*I)*a*b^2*c*x - b
^3*c*x - 6*a^3*c^2*x^2 + (6*I)*a^2*b*c^2*x^2 + a*b^2*c^2*x^2 - (4*I)*a^3*c
^3*x^3 - a^2*b*c^3*x^3 + a^3*c^4*x^4 - 21*a^2*b*ArcTan[c*x] + (12*I)*a*b^2
*ArcTan[c*x] + b^3*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] + 42*a*b^2*c
*x*ArcTan[c*x] - (12*I)*b^3*c*x*ArcTan[c*x] - 18*a^2*b*c^2*x^2*ArcTan[c*x]
 + (12*I)*a*b^2*c^2*x^2*ArcTan[c*x] + b^3*c^2*x^2*ArcTan[c*x] - (12*I)*a^2
*b*c^3*x^3*ArcTan[c*x] - 2*a*b^2*c^3*x^3*ArcTan[c*x] + 3*a^2*b*c^4*x^4*Arc
Tan[c*x] + 3*a*b^2*ArcTan[c*x]^2 - (16*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2
*c*x*ArcTan[c*x]^2 + 21*b^3*c*x*ArcTan[c*x]^2 - 18*a*b^2*c^2*x^2*ArcTan[c*
x]^2 + (6*I)*b^3*c^2*x^2*ArcTan[c*x]^2 - (12*I)*a*b^2*c^3*x^3*ArcTan[c*x]^
2 - b^3*c^3*x^3*ArcTan[c*x]^2 + 3*a*b^2*c^4*x^4*ArcTan[c*x]^2 + b^3*ArcTan
[c*x]^3 + (4*I)*b^3*c*x*ArcTan[c*x]^3 - 6*b^3*c^2*x^2*ArcTan[c*x]^3 - (4*I
)*b^3*c^3*x^3*ArcTan[c*x]^3 + b^3*c^4*x^4*ArcTan[c*x]^3 + (48*I)*a*b^2*Arc
Tan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 44*b^3*ArcTan[c*x]*Log[1 + E^((2
*I)*ArcTan[c*x])] + (24*I)*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])
] - (12*I)*a^2*b*Log[1 + c^2*x^2] - 22*a*b^2*Log[1 + c^2*x^2] + (6*I)*b^3*
Log[1 + c^2*x^2] + 2*b^2*(12*a - (11*I)*b + 12*b*ArcTan[c*x])*PolyLog[2, -
E^((2*I)*ArcTan[c*x])] + (12*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

3.120.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{3ib\int \left(c^2x^2(a+b\arctan(cx))^2d^4-4icx(a+b\arctan(cx))^2d^4-\frac{8i(i-cx)(a+b\arctan(cx))^2d^4}{c^2x^2+1}-7(a+b\arctan(cx))^2d^4\right)}{4d}$$

$$\frac{id^3(1+icx)^4(a+b\arctan(cx))^3}{4c}$$

$$\Big\downarrow \text{2009}$$

$$3ib\left(\frac{1}{3}c^2d^4x^3(a+b\arctan(cx))^2-\frac{8bd^4\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-2icd^4x^2(a+b\arctan(cx))^2-\frac{1}{3}bcd^4x^2(a\right.$$

$$\frac{id^3(1+icx)^4(a+b\arctan(cx))^3}{4c}$$

input
```
Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```

output
```
((-1/4*I)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^3)/c + (((3*I)/4)*b*((4*I)
*a*b*d^4*x + (b^2*d^4*x)/3 - (b^2*d^4*ArcTan[c*x])/(3*c) + (4*I)*b^2*d^4*x
*ArcTan[c*x] - (b*c*d^4*x^2*(a + b*ArcTan[c*x]))/3 - (((28*I)/3)*d^4*(a +
b*ArcTan[c*x])^2)/c - 7*d^4*x*(a + b*ArcTan[c*x])^2 - (2*I)*c*d^4*x^2*(a +
 b*ArcTan[c*x])^2 + (c^2*d^4*x^3*(a + b*ArcTan[c*x])^2)/3 - ((8*I)*d^4*(a
+ b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - (44*b*d^4*(a + b*ArcTan[c*x])*L
og[2/(1 + I*c*x)])/(3*c) - ((2*I)*b^2*d^4*Log[1 + c^2*x^2])/c - (8*b*d^4*(
a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c - (((22*I)/3)*b^2*d^4*
PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((4*I)*b^2*d^4*PolyLog[3, 1 - 2/(1 - I*
c*x)])/c))/d
```

3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.120.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 61.01 (sec) , antiderivative size = 1513, normalized size of antiderivative = 3.96

method	result	size
derivativedivides	Expression too large to display	1513
default	Expression too large to display	1513
parts	Expression too large to display	1521

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/4*I*d^3*a^3*(1+I*c*x)^4+d^3*b^3*(-1/4*I*arctan(c*x)^3*c^4*x^4-arct
an(c*x)^3*c^3*x^3+3/2*I*arctan(c*x)^3*c^2*x^2+arctan(c*x)^3*c*x-1/4*I*arct
an(c*x)^3+3/4*I*(1/3*I+1/3*c*x+1/3*c^3*x^3*arctan(c*x)^2-7*arctan(c*x)^2*c
*x-8*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2/3*arctan(c*x)*(c*x-
I)*(c*x+I)+1/3*arctan(c*x)*(c*x-I)^2-44/3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))-44/3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*Pi*
csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-2*Pi*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-2*Pi*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2+2*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*
x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-2*Pi*csgn(I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x
)^2+2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2
+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-4*Pi*csgn(I*(1+(1+I*c*x
)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+2*
Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*
c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+4*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(
1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2-2*Pi*csgn(I/(1+(1+I*
c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/
(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-2*I*arctan(c*x)^2
*c^2*x^2+4*I*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+16/3*I*arctan(c*x)^2-4*I*pol...
```

3.120.5 Fricas [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
-1/32*(b^3*c^3*d^3*x^4 - 4*I*b^3*c^2*d^3*x^3 - 6*b^3*c*d^3*x^2 + 4*I*b^3*d
^3*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/16*(-16*I*a^3*c^5*d^3*x^5 -
 48*a^3*c^4*d^3*x^4 + 32*I*a^3*c^3*d^3*x^3 - 32*a^3*c^2*d^3*x^2 + 48*I*a^3
*c*d^3*x + 16*a^3*d^3 - 3*(-4*I*a*b^2*c^5*d^3*x^5 - (12*a*b^2 - I*b^3)*c^4
*d^3*x^4 + 4*(2*I*a*b^2 + b^3)*c^3*d^3*x^3 - 2*(4*a*b^2 + 3*I*b^3)*c^2*d^3
*x^2 + 4*a*b^2*d^3 + 4*(3*I*a*b^2 - b^3)*c*d^3*x)*log(-(c*x + I)/(c*x - I)
)^2 + 24*(a^2*b*c^5*d^3*x^5 - 3*I*a^2*b*c^4*d^3*x^4 - 2*a^2*b*c^3*d^3*x^3
- 2*I*a^2*b*c^2*d^3*x^2 - 3*a^2*b*c*d^3*x + I*a^2*b*d^3)*log(-(c*x + I)/(c
*x - I)))/(c^2*x^2 + 1), x)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**3,x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
-1/4*I*a^3*c^3*d^3*x^4 - 24*b^3*c^5*d^3*integrate(1/128*x^5*arctan(c*x)^2*
log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 2*b^3*c^5*d^3*integrate(1/128*x^5*log
(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 12*b^3*c^5*d^3*integrate(1/128*x^5*arc
tan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c^5*d^3*integrate(1/128*x^5*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) - a^3*c^2*d^3*x^3 - 336*b^3*c^4*d^3*integrate(
1/128*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) - 36*b^3*c^4*d^3*integrate(1/128
*x^4*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 1152*a*b^2*c^4*d^3
*integrate(1/128*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) - 60*b^3*c^4*d^3*inte
grate(1/128*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 1/4*I*(3*
x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b*c^3*d
^3 + 48*b^3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^
2*x^2 + 1), x) - 4*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^3/(c^2
*x^2 + 1), x) + 120*b^3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2
 + 1), x) - 30*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2
 + 1), x) - 3/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a
^2*b*c^2*d^3 + 3/2*I*a^3*c*d^3*x^2 + 7/32*b^3*d^3*arctan(c*x)^4/c - 224*b^
3*c^2*d^3*integrate(1/128*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) - 24*b^3*c^2
*d^3*integrate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
- 768*a*b^2*c^2*d^3*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) +
120*b^3*c^2*d^3*integrate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x...
```

3.120.8 Giac [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx=\int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^3, x)
```

3.121 $\int (d+icdx)^2(a+b\arctan(cx))^3\,dx$

3.121.1 Optimal result

Integrand size = 22, antiderivative size = 298

$$\begin{aligned}\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = & -ab^2d^2x-b^3d^2x\arctan(cx)+\frac{7bd^2(a+b\arctan(cx))^2}{2c} \\ & -3ibd^2x(a+b\arctan(cx))^2 \\ & +\frac{1}{2}bcd^2x^2(a+b\arctan(cx))^2 \\ & -\frac{id^2(1+icx)^3(a+b\arctan(cx))^3}{3c} \\ & +\frac{4bd^2(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c} \\ & -\frac{6ib^2d^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} \\ & +\frac{b^3d^2\log\left(1+c^2x^2\right)}{2c} \\ & -\frac{4ib^2d^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c} \\ & +\frac{3b^3d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} \\ & +\frac{2b^3d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}\end{aligned}$$

output
```
-a*b^2*d^2*x-b^3*d^2*x*arctan(c*x)+7/2*b*d^2*(a+b*arctan(c*x))^2/c-3*I*b*d
^2*x*(a+b*arctan(c*x))^2+1/2*b*c*d^2*x^2*(a+b*arctan(c*x))^2-1/3*I*d^2*(1+
I*c*x)^3*(a+b*arctan(c*x))^3/c+4*b*d^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))
/c-6*I*b^2*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+1/2*b^3*d^2*ln(c^2*x^2+
1)/c-4*I*b^2*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/c+3*b^3*d^2*po
lylog(2,1-2/(1+I*c*x))/c+2*b^3*d^2*polylog(3,1-2/(1-I*c*x))/c
```

3.121.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.77

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx =$$
$$-\frac{d^2\left(-6a^3cx+18ia^2bcx+6ab^2cx-6ia^3c^2x^2-3a^2bc^2x^2+2a^3c^3x^3-18ia^2b\arctan(cx)-6ab^2\arctan(c\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
-1/6*(d^2*(-6*a^3*c*x + (18*I)*a^2*b*c*x + 6*a*b^2*c*x - (6*I)*a^3*c^2*x^2
 - 3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 - (18*I)*a^2*b*ArcTan[c*x] - 6*a*b^2*Ar
cTan[c*x] - 18*a^2*b*c*x*ArcTan[c*x] + (36*I)*a*b^2*c*x*ArcTan[c*x] + 6*b^
3*c*x*ArcTan[c*x] - (18*I)*a^2*b*c^2*x^2*ArcTan[c*x] - 6*a*b^2*c^2*x^2*Arc
Tan[c*x] + 6*a^2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*b^2*ArcTan[c*x]^2 + 15*b^
3*ArcTan[c*x]^2 - 18*a*b^2*c*x*ArcTan[c*x]^2 + (18*I)*b^3*c*x*ArcTan[c*x]^
2 - (18*I)*a*b^2*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^2*x^2*ArcTan[c*x]^2 + 6*a
*b^2*c^3*x^3*ArcTan[c*x]^2 + (2*I)*b^3*ArcTan[c*x]^3 - 6*b^3*c*x*ArcTan[c*
x]^3 - (6*I)*b^3*c^2*x^2*ArcTan[c*x]^3 + 2*b^3*c^3*x^3*ArcTan[c*x]^3 - 48*
a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (36*I)*b^3*ArcTan[c*x]*
Log[1 + E^((2*I)*ArcTan[c*x])] - 24*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*Arc
Tan[c*x])] + 12*a^2*b*Log[1 + c^2*x^2] - (18*I)*a*b^2*Log[1 + c^2*x^2] - 3
*b^3*Log[1 + c^2*x^2] + 6*b^2*((4*I)*a + 3*b + (4*I)*b*ArcTan[c*x])*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])] - 12*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))
/c
```

3.121.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{ib\int \left(-icx(a+b\arctan(cx))^2d^3-\frac{4i(i-cx)(a+b\arctan(cx))^2d^3}{c^2x^2+1}-3(a+b\arctan(cx))^2d^3\right)dx}{d}-\frac{id^2(1+icx)^3(a+b\arctan(cx))^3}{3c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ib\left(-\frac{4bd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-\frac{1}{2}icd^3x^2(a+b\arctan(cx))^2-3d^3x(a+b\arctan(cx))^2-\frac{7id^3(a+b\arctan(\right.}{2c}}{}-\frac{id^2(1+icx)^3(a+b\arctan(cx))^3}{3c}$$

input
```
Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
((-1/3*I)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^3)/c + (I*b*(I*a*b*d^3*x +
 I*b^2*d^3*x*ArcTan[c*x] - (((7*I)/2)*d^3*(a + b*ArcTan[c*x])^2)/c - 3*d^3
*x*(a + b*ArcTan[c*x])^2 - (I/2)*c*d^3*x^2*(a + b*ArcTan[c*x])^2 - ((4*I)*
d^3*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - (6*b*d^3*(a + b*ArcTan[c
*x])*Log[2/(1 + I*c*x)])/c - ((I/2)*b^2*d^3*Log[1 + c^2*x^2])/c - (4*b*d^3
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c - ((3*I)*b^2*d^3*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c - ((2*I)*b^2*d^3*PolyLog[3, 1 - 2/(1 - I*c*x
)])/c))/d
```

3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.121.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.64 (sec) , antiderivative size = 1384, normalized size of antiderivative = 4.64

method	result	size
derivativedivides	Expression too large to display	1384
default	Expression too large to display	1384
parts	Expression too large to display	1392

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/3*I*d^2*a^3*(1+I*c*x)^3+d^2*b^3*(-1/3*arctan(c*x)^3*c^3*x^3+I*arct
an(c*x)^3*c^2*x^2+arctan(c*x)^3*c*x-1/3*I*arctan(c*x)^3+I*(-3*arctan(c*x)^
2*c*x-6*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*ln(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*
x^2+1))-2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+5/2*I*arctan(c*x)^2+I*ln((
1+I*c*x)^2/(c^2*x^2+1)+1)+2*I*arctan(c*x)^2*ln(c^2*x^2+1)-Pi*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-4*I*arctan(c*x)
^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^
2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+2*Pi*csgn(I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2-1/2*I*arcta
n(c*x)^2*c^2*x^2+6*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*
(c*x-I)+6*I*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+Pi*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*csgn(I/((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)*arctan(c*x)^2+Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2-2*Pi*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan
(c*x)^2+Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)+1)^2)*arctan(c*x)^2-Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x
)^2-Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*...
```

3.121.5 Fricas [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = \int (i\,cdx+d)^2(b\arctan{(cx)}+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
1/24*(I*b^3*c^2*d^2*x^3 + 3*b^3*c*d^2*x^2 - 3*I*b^3*d^2*x)*log(-(c*x + I)/
(c*x - I))^3 + integral(-1/4*(4*a^3*c^4*d^2*x^4 - 8*I*a^3*c^3*d^2*x^3 - 8*
I*a^3*c*d^2*x - 4*a^3*d^2 - (3*a*b^2*c^4*d^2*x^4 + 3*I*b^3*c^2*d^2*x^2 + (
-6*I*a*b^2 - b^3)*c^3*d^2*x^3 - 3*a*b^2*d^2 - 3*(2*I*a*b^2 - b^3)*c*d^2*x)
*log(-(c*x + I)/(c*x - I))^2 + 6*(I*a^2*b*c^4*d^2*x^4 + 2*a^2*b*c^3*d^2*x^
3 + 2*a^2*b*c*d^2*x - I*a^2*b*d^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1
), x)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**3,x)
```

output
```
Timed out
```

3.121.7 Maxima [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = \int (i\,cdx+d)^2(b\arctan(cx)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
-1/3*a^3*c^2*d^2*x^3 - 28*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)^3/(c^
2*x^2 + 1), x) - 3*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*x^2 + 1), x) - 96*a*b^2*c^4*d^2*integrate(1/32*x^4*arctan(c*x)
^2/(c^2*x^2 + 1), x) - 4*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^
2*x^2 + 1)/(c^2*x^2 + 1), x) + 12*b^3*c^3*d^2*integrate(1/32*x^3*arctan(c*
x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c^3*d^2*integrate(1/32*x^3*l
og(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) + 16*b^3*c^3*d^2*integrate(1/32*x^3*ar
ctan(c*x)^2/(c^2*x^2 + 1), x) - 4*b^3*c^3*d^2*integrate(1/32*x^3*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) - 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^
2*x^2 + 1)/c^4))*a^2*b*c^2*d^2 + I*a^3*c*d^2*x^2 + 7/32*b^3*d^2*arctan(c*x
)^4/c + 24*b^3*c^2*d^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^
2*x^2 + 1), x) + 3*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b
*c*d^2 + a*b^2*d^2*arctan(c*x)^3/c + 12*b^3*c*d^2*integrate(1/32*x*arctan(
c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d^2*integrate(1/32*x*log
(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 12*b^3*c*d^2*integrate(1/32*x*arctan(c
*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*d^2*integrate(1/32*x*log(c^2*x^2 + 1)^2/
(c^2*x^2 + 1), x) + a^3*d^2*x + 3*b^3*d^2*integrate(1/32*arctan(c*x)*log(c
^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1
))*a^2*b*d^2/c - 1/24*(b^3*c^2*d^2*x^3 - 3*I*b^3*c*d^2*x^2 - 3*b^3*d^2*x)*
arctan(c*x)^3 + 1/16*(-I*b^3*c^2*d^2*x^3 - 3*b^3*c*d^2*x^2 + 3*I*b^3*d^...
```

3.121.8 Giac [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx=\int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

input
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^2, x)
```

3.122 $\int (d+icdx)(a+b\arctan(cx))^3\,dx$

3.122.1 Optimal result

Integrand size = 20, antiderivative size = 220

$$\begin{aligned}\int (d+icdx)(a+b\arctan(cx))^3\,dx &= \frac{3bd(a+b\arctan(cx))^2}{2c}-\frac{3}{2}ibdx(a+b\arctan(cx))^2\\&-\frac{id(1+icx)^2(a+b\arctan(cx))^3}{2c}\\&+\frac{3bd(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c}\\&-\frac{3ib^2d(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}\\&-\frac{3ib^2d(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}\\&+\frac{3b^3d\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c}\\&+\frac{3b^3d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2c}\end{aligned}$$

output
```
3/2*b*d*(a+b*arctan(c*x))^2/c-3/2*I*b*d*x*(a+b*arctan(c*x))^2-1/2*I*d*(1+I
*c*x)^2*(a+b*arctan(c*x))^3/c+3*b*d*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/c-
3*I*b^2*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c-3*I*b^2*d*(a+b*arctan(c*x))*
polylog(2,1-2/(1-I*c*x))/c+3/2*b^3*d*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3*d*
polylog(3,1-2/(1-I*c*x))/c
```

3.122.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.67

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx$$
$$= \frac{id\left(-2ia^3cx-3a^2bcx+a^3c^2x^2+3a^2b\arctan(cx)-6ia^2bcx\arctan(cx)-6ab^2cx\arctan(cx)+3a^2bc^2x^2\text{ a}\right.}{}$$

input
```
Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

output
```
((I/2)*d*((-2*I)*a^3*c*x - 3*a^2*b*c*x + a^3*c^2*x^2 + 3*a^2*b*ArcTan[c*x]
 - (6*I)*a^2*b*c*x*ArcTan[c*x] - 6*a*b^2*c*x*ArcTan[c*x] + 3*a^2*b*c^2*x^2
*ArcTan[c*x] - 3*a*b^2*ArcTan[c*x]^2 + (3*I)*b^3*ArcTan[c*x]^2 - (6*I)*a*b
^2*c*x*ArcTan[c*x]^2 - 3*b^3*c*x*ArcTan[c*x]^2 + 3*a*b^2*c^2*x^2*ArcTan[c*
x]^2 - b^3*ArcTan[c*x]^3 - (2*I)*b^3*c*x*ArcTan[c*x]^3 + b^3*c^2*x^2*ArcTa
n[c*x]^3 - (12*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*b^3
*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^3*ArcTan[c*x]^2*Log[
1 + E^((2*I)*ArcTan[c*x])] + (3*I)*a^2*b*Log[1 + c^2*x^2] + 3*a*b^2*Log[1
+ c^2*x^2] - 3*b^2*(2*a - I*b + 2*b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcT
an[c*x])] - (3*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

3.122.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx$$
$$\Big\downarrow \text{ 5389}$$
$$\frac{3ib\int \left(-d^2(a+b\arctan(cx))^2-\frac{2id^2(i-cx)(a+b\arctan(cx))^2}{c^2x^2+1}\right)dx}{2d}-\frac{id(1+icx)^2(a+b\arctan(cx))^3}{2c}$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{3ib\left(-\frac{2bd^2\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}+d^2(-x)(a+b\arctan(cx))^2-\frac{id^2(a+b\arctan(cx))^2}{c}-\frac{2id^2\log\left(\frac{2}{1-icx}\right)(a+b\,\text{arc}}{c}\right.}{2d}$$

$$\frac{id(1+icx)^2(a+b\arctan(cx))^3}{2c}$$

input
```
Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

output
```
((-1/2*I)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x])^3)/c + (((3*I)/2)*b*(((-I)*d
^2*(a + b*ArcTan[c*x])^2)/c - d^2*x*(a + b*ArcTan[c*x])^2 - ((2*I)*d^2*(a
+ b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - (2*b*d^2*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/c - (2*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I
*c*x)])/c - (I*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (I*b^2*d^2*PolyL
og[3, 1 - 2/(1 - I*c*x)])/c))/d
```

3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.122.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.04 (sec) , antiderivative size = 3777, normalized size of antiderivative = 17.17

method	result	size
derivativedivides	Expression too large to display	3777
default	Expression too large to display	3777
parts	Expression too large to display	3779

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-I*d*a^3*(-1/2*c^2*x^2+I*c*x)+d*b^3*(1/2*I*arctan(c*x)^3*c^2*x^2+arct
an(c*x)^3*c*x+3/2*I*(-1/4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*(2*I*arctan
(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1)))-1/4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2
+1)+1)^2)^3*(2*I*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*arctan(c*x)^2
+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))+1/4*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)^3*(2*I*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*arctan(c*x)^2+
polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1/2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)+1)^2)^3*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)
*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+1/2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x
^2+1))^3*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*
ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))-2*ln(2)*arctan(c*x)*ln(1+I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))-2*ln(2)*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))+2*ln(2)*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+2*I*ln(2)*dilog(1+I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*ln(2)*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))-2*I*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-I*ln(2)*polylog(2,-
(1+I*c*x)^2/(c^2*x^2+1))+1/2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^
2/(c^2*x^2+1)+1)^2)^3*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)...
```

3.122.5 Fricas [F]

$$\int (d + icdx)(a + b\arctan(cx))^3 \, dx = \int (i\, cdx + d)(b\arctan{(cx)} + a)^3 \, dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
1/16*(b^3*c*d*x^2 - 2*I*b^3*d*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/
8*(8*I*a^3*c^3*d*x^3 + 8*a^3*c^2*d*x^2 + 8*I*a^3*c*d*x + 8*a^3*d - 3*(2*I*
a*b^2*c^3*d*x^3 + (2*a*b^2 - I*b^3)*c^2*d*x^2 + 2*a*b^2*d + 2*(I*a*b^2 - b
^3)*c*d*x)*log(-(c*x + I)/(c*x - I))^2 - 12*(a^2*b*c^3*d*x^3 - I*a^2*b*c^2
*d*x^2 + a^2*b*c*d*x - I*a^2*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1)
, x)
```

3.122.6 Sympy [F(-2)]

Exception generated.

$$\int (d + icdx)(a + b\arctan(cx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**3,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.122.7 Maxima [F]

$$\int (d + icdx)(a + b\arctan(cx))^3\,dx = \int (i\,cdx + d)(b\arctan(cx) + a)^3\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 +
1), x) - b^3*c^3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x)
 + 12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^3
*c^3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1/2*I*a^3
*c*d*x^2 + 7/32*b^3*d*arctan(c*x)^4/c + 56*b^3*c^2*d*integrate(1/64*x^2*ar
ctan(c*x)^3/(c^2*x^2 + 1), x) + 6*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)
*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*integrate(1/64*x^2
*arctan(c*x)^2/(c^2*x^2 + 1), x) + 36*b^3*c^2*d*integrate(1/64*x^2*arctan(
c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*I*(x^2*arctan(c*x) - c*(x/c^
2 - arctan(c*x)/c^3))*a^2*b*c*d + a*b^2*d*arctan(c*x)^3/c + 12*b^3*c*d*int
egrate(1/64*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d*i
ntegrate(1/64*x*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 24*b^3*c*d*integrat
e(1/64*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^3*c*d*integrate(1/64*x*log(
c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + a^3*d*x + 6*b^3*d*integrate(1/64*arctan
(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(
c^2*x^2 + 1))*a^2*b*d/c + 1/16*(I*b^3*c*d*x^2 + 2*b^3*d*x)*arctan(c*x)^3 -
 3/32*(b^3*c*d*x^2 - 2*I*b^3*d*x)*arctan(c*x)^2*log(c^2*x^2 + 1) + 3/64*(-
I*b^3*c*d*x^2 - 2*b^3*d*x)*arctan(c*x)*log(c^2*x^2 + 1)^2 + 1/128*(b^3*c*d
*x^2 - 2*I*b^3*d*x)*log(c^2*x^2 + 1)^3 + I*integrate(1/64*(56*(b^3*c^3*d*x
^3 + b^3*c*d*x)*arctan(c*x)^3 + (b^3*c^2*d*x^2 + b^3*d)*log(c^2*x^2 + 1...
```

3.122.8 Giac [F]

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx = \int (i\,cdx+d)(b\arctan(cx)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx = \int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i), x)
```

3.123 $\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$

3.123.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = & \frac{i(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} \\ & -\frac{3b(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd} \\ & +\frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd} \\ & +\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4cd}\end{aligned}$$

output
```
I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c/d-3/2*b*(a+b*arctan(c*x))^2*polylo
g(2,1-2/(1+I*c*x))/c/d+3/2*I*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x)
)/c/d+3/4*b^3*polylog(4,1-2/(1+I*c*x))/c/d
```

3.123.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{i\left(4(a+b\arctan(cx))^3\log\left(\frac{2d}{d+icdx}\right)+3ib\left(2(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)-b\left(2i(a+b\arctan(cx)\right.\right.\right.}{4cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
 b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTa
n[c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*
x)]))))/(c*d)
```

3.123.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}dx$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{5529}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\int \frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \text{5533}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int \frac{\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd} - \frac{3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\operatorname{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - ((3*I)*b*(((-1/2*I)*(
a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b
*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1 - 2/(1 +
I*c*x)])/(4*c))))/d
```

3.123.3.1 Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.123.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.54 (sec) , antiderivative size = 1629, normalized size of antiderivative = 11.72

method	result	size
derivativedivides	Expression too large to display	1629
default	Expression too large to display	1629
parts	Expression too large to display	1640

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I*a^3/d*ln(c^2*x^2+1)+a^3/d*arctan(c*x)+b^3/d*(-I*ln(1+I*c*x)*ar
ctan(c*x)^3+3*I*(1/3*arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+1/6*I*P
i*(csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+csgn(I/((1+
I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-c
sgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))-csgn(I*(1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))+csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-1)*ar
ctan(c*x)^3-1/2*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*ar
ctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*polylog(4,-(1+I*c*x)^2
/(c^2*x^2+1))-1/6*I*arctan(c*x)^4))+3*a*b^2/d*(-I*ln(1+I*c*x)*arctan(c*x)^
2+2*I*(1/2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*Pi*(csgn((1
+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+csgn(I/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+...
```

3.123.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/
(c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x - I*d)
, x)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.123.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arc
tan(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3
*log(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16
*x*log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) +
12*a^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*a
rctan(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arc
tan(c*x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 +
 1) + b^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

3.123.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i), x)
```

3.124 $\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx$

3.124.1 Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = -\frac{3ib^3}{4cd^2(i-cx)}+\frac{3ib^3\arctan(cx)}{4cd^2}+\frac{3b^2(a+b\arctan(cx))}{2cd^2(i-cx)} -\frac{3b(a+b\arctan(cx))^2}{4cd^2}+\frac{3ib(a+b\arctan(cx))^2}{2cd^2(i-cx)} -\frac{i(a+b\arctan(cx))^3}{2cd^2}+\frac{i(a+b\arctan(cx))^3}{cd^2(1+icx)}$$

output
```
-3/4*I*b^3/c/d^2/(I-c*x)+3/4*I*b^3*arctan(c*x)/c/d^2+3/2*b^2*(a+b*arctan(c
*x))/c/d^2/(I-c*x)-3/4*b*(a+b*arctan(c*x))^2/c/d^2+3/2*I*b*(a+b*arctan(c*x
))^2/c/d^2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^3/c/d^2+I*(a+b*arctan(c*x))^3/c
/d^2/(1+I*c*x)
```

3.124.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \frac{4a^3-6ia^2b-6ab^2+3ib^3+3ib\left(-2a^2+2iab+b^2\right)(i+cx)\arctan(cx)-3b^2(2ia+b)(i+cx)\arctan(cx)^2}{4cd^2(-i+cx)}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

output
```
(4*a^3 - (6*I)*a^2*b - 6*a*b^2 + (3*I)*b^3 + (3*I)*b*(-2*a^2 + (2*I)*a*b +
 b^2)*(I + c*x)*ArcTan[c*x] - 3*b^2*((2*I)*a + b)*(I + c*x)*ArcTan[c*x]^2
+ 2*b^3*(1 - I*c*x)*ArcTan[c*x]^3)/(4*c*d^2*(-I + c*x))
```

3.124.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^3}{cd^2(1+icx)}-\frac{3ib\int\left(\frac{(a+b\arctan(cx))^2}{2d(c^2x^2+1)}-\frac{(a+b\arctan(cx))^2}{2d(i-cx)^2}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^3}{cd^2(1+icx)}-\frac{3ib\left(\frac{(a+b\arctan(cx))^3}{6bcd}-\frac{i(a+b\arctan(cx))^2}{4cd}-\frac{(a+b\arctan(cx))^2}{2cd(-cx+i)}+\frac{ib(a+b\arctan(cx))}{2cd(-cx+i)}-\frac{b^2\arctan(cx)}{4cd}+\frac{b^2}{4cd(-cx+i)}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

output
```
(I*(a + b*ArcTan[c*x])^3)/(c*d^2*(1 + I*c*x)) - ((3*I)*b*(b^2/(4*c*d*(I -
c*x)) - (b^2*ArcTan[c*x])/(4*c*d) + ((I/2)*b*(a + b*ArcTan[c*x]))/(c*d*(I
- c*x)) - ((I/4)*(a + b*ArcTan[c*x])^2)/(c*d) - (a + b*ArcTan[c*x])^2/(2*c
*d*(I - c*x)) + (a + b*ArcTan[c*x])^3/(6*b*c*d)))/d
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.124.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(161) = 322$.

Time = 0.82 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{ia^3}{d^2(icx+1)}+\frac{b^3\left(\frac{i\arctan(cx)^3}{icx+1}-\frac{i\left(-2i\arctan(cx)^3+2\arctan(cx)^3cx-3i\arctan(cx)^2cx+3\arctan(cx)^2-3i\arctan(cx)-3\arctan(cx)cx\right.}{4(cx-i)}\right.}{d^2}$
default	$\frac{ia^3}{d^2(icx+1)}+\frac{b^3\left(\frac{i\arctan(cx)^3}{icx+1}-\frac{i\left(-2i\arctan(cx)^3+2\arctan(cx)^3cx-3i\arctan(cx)^2cx+3\arctan(cx)^2-3i\arctan(cx)-3\arctan(cx)cx\right.}{4(cx-i)}\right.}{d^2}$
parts	$\frac{ia^3}{d^2(icx+1)c}+\frac{b^3\left(\frac{i\arctan(cx)^3}{icx+1}-\frac{i\left(-2i\arctan(cx)^3+2\arctan(cx)^3cx-3i\arctan(cx)^2cx+3\arctan(cx)^2-3i\arctan(cx)-3\arctan(\right.}{4(cx-i)}\right.}{d^2c}$
risch	$\frac{(b^3cx+ib^3)\ln(icx+1)^3}{16d^2(cx-i)c}+\frac{3ib^2(i\ln(-icx+1)bcx-ibcx+2acx+2ia-b\ln(-icx+1)+b)\ln(icx+1)^2}{16d^2(cx-i)c}-\frac{3ib\left(ib^2cx\ln(-icx\right.}{}$

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(I*a^3/d^2/(1+I*c*x)+b^3/d^2*(I/(1+I*c*x)*arctan(c*x)^3-1/4*I*(-2*I*ar
ctan(c*x)^3+2*arctan(c*x)^3*c*x-3*I*arctan(c*x)^2*c*x+3*arctan(c*x)^2-3*I*
arctan(c*x)-3*arctan(c*x)*c*x-3)/(c*x-I))+3*a*b^2/d^2*(I/(1+I*c*x)*arctan(
c*x)^2-2*I*(-1/4*I*arctan(c*x)*ln(c*x-I)+1/2*arctan(c*x)/(c*x-I)+1/4*I*arc
tan(c*x)*ln(I+c*x)+1/16*ln(c*x-I)^2-1/8*ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/4*I
/(c*x-I)-1/4*I*arctan(c*x)+1/16*ln(I+c*x)^2-1/8*(ln(I+c*x)-ln(-1/2*I*(I+c*
x)))*ln(-1/2*I*(I-c*x))))+3*I*a^2*b/d^2/(1+I*c*x)*arctan(c*x)-3/2*I*a^2*b/
d^2*arctan(c*x)-3/2*I*a^2*b/d^2/(c*x-I))
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx =$$
$$-\frac{(b^3cx+i\,b^3)\log\left(-\frac{cx+i}{cx-i}\right)^3-16\,a^3+24i\,a^2b+24\,ab^2-12i\,b^3+3\left(2\,ab^2-i\,b^3+(-2i\,ab^2-b^3)cx\right)\log\left(-\right.}{16\left(c^2d^2x-i\,cd^2\right)}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/16*((b^3*c*x + I*b^3)*log(-(c*x + I)/(c*x - I))^3 - 16*a^3 + 24*I*a^2*b
 + 24*a*b^2 - 12*I*b^3 + 3*(2*a*b^2 - I*b^3 + (-2*I*a*b^2 - b^3)*c*x)*log(
-(c*x + I)/(c*x - I))^2 + 6*(-2*I*a^2*b - 2*a*b^2 + I*b^3 - (2*a^2*b - 2*I
*a*b^2 - b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^2*d^2*x - I*c*d^2)
```

3.124.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(151) = 302$.

Time = 16.19 (sec) , antiderivative size = 631, normalized size of antiderivative = 3.47

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx$$
$$\begin{aligned}
&= \frac{3ib(a(1-i)-b)\,(a(1-i)-ib)\log\left(-\frac{3b(a(1-i)-b)(a(1-i)-ib)}{c}+x(6a^2b-6iab^2-3b^3)\right)}{8cd^2} \\
&- \frac{3ib(a(1-i)-b)\,(a(1-i)-ib)\log\left(\frac{3b(a(1-i)-b)(a(1-i)-ib)}{c}+x(6a^2b-6iab^2-3b^3)\right)}{8cd^2} \\
&+ \frac{(-b^3cx-ib^3)\log{(-icx+1)}^3}{16c^2d^2x-16icd^2} + \frac{(b^3cx+ib^3)\log{(icx+1)}^3}{16c^2d^2x-16icd^2} \\
&+ \frac{(6iab^2cx-6ab^2+3b^3cx+3ib^3)\log{(icx+1)}^2}{16c^2d^2x-16icd^2} \\
&+ \frac{(6iab^2cx-6ab^2+3b^3cx\log{(icx+1)}+3b^3cx+3ib^3\log{(icx+1)}+3ib^3)\log{(-icx+1)}^2}{16c^2d^2x-16icd^2} \\
&+ \frac{(24ia^2b-12iab^2cx\log{(icx+1)}+12ab^2\log{(icx+1)}+24ab^2-3b^3cx\log{(icx+1)}^2-6b^3cx\log{(icx+}}{16c^2d^2x-16icd^2} \\
&+ \frac{(-6ia^2b-6ab^2+3ib^3)\log{(icx+1)}}{4c^2d^2x-4icd^2} - \frac{-4a^3+6ia^2b+6ab^2-3ib^3}{4c^2d^2x-4icd^2}
\end{aligned}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**2,x)
```

output
```
3*I*b*(a*(1 - I) - b)*(a*(1 - I) - I*b)*log(-3*b*(a*(1 - I) - b)*(a*(1 - I
) - I*b)/c + x*(6*a**2*b - 6*I*a*b**2 - 3*b**3))/(8*c*d**2) - 3*I*b*(a*(1
- I) - b)*(a*(1 - I) - I*b)*log(3*b*(a*(1 - I) - b)*(a*(1 - I) - I*b)/c +
x*(6*a**2*b - 6*I*a*b**2 - 3*b**3))/(8*c*d**2) + (-b**3*c*x - I*b**3)*log(
-I*c*x + 1)**3/(16*c**2*d**2*x - 16*I*c*d**2) + (b**3*c*x + I*b**3)*log(I*
c*x + 1)**3/(16*c**2*d**2*x - 16*I*c*d**2) + (6*I*a*b**2*c*x - 6*a*b**2 +
3*b**3*c*x + 3*I*b**3)*log(I*c*x + 1)**2/(16*c**2*d**2*x - 16*I*c*d**2) +
(6*I*a*b**2*c*x - 6*a*b**2 + 3*b**3*c*x*log(I*c*x + 1) + 3*b**3*c*x + 3*I*
b**3*log(I*c*x + 1) + 3*I*b**3)*log(-I*c*x + 1)**2/(16*c**2*d**2*x - 16*I*
c*d**2) + (24*I*a**2*b - 12*I*a*b**2*c*x*log(I*c*x + 1) + 12*a*b**2*log(I*
c*x + 1) + 24*a*b**2 - 3*b**3*c*x*log(I*c*x + 1)**2 - 6*b**3*c*x*log(I*c*x
 + 1) - 3*I*b**3*log(I*c*x + 1)**2 - 6*I*b**3*log(I*c*x + 1) - 12*I*b**3)*
log(-I*c*x + 1)/(16*c**2*d**2*x - 16*I*c*d**2) + (-6*I*a**2*b - 6*a*b**2 +
 3*I*b**3)*log(I*c*x + 1)/(4*c**2*d**2*x - 4*I*c*d**2) - (-4*a**3 + 6*I*a*
*2*b + 6*a*b**2 - 3*I*b**3)/(4*c**2*d**2*x - 4*I*c*d**2)
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.124.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^2, x)
```

3.125 $\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$

3.125.1 Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx &= \frac{3b^3}{64cd^3(i-cx)^2}-\frac{21ib^3}{64cd^3(i-cx)}\\ &\quad+\frac{21ib^3\arctan(cx)}{64cd^3}+\frac{3ib^2(a+b\arctan(cx))}{16cd^3(i-cx)^2}\\ &\quad+\frac{9b^2(a+b\arctan(cx))}{16cd^3(i-cx)}-\frac{9b(a+b\arctan(cx))^2}{32cd^3}\\ &\quad-\frac{3b(a+b\arctan(cx))^2}{8cd^3(i-cx)^2}+\frac{3ib(a+b\arctan(cx))^2}{8cd^3(i-cx)}\\ &\quad-\frac{i(a+b\arctan(cx))^3}{8cd^3}+\frac{i(a+b\arctan(cx))^3}{2cd^3(1+icx)^2}\end{aligned}$$

output
```
3/64*b^3/c/d^3/(I-c*x)^2-21/64*I*b^3/c/d^3/(I-c*x)+21/64*I*b^3*arctan(c*x)
/c/d^3+3/16*I*b^2*(a+b*arctan(c*x))/c/d^3/(I-c*x)^2+9/16*b^2*(a+b*arctan(c
*x))/c/d^3/(I-c*x)-9/32*b*(a+b*arctan(c*x))^2/c/d^3-3/8*b*(a+b*arctan(c*x)
)^2/c/d^3/(I-c*x)^2+3/8*I*b*(a+b*arctan(c*x))^2/c/d^3/(I-c*x)-1/8*I*(a+b*a
rctan(c*x))^3/c/d^3+1/2*I*(a+b*arctan(c*x))^3/c/d^3/(1+I*c*x)^2
```

3.125.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx = -\frac{i(32a^3+3b^3(8i-7cx)+12ab^2(-4-3icx)+24a^2b(-2i+cx)+3b(i+cx)\left(b^2(9i-7cx)+4ab(-5-\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```

output
```
((-1/64*I)*(32*a^3 + 3*b^3*(8*I - 7*c*x) + 12*a*b^2*(-4 - (3*I)*c*x) + 24*
a^2*b*(-2*I + c*x) + 3*b*(I + c*x)*(b^2*(9*I - 7*c*x) + 4*a*b*(-5 - (3*I)*
c*x) + 8*a^2*(-3*I + c*x))*ArcTan[c*x] + 6*b^2*(I + c*x)*(b*(-5 - (3*I)*c*
x) + 4*a*(-3*I + c*x))*ArcTan[c*x]^2 + 8*b^3*(3 - (2*I)*c*x + c^2*x^2)*Arc
Tan[c*x]^3))/(c*d^3*(-I + c*x)^2)
```

3.125.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^3}{2cd^3(1+icx)^2}-\frac{3ib\int \left(\frac{(a+b\arctan(cx))^2}{4d^2(c^2x^2+1)}-\frac{(a+b\arctan(cx))^2}{4d^2(i-cx)^2}-\frac{i(a+b\arctan(cx))^2}{2d^2(i-cx)^3}\right)dx}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^3}{2cd^3(1+icx)^2}-\frac{3ib\left(\frac{(a+b\arctan(cx))^3}{12bcd^2}-\frac{(a+b\arctan(cx))^2}{4cd^2(-cx+i)}-\frac{3i(a+b\arctan(cx))^2}{16cd^2}-\frac{i(a+b\arctan(cx))^2}{4cd^2(-cx+i)^2}+\frac{3ib(a+b\arctan(cx))}{8cd^2(-cx+i)}-\frac{b(a+b\arctan(cx))}{8cd^2(-cx+i)^2}-\right.}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```

output
```
((I/2)*(a + b*ArcTan[c*x])^3)/(c*d^3*(1 + I*c*x)^2) - (((3*I)/2)*b*(((I/32
)*b^2)/(c*d^2*(I - c*x)^2) + (7*b^2)/(32*c*d^2*(I - c*x)) - (7*b^2*ArcTan[
c*x])/(32*c*d^2) - (b*(a + b*ArcTan[c*x]))/(8*c*d^2*(I - c*x)^2) + (((3*I)
/8)*b*(a + b*ArcTan[c*x]))/(c*d^2*(I - c*x)) - (((3*I)/16)*(a + b*ArcTan[c
*x])^2)/(c*d^2) - ((I/4)*(a + b*ArcTan[c*x])^2)/(c*d^2*(I - c*x)^2) - (a +
 b*ArcTan[c*x])^2/(4*c*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^3/(12*b*c*d^2)
))/d
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.125.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{\frac{ia^3}{2d^3(icx+1)^2}+\frac{b^3\left(\frac{i\arctan(cx)^3}{2(icx+1)^2}-\frac{i\left(-21cx+24i-16i\arctan(cx)^3cx+8\arctan(cx)^3c^2x^2-8\arctan(cx)^3-18i\arctan(cx)^2c^2x^2-30i\,\mathrm{ar}\right.}{64(cx-i)^2}\right.}{d^3}}{}$
default	$\frac{\frac{ia^3}{2d^3(icx+1)^2}+\frac{b^3\left(\frac{i\arctan(cx)^3}{2(icx+1)^2}-\frac{i\left(-21cx+24i-16i\arctan(cx)^3cx+8\arctan(cx)^3c^2x^2-8\arctan(cx)^3-18i\arctan(cx)^2c^2x^2-30i\,\mathrm{ar}\right.}{64(cx-i)^2}\right.}{d^3}}{}$
parts	$\frac{ia^3}{2d^3(icx+1)^2c}+\frac{b^3\left(\frac{i\arctan(cx)^3}{2(icx+1)^2}-\frac{i\left(-21cx+24i-16i\arctan(cx)^3cx+8\arctan(cx)^3c^2x^2-8\arctan(cx)^3-18i\arctan(cx)^2c^2x^2-\right.}{64(cx-i}\right.}{d^3c}$
risch	Expression too large to display

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*I*a^3/d^3/(1+I*c*x)^2+b^3/d^3*(1/2*I/(1+I*c*x)^2*arctan(c*x)^3-1/
64*I*(-21*c*x+24*I-16*I*arctan(c*x)^3*c*x+8*arctan(c*x)^3*c^2*x^2-8*arctan
(c*x)^3-18*I*arctan(c*x)^2*c^2*x^2-30*I*arctan(c*x)^2-12*arctan(c*x)^2*c*x
+6*I*arctan(c*x)*c*x-21*c^2*x^2*arctan(c*x)-27*arctan(c*x))/(c*x-I)^2)+3*a
*b^2/d^3*(1/2*I/(1+I*c*x)^2*arctan(c*x)^2-I*(-1/8*I*arctan(c*x)*ln(c*x-I)-
1/4*I/(c*x-I)^2*arctan(c*x)+1/4*arctan(c*x)/(c*x-I)+1/8*I*arctan(c*x)*ln(I
+c*x)+1/32*ln(I+c*x)^2-1/16*(ln(I+c*x)-ln(-1/2*I*(I+c*x)))*ln(-1/2*I*(I-c*
x))-3/16*I/(c*x-I)-1/16/(c*x-I)^2-3/16*I*arctan(c*x)+1/32*ln(c*x-I)^2-1/16
*ln(c*x-I)*ln(-1/2*I*(I+c*x))))+3/2*I*a^2*b/d^3/(1+I*c*x)^2*arctan(c*x)-3/
8*I*a^2*b/d^3*arctan(c*x)-3/8*a^2*b/d^3/(c*x-I)^2-3/8*I*a^2*b/d^3/(c*x-I))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx =$$
$$-\frac{2\left(b^3c^2x^2-2i\,b^3cx+3\,b^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^3+64i\,a^3+96\,a^2b-96i\,ab^2-48\,b^3+6\left(8i\,a^2b+12\,ab^2-7i\,b^3\right)c$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/128*(2*(b^3*c^2*x^2 - 2*I*b^3*c*x + 3*b^3)*log(-(c*x + I)/(c*x - I))^3
+ 64*I*a^3 + 96*a^2*b - 96*I*a*b^2 - 48*b^3 + 6*(8*I*a^2*b + 12*a*b^2 - 7*
I*b^3)*c*x + 3*((-4*I*a*b^2 - 3*b^3)*c^2*x^2 - 12*I*a*b^2 - 5*b^3 - 2*(4*a
*b^2 - I*b^3)*c*x)*log(-(c*x + I)/(c*x - I))^2 - 3*((8*a^2*b - 12*I*a*b^2
- 7*b^3)*c^2*x^2 + 24*a^2*b - 20*I*a*b^2 - 9*b^3 - 2*(8*I*a^2*b + 4*a*b^2
- I*b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*
d^3)
```

3.125.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(230) = 460.

Time = 119.50 (sec) , antiderivative size = 954, normalized size of antiderivative = 3.52

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$$
$$= -\frac{3b(8a^2-12iab-7b^2)\log\left(-\frac{3ib(8a^2-12iab-7b^2)}{c}+x(24a^2b-36iab^2-21b^3)\right)}{128cd^3}$$
$$+\frac{3b(8a^2-12iab-7b^2)\log\left(\frac{3ib(8a^2-12iab-7b^2)}{c}+x(24a^2b-36iab^2-21b^3)\right)}{128cd^3}$$
$$+\frac{(-b^3c^2x^2+2ib^3cx-3b^3)\log(-icx+1)^3}{64c^3d^3x^2-128ic^2d^3x-64cd^3}+\frac{(b^3c^2x^2-2ib^3cx+3b^3)\log(icx+1)^3}{64c^3d^3x^2-128ic^2d^3x-64cd^3}$$
$$+\frac{(12iab^2c^2x^2+24ab^2cx+36iab^2+9b^3c^2x^2-6ib^3cx+15b^3)\log(icx+1)^2}{128c^3d^3x^2-256ic^2d^3x-128cd^3}$$
$$+\frac{(12iab^2c^2x^2+24ab^2cx+36iab^2+6b^3c^2x^2\log(icx+1)+9b^3c^2x^2-12ib^3cx\log(icx+1)-6ib^3cx+18}{128c^3d^3x^2-256ic^2d^3x-128cd^3}$$
$$+\frac{-32ia^3-48a^2b+48iab^2+24b^3+x(-24ia^2bc-36ab^2c+21ib^3c)}{64c^3d^3x^2-128ic^2d^3x-64cd^3}$$
$$+\frac{(48a^2b-12iab^2c^2x^2\log(icx+1)-24ab^2cx\log(icx+1)+24ab^2cx-36iab^2\log(icx+1)-48iab^2-3}{}$$
$$+\frac{(-24a^2b-12ab^2cx+24iab^2+9ib^3cx+12b^3)\log(icx+1)}{32c^3d^3x^2-64ic^2d^3x-32cd^3}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**3,x)
```

3.125. $\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$

output
```
-3*b*(8*a**2 - 12*I*a*b - 7*b**2)*log(-3*I*b*(8*a**2 - 12*I*a*b - 7*b**2)/
c + x*(24*a**2*b - 36*I*a*b**2 - 21*b**3))/(128*c*d**3) + 3*b*(8*a**2 - 12
*I*a*b - 7*b**2)*log(3*I*b*(8*a**2 - 12*I*a*b - 7*b**2)/c + x*(24*a**2*b -
 36*I*a*b**2 - 21*b**3))/(128*c*d**3) + (-b**3*c**2*x**2 + 2*I*b**3*c*x -
3*b**3)*log(-I*c*x + 1)**3/(64*c**3*d**3*x**2 - 128*I*c**2*d**3*x - 64*c*d
**3) + (b**3*c**2*x**2 - 2*I*b**3*c*x + 3*b**3)*log(I*c*x + 1)**3/(64*c**3
*d**3*x**2 - 128*I*c**2*d**3*x - 64*c*d**3) + (12*I*a*b**2*c**2*x**2 + 24*
a*b**2*c*x + 36*I*a*b**2 + 9*b**3*c**2*x**2 - 6*I*b**3*c*x + 15*b**3)*log(
I*c*x + 1)**2/(128*c**3*d**3*x**2 - 256*I*c**2*d**3*x - 128*c*d**3) + (12*
I*a*b**2*c**2*x**2 + 24*a*b**2*c*x + 36*I*a*b**2 + 6*b**3*c**2*x**2*log(I*
c*x + 1) + 9*b**3*c**2*x**2 - 12*I*b**3*c*x*log(I*c*x + 1) - 6*I*b**3*c*x
+ 18*b**3*log(I*c*x + 1) + 15*b**3)*log(-I*c*x + 1)**2/(128*c**3*d**3*x**2
 - 256*I*c**2*d**3*x - 128*c*d**3) + (-32*I*a**3 - 48*a**2*b + 48*I*a*b**2
 + 24*b**3 + x*(-24*I*a**2*b*c - 36*a*b**2*c + 21*I*b**3*c))/(64*c**3*d**3
*x**2 - 128*I*c**2*d**3*x - 64*c*d**3) + (48*a**2*b - 12*I*a*b**2*c**2*x**
2*log(I*c*x + 1) - 24*a*b**2*c*x*log(I*c*x + 1) + 24*a*b**2*c*x - 36*I*a*b
**2*log(I*c*x + 1) - 48*I*a*b**2 - 3*b**3*c**2*x**2*log(I*c*x + 1)**2 - 9*
b**3*c**2*x**2*log(I*c*x + 1) + 6*I*b**3*c*x*log(I*c*x + 1)**2 + 6*I*b**3*
c*x*log(I*c*x + 1) - 18*I*b**3*c*x - 9*b**3*log(I*c*x + 1)**2 - 15*b**3*lo
g(I*c*x + 1) - 24*b**3)*log(-I*c*x + 1)/(64*c**3*d**3*x**2 - 128*I*c**2...
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$$
$$= \frac{8\left(-i\,b^3c^2x^2-2\,b^3cx-3i\,b^3\right)\arctan\left(cx\right)^3-32i\,a^3-48\,a^2b+48i\,ab^2+24\,b^3+3\left(-8i\,a^2b-12\,ab^2+7i\,b^3\right.}{}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/64*(8*(-I*b^3*c^2*x^2 - 2*b^3*c*x - 3*I*b^3)*arctan(c*x)^3 - 32*I*a^3 -
48*a^2*b + 48*I*a*b^2 + 24*b^3 + 3*(-8*I*a^2*b - 12*a*b^2 + 7*I*b^3)*c*x +
 6*((-4*I*a*b^2 - 3*b^3)*c^2*x^2 - 12*I*a*b^2 - 5*b^3 - 2*(4*a*b^2 - I*b^3
)*c*x)*arctan(c*x)^2 + 3*((-8*I*a^2*b - 12*a*b^2 + 7*I*b^3)*c^2*x^2 - 24*I
*a^2*b - 20*a*b^2 + 9*I*b^3 - 2*(8*a^2*b - 4*I*a*b^2 - b^3)*c*x)*arctan(c*
x))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

3.125.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^3, x)
```

3.126 $\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx$

3.126.1 Optimal result

Integrand size = 22, antiderivative size = 360

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx &= \frac{ib^3}{108cd^4(i-cx)^3}+\frac{19b^3}{576cd^4(i-cx)^2}-\frac{85ib^3}{576cd^4(i-cx)} \\ &+\frac{85ib^3\arctan(cx)}{576cd^4}-\frac{b^2(a+b\arctan(cx))}{18cd^4(i-cx)^3} \\ &+\frac{5ib^2(a+b\arctan(cx))}{48cd^4(i-cx)^2}+\frac{11b^2(a+b\arctan(cx))}{48cd^4(i-cx)} \\ &-\frac{11b(a+b\arctan(cx))^2}{96cd^4}-\frac{ib(a+b\arctan(cx))^2}{6cd^4(i-cx)^3} \\ &-\frac{b(a+b\arctan(cx))^2}{8cd^4(i-cx)^2}+\frac{ib(a+b\arctan(cx))^2}{8cd^4(i-cx)} \\ &-\frac{i(a+b\arctan(cx))^3}{24cd^4}+\frac{i(a+b\arctan(cx))^3}{3cd^4(1+icx)^3}\end{aligned}$$

output
```
1/108*I*b^3/c/d^4/(I-c*x)^3+19/576*b^3/c/d^4/(I-c*x)^2-85/576*I*b^3/c/d^4/
(I-c*x)+85/576*I*b^3*arctan(c*x)/c/d^4-1/18*b^2*(a+b*arctan(c*x))/c/d^4/(I
-c*x)^3+5/48*I*b^2*(a+b*arctan(c*x))/c/d^4/(I-c*x)^2+11/48*b^2*(a+b*arctan
(c*x))/c/d^4/(I-c*x)-11/96*b*(a+b*arctan(c*x))^2/c/d^4-1/6*I*b*(a+b*arctan
(c*x))^2/c/d^4/(I-c*x)^3-1/8*b*(a+b*arctan(c*x))^2/c/d^4/(I-c*x)^2+1/8*I*b
*(a+b*arctan(c*x))^2/c/d^4/(I-c*x)-1/24*I*(a+b*arctan(c*x))^3/c/d^4+1/3*I*
(a+b*arctan(c*x))^3/c/d^4/(1+I*c*x)^3
```

3.126.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx$$
$$= \frac{-576a^3+12ab^2(56+81icx-33c^2x^2)+b^3(-328i+567cx+255ic^2x^2)-72ia^2b(-10-9icx+3c^2x^2)+}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

output
```
(-576*a^3 + 12*a*b^2*(56 + (81*I)*c*x - 33*c^2*x^2) + b^3*(-328*I + 567*c*
x + (255*I)*c^2*x^2) - (72*I)*a^2*b*(-10 - (9*I)*c*x + 3*c^2*x^2) + 3*b*(I
 + c*x)*(12*a*b*(29 + (32*I)*c*x - 11*c^2*x^2) + b^2*(-139*I + 208*c*x + (
85*I)*c^2*x^2) - (72*I)*a^2*(-7 - (4*I)*c*x + c^2*x^2))*ArcTan[c*x] - (18*
I)*b^2*(I + c*x)*(b*(29*I - 32*c*x - (11*I)*c^2*x^2) + 12*a*(-7 - (4*I)*c*
x + c^2*x^2))*ArcTan[c*x]^2 - (72*I)*b^3*(-7*I - 3*c*x - (3*I)*c^2*x^2 + c
^3*x^3)*ArcTan[c*x]^3)/(1728*c*d^4*(-I + c*x)^3)
```

3.126.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx$$
$$\Big\downarrow \; 5389$$
$$\frac{i(a+b\arctan(cx))^3}{3cd^4(1+icx)^3} - \frac{ib\int \left(\frac{(a+b\arctan(cx))^2}{8d^3(c^2x^2+1)} - \frac{(a+b\arctan(cx))^2}{8d^3(i-cx)^2} - \frac{i(a+b\arctan(cx))^2}{4d^3(i-cx)^3} + \frac{(a+b\arctan(cx))^2}{2d^3(i-cx)^4}\right)dx}{d}$$
$$\Big\downarrow \; 2009$$

$$\frac{\frac{i(a+b\arctan(cx))^3}{3cd^4(1+icx)^3}-}{\quad}\qquad \frac{ib\left(\frac{(a+b\arctan(cx))^3}{24bcd^3}-\frac{(a+b\arctan(cx))^2}{8cd^3(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{8cd^3(-cx+i)^2}-\frac{11i(a+b\arctan(cx))^2}{96cd^3}+\frac{(a+b\arctan(cx))^2}{6cd^3(-cx+i)^3}+\frac{11ib(a+b\arctan(cx))}{48cd^3(-cx+i)}-\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

output
```
((I/3)*(a + b*ArcTan[c*x])^3)/(c*d^4*(1 + I*c*x)^3) - (I*b*(-1/108*b^2/(c*
d^3*(I - c*x)^3) + (((19*I)/576)*b^2)/(c*d^3*(I - c*x)^2) + (85*b^2)/(576*
c*d^3*(I - c*x)) - (85*b^2*ArcTan[c*x])/(576*c*d^3) - ((I/18)*b*(a + b*Arc
Tan[c*x]))/(c*d^3*(I - c*x)^3) - (5*b*(a + b*ArcTan[c*x]))/(48*c*d^3*(I -
c*x)^2) + (((11*I)/48)*b*(a + b*ArcTan[c*x]))/(c*d^3*(I - c*x)) - (((11*I)
/96)*(a + b*ArcTan[c*x])^2)/(c*d^3) + (a + b*ArcTan[c*x])^2/(6*c*d^3*(I -
c*x)^3) - ((I/8)*(a + b*ArcTan[c*x])^2)/(c*d^3*(I - c*x)^2) - (a + b*ArcTa
n[c*x])^2/(8*c*d^3*(I - c*x)) + (a + b*ArcTan[c*x])^3/(24*b*c*d^3)))/d
```

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

3.126.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{ia^3}{3d^4(icx+1)^3}+\frac{b^3\left(\frac{i\arctan(cx)^3}{3(icx+1)^3}-\frac{i\left(-216i\arctan(cx)^3c^2x^2+72\arctan(cx)^3c^3x^3-198i\arctan(cx)^2c^3x^3+72i\arctan(cx)^3-216\,\mathrm{arcta}\right.}{}\right.}{}}{}$
default	$\frac{\frac{ia^3}{3d^4(icx+1)^3}+\frac{b^3\left(\frac{i\arctan(cx)^3}{3(icx+1)^3}-\frac{i\left(-216i\arctan(cx)^3c^2x^2+72\arctan(cx)^3c^3x^3-198i\arctan(cx)^2c^3x^3+72i\arctan(cx)^3-216\,\mathrm{arcta}\right.}{}\right.}{}}{}$
parts	$\frac{ia^3}{3d^4(icx+1)^3c}+\frac{b^3\left(\frac{i\arctan(cx)^3}{3(icx+1)^3}-\frac{i\left(-216i\arctan(cx)^3c^2x^2+72\arctan(cx)^3c^3x^3-198i\arctan(cx)^2c^3x^3+72i\arctan(cx)^3-216\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*I*a^3/d^4/(1+I*c*x)^3+b^3/d^4*(1/3*I/(1+I*c*x)^3*arctan(c*x)^3-1/
1728*I*(-216*I*arctan(c*x)^3*c^2*x^2+72*arctan(c*x)^3*c^3*x^3-198*I*arctan
(c*x)^2*c^3*x^3+72*I*arctan(c*x)^3-216*arctan(c*x)^3*c*x-54*I*arctan(c*x)^
2*c*x-378*arctan(c*x)^2*c^2*x^2+369*I*c^2*x^2*arctan(c*x)-255*arctan(c*x)*
c^3*x^3-522*arctan(c*x)^2+417*I*arctan(c*x)-207*arctan(c*x)*c*x+567*I*c*x-
255*c^2*x^2+328)/(c*x-I)^3)+3*a*b^2/d^4*(1/3*I/(1+I*c*x)^3*arctan(c*x)^2-2
/3*I*(-1/16*I*arctan(c*x)*ln(c*x-I)-1/8*I*arctan(c*x)/(c*x-I)^2-1/6*arctan
(c*x)/(c*x-I)^3+1/8*arctan(c*x)/(c*x-I)+1/16*I*arctan(c*x)*ln(I+c*x)+1/36*
I/(c*x-I)^3-11/96*I/(c*x-I)-5/96/(c*x-I)^2-11/96*I*arctan(c*x)+1/64*ln(c*x
-I)^2-1/32*ln(c*x-I)*ln(-1/2*I*(I+c*x))+1/64*ln(I+c*x)^2-1/32*(ln(I+c*x)-l
n(-1/2*I*(I+c*x)))*ln(-1/2*I*(I-c*x))))+I*a^2*b/d^4/(1+I*c*x)^3*arctan(c*x
)-1/8*I*a^2*b/d^4*arctan(c*x)-1/8*a^2*b/d^4/(c*x-I)^2+1/6*I*a^2*b/d^4/(c*x
-I)^3-1/8*I*a^2*b/d^4/(c*x-I))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx =$$
$$-\frac{6\left(72i\,a^2b+132\,ab^2-85i\,b^3\right)c^2x^2+18\left(b^3c^3x^3-3i\,b^3c^2x^2-3\,b^3cx-7i\,b^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^3+1152\,a^3-14$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="fricas")
```

output
```
-1/3456*(6*(72*I*a^2*b + 132*a*b^2 - 85*I*b^3)*c^2*x^2 + 18*(b^3*c^3*x^3 -
 3*I*b^3*c^2*x^2 - 3*b^3*c*x - 7*I*b^3)*log(-(c*x + I)/(c*x - I))^3 + 1152
*a^3 - 1440*I*a^2*b - 1344*a*b^2 + 656*I*b^3 + 162*(8*a^2*b - 12*I*a*b^2 -
 7*b^3)*c*x + 9*((-12*I*a*b^2 - 11*b^3)*c^3*x^3 - 3*(12*a*b^2 - 7*I*b^3)*c
^2*x^2 - 84*a*b^2 + 29*I*b^3 + 3*(12*I*a*b^2 - b^3)*c*x)*log(-(c*x + I)/(c
*x - I))^2 - 3*((72*a^2*b - 132*I*a*b^2 - 85*b^3)*c^3*x^3 - 3*(72*I*a^2*b
+ 84*a*b^2 - 41*I*b^3)*c^2*x^2 - 504*I*a^2*b - 348*a*b^2 + 139*I*b^3 - 3*(
72*a^2*b + 12*I*a*b^2 + 23*b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^4*d^4*x
^3 - 3*I*c^3*d^4*x^2 - 3*c^2*d^4*x + I*c*d^4)
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**4,x)
```

output
```
Timed out
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx$$
$$= \frac{3\left(-72i\,a^2b-132\,ab^2+85i\,b^3\right)c^2x^2+72\left(-i\,b^3c^3x^3-3\,b^3c^2x^2+3i\,b^3cx-7\,b^3\right)\arctan\left(cx\right)^3-576\,a^3+72}{}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="maxima")
```

output
```
1/1728*(3*(-72*I*a^2*b - 132*a*b^2 + 85*I*b^3)*c^2*x^2 + 72*(-I*b^3*c^3*x^
3 - 3*b^3*c^2*x^2 + 3*I*b^3*c*x - 7*b^3)*arctan(c*x)^3 - 576*a^3 + 720*I*a
^2*b + 672*a*b^2 - 328*I*b^3 - 81*(8*a^2*b - 12*I*a*b^2 - 7*b^3)*c*x + 18*
((-12*I*a*b^2 - 11*b^3)*c^3*x^3 - 3*(12*a*b^2 - 7*I*b^3)*c^2*x^2 - 84*a*b^
2 + 29*I*b^3 + 3*(12*I*a*b^2 - b^3)*c*x)*arctan(c*x)^2 + 3*((-72*I*a^2*b -
 132*a*b^2 + 85*I*b^3)*c^3*x^3 - 3*(72*a^2*b - 84*I*a*b^2 - 41*b^3)*c^2*x^
2 - 504*a^2*b + 348*I*a*b^2 + 139*b^3 + 3*(72*I*a^2*b - 12*a*b^2 + 23*I*b^
3)*c*x)*arctan(c*x))/(c^4*d^4*x^3 - 3*I*c^3*d^4*x^2 - 3*c^2*d^4*x + I*c*d^
4)
```

3.126.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+c\,d\,x\,1\mathrm{i})^4}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^4,x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^4, x)
```

3.127 $\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx$

3.127.1 Optimal result

Integrand size = 25, antiderivative size = 410

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx =& -\frac{3b(a+b\arctan(cx))^2}{2c^3d}+\frac{3ibx(a+b\arctan(cx))^2}{2c^2d}\\ &+\frac{i(a+b\arctan(cx))^3}{2c^3d}+\frac{x(a+b\arctan(cx))^3}{c^2d}\\ &-\frac{ix^2(a+b\arctan(cx))^3}{2cd}+\frac{3ib^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d}\\ &+\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d}\\ &-\frac{i(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{c^3d}-\frac{3b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}\\ &+\frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d}\\ &+\frac{3b(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}\\ &+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d}\\ &-\frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d}\\ &-\frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4c^3d}\end{aligned}$$

output
```
-3/2*b*(a+b*arctan(c*x))^2/c^3/d+3/2*I*b*x*(a+b*arctan(c*x))^2/c^2/d+1/2*I
*(a+b*arctan(c*x))^3/c^3/d+x*(a+b*arctan(c*x))^3/c^2/d-1/2*I*x^2*(a+b*arct
an(c*x))^3/c/d+3*I*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d+3*b*(a+b*ar
ctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d-I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c
^3/d-3/2*b^3*polylog(2,1-2/(1+I*c*x))/c^3/d+3*I*b^2*(a+b*arctan(c*x))*poly
log(2,1-2/(1+I*c*x))/c^3/d+3/2*b*(a+b*arctan(c*x))^2*polylog(2,1-2/(1+I*c*
x))/c^3/d+3/2*b^3*polylog(3,1-2/(1+I*c*x))/c^3/d-3/2*I*b^2*(a+b*arctan(c*x
))*polylog(3,1-2/(1+I*c*x))/c^3/d-3/4*b^3*polylog(4,1-2/(1+I*c*x))/c^3/d
```

3.127.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.32

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx =$$
$$-\frac{i\left(4ia^3cx-6a^2bcx+2a^3c^2x^2-4ia^3\arctan(cx)+6a^2b\arctan(cx)+12ia^2bcx\arctan(cx)-12ab^2cx\arctan\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-1/4*I)*((4*I)*a^3*c*x - 6*a^2*b*c*x + 2*a^3*c^2*x^2 - (4*I)*a^3*ArcTan[
c*x] + 6*a^2*b*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] - 12*a*b^2*c*x*A
rcTan[c*x] + 6*a^2*b*c^2*x^2*ArcTan[c*x] - (12*I)*a^2*b*ArcTan[c*x]^2 + 18
*a*b^2*ArcTan[c*x]^2 + (6*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2*c*x*ArcTan[c
*x]^2 - 6*b^3*c*x*ArcTan[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*a*
b^2*ArcTan[c*x]^3 + 6*b^3*ArcTan[c*x]^3 + (4*I)*b^3*c*x*ArcTan[c*x]^3 + 2*
b^3*c^2*x^2*ArcTan[c*x]^3 - (2*I)*b^3*ArcTan[c*x]^4 + 12*a^2*b*ArcTan[c*x]
*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I
)*ArcTan[c*x])] - 12*b^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*a
*b^2*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*b^3*ArcTan[c*x]
^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*A
rcTan[c*x])] - 2*a^3*Log[1 + c^2*x^2] - (6*I)*a^2*b*Log[1 + c^2*x^2] + 6*a
*b^2*Log[1 + c^2*x^2] - (6*I)*b*(a + I*b + b*ArcTan[c*x])^2*PolyLog[2, -E^
((2*I)*ArcTan[c*x])] + 6*b^2*(a + I*b + b*ArcTan[c*x])*PolyLog[3, -E^((2*I
)*ArcTan[c*x])] + (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/(c^3*d)
```

3.127.3 Rubi [A] (verified)

Time = 3.74 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {5401, 27, 5361, 5401, 5345, 5379, 5451, 5345, 5419, 5455, 5379, 2849, 2752, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}dx$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))^3}{d(icx+1)}dx}{c}-\frac{i\int x(a+b\arctan(cx))^3dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\int x(a+b\arctan(cx))^3dx}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))^3dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5345}$$

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int\frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{\int(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{ 5345}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{(a+b\arctan(cx))^3}{3bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \text{ 5455}$$

3.127. $\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx$

$$i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\int\frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\arctan(\ldots}{3bc^2}\right.\right.}{c}\right.$$

$$\overline{\qquad\qquad cd \qquad\qquad}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \ 5379$$

$$i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int\frac{(a\ldots}{}}{c}\right.\right.}{c}\right.$$

$$\overline{\qquad\qquad cd \qquad\qquad}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i\ldots}{}\right.}{c^2}\right.\right.}{cd}$$

$$\Big\downarrow \ 2849$$

$$i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int\frac{(a\ldots}{}}{c}\right.\right.}{c}\right.$$

$$\overline{\qquad\qquad cd \qquad\qquad}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(c\ldots}{c}}{c}\right.}{c^2}\right.\right.}{cd}$$

$$\Big\downarrow \ 2752$$

3.127. $\int\frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int\frac{(a}{}}{c}\right.\right.}{c}\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+}{c}\right.}{c^2}\right.\right.}{cd}$$

$$\Big\downarrow \text{5529}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan\right.}{}\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+}{c}\right.}{c^2}\right.\right.}{cd}$$

$$\Big\downarrow \text{5533}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan}{2c}\right.\right.}{c}\right.}{}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+}{c}\right.}{c^2}\right.\right.}{cd}$$

3.127. $\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx$

$$\Big\downarrow \text{7164}$$

$$i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\,\mathrm{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\mathrm{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\cdots\right.\right.}{c}\cdots\right.$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\cdots}{c}\cdots\right.}{c^2}\cdots\right.\right.}{cd}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-I)*((x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x])^3/
(b*c^3) + (x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^
2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x)])/c)/c))/c^2))/2))/(c*d) + (I*(((-I)*(x*(a + b*ArcT
an[c*x])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/(b*c^2) - (((a + b*Ar
cTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a + b*ArcTan[c*x])*Po
lyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)])/(4*c)))
/c)))/c + (I*((I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/c - (3*I)*b*(((
-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + I*b*(((I/
2)*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1
- 2/(1 + I*c*x)])/(4*c)))))/c))/(c*d)
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.127. $\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx$

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.127.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.23 (sec) , antiderivative size = 1359, normalized size of antiderivative = 3.31

method	result	size
derivativedivides	Expression too large to display	1359
default	Expression too large to display	1359
parts	Expression too large to display	1406

input
```
int(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^3/d*c*x-3/2*I*a^2*b/d*arctan(c*x)*c^2*x^2+3/2*I*a^2*b/d*c*x-a^3/d
*arctan(c*x)+b^3/d*(-1/2*I*arctan(c*x)^2*(I*arctan(c*x)+arctan(c*x)*c*x-3)
*(I+c*x)-1/2*arctan(c*x)^4-I*arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-3
/2*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*arctan(c*x)*pol
ylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/4*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+3
*arctan(c*x)^2+3*I*arctan(c*x)*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+3/2*polylog(2
,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*arctan(c*x)^3+3*arctan(c*x)^2*ln((1+I*c*x)^
2/(c^2*x^2+1)+1)-3*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*p
olylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a*b^2/d*(arctan(c*x)^2*c*x-1/2*I*pol
ylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*arctan(c*x)^2+I*ln((1+I*c*x)^2/(c^2
*x^2+1)+1)-Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*
arctan(c*x)^2+1/2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/
(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+Pi*arctan(c*x)^2+
2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*arctan(c*x)*ln(1-I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*(c*x-I)-1/2*Pi*csgn((1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*Pi*csgn(I/((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^
2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+I*arctan(c*x)^2*ln(c*x
-I)-2*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*arctan(c*x)^2*c^2*x^2
-1/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+...
```

3.127.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*x^2*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x^2*log(-(c
*x + I)/(c*x - I))^2 - 12*a^2*b*x^2*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x^
2)/(c*d*x - I*d), x)
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.127.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*a^3*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/128*(1
6*b^3*arctan(c*x)^4 - b^3*log(c^2*x^2 + 1)^4 + 4*(384*b^3*c^3*integrate(1/
64*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*b^3*c^3
*integrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^4*d*x^2 + c^2*d), x) + 384*b^3*c
^3*integrate(1/64*x^3*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) - 96*b^3*c^3*i
ntegrate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - 1792*b^3*c^
2*integrate(1/64*x^2*arctan(c*x)^3/(c^4*d*x^2 + c^2*d), x) - 192*b^3*c^2*i
ntegrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) -
 6144*a*b^2*c^2*integrate(1/64*x^2*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) -
 384*b^3*c^2*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d*x^2 +
c^2*d), x) - 6144*a^2*b*c^2*integrate(1/64*x^2*arctan(c*x)/(c^4*d*x^2 + c^
2*d), x) + 384*b^3*c*integrate(1/64*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^4*
d*x^2 + c^2*d), x) + 96*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1)^3/(c^4*d*x
^2 + c^2*d), x) + 768*b^3*c*integrate(1/64*x*arctan(c*x)^2/(c^4*d*x^2 + c^
2*d), x) - 192*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*
d), x) - 192*b^3*integrate(1/64*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^4*d*x^2
+ c^2*d), x) - 3*b^3*arctan(c*x)^4/(c^3*d))*c^3*d - 128*I*c^3*d*integrate(
-1/64*(192*a^2*b*c^3*x^3*arctan(c*x) + 8*(7*b^3*c^3*x^3 - 3*b^3*c*x)*arcta
n(c*x)^3 - (b^3*c^2*x^2 + 3*b^3)*log(c^2*x^2 + 1)^3 + 12*(16*a*b^2*c^3*x^3
 + b^3*c^2*x^2)*arctan(c*x)^2 - 3*(b^3*c^2*x^2 - 2*(b^3*c^3*x^3 - b^3*c...
```

3.127.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)^3x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^3)/(d + c*d*x*1i),x)
```

output
```
int((x^2*(a + b*atan(c*x))^3)/(d + c*d*x*1i), x)
```

3.128 $\int \frac{x(a+b\arctan(cx))^3}{d+icdx} \, dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 277

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx} \, dx = \frac{(a+b\arctan(cx))^3}{c^2d} - \frac{ix(a+b\arctan(cx))^3}{cd} - \frac{3ib(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d} - \frac{(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{c^2d} + \frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d} - \frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d} - \frac{3ib^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d} - \frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d} + \frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4c^2d}$$

output
```
(a+b*arctan(c*x))^3/c^2/d-I*x*(a+b*arctan(c*x))^3/c/d-3*I*b*(a+b*arctan(c*
x))^2*ln(2/(1+I*c*x))/c^2/d-(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c^2/d+3*b^
2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^2/d-3/2*I*b*(a+b*arctan(c*x
))^2*polylog(2,1-2/(1+I*c*x))/c^2/d-3/2*I*b^3*polylog(3,1-2/(1+I*c*x))/c^2
/d-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))/c^2/d+3/4*I*b^3*poly
log(4,1-2/(1+I*c*x))/c^2/d
```

3.128.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.42

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx =$$
$$-\frac{i(4a^3cx-4a^3\arctan(cx)+12a^2bcx\arctan(cx)-12a^2b\arctan(cx)^2-12iab^2\arctan(cx)^2+12ab^2cx\operatorname{ar}}{}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-1/4*I)*(4*a^3*c*x - 4*a^3*ArcTan[c*x] + 12*a^2*b*c*x*ArcTan[c*x] - 12*a
^2*b*ArcTan[c*x]^2 - (12*I)*a*b^2*ArcTan[c*x]^2 + 12*a*b^2*c*x*ArcTan[c*x]
^2 - 8*a*b^2*ArcTan[c*x]^3 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3*c*x*ArcTan[c*
x]^3 - 2*b^3*ArcTan[c*x]^4 - (12*I)*a^2*b*ArcTan[c*x]*Log[1 + E^((2*I)*Arc
Tan[c*x])] + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (12*I)*
a*b^2*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*b^3*ArcTan[c*x]^2*
Log[1 + E^((2*I)*ArcTan[c*x])] - (4*I)*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*
ArcTan[c*x])] + (2*I)*a^3*Log[1 + c^2*x^2] - 6*a^2*b*Log[1 + c^2*x^2] - 6*
b*(a*(a + (2*I)*b) + 2*(a + I*b)*b*ArcTan[c*x] + b^2*ArcTan[c*x]^2)*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])] + 6*b^2*((-I)*a + b - I*b*ArcTan[c*x])*PolyLo
g[3, -E^((2*I)*ArcTan[c*x])] + 3*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/
(c^2*d)
```

3.128.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5401, 27, 5345, 5379, 5455, 5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}dx$$
$$\Big\downarrow \text{5401}$$
$$\frac{i\int \frac{(a+b\arctan(cx))^3}{d(icx+1)}dx}{c}-\frac{i\int (a+b\arctan(cx))^3dx}{cd}$$
$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\int (a+b\arctan(cx))^3dx}{cd}$$

$$\Big\downarrow \; 5345$$

$$\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \; 5379$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \; 5455$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\int \frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 5379$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \; 5529$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\int \frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)\right)}{cd}-$$

$$\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(\frac{1}{2}ib\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)}{c}\right.\right.}{cd}$$

$$\Big\downarrow \; 5533$$

3.128. $\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\,\text{PolyLog}\left(2\cdots\right.}{}\right.\right.}{cd}$$

$$\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)}{c}\cdots\right.\right.}{cd}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+\cdots}\right)}{2\cdots}\right.\right.}{cd}$$

$$\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{i(a+b\arctan(cx))^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{Poly}\cdots}{}\right.}{c}\right.\right.}{cd}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-I)*(x*(a + b*ArcTan[c*x])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/(
b*c^2) - (((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(
1 + I*c*x)])/(4*c)))/c)))/(c*d) + (I*((I*(a + b*ArcTan[c*x])^3*Log[2/(1 +
I*c*x)])/c - (3*I)*b*(((-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1
+ I*c*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x
)])/c + (b*PolyLog[4, 1 - 2/(1 + I*c*x)])/(4*c)))))/(c*d)
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.128. $\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx$

3.128.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.89 (sec) , antiderivative size = 3018, normalized size of antiderivative = 10.90

method	result	size
derivativedivides	Expression too large to display	3018
default	Expression too large to display	3018
parts	Expression too large to display	3054

input
```
int(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-I*a^3/d*c*x+1/2*a^3/d*ln(c^2*x^2+1)+I*a^3/d*arctan(c*x)+b^3/d*(1/2
*I*arctan(c*x)^4+ln(c*x-I)*arctan(c*x)^3-arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/
(c^2*x^2+1))-3/2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*arctan(c*x)*pol
ylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3*I*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2
+1)+1)+1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)^3+I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1))^2*arctan(c*x)^3-1/2*I*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+
I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3-I*arctan
(c*x)^3*c*x+1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^3+1/2*I*Pi*csgn(I/((1+I*c
*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*
x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^3-3/4*I*polylog(4,-(1+I*c*
x)^2/(c^2*x^2+1))+3/2*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-
I*Pi*arctan(c*x)^3-arctan(c*x)^3-3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2
*x^2+1)))+3*a*b^2/d*(-1/2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-dilog(1+I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2
*x^2+1)+1))*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*
x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/
2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))+1/4*I*Pi*csgn(I/((1+I*c*x)^...
```

3.128.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*x*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x*log(-(c*x +
 I)/(c*x - I))^2 - 12*a^2*b*x*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x)/(c*d*
x - I*d), x)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^3*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) + 1/128*(-16*I*b^3*c*x*arctan(c*
x)^3 + 12*I*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 16*I*b^3*arctan(c*x)^
4 - I*b^3*log(c^2*x^2 + 1)^4 - 4*I*(896*b^3*c^2*integrate(1/32*x^2*arctan(
c*x)^3/(c^3*d*x^2 + c*d), x) + 96*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*l
og(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x) + 3072*a*b^2*c^2*integrate(1/32*x^
2*arctan(c*x)^2/(c^3*d*x^2 + c*d), x) + 384*b^3*c^2*integrate(1/32*x^2*arc
tan(c*x)*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) + 3072*a^2*b*c^2*integrate
(1/32*x^2*arctan(c*x)/(c^3*d*x^2 + c*d), x) - 64*b^3*c*integrate(1/32*x*lo
g(c^2*x^2 + 1)^3/(c^3*d*x^2 + c*d), x) - 384*b^3*c*integrate(1/32*x*arctan
(c*x)^2/(c^3*d*x^2 + c*d), x) + 96*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)
^2/(c^3*d*x^2 + c*d), x) + 3*b^3*arctan(c*x)^4/(c^2*d) + 96*b^3*integrate(
1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x))*c^2*d + 128*c^2
*d*integrate(1/64*(80*b^3*c*x*arctan(c*x)^3 + 192*a^2*b*c*x*arctan(c*x) +
(b^3*c^2*x^2 + 3*b^3)*log(c^2*x^2 + 1)^3 - 24*(b^3*c^2*x^2 - 8*a*b^2*c*x)*
arctan(c*x)^2 + 6*(b^3*c^2*x^2 + 2*b^3*c*x*arctan(c*x))*log(c^2*x^2 + 1)^2
 - 12*(2*b^3*c*x*arctan(c*x) + (b^3*c^2*x^2 - b^3)*arctan(c*x)^2)*log(c^2*
x^2 + 1))/(c^3*d*x^2 + c*d), x) - 2*(b^3*c*x + 2*b^3*arctan(c*x))*log(c^2*
x^2 + 1)^3 + 8*(3*b^3*c*x*arctan(c*x)^2 - 2*b^3*arctan(c*x)^3)*log(c^2*x^2
 + 1))/(c^2*d)
```

3.128.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(b\arctan{(cx)}+a)^3x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x*(a + b*atan(c*x))^3)/(d + c*d*x*1i),x)
```

output
```
int((x*(a + b*atan(c*x))^3)/(d + c*d*x*1i), x)
```

3.128. $\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx$

3.129 $\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$

3.129.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{i(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd} + \frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4cd}$$

output
```
I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c/d-3/2*b*(a+b*arctan(c*x))^2*polylo
g(2,1-2/(1+I*c*x))/c/d+3/2*I*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x)
)/c/d+3/4*b^3*polylog(4,1-2/(1+I*c*x))/c/d
```

3.129.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{i\left(4(a+b\arctan(cx))^3\log\left(\frac{2d}{d+icdx}\right)+3ib\left(2(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)-b\left(2i(a+b\arctan(cx)\right.\right.\right.}{4cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
 b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTa
n[c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*
x)]))))/(c*d)
```

3.129.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}dx$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{5529}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\int \frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \text{5533}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int \frac{\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\operatorname{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - ((3*I)*b*(((-1/2*I)*(
a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b
*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1 - 2/(1 +
I*c*x)])/(4*c))))/d
```

3.129.3.1 Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.129.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.00 (sec) , antiderivative size = 1629, normalized size of antiderivative = 11.72

method	result	size
derivativedivides	Expression too large to display	1629
default	Expression too large to display	1629
parts	Expression too large to display	1640

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I*a^3/d*ln(c^2*x^2+1)+a^3/d*arctan(c*x)+b^3/d*(-I*ln(1+I*c*x)*ar
ctan(c*x)^3+3*I*(1/3*arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+1/6*I*P
i*(csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+csgn(I/((1+
I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+
I*c*x)^2/(c^2*x^2+1)+1))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-c
sgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I
*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))-csgn(I*(1+I*c*x)^2/(c^2*x
^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))+csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-1)*ar
ctan(c*x)^3-1/2*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*ar
ctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*polylog(4,-(1+I*c*x)^2
/(c^2*x^2+1))-1/6*I*arctan(c*x)^4))+3*a*b^2/d*(-I*ln(1+I*c*x)*arctan(c*x)^
2+2*I*(1/2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*Pi*(csgn((1
+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3+csgn(I/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2
-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+...
```

3.129.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/
(c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x - I*d)
, x)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arc
tan(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3
*log(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16
*x*log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) +
12*a^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*a
rctan(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arc
tan(c*x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 +
 1) + b^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

3.129.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i), x)
```

3.130 $\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx$

3.130.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx = \frac{(a+b\arctan(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d} + \frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d} - \frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)}{4d}$$

output
```
(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d+3/2*I*b*(a+b*arctan(c*x))^2*polylo
g(2,-1+2/(1+I*c*x))/d+3/2*b^2*(a+b*arctan(c*x))*polylog(3,-1+2/(1+I*c*x))/
d-3/4*I*b^3*polylog(4,-1+2/(1+I*c*x))/d
```

3.130.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 268 vs. 2(128) = 256.

Time = 0.61 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.09

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx = -\frac{i\left(8ab^2\pi^3+b^3\pi^4+64a^3\arctan(cx)+192a^2b\arctan(cx)^2+192iab^2\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)+\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

output
```
((-1/64*I)*(8*a*b^2*Pi^3 + b^3*Pi^4 + 64*a^3*ArcTan[c*x] + 192*a^2*b*ArcTa
n[c*x]^2 + (192*I)*a*b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (
64*I)*b^3*ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] + (192*I)*a^2*b*Ar
cTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (64*I)*a^3*Log[c*x] - (32*I)*a^
3*Log[1 + c^2*x^2] - 96*b^2*ArcTan[c*x]*(2*a + b*ArcTan[c*x])*PolyLog[2, E
^((-2*I)*ArcTan[c*x])] + 96*a^2*b*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (96*
I)*a*b^2*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (96*I)*b^3*ArcTan[c*x]*PolyL
og[3, E^((-2*I)*ArcTan[c*x])] + 48*b^3*PolyLog[4, E^((-2*I)*ArcTan[c*x])])
)/d
```

3.130.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5403, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}dx$$

$$\Big\downarrow \text{5403}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d}-\frac{3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{5529}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d}-\frac{3bc\left(ib\int \frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \text{5533}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d} - \frac{3bc\left(ib\left(\frac{i\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx\right)-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \; 7164$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d} - \frac{3bc\left(ib\left(\frac{i\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{PolyLog}\left(4,\frac{2}{icx+1}-1\right)}{4c}\right)-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

output
```
((a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d - (3*b*c*(((-1/2*I)*(a +
b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b*Ar
cTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])/c + (b*PolyLog[4, -1 + 2/(1 + I
*c*x)])/(4*c))))/d
```

3.130.3.1 Defintions of rubi rules used

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.130.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.54 (sec) , antiderivative size = 2795, normalized size of antiderivative = 21.84

method	result	size
parts	Expression too large to display	2795
derivativedivides	Expression too large to display	2797
default	Expression too large to display	2797

input
```
int((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/d*ln(c^2*x^2+1)-I*a^3/d*arctan(c*x)+a^3/d*ln(x)+b^3/d*(arctan(c*x
)^3*ln(c*x)-ln(c*x-I)*arctan(c*x)^3+arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(c^2*
x^2+1))-arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^3*ln(1-(1+
I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*polylog(2,(1+I*c*x)/(c^2*x^2+1
)^(1/2))+6*arctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*polylog(
4,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^3*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1
/2))-3*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*arctan(c*
x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*polylog(4,-(1+I*c*x)/(c^2*x
^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-csgn(I/((
1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/
(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))+csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1
))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+csgn(I/((1+
I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))-csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))^2-2*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-csgn((1
+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3-cs...
```

3.130.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(109) = 218$.

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.92

$$\int \frac{(a + b\arctan(cx))^3}{x(d + icdx)}\,dx$$
$$= \frac{-3i\,b^3\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 - 12\,ab^2\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right) - 12i\,a^2b\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right) + 8\,a^3\log(}{}$$

input
```
integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/8*(-3*I*b^3*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 12
*a*b^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 12*I*a^2*b*
dilog((c*x + I)/(c*x - I) + 1) + 8*a^3*log(x) - 8*a^3*log((c*x - I)/c) - 6
*I*b^3*polylog(4, -(c*x + I)/(c*x - I)) + (-I*b^3*log(-(c*x + I)/(c*x - I)
)^3 - 6*a*b^2*log(-(c*x + I)/(c*x - I))^2)*log(2*c*x/(c*x - I)) - 6*(-I*b^
3*log(-(c*x + I)/(c*x - I)) - 2*a*b^2)*polylog(3, -(c*x + I)/(c*x - I)))/d
```

3.130.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^3}{cx^2-ix}\,dx+\int \frac{b^3\operatorname{atan}^3(cx)}{cx^2-ix}\,dx+\int \frac{3ab^2\operatorname{atan}^2(cx)}{cx^2-ix}\,dx+\int \frac{3a^2b\operatorname{atan}(cx)}{cx^2-ix}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**3/x/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**3/(c*x**2 - I*x), x) + Integral(b**3*atan(c*x)**3/(c*x**2
- I*x), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**2 - I*x), x) + Integral(
3*a**2*b*atan(c*x)/(c*x**2 - I*x), x))/d
```

3.130.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-a^3*(log(I*c*x + 1)/d - log(x)/d) + 1/512*(-64*I*b^3*arctan(c*x)^4 + 64*b
^3*arctan(c*x)^3*log(c^2*x^2 + 1) + 16*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3
+ 4*I*b^3*log(c^2*x^2 + 1)^4 - I*(64*b^3*arctan(c*x)^4/d + 6144*b^3*c^2*in
tegrate(1/64*x^2*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x) + 3*
b^3*log(c^2*x^2 + 1)^4/d + 512*a*b^2*arctan(c*x)^3/d + 768*a^2*b*arctan(c*
x)^2/d + 6144*b^3*integrate(1/64*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3
 + d*x), x) - 512*b^3*integrate(1/64*log(c^2*x^2 + 1)^3/(c^2*d*x^3 + d*x),
 x))*d - 512*d*integrate(1/32*(12*b^3*c*x*arctan(c*x)^2*log(c^2*x^2 + 1) +
 b^3*c*x*log(c^2*x^2 + 1)^3 - 96*a*b^2*arctan(c*x)^2 - 96*a^2*b*arctan(c*x
) + 4*(3*b^3*c^2*x^2 - 7*b^3)*arctan(c*x)^3 + 3*(b^3*c^2*x^2 - b^3)*arctan
(c*x)*log(c^2*x^2 + 1)^2)/(c^2*d*x^3 + d*x), x))/d
```

3.130.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^3}{(icdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^3/(x*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^3/(x*(d + c*d*x*1i)), x)
```

3.131 $\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx$

3.131.1 Optimal result

Integrand size = 25, antiderivative size = 263

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx = & -\frac{ic(a+b\arctan(cx))^3}{d}-\frac{(a+b\arctan(cx))^3}{dx}\\ & +\frac{3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)}{d}\\ & -\frac{ic(a+b\arctan(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d}\\ & -\frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}\\ & +\frac{3bc(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}\\ & +\frac{3b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)}{2d}\\ & -\frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}\\ & -\frac{3b^3c\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)}{4d}\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^3/d-(a+b*arctan(c*x))^3/d/x+3*b*c*(a+b*arctan(c*x))
^2*ln(2-2/(1-I*c*x))/d-I*c*(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d-3*I*b^2
*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))/d+3/2*b*c*(a+b*arctan(c*x))
^2*polylog(2,-1+2/(1+I*c*x))/d+3/2*b^3*c*polylog(3,-1+2/(1-I*c*x))/d-3/2*I
*b^2*c*(a+b*arctan(c*x))*polylog(3,-1+2/(1+I*c*x))/d-3/4*b^3*c*polylog(4,-
1+2/(1+I*c*x))/d
```

3.131.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.67

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx =$$
$$-\frac{\frac{2a^3}{x}+2a^3c\arctan(cx)+2ia^3c\log(x)-ia^3c\log\left(1+c^2x^2\right)+\frac{3a^2b\left(2cx\arctan(cx)^2+\arctan(cx)\left(2+2icx\log\left(1-e^{2i\arctan}\right.\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```

output
```
-1/2*((2*a^3)/x + 2*a^3*c*ArcTan[c*x] + (2*I)*a^3*c*Log[x] - I*a^3*c*Log[1
 + c^2*x^2] + (3*a^2*b*(2*c*x*ArcTan[c*x]^2 + ArcTan[c*x]*(2 + (2*I)*c*x*L
og[1 - E^((2*I)*ArcTan[c*x])]) + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2]) + c*
x*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x + (6*I)*a*b^2*c*((-1/24*I)*Pi^3 +
ArcTan[c*x]^2 - (I*ArcTan[c*x]^2)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*
ArcTan[c*x])] + (2*I)*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTa
n[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x
])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2) + (2*I)*b^3*c*(Pi^3/8 - (I/64)
*Pi^4 - ArcTan[c*x]^3 - (I*ArcTan[c*x]^3)/(c*x) + (3*I)*ArcTan[c*x]^2*Log[
1 - E^((-2*I)*ArcTan[c*x])] + ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])
] + ((3*I)/2)*ArcTan[c*x]*(2*I + ArcTan[c*x])*PolyLog[2, E^((-2*I)*ArcTan[
c*x])] + (3*(I + ArcTan[c*x])*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2 - ((3*
I)/4)*PolyLog[4, E^((-2*I)*ArcTan[c*x])]))/d
```

3.131.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5405, 27, 5361, 5403, 5459, 5403, 5527, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx$$
$$\Big\downarrow \text{5405}$$
$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^2}dx}{d}-ic\int \frac{(a+b\arctan(cx))^3}{dx(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^2}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{x}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5403}$$

$$\frac{3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^3}{3b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\,\mathrm{arc}}{3}\right.}{d}$$
$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \text{5527}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int \frac{\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right.\right.\right.}{d}$$
$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \text{5529}$$

3.131. $\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right.\right.\right.}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\left(ib\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan}{2c}\right.\right.}{d}$$

$$\Big\downarrow \text{5533}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right.\right.\right.}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx\right)\right.\right.}{d}$$

$$\Big\downarrow \text{7164}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+\right.\right.}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}+\frac{b\operatorname{PolyLog}\left(4,\frac{2}{icx+1}-1\right)}{4c}\right)-\frac{i\operatorname{Poly}}{}\right.\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```

output
```
(-((a + b*ArcTan[c*x])^3/x) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/b +
I*((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + (2*I)*b*c*(((I/2)*(
a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/c - (b*PolyLog[3, -1 +
2/(1 - I*c*x)])/(4*c)))))/d - (I*c*((a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I
*c*x)] - 3*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*c
*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])/
c + (b*PolyLog[4, -1 + 2/(1 + I*c*x)])/(4*c)))))/d
```

3.131. $\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx$

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

3.131. $\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx$

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.131.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.85 (sec) , antiderivative size = 10105, normalized size of antiderivative = 38.42

method	result	size
parts	Expression too large to display	10105
derivativedivides	Expression too large to display	10106
default	Expression too large to display	10106

input
```
int((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.131.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
-1/8*(b^3*c*x*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^3 + 3*b^3*c*x
*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 6*b^3*c*x*log(-
(c*x + I)/(c*x - I))*polylog(3, -(c*x + I)/(c*x - I)) - I*b^3*log(-(c*x +
I)/(c*x - I))^3 + 6*b^3*c*x*polylog(4, -(c*x + I)/(c*x - I)) - 8*d*x*integ
ral(1/4*(-4*I*a^3*c*x + 4*a^3 - 3*(a*b^2 + (-I*a*b^2 + b^3)*c*x)*log(-(c*x
 + I)/(c*x - I))^2 + 6*(a^2*b*c*x + I*a^2*b)*log(-(c*x + I)/(c*x - I)))/(c
^2*d*x^4 + d*x^2), x))/(d*x)
```

3.131.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx=-\frac{i\left(\int \frac{a^3}{cx^3-ix^2}\,dx+\int \frac{b^3\operatorname{atan}^3(cx)}{cx^3-ix^2}\,dx+\int \frac{3ab^2\operatorname{atan}^2(cx)}{cx^3-ix^2}\,dx+\int \frac{3a^2b\operatorname{atan}(cx)}{cx^3-ix^2}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**3/x**2/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**3/(c*x**3 - I*x**2), x) + Integral(b**3*atan(c*x)**3/(c*x*
*3 - I*x**2), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**3 - I*x**2), x) +
Integral(3*a**2*b*atan(c*x)/(c*x**3 - I*x**2), x))/d
```

3.131.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx=\int \frac{(b\arctan{(cx)}+a)^3}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^3*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x)) - 1/512*(64*b^3*c*x*ar
ctan(c*x)^4 - 4*b^3*c*x*log(c^2*x^2 + 1)^4 + 64*b^3*arctan(c*x)^3 - 48*b^3
*arctan(c*x)*log(c^2*x^2 + 1)^2 - 8*(-2*I*b^3*c*x*arctan(c*x) + I*b^3)*log
(c^2*x^2 + 1)^3 - (48*b^3*c*arctan(c*x)^4/d - 6144*b^3*c^3*integrate(1/64*
x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 3*b^3*c*log(c
^2*x^2 + 1)^4/d + 3072*b^3*c^2*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*d*x^4 + d*x^2), x) - 12288*b^3*c^2*integrate(1/64*x^2*arctan(c
*x)*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 6144*b^3*c*integrate(1/64*x
*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) + 512*b^3*c*integr
ate(1/64*x*log(c^2*x^2 + 1)^3/(c^2*d*x^4 + d*x^2), x) + 12288*b^3*c*integr
ate(1/64*x*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) - 3072*b^3*c*integrate(1/
64*x*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 28672*b^3*integrate(1/64
*arctan(c*x)^3/(c^2*d*x^4 + d*x^2), x) + 3072*b^3*integrate(1/64*arctan(c*
x)*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 98304*a*b^2*integrate(1/64
*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + 98304*a^2*b*integrate(1/64*arctan
(c*x)/(c^2*d*x^4 + d*x^2), x))*d*x - 64*I*(192*b^3*c^3*integrate(1/64*x^3*
arctan(c*x)^3/(c^2*d*x^4 + d*x^2), x) + 48*b^3*c^3*integrate(1/64*x^3*arct
an(c*x)*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + b^3*c*arctan(c*x)^3/d
 + 96*b^3*c^2*integrate(1/64*x^2*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4
 + d*x^2), x) + 24*b^3*c^2*integrate(1/64*x^2*log(c^2*x^2 + 1)^3/(c^2*d...
```

3.131.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
Timed out
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^2\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^3/(x^2*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^3/(x^2*(d + c*d*x*1i)), x)
```

3.132 $\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx$

3.132.1 Optimal result

Integrand size = 25, antiderivative size = 414

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = &-\frac{3ibc^2(a+b\arctan(cx))^2}{2d} \\ &-\frac{3bc(a+b\arctan(cx))^2}{2dx}-\frac{3c^2(a+b\arctan(cx))^3}{2d} \\ &-\frac{(a+b\arctan(cx))^3}{2dx^2}+\frac{ic(a+b\arctan(cx))^3}{dx} \\ &+\frac{3b^2c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d} \\ &-\frac{3ibc^2(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)}{d} \\ &-\frac{c^2(a+b\arctan(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} \\ &-\frac{3ib^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{2d} \\ &-\frac{3b^2c^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d} \\ &-\frac{3ibc^2(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d} \\ &-\frac{3ib^3c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)}{2d} \\ &-\frac{3b^2c^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d} \\ &+\frac{3ib^3c^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)}{4d}\end{aligned}$$

output
```
-3/2*I*b*c^2*(a+b*arctan(c*x))^2/d-3/2*b*c*(a+b*arctan(c*x))^2/d/x-3/2*c^2
*(a+b*arctan(c*x))^3/d-1/2*(a+b*arctan(c*x))^3/d/x^2+I*c*(a+b*arctan(c*x))
^3/d/x+3*b^2*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d-3*I*b*c^2*(a+b*arct
an(c*x))^2*ln(2-2/(1-I*c*x))/d-c^2*(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d
-3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))/d-3*b^2*c^2*(a+b*arctan(c*x))*pol
ylog(2,-1+2/(1-I*c*x))/d-3/2*I*b*c^2*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1
+I*c*x))/d-3/2*I*b^3*c^2*polylog(3,-1+2/(1-I*c*x))/d-3/2*b^2*c^2*(a+b*arct
an(c*x))*polylog(3,-1+2/(1+I*c*x))/d+3/4*I*b^3*c^2*polylog(4,-1+2/(1+I*c*x
))/d
```

3.132.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.53

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx$$
$$= \frac{-\frac{a^3}{x^2}+\frac{2ia^3c}{x}+2ia^3c^2\arctan(cx)-2a^3c^2\log(x)+a^3c^2\log\left(1+c^2x^2\right)+\frac{3ia^2b\left(2c^2x^2\arctan(cx)^2+\arctan(cx)(i+2cx+i\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```

output
```
(-(a^3/x^2) + ((2*I)*a^3*c)/x + (2*I)*a^3*c^2*ArcTan[c*x] - 2*a^3*c^2*Log[
x] + a^3*c^2*Log[1 + c^2*x^2] + ((3*I)*a^2*b*(2*c^2*x^2*ArcTan[c*x]^2 + Ar
cTan[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c
*x])]) + c*x*(I - 2*c*x*Log[c*x] + c*x*Log[1 + c^2*x^2]) + c^2*x^2*PolyLog
[2, E^((2*I)*ArcTan[c*x])]))/x^2 + 6*a*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/
(c*x) - (3*ArcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2
)/(c*x) - ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x
]*Log[1 - E^((2*I)*ArcTan[c*x])] + Log[c*x] - Log[1 + c^2*x^2]/2 - I*ArcTa
n[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x
])] - PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2) + 2*b^3*c^2*(-1/8*Pi^3 + (I/64
)*Pi^4 - ((3*I)/2)*ArcTan[c*x]^2 - (3*ArcTan[c*x]^2)/(2*c*x) + ArcTan[c*x]
^3 + (I*ArcTan[c*x]^3)/(c*x) - ((1 + c^2*x^2)*ArcTan[c*x]^3)/(2*c^2*x^2) -
 (3*I)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^3*Log[1
 - E^((-2*I)*ArcTan[c*x])] + 3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])]
+ (3*(2 - I*ArcTan[c*x])*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])])/2
 - ((3*I)/2)*PolyLog[2, E^((2*I)*ArcTan[c*x])] - (3*(I + ArcTan[c*x])*Poly
Log[3, E^((-2*I)*ArcTan[c*x])])/2 + ((3*I)/4)*PolyLog[4, E^((-2*I)*ArcTan[
c*x])]))/(2*d)
```

3.132.3 Rubi [A] (verified)

Time = 3.82 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {5405, 27, 5361, 5405, 5361, 5403, 5453, 5361, 5419, 5459, 5403, 2897, 5527, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^3}dx}{d}-ic\int \frac{(a+b\arctan(cx))^3}{dx^2(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^3}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-\frac{ic\left(\int \frac{(a+b\arctan(cx))^3}{x^2}dx-ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-\frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx-\frac{(a+b\arctan(cx))^3}{x}\right)}{d}$$

$$\Big\downarrow \text{5403}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-$$
$$\frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+\dots}{\dots}\right)}{d}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{3}{2}bc\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx - c^2\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right) - \frac{(a+b\arctan(cx))^3}{2x^2}}{d} - \frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx - ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3 - 3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right) - (a+}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right) + 2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - \frac{(a+b\arctan(cx))^2}{x}\right) - \frac{(a+b\arctan(cx))^3}{2x^2}}{d} - \frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx - ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3 - 3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right) - (a+}{d}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{3}{2}bc\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx - \frac{c(a+b\arctan(cx))^3}{3b} - \frac{(a+b\arctan(cx))^2}{x}\right) - \frac{(a+b\arctan(cx))^3}{2x^2}}{d} - \frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx - ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3 - 3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right) - (a+}{d}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2} + \frac{3}{2}bc\left(2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx - \frac{i(a+b\arctan(cx))^2}{2b}\right) - \frac{c(a+b\arctan(cx))^3}{3b} - \frac{(a+b\arctan(cx))^2}{x}\right)}{d} - \frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3 - 3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right) + 3bc\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx - \right.\right.}{d}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2} + \frac{3}{2}bc\left(2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx - i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right) - \frac{i(a+b\arctan(cx))^2}{2b}\right) - \right.}{d}$$

$$\frac{ic\left(3bc\left(i\left(2ibc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx - i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right) - \frac{i(a+b\arctan(cx))^3}{3b}\right) - ic\left(\right.\right.}{d}$$

$$\Big\downarrow \text{2897}$$

3.132. $\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}dx$

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(c}{2b}\right.\right.}{d}$$

$$\frac{ic\left(3bc\left(i\left(2ibc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\arctan(cx))^3}{3b}\right)-ic\left(\right.\right.}{d}$$

$$\Big\downarrow \color{blue}{5527}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(c}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(\right.\right.\right.$$

$$\Big\downarrow \color{blue}{5529}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(c}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(\right.\right.\right.$$

$$\Big\downarrow \color{blue}{5533}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(c}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(\right.\right.\right.$$

$$\Big\downarrow \color{blue}{7164}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(c}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\mathrm{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```

3.132. $\int\frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx$

output
```
(-1/2*(a + b*ArcTan[c*x])^3/x^2 + (3*b*c*(-((a + b*ArcTan[c*x])^2/x) - (c*
(a + b*ArcTan[c*x])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
 I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2
/(1 - I*c*x)])/2))))/2)/d - (I*c*(-((a + b*ArcTan[c*x])^3/x) + 3*b*c*(((-1
/3*I)*(a + b*ArcTan[c*x])^3)/b + I*((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(
1 - I*c*x)] + (2*I)*b*c*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 -
 I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1 - I*c*x)])/(4*c)))) - I*c*((a + b*Ar
cTan[c*x])^3*Log[2 - 2/(1 + I*c*x)] - 3*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])
^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*Pol
yLog[3, -1 + 2/(1 + I*c*x)])/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x)])/(4*c))
))))/d
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

3.132. $\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx$

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.132.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 20.81 (sec) , antiderivative size = 2450, normalized size of antiderivative = 5.92

method	result	size
derivativedivides	Expression too large to display	2450
default	Expression too large to display	2450
parts	Expression too large to display	2457

input
```
int((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/2*a^3/d*ln(c^2*x^2+1)+I*a^3/d*arctan(c*x)-1/2*a^3/d/c^2/x^2+I*a^3/d
/c/x-a^3/d*ln(c*x)+b^3/d*(3*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+
1)^(1/2))+1/2*I*arctan(c*x)^4-arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/
2))-3*I*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^3*ln(1+(1+I*c*
x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2-2*arctan(c*x)^3-3*I*arctan(c*x)^2*
ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x
^2+1)^(1/2))-6*I*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*arctan(c*x)^2*
polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*polylog(2,-(1+I*c*x)/
(c^2*x^2+1)^(1/2))+1/2*arctan(c*x)^2*(I*arctan(c*x)+3*I*c*x+arctan(c*x)*c*
x)*(I+c*x)/c^2/x^2-6*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3
*I*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*arctan(c*x)*ln(1-(1+I*c*x)/(c^
2*x^2+1)^(1/2))-6*I*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*p
olylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*polylog(4,-(1+I*c*x)/(c^2*x^2+1)
^(1/2))-6*I*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*arctan(c*x)*ln(1+(1+
I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/
2)))+3*a*b^2/d*(1/2*I*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
*arctan(c*x)^2+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-1/2*arctan(c*x)^2/c^2/x^2
-1/2*I*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))...
```

3.132.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/16*(2*I*b^3*c^2*x^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^3 + 6
*I*b^3*c^2*x^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 1
2*I*b^3*c^2*x^2*log(-(c*x + I)/(c*x - I))*polylog(3, -(c*x + I)/(c*x - I))
 + 12*I*b^3*c^2*x^2*polylog(4, -(c*x + I)/(c*x - I)) + 16*d*x^2*integral(1
/8*(-8*I*a^3*c*x + 8*a^3 - 3*(-2*I*b^3*c^2*x^2 + 2*a*b^2 + (-2*I*a*b^2 + b
^3)*c*x)*log(-(c*x + I)/(c*x - I))^2 + 12*(a^2*b*c*x + I*a^2*b)*log(-(c*x
+ I)/(c*x - I)))/(c^2*d*x^5 + d*x^3), x) + (2*b^3*c*x + I*b^3)*log(-(c*x +
 I)/(c*x - I))^3)/(d*x^2)
```

3.132.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^3}{cx^4-ix^3}\,dx+\int \frac{b^3\,\mathrm{atan}^3\,(cx)}{cx^4-ix^3}\,dx+\int \frac{3ab^2\,\mathrm{atan}^2\,(cx)}{cx^4-ix^3}\,dx+\int \frac{3a^2b\,\mathrm{atan}\,(cx)}{cx^4-ix^3}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**3/x**3/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**3/(c*x**4 - I*x**3), x) + Integral(b**3*atan(c*x)**3/(c*x*
*4 - I*x**3), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**4 - I*x**3), x) +
Integral(3*a**2*b*atan(c*x)/(c*x**4 - I*x**3), x))/d
```

3.132.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a^3
- 1/512*(-64*I*b^3*c^2*x^2*arctan(c*x)^4 + 4*I*b^3*c^2*x^2*log(c^2*x^2 + 1
)^4 + I*(48*b^3*c^2*arctan(c*x)^4/d - 6144*b^3*c^4*integrate(1/64*x^4*arct
an(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 3*b^3*c^2*log(c^2*x^2
 + 1)^4/d + 3072*b^3*c^3*integrate(1/64*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2
/(c^2*d*x^5 + d*x^3), x) - 12288*b^3*c^3*integrate(1/64*x^3*arctan(c*x)*lo
g(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 6144*b^3*c^2*integrate(1/64*x^2*a
rctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) - 1536*b^3*c^2*integrate(1/64*x^2*log
(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 28672*b^3*c*integrate(1/64*x*arc
tan(c*x)^3/(c^2*d*x^5 + d*x^3), x) + 3072*b^3*c*integrate(1/64*x*arctan(c*
x)*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 98304*a*b^2*c*integrate(1/
64*x*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) - 6144*b^3*c*integrate(1/64*x*a
rctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 98304*a^2*b*c*integr
ate(1/64*x*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 6144*b^3*integrate(1/64*a
rctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 512*b^3*integrate(
1/64*log(c^2*x^2 + 1)^3/(c^2*d*x^5 + d*x^3), x))*d*x^2 - 64*(192*b^3*c^4*i
ntegrate(1/64*x^4*arctan(c*x)^3/(c^2*d*x^5 + d*x^3), x) + 48*b^3*c^4*integ
rate(1/64*x^4*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + b^3
*c^2*arctan(c*x)^3/d + 96*b^3*c^3*integrate(1/64*x^3*arctan(c*x)^2*log(c^2
*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 24*b^3*c^3*integrate(1/64*x^3*log(c...
```

3.132.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
Timed out
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^3}{x^3\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))^3/(x^3*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^3/(x^3*(d + c*d*x*1i)), x)
```

3.133 $\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx$

3.133.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = \mathrm{Int}\left(\frac{1}{(d+icdx)(a+b\arctan(cx))},x\right)$$

output
```
Unintegrable(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

3.133.2 Mathematica [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = \int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx$$

input
```
Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]
```

output
```
Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])), x]
```

3.133.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}dx$$

input
```
Int[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]
```

output
```
$Aborted
```

3.133.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.133.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(icdx+d)\,(a+b\arctan(cx))}dx$$

input
```
int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

output
```
int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

3.133.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx=\int \frac{1}{(i\,cdx+d)(b\arctan\left(cx\right)+a)}\,dx$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(-2/(-2*I*a*c*d*x - 2*a*d + (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x
- I))), x)
```

3.133.6 Sympy [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx=-\frac{i\int \frac{1}{acx-ia+bcx\,\mathrm{atan}\left(cx\right)-ib\,\mathrm{atan}\left(cx\right)}\,dx}{d}$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*atan(c*x)),x)
```

output
```
-I*Integral(1/(a*c*x - I*a + b*c*x*atan(c*x) - I*b*atan(c*x)), x)/d
```

3.133.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx=\int \frac{1}{(i\,cdx+d)(b\arctan\left(cx\right)+a)}\,dx$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((I*c*d*x + d)*(b*arctan(c*x) + a)), x)
```

3.133.8 Giac [N/A]

Not integrable

Time = 30.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = \int \frac{1}{(i\,cdx+d)(b\arctan{(cx)}+a)}\,dx$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.133.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{atan}\,(c\,x))\;(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int(1/((a + b*atan(c*x))*(d + c*d*x*1i)),x)
```

output
```
int(1/((a + b*atan(c*x))*(d + c*d*x*1i)), x)
```

3.134 $\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx$

3.134.1 Optimal result

Integrand size = 19, antiderivative size = 297

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx =& \frac{ad^2x}{e^3}+\frac{bdx}{2ce^2}-\frac{bx^2}{6ce}-\frac{bd\arctan(cx)}{2c^2e^2}\\ &+\frac{bd^2x\arctan(cx)}{e^3}-\frac{dx^2(a+b\arctan(cx))}{2e^2}\\ &+\frac{x^3(a+b\arctan(cx))}{3e}+\frac{d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^4}\\ &-\frac{d^3(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4}-\frac{bd^2\log(1+c^2x^2)}{2ce^3}\\ &+\frac{b\log(1+c^2x^2)}{6c^3e}-\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^4}\\ &+\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}\end{aligned}$$

output
```
a*d^2*x/e^3+1/2*b*d*x/c/e^2-1/6*b*x^2/c/e-1/2*b*d*arctan(c*x)/c^2/e^2+b*d^
2*x*arctan(c*x)/e^3-1/2*d*x^2*(a+b*arctan(c*x))/e^2+1/3*x^3*(a+b*arctan(c*
x))/e+d^3*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^4-d^3*(a+b*arctan(c*x))*ln(2
*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4-1/2*b*d^2*ln(c^2*x^2+1)/c/e^3+1/6*b*ln
(c^2*x^2+1)/c^3/e-1/2*I*b*d^3*polylog(2,1-2/(1-I*c*x))/e^4+1/2*I*b*d^3*pol
ylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4
```

3.134.2 Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.63

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx =$$

$$-\frac{\frac{be^3}{c^3}-6ad^2ex-\frac{3bde^2x}{c}+3ade^2x^2+\frac{be^3x^2}{c}-2ae^3x^3+\frac{3bde^2\arctan(cx)}{c^2}+3ibd^3\pi\arctan(cx)-6bd^2ex\arctan}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
-1/6*((b*e^3)/c^3 - 6*a*d^2*e*x - (3*b*d*e^2*x)/c + 3*a*d*e^2*x^2 + (b*e^3
*x^2)/c - 2*a*e^3*x^3 + (3*b*d*e^2*ArcTan[c*x])/c^2 + (3*I)*b*d^3*Pi*ArcTa
n[c*x] - 6*b*d^2*e*x*ArcTan[c*x] + 3*b*d*e^2*x^2*ArcTan[c*x] - 2*b*e^3*x^3
*ArcTan[c*x] - (6*I)*b*d^3*ArcTan[(c*d)/e]*ArcTan[c*x] + (3*I)*b*d^3*ArcTa
n[c*x]^2 + (3*b*d^2*e*ArcTan[c*x]^2)/c - (3*b*d^2*Sqrt[1 + (c^2*d^2)/e^2]*
e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2)/c + 3*b*d^3*Pi*Log[1 + E^((-2*I)*Ar
cTan[c*x])] - 6*b*d^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b*d^3
*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*b*
d^3*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*a*d
^3*Log[d + e*x] + (3*b*d^2*e*Log[1 + c^2*x^2])/c - (b*e^3*Log[1 + c^2*x^2]
)/c^3 + (3*b*d^3*Pi*Log[1 + c^2*x^2])/2 - 6*b*d^3*ArcTan[(c*d)/e]*Log[Sin[
ArcTan[(c*d)/e] + ArcTan[c*x]]] + (3*I)*b*d^3*PolyLog[2, -E^((2*I)*ArcTan[
c*x])] - (3*I)*b*d^3*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))]
)/e^4
```

3.134.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(-\frac{d^3(a+b\arctan(cx))}{e^3(d+ex)}+\frac{d^2(a+b\arctan(cx))}{e^3}-\frac{dx(a+b\arctan(cx))}{e^2}+\frac{x^2(a+b\arctan(cx))}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^4}-\frac{d^3(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}-\frac{dx^2(a+b\arctan(cx))}{2e^2}+\frac{x^3(a+b\arctan(cx))}{3e}+\frac{ad^2x}{e^3}-\frac{bd\arctan(cx)}{2c^2e^2}+\frac{bd^2x\arctan(cx)}{e^3}-\frac{bd^2\log\left(c^2x^2+1\right)}{2ce^3}+\frac{b\log\left(c^2x^2+1\right)}{6c^3e}-\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^4}+\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}+\frac{bdx}{2ce^2}-\frac{bx^2}{6ce}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(a*d^2*x)/e^3 + (b*d*x)/(2*c*e^2) - (b*x^2)/(6*c*e) - (b*d*ArcTan[c*x])/(2
*c^2*e^2) + (b*d^2*x*ArcTan[c*x])/e^3 - (d*x^2*(a + b*ArcTan[c*x]))/(2*e^2
) + (x^3*(a + b*ArcTan[c*x]))/(3*e) + (d^3*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/e^4 - (d^3*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(
1 - I*c*x))])/e^4 - (b*d^2*Log[1 + c^2*x^2])/(2*c*e^3) + (b*Log[1 + c^2*x^
2])/(6*c^3*e) - ((I/2)*b*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + ((I/2)*b
*d^3*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^4
```

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.134.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.27

method	result
parts	$\frac{a\,x^{3}}{3e}-\frac{ad\,x^{2}}{2e^{2}}+\frac{a\,d^{2}x}{e^{3}}-\frac{a\,d^{3}\ln\left(ex+d\right)}{e^{4}}+\frac{b\left(\frac{c^{4}\arctan\left(cx\right)x^{3}}{3e}-\frac{c^{4}\arctan\left(cx\right)x^{2}d}{2e^{2}}+\frac{c^{4}\arctan\left(cx\right)x\,d^{2}}{e^{3}}-\frac{c^{4}\arctan\left(cx\right)d^{3}\ln}{e^{4}}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^{4}d^{2}x}{e^{3}}-\frac{a\,c^{4}d\,x^{2}}{2e^{2}}+\frac{a\,c^{4}x^{3}}{3e}-\frac{a\,c^{4}d^{3}\ln\left(ecx+cd\right)}{e^{4}}+bc\left(\frac{\arctan\left(cx\right)c^{3}d^{2}x}{e^{3}}-\frac{\arctan\left(cx\right)c^{3}d\,x^{2}}{2e^{2}}+\frac{\arctan\left(cx\right)c^{3}x^{3}}{3e}-\frac{\arctan\left(cx\right)c^{3}d^{3}\ln(e}{e^{4}}\right.}{}$
default	$\frac{\frac{a\,c^{4}d^{2}x}{e^{3}}-\frac{a\,c^{4}d\,x^{2}}{2e^{2}}+\frac{a\,c^{4}x^{3}}{3e}-\frac{a\,c^{4}d^{3}\ln\left(ecx+cd\right)}{e^{4}}+bc\left(\frac{\arctan\left(cx\right)c^{3}d^{2}x}{e^{3}}-\frac{\arctan\left(cx\right)c^{3}d\,x^{2}}{2e^{2}}+\frac{\arctan\left(cx\right)c^{3}x^{3}}{3e}-\frac{\arctan\left(cx\right)c^{3}d^{3}\ln(e}{e^{4}}\right.}{}$
risch	$-\frac{b\,x^{2}}{6ce}+\frac{b\ln\left(c^{2}x^{2}+1\right)}{6c^{3}e}+\frac{bdx}{2c\,e^{2}}-\frac{bd\arctan\left(cx\right)}{2c^{2}e^{2}}-\frac{b\,d^{2}\ln\left(c^{2}x^{2}+1\right)}{2c\,e^{3}}-\frac{ad}{2c^{2}e^{2}}-\frac{a\,d^{3}\ln\left(icd-\left(-icx+1\right)e+e\right)}{e^{4}}-$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/e*x^3-1/2*a/e^2*d*x^2+a*d^2*x/e^3-a/e^4*d^3*ln(e*x+d)+b/c^4*(1/3*c^4
*arctan(c*x)/e*x^3-1/2*c^4*arctan(c*x)/e^2*x^2*d+c^4*arctan(c*x)/e^3*x*d^2
-c^4*arctan(c*x)*d^3/e^4*ln(c*e*x+c*d)-c/e*(1/2/e^2*ln(c^2*d^2-2*c*d*(c*e*
x+c*d)+e^2+(c*e*x+c*d)^2)*c^2*d^2+1/2/e*arctan(c*x)*c*d-1/6*ln(c^2*d^2-2*c
*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)-5/6/e^2*c*d*(c*e*x+c*d)+1/6/e^2*(c*e*x+c
*d)^2-1/e^2*c^3*d^3*(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-e*c*x)/(c*d+I*e))-ln((I
*e+e*c*x)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-e*c*x)/(c*d+I*e))-dilog((I*e+e*c
*x)/(I*e-c*d)))/e)))
```

3.134.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arctan(c*x) + a*x^3)/(e*x + d), x)
```

3.134.6 Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x^3(a+b\,\mathrm{atan}\,(cx))}{d+ex}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x), x)
```

3.134.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
-1/6*a*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) +
2*b*integrate(1/2*x^3*arctan(c*x)/(e*x + d), x)
```

3.134.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{d+e\,x}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x), x)
```

3.135 $\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx$

3.135.1 Optimal result

Integrand size = 19, antiderivative size = 237

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = -\frac{adx}{e^2}-\frac{bx}{2ce}+\frac{b\arctan(cx)}{2c^2e}-\frac{bdx\arctan(cx)}{e^2} +\frac{x^2(a+b\arctan(cx))}{2e}-\frac{d^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^3} +\frac{d^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3} +\frac{bd\log(1+c^2x^2)}{2ce^2}+\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3} -\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}$$

output
```
-a*d*x/e^2-1/2*b*x/c/e+1/2*b*arctan(c*x)/c^2/e-b*d*x*arctan(c*x)/e^2+1/2*x
^2*(a+b*arctan(c*x))/e-d^2*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^3+d^2*(a+b*
arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3+1/2*b*d*ln(c^2*x^2+1)
/c/e^2+1/2*I*b*d^2*polylog(2,1-2/(1-I*c*x))/e^3-1/2*I*b*d^2*polylog(2,1-2*
c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3
```

3.135.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.70

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx$$
$$= \frac{-2adex - \frac{be^2x}{c} + ae^2x^2 + \frac{be^2\arctan(cx)}{c^2} + ibd^2\pi\arctan(cx) - 2bdex\arctan(cx) + be^2x^2\arctan(cx) - 2ibd^2 a}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(-2*a*d*e*x - (b*e^2*x)/c + a*e^2*x^2 + (b*e^2*ArcTan[c*x])/c^2 + I*b*d^2*
Pi*ArcTan[c*x] - 2*b*d*e*x*ArcTan[c*x] + b*e^2*x^2*ArcTan[c*x] - (2*I)*b*d
^2*ArcTan[(c*d)/e]*ArcTan[c*x] + I*b*d^2*ArcTan[c*x]^2 + (b*d*e*ArcTan[c*x
]^2)/c - (b*d*Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^
2)/c + b*d^2*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] - 2*b*d^2*ArcTan[c*x]*Log[
1 + E^((2*I)*ArcTan[c*x])] + 2*b*d^2*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(Arc
Tan[(c*d)/e] + ArcTan[c*x]))] + 2*b*d^2*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcT
an[(c*d)/e] + ArcTan[c*x]))] + 2*a*d^2*Log[d + e*x] + (b*d*e*Log[1 + c^2*x
^2])/c + (b*d^2*Pi*Log[1 + c^2*x^2])/2 - 2*b*d^2*ArcTan[(c*d)/e]*Log[Sin[A
rcTan[(c*d)/e] + ArcTan[c*x]]] + I*b*d^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])
] - I*b*d^2*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))])/(2*e^3)
```

3.135.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{d^2(a+b\arctan(cx))}{e^2(d+ex)} - \frac{d(a+b\arctan(cx))}{e^2} + \frac{x(a+b\arctan(cx))}{e}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^3}+\frac{d^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}+\frac{x^2(a+b\arctan(cx))}{2e}-\frac{adx}{e^2}+\frac{b\arctan(cx)}{2c^2e}-\frac{bdx\arctan(cx)}{e^2}+\frac{bd\log(c^2x^2+1)}{2ce^2}+\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}-\frac{bx}{2ce}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
-((a*d*x)/e^2) - (b*x)/(2*c*e) + (b*ArcTan[c*x])/(2*c^2*e) - (b*d*x*ArcTan
[c*x])/e^2 + (x^2*(a + b*ArcTan[c*x]))/(2*e) - (d^2*(a + b*ArcTan[c*x])*Lo
g[2/(1 - I*c*x)])/e^3 + (d^2*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d
 + I*e)*(1 - I*c*x))])/e^3 + (b*d*Log[1 + c^2*x^2])/(2*c*e^2) + ((I/2)*b*d
^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^3 - ((I/2)*b*d^2*PolyLog[2, 1 - (2*c*(
d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^3
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.135.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.20

method	result
parts	$\frac{a x^2}{2e} - \frac{adx}{e^2} + \frac{a d^2 \ln(ex+d)}{e^3} + \frac{b\left(\frac{c^3 \arctan(cx)x^2}{2e} - \frac{c^3 \arctan(cx)dx}{e^2} + \frac{c^3 \arctan(cx)d^2 \ln(ecx+cd)}{e^3} - \frac{c\left(\frac{c^2d^2\left(-\frac{i\ln(ecx+c}{}\right.}{}\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{a c^3 dx}{e^2} + \frac{a c^3 x^2}{2e} + \frac{a c^3 d^2 \ln(ecx+cd)}{e^3} + bc\left(-\frac{\arctan(cx)c^2dx}{e^2} + \frac{\arctan(cx)c^2x^2}{2e} + \frac{\arctan(cx)c^2d^2\ln(ecx+cd)}{e^3} - \frac{c^2d^2\left(\frac{i\ln(ecx+cd}{}\right.}{}\right.}{}$
default	$\frac{-\frac{a c^3 dx}{e^2} + \frac{a c^3 x^2}{2e} + \frac{a c^3 d^2 \ln(ecx+cd)}{e^3} + bc\left(-\frac{\arctan(cx)c^2dx}{e^2} + \frac{\arctan(cx)c^2x^2}{2e} + \frac{\arctan(cx)c^2d^2\ln(ecx+cd)}{e^3} - \frac{c^2d^2\left(\frac{i\ln(ecx+cd}{}\right.}{}\right.}{}$
risch	$-\frac{bx}{2ce} + \frac{b\arctan(cx)}{4c^2e} + \frac{bd\ln(c^2x^2+1)}{4c\,e^2} - \frac{ib\ln(icx+1)x^2}{4e} - \frac{ib\ln(icx+1)}{4c^2e} + \frac{ib\ln(c^2x^2+1)}{8c^2e} + \frac{ib\ln(icx+1)dx}{2e^2} -$

input
```
int(x^2*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e*x^2-a*d*x/e^2+a/e^3*d^2*ln(e*x+d)+b/c^3*(1/2*c^3*arctan(c*x)/e*x^2
-c^3*arctan(c*x)*d/e^2*x+c^3*arctan(c*x)*d^2/e^3*ln(c*e*x+c*d)-c/e*(1/e*c^
2*d^2*(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-e*c*x)/(c*d+I*e))-ln((I*e+e*c*x)/(I*e
-c*d)))/e-1/2*I*(dilog((I*e-e*c*x)/(c*d+I*e))-dilog((I*e+e*c*x)/(I*e-c*d))
)/e)-1/2/e*c*d*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)-1/2*arctan(
c*x)+1/2/e*(c*e*x+c*d)))
```

3.135.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e*x + d), x)
```

3.135.6 Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x^2(a+b\operatorname{atan}(cx))}{d+ex}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/(d + e*x), x)
```

3.135.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 2*b*integrate(1/2*x
^2*arctan(c*x)/(e*x + d), x)
```

3.135.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arctan(cx))}{d + ex} \, dx = \int \frac{x^2 \, (a + b \, \mathrm{atan}(c \, x))}{d + e \, x} \, dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x), x)
```

3.136 $\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx$

3.136.1 Optimal result

Integrand size = 17, antiderivative size = 179

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \frac{ax}{e}+\frac{bx\arctan(cx)}{e}+\frac{d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2} - \frac{b\log\left(1+c^2x^2\right)}{2ce} - \frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2} + \frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}$$

output
```
a*x/e+b*x*arctan(c*x)/e+d*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^2-d*(a+b*arc
tan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2-1/2*b*ln(c^2*x^2+1)/c/e-
1/2*I*b*d*polylog(2,1-2/(1-I*c*x))/e^2+1/2*I*b*d*polylog(2,1-2*c*(e*x+d)/(
c*d+I*e)/(1-I*c*x))/e^2
```

3.136.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.84

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx$$
$$= \frac{2aex - 2ad\log(d+ex) + \frac{b\left(-icd\pi\arctan(cx)+2cex\arctan(cx)+2icd\arctan\left(\frac{cd}{e}\right)\arctan(cx)-icd\arctan(cx)^2-e\arctan(cx)^2+\sqrt{1+c\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTan[c*x] + 2*c*e*x*ArcT
an[c*x] + (2*I)*c*d*ArcTan[(c*d)/e]*ArcTan[c*x] - I*c*d*ArcTan[c*x]^2 - e*
ArcTan[c*x]^2 + Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x
]^2 - c*d*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] + 2*c*d*ArcTan[c*x]*Log[1 + E
^((2*I)*ArcTan[c*x])] - 2*c*d*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*
d)/e] + ArcTan[c*x]))] - 2*c*d*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/
e] + ArcTan[c*x]))] - e*Log[1 + c^2*x^2] - (c*d*Pi*Log[1 + c^2*x^2])/2 + 2
*c*d*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] - I*c*d*PolyL
og[2, -E^((2*I)*ArcTan[c*x])] + I*c*d*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e]
 + ArcTan[c*x]))]))/c)/(2*e^2)
```

3.136.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{a+b\arctan(cx)}{e}-\frac{d(a+b\arctan(cx))}{e(d+ex)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^2}-\frac{d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{ax}{e}+\frac{bx\arctan(cx)}{e}-\frac{b\log\left(c^2x^2+1\right)}{2ce}-\frac{ibd\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(a*x)/e + (b*x*ArcTan[c*x])/e + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])
/e^2 - (d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x)
)])/e^2 - (b*Log[1 + c^2*x^2])/(2*c*e) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1 -
I*c*x)])/e^2 + ((I/2)*b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
 I*c*x))])/e^2
```

3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.136.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.25

method	result
parts	$\frac{ax}{e} - \frac{ad\ln(ex+d)}{e^2} + \frac{b\left(\frac{c^2\arctan(cx)x}{e} - \frac{c^2\arctan(cx)d\ln(ecx+cd)}{e^2} - \frac{c\left(\frac{\ln\left(c^2d^2-2cd(ecx+cd)+e^2+(ecx+cd)^2\right)}{2} - cd\left(-\frac{i\ln(e\cdots\right.\right.}{}\right.}{c^2}$
derivativedivides	$\frac{\frac{a\,c^2x}{e} - \frac{a\,c^2d\ln(ecx+cd)}{e^2} + bc\left(\frac{\arctan(cx)cx}{e} - \frac{\arctan(cx)dc\ln(ecx+cd)}{e^2} - \frac{\frac{\ln\left(c^2d^2-2cd(ecx+cd)+e^2+(ecx+cd)^2\right)}{2} - cd\left(\frac{i\ln(ecx+c\cdots\right.}{}\right.}{c^2}$
default	$\frac{\frac{a\,c^2x}{e} - \frac{a\,c^2d\ln(ecx+cd)}{e^2} + bc\left(\frac{\arctan(cx)cx}{e} - \frac{\arctan(cx)dc\ln(ecx+cd)}{e^2} - \frac{\frac{\ln\left(c^2d^2-2cd(ecx+cd)+e^2+(ecx+cd)^2\right)}{2} - cd\left(\frac{i\ln(ecx+c\cdots\right.}{}\right.}{c^2}$
risch	$\frac{ia}{ce} + \frac{ibd\operatorname{dilog}\left(\frac{icd+(icx+1)e-e}{icd-e}\right)}{2e^2} - \frac{b\ln(c^2x^2+1)}{4ce} + \frac{ib\ln(-icx+1)x}{2e} + \frac{b}{ce} - \frac{ibd\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-\cdots}{-icd-e}\right.}{2e^2}$

input
```
int(x*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*x/e-a/e^2*d*ln(e*x+d)+b/c^2*(c^2*arctan(c*x)/e*x-c^2*arctan(c*x)*d/e^2*l
n(c*e*x+c*d)-c/e*(1/2*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)-c*d*
(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-e*c*x)/(c*d+I*e))-ln((I*e+e*c*x)/(I*e-c*d))
)/e-1/2*I*(dilog((I*e-e*c*x)/(c*d+I*e))-dilog((I*e+e*c*x)/(I*e-c*d)))/e)))
```

3.136.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x*arctan(c*x) + a*x)/(e*x + d), x)
```

3.136.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x(a+b\operatorname{atan}(cx))}{d+ex}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x), x)
```

3.136.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
a*(x/e - d*log(e*x + d)/e^2) + 2*b*integrate(1/2*x*arctan(c*x)/(e*x + d),
x)
```

3.136.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{d+e\,x}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x), x)
```

3.137 $\int \frac{a+b\arctan(cx)}{d+ex}\,dx$

3.137.1 Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}$$

output
```
-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d
+I*e)/(1-I*c*x))/e+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e-1/2*I*b*polylog(2,1-
2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e
```

3.137.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \frac{2a\log(d+ex) + ib\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right) - ib\log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right) + ib\,\text{PolyLog}\left(2,\frac{e(1-icx)}{icd+e}\right) - i}{2e}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x),x]
```

output
```
(2*a*Log[d + e*x] + I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] - I*
b*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog[2, (e*(1 - I
*c*x))/(I*c*d + e)] - I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*e)
```

3.137.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{d+ex}dx$$

$$\Big\downarrow \text{5381}$$

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2x^2+1}dx}{e}+\frac{bc\int \frac{\log\left(\frac{2}{1-icx}\right)}{c^2x^2+1}dx}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2x^2+1}dx}{e}+\frac{ib\int \frac{\log\left(\frac{2}{1-icx}\right)}{1-\frac{2}{1-icx}}d\frac{1}{1-icx}}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2x^2+1}dx}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}$$

$$\Big\downarrow \text{2897}$$

$$\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x),x]
```

output
```
-(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(
2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(
1 - I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
 I*c*x))])/e
```

3.137.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

3.137.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\left(\frac{c\ln(ecx+cd)\arctan(cx)}{e}-c\left(-\frac{i\ln(ecx+cd)\left(\ln\left(\frac{-ecx+ie}{cd+ie}\right)-\ln\left(\frac{ecx+ie}{-cd+ie}\right)\right)}{2e}-\frac{i\left(\operatorname{dilog}\left(\frac{-ecx+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{ecx+i}{-cd+i}\right.\right.}{2e}\right.\right.}{c}$
derivativedivides	$\frac{\frac{ac\ln(ecx+cd)}{e}+bc\left(\frac{\ln(ecx+cd)\arctan(cx)}{e}+\frac{i\ln(ecx+cd)\left(\ln\left(\frac{-ecx+ie}{cd+ie}\right)-\ln\left(\frac{ecx+ie}{-cd+ie}\right)\right)}{2e}+\frac{i\left(\operatorname{dilog}\left(\frac{-ecx+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{ecx+ie}{-cd+ie}\right)\right)}{2e}\right)}{c}$
default	$\frac{\frac{ac\ln(ecx+cd)}{e}+bc\left(\frac{\ln(ecx+cd)\arctan(cx)}{e}+\frac{i\ln(ecx+cd)\left(\ln\left(\frac{-ecx+ie}{cd+ie}\right)-\ln\left(\frac{ecx+ie}{-cd+ie}\right)\right)}{2e}+\frac{i\left(\operatorname{dilog}\left(\frac{-ecx+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{ecx+ie}{-cd+ie}\right)\right)}{2e}\right)}{c}$
risch	$\frac{ib\operatorname{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e}+\frac{ib\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e}+\frac{a\ln(icd-(-icx+1)e+e)}{e}-\frac{ib\operatorname{dilog}\left(\frac{icd+(}{\ }\right.}{2e}$

input
```
int((a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e+b/c*(c*ln(c*e*x+c*d)/e*arctan(c*x)-c*(-1/2*I*ln(c*e*x+c*d)*(
ln((I*e-e*c*x)/(c*d+I*e))-ln((I*e+e*c*x)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-e
*c*x)/(c*d+I*e))-dilog((I*e+e*c*x)/(I*e-c*d)))/e))
```

3.137.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx=\int \frac{b\arctan(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x + d), x)
```

3.137.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{a+b\operatorname{atan}(cx)}{d+ex}\,dx$$

input
```
integrate((a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))/(d + e*x), x)
```

3.137.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{b\arctan(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*arctan(c*x)/(e*x + d), x) + a*log(e*x + d)/e
```

3.137.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{b\arctan(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{d + ex}\, dx = \int \frac{a + b \operatorname{atan}(c\,x)}{d + e\,x}\, dx$$

input
```
int((a + b*atan(c*x))/(d + e*x),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x), x)
```

3.138 $\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx$

3.138.1 Optimal result

Integrand size = 19, antiderivative size = 181

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx &= \frac{a\log(x)}{d}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d}\\&\quad-\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}\\&\quad+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d}-\frac{ib\operatorname{PolyLog}(2,icx)}{2d}\\&\quad-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}\end{aligned}$$

output
```
a*ln(x)/d+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d-(a+b*arctan(c*x))*ln(2*c*(e*
x+d)/(c*d+I*e)/(1-I*c*x))/d+1/2*I*b*polylog(2,-I*c*x)/d-1/2*I*b*polylog(2,
I*c*x)/d-1/2*I*b*polylog(2,1-2/(1-I*c*x))/d+1/2*I*b*polylog(2,1-2*c*(e*x+d
)/(c*d+I*e)/(1-I*c*x))/d
```

3.138.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx$$
$$= \frac{2a\log(x)-2a\log(d+ex)-ib\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)+ib\log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right)+ib\,\mathrm{PolyLog}(2,-}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```

output
```
(2*a*Log[x] - 2*a*Log[d + e*x] - I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d
 - I*e)] + I*b*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog
[2, (-I)*c*x] - I*b*PolyLog[2, I*c*x] - I*b*PolyLog[2, (e*(1 - I*c*x))/(I*
c*d + e)] + I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*d)
```

3.138.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{a+b\arctan(cx)}{dx}-\frac{e(a+b\arctan(cx))}{d(d+ex)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}+\frac{a\log(x)}{d}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d}-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```

output
```
(a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan
[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + ((I/2)*b*PolyLo
g[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1
- 2/(1 - I*c*x)])/d + ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/d
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.138.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-icx+1)}{2d}-\frac{ib\,\mathrm{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d}-\frac{ib\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d}+\frac{a\ln(-icx)}{d}-\frac{a\ln$
parts	$\frac{a\ln(x)}{d}-\frac{a\ln(ex+d)}{d}+b\left(\frac{\arctan(cx)\ln(cx)}{d}-\frac{\arctan(cx)\ln(ecx+cd)}{d}-c\left(\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(cx)\ln(-icx}{2}}{c}\right.\right.$
derivativedivides	$\frac{a\ln(cx)}{d}-\frac{a\ln(ecx+cd)}{d}+bc\left(\frac{\arctan(cx)\ln(cx)}{dc}-\frac{\arctan(cx)\ln(ecx+cd)}{dc}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(cx)\ln(-icx}{2}}{c}\right.$
default	$\frac{a\ln(cx)}{d}-\frac{a\ln(ecx+cd)}{d}+bc\left(\frac{\arctan(cx)\ln(cx)}{dc}-\frac{\arctan(cx)\ln(ecx+cd)}{dc}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(cx)\ln(-icx}{2}}{c}\right.$

input
```
int((a+b*arctan(c*x))/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b/d*dilog(1-I*c*x)-1/2*I*b/d*dilog((-I*c*d+(1-I*c*x)*e-e)/(-I*c*d-e
))-1/2*I*b/d*ln(1-I*c*x)*ln((-I*c*d+(1-I*c*x)*e-e)/(-I*c*d-e))+a/d*ln(-I*c
*x)-a/d*ln(I*c*d-(1-I*c*x)*e+e)+1/2*I*b/d*dilog(1+I*c*x)+1/2*I*b/d*dilog((
I*c*d+(1+I*c*x)*e-e)/(I*c*d-e))+1/2*I*b/d*ln(1+I*c*x)*ln((I*c*d+(1+I*c*x)*
e-e)/(I*c*d-e))
```

3.138.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^2 + d*x), x)
```

3.138.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{a+b\operatorname{atan}(cx)}{x(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x)), x)
```

3.138.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="maxima")
```

output
```
-a*(log(e*x + d)/d - log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(e*x^2 + d*
x), x)
```

3.138.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x)), x)
```

3.139 $\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx$

3.139.1 Optimal result

Integrand size = 19, antiderivative size = 232

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = &-\frac{a+b\arctan(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{ae\log(x)}{d^2}\\&-\frac{e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}\\&+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2}-\frac{bc\log\left(1+c^2x^2\right)}{2d}\\&-\frac{ibe\,\mathrm{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\,\mathrm{PolyLog}(2,icx)}{2d^2}\\&+\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}\end{aligned}$$

output
```
(-a-b*arctan(c*x))/d/x+b*c*ln(x)/d-a*e*ln(x)/d^2-e*(a+b*arctan(c*x))*ln(2/
(1-I*c*x))/d^2+e*(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2
-1/2*b*c*ln(c^2*x^2+1)/d-1/2*I*b*e*polylog(2,-I*c*x)/d^2+1/2*I*b*e*polylog
(2,I*c*x)/d^2+1/2*I*b*e*polylog(2,1-2/(1-I*c*x))/d^2-1/2*I*b*e*polylog(2,1
-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2
```

3.139.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx =$$
$$-\frac{2ad+2bd\arctan(cx)-2bcdx\log(x)+2aex\log(x)-2aex\log(d+ex)-ibex\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

output
```
-1/2*(2*a*d + 2*b*d*ArcTan[c*x] - 2*b*c*d*x*Log[x] + 2*a*e*x*Log[x] - 2*a*
e*x*Log[d + e*x] - I*b*e*x*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] +
 I*b*e*x*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + b*c*d*x*Log[1 + c
^2*x^2] + I*b*e*x*PolyLog[2, (-I)*c*x] - I*b*e*x*PolyLog[2, I*c*x] - I*b*e
*x*PolyLog[2, (e*(1 - I*c*x))/(I*c*d + e)] + I*b*e*x*PolyLog[2, -((e*(-I +
 c*x))/(c*d + I*e))])/(d^2*x)
```

3.139.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{e^2(a+b\arctan(cx))}{d^2(d+ex)}-\frac{e(a+b\arctan(cx))}{d^2x}+\frac{a+b\arctan(cx)}{dx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^2}-\frac{a+b\arctan(cx)}{dx}-\frac{ae\log(x)}{d^2}-\frac{bc\log\left(c^2x^2+1\right)}{2d}-\frac{ibe\operatorname{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\operatorname{PolyLog}(2,icx)}{2d^2}+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}+\frac{bc\log(x)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

output
```
-((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
 b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])*Log[(2*c*
(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d)
- ((I/2)*b*e*PolyLog[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2
 + ((I/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b*e*PolyLog[2, 1
 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2
```

3.139.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.139.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.32

method	result
parts	$a\left(-\frac{1}{dx}-\frac{e\ln(x)}{d^2}+\frac{e\ln(ex+d)}{d^2}\right)+bc\left(-\frac{\arctan(cx)}{dcx}-\frac{\arctan(cx)e\ln(cx)}{c\,d^2}+\frac{\arctan(cx)e\ln(ecx+cd)}{c\,d^2}-c\right.$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{ae\ln(ecx+cd)}{c\,d^2}+bc\left(-\frac{\arctan(cx)}{d\,c^2x}-\frac{\arctan(cx)e\ln(cx)}{d^2c^2}+\frac{\arctan(cx)e\ln(ecx+cd)}{d^2c^2}\right.\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{ae\ln(ecx+cd)}{c\,d^2}+bc\left(-\frac{\arctan(cx)}{d\,c^2x}-\frac{\arctan(cx)e\ln(cx)}{d^2c^2}+\frac{\arctan(cx)e\ln(ecx+cd)}{d^2c^2}\right.\right.$
risch	$\frac{ibe\operatorname{dilog}(-icx+1)}{2d^2}+\frac{ibe\operatorname{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d^2}+\frac{ibe\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d^2}+\frac{cb\ln(-icx)}{2d}-cb$

input
```
int((a+b*arctan(c*x))/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x-e/d^2*ln(x)+e/d^2*ln(e*x+d))+b*c*(-arctan(c*x)/d/c/x-1/c*arctan(
c*x)*e/d^2*ln(c*x)+1/c*arctan(c*x)*e/d^2*ln(c*e*x+c*d)-c*(1/d^2/c^2*e^2*(-
1/2*I*ln(c*e*x+c*d)*(ln((I*e-e*c*x)/(c*d+I*e))-ln((I*e+e*c*x)/(I*e-c*d)))/
e-1/2*I*(dilog((I*e-e*c*x)/(c*d+I*e))-dilog((I*e+e*c*x)/(I*e-c*d)))/e)-1/d
/c*(-1/2*ln(c^2*x^2+1)+ln(c*x))-1/d^2/c^2*e*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/
2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))))
```

3.139.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx=\int \frac{b\arctan(cx)+a}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^3 + d*x^2), x)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x+d),x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 2*b*integrate(1/2*arctan
(c*x)/(e*x^3 + d*x^2), x)
```

3.139.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^2(d + ex)}\,dx = \int \frac{a + b\,\mathrm{atan}(c\,x)}{x^2\,(d + e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x)), x)
```

3.140 $\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx$

3.140.1 Optimal result

Integrand size = 19, antiderivative size = 293

$$
\begin{aligned}
\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = & -\frac{bc}{2dx} - \frac{bc^2\arctan(cx)}{2d} - \frac{a+b\arctan(cx)}{2dx^2} + \frac{e(a+b\arctan(cx))}{d^2x} \\
& - \frac{bce\log(x)}{d^2} + \frac{ae^2\log(x)}{d^3} + \frac{e^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^3} \\
& - \frac{e^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3} + \frac{bce\log(1+c^2x^2)}{2d^2} \\
& + \frac{ibe^2\operatorname{PolyLog}(2,-icx)}{2d^3} - \frac{ibe^2\operatorname{PolyLog}(2,icx)}{2d^3} \\
& - \frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3} + \frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}
\end{aligned}
$$

output
```
-1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d+1/2*(-a-b*arctan(c*x))/d/x^2+e*(a+b*a
rctan(c*x))/d^2/x-b*c*e*ln(x)/d^2+a*e^2*ln(x)/d^3+e^2*(a+b*arctan(c*x))*ln
(2/(1-I*c*x))/d^3-e^2*(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x)
)/d^3+1/2*b*c*e*ln(c^2*x^2+1)/d^2+1/2*I*b*e^2*polylog(2,-I*c*x)/d^3-1/2*I*
b*e^2*polylog(2,I*c*x)/d^3-1/2*I*b*e^2*polylog(2,1-2/(1-I*c*x))/d^3+1/2*I*
b*e^2*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = &-\frac{a+b\arctan(cx)}{2dx^2}+\frac{e(a+b\arctan(cx))}{d^2x}\\&-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2dx}+\frac{ae^2\log(x)}{d^3}\\&-\frac{ae^2\log(d+ex)}{d^3}-\frac{bce(2\log(x)-\log(1+c^2x^2))}{2d^2}\\&+\frac{ibe^2\,\text{PolyLog}(2,-icx)}{2d^3}-\frac{ibe^2\,\text{PolyLog}(2,icx)}{2d^3}\\&-\frac{ib\left(e^2\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)+e^2\,\text{PolyLog}\left(2,\frac{e(1-icx)}{icd+e}\right)\right)}{2d^3}\\&+\frac{ib\left(e^2\log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right)+e^2\,\text{PolyLog}\left(2,-\frac{e(1+icx)}{icd-e}\right)\right)}{2d^3}\end{aligned}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```

output
```
-1/2*(a + b*ArcTan[c*x])/(d*x^2) + (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*
Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*d*x) + (a*e^2*Log[x])/d^3
- (a*e^2*Log[d + e*x])/d^3 - (b*c*e*(2*Log[x] - Log[1 + c^2*x^2]))/(2*d^2)
 + ((I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, I*c*x]
)/d^3 - ((I/2)*b*(e^2*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] + e^2*
PolyLog[2, (e*(1 - I*c*x))/(I*c*d + e)]))/d^3 + ((I/2)*b*(e^2*Log[1 + I*c*
x]*Log[(c*(d + e*x))/(c*d + I*e)] + e^2*PolyLog[2, -((e*(1 + I*c*x))/(I*c*
d - e))]))/d^3
```

3.140.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{e^3(a+b\arctan(cx))}{d^3(d+ex)}+\frac{e^2(a+b\arctan(cx))}{d^3x}-\frac{e(a+b\arctan(cx))}{d^2x^2}+\frac{a+b\arctan(cx)}{dx^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{e^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}-\frac{e^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^3}+\frac{e(a+b\arctan(cx))}{d^2x}-\frac{a+b\arctan(cx)}{2dx^2}+\frac{ae^2\log(x)}{d^3}-\frac{bc^2\arctan(cx)}{2d}+\frac{bce\log\left(c^2x^2+1\right)}{2d^2}+\frac{ibe^2\operatorname{PolyLog}(2,-icx)}{2d^3}-\frac{ibe^2\operatorname{PolyLog}(2,icx)}{2d^3}-\frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}+\frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}-\frac{bce\log(x)}{d^2}-\frac{bc}{2dx}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```

output
```
-1/2*(b*c)/(d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^
2) + (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])
/d^3 + (e^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - (e^2*(a + b*ArcT
an[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3 + (b*c*e*Log[
1 + c^2*x^2])/(2*d^2) + ((I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*
e^2*PolyLog[2, I*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
^3 + ((I/2)*b*e^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))
])/d^3
```

3.140.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.140.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.22

method	result
parts	$a\left(-\frac{1}{2d\,x^{2}}+\frac{e^{2}\ln\left(x\right)}{d^{3}}+\frac{e}{d^{2}x}-\frac{e^{2}\ln\left(ex+d\right)}{d^{3}}\right)+b\,c^{2}\left(-\frac{\arctan\left(cx\right)}{2d\,c^{2}x^{2}}+\frac{\arctan\left(cx\right)e^{2}\ln\left(cx\right)}{c^{2}d^{3}}+\frac{\arctan\left(cx\right)e}{c^{2}d^{2}x}\right.$
derivativedivides	$c^{2}\left(-\frac{a}{2d\,c^{2}x^{2}}+\frac{a\,e^{2}\ln\left(cx\right)}{c^{2}d^{3}}+\frac{ae}{c^{2}d^{2}x}-\frac{a\,e^{2}\ln\left(ecx+cd\right)}{c^{2}d^{3}}+bc\left(-\frac{\arctan\left(cx\right)}{2d\,c^{3}x^{2}}+\frac{\arctan\left(cx\right)e^{2}\ln\left(cx\right)}{d^{3}c^{3}}+\right.\right.$
default	$c^{2}\left(-\frac{a}{2d\,c^{2}x^{2}}+\frac{a\,e^{2}\ln\left(cx\right)}{c^{2}d^{3}}+\frac{ae}{c^{2}d^{2}x}-\frac{a\,e^{2}\ln\left(ecx+cd\right)}{c^{2}d^{3}}+bc\left(-\frac{\arctan\left(cx\right)}{2d\,c^{3}x^{2}}+\frac{\arctan\left(cx\right)e^{2}\ln\left(cx\right)}{d^{3}c^{3}}+\right.\right.$
risch	$-\frac{ib\,e^{2}\operatorname{dilog}\left(\frac{-icd+\left(-icx+1\right)e-e}{-icd-e}\right)}{2d^{3}}-\frac{bc}{2dx}-\frac{ib\,e^{2}\operatorname{dilog}\left(-icx+1\right)}{2d^{3}}-\frac{ib\,e^{2}\ln\left(-icx+1\right)\ln\left(\frac{-icd+\left(-icx+1\right)e-e}{-icd-e}\right)}{2d^{3}}-$

input
```
int((a+b*arctan(c*x))/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x-e^2/d^3*ln(e*x+d))+b*c^2*(-1/2*arctan(
c*x)/d/c^2/x^2+1/c^2*arctan(c*x)*e^2/d^3*ln(c*x)+1/c^2*arctan(c*x)*e/d^2/x
-1/c^2*arctan(c*x)*e^2/d^3*ln(c*e*x+c*d)-1/2*c*(1/c^2/d^2*(-e*ln(c^2*x^2+1
)+d*c*arctan(c*x)+2*e*ln(c*x)+d/x)+2/d^3/c^3*e^2*(-1/2*I*ln(c*x)*ln(1+I*c*
x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))-2/
d^3/c^3*e^3*(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-e*c*x)/(c*d+I*e))-ln((I*e+e*c*x
)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-e*c*x)/(c*d+I*e))-dilog((I*e+e*c*x)/(I*e
-c*d)))/e)))
```

3.140.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^4 + d*x^3), x)
```

3.140.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atan}\,(cx)}{x^3\,(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*(d + e*x)), x)
```

3.140.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
 + 2*b*integrate(1/2*arctan(c*x)/(e*x^4 + d*x^3), x)
```

3.140.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x)), x)
```

3.141 $\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx$

3.141.1 Optimal result

Integrand size = 21, antiderivative size = 598

$$
\begin{aligned}
\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx =& \frac{abdx}{ce^2}+\frac{b^2x}{3c^2e}-\frac{b^2\arctan(cx)}{3c^3e}+\frac{b^2dx\arctan(cx)}{ce^2} \\
&-\frac{bx^2(a+b\arctan(cx))}{3ce}+\frac{id^2(a+b\arctan(cx))^2}{ce^3} \\
&-\frac{d(a+b\arctan(cx))^2}{2c^2e^2}-\frac{i(a+b\arctan(cx))^2}{3c^3e} \\
&+\frac{d^2x(a+b\arctan(cx))^2}{e^3}-\frac{dx^2(a+b\arctan(cx))^2}{2e^2} \\
&+\frac{x^3(a+b\arctan(cx))^2}{3e}+\frac{d^3(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^4} \\
&+\frac{2bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce^3} \\
&-\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3e} \\
&-\frac{d^3(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4} \\
&-\frac{b^2d\log(1+c^2x^2)}{2c^2e^2} \\
&-\frac{ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^4} \\
&+\frac{ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3e} \\
&+\frac{ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4} \\
&+\frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^4} \\
&-\frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}
\end{aligned}
$$

output
```
a*b*d*x/c/e^2+1/3*b^2*x/c^2/e-1/3*b^2*arctan(c*x)/c^3/e+b^2*d*x*arctan(c*x
)/c/e^2-1/3*b*x^2*(a+b*arctan(c*x))/c/e-1/3*I*(a+b*arctan(c*x))^2/c^3/e-1/
2*d*(a+b*arctan(c*x))^2/c^2/e^2-I*b*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1
-I*c*x))/e^4+d^2*x*(a+b*arctan(c*x))^2/e^3-1/2*d*x^2*(a+b*arctan(c*x))^2/e
^2+1/3*x^3*(a+b*arctan(c*x))^2/e+d^3*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e
^4+2*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/e^3-2/3*b*(a+b*arctan(c*x))
*ln(2/(1+I*c*x))/c^3/e-d^3*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1
-I*c*x))/e^4-1/2*b^2*d*ln(c^2*x^2+1)/c^2/e^2+I*d^2*(a+b*arctan(c*x))^2/c/e
^3-1/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^3/e+I*b*d^3*(a+b*arctan(c*x))*poly
log(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4+I*b^2*d^2*polylog(2,1-2/(1+I*
c*x))/c/e^3+1/2*b^2*d^3*polylog(3,1-2/(1-I*c*x))/e^4-1/2*b^2*d^3*polylog(3
,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4
```

3.141.2 Mathematica [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

3.141.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{d^3(a+b\arctan(cx))^2}{e^3(d+ex)}+\frac{d^2(a+b\arctan(cx))^2}{e^3}-\frac{dx(a+b\arctan(cx))^2}{e^2}+\frac{x^2(a+b\arctan(cx))^2}{e}\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{i(a+b\arctan(cx))^2}{3c^3e}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3e}-\frac{d(a+b\arctan(cx))^2}{2c^2e^2}-\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^4}+\frac{ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4}+\frac{d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e^4}-\frac{d^3(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}+\frac{d^2x(a+b\arctan(cx))^2}{e^3}+\frac{id^2(a+b\arctan(cx))^2}{ce^3}+\frac{2bd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{ce^3}-\frac{dx^2(a+b\arctan(cx))^2}{2e^2}+\frac{x^3(a+b\arctan(cx))^2}{3e}-\frac{bx^2(a+b\arctan(cx))}{3ce}+\frac{abdx}{ce^2}-\frac{b^2\arctan(cx)}{3c^3e}+\frac{b^2dx\arctan(cx)}{ce^2}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3e}-\frac{b^2d\log\left(c^2x^2+1\right)}{2c^2e^2}+\frac{b^2x}{3c^2e}+\frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^4}-\frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}+\frac{ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{ce^3}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
(a*b*d*x)/(c*e^2) + (b^2*x)/(3*c^2*e) - (b^2*ArcTan[c*x])/(3*c^3*e) + (b^2
*d*x*ArcTan[c*x])/(c*e^2) - (b*x^2*(a + b*ArcTan[c*x]))/(3*c*e) + (I*d^2*(
a + b*ArcTan[c*x])^2)/(c*e^3) - (d*(a + b*ArcTan[c*x])^2)/(2*c^2*e^2) - ((
I/3)*(a + b*ArcTan[c*x])^2)/(c^3*e) + (d^2*x*(a + b*ArcTan[c*x])^2)/e^3 -
(d*x^2*(a + b*ArcTan[c*x])^2)/(2*e^2) + (x^3*(a + b*ArcTan[c*x])^2)/(3*e)
+ (d^3*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^4 + (2*b*d^2*(a + b*Arc
Tan[c*x])*Log[2/(1 + I*c*x)])/(c*e^3) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1
+ I*c*x)])/(3*c^3*e) - (d^3*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*
d + I*e)*(1 - I*c*x))])/e^4 - (b^2*d*Log[1 + c^2*x^2])/(2*c^2*e^2) - (I*b*
d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + (I*b^2*d^2*Po
lyLog[2, 1 - 2/(1 + I*c*x)])/(c*e^3) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*
c*x)])/(c^3*e) + (I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))])/e^4 + (b^2*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)
])/(2*e^4) - (b^2*d^3*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c
*x))])/(2*e^4)
```

3.141. $\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx$

3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.141.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 38.26 (sec) , antiderivative size = 2062, normalized size of antiderivative = 3.45

method	result	size
derivativedivides	Expression too large to display	2062
default	Expression too large to display	2062
parts	Expression too large to display	2107

input
```
int(x^3*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2*c^4/e^3*d^2*x-1/2*a^2*c^4/e^2*d*x^2+1/3*a^2*c^4/e*x^3-a^2*c^4*d
^3/e^4*ln(c*e*x+c*d)+b^2*c*(1/3/e*(I+c*x)+1/3*arctan(c*x)^2/e*c^3*x^3-I/e^
4*c^3*d^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+d^3*c^3/e^3/(c*d
-I*e)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-d
^4*c^4/e^4/(c*d-I*e)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c
^2*x^2+1))+arctan(c*x)^2/e^3*c^3*d^2*x-1/2*arctan(c*x)^2/e^2*c^3*d*x^2+1/2
*I*d^3*c^3/e^3/(c*d-I*e)*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^
2+1))+1/2*I/e^4*c^3*d^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*
e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I/e^4*c^3*d^3*Pi*csgn(I*(-I*e*(1+I*c
*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*
c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*
x^2+1)+1))^2*arctan(c*x)^2-I/e^3*c^2*d^2*arctan(c*x)^2-2*I/e^3*c^2*d^2*dil
og(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2/3*I/e*dilog(1+I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))-2/3/e*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2/3/e*arcta
n(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/3*arctan(c*x)*(c*x-I)^2/e+2/3
*I/e*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/3*I/e*arctan(c*x)^2+I*d^4*c^
4/e^4/(c*d-I*e)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2
*x^2+1))+I*d^3*c^3/e^3/(c*d-I*e)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1
+I*c*x)^2/(c^2*x^2+1))-1/2*I/e^4*c^3*d^3*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c...
```

3.141.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{(b\arctan(cx)+a)^2x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e*x +
d), x)
```

3.141.6 Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x^3(a+b\operatorname{atan}(cx))^2}{d+ex}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))**2/(d + e*x), x)
```

3.141.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
-1/6*a^2*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)
+ 1/96*(96*e^3*integrate(1/48*(36*(b^2*c^2*e^3*x^5 + b^2*e^3*x^3)*arctan(c
*x)^2 + 3*(b^2*c^2*e^3*x^5 + b^2*e^3*x^3)*log(c^2*x^2 + 1)^2 + 4*(24*a*b*c
^2*e^3*x^5 - 2*b^2*c*e^3*x^4 - 3*b^2*c*d^2*e*x^2 - 6*b^2*c*d^3*x + (b^2*c*
d*e^2 + 24*a*b*e^3)*x^3)*arctan(c*x) + 2*(2*b^2*c^2*e^3*x^5 - b^2*c^2*d*e^
2*x^4 + 3*b^2*c^2*d^2*e*x^3 + 6*b^2*c^2*d^3*x^2)*log(c^2*x^2 + 1))/(c^2*e^
4*x^3 + c^2*d*e^3*x^2 + e^4*x + d*e^3), x) + 4*(2*b^2*e^2*x^3 - 3*b^2*d*e*
x^2 + 6*b^2*d^2*x)*arctan(c*x)^2 - (2*b^2*e^2*x^3 - 3*b^2*d*e*x^2 + 6*b^2*
d^2*x)*log(c^2*x^2 + 1)^2)/e^3
```

3.141.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x),x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x), x)
```

3.142 $\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx$

3.142.1 Optimal result

Integrand size = 21, antiderivative size = 430

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx &= -\frac{abx}{ce}-\frac{b^2x\arctan(cx)}{ce}-\frac{id(a+b\arctan(cx))^2}{ce^2}\\
&+\frac{(a+b\arctan(cx))^2}{2c^2e}-\frac{dx(a+b\arctan(cx))^2}{e^2}\\
&+\frac{x^2(a+b\arctan(cx))^2}{2e}-\frac{d^2(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^3}\\
&-\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce^2}\\
&+\frac{d^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3}+\frac{b^2\log(1+c^2x^2)}{2c^2e}\\
&+\frac{ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^3}\\
&-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce^2}\\
&-\frac{ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3}\\
&-\frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^3}\\
&+\frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}\end{aligned}$$

output
```
-a*b*x/c/e-b^2*x*arctan(c*x)/c/e-I*d*(a+b*arctan(c*x))^2/c/e^2+1/2*(a+b*ar
ctan(c*x))^2/c^2/e-d*x*(a+b*arctan(c*x))^2/e^2+1/2*x^2*(a+b*arctan(c*x))^2
/e-d^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^3-2*b*d*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c/e^2+d^2*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c
*x))/e^3+1/2*b^2*ln(c^2*x^2+1)/c^2/e+I*b*d^2*(a+b*arctan(c*x))*polylog(2,1
-2/(1-I*c*x))/e^3-I*b^2*d*polylog(2,1-2/(1+I*c*x))/c/e^2-I*b*d^2*(a+b*arct
an(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3-1/2*b^2*d^2*poly
log(3,1-2/(1-I*c*x))/e^3+1/2*b^2*d^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-
I*c*x))/e^3
```

3.142.2 Mathematica [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

3.142.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{e^2(d+ex)}-\frac{d(a+b\arctan(cx))^2}{e^2}+\frac{x(a+b\arctan(cx))^2}{e}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\arctan(cx))^2}{2c^2e}+\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^3}-\frac{ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3}-\frac{d^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e^3}+\frac{d^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}-\frac{dx(a+b\arctan(cx))^2}{e^2}-\frac{id(a+b\arctan(cx))^2}{ce^2}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{ce^2}+\frac{x^2(a+b\arctan(cx))^2}{2e}-\frac{abx}{ce}-\frac{b^2x\arctan(cx)}{ce}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2e}-\frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^3}+\frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{ce^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
-((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) - (I*d*(a + b*ArcTan[c*x])^2)
/(c*e^2) + (a + b*ArcTan[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTan[c*x])^2)/e
^2 + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) - (d^2*(a + b*ArcTan[c*x])^2*Log[2/
(1 - I*c*x)])/e^3 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e^2)
 + (d^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x)
)])/e^3 + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) + (I*b*d^2*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e^3 - (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)
])/(c*e^2) - (I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/(
(c*d + I*e)*(1 - I*c*x))])/e^3 - (b^2*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(
2*e^3) + (b^2*d^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))
])/(2*e^3)
```

3.142.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.142.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.05 (sec) , antiderivative size = 1710, normalized size of antiderivative = 3.98

method	result	size
parts	Expression too large to display	1710
derivativedivides	Expression too large to display	1718
default	Expression too large to display	1718

input
```
int(x^2*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2/e*x^2-a^2/e^2*x*d+a^2/e^3*d^2*ln(e*x+d)+b^2/c^3*(1/2*arctan(c*x)^2
/e*c^3*x^2-arctan(c*x)^2/e^2*c^3*d*x+c^3*arctan(c*x)^2*d^2/e^3*ln(c*e*x+c*
d)-2*c*(-1/4/e*arctan(c*x)^2-1/2*I/e^3*c^2*d^2*arctan(c*x)*polylog(2,-(1+I
*c*x)^2/(c^2*x^2+1))-1/4*I/e^3*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*e*(
1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I/((1+I*
c*x)^2/(c^2*x^2+1)+1))*Pi*c^2*d^2*arctan(c*x)^2-1/2*I/e^2*c*d*arctan(c*x)^
2+1/4*I/e^3*c^2*d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*e*(1
+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*arctan(c*x)^2-I/e^2*c*d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))+1/4*I/e^3*c^2*d^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*
x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c
*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/
2/e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/2*arctan(c*x)*(c*x-I)/e-1/4*I/e^3*Pi*c
^2*d^2*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*
e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/e^2*c*d*arctan(c*x)*
ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/e^2*c*d*arctan(c*x)*ln(1-I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))+1/2*I*d^3*c^3/e^3/(c*d-I*e)*arctan(c*x)*polylog(2,(I*e
-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/4/e^3*c^2*d^2*polylog(3,-(1+I*c
*x)^2/(c^2*x^2+1))+1/2*d^2*c^2/e^3*arctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c...
```

3.142.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e*x +
d), x)
```

3.142.6 Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{x^2(a+b\operatorname{atan}(cx))^2}{d+ex}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2/(d + e*x), x)
```

3.142.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/32*(4*(b^2*e*x^
2 - 2*b^2*d*x)*arctan(c*x)^2 + 32*e^2*integrate(1/16*(12*(b^2*c^2*e^2*x^4
+ b^2*e^2*x^2)*arctan(c*x)^2 + (b^2*c^2*e^2*x^4 + b^2*e^2*x^2)*log(c^2*x^2
 + 1)^2 + 4*(8*a*b*c^2*e^2*x^4 - b^2*c*e^2*x^3 + 2*b^2*c*d^2*x + (b^2*c*d*
e + 8*a*b*e^2)*x^2)*arctan(c*x) + 2*(b^2*c^2*e^2*x^4 - b^2*c^2*d*e*x^3 - 2
*b^2*c^2*d^2*x^2)*log(c^2*x^2 + 1))/(c^2*e^3*x^3 + c^2*d*e^2*x^2 + e^3*x +
 d*e^2), x) - (b^2*e*x^2 - 2*b^2*d*x)*log(c^2*x^2 + 1)^2)/e^2
```

3.142.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x),x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x), x)
```

3.143 $\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$

3.143.1 Optimal result

Integrand size = 19, antiderivative size = 323

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx =& \frac{i(a+b\arctan(cx))^2}{ce}+\frac{x(a+b\arctan(cx))^2}{e}\\
&+\frac{d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2}\\
&+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce}\\
&-\frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}\\
&-\frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^2}\\
&+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce}\\
&+\frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}\\
&+\frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^2}\\
&-\frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}\end{aligned}$$

output
```
I*(a+b*arctan(c*x))^2/c/e+x*(a+b*arctan(c*x))^2/e+d*(a+b*arctan(c*x))^2*ln
(2/(1-I*c*x))/e^2+2*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/e-d*(a+b*arctan(
c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2-I*b*d*(a+b*arctan(c*x))*po
lylog(2,1-2/(1-I*c*x))/e^2+I*b^2*polylog(2,1-2/(1+I*c*x))/c/e+I*b*d*(a+b*a
rctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2+1/2*b^2*d*pol
ylog(3,1-2/(1-I*c*x))/e^2-1/2*b^2*d*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I
*c*x))/e^2
```

3.143.2 Mathematica [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

3.143.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{e}-\frac{d(a+b\arctan(cx))^2}{e(d+ex)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$
-\frac{ibd\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^2}+\frac{ibd(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}+\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e^2}-\frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{x(a+b\arctan(cx))^2}{e}+\frac{i(a+b\arctan(cx))^2}{ce}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{ce}+\frac{b^2d\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^2}-\frac{b^2d\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}+\frac{ib^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{ce}
$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (d*(a + b*
ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 + (2*b*(a + b*ArcTan[c*x])*Log[2/(1
 + I*c*x)])/(c*e) - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^2 - (I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
I*c*x)])/e^2 + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e) + (I*b*d*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
^2 + (b^2*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 -
 (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^2)
```

3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.143. $\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$

3.143.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.86 (sec) , antiderivative size = 15752, normalized size of antiderivative = 48.77

method	result	size
derivativedivides	Expression too large to display	15752
default	Expression too large to display	15752
parts	Expression too large to display	15757

input
```
int(x*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.143.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{(b\arctan(cx)+a)^2x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e*x + d), x)
```

3.143.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{x(a+b\,\mathrm{atan}\,(cx))^2}{d+ex}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))**2/(d + e*x), x)
```

3.143.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*(x/e - d*log(e*x + d)/e^2) + 1/16*(4*b^2*x*arctan(c*x)^2 - b^2*x*log(c
^2*x^2 + 1)^2 + 16*e*integrate(1/16*(12*(b^2*c^2*e*x^3 + b^2*e*x)*arctan(c
*x)^2 + (b^2*c^2*e*x^3 + b^2*e*x)*log(c^2*x^2 + 1)^2 + 8*(4*a*b*c^2*e*x^3
- b^2*c*e*x^2 - (b^2*c*d - 4*a*b*e)*x)*arctan(c*x) + 4*(b^2*c^2*e*x^3 + b^
2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e),
x))/e
```

3.143.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + e*x),x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + e*x), x)
```

3.144 $\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx$

3.144.1 Optimal result

Integrand size = 18, antiderivative size = 223

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = -\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}$$

output
```
-(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/
(c*d+I*e)/(1-I*c*x))/e+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e-I*
b*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e-1/2*b^2
*polylog(3,1-2/(1-I*c*x))/e+1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I
*c*x))/e
```

3.144.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x), x]
```

3.144.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5383}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \text{5383}$$

$$-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

output
```
-(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*L
og[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(
1 - I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
 I*c*x))])/(2*e)
```

3.144.3.1 Defintions of rubi rules used

rule 5383
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

3.144.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 1199, normalized size of antiderivative = 5.38

method	result	size
derivativedivides	Expression too large to display	1199
default	Expression too large to display	1199
parts	Expression too large to display	1202

input
```
int((a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*ln(c*e*x+c*d)/e+b^2*c*(ln(c*e*x+c*d)/e*arctan(c*x)^2-2/e*(1/2*a
rctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I
*e+c*d)-1/4*I*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2
*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))*(csgn(I*(-I*e*(1+I*c*x)^2/(c
^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I/((1+I*c*x)^2/(c^2*x
^2+1)+1))-csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)
+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))
-csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)
)*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d
)/((1+I*c*x)^2/(c^2*x^2+1)+1))+csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1
+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2)*arctan(c*x)
^2-1/2*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*polylog(3,-(1
+I*c*x)^2/(c^2*x^2+1))+1/2*I*c*d/(c*d-I*e)*arctan(c*x)*polylog(2,(I*e-c*d)
/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/2*c*d/(c*d-I*e)*arctan(c*x)^2*ln(1-(
I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/4*c*d/(c*d-I*e)*polylog(3,(I
*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*e*arctan(c*x)*polylog(2,(
I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(I*c*d+e)-1/2*e*arctan(c*x)^2*
ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(I*c*d+e)-1/4*e*polylog(
3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(I*c*d+e)))+2*a*b*c*(ln(c*e
*x+c*d)/e*arctan(c*x)+1/2*I*ln(c*e*x+c*d)*(ln((I*e-e*c*x)/(c*d+I*e))-ln...
```

3.144.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x + d), x)
```

3.144.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2}{d+ex}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(d + e*x), x)
```

3.144.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*log(e*x + d)/e + integrate(1/16*(12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^
2 + 1)^2 + 32*a*b*arctan(c*x))/(e*x + d), x)
```

3.144.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{d+e\,x}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + e*x),x)
```

output
```
int((a + b*atan(c*x))^2/(d + e*x), x)
```

3.145 $\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx$

3.145.1 Optimal result

Integrand size = 21, antiderivative size = 369

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx &= \frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d}\\
&+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d}\\
&-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}\\
&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d}\\
&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d}\\
&+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d}\\
&+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}\\
&+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d}\\
&+\frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}\end{aligned}$$

output
```
-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d+(a+b*arctan(c*x))^2*ln(2/
(1-I*c*x))/d-(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d-I*b
*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d-I*b*(a+b*arctan(c*x))*polylo
g(2,1-2/(1+I*c*x))/d+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d+I*b
*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d+1/2*b^2*
polylog(3,1-2/(1-I*c*x))/d-1/2*b^2*polylog(3,1-2/(1+I*c*x))/d+1/2*b^2*poly
log(3,-1+2/(1+I*c*x))/d-1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x
))/d
```

3.145.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx=\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)), x]
```

3.145.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{(a+b\arctan(cx))^2}{dx}-\frac{e(a+b\arctan(cx))^2}{d(d+ex)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}+\frac{ib(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d}+\frac{ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d}+\frac{b^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

output
```
(2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/(
(c*d + I*e)*(1 - I*c*x))])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(
1 - I*c*x)])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d
 + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (I*b*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
d + (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 +
 I*c*x)])/(2*d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLo
g[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*d)
```

3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.145.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.36 (sec) , antiderivative size = 2337, normalized size of antiderivative = 6.33

method	result	size
parts	Expression too large to display	2337
derivativedivides	Expression too large to display	2345
default	Expression too large to display	2345

input
```
int((a+b*arctan(c*x))^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
I*b^2*c/(c*d-I*e)*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c
^2*x^2+1))+I*b^2*e*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(
c^2*x^2+1))/d/(I*c*d+e)-1/2*I*b^2/d*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1
+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*Pi*arctan(c*x)^2+1/2*I*b^2/d*csgn(I*(-I*e*(1+I*c*x)^2/(c^2
*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^
2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^2*Pi*arctan(c*x)^2-1/2*I*b^2/d*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*Pi*arctan(c*x)
^2+1/2*I*b^2/d*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*e*(1+I*c*x)^
2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1
)+1))^2*Pi*arctan(c*x)^2+1/2*I*b^2/d*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(
c^2*x^2+1)+1))*Pi*arctan(c*x)^2-1/2*b^2*c/(c*d-I*e)*polylog(3,(I*e-c*d)/(c
*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-b^2/d*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x
^2+1)-1)+b^2/d*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2/d*arcta
n(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*arctan(c*x)^2/d*ln(c*x)-b^2
*arctan(c*x)^2/d*ln(c*e*x+c*d)+b^2*arctan(c*x)^2/d*ln(-I*e*(1+I*c*x)^2/(c^
2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)+a^2/d*ln(x)-a^2/d*ln(e*x+d)+
1/2*I*b^2/d*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)...
```

3.145.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^2 + d*x), x)
```

3.145.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx=\int \frac{(a+b\operatorname{atan}(cx))^2}{x(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x*(d + e*x)), x)
```

3.145.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

output
```
-a^2*(log(e*x + d)/d - log(x)/d) + integrate(1/16*(12*b^2*arctan(c*x)^2 +
b^2*log(c^2*x^2 + 1)^2 + 32*a*b*arctan(c*x))/(e*x^2 + d*x), x)
```

3.145.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + e*x)), x)
```

3.146 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx$

3.146.1 Optimal result

Integrand size = 21, antiderivative size = 473

$$\begin{aligned}
\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx &= -\frac{ic(a+b\arctan(cx))^2}{d}-\frac{(a+b\arctan(cx))^2}{dx} \\
&\quad -\frac{2e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2} \\
&\quad -\frac{e(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^2} \\
&\quad +\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2} \\
&\quad +\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d} \\
&\quad +\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^2} \\
&\quad -\frac{ib^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d} \\
&\quad +\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2} \\
&\quad -\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2} \\
&\quad -\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2} \\
&\quad -\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^2}+\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d^2} \\
&\quad -\frac{b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2} \\
&\quad +\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}
\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*e*(a+b*arctan(c*x))^2
*arctanh(-1+2/(1+I*c*x))/d^2-e*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2+e*(
a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2+2*b*c*(a+b*arct
an(c*x))*ln(2-2/(1-I*c*x))/d+I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*
x))/d^2-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d+I*b*e*(a+b*arctan(c*x))*polylo
g(2,1-2/(1+I*c*x))/d^2-I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d
^2-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^
2-1/2*b^2*e*polylog(3,1-2/(1-I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2/(1+I*c*x)
)/d^2-1/2*b^2*e*polylog(3,-1+2/(1+I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2*c*(e
*x+d)/(c*d+I*e)/(1-I*c*x))/d^2
```

3.146.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)), x]
```

3.146.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{e^2(a+b\arctan(cx))^2}{d^2(d+ex)} - \frac{e(a+b\arctan(cx))^2}{d^2x} + \frac{(a+b\arctan(cx))^2}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{2e\,\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{ibe\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}+\frac{ibe\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d^2}-\frac{ibe\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}-\frac{ibe(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2}-\frac{e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^2}-\frac{ic(a+b\arctan(cx))^2}{d}-\frac{(a+b\arctan(cx))^2}{dx}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}-\frac{b^2e\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^2}+\frac{b^2e\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d^2}-\frac{b^2e\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^2}+\frac{b^2e\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}-\frac{ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d}
$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```

output
```
((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) - (2*e*(a +
 b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - (e*(a + b*ArcTan[c*x])
^2*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/
((c*d + I*e)*(1 - I*c*x))])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1
- I*c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
^2 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (I*b*e*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog
[2, -1 + 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (
2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(
1 - I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^
2*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(
d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*d^2)
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.146.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.14 (sec) , antiderivative size = 38040, normalized size of antiderivative = 80.42

method	result	size
parts	Expression too large to display	38040
derivativedivides	Expression too large to display	38050
default	Expression too large to display	38050

input
```
int((a+b*arctan(c*x))^2/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.146.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^3 + d*x^2), x)
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(e*x+d),x)
```

output
```
Timed out
```

3.146.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/16*(4*b^2*arctan(c*x
)^2 - b^2*log(c^2*x^2 + 1)^2 - 16*d*x*integrate(1/16*(12*(b^2*c^2*d*x^2 +
b^2*d)*arctan(c*x)^2 + (b^2*c^2*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 + 8*(b^2
*c*d*x + 4*a*b*d + (4*a*b*c^2*d + b^2*c*e)*x^2)*arctan(c*x) - 4*(b^2*c^2*e
*x^3 + b^2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*d*e*x^5 + c^2*d^2*x^4 + d*e*x
^3 + d^2*x^2), x))/(d*x)
```

3.146.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x)), x)
```

3.147 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx$

3.147.1 Optimal result

Integrand size = 21, antiderivative size = 591

$$\begin{aligned}
\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = & -\frac{bc(a+b\arctan(cx))}{dx}-\frac{c^2(a+b\arctan(cx))^2}{2d} \\
& +\frac{ice(a+b\arctan(cx))^2}{d^2} \\
& -\frac{(a+b\arctan(cx))^2}{2dx^2}+\frac{e(a+b\arctan(cx))^2}{d^2x} \\
& +\frac{2e^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^3} \\
& +\frac{b^2c^2\log(x)}{d}+\frac{e^2(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^3} \\
& -\frac{e^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3} \\
& -\frac{b^2c^2\log\left(1+c^2x^2\right)}{2d}-\frac{2bce(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^2} \\
& -\frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3} \\
& +\frac{ib^2ce\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2} \\
& -\frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^3} \\
& +\frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^3} \\
& +\frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3} \\
& +\frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^3}-\frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d^3} \\
& +\frac{b^2e^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3} \\
& -\frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}
\end{aligned}$$

output
```
-b*c*(a+b*arctan(c*x))/d/x-1/2*c^2*(a+b*arctan(c*x))^2/d+I*b^2*c*e*polylog
(2,-1+2/(1-I*c*x))/d^2-1/2*(a+b*arctan(c*x))^2/d/x^2+e*(a+b*arctan(c*x))^2
/d^2/x-2*e^2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3+b^2*c^2*ln(x)
/d+e^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^3-e^2*(a+b*arctan(c*x))^2*ln(
2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3-1/2*b^2*c^2*ln(c^2*x^2+1)/d-2*b*c*e*(
a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^2+I*c*e*(a+b*arctan(c*x))^2/d^2-I*b*e
^2*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d^3+I*b*e^2*(a+b*arctan(c*x)
)*polylog(2,-1+2/(1+I*c*x))/d^3-I*b*e^2*(a+b*arctan(c*x))*polylog(2,1-2/(1
+I*c*x))/d^3+I*b*e^2*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(
1-I*c*x))/d^3+1/2*b^2*e^2*polylog(3,1-2/(1-I*c*x))/d^3-1/2*b^2*e^2*polylog
(3,1-2/(1+I*c*x))/d^3+1/2*b^2*e^2*polylog(3,-1+2/(1+I*c*x))/d^3-1/2*b^2*e^
2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3
```

3.147.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

output
```
$Aborted
```

3.147.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{e^3(a+b\arctan(cx))^2}{d^3(d+ex)}+\frac{e^2(a+b\arctan(cx))^2}{d^3x}-\frac{e(a+b\arctan(cx))^2}{d^2x^2}+\frac{(a+b\arctan(cx))^2}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}-\frac{c^2(a+b\arctan(cx))^2}{2d}-\frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}-\frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d^3}+\frac{ibe^2\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^3}+\frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3}+\frac{e^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d^3}-\frac{e^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^3}+\frac{ice(a+b\arctan(cx))^2}{d^2}+\frac{e(a+b\arctan(cx))^2}{d^2x}-\frac{2bce\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}-\frac{(a+b\arctan(cx))^2}{2dx^2}-\frac{bc(a+b\arctan(cx))}{dx}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d}+\frac{b^2c^2\log(x)}{d}+\frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^3}-\frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d^3}+\frac{b^2e^2\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^3}-\frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}+\frac{ib^2ce\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

output
```
-((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) + (
I*c*e*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d*x^2) + (e*(a
 + b*ArcTan[c*x])^2)/(d^2*x) + (2*e^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/
(1 + I*c*x)])/d^3 + (b^2*c^2*Log[x])/d + (e^2*(a + b*ArcTan[c*x])^2*Log[2/
(1 - I*c*x)])/d^3 - (e^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d +
 I*e)*(1 - I*c*x))])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - (2*b*c*e*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - (I*b*e^2*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + (I*b^2*c*e*PolyLog[2, -1 + 2/(1 - I
*c*x)])/d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/
d^3 + (I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (
I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
 - I*c*x))])/d^3 + (b^2*e^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^3) - (b^2*
e^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^3) + (b^2*e^2*PolyLog[3, -1 + 2/(1
 + I*c*x)])/(2*d^3) - (b^2*e^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/(2*d^3)
```

3.147. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

3.147.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 53.89 (sec) , antiderivative size = 2804, normalized size of antiderivative = 4.74

method	result	size
parts	Expression too large to display	2804
derivativedivides	Expression too large to display	2853
default	Expression too large to display	2853

input
```
int((a+b*arctan(c*x))^2/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b^2/d^3*e^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*e*(1+
I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1
+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*arctan(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*
x^2+1)+1))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1
)+I*e+c*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2/d^3*e^2*
Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*
d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c
*d)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*b^2/d^3*e^2*Pi*csgn
(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2/d^3*e^2*Pi*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*b^2*c*e^2*arctan(c*x)^2*ln(1-(I*
e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(I*c*d+e)/d^2-1/2*I*b^2/d^3*e^2*
Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+a^2
*(-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x-e^2/d^3*ln(e*x+d))+1/2*I*b^2/d^3*e^2*Pi
*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)...
```

3.147.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^4 + d*x^3), x)
```

3.147.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{x^3\,(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x**3*(d + e*x)), x)
```

3.147.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2
)) + 1/32*(32*d^2*x^2*integrate(1/16*(12*(b^2*c^2*d^2*x^2 + b^2*d^2)*arcta
n(c*x)^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*log(c^2*x^2 + 1)^2 - 4*(2*b^2*c*e^2
*x^3 - b^2*c*d^2*x - 8*a*b*d^2 - (8*a*b*c^2*d^2 - b^2*c*d*e)*x^2)*arctan(c
*x) + 2*(2*b^2*c^2*e^2*x^4 + b^2*c^2*d*e*x^3 - b^2*c^2*d^2*x^2)*log(c^2*x^
2 + 1))/(c^2*d^2*e*x^6 + c^2*d^3*x^5 + d^2*e*x^4 + d^3*x^3), x) + 4*(2*b^2
*e*x - b^2*d)*arctan(c*x)^2 - (2*b^2*e*x - b^2*d)*log(c^2*x^2 + 1)^2)/(d^2
*x^2)
```

3.147.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^3\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x)), x)
```

3.148 $\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx$

3.148.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \mathrm{Int}\left(\frac{1}{(d+ex)(a+b\arctan(cx))},x\right)$$

output
```
Unintegrable(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

3.148.2 Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx$$

input
```
Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

output
```
Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

3.148.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}dx$$

input
```
Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

output
```
$Aborted
```

3.148.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.148.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\,(a+b\arctan(cx))}dx$$

input
```
int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

output
```
int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

3.148.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x + a*d + (b*e*x + b*d)*arctan(c*x)), x)
```

3.148.6 Sympy [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{atan}(cx))(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
```

output
```
Integral(1/((a + b*atan(c*x))*(d + e*x)), x)
```

3.148.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

3.148.8 Giac [N/A]

Not integrable

Time = 54.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx=\int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.148.9 Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx=\int \frac{1}{(a+b\,\mathrm{atan}\,(c\,x))\,(d+e\,x)}\,dx$$

input
```
int(1/((a + b*atan(c*x))*(d + e*x)),x)
```

output
```
int(1/((a + b*atan(c*x))*(d + e*x)), x)
```

3.149 $\int x^3(c+a^2cx^2)\arctan(ax)\,dx$

3.149.1 Optimal result

Integrand size = 18, antiderivative size = 69

$$\int x^3\left(c+a^2cx^2\right)\arctan(ax)\,dx=\frac{cx}{12a^3}-\frac{cx^3}{36a}-\frac{1}{30}acx^5-\frac{c\arctan(ax)}{12a^4}+\frac{1}{4}cx^4\arctan(ax)+\frac{1}{6}a^2cx^6\arctan(ax)$$

output
```
1/12*c*x/a^3-1/36*c*x^3/a-1/30*a*c*x^5-1/12*c*arctan(a*x)/a^4+1/4*c*x^4*ar
ctan(a*x)+1/6*a^2*c*x^6*arctan(a*x)
```

3.149.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^3\left(c+a^2cx^2\right)\arctan(ax)\,dx=\frac{cx}{12a^3}-\frac{cx^3}{36a}-\frac{1}{30}acx^5-\frac{c\arctan(ax)}{12a^4}+\frac{1}{4}cx^4\arctan(ax)+\frac{1}{6}a^2cx^6\arctan(ax)$$

input
```
Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
(c*x)/(12*a^3) - (c*x^3)/(36*a) - (a*c*x^5)/30 - (c*ArcTan[a*x])/(12*a^4)
+ (c*x^4*ArcTan[a*x])/4 + (a^2*c*x^6*ArcTan[a*x])/6
```

3.149.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5485, 5361, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax) \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5 \arctan(ax)dx+c\int x^3 \arctan(ax)dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{6}x^6 \arctan(ax)-\frac{1}{6}a\int \frac{x^6}{a^2x^2+1}dx\right)+c\left(\frac{1}{4}x^4 \arctan(ax)-\frac{1}{4}a\int \frac{x^4}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{254}$$

$$c\left(\frac{1}{4}x^4 \arctan(ax)-\frac{1}{4}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx\right)+ a^2c\left(\frac{1}{6}x^6 \arctan(ax)-\frac{1}{6}a\int \left(\frac{x^4}{a^2}-\frac{x^2}{a^4}-\frac{1}{a^6\left(a^2x^2+1\right)}+\frac{1}{a^6}\right)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{1}{4}x^4 \arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)\right)+ a^2c\left(\frac{1}{6}x^6 \arctan(ax)-\frac{1}{6}a\left(-\frac{\arctan(ax)}{a^7}+\frac{x}{a^6}-\frac{x^3}{3a^4}+\frac{x^5}{5a^2}\right)\right)$$

input
```
Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
a^2*c*((x^6*ArcTan[a*x])/6 - (a*(x/a^6 - x^3/(3*a^4) + x^5/(5*a^2) - ArcTa
n[a*x]/a^7))/6) + c*((x^4*ArcTan[a*x])/4 - (a*(-(x/a^4) + x^3/(3*a^2) + Ar
cTan[a*x]/a^5))/4)
```

3.149.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.149.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)a^6x^6}{6}+\frac{c\arctan(ax)a^4x^4}{4}-\frac{c\left(\frac{2a^5x^5}{5}+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{12}}{a^4}$
default	$\frac{\frac{c\arctan(ax)a^6x^6}{6}+\frac{c\arctan(ax)a^4x^4}{4}-\frac{c\left(\frac{2a^5x^5}{5}+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{12}}{a^4}$
parts	$\frac{a^2c\,x^6\arctan(ax)}{6}+\frac{c\,x^4\arctan(ax)}{4}-\frac{ca\left(\frac{\frac{2}{5}a^4x^5+\frac{1}{3}a^2x^3-x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{12}$
parallelrisch	$\frac{30c\arctan(ax)a^6x^6-6a^5c\,x^5+45c\arctan(ax)a^4x^4-5a^3c\,x^3+15acx-15c\arctan(ax)}{180a^4}$
risch	$-\frac{ic\,x^4\left(2a^2x^2+3\right)\ln(iax+1)}{24}+\frac{ic\,a^2x^6\ln(-iax+1)}{12}-\frac{ac\,x^5}{30}+\frac{ic\,x^4\ln(-iax+1)}{8}-\frac{c\,x^3}{36a}+\frac{cx}{12a^3}-\frac{c\arctan(ax}{12a^4}$
meijerg	$\frac{c\left(-\frac{2xa\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2xa\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4a^4}+\frac{c\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right.}{4a^4}$

input
```
int(x^3*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/6*c*arctan(a*x)*a^6*x^6+1/4*c*arctan(a*x)*a^4*x^4-1/12*c*(2/5*a^5
*x^5+1/3*a^3*x^3-a*x+arctan(a*x)))
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = -\frac{6\,a^5cx^5+5\,a^3cx^3-15\,acx-15\,(2\,a^6cx^6+3\,a^4cx^4-c)\arctan{(ax)}}{180\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/180*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x - 15*(2*a^6*c*x^6 + 3*a^4*c*x
^4 - c)*arctan(a*x))/a^4
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = \begin{cases} \frac{a^2cx^6\operatorname{atan}{(ax)}}{6}-\frac{acx^5}{30}+\frac{cx^4\operatorname{atan}{(ax)}}{4}-\frac{cx^3}{36a}+\frac{cx}{12a^3}-\frac{c\operatorname{atan}{(ax)}}{12a^4} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**6*atan(a*x)/6 - a*c*x**5/30 + c*x**4*atan(a*x)/4 - c*
x**3/(36*a) + c*x/(12*a**3) - c*atan(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = -\frac{1}{180}\,a\left(\frac{6\,a^4cx^5+5\,a^2cx^3-15\,cx}{a^4}+\frac{15\,c\arctan{(ax)}}{a^5}\right) + \frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan{(ax)}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/180*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)
 + 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)
```

3.149.8 Giac [F]

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = \int\left(a^2cx^2+c\right)x^3\arctan{(ax)}\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = -\frac{c\left(15\,\mathrm{atan}(a\,x)-15\,a\,x+5\,a^3\,x^3+6\,a^5\,x^5-45\,a^4\,x^4\,\mathrm{atan}(a\,x)-30\,a^6\,x^6\,\mathrm{atan}(a\,x)\right)}{180\,a^4}$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
-(c*(15*atan(a*x) - 15*a*x + 5*a^3*x^3 + 6*a^5*x^5 - 45*a^4*x^4*atan(a*x)
- 30*a^6*x^6*atan(a*x)))/(180*a^4)
```

3.150 $\int x^2(c+a^2cx^2)\arctan(ax)\,dx$

3.150.1 Optimal result

Integrand size = 18, antiderivative size = 66

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = -\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\arctan(ax)+\frac{1}{5}a^2cx^5\arctan(ax)+\frac{c\log(1+a^2x^2)}{15a^3}$$

output
```
-1/15*c*x^2/a-1/20*a*c*x^4+1/3*c*x^3*arctan(a*x)+1/5*a^2*c*x^5*arctan(a*x)
+1/15*c*ln(a^2*x^2+1)/a^3
```

3.150.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = -\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\arctan(ax)+\frac{1}{5}a^2cx^5\arctan(ax)+\frac{c\log(1+a^2x^2)}{15a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/15*(c*x^2)/a - (a*c*x^4)/20 + (c*x^3*ArcTan[a*x])/3 + (a^2*c*x^5*ArcTan
[a*x])/5 + (c*Log[1 + a^2*x^2])/(15*a^3)
```

3.150.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5485, 5361, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^4\arctan(ax)dx+c\int x^2\arctan(ax)dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{5}a\int \frac{x^5}{a^2x^2+1}dx\right)+c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int \frac{x^3}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{243}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int \frac{x^2}{a^2x^2+1}dx^2\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int \frac{x^4}{a^2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{49}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int \left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right)dx^2\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)\right)$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
a^2*c*((x^5*ArcTan[a*x])/5 - (a*(-(x^2/a^4) + x^4/(2*a^2) + Log[1 + a^2*x^
2]/a^6))/10) + c*((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4
))/6)
```

3.150.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.150.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)a^5x^5}{5}+\frac{c\arctan(ax)a^3x^3}{3}-\frac{c\left(\frac{3a^4x^4}{4}+a^2x^2-\ln\left(a^2x^2+1\right)\right)}{15}}{a^3}$
default	$\frac{\frac{c\arctan(ax)a^5x^5}{5}+\frac{c\arctan(ax)a^3x^3}{3}-\frac{c\left(\frac{3a^4x^4}{4}+a^2x^2-\ln\left(a^2x^2+1\right)\right)}{15}}{a^3}$
parallelrisch	$\frac{12c\arctan(ax)a^5x^5-3a^4c\,x^4+20c\arctan(ax)a^3x^3-4a^2c\,x^2+4c\ln(a^2x^2+1)}{60a^3}$
parts	$\frac{a^2c\,x^5\arctan(ax)}{5}+\frac{c\,x^3\arctan(ax)}{3}-\frac{ca\left(\frac{\frac{3}{2}a^2x^4+2x^2}{2a^2}-\frac{\ln\left(a^2x^2+1\right)}{a^4}\right)}{15}$
risch	$-\frac{ic\,x^3(3a^2x^2+5)\ln(iax+1)}{30}+\frac{ic\,a^2x^5\ln(-iax+1)}{10}-\frac{ac\,x^4}{20}+\frac{ic\,x^3\ln(-iax+1)}{6}-\frac{c\,x^2}{15a}+\frac{c\ln(-a^2x^2-1)}{15a^3}-$
meijerg	$\frac{c\left(\frac{a^2x^2\left(-3a^2x^2+6\right)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}-\frac{2\ln\left(a^2x^2+1\right)}{5}\right)}{4a^3}+\frac{c\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{3}\right)}{4a^3}$

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*c*arctan(a*x)*a^5*x^5+1/3*c*arctan(a*x)*a^3*x^3-1/15*c*(3/4*a^4
*x^4+a^2*x^2-ln(a^2*x^2+1)))
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = -\frac{3\,a^4cx^4+4\,a^2cx^2-4\,(3\,a^5cx^5+5\,a^3cx^3)\arctan(ax)-4\,c\log(a^2x^2+1)}{60\,a^3}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/60*(3*a^4*c*x^4 + 4*a^2*c*x^2 - 4*(3*a^5*c*x^5 + 5*a^3*c*x^3)*arctan(a*
x) - 4*c*log(a^2*x^2 + 1))/a^3
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx$$
$$=\begin{cases} \frac{a^2cx^5\operatorname{atan}(ax)}{5}-\frac{acx^4}{20}+\frac{cx^3\operatorname{atan}(ax)}{3}-\frac{cx^2}{15a}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{15a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**5*atan(a*x)/5 - a*c*x**4/20 + c*x**3*atan(a*x)/3 - c*
x**2/(15*a) + c*log(x**2 + a**(-2))/(15*a**3), Ne(a, 0)), (0, True))
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx=-\frac{1}{60}a\left(\frac{3a^2cx^4+4cx^2}{a^2}-\frac{4c\log(a^2x^2+1)}{a^4}\right)+\frac{1}{15}\left(3a^2cx^5+5cx^3\right)\arctan(ax)$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/60*a*((3*a^2*c*x^4 + 4*c*x^2)/a^2 - 4*c*log(a^2*x^2 + 1)/a^4) + 1/15*(3
*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)
```

3.150.8 Giac [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx=\int(a^2cx^2+c)x^2\arctan(ax)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = \frac{\frac{c\ln(a^2x^2+1)}{15}-\frac{a^2cx^2}{15}}{a^3}+\frac{c\,x^3\,\mathrm{atan}(a\,x)}{3} - \frac{a\,c\,x^4}{20}+\frac{a^2\,c\,x^5\,\mathrm{atan}(a\,x)}{5}$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
((c*log(a^2*x^2 + 1))/15 - (a^2*c*x^2)/15)/a^3 + (c*x^3*atan(a*x))/3 - (a*
c*x^4)/20 + (a^2*c*x^5*atan(a*x))/5
```

3.151 $\int x(c+a^2cx^2)\arctan(ax)\,dx$

3.151.1 Optimal result

Integrand size = 16, antiderivative size = 42

$$\int x(c+a^2cx^2)\arctan(ax)\,dx=-\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c(1+a^2x^2)^2\arctan(ax)}{4a^2}$$

output
```
-1/4*c*x/a-1/12*a*c*x^3+1/4*c*(a^2*x^2+1)^2*arctan(a*x)/a^2
```

3.151.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x(c+a^2cx^2)\arctan(ax)\,dx=-\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c\arctan(ax)}{4a^2}+\frac{1}{2}cx^2\arctan(ax)+\frac{1}{4}a^2cx^4\arctan(ax)$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/4*(c*x)/a - (a*c*x^3)/12 + (c*ArcTan[a*x])/(4*a^2) + (c*x^2*ArcTan[a*x]
)/2 + (a^2*c*x^4*ArcTan[a*x])/4
```

3.151.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)}{4a^2}-\frac{\int \left(a^2cx^2+c\right)dx}{4a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)}{4a^2}-\frac{\frac{1}{3}a^2cx^3+cx}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/4*(c*x + (a^2*c*x^3)/3)/a + (c*(1 + a^2*x^2)^2*ArcTan[a*x])/(4*a^2)
```

3.151.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.151.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

method	result
parts	$\frac{c\arctan(ax)a^2x^4}{4}+\frac{c\arctan(ax)x^2}{2}+\frac{c\arctan(ax)}{4a^2}-\frac{c\left(\frac{1}{3}a^2x^3+x\right)}{4a}$
derivativedivides	$\frac{\frac{c\arctan(ax)a^4x^4}{4}+\frac{a^2c\,x^2\arctan(ax)}{2}+\frac{c\arctan(ax)}{4}-\frac{c\left(\frac{1}{3}a^3x^3+ax\right)}{4}}{a^2}$
default	$\frac{\frac{c\arctan(ax)a^4x^4}{4}+\frac{a^2c\,x^2\arctan(ax)}{2}+\frac{c\arctan(ax)}{4}-\frac{c\left(\frac{1}{3}a^3x^3+ax\right)}{4}}{a^2}$
parallelrisch	$\frac{3c\arctan(ax)a^4x^4-a^3c\,x^3+6a^2c\,x^2\arctan(ax)-3acx+3c\arctan(ax)}{12a^2}$
meijerg	$\frac{c\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4a^2}+\frac{c\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4a^2}$
risch	$-\frac{ic\left(a^2x^2+1\right)^2\ln(iax+1)}{8a^2}+\frac{ic\,a^2x^4\ln(-iax+1)}{8}-\frac{ac\,x^3}{12}+\frac{ic\,x^2\ln(-iax+1)}{4}-\frac{cx}{4a}+\frac{ic\ln\left(a^2x^2+1\right)}{16a^2}+\frac{c\,\mathrm{arcta}}{8a}$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*arctan(a*x)*a^2*x^4+1/2*c*arctan(a*x)*x^2+1/4*c/a^2*arctan(a*x)-1/4*
c/a*(1/3*a^2*x^3+x)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int x\left(c+a^2cx^2\right)\arctan(ax)\,dx=-\frac{a^3cx^3+3\,acx-3\left(a^4cx^4+2\,a^2cx^2+c\right)\arctan\left(ax\right)}{12\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/12*(a^3*c*x^3 + 3*a*c*x - 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))/
a^2
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int x(c+a^2cx^2)\arctan(ax)\,dx$$
$$=\begin{cases} \frac{a^2cx^4\operatorname{atan}(ax)}{4}-\frac{acx^3}{12}+\frac{cx^2\operatorname{atan}(ax)}{2}-\frac{cx}{4a}+\frac{c\operatorname{atan}(ax)}{4a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**4*atan(a*x)/4 - a*c*x**3/12 + c*x**2*atan(a*x)/2 - c*
x/(4*a) + c*atan(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int x(c+a^2cx^2)\arctan(ax)\,dx=\frac{(a^2cx^2+c)^2\arctan(ax)}{4\,a^2c}-\frac{a^2c^2x^3+3\,c^2x}{12\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
1/4*(a^2*c*x^2 + c)^2*arctan(a*x)/(a^2*c) - 1/12*(a^2*c^2*x^3 + 3*c^2*x)/(
a*c)
```

3.151.8 Giac [F]

$$\int x(c+a^2cx^2)\arctan(ax)\,dx=\int(a^2cx^2+c)x\arctan(ax)\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = \frac{\frac{c\,\mathrm{atan}(a\,x)}{4}-\frac{a\,c\,x}{4}}{a^2}+\frac{c\,x^2\,\mathrm{atan}(a\,x)}{2}-\frac{a\,c\,x^3}{12}+\frac{a^2\,c\,x^4\,\mathrm{atan}(a\,x)}{4}$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
((c*atan(a*x))/4 - (a*c*x)/4)/a^2 + (c*x^2*atan(a*x))/2 - (a*c*x^3)/12 + (
a^2*c*x^4*atan(a*x))/4
```

3.152 $\int (c + a^2cx^2) \arctan(ax) \, dx$

3.152.1 Optimal result

Integrand size = 15, antiderivative size = 50

$$\int (c+a^2cx^2) \arctan(ax) \, dx = -\frac{1}{6}acx^2+cx\arctan(ax)+\frac{1}{3}a^2cx^3\arctan(ax)-\frac{c\log(1+a^2x^2)}{3a}$$

output
```
-1/6*a*c*x^2+c*x*arctan(a*x)+1/3*a^2*c*x^3*arctan(a*x)-1/3*c*ln(a^2*x^2+1)
/a
```

3.152.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2) \arctan(ax) \, dx = -\frac{1}{6}acx^2+cx\arctan(ax)+\frac{1}{3}a^2cx^3\arctan(ax)-\frac{c\log(1+a^2x^2)}{3a}$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/6*(a*c*x^2) + c*x*ArcTan[a*x] + (a^2*c*x^3*ArcTan[a*x])/3 - (c*Log[1 +
a^2*x^2])/(3*a)
```

3.152.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{2}{3}c\int \arctan(ax)dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)-\frac{c(a^2x^2+1)}{6a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{2}{3}c\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)-\frac{c(a^2x^2+1)}{6a}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{c(a^2x^2+1)}{6a}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/6*(c*(1 + a^2*x^2))/a + (c*x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*c*(x*Arc
Tan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3
```

3.152.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

3.152.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{a^{2} c \,x^{3} \arctan \left(a x \right)}{3}+c x \arctan \left(a x \right)-\frac{c a \left(\frac{x^{2}}{2}+\frac{\ln \left(a^{2} x^{2}+1\right)}{a^{2}}\right)}{3}$	46
derivativedivides	$\frac{\frac{c \arctan \left(a x \right) a^{3} x^{3}}{3}+a c x \arctan \left(a x \right)-\frac{c \left(\frac{a^{2} x^{2}}{2}+\ln \left(a^{2} x^{2}+1\right)\right)}{3}}{a}$	49
default	$\frac{\frac{c \arctan \left(a x \right) a^{3} x^{3}}{3}+a c x \arctan \left(a x \right)-\frac{c \left(\frac{a^{2} x^{2}}{2}+\ln \left(a^{2} x^{2}+1\right)\right)}{3}}{a}$	49
parallelrisch	$-\frac{-2 c \arctan \left(a x \right) a^{3} x^{3}+a^{2} c \,x^{2}-6 a c x \arctan \left(a x \right)+2 c \ln \left(a^{2} x^{2}+1\right)}{6 a}$	50
risch	$-\frac{i c x \left(a^{2} x^{2}+3\right) \ln \left(i a x +1\right)}{6}+\frac{i c \,a^{2} x^{3} \ln \left(-i a x +1\right)}{6}-\frac{a c \,x^{2}}{6}+\frac{i c x \ln \left(-i a x +1\right)}{2}-\frac{c \ln \left(-a^{2} x^{2}-1\right)}{3 a}$	79
meijerg	$\frac{c \left(-\frac{2 a^{2} x^{2}}{3}+\frac{4 a^{4} x^{4} \arctan \left(\sqrt{a^{2} x^{2}}\right)}{3 \sqrt{a^{2} x^{2}}}+\frac{2 \ln \left(a^{2} x^{2}+1\right)}{3}\right)}{4 a}+\frac{c \left(\frac{4 a^{2} x^{2} \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{4 a}$	102

input
```
int((a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*c*x^3*arctan(a*x)+c*x*arctan(a*x)-1/3*c*a*(1/2*x^2+1/a^2*ln(a^2*x^
2+1))
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \left(c+a^2cx^2\right)\arctan(ax)\,dx=-\frac{a^{2} c x^{2}-2 \,{\left(a^{3} c x^{3}+3 \, a c x\right)} \arctan \left(a x\right)+2 \, c \log \left(a^{2} x^{2}+1\right)}{6 \, a}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/6*(a^2*c*x^2 - 2*(a^3*c*x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 +
1))/a
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (c+a^2cx^2)\arctan(ax)\,dx = \begin{cases} \frac{a^2cx^3\,\mathrm{atan}\,(ax)}{3} - \frac{acx^2}{6} + cx\,\mathrm{atan}\,(ax) - \frac{c\log\left(x^2+\frac{1}{a^2}\right)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**3*atan(a*x)/3 - a*c*x**2/6 + c*x*atan(a*x) - c*log(x*
*2 + a**(-2))/(3*a), Ne(a, 0)), (0, True))
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (c+a^2cx^2)\arctan(ax)\,dx = -\frac{1}{6}\left(cx^2 + \frac{2\,c\log\left(a^2x^2+1\right)}{a^2}\right)a + \frac{1}{3}\left(a^2cx^3 + 3\,cx\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/6*(c*x^2 + 2*c*log(a^2*x^2 + 1)/a^2)*a + 1/3*(a^2*c*x^3 + 3*c*x)*arctan
(a*x)
```

3.152.8 Giac [F]

$$\int (c+a^2cx^2)\arctan(ax)\,dx = \int \left(a^2cx^2+c\right)\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (c + a^2cx^2)\arctan(ax)\,dx$$
$$= -\frac{c\,(2\,\ln(a^2\,x^2+1) + a^2\,x^2 - 2\,a^3\,x^3\,\mathrm{atan}(a\,x) - 6\,a\,x\,\mathrm{atan}(a\,x))}{6\,a}$$

input
```
int(atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
-(c*(2*log(a^2*x^2 + 1) + a^2*x^2 - 2*a^3*x^3*atan(a*x) - 6*a*x*atan(a*x))
)/(6*a)
```

3.153 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx$

3.153.1 Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = -\frac{1}{2}acx+\frac{1}{2}c\arctan(ax)+\frac{1}{2}a^2cx^2\arctan(ax)+\frac{1}{2}ic\,\mathrm{PolyLog}(2,-iax)-\frac{1}{2}ic\,\mathrm{PolyLog}(2,iax)$$

output
```
-1/2*a*c*x+1/2*c*arctan(a*x)+1/2*a^2*c*x^2*arctan(a*x)+1/2*I*c*polylog(2,-
I*a*x)-1/2*I*c*polylog(2,I*a*x)
```

3.153.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = -\frac{1}{2}acx+\frac{1}{2}c\arctan(ax)+\frac{1}{2}a^2cx^2\arctan(ax)+\frac{1}{2}ic\,\mathrm{PolyLog}(2,-iax)-\frac{1}{2}ic\,\mathrm{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

output
```
-1/2*(a*c*x) + (c*ArcTan[a*x])/2 + (a^2*c*x^2*ArcTan[a*x])/2 + (I/2)*c*Pol
yLog[2, (-I)*a*x] - (I/2)*c*PolyLog[2, I*a*x]
```

3.153.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5485, 5355, 2838, 5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\arctan(ax)dx+c\int \frac{\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{5355}$$

$$a^2c\int x\arctan(ax)dx+c\left(\frac{1}{2}i\int \frac{\log(1-iax)}{x}dx-\frac{1}{2}i\int \frac{\log(iax+1)}{x}dx\right)$$

$$\Big\downarrow \text{2838}$$

$$a^2c\int x\arctan(ax)dx+c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx\right)+c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{262}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{216}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)\right)+c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

output
```
a^2*c*((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2) + c*((I/2)*P
olyLog[2, (-I)*a*x] - (I/2)*PolyLog[2, I*a*x])
```

3.153.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.153. $\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx$

3.153.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result
risch	$\frac{ic\ln(-iax+1)x^2a^2}{4}+\frac{c\arctan(ax)}{2}-\frac{acx}{2}-\frac{ic\,\mathrm{dilog}(-iax+1)}{2}-\frac{ic\ln(iax+1)x^2a^2}{4}+\frac{ic\,\mathrm{dilog}(iax+1)}{2}$
meijerg	$\frac{c\left(-2ax+\frac{2(3a^2x^2+3)\arctan(ax)}{3}\right)}{4}+\frac{c\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}$
derivativedivides	$\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(ax)-\frac{c(ax-\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dil}}{2}$
default	$\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(ax)-\frac{c(ax-\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dil}}{2}$
parts	$\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(x)-\frac{ca\left(x-\frac{\arctan(ax)}{a}-\frac{i\ln(x)(\ln(iax+1)-\ln(-iax+1))}{a}-\frac{i(\mathrm{dilog}(iax+1)-\mathrm{dil}}{a}\right.}{2}$

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*I*c*ln(1-I*a*x)*x^2*a^2+1/2*c*arctan(a*x)-1/2*a*c*x-1/2*I*c*dilog(1-I*
a*x)-1/4*I*c*ln(1+I*a*x)*x^2*a^2+1/2*I*c*dilog(1+I*a*x)
```

3.153.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx=\int \frac{(a^2cx^2+c)\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

3.153.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx=c\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx+\int a^2x\operatorname{atan}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x,x)
```

output
```
c*(Integral(atan(a*x)/x, x) + Integral(a**2*x*atan(a*x), x))
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = -\frac{1}{2}acx - \frac{1}{4}\pi c\log\left(a^2x^2+1\right) + c\arctan\left(ax\right)\log\left(ax\right) + \frac{1}{2}\left(a^2cx^2+c\right)\arctan\left(ax\right) - \frac{1}{2}i\,c\mathrm{Li}_2(i\,ax+1) + \frac{1}{2}i\,c\mathrm{Li}_2(-i\,ax+1)$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
-1/2*a*c*x - 1/4*pi*c*log(a^2*x^2 + 1) + c*arctan(a*x)*log(a*x) + 1/2*(a^2
*c*x^2 + c)*arctan(a*x) - 1/2*I*c*dilog(I*a*x + 1) + 1/2*I*c*dilog(-I*a*x
+ 1)
```

3.153.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = \int \frac{\left(a^2cx^2+c\right)\arctan\left(ax\right)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = \begin{cases} 0 & \text{if } a=0 \\ a^2\,c\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{a\,c\,x}{2}-\frac{c\,(\mathrm{Li}_2(1-a\,x\,1\mathrm{i})-\mathrm{Li}_2(1+a\,x\,1\mathrm{i}))\,1\mathrm{i}}{2} & \text{if } a\neq 0 \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c*(dilog(- a*x*1i + 1) - dilog(a*x*1i + 1)
)*1i)/2 - (a*c*x)/2 + a^2*c*atan(a*x)*(1/(2*a^2) + x^2/2))
```

3.153. $\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx$

3.154 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx$

3.154.1 Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx = -\frac{c\arctan(ax)}{x}+a^2cx\arctan(ax)+ac\log(x)-ac\log\left(1+a^2x^2\right)$$

output
```
-c*arctan(a*x)/x+a^2*c*x*arctan(a*x)+a*c*ln(x)-a*c*ln(a^2*x^2+1)
```

3.154.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx = -\frac{c\arctan(ax)}{x}+a^2cx\arctan(ax)+ac\log(x)-ac\log\left(1+a^2x^2\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

output
```
-((c*ArcTan[a*x])/x) + a^2*c*x*ArcTan[a*x] + a*c*Log[x] - a*c*Log[1 + a^2*
x^2]
```

3.154.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5485, 5345, 240, 5361, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)}{x^2}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \arctan(ax)dx+c\int \frac{\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{ 5345}$$

$$a^2c\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+c\int \frac{\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{ 240}$$

$$c\int \frac{\arctan(ax)}{x^2}dx+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$c\left(a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{ 243}$$

$$c\left(\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{ 47}$$

$$c\left(\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{ 14}$$

$$c\left(\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{ 16}$$

$$a^2c\left(x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}\right)+c\left(\frac{1}{2}a\left(\log(x^2)-\log(a^2x^2+1)\right)-\frac{\arctan(ax)}{x}\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

output
```
c*(-(ArcTan[a*x]/x) + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) + a^2*c*(x*ArcT
an[a*x] - Log[1 + a^2*x^2]/(2*a))
```

3.154.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.154. $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.154.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

method	result	size
parts	$a^2 cx \arctan(ax) - \frac{c \arctan(ax)}{x} - ca(\ln(a^2x^2+1) - \ln(x))$	41
derivativedivides	$a\left(acx \arctan(ax) - \frac{c \arctan(ax)}{ax} - c(-\ln(ax) + \ln(a^2x^2+1))\right)$	45
default	$a\left(acx \arctan(ax) - \frac{c \arctan(ax)}{ax} - c(-\ln(ax) + \ln(a^2x^2+1))\right)$	45
parallelrisch	$\frac{a^2 c\,x^2 \arctan(ax) + ca\ln(x)x - ca\ln(a^2x^2+1)x - c\arctan(ax)}{x}$	46
risch	$-\frac{ic(a^2x^2-1)\ln(iax+1)}{2x} + \frac{ic(a^2x^2\ln(-iax+1) - 2ia\ln(x)x + 2ia\ln(-2a^2x^2-2)x - \ln(-iax+1))}{2x}$	82
meijerg	$\frac{ac\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}} - 2\ln(a^2x^2+1)\right)}{4} + \frac{ac\left(4\ln(x) + 4\ln(a) - \frac{4\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}} - 2\ln(a^2x^2+1)\right)}{4}$	92

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x*arctan(a*x)-c*arctan(a*x)/x-c*a*(ln(a^2*x^2+1)-ln(x))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^2}\,dx = -\frac{acx\log(a^2x^2+1) - acx\log(x) - (a^2cx^2 - c)\arctan(ax)}{x}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
-(a*c*x*log(a^2*x^2 + 1) - a*c*x*log(x) - (a^2*c*x^2 - c)*arctan(a*x))/x
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^2}\,dx = \begin{cases} a^2cx\,\mathrm{atan}(ax) + ac\log(x) - ac\log\left(x^2 + \frac{1}{a^2}\right) - \frac{c\,\mathrm{atan}(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x**2,x)
```

output
```
Piecewise((a**2*c*x*atan(a*x) + a*c*log(x) - a*c*log(x**2 + a**(-2)) - c*a
tan(a*x)/x, Ne(a, 0)), (0, True))
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^2}\,dx = -(c\log(a^2x^2+1) - c\log(x))a + \left(a^2cx - \frac{c}{x}\right)\arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
-(c*log(a^2*x^2 + 1) - c*log(x))*a + (a^2*c*x - c/x)*arctan(a*x)
```

3.154.8 Giac [F]

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2 + c)\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^2}\,dx = a^2\,c\,x\,\mathrm{atan}(a\,x) - \frac{c\,\mathrm{atan}(a\,x)}{x} - c\,\left(a\,\ln\left(a^2\,x^2 + 1\right) - a\,\ln(x)\right)$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x^2,x)
```

output
```
a^2*c*x*atan(a*x) - (c*atan(a*x))/x - c*(a*log(a^2*x^2 + 1) - a*log(x))
```

3.155 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx$

3.155.1 Optimal result

Integrand size = 18, antiderivative size = 70

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = -\frac{ac}{2x}-\frac{1}{2}a^2c\arctan(ax)-\frac{c\arctan(ax)}{2x^2}+\frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,-iax)-\frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,iax)$$

output
```
-1/2*a*c/x-1/2*a^2*c*arctan(a*x)-1/2*c*arctan(a*x)/x^2+1/2*I*a^2*c*polylog
(2,-I*a*x)-1/2*I*a^2*c*polylog(2,I*a*x)
```

3.155.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = -\frac{c\arctan(ax)}{2x^2}-\frac{ac\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x}+\frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,-iax)-\frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(c*ArcTan[a*x])/x^2 - (a*c*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)
])/(2*x) + (I/2)*a^2*c*PolyLog[2, (-I)*a*x] - (I/2)*a^2*c*PolyLog[2, I*a*x
]
```

3.155.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5485, 5355, 2838, 5361, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)}{x}dx+c\int \frac{\arctan(ax)}{x^3}dx$$

$$\Big\downarrow \text{5355}$$

$$c\int \frac{\arctan(ax)}{x^3}dx+a^2c\left(\frac{1}{2}i\int \frac{\log(1-iax)}{x}dx-\frac{1}{2}i\int \frac{\log(iax+1)}{x}dx\right)$$

$$\Big\downarrow \text{2838}$$

$$c\int \frac{\arctan(ax)}{x^3}dx+a^2c\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax)-\frac{1}{2}i\,\text{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{2x^2}\right)+a^2c\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax)-\frac{1}{2}i\,\text{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{264}$$

$$c\left(\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)+ a^2c\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax)-\frac{1}{2}i\,\text{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{216}$$

$$c\left(\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)+a^2c\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax)-\frac{1}{2}i\,\text{PolyLog}(2,iax)\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

output
```
c*(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2) + a^2*c*((I/2)*
PolyLog[2, (-I)*a*x] - (I/2)*PolyLog[2, I*a*x])
```

3.155.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.155. $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx$

3.155.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

method	result
parts	$c\arctan(ax)\,a^2\ln(x)-\frac{c\arctan(ax)}{2x^2}-\frac{ca\left(\frac{1}{x}+a\arctan(ax)+2a^2\left(-\frac{i\ln(x)(\ln(iax+1)-\ln(-iax+1))}{2a}-\frac{i(\mathrm{dilog}(ia\right.\right.}{2}$
derivativedivides	$a^2\left(-\frac{c\arctan(ax)}{2a^2x^2}+c\arctan(ax)\ln(ax)-\frac{c(\frac{1}{ax}+\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i}{2}\right.$
default	$a^2\left(-\frac{c\arctan(ax)}{2a^2x^2}+c\arctan(ax)\ln(ax)-\frac{c(\frac{1}{ax}+\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i}{2}\right.$
meijerg	$\frac{c\,a^2\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}+\frac{c\,a^2\left(-\frac{2}{ax}-\frac{2\left(a^2x^2+1\right)\arctan(ax)}{a^2x^2}\right)}{4}$
risch	$-\frac{ic\,a^2\,\mathrm{dilog}(-iax+1)}{2}+\frac{ic\,a^2\ln(-iax)}{4}-\frac{ac}{2x}-\frac{ic\,a^2\ln(-iax+1)}{4}-\frac{ic\ln(-iax+1)}{4x^2}+\frac{ic\,a^2\,\mathrm{dilog}(iax+1)}{2}-\frac{ic}{}$

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
c*arctan(a*x)*a^2*ln(x)-1/2*c*arctan(a*x)/x^2-1/2*c*a*(1/x+a*arctan(a*x)+2
*a^2*(-1/2*I*ln(x)*(ln(1+I*a*x)-ln(1-I*a*x))/a-1/2*I*(dilog(1+I*a*x)-dilog
(1-I*a*x))/a))
```

3.155.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx=\int \frac{(a^2cx^2+c)\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

3.155.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = c\left(\int \frac{\operatorname{atan}(ax)}{x^3}\,dx + \int \frac{a^2\operatorname{atan}(ax)}{x}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x**3,x)
```

output
```
c*(Integral(atan(a*x)/x**3, x) + Integral(a**2*atan(a*x)/x, x))
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = -\frac{\pi a^2cx^2\log(a^2x^2+1) - 4\,a^2cx^2\arctan(ax)\log(ax) + 2i\,a^2cx^2\mathrm{Li}_2(i\,ax+1) - 2i\,a^2cx^2\mathrm{Li}_2(-i\,ax+1) +}{4\,x^2}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
-1/4*(pi*a^2*c*x^2*log(a^2*x^2 + 1) - 4*a^2*c*x^2*arctan(a*x)*log(a*x) + 2
*I*a^2*c*x^2*dilog(I*a*x + 1) - 2*I*a^2*c*x^2*dilog(-I*a*x + 1) + 2*a*c*x
+ 2*(a^2*c*x^2 + c)*arctan(a*x))/x^2
```

3.155.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^3}\,dx$$
$$= \begin{cases} 0 & \text{if } a = 0 \\ -\frac{c\,\mathrm{atan}(a\,x)}{2\,x^2} - \frac{c\left(a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a} - \frac{a^2\,c\,\mathrm{Li}_2(1-a\,x\,1\mathrm{i})\,1\mathrm{i}}{2} + \frac{a^2\,c\,\mathrm{Li}_2(1+a\,x\,1\mathrm{i})\,1\mathrm{i}}{2} & \text{if } a \neq 0 \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x^3,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c*atan(a*x))/(2*x^2) - (a^2*c*dilog(- a*x*
1i + 1)*1i)/2 + (a^2*c*dilog(a*x*1i + 1)*1i)/2 - (c*(a^3*atan(a*x) + a^2/x
))/(2*a))
```

3.156 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4} dx$

3.156.1 Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4} dx = -\frac{ac}{6x^2} - \frac{c\arctan(ax)}{3x^3} - \frac{a^2c\arctan(ax)}{x} + \frac{2}{3}a^3c\log(x) - \frac{1}{3}a^3c\log\left(1+a^2x^2\right)$$

output
```
-1/6*a*c/x^2-1/3*c*arctan(a*x)/x^3-a^2*c*arctan(a*x)/x+2/3*a^3*c*ln(x)-1/3
*a^3*c*ln(a^2*x^2+1)
```

3.156.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4} dx = \frac{c(-2(1+3a^2x^2)\arctan(ax)+ax(-1+4a^2x^2\log(x)-2a^2x^2\log(1+a^2x^2)))}{6x^3}$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

output
```
(c*(-2*(1 + 3*a^2*x^2)*ArcTan[a*x] + a*x*(-1 + 4*a^2*x^2*Log[x] - 2*a^2*x^
2*Log[1 + a^2*x^2])))/(6*x^3)
```

3.156.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5485, 5361, 243, 47, 14, 16, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)}{x^2}dx+c\int \frac{\arctan(ax)}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{x}\right)+c\left(\frac{1}{3}a\int \frac{1}{x^3\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{3x^3}\right)$$

$$\Big\downarrow \text{243}$$

$$a^2c\left(\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{x}\right)+c\left(\frac{1}{6}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{3x^3}\right)$$

$$\Big\downarrow \text{47}$$

$$a^2c\left(\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+c\left(\frac{1}{6}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{3x^3}\right)$$

$$\Big\downarrow \text{14}$$

$$a^2c\left(\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+c\left(\frac{1}{6}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{3x^3}\right)$$

$$\Big\downarrow \text{16}$$

$$c\left(\frac{1}{6}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{3x^3}\right)+a^2c\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)$$

$$\Big\downarrow \text{54}$$

$$c\left(\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\arctan(ax)}{3x^3}\right)+a^2c\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$a^2c\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)+ c\left(\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

output
```
a^2*c*(-(ArcTan[a*x]/x) + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) + c*(-1/3*A
rcTan[a*x]/x^3 + (a*(-x^(-2) - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))/6)
```

3.156.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.156. $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.156.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{a^2 c \arctan(ax)}{x}-\frac{c \arctan(ax)}{3x^3}-\frac{ca\left(a^2\ln(a^2x^2+1)+\frac{1}{2x^2}-2a^2\ln(x)\right)}{3}$
derivativedivides	$a^3\left(-\frac{c\arctan(ax)}{ax}-\frac{c\arctan(ax)}{3a^3x^3}-\frac{c\left(\ln(a^2x^2+1)+\frac{1}{2a^2x^2}-2\ln(ax)\right)}{3}\right)$
default	$a^3\left(-\frac{c\arctan(ax)}{ax}-\frac{c\arctan(ax)}{3a^3x^3}-\frac{c\left(\ln(a^2x^2+1)+\frac{1}{2a^2x^2}-2\ln(ax)\right)}{3}\right)$
parallelrisch	$\frac{4c\,a^3\ln(x)x^3-2c\,a^3\ln(a^2x^2+1)x^3+a^3c\,x^3-6a^2c\,x^2\arctan(ax)-acx-2c\arctan(ax)}{6x^3}$
risch	$\frac{ic(3a^2x^2+1)\ln(iax+1)}{6x^3}+\frac{c(4\ln(x)a^3x^3-2\ln(-3a^2x^2-3)a^3x^3-3ia^2x^2\ln(-iax+1)-ax-i\ln(-iax+1))}{6x^3}$
meijerg	$\frac{a^3c\left(4\ln(x)+4\ln(a)-\frac{4\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{4}+\frac{a^3c\left(-\frac{2}{a^2x^2}+\frac{4}{9}-\frac{4\ln(x)}{3}-\frac{4\ln(a)}{3}+\frac{-\frac{4a^2x^2}{9}+\frac{4}{3}}{a^2x^2}-\frac{4\arctan(\sqrt{}}{3a^2x^2\sqrt{}}\right.}{4}$

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-a^2*c*arctan(a*x)/x-1/3*c*arctan(a*x)/x^3-1/3*c*a*(a^2*ln(a^2*x^2+1)+1/2/
x^2-2*a^2*ln(x))
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx$$
$$= -\frac{2\,a^3cx^3\log\left(a^2x^2+1\right)-4\,a^3cx^3\log\left(x\right)+acx+2\left(3\,a^2cx^2+c\right)\arctan\left(ax\right)}{6\,x^3}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a^3*c*x^3*log(a^2*x^2 + 1) - 4*a^3*c*x^3*log(x) + a*c*x + 2*(3*a^2
*c*x^2 + c)*arctan(a*x))/x^3
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx$$
$$= \begin{cases} \frac{2a^3c\log{(x)}}{3}-\frac{a^3c\log{\left(x^2+\frac{1}{a^2}\right)}}{3}-\frac{a^2c\operatorname{atan}{(ax)}}{x}-\frac{ac}{6x^2}-\frac{c\operatorname{atan}{(ax)}}{3x^3} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x**4,x)
```

output
```
Piecewise((2*a**3*c*log(x)/3 - a**3*c*log(x**2 + a**(-2))/3 - a**2*c*atan(
a*x)/x - a*c/(6*x**2) - c*atan(a*x)/(3*x**3), Ne(a, 0)), (0, True))
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx = -\frac{1}{6}\left(2\,a^2c\log\left(a^2x^2+1\right)-2\,a^2c\log\left(x^2\right)+\frac{c}{x^2}\right)a - \frac{\left(3\,a^2cx^2+c\right)\arctan\left(ax\right)}{3\,x^3}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(2*a^2*c*log(a^2*x^2 + 1) - 2*a^2*c*log(x^2) + c/x^2)*a - 1/3*(3*a^2*
c*x^2 + c)*arctan(a*x)/x^3
```

3.156.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx = \frac{c\,(4\,a^3\,\ln(x)-2\,a^3\,\ln(a^2\,x^2+1))}{6} - \frac{\frac{c\,\mathrm{atan}(a\,x)}{3}+\frac{a\,c\,x}{6}+a^2\,c\,x^2\,\mathrm{atan}(a\,x)}{x^3}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x^4,x)
```

output
```
(c*(4*a^3*log(x) - 2*a^3*log(a^2*x^2 + 1)))/6 - ((c*atan(a*x))/3 + (a*c*x)
/6 + a^2*c*x^2*atan(a*x))/x^3
```

3.157 $\int x^3(c+a^2cx^2)^2 \arctan(ax)\,dx$

3.157.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int x^3(c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7-\frac{c^2\arctan(ax)}{24a^4}+\frac{1}{4}c^2x^4\arctan(ax)+\frac{1}{3}a^2c^2x^6\arctan(ax)+\frac{1}{8}a^4c^2x^8\arctan(ax)$$

output
```
1/24*c^2*x/a^3-1/72*c^2*x^3/a-1/24*a*c^2*x^5-1/56*a^3*c^2*x^7-1/24*c^2*arc
tan(a*x)/a^4+1/4*c^2*x^4*arctan(a*x)+1/3*a^2*c^2*x^6*arctan(a*x)+1/8*a^4*c
^2*x^8*arctan(a*x)
```

3.157.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int x^3(c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7-\frac{c^2\arctan(ax)}{24a^4}+\frac{1}{4}c^2x^4\arctan(ax)+\frac{1}{3}a^2c^2x^6\arctan(ax)+\frac{1}{8}a^4c^2x^8\arctan(ax)$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 -
(c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan
[a*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

3.157.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax) \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^7 \arctan(ax)+2a^2c^2x^5 \arctan(ax)+c^2x^3 \arctan(ax)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}a^4c^2x^8 \arctan(ax)-\frac{c^2 \arctan(ax)}{24a^4}-\frac{1}{56}a^3c^2x^7+\frac{c^2x}{24a^3}+\frac{1}{3}a^2c^2x^6 \arctan(ax)+\frac{1}{4}c^2x^4 \arctan(ax)-\frac{1}{24}ac^2x^5-\frac{c^2x^3}{72a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 -
(c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan
[a*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.157.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)a^8x^8}{8}+\frac{c^2\arctan(ax)a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)}{4}-\frac{c^2\left(\frac{3a^7x^7}{7}+a^5x^5+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{24}}{a^4}$
default	$\frac{\frac{c^2\arctan(ax)a^8x^8}{8}+\frac{c^2\arctan(ax)a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)}{4}-\frac{c^2\left(\frac{3a^7x^7}{7}+a^5x^5+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{24}}{a^4}$
parts	$\frac{a^4c^2x^8\arctan(ax)}{8}+\frac{a^2c^2x^6\arctan(ax)}{3}+\frac{c^2x^4\arctan(ax)}{4}-\frac{c^2a\left(\frac{\frac{3}{7}a^6x^7+a^4x^5+\frac{1}{3}a^2x^3-x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{24}$
parallelrisch	$\frac{63c^2\arctan(ax)a^8x^8-9a^7c^2x^7+168c^2\arctan(ax)a^6x^6-21a^5c^2x^5+126a^4c^2x^4\arctan(ax)-7a^3c^2x^3+21ac^2x-21c^2\,\mathrm{arc}}{504a^4}$
risch	$-\frac{ic^2x^4(3a^4x^4+8a^2x^2+6)\ln(iax+1)}{48}+\frac{ic^2a^4x^8\ln(-iax+1)}{16}-\frac{a^3c^2x^7}{56}+\frac{ic^2a^2x^6\ln(-iax+1)}{6}-\frac{a\,c^2x^5}{24}+ic^2$
meijerg	$\frac{c^2\left(\frac{xa\left(-45a^6x^6+63a^4x^4-105a^2x^2+315\right)}{630}-\frac{xa\left(-9a^8x^8+9\right)\arctan\left(\sqrt{a^2x^2}\right)}{18\sqrt{a^2x^2}}\right)}{4a^4}+\frac{c^2\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax\left(7a^6\right.}{}\right.}{2a^4}$

input
```
int(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*c^2*arctan(a*x)*a^8*x^8+1/3*c^2*arctan(a*x)*a^6*x^6+1/4*a^4*c^2
*x^4*arctan(a*x)-1/24*c^2*(3/7*a^7*x^7+a^5*x^5+1/3*a^3*x^3-a*x+arctan(a*x)
))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)\, dx = -\frac{9\, a^7 c^2 x^7 + 21\, a^5 c^2 x^5 + 7\, a^3 c^2 x^3 - 21\, a c^2 x - 21\, (3\, a^8 c^2 x^8 + 8\, a^6 c^2 x^6 + 6\, a^4 c^2 x^4 - c^2) \arctan(ax)}{504\, a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/504*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x - 21*(
3*a^8*c^2*x^8 + 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x))/a^4
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)\, dx = \begin{cases} \frac{a^4 c^2 x^8 \operatorname{atan}(ax)}{8} - \frac{a^3 c^2 x^7}{56} + \frac{a^2 c^2 x^6 \operatorname{atan}(ax)}{3} - \frac{a c^2 x^5}{24} + \frac{c^2 x^4 \operatorname{atan}(ax)}{4} - \frac{c^2 x^3}{72a} + \frac{c^2 x}{24a^3} - \frac{c^2 \operatorname{atan}(ax)}{24a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**8*atan(a*x)/8 - a**3*c**2*x**7/56 + a**2*c**2*x**6
*atan(a*x)/3 - a*c**2*x**5/24 + c**2*x**4*atan(a*x)/4 - c**2*x**3/(72*a) +
 c**2*x/(24*a**3) - c**2*atan(a*x)/(24*a**4), Ne(a, 0)), (0, True))
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)\, dx = -\frac{1}{504}\, a \left(\frac{21\, c^2 \arctan(ax)}{a^5} + \frac{9\, a^6 c^2 x^7 + 21\, a^4 c^2 x^5 + 7\, a^2 c^2 x^3 - 21\, c^2 x}{a^4}\right) + \frac{1}{24}\, (3\, a^4 c^2 x^8 + 8\, a^2 c^2 x^6 + 6\, c^2 x^4) \arctan(ax)$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/504*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2
*c^2*x^3 - 21*c^2*x)/a^4) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^
4)*arctan(a*x)
```

3.157.8 Giac [F]

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)\, dx = \int (a^2 c x^2 + c)^2 x^3 \arctan(ax)\, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)\, dx = \mathrm{atan}(a\,x) \left(\frac{a^4\,c^2\,x^8}{8} + \frac{a^2\,c^2\,x^6}{3} + \frac{c^2\,x^4}{4}\right) + \frac{c^2\,x}{24\,a^3} - \frac{a\,c^2\,x^5}{24} - \frac{c^2\,\mathrm{atan}(a\,x)}{24\,a^4} - \frac{c^2\,x^3}{72\,a} - \frac{a^3\,c^2\,x^7}{56}$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)*((c^2*x^4)/4 + (a^2*c^2*x^6)/3 + (a^4*c^2*x^8)/8) + (c^2*x)/(24*
a^3) - (a*c^2*x^5)/24 - (c^2*atan(a*x))/(24*a^4) - (c^2*x^3)/(72*a) - (a^3
*c^2*x^7)/56
```

3.158 $\int x^2(c+a^2cx^2)^2 \arctan(ax)\,dx$

3.158.1 Optimal result

Integrand size = 20, antiderivative size = 106

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{4c^2x^2}{105a} - \frac{9}{140}ac^2x^4 - \frac{1}{42}a^3c^2x^6 + \frac{1}{3}c^2x^3\arctan(ax) + \frac{2}{5}a^2c^2x^5\arctan(ax) + \frac{1}{7}a^4c^2x^7\arctan(ax) + \frac{4c^2\log(1+a^2x^2)}{105a^3}$$

output
```
-4/105*c^2*x^2/a-9/140*a*c^2*x^4-1/42*a^3*c^2*x^6+1/3*c^2*x^3*arctan(a*x)+
2/5*a^2*c^2*x^5*arctan(a*x)+1/7*a^4*c^2*x^7*arctan(a*x)+4/105*c^2*ln(a^2*x
^2+1)/a^3
```

3.158.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{4c^2x^2}{105a} - \frac{9}{140}ac^2x^4 - \frac{1}{42}a^3c^2x^6 + \frac{1}{3}c^2x^3\arctan(ax) + \frac{2}{5}a^2c^2x^5\arctan(ax) + \frac{1}{7}a^4c^2x^7\arctan(ax) + \frac{4c^2\log(1+a^2x^2)}{105a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*Arc
Tan[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7
+ (4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

3.158.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \; 5483$$

$$\int \left(a^4c^2x^6 \arctan(ax)+2a^2c^2x^4 \arctan(ax)+c^2x^2 \arctan(ax)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}a^4c^2x^7 \arctan(ax)-\frac{1}{42}a^3c^2x^6+\frac{2}{5}a^2c^2x^5 \arctan(ax)+\frac{4c^2 \log\left(a^2x^2+1\right)}{105a^3}+\frac{1}{3}c^2x^3 \arctan(ax)-\frac{9}{140}ac^2x^4-\frac{4c^2x^2}{105a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*Arc
Tan[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7
+ (4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.158.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)a^7x^7}{7}+\frac{2c^2\arctan(ax)a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)}{3}-\frac{c^2\left(\frac{5a^6x^6}{2}+\frac{27a^4x^4}{4}+4a^2x^2-4\ln\left(a^2x^2+1\right)\right)}{105}}{a^3}$
default	$\frac{\frac{c^2\arctan(ax)a^7x^7}{7}+\frac{2c^2\arctan(ax)a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)}{3}-\frac{c^2\left(\frac{5a^6x^6}{2}+\frac{27a^4x^4}{4}+4a^2x^2-4\ln\left(a^2x^2+1\right)\right)}{105}}{a^3}$
parts	$\frac{a^4c^2x^7\arctan(ax)}{7}+\frac{2a^2c^2x^5\arctan(ax)}{5}+\frac{c^2x^3\arctan(ax)}{3}-\frac{c^2a\left(\frac{5a^4x^6+\frac{27}{2}a^2x^4+8x^2}{2a^2}-\frac{4\ln\left(a^2x^2+1\right)}{a^4}\right)}{105}$
parallelrisch	$\frac{60c^2\arctan(ax)a^7x^7-10a^6c^2x^6+168c^2\arctan(ax)a^5x^5-27a^4c^2x^4+140a^3c^2x^3\arctan(ax)-16a^2c^2x^2+16c^2\ln(a^2x^2+}{420a^3}$
risch	$-\frac{ic^2x^3\left(15a^4x^4+42a^2x^2+35\right)\ln(iax+1)}{210}+\frac{ic^2a^4x^7\ln(-iax+1)}{14}-\frac{a^3c^2x^6}{42}+\frac{ic^2a^2x^5\ln(-iax+1)}{5}-\frac{9a\,c^2x^4}{140}+$
meijerg	$\frac{c^2\left(-\frac{a^2x^2\left(4a^4x^4-6a^2x^2+12\right)}{42}+\frac{4a^8x^8\arctan\left(\sqrt{a^2x^2}\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{7}\right)}{4a^3}+\frac{c^2\left(\frac{a^2x^2\left(-3a^2x^2+6\right)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right.}{2a^3}$

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/7*c^2*arctan(a*x)*a^7*x^7+2/5*c^2*arctan(a*x)*a^5*x^5+1/3*a^3*c^2
*x^3*arctan(a*x)-1/105*c^2*(5/2*a^6*x^6+27/4*a^4*x^4+4*a^2*x^2-4*ln(a^2*x^
2+1)))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int x^2(c+a^2cx^2)^2\arctan(ax)\,dx = -\frac{10\,a^6c^2x^6+27\,a^4c^2x^4+16\,a^2c^2x^2-16\,c^2\log\left(a^2x^2+1\right)-4\left(15\,a^7c^2x^7+42\,a^5c^2x^5+35\,a^3c^2x^3\right)\arctan}{420\,a^3}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/420*(10*a^6*c^2*x^6 + 27*a^4*c^2*x^4 + 16*a^2*c^2*x^2 - 16*c^2*log(a^2*
x^2 + 1) - 4*(15*a^7*c^2*x^7 + 42*a^5*c^2*x^5 + 35*a^3*c^2*x^3)*arctan(a*x
))/a^3
```

3.158.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int x^2(c+a^2cx^2)^2\arctan(ax)\,dx$$
$$=\begin{cases}\frac{a^4c^2x^7\,\mathrm{atan}\,(ax)}{7}-\frac{a^3c^2x^6}{42}+\frac{2a^2c^2x^5\,\mathrm{atan}\,(ax)}{5}-\frac{9ac^2x^4}{140}+\frac{c^2x^3\,\mathrm{atan}\,(ax)}{3}-\frac{4c^2x^2}{105a}+\frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{105a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**7*atan(a*x)/7 - a**3*c**2*x**6/42 + 2*a**2*c**2*x*
*5*atan(a*x)/5 - 9*a*c**2*x**4/140 + c**2*x**3*atan(a*x)/3 - 4*c**2*x**2/(
105*a) + 4*c**2*log(x**2 + a**(-2))/(105*a**3), Ne(a, 0)), (0, True))
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int x^2(c+a^2cx^2)^2\arctan(ax)\,dx$$
$$=-\frac{1}{420}\,a\left(\frac{10\,a^4c^2x^6+27\,a^2c^2x^4+16\,c^2x^2}{a^2}-\frac{16\,c^2\log\left(a^2x^2+1\right)}{a^4}\right)$$
$$+\frac{1}{105}\left(15\,a^4c^2x^7+42\,a^2c^2x^5+35\,c^2x^3\right)\arctan\left(ax\right)$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/420*a*((10*a^4*c^2*x^6 + 27*a^2*c^2*x^4 + 16*c^2*x^2)/a^2 - 16*c^2*log(
a^2*x^2 + 1)/a^4) + 1/105*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*a
rctan(a*x)
```

3.158.8 Giac [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int x^2(c+a^2cx^2)^2\arctan(ax)\,dx$$
$$=\frac{c^2\left(16\,\ln\left(a^2\,x^2+1\right)-16\,a^2\,x^2-27\,a^4\,x^4-10\,a^6\,x^6+140\,a^3\,x^3\,\mathrm{atan}(a\,x)+168\,a^5\,x^5\,\mathrm{atan}(a\,x)+60\,a^7\,x^7\right.}{420\,a^3}$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
(c^2*(16*log(a^2*x^2 + 1) - 16*a^2*x^2 - 27*a^4*x^4 - 10*a^6*x^6 + 140*a^3
*x^3*atan(a*x) + 168*a^5*x^5*atan(a*x) + 60*a^7*x^7*atan(a*x)))/(420*a^3)
```

3.159 $\int x(c+a^2cx^2)^2 \arctan(ax)\,dx$

3.159.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{c^2x}{6a} - \frac{1}{9}ac^2x^3 - \frac{1}{30}a^3c^2x^5 + \frac{c^2(1+a^2x^2)^3\arctan(ax)}{6a^2}$$

output
```
-1/6*c^2*x/a-1/9*a*c^2*x^3-1/30*a^3*c^2*x^5+1/6*c^2*(a^2*x^2+1)^3*arctan(a
*x)/a^2
```

3.159.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

$$\begin{aligned}\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = &-\frac{c^2x}{6a} - \frac{1}{9}ac^2x^3 - \frac{1}{30}a^3c^2x^5 \\ &+ \frac{c^2\arctan(ax)}{6a^2} + \frac{1}{2}c^2x^2\arctan(ax) \\ &+ \frac{1}{2}a^2c^2x^4\arctan(ax) + \frac{1}{6}a^4c^2x^6\arctan(ax)\end{aligned}$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
-1/6*(c^2*x)/a - (a*c^2*x^3)/9 - (a^3*c^2*x^5)/30 + (c^2*ArcTan[a*x])/(6*a
^2) + (c^2*x^2*ArcTan[a*x])/2 + (a^2*c^2*x^4*ArcTan[a*x])/2 + (a^4*c^2*x^6
*ArcTan[a*x])/6
```

3.159.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5465, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)}{6a^2}-\frac{\int \left(a^2cx^2+c\right)^2dx}{6a}$$

$$\Big\downarrow \text{210}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)}{6a^2}-\frac{\int \left(a^4c^2x^4+2a^2c^2x^2+c^2\right)dx}{6a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)}{6a^2}-\frac{\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x}{6a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
-1/6*(c^2*x + (2*a^2*c^2*x^3)/3 + (a^4*c^2*x^5)/5)/a + (c^2*(1 + a^2*x^2)^
3*ArcTan[a*x])/(6*a^2)
```

3.159.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.159.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

method	result
parts	$\frac{c^2 \arctan(ax) a^4 x^6}{6} + \frac{c^2 \arctan(ax) a^2 x^4}{2} + \frac{c^2 \arctan(ax) x^2}{2} + \frac{c^2 \arctan(ax)}{6a^2} - \frac{c^2\left(\frac{1}{5}a^4x^5+\frac{2}{3}a^2x^3+x\right)}{6a}$
derivativedivides	$\frac{\frac{c^2 \arctan(ax) a^6 x^6}{6} + \frac{a^4 c^2 x^4 \arctan(ax)}{2} + \frac{a^2 c^2 x^2 \arctan(ax)}{2} + \frac{c^2 \arctan(ax)}{6} - \frac{c^2\left(\frac{1}{5}a^5x^5+\frac{2}{3}a^3x^3+ax\right)}{6}}{a^2}$
default	$\frac{\frac{c^2 \arctan(ax) a^6 x^6}{6} + \frac{a^4 c^2 x^4 \arctan(ax)}{2} + \frac{a^2 c^2 x^2 \arctan(ax)}{2} + \frac{c^2 \arctan(ax)}{6} - \frac{c^2\left(\frac{1}{5}a^5x^5+\frac{2}{3}a^3x^3+ax\right)}{6}}{a^2}$
parallelrisch	$\frac{15c^2 \arctan(ax) a^6 x^6 - 3a^5c^2x^5 + 45a^4c^2x^4 \arctan(ax) - 10a^3c^2x^3 + 45a^2c^2x^2 \arctan(ax) - 15a\,c^2x + 15c^2 \arctan(ax)}{90a^2}$
risch	$-\frac{ic^2\left(a^2x^2+1\right)^3 \ln(iax+1)}{12a^2} + \frac{ic^2a^4x^6 \ln(-iax+1)}{12} - \frac{a^3c^2x^5}{30} + \frac{ic^2a^2x^4 \ln(-iax+1)}{4} - \frac{a\,c^2x^3}{9} + \frac{ic^2x^2 \ln(-iax}{4}$
meijerg	$\frac{c^2\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4a^2} + \frac{c^2\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right.}{5\sqrt{a^2x^2}}\right.}{2a^2}$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*arctan(a*x)*a^4*x^6+1/2*c^2*arctan(a*x)*a^2*x^4+1/2*c^2*arctan(a*x
)*x^2+1/6*c^2/a^2*arctan(a*x)-1/6*c^2/a*(1/5*a^4*x^5+2/3*a^2*x^3+x)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{3\,a^5c^2x^5 + 10\,a^3c^2x^3 + 15\,ac^2x - 15\,(a^6c^2x^6 + 3\,a^4c^2x^4 + 3\,a^2c^2x^2 + c^2)\arctan(ax)}{90\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/90*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x - 15*(a^6*c^2*x^6 + 3*a
^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))/a^2
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.51

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = \begin{cases} \frac{a^4c^2x^6 \operatorname{atan}(ax)}{6} - \frac{a^3c^2x^5}{30} + \frac{a^2c^2x^4 \operatorname{atan}(ax)}{2} - \frac{ac^2x^3}{9} + \frac{c^2x^2 \operatorname{atan}(ax)}{2} - \frac{c^2x}{6a} + \frac{c^2 \operatorname{atan}(ax)}{6a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**6*atan(a*x)/6 - a**3*c**2*x**5/30 + a**2*c**2*x**4
*atan(a*x)/2 - a*c**2*x**3/9 + c**2*x**2*atan(a*x)/2 - c**2*x/(6*a) + c**2
*atan(a*x)/(6*a**2), Ne(a, 0)), (0, True))
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{(a^2cx^2+c)^3 \arctan(ax)}{6\,a^2c} - \frac{3\,a^4c^3x^5 + 10\,a^2c^3x^3 + 15\,c^3x}{90\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
1/6*(a^2*c*x^2 + c)^3*arctan(a*x)/(a^2*c) - 1/90*(3*a^4*c^3*x^5 + 10*a^2*c
^3*x^3 + 15*c^3*x)/(a*c)
```

3.159.8 Giac [F]

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = \int (a^2cx^2+c)^2 x \arctan(ax)\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{c^2\,(15\,\mathrm{atan}(a\,x)-15\,a\,x-10\,a^3\,x^3-3\,a^5\,x^5+45\,a^2\,x^2\,\mathrm{atan}(a\,x)+45\,a^4\,x^4\,\mathrm{atan}(a\,x)+15\,a^6\,x^6\,\mathrm{atan}(a\,x)}{90\,a^2}$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
(c^2*(15*atan(a*x) - 15*a*x - 10*a^3*x^3 - 3*a^5*x^5 + 45*a^2*x^2*atan(a*x
) + 45*a^4*x^4*atan(a*x) + 15*a^6*x^6*atan(a*x)))/(90*a^2)
```

3.160 $\int (c+a^2cx^2)^2 \arctan(ax)\,dx$

3.160.1 Optimal result

Integrand size = 17, antiderivative size = 117

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{2c^2(1+a^2x^2)}{15a} - \frac{c^2(1+a^2x^2)^2}{20a} + \frac{8}{15}c^2x\arctan(ax) + \frac{4}{15}c^2x(1+a^2x^2)\arctan(ax) + \frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax) - \frac{4c^2\log(1+a^2x^2)}{15a}$$

output
```
-2/15*c^2*(a^2*x^2+1)/a-1/20*c^2*(a^2*x^2+1)^2/a+8/15*c^2*x*arctan(a*x)+4/
15*c^2*x*(a^2*x^2+1)*arctan(a*x)+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)-4/15*
c^2*ln(a^2*x^2+1)/a
```

3.160.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{c^2(-14a^2x^2-3a^4x^4+4ax(15+10a^2x^2+3a^4x^4)\arctan(ax)-16\log(1+a^2x^2))}{60a}$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*(-14*a^2*x^2 - 3*a^4*x^4 + 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan
[a*x] - 16*Log[1 + a^2*x^2]))/(60*a)
```

3.160.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5413, 27, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)^2dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
-1/20*(c^2*(1 + a^2*x^2)^2)/a + (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4
*c^2*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTa
n[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/5
```

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

3.160.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

method	result
parts	$\frac{c^2 \arctan(ax)a^4x^5}{5}+\frac{2c^2 \arctan(ax)a^2x^3}{3}+c^2x\arctan(ax)-\frac{c^2a\left(\frac{3a^2x^4}{4}+\frac{7x^2}{2}+\frac{4\ln\left(a^2x^2+1\right)}{a^2}\right)}{15}$
derivativedivides	$\frac{\frac{c^2\arctan(ax)a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)}{3}+a\,c^2x\arctan(ax)-\frac{c^2\left(\frac{3a^4x^4}{4}+\frac{7a^2x^2}{2}+4\ln\left(a^2x^2+1\right)\right)}{15}}{a}$
default	$\frac{\frac{c^2\arctan(ax)a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)}{3}+a\,c^2x\arctan(ax)-\frac{c^2\left(\frac{3a^4x^4}{4}+\frac{7a^2x^2}{2}+4\ln\left(a^2x^2+1\right)\right)}{15}}{a}$
parallelrisch	$-\frac{-12c^2\arctan(ax)a^5x^5+3a^4c^2x^4-40a^3c^2x^3\arctan(ax)+14a^2c^2x^2-60a\,c^2x\arctan(ax)+16c^2\ln(a^2x^2+1)}{60a}$
risch	$-\frac{ic^2x(3a^4x^4+10a^2x^2+15)\ln(iax+1)}{30}+\frac{ic^2a^4x^5\ln(-iax+1)}{10}-\frac{c^2a^3x^4}{20}+\frac{ic^2a^2x^3\ln(-iax+1)}{3}-\frac{7c^2a\,x^2}{30}+ic$
meijerg	$\frac{c^2\left(\frac{a^2x^2\left(-3a^2x^2+6\right)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}-\frac{2\ln\left(a^2x^2+1\right)}{5}\right)}{4a}+\frac{c^2\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{3}\right)}{2a}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*c^2*arctan(a*x)*a^4*x^5+2/3*c^2*arctan(a*x)*a^2*x^3+c^2*x*arctan(a*x)-
1/15*c^2*a*(3/4*a^2*x^4+7/2*x^2+4/a^2*ln(a^2*x^2+1))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \left(c+a^2cx^2\right)^2\arctan(ax)\,dx = -\frac{3\,a^4c^2x^4+14\,a^2c^2x^2+16\,c^2\log\left(a^2x^2+1\right)-4\left(3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x\right)\arctan\left(ax\right)}{60\,a}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/60*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5
*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx$$
$$= \begin{cases} \frac{a^4c^2x^5\,\mathrm{atan}\,(ax)}{5} - \frac{a^3c^2x^4}{20} + \frac{2a^2c^2x^3\,\mathrm{atan}\,(ax)}{3} - \frac{7ac^2x^2}{30} + c^2x\,\mathrm{atan}\,(ax) - \frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{15a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**5*atan(a*x)/5 - a**3*c**2*x**4/20 + 2*a**2*c**2*x**
*3*atan(a*x)/3 - 7*a*c**2*x**2/30 + c**2*x*atan(a*x) - 4*c**2*log(x**2 + a
**(-2))/(15*a), Ne(a, 0)), (0, True))
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = -\frac{1}{60}\left(3\,a^2c^2x^4 + 14\,c^2x^2 + \frac{16\,c^2\log\left(a^2x^2+1\right)}{a^2}\right)a + \frac{1}{15}\left(3\,a^4c^2x^5 + 10\,a^2c^2x^3 + 15\,c^2x\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/60*(3*a^2*c^2*x^4 + 14*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1)/a^2)*a + 1/15*
(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)
```

3.160.8 Giac [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int (c+a^2cx^2)^2 \arctan(ax) \, dx = -\frac{c^2 \left(16 \ln \left(a^2 x^2+1\right)+14 a^2 x^2+3 a^4 x^4-40 a^3 x^3 \operatorname{atan}(a x)-12 a^5 x^5 \operatorname{atan}(a x)-60 a x \operatorname{atan}(a x)\right)}{60 a}$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
-(c^2*(16*log(a^2*x^2 + 1) + 14*a^2*x^2 + 3*a^4*x^4 - 40*a^3*x^3*atan(a*x)
 - 12*a^5*x^5*atan(a*x) - 60*a*x*atan(a*x)))/(60*a)
```

3.161 $\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x}\,dx$

3.161.1 Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x}\,dx = -\frac{3}{4}ac^2x-\frac{1}{12}a^3c^2x^3+\frac{3}{4}c^2\arctan(ax)+a^2c^2x^2\arctan(ax)+\frac{1}{4}a^4c^2x^4\arctan(ax)+\frac{1}{2}ic^2\,\mathrm{PolyLog}(2,-iax)-\frac{1}{2}ic^2\,\mathrm{PolyLog}(2,iax)$$

output
```
-3/4*a*c^2*x-1/12*a^3*c^2*x^3+3/4*c^2*arctan(a*x)+a^2*c^2*x^2*arctan(a*x)+
1/4*a^4*c^2*x^4*arctan(a*x)+1/2*I*c^2*polylog(2,-I*a*x)-1/2*I*c^2*polylog(
2,I*a*x)
```

3.161.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x}\,dx = -\frac{3}{4}ac^2x-\frac{1}{12}a^3c^2x^3+\frac{3}{4}c^2\arctan(ax)+a^2c^2x^2\arctan(ax)+\frac{1}{4}a^4c^2x^4\arctan(ax)+\frac{1}{2}ic^2\,\mathrm{PolyLog}(2,-iax)-\frac{1}{2}ic^2\,\mathrm{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```

output
```
(-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Ar
cTan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] -
 (I/2)*c^2*PolyLog[2, I*a*x]
```

3.161.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x}dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^4c^2x^3\arctan(ax)+2a^2c^2x\arctan(ax)+\frac{c^2\arctan(ax)}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}a^4c^2x^4\arctan(ax)-\frac{1}{12}a^3c^2x^3+a^2c^2x^2\arctan(ax)+\frac{3}{4}c^2\arctan(ax)+\frac{1}{2}ic^2\operatorname{PolyLog}(2,-iax)-\frac{1}{2}ic^2\operatorname{PolyLog}(2,iax)-\frac{3}{4}ac^2x$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```

output
```
(-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Ar
cTan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] -
 (I/2)*c^2*PolyLog[2, I*a*x]
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.161.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(ax)-\frac{c^2\left(\frac{a^3x^3}{3}+3ax-3\arctan(ax)-2i\,\mathrm{l}\right.}{}$
default	$\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(ax)-\frac{c^2\left(\frac{a^3x^3}{3}+3ax-3\arctan(ax)-2i\,\mathrm{l}\right.}{}$
parts	$\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(x)-\frac{c^2a\left(\frac{a^2x^3}{3}+3x-\frac{3\arctan(ax)}{a}-\frac{2i\ln(x}{}\right.}{}$
risch	$\frac{ic^2\ln(-iax+1)x^4a^4}{8}+\frac{ic^2\ln(-iax+1)x^2a^2}{2}+\frac{3c^2\arctan(ax)}{4}-\frac{a^3c^2x^3}{12}-\frac{3a\,c^2x}{4}-\frac{ic^2\,\mathrm{dilog}(-iax+1)}{2}-\frac{ic^2\,\mathrm{l}}{}$
meijerg	$\frac{c^2\left(\frac{ax(-5a^2x^2+15)}{15}-\frac{ax(-5a^4x^4+5)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4}+\frac{c^2\left(-2ax+\frac{2(3a^2x^2+3)\arctan(ax)}{3}\right)}{2}+\frac{c^2\left(-\frac{2iax\,\mathrm{polylog}(2,i}{\sqrt{a^2x^2}}\right.}{}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^4*c^2*x^4*arctan(a*x)+a^2*c^2*x^2*arctan(a*x)+c^2*arctan(a*x)*ln(a*x
)-1/4*c^2*(1/3*a^3*x^3+3*a*x-3*arctan(a*x)-2*I*ln(a*x)*ln(1+I*a*x)+2*I*ln(
a*x)*ln(1-I*a*x)-2*I*dilog(1+I*a*x)+2*I*dilog(1-I*a*x))
```

3.161.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x, x)
```

3.161.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx = c^2\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx + \int 2a^2x\operatorname{atan}(ax)\,dx + \int a^4x^3\operatorname{atan}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x,x)
```

output
```
c**2*(Integral(atan(a*x)/x, x) + Integral(2*a**2*x*atan(a*x), x) + Integra
l(a**4*x**3*atan(a*x), x))
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx = -\frac{1}{12}a^3c^2x^3 - \frac{3}{4}ac^2x - \frac{1}{4}\pi c^2\log(a^2x^2+1) + c^2\arctan(ax)\log(ax) - \frac{1}{2}ic^2\mathrm{Li}_2(iax+1) + \frac{1}{2}ic^2\mathrm{Li}_2(-iax+1) + \frac{1}{4}(a^4c^2x^4+4a^2c^2x^2+3c^2)\arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
-1/12*a^3*c^2*x^3 - 3/4*a*c^2*x - 1/4*pi*c^2*log(a^2*x^2 + 1) + c^2*arctan
(a*x)*log(a*x) - 1/2*I*c^2*dilog(I*a*x + 1) + 1/2*I*c^2*dilog(-I*a*x + 1)
+ 1/4*(a^4*c^2*x^4 + 4*a^2*c^2*x^2 + 3*c^2)*arctan(a*x)
```

3.161.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan{(ax)}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx$$
$$= \begin{cases} 0 \\ 2\,a^2\,c^2\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-a\,c^2\,x-\frac{c^2\,\left(3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3\right)}{12}+\frac{a^4\,c^2\,x^4\,\mathrm{atan}(a\,x)}{4}-\frac{c^2\,\mathrm{Li}_2(1-a\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+c^2\,\mathrm{Li}_2(\end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c^2*dilog(- a*x*1i + 1)*1i)/2 + (c^2*dilog
(a*x*1i + 1)*1i)/2 - (c^2*(3*atan(a*x) - 3*a*x + a^3*x^3))/12 - a*c^2*x +
2*a^2*c^2*atan(a*x)*(1/(2*a^2) + x^2/2) + (a^4*c^2*x^4*atan(a*x))/4)
```

3.162 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx$

3.162.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx = -\frac{1}{6}a^3c^2x^2 - \frac{c^2\arctan(ax)}{x} + 2a^2c^2x\arctan(ax) + \frac{1}{3}a^4c^2x^3\arctan(ax) + ac^2\log(x) - \frac{4}{3}ac^2\log\left(1+a^2x^2\right)$$

output
```
-1/6*a^3*c^2*x^2-c^2*arctan(a*x)/x+2*a^2*c^2*x*arctan(a*x)+1/3*a^4*c^2*x^3
*arctan(a*x)+a*c^2*ln(x)-4/3*a*c^2*ln(a^2*x^2+1)
```

3.162.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx = \frac{c^2(2(-3+6a^2x^2+a^4x^4)\arctan(ax) - ax(a^2x^2 - 6\log(x) + 8\log(1+a^2x^2)))}{6x}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

output
```
(c^2*(2*(-3 + 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] - a*x*(a^2*x^2 - 6*Log[x] +
 8*Log[1 + a^2*x^2])))/(6*x)
```

3.162.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x^2}\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^2\arctan(ax)+2a^2c^2\arctan(ax)+\frac{c^2\arctan(ax)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^4c^2x^3\arctan(ax)-\frac{1}{6}a^3c^2x^2+2a^2c^2x\arctan(ax)-\frac{4}{3}ac^2\log\left(a^2x^2+1\right)-\frac{c^2\arctan(ax)}{x}+ac^2\log(x)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

output
```
-1/6*(a^3*c^2*x^2) - (c^2*ArcTan[a*x])/x + 2*a^2*c^2*x*ArcTan[a*x] + (a^4*
c^2*x^3*ArcTan[a*x])/3 + a*c^2*Log[x] - (4*a*c^2*Log[1 + a^2*x^2])/3
```

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.162.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

method	result
parts	$\frac{a^4c^2x^3\arctan(ax)}{3}+2a^2c^2x\arctan(ax)-\frac{c^2\arctan(ax)}{x}-\frac{c^2a\left(\frac{a^2x^2}{2}+4\ln(a^2x^2+1)-3\ln(x)\right)}{3}$
derivativedivides	$a\left(\frac{a^3c^2x^3\arctan(ax)}{3}+2a\,c^2x\arctan(ax)-\frac{c^2\arctan(ax)}{ax}-\frac{c^2\left(\frac{a^2x^2}{2}-3\ln(ax)+4\ln(a^2x^2+1)\right)}{3}\right)$
default	$a\left(\frac{a^3c^2x^3\arctan(ax)}{3}+2a\,c^2x\arctan(ax)-\frac{c^2\arctan(ax)}{ax}-\frac{c^2\left(\frac{a^2x^2}{2}-3\ln(ax)+4\ln(a^2x^2+1)\right)}{3}\right)$
parallelrisch	$\frac{2a^4c^2x^4\arctan(ax)-a^3c^2x^3+12a^2c^2x^2\arctan(ax)+6c^2a\ln(x)x-8c^2a\ln(a^2x^2+1)x-6c^2\arctan(ax)}{6x}$
risch	$-\frac{ic^2(a^4x^4+6a^2x^2-3)\ln(iax+1)}{6x}+\frac{ic^2(x^4\ln(-iax+1)a^4+ia^3x^3+6a^2x^2\ln(-iax+1)-6ia\ln(x)x+8ia\ln(7a^2x^2+7)\cdots}{6x}$
meijerg	$\frac{a\,c^2\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{3}\right)}{4}+\frac{a\,c^2\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{2}+\frac{a\,c^2\left(4\ln(x)\cdots\right.}{\cdots}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^4*c^2*x^3*arctan(a*x)+2*a^2*c^2*x*arctan(a*x)-c^2*arctan(a*x)/x-1/3*
c^2*a*(1/2*a^2*x^2+4*ln(a^2*x^2+1)-3*ln(x))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int\frac{(c+a^2cx^2)^2\arctan(ax)}{x^2}\,dx$$
$$=-\frac{a^3c^2x^3+8\,ac^2x\log(a^2x^2+1)-6\,ac^2x\log(x)-2\,(a^4c^2x^4+6\,a^2c^2x^2-3\,c^2)\arctan(ax)}{6\,x}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/6*(a^3*c^2*x^3 + 8*a*c^2*x*log(a^2*x^2 + 1) - 6*a*c^2*x*log(x) - 2*(a^4
*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x))/x
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx$$
$$= \begin{cases} \frac{a^4c^2x^3 \operatorname{atan}(ax)}{3} - \frac{a^3c^2x^2}{6} + 2a^2c^2x \operatorname{atan}(ax) + ac^2 \log(x) - \frac{4ac^2 \log\left(x^2+\frac{1}{a^2}\right)}{3} - \frac{c^2 \operatorname{atan}(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x**2,x)
```

output
```
Piecewise((a**4*c**2*x**3*atan(a*x)/3 - a**3*c**2*x**2/6 + 2*a**2*c**2*x*a
tan(a*x) + a*c**2*log(x) - 4*a*c**2*log(x**2 + a**(-2))/3 - c**2*atan(a*x)
/x, Ne(a, 0)), (0, True))
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx = -\frac{1}{6} \left(a^2c^2x^2 + 8\,c^2 \log\left(a^2x^2+1\right) - 6\,c^2 \log(x)\right)a + \frac{1}{3}\left(a^4c^2x^3 + 6\,a^2c^2x - \frac{3\,c^2}{x}\right)\arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
-1/6*(a^2*c^2*x^2 + 8*c^2*log(a^2*x^2 + 1) - 6*c^2*log(x))*a + 1/3*(a^4*c^
2*x^3 + 6*a^2*c^2*x - 3*c^2/x)*arctan(a*x)
```

3.162.8 Giac [F]

$$\int \frac{\left(c+a^2cx^2\right)^2 \arctan(ax)}{x^2} \, dx = \int \frac{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{\left(c+a^2cx^2\right)^2 \arctan(ax)}{x^2} \, dx = \frac{a^4\,c^2\,x^3\,\mathrm{atan}(a\,x)}{3} - \frac{c^2\,\mathrm{atan}(a\,x)}{x} - \frac{a^3\,c^2\,x^2}{6} - \frac{c^2\,\left(8\,a\,\ln\left(a^2\,x^2+1\right)-6\,a\,\ln\left(x\right)\right)}{6} + 2\,a^2\,c^2\,x\,\mathrm{atan}(a\,x)$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
(a^4*c^2*x^3*atan(a*x))/3 - (c^2*atan(a*x))/x - (a^3*c^2*x^2)/6 - (c^2*(8*
a*log(a^2*x^2 + 1) - 6*a*log(x)))/6 + 2*a^2*c^2*x*atan(a*x)
```

3.163 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} dx$

3.163.1 Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} dx = -\frac{ac^2}{2x} - \frac{1}{2}a^3c^2x - \frac{c^2\arctan(ax)}{2x^2} + \frac{1}{2}a^4c^2x^2\arctan(ax) + ia^2c^2\,\mathrm{PolyLog}(2,-iax) - ia^2c^2\,\mathrm{PolyLog}(2,iax)$$

output
```
-1/2*a*c^2/x-1/2*a^3*c^2*x-1/2*c^2*arctan(a*x)/x^2+1/2*a^4*c^2*x^2*arctan(
a*x)+I*a^2*c^2*polylog(2,-I*a*x)-I*a^2*c^2*polylog(2,I*a*x)
```

3.163.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} dx &= -\frac{1}{2}a^3c^2x + \frac{1}{2}a^2c^2\arctan(ax) \\ &- \frac{c^2\arctan(ax)}{2x^2} + \frac{1}{2}a^4c^2x^2\arctan(ax) \\ &- \frac{ac^2\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x} \\ &+ ia^2c^2\,\mathrm{PolyLog}(2,-iax) - ia^2c^2\,\mathrm{PolyLog}(2,iax)\end{aligned}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(a^3*c^2*x) + (a^2*c^2*ArcTan[a*x])/2 - (c^2*ArcTan[a*x])/(2*x^2) + (
a^4*c^2*x^2*ArcTan[a*x])/2 - (a*c^2*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*
x^2)])/(2*x) + I*a^2*c^2*PolyLog[2, (-I)*a*x] - I*a^2*c^2*PolyLog[2, I*a*x
]
```

3.163.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{5483, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x^3}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x\arctan(ax)+\frac{2a^2c^2\arctan(ax)}{x}+\frac{c^2\arctan(ax)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^4c^2x^2\arctan(ax)-\frac{1}{2}a^3c^2x+ia^2c^2\operatorname{PolyLog}(2,-iax)-ia^2c^2\operatorname{PolyLog}(2,iax)-\frac{c^2\arctan(ax)}{2x^2}-\frac{ac^2}{2x}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(a*c^2)/x - (a^3*c^2*x)/2 - (c^2*ArcTan[a*x])/(2*x^2) + (a^4*c^2*x^2*
ArcTan[a*x])/2 + I*a^2*c^2*PolyLog[2, (-I)*a*x] - I*a^2*c^2*PolyLog[2, I*a
*x]
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.163.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

method	result
parts	$\frac{a^4c^2x^2\arctan(ax)}{2}+2c^2\arctan(ax)\,a^2\ln(x)-\frac{c^2\arctan(ax)}{2x^2}-\frac{c^2a\left(a^2x+\frac{1}{x}+4a^2\left(-\frac{i\ln(x)(\ln(iax+1)-\ln(-}{2a}\right.\right.}{}$
derivativedivides	$a^2\left(\frac{a^2c^2x^2\arctan(ax)}{2}-\frac{c^2\arctan(ax)}{2a^2x^2}+2c^2\arctan(ax)\ln(ax)-\frac{c^2\left(ax+\frac{1}{ax}-2i\ln(ax)\ln(iax+1)+2i\ln\right.}{}\right.$
default	$a^2\left(\frac{a^2c^2x^2\arctan(ax)}{2}-\frac{c^2\arctan(ax)}{2a^2x^2}+2c^2\arctan(ax)\ln(ax)-\frac{c^2\left(ax+\frac{1}{ax}-2i\ln(ax)\ln(iax+1)+2i\ln\right.}{}\right.$
meijerg	$\frac{a^2c^2\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4}+\frac{a^2c^2\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{2}+\frac{a^2c^2\left(-\frac{2}{ax}-\right.}{}$
risch	$\frac{ic^2a^4\ln(-iax+1)x^2}{4}-\frac{a^3c^2x}{2}+\frac{ic^2a^2\ln(-iax)}{4}-\frac{a\,c^2}{2x}-\frac{ic^2\ln(-iax+1)}{4x^2}-ic^2a^2\,\mathrm{dilog}\left(-iax+1\right)-$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^4*c^2*x^2*arctan(a*x)+2*c^2*arctan(a*x)*a^2*ln(x)-1/2*c^2*arctan(a*x
)/x^2-1/2*c^2*a*(a^2*x+1/x+4*a^2*(-1/2*I*ln(x)*(ln(1+I*a*x)-ln(1-I*a*x))/a
-1/2*I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a))
```

3.163.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x^3, x)
```

3.163.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} \, dx = c^2 \left(\int \frac{\operatorname{atan}(ax)}{x^3} \, dx + \int \frac{2a^2 \operatorname{atan}(ax)}{x} \, dx + \int a^4 x \operatorname{atan}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x**3,x)
```

output
```
c**2*(Integral(atan(a*x)/x**3, x) + Integral(2*a**2*atan(a*x)/x, x) + Inte
gral(a**4*x*atan(a*x), x))
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} \, dx = -\frac{a^3c^2x^3 + \pi a^2c^2x^2 \log(a^2x^2+1) - 4\,a^2c^2x^2 \arctan(ax) \log(ax) + 2i\,a^2c^2x^2 \mathrm{Li}_2(i\,ax+1) - 2i\,a^2c^2x^2 \mathrm{Li}_2(}{2\,x^2}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(a^3*c^2*x^3 + pi*a^2*c^2*x^2*log(a^2*x^2 + 1) - 4*a^2*c^2*x^2*arctan
(a*x)*log(a*x) + 2*I*a^2*c^2*x^2*dilog(I*a*x + 1) - 2*I*a^2*c^2*x^2*dilog(
-I*a*x + 1) + a*c^2*x - (a^4*c^2*x^4 - c^2)*arctan(a*x))/x^2
```

3.163.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^2\arctan{(ax)}}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^3}\,dx$$
$$= \begin{cases} 0 \\ a^4\,c^2\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{c^2\,\mathrm{atan}(a\,x)}{2\,x^2}-\frac{c^2\,\left(a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a}-\frac{a^3\,c^2\,x}{2}-a^2\,c^2\,\mathrm{Li_2}(1-a\,x\,1\mathrm{i})\,1\mathrm{i}+a^2\,c^2\,\mathrm{Li_2} \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x^3,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (a^3*c^2*x)/2 - (c^2*atan(a*x))/(2*x^2) - a
^2*c^2*dilog(- a*x*1i + 1)*1i + a^2*c^2*dilog(a*x*1i + 1)*1i - (c^2*(a^3*a
tan(a*x) + a^2/x))/(2*a) + a^4*c^2*atan(a*x)*(1/(2*a^2) + x^2/2))
```

3.164 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} dx$

3.164.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} dx = -\frac{ac^2}{6x^2} - \frac{c^2\arctan(ax)}{3x^3} - \frac{2a^2c^2\arctan(ax)}{x} + a^4c^2x\arctan(ax) + \frac{5}{3}a^3c^2\log(x) - \frac{4}{3}a^3c^2\log\left(1+a^2x^2\right)$$

output
```
-1/6*a*c^2/x^2-1/3*c^2*arctan(a*x)/x^3-2*a^2*c^2*arctan(a*x)/x+a^4*c^2*x*a
rctan(a*x)+5/3*a^3*c^2*ln(x)-4/3*a^3*c^2*ln(a^2*x^2+1)
```

3.164.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} dx = \frac{c^2(2(-1-6a^2x^2+3a^4x^4)\arctan(ax) + ax(-1+10a^2x^2\log(x) - 8a^2x^2\log(1+a^2x^2)))}{6x^3}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
```

output
```
(c^2*(2*(-1 - 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + a*x*(-1 + 10*a^2*x^2*Lo
g[x] - 8*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

3.164.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x^4}\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2\arctan(ax)+\frac{2a^2c^2\arctan(ax)}{x^2}+\frac{c^2\arctan(ax)}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^4c^2x\arctan(ax)+\frac{5}{3}a^3c^2\log(x)-\frac{2a^2c^2\arctan(ax)}{x}-\frac{4}{3}a^3c^2\log\left(a^2x^2+1\right)-\frac{c^2\arctan(ax)}{3x^3}-\frac{ac^2}{6x^2}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
```

output
```
-1/6*(a*c^2)/x^2 - (c^2*ArcTan[a*x])/(3*x^3) - (2*a^2*c^2*ArcTan[a*x])/x +
 a^4*c^2*x*ArcTan[a*x] + (5*a^3*c^2*Log[x])/3 - (4*a^3*c^2*Log[1 + a^2*x^2
])/3
```

3.164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.164.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

method	result
parts	$a^{4} c^{2} x \arctan \left(a x \right)-\frac{2 a^{2} c^{2} \arctan \left(a x \right)}{x}-\frac{c^{2} \arctan \left(a x \right)}{3 x^{3}}-\frac{c^{2} a \left(4 a^{2} \ln \left(a^{2} x^{2}+1\right)+\frac{1}{2 x^{2}}-5 a^{2} \ln \left(x \right)\right)}{3}$
derivativedivides	$a^{3} \left(a \,c^{2} x \arctan \left(a x \right)-\frac{2 c^{2} \arctan \left(a x \right)}{a x}-\frac{c^{2} \arctan \left(a x \right)}{3 a^{3} x^{3}}-\frac{c^{2} \left(4 \ln \left(a^{2} x^{2}+1\right)+\frac{1}{2 a^{2} x^{2}}-5 \ln \left(a x \right)\right)}{3}\right)$
default	$a^{3} \left(a \,c^{2} x \arctan \left(a x \right)-\frac{2 c^{2} \arctan \left(a x \right)}{a x}-\frac{c^{2} \arctan \left(a x \right)}{3 a^{3} x^{3}}-\frac{c^{2} \left(4 \ln \left(a^{2} x^{2}+1\right)+\frac{1}{2 a^{2} x^{2}}-5 \ln \left(a x \right)\right)}{3}\right)$
parallelrisch	$\frac{6 a^{4} c^{2} x^{4} \arctan \left(a x \right)+10 a^{3} c^{2} \ln \left(x \right) x^{3}-8 a^{3} c^{2} \ln \left(a^{2} x^{2}+1\right) x^{3}+a^{3} c^{2} x^{3}-12 a^{2} c^{2} x^{2} \arctan \left(a x \right)-a \,c^{2} x-2 c^{2} \arctan \left(a x \right)}{6 x^{3}}$
risch	$-\frac{i c^{2} \left(3 a^{4} x^{4}-6 a^{2} x^{2}-1\right) \ln \left(i a x+1\right)}{6 x^{3}}+\frac{i c^{2} \left(3 x^{4} \ln \left(-i a x+1\right) a^{4}-10 i \ln \left(x \right) a^{3} x^{3}+8 i \ln \left(-9 a^{2} x^{2}-9\right) a^{3} x^{3}-6 a^{2} x^{2} \ln (-iax-}{6 x^{3}}$
meijerg	$\frac{a^{3} c^{2} \left(\frac{4 a^{2} x^{2} \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{4}+\frac{a^{3} c^{2} \left(4 \ln \left(x \right)+4 \ln \left(a \right)-\frac{4 \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{2}+\frac{a^{3} c^{2} (-}{}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*c^2*x*arctan(a*x)-2*a^2*c^2*arctan(a*x)/x-1/3*c^2*arctan(a*x)/x^3-1/3*
c^2*a*(4*a^2*ln(a^2*x^2+1)+1/2/x^2-5*a^2*ln(x))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{\left(c+a^2cx^2\right)^2\arctan(ax)}{x^4}\,dx = -\frac{8\,a^{3}c^{2}x^{3}\log\left(a^{2}x^{2}+1\right)-10\,a^{3}c^{2}x^{3}\log\left(x\right)+ac^{2}x-2\left(3\,a^{4}c^{2}x^{4}-6\,a^{2}c^{2}x^{2}-c^{2}\right)\arctan\left(ax\right)}{6\,x^{3}}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(8*a^3*c^2*x^3*log(a^2*x^2 + 1) - 10*a^3*c^2*x^3*log(x) + a*c^2*x - 2
*(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x))/x^3
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^4}\,dx$$
$$= \begin{cases} a^4c^2x\,\mathrm{atan}\,(ax) + \frac{5a^3c^2\log(x)}{3} - \frac{4a^3c^2\log\left(x^2+\frac{1}{a^2}\right)}{3} - \frac{2a^2c^2\,\mathrm{atan}\,(ax)}{x} - \frac{ac^2}{6x^2} - \frac{c^2\,\mathrm{atan}\,(ax)}{3x^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x**4,x)
```

output
```
Piecewise((a**4*c**2*x*atan(a*x) + 5*a**3*c**2*log(x)/3 - 4*a**3*c**2*log(
x**2 + a**(-2))/3 - 2*a**2*c**2*atan(a*x)/x - a*c**2/(6*x**2) - c**2*atan(
a*x)/(3*x**3), Ne(a, 0)), (0, True))
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^4}\,dx = -\frac{1}{6}\left(8\,a^2c^2\log\left(a^2x^2+1\right) - 10\,a^2c^2\log(x) + \frac{c^2}{x^2}\right)a + \frac{1}{3}\left(3\,a^4c^2x - \frac{6\,a^2c^2x^2+c^2}{x^3}\right)\arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(8*a^2*c^2*log(a^2*x^2 + 1) - 10*a^2*c^2*log(x) + c^2/x^2)*a + 1/3*(3
*a^4*c^2*x - (6*a^2*c^2*x^2 + c^2)/x^3)*arctan(a*x)
```

3.164.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4}\,dx = \frac{c^2\,(10\,a^3\,\ln(x)-8\,a^3\,\ln(a^2\,x^2+1))}{6} - \frac{\frac{c^2\,\mathrm{atan}(a\,x)}{3}+\frac{a\,c^2\,x}{6}+2\,a^2\,c^2\,x^2\,\mathrm{atan}(a\,x)}{x^3} + a^4\,c^2\,x\,\mathrm{atan}(a\,x)$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x^4,x)
```

output
```
(c^2*(10*a^3*log(x) - 8*a^3*log(a^2*x^2 + 1)))/6 - ((c^2*atan(a*x))/3 + (a
*c^2*x)/6 + 2*a^2*c^2*x^2*atan(a*x))/x^3 + a^4*c^2*x*atan(a*x)
```

3.165 $\int x^3(c+a^2cx^2)^3 \arctan(ax)\,dx$

3.165.1 Optimal result

Integrand size = 20, antiderivative size = 141

$$\begin{aligned}\int x^3(c+a^2cx^2)^3 \arctan(ax)\,dx = {} & \frac{c^3x}{40a^3}-\frac{c^3x^3}{120a}-\frac{9}{200}ac^3x^5-\frac{11}{280}a^3c^3x^7-\frac{1}{90}a^5c^3x^9 \\ & -\frac{c^3\arctan(ax)}{40a^4}+\frac{1}{4}c^3x^4\arctan(ax)+\frac{1}{2}a^2c^3x^6\arctan(ax) \\ & +\frac{3}{8}a^4c^3x^8\arctan(ax)+\frac{1}{10}a^6c^3x^{10}\arctan(ax)\end{aligned}$$

output
```
1/40*c^3*x/a^3-1/120*c^3*x^3/a-9/200*a*c^3*x^5-11/280*a^3*c^3*x^7-1/90*a^5
*c^3*x^9-1/40*c^3*arctan(a*x)/a^4+1/4*c^3*x^4*arctan(a*x)+1/2*a^2*c^3*x^6*
arctan(a*x)+3/8*a^4*c^3*x^8*arctan(a*x)+1/10*a^6*c^3*x^10*arctan(a*x)
```

3.165.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3(c+a^2cx^2)^3 \arctan(ax)\,dx = {} & \frac{c^3x}{40a^3}-\frac{c^3x^3}{120a}-\frac{9}{200}ac^3x^5-\frac{11}{280}a^3c^3x^7-\frac{1}{90}a^5c^3x^9 \\ & -\frac{c^3\arctan(ax)}{40a^4}+\frac{1}{4}c^3x^4\arctan(ax)+\frac{1}{2}a^2c^3x^6\arctan(ax) \\ & +\frac{3}{8}a^4c^3x^8\arctan(ax)+\frac{1}{10}a^6c^3x^{10}\arctan(ax)\end{aligned}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7
)/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*
x])/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6
*c^3*x^10*ArcTan[a*x])/10
```

3.165.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax) \left(a^2cx^2+c\right)^3 \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^9 \arctan(ax)+3a^4c^3x^7 \arctan(ax)+3a^2c^3x^5 \arctan(ax)+c^3x^3 \arctan(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}a^6c^3x^{10}\arctan(ax)-\frac{1}{90}a^5c^3x^9+\frac{3}{8}a^4c^3x^8\arctan(ax)-\frac{c^3\arctan(ax)}{40a^4}-\frac{11}{280}a^3c^3x^7+\frac{c^3x}{40a^3}+\frac{1}{2}a^2c^3x^6\arctan(ax)+\frac{1}{4}c^3x^4\arctan(ax)-\frac{9}{200}ac^3x^5-\frac{c^3x^3}{120a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7
)/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*
x])/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6
*c^3*x^10*ArcTan[a*x])/10
```

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.165.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{c^3 \arctan(ax) a^{10} x^{10}}{10}+\frac{3c^3 \arctan(ax) a^8 x^8}{8}+\frac{a^6 c^3 x^6 \arctan(ax)}{2}+\frac{a^4 c^3 x^4 \arctan(ax)}{4}-\frac{c^3 \left(\frac{4a^9x^9}{9}+\frac{11a^7x^7}{7}+\frac{9a^5x^5}{5}+\frac{a^3x^3}{3}-ax+a\right.}{40}}{a^4}$
default	$\frac{\frac{c^3 \arctan(ax) a^{10} x^{10}}{10}+\frac{3c^3 \arctan(ax) a^8 x^8}{8}+\frac{a^6 c^3 x^6 \arctan(ax)}{2}+\frac{a^4 c^3 x^4 \arctan(ax)}{4}-\frac{c^3 \left(\frac{4a^9x^9}{9}+\frac{11a^7x^7}{7}+\frac{9a^5x^5}{5}+\frac{a^3x^3}{3}-ax+a\right.}{40}}{a^4}$
parts	$\frac{a^6 c^3 x^{10} \arctan(ax)}{10}+\frac{3a^4 c^3 x^8 \arctan(ax)}{8}+\frac{a^2 c^3 x^6 \arctan(ax)}{2}+\frac{c^3 x^4 \arctan(ax)}{4}-\frac{c^3 a \left(\frac{\frac{4}{9}a^8x^9+\frac{11}{7}a^6x^7+\frac{9}{5}a^4x\right.}{a^4}}{40}$
parallelrisch	$\frac{1260c^3 \arctan(ax) a^{10} x^{10}-140a^9c^3x^9+4725c^3 \arctan(ax) a^8 x^8-495a^7c^3x^7+6300a^6c^3x^6 \arctan(ax)-567a^5c^3x^5+3150}{12600a^4}$
risch	$-\frac{ic^3x^4\left(4a^6x^6+15a^4x^4+20a^2x^2+10\right)\ln(iax+1)}{80}+\frac{ic^3a^6x^{10}\ln(-iax+1)}{20}-\frac{a^5c^3x^9}{90}+\frac{3ic^3a^4x^8\ln(-iax+1)}{16}-1$
meijerg	$\frac{c^3\left(-\frac{2xa\left(385a^8x^8-495a^6x^6+693a^4x^4-1155a^2x^2+3465\right)}{17325}+\frac{2xa\left(11a^{10}x^{10}+11\right)\arctan\left(\sqrt{a^2x^2}\right)}{55\sqrt{a^2x^2}}\right)}{4a^4}+\frac{3c^3\left(\frac{xa\left(-45a^6x^6+63a^4\right.}{63}\right.}{}$

input
```
int(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/10*c^3*arctan(a*x)*a^10*x^10+3/8*c^3*arctan(a*x)*a^8*x^8+1/2*a^6*
c^3*x^6*arctan(a*x)+1/4*a^4*c^3*x^4*arctan(a*x)-1/40*c^3*(4/9*a^9*x^9+11/7
*a^7*x^7+9/5*a^5*x^5+1/3*a^3*x^3-a*x+arctan(a*x)))
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int x^3\left(c+a^2cx^2\right)^3\arctan(ax)\,dx =$$
$$-\frac{140\,a^9c^3x^9+495\,a^7c^3x^7+567\,a^5c^3x^5+105\,a^3c^3x^3-315\,ac^3x-315\,(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8+20\,a^6c^3x}{12600\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/12600*(140*a^9*c^3*x^9 + 495*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^
3*x^3 - 315*a*c^3*x - 315*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 + 20*a^6*c^3*x
^6 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x))/a^4
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$\int x^3\left(c+a^2cx^2\right)^3\arctan(ax)\,dx$$
$$=\begin{cases}\frac{a^6c^3x^{10}\operatorname{atan}(ax)}{10}-\frac{a^5c^3x^9}{90}+\frac{3a^4c^3x^8\operatorname{atan}(ax)}{8}-\frac{11a^3c^3x^7}{280}+\frac{a^2c^3x^6\operatorname{atan}(ax)}{2}-\frac{9ac^3x^5}{200}+\frac{c^3x^4\operatorname{atan}(ax)}{4}-\frac{c^3x^3}{120a}+\frac{c^3x}{40a^3}- \\ 0\end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**10*atan(a*x)/10 - a**5*c**3*x**9/90 + 3*a**4*c**3*
x**8*atan(a*x)/8 - 11*a**3*c**3*x**7/280 + a**2*c**3*x**6*atan(a*x)/2 - 9*
a*c**3*x**5/200 + c**3*x**4*atan(a*x)/4 - c**3*x**3/(120*a) + c**3*x/(40*a
**3) - c**3*atan(a*x)/(40*a**4), Ne(a, 0)), (0, True))
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int x^3 (c + a^2 c x^2)^3 \arctan(ax)\, dx = -\frac{1}{12600} a \left(\frac{315 c^3 \arctan(ax)}{a^5} + \frac{140 a^8 c^3 x^9 + 495 a^6 c^3 x^7 + 567 a^4 c^3 x^5 + 105 a^2 c^3 x^3 - 315 c^3 x}{a^4} \right) + \frac{1}{40} \left(4 a^6 c^3 x^{10} + 15 a^4 c^3 x^8 + 20 a^2 c^3 x^6 + 10 c^3 x^4 \right) \arctan(ax)$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/12600*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 +
 567*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4) + 1/40*(4*a^6*c^3*x^1
0 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)
```

3.165.8 Giac [F]

$$\int x^3 (c + a^2 c x^2)^3 \arctan(ax)\, dx = \int (a^2 c x^2 + c)^3 x^3 \arctan(ax)\, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int x^3 (c+a^2 c x^2)^3 \arctan(ax)\, dx = \mathrm{atan}(a\, x) \left(\frac{a^6\, c^3\, x^{10}}{10} + \frac{3\, a^4\, c^3\, x^8}{8} + \frac{a^2\, c^3\, x^6}{2} + \frac{c^3\, x^4}{4} \right) + \frac{c^3\, x}{40\, a^3} - \frac{9\, a\, c^3\, x^5}{200} - \frac{c^3\, \mathrm{atan}(a\, x)}{40\, a^4} - \frac{c^3\, x^3}{120\, a} - \frac{11\, a^3\, c^3\, x^7}{280} - \frac{a^5\, c^3\, x^9}{90}$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)*((c^3*x^4)/4 + (a^2*c^3*x^6)/2 + (3*a^4*c^3*x^8)/8 + (a^6*c^3*x^
10)/10) + (c^3*x)/(40*a^3) - (9*a*c^3*x^5)/200 - (c^3*atan(a*x))/(40*a^4)
- (c^3*x^3)/(120*a) - (11*a^3*c^3*x^7)/280 - (a^5*c^3*x^9)/90
```

3.165. $\int x^3 (c + a^2 c x^2)^3 \arctan(ax) \, dx$

3.166 $\int x^2(c+a^2cx^2)^3 \arctan(ax)\,dx$

3.166.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10}{189}a^3c^3x^6-\frac{1}{72}a^5c^3x^8+\frac{1}{3}c^3x^3\arctan(ax)+\frac{3}{5}a^2c^3x^5\arctan(ax)+\frac{3}{7}a^4c^3x^7\arctan(ax)+\frac{1}{9}a^6c^3x^9\arctan(ax)+\frac{8c^3\log(1+a^2x^2)}{315a^3}$$

output
```
-8/315*c^3*x^2/a-89/1260*a*c^3*x^4-10/189*a^3*c^3*x^6-1/72*a^5*c^3*x^8+1/3
*c^3*x^3*arctan(a*x)+3/5*a^2*c^3*x^5*arctan(a*x)+3/7*a^4*c^3*x^7*arctan(a*
x)+1/9*a^6*c^3*x^9*arctan(a*x)+8/315*c^3*ln(a^2*x^2+1)/a^3
```

3.166.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10}{189}a^3c^3x^6-\frac{1}{72}a^5c^3x^8+\frac{1}{3}c^3x^3\arctan(ax)+\frac{3}{5}a^2c^3x^5\arctan(ax)+\frac{3}{7}a^4c^3x^7\arctan(ax)+\frac{1}{9}a^6c^3x^9\arctan(ax)+\frac{8c^3\log(1+a^2x^2)}{315a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c
^3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*
a^4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 +
a^2*x^2])/(315*a^3)
```

3.166.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right)^3 \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^8 \arctan(ax)+3a^4c^3x^6 \arctan(ax)+3a^2c^3x^4 \arctan(ax)+c^3x^2 \arctan(ax)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}a^6c^3x^9 \arctan(ax)-\frac{1}{72}a^5c^3x^8+\frac{3}{7}a^4c^3x^7 \arctan(ax)-\frac{10}{189}a^3c^3x^6+\frac{3}{5}a^2c^3x^5 \arctan(ax)+\frac{8c^3 \log\left(a^2x^2+1\right)}{315a^3}+\frac{1}{3}c^3x^3 \arctan(ax)-\frac{89ac^3x^4}{1260}-\frac{8c^3x^2}{315a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c
^3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*
a^4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 +
a^2*x^2])/(315*a^3)
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.166.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)a^9x^9}{9}+\frac{3c^3\arctan(ax)a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)}{5}+\frac{a^3c^3x^3\arctan(ax)}{3}-\frac{c^3\left(\frac{35a^8x^8}{8}+\frac{50a^6x^6}{3}+\frac{89a^4x^4}{4}+8a^2x^2-8\ln\right.}{315}}{a^3}$
default	$\frac{\frac{c^3\arctan(ax)a^9x^9}{9}+\frac{3c^3\arctan(ax)a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)}{5}+\frac{a^3c^3x^3\arctan(ax)}{3}-\frac{c^3\left(\frac{35a^8x^8}{8}+\frac{50a^6x^6}{3}+\frac{89a^4x^4}{4}+8a^2x^2-8\ln\right.}{315}}{a^3}$
parts	$\frac{a^6c^3x^9\arctan(ax)}{9}+\frac{3a^4c^3x^7\arctan(ax)}{7}+\frac{3a^2c^3x^5\arctan(ax)}{5}+\frac{c^3x^3\arctan(ax)}{3}-\frac{c^3a\left(\frac{\frac{35}{4}a^6x^8+\frac{100}{3}a^4x^6+\frac{89}{2}\right.}{2a^2}}{3}$
parallelrisch	$\frac{840c^3\arctan(ax)a^9x^9-105a^8c^3x^8+3240c^3\arctan(ax)a^7x^7-400a^6c^3x^6+4536a^5c^3x^5\arctan(ax)-534a^4c^3x^4+2520a^3c}{7560a^3}$
risch	$-\frac{ic^3x^3(35a^6x^6+135a^4x^4+189a^2x^2+105)\ln(iax+1)}{630}+\frac{ic^3a^6x^9\ln(-iax+1)}{18}-\frac{a^5c^3x^8}{72}+\frac{3ic^3a^4x^7\ln(-iax+1)}{14}-$
meijerg	$\frac{c^3\left(\frac{a^2x^2\left(-15a^6x^6+20a^4x^4-30a^2x^2+60\right)}{270}+\frac{4a^{10}x^{10}\arctan\left(\sqrt{a^2x^2}\right)}{9\sqrt{a^2x^2}}-\frac{2\ln\left(a^2x^2+1\right)}{9}\right)}{4a^3}+\frac{3c^3\left(-\frac{a^2x^2\left(4a^4x^4-6a^2x^2+12\right)}{42}+\right.}{}$

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/9*c^3*arctan(a*x)*a^9*x^9+3/7*c^3*arctan(a*x)*a^7*x^7+3/5*a^5*c^3
*x^5*arctan(a*x)+1/3*a^3*c^3*x^3*arctan(a*x)-1/315*c^3*(35/8*a^8*x^8+50/3*
a^6*x^6+89/4*a^4*x^4+8*a^2*x^2-8*ln(a^2*x^2+1)))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx = -\frac{105\,a^8c^3x^8+400\,a^6c^3x^6+534\,a^4c^3x^4+192\,a^2c^3x^2-192\,c^3\log\left(a^2x^2+1\right)-24\left(35\,a^9c^3x^9+135\,a^7c^3x^7\right.}{7560\,a^3}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/7560*(105*a^8*c^3*x^8 + 400*a^6*c^3*x^6 + 534*a^4*c^3*x^4 + 192*a^2*c^3
*x^2 - 192*c^3*log(a^2*x^2 + 1) - 24*(35*a^9*c^3*x^9 + 135*a^7*c^3*x^7 + 1
89*a^5*c^3*x^5 + 105*a^3*c^3*x^3)*arctan(a*x))/a^3
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx$$
$$= \begin{cases} \frac{a^6c^3x^9\operatorname{atan}(ax)}{9} - \frac{a^5c^3x^8}{72} + \frac{3a^4c^3x^7\operatorname{atan}(ax)}{7} - \frac{10a^3c^3x^6}{189} + \frac{3a^2c^3x^5\operatorname{atan}(ax)}{5} - \frac{89ac^3x^4}{1260} + \frac{c^3x^3\operatorname{atan}(ax)}{3} - \frac{8c^3x^2}{315a} + \frac{8c^3\log}{3} \\ 0 \end{cases}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**9*atan(a*x)/9 - a**5*c**3*x**8/72 + 3*a**4*c**3*x*
*7*atan(a*x)/7 - 10*a**3*c**3*x**6/189 + 3*a**2*c**3*x**5*atan(a*x)/5 - 89
*a*c**3*x**4/1260 + c**3*x**3*atan(a*x)/3 - 8*c**3*x**2/(315*a) + 8*c**3*l
og(x**2 + a**(-2))/(315*a**3), Ne(a, 0)), (0, True))
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx$$
$$=\frac{1}{7560}a\left(\frac{192\,c^3\log\left(a^2x^2+1\right)}{a^4}-\frac{105\,a^6c^3x^8+400\,a^4c^3x^6+534\,a^2c^3x^4+192\,c^3x^2}{a^2}\right)$$
$$+\frac{1}{315}\left(35\,a^6c^3x^9+135\,a^4c^3x^7+189\,a^2c^3x^5+105\,c^3x^3\right)\arctan\left(ax\right)$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
1/7560*a*(192*c^3*log(a^2*x^2 + 1)/a^4 - (105*a^6*c^3*x^8 + 400*a^4*c^3*x^
6 + 534*a^2*c^3*x^4 + 192*c^3*x^2)/a^2) + 1/315*(35*a^6*c^3*x^9 + 135*a^4*
c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*arctan(a*x)
```

3.166.8 Giac [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.166.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx=\mathrm{atan}(a\,x)\left(\frac{a^6\,c^3\,x^9}{9}+\frac{3\,a^4\,c^3\,x^7}{7}+\frac{3\,a^2\,c^3\,x^5}{5}+\frac{c^3\,x^3}{3}\right)-\frac{89\,a\,c^3\,x^4}{1260}+\frac{8\,c^3\,\ln\left(a^2\,x^2+1\right)}{315\,a^3}-\frac{8\,c^3\,x^2}{315\,a}-\frac{10\,a^3\,c^3\,x^6}{189}-\frac{a^5\,c^3\,x^8}{72}$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)*((c^3*x^3)/3 + (3*a^2*c^3*x^5)/5 + (3*a^4*c^3*x^7)/7 + (a^6*c^3*
x^9)/9) - (89*a*c^3*x^4)/1260 + (8*c^3*log(a^2*x^2 + 1))/(315*a^3) - (8*c^
3*x^2)/(315*a) - (10*a^3*c^3*x^6)/189 - (a^5*c^3*x^8)/72
```

3.166. $\int x^2(c+a^2cx^2)^3 \arctan(ax)\,dx$

3.167 $\int x(c+a^2cx^2)^3 \arctan(ax)\,dx$

3.167.1 Optimal result

Integrand size = 18, antiderivative size = 74

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{c^3x}{8a}-\frac{1}{8}ac^3x^3-\frac{3}{40}a^3c^3x^5-\frac{1}{56}a^5c^3x^7+\frac{c^3(1+a^2x^2)^4\arctan(ax)}{8a^2}$$

output
```
-1/8*c^3*x/a-1/8*a*c^3*x^3-3/40*a^3*c^3*x^5-1/56*a^5*c^3*x^7+1/8*c^3*(a^2*
x^2+1)^4*arctan(a*x)/a^2
```

3.167.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{c^3x}{8a}-\frac{1}{8}ac^3x^3-\frac{3}{40}a^3c^3x^5-\frac{1}{56}a^5c^3x^7+\frac{c^3\arctan(ax)}{8a^2}+\frac{1}{2}c^3x^2\arctan(ax)+\frac{3}{4}a^2c^3x^4\arctan(ax)+\frac{1}{2}a^4c^3x^6\arctan(ax)+\frac{1}{8}a^6c^3x^8\arctan(ax)$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
-1/8*(c^3*x)/a - (a*c^3*x^3)/8 - (3*a^3*c^3*x^5)/40 - (a^5*c^3*x^7)/56 + (
c^3*ArcTan[a*x])/(8*a^2) + (c^3*x^2*ArcTan[a*x])/2 + (3*a^2*c^3*x^4*ArcTan
[a*x])/4 + (a^4*c^3*x^6*ArcTan[a*x])/2 + (a^6*c^3*x^8*ArcTan[a*x])/8
```

3.167.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5465, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)}{8a^2}-\frac{\int \left(a^2cx^2+c\right)^3dx}{8a}$$

$$\Big\downarrow \text{210}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)}{8a^2}-\frac{\int \left(a^6c^3x^6+3a^4c^3x^4+3a^2c^3x^2+c^3\right)dx}{8a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)}{8a^2}-\frac{\frac{1}{7}a^6c^3x^7+\frac{3}{5}a^4c^3x^5+a^2c^3x^3+c^3x}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
-1/8*(c^3*x + a^2*c^3*x^3 + (3*a^4*c^3*x^5)/5 + (a^6*c^3*x^7)/7)/a + (c^3*
(1 + a^2*x^2)^4*ArcTan[a*x])/(8*a^2)
```

3.167.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.167.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

method	result
parts	$\frac{c^3\arctan(ax)a^6x^8}{8}+\frac{c^3\arctan(ax)a^4x^6}{2}+\frac{3c^3\arctan(ax)a^2x^4}{4}+\frac{c^3\arctan(ax)x^2}{2}+\frac{c^3\arctan(ax)}{8a^2}-\frac{c^3\left(\frac{1}{7}a^6x^7\right.}{}$
derivativedivides	$\frac{\frac{c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{a^2c^3x^2\arctan(ax)}{2}+\frac{c^3\arctan(ax)}{8}-\frac{c^3\left(\frac{1}{7}a^7x^7+\frac{3}{5}a^5x^5+a^3x^3+ax\right.}{8}}{a^2}$
default	$\frac{\frac{c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{a^2c^3x^2\arctan(ax)}{2}+\frac{c^3\arctan(ax)}{8}-\frac{c^3\left(\frac{1}{7}a^7x^7+\frac{3}{5}a^5x^5+a^3x^3+ax\right.}{8}}{a^2}$
parallelrisch	$\frac{35c^3\arctan(ax)a^8x^8-5a^7c^3x^7+140a^6c^3x^6\arctan(ax)-21a^5c^3x^5+210a^4c^3x^4\arctan(ax)-35a^3c^3x^3+140a^2c^3x^2\,\mathrm{arcta}}{280a^2}$
risch	$-\frac{ic^3\left(a^2x^2+1\right)^4\ln(iax+1)}{16a^2}+\frac{ic^3a^6x^8\ln(-iax+1)}{16}-\frac{a^5c^3x^7}{56}+\frac{ic^3a^4x^6\ln(-iax+1)}{4}-\frac{3a^3c^3x^5}{40}+\frac{3ic^3a^2x^4\ln(}{8}$
meijerg	$\frac{c^3\left(\frac{xa\left(-45a^6x^6+63a^4x^4-105a^2x^2+315\right)}{630}-\frac{xa\left(-9a^8x^8+9\right)\arctan\left(\sqrt{a^2x^2}\right)}{18\sqrt{a^2x^2}}\right)}{4a^2}+\frac{3c^3\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax(7a}{}\right.}{4a^2}$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*arctan(a*x)*a^6*x^8+1/2*c^3*arctan(a*x)*a^4*x^6+3/4*c^3*arctan(a*x
)*a^2*x^4+1/2*c^3*arctan(a*x)*x^2+1/8*c^3/a^2*arctan(a*x)-1/8*c^3/a*(1/7*a
^6*x^7+3/5*a^4*x^5+a^2*x^3+x)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{5\,a^7c^3x^7 + 21\,a^5c^3x^5 + 35\,a^3c^3x^3 + 35\,ac^3x - 35\,(a^8c^3x^8 + 4\,a^6c^3x^6 + 6\,a^4c^3x^4 + 4\,a^2c^3x^2 + c^3)\arctan(}{280\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/280*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x - 35*
(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan
(a*x))/a^2
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = \begin{cases} \frac{a^6c^3x^8\operatorname{atan}(ax)}{8} - \frac{a^5c^3x^7}{56} + \frac{a^4c^3x^6\operatorname{atan}(ax)}{2} - \frac{3a^3c^3x^5}{40} + \frac{3a^2c^3x^4\operatorname{atan}(ax)}{4} - \frac{ac^3x^3}{8} + \frac{c^3x^2\operatorname{atan}(ax)}{2} - \frac{c^3x}{8a} + \frac{c^3\operatorname{atan}(ax)}{8a^2} \\ 0 \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**8*atan(a*x)/8 - a**5*c**3*x**7/56 + a**4*c**3*x**6
*atan(a*x)/2 - 3*a**3*c**3*x**5/40 + 3*a**2*c**3*x**4*atan(a*x)/4 - a*c**3
*x**3/8 + c**3*x**2*atan(a*x)/2 - c**3*x/(8*a) + c**3*atan(a*x)/(8*a**2),
Ne(a, 0)), (0, True))
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = \frac{(a^2cx^2+c)^4 \arctan(ax)}{8\,a^2c} - \frac{5\,a^6c^4x^7 + 21\,a^4c^4x^5 + 35\,a^2c^4x^3 + 35\,c^4x}{280\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
1/8*(a^2*c*x^2 + c)^4*arctan(a*x)/(a^2*c) - 1/280*(5*a^6*c^4*x^7 + 21*a^4*
c^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)/(a*c)
```

3.167.8 Giac [F]

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = \int (a^2cx^2+c)^3 x \arctan(ax)\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = \operatorname{atan}(a\,x)\left(\frac{a^6\,c^3\,x^8}{8} + \frac{a^4\,c^3\,x^6}{2} + \frac{3\,a^2\,c^3\,x^4}{4} + \frac{c^3\,x^2}{2}\right) - \frac{c^3\,x}{8\,a} - \frac{a\,c^3\,x^3}{8} + \frac{c^3\operatorname{atan}(a\,x)}{8\,a^2} - \frac{3\,a^3\,c^3\,x^5}{40} - \frac{a^5\,c^3\,x^7}{56}$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)*((c^3*x^2)/2 + (3*a^2*c^3*x^4)/4 + (a^4*c^3*x^6)/2 + (a^6*c^3*x^
8)/8) - (c^3*x)/(8*a) - (a*c^3*x^3)/8 + (c^3*atan(a*x))/(8*a^2) - (3*a^3*c
^3*x^5)/40 - (a^5*c^3*x^7)/56
```

3.168 $\int (c+a^2cx^2)^3 \arctan(ax)\,dx$

3.168.1 Optimal result

Integrand size = 17, antiderivative size = 161

$$\begin{aligned}\int (c+a^2cx^2)^3 \arctan(ax)\,dx = & -\frac{4c^3(1+a^2x^2)}{35a}-\frac{3c^3(1+a^2x^2)^2}{70a}-\frac{c^3(1+a^2x^2)^3}{42a}\\ & +\frac{16}{35}c^3x\arctan(ax)+\frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)\\ & +\frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)\\ & +\frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)-\frac{8c^3\log(1+a^2x^2)}{35a}\end{aligned}$$

output
```
-4/35*c^3*(a^2*x^2+1)/a-3/70*c^3*(a^2*x^2+1)^2/a-1/42*c^3*(a^2*x^2+1)^3/a+
16/35*c^3*x*arctan(a*x)+8/35*c^3*x*(a^2*x^2+1)*arctan(a*x)+6/35*c^3*x*(a^2
*x^2+1)^2*arctan(a*x)+1/7*c^3*x*(a^2*x^2+1)^3*arctan(a*x)-8/35*c^3*ln(a^2*
x^2+1)/a
```

3.168.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\begin{aligned}&\int (c+a^2cx^2)^3 \arctan(ax)\,dx\\ &=\frac{c^3(-a^2x^2(57+24a^2x^2+5a^4x^4)+6ax(35+35a^2x^2+21a^4x^4+5a^6x^6)\arctan(ax)-48\log(1+a^2x^2))}{210a}\end{aligned}$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*(-(a^2*x^2*(57 + 24*a^2*x^2 + 5*a^4*x^4)) + 6*a*x*(35 + 35*a^2*x^2 +
21*a^4*x^4 + 5*a^6*x^6)*ArcTan[a*x] - 48*Log[1 + a^2*x^2]))/(210*a)
```

3.168.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5413, 27, 5413, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)^3dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(\right.$$

$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
-1/42*(c^3*(1 + a^2*x^2)^3)/a + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x])/7 + (6
*c^3*(-1/20*(1 + a^2*x^2)^2/a + (x*(1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4*(-1
/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] -
 Log[1 + a^2*x^2]/(2*a)))/3))/5))/7
```

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

3.168.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.61

method	result
parts	$\frac{c^3 \arctan(ax) a^6 x^7}{7}+\frac{3 c^3 \arctan(ax) a^4 x^5}{5}+c^3 \arctan(ax) a^2 x^3+c^3 x \arctan(ax)-\frac{c^3 a\left(\frac{5 a^4 x^6}{6}+4 a^2\right.}{}$
derivativedivides	$\frac{\frac{c^3 \arctan(ax) a^7 x^7}{7}+\frac{3 a^5 c^3 x^5 \arctan(ax)}{5}+a^3 c^3 x^3 \arctan(ax)+a c^3 x \arctan(ax)-\frac{c^3\left(\frac{5 a^6 x^6}{6}+4 a^4 x^4+\frac{19 a^2 x^2}{2}+8 \ln\left(a^2 x^2+1\right)\right)}{35}}{a}$
default	$\frac{\frac{c^3 \arctan(ax) a^7 x^7}{7}+\frac{3 a^5 c^3 x^5 \arctan(ax)}{5}+a^3 c^3 x^3 \arctan(ax)+a c^3 x \arctan(ax)-\frac{c^3\left(\frac{5 a^6 x^6}{6}+4 a^4 x^4+\frac{19 a^2 x^2}{2}+8 \ln\left(a^2 x^2+1\right)\right)}{35}}{a}$
parallelrisch	$-\frac{-30 c^3 \arctan(ax) a^7 x^7+5 a^6 c^3 x^6-126 a^5 c^3 x^5 \arctan(ax)+24 a^4 c^3 x^4-210 a^3 c^3 x^3 \arctan(ax)+57 a^2 c^3 x^2-210 a c^3 x \operatorname{arc}}{210 a}$
risch	$-\frac{i c^3 x\left(5 a^6 x^6+21 a^4 x^4+35 a^2 x^2+35\right) \ln(iax+1)}{70}+\frac{i c^3 a^6 x^7 \ln(-iax+1)}{14}-\frac{a^5 c^3 x^6}{42}+\frac{3 i c^3 a^4 x^5 \ln(-iax+1)}{10}-\frac{4 a^3}{3}$
meijerg	$\frac{c^3\left(-\frac{a^2 x^2\left(4 a^4 x^4-6 a^2 x^2+12\right)}{42}+\frac{4 a^8 x^8 \arctan\left(\sqrt{a^2 x^2}\right)}{7 \sqrt{a^2 x^2}}+\frac{2 \ln\left(a^2 x^2+1\right)}{7}\right)}{4 a}+\frac{3 c^3\left(\frac{a^2 x^2\left(-3 a^2 x^2+6\right)}{15}+\frac{4 a^6 x^6 \arctan\left(\sqrt{a^2 x^2}\right)}{5 \sqrt{a^2 x^2}}\right.}{4 a}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*c^3*arctan(a*x)*a^6*x^7+3/5*c^3*arctan(a*x)*a^4*x^5+c^3*arctan(a*x)*a^
2*x^3+c^3*x*arctan(a*x)-1/35*c^3*a*(5/6*a^4*x^6+4*a^2*x^4+19/2*x^2+8/a^2*l
n(a^2*x^2+1))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.63

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{5\,a^6c^3x^6+24\,a^4c^3x^4+57\,a^2c^3x^2+48\,c^3\log\left(a^2x^2+1\right)-6\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x\right)}{210\,a}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/210*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x
^2 + 1) - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x)
*arctan(a*x))/a
```

3.168.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = \begin{cases} \frac{a^6c^3x^7\operatorname{atan}(ax)}{7} - \frac{a^5c^3x^6}{42} + \frac{3a^4c^3x^5\operatorname{atan}(ax)}{5} - \frac{4a^3c^3x^4}{35} + a^2c^3x^3\operatorname{atan}(ax) - \frac{19ac^3x^2}{70} + c^3x\operatorname{atan}(ax) - \frac{8c^3\log\left(x^2\right.}{35a} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**7*atan(a*x)/7 - a**5*c**3*x**6/42 + 3*a**4*c**3*x*
*5*atan(a*x)/5 - 4*a**3*c**3*x**4/35 + a**2*c**3*x**3*atan(a*x) - 19*a*c**
3*x**2/70 + c**3*x*atan(a*x) - 8*c**3*log(x**2 + a**(-2))/(35*a), Ne(a, 0)
), (0, True))
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx$$
$$= -\frac{1}{210}\left(5\,a^4c^3x^6 + 24\,a^2c^3x^4 + 57\,c^3x^2 + \frac{48\,c^3\log\left(a^2x^2+1\right)}{a^2}\right)a$$
$$+ \frac{1}{35}\left(5\,a^6c^3x^7 + 21\,a^4c^3x^5 + 35\,a^2c^3x^3 + 35\,c^3x\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/210*(5*a^4*c^3*x^6 + 24*a^2*c^3*x^4 + 57*c^3*x^2 + 48*c^3*log(a^2*x^2 +
 1)/a^2)*a + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^
3*x)*arctan(a*x)
```

3.168.8 Giac [F]

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx =$$
$$-\frac{c^3\left(48\,\ln\left(a^2\,x^2+1\right)+57\,a^2\,x^2+24\,a^4\,x^4+5\,a^6\,x^6-210\,a^3\,x^3\,\mathrm{atan}(a\,x)-126\,a^5\,x^5\,\mathrm{atan}(a\,x)-30\,a^7\,x\right.}{210\,a}$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
-(c^3*(48*log(a^2*x^2 + 1) + 57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 210*a^3
*x^3*atan(a*x) - 126*a^5*x^5*atan(a*x) - 30*a^7*x^7*atan(a*x) - 210*a*x*at
an(a*x)))/(210*a)
```

3.168. $\int \left(c+a^2cx^2\right)^3 \arctan(ax)\,dx$

3.169 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} \, dx$

3.169.1 Optimal result

Integrand size = 20, antiderivative size = 132

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} \, dx = &-\frac{11}{12}ac^3x-\frac{7}{36}a^3c^3x^3-\frac{1}{30}a^5c^3x^5 \\ &+\frac{11}{12}c^3\arctan(ax)+\frac{3}{2}a^2c^3x^2\arctan(ax) \\ &+\frac{3}{4}a^4c^3x^4\arctan(ax)+\frac{1}{6}a^6c^3x^6\arctan(ax) \\ &+\frac{1}{2}ic^3\operatorname{PolyLog}(2,-iax)-\frac{1}{2}ic^3\operatorname{PolyLog}(2,iax)\end{aligned}$$

output
```
-11/12*a*c^3*x-7/36*a^3*c^3*x^3-1/30*a^5*c^3*x^5+11/12*c^3*arctan(a*x)+3/2
*a^2*c^3*x^2*arctan(a*x)+3/4*a^4*c^3*x^4*arctan(a*x)+1/6*a^6*c^3*x^6*arcta
n(a*x)+1/2*I*c^3*polylog(2,-I*a*x)-1/2*I*c^3*polylog(2,I*a*x)
```

3.169.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} \, dx = &-\frac{11}{12}ac^3x-\frac{7}{36}a^3c^3x^3-\frac{1}{30}a^5c^3x^5 \\ &+\frac{11}{12}c^3\arctan(ax)+\frac{3}{2}a^2c^3x^2\arctan(ax) \\ &+\frac{3}{4}a^4c^3x^4\arctan(ax)+\frac{1}{6}a^6c^3x^6\arctan(ax) \\ &+\frac{1}{2}ic^3\operatorname{PolyLog}(2,-iax)-\frac{1}{2}ic^3\operatorname{PolyLog}(2,iax)\end{aligned}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

output
```
(-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[
a*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 +
 (a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*
PolyLog[2, I*a*x]
```

3.169.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x}\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^5\arctan(ax)+3a^4c^3x^3\arctan(ax)+3a^2c^3x\arctan(ax)+\frac{c^3\arctan(ax)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a^6c^3x^6\arctan(ax)-\frac{1}{30}a^5c^3x^5+\frac{3}{4}a^4c^3x^4\arctan(ax)-\frac{7}{36}a^3c^3x^3+\frac{3}{2}a^2c^3x^2\arctan(ax)+\frac{11}{12}c^3\arctan(ax)+\frac{1}{2}ic^3\operatorname{PolyLog}(2,-iax)-\frac{1}{2}ic^3\operatorname{PolyLog}(2,iax)-\frac{11}{12}ac^3x$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

output
```
(-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[
a*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 +
 (a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*
PolyLog[2, I*a*x]
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.169.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{2a^5x^5}{5}+\frac{7a^{\cdots}}{}\right.}{\cdots}$
default	$\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{2a^5x^5}{5}+\frac{7a^{\cdots}}{}\right.}{\cdots}$
parts	$\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(x)-\frac{c^3a\left(\frac{2a^4x^5}{5}+\frac{7a^{\cdots}}{}\right.}{\cdots}$
risch	$-\frac{11a\,c^3x}{12}+\frac{3ic^3\ln(-iax+1)x^4a^4}{8}+\frac{3ic^3\ln(-iax+1)x^2a^2}{4}+\frac{ic^3\ln(-iax+1)x^6a^6}{12}-\frac{ic^3\operatorname{dilog}(-iax+1)}{2}+\frac{11c^{\cdots}}{}$
meijerg	$\frac{c^3\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4}+\frac{3c^3\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^{\cdots}}\right.}{5\sqrt{a^2x^2}}\right.}{4}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^6*c^3*x^6*arctan(a*x)+3/4*a^4*c^3*x^4*arctan(a*x)+3/2*a^2*c^3*x^2*ar
ctan(a*x)+c^3*arctan(a*x)*ln(a*x)-1/12*c^3*(2/5*a^5*x^5+7/3*a^3*x^3+11*a*x
-11*arctan(a*x)-6*I*ln(a*x)*ln(1+I*a*x)+6*I*ln(a*x)*ln(1-I*a*x)-6*I*dilog(
1+I*a*x)+6*I*dilog(1-I*a*x))
```

3.169.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x
, x)
```

3.169.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x}\,dx = c^3\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}(ax)\,dx + \int 3a^4x^3\operatorname{atan}(ax)\,dx + \int a^6x^5\operatorname{atan}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x,x)
```

output
```
c**3*(Integral(atan(a*x)/x, x) + Integral(3*a**2*x*atan(a*x), x) + Integra
l(3*a**4*x**3*atan(a*x), x) + Integral(a**6*x**5*atan(a*x), x))
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x}\,dx = &-\frac{1}{30}\,a^5c^3x^5 - \frac{7}{36}\,a^3c^3x^3 - \frac{11}{12}\,ac^3x \\ &- \frac{1}{4}\,\pi c^3\log\left(a^2x^2+1\right) + c^3\arctan(ax)\log(ax) \\ &- \frac{1}{2}i\,c^3\mathrm{Li}_2(i\,ax+1) + \frac{1}{2}i\,c^3\mathrm{Li}_2(-i\,ax+1) \\ &+ \frac{1}{12}\left(2\,a^6c^3x^6 + 9\,a^4c^3x^4 + 18\,a^2c^3x^2 + 11\,c^3\right)\arctan(ax)\end{aligned}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
-1/30*a^5*c^3*x^5 - 7/36*a^3*c^3*x^3 - 11/12*a*c^3*x - 1/4*pi*c^3*log(a^2*
x^2 + 1) + c^3*arctan(a*x)*log(a*x) - 1/2*I*c^3*dilog(I*a*x + 1) + 1/2*I*c
^3*dilog(-I*a*x + 1) + 1/12*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^
2 + 11*c^3)*arctan(a*x)
```

3.169.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} \, dx$$
$$= \begin{cases} 0 \\ 3\,a^2\,c^3\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{a^5\,c^3\,\left(\frac{x}{a^4}-\frac{\mathrm{atan}(a\,x)}{a^5}+\frac{x^5}{5}-\frac{x^3}{3\,a^2}\right)}{6}-\frac{3\,a\,c^3\,x}{2}-\frac{c^3\,(3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3)}{4}+\frac{3\,a^4\,c^3\,x^4\,\mathrm{atan}}{4} \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c^3*dilog(- a*x*1i + 1)*1i)/2 + (c^3*dilog
(a*x*1i + 1)*1i)/2 - (c^3*(3*atan(a*x) - 3*a*x + a^3*x^3))/4 - (a^5*c^3*(x
/a^4 - atan(a*x)/a^5 + x^5/5 - x^3/(3*a^2)))/6 - (3*a*c^3*x)/2 + 3*a^2*c^3
*atan(a*x)*(1/(2*a^2) + x^2/2) + (3*a^4*c^3*x^4*atan(a*x))/4 + (a^6*c^3*x^
6*atan(a*x))/6)
```

3.170 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} \, dx$

3.170.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} \, dx = -\frac{2}{5}a^3c^3x^2 - \frac{1}{20}a^5c^3x^4 - \frac{c^3 \arctan(ax)}{x} + 3a^2c^3x \arctan(ax) + a^4c^3x^3 \arctan(ax) + \frac{1}{5}a^6c^3x^5 \arctan(ax) + ac^3 \log(x) - \frac{8}{5}ac^3 \log(1+a^2x^2)$$

output
```
-2/5*a^3*c^3*x^2-1/20*a^5*c^3*x^4-c^3*arctan(a*x)/x+3*a^2*c^3*x*arctan(a*x
)+a^4*c^3*x^3*arctan(a*x)+1/5*a^6*c^3*x^5*arctan(a*x)+a*c^3*ln(x)-8/5*a*c^
3*ln(a^2*x^2+1)
```

3.170.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} \, dx = \frac{c^3(4(-5+15a^2x^2+5a^4x^4+a^6x^6) \arctan(ax) - ax(8a^2x^2+a^4x^4-20\log(x)+32\log(1+a^2x^2)))}{20x}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

output
```
(c^3*(4*(-5 + 15*a^2*x^2 + 5*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(8*a^2*x
^2 + a^4*x^4 - 20*Log[x] + 32*Log[1 + a^2*x^2])))/(20*x)
```

3.170.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x^2}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^4\arctan(ax)+3a^4c^3x^2\arctan(ax)+3a^2c^3\arctan(ax)+\frac{c^3\arctan(ax)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^6c^3x^5\arctan(ax)-\frac{1}{20}a^5c^3x^4+a^4c^3x^3\arctan(ax)-\frac{2}{5}a^3c^3x^2+3a^2c^3x\arctan(ax)-\frac{8}{5}ac^3\log\left(a^2x^2+1\right)-\frac{c^3\arctan(ax)}{x}+ac^3\log(x)$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

output
```
(-2*a^3*c^3*x^2)/5 - (a^5*c^3*x^4)/20 - (c^3*ArcTan[a*x])/x + 3*a^2*c^3*x*
ArcTan[a*x] + a^4*c^3*x^3*ArcTan[a*x] + (a^6*c^3*x^5*ArcTan[a*x])/5 + a*c^
3*Log[x] - (8*a*c^3*Log[1 + a^2*x^2])/5
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.170.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

method	result
parts	$\frac{a^6c^3x^5\arctan(ax)}{5}+a^4c^3x^3\arctan(ax)+3a^2c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{x}-\frac{c^3a\left(\frac{a^4x^4}{4}+2a^2x^2+\right.}{}$
derivativedivides	$a\left(\frac{a^5c^3x^5\arctan(ax)}{5}+a^3c^3x^3\arctan(ax)+3a\,c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{ax}-\frac{c^3\left(\frac{a^4x^4}{4}+2a^2x\right.}{}\right.$
default	$a\left(\frac{a^5c^3x^5\arctan(ax)}{5}+a^3c^3x^3\arctan(ax)+3a\,c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{ax}-\frac{c^3\left(\frac{a^4x^4}{4}+2a^2x\right.}{}\right.$
parallelrisch	$\frac{4a^6c^3x^6\arctan(ax)-a^5c^3x^5+20a^4c^3x^4\arctan(ax)-8a^3c^3x^3+60a^2c^3x^2\arctan(ax)+20c^3a\ln(x)x-32c^3a\ln(a^2x^2+1)x}{20x}$
risch	$-\frac{ic^3(a^6x^6+5a^4x^4+15a^2x^2-5)\ln(iax+1)}{10x}+\frac{ic^3(2a^6x^6\ln(-iax+1)+ia^5x^5+10x^4\ln(-iax+1)a^4+8ia^3x^3+30a^2x^2\ln}{20x}$
meijerg	$\frac{a\,c^3\left(\frac{a^2x^2(-3a^2x^2+6)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}-\frac{2\ln(a^2x^2+1)}{5}\right)}{4}+\frac{3a\,c^3\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1}{3}\right.}{4}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^6*c^3*x^5*arctan(a*x)+a^4*c^3*x^3*arctan(a*x)+3*a^2*c^3*x*arctan(a*x
)-c^3*arctan(a*x)/x-1/5*c^3*a*(1/4*a^4*x^4+2*a^2*x^2+8*ln(a^2*x^2+1)-5*ln(
x))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^2}\,dx =$$
$$-\frac{a^5c^3x^5+8\,a^3c^3x^3+32\,ac^3x\log(a^2x^2+1)-20\,ac^3x\log(x)-4\,(a^6c^3x^6+5\,a^4c^3x^4+15\,a^2c^3x^2-5\,c^3)\,a}{20\,x}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/20*(a^5*c^3*x^5 + 8*a^3*c^3*x^3 + 32*a*c^3*x*log(a^2*x^2 + 1) - 20*a*c^
3*x*log(x) - 4*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arct
an(a*x))/x
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} \, dx$$
$$= \begin{cases} \frac{a^6c^3x^5 \operatorname{atan}(ax)}{5} - \frac{a^5c^3x^4}{20} + a^4c^3x^3 \operatorname{atan}(ax) - \frac{2a^3c^3x^2}{5} + 3a^2c^3x \operatorname{atan}(ax) + ac^3 \log(x) - \frac{8ac^3 \log\left(x^2+\frac{1}{a^2}\right)}{5} - \frac{c}{} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x**2,x)
```

output
```
Piecewise((a**6*c**3*x**5*atan(a*x)/5 - a**5*c**3*x**4/20 + a**4*c**3*x**3
*atan(a*x) - 2*a**3*c**3*x**2/5 + 3*a**2*c**3*x*atan(a*x) + a*c**3*log(x)
- 8*a*c**3*log(x**2 + a**(-2))/5 - c**3*atan(a*x)/x, Ne(a, 0)), (0, True))
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} \, dx$$
$$= -\frac{1}{20} \left(a^4c^3x^4 + 8\,a^2c^3x^2 + 32\,c^3 \log\left(a^2x^2+1\right) - 20\,c^3 \log(x)\right)a + \frac{1}{5}\left(a^6c^3x^5 + 5\,a^4c^3x^3 + 15\,a^2c^3x - \frac{5\,c^3}{x}\right) \arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
-1/20*(a^4*c^3*x^4 + 8*a^2*c^3*x^2 + 32*c^3*log(a^2*x^2 + 1) - 20*c^3*log(
x))*a + 1/5*(a^6*c^3*x^5 + 5*a^4*c^3*x^3 + 15*a^2*c^3*x - 5*c^3/x)*arctan(
a*x)
```

3.170.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2}\,dx = -\frac{c^3\left(\operatorname{atan}(a\,x)+\frac{2\,a^3\,x^3}{5}+\frac{a^5\,x^5}{20}-a\,x\,\ln(x)-3\,a^2\,x^2\,\operatorname{atan}(a\,x)-a^4\,x^4\,\operatorname{atan}(a\,x)-\frac{a^6\,x^6\,\operatorname{atan}(a\,x)}{5}+\frac{8\,a\,x\,\ln(a}{5}\right.}{x}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
-(c^3*(atan(a*x) + (2*a^3*x^3)/5 + (a^5*x^5)/20 - a*x*log(x) - 3*a^2*x^2*a
tan(a*x) - a^4*x^4*atan(a*x) - (a^6*x^6*atan(a*x))/5 + (8*a*x*log(a^2*x^2
+ 1))/5))/x
```

3.171 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} dx$

3.171.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} dx = &-\frac{ac^3}{2x} - \frac{5}{4}a^3c^3x - \frac{1}{12}a^5c^3x^3 + \frac{3}{4}a^2c^3 \arctan(ax) \\ &- \frac{c^3 \arctan(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2 \arctan(ax) + \frac{1}{4}a^6c^3x^4 \arctan(ax) \\ &+ \frac{3}{2}ia^2c^3 \operatorname{PolyLog}(2,-iax) - \frac{3}{2}ia^2c^3 \operatorname{PolyLog}(2,iax)\end{aligned}$$

output
```
-1/2*a*c^3/x-5/4*a^3*c^3*x-1/12*a^5*c^3*x^3+3/4*a^2*c^3*arctan(a*x)-1/2*c^
3*arctan(a*x)/x^2+3/2*a^4*c^3*x^2*arctan(a*x)+1/4*a^6*c^3*x^4*arctan(a*x)+
3/2*I*a^2*c^3*polylog(2,-I*a*x)-3/2*I*a^2*c^3*polylog(2,I*a*x)
```

3.171.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} dx = &-\frac{5}{4}a^3c^3x - \frac{1}{12}a^5c^3x^3 + \frac{5}{4}a^2c^3 \arctan(ax) - \frac{c^3 \arctan(ax)}{2x^2} \\ &+ \frac{3}{2}a^4c^3x^2 \arctan(ax) + \frac{1}{4}a^6c^3x^4 \arctan(ax) \\ &- \frac{ac^3 \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x} \\ &+ \frac{3}{2}ia^2c^3 \operatorname{PolyLog}(2,-iax) - \frac{3}{2}ia^2c^3 \operatorname{PolyLog}(2,iax)\end{aligned}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```

output
```
(-5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (5*a^2*c^3*ArcTan[a*x])/4 - (c^3*Arc
Tan[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c^3*x^4*ArcTan[a*
x])/4 - (a*c^3*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2*x) + ((3*I)
/2)*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)*a^2*c^3*PolyLog[2, I*a*x]
```

3.171.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x^3}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^3\arctan(ax)+3a^4c^3x\arctan(ax)+\frac{3a^2c^3\arctan(ax)}{x}+\frac{c^3\arctan(ax)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^6c^3x^4\arctan(ax)-\frac{1}{12}a^5c^3x^3+\frac{3}{2}a^4c^3x^2\arctan(ax)-\frac{5}{4}a^3c^3x+\frac{3}{4}a^2c^3\arctan(ax)+\frac{3}{2}ia^2c^3\,\text{PolyLog}(2,-iax)-\frac{3}{2}ia^2c^3\,\text{PolyLog}(2,iax)-\frac{c^3\arctan(ax)}{2x^2}-\frac{ac^3}{2x}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(a*c^3)/x - (5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (3*a^2*c^3*ArcTan[a*
x])/4 - (c^3*ArcTan[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c
^3*x^4*ArcTan[a*x])/4 + ((3*I)/2)*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)
*a^2*c^3*PolyLog[2, I*a*x]
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.171.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

method	result
derivativedivides	$a^2\left(\frac{a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}-\frac{c^3\arctan(ax)}{2a^2x^2}+3c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{a^3x^3}{3}+5a\right.}{}\right.$
default	$a^2\left(\frac{a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}-\frac{c^3\arctan(ax)}{2a^2x^2}+3c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{a^3x^3}{3}+5a\right.}{}\right.$
parts	$\frac{a^6c^3x^4\arctan(ax)}{4}+\frac{3a^4c^3x^2\arctan(ax)}{2}+3c^3\arctan(ax)a^2\ln(x)-\frac{c^3\arctan(ax)}{2x^2}-\frac{c^3a\left(\frac{a^4x^3}{3}+5a^2x\right.}{}$
meijerg	$\frac{a^2c^3\left(\frac{ax(-5a^2x^2+15)}{15}-\frac{ax(-5a^4x^4+5)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4}+\frac{3a^2c^3\left(-2ax+\frac{2(3a^2x^2+3)\arctan(ax)}{3}\right)}{4}+\frac{3a^2c^3\left(-\frac{2iax}{}\right.}{}$
risch	$-\frac{ic^3a^2\ln(iax)}{4}+\frac{ic^3\ln(iax+1)}{4x^2}+\frac{3ic^3a^4\ln(-iax+1)x^2}{4}+\frac{3a^2c^3\arctan(ax)}{4}-\frac{a^5c^3x^3}{12}-\frac{5a^3c^3x}{4}+\frac{ic^3a^6\ln(}{}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/4*a^4*c^3*x^4*arctan(a*x)+3/2*a^2*c^3*x^2*arctan(a*x)-1/2*c^3*arcta
n(a*x)/a^2/x^2+3*c^3*arctan(a*x)*ln(a*x)-1/4*c^3*(1/3*a^3*x^3+5*a*x-3*arct
an(a*x)+2/a/x-6*I*ln(a*x)*ln(1+I*a*x)+6*I*ln(a*x)*ln(1-I*a*x)-6*I*dilog(1+
I*a*x)+6*I*dilog(1-I*a*x)))
```

3.171.5 Fricas [F]

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)}{x^3}\,dx = \int \frac{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x
^3, x)
```

3.171.6 Sympy [F]

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)}{x^3}\,dx = c^3\left(\int \frac{\operatorname{atan}\left(ax\right)}{x^3}\,dx + \int \frac{3a^2\operatorname{atan}\left(ax\right)}{x}\,dx + \int 3a^4x\operatorname{atan}\left(ax\right)dx + \int a^6x^3\operatorname{atan}\left(ax\right)dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x**3,x)
```

output
```
c**3*(Integral(atan(a*x)/x**3, x) + Integral(3*a**2*atan(a*x)/x, x) + Inte
gral(3*a**4*x*atan(a*x), x) + Integral(a**6*x**3*atan(a*x), x))
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)}{x^3}\,dx = -\frac{a^5c^3x^5 + 15\,a^3c^3x^3 + 9\,\pi a^2c^3x^2\log\left(a^2x^2+1\right) - 36\,a^2c^3x^2\arctan\left(ax\right)\log\left(ax\right) + 18i\,a^2c^3x^2\mathrm{Li}_2(i\,ax+1)}{12\,x^2}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
-1/12*(a^5*c^3*x^5 + 15*a^3*c^3*x^3 + 9*pi*a^2*c^3*x^2*log(a^2*x^2 + 1) -
36*a^2*c^3*x^2*arctan(a*x)*log(a*x) + 18*I*a^2*c^3*x^2*dilog(I*a*x + 1) -
18*I*a^2*c^3*x^2*dilog(-I*a*x + 1) + 6*a*c^3*x - 3*(a^6*c^3*x^6 + 6*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 - 2*c^3)*arctan(a*x))/x^2
```

3.171.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} \, dx = \begin{cases} 0 \\ 3\,a^4\,c^3\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{a^2\,c^3\,\left(3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3\right)}{12}-\frac{c^3\,\mathrm{atan}(a\,x)}{2\,x^2}-\frac{c^3\,\left(a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a}-\frac{3\,a^3\,c^3\,x}{2}+\frac{a^6\,c^3}{} \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x^3,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (3*a^3*c^3*x)/2 - (a^2*c^3*(3*atan(a*x) - 3
*a*x + a^3*x^3))/12 - (c^3*atan(a*x))/(2*x^2) - (a^2*c^3*dilog(- a*x*1i +
1)*3i)/2 + (a^2*c^3*dilog(a*x*1i + 1)*3i)/2 - (c^3*(a^3*atan(a*x) + a^2/x)
)/(2*a) + 3*a^4*c^3*atan(a*x)*(1/(2*a^2) + x^2/2) + (a^6*c^3*x^4*atan(a*x)
)/4)
```

3.172 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4} dx$

3.172.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4} dx = -\frac{ac^3}{6x^2} - \frac{1}{6}a^5c^3x^2 - \frac{c^3\arctan(ax)}{3x^3} - \frac{3a^2c^3\arctan(ax)}{x} + 3a^4c^3x\arctan(ax) + \frac{1}{3}a^6c^3x^3\arctan(ax) + \frac{8}{3}a^3c^3\log(x) - \frac{8}{3}a^3c^3\log\left(1+a^2x^2\right)$$

output
```
-1/6*a*c^3/x^2-1/6*a^5*c^3*x^2-1/3*c^3*arctan(a*x)/x^3-3*a^2*c^3*arctan(a*
x)/x+3*a^4*c^3*x*arctan(a*x)+1/3*a^6*c^3*x^3*arctan(a*x)+8/3*a^3*c^3*ln(x)
-8/3*a^3*c^3*ln(a^2*x^2+1)
```

3.172.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4} dx = \frac{c^3(2(-1-9a^2x^2+9a^4x^4+a^6x^6)\arctan(ax)-ax(1+a^4x^4-16a^2x^2\log(x)+16a^2x^2\log(1+a^2x^2)))}{6x^3}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

output
```
(c^3*(2*(-1 - 9*a^2*x^2 + 9*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(1 + a^4*
x^4 - 16*a^2*x^2*Log[x] + 16*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

3.172.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x^4}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^2\arctan(ax)+3a^4c^3\arctan(ax)+\frac{3a^2c^3\arctan(ax)}{x^2}+\frac{c^3\arctan(ax)}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^6c^3x^3\arctan(ax)-\frac{1}{6}a^5c^3x^2+3a^4c^3x\arctan(ax)+\frac{8}{3}a^3c^3\log(x)-\frac{3a^2c^3\arctan(ax)}{x}-\frac{8}{3}a^3c^3\log\left(a^2x^2+1\right)-\frac{c^3\arctan(ax)}{3x^3}-\frac{ac^3}{6x^2}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

output
```
-1/6*(a*c^3)/x^2 - (a^5*c^3*x^2)/6 - (c^3*ArcTan[a*x])/(3*x^3) - (3*a^2*c^
3*ArcTan[a*x])/x + 3*a^4*c^3*x*ArcTan[a*x] + (a^6*c^3*x^3*ArcTan[a*x])/3 +
 (8*a^3*c^3*Log[x])/3 - (8*a^3*c^3*Log[1 + a^2*x^2])/3
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.172.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result
parts	$\frac{a^6c^3x^3\arctan(ax)}{3}+3a^4c^3x\arctan(ax)-\frac{3a^2c^3\arctan(ax)}{x}-\frac{c^3\arctan(ax)}{3x^3}-\frac{c^3a\left(\frac{a^4x^2}{2}+8a^2\ln(a^2x^2+1\right.}{3}$
derivativedivides	$a^3\left(\frac{a^3c^3x^3\arctan(ax)}{3}+3a\,c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{3a^3x^3}-\frac{3c^3\arctan(ax)}{ax}-\frac{c^3\left(8\ln(a^2x^2+1)+\frac{a^2x^2}{2}\right.}{3}\right.$
default	$a^3\left(\frac{a^3c^3x^3\arctan(ax)}{3}+3a\,c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{3a^3x^3}-\frac{3c^3\arctan(ax)}{ax}-\frac{c^3\left(8\ln(a^2x^2+1)+\frac{a^2x^2}{2}\right.}{3}\right.$
parallelrisch	$\frac{2a^6c^3x^6\arctan(ax)-a^5c^3x^5+18a^4c^3x^4\arctan(ax)+16a^3c^3\ln(x)x^3-16a^3c^3\ln(a^2x^2+1)x^3+a^3c^3x^3-18a^2c^3x^2\arctan(}{6x^3}$
risch	$-\frac{ic^3(a^6x^6+9a^4x^4-9a^2x^2-1)\ln(iax+1)}{6x^3}+\frac{ic^3(a^6x^6\ln(-iax+1)+ia^5x^5+9x^4\ln(-iax+1)a^4-16i\ln(x)a^3x^3+16i\ln}{6x^3}$
meijerg	$\frac{a^3c^3\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{3}\right)}{4}+\frac{3a^3c^3\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln\left(a^2x^2+1\right)\right)}{4}+\frac{3a^3c^3\left(4\right.}{}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^6*c^3*x^3*arctan(a*x)+3*a^4*c^3*x*arctan(a*x)-3*a^2*c^3*arctan(a*x)/
x-1/3*c^3*arctan(a*x)/x^3-1/3*c^3*a*(1/2*a^4*x^2+8*a^2*ln(a^2*x^2+1)+1/2/x
^2-8*a^2*ln(x))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^4}\,dx = -\frac{a^5c^3x^5+16\,a^3c^3x^3\log\left(a^2x^2+1\right)-16\,a^3c^3x^3\log\left(x\right)+ac^3x-2\left(a^6c^3x^6+9\,a^4c^3x^4-9\,a^2c^3x^2-c^3\right)\mathrm{arc}}{6\,x^3}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a^5*c^3*x^5 + 16*a^3*c^3*x^3*log(a^2*x^2 + 1) - 16*a^3*c^3*x^3*log(x
) + a*c^3*x - 2*(a^6*c^3*x^6 + 9*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan
(a*x))/x^3
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4}\,dx$$
$$= \begin{cases} \frac{a^6c^3x^3\operatorname{atan}(ax)}{3} - \frac{a^5c^3x^2}{6} + 3a^4c^3x\operatorname{atan}(ax) + \frac{8a^3c^3\log(x)}{3} - \frac{8a^3c^3\log\left(x^2+\frac{1}{a^2}\right)}{3} - \frac{3a^2c^3\operatorname{atan}(ax)}{x} - \frac{ac^3}{6x^2} - \frac{c^3\operatorname{atan}(ax)}{3x^3} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x**4,x)
```

output
```
Piecewise((a**6*c**3*x**3*atan(a*x)/3 - a**5*c**3*x**2/6 + 3*a**4*c**3*x*a
tan(a*x) + 8*a**3*c**3*log(x)/3 - 8*a**3*c**3*log(x**2 + a**(-2))/3 - 3*a*
*2*c**3*atan(a*x)/x - a*c**3/(6*x**2) - c**3*atan(a*x)/(3*x**3), Ne(a, 0))
, (0, True))
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4}\,dx$$
$$= -\frac{1}{6}\left(a^4c^3x^2 + 16\,a^2c^3\log\left(a^2x^2+1\right) - 16\,a^2c^3\log(x) + \frac{c^3}{x^2}\right)a$$
$$+ \frac{1}{3}\left(a^6c^3x^3 + 9\,a^4c^3x - \frac{9\,a^2c^3x^2 + c^3}{x^3}\right)\arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(a^4*c^3*x^2 + 16*a^2*c^3*log(a^2*x^2 + 1) - 16*a^2*c^3*log(x) + c^3/
x^2)*a + 1/3*(a^6*c^3*x^3 + 9*a^4*c^3*x - (9*a^2*c^3*x^2 + c^3)/x^3)*arcta
n(a*x)
```

3.172.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.172.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4}\,dx =$$
$$-\frac{c^3\,(2\,\mathrm{atan}(a\,x)+a\,x-a^3\,x^3+a^5\,x^5+18\,a^2\,x^2\,\mathrm{atan}(a\,x)-18\,a^4\,x^4\,\mathrm{atan}(a\,x)-2\,a^6\,x^6\,\mathrm{atan}(a\,x)+16\,a}{6\,x^3}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x^4,x)
```

output
```
-(c^3*(2*atan(a*x) + a*x - a^3*x^3 + a^5*x^5 + 18*a^2*x^2*atan(a*x) - 18*a
^4*x^4*atan(a*x) - 2*a^6*x^6*atan(a*x) + 16*a^3*x^3*log(a^2*x^2 + 1) - 16*
a^3*x^3*log(x)))/(6*x^3)
```

3.173 $\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx$

3.173.1 Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx=-\frac{x^2}{6a^3c}-\frac{x\arctan(ax)}{a^4c}+\frac{x^3\arctan(ax)}{3a^2c}+\frac{\arctan(ax)^2}{2a^5c}+\frac{2\log(1+a^2x^2)}{3a^5c}$$

output
```
-1/6*x^2/a^3/c-x*arctan(a*x)/a^4/c+1/3*x^3*arctan(a*x)/a^2/c+1/2*arctan(a*
x)^2/a^5/c+2/3*ln(a^2*x^2+1)/a^5/c
```

3.173.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx =\frac{-a^2x^2+2ax(-3+a^2x^2)\arctan(ax)+3\arctan(ax)^2+4\log(1+a^2x^2)}{6a^5c}$$

input
```
Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
(-(a^2*x^2) + 2*a*x*(-3 + a^2*x^2)*ArcTan[a*x] + 3*ArcTan[a*x]^2 + 4*Log[1
 + a^2*x^2])/(6*a^5*c)
```

3.173.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5451, 27, 5361, 243, 49, 2009, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \ 5451$$

$$\frac{\int x^2 \arctan(ax)dx}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int x^2 \arctan(ax)dx}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 5361$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{3}a\int \frac{x^3}{a^2x^2+1}dx}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{6}a\int \frac{x^2}{a^2x^2+1}dx^2}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 49$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{6}a\int \left(\frac{1}{a^2} - \frac{1}{a^2(a^2x^2+1)}\right)dx^2}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{6}a\left(\frac{x^2}{a^2} - \frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 5451$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{6}a\left(\frac{x^2}{a^2} - \frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c} - \frac{\frac{\int \arctan(ax)dx}{a^2} - \frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \ 5345$$

$$\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c}-\frac{\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6)/(a^2*c) - (
-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2)/(a^
2*c)
```

3.173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.173. $\int \frac{x^4\arctan(ax)}{c+a^2cx^2}\,dx$

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.173.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{2 \arctan(ax) x^3 a^3 - a^2 x^2 - 6x \arctan(ax) a + 3 \arctan(ax)^2 + 4 \ln\left(a^2 x^2 + 1\right)}{6c\, a^5}$
derivativedivides	$\frac{\frac{\arctan(ax) a^3 x^3}{3c} - \frac{\arctan(ax) a x}{c} + \frac{\arctan(ax)^2}{c} - \frac{\frac{a^2 x^2}{2} - 2\ln\left(a^2 x^2 + 1\right) + \frac{3 \arctan(ax)^2}{2}}{3c}}{a^5}$
default	$\frac{\frac{\arctan(ax) a^3 x^3}{3c} - \frac{\arctan(ax) a x}{c} + \frac{\arctan(ax)^2}{c} - \frac{\frac{a^2 x^2}{2} - 2\ln\left(a^2 x^2 + 1\right) + \frac{3 \arctan(ax)^2}{2}}{3c}}{a^5}$
parts	$\frac{x^3 \arctan(ax)}{3a^2 c} - \frac{x \arctan(ax)}{a^4 c} + \frac{\arctan(ax)^2}{a^5 c} - \frac{\frac{\arctan(ax)^2}{2a^5} + \frac{\frac{a^2 x^2}{2} - 2\ln\left(a^2 x^2 + 1\right)}{3a^5}}{c}$
risch	$-\frac{\ln(iax+1)^2}{8a^5 c} - \frac{i\left(2a^3 x^3 + 3i \ln(-iax+1) - 6ax\right) \ln(iax+1)}{12c\, a^5} + \frac{i x^3 \ln(-iax+1)}{6a^2 c} - \frac{i x \ln(-iax+1)}{2a^4 c} - \frac{x^2}{6a^3 c} -$

input
```
int(x^4*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arctan(a*x)*x^3*a^3-a^2*x^2-6*x*arctan(a*x)*a+3*arctan(a*x)^2+4*ln(
a^2*x^2+1))/c/a^5
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = -\frac{a^2x^2-2\,(a^3x^3-3\,ax)\arctan{(ax)}-3\,\arctan{(ax)}^2-4\,\log{(a^2x^2+1)}}{6\,a^5c}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/6*(a^2*x^2 - 2*(a^3*x^3 - 3*a*x)*arctan(a*x) - 3*arctan(a*x)^2 - 4*log(
a^2*x^2 + 1))/(a^5*c)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = \begin{cases} \frac{x^3\operatorname{atan}{(ax)}}{3a^2c} - \frac{x^2}{6a^3c} - \frac{x\operatorname{atan}{(ax)}}{a^4c} + \frac{2\log{\left(x^2+\frac{1}{a^2}\right)}}{3a^5c} + \frac{\operatorname{atan}^2{(ax)}}{2a^5c} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((x**3*atan(a*x)/(3*a**2*c) - x**2/(6*a**3*c) - x*atan(a*x)/(a**4
*c) + 2*log(x**2 + a**(-2))/(3*a**5*c) + atan(a*x)**2/(2*a**5*c), Ne(a, 0)
), (0, True))
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = \frac{1}{3}\left(\frac{a^2x^3-3\,x}{a^4c}+\frac{3\,\arctan\left(ax\right)}{a^5c}\right)\arctan\left(ax\right) - \frac{a^2x^2+3\,\arctan\left(ax\right)^2-4\,\log\left(a^2x^2+1\right)}{6\,a^5c}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3*((a^2*x^3 - 3*x)/(a^4*c) + 3*arctan(a*x)/(a^5*c))*arctan(a*x) - 1/6*(a
^2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/(a^5*c)
```

3.173.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x^4\arctan\left(ax\right)}{a^2cx^2+c}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = \frac{2\,\ln\left(a^2\,x^2+1\right)}{3\,a^5\,c} - a^2\,\mathrm{atan}(a\,x)\,\left(\frac{x}{a^6\,c}-\frac{x^3}{3\,a^4\,c}\right) - \frac{x^2}{6\,a^3\,c} + \frac{\mathrm{atan}(a\,x)^2}{2\,a^5\,c}$$

input
```
int((x^4*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
(2*log(a^2*x^2 + 1))/(3*a^5*c) - a^2*atan(a*x)*(x/(a^6*c) - x^3/(3*a^4*c))
 - x^2/(6*a^3*c) + atan(a*x)^2/(2*a^5*c)
```

3.174 $\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} dx$

3.174.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} dx = -\frac{x}{2a^3c} + \frac{\arctan(ax)}{2a^4c} + \frac{x^2 \arctan(ax)}{2a^2c} + \frac{i \arctan(ax)^2}{2a^4c} + \frac{\arctan(ax) \log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{i \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{2a^4c}$$

output
```
-1/2*x/a^3/c+1/2*arctan(a*x)/a^4/c+1/2*x^2*arctan(a*x)/a^2/c+1/2*I*arctan(
a*x)^2/a^4/c+arctan(a*x)*ln(2/(1+I*a*x))/a^4/c+1/2*I*polylog(2,1-2/(1+I*a*
x))/a^4/c
```

3.174.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} dx = -\frac{x}{2a^3c} + \frac{\arctan(ax)}{2a^4c} + \frac{x^2 \arctan(ax)}{2a^2c} + \frac{i \arctan(ax)^2}{2a^4c} + \frac{\arctan(ax) \log\left(\frac{2i}{i-ax}\right)}{a^4c} + \frac{i \operatorname{PolyLog}\left(2, -\frac{i+ax}{i-ax}\right)}{2a^4c}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
-1/2*x/(a^3*c) + ArcTan[a*x]/(2*a^4*c) + (x^2*ArcTan[a*x])/(2*a^2*c) + ((I
/2)*ArcTan[a*x]^2)/(a^4*c) + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^4*c) +
((I/2)*PolyLog[2, -((I + a*x)/(I - a*x))])/(a^4*c)
```

3.174.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5451, 27, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x \arctan(ax)dx}{a^2c} - \frac{\int \frac{x \arctan(ax)}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x \arctan(ax)dx}{a^2c} - \frac{\int \frac{x \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2 \arctan(ax) - \frac{1}{2}a \int \frac{x^2}{a^2x^2+1}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}x^2 \arctan(ax) - \frac{1}{2}a \left(\frac{x}{a^2} - \frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2c} - \frac{\int \frac{x \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}x^2 \arctan(ax) - \frac{1}{2}a \left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right)}{a^2c} - \frac{\int \frac{x \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5455}$$

$$\frac{\frac{1}{2}x^2 \arctan(ax) - \frac{1}{2}a \left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right)}{a^2c} - \frac{-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a} - \frac{i \arctan(ax)^2}{2a^2}}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$\frac{\frac{1}{2}x^2 \arctan(ax) - \frac{1}{2}a \left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right)}{a^2c} - \frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a} - \int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a} - \frac{i \arctan(ax)^2}{2a^2}}{a^2c}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2c}-\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c}$$

$$\Big\downarrow \; 2752$$

$$\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2c}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/(a^2*c) - (((-1/2*
I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyL
og[2, 1 - 2/(1 + I*a*x)])/a)/a)/(a^2*c)
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.174.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

method	result
parts	$\frac{x^{2}\arctan(ax)}{2a^{2}c}-\frac{\arctan(ax)\ln(a^{2}x^{2}+1)}{2c\,a^{4}}-\frac{a\left(\frac{x}{a^{4}}-\frac{\arctan(ax)}{a^{5}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^{2}\_Z^{2}+1\right)}\frac{2\ln(x-\_\alpha)\ln\left(a^{2}x^{2}+1\right)-a^{2}\left(\frac{1}{}\right.}{}}{}\right)}{2c}$
derivativedivides	$\frac{\frac{\arctan(ax)a^{2}x^{2}}{2c}-\frac{\arctan(ax)\ln\left(a^{2}x^{2}+1\right)}{2c}-\frac{ax-\arctan(ax)+\frac{i\left(\ln(ax-i)\ln\left(a^{2}x^{2}+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)\right.}{2}}{}}{a^{4}}$
default	$\frac{\frac{\arctan(ax)a^{2}x^{2}}{2c}-\frac{\arctan(ax)\ln\left(a^{2}x^{2}+1\right)}{2c}-\frac{ax-\arctan(ax)+\frac{i\left(\ln(ax-i)\ln\left(a^{2}x^{2}+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)\right.}{2}}{}}{a^{4}}$
risch	$\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c\,a^{4}}+\frac{i\ln(-iax+1)x^{2}}{4c\,a^{2}}+\frac{i\ln(-iax+1)}{4c\,a^{4}}-\frac{x}{2a^{3}c}-\frac{i\ln(iax+1)x^{2}}{4c\,a^{2}}-\frac{i\ln(iax+1)}{4c\,a^{4}}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}\right.}{4c\,a}$

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(a*x)/a^2/c-1/2/c*arctan(a*x)/a^4*ln(a^2*x^2+1)-1/2/c*a*(1/a
^4*x-1/a^5*arctan(a*x)-1/4/a^6*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-
a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_
alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha)),_alpha=RootOf(_Z^2*a^2+1)))
```

3.174.5 Fricas [F]

$$\int \frac{x^{3}\arctan(ax)}{c+a^{2}cx^{2}}\,dx=\int \frac{x^{3}\arctan\left(ax\right)}{a^{2}cx^{2}+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.174.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{c + a^2cx^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)/(a**2*x**2 + 1), x)/c
```

3.174.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.174.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\, x)}{c\, a^2\, x^2 + c} \, dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2), x)
```

3.175 $\int \frac{x^2 \arctan(ax)}{c+a^2cx^2} dx$

3.175.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2} dx = \frac{x \arctan(ax)}{a^2c} - \frac{\arctan(ax)^2}{2a^3c} - \frac{\log(1+a^2x^2)}{2a^3c}$$

output
```
x*arctan(a*x)/a^2/c-1/2*arctan(a*x)^2/a^3/c-1/2*ln(a^2*x^2+1)/a^3/c
```

3.175.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2} dx = \frac{x \arctan(ax)}{a^2c} - \frac{\arctan(ax)^2}{2a^3c} - \frac{\log(1+a^2x^2)}{2a^3c}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
(x*ArcTan[a*x])/(a^2*c) - ArcTan[a*x]^2/(2*a^3*c) - Log[1 + a^2*x^2]/(2*a^
3*c)
```

3.175.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5451, 27, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{a^2cx^2+c}dx$$
$$\Big\downarrow \text{5451}$$
$$\frac{\int \arctan(ax)dx}{a^2c}-\frac{\int \frac{\arctan(ax)}{c(a^2x^2+1)}dx}{a^2}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \arctan(ax)dx}{a^2c}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \text{5345}$$
$$\frac{x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx}{a^2c}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \text{240}$$
$$\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2c}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \text{5419}$$
$$\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2c}-\frac{\arctan(ax)^2}{2a^3c}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
-1/2*ArcTan[a*x]^2/(a^3*c) + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/(a^2
*c)
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.175.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{2x\arctan(ax)a-\arctan(ax)^2-\ln(a^2x^2+1)}{2c\,a^3}$	38
derivativedivides	$\frac{\frac{\arctan(ax)ax}{c}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{c}}{a^3}$	53
default	$\frac{\frac{\arctan(ax)ax}{c}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{c}}{a^3}$	53
parts	$\frac{x\arctan(ax)}{a^2c}-\frac{\arctan(ax)^2}{a^3c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2a^3}-\frac{\arctan(ax)^2}{2a^3}}{c}$	60
risch	$\frac{\ln(iax+1)^2}{8a^3c}-\frac{i(-i\ln(-iax+1)+2ax)\ln(iax+1)}{4c\,a^3}+\frac{\ln(-iax+1)^2}{8c\,a^3}+\frac{ix\ln(-iax+1)}{2c\,a^2}-\frac{\ln(-a^2x^2-1)}{2c\,a^3}$	108

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*x*arctan(a*x)*a-arctan(a*x)^2-ln(a^2*x^2+1))/c/a^3
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2} \, dx = \frac{2\, ax \arctan{(ax)} - \arctan{(ax)}^2 - \log{(a^2x^2+1)}}{2\, a^3c}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*x*arctan(a*x) - arctan(a*x)^2 - log(a^2*x^2 + 1))/(a^3*c)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2} \, dx = \begin{cases} \frac{x \operatorname{atan}{(ax)}}{a^2c} - \frac{\log{\left(x^2+\frac{1}{a^2}\right)}}{2a^3c} - \frac{\operatorname{atan}^2{(ax)}}{2a^3c} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((x*atan(a*x)/(a**2*c) - log(x**2 + a**(-2))/(2*a**3*c) - atan(a*
x)**2/(2*a**3*c), Ne(a, 0)), (0, True))
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2}\,dx = \left(\frac{x}{a^2c} - \frac{\arctan(ax)}{a^3c}\right)\arctan(ax) + \frac{\arctan(ax)^2 - \log(a^2x^2+1)}{2\,a^3c}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
(x/(a^2*c) - arctan(a*x)/(a^3*c))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a
^2*x^2 + 1))/(a^3*c)
```

3.175.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x^2 \arctan(ax)}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.175.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2}\,dx = -\frac{\mathrm{atan}(a\,x)^2 - 2\,a\,x\,\mathrm{atan}(a\,x) + \ln(a^2\,x^2+1)}{2\,a^3\,c}$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
-(log(a^2*x^2 + 1) + atan(a*x)^2 - 2*a*x*atan(a*x))/(2*a^3*c)
```

3.176 $\int \frac{x \arctan(ax)}{c+a^2cx^2} dx$

3.176.1 Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{x \arctan(ax)}{c+a^2cx^2} dx = -\frac{i \arctan(ax)^2}{2a^2c} - \frac{\arctan(ax) \log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{i \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{2a^2c}$$

output
```
-1/2*I*arctan(a*x)^2/a^2/c-arctan(a*x)*ln(2/(1+I*a*x))/a^2/c-1/2*I*polylog
(2,1-2/(1+I*a*x))/a^2/c
```

3.176.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{x \arctan(ax)}{c+a^2cx^2} dx = -\frac{i \arctan(ax)^2}{2a^2c} - \frac{\arctan(ax) \log\left(\frac{2i}{i-ax}\right)}{a^2c} - \frac{i \operatorname{PolyLog}\left(2, \frac{i+ax}{-i+ax}\right)}{2a^2c}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((-1/2*I)*ArcTan[a*x]^2)/(a^2*c) - (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^2
*c) - ((I/2)*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c)
```

3.176.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{ac}-\frac{i\arctan(ax)^2}{2a^2c}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{ac}-\frac{i\arctan(ax)^2}{2a^2c}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{ac}-\frac{i\arctan(ax)^2}{2a^2c}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{i\arctan(ax)^2}{2a^2c}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{ac}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((-1/2*I)*ArcTan[a*x]^2)/(a^2*c) - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + (
(I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/(a*c)
```

3.176.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.176.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.65

method	result
parts	$\frac{\ln\left(a^2x^2+1\right)\arctan\left(ax\right)}{2a^2c}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^2+1\right)}\frac{2\ln\left(x-\_\alpha\right)\ln\left(a^2x^2+1\right)-a^2\left(\frac{\ln\left(x-\_\alpha\right)^2}{a^2\_\alpha}+2\_\alpha\ln\left(x-\_\alpha\right)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)+2\right)}{\_\alpha}}{8a^3c}$
risch	$\frac{i\ln\left(-iax+1\right)^2}{8c\,a^2}+\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(-iax+1\right)}{4c\,a^2}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c\,a^2}-\frac{i\ln\left(iax+1\right)^2}{8c\,a^2}-\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln\left(iax+1\right)}{4c\,a^2}+i\,\mathrm{dilo}$
derivativedivides	$\frac{\frac{\arctan\left(ax\right)\ln\left(a^2x^2+1\right)}{2c}-\frac{-\frac{i\left(\ln\left(ax-i\right)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i\left(ax+i\right)}{2}\right)-\ln\left(ax-i\right)\ln\left(-\frac{i\left(ax+i\right)}{2}\right)-\frac{\ln\left(ax-i\right)^2}{2}\right)}{2}+i\left(\ln\left(ax+i\right)\ln\left(a\right)\right)}{2c}}{a^2}$
default	$\frac{\frac{\arctan\left(ax\right)\ln\left(a^2x^2+1\right)}{2c}-\frac{-\frac{i\left(\ln\left(ax-i\right)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i\left(ax+i\right)}{2}\right)-\ln\left(ax-i\right)\ln\left(-\frac{i\left(ax+i\right)}{2}\right)-\frac{\ln\left(ax-i\right)^2}{2}\right)}{2}+i\left(\ln\left(ax+i\right)\ln\left(a\right)\right)}{2c}}{a^2}$

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/c*ln(a^2*x^2+1)*arctan(a*x)-1/8/a^3/c*sum(1/_alpha*(2*ln(x-_alpha)
*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1
/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootO
f(_Z^2*a^2+1))
```

3.176.5 Fricas [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x\arctan{(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.176.6 Sympy [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}\,(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)/(a**2*x**2 + 1), x)/c
```

3.176.7 Maxima [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x\arctan{(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.176.8 Giac [F]

$$\int \frac{x \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x \arctan(ax)}{a^2cx^2 + c} \, dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x \operatorname{atan}(a\,x)}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2), x)
```

3.177 $\int \frac{\arctan(ax)}{c+a^2cx^2}\,dx$

3.177.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{\arctan(ax)}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^2}{2ac}$$

output
```
1/2*arctan(a*x)^2/a/c
```

3.177.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^2}{2ac}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^2/(2*a*c)
```

3.177.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5419}$$

$$\frac{\arctan(ax)^2}{2ac}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^2/(2*a*c)
```

3.177.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.177.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\arctan(ax)^2}{2ac}$	15
default	$\frac{\arctan(ax)^2}{2ac}$	15
parallelrisch	$\frac{\arctan(ax)^2}{2ac}$	15
parts	$\frac{\arctan(ax)^2}{2ac}$	15
risch	$-\frac{\ln(iax+1)^2}{8ca}+\frac{\ln(-iax+1)\ln(iax+1)}{4ca}-\frac{\ln(-iax+1)^2}{8ca}$	62

input
```
int(arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(a*x)^2/a/c
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^2}{2\,ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/2*arctan(a*x)^2/(a*c)
```

3.177.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{\arctan(ax)}{c+a^2cx^2}\,dx = \begin{cases} 0 & \text{for}\ a=0 \\ \tilde{\infty}\left(\begin{cases} 0 & \text{for}\ a=0 \\ \frac{ax\operatorname{atan}{(ax)}-\frac{\log{\left(a^2x^2+1\right)}}{2}}{a} & \text{otherwise} \end{cases}\right) & \text{for}\ c=0 \\ \frac{\operatorname{atan}^2{(ax)}}{2ac} & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((0, Eq(a, 0)), (zoo*Piecewise((0, Eq(a, 0)), ((a*x*atan(a*x) - l
og(a**2*x**2 + 1)/2)/a, True)), Eq(c, 0)), (atan(a*x)**2/(2*a*c), True))
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c + a^2cx^2} \, dx = \frac{\arctan{(ax)}^2}{2\,ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/2*arctan(a*x)^2/(a*c)
```

3.177.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{\arctan(ax)}{c + a^2cx^2} \, dx = -\frac{2\,\pi \arctan{(ax)} \left\lfloor \frac{\arctan(ax)}{\pi} + \frac{1}{2} \right\rfloor - \arctan{(ax)}^2}{2\,ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
-1/2*(2*pi*arctan(a*x)*floor(arctan(a*x)/pi + 1/2) - arctan(a*x)^2)/(a*c)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c + a^2cx^2} \, dx = \frac{\mathrm{atan}(a\,x)^2}{2\,a\,c}$$

input
```
int(atan(a*x)/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^2/(2*a*c)
```

3.178 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx$

3.178.1 Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx = -\frac{i\arctan(ax)^2}{2c}+\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{i\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-1/2*I*arctan(a*x)^2/c+arctan(a*x)*ln(2-2/(1-I*a*x))/c-1/2*I*polylog(2,-1+
2/(1-I*a*x))/c
```

3.178.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.61

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx = \frac{i\arctan(ax)^2}{2c}+\frac{\arctan(ax)\log\left(\frac{2i}{i-ax}\right)}{c}+\frac{i\,\mathrm{PolyLog}(2,-iax)}{2c}-\frac{i\,\mathrm{PolyLog}(2,iax)}{2c}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2c}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```

output
```
((I/2)*ArcTan[a*x]^2)/c + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/c + ((I/2)*Po
lyLog[2, (-I)*a*x])/c - ((I/2)*PolyLog[2, I*a*x])/c + ((I/2)*PolyLog[2, -(
(I + a*x)/(I - a*x))])/c
```

3.178.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5459}$$

$$\frac{i\int \frac{\arctan(ax)}{x(ax+i)}dx}{c}-\frac{i\arctan(ax)^2}{2c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^2}{2c}$$

$$\Big\downarrow \text{2897}$$

$$\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)}{c}-\frac{i\arctan(ax)^2}{2c}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/2*I)*ArcTan[a*x]^2)/c + (I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] -
 PolyLog[2, -1 + 2/(1 - I*a*x)]/2))/c
```

3.178.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.178.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.09

method	result
risch	$-\frac{i\,\mathrm{dilog}(-iax+1)}{2c}-\frac{i\ln(-iax+1)^2}{8c}-\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c}+\frac{i\,\mathrm{dilog}(iax+1)}{2c}+\frac{i\ln(iax}{8c}$
parts	$-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}+\frac{\arctan(ax)\ln(x)}{c}-\frac{a\left(-\frac{i\ln(x)(\ln(iax+1)-\ln(-iax+1))}{a}-\frac{i(\mathrm{dilog}(iax+1)-\mathrm{dilog}(-iax+1))}{a}-\right.}{}$
derivativedivides	$\frac{\arctan(ax)\ln(ax)}{c}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}(iax+1)+i\,\mathrm{dilog}(-}{}$
default	$\frac{\arctan(ax)\ln(ax)}{c}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}(iax+1)+i\,\mathrm{dilog}(-}{}$

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/c*dilog(1-I*a*x)-1/8*I/c*ln(1-I*a*x)^2-1/4*I/c*ln(1/2+1/2*I*a*x)*ln
(1-I*a*x)+1/4*I/c*dilog(1/2-1/2*I*a*x)+1/2*I/c*dilog(1+I*a*x)+1/8*I/c*ln(1
+I*a*x)^2+1/4*I/c*ln(1/2-1/2*I*a*x)*ln(1+I*a*x)-1/4*I/c*dilog(1/2+1/2*I*a*
x)
```

3.178.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\,(ax)}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

3.178.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}\,(ax)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)/(a**2*x**3 + x), x)/c
```

3.178.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\,(ax)}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x), x)
```

3.178.8 Giac [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)), x)
```

3.179 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)} dx$

3.179.1 Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)} dx = -\frac{\arctan(ax)}{cx} - \frac{a\arctan(ax)^2}{2c} + \frac{a\log(x)}{c} - \frac{a\log(1+a^2x^2)}{2c}$$

output
```
-arctan(a*x)/c/x-1/2*a*arctan(a*x)^2/c+a*ln(x)/c-1/2*a*ln(a^2*x^2+1)/c
```

3.179.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)} dx = -\frac{\arctan(ax)}{cx} - \frac{a\arctan(ax)^2}{2c} + \frac{a\log(x)}{c} - \frac{a\log(1+a^2x^2)}{2c}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-(ArcTan[a*x]/(c*x)) - (a*ArcTan[a*x]^2)/(2*c) + (a*Log[x])/c - (a*Log[1 +
 a^2*x^2])/(2*c)
```

3.179.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5453, 27, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2(a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5453}$$
$$\frac{\int \frac{\arctan(ax)}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)}{c(a^2x^2+1)}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\arctan(ax)}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \text{5361}$$
$$\frac{a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \text{243}$$
$$\frac{\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \text{47}$$
$$\frac{\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \text{14}$$
$$\frac{\frac{1}{2}a\left(\log(x^2)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \text{16}$$
$$\frac{\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \text{5419}$$
$$\frac{\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{\arctan(ax)}{x}}{c}-\frac{a\arctan(ax)^2}{2c}$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-1/2*(a*ArcTan[a*x]^2)/c + (-(ArcTan[a*x]/x) + (a*(Log[x^2] - Log[1 + a^2*
x^2]))/2)/c
```

3.179.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.179. $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx$

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.179.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{-a\arctan(ax)^2x+2ax\ln(x)-a\ln(a^2x^2+1)x-2\arctan(ax)}{2cx}$
parts	$-\frac{a\arctan(ax)^2}{c}-\frac{\arctan(ax)}{cx}-\frac{-\frac{a\arctan(ax)^2}{2}-a\left(\ln(ax)-\frac{\ln\left(a^2x^2+1\right)}{2}\right)}{c}$
derivativedivides	$a\left(-\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{cax}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\ln(ax)-\frac{\arctan(ax)^2}{2}}{c}\right)$
default	$a\left(-\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{cax}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\ln(ax)-\frac{\arctan(ax)^2}{2}}{c}\right)$
risch	$\frac{a\ln(iax+1)^2}{8c}-\frac{(ax\ln(-iax+1)-2i)\ln(iax+1)}{4cx}-\frac{-a\ln(-iax+1)^2x-8ax\ln(x)+4ax\ln(-3a^2x^2-3)+4i\ln(-iax+1}{8cx}$

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*arctan(a*x)^2*x+2*a*x*ln(x)-a*ln(a^2*x^2+1)*x-2*arctan(a*x))/c/x
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)}\,dx=-\frac{ax\arctan\left(ax\right)^2+ax\log\left(a^2x^2+1\right)-2\,ax\log\left(x\right)+2\,\arctan\left(ax\right)}{2\,cx}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(a*x*arctan(a*x)^2 + a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) + 2*arctan(a
*x))/(c*x)
```

3.179.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = \begin{cases} \frac{a\log(x)}{c} - \frac{a\log\left(x^2+\frac{1}{a^2}\right)}{2c} - \frac{a\operatorname{atan}^2(ax)}{2c} - \frac{\operatorname{atan}(ax)}{cx} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c),x)
```

output
```
Piecewise((a*log(x)/c - a*log(x**2 + a**(-2))/(2*c) - a*atan(a*x)**2/(2*c)
 - atan(a*x)/(c*x), Ne(a, 0)), (0, True))
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = -\left(\frac{a\arctan(ax)}{c} + \frac{1}{cx}\right)\arctan(ax) + \frac{\left(\arctan(ax)^2 - \log(a^2x^2+1) + 2\log(x)\right)a}{2c}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-(a*arctan(a*x)/c + 1/(c*x))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a^2*x^
2 + 1) + 2*log(x))*a/c
```

3.179.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)}\,dx = \frac{a\,\ln\left(x\right)}{c}-\frac{a\,\ln\left(a^2\,x^2+1\right)}{2\,c}-\frac{a\,\mathrm{atan}(a\,x)^2}{2\,c}-\frac{\mathrm{atan}(a\,x)}{c\,x}$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
(a*log(x))/c - (a*log(a^2*x^2 + 1))/(2*c) - (a*atan(a*x)^2)/(2*c) - atan(a
*x)/(c*x)
```

3.180 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx$

3.180.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx = -\frac{a}{2cx} - \frac{a^2\arctan(ax)}{2c} - \frac{\arctan(ax)}{2cx^2} + \frac{ia^2\arctan(ax)^2}{2c} - \frac{a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c} + \frac{ia^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-1/2*a/c/x-1/2*a^2*arctan(a*x)/c-1/2*arctan(a*x)/c/x^2+1/2*I*a^2*arctan(a*
x)^2/c-a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c+1/2*I*a^2*polylog(2,-1+2/(1-I*a
*x))/c
```

3.180.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx = -\frac{\frac{\arctan(ax)}{x^2} + \frac{a\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{x} + ia^2\left(\arctan(ax)^2 - 2i\arctan(ax)\log\left(\frac{2i}{i-ax}\right) + \mathrm{PolyLog}(2,-a\right.}{2c}$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
-1/2*(ArcTan[a*x]/x^2 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/x
+ I*a^2*(ArcTan[a*x]^2 - (2*I)*ArcTan[a*x]*Log[(2*I)/(I - a*x)] + PolyLog[
2, (-I)*a*x] - PolyLog[2, I*a*x] + PolyLog[2, (I + a*x)/(-I + a*x)]))/c
```

3.180.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5453, 27, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^3}dx}{c}-a^2\int \frac{\arctan(ax)}{cx(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2)/c - (a^2*((-1/2*I
)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2,
-1 + 2/(1 - I*a*x)]/2)))/c
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.180. $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.180.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.88

method	result
parts	$\frac{\arctan(ax)a^2\ln(a^2x^2+1)}{2c}-\frac{\arctan(ax)}{2c\,x^2}-\frac{\arctan(ax)a^2\ln(x)}{c}-\frac{a\left(\left(\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^2+1\right)}\frac{2\ln(x-\_\alpha)\ln\left(a^2x^2+\cdots\right.}{\cdots}\right.\right.}{\cdots}$
derivativedivides	$a^2\left(\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{\arctan(ax)}{2c\,a^2x^2}-\frac{\arctan(ax)\ln(ax)}{c}-\frac{-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(a\cdots\right.}{2}}{\cdots}\right.$
default	$a^2\left(\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{\arctan(ax)}{2c\,a^2x^2}-\frac{\arctan(ax)\ln(ax)}{c}-\frac{-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(a\cdots\right.}{2}}{\cdots}\right.$
risch	$-\frac{ia^2\ln(iax+1)^2}{8c}+\frac{ia^2\,\mathrm{dilog}(-iax+1)}{2c}-\frac{ia^2\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c}-\frac{a}{2cx}+\frac{ia^2\ln(iax+1)}{4c}-\frac{ia^2\ln(iax)}{4c}-\cdots$

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*arctan(a*x)*a^2*ln(a^2*x^2+1)-1/2*arctan(a*x)/c/x^2-1/c*arctan(a*x)*
a^2*ln(x)-1/2*a/c*(1/4*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a
^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2
*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*a^2+1))+1/x+a*ar
ctan(a*x)-2*a^2*(-1/2*I*ln(x)*(ln(1+I*a*x)-ln(1-I*a*x))/a-1/2*I*(dilog(1+I
*a*x)-dilog(1-I*a*x))/a))
```

3.180.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

3.180.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\operatorname{atan}(ax)}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)/(a**2*x**5 + x**3), x)/c
```

3.180.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^3), x)
```

3.180.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3 (c + a^2 c x^2)} \, dx = \int \frac{\mathrm{atan}(a\,x)}{x^3\,(c\,a^2\,x^2 + c)} \, dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)), x)
```

3.181 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx$

3.181.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx = -\frac{a}{6cx^2}-\frac{\arctan(ax)}{3cx^3}+\frac{a^2\arctan(ax)}{cx}+\frac{a^3\arctan(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{2a^3\log(1+a^2x^2)}{3c}$$

output
```
-1/6*a/c/x^2-1/3*arctan(a*x)/c/x^3+a^2*arctan(a*x)/c/x+1/2*a^3*arctan(a*x)
^2/c-4/3*a^3*ln(x)/c+2/3*a^3*ln(a^2*x^2+1)/c
```

3.181.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx = -\frac{a}{6cx^2}-\frac{\arctan(ax)}{3cx^3}+\frac{a^2\arctan(ax)}{cx}+\frac{a^3\arctan(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{2a^3\log(1+a^2x^2)}{3c}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
-1/6*a/(c*x^2) - ArcTan[a*x]/(3*c*x^3) + (a^2*ArcTan[a*x])/(c*x) + (a^3*Ar
cTan[a*x]^2)/(2*c) - (4*a^3*Log[x])/(3*c) + (2*a^3*Log[1 + a^2*x^2])/(3*c)
```

3.181.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5453, 27, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4\,(a^2cx^2+c)}\,dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^4}dx}{c} - a^2\int \frac{\arctan(ax)}{cx^2\,(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^4}dx}{c} - \frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{3}a\int \frac{1}{x^3(a^2x^2+1)}dx - \frac{\arctan(ax)}{3x^3}}{c} - \frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{6}a\int \frac{1}{x^4(a^2x^2+1)}dx^2 - \frac{\arctan(ax)}{3x^3}}{c} - \frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{54}$$

$$\frac{\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1} - \frac{a^2}{x^2} + \frac{1}{x^4}\right)dx^2 - \frac{\arctan(ax)}{3x^3}}{c} - \frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right) + a^2\log\left(a^2x^2+1\right) - \frac{1}{x^2}\right) - \frac{\arctan(ax)}{3x^3}}{c} - \frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right) + a^2\log\left(a^2x^2+1\right) - \frac{1}{x^2}\right) - \frac{\arctan(ax)}{3x^3}}{c} - \frac{a^2\left(\int \frac{\arctan(ax)}{x^2}dx - a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int\frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int\frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)}{c}$$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
-((a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^
2*x^2]))/2))/c) + (-1/3*ArcTan[a*x]/x^3 + (a*(-x^(-2) - a^2*Log[x^2] + a^2
*Log[1 + a^2*x^2]))/6)/c
```

3.181. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx$

3.181.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.181. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx$

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.181.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

method	result
parallelrisch	$-\frac{-3a^3\arctan(ax)^2x^3+8\ln(x)a^3x^3-4a^3\ln(a^2x^2+1)x^3-2a^3x^3-6a^2\arctan(ax)x^2+ax+2\arctan(ax)}{6c\,x^3}$
derivativedivides	$a^3\left(\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,a^3x^3}+\frac{\arctan(ax)}{cax}-\frac{-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)+\frac{3\arctan(ax)^2}{2}}{3c}\right)$
default	$a^3\left(\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,a^3x^3}+\frac{\arctan(ax)}{cax}-\frac{-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)+\frac{3\arctan(ax)^2}{2}}{3c}\right)$
parts	$\frac{a^3\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,x^3}+\frac{a^2\arctan(ax)}{cx}-\frac{a^3\left(-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)\right)+\frac{3a^3\arctan(ax)^2}{2}}{3c}$
risch	$-\frac{a^3\ln(iax+1)^2}{8c}+\frac{(3a^3x^3\ln(-iax+1)-6ia^2x^2+2i)\ln(iax+1)}{12c\,x^3}-\frac{3a^3\ln(-iax+1)^2x^3+32\ln(x)a^3x^3-16\ln(3a^2x^2+}{24}$

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3*a^3*arctan(a*x)^2*x^3+8*ln(x)*a^3*x^3-4*a^3*ln(a^2*x^2+1)*x^3-2*a
^3*x^3-6*a^2*arctan(a*x)*x^2+a*x+2*arctan(a*x))/c/x^3
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx$$
$$=\frac{3\,a^3x^3\arctan\left(ax\right)^2+4\,a^3x^3\log\left(a^2x^2+1\right)-8\,a^3x^3\log\left(x\right)-ax+2\left(3\,a^2x^2-1\right)\arctan\left(ax\right)}{6\,cx^3}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/6*(3*a^3*x^3*arctan(a*x)^2 + 4*a^3*x^3*log(a^2*x^2 + 1) - 8*a^3*x^3*log(
x) - a*x + 2*(3*a^2*x^2 - 1)*arctan(a*x))/(c*x^3)
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = \begin{cases} -\frac{4a^3\log\left(x\right)}{3c} + \frac{2a^3\log\left(x^2+\frac{1}{a^2}\right)}{3c} + \frac{a^3\,\mathrm{atan}^2\left(ax\right)}{2c} + \frac{a^2\,\mathrm{atan}\left(ax\right)}{cx} - \frac{a}{6cx^2} - \frac{\mathrm{atan}\left(ax\right)}{3cx^3} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c),x)
```

output
```
Piecewise((-4*a**3*log(x)/(3*c) + 2*a**3*log(x**2 + a**(-2))/(3*c) + a**3*
atan(a*x)**2/(2*c) + a**2*atan(a*x)/(c*x) - a/(6*c*x**2) - atan(a*x)/(3*c*
x**3), Ne(a, 0)), (0, True))
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = \frac{1}{3}\left(\frac{3\,a^3\arctan\left(ax\right)}{c} + \frac{3\,a^2x^2-1}{cx^3}\right)\arctan\left(ax\right) - \frac{\left(3\,a^2x^2\arctan\left(ax\right)^2 - 4\,a^2x^2\log\left(a^2x^2+1\right) + 8\,a^2x^2\log\left(x\right) + 1\right)a}{6\,cx^2}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3*(3*a^3*arctan(a*x)/c + (3*a^2*x^2 - 1)/(c*x^3))*arctan(a*x) - 1/6*(3*a
^2*x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*
a/(c*x^2)
```

3.181.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.181.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx = \frac{2\,a^3\,\ln(a^2\,x^2+1)}{3\,c} - \frac{\mathrm{atan}(a\,x)}{3\,c\,x^3} - \frac{a}{6\,c\,x^2} - \frac{4\,a^3\,\ln(x)}{3\,c} + \frac{a^3\,\mathrm{atan}(a\,x)^2}{2\,c} + \frac{a^2\,\mathrm{atan}(a\,x)}{c\,x}$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
(2*a^3*log(a^2*x^2 + 1))/(3*c) - atan(a*x)/(3*c*x^3) - a/(6*c*x^2) - (4*a^
3*log(x))/(3*c) + (a^3*atan(a*x)^2)/(2*c) + (a^2*atan(a*x))/(c*x)
```

3.182 $\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2} \, dx$

3.182.1 Optimal result

Integrand size = 20, antiderivative size = 157

$$\begin{aligned}\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2} \, dx &= -\frac{x}{2a^5c^2} + \frac{x}{4a^5c^2(1+a^2x^2)} + \frac{3\arctan(ax)}{4a^6c^2} \\ &\quad + \frac{x^2\arctan(ax)}{2a^4c^2} - \frac{\arctan(ax)}{2a^6c^2(1+a^2x^2)} + \frac{i\arctan(ax)^2}{a^6c^2} \\ &\quad + \frac{2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^6c^2} + \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^6c^2}\end{aligned}$$

output
```
-1/2*x/a^5/c^2+1/4*x/a^5/c^2/(a^2*x^2+1)+3/4*arctan(a*x)/a^6/c^2+1/2*x^2*a
rctan(a*x)/a^4/c^2-1/2*arctan(a*x)/a^6/c^2/(a^2*x^2+1)+I*arctan(a*x)^2/a^6
/c^2+2*arctan(a*x)*ln(2/(1+I*a*x))/a^6/c^2+I*polylog(2,1-2/(1+I*a*x))/a^6/
c^2
```

3.182.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2} \, dx$$
$$= \frac{-4ax - 8i\arctan(ax)^2 + 2\arctan(ax)\left(2 + 2a^2x^2 - \cos(2\arctan(ax)) + 8\log\left(1+e^{2i\arctan(ax)}\right)\right) - 8i\,\mathrm{Pol}}{8a^6c^2}$$

input
```
Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(-4*a*x - (8*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(2 + 2*a^2*x^2 - Cos[2*ArcTa
n[a*x]] + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) - (8*I)*PolyLog[2, -E^((2*I)*A
rcTan[a*x])] + Sin[2*ArcTan[a*x]])/(8*a^6*c^2)
```

3.182.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.67, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5499, 27, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752, 5499, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \ 5499$$

$$\frac{\int \frac{x^3\arctan(ax)}{c(a^2x^2+1)}dx}{a^2c}-\frac{\int \frac{x^3\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \ 5451$$

$$\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \ 5361$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int\frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int\frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\int\frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\int\frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\int\frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5499}$$

$$-\frac{\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a^2}}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5455}$$

3.182. $\displaystyle\int\frac{x^5\arctan(ax)}{(c+a^2cx^2)^2}\,dx$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a^2}+\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5379$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a^2}+\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 2849$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a^2}+\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 2752$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 215$$

3.182. $\int \frac{x^5\arctan(ax)}{\left(c+a^2cx^2\right)^2}\,dx$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

input
```
Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*
ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[
2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/(a^2*c^2) - (-((-1/2*ArcTan[a*x]/(a^2*(1
 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))/a^2) + (((
-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*
PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/(a^2*c^2)
```

3.182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.182. $\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2} \, dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.182.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.21

method	result
parts	$\frac{x^2\arctan(ax)}{2a^4c^2}-\frac{\arctan(ax)\ln(a^2x^2+1)}{c^2a^6}-\frac{\arctan(ax)}{2a^6c^2(a^2x^2+1)}-\frac{a\left(\frac{x-\frac{x}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2a}}{a^6}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^2\right.}}{}\right.}{}$
derivativedivides	$\frac{\frac{\arctan(ax)a^2x^2}{2c^2}-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{ax-\frac{ax}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2}+i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(a}{}\right.\right.}{}}{a^6}$
default	$\frac{\frac{\arctan(ax)a^2x^2}{2c^2}-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{ax-\frac{ax}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2}+i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(a}{}\right.\right.}{}}{a^6}$
risch	$-\frac{x}{2a^5c^2}+\frac{\arctan(ax)}{8a^6c^2}+\frac{i\ln(-iax+1)}{4c^2a^6}-\frac{i\ln(-iax+1)}{8c^2a^6(-iax+1)}-\frac{i\ln(iax+1)}{4c^2a^6}+\frac{i}{8c^2a^6(iax+1)}+\frac{i\ln(iax+1)^2}{4c^2a^6}-$

input
```
int(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(a*x)/a^4/c^2-1/c^2*arctan(a*x)/a^6*ln(a^2*x^2+1)-1/2*arctan
(a*x)/a^6/c^2/(a^2*x^2+1)-1/2/c^2*a*(1/a^6*(x-1/2*x/(a^2*x^2+1)-3/2/a*arct
an(a*x))-1/2/a^8*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_al
pha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alph
a*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*a^2+1)))
```

3.182.5 Fricas [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^5*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.182.6 Sympy [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^5 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**5*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.182.7 Maxima [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.182.8 Giac [F]

$$\int \frac{x^5 \arctan(ax)}{(c + a^2cx^2)^2} \, dx = \int \frac{x^5 \arctan(ax)}{(a^2cx^2 + c)^2} \, dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)}{(c + a^2cx^2)^2} \, dx = \int \frac{x^5 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

3.183 $\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2} dx$

3.183.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{1}{4a^5c^2(1+a^2x^2)} + \frac{x\arctan(ax)}{a^4c^2} + \frac{x\arctan(ax)}{2a^4c^2(1+a^2x^2)} - \frac{3\arctan(ax)^2}{4a^5c^2} - \frac{\log(1+a^2x^2)}{2a^5c^2}$$

output
```
1/4/a^5/c^2/(a^2*x^2+1)+x*arctan(a*x)/a^4/c^2+1/2*x*arctan(a*x)/a^4/c^2/(a
^2*x^2+1)-3/4*arctan(a*x)^2/a^5/c^2-1/2*ln(a^2*x^2+1)/a^5/c^2
```

3.183.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{1 + (6ax + 4a^3x^3)\arctan(ax) - 3(1+a^2x^2)\arctan(ax)^2 - 2(1+a^2x^2)\log(1+a^2x^2)}{4a^5c^2(1+a^2x^2)}$$

input
```
Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(1 + (6*a*x + 4*a^3*x^3)*ArcTan[a*x] - 3*(1 + a^2*x^2)*ArcTan[a*x]^2 - 2*(
1 + a^2*x^2)*Log[1 + a^2*x^2])/(4*a^5*c^2*(1 + a^2*x^2))
```

3.183.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5499, 27, 5451, 5345, 240, 5419, 5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \textbf{5499}$$

$$\frac{\int \frac{x^2\arctan(ax)}{c(a^2x^2+1)}dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \textbf{5451}$$

$$\frac{\frac{\int \arctan(ax)dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \textbf{5345}$$

$$\frac{\frac{x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \textbf{240}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \textbf{5419}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \textbf{5469}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c^2}-\frac{\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{2a^2}-\frac{x\arctan(ax)}{2a^2(a^2x^2+1)}-\frac{1}{4a^3(a^2x^2+1)}}{a^2c^2}$$

$$\Big\downarrow \textbf{5419}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c^2}-\frac{\frac{\arctan(ax)^2}{4a^3}-\frac{x\arctan(ax)}{2a^2(a^2x^2+1)}-\frac{1}{4a^3(a^2x^2+1)}}{a^2c^2}$$

input
```
Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-((-1/4*1/(a^3*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^2*(1 + a^2*x^2)) + Ar
cTan[a*x]^2/(4*a^3))/(a^2*c^2)) + (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x]
 - Log[1 + a^2*x^2]/(2*a))/a^2)/(a^2*c^2)
```

3.183.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.183. $\int \frac{x^4\arctan(ax)}{(c+a^2cx^2)^2}\,dx$

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.183.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{\arctan(ax)ax}{c^2}+\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\ln\left(a^2x^2+1\right)-\frac{3\arctan(ax)^2}{2}}{2c^2}}{a^5}$
default	$\frac{\frac{\arctan(ax)ax}{c^2}+\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\ln\left(a^2x^2+1\right)-\frac{3\arctan(ax)^2}{2}}{2c^2}}{a^5}$
parts	$\frac{x\arctan(ax)}{a^4c^2}+\frac{x\arctan(ax)}{2a^4c^2(a^2x^2+1)}-\frac{3\arctan(ax)^2}{2a^5c^2}-\frac{-\frac{3\arctan(ax)^2}{4a^5}+\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\ln\left(a^2x^2+1\right)}{2a^5}}{c^2}$
parallelrisch	$\frac{4\arctan(ax)x^3a^3-3x^2\arctan(ax)^2a^2-2a^2\ln(a^2x^2+1)x^2-a^2x^2+6x\arctan(ax)a-3\arctan(ax)^2-2\ln(a^2x^2+1)}{4c^2(a^2x^2+1)a^5}$
risch	$\frac{3\ln(iax+1)^2}{16a^5c^2}-\frac{i(-3ia^2x^2\ln(-iax+1)+4a^3x^3-3i\ln(-iax+1)+6ax)\ln(iax+1)}{8a^5c^2(a^2x^2+1)}+\frac{i\left(-3ia^2x^2\ln(-iax+1)^2-3i\ln(-\right.}{}$

input
```
int(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/c^2*arctan(a*x)*a*x+1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/2*arcta
n(a*x)^2/c^2-1/2/c^2*(-1/2/(a^2*x^2+1)+ln(a^2*x^2+1)-3/2*arctan(a*x)^2))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{3\left(a^2x^2+1\right)\arctan\left(ax\right)^2-2\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)+2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right)-1}{4\left(a^7c^2x^2+a^5c^2\right)}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(2*a^3*x^3 + 3*a*x)*arctan(a*x) +
2*(a^2*x^2 + 1)*log(a^2*x^2 + 1) - 1)/(a^7*c^2*x^2 + a^5*c^2)
```

3.183.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(90) = 180$.

Time = 0.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.32

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{4a^3x^3\,\mathrm{atan}\,(ax)}{4a^7c^2x^2+4a^5c^2} - \frac{2a^2x^2\log\left(x^2+\frac{1}{a^2}\right)}{4a^7c^2x^2+4a^5c^2} - \frac{3a^2x^2\,\mathrm{atan}^2\,(ax)}{4a^7c^2x^2+4a^5c^2} + \frac{6ax\,\mathrm{atan}\,(ax)}{4a^7c^2x^2+4a^5c^2} - \frac{2\log\left(x^2+\frac{1}{a^2}\right)}{4a^7c^2x^2+4a^5c^2} - \frac{3\,\mathrm{atan}^2\,(ax)}{4a^7c^2x^2+4a^5c^2} + \frac{1}{4a^7c^2x^2+4a^5} \\ 0 \end{cases}$$

input
```
integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((4*a**3*x**3*atan(a*x)/(4*a**7*c**2*x**2 + 4*a**5*c**2) - 2*a**2
*x**2*log(x**2 + a**(-2))/(4*a**7*c**2*x**2 + 4*a**5*c**2) - 3*a**2*x**2*a
tan(a*x)**2/(4*a**7*c**2*x**2 + 4*a**5*c**2) + 6*a*x*atan(a*x)/(4*a**7*c**
2*x**2 + 4*a**5*c**2) - 2*log(x**2 + a**(-2))/(4*a**7*c**2*x**2 + 4*a**5*c
**2) - 3*atan(a*x)**2/(4*a**7*c**2*x**2 + 4*a**5*c**2) + 1/(4*a**7*c**2*x*
*2 + 4*a**5*c**2), Ne(a, 0)), (0, True))
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{a^6c^2x^2+a^4c^2}+\frac{2x}{a^4c^2}-\frac{3\arctan(ax)}{a^5c^2}\right)\arctan(ax) + \frac{(3(a^2x^2+1)\arctan(ax)^2-2(a^2x^2+1)\log(a^2x^2+1)+1)a}{4(a^8c^2x^2+a^6c^2)}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^6*c^2*x^2 + a^4*c^2) + 2*x/(a^4*c^2) - 3*arctan(a*x)/(a^5*c^2))*
arctan(a*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2
*x^2 + 1) + 1)*a/(a^8*c^2*x^2 + a^6*c^2)
```

3.183.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^4\arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1}{2\,a^2\,(2\,a^5\,c^2\,x^2+2\,a^3\,c^2)} - \frac{\ln(a^2\,x^2+1)}{2\,a^5\,c^2} + \frac{\mathrm{atan}(a\,x)\,\left(\frac{3\,x}{2\,a^6\,c^2}+\frac{x^3}{a^4\,c^2}\right)}{\frac{1}{a^2}+x^2} - \frac{3\,\mathrm{atan}(a\,x)^2}{4\,a^5\,c^2}$$

input
```
int((x^4*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
1/(2*a^2*(2*a^3*c^2 + 2*a^5*c^2*x^2)) - log(a^2*x^2 + 1)/(2*a^5*c^2) + (at
an(a*x)*((3*x)/(2*a^6*c^2) + x^3/(a^4*c^2)))/(1/a^2 + x^2) - (3*atan(a*x)^
2)/(4*a^5*c^2)
```

3.183. $\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2} \, dx$

3.184 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2} \, dx$

3.184.1 Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2} \, dx = -\frac{x}{4a^3c^2(1+a^2x^2)} - \frac{\arctan(ax)}{4a^4c^2} + \frac{\arctan(ax)}{2a^4c^2(1+a^2x^2)} - \frac{i\arctan(ax)^2}{2a^4c^2} - \frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2} - \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2}$$

output
```
-1/4*x/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a^4/c^2+1/2*arctan(a*x)/a^4/c^2
/(a^2*x^2+1)-1/2*I*arctan(a*x)^2/a^4/c^2-arctan(a*x)*ln(2/(1+I*a*x))/a^4/c
^2-1/2*I*polylog(2,1-2/(1+I*a*x))/a^4/c^2
```

3.184.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2} \, dx = \frac{4i\arctan(ax)^2 + 2\arctan(ax)\left(\cos(2\arctan(ax)) - 4\log\left(1+e^{2i\arctan(ax)}\right)\right) + 4i\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(ax)}\right.}{8a^4c^2}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
((4*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 4*Log[1 + E^((2
*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - Sin[2*ArcT
an[a*x]])/(8*a^4*c^2)
```

3.184.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5499, 27, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x\arctan(ax)}{c(a^2x^2+1)}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c^2}-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c^2}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c^2}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

$$\Big\downarrow \text{215}$$

$$-\frac{\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-((-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*
x]/(2*a))/(2*a))/(a^2*c^2)) + (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x
]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/(a^2
*c^2)
```

3.184.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.184. $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2} \, dx$

3.184.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

method	result
parts	$\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2a^4}+\frac{\arctan(ax)}{2a^4c^2(a^2x^2+1)}-\frac{a\left(\frac{x}{2a^4\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a^5}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^2+1\right)}\frac{2\ln(x-\_\alpha)\ln\left(a^2\right.}{}}{}\right.}{2}$
derivativedivides	$\frac{\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c^2}+\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(a}{}\right.\right.}{2}}{}}{a^4}$
default	$\frac{\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c^2}+\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(a}{}\right.\right.}{2}}{}}{a^4}$
risch	$-\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c^2a^4}-\frac{i\ln(-iax+1)}{16c^2a^4(-iax-1)}+\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c^2a^4}+\frac{i}{8c^2a^4(-iax+1)}-\frac{\arctan(ax)}{8a^4c^2}-\frac{}{1}$

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c^2*arctan(a*x)/a^4*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^4/c^2/(a^2*x^2+1)-
1/2/c^2*a*(1/2/a^4*x/(a^2*x^2+1)+1/2/a^5*arctan(a*x)+1/4/a^6*sum(1/_alpha*
(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln
(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha)
)),_alpha=RootOf(_Z^2*a^2+1)))
```

3.184.5 Fricas [F]

$$\int \frac{x^3\arctan(ax)}{(c+a^2cx^2)^2}\,dx=\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.184.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.184.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.184.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{(c + a^2cx^2)^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)}{{(c\,a^2\,x^2 + c)}^2} \,dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

3.185 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2} dx$

3.185.1 Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2} dx = -\frac{1}{4a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^2}{4a^3c^2}$$

output
```
-1/4/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+1/4*arctan(
a*x)^2/a^3/c^2
```

3.185.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{-1-2ax\arctan(ax)+(1+a^2x^2)\arctan(ax)^2}{4a^3c^2(1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(-1 - 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2
*x^2))
```

3.185.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5469, 27, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5469}$$

$$\frac{\int \frac{\arctan(ax)}{c(a^2x^2+1)}dx}{2a^2c} - \frac{x\arctan(ax)}{2a^2c^2(a^2x^2+1)} - \frac{1}{4a^3c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{2a^2c^2} - \frac{x\arctan(ax)}{2a^2c^2(a^2x^2+1)} - \frac{1}{4a^3c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\arctan(ax)^2}{4a^3c^2} - \frac{x\arctan(ax)}{2a^2c^2(a^2x^2+1)} - \frac{1}{4a^3c^2(a^2x^2+1)}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/4*1/(a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^2*c^2*(1 + a^2*x^2))
 + ArcTan[a*x]^2/(4*a^3*c^2)
```

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

3.185.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{x^{2} \arctan \left(a x \right)^{2} a^{2}+a^{2} x^{2}-2 x \arctan \left(a x \right) a +\arctan \left(a x \right)^{2}}{4 c^{2} \left(a^{2} x^{2}+1\right) a^{3}}$
derivativedivides	$\frac{-\frac{a x \arctan \left(a x \right)}{2 c^{2} \left(a^{2} x^{2}+1\right)}+\frac{\arctan \left(a x \right)^{2}}{2 c^{2}}-\frac{\frac{\arctan \left(a x \right)^{2}}{2}+\frac{1}{2 a^{2} x^{2}+2}}{2 c^{2}}}{a^{3}}$
default	$\frac{-\frac{a x \arctan \left(a x \right)}{2 c^{2} \left(a^{2} x^{2}+1\right)}+\frac{\arctan \left(a x \right)^{2}}{2 c^{2}}-\frac{\frac{\arctan \left(a x \right)^{2}}{2}+\frac{1}{2 a^{2} x^{2}+2}}{2 c^{2}}}{a^{3}}$
parts	$-\frac{x \arctan \left(a x \right)}{2 a^{2} c^{2} \left(a^{2} x^{2}+1\right)}+\frac{\arctan \left(a x \right)^{2}}{2 a^{3} c^{2}}-\frac{\frac{\arctan \left(a x \right)^{2}}{2 a^{3}}+\frac{1}{2 a^{3} \left(a^{2} x^{2}+1\right)}}{2 c^{2}}$
risch	$-\frac{\ln \left(i a x +1\right)^{2}}{16 a^{3} c^{2}}+\frac{\left(a^{2} x^{2} \ln \left(-i a x +1\right)+\ln \left(-i a x +1\right)+2 i a x \right) \ln \left(i a x +1\right)}{8 a^{3} c^{2} \left(a^{2} x^{2}+1\right)}-\frac{a^{2} x^{2} \ln \left(-i a x +1\right)^{2}+\ln \left(-i a x +1\right)^{2}+4 i a x \ln (-}{16 a^{3} c^{2} \left(a x +i\right) \left(a x -i\right)}$

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*arctan(a*x)^2*a^2+a^2*x^2-2*x*arctan(a*x)*a+arctan(a*x)^2)/c^2/(a
^2*x^2+1)/a^3
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{2\,ax\arctan\left(ax\right)-\left(a^2x^2+1\right)\arctan\left(ax\right)^2+1}{4\,\left(a^5c^2x^2+a^3c^2\right)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a*x*arctan(a*x) - (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^5*c^2*x^2 +
a^3*c^2)
```

3.185.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2}-\frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax) - \frac{\left((a^2x^2+1)\arctan(ax)^2+1\right)a}{4\,(a^6c^2x^2+a^4c^2)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x) - 1/4
*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a/(a^6*c^2*x^2 + a^4*c^2)
```

3.185.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.185.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{a^2\,x^2\,\mathrm{atan}(a\,x)^2 - 2\,a\,x\,\mathrm{atan}(a\,x) + \mathrm{atan}(a\,x)^2 - 1}{4\,a^3\,c^2\,\left(a^2\,x^2+1\right)}$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
(atan(a*x)^2 - 2*a*x*atan(a*x) + a^2*x^2*atan(a*x)^2 - 1)/(4*a^3*c^2*(a^2*
x^2 + 1))
```

3.186 $\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2} dx$

3.186.1 Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{x}{4ac^2(1+a^2x^2)} + \frac{\arctan(ax)}{4a^2c^2} - \frac{\arctan(ax)}{2a^2c^2(1+a^2x^2)}$$

output
```
1/4*x/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^2/c^2-1/2*arctan(a*x)/a^2/c^2/(a
^2*x^2+1)
```

3.186.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{ax + (-1+a^2x^2)\arctan(ax)}{4a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(a*x + (-1 + a^2*x^2)*ArcTan[a*x])/(4*a^2*c^2*(1 + a^2*x^2))
```

3.186.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5465, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \color{blue}{5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \color{blue}{215}$$

$$\frac{\frac{\int \frac{1}{a^2cx^2+c}dx}{2c}+\frac{x}{2c^2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \color{blue}{218}$$

$$\frac{\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}}{2a}-\frac{\arctan(ax)}{2a^2c^2(a^2x^2+1)}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]/(a^2*c^2*(1 + a^2*x^2)) + (x/(2*c^2*(1 + a^2*x^2)) + ArcT
an[a*x]/(2*a*c^2))/(2*a)
```

3.186.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.186.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

method	result	size
parallelrisch	$\frac{a^{2} \arctan \left(a x \right) x^{2}+a x -\arctan \left(a x \right)}{4 c^{2} \left(a^{2} x^{2}+1\right) a^{2}}$	41
derivativedivides	$\frac{-\frac{\arctan \left(a x \right)}{2 c^{2} \left(a^{2} x^{2}+1\right)}+\frac{\frac{a x}{2 a^{2} x^{2}+2}+\frac{\arctan \left(a x \right)}{2}}{2 c^{2}}}{a^{2}}$	53
default	$\frac{-\frac{\arctan \left(a x \right)}{2 c^{2} \left(a^{2} x^{2}+1\right)}+\frac{\frac{a x}{2 a^{2} x^{2}+2}+\frac{\arctan \left(a x \right)}{2}}{2 c^{2}}}{a^{2}}$	53
parts	$-\frac{\arctan \left(a x \right)}{2 a^{2} c^{2} \left(a^{2} x^{2}+1\right)}+\frac{\frac{x}{2 a^{2} x^{2}+2}+\frac{\arctan \left(a x \right)}{2 a}}{2 a \,c^{2}}$	57
risch	$\frac{i \ln \left(i a x +1\right)}{4 a^{2} c^{2} \left(a^{2} x^{2}+1\right)}-\frac{i \left(2 \ln \left(-i a x +1\right)+\ln \left(a x -i\right) a^{2} x^{2}+\ln \left(a x -i\right)-\ln \left(-a x -i\right) a^{2} x^{2}-\ln \left(-a x -i\right)+2 i a x \right)}{8 \left(a x +i\right) a^{2} c^{2} \left(a x -i\right)}$	118

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^2*arctan(a*x)*x^2+a*x-arctan(a*x))/c^2/(a^2*x^2+1)/a^2
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{ax + (a^2x^2-1)\arctan(ax)}{4(a^4c^2x^2+a^2c^2)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/4*(a*x + (a^2*x^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

3.186.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{a^2x^2\operatorname{atan}(ax)}{4a^4c^2x^2+4a^2c^2} + \frac{ax}{4a^4c^2x^2+4a^2c^2} - \frac{\operatorname{atan}(ax)}{4a^4c^2x^2+4a^2c^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((a**2*x**2*atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2) + a*x/(4*a
**4*c**2*x**2 + 4*a**2*c**2) - atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2),
 Ne(a, 0)), (0, True))
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\frac{x}{a^2cx^2+c} + \frac{\arctan(ax)}{ac}}{4\,ac} - \frac{\arctan(ax)}{2\,(a^2cx^2+c)a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))/(a*c) - 1/2*arctan(a*x)/((a^2*
c*x^2 + c)*a^2*c)
```

3.186.8 Giac [F]

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x\arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{x \arctan(ax)}{(c + a^2 c x^2)^2} \, dx = \frac{a\,x - \operatorname{atan}(a\,x) + a^2\,x^2\,\operatorname{atan}(a\,x)}{4\,a^2\,c^2\,\left(a^2\,x^2 + 1\right)}$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
(a*x - atan(a*x) + a^2*x^2*atan(a*x))/(4*a^2*c^2*(a^2*x^2 + 1))
```

3.187 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^2} dx$

3.187.1 Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{1}{4ac^2(1+a^2x^2)} + \frac{x\arctan(ax)}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^2}{4ac^2}$$

output
```
1/4/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^2/
a/c^2
```

3.187.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{1+2ax\arctan(ax)+(1+a^2x^2)\arctan(ax)^2}{4c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

output
```
(1 + 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*c^2*(a + a^3*x^2)
)
```

3.187.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5427, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \; 5427$$

$$-\frac{1}{2}a\int \frac{x}{c^2\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4ac^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\int \frac{x}{\left(a^2x^2+1\right)^2}dx}{2c^2}+\frac{x\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4ac^2}$$

$$\Big\downarrow \; 241$$

$$\frac{x\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{1}{4ac^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4ac^2}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

output
```
1/(4*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + ArcTan
[a*x]^2/(4*a*c^2)
```

3.187.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.187.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{x^2\arctan(ax)^2a^2-a^2x^2+2x\arctan(ax)a+\arctan(ax)^2}{4c^2(a^2x^2+1)a}$
derivativedivides	$\frac{\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2}}{2c^2}}{a}$
default	$\frac{\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2}}{2c^2}}{a}$
parts	$\frac{x\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^2}{2a\,c^2}-\frac{\frac{\arctan(ax)^2}{2a}-\frac{1}{2a\left(a^2x^2+1\right)}}{2c^2}$
risch	$-\frac{\ln(iax+1)^2}{16c^2a}+\frac{(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax)\ln(iax+1)}{8c^2(a^2x^2+1)a}-\frac{a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2-4iax\ln(-}{16c^2(ax+i)(ax-i)a}$

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*arctan(a*x)^2*a^2-a^2*x^2+2*x*arctan(a*x)*a+arctan(a*x)^2)/c^2/(a
^2*x^2+1)/a
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{2\,ax\arctan\left(ax\right)+\left(a^2x^2+1\right)\arctan\left(ax\right)^2+1}{4\left(a^3c^2x^2+ac^2\right)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/4*(2*a*x*arctan(a*x) + (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^3*c^2*x^2 + a
*c^2)
```

3.187.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded while
 calling a Python object
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2} + \frac{\arctan(ax)}{ac^2}\right)\arctan(ax) - \frac{\left((a^2x^2+1)\arctan(ax)^2-1\right)a}{4\left(a^4c^2x^2+a^2c^2\right)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x) - 1/4*((a^2*
x^2 + 1)*arctan(a*x)^2 - 1)*a/(a^4*c^2*x^2 + a^2*c^2)
```

3.187.8 Giac [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.187.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{a^2\,x^2\,\mathrm{atan}(a\,x)^2+2\,a\,x\,\mathrm{atan}(a\,x)+\mathrm{atan}(a\,x)^2+1}{4\,a\,c^2\,\left(a^2\,x^2+1\right)}$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^2,x)
```

output
```
(atan(a*x)^2 + 2*a*x*atan(a*x) + a^2*x^2*atan(a*x)^2 + 1)/(4*a*c^2*(a^2*x^
2 + 1))
```

3.188 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^2}\,dx$

3.188.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx = -\frac{ax}{4c^2\left(1+a^2x^2\right)} - \frac{\arctan(ax)}{4c^2} + \frac{\arctan(ax)}{2c^2\left(1+a^2x^2\right)} - \frac{i\arctan(ax)^2}{2c^2} + \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2} - \frac{i\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}$$

output
```
-1/4*a*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/c^2+1/2*arctan(a*x)/c^2/(a^2*x^2+
1)-1/2*I*arctan(a*x)^2/c^2+arctan(a*x)*ln(2-2/(1-I*a*x))/c^2-1/2*I*polylog
(2,-1+2/(1-I*a*x))/c^2
```

3.188.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx = -\frac{4i\arctan(ax)^2 - 2\arctan(ax)\left(\cos(2\arctan(ax)) + 4\log\left(1-e^{2i\arctan(ax)}\right)\right) + 4i\,\mathrm{PolyLog}\left(2,e^{2i\arctan(ax}\right.}{8c^2}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-1/8*((4*I)*ArcTan[a*x]^2 - 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] + 4*Log[1 -
E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + Sin[2*
ArcTan[a*x]])/c^2
```

3.188.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5501, 27, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)}{cx(a^2x^2+1)}dx}{c}-a^2\int \frac{x\arctan(ax)}{c^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \; 5459$$

$$-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \; 2897$$

$$-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \; 5465$$

$$-\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a^2\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcT
an[a*x]/(2*a))/(2*a)))/c^2) + ((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x
]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))/c^2
```

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.188. $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^2}\,dx$

3.188.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.92

method	result
parts	$-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2}+\frac{\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)\ln(x)}{c^2}-\frac{a\left(\frac{x}{2a^2x^2+2}+\frac{\arctan(ax)}{2a}-\frac{i\ln(x)(\ln(iax+1)-\ln(-ia\ldots}{a}\right.}{}$
derivativedivides	$\frac{\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2}+\frac{\arctan(ax)\ln(ax)}{c^2}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-i\ln(ax)\ln(iax+1)+i\ln(ax)}{}$
default	$\frac{\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2}+\frac{\arctan(ax)\ln(ax)}{c^2}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-i\ln(ax)\ln(iax+1)+i\ln(ax)}{}$
risch	$-\frac{i\ln(iax+1)}{8c^2(iax+1)}-\frac{i\ln(-iax+1)^2}{8c^2}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{4c^2}+\frac{i\ln(iax+1)^2}{8c^2}+\frac{i\,\mathrm{dilog}(iax+1)}{2c^2}+\frac{i\ln(iax+1)}{16c^2(iax-1)}+\frac{i\ln\left(\frac{1}{2}-\ldots\right.}{}$

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/c^2/(a^2*x^2+1)+1/c^2*a
rctan(a*x)*ln(x)-1/2/c^2*a*(1/2*x/(a^2*x^2+1)+1/2/a*arctan(a*x)-I*ln(x)*(l
n(1+I*a*x)-ln(1-I*a*x))/a-I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a-1/4/a^2*sum(
1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*
_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha
)/_alpha))),_alpha=RootOf(_Z^2*a^2+1)))
```

3.188.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)^2}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

3.188.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)^2}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

3.188.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^2x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x), x)
```

3.188.8 Giac [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^2x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^2), x)
```

3.189 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx$

3.189.1 Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx = -\frac{a}{4c^2(1+a^2x^2)} - \frac{\arctan(ax)}{c^2x} - \frac{a^2x\arctan(ax)}{2c^2(1+a^2x^2)} - \frac{3a\arctan(ax)^2}{4c^2} + \frac{a\log(x)}{c^2} - \frac{a\log(1+a^2x^2)}{2c^2}$$

output
```
-1/4*a/c^2/(a^2*x^2+1)-arctan(a*x)/c^2/x-1/2*a^2*x*arctan(a*x)/c^2/(a^2*x^
2+1)-3/4*a*arctan(a*x)^2/c^2+a*ln(x)/c^2-1/2*a*ln(a^2*x^2+1)/c^2
```

3.189.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx = -\frac{a}{4c^2(1+a^2x^2)} - \frac{(2+3a^2x^2)\arctan(ax)}{2c^2x(1+a^2x^2)} - \frac{3a\arctan(ax)^2}{4c^2} + \frac{a\log(x)}{c^2} - \frac{a\log(1+a^2x^2)}{2c^2}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-1/4*a/(c^2*(1 + a^2*x^2)) - ((2 + 3*a^2*x^2)*ArcTan[a*x])/(2*c^2*x*(1 + a
^2*x^2)) - (3*a*ArcTan[a*x]^2)/(4*c^2) + (a*Log[x])/c^2 - (a*Log[1 + a^2*x
^2])/(2*c^2)
```

3.189.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5501, 27, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{cx^2(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)}{c^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 14$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 16$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 5419$$

$$\frac{\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan
[a*x]^2/(4*a)))/c^2) + (-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x
^2] - Log[1 + a^2*x^2]))/2)/c^2
```

3.189.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.189. $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.189.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

method	result
derivativedivides	$a\left(-\frac{\arctan(ax)}{c^2ax}-\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{-\frac{3\arctan(ax)^2}{2}-2\ln(ax)+\frac{1}{2a^2x^2+2}+\ln(a^2x^2+1)}{2c^2}\right)$
default	$a\left(-\frac{\arctan(ax)}{c^2ax}-\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{-\frac{3\arctan(ax)^2}{2}-2\ln(ax)+\frac{1}{2a^2x^2+2}+\ln(a^2x^2+1)}{2c^2}\right)$
parts	$-\frac{a^2x\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{3a\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{c^2x}-\frac{-\frac{3a\arctan(ax)^2}{4}-\frac{a\left(2\ln(ax)-\frac{1}{2\left(a^2x^2+1\right)}-\ln\left(a^2x^2+1\right)\right)}{2}}{c^2}$
parallelrisch	$\frac{-3a^3\arctan(ax)^2x^3+4\ln(x)a^3x^3-2a^3\ln(a^2x^2+1)x^3+a^3x^3-6a^2\arctan(ax)x^2-3a\arctan(ax)^2x+4ax\ln(x)-2a\ln(a^2}{4x\,c^2(a^2x^2+1)}$
risch	$\frac{3a\ln(iax+1)^2}{16c^2}-\frac{(3a^3x^3\ln(-iax+1)+3ax\ln(-iax+1)-6ia^2x^2-4i)\ln(iax+1)}{8x\,c^2(a^2x^2+1)}-\frac{-3a^3\ln(-iax+1)^2x^3-16\ln(x)a^3}{}$

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c^2*arctan(a*x)/a/x-1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/2*arctan(a
*x)^2/c^2-1/2/c^2*(-3/2*arctan(a*x)^2-2*ln(a*x)+1/2/(a^2*x^2+1)+ln(a^2*x^2
+1)))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx =$$
$$-\frac{3\left(a^3x^3+ax\right)\arctan\left(ax\right)^2+ax+2\left(3\,a^2x^2+2\right)\arctan\left(ax\right)+2\left(a^3x^3+ax\right)\log\left(a^2x^2+1\right)-4\left(a^3x^3+\right.}{4\left(a^2c^2x^3+c^2x\right)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(3*(a^3*x^3 + a*x)*arctan(a*x)^2 + a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x
) + 2*(a^3*x^3 + a*x)*log(a^2*x^2 + 1) - 4*(a^3*x^3 + a*x)*log(x))/(a^2*c^
2*x^3 + c^2*x)
```

3.189.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(88) = 176$.

Time = 0.64 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.82

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{4a^3x^3\log(x)}{4a^2c^2x^3+4c^2x} - \frac{2a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{4a^2c^2x^3+4c^2x} - \frac{3a^3x^3\operatorname{atan}^2(ax)}{4a^2c^2x^3+4c^2x} - \frac{6a^2x^2\operatorname{atan}(ax)}{4a^2c^2x^3+4c^2x} + \frac{4ax\log(x)}{4a^2c^2x^3+4c^2x} - \frac{2ax\log\left(x^2+\frac{1}{a^2}\right)}{4a^2c^2x^3+4c^2x} - \frac{3ax\operatorname{atan}^2(ax}{4a^2c^2x^3+4c^2} \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((4*a**3*x**3*log(x)/(4*a**2*c**2*x**3 + 4*c**2*x) - 2*a**3*x**3*
log(x**2 + a**(-2))/(4*a**2*c**2*x**3 + 4*c**2*x) - 3*a**3*x**3*atan(a*x)*
*2/(4*a**2*c**2*x**3 + 4*c**2*x) - 6*a**2*x**2*atan(a*x)/(4*a**2*c**2*x**3
 + 4*c**2*x) + 4*a*x*log(x)/(4*a**2*c**2*x**3 + 4*c**2*x) - 2*a*x*log(x**2
 + a**(-2))/(4*a**2*c**2*x**3 + 4*c**2*x) - 3*a*x*atan(a*x)**2/(4*a**2*c**
2*x**3 + 4*c**2*x) - a*x/(4*a**2*c**2*x**3 + 4*c**2*x) - 4*atan(a*x)/(4*a*
*2*c**2*x**3 + 4*c**2*x), Ne(a, 0)), (0, True))
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx = -\frac{1}{2}\left(\frac{3\,a^2x^2+2}{a^2c^2x^3+c^2x} + \frac{3\,a\arctan(ax)}{c^2}\right)\arctan(ax) + \frac{\left(3\left(a^2x^2+1\right)\arctan(ax)^2 - 2\left(a^2x^2+1\right)\log\left(a^2x^2+1\right) + 4\left(a^2x^2+1\right)\log(x) - 1\right)a}{4\left(a^2c^2x^2+c^2\right)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*a^2*x^2 + 2)/(a^2*c^2*x^3 + c^2*x) + 3*a*arctan(a*x)/c^2)*arctan(
a*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2*x^2 +
1) + 4*(a^2*x^2 + 1)*log(x) - 1)*a/(a^2*c^2*x^2 + c^2)
```

3.189.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.189.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx = \frac{a\,\ln(x)}{c^2} - \frac{a\,\ln\left(a^2\,x^2+1\right)}{2\,c^2} - \frac{\mathrm{atan}(a\,x)\,\left(\frac{1}{a^2\,c^2}+\frac{3\,x^2}{2\,c^2}\right)}{\frac{x}{a^2}+x^3} - \frac{a}{2\,\left(2\,a^2\,c^2\,x^2+2\,c^2\right)} - \frac{3\,a\,\mathrm{atan}(a\,x)^2}{4\,c^2}$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^2),x)
```

output
```
(a*log(x))/c^2 - (a*log(a^2*x^2 + 1))/(2*c^2) - (atan(a*x)*(1/(a^2*c^2) +
(3*x^2)/(2*c^2)))/(x/a^2 + x^3) - a/(2*(2*c^2 + 2*a^2*c^2*x^2)) - (3*a*ata
n(a*x)^2)/(4*c^2)
```

3.190 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^2}\,dx$

3.190.1 Optimal result

Integrand size = 20, antiderivative size = 156

$$
\begin{aligned}
\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx = &-\frac{a}{2c^2x}+\frac{a^3x}{4c^2\left(1+a^2x^2\right)}-\frac{a^2\arctan(ax)}{4c^2}\\
&-\frac{\arctan(ax)}{2c^2x^2}-\frac{a^2\arctan(ax)}{2c^2\left(1+a^2x^2\right)}+\frac{ia^2\arctan(ax)^2}{c^2}\\
&-\frac{2a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{ia^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}
\end{aligned}
$$

output
```
-1/2*a/c^2/x+1/4*a^3*x/c^2/(a^2*x^2+1)-1/4*a^2*arctan(a*x)/c^2-1/2*arctan(
a*x)/c^2/x^2-1/2*a^2*arctan(a*x)/c^2/(a^2*x^2+1)+I*a^2*arctan(a*x)^2/c^2-2
*a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c^2+I*a^2*polylog(2,-1+2/(1-I*a*x))/c^2
```

3.190.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.60

$$
\begin{aligned}
&\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx\\
&=\frac{a^2\left(-\frac{4}{ax}+8i\arctan(ax)^2+\arctan(ax)\left(-4-\frac{4}{a^2x^2}-2\cos(2\arctan(ax))-16\log\left(1-e^{2i\arctan(ax)}\right)\right)+8i\right.}{8c^2}
\end{aligned}
$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(a^2*(-4/(a*x) + (8*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-4 - 4/(a^2*x^2) - 2*C
os[2*ArcTan[a*x]] - 16*Log[1 - E^((2*I)*ArcTan[a*x])]) + (8*I)*PolyLog[2,
E^((2*I)*ArcTan[a*x])] + Sin[2*ArcTan[a*x]]))/(8*c^2)
```

3.190.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.49, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5501, 27, 5453, 5361, 264, 216, 5459, 5403, 2897, 5501, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)}{cx^3(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)}{c^2x\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^3}dx-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \; 5459$$

$$-\frac{a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx}{c^2}+\frac{-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)}{c^2}$$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx}{c^2}+\frac{-\left(a^2\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(\right.}{c^2}$$

$$\Big\downarrow \; 2897$$

$$-\frac{a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx}{c^2}+\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\,\text{ar}\right.}{c^2}$$

$$\Big\downarrow \; 5501$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-a^2\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx\right)}{c^2}+\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\,\text{ar}\right.}{c^2}$$

$$\Big\downarrow \; 5459$$

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\,\text{ar}\right.}{c^2}$$
$$\frac{a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx\right)+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)}{c^2}$$

$$\Big\downarrow \; 5403$$

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\,\text{ar}\right.}{c^2}$$
$$\frac{a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx\right)+i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c^2}$$

$$\Big\downarrow \; 2897$$

3.190. $\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx$

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\text{ arc}}{c^2}$$

$$\frac{a^2\left(a^2\left(-\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c^2}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\text{ arc}}{c^2}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\right.}{c^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\text{ arc}}{c^2}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\text{ arc}}{c^2}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right.}{c^2}$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2 - a^2*((-1/2*I)*Ar
cTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 +
 2/(1 - I*a*x)]/2)))/c^2 - (a^2*((-1/2*I)*ArcTan[a*x]^2 - a^2*(-1/2*ArcTan
[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a
)) + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I
*a*x)]/2)))/c^2
```

3.190. $\int\frac{\arctan(ax)}{x^3(c+a^2cx^2)^2}\,dx$

3.190.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

3.190. $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^2}\,dx$

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.190.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

method	result
parts	$\frac{\arctan(ax)a^2\ln(a^2x^2+1)}{c^2} - \frac{a^2\arctan(ax)}{2c^2(a^2x^2+1)} - \frac{\arctan(ax)}{2c^2x^2} - \frac{2\arctan(ax)a^2\ln(x)}{c^2} - \frac{a\left(-4a^2\left(-\frac{i\ln(x)(\ln(iax+1)-}{2a}\right.\right.}{}$
derivativedivides	$a^2\left(-\frac{\arctan(ax)}{2c^2a^2x^2} - \frac{2\arctan(ax)\ln(ax)}{c^2} + \frac{\arctan(ax)\ln(a^2x^2+1)}{c^2} - \frac{\arctan(ax)}{2c^2(a^2x^2+1)} - \frac{2i\ln(ax)\ln(iax+1)-2i\ln}{}\right.$
default	$a^2\left(-\frac{\arctan(ax)}{2c^2a^2x^2} - \frac{2\arctan(ax)\ln(ax)}{c^2} + \frac{\arctan(ax)\ln(a^2x^2+1)}{c^2} - \frac{\arctan(ax)}{2c^2(a^2x^2+1)} - \frac{2i\ln(ax)\ln(iax+1)-2i\ln}{}\right.$
risch	$-\frac{a}{2c^2x} + \frac{a^2\arctan(ax)}{8c^2} + \frac{ia^2}{8c^2(iax+1)} - \frac{ia^2\operatorname{dilog}(iax+1)}{c^2} - \frac{ia^2\ln(iax+1)^2}{4c^2} + \frac{ia^2\operatorname{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{2c^2} - \frac{ia^2\ln(ia}{4c^2}$

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*arctan(a*x)*a^2*ln(a^2*x^2+1)-1/2*a^2*arctan(a*x)/c^2/(a^2*x^2+1)-1/
2*arctan(a*x)/c^2/x^2-2/c^2*arctan(a*x)*a^2*ln(x)-1/2*a/c^2*(-4*a^2*(-1/2*
I*ln(x)*(ln(1+I*a*x)-ln(1-I*a*x))/a-1/2*I*(dilog(1+I*a*x)-dilog(1-I*a*x))/
a)-1/2*a^2*x/(a^2*x^2+1)+1/2*a*arctan(a*x)+1/x+1/2*sum(1/_alpha*(2*ln(x-_a
lpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)
*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=
RootOf(_Z^2*a^2+1))
```

3.190.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

3.190.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^2} \, dx = \frac{\int \frac{\operatorname{atan}(ax)}{a^4x^7+2a^2x^5+x^3} \, dx}{c^2}$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

3.190.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^3), x)
```

3.190.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\mathrm{atan}(a\,x)}{x^3 \left(c\,a^2\,x^2 + c\right)^2} \, dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^2), x)
```

3.191 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx$

3.191.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx = -\frac{a}{6c^2x^2}+\frac{a^3}{4c^2(1+a^2x^2)}-\frac{\arctan(ax)}{3c^2x^3}+\frac{2a^2\arctan(ax)}{c^2x}+\frac{a^4x\arctan(ax)}{2c^2(1+a^2x^2)}+\frac{5a^3\arctan(ax)^2}{4c^2}-\frac{7a^3\log(x)}{3c^2}+\frac{7a^3\log(1+a^2x^2)}{6c^2}$$

output
```
-1/6*a/c^2/x^2+1/4*a^3/c^2/(a^2*x^2+1)-1/3*arctan(a*x)/c^2/x^3+2*a^2*arcta
n(a*x)/c^2/x+1/2*a^4*x*arctan(a*x)/c^2/(a^2*x^2+1)+5/4*a^3*arctan(a*x)^2/c
^2-7/3*a^3*ln(x)/c^2+7/6*a^3*ln(a^2*x^2+1)/c^2
```

3.191.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx = -\frac{a}{6c^2x^2}+\frac{a^3}{4c^2(1+a^2x^2)}+\frac{(-2+10a^2x^2+15a^4x^4)\arctan(ax)}{6c^2x^3(1+a^2x^2)}+\frac{5a^3\arctan(ax)^2}{4c^2}-\frac{7a^3\log(x)}{3c^2}+\frac{7a^3\log(1+a^2x^2)}{6c^2}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
-1/6*a/(c^2*x^2) + a^3/(4*c^2*(1 + a^2*x^2)) + ((-2 + 10*a^2*x^2 + 15*a^4*
x^4)*ArcTan[a*x])/(6*c^2*x^3*(1 + a^2*x^2)) + (5*a^3*ArcTan[a*x]^2)/(4*c^2
) - (7*a^3*Log[x])/(3*c^2) + (7*a^3*Log[1 + a^2*x^2])/(6*c^2)
```

3.191.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.52, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {5501, 27, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419, 5501, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4 \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{cx^4(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)}{c^2x^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^4}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{3}a\int \frac{1}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{6}a\int \frac{1}{x^4(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow 2009$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow 5453$$

$$\frac{-\left(a^2\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow 5361$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\text{arc}}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow 243$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow 47$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow 14$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

3.191. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx$

$$\Big\downarrow \; 16$$

$$-\frac{\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \; 5419$$

$$-\frac{\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \; 5501$$

$$-\frac{\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx\right)}{c^2}$$

$$\Big\downarrow \; 5427$$

$$-\frac{\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)}{c^2}$$

$$\Big\downarrow \; 241$$

$$-\frac{\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)}{c^2}$$

$$\Big\downarrow \; 5453$$

$$-\frac{\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx+\int \frac{\arctan(ax)}{x^2}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)\right)}{c^2}$$

3.191. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx$

$$\Big\downarrow \textbf{5361}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$
$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+a\int\frac{1}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}\right)}{c^2}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$
$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\int\frac{1}{x^2(a^2x^2+1)}dx^2-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}\right)}{c^2}$$

$$\Big\downarrow \textbf{47}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$
$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax}{x}\right.}{c^2}$$

$$\Big\downarrow \textbf{14}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$
$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax}{x}\right.}{c^2}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$
$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right.}{c^2}$$

$$\Big\downarrow \textbf{5419}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$
$$\frac{a^2\left(-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)}{c^2}$$

3.191. $\int\frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*a*(1 + a^2*x^2)
) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)) + (a*(Log[x^2
] - Log[1 + a^2*x^2]))/2))/c^2) + (-1/3*ArcTan[a*x]/x^3 - a^2*(-(ArcTan[a*
x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) + (a*(-
x^(-2) - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))/6)/c^2
```

3.191.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.191. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx$

3.191.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

method	result
derivativedivides	$a^3\left(\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2a^3x^3}+\frac{2\arctan(ax)}{c^2ax}-\frac{-7\ln(a^2x^2+1)-\frac{3}{2(a^2x^2+1)}+\frac{1}{a^2x^2}+14\ln(}{6c^2}\right.$
default	$a^3\left(\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2a^3x^3}+\frac{2\arctan(ax)}{c^2ax}-\frac{-7\ln(a^2x^2+1)-\frac{3}{2(a^2x^2+1)}+\frac{1}{a^2x^2}+14\ln(}{6c^2}\right.$
parts	$\frac{a^4x\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5a^3\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2x^3}+\frac{2a^2\arctan(ax)}{c^2x}-\frac{\frac{15a^3\arctan(ax)^2}{4}+\frac{a^3\left(-7\ln(a^2x^2+1)-\frac{}{2(a^2}\right.}{2}}{3c^2}$
parallelrisch	$-\frac{-15a^5\arctan(ax)^2x^5+28\ln(x)x^5a^5-14\ln(a^2x^2+1)x^5a^5-3a^5x^5-30\arctan(ax)a^4x^4-15a^3\arctan(ax)^2x^3+28\ln(x}{12x^3c^2(a^2x^2+1)}$
risch	$\frac{ia^2\ln(-iax+1)}{c^2x}-\frac{ia^2\ln(iax+1)}{c^2x}+\frac{a^3\ln(-iax+1)}{8c^2(-iax+1)}+\frac{a^3\ln(-iax+1)}{16c^2(-iax-1)}+\frac{i\ln(iax+1)}{6c^2x^3}+\frac{5a^3\ln(\frac{1}{2}-\frac{iax}{2})\ln(iax+1}{8c^2}$

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)+5/2*arctan(a*x)^2/c^2-1/3/c^2*arc
tan(a*x)/a^3/x^3+2/c^2*arctan(a*x)/a/x-1/6/c^2*(-7*ln(a^2*x^2+1)-3/2/(a^2*
x^2+1)+1/a^2/x^2+14*ln(a*x)+15/2*arctan(a*x)^2))
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx$$
$$=\frac{a^3x^3+15(a^5x^5+a^3x^3)\arctan(ax)^2-2ax+2(15a^4x^4+10a^2x^2-2)\arctan(ax)+14(a^5x^5+a^3x^3)\,\mathrm{lo}}{12(a^2c^2x^5+c^2x^3)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/12*(a^3*x^3 + 15*(a^5*x^5 + a^3*x^3)*arctan(a*x)^2 - 2*a*x + 2*(15*a^4*x
^4 + 10*a^2*x^2 - 2)*arctan(a*x) + 14*(a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1)
 - 28*(a^5*x^5 + a^3*x^3)*log(x))/(a^2*c^2*x^5 + c^2*x^3)
```

3.191.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(129) = 258$.

Time = 0.93 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.66

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^2} dx$$
$$= \begin{cases} -\frac{28a^5x^5\log(x)}{12a^2c^2x^5+12c^2x^3} + \frac{14a^5x^5\log\left(x^2+\frac{1}{a^2}\right)}{12a^2c^2x^5+12c^2x^3} + \frac{15a^5x^5\operatorname{atan}^2(ax)}{12a^2c^2x^5+12c^2x^3} + \frac{30a^4x^4\operatorname{atan}(ax)}{12a^2c^2x^5+12c^2x^3} - \frac{28a^3x^3\log(x)}{12a^2c^2x^5+12c^2x^3} + \frac{14a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{12a^2c^2x^5+12c^2x^3} \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((-28*a**5*x**5*log(x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 14*a*
*5*x**5*log(x**2 + a**(-2))/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 15*a**5*x
**5*atan(a*x)**2/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 30*a**4*x**4*atan(a*
x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) - 28*a**3*x**3*log(x)/(12*a**2*c**2*
x**5 + 12*c**2*x**3) + 14*a**3*x**3*log(x**2 + a**(-2))/(12*a**2*c**2*x**5
 + 12*c**2*x**3) + 15*a**3*x**3*atan(a*x)**2/(12*a**2*c**2*x**5 + 12*c**2*
x**3) + a**3*x**3/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 20*a**2*x**2*atan(a
*x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) - 2*a*x/(12*a**2*c**2*x**5 + 12*c**
2*x**3) - 4*atan(a*x)/(12*a**2*c**2*x**5 + 12*c**2*x**3), Ne(a, 0)), (0, T
rue))
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.18

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^2} dx = \frac{1}{6}\left(\frac{15\,a^3\arctan(ax)}{c^2} + \frac{15\,a^4x^4 + 10\,a^2x^2 - 2}{a^2c^2x^5 + c^2x^3}\right)\arctan(ax)$$
$$+ \frac{\left(a^2x^2 - 15\left(a^4x^4 + a^2x^2\right)\arctan(ax)^2 + 14\left(a^4x^4 + a^2x^2\right)\log\left(a^2x^2 + 1\right) - 28\left(a^4x^4 + a^2x^2\right)\log(x) - 2\right)}{12\left(a^2c^2x^4 + c^2x^2\right)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/6*(15*a^3*arctan(a*x)/c^2 + (15*a^4*x^4 + 10*a^2*x^2 - 2)/(a^2*c^2*x^5 +
 c^2*x^3))*arctan(a*x) + 1/12*(a^2*x^2 - 15*(a^4*x^4 + a^2*x^2)*arctan(a*x
)^2 + 14*(a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 28*(a^4*x^4 + a^2*x^2)*log
(x) - 2)*a/(a^2*c^2*x^4 + c^2*x^2)
```

3.191.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^2 x^4} \, dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.191.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^2} \, dx = \frac{\operatorname{atan}(a\,x)\,\left(\frac{5\,x^2}{3\,c^2}-\frac{1}{3\,a^2\,c^2}+\frac{5\,a^2\,x^4}{2\,c^2}\right)}{x^5+\frac{x^3}{a^2}} - \frac{a-\frac{a^3\,x^2}{2}}{6\,a^2\,c^2\,x^4+6\,c^2\,x^2} + \frac{7\,a^3\,\ln\left(a^2\,x^2+1\right)}{6\,c^2} - \frac{7\,a^3\,\ln\left(x\right)}{3\,c^2} + \frac{5\,a^3\,\operatorname{atan}(a\,x)^2}{4\,c^2}$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^2),x)
```

output
```
(atan(a*x)*((5*x^2)/(3*c^2) - 1/(3*a^2*c^2) + (5*a^2*x^4)/(2*c^2)))/(x^5 +
 x^3/a^2) - (a - (a^3*x^2)/2)/(6*c^2*x^2 + 6*a^2*c^2*x^4) + (7*a^3*log(a^2
*x^2 + 1))/(6*c^2) - (7*a^3*log(x))/(3*c^2) + (5*a^3*atan(a*x)^2)/(4*c^2)
```

3.192 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3} dx$

3.192.1 Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{x^3}{16ac^3(1+a^2x^2)^2} + \frac{3x}{32a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)}{32a^4c^3} + \frac{x^4\arctan(ax)}{4c^3(1+a^2x^2)^2}$$

output
```
1/16*x^3/a/c^3/(a^2*x^2+1)^2+3/32*x/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)/a
^4/c^3+1/4*x^4*arctan(a*x)/c^3/(a^2*x^2+1)^2
```

3.192.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{ax(3+5a^2x^2) + (-3-6a^2x^2+5a^4x^4)\arctan(ax)}{32a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(a*x*(3 + 5*a^2*x^2) + (-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c
^3*(1 + a^2*x^2)^2)
```

3.192.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5479, 27, 252, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \ 5479$$

$$\frac{x^4 \arctan(ax)}{4c^3 (a^2x^2+1)^2} - \frac{1}{4} a \int \frac{x^4}{c^3 (a^2x^2+1)^3} dx$$

$$\Big\downarrow \ 27$$

$$\frac{x^4 \arctan(ax)}{4c^3 (a^2x^2+1)^2} - \frac{a \int \frac{x^4}{(a^2x^2+1)^3} dx}{4c^3}$$

$$\Big\downarrow \ 252$$

$$\frac{x^4 \arctan(ax)}{4c^3 (a^2x^2+1)^2} - \frac{a \left(\frac{3 \int \frac{x^2}{(a^2x^2+1)^2} dx}{4a^2} - \frac{x^3}{4a^2 (a^2x^2+1)^2} \right)}{4c^3}$$

$$\Big\downarrow \ 252$$

$$\frac{x^4 \arctan(ax)}{4c^3 (a^2x^2+1)^2} - \frac{a \left(\frac{3 \left(\frac{\int \frac{1}{a^2x^2+1} dx}{2a^2} - \frac{x}{2a^2 (a^2x^2+1)} \right)}{4a^2} - \frac{x^3}{4a^2 (a^2x^2+1)^2} \right)}{4c^3}$$

$$\Big\downarrow \ 216$$

$$\frac{x^4 \arctan(ax)}{4c^3 (a^2x^2+1)^2} - \frac{a \left(\frac{3 \left(\frac{\arctan(ax)}{2a^3} - \frac{x}{2a^2 (a^2x^2+1)} \right)}{4a^2} - \frac{x^3}{4a^2 (a^2x^2+1)^2} \right)}{4c^3}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (a*(-1/4*x^3/(a^2*(1 + a^2*x^2
)^2) + (3*(-1/2*x/(a^2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a^3)))/(4*a^2)))/(4
*c^3)
```

3.192.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.192. $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3} \, dx$

3.192.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{5\arctan(ax)a^4x^4+5a^3x^3-6a^2\arctan(ax)x^2+3ax-3\arctan(ax)}{32c^3(a^2x^2+1)^2a^4}$
derivativedivides	$\frac{-\frac{\arctan(ax)}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{-\frac{\frac{5}{8}a^3x^3+\frac{3}{8}ax}{(a^2x^2+1)^2}-\frac{5\arctan(ax)}{8}}{4c^3}}{a^4}$
default	$\frac{-\frac{\arctan(ax)}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{-\frac{\frac{5}{8}a^3x^3+\frac{3}{8}ax}{(a^2x^2+1)^2}-\frac{5\arctan(ax)}{8}}{4c^3}}{a^4}$
parts	$-\frac{\arctan(ax)}{2a^4c^3(a^2x^2+1)}+\frac{\arctan(ax)}{4c^3a^4(a^2x^2+1)^2}-\frac{-\frac{\frac{5}{8}a^2x^3+\frac{3}{8}x}{(a^2x^2+1)^2}-\frac{5\arctan(ax)}{8a}}{4c^3a^3}$
risch	$\frac{i(2a^2x^2+1)\ln(iax+1)}{8a^4c^3(a^2x^2+1)^2}-\frac{i(16a^2x^2\ln(-iax+1)+8\ln(-iax+1)-5\ln(ax+i)a^4x^4-10\ln(ax+i)a^2x^2-5\ln(ax+i)+5\ln(}{64a^4(ax+i)^2(ax-i)^2c^3}$

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(5*arctan(a*x)*a^4*x^4+5*a^3*x^3-6*a^2*arctan(a*x)*x^2+3*a*x-3*arctan
(a*x))/c^3/(a^2*x^2+1)^2/a^4
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{x^3\arctan(ax)}{(c+a^2cx^2)^3}\,dx=\frac{5\,a^3x^3+3\,ax+(5\,a^4x^4-6\,a^2x^2-3)\arctan{(ax)}}{32\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(5*a^3*x^3 + 3*a*x + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x))/(a^8*c^
3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.192.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(78) = 156$.

Time = 0.60 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.43

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \begin{cases} \frac{5a^4x^4\operatorname{atan}(ax)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} + \frac{5a^3x^3}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} - \frac{6a^2x^2\operatorname{atan}(ax)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} + \frac{3ax}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} - \frac{}{32a^8c} \\ 0 \end{cases}$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((5*a**4*x**4*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 +
32*a**4*c**3) + 5*a**3*x**3/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a*
*4*c**3) - 6*a**2*x**2*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 +
32*a**4*c**3) + 3*a*x/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**
3) - 3*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**3), N
e(a, 0)), (0, True))
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{1}{32}\,a\left(\frac{5\,a^2x^3+3\,x}{a^8c^3x^4+2\,a^6c^3x^2+a^4c^3} + \frac{5\,\arctan(ax)}{a^5c^3}\right) - \frac{(2\,a^2x^2+1)\arctan(ax)}{4\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arct
an(a*x)/(a^5*c^3)) - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*
c^3*x^2 + a^4*c^3)
```

3.192.8 Giac [F]

$$\int \frac{x^3\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.192.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{x^3\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\,a\,x-3\,\mathrm{atan}(a\,x)+5\,a^3\,x^3-6\,a^2\,x^2\,\mathrm{atan}(a\,x)+5\,a^4\,x^4\,\mathrm{atan}(a\,x)}{32\,a^4\,c^3\,(a^2\,x^2+1)^2}$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(3*a*x - 3*atan(a*x) + 5*a^3*x^3 - 6*a^2*x^2*atan(a*x) + 5*a^4*x^4*atan(a*
x))/(32*a^4*c^3*(a^2*x^2 + 1)^2)
```

3.193 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx$

3.193.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx = -\frac{1}{16a^3c^3(1+a^2x^2)^2} + \frac{1}{16a^3c^3(1+a^2x^2)} - \frac{x\arctan(ax)}{4a^2c^3(1+a^2x^2)^2} + \frac{x\arctan(ax)}{8a^2c^3(1+a^2x^2)} + \frac{\arctan(ax)^2}{16a^3c^3}$$

output
```
-1/16/a^3/c^3/(a^2*x^2+1)^2+1/16/a^3/c^3/(a^2*x^2+1)-1/4*x*arctan(a*x)/a^2
/c^3/(a^2*x^2+1)^2+1/8*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)+1/16*arctan(a*x)^
2/a^3/c^3
```

3.193.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{a^2x^2 + 2ax(-1+a^2x^2)\arctan(ax) + (1+a^2x^2)^2\arctan(ax)^2}{16a^3c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(a^2*x^2 + 2*a*x*(-1 + a^2*x^2)*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a*x]^
2)/(16*a^3*c^3*(1 + a^2*x^2)^2)
```

3.193.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5469, 27, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5469}$$

$$\frac{\int \frac{\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{4a^2c}-\frac{x\arctan(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{1}{16a^3c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{4a^2c^3}-\frac{x\arctan(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{1}{16a^3c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{4a^2c^3}-\frac{x\arctan(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}-\frac{1}{16a^3c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{241}$$

$$-\frac{x\arctan(ax)}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{4a^2c^3}-\frac{1}{16a^3c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
-1/16*1/(a^3*c^3*(1 + a^2*x^2)^2) - (x*ArcTan[a*x])/(4*a^2*c^3*(1 + a^2*x^
2)^2) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTa
n[a*x]^2/(4*a))/(4*a^2*c^3)
```

3.193.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

3.193.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{a^4 \arctan(ax)^2 x^4+2\arctan(ax) x^3 a^3+2x^2 \arctan(ax)^2 a^2+a^2x^2-2x\arctan(ax) a+\arctan(ax)^2}{16c^3(a^2x^2+1)^2 a^3}$
derivativedivides	$\frac{\frac{\arctan(ax)a^3x^3}{8c^3\left(a^2x^2+1\right)^2}-\frac{ax\arctan(ax)}{8c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)^2}{8c^3}-\frac{\frac{\arctan(ax)^2}{2}-\frac{1}{2\left(a^2x^2+1\right)}+\frac{1}{2\left(a^2x^2+1\right)^2}}{8c^3}}{a^3}$
default	$\frac{\frac{\arctan(ax)a^3x^3}{8c^3\left(a^2x^2+1\right)^2}-\frac{ax\arctan(ax)}{8c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)^2}{8c^3}-\frac{\frac{\arctan(ax)^2}{2}-\frac{1}{2\left(a^2x^2+1\right)}+\frac{1}{2\left(a^2x^2+1\right)^2}}{8c^3}}{a^3}$
parts	$\frac{\arctan(ax)x^3}{8c^3(a^2x^2+1)^2}-\frac{x\arctan(ax)}{8a^2c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^2}{8a^3c^3}-\frac{\frac{\arctan(ax)^2}{2a^3}+\frac{-\frac{1}{2\left(a^2x^2+1\right)}+\frac{1}{2\left(a^2x^2+1\right)^2}}{a^3}}{8c^3}$
risch	$-\frac{\ln(iax+1)^2}{64a^3c^3}+\frac{(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)}{32a^3c^3(a^2x^2+1)^2}-\frac{a^4x^4\ln(-iax+1}{}$

3.193. $\int \frac{x^2\arctan(ax)}{(c+a^2cx^2)^3}\,dx$

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(a^4*arctan(a*x)^2*x^4+2*arctan(a*x)*x^3*a^3+2*x^2*arctan(a*x)^2*a^2+
a^2*x^2-2*x*arctan(a*x)*a+arctan(a*x)^2)/c^3/(a^2*x^2+1)^2/a^3
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{x^2\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{a^2x^2 + (a^4x^4 + 2\,a^2x^2 + 1)\arctan{(ax)}^2 + 2\,(a^3x^3 - ax)\arctan{(ax)}}{16\,(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/16*(a^2*x^2 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(a^3*x^3 - a*x
)*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

3.193.6 Sympy [F]

$$\int \frac{x^2\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2\operatorname{atan}{(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan(ax)}{a^3c^3}\right)\arctan(ax) + \frac{\left(a^2x^2-\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)^2\right)a}{16\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(
a^3*c^3))*arctan(a*x) + 1/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a
*x)^2)*a/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.193.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.193.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{a^4\,x^4\,\mathrm{atan}(a\,x)^2+2\,a^3\,x^3\,\mathrm{atan}(a\,x)+2\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+a^2\,x^2-2\,a\,x\,\mathrm{atan}(a\,x)+\mathrm{atan}(a\,x)^2}{16\,a^3\,c^3\,\left(a^2\,x^2+1\right)^2}$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(a^2*x^2 + atan(a*x)^2 + 2*a^3*x^3*atan(a*x) - 2*a*x*atan(a*x) + 2*a^2*x^2
*atan(a*x)^2 + a^4*x^4*atan(a*x)^2)/(16*a^3*c^3*(a^2*x^2 + 1)^2)
```

3.194 $\int \frac{x \arctan(ax)}{(c+a^2cx^2)^3} dx$

3.194.1 Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{x}{16ac^3(1+a^2x^2)^2} + \frac{3x}{32ac^3(1+a^2x^2)} + \frac{3\arctan(ax)}{32a^2c^3} - \frac{\arctan(ax)}{4a^2c^3(1+a^2x^2)^2}$$

output
```
1/16*x/a/c^3/(a^2*x^2+1)^2+3/32*x/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)/a^2/c
^3-1/4*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2
```

3.194.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{ax(5+3a^2x^2) + (-5+6a^2x^2+3a^4x^4)\arctan(ax)}{32c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(a*x*(5 + 3*a^2*x^2) + (-5 + 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x])/(32*c^3*(
a + a^3*x^2)^2)
```

3.194.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5465, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^3}dx}{4a}-\frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{(a^2cx^2+c)^2}dx}{4c}+\frac{x}{4c^3(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{a^2cx^2+c}dx}{2c}+\frac{x}{2c^2(a^2x^2+1)}\right)}{4c}+\frac{x}{4c^3(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}+\frac{x}{4c^3(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]/(a^2*c^3*(1 + a^2*x^2)^2) + (x/(4*c^3*(1 + a^2*x^2)^2) +
(3*(x/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a*c^2)))/(4*c))/(4*a)
```

3.194.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.194.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{3\arctan(ax)a^4x^4+3a^3x^3+6a^2\arctan(ax)x^2+5ax-5\arctan(ax)}{32c^3(a^2x^2+1)^2a^2}$
derivativedivides	$\frac{-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{\frac{ax}{4(a^2x^2+1)^2}+\frac{3ax}{8(a^2x^2+1)}+\frac{3\arctan(ax)}{8}}{4c^3}}{a^2}$
default	$\frac{-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{\frac{ax}{4(a^2x^2+1)^2}+\frac{3ax}{8(a^2x^2+1)}+\frac{3\arctan(ax)}{8}}{4c^3}}{a^2}$
parts	$-\frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}+\frac{\frac{x}{4(a^2x^2+1)^2}+\frac{3x}{8(a^2x^2+1)}+\frac{3\arctan(ax)}{8a}}{4a\,c^3}$
risch	$\frac{i\ln(iax+1)}{8a^2c^3(a^2x^2+1)^2}-\frac{i(8\ln(-iax+1)+3\ln(-ax+i)a^4x^4+6\ln(-ax+i)a^2x^2+3\ln(-ax+i)-3\ln(ax+i)a^4x^4-6\ln(ax+i)}{64a^2(ax+i)^2(ax-i)^2c^3}$

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(3*arctan(a*x)*a^4*x^4+3*a^3*x^3+6*a^2*arctan(a*x)*x^2+5*a*x-5*arctan
(a*x))/c^3/(a^2*x^2+1)^2/a^2
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^3x^3+5\,ax+(3\,a^4x^4+6\,a^2x^2-5)\arctan{(ax)}}{32\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(3*a^3*x^3 + 5*a*x + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x))/(a^6*c^
3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(75) = 150.

Time = 0.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.49

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx$$
$$= \begin{cases} \frac{3a^4x^4\operatorname{atan}{(ax)}}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} + \frac{3a^3x^3}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} + \frac{6a^2x^2\operatorname{atan}{(ax)}}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} + \frac{5ax}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} - \frac{}{32a^6c} \\ 0 \end{cases}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((3*a**4*x**4*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 +
32*a**2*c**3) + 3*a**3*x**3/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a*
*2*c**3) + 6*a**2*x**2*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 +
32*a**2*c**3) + 5*a*x/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**
3) - 5*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**3), N
e(a, 0)), (0, True))
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2}+\frac{3\,\arctan(ax)}{ac^2}}{32\,ac} - \frac{\arctan\left(ax\right)}{4\left(a^2cx^2+c\right)^2a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x
)/(a*c^2))/(a*c) - 1/4*arctan(a*x)/((a^2*c*x^2 + c)^2*a^2*c)
```

3.194.8 Giac [F]

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \int \frac{x\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.194.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.23

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{\frac{5\,x}{32\,a}+\frac{a\,x^3}{4}-\frac{\mathrm{atan}(a\,x)}{4\,a^2}-\frac{x^2\,\mathrm{atan}(a\,x)}{4}+\frac{3\,a^3\,x^5}{32}}{a^6\,c^3\,x^6+3\,a^4\,c^3\,x^4+3\,a^2\,c^3\,x^2+c^3} + \frac{3\,\mathrm{atan}\left(\frac{a^2\,x}{\sqrt{a^2}}\right)}{32\,a\,c^3\,\sqrt{a^2}}$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
((5*x)/(32*a) + (a*x^3)/4 - atan(a*x)/(4*a^2) - (x^2*atan(a*x))/4 + (3*a^3
*x^5)/32)/(c^3 + 3*a^2*c^3*x^2 + 3*a^4*c^3*x^4 + a^6*c^3*x^6) + (3*atan((a
^2*x)/(a^2)^(1/2)))/(32*a*c^3*(a^2)^(1/2))
```

3.195 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^3} dx$

3.195.1 Optimal result

Integrand size = 17, antiderivative size = 105

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{1}{16ac^3(1+a^2x^2)^2} + \frac{3}{16ac^3(1+a^2x^2)} + \frac{x\arctan(ax)}{4c^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)}{8c^3(1+a^2x^2)} + \frac{3\arctan(ax)^2}{16ac^3}$$

output
```
1/16/a/c^3/(a^2*x^2+1)^2+3/16/a/c^3/(a^2*x^2+1)+1/4*x*arctan(a*x)/c^3/(a^2
*x^2+1)^2+3/8*x*arctan(a*x)/c^3/(a^2*x^2+1)+3/16*arctan(a*x)^2/a/c^3
```

3.195.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{4+3a^2x^2+2ax(5+3a^2x^2)\arctan(ax)+3(1+a^2x^2)^2\arctan(ax)^2}{16ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

output
```
(4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*Arc
Tan[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)^2)
```

3.195.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5431, 27, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5431}$$

$$\frac{3\int \frac{\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{4c^3}+\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{4c^3}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

output
```
1/(16*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) + (
3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]
^2/(4*a)))/(4*c^3)
```

3.195.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.195.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result
parallelrisch	$\frac{3a^4\arctan(ax)^2x^4-4a^4x^4+6\arctan(ax)x^3a^3+6x^2\arctan(ax)^2a^2-5a^2x^2+10x\arctan(ax)a+3\arctan(ax)^2}{16c^3(a^2x^2+1)^2a}$
derivativedivides	$\frac{\frac{ax\arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{3ax\arctan(ax)}{8c^3\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^2}{8c^3}-\frac{-\frac{1}{2\left(a^2x^2+1\right)^2}-\frac{3}{2\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^2}{2}}{8c^3}}{a}$
default	$\frac{\frac{ax\arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{3ax\arctan(ax)}{8c^3\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^2}{8c^3}-\frac{-\frac{1}{2\left(a^2x^2+1\right)^2}-\frac{3}{2\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^2}{2}}{8c^3}}{a}$
parts	$\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3x\arctan(ax)}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^2}{8a\,c^3}-\frac{\frac{-\frac{1}{2\left(a^2x^2+1\right)^2}-\frac{3}{2\left(a^2x^2+1\right)}}{a}+\frac{3\arctan(ax)^2}{2a}}{8c^3}$
risch	$-\frac{3\ln(iax+1)^2}{64c^3a}+\frac{(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax)\ln(iax+1)}{32c^3(a^2x^2+1)^2a}-\frac{3a^4x^4\ln(-$

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(3*a^4*arctan(a*x)^2*x^4-4*a^4*x^4+6*arctan(a*x)*x^3*a^3+6*x^2*arctan
(a*x)^2*a^2-5*a^2*x^2+10*x*arctan(a*x)*a+3*arctan(a*x)^2)/c^3/(a^2*x^2+1)^
2/a
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^2x^2+3\,(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}^2+2\,(3\,a^3x^3+5\,ax)\arctan{(ax)}+4}{16\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/16*(3*a^2*x^2 + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(3*a^3*x^3
 + 5*a*x)*arctan(a*x) + 4)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

3.195.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan\left(ax\right)}{ac^3}\right)\arctan\left(ax\right) + \frac{\left(3\,a^2x^2-3\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+4\right)a}{16\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)
/(a*c^3))*arctan(a*x) + 1/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arct
an(a*x)^2 + 4)*a/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

3.195.8 Giac [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.195.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+6\,a^3\,x^3\,\mathrm{atan}(a\,x)+6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+3\,a^2\,x^2+10\,a\,x\,\mathrm{atan}(a\,x)+3\,\mathrm{atan}(a\,x)^2+4}{16\,a\,c^3\,\left(a^2\,x^2+1\right)^2}$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^3,x)
```

output
```
(3*a^2*x^2 + 3*atan(a*x)^2 + 6*a^3*x^3*atan(a*x) + 10*a*x*atan(a*x) + 6*a^
2*x^2*atan(a*x)^2 + 3*a^4*x^4*atan(a*x)^2 + 4)/(16*a*c^3*(a^2*x^2 + 1)^2)
```

3.196 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^3}\,dx$

3.196.1 Optimal result

Integrand size = 20, antiderivative size = 159

$$\begin{aligned}\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = &-\frac{ax}{16c^3\left(1+a^2x^2\right)^2}-\frac{11ax}{32c^3\left(1+a^2x^2\right)}-\frac{11\arctan(ax)}{32c^3}\\ &+\frac{\arctan(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{\arctan(ax)}{2c^3\left(1+a^2x^2\right)}-\frac{i\arctan(ax)^2}{2c^3}\\ &+\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{i\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
-1/16*a*x/c^3/(a^2*x^2+1)^2-11/32*a*x/c^3/(a^2*x^2+1)-11/32*arctan(a*x)/c^
3+1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)/c^3/(a^2*x^2+1)-1/2*I*
arctan(a*x)^2/c^3+arctan(a*x)*ln(2-2/(1-I*a*x))/c^3-1/2*I*polylog(2,-1+2/(
1-I*a*x))/c^3
```

3.196.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = -\frac{64i\arctan(ax)^2-4\arctan(ax)\left(12\cos(2\arctan(ax))+\cos(4\arctan(ax))+32\log\left(1-e^{2i\arctan(ax)}\right)\right)+}{128c^3}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-1/128*((64*I)*ArcTan[a*x]^2 - 4*ArcTan[a*x]*(12*Cos[2*ArcTan[a*x]] + Cos[
4*ArcTan[a*x]] + 32*Log[1 - E^((2*I)*ArcTan[a*x])]) + (64*I)*PolyLog[2, E^
((2*I)*ArcTan[a*x])] + 24*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a*x]])/c^3
```

3.196.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5501, 27, 5465, 215, 215, 216, 5501, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \ 5501$$

$$\frac{\int \frac{\arctan(ax)}{c^2x(a^2x^2+1)^2}dx}{c}-a^2\int \frac{x\arctan(ax)}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \ 5465$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3}dx}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 215$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\int \frac{1}{\left(a^2x^2+1\right)^2}dx+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 215$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 216$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 5501$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 5459$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2}{c^3}$$

$$\Big\downarrow \ 5403$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^3}$$

$$\Big\downarrow \ 2897$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^3}$$

3.196. $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^3}\,dx$

$$\Big\downarrow \; 5465$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{-\left(a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\,\text{arct}}{c^3}$$

$$\Big\downarrow \; 215$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{-\left(a^2\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-}{c^3}$$

$$\Big\downarrow \; 216$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{-\left(a^2\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}}{c^3}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-1/4*ArcTan[a*x]/(a^2*(1 + a^2*x^2)^2) + (x/(4*(1 + a^2*x^2)^2) +
(3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/(4*a)))/c^3) + ((-1/2*I)*
ArcTan[a*x]^2 - a^2*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2
*x^2)) + ArcTan[a*x]/(2*a))/(2*a)) + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*
a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))/c^3
```

3.196. $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^3}\,dx$

3.196.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.196. $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^3}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.196.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.61

method	result
parts	$\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2} - \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3} + \frac{\arctan(ax)}{2c^3(a^2x^2+1)} + \frac{\arctan(ax)\ln(x)}{c^3} - \frac{a\left(-\frac{i\ln(x)(\ln(iax+1)-\ln(-iax+1)}{a}\right.}{}$
derivativedivides	$\frac{\arctan(ax)\ln(ax)}{c^3} + \frac{\arctan(ax)}{2c^3(a^2x^2+1)} - \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3} + \frac{\arctan(ax)}{4c^3(a^2x^2+1)^2} - \frac{\frac{\frac{11}{8}a^3x^3+\frac{13}{8}ax}{(a^2x^2+1)^2} + \frac{11\arctan(ax)}{8} - 2}{}$
default	$\frac{\arctan(ax)\ln(ax)}{c^3} + \frac{\arctan(ax)}{2c^3(a^2x^2+1)} - \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3} + \frac{\arctan(ax)}{4c^3(a^2x^2+1)^2} - \frac{\frac{\frac{11}{8}a^3x^3+\frac{13}{8}ax}{(a^2x^2+1)^2} + \frac{11\arctan(ax)}{8} - 2}{}$
risch	$-\frac{i}{64c^3(iax+1)^2} + \frac{i\,\mathrm{dilog}(iax+1)}{2c^3} + \frac{i\ln(iax+1)^2}{8c^3} - \frac{i}{64c^3(iax-1)} - \frac{i\,\mathrm{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{4c^3} - \frac{5i}{32c^3(iax+1)} + \frac{}{32c^3(-}$

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2-1/2/c^3*arctan(a*x)*ln(a^2*x^2+1)+1/2*ar
ctan(a*x)/c^3/(a^2*x^2+1)+1/c^3*arctan(a*x)*ln(x)-1/2/c^3*a*(-I*ln(x)*(ln(
1+I*a*x)-ln(1-I*a*x))/a-I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a-1/4/a^2*sum(1/
_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_a
lpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/
_alpha))),_alpha=RootOf(_Z^2*a^2+1))+1/2*(11/8*a^2*x^3+13/8*x)/(a^2*x^2+1)
^2+11/16/a*arctan(a*x))
```

3.196.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)
, x)
```

3.196.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

3.196.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x), x)
```

3.196.8 Giac [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\mathrm{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^3), x)
```

3.197 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx$

3.197.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^3}\,dx=-\frac{a}{16c^3\left(1+a^2x^2\right)^2}-\frac{7a}{16c^3\left(1+a^2x^2\right)}-\frac{\arctan(ax)}{c^3x}-\frac{a^2x\arctan(ax)}{4c^3\left(1+a^2x^2\right)^2}-\frac{7a^2x\arctan(ax)}{8c^3\left(1+a^2x^2\right)}-\frac{15a\arctan(ax)^2}{16c^3}+\frac{a\log(x)}{c^3}-\frac{a\log\left(1+a^2x^2\right)}{2c^3}$$

output
```
-1/16*a/c^3/(a^2*x^2+1)^2-7/16*a/c^3/(a^2*x^2+1)-arctan(a*x)/c^3/x-1/4*a^2
*x*arctan(a*x)/c^3/(a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)/c^3/(a^2*x^2+1)-15/
16*a*arctan(a*x)^2/c^3+a*ln(x)/c^3-1/2*a*ln(a^2*x^2+1)/c^3
```

3.197.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^3}\,dx$$
$$=\frac{-2(8+25a^2x^2+15a^4x^4)\arctan(ax)-15ax(1+a^2x^2)^2\arctan(ax)^2+ax\Big(-8-7a^2x^2+16(1+a^2x^2)^2\,\mathrm{l}\cdots}{16c^3x\left(1+a^2x^2\right)^2}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
(-2*(8 + 25*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] - 15*a*x*(1 + a^2*x^2)^2*Arc
Tan[a*x]^2 + a*x*(-8 - 7*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[x] - 8*(1 + a^2*
x^2)^2*Log[1 + a^2*x^2]))/(16*c^3*x*(1 + a^2*x^2)^2)
```

3.197.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.46, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5501, 27, 5431, 5427, 241, 5501, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{c^2x^2(a^2x^2+1)^2}dx}{c}-a^2\int \frac{\arctan(ax)}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5427$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 241$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5453$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \int \frac{\arctan(ax)}{x^2}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right)}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5361$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + a\int \frac{1}{x(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \frac{\arctan(ax)}{x}}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 243$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2 - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \frac{\arctan(ax)}{x}}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 47$$

3.197. $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 14$$

$$\frac{-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 16$$

$$\frac{-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5419$$

$$\frac{-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (
3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]
^2/(4*a)))/4))/c^3) + (-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*
a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)
) + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2)/c^3
```

3.197. $\int\frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx$

3.197.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.197. $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx$

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.197.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

method	result
derivativedivides	$a\left(-\frac{\arctan(ax)}{c^3ax}-\frac{7\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^2}{8c^3}-\frac{-\frac{15\arctan(ax)^2}{2}-8\ln(ax)+\frac{7}{2(a^2x^2}}{8c}\right.$
default	$a\left(-\frac{\arctan(ax)}{c^3ax}-\frac{7\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^2}{8c^3}-\frac{-\frac{15\arctan(ax)^2}{2}-8\ln(ax)+\frac{7}{2(a^2x^2}}{8c}\right.$
parts	$-\frac{7\arctan(ax)a^4x^3}{8c^3(a^2x^2+1)^2}-\frac{9a^2x\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15a\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{c^3x}-\frac{-\frac{15a\arctan(ax)^2}{16}-\frac{a\left(8\ln(ax)-\frac{7}{2(a^2x^2}\right.}{}}{c^3}$
parallelrisch	$\frac{-15a^5\arctan(ax)^2x^5+16\ln(x)x^5a^5-8\ln(a^2x^2+1)x^5a^5+8a^5x^5-30\arctan(ax)a^4x^4-30a^3\arctan(ax)^2x^3+32\ln(x)a^3}{16x\,c^3(a^2}$
risch	$-\frac{7a}{32c^3(iax+1)}+\frac{a\ln(iax)}{2c^3}-\frac{a\ln(iax+1)}{2c^3}-\frac{a}{64c^3(iax+1)^2}+\frac{15a\ln(iax+1)^2}{64c^3}+\frac{a}{64c^3(iax-1)}+\frac{15a\,\mathrm{dilog}(\frac{1}{2}+}{32c^3}$

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c^3*arctan(a*x)/a/x-7/8/c^3*arctan(a*x)/(a^2*x^2+1)^2*a^3*x^3-9/8*a*
x*arctan(a*x)/c^3/(a^2*x^2+1)^2-15/8*arctan(a*x)^2/c^3-1/8/c^3*(-15/2*arct
an(a*x)^2-8*ln(a*x)+7/2/(a^2*x^2+1)+1/2/(a^2*x^2+1)^2+4*ln(a^2*x^2+1)))
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx = -\frac{7a^3x^3+15(a^5x^5+2a^3x^3+ax)\arctan(ax)^2+8ax+2(15a^4x^4+25a^2x^2+8)\arctan(ax)+8(a^5x^5}{16(a^4c^3x^5+2a^2c^3x^3+c^3x)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/16*(7*a^3*x^3 + 15*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^2 + 8*a*x +
2*(15*a^4*x^4 + 25*a^2*x^2 + 8)*arctan(a*x) + 8*(a^5*x^5 + 2*a^3*x^3 + a*x
)*log(a^2*x^2 + 1) - 16*(a^5*x^5 + 2*a^3*x^3 + a*x)*log(x))/(a^4*c^3*x^5 +
 2*a^2*c^3*x^3 + c^3*x)
```

3.197.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(134) = 268$.

Time = 1.15 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.25

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx = \begin{cases} \frac{16a^5x^5\log(x)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{8a^5x^5\log\left(x^2+\frac{1}{a^2}\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{15a^5x^5\operatorname{atan}^2(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{30a^4x^4\operatorname{atan}(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} + \frac{32}{16a^4c^3x^5} \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((16*a**5*x**5*log(x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16
*c**3*x) - 8*a**5*x**5*log(x**2 + a**(-2))/(16*a**4*c**3*x**5 + 32*a**2*c*
*3*x**3 + 16*c**3*x) - 15*a**5*x**5*atan(a*x)**2/(16*a**4*c**3*x**5 + 32*a
**2*c**3*x**3 + 16*c**3*x) - 30*a**4*x**4*atan(a*x)/(16*a**4*c**3*x**5 + 3
2*a**2*c**3*x**3 + 16*c**3*x) + 32*a**3*x**3*log(x)/(16*a**4*c**3*x**5 + 3
2*a**2*c**3*x**3 + 16*c**3*x) - 16*a**3*x**3*log(x**2 + a**(-2))/(16*a**4*
c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 30*a**3*x**3*atan(a*x)**2/(16
*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 7*a**3*x**3/(16*a**4*c*
*3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 50*a**2*x**2*atan(a*x)/(16*a**4
*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) + 16*a*x*log(x)/(16*a**4*c**3*
x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 8*a*x*log(x**2 + a**(-2))/(16*a**4
*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 15*a*x*atan(a*x)**2/(16*a**4
*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 8*a*x/(16*a**4*c**3*x**5 + 3
2*a**2*c**3*x**3 + 16*c**3*x) - 16*atan(a*x)/(16*a**4*c**3*x**5 + 32*a**2*
c**3*x**3 + 16*c**3*x), Ne(a, 0)), (0, True))
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx = -\frac{1}{8}\left(\frac{15\,a^4x^4+25\,a^2x^2+8}{a^4c^3x^5+2\,a^2c^3x^3+c^3x} + \frac{15\,a\arctan(ax)}{c^3}\right)\arctan(ax) - \frac{(7\,a^2x^2 - 15\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2 + 8\,(a^4x^4+2\,a^2x^2+1)\log(a^2x^2+1) - 16\,(a^4x^4+2\,a^2x^2}{16\,(a^4c^3x^4+2\,a^2c^3x^2+c^3)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^4*x^4 + 25*a^2*x^2 + 8)/(a^4*c^3*x^5 + 2*a^2*c^3*x^3 + c^3*x)
+ 15*a*arctan(a*x)/c^3)*arctan(a*x) - 1/16*(7*a^2*x^2 - 15*(a^4*x^4 + 2*a^
2*x^2 + 1)*arctan(a*x)^2 + 8*(a^4*x^4 + 2*a^2*x^2 + 1)*log(a^2*x^2 + 1) -
16*(a^4*x^4 + 2*a^2*x^2 + 1)*log(x) + 8)*a/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 +
c^3)
```

3.197.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.197.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^3}\,dx=\frac{a\,\ln\left(x\right)}{c^3}-\frac{a\,\ln\left(a^2\,x^2+1\right)}{2\,c^3}-\frac{\frac{7\,a^3\,x^2}{2}+4\,a}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3}-\frac{\mathrm{atan}(a\,x)\,\left(\frac{1}{a^2\,c^3}+\frac{25\,x^2}{8\,c^3}+\frac{15\,a^2\,x^4}{8\,c^3}\right)}{\frac{x}{a^2}+2\,x^3+a^2\,x^5}-\frac{15\,a\,\mathrm{atan}(a\,x)^2}{16\,c^3}$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^3),x)
```

output
```
(a*log(x))/c^3 - (a*log(a^2*x^2 + 1))/(2*c^3) - (4*a + (7*a^3*x^2)/2)/(8*c
^3 + 16*a^2*c^3*x^2 + 8*a^4*c^3*x^4) - (atan(a*x)*(1/(a^2*c^3) + (25*x^2)/
(8*c^3) + (15*a^2*x^4)/(8*c^3)))/(x/a^2 + 2*x^3 + a^2*x^5) - (15*a*atan(a*
x)^2)/(16*c^3)
```

3.198 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$

3.198.1 Optimal result

Integrand size = 20, antiderivative size = 205

$$\begin{aligned}\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx = &-\frac{a}{2c^3x}+\frac{a^3x}{16c^3(1+a^2x^2)^2}+\frac{19a^3x}{32c^3(1+a^2x^2)}+\frac{3a^2\arctan(ax)}{32c^3}\\ &-\frac{\arctan(ax)}{2c^3x^2}-\frac{a^2\arctan(ax)}{4c^3(1+a^2x^2)^2}-\frac{a^2\arctan(ax)}{c^3(1+a^2x^2)}+\frac{3ia^2\arctan(ax)^2}{2c^3}\\ &-\frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}+\frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
-1/2*a/c^3/x+1/16*a^3*x/c^3/(a^2*x^2+1)^2+19/32*a^3*x/c^3/(a^2*x^2+1)+3/32
*a^2*arctan(a*x)/c^3-1/2*arctan(a*x)/c^3/x^2-1/4*a^2*arctan(a*x)/c^3/(a^2*
x^2+1)^2-a^2*arctan(a*x)/c^3/(a^2*x^2+1)+3/2*I*a^2*arctan(a*x)^2/c^3-3*a^2
*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3+3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^3
```

3.198.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$$
$$= \frac{a^2\left(-\frac{64}{ax}+192i\arctan(ax)^2+\arctan(ax)\left(-64-\frac{64}{a^2x^2}-80\cos(2\arctan(ax))-4\cos(4\arctan(ax))-384\right.\right.}{128c^3}$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(a^2*(-64/(a*x) + (192*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-64 - 64/(a^2*x^2)
- 80*Cos[2*ArcTan[a*x]] - 4*Cos[4*ArcTan[a*x]] - 384*Log[1 - E^((2*I)*ArcT
an[a*x])]) + (192*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 40*Sin[2*ArcTan[a
*x]] + Sin[4*ArcTan[a*x]]))/(128*c^3)
```

3.198.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 446 vs. 2(205) = 410.

Time = 2.52 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.18, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.050, Rules used = {5501, 27, 5501, 5453, 5361, 264, 216, 5459, 5403, 2897, 5465, 215, 215, 216, 5501, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3 \left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \textbf{5501}$$

$$\frac{\int \frac{\arctan(ax)}{c^2x^3\left(a^2x^2+1\right)^2}dx}{c}-a^2\int \frac{\arctan(ax)}{c^3x\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^3\left(a^2x^2+1\right)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^3}dx}{c^3}$$

$$\Big\downarrow \textbf{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^3\left(a^2x^2+1\right)}dx-a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx-a^2\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^3}dx\right)}{c^3}$$

$$\Big\downarrow \textbf{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx+\int \frac{\arctan(ax)}{x^3}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx-a^2\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^3}dx\right)}{c^3}$$

$$\Big\downarrow \; 5361$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx - \frac{\arctan(ax)}{2x^2}}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \; 264$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx + \frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right) - \frac{1}{x}\right) - \frac{\arctan(ax)}{2x^2}}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \; 216$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)}{2x^2} + \frac{1}{2}a\left(-a\arctan(ax) - \frac{1}{x}\right)}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \; 5459$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx - \frac{1}{2}i\arctan(ax)^2\right)\right) - \frac{\arctan(ax)}{2x^2} + \frac{1}{2}a\left(-a\arctan(ax) - \frac{1}{x}\right)}{c^3}$$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right) - a^2\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{2}i\arctan(ax)^2\right) - \frac{\arctan(a}{2x^2}}{c^3}$$

$$\Big\downarrow \; 2897$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) - a}{c^3}$$

$$\Big\downarrow \; 5465$$

3.198. $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^3}dx}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) - \text{a}}{c^3}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\left(\frac{\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx + \frac{x}{4(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) - \text{a}}{c^3}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2(a^2x^2+1)}\right) + \frac{x}{4(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) - \text{a}}{c^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) + \frac{x}{4(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) - \text{a}}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$-\frac{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) + \frac{x}{4(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)\right)}{c^3} + \frac{-a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right) - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \right.\right.}{c^3}$$

$$\Big\downarrow \text{5459}$$

3.198. $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{Poly}\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)-\frac{1}{2}i\,\text{arcta}\right.}{c^3}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\left(a^2\left(\right.\right.}{}$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)}{4a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{2897}$$

$$\frac{-a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-}{}$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{-a^2\left(-\left(a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}\right.}{}$$

$$\frac{a^2\left(-a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)+i\left(-i\,\text{arcta}\right.\right.}{c^3}$$

$$\Big\downarrow \text{215}$$

3.198. $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(-\left(a^2\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.}{}$$

$$\frac{a^2\left(-a^2\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)+i\right.}{c^3}$$

$$\Big\downarrow \ 216$$

$$\frac{-a^2\left(-\left(a^2\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right.\right.}{}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)-a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)+i\left(-\right.\right.}{c^3}$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2 - a^2*((-1/2*I)*Ar
cTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 +
 2/(1 - I*a*x)]/2)) - a^2*((-1/2*I)*ArcTan[a*x]^2 - a^2*(-1/2*ArcTan[a*x]/
(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a)) + I
*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]
/2)))/c^3 - (a^2*((-1/2*I)*ArcTan[a*x]^2 - a^2*(-1/2*ArcTan[a*x]/(a^2*(1 +
 a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a)) - a^2*(-1/4*
ArcTan[a*x]/(a^2*(1 + a^2*x^2)^2) + (x/(4*(1 + a^2*x^2)^2) + (3*(x/(2*(1 +
 a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/(4*a)) + I*((-I)*ArcTan[a*x]*Log[2 - 2
/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2)))/c^3
```

3.198. $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$

3.198.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

3.198. $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.198.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.42

method	result
parts	$-\frac{a^2\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)a^2\ln(a^2x^2+1)}{2c^3}-\frac{a^2\arctan(ax)}{c^3(a^2x^2+1)}-\frac{\arctan(ax)}{2c^3x^2}-\frac{3\arctan(ax)a^2\ln(x)}{c^3}-\frac{a\left(-6\right.}{}$
derivativedivides	$a^2\left(-\frac{\arctan(ax)}{2c^3a^2x^2}-\frac{3\arctan(ax)\ln(ax)}{c^3}-\frac{\arctan(ax)}{c^3(a^2x^2+1)}-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)\ln(a^2x^2+1)}{2c^3}-\frac{6i\ln(}{}\right.$
default	$a^2\left(-\frac{\arctan(ax)}{2c^3a^2x^2}-\frac{3\arctan(ax)\ln(ax)}{c^3}-\frac{\arctan(ax)}{c^3(a^2x^2+1)}-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)\ln(a^2x^2+1)}{2c^3}-\frac{6i\ln(}{}\right.$
risch	$-\frac{a}{2c^3x}+\frac{19a^2\arctan(ax)}{64c^3}+\frac{ia^2}{64c^3(iax-1)}-\frac{ia^2\ln(iax)}{4c^3}+\frac{ia^2\ln(iax+1)}{4c^3}+\frac{i\ln(iax+1)}{4c^3x^2}+\frac{3ia^2\operatorname{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{4c^3}$

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*arctan(a*x)/c^3/(a^2*x^2+1)^2+3/2/c^3*arctan(a*x)*a^2*ln(a^2*x^2+
1)-a^2*arctan(a*x)/c^3/(a^2*x^2+1)-1/2*arctan(a*x)/c^3/x^2-3/c^3*arctan(a*
x)*a^2*ln(x)-1/2/c^3*a*(-6*a^2*(-1/2*I*ln(x)*(ln(1+I*a*x)-ln(1-I*a*x))/a-1
/2*I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a)+1/2*a^2*((-19/8*a^2*x^3-21/8*x)/(a
^2*x^2+1)^2-3/8/a*arctan(a*x))+1/x+3/4*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^2
*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+_
alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*
a^2+1)))
```

3.198.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^
3), x)
```

3.198.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3}\,dx}{c^3}$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c**3
```

3.198.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^3 x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^3), x)
```

3.198.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^3 x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^3} \, dx = \int \frac{\operatorname{atan}(a\,x)}{x^3 \left(c\,a^2\,x^2 + c\right)^3} \, dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^3), x)
```

3.199 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$

3.199.1 Optimal result

Integrand size = 20, antiderivative size = 183

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)^3}\,dx=-\frac{a}{6c^3x^2}+\frac{a^3}{16c^3\left(1+a^2x^2\right)^2}+\frac{11a^3}{16c^3\left(1+a^2x^2\right)}-\frac{\arctan(ax)}{3c^3x^3}+\frac{3a^2\arctan(ax)}{c^3x}+\frac{a^4x\arctan(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{11a^4x\arctan(ax)}{8c^3\left(1+a^2x^2\right)}+\frac{35a^3\arctan(ax)^2}{16c^3}-\frac{10a^3\log(x)}{3c^3}+\frac{5a^3\log\left(1+a^2x^2\right)}{3c^3}$$

output
```
-1/6*a/c^3/x^2+1/16*a^3/c^3/(a^2*x^2+1)^2+11/16*a^3/c^3/(a^2*x^2+1)-1/3*ar
ctan(a*x)/c^3/x^3+3*a^2*arctan(a*x)/c^3/x+1/4*a^4*x*arctan(a*x)/c^3/(a^2*x
^2+1)^2+11/8*a^4*x*arctan(a*x)/c^3/(a^2*x^2+1)+35/16*a^3*arctan(a*x)^2/c^3
-10/3*a^3*ln(x)/c^3+5/3*a^3*ln(a^2*x^2+1)/c^3
```

3.199.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)^3}\,dx =\frac{2(-8+56a^2x^2+175a^4x^4+105a^6x^6)\arctan(ax)+105a^3x^3(1+a^2x^2)^2\arctan(ax)^2+ax\Big(-8+20a^2x^2+}{48c^3x^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
(2*(-8 + 56*a^2*x^2 + 175*a^4*x^4 + 105*a^6*x^6)*ArcTan[a*x] + 105*a^3*x^3
*(1 + a^2*x^2)^2*ArcTan[a*x]^2 + a*x*(-8 + 20*a^2*x^2 + 25*a^4*x^4 - 160*(
a*x + a^3*x^3)^2*Log[x] + 80*(a*x + a^3*x^3)^2*Log[1 + a^2*x^2]))/(48*c^3*
x^3*(1 + a^2*x^2)^2)
```

3.199.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 412 vs. $2(183) = 366$.

Time = 2.99 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.25, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {5501, 27, 5501, 5431, 5427, 241, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419, 5501, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)}{c^2x^4(a^2x^2+1)^2}dx}{c}-a^2\int \frac{\arctan(ax)}{c^3x^2\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \; 5431$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 5427$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 241$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 5453$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx + \int \frac{\arctan(ax)}{x^4}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 5361$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx + \frac{1}{3}a\int \frac{1}{x^3(a^2x^2+1)}dx - \frac{\arctan(ax)}{3x^3}}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 243$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx + \frac{1}{6}a\int \frac{1}{x^4(a^2x^2+1)}dx^2 - \frac{\arctan(ax)}{3x^3}}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 54$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx + \frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1} - \frac{a^2}{x^2} + \frac{1}{x^4}\right)dx^2 - \frac{\arctan(ax)}{3x^3}}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 2009$$

3.199. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a\right.\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\text{lo}\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\right.\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\right.\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 16}$$

3.199. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right) - a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{\arctan(ax)}{x}\right) + \frac{1}{6}a\left(a^2\left(-\log\left(\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5419}$$

$$\frac{-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - \left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a\arctan(ax)^2 - \frac{\arctan(ax)}{x}\right)\right) + \frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right) + \right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx\right) - \left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a\arctan(ax)^2 - \frac{\arctan(ax)}{x}\right)\right) + }{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx + \int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \right.\right.}{c^3}$$

$$\frac{a^2\left(-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{}{4a(a}\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{-a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a\,\mathrm{arcta}\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{-a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \int \frac{\arctan(ax)}{x^2}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right)\right) - \left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \int \frac{\arctan(ax)}{x^2}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\,\mathrm{arctar}}{2(a^2x^2}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5361}$$

3.199. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+a\int\frac{1}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}\right)-\left(a^2\left(\frac{1}{2}a(\right.\right.}{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+a\int\frac{1}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan}{2(a^2}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \ 243$$

$$\frac{-a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\int\frac{1}{x^2(a^2x^2+1)}dx^2-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}\right)-\left(a^2\left(\right.\right.}{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\int\frac{1}{x^2(a^2x^2+1)}dx^2-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\right.\right.\right.}{c^3}$$

$$\Big\downarrow \ 47$$

$$\frac{-a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(}{x}\right.}{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan}{4(a^2}\right.\right.}{c^3}$$

$$\Big\downarrow \ 14$$

$$\frac{-a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(}{x}\right.}{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan}{4(a^2}\right.\right.}{c^3}$$

$$\Big\downarrow \ 16$$

$$\frac{-a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(a}{x}\right.}{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \ 5419$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)-a^2\left(-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan}{4}\right.\right.\right.}{a^2\left(-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{}{1}\right.\right.}{c^3}$$

3.199. $\int\frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*a*(1 + a^2*x^2)
) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)) - a^2*(1/(16*
a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 +
 a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)
+ (a*(Log[x^2] - Log[1 + a^2*x^2]))/2))/c^3) + (-1/3*ArcTan[a*x]/x^3 - a^2
*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]
))/2) - a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*a*(1 + a^2
*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)) + (a*(Lo
g[x^2] - Log[1 + a^2*x^2]))/2) + (a*(-x^(-2) - a^2*Log[x^2] + a^2*Log[1 +
a^2*x^2]))/6)/c^3
```

3.199.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.199. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.199. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.199.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

method	result
derivativedivides	$a^3\left(\frac{11\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3a^3x^3}+\frac{3\arctan(ax)}{c^3ax}-\frac{-40\ln(a^2x^2+1)}{}\right.$
default	$a^3\left(\frac{11\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3a^3x^3}+\frac{3\arctan(ax)}{c^3ax}-\frac{-40\ln(a^2x^2+1)}{}\right.$
parts	$\frac{11\arctan(ax)a^6x^3}{8c^3(a^2x^2+1)^2}+\frac{13a^4x\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35a^3\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3x^3}+\frac{3a^2\arctan(ax)}{c^3x}-\frac{\frac{105a^3\arctan(ax)^2}{16}}{}$
parallelrisch	$-\frac{-105a^7\arctan(ax)^2x^7+160\ln(x)a^7x^7-80\ln(a^2x^2+1)x^7a^7+4a^7x^7-210a^6\arctan(ax)x^6-210a^5\arctan(ax)^2x^5+32}{}$
risch	$-\frac{35a^3\ln(iax+1)^2}{64c^3}+\frac{(105a^7x^7\ln(-iax+1)-210ia^6x^6+210a^5x^5\ln(-iax+1)-350ia^4x^4+105a^3x^3\ln(-iax+1)-112i}{96x^3c^3(a^2x^2+1)^2}$

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(11/8/c^3*arctan(a*x)/(a^2*x^2+1)^2*a^3*x^3+13/8*a*x*arctan(a*x)/c^3/(
a^2*x^2+1)^2+35/8*arctan(a*x)^2/c^3-1/3/c^3*arctan(a*x)/a^3/x^3+3/c^3*arct
an(a*x)/a/x-1/24/c^3*(-40*ln(a^2*x^2+1)-3/2/(a^2*x^2+1)^2-33/2/(a^2*x^2+1)
+4/a^2/x^2+80*ln(a*x)+105/2*arctan(a*x)^2))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^3} \, dx$$
$$= \frac{25\,a^5x^5 + 20\,a^3x^3 + 105\,(a^7x^7 + 2\,a^5x^5 + a^3x^3)\arctan{(ax)}^2 - 8\,ax + 2\,(105\,a^6x^6 + 175\,a^4x^4 + 56\,a^2x^2 -}{48\,(a^4c^3x^7 + 2\,a^2c^3x^5}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/48*(25*a^5*x^5 + 20*a^3*x^3 + 105*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*arctan
(a*x)^2 - 8*a*x + 2*(105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 - 8)*arctan(a*
x) + 80*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1) - 160*(a^7*x^7 +
2*a^5*x^5 + a^3*x^3)*log(x))/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3)
```

3.199.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(177) = 354.

Time = 1.72 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.96

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^3} \, dx$$
$$= \begin{cases} -\frac{160a^7x^7\log{(x)}}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{80a^7x^7\log{\left(x^2+\frac{1}{a^2}\right)}}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{105a^7x^7\operatorname{atan}^2{(ax)}}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{210a^6x^6\operatorname{atan}{(ax)}}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} - \frac{}{48} \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((-160*a**7*x**7*log(x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 +
48*c**3*x**3) + 80*a**7*x**7*log(x**2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a
**2*c**3*x**5 + 48*c**3*x**3) + 105*a**7*x**7*atan(a*x)**2/(48*a**4*c**3*x
**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 210*a**6*x**6*atan(a*x)/(48*a**4
*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) - 320*a**5*x**5*log(x)/(48*
a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 160*a**5*x**5*log(x**
2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 210*
a**5*x**5*atan(a*x)**2/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x*
*3) + 25*a**5*x**5/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3)
+ 350*a**4*x**4*atan(a*x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3
*x**3) - 160*a**3*x**3*log(x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*
c**3*x**3) + 80*a**3*x**3*log(x**2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a**2
*c**3*x**5 + 48*c**3*x**3) + 105*a**3*x**3*atan(a*x)**2/(48*a**4*c**3*x**7
 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 20*a**3*x**3/(48*a**4*c**3*x**7 + 9
6*a**2*c**3*x**5 + 48*c**3*x**3) + 112*a**2*x**2*atan(a*x)/(48*a**4*c**3*x
**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) - 8*a*x/(48*a**4*c**3*x**7 + 96*a*
*2*c**3*x**5 + 48*c**3*x**3) - 16*atan(a*x)/(48*a**4*c**3*x**7 + 96*a**2*c
**3*x**5 + 48*c**3*x**3), Ne(a, 0)), (0, True))
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.22

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^3} \, dx$$
$$= \frac{1}{24} \left(\frac{105\, a^3 \arctan{(ax)}}{c^3} + \frac{105\, a^6 x^6 + 175\, a^4 x^4 + 56\, a^2 x^2 - 8}{a^4 c^3 x^7 + 2\, a^2 c^3 x^5 + c^3 x^3} \right) \arctan{(ax)}$$
$$+ \frac{(25\, a^4 x^4 + 20\, a^2 x^2 - 105\, (a^6 x^6 + 2\, a^4 x^4 + a^2 x^2) \arctan{(ax)}^2 + 80\, (a^6 x^6 + 2\, a^4 x^4 + a^2 x^2) \log{(a^2 x^2 + 1}}{48\, (a^4 c^3 x^6 + 2\, a^2 c^3 x^4 + c^3 x^2)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/24*(105*a^3*arctan(a*x)/c^3 + (105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 -
8)/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3))*arctan(a*x) + 1/48*(25*a^4*x^4
 + 20*a^2*x^2 - 105*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*arctan(a*x)^2 + 80*(a^
6*x^6 + 2*a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 160*(a^6*x^6 + 2*a^4*x^4 +
 a^2*x^2)*log(x) - 8)*a/(a^4*c^3*x^6 + 2*a^2*c^3*x^4 + c^3*x^2)
```

3.199.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^3}\,dx = \int \frac{\arctan{(ax)}}{(a^2cx^2+c)^3x^4}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.199.9 Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^3}\,dx = \frac{\frac{25\,a^5\,x^4}{2}+10\,a^3\,x^2-4\,a}{24\,a^4\,c^3\,x^6+48\,a^2\,c^3\,x^4+24\,c^3\,x^2} + \frac{\mathrm{atan}(a\,x)\,\left(\frac{7\,x^2}{3\,c^3}-\frac{1}{3\,a^2\,c^3}+\frac{175\,a^2\,x^4}{24\,c^3}+\frac{35\,a^4\,x^6}{8\,c^3}\right)}{2\,x^5+\frac{x^3}{a^2}+a^2\,x^7} + \frac{5\,a^3\,\ln\left(a^2\,x^2+1\right)}{3\,c^3} - \frac{10\,a^3\,\ln\left(x\right)}{3\,c^3} + \frac{35\,a^3\,\mathrm{atan}(a\,x)^2}{16\,c^3}$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^3),x)
```

output
```
(10*a^3*x^2 - 4*a + (25*a^5*x^4)/2)/(24*c^3*x^2 + 48*a^2*c^3*x^4 + 24*a^4*
c^3*x^6) + (atan(a*x)*((7*x^2)/(3*c^3) - 1/(3*a^2*c^3) + (175*a^2*x^4)/(24
*c^3) + (35*a^4*x^6)/(8*c^3)))/(2*x^5 + x^3/a^2 + a^2*x^7) + (5*a^3*log(a^
2*x^2 + 1))/(3*c^3) - (10*a^3*log(x))/(3*c^3) + (35*a^3*atan(a*x)^2)/(16*c
^3)
```

3.200 $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx$

3.200.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \frac{x\sqrt{c+a^2cx^2}}{24a^3}-\frac{x^3\sqrt{c+a^2cx^2}}{20a} - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)}{15a^4}+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{15a^2} + \frac{1}{5}x^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{11\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{120a^4}$$

output
```
11/120*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+1/24*x*(a^2*c*
x^2+c)^(1/2)/a^3-1/20*x^3*(a^2*c*x^2+c)^(1/2)/a-2/15*arctan(a*x)*(a^2*c*x^
2+c)^(1/2)/a^4+1/15*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+1/5*x^4*arctan
(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \frac{ax(5-6a^2x^2)\sqrt{c+a^2cx^2}+8\sqrt{c+a^2cx^2}(-2+a^2x^2+3a^4x^4)\arctan(ax)+11\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a^2c}\right.}{120a^4}$$

input
```
Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(a*x*(5 - 6*a^2*x^2)*Sqrt[c + a^2*c*x^2] + 8*Sqrt[c + a^2*c*x^2]*(-2 + a^2
*x^2 + 3*a^4*x^4)*ArcTan[a*x] + 11*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^
2*c*x^2]])/(120*a^4)
```

3.200.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.87, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5481, 262, 262, 224, 219, 5487, 262, 224, 219, 5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \ 5481$$

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\int \frac{x^4}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \ 262$$

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \ 262$$

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \ 224$$

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \text{5487}$$

$$\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \text{224}$$

3.200. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx$

$$\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \; 5465$$

$$\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \; 224$$

3.200. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx$

$$\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right.$$

$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+$$

$$\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right.$$

$$\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

input
```
Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/5 - (a*c*((x^3*Sqrt[c + a^2*c*x^2])/
(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTanh[(a*Sqrt[c]*x)/
Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2)))/5 + (c*((x^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcT
anh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*a) - (2*((Sqrt[
c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c
*x^2]]/(a^2*Sqrt[c])))/(3*a^2)))/5
```

3.200. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx$

3.200.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.200.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.10

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,(24\arctan(ax)a^4x^4-6a^3x^3+8a^2\arctan(ax)x^2+5ax-16\arctan(ax))}{120a^4}-\frac{11\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)}{120a^4\sqrt{a^2x^2+1}}$

input
```
int(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(24*arctan(a*x)*a^4*x^4-6*a^3*x^3+8*a^
2*arctan(a*x)*x^2+5*a*x-16*arctan(a*x))-11/120/a^4*(c*(a*x-I)*(I+a*x))^(1/
2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)+11/120/a^4*(c*(a*x-
I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{2\left(6\,a^3x^3-5\,ax-8\left(3\,a^4x^4+a^2x^2-2\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}-11\,\sqrt{c}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}\right.}{240\,a^4}$$

input
```
integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/240*(2*(6*a^3*x^3 - 5*a*x - 8*(3*a^4*x^4 + a^2*x^2 - 2)*arctan(a*x))*sq
rt(a^2*c*x^2 + c) - 11*sqrt(c)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*
sqrt(c)*x - c))/a^4
```

3.200.6 Sympy [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x^3\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate(x**3*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx =$$

$$-\frac{1}{120}\left(a\left(\frac{3\left(\frac{2\left(a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{a^2x^2+1}x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}\right)}{a^2}-\frac{8\left(\sqrt{a^2x^2+1}x+\frac{\operatorname{arsinh}(ax)}{a}\right)}{a^4}\right)-8\left(\frac{3\left(a^2x^2+1\right)^{\frac{3}{2}}}{a^2}\right.\right.$$

input
```
integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/120*(a*(3*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(
a^2*x^2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*sqrt(
c)
```

3.200.8 Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int x^3\operatorname{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.201 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx$

3.201.1 Optimal result

Integrand size = 22, antiderivative size = 298

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\frac{\sqrt{c+a^2cx^2}}{8a^3}-\frac{(c+a^2cx^2)^{3/2}}{12a^3c}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a^3\sqrt{c+a^2cx^2}}-\frac{ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{c+a^2cx^2}}+\frac{ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{c+a^2cx^2}}$$

output
```
-1/12*(a^2*c*x^2+c)^(3/2)/a^3/c+1/4*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-1/8*I*c*polylo
g(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c
)^(1/2)+1/8*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/8*(a^2*c*x^2+c)^(1/2)/a^3+1/8*x*arctan(a*x)
*(a^2*c*x^2+c)^(1/2)/a^2+1/4*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.93

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$= \frac{\sqrt{c(1+a^2x^2)}\left(-6i\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)+6i\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\frac{1}{4}(1+a^2x^2)^2\left(-\frac{2}{\sqrt{1+a^2x^2}}-\right.\right.}{}$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*((-6*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*
PolyLog[2, I*E^(I*ArcTan[a*x])] - ((1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] -
 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log
[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*
x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*
ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*
x])] + 2*Sin[3*ArcTan[a*x]])))/4))/(48*a^3*Sqrt[1 + a^2*x^2])
```

3.201.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5481, 243, 53, 2009, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5481}$$

$$\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{4}ac\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int \left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{5487}$$

$$\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{5425}$$

$$\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{5421}$$

$$\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right.$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

3.201. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx$

output
```
-1/8*(a*c*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3
*a^4*c^2))) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 + (c*(-1/2*Sqrt[c +
a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt
[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLo
g[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])
))/4
```

3.201.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.201.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,(6\arctan(ax)x^3a^3-2a^2x^2+3x\arctan(ax)a+1)}{24a^3}+\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\right.}{8a^3}$

input
```
int(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)*x^3*a^3-2*a^2*x^2+3*x*ar
ctan(a*x)*a+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilo
g(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/a^3/(a^2*x^2+1)^(1/2)
```

3.201.5 Fricas [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan(ax)\,dx$$

input
```
integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

3.201.6 Sympy [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int x^2\sqrt{c(a^2x^2+1)}\,\mathrm{atan}(ax)\,dx$$

input
```
integrate(x**2*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

3.201.7 Maxima [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan(ax)\,dx$$

input
```
integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

3.201.8 Giac [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int \sqrt{a^2cx^2+cx^2}\arctan(ax)\,dx$$

input
```
integrate(x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x^2\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.202 $\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx$

3.202.1 Optimal result

Integrand size = 20, antiderivative size = 86

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx=-\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{3a^2c}-\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^2}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^2/c-1/6*arctanh(a*x*c^(1/2)/(a^2*c*x
^2+c)^(1/2))*c^(1/2)/a^2-1/6*x*(a^2*c*x^2+c)^(1/2)/a
```

3.202.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx=-\frac{ax\sqrt{c+a^2cx^2}-2(1+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax)+\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{6a^2}$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
-1/6*(a*x*Sqrt[c + a^2*c*x^2] - 2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x] + Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/a^2
```

3.202.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5465, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}dx}{3a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/2
 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a))/(3*a)
```

3.202.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.202.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,(2a^2\arctan(ax)x^2-ax+2\arctan(ax))}{6a^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)}{6a^2\sqrt{a^2x^2+1}}+\frac{\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+}}\right.}{6a^2\sqrt{a^2x^2+1}}$

input
```
int(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(2*a^2*arctan(a*x)*x^2-a*x+2*arctan(a*x)
)-1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2
*x^2+1)^(1/2)+1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)-I)/(a^2*x^2+1)^(1/2)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{2\sqrt{a^2cx^2+c}(ax-2(a^2x^2+1)\arctan(ax))-\sqrt{c}\log\left(-2a^2cx^2+2\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)}{12a^2}$$

input
```
integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 + 1)*arctan(a*x)) - sqrt(c)
*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/a^2
```

3.202.6 Sympy [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x\sqrt{c(a^2x^2+1)}\operatorname{atan}(ax)\,dx$$

input
```
integrate(x*atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

3.202.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(70) = 140.

Time = 0.35 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.02

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$= \frac{4(a^2x^2+1)^{\frac{3}{2}}\sqrt{c}\arctan(ax)-2(a^4x^4+10a^2x^2+9)^{\frac{1}{4}}\left(ax\cos\left(\frac{1}{2}\arctan(4ax,-a^2x^2+3)\right)+2\sin\left(\frac{1}{2}\text{arc}\right.\right.}{}$$

input
```
integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(4*(a^2*x^2 + 1)^(3/2)*sqrt(c)*arctan(a*x) - 2*(a^4*x^4 + 10*a^2*x^2
+ 9)^(1/4)*(a*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*sin(1/2*arctan2(
4*a*x, -a^2*x^2 + 3)))*sqrt(c) + sqrt(c)*(arctan2((a^4*x^4 + 10*a^2*x^2 +
9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2
*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + arctan2((a^4*x^4 +
 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a
^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3)))))/a^2
```

3.202.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.203 $\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx$

3.203.1 Optimal result

Integrand size = 19, antiderivative size = 244

$$\begin{aligned}\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = &-\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)\\ &-\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}\\ &+\frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{c+a^2cx^2}}\\ &-\frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/a/(a^2*c*x^2+c)^(1/2)+1/2*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-1/2*I*c*polylog(2,I*(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-1/2*(a^2*c*x
^2+c)^(1/2)/a+1/2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$=\frac{\sqrt{c\,(1+a^2x^2)}\left(\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+\arctan(ax)\left(\log\left(1-ie^{i\arctan(ax)}\right)-\log\left(1+ie^{i\arctan(ax}\right.\right.\right.}{2a\sqrt{1+a^2x^2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2]*(-1 + a*x*ArcTan[a*x]) + ArcTan[
a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(2*a
*Sqrt[1 + a^2*x^2])
```

3.203.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5413}$$
$$\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}$$
$$\Big\downarrow \text{5425}$$
$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}$$
$$\Big\downarrow \text{5421}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sq
rt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Poly
Log[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])
```

3.203.3.1 Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.203.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,(x\arctan(ax)a-1)}{2a}-\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\left(\right.\right.}{2a\sqrt{a^2x^2+1}}$

input
```
int(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(x*arctan(a*x)*a-1)-1/2*(c*(a*x-I)*(I+a*x)
)^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*
dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

3.203.5 Fricas [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}\arctan(ax)\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

3.203.6 Sympy [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}(ax)\,dx$$

input
```
integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

3.203.7 Maxima [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int \sqrt{a^2cx^2+c}\arctan(ax)\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

3.203.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int \mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.204 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx$

3.204.1 Optimal result

Integrand size = 22, antiderivative size = 229

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx &= \sqrt{c+a^2cx^2}\arctan(ax)\\ &\quad-\frac{2c\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\ &\quad-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)\\ &\quad+\frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\ &\quad-\frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)-2*c*arctan(a*x)*arctanh(
(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*c
*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+
c)^(1/2)-I*c*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)+arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(\sqrt{1+a^2x^2}\arctan(ax)+\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)-\arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)+\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x] + ArcTan[a*x]*Log[1 -
E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + Log[Cos[ArcT
an[a*x]/2] - Sin[ArcTan[a*x]/2]] - Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x
]/2]] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan[a*x])]
))/Sqrt[1 + a^2*x^2]
```

3.204.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5481, 224, 219, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x}dx$$
$$\Big\downarrow \text{5481}$$
$$c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}$$
$$\Big\downarrow \text{224}$$
$$c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}$$
$$\Big\downarrow \text{219}$$
$$c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5489}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

output
```
Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a
^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/
Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Po
lyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]
```

3.204.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.204.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.66

method	result
default	$\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan\left(ax\right)-\frac{\sqrt{c(ax-i)(ax+i)}\,\big(\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-2i\arctan\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\big(}{\sqrt{a^2x^2+1}}$

input
```
int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)-(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*
x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2
))-I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2
)+1))/(a^2*x^2+1)^(1/2)
```

3.204.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)}{x}\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

3.204.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}\left(ax\right)}{x}\,dx$$

input
```
integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x, x)
```

3.204.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)}{x}\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

3.204.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.205 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx$

3.205.1 Optimal result

Integrand size = 22, antiderivative size = 242

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = &-\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x} \\ &-\frac{2iac\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &-a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) \\ &+\frac{iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &-\frac{iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-2*I*a*c*arctan(a*x)*arctan
((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*
a*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)-I*a*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x
```

3.205.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx =$$
$$-\frac{a\sqrt{c\left(1+a^2x^2\right)}\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)}{ax}-\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)+\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```

output
```
-((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x) - ArcTan
[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x
])] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]] - I*PolyLog[2, (-I
)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^
2])
```

3.205.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5421}$$

$$\frac{c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+ \\ a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5479}$$

$$c\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+ \\ \frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{243}$$

$$c\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+ \\ \frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{73}$$

$$c\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+ \\ \frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{221}$$

$$c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+ \\ \frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```

3.205. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx$

output
```
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*
x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*Ar
cTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a
*x]])/a))/Sqrt[c + a^2*c*x^2]
```

3.205.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.205. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx$

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.205.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(199) = 398$.

Time = 0.60 (sec) , antiderivative size = 930, normalized size of antiderivative = 3.84

method	result
default	$\frac{\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)\left(i a^2 x^2+2ax-i\right)\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,x}-\frac{\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)\left(i a^2 x^2+2ax-i\right)\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,x}+\frac{\mathrm{dilog}\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right.}{}$

input
```
int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(I*a^2*x^2+2*a*x-I)/(a^2*x^2+1)^(1/2
)*(c*(a*x-I)*(I+a*x))^(1/2)/x-1/4*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*(I*a^2
*x^2+2*a*x-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x+1/4*dilog(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2-2*I*a*x-1)/(a^2*x^2+1)^(1/2)*(c*(a*x
-I)*(I+a*x))^(1/2)/x-1/4*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
I*a^2*x^2+2*a*x-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x+1/4*arcta
n(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(I*a^2*x^2+2*a*x-I)/(a^2*x^2+1)
^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x-1/4*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2-2*I*a*x-1)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x+1/4/
(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(a^2*x^2+2*I*a*x-1)*dilog(1+I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))/x+1/4/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(
1/2)*(I*a^2*x^2-2*a*x-I)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)/x
-1/4/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^2*x^2-2*a*x-I)*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)/x-1/4/(a^2*x^2+1)^(1/2)*(c*(a*x
-I)*(I+a*x))^(1/2)*(a^2*x^2+2*I*a*x-1)*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))/x-1/2*arctan(a*x)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/x-1/4/(a^2*x^2+1
)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^2*x^2-2*a*x-I)*ln((1+I*a*x)/(a^2*x^
2+1)^(1/2)-1)/x+1/4/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^2*x^2
-2*a*x-I)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)/x+1/2*(c*(a*x-I)*(I+a*x))^(1/2
)*(I*a*x-1)*arctan(a*x)/x
```

3.205.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}}{x^2}\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

3.205.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx=\int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}\left(ax\right)}{x^2}\,dx$$

input
```
integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**2, x)
```

3.205.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)}{x^2}\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

3.205.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

3.206 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx$

3.206.1 Optimal result

Integrand size = 22, antiderivative size = 240

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx =& -\frac{a\sqrt{c+a^2cx^2}}{2x}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{2x^2}\\ &-\frac{a^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\ &+\frac{ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\\ &-\frac{ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a^2*c*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)+1/2*I*a^2*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*I*a^2*c*polylog(2,(1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*
c*x^2+c)^(1/2)/x-1/2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2
```

3.206.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx$$
$$= \frac{a^2\sqrt{c\,(1+a^2x^2)}\left(-2\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)+4\arctan(ax)\log\left(1-e^{i\arctan(}\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```

output
```
(a^2*Sqrt[c*(1 + a^2*x^2)]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan
[a*x]/2]^2 + 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 4*ArcTan[a*x]*Log[
1 + E^(I*ArcTan[a*x])] + (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*Poly
Log[2, E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTa
n[a*x]/2]))/(8*Sqrt[1 + a^2*x^2])
```

3.206.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5481, 242, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^3}\,dx$$

$$\Big\downarrow \text{5481}$$

$$-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}$$

$$\Big\downarrow \text{242}$$

$$-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$$\Big\downarrow \text{5497}$$

$$-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$$\Big\downarrow \text{242}$$

$$-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$$\Big\downarrow \text{5493}$$

$$-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$$\Big\downarrow \text{5489}$$

$$-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\text{arct}\right.$$
$$-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```

output
```
-((a*Sqrt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(
-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*
c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Poly
Log[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))
```

3.206. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx$

3.206.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.206. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx$

3.206.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+\arctan(ax))}{2x^2}+\frac{i a^2\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\mathrm{po}\cdots\right.}{2\sqrt{a^2x^2+1}}$

input
```
int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(a*x+arctan(a*x))/x^2+1/2*I*a^2*(c*(a*x-I)*
(I+a*x))^(1/2)*(I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a
*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))-polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.206.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}}{x^3}\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

3.206.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx=\int \frac{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}\,(ax)}{x^3}\,dx$$

input
```
integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**3, x)
```

3.206.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}\,dx$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

3.206.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x^3}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

3.207 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx$

3.207.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = -\frac{a\sqrt{c+a^2cx^2}}{6x^2}-\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}{3cx^3} - \frac{1}{6}a^3\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)$$

output
```
-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/c/x^3-1/6*a^3*arctanh((a^2*c*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)-1/6*a*(a^2*c*x^2+c)^(1/2)/x^2
```

3.207.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \frac{-2(1+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax)+a^3\sqrt{c}x^3\log(x)-ax\left(\sqrt{c+a^2cx^2}+a^2\sqrt{c}x^2\log\left(c+\sqrt{c}\sqrt{c+a^2cx^2}\right)\right)}{6x^3}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

output
```
(-2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + a^3*Sqrt[c]*x^3*Log[x]
 - a*x*(Sqrt[c + a^2*c*x^2] + a^2*Sqrt[c]*x^2*Log[c + Sqrt[c]*Sqrt[c + a^2
*c*x^2]]))/(6*x^3)
```

3.207.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5479, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{1}{3}a\int \frac{\sqrt{a^2cx^2+c}}{x^3}dx-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}a\int \frac{\sqrt{a^2cx^2+c}}{x^4}dx^2-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{6}a\left(\frac{1}{2}a^2c\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}a\left(\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(c*x^3) + (a*(-(Sqrt[c + a^2*c*x^
2]/x^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]))/6
```

3.207.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.207.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.82

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,(2a^2\arctan(ax)x^2+ax+2\arctan(ax))}{6x^3}+\frac{a^3\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)}{6\sqrt{a^2x^2+1}}-\frac{a^3\sqrt{c(ax-i)(ax+i)}\,\ln\left(\right.}{6\sqrt{a^2x^2+1}}$

input
```
int(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*(a*x-I)*(I+a*x))^(1/2)*(2*a^2*arctan(a*x)*x^2+a*x+2*arctan(a*x))/x
^3+1/6*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)/(a^
2*x^2+1)^(1/2)-1/6*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^
(1/2)+1)/(a^2*x^2+1)^(1/2)
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \frac{a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}(ax+2(a^2x^2+1)\arctan(ax))}{12x^3}$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/12*(a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*
c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + 2*(a^2*x^2 + 1)*arctan(a*x)))/x^3
```

3.207.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \int \frac{\sqrt{c(a^2x^2+1)}\operatorname{atan}(ax)}{x^4}\,dx$$

input
```
integrate(atan(a*x)*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**4, x)
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = -\frac{1}{6}\left(\left(a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\sqrt{a^2x^2+1}a^2+\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{x^2}\right)a+\frac{2\left(a^2x^2+1\right)^{\frac{3}{2}}\arctan\left(ax\right)}{x^3}\right)\sqrt{c}$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/6*((a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)*a^2 + (a^2*x^2 + 1)^(
3/2)/x^2)*a + 2*(a^2*x^2 + 1)^(3/2)*arctan(a*x)/x^3)*sqrt(c)
```

3.207.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \int \frac{\operatorname{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

3.208 $\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)\,dx$

3.208.1 Optimal result

Integrand size = 22, antiderivative size = 217

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)\,dx = \frac{3cx\sqrt{c + a^2 c x^2}}{112a^3} - \frac{23cx^3\sqrt{c + a^2 c x^2}}{840a} - \frac{1}{42} acx^5\sqrt{c + a^2 c x^2} - \frac{2c\sqrt{c + a^2 c x^2}\arctan(ax)}{35a^4} + \frac{cx^2\sqrt{c + a^2 c x^2}\arctan(ax)}{35a^2} + \frac{8}{35} cx^4\sqrt{c + a^2 c x^2}\arctan(ax) + \frac{1}{7} a^2 cx^6\sqrt{c + a^2 c x^2}\arctan(ax) + \frac{17c^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c + a^2 c x^2}}\right)}{560a^4}$$

output
```
17/560*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4+3/112*c*x*(a^2
*c*x^2+c)^(1/2)/a^3-23/840*c*x^3*(a^2*c*x^2+c)^(1/2)/a-1/42*a*c*x^5*(a^2*c
*x^2+c)^(1/2)-2/35*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4+1/35*c*x^2*arctan
(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+8/35*c*x^4*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1
/7*a^2*c*x^6*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.55

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{acx\sqrt{c+a^2cx^2}(45-46a^2x^2-40a^4x^4)+48c(1+a^2x^2)^2\left(-2+5a^2x^2\right)\sqrt{c+a^2c}}{1680a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
(a*c*x*Sqrt[c + a^2*c*x^2]*(45 - 46*a^2*x^2 - 40*a^4*x^4) + 48*c*(1 + a^2*
x^2)^2*(-2 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + 51*c^(3/2)*Log[a
*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(1680*a^4)
```

3.208.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 783 vs. 2(217) = 434.

Time = 2.31 (sec) , antiderivative size = 783, normalized size of antiderivative = 3.61, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {5485, 5481, 262, 262, 224, 219, 262, 224, 219, 5487, 262, 224, 219, 262, 224, 219, 5465, 224, 219, 5487, 262, 224, 219, 5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)dx$$

$$\Big\downarrow \text{5481}$$

$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{262}$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{262}$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right.$$
$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{\right.$$
$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx\right.$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx\right.$$
$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$\Big\downarrow \text{262}$$

$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\,\mathrm{arct}$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\mathrm{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)+\right.$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\mathrm{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.$$

$$\Big\downarrow \text{219}$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.$$

$$a^2c\left(\frac{1}{7}c\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}\right.}{}\right.}{6a^2}\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\right.\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\int \frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{7}ac\right.$$

$$\Big\downarrow \text{262}$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2\right.$$

$$\downarrow \color{blue}{224}$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2\right.$$

$$\downarrow \color{blue}{219}$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2\right.$$

$$\Big\downarrow \; 262$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\operatorname{ar}\right.$$

$$\Big\downarrow \; 224$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)\right.$$

$$\Big\downarrow \; 219$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)+\right.$$

$$\Big\downarrow \; 5465$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$c\left(\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)+\right.$$

$$\Big\downarrow \; 224$$

$$c\left(\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a}{\sqrt{a^2}}\right.}{2a^3\sqrt{c}}}{3a}\right.\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)+\right.$$

$$\Big\downarrow \; 219$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)+\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}}{}\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{262}$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx}}{5a^2c}\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{219}$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2}}{5a^2c}\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{ 5465}$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)}{5a^2}+\frac{x^4}{}\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{ 224}$$

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)+\right.$$

$$c\left(\frac{1}{5}\sqrt{a^2cx^2+c}\arctan(ax)x^4-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)+\frac{1}{5}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.$$

$$\Big\downarrow \; 219$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right.\right.$$

$$a^2c\left(\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{7}c\left(\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}\right.}{}\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

output
```
c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/5 - (a*c*((x^3*Sqrt[c + a^2*c*x^2
])/(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2)))/5 + (c*((x^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - A
rcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*a) - (2*((Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^
2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2)))/5) + a^2*c*((x^6*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x])/7 - (a*c*((x^5*Sqrt[c + a^2*c*x^2])/(6*a^2*c) - (5*((x^3*Sqrt[
c + a^2*c*x^2])/(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTan
h[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2)))/(6*a^2)))
/7 + (c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(5*a^2*c) - ((x^3*Sqrt[c +
a^2*c*x^2])/(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTanh[(a
*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2))/(5*a) - (4*((x
^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(
2*a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*
a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)
/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2)))/(5*a^2)))/7)
```

3.208.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.208. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.208.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.92

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,(240a^6\arctan(ax)x^6-40a^5x^5+384\arctan(ax)a^4x^4-46a^3x^3+48a^2\arctan(ax)x^2+45ax-96\arctan(ax))}{1680a^4}+\frac{17c}{}$

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/1680*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(240*a^6*arctan(a*x)*x^6-40*a^5*x^5
+384*arctan(a*x)*a^4*x^4-46*a^3*x^3+48*a^2*arctan(a*x)*x^2+45*a*x-96*arcta
n(a*x))+17/560*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)+I)/(a^2*x^2+1)^(1/2)-17/560*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.54

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{51\,c^{\frac{3}{2}}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(40\,a^5cx^5+46\,a^3cx^3-45\,acx\right.}{3360\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/3360*(51*c^(3/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x -
c) - 2*(40*a^5*c*x^5 + 46*a^3*c*x^3 - 45*a*c*x - 48*(5*a^6*c*x^6 + 8*a^4*c
*x^4 + a^2*c*x^2 - 2*c)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

3.208.6 Sympy [F]

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}(ax)\,dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx =$$

$$-\frac{1}{1680}\left(\left(\left(5\left(\frac{8\,(a^2x^2+1)^{\frac{3}{2}}x^3}{a^2}-\frac{6\,(a^2x^2+1)^{\frac{3}{2}}x}{a^4}+\frac{3\sqrt{a^2x^2+1}x}{a^4}+\frac{3\,\operatorname{arsinh}(ax)}{a^5}\right)c+\frac{18\,c\left(\frac{2\,(a^2x^2+1)^{\frac{3}{2}}x}{a^2}-\right.}{}\right.\right.\right.$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/1680*((5*(8*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3/2)*x/a^4 +
 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*c + 18*c*(2*(a^2*x^2 + 1)
^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 48*(sqrt(
a^2*x^2 + 1)*x + arcsinh(a*x)/a)*c/a^4)*a - 48*(5*(a^2*x^2 + 1)^(3/2)*c*x^
4 + 3*(a^2*x^2 + 1)^(3/2)*c*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)*c/a^4)*arctan(
a*x))*sqrt(c)
```

3.208.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (c + a^2cx^2)^{3/2} \arctan(ax)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (c + a^2cx^2)^{3/2} \arctan(ax)\, dx = \int x^3 \operatorname{atan}(a\,x)\, \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.209 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

3.209.1 Optimal result

Integrand size = 22, antiderivative size = 357

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\frac{c\sqrt{c+a^2cx^2}}{16a^3}+\frac{(c+a^2cx^2)^{3/2}}{72a^3}-\frac{(c+a^2cx^2)^{5/2}}{30a^3c}+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)}{16a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\arctan(ax)+\frac{ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{c+a^2cx^2}}$$

output

```
1/72*(a^2*c*x^2+c)^(3/2)/a^3-1/30*(a^2*c*x^2+c)^(5/2)/a^3/c+1/8*I*c^2*arct
an(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2
*c*x^2+c)^(1/2)-1/16*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/16*I*c^2*polylog(2,I*(1+I*a*x)^
(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/16*c*(a
^2*c*x^2+c)^(1/2)/a^3+1/16*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+7/24*c*
x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/6*a^2*c*x^5*arctan(a*x)*(a^2*c*x^2+c
)^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.61

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{c\sqrt{c+a^2cx^2}\left(\frac{3}{4}(1+a^2x^2)^{5/2}+\frac{55}{8}(1+a^2x^2)^3\cos(3\arctan(ax))-\frac{45}{8}(1+a^2x^2)^3\right.}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + (55*(1 + a^2*x^2)^3*Co
s[3*ArcTan[a*x]])/8 - (45*(1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 - (90*I)*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] + (90*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
 - (15*(1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*Ar
cTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] +
4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan
[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^
(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])
))/2 + (15*(1 + a^2*x^2)^3*ArcTan[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*L
og[1 - I*E^(I*ArcTan[a*x])] + 3*Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a
*x])] + 45*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^
(I*ArcTan[a*x])]) + 18*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] -
Log[1 + I*E^(I*ArcTan[a*x])]) - 30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*
ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[
5*ArcTan[a*x]]))/16))/(1440*a^3*Sqrt[1 + a^2*x^2])
```

3.209.3 Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5481, 243, 53, 2009, 5487, 241, 243, 53, 2009, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5485}$$

$$c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)dx$$

$$\Big\downarrow \; 5481$$

$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{4}ac\int \frac{x^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{6}ac\int \frac{x^5}{\sqrt{a^2cx^2+c}}dx+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \; 243$$

$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{12}ac\int \frac{x^4}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \; 53$$

$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int \left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{12}ac\int \left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\frac{1}{a^4\sqrt{a^2cx^2+c}}\right)dx^2+\frac{1}{6}x^5\arctan(ax)\cdots\right.$$

$$\Big\downarrow \; 2009$$

$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\cdots\right.\right.$$
$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)\right)$$

$$\Big\downarrow \; 5487$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}\cdots\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 241$$

3.209. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}c$$
$$c\left(\frac{1}{4}c\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx}{3}\right.\right.$$

$\big\downarrow$ 243

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx}{3}\right.\right.$$

$\big\downarrow$ 53

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax)\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx}{3}\right.\right.$$

$\big\downarrow$ 2009

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx}\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx}{3}\right.\right.$$

$\big\downarrow$ 5425

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx}\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}\right.$$

$\big\downarrow$ 5421

3.209. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan}{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-}{8a}\right.\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan}{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{241}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-2}{8a}\right.\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan}{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

3.209. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\left(-\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{2(a^2cx^2+c)}{3a^4c^2}\right.\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan}{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan}{a}\right.}{}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{x^3\arctan(}{}\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

3.209. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

output
```
c*(-1/8*(a*c*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))
/(3*a^4*c^2))) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 + (c*(-1/2*Sqrt[c
 + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (S
qrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Pol
yLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^
2])))/4) + a^2*c*(-1/12*(a*c*((2*Sqrt[c + a^2*c*x^2])/(a^6*c) - (4*(c + a^
2*c*x^2)^(3/2))/(3*a^6*c^2) + (2*(c + a^2*c*x^2)^(5/2))/(5*a^6*c^3))) + (x
^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/6 + (c*(-1/8*((-2*Sqrt[c + a^2*c*x^2])
/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a + (x^3*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*Arc
Tan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(4*a^2)))/6)
```

3.209.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.209. $\int x^2(c + a^2cx^2)^{3/2} \arctan(ax)\, dx$

3.209.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.62

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,(120\arctan(ax)a^5x^5-24a^4x^4+210\arctan(ax)x^3a^3-38a^2x^2+45x\arctan(ax)a+31)}{720a^3}+\frac{c\sqrt{c(ax-i)(ax+i)}\,\Big(\mathrm{arct}$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/720*c/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(120*arctan(a*x)*a^5*x^5-24*a^4*x^4+
210*arctan(a*x)*x^3*a^3-38*a^2*x^2+45*x*arctan(a*x)*a+31)+1/16*c*(c*(a*x-I
)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a
*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

3.209.5 Fricas [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx=\int (a^2cx^2+c)^{\frac{3}{2}}x^2\arctan(ax)\ dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

3.209.6 Sympy [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx=\int x^2\big(c(a^2x^2+1)\big)^{\frac{3}{2}}\operatorname{atan}(ax)\,dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

3.209.7 Maxima [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \int (a^2cx^2+c)^{\frac{3}{2}}x^2\arctan(ax)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x), x)
```

3.209.8 Giac [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \int (a^2cx^2+c)^{\frac{3}{2}}x^2\arctan(ax)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \int x^2\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.210 $\int x(c + a^2cx^2)^{3/2} \arctan(ax)\, dx$

3.210.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = -\frac{3cx\sqrt{c+a^2cx^2}}{40a} - \frac{x(c+a^2cx^2)^{3/2}}{20a} + \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{5a^2c} - \frac{3c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{40a^2}$$

output
```
-1/20*x*(a^2*c*x^2+c)^(3/2)/a+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a^2/c-3/
40*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^2-3/40*c*x*(a^2*c*x^
2+c)^(1/2)/a
```

3.210.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = -\frac{acx(5+2a^2x^2)\sqrt{c+a^2cx^2} - 8c(1+a^2x^2)^2\sqrt{c+a^2cx^2}\arctan(ax) + 3c^{3/2}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{40a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
-1/40*(a*c*x*(5 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 8*c*(1 + a^2*x^2)^2*Sqr
t[c + a^2*c*x^2]*ArcTan[a*x] + 3*c^(3/2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*
c*x^2]])/a^2
```

3.210.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5465, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \ 5465$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \left(a^2cx^2+c\right)^{3/2}dx}{5a}$$

$$\Big\downarrow \ 211$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

$$\Big\downarrow \ 211$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

$$\Big\downarrow \ 224$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

$$\Big\downarrow \ 219$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(5*a^2*c) - ((x*(c + a^2*c*x^2)^(3/2))
/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]])/(2*a)))/4)/(5*a)
```

3.210.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.210.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.64

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,(8\arctan(ax)a^4x^4-2a^3x^3+16a^2\arctan(ax)x^2-5ax+8\arctan(ax))}{40a^2}-\frac{3c\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)}{40a^2\sqrt{a^2x^2+1}}$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/40*c/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(8*arctan(a*x)*a^4*x^4-2*a^3*x^3+16*a
^2*arctan(a*x)*x^2-5*a*x+8*arctan(a*x))-3/40*c/a^2*(c*(a*x-I)*(I+a*x))^(1/
2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)+3/40*c/a^2*(c*(a*x-
I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{3\,c^{\frac{3}{2}}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(2\,a^3cx^3+5\,acx-8\left(a^4cx^4+2\right.\right.}{80\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/80*(3*c^(3/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c)
- 2*(2*a^3*c*x^3 + 5*a*c*x - 8*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*
sqrt(a^2*c*x^2 + c))/a^2
```

3.210.6 Sympy [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

3.210.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(89) = 178.

Time = 0.40 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.72

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx = \frac{40\,(a^2cx^2+c)\sqrt{a^2x^2+1}\sqrt{c}\arctan{(ax)} - 20\,(a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\left(acx\cos\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
1/120*(40*(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 20*(a^4*
x^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) +
 2*c*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(3*(2*(a^2*x^2 +
 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 8*(sqr
t(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2)*x^2/a^2
 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c - 10*c*arctan2((a^4*x^4 +
 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^
4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) - 10*c
*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 -
 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a
^2*x^2 - 3))))*sqrt(c))/a^2
```

3.210.8 Giac [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \int x\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.211 $\int (c+a^2cx^2)^{3/2} \arctan(ax)\,dx$

3.211.1 Optimal result

Integrand size = 19, antiderivative size = 298

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)\,dx = -\frac{3c\sqrt{c+a^2cx^2}}{8a} - \frac{(c+a^2cx^2)^{3/2}}{12a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a\sqrt{c+a^2cx^2}} + \frac{3ic^2\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\cdots}{\cdots}\right.}{8a\sqrt{c+a^2cx^2}}$$

output
```
-1/12*(a^2*c*x^2+c)^(3/2)/a+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-3/4*I*c^
2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/
(a^2*c*x^2+c)^(1/2)+3/8*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2)
)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/8*I*c^2*polylog(2,I*(1+I*a*x)^
(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/8*c*(a^2*
c*x^2+c)^(1/2)/a+3/8*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.18

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{c\sqrt{c+a^2cx^2}\left(2(1+a^2x^2)^{3/2}+96\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+6(1+a^2x^2)\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(2*(1 + a^2*x^2)^(3/2) + 96*Sqrt[1 + a^2*x^2]*(-1 +
 a*x*ArcTan[a*x]) + 6*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]] + 96*ArcTan[a*x]*
(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (72*I)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] -
 3*(1 + a^2*x^2)^2*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*
E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] -
Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[
a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] +
2*Sin[3*ArcTan[a*x]])))/(192*a*Sqrt[1 + a^2*x^2])
```

3.211.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \; 5413$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

$$\Big\downarrow \; 5413$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$
$$\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 +
(3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 +
 (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 -
 I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (
I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x
^2])))/4
```

3.211.3.1 Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.211.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.67

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,(6\arctan(ax)x^3a^3-2a^2x^2+15x\arctan(ax)a-11)}{24a}-\frac{3c\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\text{arcta}\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/24*c/a*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)*x^3*a^3-2*a^2*x^2+15*x*a
rctan(a*x)*a-11)-3/8*c*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*
dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2)))/a/(a^2*x^2+1)^(1/2)
```

3.211.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x), x)
```

3.211.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

3.211.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x), x)
```

3.211.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.212 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx$

3.212.1 Optimal result

Integrand size = 22, antiderivative size = 281

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = -\frac{1}{6}acx\sqrt{c+a^2cx^2}+c\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1}{3}(c+a^2cx^2)^{3/2}\arctan(ax)-\frac{2c^2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\frac{7}{6}c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)+\frac{ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\frac{ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-7/6*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c
*x^2+c)^(1/2))-2*c^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*c^2*polylog(2,(1+I*a
*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/6*a*c*x
*(a^2*c*x^2+c)^(1/2)+c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}+8\sqrt{1+a^2x^2}\arctan(ax)+2a^2x^2\sqrt{1+a^2x^2}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) + 8*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 6*ArcTan[a*x]*Log[1 -
 E^(I*ArcTan[a*x])] - 6*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + Log[-(a*x
) + Sqrt[1 + a^2*x^2]] + 6*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] -
6*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (6*I)*PolyLog[2, -E^(I*Ar
cTan[a*x])] - (6*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/(6*Sqrt[1 + a^2*x^2])
```

3.212.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5485, 5465, 211, 224, 219, 5481, 224, 219, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}dx}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{211}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \; 224$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \; 219$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$$\Big\downarrow \; 5481$$

$$c\left(c\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$$\Big\downarrow \; 224$$

$$c\left(c\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$$\Big\downarrow \; 219$$

$$c\left(c\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

3.212. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}{x}dx$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$$\Big\downarrow \text{5489}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\arctan(a\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```

output
```
a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*
x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a))/(3*a
)) + c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sq
rt[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])
] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2])
```

3.212.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.212. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx$

3.212.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,\left(-2\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}\,ax+6\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-8\arctan(ax)\sqrt{a^2x^2+1}-14i\,\mathrm{arc}\right.}{6\sqrt{a^2x^2+1}}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/6/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(-2*arctan(a*x)*(a^2*x^2+
1)^(1/2)*a^2*x^2+(a^2*x^2+1)^(1/2)*a*x+6*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2)+1)-8*arctan(a*x)*(a^2*x^2+1)^(1/2)-14*I*arctan((1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-6*I*dilog((1+I*a*x)/(
a^2*x^2+1)^(1/2)))*c
```

3.212.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x, x)
```

3.212.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\,\mathrm{atan}(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x, x)
```

3.212.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
1/3*(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/6*(a^4*x^4 +
 10*a^2*x^2 + 9)^(1/4)*(a*c*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*c*
sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) + 1/12*(c*arctan2((a^4*x^4
+ 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a
^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + c*a
rctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3
)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2
*x^2 - 3))) + 12*c*integrate(sqrt(a^2*x^2 + 1)*arctan(a*x)/x, x))*sqrt(c)
```

3.212.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.212. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx$

3.213 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx$

3.213.1 Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\arctan(ax) - \frac{3iac^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - ac^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{3iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} - \frac{3iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}$$

output
```
-a*c^(3/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))-3*I*a*c^2*arctan(a*x)*arct
an((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+
3/2*I*a*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-3/2*I*a*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*c*(a^2*c*x^2+c)^(1/2)-c*a
rctan(a*x)*(a^2*c*x^2+c)^(1/2)/x+1/2*a^2*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/
2)
```

3.213.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.73

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}-2\sqrt{1+a^2x^2}\arctan(ax)+a^2x^2\sqrt{1+a^2x^2}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 2*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x] + a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 3*a*x*ArcTan[a*x]*Log[1
 - I*E^(I*ArcTan[a*x])] - 3*a*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] -
 2*a*x*Log[Cos[ArcTan[a*x]/2]] + 2*a*x*Log[Sin[ArcTan[a*x]/2]] + (3*I)*a*x
*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (3*I)*a*x*PolyLog[2, I*E^(I*ArcTan[a
*x])]))/(2*x*Sqrt[1 + a^2*x^2])
```

3.213.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.33, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5413, 5425, 5421, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+ c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{ 5421}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(a^2c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

$$\Big\downarrow \text{ 5421}$$

$$c\left(c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

3.213. $\int\frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}dx$

$$\Big\downarrow \text{5479}$$

$$c\left(c\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

$$\Big\downarrow \text{243}$$

$$c\left(c\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

$$\Big\downarrow \text{73}$$

$$c\left(c\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(\right.}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

$$\Big\downarrow \text{221}$$

3.213. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx$

$$c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}(\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\cdots}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

output
```
a^2*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2
+ (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1
- I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a -
(I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*
x^2])) + c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqr
t[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*Ar
cTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2])
```

3.213.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.213. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx$

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.213. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx$

3.213.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2+3\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)ax-3\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)ax+3i\operatorname{dilog}\left(1+\frac{i}{\sqrt{}}\right.\right.}{2\sqrt{a^2x}}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+3*arc
tan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-3*arctan(a*x)*ln(1+I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))*a*x+3*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*
x-3*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-(a^2*x^2+1)^(1/2)*a*x-2*l
n((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a*x+2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*a
*x-2*arctan(a*x)*(a^2*x^2+1)^(1/2))*c/(a^2*x^2+1)^(1/2)/x
```

3.213.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)
```

3.213.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**2, x)
```

3.213.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)
```

3.213.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^2}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

3.214 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx$

3.214.1 Optimal result

Integrand size = 22, antiderivative size = 304

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = -\frac{ac\sqrt{c+a^2cx^2}}{2x} + a^2c\sqrt{c+a^2cx^2}\arctan(ax) - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)}{2x^2} - \frac{3a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - a^2c^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) + \frac{3ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} - \frac{3ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}$$

output
```
-a^2*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-3*a^2*c^2*arctan(a*x
)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)
^(1/2)+3/2*I*a^2*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*I*a^2*c^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*c*(a^2*c*x^2+c)^(1
/2)/x+a^2*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/2*c*arctan(a*x)*(a^2*c*x^2+c
)^(1/2)/x^2
```

3.214.2 Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.99

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \frac{a^2c\sqrt{c+a^2cx^2}\left(-2-2\cot^2\left(\frac{1}{2}\arctan(ax)\right)+4ax\arctan(ax)\csc^2\left(\frac{1}{2}\arctan\right.\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```

output
```
(a^2*c*Sqrt[c + a^2*c*x^2]*(-2 - 2*Cot[ArcTan[a*x]/2]^2 + 4*a*x*ArcTan[a*x
]*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]
^2 + 12*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 12*Arc
Tan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*Cot[ArcTan[a*x]
/2]*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 8*Cot[ArcTan[a*x]/2]*Lo
g[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (12*I)*Cot[ArcTan[a*x]/2]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - (12*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*Ar
cTan[a*x])] + ArcTan[a*x]*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTa
n[a*x]/2])/(8*Sqrt[1 + a^2*x^2])
```

3.214.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5481, 224, 219, 242, 5493, 5489, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx$$

$$\Big\downarrow \text{5481}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+ c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+ c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+ c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)$$

$$\Big\downarrow \text{242}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+ c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)$$

$$\Big\downarrow \text{5493}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+ c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)$$

$$\Big\downarrow \text{5489}$$

$$c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)+ a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{arctan}\right.$$

$$\Big\downarrow \text{5497}$$

$$c\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\right. a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{arctan}\right.$$

$$\Big\downarrow \text{242}$$

3.214. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx$

$$c\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2}}{x}\right.$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\arctan\right.$$

$$\Big\downarrow \text{ 5493}$$

$$c\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-a\sqrt{}\right.$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\arctan\right.$$

$$\Big\downarrow \text{ 5489}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\arctan\right.$$
$$c\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```

output
```
c*(-((a*Sqrt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 -
c*(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(2*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]
/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*P
olyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))) + a
^2*c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])]
- I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2])
```

3.214. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx$

3.214.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.214. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx$

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.214.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\left(3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)\arctan(ax)a^2x^2-3i\,\mathrm{dilog}\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^2x^2-3i\,\mathrm{dilog}\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)a^2x^2-4i\arctan\left(\frac{ia}{\sqrt{a^2}}\right.}{2\sqrt{a^2x^2+1}\,x^2}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(3*ln((1+I*a*x)/(a^2*x^2+
1)^(1/2)+1)*arctan(a*x)*a^2*x^2-3*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))*a^2
*x^2-3*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^2*x^2-4*I*arctan((1+I*a*x)
/(a^2*x^2+1)^(1/2))*a^2*x^2-2*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+(a^2*x
^2+1)^(1/2)*a*x+arctan(a*x)*(a^2*x^2+1)^(1/2))*c/x^2
```

3.214.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

3.214.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**3, x)
```

3.214.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

3.214.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)}{x^3} \, dx = \int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2 + c\right)^{3/2}}{x^3} \, dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

3.215 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx$

3.215.1 Optimal result

Integrand size = 22, antiderivative size = 310

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)}{x} - \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{3x^3} - \frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{7}{6}a^3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{ia^3c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{ia^3c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{}}{\sqrt{}}\right.}{\sqrt{c+a^2cx^2}}$$

output
```
-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3-7/6*a^3*c^(3/2)*arctanh((a^2*c*x^
2+c)^(1/2)/c^(1/2))-2*I*a^3*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*
x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*a^3*c^2*polylog(2,-I*(1+
I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*a^3*
c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*
x^2+c)^(1/2)-1/6*a*c*(a^2*c*x^2+c)^(1/2)/x^2-a^2*c*arctan(a*x)*(a^2*c*x^2+
c)^(1/2)/x
```

3.215.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.85

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx =$$
$$-\frac{c\sqrt{c+a^2cx^2}\left(ax\sqrt{1+a^2x^2}+2\sqrt{1+a^2x^2}\arctan(ax)+8a^2x^2\sqrt{1+a^2x^2}\arctan(ax)+a^3x^3\text{arctanh}\left(\sqrt{1+\right.\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```

output
```
-1/6*(c*Sqrt[c + a^2*c*x^2]*(a*x*Sqrt[1 + a^2*x^2] + 2*Sqrt[1 + a^2*x^2]*A
rcTan[a*x] + 8*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + a^3*x^3*ArcTanh[Sqr
t[1 + a^2*x^2]] - 6*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 6*a
^3*x^3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 6*a^3*x^3*Log[Cos[ArcTan
[a*x]/2]] - 6*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] - (6*I)*a^3*x^3*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] + (6*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(
x^3*Sqrt[1 + a^2*x^2])
```

3.215.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5485, 5479, 243, 51, 73, 221, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{3}a\int \frac{\sqrt{a^2cx^2+c}}{x^3}dx-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{243}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\int \frac{\sqrt{a^2cx^2+c}}{x^4}dx^2-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \ 51$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+ c\left(\frac{1}{6}a\left(\frac{1}{2}a^2c\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \ 73$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+ c\left(\frac{1}{6}a\left(\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \ 221$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+ c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \ 5485$$

$$a^2c\left(a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \ 5425$$

$$a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \ 5421$$

3.215. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}{x^4}\,dx$

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}(2\ldots}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{243}$$

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyL}\ldots}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{73}$$

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLo}\ldots}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{221}$$

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyL}\ldots}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

3.215. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}{x^4}\,dx$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```

output
```
c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(c*x^3) + (a*(-(Sqrt[c + a^2*c
*x^2]/x^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]))/6) + a^2*c
*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*
c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*
ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 +
I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I
*a*x]])/a))/Sqrt[c + a^2*c*x^2])
```

3.215.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.215.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2334 vs. 2(257) = 514.

Time = 0.79 (sec) , antiderivative size = 2335, normalized size of antiderivative = 7.53

method	result	size
default	Expression too large to display	2335

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*arctan(a*x)*
c/x^3+7/48*c*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*(I*a^2*x^2-2*a*x-I)*(a^2*x^
2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x^3-7/48*c*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)-1)*(I*a^2*x^2-2*a*x-I)*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x^
3+7/96/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^4*x^4-4*a^3*x^3-6*
I*a^2*x^2+4*a*x+I)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*c/x^3-7/96/(a^2*x^2+1
)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^4*x^4-4*a^3*x^3-6*I*a^2*x^2+4*a*x+I
)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*c/x^3-7/96*c*ln((1+I*a*x)/(a^2*x^2+1)^
(1/2)+1)*(I*a^4*x^4+4*a^3*x^3-6*I*a^2*x^2-4*a*x+I)/(a^2*x^2+1)^(1/2)*(c*(a
*x-I)*(I+a*x))^(1/2)/x^3-1/48*c*(a^3*x^3+I*a^2*x^2+a*x+I)*(c*(a*x-I)*(I+a*
x))^(1/2)/x^3-1/48*c*(a^3*x^3-I*a^2*x^2+a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/x
^3+1/48*c*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/x^3+1/48
*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*c/x^3+7/96*c*ln((
1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(I*a^4*x^4+4*a^3*x^3-6*I*a^2*x^2-4*a*x+I)/(a
^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x^3-7/48*c*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2)+1)*(I*a^2*x^2+2*a*x-I)*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/
2)/x^3+7/48*c*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(I*a^2*x^2+2*a*x-I)*(a^2*x
^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x^3+1/16*c*dilog(1-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^4*x^4-4*I*a^3*x^3-6*a^2*x^2+4*I*a*x+1)/(a^2*x^2+1)^(1/2)*
(c*(a*x-I)*(I+a*x))^(1/2)/x^3-1/16*c*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1...
```

3.215.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

3.215.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**4, x)
```

3.215.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

3.215.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)}{x^4} \, dx = \int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2 + c\right)^{3/2}}{x^4} \, dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

3.216 $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

3.216.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\begin{aligned}\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx &= \frac{47c^2x\sqrt{c+a^2cx^2}}{2688a^3}\\ &-\frac{205c^2x^3\sqrt{c+a^2cx^2}}{12096a}-\frac{103ac^2x^5\sqrt{c+a^2cx^2}}{3024}-\frac{1}{72}a^3c^2x^7\sqrt{c+a^2cx^2}\\ &-\frac{2c^2\sqrt{c+a^2cx^2}\arctan(ax)}{63a^4}+\frac{c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)}{63a^2}\\ &+\frac{5}{21}c^2x^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{19}{63}a^2c^2x^6\sqrt{c+a^2cx^2}\arctan(ax)\\ &+\frac{1}{9}a^4c^2x^8\sqrt{c+a^2cx^2}\arctan(ax)+\frac{115c^{5/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{8064a^4}\end{aligned}$$

output
```
115/8064*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4+47/2688*c^2*
x*(a^2*c*x^2+c)^(1/2)/a^3-205/12096*c^2*x^3*(a^2*c*x^2+c)^(1/2)/a-103/3024
*a*c^2*x^5*(a^2*c*x^2+c)^(1/2)-1/72*a^3*c^2*x^7*(a^2*c*x^2+c)^(1/2)-2/63*c
^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4+1/63*c^2*x^2*arctan(a*x)*(a^2*c*x^2
+c)^(1/2)/a^2+5/21*c^2*x^4*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+19/63*a^2*c^2*x
^6*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/9*a^4*c^2*x^8*arctan(a*x)*(a^2*c*x^2+
c)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.45

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{c^2\left(-ax\sqrt{c+a^2cx^2}(-423+410a^2x^2+824a^4x^4+336a^6x^6)+384(1+a^2x^2)^3(-\right.}{24192a^{}}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(-423 + 410*a^2*x^2 + 824*a^4*x^4 + 336*a^
6*x^6)) + 384*(1 + a^2*x^2)^3*(-2 + 7*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x] + 345*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(24192*a^4)
```

3.216.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)\left(a^2cx^2+c\right)^{5/2}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^5\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+c\int x^3\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int x^7\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)dx\right)+ c\left(a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)dx\right)$$

$$\Big\downarrow \text{ 5481}$$

$$a^2c\left(a^2c\left(\frac{1}{9}c\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\int\frac{x^8}{\sqrt{a^2cx^2+c}}dx+\frac{1}{9}x^8\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{7}c\int\frac{x^5\arctan(a}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}d\right.\right.$$

$$\Big\downarrow \text{ 262}$$

$$c\left(c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\right.\right.$$

$$a^2c\left(a^2c\left(\frac{1}{9}c\int \frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\int \frac{x^6}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)+\frac{1}{9}x^8\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\right.$$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2\right.\right.$$

$$a^2c\left(c\left(\frac{1}{7}c\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a\right.\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax\right.\right.$$

$$a^2c\left(c\left(\frac{1}{7}c\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a\right.\right.$$

$$\Big\downarrow \text{219}$$

3.216. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$a^2c\left(c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a\right.\right.$$

$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a\right.\right.$$

$$\Big\downarrow 262$$

$$a^2c\left(c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\text{a}\right.\right.$$

$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\text{a}\right.\right.$$

$$\Big\downarrow 224$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$a^2c\left(c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow \color{blue}{219}$$

$$c\left(c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a}{\sqrt{a}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.\right.$$

$$a^2c\left(a^2c\left(\frac{1}{9}c\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)}{8a^2}\right)+\frac{1}{9}\right.\right.$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a}{\sqrt{a}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.\right.$$

$$a^2c\left(a^2c\left(\frac{1}{9}c\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}d}{2a^2}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$\Big\downarrow \text{224}$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow \; 219$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)}{8a^2}\right)\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow \text{ 5487}$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)}{8a^2}\right)\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow \text{ 262}$$

3.216. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\right.}{2a^3}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow \; 224$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\cdots\right.}{2a^3\cdots}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 219

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\right.}{2a^3}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \; 262$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\,\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\right.}{2a^3}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\,\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow\ 224$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\cdots\right)}{2a^3\cdots}\right)}{4a^2}\right)}{6a^2}\cdots\right)}{8a^2}\cdots\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\cdots\right.\right.$$

$$\downarrow \; 219$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\ldots\right.}{2a^3\ldots}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \; 262$$

3.216. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\cdots\right)}{2a^3\cdots}\cdots\right.}{4a^2}\cdots\right.}{6a^2}\cdots\right.}{8a^2}\cdots\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\cdots\right.\right.$$

$$\Big\downarrow \ 224$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{}{\sqrt{}}\right)}{2a^3\sqrt{}}\right)}{4a^2}\right)}{6a^2}\right)}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow \; 219$$

3.216. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\right.}{2a^3}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 5465

3.216. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\right.}{2a^3}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \; 224$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\right.}{2a^3}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \text{219}$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\cdots\right.}{2a^3\cdots}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$$\Big\downarrow \; 5487$$

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\cdots\right.}{2a^3\cdots}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

$\downarrow$ 262

3.216. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\cdots\right)}{2a^3\cdots}\right)}{4a^2}\cdots\right)}{6a^2}\cdots\right)}{8a^2}\cdots\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\cdots\right.\right.$$

$$\Big\downarrow 224$$

3.216. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\,\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\cdots\right.}{2a^3\cdots}\right.}{4a^2}\cdots\right.}{6a^2}\cdots\right.}{8a^2}\cdots\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\,\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\cdots\right.\right.\right.$$

$\downarrow$ 219

3.216. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\right.}{2a^3}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
$Aborted
```

3.216.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.216. $\int x^3(c + a^2cx^2)^{5/2} \arctan(ax)\, dx$

3.216.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.78

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(2688\arctan(ax)a^8x^8-336a^7x^7+7296a^6\arctan(ax)x^6-824a^5x^5+5760\arctan(ax)a^4x^4-410a^3x^3+384a^2\arctan$ … $}{24192a^4}$

input
```
int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/24192*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(2688*arctan(a*x)*a^8*x^8-336*a^
7*x^7+7296*a^6*arctan(a*x)*x^6-824*a^5*x^5+5760*arctan(a*x)*a^4*x^4-410*a^
3*x^3+384*a^2*arctan(a*x)*x^2+423*a*x-768*arctan(a*x))+115/8064*c^2/a^4*(c
*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2
)-115/8064*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
)-I)/(a^2*x^2+1)^(1/2)
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.53

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{345\,c^{\frac{5}{2}}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\,(336\,a^7c^2x^7+824\,a^5c^2x^5+ \cdots}{}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/48384*(345*c^(5/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x
- c) - 2*(336*a^7*c^2*x^7 + 824*a^5*c^2*x^5 + 410*a^3*c^2*x^3 - 423*a*c^2*
x - 384*(7*a^8*c^2*x^8 + 19*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + a^2*c^2*x^2 - 2
*c^2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

3.216.6 Sympy [F]

$$\int x^3 (c + a^2 c x^2)^{5/2} \arctan(ax)\, dx = \int x^3 \left(c\left(a^2 x^2 + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}(ax)\, dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.17

$$\int x^3 (c + a^2 c x^2)^{5/2} \arctan(ax)\, dx =$$

$$-\frac{1}{24192}\left(\left(\left(7\left(\frac{48\,(a^2x^2+1)^{\frac{3}{2}}x^5}{a^2} - \frac{40\,(a^2x^2+1)^{\frac{3}{2}}x^3}{a^4} + \frac{30\,(a^2x^2+1)^{\frac{3}{2}}x}{a^6} - \frac{15\sqrt{a^2x^2+1}\,x}{a^6} - \frac{15\operatorname{arsinh}(ax}{a^7}\right.\right.\right.\right.$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/24192*((7*(48*(a^2*x^2 + 1)^(3/2)*x^5/a^2 - 40*(a^2*x^2 + 1)^(3/2)*x^3/
a^4 + 30*(a^2*x^2 + 1)^(3/2)*x/a^6 - 15*sqrt(a^2*x^2 + 1)*x/a^6 - 15*arcsi
nh(a*x)/a^7)*a^2*c^2 + 96*(8*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)
^(3/2)*x/a^4 + 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*c^2 + 144*c
^2*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a
^3)/a^2 - 384*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)*c^2/a^4)*a - 384*(7*(
a^2*x^2 + 1)^(3/2)*a^2*c^2*x^6 + 12*(a^2*x^2 + 1)^(3/2)*c^2*x^4 + 3*(a^2*x
^2 + 1)^(3/2)*c^2*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)*c^2/a^4)*arctan(a*x))*sq
rt(c)
```

3.216.8 Giac [F(-2)]

Exception generated.

$$\int x^3 (c + a^2 cx^2)^{5/2} \arctan(ax)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (c + a^2 cx^2)^{5/2} \arctan(ax)\, dx = \int x^3\, \mathrm{atan}(a\, x)\, \left(c\, a^2\, x^2 + c\right)^{5/2} dx$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.217 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

3.217.1 Optimal result

Integrand size = 22, antiderivative size = 418

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{5c^2\sqrt{c+a^2cx^2}}{128a^3}+\frac{5c(c+a^2cx^2)^{3/2}}{576a^3}$$
$$+\frac{(c+a^2cx^2)^{5/2}}{240a^3}-\frac{(c+a^2cx^2)^{7/2}}{56a^3c}+\frac{5c^2x\sqrt{c+a^2cx^2}\arctan(ax)}{128a^2}$$
$$+\frac{59}{192}c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{17}{48}a^2c^2x^5\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1}{8}a^4c^2x^7\sqrt{c+a^2cx^2}\arctan(ax)+\frac{5ic^3\sqrt{}}{}$$

output
```
5/576*c*(a^2*c*x^2+c)^(3/2)/a^3+1/240*(a^2*c*x^2+c)^(5/2)/a^3-1/56*(a^2*c*
x^2+c)^(7/2)/a^3/c+5/64*I*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-5/128*I*c^3*polylog(2,-I
*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2
)+5/128*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/
2)/a^3/(a^2*c*x^2+c)^(1/2)+5/128*c^2*(a^2*c*x^2+c)^(1/2)/a^3+5/128*c^2*x*a
rctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+59/192*c^2*x^3*arctan(a*x)*(a^2*c*x^2+c
)^(1/2)+17/48*a^2*c^2*x^5*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/8*a^4*c^2*x^7*
arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.217.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 907 vs. 2(418) = 836.

Time = 12.47 (sec) , antiderivative size = 907, normalized size of antiderivative = 2.17

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(-\frac{19067}{32}(1+a^2x^2)^{7/2}-\frac{3829}{32}(1+a^2x^2)^4\cos(3\arctan(ax))-3150i\right.}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*((-19067*(1 + a^2*x^2)^(7/2))/32 - (3829*(1 + a^2
*x^2)^4*Cos[3*ArcTan[a*x]])/32 - (3150*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] + (3150*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 420*(1 + a^2*x^2)^2*(-2/Sq
rt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 +
 a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 -
 I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]
*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 +
 I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])) + 7*(1 + a^2*x^2)^3*(12/Sqr
t[1 + a^2*x^2] + 110*Cos[3*ArcTan[a*x]] - 90*Cos[5*ArcTan[a*x]] + 15*ArcTa
n[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[a*x])] + 3*
Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcTan[a*x]]*(L
og[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 18*Cos[4*Arc
Tan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) -
30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*Log[1 + I*E^(I*ArcT
an[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]])) - (35*(1 + a^2*
x^2)^4*(314*Cos[5*ArcTan[a*x]] - 90*Cos[7*ArcTan[a*x]] + 3*ArcTan[a*x]*((-
3530*a*x)/Sqrt[1 + a^2*x^2] + 525*Log[1 - I*E^(I*ArcTan[a*x])] + 120*Cos[6
*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 15*Cos[8*ArcTan[a*x]]*Log[1 -
 I*E^(I*ArcTan[a*x])] + 840*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x]
)] - Log[1 + I*E^(I*ArcTan[a*x])]) + 420*Cos[4*ArcTan[a*x]]*(Log[1 - I*...
```

3.217.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1650 vs. 2(418) = 836.

Time = 6.86 (sec) , antiderivative size = 1650, normalized size of antiderivative = 3.95, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {5485, 5485, 5481, 243, 53, 2009, 5487, 241, 243, 53, 2009, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5485}$$

$$c\int x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+a^2c\int x^4\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx$$

$$\Big\downarrow \text{5485}$$

$$c\left(c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)dx\right)+ a^2c\left(a^2c\int x^6\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)dx\right)$$

$$\Big\downarrow \text{5481}$$

$$c\left(c\left(\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{4}ac\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{1}{6}c\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}d\right.\right.$$
$$a^2c\left(a^2c\left(\frac{1}{8}c\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int\frac{x^7}{\sqrt{a^2cx^2+c}}dx+\frac{1}{8}x^7\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{6}c\int\frac{x^4\arctan(ax}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$c\left(c\left(\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{1}{6}c\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$a^2c\left(a^2c\left(\frac{1}{8}c\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{16}ac\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{8}x^7\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{6}c\int\frac{x^4\arctan(}{\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \text{53}$$

$$c\Bigg(c\Bigg(\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int\Bigg(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\Bigg)dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\Bigg)+a^2c$$
$$a^2c\Bigg(a^2c\Bigg(\frac{1}{8}c\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{16}ac\int\Bigg(\frac{\left(a^2cx^2+c\right)^{5/2}}{a^6c^3}-\frac{3\left(a^2cx^2+c\right)^{3/2}}{a^6c^2}+\frac{3\sqrt{a^2cx^2+c}}{a^6c}-\frac{1}{a^6\sqrt{a^2cx^2+c}}$$

$\downarrow$ 2009

$$a^2c\Bigg(a^2c\Bigg(\frac{1}{8}c\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{8}x^7\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{16}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2(}{}$$
$$c\Bigg(a^2c\Bigg(\frac{1}{6}c\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a}}{}$$

$\downarrow$ 5487

$$c\Bigg(c\Bigg(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}$$
$$c\Bigg(c\Bigg(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\Bigg)+\frac{1}{6}c\Bigg(\frac{\sqrt{a^2cx^2+}}{}$$

$\downarrow$ 241

$$c\Bigg(c\Bigg(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}$$
$$c\Bigg(c\Bigg(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\Bigg)+\frac{1}{6}c\Bigg(\frac{\sqrt{a^2cx^2+}}{}$$

$\downarrow$ 243

$$c\Bigg(c\Bigg(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}$$
$$c\Bigg(c\Bigg(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\Bigg)+\frac{1}{6}c\Bigg(\frac{\sqrt{a^2cx^2+}}{}$$

$\downarrow$ 53

3.217. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.\right.$$

$$c\left(c\left(\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$c\left(c\left(\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.\right.$$

$$c\left(c\left(\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.\right.$$

$$c\left(c\left(\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\left(\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.\right.$$

$$c\left(c\left(\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5487}$$

3.217. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\Bigg(c\Bigg(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}$$

$$c\Bigg(c\Bigg(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\Bigg)+\frac{1}{6}c\Bigg(\frac{\sqrt{a^2cx^2+}}{}$$

$$\Big\downarrow 241$$

$$c\Bigg(c\Bigg(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}$$

$$c\Bigg(c\Bigg(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\Bigg)+\frac{1}{6}c\Bigg(\frac{\sqrt{a^2cx^2+}}{}$$

$$\Big\downarrow 243$$

$$c\Bigg(c\Bigg(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}$$

$$c\Bigg(c\Bigg(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\Bigg(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\Bigg)+\frac{1}{6}c\Bigg(\frac{\sqrt{a^2cx^2+}}{}$$

$$\Big\downarrow 53$$

3.217. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\sqrt{a^2cx^2+}\right.\right.\right.$$

$$\Big\downarrow \ 2009$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\sqrt{a^2cx^2+}\right.\right.\right.$$

$$\Big\downarrow \ 5425$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\sqrt{a^2cx^2+}\right.\right.\right.$$

$$\Big\downarrow \ 5421$$

3.217. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.$$

$$\Big\downarrow \text{241}$$

3.217. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+}}{}\right.\right.\right.$$

$$\Big\downarrow \text{5421}$$

3.217. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2+\cdots}}{}\right.\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

3.217. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

output
```
c*(c*(-1/8*(a*c*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/
2))/(3*a^4*c^2))) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 + (c*(-1/2*Sqr
t[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) -
 (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*
PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c
*x^2])))/4) + a^2*c*(-1/12*(a*c*((2*Sqrt[c + a^2*c*x^2])/(a^6*c) - (4*(c +
 a^2*c*x^2)^(3/2))/(3*a^6*c^2) + (2*(c + a^2*c*x^2)^(5/2))/(5*a^6*c^3))) +
 (x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/6 + (c*(-1/8*((-2*Sqrt[c + a^2*c*x^
2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a + (x^3*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*
ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-
I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(4*a^2)))/6)) + a^2*
c*(c*(-1/12*(a*c*((2*Sqrt[c + a^2*c*x^2])/(a^6*c) - (4*(c + a^2*c*x^2)^(3/
2))/(3*a^6*c^2) + (2*(c + a^2*c*x^2)^(5/2))/(5*a^6*c^3))) + (x^5*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x])/6 + (c*(-1/8*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (
2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^...
```

3.217.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.217. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.217. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

3.217.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.59

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(5040\arctan(ax)a^7x^7-720a^6x^6+14280\arctan(ax)a^5x^5-1992a^4x^4+12390\arctan(ax)x^3a^3-1474a^2x^2+1575x\,a}{40320a^3}$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/40320*c^2/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(5040*arctan(a*x)*a^7*x^7-720*a^
6*x^6+14280*arctan(a*x)*a^5*x^5-1992*a^4*x^4+12390*arctan(a*x)*x^3*a^3-147
4*a^2*x^2+1575*x*arctan(a*x)*a+1373)+5/128*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(
arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

3.217.5 Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x), x)
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Timed out
```

3.217.7 Maxima [F]

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \int \left(a^2 c x^2 + c\right)^{\frac{5}{2}} x^2 \arctan\left(ax\right)\, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x), x)
```

3.217.8 Giac [F]

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \int \left(a^2 c x^2 + c\right)^{\frac{5}{2}} x^2 \arctan\left(ax\right)\, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \int x^2 \operatorname{atan}(a\,x)\, \left(c\,a^2\,x^2 + c\right)^{5/2}\, dx$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.218 $\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$

3.218.1 Optimal result

Integrand size = 20, antiderivative size = 134

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=-\frac{5c^2x\sqrt{c+a^2cx^2}}{112a}-\frac{5cx(c+a^2cx^2)^{3/2}}{168a}-\frac{x(c+a^2cx^2)^{5/2}}{42a}+\frac{(c+a^2cx^2)^{7/2}\arctan(ax)}{7a^2c}-\frac{5c^{5/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{112a^2}$$

output
```
-5/168*c*x*(a^2*c*x^2+c)^(3/2)/a-1/42*x*(a^2*c*x^2+c)^(5/2)/a+1/7*(a^2*c*x
^2+c)^(7/2)*arctan(a*x)/a^2/c-5/112*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2
+c)^(1/2))/a^2-5/112*c^2*x*(a^2*c*x^2+c)^(1/2)/a
```

3.218.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\frac{c^2\left(-ax\sqrt{c+a^2cx^2}(33+26a^2x^2+8a^4x^4)+48(1+a^2x^2)^3\sqrt{c+a^2cx^2}\arctan(a\right.}{336a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(33 + 26*a^2*x^2 + 8*a^4*x^4)) + 48*(1 + a
^2*x^2)^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - 15*Sqrt[c]*Log[a*c*x + Sqrt[c]
*Sqrt[c + a^2*c*x^2]]))/(336*a^2)
```

3.218.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5465, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^{5/2} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\int \left(a^2cx^2+c\right)^{5/2}dx}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}dx+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
((c + a^2*c*x^2)^(7/2)*ArcTan[a*x])/(7*a^2*c) - ((x*(c + a^2*c*x^2)^(5/2))
/6 + (5*c*((x*(c + a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 +
 (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4))/6)/(7*a)
```

3.218.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.218.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.53

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(48a^6\arctan(ax)x^6-8a^5x^5+144\arctan(ax)a^4x^4-26a^3x^3+144a^2\arctan(ax)x^2-33ax+48\arctan(ax))}{336a^2} - \frac{5c^2}{}$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/336*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(48*a^6*arctan(a*x)*x^6-8*a^5*x^5+
144*arctan(a*x)*a^4*x^4-26*a^3*x^3+144*a^2*arctan(a*x)*x^2-33*a*x+48*arcta
n(a*x))-5/112*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)+I)/(a^2*x^2+1)^(1/2)+5/112*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{15\,c^{\frac{5}{2}}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(8\,a^5c^2x^5+26\,a^3c^2x^3+33\,ac^2\right.}{672\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/672*(15*c^(5/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c
) - 2*(8*a^5*c^2*x^5 + 26*a^3*c^2*x^3 + 33*a*c^2*x - 48*(a^6*c^2*x^6 + 3*a
^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^2
```

3.218.6 Sympy [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\int x\big(c(a^2x^2+1)\big)^{\frac{5}{2}}\operatorname{atan}(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

3.218.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(110) = 220$.

Time = 0.48 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.75

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\frac{560\,(a^2c^2x^2+c^2)\sqrt{a^2x^2+1}\sqrt{c}\arctan(ax)-280\,(a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\,(ac^2x\cos\cdots}{\cdots}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
1/1680*(560*(a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 28
0*(a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^
2 + 3)) + 2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(5*(8
*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3/2)*x/a^4 + 3*sqrt(a^2*x^
2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)/a^2 - 24*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 -
 sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^4 + 64*(sqrt(a^2*x^2 + 1)*x
 + arcsinh(a*x)/a)/a^6) - 16*(15*(a^2*x^2 + 1)^(3/2)*x^4/a^2 - 12*(a^2*x^2
 + 1)^(3/2)*x^2/a^4 + 8*(a^2*x^2 + 1)^(3/2)/a^6)*arctan(a*x))*a^6*c^2 + 28
*(a*(3*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*
x)/a^3)/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^
2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c^2 - 1
40*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2
*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a
*x, a^2*x^2 - 3))) - 140*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(
1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1
/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))))*sqrt(c))/a^2
```

3.218.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \int x\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.219 $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

3.219.1 Optimal result

Integrand size = 19, antiderivative size = 348

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx = -\frac{5c^2\sqrt{c+a^2cx^2}}{16a}-\frac{5c\left(c+a^2cx^2\right)^{3/2}}{72a}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a}$$

$$+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)+\frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\arctan(ax)+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)-\frac{5ic^3\sqrt{1+a^2\cdots}}{\cdots}$$

output
```
-5/72*c*(a^2*c*x^2+c)^(3/2)/a-1/30*(a^2*c*x^2+c)^(5/2)/a+5/24*c*x*(a^2*c*x
^2+c)^(3/2)*arctan(a*x)+1/6*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)-5/8*I*c^3*ar
ctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2
*c*x^2+c)^(1/2)+5/16*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/16*I*c^3*polylog(2,I*(1+I*a*x)^(1
/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/16*c^2*(a^2
*c*x^2+c)^(1/2)/a+5/16*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.219.2 Mathematica [A] (warning: unable to verify)

Time = 5.36 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.85

$$\int (c +a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(\frac{3}{4}(1+a^2x^2)^{5/2}+720\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+\frac{55}{8}(1+a\cdots\right.}{\cdots}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + 720*Sqrt[1 + a^2*x^2
]*(-1 + a*x*ArcTan[a*x]) + (55*(1 + a^2*x^2)^3*Cos[3*ArcTan[a*x]])/8 - (45
*(1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 + 720*ArcTan[a*x]*(Log[1 - I*E^(I*A
rcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (450*I)*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])] - (450*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 15*(1 + a^2*x^
2)^2*(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*
x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x
]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*A
rcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])])
- 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])) + (15*(1 + a^2*x
^2)^3*ArcTan[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[
a*x])] + 3*Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcT
an[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 1
8*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan
[a*x])]) - 30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*Log[1 +
I*E^(I*ArcTan[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]]))/16))
/(1440*a*Sqrt[1 + a^2*x^2])
```

3.219.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5413, 5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2\right.\right.$$
$$\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}\right.$$
$$\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$
$$\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
-1/30*(c + a^2*c*x^2)^(5/2)/a + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/6 +
(5*c*(-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]
)/4 + (3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])
/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a
^2*c*x^2])))/4))/6
```

3.219. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$

3.219.3.1 Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.219.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.65

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(120\arctan(ax)a^5x^5-24a^4x^4+390\arctan(ax)x^3a^3-98a^2x^2+495x\arctan(ax)a-299)}{720a}-\frac{5c^2\sqrt{c(ax-i)(ax+i)}}{}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/720*c^2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(120*arctan(a*x)*a^5*x^5-24*a^4*x^4+
390*arctan(a*x)*x^3*a^3-98*a^2*x^2+495*x*arctan(a*x)*a-299)-5/16*c^2*(c*(a
*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arct
an(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

3.219.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x), x)
```

3.219.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

3.219.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x), x)
```

3.219.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int \mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.220 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx$

3.220.1 Optimal result

Integrand size = 22, antiderivative size = 329

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx(c+a^2cx^2)^{3/2} + c^2\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{3}c(c+a^2cx^2)^{3/2}\arctan(ax)+\frac{1}{5}(c+a^2cx^2)^{5/2}\arctan(ax)-\frac{2c^3\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}-\frac{14}{12}$$

output
```
-1/20*a*c*x*(a^2*c*x^2+c)^(3/2)+1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+1/5*
(a^2*c*x^2+c)^(5/2)*arctan(a*x)-149/120*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c
*x^2+c)^(1/2))-2*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*c^3*polylog(2,-(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*c^3*polylog(2,(1+I*a
*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-29/120*a*
c^2*x*(a^2*c*x^2+c)^(1/2)+c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(-35ax\sqrt{1+a^2x^2}-6a^3x^3\sqrt{1+a^2x^2}+184\sqrt{1+a^2x^2}\,\mathrm{arcta}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(-35*a*x*Sqrt[1 + a^2*x^2] - 6*a^3*x^3*Sqrt[1 + a
^2*x^2] + 184*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 88*a^2*x^2*Sqrt[1 + a^2*x^2]
*ArcTan[a*x] + 24*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 120*ArcTan[a*x]*
Log[1 - E^(I*ArcTan[a*x])] - 120*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] +
29*Log[-(a*x) + Sqrt[1 + a^2*x^2]] + 120*Log[Cos[ArcTan[a*x]/2] - Sin[ArcT
an[a*x]/2]] - 120*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (120*I)*P
olyLog[2, -E^(I*ArcTan[a*x])] - (120*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/(1
20*Sqrt[1 + a^2*x^2])
```

3.220.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5465, 211, 211, 224, 219, 5485, 5465, 211, 224, 219, 5481, 224, 219, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{x}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{ 5465}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \left(a^2cx^2+c\right)^{3/2}dx}{5a}\right)+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{ 211}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{211}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{219}$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{5465}$$

3.220. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}dx$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}dx}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{211}$$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{224}$$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}d\right.$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{219}$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)\right)$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

3.220. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}dx$

$$\Big\downarrow \text{5481}$$

$$c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\right.\right.$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{224}$$

$$c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2\right.}{3a^2c}\right.\right.$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{219}$$

$$c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right.}{3a^2c}\right.\right.$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{5493}$$

3.220. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}\,dx$

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\left(a^2c\right.}{3a^2c}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{5489}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+$$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{a}\right.}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```

output
```
a^2*c*(((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(5*a^2*c) - ((x*(c + a^2*c*x^2)
^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4)/(5*a)) + c*(a^2*c*(((c + a^2*c*x^2)^(3
/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh
[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a))/(3*a)) + c*(Sqrt[c + a^2*c*x^2
]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (c*Sq
rt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] +
 I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]))
```

3.220. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}\,dx$

3.220.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.220. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx$

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.220.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.60

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(24\arctan(ax)a^4x^4-6a^3x^3+88a^2\arctan(ax)x^2-35ax+184\arctan(ax))}{120}-\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\big(60\arctan(ax}{}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/120*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(24*arctan(a*x)*a^4*x^4-6*a^3*x^3+88*a
^2*arctan(a*x)*x^2-35*a*x+184*arctan(a*x))-1/60*c^2*(c*(a*x-I)*(I+a*x))^(1
/2)*(60*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-149*I*arctan((1+I*a*
x)/(a^2*x^2+1)^(1/2))-60*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-60*I*dilog
((1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.220.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x, x)
```

3.220.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x, x)
```

3.220.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
2/3*(a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/3*(a^4*x
^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3))
+ 2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - 1/120*((a*(3*(2*(
a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2
 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2
)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c^2 - 20*c^2*arcta
n2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) +
 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2
- 3))) - 20*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4
*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*a
rctan2(4*a*x, a^2*x^2 - 3))) - 120*c^2*integrate(sqrt(a^2*x^2 + 1)*arctan(
a*x)/x, x))*sqrt(c)
```

3.220.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.221 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx$

3.221.1 Optimal result

Integrand size = 22, antiderivative size = 355

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac(c+a^2cx^2)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{4}a^2cx(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{15iac^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4\sqrt{c+a^2cx^2}} - ac^{5/2}\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right.$$

output
```
-1/12*a*c*(a^2*c*x^2+c)^(3/2)+1/4*a^2*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-
a*c^(5/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))-15/4*I*a*c^3*arctan(a*x)*ar
ctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)+15/8*I*a*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)-15/8*I*a*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7/8*a*c^2*(a^2*c*x^2+c)^(1/
2)-c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x+7/8*a^2*c^2*x*arctan(a*x)*(a^2*c*
x^2+c)^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 3.39 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.38

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \frac{ac^2\sqrt{c+a^2cx^2}\left(\frac{1}{2}(1+a^2x^2)^{3/2}+48\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+\frac{3}{2}(\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```

output
```
(a*c^2*Sqrt[c + a^2*c*x^2]*((1 + a^2*x^2)^(3/2)/2 + 48*Sqrt[1 + a^2*x^2]*(
-1 + a*x*ArcTan[a*x]) + (3*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]])/2 + 48*ArcT
an[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (4
2*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 48*((Sqrt[1 + a^2*x^2]*ArcTan[a*
x])/(a*x) - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 +
 I*E^(I*ArcTan[a*x])] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]]
- I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]
) - (42*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (3*(1 + a^2*x^2)^2*ArcTan[a*x
]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*
ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])])
 + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTa
n[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]]))/4))/(4
8*Sqrt[1 + a^2*x^2])
```

3.221.3 Rubi [A] (verified)

Time = 2.96 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.82, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5413, 5413, 5425, 5421, 5485, 5413, 5425, 5421, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \; 5485$$

$$a^2c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/}$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right)+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)$$

$$\Big\downarrow \text{5413}$$

3.221. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^2}\,dx$

$$c\left(a^2c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right)+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLo}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(c\left(a^2c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

3.221. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^2}\,dx$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{F}}{}\right.}{2\sqrt{a^2cx^2}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\left(c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(c\left(c\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}(}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{243}$$

3.221. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^2}dx$

$$c\left(c\left(c\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{73}$$

$$c\left(c\left(c\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyL}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{221}$$

$$c\left(c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```

3.221. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx$

output
```
a^2*c*(-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x
])/4 + (3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]]
)/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c +
a^2*c*x^2])))/4) + c*(a^2*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/
a))/(2*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(
c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1
+ a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
 + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2
, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2]))
```

3.221.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

3.221. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.221.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.75

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(6\arctan(ax)a^4x^4-2a^3x^3+27a^2\arctan(ax)x^2-23ax-24\arctan(ax))}{24x}-\frac{\sqrt{c(ax-i)(ax+i)}\left(15\arctan(ax)\ln\left(\right.\right.}{}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(6*arctan(a*x)*a^4*x^4-2*a^3*x^3+27*a^2
*arctan(a*x)*x^2-23*a*x-24*arctan(a*x))/x-1/8*(c*(a*x-I)*(I+a*x))^(1/2)/(a
^2*x^2+1)^(1/2)*(15*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arc
tan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-8*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)-1)+8*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-15*I*dilog(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+15*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c^2*a
```

3.221.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x^2, x)
```

3.221.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**2, x)
```

3.221.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^2, x)
```

3.221.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x^2}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

3.222 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx$

3.222.1 Optimal result

Integrand size = 22, antiderivative size = 364

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = -\frac{ac^2\sqrt{c+a^2cx^2}}{2x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} + 2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax) - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)}{2x^2} + \frac{1}{3}a^2c(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{5a^2c^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{13}{6}a^2c^{5/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) + \frac{5ia^2c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} - \frac{5ia^2c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2\right.}{2\sqrt{c+a^2cx^2}}$$

output
```
1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-13/6*a^2*c^(5/2)*arctanh(a*x*c^(
1/2)/(a^2*c*x^2+c)^(1/2))-5*a^2*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5/2*I*a^2*c^3*polylog
(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
-5/2*I*a^2*c^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-1/2*a*c^2*(a^2*c*x^2+c)^(1/2)/x-1/6*a^3*c^2*x*(a^2*c
*x^2+c)^(1/2)+2*a^2*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/2*c^2*arctan(a*x
)*(a^2*c*x^2+c)^(1/2)/x^2
```

3.222.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.03

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \frac{a^2c^2\sqrt{c+a^2cx^2}\left(-6-6\cot^2\left(\frac{1}{2}\arctan(ax)\right)-2a^2x^2\csc^2\left(\frac{1}{2}\arctan(ax)\right)\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```

output
```
(a^2*c^2*Sqrt[c + a^2*c*x^2]*(-6 - 6*Cot[ArcTan[a*x]/2]^2 - 2*a^2*x^2*Csc[
ArcTan[a*x]/2]^2 + 28*a*x*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 4*a^3*x^3*Arc
Tan[a*x]*Csc[ArcTan[a*x]/2]^2 - 3*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTa
n[a*x]/2]^2 + 60*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])]
 - 60*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 4*Cot[Ar
cTan[a*x]/2]*Log[-(a*x) + Sqrt[1 + a^2*x^2]] + 48*Cot[ArcTan[a*x]/2]*Log[C
os[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 48*Cot[ArcTan[a*x]/2]*Log[Cos[Ar
cTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (60*I)*Cot[ArcTan[a*x]/2]*PolyLog[2,
-E^(I*ArcTan[a*x])] - (60*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x
])] + 3*ArcTan[a*x]*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]
/2])/(24*Sqrt[1 + a^2*x^2])
```

3.222.3 Rubi [A] (verified)

Time = 2.85 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.91, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5485, 5465, 211, 224, 219, 5481, 224, 219, 242, 5493, 5489, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{ 5465}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}dx}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{ 211}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{ 224}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{ 219}$$

$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)\right)+c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{ 5481}$$

3.222. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^3}\,dx$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx - ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx + \arctan(ax)\sqrt{a^2cx^2+c}\right) + a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx - ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx + \arctan(ax)\sqrt{a^2cx^2+c}\right) + c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx + ac\int\right.\right.$$

$$\Big\downarrow 224$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx - ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}} + \arctan(ax)\sqrt{a^2cx^2+c}\right) + a^2c\left(\frac{\arctan(ax)\left(a^2c\right.}{3a^2c}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx - ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}} + \arctan(ax)\sqrt{a^2cx^2+c}\right) + c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow 219$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx + \arctan(ax)\sqrt{a^2cx^2+c} - \sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right) + a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+\right.}{3a^2c}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx + \arctan(ax)\sqrt{a^2cx^2+c} - \sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right) + c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx\right.\right.$$

$$\Big\downarrow 242$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx + \arctan(ax)\sqrt{a^2cx^2+c} - \sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right) + a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+\right.}{3a^2c}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx + \arctan(ax)\sqrt{a^2cx^2+c} - \sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right) + c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx\right.\right.$$

$$\Big\downarrow 5493$$

$$a^2c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} + \arctan(ax)\sqrt{a^2cx^2+c} - \sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right) + a^2c\left(\frac{\arctan(ax)\left(a\right.}{3a^2}\right.\right.$$

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} + \arctan(ax)\sqrt{a^2cx^2+c} - \sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right) + c\left(-c\int \frac{\arctan(a}{x^3\sqrt{a^2cx^2}}\right.\right.$$

3.222. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^3}\,dx$

$$\Big\downarrow \text{5489}$$

$$c\left(c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arcta}\right.}{}\right.\right.$$
$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax\right.}{}\right.\right.$$

$$\Big\downarrow \text{5497}$$

$$c\left(c\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+}}{x^2}\right.\right.$$
$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax\right.}{}\right.\right.$$

$$\Big\downarrow \text{242}$$

$$c\left(c\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2}}{}\right.\right.$$
$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax\right.}{}\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$c\left(c\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\right.\right.$$
$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax\right.}{}\right.\right.$$

$$\Big\downarrow \text{5489}$$

3.222. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^3}\,dx$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax\right.}{}\right.\right.$$

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\operatorname{arct}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```

output
```
a^2*c*(a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c +
 a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a
))/(3*a)) + c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c
]*x)/Sqrt[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*
x^2])) + c*(c*(-((a*Sqrt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/x^2 - c*(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*A
rcTan[a*x])/(2*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c
*x^2]))) + a^2*c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqr
t[c]*x)/Sqrt[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2
*c*x^2]))
```

3.222.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

3.222. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.222.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.70

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\left(-2\arctan(ax)\sqrt{a^2x^2+1}\,a^4x^4+\sqrt{a^2x^2+1}\,a^3x^3+15\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)\arctan(ax)a^2x^2-14\arctan(ax)\sqrt{a^2x^2+\right.}{6}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)*(-2*arctan(a*x)*(a^2*x^2+
1)^(1/2)*a^4*x^4+(a^2*x^2+1)^(1/2)*a^3*x^3+15*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2)+1)*arctan(a*x)*a^2*x^2-14*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-15*I*di
log((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^2*x^2-26*I*arctan((1+I*a*x)/(a^2*x^2+
1)^(1/2))*a^2*x^2-15*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))*a^2*x^2+3*(a^2*x
^2+1)^(1/2)*a*x+3*arctan(a*x)*(a^2*x^2+1)^(1/2))*c^2/x^2
```

3.222.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x^3, x)
```

3.222.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}{(ax)}}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**3, x)
```

3.222.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/3*(a^4*c^2*x^2 + a^2*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/6*(a
^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*(a^3*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2
+ 3)) + 2*a^2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) + 1/12*(a
^2*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2
*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a
*x, a^2*x^2 - 3))) + a^2*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(
1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1
/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + 24*a^2*c^2*integrate(sqrt(a^2*
x^2 + 1)*arctan(a*x)/x, x) + 12*c^2*integrate(sqrt(a^2*x^2 + 1)*arctan(a*x
)/x^3, x))*sqrt(c)
```

3.222.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)}{x^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)}{x^3} \, dx = \int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2 + c\right)^{5/2}}{x^3} \, dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^3,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

3.223 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx$

3.223.1 Optimal result

Integrand size = 22, antiderivative size = 372

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} - \frac{2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)}{x} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\arctan(ax) - \frac{c(c+a^2cx^2)^{3/2}\arctan(ax)}{3x^3} - \frac{5ia^3c^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{13}{6}a^3c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{5ia^3c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} - \frac{5ia^3c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2\right.}{2\sqrt{c+a^2cx^2}}$$

output
```
-1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3-13/6*a^3*c^(5/2)*arctanh((a^2*c
*x^2+c)^(1/2)/c^(1/2))-5*I*a^3*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I
*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5/2*I*a^3*c^3*polylog(2
,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
-5/2*I*a^3*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)-1/2*a^3*c^2*(a^2*c*x^2+c)^(1/2)-1/6*a*c^2*(a^2*c*x
^2+c)^(1/2)/x^2-2*a^2*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x+1/2*a^4*c^2*x*
arctan(a*x)*(a^2*c*x^2+c)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.84

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}-3a^3x^3\sqrt{1+a^2x^2}-2\sqrt{1+a^2x^2}\arctan(a\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 3*a^3*x^3*Sqrt[1 + a^
2*x^2] - 2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 14*a^2*x^2*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x] + 3*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - a^3*x^3*ArcTanh[Sqrt
[1 + a^2*x^2]] + 15*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - 15*
a^3*x^3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 12*a^3*x^3*Log[Cos[ArcT
an[a*x]/2]] + 12*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] + (15*I)*a^3*x^3*PolyLog[
2, (-I)*E^(I*ArcTan[a*x])] - (15*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])
]))/(6*x^3*Sqrt[1 + a^2*x^2])
```

3.223.3 Rubi [A] (verified)

Time = 3.10 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {5485, 5485, 5413, 5425, 5421, 5479, 243, 51, 73, 221, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\,(a^2cx^2+c)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{(a^2cx^2+c)^{3/2}\arctan(ax)}{x^2}dx + c\int \frac{(a^2cx^2+c)^{3/2}\arctan(ax)}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)dx + c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right) + c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx + c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right)+ c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}d\right. c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)$$

$$\Big\downarrow \text{5421}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)+ a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{Poly}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{3}a\int \frac{\sqrt{a^2cx^2+c}}{x^3}dx-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+ a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{Poly}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\int \frac{\sqrt{a^2cx^2+c}}{x^4}dx^2-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+ a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{Poly}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

3.223. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^4}\,dx$

$$\Big\downarrow \text{ 51}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\left(\frac{1}{2}a^2c\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right.}{3cx^3}\right.\right.$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{Poly}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{ 73}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\left(\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)}{3cx^3}\right.\right.$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{Poly}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{ 221}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2\right.}{3cx^3}\right.\right.$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{Poly}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(a^2c\left(a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right.\right.\right.$$
$$a^2c\left(c\left(a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{ 5425}$$

3.223. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^4}\,dx$

$$c\left(a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2}}{}\right.\right.\right.$$

$$a^2c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{2\sqrt{a^2c}}\right.\right.$$

$$\Big\downarrow \ 5421$$

$$c\left(c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\int \frac{\arctan(}{x^2\sqrt{a^2cx}}\right.\right.$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i}}{\sqrt{1-}}\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \ 5479$$

$$c\left(c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(a\int \frac{}{x\sqrt{a^2c}}\right.\right.\right.$$

$$a^2c\left(c\left(c\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyL}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \ 243$$

3.223. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^4}\,dx$

$$c\left(c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(\frac{1}{2}a\int\frac{}{x^2\sqrt{}}\right.\right.\right.$$

$$a^2c\left(c\left(c\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{P}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \textbf{73}$$

$$c\left(c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d}{c}\right.\right.\right.$$

$$a^2c\left(c\left(c\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Pol}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \textbf{221}$$

$$a^2c\left(c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(-\frac{\arctan(a}{}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]
```

3.223. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x^4}\,dx$

output
```
a^2*c*(a^2*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/
Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]
])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c +
 a^2*c*x^2])) + c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcT
anh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-
2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2
, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2])) + c*(c*(-1/3*((c + a^2*
c*x^2)^(3/2)*ArcTan[a*x])/(c*x^3) + (a*(-(Sqrt[c + a^2*c*x^2]/x^2) - a^2*S
qrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]))/6) + a^2*c*(c*(-((Sqrt[c + a
^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/S
qrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*
a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I
*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c
 + a^2*c*x^2]))
```

3.223.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.223. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx$

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.223. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx$

3.223.4 Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.73

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(3\arctan(ax)a^4x^4-3a^3x^3-14a^2\arctan(ax)x^2-ax-2\arctan(ax))}{6x^3}+\frac{ic^2a^3\sqrt{c(ax-i)(ax+i)}\,\left(15i\arctan(ax)\,\mathrm{l}\right.}{}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(3*arctan(a*x)*a^4*x^4-3*a^3*x^3-14*a^2*
arctan(a*x)*x^2-a*x-2*arctan(a*x))/x^3+1/6*I*c^2*a^3*(c*(a*x-I)*(I+a*x))^(
1/2)*(15*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x
)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-13*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-
1)+13*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+15*dilog(1+I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-15*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.223.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x^4, x)
```

3.223.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**4, x)
```

3.223.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^4, x)
```

3.223.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x^4}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

3.224 $\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}} \, dx$

3.224.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}} \, dx = -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{3a^2c} + \frac{5\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^4\sqrt{c}}$$

output
```
5/6*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4/c^(1/2)-1/6*x*(a^2*c*x^2+
c)^(1/2)/a^3/c-2/3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*
x)*(a^2*c*x^2+c)^(1/2)/a^2/c
```

3.224.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}} \, dx = \frac{-ax\sqrt{c+a^2cx^2} + 2(-2+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax) + 5\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{6a^4c}$$

input
```
Integrate[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x] + 5*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6*a^4*c)
```

3.224.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5487, 262, 224, 219, 5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{262}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{224}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

$$\Big\downarrow \text{224}$$

$$-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

input
```
Int[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])
/(2*a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(
3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2)
```

3.224.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.224.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.38

method	result
default	$\frac{(2a^2\arctan(ax)x^2-ax-4\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{6a^4c}-\frac{5\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)\sqrt{c(ax-i)(ax+i)}}{6\sqrt{a^2x^2+1}\,a^4c}+\frac{5\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)\sqrt{c(ax-i)}}{6\sqrt{a^2x^2+1}\,a^4c}$

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*a^2*arctan(a*x)*x^2-a*x-4*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/a^
4/c-5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x
^2+1)^(1/2)/a^4/c+5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*(c*(a*x-I)*(I+a*x)
)^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{x^3\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{a^2cx^2+c}(ax-2(a^2x^2-2)\arctan(ax))-5\sqrt{c}\log\left(-2a^2cx^2-2\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)}{12a^4c}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 - 2)*arctan(a*x)) - 5*sqrt(
c)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^4*c)
```

3.224.6 Sympy [F]

$$\int \frac{x^3\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\operatorname{atan}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x^3\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{a\left(\frac{\frac{\sqrt{a^2x^2+1}x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}}{a^2}-\frac{4\operatorname{arsinh}(ax)}{a^5}\right)-2\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\arctan(ax)}{6\sqrt{c}}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(a*((sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 4*arcsinh(a*x)
/a^5) - 2*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arctan(a*x
))/sqrt(c)
```

3.224.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.225 $\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$

3.225.1 Optimal result

Integrand size = 22, antiderivative size = 250

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}}{2a^3c}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}}$$

output
```
I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^
3/(a^2*c*x^2+c)^(1/2)-1/2*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/2*I*polylog(2,I*(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-1/2*(a^2*c*x^
2+c)^(1/2)/a^3/c+1/2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2/c
```

3.225.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.63

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c(1+a^2x^2)}\left(\sqrt{1+a^2x^2}-ax\sqrt{1+a^2x^2}\arctan(ax)+\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)-\arctan(ax)\right.}{2a^3c\sqrt{1+a^2x^2}}$$

input
```
Integrate[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
-1/2*(Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2] - a*x*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x] + ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 +
I*E^(I*ArcTan[a*x])] + I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2,
 I*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[1 + a^2*x^2])
```

3.225.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{ 5487}$$

$$-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}$$

$$\Big\downarrow \text{ 5425}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}$$

$$\Big\downarrow \text{5421}$$

$$-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}$$

input
```
Int[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*
a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])
/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c
 + a^2*c*x^2])
```

3.225.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.225. $\int \frac{x^2\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.225.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74

method	result
default	$\frac{(x\arctan(ax)a-1)\sqrt{c(ax-i)(ax+i)}}{2c\,a^3}+\frac{\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{d}\right.}{2\sqrt{a^2x^2+1}\,a^3c}$

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x*arctan(a*x)*a-1)*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^3+1/2*(arctan(a*x)*l
n(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

3.225.5 Fricas [F]

$$\int \frac{x^2\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.225.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c + a^2cx^2}}\, dx = \int \frac{x^2 \operatorname{atan}(ax)}{\sqrt{c(a^2x^2 + 1)}}\, dx$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.225.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c + a^2cx^2}}\, dx = \int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2 + c}}\, dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.225.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c + a^2cx^2}}\, dx = \int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2 + c}}\, dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2 + c}}\,dx$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.226 $\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}} dx$

3.226.1 Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}} dx = \frac{\sqrt{c+a^2cx^2} \arctan(ax)}{a^2c} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^2\sqrt{c}}$$

output
```
-arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^2/c^(1/2)+arctan(a*x)*(a^2*c*x
^2+c)^(1/2)/a^2/c
```

3.226.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}} dx = \frac{\sqrt{c+a^2cx^2} \arctan(ax) - \sqrt{c} \log \left(acx + \sqrt{c}\sqrt{c+a^2cx^2}\right)}{a^2c}$$

input
```
Integrate[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^
2*c*x^2]])/(a^2*c)
```

3.226.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}$$
$$\Big\downarrow \text{224}$$
$$\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}$$
$$\Big\downarrow \text{219}$$
$$\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}$$

input
```
Int[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c +
 a^2*c*x^2]]/(a^2*Sqrt[c])
```

3.226.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.226.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

method	result	size
default	$\frac{\left(\arctan(ax)\sqrt{a^2x^2+1}-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)+\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)\right)\sqrt{c(ax-i)(ax+i)}}{\sqrt{a^2x^2+1}\,a^2c}$	100

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(arctan(a*x)*(a^2*x^2+1)^(1/2)-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)+ln((1+I*a
*x)/(a^2*x^2+1)^(1/2)-I))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/
c
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{x\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{a^2cx^2+c}\arctan(ax)+\sqrt{c}\log\left(-2\,a^2cx^2+2\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)}{2\,a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(a^2*c*x^2 + c)*arctan(a*x) + sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt
(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^2*c)
```

3.226.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{a^2x^2+1}\arctan(ax) - \log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}\right)}{2\,a^2\sqrt{c}}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a^2*x^2 + 1)*arctan(a*x) - log(a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1)))/(a^2*sqrt(c))
```

3.226.8 Giac [F]

$$\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x \operatorname{atan}(a\,x)}{\sqrt{c\,a^2\,x^2 + c}} \, dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.227 $\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$

3.227.1 Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}$$

output
```
-2*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/a/(a^2*c*x^2+c)^(1/2)+I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^
2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*
x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{c(1+a^2x^2)}\left(2\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax) - \mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right) + \mathrm{PolyLog}\left(2,ie^{i\arctan(ax)}\right)\right)}{ac\sqrt{1+a^2x^2}}$$

input
```
Integrate[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

output
```
((-I)*Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x] - Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] + PolyLog[2, I*E^(I*ArcTan[a*x])]))/(a*c*S
qrt[1 + a^2*x^2])
```

3.227.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.73, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5421}$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*P
olyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2]
```

3.227.3.1 Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.227.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{dilog}\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)\right)\sqrt{c(ax-i)(ax+i}}{\sqrt{a^2x^2+1}\,ca}$

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/
2)/c/a
```

3.227.5 Fricas [F]

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.227.6 Sympy [F]

$$\int \frac{\arctan(ax)}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{\sqrt{c(a^2x^2 + 1)}}\,dx$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.227.7 Maxima [F]

$$\int \frac{\arctan(ax)}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2 + c}}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.227.8 Giac [F]

$$\int \frac{\arctan(ax)}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2 + c}}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)/(c + a^2*c*x^2)^(1/2), x)
```

3.228 $\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx$

3.228.1 Optimal result

Integrand size = 22, antiderivative size = 177

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)+I*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2
+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{1+a^2x^2}\left(\arctan(ax)\left(\log\left(1-e^{i\arctan(ax)}\right)-\log\left(1+e^{i\arctan(ax)}\right)\right)+i\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\operatorname{Pol}\right.}{\sqrt{c\left(1+a^2x^2\right)}}$$

input
```
Integrate[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I
*ArcTan[a*x])]) + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*Arc
Tan[a*x])]))/Sqrt[c*(1 + a^2*x^2)]
```

3.228.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5493}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{5489}$$
$$\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]
```

3.228.3.1 Defintions of rubi rules used

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.228.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

method	result
default	$\frac{i\left(i\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\right)\sqrt{c(ax-i)(ax}}{\sqrt{a^2x^2+1}\,c}$

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I
*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2
,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)
/c
```

3.228.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

3.228.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

3.228.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}\,x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.228.8 Giac [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}\,x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.229 $\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx$

3.229.1 Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{cx} - \frac{a\,\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
-a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^
(1/2)/c/x
```

3.229.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{cx} + \frac{a\left(\log(x) - \log\left(c+\sqrt{c}\sqrt{c+a^2cx^2}\right)\right)}{\sqrt{c}}$$

input
```
Integrate[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) + (a*(Log[x] - Log[c + Sqrt[c]*
Sqrt[c + a^2*c*x^2]]))/Sqrt[c]
```

3.229.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5479}$$

$$a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2
]/Sqrt[c]])/Sqrt[c]
```

3.229.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.229.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.23

method	result	size
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,\left(\ln\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}+1\right)\sqrt{a^{2}x^{2}+1}\,ax-\ln\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}-1\right)\sqrt{a^{2}x^{2}+1}\,ax+a^{2}\arctan(ax)x^{2}+\arctan(ax)\right)}{cx(a^{2}x^{2}+1)}$	125

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*(a*x-I)*(I+a*x))^(1/2)*(ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*(a^2*x^2+1)^
(1/2)*a*x-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(a^2*x^2+1)^(1/2)*a*x+a^2*arct
an(a*x)*x^2+arctan(a*x))/c/x/(a^2*x^2+1)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\sqrt{a^2cx^2+c}\arctan(ax)}{2\,cx}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(a*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*c)/x^
2) - 2*sqrt(a^2*c*x^2 + c)*arctan(a*x))/(c*x)
```

3.229.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{a\,\operatorname{arsinh}\left(\frac{1}{a|x|}\right)+\frac{\sqrt{a^2x^2+1}\arctan(ax)}{x}}{\sqrt{c}}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-(a*arcsinh(1/(a*abs(x))) + sqrt(a^2*x^2 + 1)*arctan(a*x)/x)/sqrt(c)
```

3.229.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.230 $\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx$

3.230.1 Optimal result

Integrand size = 22, antiderivative size = 242

$$\begin{aligned}\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = &-\frac{a\sqrt{c+a^2cx^2}}{2cx}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{2cx^2}\\ &+\frac{a^2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\ &-\frac{ia^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\\ &+\frac{ia^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
a^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)-1/2*I*a^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/2*I*a^2*polylog(2,(1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*c*x^2+c
)^(1/2)/c/x-1/2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c/x^2
```

3.230.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.68

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{a^2\sqrt{1+a^2x^2}\left(-2\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)-4\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^2*Sqrt[1 + a^2*x^2]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan[a*x
]/2]^2 - 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcTan[a*x]*Log[1 +
E^(I*ArcTan[a*x])] - (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + (4*I)*PolyLog[
2, E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTan[a*
x]/2]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

3.230.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}$$

$$\Big\downarrow \text{5493}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}$$

$$\Big\downarrow \text{5489}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}$$

input
```
Int[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*
c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Poly
Log[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2])
```

3.230.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.230.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

method	result
default	$-\frac{(ax+\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{2c\,x^2}-\frac{ia^2\left(i\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\mathrm{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.}{2\sqrt{a^2x^2+1}\,c}$

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^2-1/2*I*a^2*(I*arctan
(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*
x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/
(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

3.230.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}}{\sqrt{a^2cx^2+c}\,x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

3.230.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^3\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

3.230.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.230.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.231 $\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.231.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{a\sqrt{c+a^2cx^2}}{6cx^2} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)}{3cx} + \frac{5a^3\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

output
```
5/6*a^3*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)-1/6*a*(a^2*c*x^2+c)^(
1/2)/c/x^2-1/3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)*(
a^2*c*x^2+c)^(1/2)/c/x
```

3.231.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{-ax\sqrt{c+a^2cx^2} + 2(-1+2a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax) - 5a^3\sqrt{c}x^3\log(x) + 5a^3\sqrt{c}x^3\log\left(c+\sqrt{c}\sqrt{c+a\right.}{6cx^3}$$

input
```
Integrate[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-1 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x] - 5*a^3*Sqrt[c]*x^3*Log[x] + 5*a^3*Sqrt[c]*x^3*Log[c + Sqrt[c]*Sqrt
[c + a^2*c*x^2]])/(6*c*x^3)
```

3.231.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5497, 243, 52, 73, 221, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{3}a\int \frac{1}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\int \frac{1}{x^4\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\left(-\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\left(-\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{c}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \; 243$$

$$-\frac{2}{3}a^2\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \; 73$$

$$-\frac{2}{3}a^2\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \; 221$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

input
```
Int[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[
c]))/3 + (a*(-(Sqrt[c + a^2*c*x^2]/(c*x^2)) + (a^2*ArcTanh[Sqrt[c + a^2*c*
x^2]/Sqrt[c]])/Sqrt[c]))/6
```

3.231. $\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.231.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.231. $\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.231.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.38

method	result
default	$\frac{\left(4 a^{2} \arctan \left(a x\right) x^{2}-a x-2 \arctan \left(a x\right)\right) \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{6 c \,x^{3}}-\frac{5 a^{3} \ln \left(\frac{i a x+1}{\sqrt{a^{2} x^{2}+1}}-1\right) \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{6 \sqrt{a^{2} x^{2}+1}\, c}+\frac{5 a^{3} \ln \left(\frac{i a x+1}{\sqrt{a^{2} x^{2}+1}}+1\right) \sqrt{c(}}{6 \sqrt{a^{2} x^{2}+1}\, c}$

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*a^2*arctan(a*x)*x^2-a*x-2*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c/
x^3-5/6*a^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a
^2*x^2+1)^(1/2)/c+5/6*a^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*(c*(a*x-I)*(I+
a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{5\,a^{3}\sqrt{c}x^{3}\log\left(-\frac{a^{2}cx^{2}+2\,\sqrt{a^{2}cx^{2}+c}\sqrt{c}+2\,c}{x^{2}}\right)-2\,\sqrt{a^{2}cx^{2}+c}(ax-2\,(2\,a^{2}x^{2}-1)\arctan\left(ax\right))}{12\,cx^{3}}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(5*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*sqrt(c) +
2*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(2*a^2*x^2 - 1)*arctan(a*x)))/(
c*x^3)
```

3.231.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^4\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{\left(5a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{\sqrt{a^2x^2+1}}{x^2}\right)a + 2\left(\frac{2\sqrt{a^2x^2+1}a^2}{x} - \frac{\sqrt{a^2x^2+1}}{x^3}\right)\arctan(ax)}{6\sqrt{c}}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/6*((5*a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x^2)*a + 2*(2*sqrt(a
^2*x^2 + 1)*a^2/x - sqrt(a^2*x^2 + 1)/x^3)*arctan(a*x))/sqrt(c)
```

3.231.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(a\,x)}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.232 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

3.232.1 Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{x}{a^3c\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^4c^2} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^4c^{3/2}}$$

output
```
-arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4/c^(3/2)-x/a^3/c/(a^2*c*x^2+c
)^(1/2)+arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)*(a^2*c*x^2+c)^(1
/2)/a^4/c^2
```

3.232.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{-ax\sqrt{c+a^2cx^2} + (2+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax) - \sqrt{c}(1+a^2x^2)\log\left(acx+\sqrt{c}\sqrt{\right.}{a^4c^2(1+a^2x^2)}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(-(a*x*Sqrt[c + a^2*c*x^2]) + (2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
] - Sqrt[c]*(1 + a^2*x^2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(a^4*c
^2*(1 + a^2*x^2))
```

3.232.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5499, 5465, 208, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \; 5499$$

$$\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2]))/
a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/
Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/(a^2*c)
```

3.232.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.232.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.04

method	result
default	$\frac{\left(\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)a^2x^2+\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)a^2x^2-\sqrt{a^2x^2+1}\,ax+2\arctan(ax)\sqrt{a^2x^2+1}-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+\right.\right.}{a^4c^2(a^4x^4+2a^2x^2+1)}$

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*a
^2*x^2+ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*a^2*x^2-(a^2*x^2+1)^(1/2)*a*x+2*a
rctan(a*x)*(a^2*x^2+1)^(1/2)-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)+ln((1+I*a*x
)/(a^2*x^2+1)^(1/2)-I))*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a^4/c^
2/(a^4*x^4+2*a^2*x^2+1)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(a^2x^2+1)\sqrt{c}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\,\sqrt{a^2cx^2+c}(ax-(a^2x^2+}{2\,(a^6c^2x^2+a^4c^2)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt
(c)*x - c) - 2*sqrt(a^2*c*x^2 + c)*(a*x - (a^2*x^2 + 2)*arctan(a*x)))/(a^6
*c^2*x^2 + a^4*c^2)
```

3.232.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.232.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan{(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

3.232.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.233 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

3.233.1 Optimal result

Integrand size = 22, antiderivative size = 251

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{1}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}}$$

output
```
-1/a^3/c/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)-2*I*a
rctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c
/(a^2*c*x^2+c)^(1/2)+I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*
x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.62

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{1+a^2x^2}\left(\frac{1}{\sqrt{1+a^2x^2}}+\frac{ax\arctan(ax)}{\sqrt{1+a^2x^2}}-\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)+\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)-\right.}{a^3c\sqrt{c\,(1+a^2x^2)}}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*(1/Sqrt[1 + a^2*x^2] + (a*x*ArcTan[a*x])/Sqrt[1 + a^2
*x^2] - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E
^(I*ArcTan[a*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*
E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

3.233.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5469, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \; 5469$$

$$\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5425$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5421$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(1/(a^3*c*Sqrt[c + a^2*c*x^2])) - (x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x
^2]) + (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a
 - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a^2*c*Sqrt[c +
 a^2*c*x^2])
```

3.233.3.1 Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

3.233.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.98

method	result
default	$-\frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^3c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)(\arctan(ax)-i)}{2(a^2x^2+1)a^3c^2}-\frac{\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan\right.}{}$

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/a^3/c^2
-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)/(a^2*x^2+1)/a^3/c^2
-(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/
2)/a^3/c^2
```

3.233.5 Fricas [F]

$$\int \frac{x^2\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
+ c^2), x)
```

3.233.6 Sympy [F]

$$\int \frac{x^2\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^2\operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.233.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

3.233.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.234 $\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx$

3.234.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx = \frac{x}{ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
x/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{c+a^2cx^2}(ax - \arctan(ax))}{a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(a*x - ArcTan[a*x]))/(a^2*c^2*(1 + a^2*x^2))
```

3.234.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])
```

3.234.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.234.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

method	result	size
default	$-\frac{(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)(\arctan(ax)-i)}{2(a^2x^2+1)a^2c^2}$	100

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/a^2/c
^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)/(a^2*x^2+1)/a^2
/c^2
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax-\arctan(ax))}{a^4c^2x^2+a^2c^2}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x - arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

3.234.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{ax-\arctan(ax)}{\sqrt{a^2x^2+1}a^2c^{\frac{3}{2}}}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
(a*x - arctan(a*x))/(sqrt(a^2*x^2 + 1)*a^2*c^(3/2))
```

3.234.8 Giac [F]

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.235 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

3.235.1 Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{1}{ac\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)}{c\sqrt{c+a^2cx^2}}$$

output
```
1/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(1+ax\arctan(ax))}{c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(1 + a*x*ArcTan[a*x]))/(c^2*(a + a^3*x^2))
```

3.235.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \; 5429$$

$$\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
```

3.235.3.1 Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.235.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

method	result	size
default	$\frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2a}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)(\arctan(ax)-i)}{2(a^2x^2+1)c^2a}$	98

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2/a+1/
2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2/a
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(ax\arctan{(ax)}+1)}{a^3c^2x^2+ac^2}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x) + 1)/(a^3*c^2*x^2 + a*c^2)
```

3.235.6 Sympy [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}{(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan{(ax)}}{\sqrt{a^2cx^2+c}c} + \frac{1}{\sqrt{a^2cx^2+c}ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*c) + 1/(sqrt(a^2*c*x^2 + c)*a*c)
```

3.235.8 Giac [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)/(c + a^2*c*x^2)^(3/2), x)
```

3.236 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{3/2}}\,dx$

3.236.1 Optimal result

Integrand size = 22, antiderivative size = 229

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{3/2}}\,dx = -\frac{ax}{c\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}}$$

output
```
-a*x/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x)
*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c
)^(1/2)+I*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/
(a^2*c*x^2+c)^(1/2)-I*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+
1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{ax}{\sqrt{1+a^2x^2}}+\frac{\arctan(ax)}{\sqrt{1+a^2x^2}}+\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)-\arctan(ax)\,\mathrm{lo}\right.}{c\sqrt{c\,(1+a}}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-((a*x)/Sqrt[1 + a^2*x^2]) + ArcTan[a*x]/Sqrt[1 + a^2*
x^2] + ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*A
rcTan[a*x])] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan
[a*x])]))/(c*Sqrt[c*(1 + a^2*x^2)])
```

3.236.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5501, 5465, 208, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \; 5465$$

$$\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 208$$

$$\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 5493$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5489}$$

$$-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right) + \frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right) + i\,\text{PolyLog}\left(2, -\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right) - i\,\text{PolyLog}\left(2, \frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2
]))) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 -
 I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2,
 Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(c*Sqrt[c + a^2*c*x^2])
```

3.236.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.236.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01

method	result
default	$\frac{(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2} - \frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)(\arctan(ax)-i)}{2(a^2x^2+1)c^2} + \frac{i\left(i\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\,\mathrm{arct}\right.}{}$

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2-1/
2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2+I*(I
*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*a*x
)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+
I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

3.236.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^
2*x), x)
```

3.236.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.236.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.236.8 Giac [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}(a\,x)}{x\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.237 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx$

3.237.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx = -\frac{a}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{c^2x}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

output
```
-a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2)-a/c/(a^2*c*x^2+c)^(1/2)-a^
2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/
x
```

3.237.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx = -\frac{a\sqrt{c(1+a^2x^2)}}{c^2(1+a^2x^2)}-\frac{\sqrt{c(1+a^2x^2)}(1+2a^2x^2)\arctan(ax)}{c^2x(1+a^2x^2)} + \frac{a\log(x)}{c^{3/2}}-\frac{a\log\left(c+\sqrt{c}\sqrt{c(1+a^2x^2)}\right)}{c^{3/2}}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-((a*Sqrt[c*(1 + a^2*x^2)])/(c^2*(1 + a^2*x^2))) - (Sqrt[c*(1 + a^2*x^2)]*
(1 + 2*a^2*x^2)*ArcTan[a*x])/(c^2*x*(1 + a^2*x^2)) + (a*Log[x])/c^(3/2) -
(a*Log[c + Sqrt[c]*Sqrt[c*(1 + a^2*x^2)]])/c^(3/2)
```

3.237.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5501, 5429, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2 \int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2 \left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5479}$$

$$\frac{a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c} - a^2 \left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2 - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c} - a^2 \left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac} - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c} - a^2 \left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a
^2*c*x^2]/Sqrt[c]])/Sqrt[c])/c
```

3.237.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.237. $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.237.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.13

method	result
default	$-\frac{\left(\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)a^3x^3-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)a^3x^3+2\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2+\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)ax-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)ax+\sqrt{a}\right.}{x\,c^2(a^4x^4+2a^2x^2+1)}$

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^3*x^3-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
-1)*a^3*x^3+2*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+ln((1+I*a*x)/(a^2*x^2+
1)^(1/2)+1)*a*x-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*a*x+(a^2*x^2+1)^(1/2)*a*
x+arctan(a*x)*(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/
2)/x/c^2/(a^4*x^4+2*a^2*x^2+1)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{(a^3x^3+ax)\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}(ax+(2a^2x^2+1)\,\mathrm{arc}}{2\left(a^2c^2x^3+c^2x\right)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(
c) + 2*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + (2*a^2*x^2 + 1)*arctan(a*x))
)/(a^2*c^2*x^3 + c^2*x)
```

3.237.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\operatorname{atan}(ax)}{x^2 (c (a^2x^2 + 1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.237.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.237.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\operatorname{atan}(a\,x)}{x^2 \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.238 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^{3/2}}\,dx$

3.238.1 Optimal result

Integrand size = 22, antiderivative size = 300

$$\begin{aligned}\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^{3/2}}\,dx &= \frac{a^3x}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{2c^2x} - \frac{a^2\arctan(ax)}{c\sqrt{c+a^2cx^2}} \\ &- \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{2c^2x^2} + \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} \\ &- \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{c+a^2cx^2}} + \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
a^3*x/c/(a^2*c*x^2+c)^(1/2)-a^2*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+3*a^2*ar
ctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^
2*c*x^2+c)^(1/2)-3/2*I*a^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^
2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+3/2*I*a^2*polylog(2,(1+I*a*x)^(1/2)/(
1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*c*x^2+c
)^(1/2)/c^2/x-1/2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x^2
```

3.238.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx =$$
$$-\frac{a^2\left(-8ax+8\arctan(ax)+ax\csc^2\left(\frac{1}{2}\arctan(ax)\right)+\sqrt{1+a^2x^2}\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)+12\sqrt{1+\right.}{}$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-1/8*(a^2*(-8*a*x + 8*ArcTan[a*x] + a*x*Csc[ArcTan[a*x]/2]^2 + Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*Log[1 - E^(I*ArcTan[a*x])] - 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(
I*ArcTan[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])]
- (12*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, E^(I*ArcTan[a*x])] - Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 + 2*Sqrt[1 + a^2*x^2]*Tan[ArcTan[a*x]/
2]))/(c*Sqrt[c + a^2*c*x^2])
```

3.238.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5501, 5497, 242, 5493, 5489, 5501, 5465, 208, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{242}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5493}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5489}$$

$$-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx+\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}$$

$$\Big\downarrow \text{5501}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx\right)+\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}$$

$$\Big\downarrow \text{5465}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}$$

$$\Big\downarrow \text{208}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}$$

$$\Big\downarrow \text{5493}$$

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}$$

3.238. $\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx$

$$\Big\downarrow \text{5489}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}$$

$$a^2\left(-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax}}{\sqrt{1-}}\right.\right.}{c\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2
*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Pol
yLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))/c - a^
2*(-(a^2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*
x^2]))) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog
[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(c*Sqrt[c + a^2*c*x^2]))
```

3.238.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.238.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^{2}(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^{2}x^{2}+1)c^{2}}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)(\arctan(ax)-i)a^{2}}{2(a^{2}x^{2}+1)c^{2}}-\frac{(ax+\arctan(ax))\sqrt{c(ax-i)(ax+}}{2c^{2}x^{2}}$

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c
^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)*a^2/(a^2*x^2+1)
/c^2-1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x^2-3/2*I*a^2*(I*
arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*a*x)
/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I
*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

3.238.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x^3 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^
2*x^3), x)
```

3.238.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^3 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}\left(ax\right)}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.238.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^3 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

3.238.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^3\,(c\,a^2\,x^2 + c)^{3/2}}\,dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.239 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx$

3.239.1 Optimal result

Integrand size = 22, antiderivative size = 165

$$\int \frac{\arctan(ax)}{x^4\,(c+a^2cx^2)^{3/2}}\,dx = \frac{a^3}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{6c^2x^2} + \frac{a^4x\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{3c^2x^3} + \frac{5a^2\sqrt{c+a^2cx^2}\arctan(ax)}{3c^2x} + \frac{11a^3\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{6c^{3/2}}$$

output
```
11/6*a^3*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+a^3/c/(a^2*c*x^2+c)^
(1/2)+a^4*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-1/6*a*(a^2*c*x^2+c)^(1/2)/c^
2/x^2-1/3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x^3+5/3*a^2*arctan(a*x)*(a^2
*c*x^2+c)^(1/2)/c^2/x
```

3.239.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(ax)}{x^4\,(c+a^2cx^2)^{3/2}}\,dx = \frac{\frac{a(-1+5a^2x^2)\sqrt{c+a^2cx^2}}{x^2+a^2x^4} + \frac{2\sqrt{c+a^2cx^2}(-1+4a^2x^2+8a^4x^4)\arctan(ax)}{x^3+a^2x^5} - 11a^3\sqrt{c}\log(x) + 11a^3\sqrt{c}}{6c^2}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
((a*(-1 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2])/(x^2 + a^2*x^4) + (2*Sqrt[c + a^
2*c*x^2]*(-1 + 4*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x])/(x^3 + a^2*x^5) - 11*a^
3*Sqrt[c]*Log[x] + 11*a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6
*c^2)
```

3.239.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.64, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5501, 5497, 243, 52, 73, 221, 5479, 243, 73, 221, 5501, 5429, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^4\sqrt{a^2cx^2+c}}dx}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{-\frac{2}{3}a^2 \int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx + \frac{1}{3}a \int \frac{1}{x^3\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{2}{3}a^2 \int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx + \frac{1}{6}a \int \frac{1}{x^4\sqrt{a^2cx^2+c}}dx^2 - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{2}{3}a^2 \int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx + \frac{1}{6}a\left(-\frac{1}{2}a^2 \int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2 - \frac{\sqrt{a^2cx^2+c}}{cx^2}\right) - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\left(-\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{c}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{-\frac{2}{3}a^2\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{2}{3}a^2\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{2}{3}a^2\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{221}$$

3.239. $\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\int\frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5479}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

3.239. $\int\frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx$

$$\Big\downarrow \color{blue}{73}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}$$

$$a^2\left(\frac{\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}$$

$$a^2\left(\frac{-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(-(a^2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^
2*c*x^2]))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqr
t[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c])/c)) + (-1/3*(Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x])/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*
ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/3 + (a*(-(Sqrt[c + a^2*c*x
^2]/(c*x^2)) + (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/6)/c
```

3.239.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

3.239. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.239. $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.239.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.65

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\sqrt{a^2x^2+1}\left(-11\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)a^5x^5+11\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)a^5x^5+16\arctan(ax)\sqrt{a^2x^2+1}\,a^4x^4+5\sqrt{a^2x^2+1}\,a^3x}{6x^3c^2(a^4x^4+2}$

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*(a*x-I)*(I+a*x))^(1/2)*(a^2*x^2+1)^(1/2)*(-11*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2)-1)*a^5*x^5+11*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^5*x^5+16*arcta
n(a*x)*(a^2*x^2+1)^(1/2)*a^4*x^4+5*(a^2*x^2+1)^(1/2)*a^3*x^3-11*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2)-1)*a^3*x^3+11*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^3*x
^3+8*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-(a^2*x^2+1)^(1/2)*a*x-2*arctan(
a*x)*(a^2*x^2+1)^(1/2))/x^3/c^2/(a^4*x^4+2*a^2*x^2+1)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)}{x^4\,(c+a^2cx^2)^{3/2}}\,dx = \frac{11\,(a^5x^5+a^3x^3)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(5\,a^3x^3-ax+2\,(8\,a^4x^4+4\,a}{12\,(a^2c^2x^5+c^2x^3)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(11*(a^5*x^5 + a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 +
c)*sqrt(c) + 2*c)/x^2) + 2*(5*a^3*x^3 - a*x + 2*(8*a^4*x^4 + 4*a^2*x^2 - 1
)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^2*c^2*x^5 + c^2*x^3)
```

3.239.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^4(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.239.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^4), x)
```

3.239.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^4\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(3/2)), x)
```

3.240 $\int \frac{x^5\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

3.240.1 Optimal result

Integrand size = 22, antiderivative size = 170

$$\int \frac{x^5\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{x^3}{9a^3c\,(c+a^2cx^2)^{3/2}} - \frac{5x}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2\arctan(ax)}{3a^4c\,(c+a^2cx^2)^{3/2}} + \frac{5\arctan(ax)}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^6c^3} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^6c^{5/2}}$$

output
```
-1/9*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)+1/3*x^2*arctan(a*x)/a^4/c/(a^2*c*x^2+c)
^(3/2)-arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^6/c^(5/2)-5/3*x/a^5/c^2/
(a^2*c*x^2+c)^(1/2)+5/3*arctan(a*x)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x
)*(a^2*c*x^2+c)^(1/2)/a^6/c^3
```

3.240.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.77

$$\int \frac{x^5\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{ax(15+16a^2x^2)\sqrt{c+a^2cx^2} - 3\sqrt{c+a^2cx^2}(8+12a^2x^2+3a^4x^4)\arctan(ax) + 9\sqrt{c}(1+a^2x^2)^2\log\left(acx + \right.}{9a^6c^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/9*(a*x*(15 + 16*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 3*Sqrt[c + a^2*c*x^2]*(8
 + 12*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[a*c
*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(a^6*c^3*(1 + a^2*x^2)^2)
```

3.240.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5499, 5473, 5465, 208, 5499, 5465, 208, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5473}$$

$$\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{\int\frac{1}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

input
```
Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-((x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2
*c*x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt
[c + a^2*c*x^2])))/(3*a^2*c))/a^2) + (-((x/(a*c*Sqrt[c + a^2*c*x^2]) - Arc
Tan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2]))/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/
(a^2*c))/(a^2*c)
```

3.240. $\int \frac{x^5\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

3.240.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.240. $\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

3.240.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.92

method	result
default	$\frac{\left(9 \arctan(ax)\sqrt{a^2x^2+1}\,a^4x^4+9\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)a^4x^4-9\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)a^4x^4-16\sqrt{a^2x^2+1}\,a^3x^3+36\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2\cdots\right.}{9\sqrt{c\cdots}}$

input
```
int(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^4*x^4+9*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)-I)*a^4*x^4-9*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*a^4*x^4-16*(a^2*x^2+1)^
(1/2)*a^3*x^3+36*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+18*ln((1+I*a*x)/(a^
2*x^2+1)^(1/2)-I)*a^2*x^2-18*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*a^2*x^2-15*
(a^2*x^2+1)^(1/2)*a*x+24*arctan(a*x)*(a^2*x^2+1)^(1/2)+9*ln((1+I*a*x)/(a^2
*x^2+1)^(1/2)-I)-9*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I))/(a^2*x^2+1)^(1/2)*(c
*(a*x-I)*(I+a*x))^(1/2)/a^6/c^3/(a^4*x^4+2*a^2*x^2+1)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{9\,(a^4x^4+2\,a^2x^2+1)\sqrt{c}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}\,a\sqrt{c}x-c\right)-2\,(16\,a^3x^3+15\,a\cdots}{18\,(a^{10}c^3x^4+2\,a^8c^3x^2+a^6c^3)}$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(9*(a^4*x^4 + 2*a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*
x^2 + c)*a*sqrt(c)*x - c) - 2*(16*a^3*x^3 + 15*a*x - 3*(3*a^4*x^4 + 12*a^2
*x^2 + 8)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^10*c^3*x^4 + 2*a^8*c^3*x^2
+ a^6*c^3)
```

3.240.6 Sympy [F]

$$\int \frac{x^5 \arctan(ax)}{(c + a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**5*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.240.7 Maxima [F]

$$\int \frac{x^5 \arctan(ax)}{(c + a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

3.240.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)}{(c + a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)}{(c + a^2cx^2)^{5/2}} \, dx = \int \frac{x^5 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2 + c)^{5/2}} \, dx$$

input
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.241 $\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

3.241.1 Optimal result

Integrand size = 22, antiderivative size = 308

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{9a^5c(c+a^2cx^2)^{3/2}} - \frac{4}{3a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\arctan(ax)}{3a^2c(c+a^2cx^2)^{3/2}} - \frac{x\arctan(ax)}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{c+a^2cx^2}}$$

output
```
1/9/a^5/c/(a^2*c*x^2+c)^(3/2)-1/3*x^3*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2
)-4/3/a^5/c^2/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)/a^4/c^2/(a^2*c*x^2+c)^(1/2
)-2*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-I*polylog(2,I*(1+I*a*x)^
(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.57

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(-\frac{45}{\sqrt{1+a^2x^2}} - \frac{45ax\arctan(ax)}{\sqrt{1+a^2x^2}} + \cos(3\arctan(ax)) + 36\arctan(ax)\,(\log(1$$

input
```
Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(-45/Sqrt[1 + a^2*x^2] - (45*a*x*ArcTan[a*x])/Sqrt[
1 + a^2*x^2] + Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[
a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (36*I)*(PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) + 3*ArcTan[a*x]*Sin[3*ArcTan[
a*x]]))/(36*a^5*c^3*Sqrt[1 + a^2*x^2])
```

3.241.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5499, 5469, 5425, 5421, 5479, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5469}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c} - \frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c} - \frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}} - \frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c} - \frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \; 5421$$

$$\frac{-\frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \; 5479$$

$$\frac{-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{3}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \; 243$$

$$\frac{-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \; 53$$

$$\frac{-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}}-\frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\left(\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}-\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

3.241. $\int \frac{x^4\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

output
```
-((-1/6*(a*(2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) - 2/(a^4*c^2*Sqrt[c + a^2*c*
x^2]))) + (x^3*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)))/a^2) + (-(1/(a^3*
c*Sqrt[c + a^2*c*x^2])) - (x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^2]) + (S
qrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Pol
yLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a^2*c*Sqrt[c + a^2*c*x^
2]))/(a^2*c)
```

3.241.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.241. $\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.241.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.26

method	result
default	$-\frac{(i+3\arctan(ax))(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^5c^3}-\frac{5(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8c^3a^5(a^2x^2+1)}-\frac{5\sqrt{c(ax-i)(ax+i)}\,(}{8c^3a^5(}$

input
```
int(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^
(1/2)/(a^2*x^2+1)^2/a^5/c^3-5/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x)
)^(1/2)/c^3/a^5/(a^2*x^2+1)-5/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(
a*x)-I)/c^3/a^5/(a^2*x^2+1)-1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1
/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a^5/c^3-(arctan(a*
x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^3/a^5
```

3.241.5 Fricas [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 + c^3), x)
```

3.241.6 Sympy [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.241.7 Maxima [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

3.241.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.242 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

3.242.1 Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{2x}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\arctan(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
1/9*x^3/a/c/(a^2*c*x^2+c)^(3/2)-1/3*x^2*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3
/2)+2/3*x/a^3/c^2/(a^2*c*x^2+c)^(1/2)-2/3*arctan(a*x)/a^4/c^2/(a^2*c*x^2+c
)^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(ax(6+7a^2x^2)-3(2+3a^2x^2)\arctan(ax))}{9a^4c^3\,(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(a*x*(6 + 7*a^2*x^2) - 3*(2 + 3*a^2*x^2)*ArcTan[a*x])
)/(9*a^4*c^3*(1 + a^2*x^2)^2)
```

3.242.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5473, 5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5473}$$

$$\frac{2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*
x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c
+ a^2*c*x^2])))/(3*a^2*c)
```

3.242.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

3.242.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.18

method	result
default	$-\frac{(i+3\arctan(ax))(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^4c^3}-\frac{3(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{8c^3a^4(a^2x^2+1)}+\frac{3\sqrt{c(ax-i)(ax+i)}}{8c^3a}$

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(I+3*arctan(a*x))*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x)
)^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+
a*x))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(a
rctan(a*x)-I)/c^3/a^4/(a^2*x^2+1)+1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^
3-3*a^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan{(ax)})\sqrt{a^2cx^2+c}}{9\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*sqrt(a^2*c*x^2 + c
)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.242.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\mathrm{atan}\,(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan{(ax)}}{9\,(a^6c^2x^2+a^4c^2)\sqrt{a^2x^2+1}\sqrt{c}}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))/((a^6*c^2*x^2 + a^
4*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

3.242.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\, dx = \int \frac{x^3 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\, dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.243 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx$

3.243.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx = -\frac{1}{9a^3c(c+a^2cx^2)^{3/2}} + \frac{1}{3a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3 \arctan(ax)}{3c(c+a^2cx^2)^{3/2}}$$

output
```
-1/9/a^3/c/(a^2*c*x^2+c)^(3/2)+1/3*x^3*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1
/3/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx = \frac{\sqrt{c+a^2cx^2}(2+3a^2x^2+3a^3x^3\arctan(ax))}{9a^3c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(2 + 3*a^2*x^2 + 3*a^3*x^3*ArcTan[a*x]))/(9*a^3*c^3*(
1 + a^2*x^2)^2)
```

3.243.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5479, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{3}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}}-\frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\left(\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}-\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/6*(a*(2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) - 2/(a^4*c^2*Sqrt[c + a^2*c*x^2
])) + (x^3*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2))
```

3.243.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.243.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.12

method	result
default	$\frac{(i+3\arctan(ax))(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^3c^3}+\frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8c^3a^3(a^2x^2+1)}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i}{8c^3a^3(a^2x^2}$

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/72*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(
1/2)/(a^2*x^2+1)^2/a^3/c^3+1/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))
^(1/2)/c^3/a^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a
*x)-I)/c^3/a^3/(a^2*x^2+1)+1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/
2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a^3/c^3
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(3\,a^3x^3 \arctan{(ax)} + 3\,a^2x^2 + 2)\sqrt{a^2cx^2+c}}{9\,(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(3*a^3*x^3*arctan(a*x) + 3*a^2*x^2 + 2)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x
^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

3.243.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}{(ax)}}{\left(c\,(a^2x^2+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{9}\,a\left(\frac{3}{\sqrt{a^2cx^2+c}a^4c^2} - \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}a^4c}\right) + \frac{1}{3}\left(\frac{x}{\sqrt{a^2cx^2+c}a^2c^2} - \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right)\arctan{(ax)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*a*(3/(sqrt(a^2*c*x^2 + c)*a^4*c^2) - 1/((a^2*c*x^2 + c)^(3/2)*a^4*c))
+ 1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*
arctan(a*x)
```

3.243.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.244 $\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

3.244.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{2x}{9ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}}$$

output
```
1/9*x/a/c/(a^2*c*x^2+c)^(3/2)-1/3*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)+2/
9*x/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(3ax+2a^3x^3-3\arctan(ax))}{9c^3\,(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(3*a*x + 2*a^3*x^3 - 3*ArcTan[a*x]))/(9*c^3*(a + a^3*
x^2)^2)
```

3.244.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5465, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}}dx}{3a}-\frac{\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{\frac{2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a}-\frac{\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a}-\frac{\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2]))/(3*a)
- ArcTan[a*x]/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

3.244.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.244.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.09

method	result
default	$\frac{(i+3\arctan(ax))(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^2c^3} - \frac{(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{8c^3a^2(a^2x^2+1)} + \frac{\sqrt{c(ax-i)(ax+i)}\,(iax}{8c^3a^2(a^2}$

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/72*(I+3*arctan(a*x))*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))
^(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a
*x))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(ar
ctan(a*x)-I)/c^3/a^2/(a^2*x^2+1)-1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3
-3*a^2*x^2-3*I*a*x+1)*(-I+3*arctan(a*x))/c^3/a^2/(a^4*x^4+2*a^2*x^2+1)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3 + 3\,ax - 3\,\arctan(ax))\sqrt{a^2cx^2+c}}{9\,(a^6c^3x^4 + 2\,a^4c^3x^2 + a^2c^3)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4 +
 2*a^4*c^3*x^2 + a^2*c^3)
```

3.244.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3+3\,ax-3\,\arctan(ax))\sqrt{a^2x^2+1}\sqrt{c}}{9\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*x^2 + 1)*sqrt(c)/(a^6*c^3
*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

3.244.8 Giac [F]

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

3.245 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$

3.245.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{9ac(c+a^2cx^2)^{3/2}} + \frac{2}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)}{3c(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/9/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+2/3/a/
c^2/(a^2*c*x^2+c)^(1/2)+2/3*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(7+6a^2x^2+(9ax+6a^3x^3)\arctan(ax))}{9ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(7 + 6*a^2*x^2 + (9*a*x + 6*a^3*x^3)*ArcTan[a*x]))/(9
*a*c^3*(1 + a^2*x^2)^2)
```

3.245.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5431, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$
$$\Big\downarrow \text{5431}$$
$$\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}$$
$$\Big\downarrow \text{5429}$$
$$\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
1/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/
2)) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*
x^2])))/(3*c)
```

3.245.3.1 Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.245.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.38

method	result
default	$-\frac{(i+3\arctan(ax))(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2c^3a}+\frac{3(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8a\,c^3(a^2x^2+1)}+\frac{3\sqrt{c(ax-i)(ax+i)}\,(}{8a\,c^3(a}$

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^
(1/2)/(a^2*x^2+1)^2/c^3/a+3/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^
(1/2)/a/c^3/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)
-I)/a/c^3/(a^2*x^2+1)-1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)*(a
^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx=\frac{\sqrt{a^2cx^2+c}(6\,a^2x^2+3\,(2\,a^3x^3+3\,ax)\arctan\,(ax)+7)}{9\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(a^2*c*x^2 + c)*(6*a^2*x^2 + 3*(2*a^3*x^3 + 3*a*x)*arctan(a*x) + 7
)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

3.245.6 Sympy [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{\operatorname{atan}\,(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{9}a\left(\frac{6}{\sqrt{a^2cx^2+c}a^2c^2}+\frac{1}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right) + \frac{1}{3}\left(\frac{2x}{\sqrt{a^2cx^2+c}c^2}+\frac{x}{(a^2cx^2+c)^{\frac{3}{2}}c}\right)\arctan(ax)$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*a*(6/(sqrt(a^2*c*x^2 + c)*a^2*c^2) + 1/((a^2*c*x^2 + c)^(3/2)*a^2*c))
+ 1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan
(a*x)
```

3.245.8 Giac [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)/(c + a^2*c*x^2)^(5/2), x)
```

3.246 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{5/2}} dx$

3.246.1 Optimal result

Integrand size = 22, antiderivative size = 279

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = -\frac{ax}{9c\left(c+a^2cx^2\right)^{3/2}} - \frac{11ax}{9c^2\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)}{3c\left(c+a^2cx^2\right)^{3/2}} + \frac{\arctan(ax)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}$$

output
```
-1/9*a*x/c/(a^2*c*x^2+c)^(3/2)+1/3*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-11/9*
a*x/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-2*arctan(a
*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*
x^2+c)^(1/2)+I*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/
2)/c^2/(a^2*c*x^2+c)^(1/2)-I*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\left(1+a^2x^2\right)^{3/2}\left(-\frac{45ax}{\sqrt{1+a^2x^2}}+\frac{45\arctan(ax)}{\sqrt{1+a^2x^2}}+3\arctan(ax)\cos(3\arctan(ax))+36\arctan\right.}{}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
((1 + a^2*x^2)^(3/2)*((-45*a*x)/Sqrt[1 + a^2*x^2] + (45*ArcTan[a*x])/Sqrt[
1 + a^2*x^2] + 3*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*Log[1 - E
^(I*ArcTan[a*x])] - 36*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (36*I)*Pol
yLog[2, -E^(I*ArcTan[a*x])] - (36*I)*PolyLog[2, E^(I*ArcTan[a*x])] - Sin[3
*ArcTan[a*x]]))/(36*c*(c*(1 + a^2*x^2))^(3/2))
```

3.246.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5501, 5465, 209, 208, 5501, 5465, 208, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx}{c}-a^2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}}dx}{3a}-\frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 208$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5501$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5493$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5489$$

3.246. $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{-a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a}-\frac{\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}\right)+\frac{-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{c\sqrt{a^2cx^2+c}}}{c}}{}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*((x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2]))
/(3*a) - ArcTan[a*x]/(3*a^2*c*(c + a^2*c*x^2)^(3/2)))) + (-(a^2*(x/(a*c*Sq
rt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2]))) + (Sqrt[1 +
 a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*Pol
yLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]
/Sqrt[1 - I*a*x]]))/(c*Sqrt[c + a^2*c*x^2]))/c
```

3.246.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.246.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.33

method	result
default	$-\frac{(i+3\arctan(ax))(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2c^3}+\frac{5(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{8c^3(a^2x^2+1)}-\frac{5\sqrt{c(ax-i)(ax+i}}{8c^3}$

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(I+3*arctan(a*x))*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x)
)^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x)
)^(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*
x)-I)/c^3/(a^2*x^2+1)+1/72*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-
3*I*a*x+1)*(-I+3*arctan(a*x))/c^3/(a^4*x^4+2*a^2*x^2+1)+I*(I*arctan(a*x)*l
n((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^
(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/(a^2*x^
2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

3.246.5 Fricas [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*
a^2*c^3*x^3 + c^3*x), x)
```

3.246.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.246.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.246.8 Giac [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\operatorname{atan}(a\,x)}{x\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.247 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx$

3.247.1 Optimal result

Integrand size = 22, antiderivative size = 158

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=-\frac{a}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{5a}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{5a^2x\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{c^3x}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

output
```
-1/9*a/c/(a^2*c*x^2+c)^(3/2)-1/3*a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-a
*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(5/2)-5/3*a/c^2/(a^2*c*x^2+c)^(1/2
)-5/3*a^2*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-arctan(a*x)*(a^2*c*x^2+c)^
(1/2)/c^3/x
```

3.247.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{-3\sqrt{c+a^2cx^2}(3+12a^2x^2+8a^4x^4)\arctan(ax)+ax\left(-\left((16+15a^2x^2\right)\sqrt{c+a^2cx\right.}{9c^3x\left(1+a^2x\right.}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
(-3*Sqrt[c + a^2*c*x^2]*(3 + 12*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x] + a*x*(-(
(16 + 15*a^2*x^2)*Sqrt[c + a^2*c*x^2]) + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[x]
- 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(9*c^3*
x*(1 + a^2*x^2)^2)
```

3.247.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5501, 5431, 5429, 5501, 5429, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5479$$

$$\frac{\frac{a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

3.247. $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx$

output
```
-(a^2*(1/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x
^2)^(3/2)) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c +
 a^2*c*x^2])))/(3*c))) + (-(a^2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a
*x])/(c*Sqrt[c + a^2*c*x^2]))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x
)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c])/c)/c
```

3.247.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.247.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.06

method	result
default	$-\frac{\left(9\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)a^5x^5-9\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)a^5x^5+24\arctan(ax)\sqrt{a^2x^2+1}\,a^4x^4+15\sqrt{a^2x^2+1}\,a^3x^3+18\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)a^3x}{}$

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(9*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^5*x^5-9*ln((1+I*a*x)/(a^2*x^2+
1)^(1/2)-1)*a^5*x^5+24*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^4*x^4+15*(a^2*x^2+1
)^(1/2)*a^3*x^3+18*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^3*x^3-18*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2)-1)*a^3*x^3+36*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+1
6*(a^2*x^2+1)^(1/2)*a*x+9*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a*x-9*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2)-1)*a*x+9*arctan(a*x)*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)
^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x/c^3/(a^4*x^4+2*a^2*x^2+1)
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx = \frac{9\,(a^5x^5+2\,a^3x^3+ax)\sqrt{c}\log\left(-\frac{a^2cx^2-2\,\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(15\,a^3x^3+16\,ax+3\,(8}{18\,(a^4c^3x^5+2\,a^2c^3x^3+c^3x)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(9*(a^5*x^5 + 2*a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c
*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(15*a^3*x^3 + 16*a*x + 3*(8*a^4*x^4 + 12
*a^2*x^2 + 3)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^4*c^3*x^5 + 2*a^2*c^3*x
^3 + c^3*x)
```

3.247.6 Sympy [F]

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^2\left(c\,(a^2x^2+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.247.7 Maxima [F]

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

3.247.8 Giac [F]

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{5/2}} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{5/2}} \, dx = \int \frac{\operatorname{atan}(a\,x)}{x^2\,(c\,a^2\,x^2 + c)^{5/2}} \, dx$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.248 $\int x^m(c+a^2cx^2)^3 \arctan(ax)\,dx$

3.248.1 Optimal result

Integrand size = 20, antiderivative size = 270

$$\begin{aligned}\int x^m(c+a^2cx^2)^3 \arctan(ax)\,dx &= \frac{c^3x^{1+m}\arctan(ax)}{1+m}+\frac{3a^2c^3x^{3+m}\arctan(ax)}{3+m}\\ &+\frac{3a^4c^3x^{5+m}\arctan(ax)}{5+m}+\frac{a^6c^3x^{7+m}\arctan(ax)}{7+m}\\ &-\frac{ac^3x^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}\\ &-\frac{3a^3c^3x^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}\\ &-\frac{3a^5c^3x^{6+m}\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-a^2x^2\right)}{(5+m)(6+m)}\\ &-\frac{a^7c^3x^{8+m}\,\text{Hypergeometric2F1}\left(1,\frac{8+m}{2},\frac{10+m}{2},-a^2x^2\right)}{(7+m)(8+m)}\end{aligned}$$

output
```
c^3*x^(1+m)*arctan(a*x)/(1+m)+3*a^2*c^3*x^(3+m)*arctan(a*x)/(3+m)+3*a^4*c^
3*x^(5+m)*arctan(a*x)/(5+m)+a^6*c^3*x^(7+m)*arctan(a*x)/(7+m)-a*c^3*x^(2+m
)*hypergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)-3*a^3*c^3*x^(4+m)
*hypergeom([1, 2+1/2*m],[3+1/2*m],-a^2*x^2)/(m^2+7*m+12)-3*a^5*c^3*x^(6+m)
*hypergeom([1, 3+1/2*m],[4+1/2*m],-a^2*x^2)/(5+m)/(6+m)-a^7*c^3*x^(8+m)*hy
pergeom([1, 4+1/2*m],[5+1/2*m],-a^2*x^2)/(7+m)/(8+m)
```

3.248.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.87

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)\,dx = c^3x^{1+m}\left(\frac{\arctan(ax)}{1+m}+\frac{3a^2x^2\arctan(ax)}{3+m}+\frac{3a^4x^4\arctan(ax)}{5+m}+\frac{a^6x^6\arctan(ax)}{7+m}-\frac{a^7x^7\,\text{Hypergeometric2F1}\left(1,4+\frac{m}{2},5+\frac{m}{2},-a^2x^2\right)}{(7+m)(8+m)}-\frac{ax\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{3a^3x^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}-\frac{3a^5x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-a^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
c^3*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (3*a^2*x^2*ArcTan[a*x])/(3 + m) + (3*
a^4*x^4*ArcTan[a*x])/(5 + m) + (a^6*x^6*ArcTan[a*x])/(7 + m) - (a^7*x^7*Hy
pergeometric2F1[1, 4 + m/2, 5 + m/2, -(a^2*x^2)])/((7 + m)*(8 + m)) - (a*x
*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) -
 (3*a^3*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 +
7*m + m^2) - (3*a^5*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x
^2)])/((5 + m)*(6 + m)))
```

3.248.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)\left(a^2cx^2+c\right)^3dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^6c^3x^{m+6}\arctan(ax)+3a^4c^3x^{m+4}\arctan(ax)+3a^2c^3x^{m+2}\arctan(ax)+c^3x^m\arctan(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6c^3x^{m+7}\arctan(ax)}{m+7}+\frac{3a^4c^3x^{m+5}\arctan(ax)}{m+5}+\frac{3a^2c^3x^{m+3}\arctan(ax)}{m+3}-\frac{ac^3x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{a^7c^3x^{m+8}\,\text{Hypergeometric2F1}\left(1,\frac{m+8}{2},\frac{m+10}{2},-a^2x^2\right)}{(m+7)(m+8)}-\frac{3a^5c^3x^{m+6}\,\text{Hypergeometric2F1}\left(1,\frac{m+6}{2},\frac{m+8}{2},-a^2x^2\right)}{(m+5)(m+6)}-\frac{3a^3c^3x^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}+\frac{c^3x^{m+1}\arctan(ax)}{m+1}$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*x^(1 + m)*ArcTan[a*x])/(1 + m) + (3*a^2*c^3*x^(3 + m)*ArcTan[a*x])/(3
 + m) + (3*a^4*c^3*x^(5 + m)*ArcTan[a*x])/(5 + m) + (a^6*c^3*x^(7 + m)*Arc
Tan[a*x])/(7 + m) - (a*c^3*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 +
m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (3*a^3*c^3*x^(4 + m)*Hypergeometric2F
1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (3*a^5*c^3*x^(6
 + m)*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6
+ m)) - (a^7*c^3*x^(8 + m)*Hypergeometric2F1[1, (8 + m)/2, (10 + m)/2, -(a
^2*x^2)])/((7 + m)*(8 + m))
```

3.248.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.248.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 167.30 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.22

method	result
meijerg	$\frac{a^{-m-1}c^3\left(-\frac{4x^m a^m\left(a^6m^3x^6+6a^6m^2x^6+8a^6m\,x^6-a^4m^3x^4-8a^4m^2x^4-12a^4m\,x^4+a^2m^3x^2+10a^2m^2x^2+24a^2m\,x^2-m^3-12m^2-44m-48\right)}{(7+m)m(2+m)(4+m)(6+m)}\right.}{4}$

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^6*m^3*x^6+6*a^6*m^2*x^6+8*a^6*m*x^6-a^4*m^
3*x^4-8*a^4*m^2*x^4-12*a^4*m*x^4+a^2*m^3*x^2+10*a^2*m^2*x^2+24*a^2*m*x^2-m
^3-12*m^2-44*m-48)/(7+m)/m/(2+m)/(4+m)/(6+m)+8*x^(8+m)*a^(8+m)/(14+2*m)/(a
^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(8+m)*x^m*a^m*(-8-m)/(7+m)*LerchPh
i(-a^2*x^2,1,1/2*m))+3/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4
-a^2*m^2*x^2-4*a^2*m*x^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/
(10+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(
-a^2*x^2,1,1/2*m))+3/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2
+m)+8*x^(4+m)*a^(4+m)/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4
+m)*x^m*a^m*(-m-4)/(3+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c^3*(4/(
2+m)*x^m*a^m*(-m-2)/(1+m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arct
an((a^2*x^2)^(1/2))+2*x^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

3.248.5 Fricas [F]

$$\int x^m\left(c+a^2cx^2\right)^3\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)^3x^m\arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*
x), x)
```

3.248.6 Sympy [F]

$$\int x^m\left(c+a^2cx^2\right)^3\arctan(ax)\,dx = c^3\left(\int x^m\,\mathrm{atan}\,(ax)\,dx+\int 3a^2x^2x^m\,\mathrm{atan}\,(ax)\,dx+\int 3a^4x^4x^m\,\mathrm{atan}\,(ax)\,dx+\int a^6x^6x^m\,\mathrm{atan}\,(ax)\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
c**3*(Integral(x**m*atan(a*x), x) + Integral(3*a**2*x**2*x**m*atan(a*x), x
) + Integral(3*a**4*x**4*x**m*atan(a*x), x) + Integral(a**6*x**6*x**m*atan
(a*x), x))
```

3.248.7 Maxima [F]

$$\int x^m\left(c+a^2cx^2\right)^3\arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^3x^m\arctan\,(ax)\;dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
(((a^6*c^3*m^3 + 9*a^6*c^3*m^2 + 23*a^6*c^3*m + 15*a^6*c^3)*x^7 + 3*(a^4*c
^3*m^3 + 11*a^4*c^3*m^2 + 31*a^4*c^3*m + 21*a^4*c^3)*x^5 + 3*(a^2*c^3*m^3
+ 13*a^2*c^3*m^2 + 47*a^2*c^3*m + 35*a^2*c^3)*x^3 + (c^3*m^3 + 15*c^3*m^2
+ 71*c^3*m + 105*c^3)*x)*x^m*arctan(a*x) - (m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)*integrate(((a^7*c^3*m^3 + 9*a^7*c^3*m^2 + 23*a^7*c^3*m + 15*a^7*c^3
)*x^7 + 3*(a^5*c^3*m^3 + 11*a^5*c^3*m^2 + 31*a^5*c^3*m + 21*a^5*c^3)*x^5 +
 3*(a^3*c^3*m^3 + 13*a^3*c^3*m^2 + 47*a^3*c^3*m + 35*a^3*c^3)*x^3 + (a*c^3
*m^3 + 15*a*c^3*m^2 + 71*a*c^3*m + 105*a*c^3)*x)*x^m/(m^4 + 16*m^3 + (a^2*
m^4 + 16*a^2*m^3 + 86*a^2*m^2 + 176*a^2*m + 105*a^2)*x^2 + 86*m^2 + 176*m
+ 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.248.8 Giac [F]

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^3 x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)\,dx = \int x^m\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^3, x)
```

3.249 $\int x^m(c+a^2cx^2)^2 \arctan(ax)\,dx$

3.249.1 Optimal result

Integrand size = 20, antiderivative size = 201

$$\int x^m(c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{c^2x^{1+m}\arctan(ax)}{1+m} + \frac{2a^2c^2x^{3+m}\arctan(ax)}{3+m} + \frac{a^4c^2x^{5+m}\arctan(ax)}{5+m} - \frac{ac^2x^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2} - \frac{2a^3c^2x^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2} - \frac{a^5c^2x^{6+m}\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-a^2x^2\right)}{(5+m)(6+m)}$$

output
```
c^2*x^(1+m)*arctan(a*x)/(1+m)+2*a^2*c^2*x^(3+m)*arctan(a*x)/(3+m)+a^4*c^2*
x^(5+m)*arctan(a*x)/(5+m)-a*c^2*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-
a^2*x^2)/(m^2+3*m+2)-2*a^3*c^2*x^(4+m)*hypergeom([1, 2+1/2*m],[3+1/2*m],-a
^2*x^2)/(m^2+7*m+12)-a^5*c^2*x^(6+m)*hypergeom([1, 3+1/2*m],[4+1/2*m],-a^2
*x^2)/(5+m)/(6+m)
```

3.249.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = c^2x^{1+m}\left(\frac{\arctan(ax)}{1+m}+\frac{2a^2x^2\arctan(ax)}{3+m}+\frac{a^4x^4\arctan(ax)}{5+m}-\frac{ax\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{2a^3x^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}-\frac{a^5x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-a^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
c^2*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (2*a^2*x^2*ArcTan[a*x])/(3 + m) + (a^
4*x^4*ArcTan[a*x])/(5 + m) - (a*x*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*x^3*Hypergeometric2F1[1, (4 + m)/
2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (a^5*x^5*Hypergeometric2F1[1
, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6 + m)))
```

3.249.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \; 5483$$

$$\int \left(a^4c^2x^{m+4}\arctan(ax)+2a^2c^2x^{m+2}\arctan(ax)+c^2x^m\arctan(ax)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^4c^2x^{m+5}\arctan(ax)}{m+5}+\frac{2a^2c^2x^{m+3}\arctan(ax)}{m+3}-\frac{ac^2x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{a^5c^2x^{m+6}\,\text{Hypergeometric2F1}\left(1,\frac{m+6}{2},\frac{m+8}{2},-a^2x^2\right)}{(m+5)(m+6)}-\frac{2a^3c^2x^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}+\frac{c^2x^{m+1}\arctan(ax)}{m+1}$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*x^(1 + m)*ArcTan[a*x])/(1 + m) + (2*a^2*c^2*x^(3 + m)*ArcTan[a*x])/(3
 + m) + (a^4*c^2*x^(5 + m)*ArcTan[a*x])/(5 + m) - (a*c^2*x^(2 + m)*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*
c^2*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12
+ 7*m + m^2) - (a^5*c^2*x^(6 + m)*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/
2, -(a^2*x^2)])/((5 + m)*(6 + m))
```

3.249.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.249.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 57.86 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.87

method	result
meijerg	$\frac{a^{-m-1}c^2\left(-\frac{4x^m a^m\left(a^4m^2x^4+2a^4m\,x^4-a^2m^2x^2-4a^2m\,x^2+m^2+6m+8\right)}{(5+m)m(2+m)(4+m)}+\frac{8x^{6+m}a^{6+m}\arctan\left(\sqrt{a^2x^2}\right)}{(10+2m)\sqrt{a^2x^2}}+\frac{2x^m a^m\,\text{LerchPhi}\left(-a^2x^2,1,\frac{m}{2}\right)}{5+m}\right.}{4}$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(-m-1)*c^2*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4-a^2*m^2*x^2-4*a^2*m*
x^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/(10+2*m)/(a^2*x^2)^(1
/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/
2*a^(-m-1)*c^2*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)
/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-m-4)/(3
+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c^2*(4/(2+m)*x^m*a^m*(-m-2)/(
1+m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2
*x^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

3.249.5 Fricas [F]

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x), x)
```

3.249.6 Sympy [F]

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)\,dx = c^2\left(\int x^m \operatorname{atan}\left(ax\right)dx + \int 2a^2x^2x^m \operatorname{atan}\left(ax\right)dx + \int a^4x^4x^m \operatorname{atan}\left(ax\right)dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
c**2*(Integral(x**m*atan(a*x), x) + Integral(2*a**2*x**2*x**m*atan(a*x), x
) + Integral(a**4*x**4*x**m*atan(a*x), x))
```

3.249.7 Maxima [F]

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
(((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a^2*c^2
*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*x) - (m
^3 + 9*m^2 + 23*m + 15)*integrate(((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*c^2)
*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*a*c^
2*m + 15*a*c^2)*x)*x^m/(m^3 + (a^2*m^3 + 9*a^2*m^2 + 23*a^2*m + 15*a^2)*x^
2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

3.249.8 Giac [F]

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = \int x^m \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^2, x)
```

3.250 $\int x^m(c+a^2cx^2)\arctan(ax)\,dx$

3.250.1 Optimal result

Integrand size = 18, antiderivative size = 124

$$\int x^m(c+a^2cx^2)\arctan(ax)\,dx = \frac{cx^{1+m}\arctan(ax)}{1+m}+\frac{a^2cx^{3+m}\arctan(ax)}{3+m}-\frac{acx^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{a^3cx^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}$$

output
```
c*x^(1+m)*arctan(a*x)/(1+m)+a^2*c*x^(3+m)*arctan(a*x)/(3+m)-a*c*x^(2+m)*hy
pergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)-a^3*c*x^(4+m)*hyperge
om([1, 2+1/2*m],[3+1/2*m],-a^2*x^2)/(m^2+7*m+12)
```

3.250.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x^m(c+a^2cx^2)\arctan(ax)\,dx = cx^{1+m}\left(\left(\frac{1}{1+m}+\frac{a^2x^2}{3+m}\right)\arctan(ax)-\frac{ax\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{a^3x^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}\right)$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
c*x^(1 + m)*(((1 + m)^(-1) + (a^2*x^2)/(3 + m))*ArcTan[a*x] - (a*x*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (a^3*x^
3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2)
)
```

3.250.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5485, 5361, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax) \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^{m+2}\arctan(ax)dx+c\int x^m\arctan(ax)dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{x^{m+3}\arctan(ax)}{m+3}-\frac{a\int \frac{x^{m+3}}{a^2x^2+1}dx}{m+3}\right)+c\left(\frac{x^{m+1}\arctan(ax)}{m+1}-\frac{a\int \frac{x^{m+1}}{a^2x^2+1}dx}{m+1}\right)$$

$$\Big\downarrow \text{278}$$

$$a^2c\left(\frac{x^{m+3}\arctan(ax)}{m+3}-\frac{ax^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{(m+3)(m+4)}\right)+ c\left(\frac{x^{m+1}\arctan(ax)}{m+1}-\frac{ax^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+1)(m+2)}\right)$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
c*((x^(1 + m)*ArcTan[a*x])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[1, (2
+ m)/2, (4 + m)/2, -(a^2*x^2)])/((1 + m)*(2 + m))) + a^2*c*((x^(3 + m)*Arc
Tan[a*x])/(3 + m) - (a*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2
, -(a^2*x^2)])/((3 + m)*(4 + m)))
```

3.250.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.250.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 12.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.79

method	result
meijerg	$\frac{a^{-m-1}c\left(-\frac{4x^m a^m\left(a^2 m\,x^2-m-2\right)}{(3+m)m(2+m)}+\frac{8x^{4+m}a^{4+m}\arctan\left(\sqrt{a^2x^2}\right)}{(6+2m)\sqrt{a^2x^2}}+\frac{2x^m a^m(-m-4)\,\mathrm{LerchPhi}\left(-a^2x^2,1,\frac{m}{2}\right)}{(4+m)(3+m)}\right)}{4}+\frac{a^{-m-1}c\left(\frac{4x^m a^m(-m}{(2+m)(1+m}\right.}{}$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(-m-1)*c*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)
/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-m-4)/(3
+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c*(4/(2+m)*x^m*a^m*(-m-2)/(1+
m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x
^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

3.250.5 Fricas [F]

$$\int x^m (c + a^2 c x^2) \arctan(ax) \, dx = \int (a^2 c x^2 + c) x^m \arctan(ax) \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

3.250.6 Sympy [F]

$$\int x^m (c + a^2 c x^2) \arctan(ax) \, dx = c \left(\int x^m \operatorname{atan}(ax) \, dx + \int a^2 x^2 x^m \operatorname{atan}(ax) \, dx \right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
c*(Integral(x**m*atan(a*x), x) + Integral(a**2*x**2*x**m*atan(a*x), x))
```

3.250.7 Maxima [F]

$$\int x^m (c + a^2 c x^2) \arctan(ax) \, dx = \int (a^2 c x^2 + c) x^m \arctan(ax) \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
(((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x) - (m^2 + 4*m + 3)
*integrate(((a^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m/((a^2*m^2 + 4*a
^2*m + 3*a^2)*x^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

3.250.8 Giac [F]

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)x^m\arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)\,dx=\int x^m\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2), x)
```

3.251 $\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx$

3.251.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)}{c+a^2cx^2},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

3.251.2 Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2), x]
```

3.251.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.251.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.251.4 Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan\left(ax\right)}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

3.251.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.251.6 Sympy [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)/(a**2*x**2 + 1), x)/c
```

3.251.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

3.251.8 Giac [N/A]

Not integrable

Time = 58.95 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x^m \arctan(ax)}{c + a^2cx^2}\, dx = \int \frac{x^m \arctan(ax)}{a^2cx^2 + c}\, dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.251.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c + a^2cx^2}\, dx = \int \frac{x^m \operatorname{atan}(a\,x)}{c\,a^2\,x^2 + c}\, dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2), x)
```

3.252 $\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} \, dx$

3.252.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} \, dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

3.252.2 Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2, x]
```

3.252.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.252.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.252.4 Maple [N/A] (verified)

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(a^2c\,x^2+c)^2}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

3.252.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.252.6 Sympy [N/A]

Not integrable

Time = 4.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}{(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.252.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

3.252.8 Giac [N/A]

Not integrable

Time = 73.96 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.252.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

3.253 $\int x^m (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx$

3.253.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax) \, dx = \text{Int}\left(x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax), x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

3.253.2 Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax) \, dx = \int x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax) \, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x], x]
```

3.253.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
$Aborted
```

3.253.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.253.4 Maple [N/A] (verified)

Not integrable

Time = 0.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

3.253.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arcta
n(a*x), x)
```

3.253.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Timed out
```

3.253.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m*arctan(a*x), x)
```

3.253.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.253.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \int x^m \operatorname{atan}(a\,x)\, \left(c\,a^2\,x^2 + c\right)^{5/2} dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

3.254 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\mathrm{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax),x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

3.254.2 Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x], x]
```

3.254.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)\left(a^2cx^2+c\right)^{3/2}dx$$
$$\Big\downarrow \textcolor{blue}{5560}$$
$$\int x^m \arctan(ax)\left(a^2cx^2+c\right)^{3/2}dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
$Aborted
```

3.254.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.254.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^m \left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

3.254.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} x^m \arctan(ax)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

3.254.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Timed out
```

3.254.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} x^m \arctan(ax)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

3.254.8 Giac [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2)^{3/2} \arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.254.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2)^{3/2} \arctan(ax)\,dx = \int x^m \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2 + c\right)^{3/2}\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

3.255 $\int x^m \sqrt{c+a^2 c x^2} \arctan(ax) \, dx$

3.255.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int x^m \sqrt{c+a^2 c x^2} \arctan(ax) \, dx &= \frac{x^{1+m} \sqrt{c+a^2 c x^2} \arctan(ax)}{2+m} \\ &\quad - \frac{a x^{2+m} \sqrt{c+a^2 c x^2} \, \text{Hypergeometric2F1}\left(1, \frac{3+m}{2}, \frac{4+m}{2}, -a^2 x^2\right)}{(2+m)^2} + \frac{c \text{Int}\left(\frac{x^m \arctan(ax)}{\sqrt{c+a^2 c x^2}}, x\right)}{2+m}\end{aligned}$$

output
```
x^(1+m)*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/(2+m)-a*x^(2+m)*hypergeom([1, 3/2+
1/2*m],[2+1/2*m],-a^2*x^2)*(a^2*c*x^2+c)^(1/2)/(2+m)^2+c*Unintegrable(x^m*
arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)/(2+m)
```

3.255.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax) \, dx = \int x^m \sqrt{c+a^2 c x^2} \arctan(ax) \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x], x]
```

3.255.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5481, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5481}$$

$$\frac{c\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}-\frac{ac\int \frac{x^{m+1}}{\sqrt{a^2cx^2+c}}dx}{m+2}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}$$

$$\Big\downarrow \text{ 279}$$

$$\frac{c\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}-\frac{ac\sqrt{a^2x^2+1}\int \frac{x^{m+1}}{\sqrt{a^2x^2+1}}dx}{(m+2)\sqrt{a^2cx^2+c}}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{c\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}-\frac{ac\sqrt{a^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+2)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{c\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}-\frac{ac\sqrt{a^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+2)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
$Aborted
```

3.255.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.255.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \sqrt{a^{2} c \,x^{2}+c}\, \arctan \left(a x \right)d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
```

3.255.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^m\arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

3.255.6 Sympy [N/A]

Not integrable

Time = 18.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int x^m\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

3.255.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^m\arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

3.255.8 Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.255.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x^m\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

3.256 $\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$

3.256.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.256.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x]
```

3.256.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}} dx$$

input
```
Int[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.256.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.256.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan\left(ax\right)}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.256.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.256.6 Sympy [N/A]

Not integrable

Time = 10.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.256.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.256.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan{(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

3.256.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

3.257 $\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$

3.257.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

3.257.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2), x]
```

3.257.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.257.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.257.4 Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan\left(ax\right)}{\left(a^2 c\, x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

3.257.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
+ c^2), x)
```

3.257.6 Sympy [N/A]

Not integrable

Time = 24.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.257.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

3.257.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

3.257.9 Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

3.258 $\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$

3.258.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\begin{aligned}\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx = &-\frac{cx^2}{180a^2}+\frac{cx^4}{60}+\frac{cx\arctan(ax)}{6a^3}-\frac{cx^3\arctan(ax)}{18a}\\ &-\frac{1}{15}acx^5\arctan(ax)-\frac{c\arctan(ax)^2}{12a^4}+\frac{1}{4}cx^4\arctan(ax)^2\\ &+\frac{1}{6}a^2cx^6\arctan(ax)^2-\frac{7c\log(1+a^2x^2)}{90a^4}\end{aligned}$$

output
```
-1/180*c*x^2/a^2+1/60*c*x^4+1/6*c*x*arctan(a*x)/a^3-1/18*c*x^3*arctan(a*x)
/a-1/15*a*c*x^5*arctan(a*x)-1/12*c*arctan(a*x)^2/a^4+1/4*c*x^4*arctan(a*x)
^2+1/6*a^2*c*x^6*arctan(a*x)^2-7/90*c*ln(a^2*x^2+1)/a^4
```

3.258.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\frac{c(-a^2x^2+3a^4x^4-2ax(-15+5a^2x^2+6a^4x^4)\arctan(ax)+15(-1+3a^4x^4+2a^6x^6)\arctan(ax)^2-14\,\mathrm{lo}}{180a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(-(a^2*x^2) + 3*a^4*x^4 - 2*a*x*(-15 + 5*a^2*x^2 + 6*a^4*x^4)*ArcTan[a*
x] + 15*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^2 - 14*Log[1 + a^2*x^2]))
/(180*a^4)
```

3.258.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 290 vs. 2(124) = 248.

Time = 1.92 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.34, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {5485, 5361, 5451, 5361, 243, 49, 2009, 5451, 5345, 240, 5361, 243, 49, 2009, 5419, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2+c\right) dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5 \arctan(ax)^2dx+c\int x^3 \arctan(ax)^2dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\int \frac{x^6\arctan(ax)}{a^2x^2+1}dx\right)+c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5451}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\int x^4\arctan(ax)dx}{a^2}-\frac{\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int \frac{x^3}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{5}a\int \frac{x^5}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int\frac{x^4}{a^2x^2+1}dx^2}{a^2}-\frac{\int\frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 49}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2(a^2x^2+1)}\right)dx^2}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx^2}{a^2}-\frac{\int\frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\int\frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 5451}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right.\right.$$

$$\Big\downarrow \text{ 5345}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right.\right.$$

$$\Big\downarrow \text{ 240}$$

3.258. $\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right.\right.$$

$$\Big\downarrow \; 5361$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx}{a^2}-\frac{\int x}{}}{a^2}\right.\right.$$

$$\Big\downarrow \; 243$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2}{a^2}-\frac{\int}{}}{a^2}\right.\right.$$

$$\Big\downarrow \; 49$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right.}{a^2}}{a^2}\right.\right.$$

$$\Big\downarrow \; 2009$$

3.258. $\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}}{a^2}\right.\right.$$

$$\Big\downarrow \text{5419}$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$$\Big\downarrow \text{5451}$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$$\Big\downarrow \text{240}$$

3.258. $\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$$\Big\downarrow \; 5419$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
c*((x^4*ArcTan[a*x]^2)/4 - (a*((((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1
+ a^2*x^2]/a^4))/6)/a^2 - (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1
 + a^2*x^2]/(2*a))/a^2)/a^2))/2) + a^2*c*((x^6*ArcTan[a*x]^2)/6 - (a*(((x^
5*ArcTan[a*x])/5 - (a*(-(x^2/a^4) + x^4/(2*a^2) + Log[1 + a^2*x^2]/a^6))/1
0)/a^2 - (((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6)/a
^2 - (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^
2)/a^2)/a^2))/3)
```

3.258.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.258. $\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$

3.258.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)^2a^6x^6}{6}+\frac{c\arctan(ax)^2a^4x^4}{4}-\frac{c\left(\frac{2\arctan(ax)a^5x^5}{5}+\frac{\arctan(ax)x^3a^3}{3}-x\arctan(ax)a+\frac{\arctan(ax)^2}{2}-\frac{a^4x^4}{10}+\frac{a^2x^2}{30}+\frac{7\ln}{}\right.}{6}}{a^4}$
default	$\frac{\frac{c\arctan(ax)^2a^6x^6}{6}+\frac{c\arctan(ax)^2a^4x^4}{4}-\frac{c\left(\frac{2\arctan(ax)a^5x^5}{5}+\frac{\arctan(ax)x^3a^3}{3}-x\arctan(ax)a+\frac{\arctan(ax)^2}{2}-\frac{a^4x^4}{10}+\frac{a^2x^2}{30}+\frac{7\ln}{}\right.}{6}}{a^4}$
parallelrisch	$-\frac{-30c\arctan(ax)^2a^6x^6+12c\arctan(ax)a^5x^5-45c\arctan(ax)^2a^4x^4-3a^4cx^4+10c\arctan(ax)a^3x^3+a^2cx^2-30acx\,\mathrm{arc}}{180a^4}$
parts	$\frac{a^2cx^6\arctan(ax)^2}{6}+\frac{cx^4\arctan(ax)^2}{4}-\frac{c\left(\frac{2a\arctan(ax)x^5}{5}+\frac{\arctan(ax)x^3}{3a}-\frac{\arctan(ax)x}{a^3}+\frac{\arctan(ax)^2}{a^4}-\frac{\frac{3a^4x^4}{2}-\frac{a^2x^2}{2}}{}\right.}{6}$
risch	$-\frac{c(2a^6x^6+3a^4x^4-1)\ln(iax+1)^2}{48a^4}+\frac{c(30a^6x^6\ln(-iax+1)+12ia^5x^5+45x^4\ln(-iax+1)a^4+10ia^3x^3-30iax-15\ln(-}{360a^4}$

input
```
int(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/6*c*arctan(a*x)^2*a^6*x^6+1/4*c*arctan(a*x)^2*a^4*x^4-1/6*c*(2/5*
arctan(a*x)*a^5*x^5+1/3*arctan(a*x)*x^3*a^3-x*arctan(a*x)*a+1/2*arctan(a*x
)^2-1/10*a^4*x^4+1/30*a^2*x^2+7/15*ln(a^2*x^2+1)))
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$$

$$=\frac{3\,a^4cx^4-a^2cx^2+15\,(2\,a^6cx^6+3\,a^4cx^4-c)\arctan\left(ax\right)^2-2\,(6\,a^5cx^5+5\,a^3cx^3-15\,acx)\arctan\left(ax\right)-1}{180\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/180*(3*a^4*c*x^4 - a^2*c*x^2 + 15*(2*a^6*c*x^6 + 3*a^4*c*x^4 - c)*arctan
(a*x)^2 - 2*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x)*arctan(a*x) - 14*c*log(
a^2*x^2 + 1))/a^4
```

3.258.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\begin{cases}\frac{a^2cx^6\operatorname{atan}^2(ax)}{6}-\frac{acx^5\operatorname{atan}(ax)}{15}+\frac{cx^4\operatorname{atan}^2(ax)}{4}+\frac{cx^4}{60}-\frac{cx^3\operatorname{atan}(ax)}{18a}-\frac{cx^2}{180a^2}+\frac{cx\operatorname{atan}(ax)}{6a^3}-\frac{7c\log\left(x^2+\frac{1}{a^2}\right)}{90a^4}-\frac{c\operatorname{atan}^2}{12a} \\ 0\end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
Piecewise((a**2*c*x**6*atan(a*x)**2/6 - a*c*x**5*atan(a*x)/15 + c*x**4*ata
n(a*x)**2/4 + c*x**4/60 - c*x**3*atan(a*x)/(18*a) - c*x**2/(180*a**2) + c*
x*atan(a*x)/(6*a**3) - 7*c*log(x**2 + a**(-2))/(90*a**4) - c*atan(a*x)**2/
(12*a**4), Ne(a, 0)), (0, True))
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\begin{aligned}\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx &= -\frac{1}{90}\,a\left(\frac{6\,a^4cx^5+5\,a^2cx^3-15\,cx}{a^4}+\frac{15\,c\arctan(ax)}{a^5}\right)\arctan(ax)\\ &\quad+\frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan(ax)^2\\ &\quad+\frac{3\,a^4cx^4-a^2cx^2+15\,c\arctan(ax)^2-14\,c\log(a^2x^2+1)}{180\,a^4}\end{aligned}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/90*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)*
arctan(a*x) + 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)^2 + 1/180*(3*a^4*c*
x^4 - a^2*c*x^2 + 15*c*arctan(a*x)^2 - 14*c*log(a^2*x^2 + 1))/a^4
```

3.258.8 Giac [F]

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx = \int (a^2cx^2+c)x^3\arctan(ax)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.258.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx =$$
$$-\frac{c\left(14\ln\left(a^2\,x^2+1\right)+a^2\,x^2-3\,a^4\,x^4+15\,\mathrm{atan}(a\,x)^2+10\,a^3\,x^3\,\mathrm{atan}(a\,x)+12\,a^5\,x^5\,\mathrm{atan}(a\,x)-30\,a\,x\,a}{180\,a^4}$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
-(c*(14*log(a^2*x^2 + 1) + a^2*x^2 - 3*a^4*x^4 + 15*atan(a*x)^2 + 10*a^3*x
^3*atan(a*x) + 12*a^5*x^5*atan(a*x) - 30*a*x*atan(a*x) - 45*a^4*x^4*atan(a
*x)^2 - 30*a^6*x^6*atan(a*x)^2))/(180*a^4)
```

3.259 $\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$

3.259.1 Optimal result

Integrand size = 20, antiderivative size = 156

$$\begin{aligned}\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx &= \frac{cx}{30a^2}+\frac{cx^3}{30}-\frac{c\arctan(ax)}{30a^3}-\frac{2cx^2\arctan(ax)}{15a}\\&\quad-\frac{1}{10}acx^4\arctan(ax)-\frac{2ic\arctan(ax)^2}{15a^3}\\&\quad+\frac{1}{3}cx^3\arctan(ax)^2+\frac{1}{5}a^2cx^5\arctan(ax)^2\\&\quad-\frac{4c\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a^3}-\frac{2ic\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^3}\end{aligned}$$

output
```
1/30*c*x/a^2+1/30*c*x^3-1/30*c*arctan(a*x)/a^3-2/15*c*x^2*arctan(a*x)/a-1/
10*a*c*x^4*arctan(a*x)-2/15*I*c*arctan(a*x)^2/a^3+1/3*c*x^3*arctan(a*x)^2+
1/5*a^2*c*x^5*arctan(a*x)^2-4/15*c*arctan(a*x)*ln(2/(1+I*a*x))/a^3-2/15*I*
c*polylog(2,1-2/(1+I*a*x))/a^3
```

3.259.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

$$\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\frac{c\left(ax+a^3x^3+2(2i+5a^3x^3+3a^5x^5)\arctan(ax)^2-\arctan(ax)\left(1+4a^2x^2+3a^4x^4+8\log\left(1+e^{2i\arctan(\ldots}\right.\right.\right.}{30a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(a*x + a^3*x^3 + 2*(2*I + 5*a^3*x^3 + 3*a^5*x^5)*ArcTan[a*x]^2 - ArcTan
[a*x]*(1 + 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*
I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a^3)
```

3.259.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 318 vs. 2(156) = 312.

Time = 1.49 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5485, 5361, 5451, 5361, 254, 262, 216, 2009, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^4 \arctan(ax)^2dx+c\int x^2 \arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\int \frac{x^5\arctan(ax)}{a^2x^2+1}dx\right)+c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\int \frac{x^3\arctan(ax)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{ 5451}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\int x^3\arctan(ax)dx}{a^2}-\frac{\int \frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 5361}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\frac{x^4}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 254}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 262}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 216}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{ 5451}$$

3.259. $\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)$$

$$\Big\downarrow \ 5361$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan \cdots}{a^2x^2 \cdots}}{a^2}}{a^2}\cdots$$

$$\Big\downarrow \ 262$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\int\cdots}{a^2}\cdots$$

$$\Big\downarrow \ 216$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\int\frac{x \cdots}{\cdots}}{a^2}\cdots$$

$$\Big\downarrow \ 5455$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\int\cdots}{\cdots}}{a^2}\cdots$$

$$\Big\downarrow \ 5379$$

3.259. $\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax}{2a^2}\right.\right.}{a^2}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\right.\right.$$

$$\Big\downarrow \; 2849$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan}{}}{a^2}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\right.\right.$$

$$\Big\downarrow \; 2752$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\right.}{2a}}{a}}{a^2}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

3.259. $\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$

output
```
c*((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan
[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(
1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2))/3) + a^
2*c*((x^5*ArcTan[a*x]^2)/5 - (2*a*(((x^4*ArcTan[a*x])/4 - (a*(-(x/a^4) + x
^3/(3*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*ArcTan[a*x])/2 - (a*(x/a^2
- ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]
*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/
a^2))/5)
```

3.259.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.259. $\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$

3.259.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

method	result
parts	$\frac{a^2 c x^5 \arctan(ax)^2}{5} + \frac{c x^3 \arctan(ax)^2}{3} - \frac{2c\left(\frac{3a\arctan(ax)x^4}{4} + \frac{\arctan(ax)x^2}{a} - \frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{a^3} - \frac{a^3x^3+ax-\arctan}{}\right.}{}$
derivativedivides	$\frac{\frac{c\arctan(ax)^2 a^5 x^5}{5} + \frac{c\arctan(ax)^2 a^3 x^3}{3} - 2c\left(\frac{3\arctan(ax)a^4x^4}{4} + a^2\arctan(ax)x^2 - \arctan(ax)\ln\left(a^2x^2+1\right) - \frac{a^3x^3}{4} - \frac{ax}{4} + \frac{\arctan(ax}{4}\right.}{}$
default	$\frac{\frac{c\arctan(ax)^2 a^5 x^5}{5} + \frac{c\arctan(ax)^2 a^3 x^3}{3} - 2c\left(\frac{3\arctan(ax)a^4x^4}{4} + a^2\arctan(ax)x^2 - \arctan(ax)\ln\left(a^2x^2+1\right) - \frac{a^3x^3}{4} - \frac{ax}{4} + \frac{\arctan(ax}{4}\right.}{}$
risch	$\frac{cx}{30a^2} - \frac{c\arctan(ax)}{30a^3} + \frac{cx^3}{30} - \frac{2ic\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{15a^3} - \frac{c\,a^2\ln(iax+1)^2x^5}{20} - \frac{c\,a^2\ln(-iax+1)^2x^5}{20} - ic\ln($

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*c*x^5*arctan(a*x)^2+1/3*c*x^3*arctan(a*x)^2-2/15*c*(3/4*a*arctan(a
*x)*x^4+1/a*arctan(a*x)*x^2-1/a^3*arctan(a*x)*ln(a^2*x^2+1)-1/4/a^3*(a^3*x
^3+a*x-arctan(a*x)+2*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a
*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dil
og(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.259.5 Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^2, x)
```

3.259.6 Sympy [F]

$$\int x^2 (c + a^2 c x^2) \arctan(ax)^2 \, dx = c \left(\int x^2 \operatorname{atan}^2(ax) \, dx + \int a^2 x^4 \operatorname{atan}^2(ax) \, dx \right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
c*(Integral(x**2*atan(a*x)**2, x) + Integral(a**2*x**4*atan(a*x)**2, x))
```

3.259.7 Maxima [F]

$$\int x^2 (c + a^2 c x^2) \arctan(ax)^2 \, dx = \int (a^2 c x^2 + c) x^2 \arctan(ax)^2 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/60*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^2 - 1/240*(3*a^2*c*x^5 + 5*c*x^3)
*log(a^2*x^2 + 1)^2 + integrate(1/240*(180*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^
2)*arctan(a*x)^2 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*log(a^2*x^2 + 1)^2
 - 8*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x) + 4*(3*a^4*c*x^6 + 5*a^2*c*x^4)
*log(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

3.259.8 Giac [F]

$$\int x^2 (c + a^2 c x^2) \arctan(ax)^2 \, dx = \int (a^2 c x^2 + c) x^2 \arctan(ax)^2 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx = \int x^2\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2), x)
```

3.260 $\int x(c+a^2cx^2)\arctan(ax)^2\,dx$

3.260.1 Optimal result

Integrand size = 18, antiderivative size = 96

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx=\frac{c(1+a^2x^2)}{12a^2}-\frac{cx\arctan(ax)}{3a}-\frac{cx(1+a^2x^2)\arctan(ax)}{6a}+\frac{c(1+a^2x^2)^2\arctan(ax)^2}{4a^2}+\frac{c\log(1+a^2x^2)}{6a^2}$$

output
```
1/12*c*(a^2*x^2+1)/a^2-1/3*c*x*arctan(a*x)/a-1/6*c*x*(a^2*x^2+1)*arctan(a*
x)/a+1/4*c*(a^2*x^2+1)^2*arctan(a*x)^2/a^2+1/6*c*ln(a^2*x^2+1)/a^2
```

3.260.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx=\frac{c\left(a^2x^2-2ax(3+a^2x^2)\arctan(ax)+3(1+a^2x^2)^2\arctan(ax)^2+2\log(1+a^2x^2)\right)}{12a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(a^2*x^2 - 2*a*x*(3 + a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*ArcTan[a
*x]^2 + 2*Log[1 + a^2*x^2]))/(12*a^2)
```

3.260.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5465, 27, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\left(a^2cx^2+c\right)dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{\int c\left(a^2x^2+1\right)\arctan(ax)dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\int\left(a^2x^2+1\right)\arctan(ax)dx}{2a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\left(\frac{2}{3}\int\arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)}{2a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)}{2a}$$

$$\Big\downarrow \text{240}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)}{2a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(4*a^2) - (c*(-1/6*(1 + a^2*x^2)/a + (x*
(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))
)/3))/(2*a)
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.260.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

method	result
parts	$\frac{a^2 c x^4 \arctan(ax)^2}{4} + \frac{c x^2 \arctan(ax)^2}{2} + \frac{c \arctan(ax)^2}{4a^2} - \frac{c\left(\frac{\arctan(ax) x^3 a^3}{3} + x \arctan(ax) a - \frac{a^2 x^2}{6} - \frac{\ln(a^2x^2+1)}{3}\right)}{2a^2}$
derivativedivides	$\frac{\frac{c \arctan(ax)^2 a^4 x^4}{4} + \frac{a^2 c x^2 \arctan(ax)^2}{2} + \frac{c \arctan(ax)^2}{4} - \frac{c\left(\frac{\arctan(ax) x^3 a^3}{3} + x \arctan(ax) a - \frac{a^2 x^2}{6} - \frac{\ln(a^2x^2+1)}{3}\right)}{2}}{a^2}$
default	$\frac{\frac{c \arctan(ax)^2 a^4 x^4}{4} + \frac{a^2 c x^2 \arctan(ax)^2}{2} + \frac{c \arctan(ax)^2}{4} - \frac{c\left(\frac{\arctan(ax) x^3 a^3}{3} + x \arctan(ax) a - \frac{a^2 x^2}{6} - \frac{\ln(a^2x^2+1)}{3}\right)}{2}}{a^2}$
parallelrisch	$\frac{3c \arctan(ax)^2 a^4 x^4 - 2c \arctan(ax) a^3 x^3 + 6a^2 c x^2 \arctan(ax)^2 + a^2 c x^2 - 6acx \arctan(ax) + 3c \arctan(ax)^2 + 2c \ln(a^2x^2+1}{12a^2}$
risch	$-\frac{c(a^2x^2+1)^2 \ln(iax+1)^2}{16a^2} + \frac{c(3x^4 \ln(-iax+1)a^4 + 2ia^3x^3 + 6a^2x^2 \ln(-iax+1) + 6iax + 3\ln(-iax+1)) \ln(iax+1)}{24a^2} -$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*c*x^4*arctan(a*x)^2+1/2*c*x^2*arctan(a*x)^2+1/4*c*arctan(a*x)^2/a^
2-1/2*c/a^2*(1/3*arctan(a*x)*x^3*a^3+x*arctan(a*x)*a-1/6*a^2*x^2-1/3*ln(a^
2*x^2+1))
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int x(c + a^2cx^2) \arctan(ax)^2 \, dx$$
$$= \frac{a^2cx^2 + 3(a^4cx^4 + 2a^2cx^2 + c)\arctan(ax)^2 - 2(a^3cx^3 + 3acx)\arctan(ax) + 2c\log(a^2x^2 + 1)}{12a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/12*(a^2*c*x^2 + 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^2 - 2*(a^3*c
*x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 + 1))/a^2
```

3.260.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\begin{cases}\frac{a^2cx^4\operatorname{atan}^2(ax)}{4}-\frac{acx^3\operatorname{atan}(ax)}{6}+\frac{cx^2\operatorname{atan}^2(ax)}{2}+\frac{cx^2}{12}-\frac{cx\operatorname{atan}(ax)}{2a}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{6a^2}+\frac{c\operatorname{atan}^2(ax)}{4a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
Piecewise((a**2*c*x**4*atan(a*x)**2/4 - a*c*x**3*atan(a*x)/6 + c*x**2*atan
(a*x)**2/2 + c*x**2/12 - c*x*atan(a*x)/(2*a) + c*log(x**2 + a**(-2))/(6*a*
*2) + c*atan(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx=\frac{(a^2cx^2+c)^2\arctan(ax)^2}{4\,a^2c}+\frac{\left(c^2x^2+\frac{2\,c^2\log(a^2x^2+1)}{a^2}\right)a-2\,(a^2c^2x^3+3\,c^2x)\arctan(ax)}{12\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*(a^2*c*x^2 + c)^2*arctan(a*x)^2/(a^2*c) + 1/12*((c^2*x^2 + 2*c^2*log(a
^2*x^2 + 1)/a^2)*a - 2*(a^2*c^2*x^3 + 3*c^2*x)*arctan(a*x))/(a*c)
```

3.260.8 Giac [F]

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx = \int (a^2cx^2+c)x\arctan{(ax)}^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.260.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx = \frac{c\left(6\,x^2\,\mathrm{atan}(a\,x)^2+x^2\right)}{12} + \frac{\frac{c\left(3\,\mathrm{atan}(a\,x)^2+2\,\ln\left(a^2\,x^2+1\right)\right)}{12}-\frac{a\,c\,x\,\mathrm{atan}(a\,x)}{2}}{a^2} + \frac{a^2\,c\,x^4\,\mathrm{atan}(a\,x)^2}{4} - \frac{a\,c\,x^3\,\mathrm{atan}(a\,x)}{6}$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
(c*(6*x^2*atan(a*x)^2 + x^2))/12 + ((c*(2*log(a^2*x^2 + 1) + 3*atan(a*x)^2
))/12 - (a*c*x*atan(a*x))/2)/a^2 + (a^2*c*x^4*atan(a*x)^2)/4 - (a*c*x^3*at
an(a*x))/6
```

3.261 $\int (c+a^2cx^2) \arctan(ax)^2 \, dx$

3.261.1 Optimal result

Integrand size = 17, antiderivative size = 128

$$\int (c+a^2cx^2) \arctan(ax)^2 \, dx = \frac{cx}{3} - \frac{c(1+a^2x^2)\arctan(ax)}{3a} + \frac{2ic\arctan(ax)^2}{3a} + \frac{2}{3}cx\arctan(ax)^2 + \frac{1}{3}cx(1+a^2x^2)\arctan(ax)^2 + \frac{4c\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{3a} + \frac{2ic\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a}$$

output
```
1/3*c*x-1/3*c*(a^2*x^2+1)*arctan(a*x)/a+2/3*I*c*arctan(a*x)^2/a+2/3*c*x*ar
ctan(a*x)^2+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^2+4/3*c*arctan(a*x)*ln(2/(1+I*
a*x))/a+2/3*I*c*polylog(2,1-2/(1+I*a*x))/a
```

3.261.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

$$\int (c+a^2cx^2) \arctan(ax)^2 \, dx = \frac{c\left(ax+(-2i+3ax+a^3x^3)\arctan(ax)^2-\arctan(ax)\left(1+a^2x^2-4\log\left(1+e^{2i\arctan(ax)}\right)\right)-2i\operatorname{PolyLog}\left(\right.\right.}{3a}$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(a*x + (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 - ArcTan[a*x]*(1 + a^2*x^
2 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) - (2*I)*PolyLog[2, -E^((2*I)*ArcTan[
a*x])]))/(3*a)
```

3.261.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2 \left(a^2cx^2+c\right) dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{2}{3}c\int \arctan(ax)^2dx+\frac{c\int 1dx}{3}+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{2}{3}c\int \arctan(ax)^2dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$$\Big\downarrow \text{5345}$$

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$$\Big\downarrow \text{5455}$$

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$$\Big\downarrow \text{5379}$$

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$$\Big\downarrow \text{2849}$$

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+$$
$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$$\Big\downarrow \text{2752}$$

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+$$
$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*x)/3 - (c*(1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (c*x*(1 + a^2*x^2)*ArcTan[
a*x]^2)/3 + (2*c*(x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((
ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/
a)/a)))/3
```

3.261.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.261.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)^2 a^3 x^3}{3}+acx\arctan(ax)^2-\frac{2c\left(\frac{a^2\arctan(ax)x^2}{2}+\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)\right.}{}\right.}{}}{}$
default	$\frac{\frac{c\arctan(ax)^2 a^3 x^3}{3}+acx\arctan(ax)^2-\frac{2c\left(\frac{a^2\arctan(ax)x^2}{2}+\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)\right.}{}\right.}{}}{}$
parts	$\frac{a^2cx^3\arctan(ax)^2}{3}+cx\arctan(ax)^2-\frac{2c\left(\frac{a\arctan(ax)x^2}{2}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{a}-\frac{ax-\arctan(ax)-i\left(\ln(ax-i)\ln\right.}{}\right.}{}$
risch	$\frac{ica\ln(iax+1)x^2}{6}-\frac{c\ln(iax+1)^2x}{4}-\frac{c\ln(-iax+1)^2x}{4}+\frac{cx}{3}-\frac{ic\ln(-iax+1)^2}{6a}-\frac{c\arctan(ax)}{3a}+\frac{2ic\ln\left(\frac{1}{2}+\frac{iax}{2}\right)}{3a}$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/3*c*arctan(a*x)^2*a^3*x^3+a*c*x*arctan(a*x)^2-2/3*c*(1/2*a^2*arctan
(a*x)*x^2+arctan(a*x)*ln(a^2*x^2+1)-1/2*a*x+1/2*arctan(a*x)+1/2*I*(ln(a*x-
I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln
(a*x-I)^2)-1/2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*l
n(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.261.5 Fricas [F]

$$\int\left(c+a^2cx^2\right)\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

3.261.6 Sympy [F]

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx = c\left(\int a^2x^2\operatorname{atan}^2(ax)\,dx + \int \operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**2, x) + Integral(atan(a*x)**2, x))
```

3.261.7 Maxima [F]

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx = \int (a^2cx^2+c)\arctan(ax)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
36*a^4*c*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a^4*c*inte
grate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*integrate(1/
48*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 8*a^3*c*integrate(1/48*x^3*arc
tan(a*x)/(a^2*x^2 + 1), x) + 72*a^2*c*integrate(1/48*x^2*arctan(a*x)^2/(a^
2*x^2 + 1), x) + 6*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 +
1), x) + 12*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) +
1/12*(a^2*c*x^3 + 3*c*x)*arctan(a*x)^2 + 1/4*c*arctan(a*x)^3/a - 24*a*c*in
tegrate(1/48*x*arctan(a*x)/(a^2*x^2 + 1), x) - 1/48*(a^2*c*x^3 + 3*c*x)*lo
g(a^2*x^2 + 1)^2 + 3*c*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

3.261.8 Giac [F]

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx = \int (a^2cx^2+c)\arctan(ax)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int (c + a^2cx^2) \arctan(ax)^2 \, dx = \int \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2), x)
```

3.262 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx$

3.262.1 Optimal result

Integrand size = 20, antiderivative size = 169

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx ={}& -acx\arctan(ax)+\frac{1}{2}c\arctan(ax)^2+\frac{1}{2}a^2cx^2\arctan(ax)^2\\&+2c\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)+\frac{1}{2}c\log\left(1+a^2x^2\right)\\&-ic\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\&+ic\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\&-\frac{1}{2}c\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\&+\frac{1}{2}c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-a*c*x*arctan(a*x)+1/2*c*arctan(a*x)^2+1/2*a^2*c*x^2*arctan(a*x)^2-2*c*arc
tan(a*x)^2*arctanh(-1+2/(1+I*a*x))+1/2*c*ln(a^2*x^2+1)-I*c*arctan(a*x)*pol
ylog(2,1-2/(1+I*a*x))+I*c*arctan(a*x)*polylog(2,-1+2/(1+I*a*x))-1/2*c*poly
log(3,1-2/(1+I*a*x))+1/2*c*polylog(3,-1+2/(1+I*a*x))
```

3.262.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = -acx\arctan(ax)+\frac{1}{2}c(1+a^2x^2)\arctan(ax)^2 + \frac{2}{3}ic\arctan(ax)^3 + c\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) - c\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)+\frac{1}{2}c\log\left(1+a^2x^2\right) + ic\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + ic\arctan(ax)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + \frac{1}{2}c\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - \frac{1}{2}c\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

output
```
-(a*c*x*ArcTan[a*x]) + (c*(1 + a^2*x^2)*ArcTan[a*x]^2)/2 + ((2*I)/3)*c*Arc
Tan[a*x]^3 + c*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - c*ArcTan[a*
x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + (c*Log[1 + a^2*x^2])/2 + I*c*ArcTan[
a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + I*c*ArcTan[a*x]*PolyLog[2, -E^((
2*I)*ArcTan[a*x])] + (c*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 - (c*PolyLog
[3, -E^((2*I)*ArcTan[a*x])])/2
```

3.262.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5357, 5361, 5451, 5345, 240, 5419, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)}{x}\,dx$$

$$\Big\downarrow \ 5485$$

$$a^2c\int x\arctan(ax)^2dx + c\int \frac{\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5357}$$

$$a^2c\int x\arctan(ax)^2dx+c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx\right)+c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5451}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{240}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5419}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

3.262. $\int\frac{(c+a^2cx^2)\arctan(ax)^2}{x}dx$

$$\Big\downarrow \; 5523$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\int\frac{\arctan(ax)\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}\right.\right.$$

$$\Big\downarrow \; 5529$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{i}\right.}{a^2x^2+1}\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}+\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

output
```
a^2*c*((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x]
- Log[1 + a^2*x^2]/(2*a))/a^2)) + c*(2*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*
a*x)] - 4*a*((((I/2)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + PolyLo
g[3, 1 - 2/(1 + I*a*x)]/(4*a))/2 + (((-1/2*I)*ArcTan[a*x]*PolyLog[2, -1 +
2/(1 + I*a*x)])/a - PolyLog[3, -1 + 2/(1 + I*a*x)]/(4*a))/2))
```

3.262. $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx$

3.262.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.262. $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx$

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.262.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.39 (sec) , antiderivative size = 1055, normalized size of antiderivative = 6.24

method	result	size
derivativedivides	Expression too large to display	1055
default	Expression too large to display	1055
parts	Expression too large to display	1545

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*c*x^2*arctan(a*x)^2+c*arctan(a*x)^2*ln(a*x)-c*(arctan(a*x)^2*ln((1
+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+1
/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*ar
ctan(a*x)^2-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-2*polylog(3,(1+I*a*x)/(a^2*x^2+
1)^(1/2))-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arc
tan(a*x)^2+1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))^2*arctan(a*x)^2+ln((1+I*a*x)^2/(a^2*x^2+1)+1)-1/2*arctan(a*x)^
2+arctan(a*x)*(a*x-I)+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*
x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^
2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2
*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2
*I*Pi*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))...
```

3.262.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

3.262.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = c\left(\int \frac{\operatorname{atan}^2(ax)}{x}\,dx + \int a^2x\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x,x)
```

output
```
c*(Integral(atan(a*x)**2/x, x) + Integral(a**2*x*atan(a*x)**2, x))
```

3.262.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
1/8*a^2*c*x^2*arctan(a*x)^2 - 1/32*a^2*c*x^2*log(a^2*x^2 + 1)^2 + 12*a^4*c
*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + a^4*c*integrate(1/16
*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^4*c*integrate(1/16*x^4*log
(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^3*c*integrate(1/16*x^3*arctan(a*x)/(
a^2*x^3 + x), x) + 24*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x)
, x) + 1/48*c*log(a^2*x^2 + 1)^3 + 12*c*integrate(1/16*arctan(a*x)^2/(a^2*
x^3 + x), x) + c*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x)
```

3.262.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x, x)
```

3.263 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx$

3.263.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = &-\frac{c\arctan(ax)^2}{x}+a^2cx\arctan(ax)^2\\&+2ac\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\&+2ac\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\&-iac\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\&+iac\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-c*arctan(a*x)^2/x+a^2*c*x*arctan(a*x)^2+2*a*c*arctan(a*x)*ln(2/(1+I*a*x))
+2*a*c*arctan(a*x)*ln(2-2/(1-I*a*x))-I*a*c*polylog(2,-1+2/(1-I*a*x))+I*a*c
*polylog(2,1-2/(1+I*a*x))
```

3.263.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = c\left(\frac{(-i+ax)^2\arctan(ax)^2}{x} + 2a\arctan(ax)\left(\log\left(1-e^{2i\arctan(ax)}\right) + \log\left(1+e^{2i\arctan(ax)}\right)\right) - ia\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) - ia\,\mathrm{PolyLog}\left(2,e^{2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
c*(((-I + a*x)^2*ArcTan[a*x]^2)/x + 2*a*ArcTan[a*x]*(Log[1 - E^((2*I)*ArcT
an[a*x])] + Log[1 + E^((2*I)*ArcTan[a*x])]) - I*a*PolyLog[2, -E^((2*I)*Arc
Tan[a*x])] - I*a*PolyLog[2, E^((2*I)*ArcTan[a*x])])
```

3.263.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5345, 5361, 5455, 5379, 2849, 2752, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)}{x^2}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \arctan(ax)^2dx + c\int \frac{\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(x\arctan(ax)^2 - 2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx\right) + c\int \frac{\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(x\arctan(ax)^2 - 2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx\right) + c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx - \frac{\arctan(ax)^2}{x}\right)$$

$$\Big\downarrow \text{5455}$$

$$c\left(2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$c\left(2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)$$

$$\Big\downarrow \text{2849}$$

$$c\left(2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)$$

$$\Big\downarrow \text{2752}$$

$$c\left(2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)$$

$$\Big\downarrow \text{5459}$$

$$c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)$$

$$\Big\downarrow \text{5403}$$

3.263. $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx$

$$c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)+$$
$$a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)$$

$$\Big\downarrow \; 2897$$

$$a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+$$
$$c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
c*(-(ArcTan[a*x]^2/x) + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*
Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))) + a^2*c*(x*Ar
cTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 +
 I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))
```

3.263.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.263. $\int\frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx$

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.263. $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.263.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(107) = 214.

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.11

method	result
derivativedivides	$a\left(acx\arctan\left(ax\right)^{2}-\frac{c\arctan(ax)^{2}}{ax}-2c\left(-\arctan\left(ax\right)\ln\left(ax\right)+\arctan\left(ax\right)\ln\left(a^{2}x^{2}+1\right.\right.\right.$
default	$a\left(acx\arctan\left(ax\right)^{2}-\frac{c\arctan(ax)^{2}}{ax}-2c\left(-\arctan\left(ax\right)\ln\left(ax\right)+\arctan\left(ax\right)\ln\left(a^{2}x^{2}+1\right.\right.\right.$
parts	$a^{2}cx\arctan\left(ax\right)^{2}-\frac{c\arctan(ax)^{2}}{x}-2c\left(-a\arctan\left(ax\right)\ln\left(ax\right)+a\arctan\left(ax\right)\ln\left(a^{2}x^{2}+\right.\right.$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(a*c*x*arctan(a*x)^2-c*arctan(a*x)^2/a/x-2*c*(-arctan(a*x)*ln(a*x)+arcta
n(a*x)*ln(a^2*x^2+1)+1/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-
ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-1/2*I*(ln(I+a*x)*ln(a^2*x^2+
1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)-1/2*I
*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/2*I*
dilog(1-I*a*x)))
```

3.263.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

3.263.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = c\left(\int a^2\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x))
```

3.263.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/16*(4*(a^2*c*x^2 - c)*arctan(a*x)^2 - (a^2*c*x^2 - c)*log(a^2*x^2 + 1)^2
 + 8*(24*a^4*c*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 2*a^
4*c*integrate(1/16*x^4*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 8*a^4*c*in
tegrate(1/16*x^4*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + a*c*arctan(a*x)^3
- 16*a^3*c*integrate(1/16*x^3*arctan(a*x)/(a^2*x^4 + x^2), x) + 4*a^2*c*in
tegrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 8*a^2*c*integrat
e(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 16*a*c*integrate(1/16*x*
arctan(a*x)/(a^2*x^4 + x^2), x) + 24*c*integrate(1/16*arctan(a*x)^2/(a^2*x
^4 + x^2), x) + 2*c*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))
*x)/x
```

3.263.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^2, x)
```

3.264 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx$

3.264.1 Optimal result

Integrand size = 20, antiderivative size = 196

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = & -\frac{ac\arctan(ax)}{x}-\frac{1}{2}a^2c\arctan(ax)^2-\frac{c\arctan(ax)^2}{2x^2}\\
& +2a^2c\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)\\
& +a^2c\log(x)-\frac{1}{2}a^2c\log\left(1+a^2x^2\right)\\
& -ia^2c\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\
& +ia^2c\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\
& -\frac{1}{2}a^2c\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\
& +\frac{1}{2}a^2c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-a*c*arctan(a*x)/x-1/2*a^2*c*arctan(a*x)^2-1/2*c*arctan(a*x)^2/x^2-2*a^2*c
*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+a^2*c*ln(x)-1/2*a^2*c*ln(a^2*x^2+1)
-I*a^2*c*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+I*a^2*c*arctan(a*x)*polylog(
2,-1+2/(1+I*a*x))-1/2*a^2*c*polylog(3,1-2/(1+I*a*x))+1/2*a^2*c*polylog(3,-
1+2/(1+I*a*x))
```

3.264.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.12

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = -\frac{ac\arctan(ax)}{x}+\frac{c(-1-a^2x^2)\arctan(ax)^2}{2x^2} + \frac{2}{3}ia^2c\arctan(ax)^3+a^2c\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) - a^2c\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) + a^2c\log(x)-\frac{1}{2}a^2c\log\left(1+a^2x^2\right) + ia^2c\arctan(ax)\,\text{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + ia^2c\arctan(ax)\,\text{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + \frac{1}{2}a^2c\,\text{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - \frac{1}{2}a^2c\,\text{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
-((a*c*ArcTan[a*x])/x) + (c*(-1 - a^2*x^2)*ArcTan[a*x]^2)/(2*x^2) + ((2*I)
/3)*a^2*c*ArcTan[a*x]^3 + a^2*c*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x
])] - a^2*c*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + a^2*c*Log[x] -
(a^2*c*Log[1 + a^2*x^2])/2 + I*a^2*c*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcT
an[a*x])] + I*a^2*c*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + (a^2*
c*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 - (a^2*c*PolyLog[3, -E^((2*I)*ArcT
an[a*x])])/2
```

3.264.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5485, 5357, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)}{x^3}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{x}dx+c\int \frac{\arctan(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{5357}$$

$$c\int \frac{\arctan(ax)^2}{x^3}dx+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(a\int \frac{\arctan(ax)}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5453}$$

$$c\left(a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{243}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{47}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

3.264. $\int \frac{\left(c+a^2cx^2\right)\arctan(ax)^2}{x^3}\,dx$

$$\Big\downarrow \text{14}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{16}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5419}$$

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5523}$$

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\int \frac{\arctan(ax)\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right.}{a^2x^2+1}\right.\right.$$

$$\Big\downarrow \text{5529}$$

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,1-\right.}{a^2x^2+}\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}+\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+}\right.}{4a}\right.\right.\right.$$

3.264. $\int \frac{\left(c+a^2cx^2\right)\arctan(ax)^2}{x^3}\,dx$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
c*(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a
*(Log[x^2] - Log[1 + a^2*x^2]))/2)) + a^2*c*(2*ArcTan[a*x]^2*ArcTanh[1 - 2
/(1 + I*a*x)] - 4*a*((((I/2)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
+ PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a))/2 + (((-1/2*I)*ArcTan[a*x]*PolyLog[
2, -1 + 2/(1 + I*a*x)])/a - PolyLog[3, -1 + 2/(1 + I*a*x)]/(4*a))/2))
```

3.264.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.264. $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx$

3.264.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 45.98 (sec) , antiderivative size = 1134, normalized size of antiderivative = 5.79

method	result	size
derivativedivides	Expression too large to display	1134
default	Expression too large to display	1134
parts	Expression too large to display	1561

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(c*arctan(a*x)^2*ln(a*x)-1/2*c*arctan(a*x)^2/a^2/x^2-c*(arctan(a*x)^2*
ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
+1)+1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^2*arctan(a*x)^2-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*l
n(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(1+I*a*x
)^2/(a^2*x^2+1))+1/2*arctan(a*x)^2+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^
2*x^2+1)^(1/2))-1/2*I*Pi*arctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-1
/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*ar
ctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-1/2*I*Pi*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-1/2*I*Pi*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/
2*arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+1/2*arctan(a*x)*(I*a*x+(a^2*
x^2+1)^(1/2)+1)/a/x+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2
*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+2*I*arctan(a*x)*polylog...
```

3.264.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

3.264.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = c\left(\int \frac{\operatorname{atan}^2{(ax)}}{x^3}\,dx + \int \frac{a^2\operatorname{atan}^2{(ax)}}{x}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x**3,x)
```

output
```
c*(Integral(atan(a*x)**2/x**3, x) + Integral(a**2*atan(a*x)**2/x, x))
```

3.264.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/96*((1152*a^4*c*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^5 + x^3), x) + a
^2*c*log(a^2*x^2 + 1)^3 + 2304*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2
*x^5 + x^3), x) + 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^5
 + x^3), x) - 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^5 + x^3
), x) + 384*a*c*integrate(1/16*x*arctan(a*x)/(a^2*x^5 + x^3), x) + 1152*c*
integrate(1/16*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 96*c*integrate(1/16*log
(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x))*x^2 - 12*c*arctan(a*x)^2 + 3*c*log(a^
2*x^2 + 1)^2)/x^2
```

3.264.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^3, x)
```

3.265 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx$

3.265.1 Optimal result

Integrand size = 20, antiderivative size = 135

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = &-\frac{a^2c}{3x}-\frac{1}{3}a^3c\arctan(ax)-\frac{ac\arctan(ax)}{3x^2}\\ &-\frac{2}{3}ia^3c\arctan(ax)^2-\frac{c\arctan(ax)^2}{3x^3}-\frac{a^2c\arctan(ax)^2}{x}\\ &+\frac{4}{3}a^3c\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\ &-\frac{2}{3}ia^3c\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\end{aligned}$$

output
```
-1/3*a^2*c/x-1/3*a^3*c*arctan(a*x)-1/3*a*c*arctan(a*x)/x^2-2/3*I*a^3*c*arc
tan(a*x)^2-1/3*c*arctan(a*x)^2/x^3-a^2*c*arctan(a*x)^2/x+4/3*a^3*c*arctan(
a*x)*ln(2-2/(1-I*a*x))-2/3*I*a^3*c*polylog(2,-1+2/(1-I*a*x))
```

3.265.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.76

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = \frac{c\left(-a^2x^2+(1-2iax)(-i+ax)^2\arctan(ax)^2+ax\arctan(ax)\left(-1-a^2x^2+4a^2x^2\log\left(1-e^{2i\arctan(ax)}\right)\right)\right)}{3x^3}$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
(c*(-(a^2*x^2) + (1 - (2*I)*a*x)*(-I + a*x)^2*ArcTan[a*x]^2 + a*x*ArcTan[a
*x]*(-1 - a^2*x^2 + 4*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) - (2*I)*a^3*
x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

3.265.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.47, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5485, 5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{x^2}dx+c\int \frac{\arctan(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+c\left(\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{5453}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+c\left(\frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx-a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+c\left(\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx\right)+\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{264}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+$$
$$c\left(\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{216}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+$$
$$c\left(\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{5459}$$

$$c\left(-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right.\right.$$
$$a^2c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$a^2c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)+$$
$$c\left(-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)\right.\right.\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$c\left(-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan\right.\right.\right.\right.$$
$$a^2c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
c*(-1/3*ArcTan[a*x]^2/x^3 + (2*a*(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*A
rcTan[a*x]))/2 - a^2*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 -
 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3) + a^2*c*(-(ArcTa
n[a*x]^2/x) + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/
(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2)))
```

3.265. $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx$

3.265.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.265. $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.265.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(117) = 234$.

Time = 0.75 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.01

method	result
derivativedivides	$a^3\left(-\frac{c\arctan(ax)^2}{3a^3x^3}-\frac{c\arctan(ax)^2}{ax}-\frac{2c\left(\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-2\arctan(ax)\ln(ax)+\frac{\arctan(ax)}{2}+\cdots\right.}{\cdots}\right.$
default	$a^3\left(-\frac{c\arctan(ax)^2}{3a^3x^3}-\frac{c\arctan(ax)^2}{ax}-\frac{2c\left(\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-2\arctan(ax)\ln(ax)+\frac{\arctan(ax)}{2}+\cdots\right.}{\cdots}\right.$
parts	$-\frac{a^2c\arctan(ax)^2}{x}-\frac{c\arctan(ax)^2}{3x^3}-\frac{2c\left(a^3\arctan(ax)\ln(a^2x^2+1)+\frac{a\arctan(ax)}{2x^2}-2a^3\arctan(ax)\ln(ax)+\frac{a^3\left(\frac{1}{ax}+\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3*c*arctan(a*x)^2/a^3/x^3-c*arctan(a*x)^2/a/x-2/3*c*(arctan(a*x)*l
n(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2-2*arctan(a*x)*ln(a*x)+1/2*arctan(a*x)
+1/2/a/x-I*ln(a*x)*ln(1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*di
log(1-I*a*x)+1/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I
)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-1/2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog
(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.265.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

3.265.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = c\left(\int \frac{\operatorname{atan}^2{(ax)}}{x^4}\,dx + \int \frac{a^2\operatorname{atan}^2{(ax)}}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x**4,x)
```

output
```
c*(Integral(atan(a*x)**2/x**4, x) + Integral(a**2*atan(a*x)**2/x**2, x))
```

3.265.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(12*(a^3*c*arctan(a*x)^3 + 12*a^4*c*integrate(1/48*x^4*log(a^2*x^2 +
1)^2/(a^2*x^6 + x^4), x) - 48*a^4*c*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a
^2*x^6 + x^4), x) + 96*a^3*c*integrate(1/48*x^3*arctan(a*x)/(a^2*x^6 + x^4
), x) + 288*a^2*c*integrate(1/48*x^2*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 2
4*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 16*a^2
*c*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 32*a*c*integr
ate(1/48*x*arctan(a*x)/(a^2*x^6 + x^4), x) + 144*c*integrate(1/48*arctan(a
*x)^2/(a^2*x^6 + x^4), x) + 12*c*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^
6 + x^4), x))*x^3 - 4*(3*a^2*c*x^2 + c)*arctan(a*x)^2 + (3*a^2*c*x^2 + c)*
log(a^2*x^2 + 1)^2)/x^3
```

3.265.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx=\int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^4, x)
```

3.266 $\int x^3(c+a^2cx^2)^2 \arctan(ax)^2\,dx$

3.266.1 Optimal result

Integrand size = 22, antiderivative size = 191

$$\begin{aligned}\int x^3(c+a^2cx^2)^2 \arctan(ax)^2\,dx = &-\frac{5c^2x^2}{504a^2}+\frac{c^2x^4}{84}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\arctan(ax)}{12a^3}\\ &-\frac{c^2x^3\arctan(ax)}{36a}-\frac{1}{12}ac^2x^5\arctan(ax)\\ &-\frac{1}{28}a^3c^2x^7\arctan(ax)-\frac{c^2\arctan(ax)^2}{24a^4}\\ &+\frac{1}{4}c^2x^4\arctan(ax)^2+\frac{1}{3}a^2c^2x^6\arctan(ax)^2\\ &+\frac{1}{8}a^4c^2x^8\arctan(ax)^2-\frac{2c^2\log(1+a^2x^2)}{63a^4}\end{aligned}$$

output
```
-5/504*c^2*x^2/a^2+1/84*c^2*x^4+1/168*a^2*c^2*x^6+1/12*c^2*x*arctan(a*x)/a
^3-1/36*c^2*x^3*arctan(a*x)/a-1/12*a*c^2*x^5*arctan(a*x)-1/28*a^3*c^2*x^7*
arctan(a*x)-1/24*c^2*arctan(a*x)^2/a^4+1/4*c^2*x^4*arctan(a*x)^2+1/3*a^2*c
^2*x^6*arctan(a*x)^2+1/8*a^4*c^2*x^8*arctan(a*x)^2-2/63*c^2*ln(a^2*x^2+1)/
a^4
```

3.266.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.58

$$\int x^3\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$$
$$=\frac{c^2\left(-5a^2x^2+6a^4x^4+3a^6x^6-2ax(-21+7a^2x^2+21a^4x^4+9a^6x^6)\arctan(ax)+21(1+a^2x^2)^3\,(-1+3a\right.}{504a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(-5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 - 2*a*x*(-21 + 7*a^2*x^2 + 21*a^4
*x^4 + 9*a^6*x^6)*ArcTan[a*x] + 21*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*ArcTan
[a*x]^2 - 16*Log[1 + a^2*x^2]))/(504*a^4)
```

3.266.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)^2\left(a^2cx^2+c\right)^2\,dx$$
$$\Big\downarrow \; 5483$$
$$\int \left(a^4c^2x^7\arctan(ax)^2+2a^2c^2x^5\arctan(ax)^2+c^2x^3\arctan(ax)^2\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{8}a^4c^2x^8\arctan(ax)^2-\frac{c^2\arctan(ax)^2}{24a^4}-\frac{1}{28}a^3c^2x^7\arctan(ax)+\frac{c^2x\arctan(ax)}{12a^3}+\frac{1}{3}a^2c^2x^6\arctan(ax)^2+\frac{1}{168}a^2c^2x^6-\frac{5c^2x^2}{504a^2}-\frac{2c^2\log\left(a^2x^2+1\right)}{63a^4}-\frac{1}{12}ac^2x^5\arctan(ax)+\frac{1}{4}c^2x^4\arctan(ax)^2-\frac{c^2x^3\arctan(ax)}{36a}+\frac{c^2x^4}{84}$$

input
```
Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(-5*c^2*x^2)/(504*a^2) + (c^2*x^4)/84 + (a^2*c^2*x^6)/168 + (c^2*x*ArcTan[
a*x])/(12*a^3) - (c^2*x^3*ArcTan[a*x])/(36*a) - (a*c^2*x^5*ArcTan[a*x])/12
 - (a^3*c^2*x^7*ArcTan[a*x])/28 - (c^2*ArcTan[a*x]^2)/(24*a^4) + (c^2*x^4*
ArcTan[a*x]^2)/4 + (a^2*c^2*x^6*ArcTan[a*x]^2)/3 + (a^4*c^2*x^8*ArcTan[a*x
]^2)/8 - (2*c^2*Log[1 + a^2*x^2])/(63*a^4)
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.266.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{c^2\arctan(ax)^2a^8x^8}{8}+\frac{c^2\arctan(ax)^2a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^2}{4}-\frac{c^2\left(\frac{3\arctan(ax)a^7x^7}{7}+\arctan(ax)a^5x^5+\frac{\arctan(ax)x^3a^3}{3}-x\,\mathrm{arc}\cdots\right.}{\cdots}}{a^4}$
default	$\frac{\frac{c^2\arctan(ax)^2a^8x^8}{8}+\frac{c^2\arctan(ax)^2a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^2}{4}-\frac{c^2\left(\frac{3\arctan(ax)a^7x^7}{7}+\arctan(ax)a^5x^5+\frac{\arctan(ax)x^3a^3}{3}-x\,\mathrm{arc}\cdots\right.}{\cdots}}{a^4}$
parts	$\frac{a^4c^2x^8\arctan(ax)^2}{8}+\frac{a^2c^2x^6\arctan(ax)^2}{3}+\frac{c^2x^4\arctan(ax)^2}{4}-\frac{c^2\left(\frac{3a^3\arctan(ax)x^7}{7}+a\arctan(ax)x^5+\frac{\arctan(ax)\cdots}{3a}\right.}{\cdots}$
parallelrisch	$-\frac{-63c^2\arctan(ax)^2a^8x^8+18a^7c^2\arctan(ax)x^7-168c^2\arctan(ax)^2a^6x^6-3a^6c^2x^6+42c^2\arctan(ax)a^5x^5-126a^4c^2x^4\cdots}{50\cdots}$
risch	$-\frac{c^2(3a^8x^8+8a^6x^6+6a^4x^4-1)\ln(iax+1)^2}{96a^4}+\frac{c^2(63a^8x^8\ln(-iax+1)+18ia^7x^7+168a^6x^6\ln(-iax+1)+42ia^5x^5+126\cdots}{1008a^4\cdots}$

input
```
int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*c^2*arctan(a*x)^2*a^8*x^8+1/3*c^2*arctan(a*x)^2*a^6*x^6+1/4*a^4
*c^2*x^4*arctan(a*x)^2-1/12*c^2*(3/7*arctan(a*x)*a^7*x^7+arctan(a*x)*a^5*x
^5+1/3*arctan(a*x)*x^3*a^3-x*arctan(a*x)*a+1/2*arctan(a*x)^2-1/14*a^6*x^6-
1/7*a^4*x^4+5/42*a^2*x^2+8/21*ln(a^2*x^2+1)))
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

$$\int x^3(c+a^2cx^2)^2 \arctan(ax)^2\,dx$$
$$= \frac{3\,a^6c^2x^6 + 6\,a^4c^2x^4 - 5\,a^2c^2x^2 + 21\,(3\,a^8c^2x^8 + 8\,a^6c^2x^6 + 6\,a^4c^2x^4 - c^2)\arctan(ax)^2 - 16\,c^2\log(a^2x^2 +}{504\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^2*c^2*x^2 + 21*(3*a^8*c^2*x^8 +
 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 +
 1) - 2*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x)*arct
an(a*x))/a^4
```

3.266.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.97

$$\int x^3(c+a^2cx^2)^2 \arctan(ax)^2\,dx$$
$$= \begin{cases} \frac{a^4c^2x^8\operatorname{atan}^2(ax)}{8} - \frac{a^3c^2x^7\operatorname{atan}(ax)}{28} + \frac{a^2c^2x^6\operatorname{atan}^2(ax)}{3} + \frac{a^2c^2x^6}{168} - \frac{ac^2x^5\operatorname{atan}(ax)}{12} + \frac{c^2x^4\operatorname{atan}^2(ax)}{4} + \frac{c^2x^4}{84} - \frac{c^2x^3\operatorname{atan}(ax}{36a} \\ 0 \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
Piecewise((a**4*c**2*x**8*atan(a*x)**2/8 - a**3*c**2*x**7*atan(a*x)/28 + a
**2*c**2*x**6*atan(a*x)**2/3 + a**2*c**2*x**6/168 - a*c**2*x**5*atan(a*x)/
12 + c**2*x**4*atan(a*x)**2/4 + c**2*x**4/84 - c**2*x**3*atan(a*x)/(36*a)
- 5*c**2*x**2/(504*a**2) + c**2*x*atan(a*x)/(12*a**3) - 2*c**2*log(x**2 +
a**(-2))/(63*a**4) - c**2*atan(a*x)**2/(24*a**4), Ne(a, 0)), (0, True))
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int x^3(c+a^2cx^2)^2\arctan(ax)^2\,dx$$
$$= -\frac{1}{252}a\left(\frac{21\,c^2\arctan(ax)}{a^5}+\frac{9\,a^6c^2x^7+21\,a^4c^2x^5+7\,a^2c^2x^3-21\,c^2x}{a^4}\right)\arctan(ax)$$
$$+\frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan(ax)^2$$
$$+\frac{3\,a^6c^2x^6+6\,a^4c^2x^4-5\,a^2c^2x^2+21\,c^2\arctan(ax)^2-16\,c^2\log(a^2x^2+1)}{504\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/252*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2
*c^2*x^3 - 21*c^2*x)/a^4)*arctan(a*x) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^
6 + 6*c^2*x^4)*arctan(a*x)^2 + 1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^
2*c^2*x^2 + 21*c^2*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 + 1))/a^4
```

3.266.8 Giac [F]

$$\int x^3(c+a^2cx^2)^2\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^2x^3\arctan(ax)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.266.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int x^3(c+a^2cx^2)^2\arctan(ax)^2\,dx=\mathrm{atan}(a\,x)^2\left(\frac{c^2\,x^4}{4}-\frac{c^2}{24\,a^4}+\frac{a^2\,c^2\,x^6}{3}+\frac{a^4\,c^2\,x^8}{8}\right)+\frac{c^2\,x^4}{84}$$
$$-a^2\,\mathrm{atan}(a\,x)\left(\frac{a\,c^2\,x^7}{28}-\frac{c^2\,x}{12\,a^5}+\frac{c^2\,x^5}{12\,a}+\frac{c^2\,x^3}{36\,a^3}\right)$$
$$-\frac{2\,c^2\,\ln(a^2\,x^2+1)}{63\,a^4}-\frac{5\,c^2\,x^2}{504\,a^2}+\frac{a^2\,c^2\,x^6}{168}$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)^2*((c^2*x^4)/4 - c^2/(24*a^4) + (a^2*c^2*x^6)/3 + (a^4*c^2*x^8)/
8) + (c^2*x^4)/84 - a^2*atan(a*x)*((a*c^2*x^7)/28 - (c^2*x)/(12*a^5) + (c^
2*x^5)/(12*a) + (c^2*x^3)/(36*a^3)) - (2*c^2*log(a^2*x^2 + 1))/(63*a^4) -
(5*c^2*x^2)/(504*a^2) + (a^2*c^2*x^6)/168
```

3.266. $\int x^3 (c + a^2 c x^2)^2 \arctan(ax)^2 \, dx$

3.267 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^2 \, dx$

3.267.1 Optimal result

Integrand size = 22, antiderivative size = 225

$$\begin{aligned}\int x^2(c+a^2cx^2)^2 \arctan(ax)^2 \, dx = & -\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{c^2\arctan(ax)}{210a^3} \\ & -\frac{8c^2x^2\arctan(ax)}{105a}-\frac{9}{70}ac^2x^4\arctan(ax) \\ & -\frac{1}{21}a^3c^2x^6\arctan(ax)-\frac{8ic^2\arctan(ax)^2}{105a^3} \\ & +\frac{1}{3}c^2x^3\arctan(ax)^2+\frac{2}{5}a^2c^2x^5\arctan(ax)^2 \\ & +\frac{1}{7}a^4c^2x^7\arctan(ax)^2-\frac{16c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{105a^3} \\ & -\frac{8ic^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{105a^3}\end{aligned}$$

output
```
-1/210*c^2*x/a^2+17/630*c^2*x^3+1/105*a^2*c^2*x^5+1/210*c^2*arctan(a*x)/a^
3-8/105*c^2*x^2*arctan(a*x)/a-9/70*a*c^2*x^4*arctan(a*x)-1/21*a^3*c^2*x^6*
arctan(a*x)-8/105*I*c^2*arctan(a*x)^2/a^3+1/3*c^2*x^3*arctan(a*x)^2+2/5*a^
2*c^2*x^5*arctan(a*x)^2+1/7*a^4*c^2*x^7*arctan(a*x)^2-16/105*c^2*arctan(a*
x)*ln(2/(1+I*a*x))/a^3-8/105*I*c^2*polylog(2,1-2/(1+I*a*x))/a^3
```

3.267.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.59

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$$
$$=\frac{c^2\left(ax(-3+17a^2x^2+6a^4x^4)+6(8i+35a^3x^3+42a^5x^5+15a^7x^7)\arctan(ax)^2-3\arctan(ax)\left(-1+16a\right.\right.}{630a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(a*x*(-3 + 17*a^2*x^2 + 6*a^4*x^4) + 6*(8*I + 35*a^3*x^3 + 42*a^5*x^5
 + 15*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(-1 + 16*a^2*x^2 + 27*a^4*x^4
 + 10*a^6*x^6 + 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (48*I)*PolyLog[2, -E^
((2*I)*ArcTan[a*x])]))/(630*a^3)
```

3.267.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)^2\left(a^2cx^2+c\right)^2\,dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^4c^2x^6\arctan(ax)^2+2a^2c^2x^4\arctan(ax)^2+c^2x^2\arctan(ax)^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{7}a^4c^2x^7\arctan(ax)^2-\frac{1}{21}a^3c^2x^6\arctan(ax)-\frac{8ic^2\arctan(ax)^2}{105a^3}+\frac{c^2\arctan(ax)}{210a^3}-\frac{16c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{105a^3}-\frac{8ic^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{105a^3}+\frac{2}{5}a^2c^2x^5\arctan(ax)^2+\frac{1}{105}a^2c^2x^5-\frac{c^2x}{210a^2}-\frac{9}{70}ac^2x^4\arctan(ax)+\frac{1}{3}c^2x^3\arctan(ax)^2-\frac{8c^2x^2\arctan(ax)}{105a}+\frac{17c^2x^3}{630}$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
-1/210*(c^2*x)/a^2 + (17*c^2*x^3)/630 + (a^2*c^2*x^5)/105 + (c^2*ArcTan[a*
x])/(210*a^3) - (8*c^2*x^2*ArcTan[a*x])/(105*a) - (9*a*c^2*x^4*ArcTan[a*x]
)/70 - (a^3*c^2*x^6*ArcTan[a*x])/21 - (((8*I)/105)*c^2*ArcTan[a*x]^2)/a^3
+ (c^2*x^3*ArcTan[a*x]^2)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^2)/5 + (a^4*c^2*x
^7*ArcTan[a*x]^2)/7 - (16*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(105*a^3) -
(((8*I)/105)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3
```

3.267.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.267.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.19

method	result
parts	$\frac{a^4c^2x^7\arctan(ax)^2}{7}+\frac{2a^2c^2x^5\arctan(ax)^2}{5}+\frac{c^2x^3\arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5a^3\arctan(ax)x^6}{2}+\frac{27a\arctan(ax)x^4}{4}+\frac{4\arcta\cdots}{}\right.}{}$
derivativedivides	$\frac{\frac{c^2\arctan(ax)^2a^7x^7}{7}+\frac{2c^2\arctan(ax)^2a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5a^6\arctan(ax)x^6}{2}+\frac{27\arctan(ax)a^4x^4}{4}+4a^2\arctan(ax)x^2\cdots\right.}{}}{}$
default	$\frac{\frac{c^2\arctan(ax)^2a^7x^7}{7}+\frac{2c^2\arctan(ax)^2a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5a^6\arctan(ax)x^6}{2}+\frac{27\arctan(ax)a^4x^4}{4}+4a^2\arctan(ax)x^2\cdots\right.}{}}{}$
risch	$-\frac{c^2a^2\ln(iax+1)^2x^5}{10}-\frac{c^2a^4\ln(iax+1)^2x^7}{28}-\frac{c^2a^4\ln(-iax+1)^2x^7}{28}-\frac{c^2a^2\ln(-iax+1)^2x^5}{10}-\frac{c^2x}{210a^2}+\frac{a^2c^2x^5}{105}$

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^4*c^2*x^7*arctan(a*x)^2+2/5*a^2*c^2*x^5*arctan(a*x)^2+1/3*c^2*x^3*ar
ctan(a*x)^2-2/105*c^2*(5/2*a^3*arctan(a*x)*x^6+27/4*a*arctan(a*x)*x^4+4/a*
arctan(a*x)*x^2-4/a^3*arctan(a*x)*ln(a^2*x^2+1)-1/4/a^3*(2*a^5*x^5+17/3*a^
3*x^3-a*x+arctan(a*x)+8*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-l
n(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-8*I*(ln(I+a*x)*ln(a^2*x^2+1)-
dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.267.5 Fricas [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2, x)
```

3.267.6 Sympy [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx=c^2\left(\int x^2\operatorname{atan}^2\left(ax\right)dx+\int 2a^2x^4\operatorname{atan}^2\left(ax\right)dx+\int a^4x^6\operatorname{atan}^2\left(ax\right)dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**2, x) + Integral(2*a**2*x**4*atan(a*x)**2,
x) + Integral(a**4*x**6*atan(a*x)**2, x))
```

3.267.7 Maxima [F]

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/420*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^2 - 1/168
0*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*log(a^2*x^2 + 1)^2 + inte
grate(1/1680*(1260*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)
*arctan(a*x)^2 + 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^
2)*log(a^2*x^2 + 1)^2 - 8*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3)
*arctan(a*x) + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*log(a^
2*x^2 + 1))/(a^2*x^2 + 1), x)
```

3.267.8 Giac [F]

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\int x^2\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

3.268 $\int x(c+a^2cx^2)^2 \arctan(ax)^2\,dx$

3.268.1 Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx = \frac{2c^2(1+a^2x^2)}{45a^2}+\frac{c^2(1+a^2x^2)^2}{60a^2}-\frac{8c^2x\arctan(ax)}{45a} - \frac{4c^2x(1+a^2x^2)\arctan(ax)}{45a}-\frac{c^2x(1+a^2x^2)^2\arctan(ax)}{15a} + \frac{c^2(1+a^2x^2)^3\arctan(ax)^2}{6a^2}+\frac{4c^2\log(1+a^2x^2)}{45a^2}$$

output
```
2/45*c^2*(a^2*x^2+1)/a^2+1/60*c^2*(a^2*x^2+1)^2/a^2-8/45*c^2*x*arctan(a*x)
/a-4/45*c^2*x*(a^2*x^2+1)*arctan(a*x)/a-1/15*c^2*x*(a^2*x^2+1)^2*arctan(a*
x)/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^2/a^2+4/45*c^2*ln(a^2*x^2+1)/a^2
```

3.268.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx = \frac{c^2\left(14a^2x^2+3a^4x^4-4ax(15+10a^2x^2+3a^4x^4)\arctan(ax)+30(1+a^2x^2)^3\arctan(ax)^2+16\log\left(1+a^2\right.\right.}{180a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(14*a^2*x^2 + 3*a^4*x^4 - 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan[
a*x] + 30*(1 + a^2*x^2)^3*ArcTan[a*x]^2 + 16*Log[1 + a^2*x^2]))/(180*a^2)
```

3.268.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5465, 27, 5413, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^2 \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \; 5465$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{\int c^2\left(a^2x^2+1\right)^2\arctan(ax)dx}{3a}$$

$$\Big\downarrow \; 27$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\int \left(a^2x^2+1\right)^2\arctan(ax)dx}{3a}$$

$$\Big\downarrow \; 5413$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)}{3a}$$

$$\Big\downarrow \; 5413$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)}{3a}$$

$$\Big\downarrow \; 5345$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\right.}{3a}$$

$$\Big\downarrow \; 240$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)-\frac{(a^2x^2+}{20a}}{3a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(6*a^2) - (c^2*(-1/20*(1 + a^2*x^2)^2/
a + (x*(1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4*(-1/6*(1 + a^2*x^2)/a + (x*(1 +
 a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3)
)/5))/(3*a)
```

3.268.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x]))/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

3.268. $\int x(c+a^2cx^2)^2\arctan(ax)^2\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.268.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a^4c^2x^6\arctan(ax)^2}{6}+\frac{a^2c^2x^4\arctan(ax)^2}{2}+\frac{c^2x^2\arctan(ax)^2}{2}+\frac{c^2\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan(\ldots}{3}\right.}{\ldots}$
derivativedivides	$\frac{\frac{c^2\arctan(ax)^2a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^2}{2}+\frac{a^2c^2x^2\arctan(ax)^2}{2}+\frac{c^2\arctan(ax)^2}{6}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan(ax)x^3a^3}{3}+x\arctan\ldots\right.}{3}}{a^2}$
default	$\frac{\frac{c^2\arctan(ax)^2a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^2}{2}+\frac{a^2c^2x^2\arctan(ax)^2}{2}+\frac{c^2\arctan(ax)^2}{6}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan(ax)x^3a^3}{3}+x\arctan\ldots\right.}{3}}{a^2}$
parallelrisch	$\frac{30c^2\arctan(ax)^2a^6x^6-12c^2\arctan(ax)a^5x^5+90a^4c^2x^4\arctan(ax)^2+3a^4c^2x^4-40a^3c^2x^3\arctan(ax)+90a^2c^2x^2\arctan\ldots}{180a^2}$
risch	$-\frac{c^2(a^2x^2+1)^3\ln(iax+1)^2}{24a^2}+\frac{c^2(15a^6x^6\ln(-iax+1)+6ia^5x^5+45x^4\ln(-iax+1)a^4+20ia^3x^3+45a^2x^2\ln(-iax+1)+\ldots}{180a^2}$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^4*c^2*x^6*arctan(a*x)^2+1/2*a^2*c^2*x^4*arctan(a*x)^2+1/2*c^2*x^2*ar
ctan(a*x)^2+1/6*c^2*arctan(a*x)^2/a^2-1/3*c^2/a^2*(1/5*arctan(a*x)*a^5*x^5
+2/3*arctan(a*x)*x^3*a^3+x*arctan(a*x)*a-1/20*a^4*x^4-7/30*a^2*x^2-4/15*ln
(a^2*x^2+1))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$$
$$=\frac{3\,a^4c^2x^4+14\,a^2c^2x^2+30\,(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2)\arctan{(ax)}^2+16\,c^2\log{(a^2x^2+1)}-4\,(3\,a^5\ldots}{180\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/180*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 30*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 +
3*a^2*c^2*x^2 + c^2)*arctan(a*x)^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5*c^
2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a^2
```

3.268.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int x(c+a^2cx^2)^2 \arctan(ax)^2 \, dx$$
$$= \begin{cases} \frac{a^4c^2x^6 \operatorname{atan}^2(ax)}{6} - \frac{a^3c^2x^5 \operatorname{atan}(ax)}{15} + \frac{a^2c^2x^4 \operatorname{atan}^2(ax)}{2} + \frac{a^2c^2x^4}{60} - \frac{2ac^2x^3 \operatorname{atan}(ax)}{9} + \frac{c^2x^2 \operatorname{atan}^2(ax)}{2} + \frac{7c^2x^2}{90} - \frac{c^2x \operatorname{atan}(a}{3a} \\ 0 \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
Piecewise((a**4*c**2*x**6*atan(a*x)**2/6 - a**3*c**2*x**5*atan(a*x)/15 + a
**2*c**2*x**4*atan(a*x)**2/2 + a**2*c**2*x**4/60 - 2*a*c**2*x**3*atan(a*x)
/9 + c**2*x**2*atan(a*x)**2/2 + 7*c**2*x**2/90 - c**2*x*atan(a*x)/(3*a) +
4*c**2*log(x**2 + a**(-2))/(45*a**2) + c**2*atan(a*x)**2/(6*a**2), Ne(a, 0
)), (0, True))
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.73

$$\int x(c+a^2cx^2)^2 \arctan(ax)^2 \, dx$$
$$= \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{6\,a^2c} + \frac{\left(3\,a^2c^3x^4 + 14\,c^3x^2 + \frac{16\,c^3 \log(a^2x^2+1)}{a^2}\right)a - 4\left(3\,a^4c^3x^5 + 10\,a^2c^3x^3 + 15\,c^3x\right)\arctan(ax)}{180\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/6*(a^2*c*x^2 + c)^3*arctan(a*x)^2/(a^2*c) + 1/180*((3*a^2*c^3*x^4 + 14*c
^3*x^2 + 16*c^3*log(a^2*x^2 + 1)/a^2)*a - 4*(3*a^4*c^3*x^5 + 10*a^2*c^3*x^
3 + 15*c^3*x)*arctan(a*x))/(a*c)
```

3.268.8 Giac [F]

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.268.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\frac{\frac{c^2\left(30\,\mathrm{atan}(a\,x)^2+16\,\ln\left(a^2\,x^2+1\right)\right)}{180}-\frac{a\,c^2\,x\,\mathrm{atan}(a\,x)}{3}}{a^2}+\frac{c^2\,\left(90\,x^2\,\mathrm{atan}(a\,x)^2+14\,x^2\right)}{180}+\frac{a^2\,c^2\,\left(90\,x^4\,\mathrm{atan}(a\,x)^2+3\,x^4\right)}{180}-\frac{a^3\,c^2\,x^5\,\mathrm{atan}(a\,x)}{15}+\frac{a^4\,c^2\,x^6\,\mathrm{atan}(a\,x)^2}{6}-\frac{2\,a\,c^2\,x^3\,\mathrm{atan}(a\,x)}{9}$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
((c^2*(16*log(a^2*x^2 + 1) + 30*atan(a*x)^2))/180 - (a*c^2*x*atan(a*x))/3)
/a^2 + (c^2*(90*x^2*atan(a*x)^2 + 14*x^2))/180 + (a^2*c^2*(90*x^4*atan(a*x
)^2 + 3*x^4))/180 - (a^3*c^2*x^5*atan(a*x))/15 + (a^4*c^2*x^6*atan(a*x)^2)
/6 - (2*a*c^2*x^3*atan(a*x))/9
```

3.269 $\int (c+a^2cx^2)^2 \arctan(ax)^2\,dx$

3.269.1 Optimal result

Integrand size = 19, antiderivative size = 205

$$\begin{aligned}\int (c+a^2cx^2)^2 \arctan(ax)^2\,dx &= \frac{11c^2x}{30}+\frac{1}{30}a^2c^2x^3-\frac{4c^2(1+a^2x^2)\arctan(ax)}{15a}\\ &\quad-\frac{c^2(1+a^2x^2)^2\arctan(ax)}{10a}+\frac{8ic^2\arctan(ax)^2}{15a}\\ &\quad+\frac{8}{15}c^2x\arctan(ax)^2+\frac{4}{15}c^2x(1+a^2x^2)\arctan(ax)^2\\ &\quad+\frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax)^2\\ &\quad+\frac{16c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a}+\frac{8ic^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a}\end{aligned}$$

output
```
11/30*c^2*x+1/30*a^2*c^2*x^3-4/15*c^2*(a^2*x^2+1)*arctan(a*x)/a-1/10*c^2*(
a^2*x^2+1)^2*arctan(a*x)/a+8/15*I*c^2*arctan(a*x)^2/a+8/15*c^2*x*arctan(a*
x)^2+4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^2+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a
*x)^2+16/15*c^2*arctan(a*x)*ln(2/(1+I*a*x))/a+8/15*I*c^2*polylog(2,1-2/(1+
I*a*x))/a
```

3.269.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.55

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx$$
$$= \frac{c^2\left(ax(11+a^2x^2)+2(-8i+15ax+10a^3x^3+3a^5x^5)\arctan(ax)^2-\arctan(ax)\left(11+14a^2x^2+3a^4x^4-3\right.\right.}{30a}$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(a*x*(11 + a^2*x^2) + 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*ArcT
an[a*x]^2 - ArcTan[a*x]*(11 + 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)
*ArcTan[a*x])]) - (16*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a)
```

3.269.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5415, 27, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2\left(a^2cx^2+c\right)^2\,dx$$
$$\Big\downarrow \; 5415$$
$$\frac{4}{5}c\int c(a^2x^2+1)\arctan(ax)^2dx+\frac{1}{10}c\int\left(a^2cx^2+c\right)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}$$
$$\Big\downarrow \; 27$$
$$\frac{4}{5}c^2\int(a^2x^2+1)\arctan(ax)^2dx+\frac{1}{10}c\int\left(a^2cx^2+c\right)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}$$
$$\Big\downarrow \; 2009$$

$$\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \ 5415$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \ 24$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \ 5345$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \ 5455$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\text{arct}}{3a}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \ 5379$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{arctan}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \ 2849$$

3.269. $\int \left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{arc}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \; 2752$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{ar}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c*(c*x + (a^2*c*x^3)/3))/10 - (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) +
(c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/5 + (4*c^2*(x/3 - ((1 + a^2*x^2)*Arc
Tan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a*x]^2
- 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a
+ ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)))/3))/5
```

3.269.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

3.269. $\int\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.269. $\int (c + a^2cx^2)^2 \arctan(ax)^2 \, dx$

3.269.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{c^2\arctan(ax)^2a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)^2}{3}+a\,c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3\arctan(ax)a^4x^4}{4}+\frac{7a^2\arctan(ax)x^2}{2}+4\arctan(ax)\ln\left(a^2x\right.\cdots\right.}{\cdots}$
default	$\frac{c^2\arctan(ax)^2a^5x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)^2}{3}+a\,c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3\arctan(ax)a^4x^4}{4}+\frac{7a^2\arctan(ax)x^2}{2}+4\arctan(ax)\ln\left(a^2x\right.\cdots\right.}{\cdots}$
parts	$\frac{a^4c^2x^5\arctan(ax)^2}{5}+\frac{2a^2c^2x^3\arctan(ax)^2}{3}+c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3a^3\arctan(ax)x^4}{4}+\frac{7a\arctan(ax)x^2}{2}+\frac{4a\cdots}{}\right.}{\cdots}$
risch	$\frac{7ic^2a\ln(iax+1)x^2}{30}+\frac{c^2\ln(iax+1)\ln(-iax+1)x}{2}+\frac{a^2c^2x^3}{30}+\frac{11c^2x}{30}-\frac{8ic^2\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{15a}+\frac{ic^2a^3\ln(i\cdots}{2\cdots}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*c^2*arctan(a*x)^2*a^5*x^5+2/3*a^3*c^2*x^3*arctan(a*x)^2+a*c^2*x*a
rctan(a*x)^2-2/15*c^2*(3/4*arctan(a*x)*a^4*x^4+7/2*a^2*arctan(a*x)*x^2+4*a
rctan(a*x)*ln(a^2*x^2+1)-1/4*a^3*x^3-11/4*a*x+11/4*arctan(a*x)+2*I*(ln(a*x
-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*l
n(a*x-I)^2)-2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln
(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.269.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2, x)
```

3.269.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = c^2\left(\int 2a^2x^2\,\mathrm{atan}^2\,(ax)\,dx + \int a^4x^4\,\mathrm{atan}^2\,(ax)\,dx + \int \mathrm{atan}^2\,(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**2, x) + Integral(a**4*x**4*atan(a*x)
**2, x) + Integral(atan(a*x)**2, x))
```

3.269.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
180*a^6*c^2*integrate(1/240*x^6*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*a^6*c
^2*integrate(1/240*x^6*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^6*c^2*i
ntegrate(1/240*x^6*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 24*a^5*c^2*integra
te(1/240*x^5*arctan(a*x)/(a^2*x^2 + 1), x) + 540*a^4*c^2*integrate(1/240*x
^4*arctan(a*x)^2/(a^2*x^2 + 1), x) + 45*a^4*c^2*integrate(1/240*x^4*log(a^
2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c^2*integrate(1/240*x^4*log(a^2*x^
2 + 1)/(a^2*x^2 + 1), x) - 80*a^3*c^2*integrate(1/240*x^3*arctan(a*x)/(a^2
*x^2 + 1), x) + 540*a^2*c^2*integrate(1/240*x^2*arctan(a*x)^2/(a^2*x^2 + 1
), x) + 45*a^2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 60*a^2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/
4*c^2*arctan(a*x)^3/a - 120*a*c^2*integrate(1/240*x*arctan(a*x)/(a^2*x^2 +
 1), x) + 1/60*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)^2 +
 15*c^2*integrate(1/240*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/240*(3*a^
4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*log(a^2*x^2 + 1)^2
```

3.269.8 Giac [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2 \, dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^2 \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2 \, dx = \int \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^2 dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

3.270 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x} \, dx$

3.270.1 Optimal result

Integrand size = 22, antiderivative size = 235

$$\begin{aligned}
\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x} \, dx = {} & \frac{1}{12}a^2c^2x^2 - \frac{3}{2}ac^2x\arctan(ax) \\
& - \frac{1}{6}a^3c^2x^3\arctan(ax) + \frac{3}{4}c^2\arctan(ax)^2 \\
& + a^2c^2x^2\arctan(ax)^2 + \frac{1}{4}a^4c^2x^4\arctan(ax)^2 \\
& + 2c^2\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) \\
& + \frac{2}{3}c^2\log\left(1+a^2x^2\right) \\
& - ic^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
& + ic^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) \\
& - \frac{1}{2}c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) \\
& + \frac{1}{2}c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)
\end{aligned}$$

output
```
1/12*a^2*c^2*x^2-3/2*a*c^2*x*arctan(a*x)-1/6*a^3*c^2*x^3*arctan(a*x)+3/4*c
^2*arctan(a*x)^2+a^2*c^2*x^2*arctan(a*x)^2+1/4*a^4*c^2*x^4*arctan(a*x)^2-2
*c^2*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+2/3*c^2*ln(a^2*x^2+1)-I*c^2*arc
tan(a*x)*polylog(2,1-2/(1+I*a*x))+I*c^2*arctan(a*x)*polylog(2,-1+2/(1+I*a*
x))-1/2*c^2*polylog(3,1-2/(1+I*a*x))+1/2*c^2*polylog(3,-1+2/(1+I*a*x))
```

3.270.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x}\,dx = \frac{1}{24}c^2\big(2-i\pi^3+2a^2x^2-36ax\arctan(ax)-4a^3x^3\arctan(ax) +18\arctan(ax)^2+24a^2x^2\arctan(ax)^2+6a^4x^4\arctan(ax)^2 +16i\arctan(ax)^3+24\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) -24\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)+16\log\left(1+a^2x^2\right) +24i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) +24i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) +12\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) -12\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\big)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```

output
```
(c^2*(2 - I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcTan[a*x] - 4*a^3*x^3*ArcTan[a*x]
+ 18*ArcTan[a*x]^2 + 24*a^2*x^2*ArcTan[a*x]^2 + 6*a^4*x^4*ArcTan[a*x]^2 +
(16*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] -
24*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 16*Log[1 + a^2*x^2] + (2
4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*P
olyLog[2, -E^((2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[a*x])]
- 12*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/24
```

3.270.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)^2}{x}\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^3\arctan(ax)^2+2a^2c^2x\arctan(ax)^2+\frac{c^2\arctan(ax)^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^4c^2x^4\arctan(ax)^2-\frac{1}{6}a^3c^2x^3\arctan(ax)+a^2c^2x^2\arctan(ax)^2+\frac{1}{12}a^2c^2x^2+\frac{2}{3}c^2\log\left(a^2x^2+1\right)+ 2c^2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-ic^2\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+ ic^2\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{3}{2}ac^2x\arctan(ax)+\frac{3}{4}c^2\arctan(ax)^2- \frac{1}{2}c^2\,\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{1}{2}c^2\,\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```

output
```
(a^2*c^2*x^2)/12 - (3*a*c^2*x*ArcTan[a*x])/2 - (a^3*c^2*x^3*ArcTan[a*x])/6
 + (3*c^2*ArcTan[a*x]^2)/4 + a^2*c^2*x^2*ArcTan[a*x]^2 + (a^4*c^2*x^4*ArcT
an[a*x]^2)/4 + 2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (2*c^2*Log
[1 + a^2*x^2])/3 - I*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c^2
*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^2*PolyLog[3, 1 - 2/(1 + I
*a*x)])/2 + (c^2*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

3.270.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.270.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 50.12 (sec) , antiderivative size = 1185, normalized size of antiderivative = 5.04

method	result	size
derivativedivides	Expression too large to display	1185
default	Expression too large to display	1185
parts	Expression too large to display	1696

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^4*c^2*x^4*arctan(a*x)^2+a^2*c^2*x^2*arctan(a*x)^2+c^2*arctan(a*x)^2*
ln(a*x)-1/2*c^2*(-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*
x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))*arctan(a*x)^2+1/3*arctan(a*x)*(a*x-I)^3-3/2*arctan(a*x)^2+2*arc
tan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-4*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*Pi*csgn(((1+I*a*x)^2/(
a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-I*Pi*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+I*Pi
*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*
x)^2-2*I*arctan(a*x)*(a*x-I)*(I+a*x)+polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+8
/3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))*arctan(a*x)^2-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+2*arctan(a*x)*(a*x
-I)+1/3*I*(I+a*x)-1/6*(I+a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(
a^2*x^2+1)+1))^2*arctan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x
)^2+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-arctan(a*x)*(a*x-I)^...
```

3.270.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^2\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x, x)
```

3.270.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x}\,dx = c^2\left(\int \frac{\operatorname{atan}^2(ax)}{x}\,dx + \int 2a^2x\operatorname{atan}^2(ax)\,dx + \int a^4x^3\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x,x)
```

output
```
c**2*(Integral(atan(a*x)**2/x, x) + Integral(2*a**2*x*atan(a*x)**2, x) + I
ntegral(a**4*x**3*atan(a*x)**2, x))
```

3.270.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
12*a^6*c^2*integrate(1/16*x^6*arctan(a*x)^2/(a^2*x^3 + x), x) + a^6*c^2*in
tegrate(1/16*x^6*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + a^6*c^2*integrate(
1/16*x^6*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 2*a^5*c^2*integrate(1/16*x^5
*arctan(a*x)/(a^2*x^3 + x), x) + 36*a^4*c^2*integrate(1/16*x^4*arctan(a*x)
^2/(a^2*x^3 + x), x) + 3*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)^2/(a^
2*x^3 + x), x) + 4*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)/(a^2*x^3 +
x), x) - 8*a^3*c^2*integrate(1/16*x^3*arctan(a*x)/(a^2*x^3 + x), x) + 36*a
^2*c^2*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x), x) + 1/32*c^2*log(a
^2*x^2 + 1)^3 + 1/16*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^2 + 12*c^2*
integrate(1/16*arctan(a*x)^2/(a^2*x^3 + x), x) + c^2*integrate(1/16*log(a^
2*x^2 + 1)^2/(a^2*x^3 + x), x) - 1/64*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*log(a^
2*x^2 + 1)^2
```

3.270.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan{(ax)}^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,{(c\,a^2\,x^2+c)}^2}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x, x)
```

3.271 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} dx$

3.271.1 Optimal result

Integrand size = 22, antiderivative size = 205

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} dx &= \frac{1}{3}a^2c^2x - \frac{1}{3}ac^2\arctan(ax) - \frac{1}{3}a^3c^2x^2\arctan(ax) \\ &\quad + \frac{2}{3}iac^2\arctan(ax)^2 - \frac{c^2\arctan(ax)^2}{x} + 2a^2c^2x\arctan(ax)^2 \\ &\quad + \frac{1}{3}a^4c^2x^3\arctan(ax)^2 + \frac{10}{3}ac^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) \\ &\quad + 2ac^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) \\ &\quad - iac^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ &\quad + \frac{5}{3}iac^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
1/3*a^2*c^2*x-1/3*a*c^2*arctan(a*x)-1/3*a^3*c^2*x^2*arctan(a*x)+2/3*I*a*c^
2*arctan(a*x)^2-c^2*arctan(a*x)^2/x+2*a^2*c^2*x*arctan(a*x)^2+1/3*a^4*c^2*
x^3*arctan(a*x)^2+10/3*a*c^2*arctan(a*x)*ln(2/(1+I*a*x))+2*a*c^2*arctan(a*
x)*ln(2-2/(1-I*a*x))-I*a*c^2*polylog(2,-1+2/(1-I*a*x))+5/3*I*a*c^2*polylog
(2,1-2/(1+I*a*x))
```

3.271.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} dx$$
$$= \frac{c^2(a^2x^2 - ax\arctan(ax) - a^3x^3\arctan(ax) - 3\arctan(ax)^2 - 8iax\arctan(ax)^2 + 6a^2x^2\arctan(ax)^2 + a^4}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c^2*(a^2*x^2 - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - 3*ArcTan[a*x]^2 -
(8*I)*a*x*ArcTan[a*x]^2 + 6*a^2*x^2*ArcTan[a*x]^2 + a^4*x^4*ArcTan[a*x]^2
+ 6*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 10*a*x*ArcTan[a*x]*Lo
g[1 + E^((2*I)*ArcTan[a*x])] - (5*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a*x])
] - (3*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x)
```

3.271.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 (a^2cx^2+c)^2}{x^2} dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^4c^2x^2\arctan(ax)^2 + 2a^2c^2\arctan(ax)^2 + \frac{c^2\arctan(ax)^2}{x^2}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}a^4c^2x^3\arctan(ax)^2 - \frac{1}{3}a^3c^2x^2\arctan(ax) + 2a^2c^2x\arctan(ax)^2 + \frac{1}{3}a^2c^2x + \frac{2}{3}iac^2\arctan(ax)^2 - \frac{1}{3}ac^2\arctan(ax) - \frac{c^2\arctan(ax)^2}{x} + \frac{10}{3}ac^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) + 2ac^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - iac^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right) + \frac{5}{3}iac^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```

output
```
(a^2*c^2*x)/3 - (a*c^2*ArcTan[a*x])/3 - (a^3*c^2*x^2*ArcTan[a*x])/3 + ((2*
I)/3)*a*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/x + 2*a^2*c^2*x*ArcTan[a*x
]^2 + (a^4*c^2*x^3*ArcTan[a*x]^2)/3 + (10*a*c^2*ArcTan[a*x]*Log[2/(1 + I*a
*x)])/3 + 2*a*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^2*PolyLog[2,
-1 + 2/(1 - I*a*x)] + ((5*I)/3)*a*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

3.271.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.271.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.39

method	result
derivativedivides	$a\left(\frac{a^3c^2x^3\arctan(ax)^2}{3}+2a\,c^2x\arctan(ax)^2-\frac{c^2\arctan(ax)^2}{ax}-\frac{2c^2\left(\frac{a^2\arctan(ax)x^2}{2}-3\arctan(ax)\ln(ax)\right.}{}\right.$
default	$a\left(\frac{a^3c^2x^3\arctan(ax)^2}{3}+2a\,c^2x\arctan(ax)^2-\frac{c^2\arctan(ax)^2}{ax}-\frac{2c^2\left(\frac{a^2\arctan(ax)x^2}{2}-3\arctan(ax)\ln(ax)\right.}{}\right.$
parts	$\frac{a^4c^2x^3\arctan(ax)^2}{3}+2a^2c^2x\arctan(ax)^2-\frac{c^2\arctan(ax)^2}{x}-\frac{2c^2\left(\frac{\arctan(ax)x^2a^3}{2}-3a\arctan(ax)\ln(ax)+\right.}{}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*a^3*c^2*x^3*arctan(a*x)^2+2*a*c^2*x*arctan(a*x)^2-c^2*arctan(a*x)^2
/a/x-2/3*c^2*(1/2*a^2*arctan(a*x)*x^2-3*arctan(a*x)*ln(a*x)+4*arctan(a*x)*
ln(a^2*x^2+1)-1/2*a*x+1/2*arctan(a*x)-3/2*I*ln(a*x)*ln(1+I*a*x)+3/2*I*ln(a
*x)*ln(1-I*a*x)-3/2*I*dilog(1+I*a*x)+3/2*I*dilog(1-I*a*x)+2*I*(ln(a*x-I)*l
n(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x
-I)^2)-2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*
I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.271.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^2\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^2, x)
```

3.271.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^2}\,dx = c^2\left(\int 2a^2\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^2}\,dx + \int a^4x^2\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) +
Integral(a**4*x**2*atan(a*x)**2, x))
```

3.271.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/48*(4*(a^4*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)^2 - (a^4*c^2*x^4
 + 6*a^2*c^2*x^2 - 3*c^2)*log(a^2*x^2 + 1)^2 + 12*(144*a^6*c^2*integrate(1
/48*x^6*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 12*a^6*c^2*integrate(1/48*x^6*
log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 16*a^6*c^2*integrate(1/48*x^6*log
(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 32*a^5*c^2*integrate(1/48*x^5*arctan(a
*x)/(a^2*x^4 + x^2), x) + 432*a^4*c^2*integrate(1/48*x^4*arctan(a*x)^2/(a^
2*x^4 + x^2), x) + 36*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x
^4 + x^2), x) + 96*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^4 +
x^2), x) + 3*a*c^2*arctan(a*x)^3 - 192*a^3*c^2*integrate(1/48*x^3*arctan(a
*x)/(a^2*x^4 + x^2), x) + 36*a^2*c^2*integrate(1/48*x^2*log(a^2*x^2 + 1)^2
/(a^2*x^4 + x^2), x) - 48*a^2*c^2*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2
*x^4 + x^2), x) + 96*a*c^2*integrate(1/48*x*arctan(a*x)/(a^2*x^4 + x^2), x
) + 144*c^2*integrate(1/48*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 12*c^2*inte
grate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

3.271.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} \, dx = \int \frac{\operatorname{atan}(a\,x)^2 \,(c\,a^2\,x^2+c)^2}{x^2} \, dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^2, x)
```

3.272 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} dx$

3.272.1 Optimal result

Integrand size = 22, antiderivative size = 207

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} dx = & -\frac{ac^2 \arctan(ax)}{x} - a^3c^2x \arctan(ax) \\ & -\frac{c^2 \arctan(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2 \arctan(ax)^2 \\ & + 4a^2c^2 \arctan(ax)^2 \operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) + a^2c^2 \log(x) \\ & - 2ia^2c^2 \arctan(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right) \\ & + 2ia^2c^2 \arctan(ax) \operatorname{PolyLog}\left(2, -1+\frac{2}{1+iax}\right) \\ & - a^2c^2 \operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right) \\ & + a^2c^2 \operatorname{PolyLog}\left(3, -1+\frac{2}{1+iax}\right)\end{aligned}$$

output

```
-a*c^2*arctan(a*x)/x-a^3*c^2*x*arctan(a*x)-1/2*c^2*arctan(a*x)^2/x^2+1/2*a
^4*c^2*x^2*arctan(a*x)^2-4*a^2*c^2*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+a
^2*c^2*ln(x)-2*I*a^2*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+2*I*a^2*c^2*
arctan(a*x)*polylog(2,-1+2/(1+I*a*x))-a^2*c^2*polylog(3,1-2/(1+I*a*x))+a^2
*c^2*polylog(3,-1+2/(1+I*a*x))
```

3.272.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} \, dx = a^2c^2\left(-\frac{i\pi^3}{12} - \frac{\arctan(ax)}{ax} - ax\arctan(ax) - \frac{\arctan(ax)^2}{2a^2x^2} + \frac{1}{2}a^2x^2\arctan(ax)^2 + \frac{4}{3}i\arctan(ax)^3 + 2\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) - 2\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) + \log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) + \frac{1}{2}\log\left(1+a^2x^2\right) + 2i\arctan(ax)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + 2i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + \mathrm{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - \mathrm{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```

output
```
a^2*c^2*((-1/12*I)*Pi^3 - ArcTan[a*x]/(a*x) - a*x*ArcTan[a*x] - ArcTan[a*x
]^2/(2*a^2*x^2) + (a^2*x^2*ArcTan[a*x]^2)/2 + ((4*I)/3)*ArcTan[a*x]^3 + 2*
ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - 2*ArcTan[a*x]^2*Log[1 + E^
((2*I)*ArcTan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] + Log[1 + a^2*x^2]/2 +
 (2*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*
PolyLog[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, E^((-2*I)*ArcTan[a*x])] -
PolyLog[3, -E^((2*I)*ArcTan[a*x])])
```

3.272.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^2}{x^3} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x\arctan(ax)^2+\frac{2a^2c^2\arctan(ax)^2}{x}+\frac{c^2\arctan(ax)^2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^4c^2x^2\arctan(ax)^2-a^3c^2x\arctan(ax)+4a^2c^2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-2ia^2c^2\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+2ia^2c^2\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-a^2c^2\,\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+a^2c^2\,\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)+a^2c^2\log(x)-\frac{c^2\arctan(ax)^2}{2x^2}-\frac{ac^2\arctan(ax)}{x}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```

output
```
-((a*c^2*ArcTan[a*x])/x) - a^3*c^2*x*ArcTan[a*x] - (c^2*ArcTan[a*x]^2)/(2*
x^2) + (a^4*c^2*x^2*ArcTan[a*x]^2)/2 + 4*a^2*c^2*ArcTan[a*x]^2*ArcTanh[1 -
 2/(1 + I*a*x)] + a^2*c^2*Log[x] - (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, 1
- 2/(1 + I*a*x)] + (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)
] - a^2*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)] + a^2*c^2*PolyLog[3, -1 + 2/(1 +
 I*a*x)]
```

3.272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.272.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 58.06 (sec) , antiderivative size = 1184, normalized size of antiderivative = 5.72

method	result	size
derivativedivides	Expression too large to display	1184
default	Expression too large to display	1184
parts	Expression too large to display	1614

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*a^2*c^2*x^2*arctan(a*x)^2+2*c^2*arctan(a*x)^2*ln(a*x)-1/2*c^2*arc
tan(a*x)^2/a^2/x^2-c^2*(2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2*ar
ctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*Pi*csgn(((1+I*a*x)^2/(a^2*
x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-4*polylog(3,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+I
*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-4*polylog(3,(1+I*a*x)/(a^2*x
^2+1)^(1/2))+1/2*arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+polylog(3,-(1
+I*a*x)^2/(a^2*x^2+1))+ln((1+I*a*x)^2/(a^2*x^2+1)+1)-ln((1+I*a*x)/(a^2*x^2
+1)^(1/2)+1)-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-I*Pi*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+1/2*arctan(a*x)*
(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^
2*x^2+1))+arctan(a*x)*(a*x-I)+4*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))+I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*arctan(a*x)^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))*arctan(a*x)^2+4*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-I
*Pi*arctan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^
2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^2*arctan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*...
```

3.272.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^3, x)
```

3.272.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} \, dx = c^2\left(\int \frac{\operatorname{atan}^2(ax)}{x^3} \, dx + \int \frac{2a^2 \operatorname{atan}^2(ax)}{x} \, dx + \int a^4 x \operatorname{atan}^2(ax) \, dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**3,x)
```

output
```
c**2*(Integral(atan(a*x)**2/x**3, x) + Integral(2*a**2*atan(a*x)**2/x, x)
+ Integral(a**4*x*atan(a*x)**2, x))
```

3.272.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
-1/32*(2*a^4*c^2*x^4 - 4*a^4*c^2*x^2*integrate(4*x*arctan(a*x)^2 + x*log(a
^2*x^2 + 1)^2, x) - 8*a^3*c^2*x^2*integrate(-1/4*(12*(a^2*x^2 + 1)*a*x*arc
tan(a*x)^2 - 3*(a^2*x^2 + 1)*a*x*log(a^2*x^2 + 1)^2 + 12*(a^2*x^2 + 1)*arc
tan(a*x)*log(a^2*x^2 + 1) + (4*(a^2*x^2 + 1)^2*arctan(a*x)*cos(3*arctan(a*
x))*log(a^2*x^2 + 1) - 12*(a^2*x^2 + 1)^(3/2)*arctan(a*x)*cos(2*arctan(a*x
))*log(a^2*x^2 + 1) - 4*sqrt(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1) + (
4*(a^2*x^2 + 1)^2*arctan(a*x)^2 - (a^2*x^2 + 1)^2*log(a^2*x^2 + 1)^2)*sin(
3*arctan(a*x)) - 3*(4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^2 - (a^2*x^2 + 1)^(3
/2)*log(a^2*x^2 + 1)^2)*sin(2*arctan(a*x)))*sqrt(a^2*x^2 + 1))/((a^2*x^2 +
 1)^4*cos(3*arctan(a*x))^2 + (a^2*x^2 + 1)^4*sin(3*arctan(a*x))^2 - 6*(a^2
*x^2 + 1)^(7/2)*sin(3*arctan(a*x))*sin(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^3*
cos(2*arctan(a*x))^2 + 9*(a^2*x^2 + 1)^3*sin(2*arctan(a*x))^2 + a^2*x^2 +
6*(a^2*x^2 + 1)^2*cos(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^2 - 2*(3*(a^2*x^2 +
 1)^(7/2)*cos(2*arctan(a*x)) + (a^2*x^2 + 1)^(5/2))*cos(3*arctan(a*x)) + 6
*((a^2*x^2 + 1)^2*a*x*sin(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*a*x*sin(2
*arctan(a*x)) + (a^2*x^2 + 1)^2*cos(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)
*cos(2*arctan(a*x)) - sqrt(a^2*x^2 + 1))*sqrt(a^2*x^2 + 1) + 1), x) - 8*a^
3*c^2*x^2*integrate(1/4*(4*(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1) - (4
*(a^2*x^2 + 1)*a*x*arctan(a*x)^2 - (a^2*x^2 + 1)*a*x*log(a^2*x^2 + 1)^2 +
4*(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1))*cos(2*arctan(a*x)) - (4*(...
```

3.272.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^3}\,dx=\int \frac{(a^2cx^2+c)^2\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^2\arctan(ax)^2}{x^3}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^2}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^3, x)
```

3.273 $\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^4}\,dx$

3.273.1 Optimal result

Integrand size = 22, antiderivative size = 216

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^4}\,dx = &-\frac{a^2c^2}{3x}-\frac{1}{3}a^3c^2\arctan(ax)-\frac{ac^2\arctan(ax)}{3x^2}\\&-\frac{2}{3}ia^3c^2\arctan(ax)^2-\frac{c^2\arctan(ax)^2}{3x^3}-\frac{2a^2c^2\arctan(ax)^2}{x}\\&+a^4c^2x\arctan(ax)^2+2a^3c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\&+\frac{10}{3}a^3c^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\&-\frac{5}{3}ia^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\&+ia^3c^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-1/3*a^2*c^2/x-1/3*a^3*c^2*arctan(a*x)-1/3*a*c^2*arctan(a*x)/x^2-2/3*I*a^3
*c^2*arctan(a*x)^2-1/3*c^2*arctan(a*x)^2/x^3-2*a^2*c^2*arctan(a*x)^2/x+a^4
*c^2*x*arctan(a*x)^2+2*a^3*c^2*arctan(a*x)*ln(2/(1+I*a*x))+10/3*a^3*c^2*ar
ctan(a*x)*ln(2-2/(1-I*a*x))-5/3*I*a^3*c^2*polylog(2,-1+2/(1-I*a*x))+I*a^3*
c^2*polylog(2,1-2/(1+I*a*x))
```

3.273.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4}\,dx$$
$$= \frac{c^2(-a^2x^2 - ax\arctan(ax) - a^3x^3\arctan(ax) - \arctan(ax)^2 - 6a^2x^2\arctan(ax)^2 - 8ia^3x^3\arctan(ax)^2 +}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```

output
```
(c^2*(-(a^2*x^2) - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - ArcTan[a*x]^2 -
 6*a^2*x^2*ArcTan[a*x]^2 - (8*I)*a^3*x^3*ArcTan[a*x]^2 + 3*a^4*x^4*ArcTan[
a*x]^2 + 10*a^3*x^3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 6*a^3*x^3
*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - (3*I)*a^3*x^3*PolyLog[2, -E^
((2*I)*ArcTan[a*x])] - (5*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(
3*x^3)
```

3.273.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 (a^2cx^2+c)^2}{x^4}\,dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^4c^2\arctan(ax)^2 + \frac{2a^2c^2\arctan(ax)^2}{x^2} + \frac{c^2\arctan(ax)^2}{x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a^4c^2x\arctan(ax)^2 - \frac{2}{3}ia^3c^2\arctan(ax)^2 - \frac{1}{3}a^3c^2\arctan(ax) + 2a^3c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) + \frac{10}{3}a^3c^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{5}{3}ia^3c^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right) + ia^3c^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right) - \frac{2a^2c^2\arctan(ax)^2}{x} - \frac{a^2c^2}{3x} - \frac{c^2\arctan(ax)^2}{3x^3} - \frac{ac^2\arctan(ax)}{3x^2}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/3*(a^2*c^2)/x - (a^3*c^2*ArcTan[a*x])/3 - (a*c^2*ArcTan[a*x])/(3*x^2) -
 ((2*I)/3)*a^3*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/(3*x^3) - (2*a^2*c^
2*ArcTan[a*x]^2)/x + a^4*c^2*x*ArcTan[a*x]^2 + 2*a^3*c^2*ArcTan[a*x]*Log[2
/(1 + I*a*x)] + (10*a^3*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((5*I)
/3)*a^3*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] + I*a^3*c^2*PolyLog[2, 1 - 2/(1
 + I*a*x)]
```

3.273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.273.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.34

method	result
derivativedivides	$a^3\left(a\,c^2 x \arctan\left(ax\right)^2-\frac{c^2\arctan(ax)^2}{3a^3x^3}-\frac{2c^2\arctan(ax)^2}{ax}-\frac{2c^2\left(4\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-5\right.}{}\right.$
default	$a^3\left(a\,c^2 x \arctan\left(ax\right)^2-\frac{c^2\arctan(ax)^2}{3a^3x^3}-\frac{2c^2\arctan(ax)^2}{ax}-\frac{2c^2\left(4\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}-5\right.}{}\right.$
parts	$a^4c^2x\arctan\left(ax\right)^2-\frac{2a^2c^2\arctan(ax)^2}{x}-\frac{c^2\arctan(ax)^2}{3x^3}-\frac{2c^2\left(4a^3\arctan(ax)\ln(a^2x^2+1)+\frac{a\arctan(ax)}{2x^2}-\right.}{}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(a*c^2*x*arctan(a*x)^2-1/3*c^2*arctan(a*x)^2/a^3/x^3-2*c^2*arctan(a*x)
^2/a/x-2/3*c^2*(4*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2-5*arct
an(a*x)*ln(a*x)+1/2*arctan(a*x)+1/2/a/x-5/2*I*ln(a*x)*ln(1+I*a*x)+5/2*I*ln
(a*x)*ln(1-I*a*x)-5/2*I*dilog(1+I*a*x)+5/2*I*dilog(1-I*a*x)+2*I*(ln(a*x-I)
*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a
*x-I)^2)-2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/
2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.273.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^2\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^4, x)
```

3.273.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^4}\,dx = c^2\left(\int a^4\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^4}\,dx + \int \frac{2a^2\operatorname{atan}^2(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**4,x)
```

output
```
c**2*(Integral(a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) + In
tegral(2*a**2*atan(a*x)**2/x**2, x))
```

3.273.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(12*(144*a^6*c^2*integrate(1/48*x^6*arctan(a*x)^2/(a^2*x^6 + x^4), x)
 + 12*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) +
48*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 3*a^3
*c^2*arctan(a*x)^3 - 96*a^5*c^2*integrate(1/48*x^5*arctan(a*x)/(a^2*x^6 +
x^4), x) + 36*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4
), x) - 96*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x)
 + 192*a^3*c^2*integrate(1/48*x^3*arctan(a*x)/(a^2*x^6 + x^4), x) + 432*a^
2*c^2*integrate(1/48*x^2*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 36*a^2*c^2*in
tegrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 16*a^2*c^2*integ
rate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 32*a*c^2*integrate(1/
48*x*arctan(a*x)/(a^2*x^6 + x^4), x) + 144*c^2*integrate(1/48*arctan(a*x)^
2/(a^2*x^6 + x^4), x) + 12*c^2*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^6
+ x^4), x))*x^3 + 4*(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x)^2 -
(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*log(a^2*x^2 + 1)^2)/x^3
```

3.273.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4} \, dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2}{x^4} \, dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^4, x)
```

3.274 $\int x^3(c+a^2cx^2)^3 \arctan(ax)^2\,dx$

3.274.1 Optimal result

Integrand size = 22, antiderivative size = 240

$$\begin{aligned}\int x^3(c+a^2cx^2)^3 \arctan(ax)^2\,dx = & -\frac{107c^3x^2}{12600a^2}+\frac{53c^3x^4}{6300}+\frac{71a^2c^3x^6}{7560} \\ & +\frac{1}{360}a^4c^3x^8+\frac{c^3x\arctan(ax)}{20a^3} \\ & -\frac{c^3x^3\arctan(ax)}{60a}-\frac{9}{100}ac^3x^5\arctan(ax) \\ & -\frac{11}{140}a^3c^3x^7\arctan(ax)-\frac{1}{45}a^5c^3x^9\arctan(ax) \\ & -\frac{c^3\arctan(ax)^2}{40a^4}+\frac{1}{4}c^3x^4\arctan(ax)^2 \\ & +\frac{1}{2}a^2c^3x^6\arctan(ax)^2+\frac{3}{8}a^4c^3x^8\arctan(ax)^2 \\ & +\frac{1}{10}a^6c^3x^{10}\arctan(ax)^2-\frac{26c^3\log(1+a^2x^2)}{1575a^4}\end{aligned}$$

output
```
-107/12600*c^3*x^2/a^2+53/6300*c^3*x^4+71/7560*a^2*c^3*x^6+1/360*a^4*c^3*x
^8+1/20*c^3*x*arctan(a*x)/a^3-1/60*c^3*x^3*arctan(a*x)/a-9/100*a*c^3*x^5*a
rctan(a*x)-11/140*a^3*c^3*x^7*arctan(a*x)-1/45*a^5*c^3*x^9*arctan(a*x)-1/4
0*c^3*arctan(a*x)^2/a^4+1/4*c^3*x^4*arctan(a*x)^2+1/2*a^2*c^3*x^6*arctan(a
*x)^2+3/8*a^4*c^3*x^8*arctan(a*x)^2+1/10*a^6*c^3*x^10*arctan(a*x)^2-26/157
5*c^3*ln(a^2*x^2+1)/a^4
```

3.274.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.52

$$\int x^3\left(c+a^2cx^2\right)^3\arctan(ax)^2\,dx$$
$$=\frac{c^3\left(-321a^2x^2+318a^4x^4+355a^6x^6+105a^8x^8-6ax(-315+105a^2x^2+567a^4x^4+495a^6x^6+140a^8x^8)\,a\right.}{37800a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(-321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 - 6*a*x*(-315
 + 105*a^2*x^2 + 567*a^4*x^4 + 495*a^6*x^6 + 140*a^8*x^8)*ArcTan[a*x] + 94
5*(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^2 - 624*Log[1 + a^2*x^2]))/
(37800*a^4)
```

3.274.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)^2\left(a^2cx^2+c\right)^3\,dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^6c^3x^9\arctan(ax)^2+3a^4c^3x^7\arctan(ax)^2+3a^2c^3x^5\arctan(ax)^2+c^3x^3\arctan(ax)^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{10}a^6c^3x^{10}\arctan(ax)^2-\frac{1}{45}a^5c^3x^9\arctan(ax)+\frac{3}{8}a^4c^3x^8\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{40a^4}+\frac{1}{360}a^4c^3x^8-\frac{11}{140}a^3c^3x^7\arctan(ax)+\frac{c^3x\arctan(ax)}{20a^3}+\frac{1}{2}a^2c^3x^6\arctan(ax)^2+\frac{71a^2c^3x^6}{7560}-\frac{107c^3x^2}{12600a^2}-\frac{26c^3\log\left(a^2x^2+1\right)}{1575a^4}-\frac{9}{100}ac^3x^5\arctan(ax)+\frac{1}{4}c^3x^4\arctan(ax)^2-\frac{c^3x^3\arctan(ax)}{60a}+\frac{53c^3x^4}{6300}$$

input
```
Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(-107*c^3*x^2)/(12600*a^2) + (53*c^3*x^4)/6300 + (71*a^2*c^3*x^6)/7560 + (
a^4*c^3*x^8)/360 + (c^3*x*ArcTan[a*x])/(20*a^3) - (c^3*x^3*ArcTan[a*x])/(6
0*a) - (9*a*c^3*x^5*ArcTan[a*x])/100 - (11*a^3*c^3*x^7*ArcTan[a*x])/140 -
(a^5*c^3*x^9*ArcTan[a*x])/45 - (c^3*ArcTan[a*x]^2)/(40*a^4) + (c^3*x^4*Arc
Tan[a*x]^2)/4 + (a^2*c^3*x^6*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]
^2)/8 + (a^6*c^3*x^10*ArcTan[a*x]^2)/10 - (26*c^3*Log[1 + a^2*x^2])/(1575*
a^4)
```

3.274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.274.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{c^3 \arctan(ax)^2 a^{10} x^{10}}{10}+\frac{3c^3 \arctan(ax)^2 a^8 x^8}{8}+\frac{a^6 c^3 x^6 \arctan(ax)^2}{2}+\frac{a^4 c^3 x^4 \arctan(ax)^2}{4}-\frac{c^3\left(\frac{4\arctan(ax) a^9 x^9}{9}+\frac{11\arctan(ax) a}{7}\right.}{}}{a^4}$
default	$\frac{\frac{c^3 \arctan(ax)^2 a^{10} x^{10}}{10}+\frac{3c^3 \arctan(ax)^2 a^8 x^8}{8}+\frac{a^6 c^3 x^6 \arctan(ax)^2}{2}+\frac{a^4 c^3 x^4 \arctan(ax)^2}{4}-\frac{c^3\left(\frac{4\arctan(ax) a^9 x^9}{9}+\frac{11\arctan(ax) a}{7}\right.}{}}{a^4}$
parts	$\frac{a^6 c^3 x^{10} \arctan(ax)^2}{10}+\frac{3a^4 c^3 x^8 \arctan(ax)^2}{8}+\frac{a^2 c^3 x^6 \arctan(ax)^2}{2}+\frac{c^3 x^4 \arctan(ax)^2}{4}-\frac{c^3\left(\frac{4a^5 \arctan(ax) x^9}{9}+\right.}{}$
parallelrisch	$-\frac{-3780c^3 \arctan(ax)^2 a^{10} x^{10}+840c^3 \arctan(ax) a^9 x^9-14175c^3 \arctan(ax)^2 a^8 x^8-105a^8 c^3 x^8+2970c^3 \arctan(ax) a^7 x^7}{}$
risch	$-\frac{c^3\left(4a^{10}x^{10}+15a^8x^8+20a^6x^6+10a^4x^4-1\right)\ln(iax+1)^2}{160a^4}+\frac{c^3\left(1260a^{10}x^{10}\ln(-iax+1)+280ia^9x^9+4725a^8x^8\ln(-iax\right.}{}$

input
```
int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/10*c^3*arctan(a*x)^2*a^10*x^10+3/8*c^3*arctan(a*x)^2*a^8*x^8+1/2*
a^6*c^3*x^6*arctan(a*x)^2+1/4*a^4*c^3*x^4*arctan(a*x)^2-1/20*c^3*(4/9*arct
an(a*x)*a^9*x^9+11/7*arctan(a*x)*a^7*x^7+9/5*arctan(a*x)*a^5*x^5+1/3*arcta
n(a*x)*x^3*a^3-x*arctan(a*x)*a+1/2*arctan(a*x)^2-1/18*a^8*x^8-71/378*a^6*x
^6-53/315*a^4*x^4+107/630*a^2*x^2+104/315*ln(a^2*x^2+1)))
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.75

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^2\,dx$$
$$= \frac{105\,a^8c^3x^8+355\,a^6c^3x^6+318\,a^4c^3x^4-321\,a^2c^3x^2-624\,c^3\log\left(a^2x^2+1\right)+945\left(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8+\right.}{}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*c^3
*x^2 - 624*c^3*log(a^2*x^2 + 1) + 945*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 +
20*a^6*c^3*x^6 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x)^2 - 6*(140*a^9*c^3*x^9
+ 495*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3*x^3 - 315*a*c^3*x)*arcta
n(a*x))/a^4
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^2\,dx$$
$$= \begin{cases} \frac{a^6c^3x^{10}\operatorname{atan}^2(ax)}{10} - \frac{a^5c^3x^9\operatorname{atan}(ax)}{45} + \frac{3a^4c^3x^8\operatorname{atan}^2(ax)}{8} + \frac{a^4c^3x^8}{360} - \frac{11a^3c^3x^7\operatorname{atan}(ax)}{140} + \frac{a^2c^3x^6\operatorname{atan}^2(ax)}{2} + \frac{71a^2c^3x^6}{7560} - \\ 0 \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
Piecewise((a**6*c**3*x**10*atan(a*x)**2/10 - a**5*c**3*x**9*atan(a*x)/45 +
 3*a**4*c**3*x**8*atan(a*x)**2/8 + a**4*c**3*x**8/360 - 11*a**3*c**3*x**7*
atan(a*x)/140 + a**2*c**3*x**6*atan(a*x)**2/2 + 71*a**2*c**3*x**6/7560 - 9
*a*c**3*x**5*atan(a*x)/100 + c**3*x**4*atan(a*x)**2/4 + 53*c**3*x**4/6300
- c**3*x**3*atan(a*x)/(60*a) - 107*c**3*x**2/(12600*a**2) + c**3*x*atan(a*
x)/(20*a**3) - 26*c**3*log(x**2 + a**(-2))/(1575*a**4) - c**3*atan(a*x)**2
/(40*a**4), Ne(a, 0)), (0, True))
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.84

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^2\,dx =$$
$$-\frac{1}{6300}a\left(\frac{315\,c^3\arctan(ax)}{a^5}+\frac{140\,a^8c^3x^9+495\,a^6c^3x^7+567\,a^4c^3x^5+105\,a^2c^3x^3-315\,c^3x}{a^4}\right)\arctan(ax)$$
$$+\frac{1}{40}\left(4\,a^6c^3x^{10}+15\,a^4c^3x^8+20\,a^2c^3x^6+10\,c^3x^4\right)\arctan(ax)^2$$
$$+\frac{105\,a^8c^3x^8+355\,a^6c^3x^6+318\,a^4c^3x^4-321\,a^2c^3x^2+945\,c^3\arctan(ax)^2-624\,c^3\log(a^2x^2+1)}{37800\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/6300*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 +
567*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4)*arctan(a*x) + 1/40*(4*
a^6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)^2
 + 1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*
c^3*x^2 + 945*c^3*arctan(a*x)^2 - 624*c^3*log(a^2*x^2 + 1))/a^4
```

3.274.8 Giac [F]

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^3x^3\arctan(ax)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.274.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^2\,dx = \operatorname{atan}(a\,x)^2\left(\frac{c^3\,x^4}{4}-\frac{c^3}{40\,a^4}+\frac{a^2\,c^3\,x^6}{2}+\frac{3\,a^4\,c^3\,x^8}{8}+\frac{a^6\,c^3\,x^{10}}{10}\right)+\frac{53\,c^3\,x^4}{6300}-\frac{26\,c^3\,\ln\left(a^2\,x^2+1\right)}{1575\,a^4}-\frac{107\,c^3\,x^2}{12600\,a^2}+\frac{71\,a^2\,c^3\,x^6}{7560}+\frac{a^4\,c^3\,x^8}{360}-a^2\operatorname{atan}(a\,x)\left(\frac{11\,a\,c^3\,x^7}{140}-\frac{c^3\,x}{20\,a^5}+\frac{9\,c^3\,x^5}{100\,a}+\frac{c^3\,x^3}{60\,a^3}+\frac{a^3\,c^3\,x^9}{45}\right)$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^2*((c^3*x^4)/4 - c^3/(40*a^4) + (a^2*c^3*x^6)/2 + (3*a^4*c^3*x^8
)/8 + (a^6*c^3*x^10)/10) + (53*c^3*x^4)/6300 - (26*c^3*log(a^2*x^2 + 1))/(
1575*a^4) - (107*c^3*x^2)/(12600*a^2) + (71*a^2*c^3*x^6)/7560 + (a^4*c^3*x
^8)/360 - a^2*atan(a*x)*((11*a*c^3*x^7)/140 - (c^3*x)/(20*a^5) + (9*c^3*x^
5)/(100*a) + (c^3*x^3)/(60*a^3) + (a^3*c^3*x^9)/45)
```

3.275 $\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^2 \, dx$

3.275.1 Optimal result

Integrand size = 22, antiderivative size = 274

$$\begin{aligned}\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^2 \, dx = & -\frac{47c^3x}{3780a^2} + \frac{239c^3x^3}{11340} + \frac{59a^2c^3x^5}{3780} + \frac{1}{252}a^4c^3x^7 \\ & + \frac{47c^3 \arctan(ax)}{3780a^3} - \frac{16c^3x^2 \arctan(ax)}{315a} \\ & - \frac{89}{630}ac^3x^4 \arctan(ax) - \frac{20}{189}a^3c^3x^6 \arctan(ax) \\ & - \frac{1}{36}a^5c^3x^8 \arctan(ax) - \frac{16ic^3 \arctan(ax)^2}{315a^3} \\ & + \frac{1}{3}c^3x^3 \arctan(ax)^2 + \frac{3}{5}a^2c^3x^5 \arctan(ax)^2 \\ & + \frac{3}{7}a^4c^3x^7 \arctan(ax)^2 + \frac{1}{9}a^6c^3x^9 \arctan(ax)^2 \\ & - \frac{32c^3 \arctan(ax) \log\left(\frac{2}{1+iax}\right)}{315a^3} \\ & - \frac{16ic^3 \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{315a^3}\end{aligned}$$

output
```
-47/3780*c^3*x/a^2+239/11340*c^3*x^3+59/3780*a^2*c^3*x^5+1/252*a^4*c^3*x^7
+47/3780*c^3*arctan(a*x)/a^3-16/315*c^3*x^2*arctan(a*x)/a-89/630*a*c^3*x^4
*arctan(a*x)-20/189*a^3*c^3*x^6*arctan(a*x)-1/36*a^5*c^3*x^8*arctan(a*x)-1
6/315*I*c^3*arctan(a*x)^2/a^3+1/3*c^3*x^3*arctan(a*x)^2+3/5*a^2*c^3*x^5*ar
ctan(a*x)^2+3/7*a^4*c^3*x^7*arctan(a*x)^2+1/9*a^6*c^3*x^9*arctan(a*x)^2-32
/315*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a^3-16/315*I*c^3*polylog(2,1-2/(1+I*a
*x))/a^3
```

3.275.2 Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.57

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)^2\,dx$$
$$= \frac{c^3(ax(-141+239a^2x^2+177a^4x^4+45a^6x^6)+36(16i+105a^3x^3+189a^5x^5+135a^7x^7+35a^9x^9)\arctan(}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(a*x*(-141 + 239*a^2*x^2 + 177*a^4*x^4 + 45*a^6*x^6) + 36*(16*I + 105
*a^3*x^3 + 189*a^5*x^5 + 135*a^7*x^7 + 35*a^9*x^9)*ArcTan[a*x]^2 - 3*ArcTa
n[a*x]*(-47 + 192*a^2*x^2 + 534*a^4*x^4 + 400*a^6*x^6 + 105*a^8*x^8 + 384*
Log[1 + E^((2*I)*ArcTan[a*x])]) + (576*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x]
)]))/(11340*a^3)
```

3.275.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^3 dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^6c^3x^8\arctan(ax)^2+3a^4c^3x^6\arctan(ax)^2+3a^2c^3x^4\arctan(ax)^2+c^3x^2\arctan(ax)^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{9}a^6c^3x^9\arctan(ax)^2-\frac{1}{36}a^5c^3x^8\arctan(ax)+\frac{3}{7}a^4c^3x^7\arctan(ax)^2+\frac{1}{252}a^4c^3x^7-\frac{20}{189}a^3c^3x^6\arctan(ax)-\frac{16ic^3\arctan(ax)^2}{315a^3}+\frac{47c^3\arctan(ax)}{3780a^3}-\frac{32c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{315a^3}-\frac{16ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{315a^3}+\frac{3}{5}a^2c^3x^5\arctan(ax)^2+\frac{59a^2c^3x^5}{3780}-\frac{47c^3x}{3780a^2}-\frac{89}{630}ac^3x^4\arctan(ax)+\frac{1}{3}c^3x^3\arctan(ax)^2-\frac{16c^3x^2\arctan(ax)}{315a}+\frac{239c^3x^3}{11340}$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(-47*c^3*x)/(3780*a^2) + (239*c^3*x^3)/11340 + (59*a^2*c^3*x^5)/3780 + (a^
4*c^3*x^7)/252 + (47*c^3*ArcTan[a*x])/(3780*a^3) - (16*c^3*x^2*ArcTan[a*x]
)/(315*a) - (89*a*c^3*x^4*ArcTan[a*x])/630 - (20*a^3*c^3*x^6*ArcTan[a*x])/
189 - (a^5*c^3*x^8*ArcTan[a*x])/36 - (((16*I)/315)*c^3*ArcTan[a*x]^2)/a^3
+ (c^3*x^3*ArcTan[a*x]^2)/3 + (3*a^2*c^3*x^5*ArcTan[a*x]^2)/5 + (3*a^4*c^3
*x^7*ArcTan[a*x]^2)/7 + (a^6*c^3*x^9*ArcTan[a*x]^2)/9 - (32*c^3*ArcTan[a*x
]*Log[2/(1 + I*a*x)])/(315*a^3) - (((16*I)/315)*c^3*PolyLog[2, 1 - 2/(1 +
I*a*x)])/a^3
```

3.275.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.275.4 Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{c^3 \arctan(ax)^2 a^9 x^9}{9}+\frac{3c^3 \arctan(ax)^2 a^7 x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+\frac{a^3c^3x^3\arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35\arctan(ax)a^8x^8}{8}+\frac{50a^6\arctan(a}{3}\right.}{}}{}$
default	$\frac{\frac{c^3 \arctan(ax)^2 a^9 x^9}{9}+\frac{3c^3 \arctan(ax)^2 a^7 x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+\frac{a^3c^3x^3\arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35\arctan(ax)a^8x^8}{8}+\frac{50a^6\arctan(a}{3}\right.}{}}{}$
parts	$\frac{a^6c^3x^9\arctan(ax)^2}{9}+\frac{3a^4c^3x^7\arctan(ax)^2}{7}+\frac{3a^2c^3x^5\arctan(ax)^2}{5}+\frac{c^3x^3\arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35a^5\arctan(ax)x^8}{8}\right.}{}$
risch	$-\frac{8ic^3\ln(-iax+1)x^2}{315a}-\frac{16ic^3\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{315a^3}+\frac{16ic^3\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{315a^3}-\frac{8ic^3\ln(iax+1)\ln(-iax+}{315a^3}$

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/9*c^3*arctan(a*x)^2*a^9*x^9+3/7*c^3*arctan(a*x)^2*a^7*x^7+3/5*a^5
*c^3*x^5*arctan(a*x)^2+1/3*a^3*c^3*x^3*arctan(a*x)^2-2/315*c^3*(35/8*arcta
n(a*x)*a^8*x^8+50/3*a^6*arctan(a*x)*x^6+89/4*arctan(a*x)*a^4*x^4+8*a^2*arc
tan(a*x)*x^2-8*arctan(a*x)*ln(a^2*x^2+1)-5/8*a^7*x^7-59/24*a^5*x^5-239/72*
a^3*x^3+47/24*a*x-47/24*arctan(a*x)-4*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/
2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+4*I*(ln(I+a*x)*
ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*
x)^2)))
```

3.275.5 Fricas [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*
x)^2, x)
```

3.275.6 Sympy [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^2\,dx=c^3\left(\int x^2\operatorname{atan}^2(ax)\,dx+\int 3a^2x^4\operatorname{atan}^2(ax)\,dx+\int 3a^4x^6\operatorname{atan}^2(ax)\,dx+\int a^6x^8\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
c**3*(Integral(x**2*atan(a*x)**2, x) + Integral(3*a**2*x**4*atan(a*x)**2,
x) + Integral(3*a**4*x**6*atan(a*x)**2, x) + Integral(a**6*x**8*atan(a*x)*
*2, x))
```

3.275.7 Maxima [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^2\,dx=\int (a^2cx^2+c)^3x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/1260*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*
arctan(a*x)^2 - 1/5040*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5
 + 105*c^3*x^3)*log(a^2*x^2 + 1)^2 + integrate(1/5040*(3780*(a^8*c^3*x^10
+ 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x)^2 +
 315*(a^8*c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x
^2)*log(a^2*x^2 + 1)^2 - 8*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3
*x^5 + 105*a*c^3*x^3)*arctan(a*x) + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 +
 189*a^4*c^3*x^6 + 105*a^2*c^3*x^4)*log(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

3.275.8 Giac [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^2\,dx=\int (a^2cx^2+c)^3x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^2\,dx=\int x^2\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3, x)
```

3.276 $\int x(c+a^2cx^2)^3 \arctan(ax)^2\,dx$

3.276.1 Optimal result

Integrand size = 20, antiderivative size = 200

$$\int x(c+a^2cx^2)^3 \arctan(ax)^2\,dx = \frac{c^3(1+a^2x^2)}{35a^2}+\frac{3c^3(1+a^2x^2)^2}{280a^2}+\frac{c^3(1+a^2x^2)^3}{168a^2} - \frac{4c^3x\arctan(ax)}{35a}-\frac{2c^3x(1+a^2x^2)\arctan(ax)}{35a} - \frac{3c^3x(1+a^2x^2)^2\arctan(ax)}{70a} - \frac{c^3x(1+a^2x^2)^3\arctan(ax)}{28a} + \frac{c^3(1+a^2x^2)^4\arctan(ax)^2}{8a^2}+\frac{2c^3\log(1+a^2x^2)}{35a^2}$$

output
```
1/35*c^3*(a^2*x^2+1)/a^2+3/280*c^3*(a^2*x^2+1)^2/a^2+1/168*c^3*(a^2*x^2+1)
^3/a^2-4/35*c^3*x*arctan(a*x)/a-2/35*c^3*x*(a^2*x^2+1)*arctan(a*x)/a-3/70*
c^3*x*(a^2*x^2+1)^2*arctan(a*x)/a-1/28*c^3*x*(a^2*x^2+1)^3*arctan(a*x)/a+1
/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^2/a^2+2/35*c^3*ln(a^2*x^2+1)/a^2
```

3.276.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.50

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^2\,dx$$
$$=\frac{c^3\left(57a^2x^2+24a^4x^4+5a^6x^6-6ax(35+35a^2x^2+21a^4x^4+5a^6x^6)\arctan(ax)+105(1+a^2x^2)^4\arctan(\right.}{840a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 6*a*x*(35 + 35*a^2*x^2 + 21*a^
4*x^4 + 5*a^6*x^6)*ArcTan[a*x] + 105*(1 + a^2*x^2)^4*ArcTan[a*x]^2 + 48*Lo
g[1 + a^2*x^2]))/(840*a^2)
```

3.276.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5465, 27, 5413, 5413, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\left(a^2cx^2+c\right)^3\,dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{\int c^3\left(a^2x^2+1\right)^3\arctan(ax)dx}{4a}$$
$$\Big\downarrow \text{27}$$
$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\int\left(a^2x^2+1\right)^3\arctan(ax)dx}{4a}$$
$$\Big\downarrow \text{5413}$$
$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-$$
$$\frac{c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{\left(a^2x^2+1\right)^3}{42a}\right)}{4a}$$

$$\Big\downarrow \ 5413$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{6}{7}\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{(a^2x}{}\right.}{4a}$$

$$\Big\downarrow \ 5413$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\frac{1}{7}\right.}{4a}$$

$$\Big\downarrow \ 5345$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\right.\right.\right.}{4a}$$

$$\Big\downarrow \ 240$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)+\frac{6}{7}\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\right.\right.\right.\right.}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^2)/(8*a^2) - (c^3*(-1/42*(1 + a^2*x^2)^3/
a + (x*(1 + a^2*x^2)^3*ArcTan[a*x])/7 + (6*(-1/20*(1 + a^2*x^2)^2/a + (x*(
1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2
)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/5))/7)
)/(4*a)
```

3.276. $\int x\left(c+a^2cx^2\right)^3\arctan(ax)^2\,dx$

3.276.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.276.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.84

method	result
parts	$\frac{c^3 \arctan(ax)^2 a^6 x^8}{8} + \frac{c^3 \arctan(ax)^2 a^4 x^6}{2} + \frac{3c^3 \arctan(ax)^2 a^2 x^4}{4} + \frac{c^3 \arctan(ax)^2 x^2}{2} + \frac{c^3 \arctan(ax)^2}{8a^2} - \frac{c^3 \Big(}{}$
derivativedivides	$\frac{\frac{c^3 \arctan(ax)^2 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^2}{2} + \frac{3a^4 c^3 x^4 \arctan(ax)^2}{4} + \frac{a^2 c^3 x^2 \arctan(ax)^2}{2} + \frac{c^3 \arctan(ax)^2}{8} - \frac{c^3\left(\frac{\arctan(ax) a^7 x^7}{7} + \frac{3}{}\right.}{}}{a^2}$
default	$\frac{\frac{c^3 \arctan(ax)^2 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^2}{2} + \frac{3a^4 c^3 x^4 \arctan(ax)^2}{4} + \frac{a^2 c^3 x^2 \arctan(ax)^2}{2} + \frac{c^3 \arctan(ax)^2}{8} - \frac{c^3\left(\frac{\arctan(ax) a^7 x^7}{7} + \frac{3}{}\right.}{}}{a^2}$
parallelrisch	$105c^3 \arctan(ax)^2 a^8 x^8 - 30c^3 \arctan(ax) a^7 x^7 + 420a^6 c^3 x^6 \arctan(ax)^2 + 5a^6 c^3 x^6 - 126a^5 c^3 x^5 \arctan(ax) + 630a^4 c^3 x^4 \mathrm{ar}$
risch	$-\frac{c^3 (a^2x^2+1)^4 \ln(iax+1)^2}{32a^2} + \frac{c^3 (35a^8x^8 \ln(-iax+1) + 10ia^7x^7 + 140a^6x^6 \ln(-iax+1) + 42ia^5x^5 + 210x^4 \ln(-iax+1)a}{560a^2}$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*arctan(a*x)^2*a^6*x^8+1/2*c^3*arctan(a*x)^2*a^4*x^6+3/4*c^3*arctan
(a*x)^2*a^2*x^4+1/2*c^3*arctan(a*x)^2*x^2+1/8*c^3*arctan(a*x)^2/a^2-1/4*c^
3/a^2*(1/7*arctan(a*x)*a^7*x^7+3/5*arctan(a*x)*a^5*x^5+arctan(a*x)*x^3*a^3
+x*arctan(a*x)*a-1/42*a^6*x^6-4/35*a^4*x^4-19/70*a^2*x^2-8/35*ln(a^2*x^2+1
))
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int x(c + a^2cx^2)^3 \arctan(ax)^2 \, dx$$
$$= \frac{5\, a^6c^3x^6 + 24\, a^4c^3x^4 + 57\, a^2c^3x^2 + 48\, c^3 \log\left(a^2x^2+1\right) + 105\left(a^8c^3x^8 + 4\, a^6c^3x^6 + 6\, a^4c^3x^4 + 4\, a^2c^3x^2 + c\right.}{840\, a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/840*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x^
2 + 1) + 105*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2
+ c^3)*arctan(a*x)^2 - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3
+ 35*a*c^3*x)*arctan(a*x))/a^2
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.04

$$\int x(c+a^2cx^2)^3 \arctan(ax)^2\,dx$$
$$= \begin{cases} \frac{a^6c^3x^8\operatorname{atan}^2(ax)}{8} - \frac{a^5c^3x^7\operatorname{atan}(ax)}{28} + \frac{a^4c^3x^6\operatorname{atan}^2(ax)}{2} + \frac{a^4c^3x^6}{168} - \frac{3a^3c^3x^5\operatorname{atan}(ax)}{20} + \frac{3a^2c^3x^4\operatorname{atan}^2(ax)}{4} + \frac{a^2c^3x^4}{35} - \frac{ac^3x \\ 0 \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
Piecewise((a**6*c**3*x**8*atan(a*x)**2/8 - a**5*c**3*x**7*atan(a*x)/28 + a
**4*c**3*x**6*atan(a*x)**2/2 + a**4*c**3*x**6/168 - 3*a**3*c**3*x**5*atan(
a*x)/20 + 3*a**2*c**3*x**4*atan(a*x)**2/4 + a**2*c**3*x**4/35 - a*c**3*x**
3*atan(a*x)/4 + c**3*x**2*atan(a*x)**2/2 + 19*c**3*x**2/280 - c**3*x*atan(
a*x)/(4*a) + 2*c**3*log(x**2 + a**(-2))/(35*a**2) + c**3*atan(a*x)**2/(8*a
**2), Ne(a, 0)), (0, True))
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int x(c+a^2cx^2)^3 \arctan(ax)^2\,dx = \frac{(a^2cx^2+c)^4\arctan(ax)^2}{8\,a^2c}$$
$$+ \frac{\left(5\,a^4c^4x^6 + 24\,a^2c^4x^4 + 57\,c^4x^2 + \frac{48\,c^4\log(a^2x^2+1)}{a^2}\right)a - 6\,(5\,a^6c^4x^7 + 21\,a^4c^4x^5 + 35\,a^2c^4x^3 + 35\,c^4x)\,\mathrm{arct}}{840\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/8*(a^2*c*x^2 + c)^4*arctan(a*x)^2/(a^2*c) + 1/840*((5*a^4*c^4*x^6 + 24*a
^2*c^4*x^4 + 57*c^4*x^2 + 48*c^4*log(a^2*x^2 + 1)/a^2)*a - 6*(5*a^6*c^4*x^
7 + 21*a^4*c^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)*arctan(a*x))/(a*c)
```

3.276.8 Giac [F]

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.276.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\begin{aligned}\int x\left(c+a^2cx^2\right)^3\arctan(ax)^2\,dx=\ &\mathrm{atan}(a\,x)^2\left(\frac{c^3}{8\,a^2}+\frac{c^3\,x^2}{2}+\frac{3\,a^2\,c^3\,x^4}{4}+\frac{a^4\,c^3\,x^6}{2}+\frac{a^6\,c^3\,x^8}{8}\right)\\&+\frac{19\,c^3\,x^2}{280}\\&-a^2\,\mathrm{atan}(a\,x)\left(\frac{c^3\,x}{4\,a^3}+\frac{3\,a\,c^3\,x^5}{20}+\frac{c^3\,x^3}{4\,a}+\frac{a^3\,c^3\,x^7}{28}\right)\\&+\frac{2\,c^3\,\ln\left(a^2\,x^2+1\right)}{35\,a^2}+\frac{a^2\,c^3\,x^4}{35}+\frac{a^4\,c^3\,x^6}{168}\end{aligned}$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^2*(c^3/(8*a^2) + (c^3*x^2)/2 + (3*a^2*c^3*x^4)/4 + (a^4*c^3*x^6)
/2 + (a^6*c^3*x^8)/8) + (19*c^3*x^2)/280 - a^2*atan(a*x)*((c^3*x)/(4*a^3)
+ (3*a*c^3*x^5)/20 + (c^3*x^3)/(4*a) + (a^3*c^3*x^7)/28) + (2*c^3*log(a^2*
x^2 + 1))/(35*a^2) + (a^2*c^3*x^4)/35 + (a^4*c^3*x^6)/168
```

3.277 $\int (c+a^2cx^2)^3 \arctan(ax)^2\,dx$

3.277.1 Optimal result

Integrand size = 19, antiderivative size = 268

$$
\begin{aligned}
\int (c+a^2cx^2)^3 \arctan(ax)^2\,dx ={}& \frac{38c^3x}{105}+\frac{19}{315}a^2c^3x^3+\frac{1}{105}a^4c^3x^5 \\
&-\frac{8c^3(1+a^2x^2)\arctan(ax)}{35a}-\frac{3c^3(1+a^2x^2)^2\arctan(ax)}{35a} \\
&-\frac{c^3(1+a^2x^2)^3\arctan(ax)}{21a}+\frac{16ic^3\arctan(ax)^2}{35a} \\
&+\frac{16}{35}c^3x\arctan(ax)^2+\frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^2 \\
&+\frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^2 \\
&+\frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^2 \\
&+\frac{32c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{35a} \\
&+\frac{16ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a}
\end{aligned}
$$

output
```
38/105*c^3*x+19/315*a^2*c^3*x^3+1/105*a^4*c^3*x^5-8/35*c^3*(a^2*x^2+1)*arc
tan(a*x)/a-3/35*c^3*(a^2*x^2+1)^2*arctan(a*x)/a-1/21*c^3*(a^2*x^2+1)^3*arc
tan(a*x)/a+16/35*I*c^3*arctan(a*x)^2/a+16/35*c^3*x*arctan(a*x)^2+8/35*c^3*
x*(a^2*x^2+1)*arctan(a*x)^2+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^2+1/7*c^3
*x*(a^2*x^2+1)^3*arctan(a*x)^2+32/35*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a+16/
35*I*c^3*polylog(2,1-2/(1+I*a*x))/a
```

3.277.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.51

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^2\,dx$$
$$=\frac{c^3\left(ax(114+19a^2x^2+3a^4x^4)+9(-16i+35ax+35a^3x^3+21a^5x^5+5a^7x^7)\arctan(ax)^2-3\arctan(ax)\right.}{315a}$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4) + 9*(-16*I + 35*a*x + 35*a^3*x^3
+ 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(38 + 57*a^2*x^2 +
 24*a^4*x^4 + 5*a^6*x^6 - 96*Log[1 + E^((2*I)*ArcTan[a*x])]) - (144*I)*Pol
yLog[2, -E^((2*I)*ArcTan[a*x])]))/(315*a)
```

3.277.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {5415, 27, 210, 2009, 5415, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}c\int \left(a^2cx^2+c\right)^2dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}c\int \left(a^2cx^2+c\right)^2dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}$$

$$\Big\downarrow \text{210}$$

$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)^2dx+\frac{1}{21}c\int \left(a^4c^2x^4+2a^2c^2x^2+c^2\right)dx+$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)^2dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+$$
$$\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{5415}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}\int \left(a^2x^2+1\right)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(a\right.}{10a}$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}+x\right)\right)$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{5415}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2 a\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2 \text{arcta}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{5345}$$

3.277. $\int \left(c+a^2cx^2\right)^3\arctan(ax)^2\,dx$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^2-\frac{c^3(a^2x^2+1)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\,a}{3a}\right.\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^2-\frac{c^3(a^2x^2+1)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)\right)+\frac{1}{3}x(a^2x^2+1)\,\mathrm{arc}\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^2-\frac{c^3(a^2x^2+1)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)\right)+\frac{1}{3}x(a^2x^2+1)\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^2-\frac{c^3(a^2x^2+1)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)\right)+\frac{1}{3}x(a^2x^2+1\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^2-\frac{c^3(a^2x^2+1)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

3.277. $\int(c+a^2cx^2)^3\arctan(ax)^2\,dx$

output
```
(c*(c^2*x + (2*a^2*c^2*x^3)/3 + (a^4*c^2*x^5)/5))/21 - (c^3*(1 + a^2*x^2)^
3*ArcTan[a*x])/(21*a) + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/7 + (6*c^3*(
(x + (a^2*x^3)/3)/10 - ((1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) + (x*(1 + a^2*
x^2)^2*ArcTan[a*x]^2)/5 + (4*(x/3 - ((1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (x
*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*Arc
Tan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2,
1 - 2/(1 + I*a*x)])/a)/a)))/3))/5))/7
```

3.277.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.277.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{c^3\arctan(ax)^2a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+a\,c^3x\arctan(ax)^2-\frac{2c^3\left(\frac{5a^6\arctan(ax)x^6}{6}+4\arctan(ax)a^4x\right.}{}}{}$
default	$\frac{\frac{c^3\arctan(ax)^2a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+a\,c^3x\arctan(ax)^2-\frac{2c^3\left(\frac{5a^6\arctan(ax)x^6}{6}+4\arctan(ax)a^4x\right.}{}}{}$
parts	$\frac{c^3\arctan(ax)^2a^6x^7}{7}+\frac{3c^3\arctan(ax)^2a^4x^5}{5}+c^3\arctan(ax)^2a^2x^3+c^3x\arctan(ax)^2-\frac{2c^3\left(\frac{5a^5\mathrm{arc}}{}\right.}{}$
risch	$-\frac{38c^3\arctan(ax)}{105a}+\frac{19a^2c^3x^3}{315}+\frac{a^4c^3x^5}{105}+\frac{38c^3x}{105}-\frac{c^3\ln(iax+1)^2x}{4}-\frac{c^3\ln(-iax+1)^2x}{4}+\frac{20469ic^3}{42875a}+\frac{16ic}{}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/7*c^3*arctan(a*x)^2*a^7*x^7+3/5*a^5*c^3*x^5*arctan(a*x)^2+a^3*c^3*x
^3*arctan(a*x)^2+a*c^3*x*arctan(a*x)^2-2/35*c^3*(5/6*a^6*arctan(a*x)*x^6+4
*arctan(a*x)*a^4*x^4+19/2*a^2*arctan(a*x)*x^2+8*arctan(a*x)*ln(a^2*x^2+1)-
1/6*a^5*x^5-19/18*a^3*x^3-19/3*a*x+19/3*arctan(a*x)+4*I*(ln(a*x-I)*ln(a^2*
x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)
-4*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x
-I))-1/2*ln(I+a*x)^2)))
```

3.277.5 Fricas [F]

$$\int (c+a^2cx^2)^3 \arctan(ax)^2 \, dx = \int (a^2cx^2+c)^3 \arctan(ax)^2 \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
, x)
```

3.277.6 Sympy [F]

$$\int (c+a^2cx^2)^3 \arctan(ax)^2 \, dx = c^3 \left(\int 3a^2x^2 \operatorname{atan}^2(ax) \, dx + \int 3a^4x^4 \operatorname{atan}^2(ax) \, dx + \int a^6x^6 \operatorname{atan}^2(ax) \, dx + \int \operatorname{atan}^2(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**2, x) + Integral(3*a**4*x**4*atan(a*
x)**2, x) + Integral(a**6*x**6*atan(a*x)**2, x) + Integral(atan(a*x)**2, x
))
```

3.277.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
420*a^8*c^3*integrate(1/560*x^8*arctan(a*x)^2/(a^2*x^2 + 1), x) + 35*a^8*c
^3*integrate(1/560*x^8*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 20*a^8*c^3*i
ntegrate(1/560*x^8*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40*a^7*c^3*integra
te(1/560*x^7*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a^6*c^3*integrate(1/560*
x^6*arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^6*c^3*integrate(1/560*x^6*log(
a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 84*a^6*c^3*integrate(1/560*x^6*log(a^2*
x^2 + 1)/(a^2*x^2 + 1), x) - 168*a^5*c^3*integrate(1/560*x^5*arctan(a*x)/(
a^2*x^2 + 1), x) + 2520*a^4*c^3*integrate(1/560*x^4*arctan(a*x)^2/(a^2*x^2
 + 1), x) + 210*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 +
1), x) + 140*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x
) - 280*a^3*c^3*integrate(1/560*x^3*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a
^2*c^3*integrate(1/560*x^2*arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*i
ntegrate(1/560*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*inte
grate(1/560*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/4*c^3*arctan(a*x)^3
/a - 280*a*c^3*integrate(1/560*x*arctan(a*x)/(a^2*x^2 + 1), x) + 35*c^3*in
tegrate(1/560*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/140*(5*a^6*c^3*x^7
+ 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*arctan(a*x)^2 - 1/560*(5*a^6
*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*log(a^2*x^2 + 1)^2
```

3.277.8 Giac [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int (c+a^2cx^2)^3 \arctan(ax)^2 \, dx = \int \operatorname{atan}(a\,x)^2 \, (c\,a^2\,x^2+c)^3 \, dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^3, x)
```

3.278 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x} dx$

3.278.1 Optimal result

Integrand size = 22, antiderivative size = 287

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x} dx &= \frac{29}{180}a^2c^3x^2 + \frac{1}{60}a^4c^3x^4 - \frac{11}{6}ac^3x\arctan(ax) \\ &- \frac{7}{18}a^3c^3x^3\arctan(ax) - \frac{1}{15}a^5c^3x^5\arctan(ax) \\ &+ \frac{11}{12}c^3\arctan(ax)^2 + \frac{3}{2}a^2c^3x^2\arctan(ax)^2 \\ &+ \frac{3}{4}a^4c^3x^4\arctan(ax)^2 + \frac{1}{6}a^6c^3x^6\arctan(ax)^2 \\ &+ 2c^3\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) \\ &+ \frac{34}{45}c^3\log\left(1+a^2x^2\right) \\ &- ic^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\ &+ ic^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) \\ &- \frac{1}{2}c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) \\ &+ \frac{1}{2}c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
29/180*a^2*c^3*x^2+1/60*a^4*c^3*x^4-11/6*a*c^3*x*arctan(a*x)-7/18*a^3*c^3*
x^3*arctan(a*x)-1/15*a^5*c^3*x^5*arctan(a*x)+11/12*c^3*arctan(a*x)^2+3/2*a
^2*c^3*x^2*arctan(a*x)^2+3/4*a^4*c^3*x^4*arctan(a*x)^2+1/6*a^6*c^3*x^6*arc
tan(a*x)^2-2*c^3*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+34/45*c^3*ln(a^2*x^
2+1)-I*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+I*c^3*arctan(a*x)*polylog(
2,-1+2/(1+I*a*x))-1/2*c^3*polylog(3,1-2/(1+I*a*x))+1/2*c^3*polylog(3,-1+2/
(1+I*a*x))
```

3.278.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x}\,dx &= \frac{1}{360}c^3\big(52-15i\pi^3+58a^2x^2+6a^4x^4-660ax\arctan(ax)\\&\quad-140a^3x^3\arctan(ax)-24a^5x^5\arctan(ax)\\&\quad+330\arctan(ax)^2+540a^2x^2\arctan(ax)^2\\&\quad+270a^4x^4\arctan(ax)^2+60a^6x^6\arctan(ax)^2\\&\quad+240i\arctan(ax)^3+360\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)\\&\quad-360\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)+272\log\left(1+a^2x^2\right)\\&\quad+360i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)\\&\quad+360i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)\\&\quad+180\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)\\&\quad-180\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\big)\end{aligned}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```

output
```
(c^3*(52 - (15*I)*Pi^3 + 58*a^2*x^2 + 6*a^4*x^4 - 660*a*x*ArcTan[a*x] - 14
0*a^3*x^3*ArcTan[a*x] - 24*a^5*x^5*ArcTan[a*x] + 330*ArcTan[a*x]^2 + 540*a
^2*x^2*ArcTan[a*x]^2 + 270*a^4*x^4*ArcTan[a*x]^2 + 60*a^6*x^6*ArcTan[a*x]^
2 + (240*I)*ArcTan[a*x]^3 + 360*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x
])] - 360*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 272*Log[1 + a^2*x
^2] + (360*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (360*I)*Arc
Tan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 180*PolyLog[3, E^((-2*I)*Arc
Tan[a*x])] - 180*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/360
```

3.278.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^3}{x} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^5\arctan(ax)^2+3a^4c^3x^3\arctan(ax)^2+3a^2c^3x\arctan(ax)^2+\frac{c^3\arctan(ax)^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a^6c^3x^6\arctan(ax)^2-\frac{1}{15}a^5c^3x^5\arctan(ax)+\frac{3}{4}a^4c^3x^4\arctan(ax)^2+\frac{1}{60}a^4c^3x^4-\frac{7}{18}a^3c^3x^3\arctan(ax)+\frac{3}{2}a^2c^3x^2\arctan(ax)^2+\frac{29}{180}a^2c^3x^2+\frac{34}{45}c^3\log\left(a^2x^2+1\right)+2c^3\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-ic^3\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+ic^3\arctan(ax)\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{11}{6}ac^3x\arctan(ax)+\frac{11}{12}c^3\arctan(ax)^2-\frac{1}{2}c^3\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{1}{2}c^3\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```

output
```
(29*a^2*c^3*x^2)/180 + (a^4*c^3*x^4)/60 - (11*a*c^3*x*ArcTan[a*x])/6 - (7*
a^3*c^3*x^3*ArcTan[a*x])/18 - (a^5*c^3*x^5*ArcTan[a*x])/15 + (11*c^3*ArcTa
n[a*x]^2)/12 + (3*a^2*c^3*x^2*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^4*ArcTan[a*x
]^2)/4 + (a^6*c^3*x^6*ArcTan[a*x]^2)/6 + 2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2
/(1 + I*a*x)] + (34*c^3*Log[1 + a^2*x^2])/45 - I*c^3*ArcTan[a*x]*PolyLog[2
, 1 - 2/(1 + I*a*x)] + I*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] -
(c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c^3*PolyLog[3, -1 + 2/(1 + I*a*x)
])/2
```

3.278.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.278.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 68.15 (sec) , antiderivative size = 1405, normalized size of antiderivative = 4.90

method	result	size
derivativedivides	Expression too large to display	1405
default	Expression too large to display	1405
parts	Expression too large to display	1948

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^6*c^3*x^6*arctan(a*x)^2+3/4*a^4*c^3*x^4*arctan(a*x)^2+3/2*a^2*c^3*x^
2*arctan(a*x)^2+c^3*arctan(a*x)^2*ln(a*x)-1/6*c^3*(-3*I*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+2*arctan(a*x)
*(a*x-I)*(I+a*x)^4+3*I*arctan(a*x)*(a*x-I)^2+4*arctan(a*x)*(a*x-I)^3*(I+a*
x)^2-11/2*arctan(a*x)^2-5/3*arctan(a*x)*(a*x-I)^3+23/15*I*(I+a*x)+2/5*I*(I
+a*x)^3+6*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-6*arctan(a*x)^2*ln(1
-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2)+1)+6*arctan(a*x)*(a*x-I)+2/5*arctan(a*x)*(a*x-I)^5-12*polylog(3,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-12*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/10*(I+a
*x)^4-4*arctan(a*x)*(a*x-I)^2*(I+a*x)^3+3*polylog(3,-(1+I*a*x)^2/(a^2*x^2+
1))+136/15*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-2*arctan(a*x)*(a*x-I)^4*(I+a*x)-5
*arctan(a*x)*(a*x-I)*(I+a*x)^2-8*I*arctan(a*x)*(a*x-I)*(I+a*x)^3+3*I*Pi*cs
gn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^
2-3*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a
rctan(a*x)^2-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x
^2+1)+1))^3*arctan(a*x)^2-8*I*arctan(a*x)*(a*x-I)^3*(I+a*x)-6*I*arctan(a*x
)*(a*x-I)*(I+a*x)+12*I*arctan(a*x)*(a*x-I)^2*(I+a*x)^2+3*I*Pi*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))^2*arctan(a*x)^2+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)...
```

3.278.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x, x)
```

3.278.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x} \, dx = c^3 \left(\int \frac{\operatorname{atan}^2(ax)}{x} \, dx + \int 3a^2x \operatorname{atan}^2(ax) \, dx + \int 3a^4x^3 \operatorname{atan}^2(ax) \, dx + \int a^6x^5 \operatorname{atan}^2(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x,x)
```

output
```
c**3*(Integral(atan(a*x)**2/x, x) + Integral(3*a**2*x*atan(a*x)**2, x) + I
ntegral(3*a**4*x**3*atan(a*x)**2, x) + Integral(a**6*x**5*atan(a*x)**2, x)
)
```

3.278.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
36*a^8*c^3*integrate(1/48*x^8*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*a^8*c^3*
integrate(1/48*x^8*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^8*c^3*integr
ate(1/48*x^8*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^7*c^3*integrate(1/48
*x^7*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^6*c^3*integrate(1/48*x^6*arctan
(a*x)^2/(a^2*x^3 + x), x) + 12*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)
^2/(a^2*x^3 + x), x) + 9*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*
x^3 + x), x) - 18*a^5*c^3*integrate(1/48*x^5*arctan(a*x)/(a^2*x^3 + x), x)
 + 216*a^4*c^3*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + 18*a^4
*c^3*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 18*a^4*c^3*
integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 36*a^3*c^3*integra
te(1/48*x^3*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^2*c^3*integrate(1/48*x^2
*arctan(a*x)^2/(a^2*x^3 + x), x) + 1/24*c^3*log(a^2*x^2 + 1)^3 + 36*c^3*in
tegrate(1/48*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*c^3*integrate(1/48*log(a^
2*x^2 + 1)^2/(a^2*x^3 + x), x) + 1/48*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*
a^2*c^3*x^2)*arctan(a*x)^2 - 1/192*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2
*c^3*x^2)*log(a^2*x^2 + 1)^2
```

3.278.8 Giac [F]

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)^2}{x}\,dx = \int \frac{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^3}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x, x)
```

3.279 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2} \, dx$

3.279.1 Optimal result

Integrand size = 22, antiderivative size = 251

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2} \, dx = {} & \frac{7}{10}a^2c^3x + \frac{1}{30}a^4c^3x^3 - \frac{7}{10}ac^3\arctan(ax) \\ & - \frac{4}{5}a^3c^3x^2\arctan(ax) - \frac{1}{10}a^5c^3x^4\arctan(ax) \\ & + \frac{6}{5}iac^3\arctan(ax)^2 - \frac{c^3\arctan(ax)^2}{x} \\ & + 3a^2c^3x\arctan(ax)^2 + a^4c^3x^3\arctan(ax)^2 \\ & + \frac{1}{5}a^6c^3x^5\arctan(ax)^2 + \frac{22}{5}ac^3\arctan(ax)\log\left(\frac{2}{1+iax}\right) \\ & + 2ac^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) \\ & - iac^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ & + \frac{11}{5}iac^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
7/10*a^2*c^3*x+1/30*a^4*c^3*x^3-7/10*a*c^3*arctan(a*x)-4/5*a^3*c^3*x^2*arc
tan(a*x)-1/10*a^5*c^3*x^4*arctan(a*x)+6/5*I*a*c^3*arctan(a*x)^2-c^3*arctan
(a*x)^2/x+3*a^2*c^3*x*arctan(a*x)^2+a^4*c^3*x^3*arctan(a*x)^2+1/5*a^6*c^3*
x^5*arctan(a*x)^2+22/5*a*c^3*arctan(a*x)*ln(2/(1+I*a*x))+2*a*c^3*arctan(a*
x)*ln(2-2/(1-I*a*x))-I*a*c^3*polylog(2,-1+2/(1-I*a*x))+11/5*I*a*c^3*polylo
g(2,1-2/(1+I*a*x))
```

3.279.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.80

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2}\,dx$$
$$= \frac{c^3\left(21a^2x^2+a^4x^4-21ax\arctan(ax)-24a^3x^3\arctan(ax)-3a^5x^5\arctan(ax)-30\arctan(ax)^2-96iax\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c^3*(21*a^2*x^2 + a^4*x^4 - 21*a*x*ArcTan[a*x] - 24*a^3*x^3*ArcTan[a*x] -
 3*a^5*x^5*ArcTan[a*x] - 30*ArcTan[a*x]^2 - (96*I)*a*x*ArcTan[a*x]^2 + 90*
a^2*x^2*ArcTan[a*x]^2 + 30*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2
 + 60*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 132*a*x*ArcTan[a*x]
*Log[1 + E^((2*I)*ArcTan[a*x])] - (66*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a
*x])] - (30*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(30*x)
```

3.279.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)^3}{x^2}\,dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^6c^3x^4\arctan(ax)^2+3a^4c^3x^2\arctan(ax)^2+3a^2c^3\arctan(ax)^2+\frac{c^3\arctan(ax)^2}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^6c^3x^5\arctan(ax)^2-\frac{1}{10}a^5c^3x^4\arctan(ax)+a^4c^3x^3\arctan(ax)^2+\frac{1}{30}a^4c^3x^3-\frac{4}{5}a^3c^3x^2\arctan(ax)+3a^2c^3x\arctan(ax)^2+\frac{7}{10}a^2c^3x+\frac{6}{5}iac^3\arctan(ax)^2-\frac{7}{10}ac^3\arctan(ax)-\frac{c^3\arctan(ax)^2}{x}+\frac{22}{5}ac^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)+2ac^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-iac^3\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{11}{5}iac^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```

output
```
(7*a^2*c^3*x)/10 + (a^4*c^3*x^3)/30 - (7*a*c^3*ArcTan[a*x])/10 - (4*a^3*c^
3*x^2*ArcTan[a*x])/5 - (a^5*c^3*x^4*ArcTan[a*x])/10 + ((6*I)/5)*a*c^3*ArcT
an[a*x]^2 - (c^3*ArcTan[a*x]^2)/x + 3*a^2*c^3*x*ArcTan[a*x]^2 + a^4*c^3*x^
3*ArcTan[a*x]^2 + (a^6*c^3*x^5*ArcTan[a*x]^2)/5 + (22*a*c^3*ArcTan[a*x]*Lo
g[2/(1 + I*a*x)])/5 + 2*a*c^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^3
*PolyLog[2, -1 + 2/(1 - I*a*x)] + ((11*I)/5)*a*c^3*PolyLog[2, 1 - 2/(1 + I
*a*x)]
```

3.279.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.279.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.28

method	result
derivativedivides	$a\left(\frac{a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\arctan\right.}{}\right.$
default	$a\left(\frac{a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\arctan\right.}{}\right.$
parts	$\frac{a^6c^3x^5\arctan(ax)^2}{5}+a^4c^3x^3\arctan(ax)^2+3a^2c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{x}-\frac{2c^3\left(\frac{\arctan(a}{4}\right.}{}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*a^5*c^3*x^5*arctan(a*x)^2+a^3*c^3*x^3*arctan(a*x)^2+3*a*c^3*x*arcta
n(a*x)^2-c^3*arctan(a*x)^2/a/x-2/5*c^3*(1/4*arctan(a*x)*a^4*x^4+2*a^2*arct
an(a*x)*x^2+8*arctan(a*x)*ln(a^2*x^2+1)-5*arctan(a*x)*ln(a*x)-1/12*a^3*x^3
-7/4*a*x+7/4*arctan(a*x)-5/2*I*ln(a*x)*ln(1+I*a*x)+5/2*I*ln(a*x)*ln(1-I*a*
x)-5/2*I*dilog(1+I*a*x)+5/2*I*dilog(1-I*a*x)+4*I*(ln(a*x-I)*ln(a^2*x^2+1)-
dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-4*I*(l
n(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/
2*ln(I+a*x)^2)))
```

3.279.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x^2, x)
```

3.279.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx = c^3\left(\int 3a^2\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^2}\,dx + \int 3a^4x^2\operatorname{atan}^2(ax)\,dx + \int a^6x^4\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) +
Integral(3*a**4*x**2*atan(a*x)**2, x) + Integral(a**6*x**4*atan(a*x)**2, x
))
```

3.279.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/80*(4*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x)
^2 - (a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*log(a^2*x^2 +
1)^2 + 80*(60*a^8*c^3*integrate(1/80*x^8*arctan(a*x)^2/(a^2*x^4 + x^2), x)
 + 5*a^8*c^3*integrate(1/80*x^8*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 4
*a^8*c^3*integrate(1/80*x^8*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 8*a^7*c
^3*integrate(1/80*x^7*arctan(a*x)/(a^2*x^4 + x^2), x) + 240*a^6*c^3*integr
ate(1/80*x^6*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 20*a^6*c^3*integrate(1/80
*x^6*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 20*a^6*c^3*integrate(1/80*x^
6*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 40*a^5*c^3*integrate(1/80*x^5*arc
tan(a*x)/(a^2*x^4 + x^2), x) + 360*a^4*c^3*integrate(1/80*x^4*arctan(a*x)^
2/(a^2*x^4 + x^2), x) + 30*a^4*c^3*integrate(1/80*x^4*log(a^2*x^2 + 1)^2/(
a^2*x^4 + x^2), x) + 60*a^4*c^3*integrate(1/80*x^4*log(a^2*x^2 + 1)/(a^2*x
^4 + x^2), x) + a*c^3*arctan(a*x)^3 - 120*a^3*c^3*integrate(1/80*x^3*arcta
n(a*x)/(a^2*x^4 + x^2), x) + 20*a^2*c^3*integrate(1/80*x^2*log(a^2*x^2 + 1
)^2/(a^2*x^4 + x^2), x) - 20*a^2*c^3*integrate(1/80*x^2*log(a^2*x^2 + 1)/(
a^2*x^4 + x^2), x) + 40*a*c^3*integrate(1/80*x*arctan(a*x)/(a^2*x^4 + x^2)
, x) + 60*c^3*integrate(1/80*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 5*c^3*int
egrate(1/80*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

3.279.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2} \, dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3}{x^2} \, dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^2, x)
```

3.280 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx$

3.280.1 Optimal result

Integrand size = 22, antiderivative size = 299

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx &= \frac{1}{12}a^4c^3x^2 - \frac{ac^3\arctan(ax)}{x} - \frac{5}{2}a^3c^3x\arctan(ax) \\ &\quad - \frac{1}{6}a^5c^3x^3\arctan(ax) + \frac{3}{4}a^2c^3\arctan(ax)^2 - \frac{c^3\arctan(ax)^2}{2x^2} \\ &\quad + \frac{3}{2}a^4c^3x^2\arctan(ax)^2 + \frac{1}{4}a^6c^3x^4\arctan(ax)^2 \\ &\quad + 6a^2c^3\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) \\ &\quad + a^2c^3\log(x) + \frac{2}{3}a^2c^3\log\left(1+a^2x^2\right) \\ &\quad - 3ia^2c^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\ &\quad + 3ia^2c^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) \\ &\quad - \frac{3}{2}a^2c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) \\ &\quad + \frac{3}{2}a^2c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
1/12*a^4*c^3*x^2-a*c^3*arctan(a*x)/x-5/2*a^3*c^3*x*arctan(a*x)-1/6*a^5*c^3
*x^3*arctan(a*x)+3/4*a^2*c^3*arctan(a*x)^2-1/2*c^3*arctan(a*x)^2/x^2+3/2*a
^4*c^3*x^2*arctan(a*x)^2+1/4*a^6*c^3*x^4*arctan(a*x)^2-6*a^2*c^3*arctan(a*
x)^2*arctanh(-1+2/(1+I*a*x))+a^2*c^3*ln(x)+2/3*a^2*c^3*ln(a^2*x^2+1)-3*I*a
^2*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+3*I*a^2*c^3*arctan(a*x)*polylo
g(2,-1+2/(1+I*a*x))-3/2*a^2*c^3*polylog(3,1-2/(1+I*a*x))+3/2*a^2*c^3*polyl
og(3,-1+2/(1+I*a*x))
```

3.280.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx$$
$$= \frac{c^3\left(2a^2x^2 - 3ia^2\pi^3x^2 + 2a^4x^4 - 24ax\arctan(ax) - 60a^3x^3\arctan(ax) - 4a^5x^5\arctan(ax) - 12\arctan(ax\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```

output
```
(c^3*(2*a^2*x^2 - (3*I)*a^2*Pi^3*x^2 + 2*a^4*x^4 - 24*a*x*ArcTan[a*x] - 60
*a^3*x^3*ArcTan[a*x] - 4*a^5*x^5*ArcTan[a*x] - 12*ArcTan[a*x]^2 + 18*a^2*x
^2*ArcTan[a*x]^2 + 36*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2 + (4
8*I)*a^2*x^2*ArcTan[a*x]^3 + 72*a^2*x^2*ArcTan[a*x]^2*Log[1 - E^((-2*I)*Ar
cTan[a*x])] - 72*a^2*x^2*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 24
*a^2*x^2*Log[(a*x)/Sqrt[1 + a^2*x^2]] + 28*a^2*x^2*Log[1 + a^2*x^2] + (72*
I)*a^2*x^2*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (72*I)*a^2*x^2
*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 36*a^2*x^2*PolyLog[3, E^
((-2*I)*ArcTan[a*x])] - 36*a^2*x^2*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(2
4*x^2)
```

3.280.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^3}{x^3}\,dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^6c^3x^3\arctan(ax)^2+3a^4c^3x\arctan(ax)^2+\frac{3a^2c^3\arctan(ax)^2}{x}+\frac{c^3\arctan(ax)^2}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}a^6c^3x^4\arctan(ax)^2-\frac{1}{6}a^5c^3x^3\arctan(ax)+\frac{3}{2}a^4c^3x^2\arctan(ax)^2+\frac{1}{12}a^4c^3x^2-\frac{5}{2}a^3c^3x\arctan(ax)+6a^2c^3\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)-3ia^2c^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+3ia^2c^3\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{iax+1}-1\right)+\frac{3}{4}a^2c^3\arctan(ax)^2-\frac{3}{2}a^2c^3\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{3}{2}a^2c^3\operatorname{PolyLog}\left(3,\frac{2}{iax+1}-1\right)+\frac{2}{3}a^2c^3\log\left(a^2x^2+1\right)+a^2c^3\log(x)-\frac{c^3\arctan(ax)^2}{2x^2}-\frac{ac^3\arctan(ax)}{x}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```

output
```
(a^4*c^3*x^2)/12 - (a*c^3*ArcTan[a*x])/x - (5*a^3*c^3*x*ArcTan[a*x])/2 - (
a^5*c^3*x^3*ArcTan[a*x])/6 + (3*a^2*c^3*ArcTan[a*x]^2)/4 - (c^3*ArcTan[a*x
]^2)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^2)/2 + (a^6*c^3*x^4*ArcTan[a*x]^
2)/4 + 6*a^2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c^3*Log[x]
 + (2*a^2*c^3*Log[1 + a^2*x^2])/3 - (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, 1
 - 2/(1 + I*a*x)] + (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x
)] - (3*a^2*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*c^3*PolyLog[3, -
1 + 2/(1 + I*a*x)])/2
```

3.280.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.280.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 67.32 (sec) , antiderivative size = 1318, normalized size of antiderivative = 4.41

method	result	size
derivativedivides	Expression too large to display	1318
default	Expression too large to display	1318
parts	Expression too large to display	1754

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/4*a^4*c^3*x^4*arctan(a*x)^2+3/2*a^2*c^3*x^2*arctan(a*x)^2-1/2*c^3*a
rctan(a*x)^2/a^2/x^2+3*c^3*arctan(a*x)^2*ln(a*x)-1/2*c^3*(-3*I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-3/2*ar
ctan(a*x)^2+1/3*arctan(a*x)*(a*x-I)^3+6*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*
x^2+1)-1)-6*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^
2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+4*arctan(a*x)*(a*x-I)-12*polylog(3,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-12*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*ln(
(1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+3*polyl
og(3,-(1+I*a*x)^2/(a^2*x^2+1))+14/3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+arctan(a
*x)*(a*x-I)*(I+a*x)^2+3*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+arctan(a*x)*(I*a*
x-(a^2*x^2+1)^(1/2)+1)/a/x-2*I*arctan(a*x)*(a*x-I)*(I+a*x)+arctan(a*x)*(I*
a*x+(a^2*x^2+1)^(1/2)+1)/a/x+3*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x
)^2+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*
x)^2+I*arctan(a*x)*(a*x-I)^2-1/6*(I+a*x)^2-arctan(a*x)*(a*x-I)^2*(I+a*x...
```

3.280.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^3}\,dx=\int \frac{(a^2cx^2+c)^3\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x^3, x)
```

3.280.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^3}\,dx=c^3\left(\int \frac{\operatorname{atan}^2{(ax)}}{x^3}\,dx+\int \frac{3a^2\operatorname{atan}^2{(ax)}}{x}\,dx+\int 3a^4x\operatorname{atan}^2{(ax)}\,dx+\int a^6x^3\operatorname{atan}^2{(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**3,x)
```

output
```
c**3*(Integral(atan(a*x)**2/x**3, x) + Integral(3*a**2*atan(a*x)**2/x, x)
+ Integral(3*a**4*x*atan(a*x)**2, x) + Integral(a**6*x**3*atan(a*x)**2, x)
)
```

3.280.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^3}\,dx=\int \frac{(a^2cx^2+c)^3\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/64*(4*(192*a^8*c^3*integrate(1/16*x^8*arctan(a*x)^2/(a^2*x^5 + x^3), x)
+ 16*a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 1
6*a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 32*a^7
*c^3*integrate(1/16*x^7*arctan(a*x)/(a^2*x^5 + x^3), x) + 768*a^6*c^3*inte
grate(1/16*x^6*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^6*c^3*integrate(1/
16*x^6*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 96*a^6*c^3*integrate(1/16*
x^6*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 192*a^5*c^3*integrate(1/16*x^5*
arctan(a*x)/(a^2*x^5 + x^3), x) + 1152*a^4*c^3*integrate(1/16*x^4*arctan(a
*x)^2/(a^2*x^5 + x^3), x) + a^2*c^3*log(a^2*x^2 + 1)^3 + 768*a^2*c^3*integ
rate(1/16*x^2*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^2*c^3*integrate(1/1
6*x^2*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 32*a^2*c^3*integrate(1/16*x
^2*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) + 64*a*c^3*integrate(1/16*x*arctan
(a*x)/(a^2*x^5 + x^3), x) + 192*c^3*integrate(1/16*arctan(a*x)^2/(a^2*x^5
+ x^3), x) + 16*c^3*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x))
*x^2 + 4*(a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^2 - (a^6*c^3*x^
6 + 6*a^4*c^3*x^4 - 2*c^3)*log(a^2*x^2 + 1)^2)/x^2
```

3.280.8 Giac [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^3}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^3}\,dx=\int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^3, x)
```

3.281 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4} dx$

3.281.1 Optimal result

Integrand size = 22, antiderivative size = 250

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4} dx = &-\frac{a^2c^3}{3x}+\frac{1}{3}a^4c^3x-\frac{2}{3}a^3c^3\arctan(ax)-\frac{ac^3\arctan(ax)}{3x^2}\\&-\frac{1}{3}a^5c^3x^2\arctan(ax)-\frac{c^3\arctan(ax)^2}{3x^3}\\&-\frac{3a^2c^3\arctan(ax)^2}{x}+3a^4c^3x\arctan(ax)^2\\&+\frac{1}{3}a^6c^3x^3\arctan(ax)^2+\frac{16}{3}a^3c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\&+\frac{16}{3}a^3c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\&-\frac{8}{3}ia^3c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\&+\frac{8}{3}ia^3c^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-1/3*a^2*c^3/x+1/3*a^4*c^3*x-2/3*a^3*c^3*arctan(a*x)-1/3*a*c^3*arctan(a*x)
/x^2-1/3*a^5*c^3*x^2*arctan(a*x)-1/3*c^3*arctan(a*x)^2/x^3-3*a^2*c^3*arcta
n(a*x)^2/x+3*a^4*c^3*x*arctan(a*x)^2+1/3*a^6*c^3*x^3*arctan(a*x)^2+16/3*a^
3*c^3*arctan(a*x)*ln(2/(1+I*a*x))+16/3*a^3*c^3*arctan(a*x)*ln(2-2/(1-I*a*x
))-8/3*I*a^3*c^3*polylog(2,-1+2/(1-I*a*x))+8/3*I*a^3*c^3*polylog(2,1-2/(1+
I*a*x))
```

3.281.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4}\,dx$$
$$= \frac{c^3\left(-a^2x^2+a^4x^4-ax\arctan(ax)-2a^3x^3\arctan(ax)-a^5x^5\arctan(ax)-\arctan(ax)^2-9a^2x^2\arctan(a\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

output
```
(c^3*(-(a^2*x^2) + a^4*x^4 - a*x*ArcTan[a*x] - 2*a^3*x^3*ArcTan[a*x] - a^5
*x^5*ArcTan[a*x] - ArcTan[a*x]^2 - 9*a^2*x^2*ArcTan[a*x]^2 - (16*I)*a^3*x^
3*ArcTan[a*x]^2 + 9*a^4*x^4*ArcTan[a*x]^2 + a^6*x^6*ArcTan[a*x]^2 + 16*a^3
*x^3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 16*a^3*x^3*ArcTan[a*x]*L
og[1 + E^((2*I)*ArcTan[a*x])] - (8*I)*a^3*x^3*PolyLog[2, -E^((2*I)*ArcTan[
a*x])] - (8*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

3.281.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)^3}{x^4}\,dx$$
$$\Big\downarrow \; 5483$$
$$\int \left(a^6c^3x^2\arctan(ax)^2+3a^4c^3\arctan(ax)^2+\frac{3a^2c^3\arctan(ax)^2}{x^2}+\frac{c^3\arctan(ax)^2}{x^4}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{1}{3}a^6c^3x^3\arctan(ax)^2-\frac{1}{3}a^5c^3x^2\arctan(ax)+3a^4c^3x\arctan(ax)^2+\frac{1}{3}a^4c^3x-\frac{2}{3}a^3c^3\arctan(ax)+\frac{16}{3}a^3c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)+\frac{16}{3}a^3c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{8}{3}ia^3c^3\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{8}{3}ia^3c^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)-\frac{3a^2c^3\arctan(ax)^2}{x}-\frac{a^2c^3}{3x}-\frac{c^3\arctan(ax)^2}{3x^3}-\frac{ac^3\arctan(ax)}{3x^2}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/3*(a^2*c^3)/x + (a^4*c^3*x)/3 - (2*a^3*c^3*ArcTan[a*x])/3 - (a*c^3*ArcT
an[a*x])/(3*x^2) - (a^5*c^3*x^2*ArcTan[a*x])/3 - (c^3*ArcTan[a*x]^2)/(3*x^
3) - (3*a^2*c^3*ArcTan[a*x]^2)/x + 3*a^4*c^3*x*ArcTan[a*x]^2 + (a^6*c^3*x^
3*ArcTan[a*x]^2)/3 + (16*a^3*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/3 + (16*a
^3*c^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((8*I)/3)*a^3*c^3*PolyLog[2
, -1 + 2/(1 - I*a*x)] + ((8*I)/3)*a^3*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

3.281.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.281.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.29

method	result
derivativedivides	$a^3\left(\frac{a^3c^3x^3\arctan(ax)^2}{3}+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{3a^3x^3}-\frac{3c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\frac{a^2\arctan(ax)x^2}{2}\right.}{}\right.$
default	$a^3\left(\frac{a^3c^3x^3\arctan(ax)^2}{3}+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{3a^3x^3}-\frac{3c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\frac{a^2\arctan(ax)x^2}{2}\right.}{}\right.$
parts	$\frac{a^6c^3x^3\arctan(ax)^2}{3}+3a^4c^3x\arctan(ax)^2-\frac{3a^2c^3\arctan(ax)^2}{x}-\frac{c^3\arctan(ax)^2}{3x^3}-\frac{2c^3\left(\frac{a^5\arctan(ax)x^2}{2}+\right.}{}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/3*a^3*c^3*x^3*arctan(a*x)^2+3*a*c^3*x*arctan(a*x)^2-1/3*c^3*arctan(
a*x)^2/a^3/x^3-3*c^3*arctan(a*x)^2/a/x-2/3*c^3*(1/2*a^2*arctan(a*x)*x^2+8*
arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2-8*arctan(a*x)*ln(a*x)-1/
2*a*x+arctan(a*x)+1/2/a/x-4*I*ln(a*x)*ln(1+I*a*x)+4*I*ln(a*x)*ln(1-I*a*x)-
4*I*dilog(1+I*a*x)+4*I*dilog(1-I*a*x)+4*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-
1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-4*I*(ln(I+a*x
)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+
a*x)^2)))
```

3.281.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^4}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x^4, x)
```

3.281.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4}\,dx = c^3\left(\int 3a^4\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^4}\,dx + \int \frac{3a^2\operatorname{atan}^2(ax)}{x^2}\,dx + \int a^6x^2\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**4,x)
```

output
```
c**3*(Integral(3*a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) +
Integral(3*a**2*atan(a*x)**2/x**2, x) + Integral(a**6*x**2*atan(a*x)**2, x
))
```

3.281.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(24*(72*a^8*c^3*integrate(1/48*x^8*arctan(a*x)^2/(a^2*x^6 + x^4), x)
+ 6*a^8*c^3*integrate(1/48*x^8*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 8*
a^8*c^3*integrate(1/48*x^8*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 16*a^7*c
^3*integrate(1/48*x^7*arctan(a*x)/(a^2*x^6 + x^4), x) + 288*a^6*c^3*integr
ate(1/48*x^6*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 24*a^6*c^3*integrate(1/48
*x^6*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 72*a^6*c^3*integrate(1/48*x^
6*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 3*a^3*c^3*arctan(a*x)^3 - 144*a^5
*c^3*integrate(1/48*x^5*arctan(a*x)/(a^2*x^6 + x^4), x) + 36*a^4*c^3*integ
rate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 72*a^4*c^3*integrat
e(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 144*a^3*c^3*integrate(1/
48*x^3*arctan(a*x)/(a^2*x^6 + x^4), x) + 288*a^2*c^3*integrate(1/48*x^2*ar
ctan(a*x)^2/(a^2*x^6 + x^4), x) + 24*a^2*c^3*integrate(1/48*x^2*log(a^2*x^
2 + 1)^2/(a^2*x^6 + x^4), x) - 8*a^2*c^3*integrate(1/48*x^2*log(a^2*x^2 +
1)/(a^2*x^6 + x^4), x) + 16*a*c^3*integrate(1/48*x*arctan(a*x)/(a^2*x^6 +
x^4), x) + 72*c^3*integrate(1/48*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 6*c^3
*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 + 4*(a^6*c^3*x
^6 + 9*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan(a*x)^2 - (a^6*c^3*x^6 + 9
*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*log(a^2*x^2 + 1)^2)/x^3
```

3.281.8 Giac [F]

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)^2}{x^4}\,dx=\int \frac{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)^2}{x^4}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^3}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^4, x)
```

3.282 $\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2}\,dx$

3.282.1 Optimal result

Integrand size = 22, antiderivative size = 166

$$\begin{aligned}\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2}\,dx &= \frac{x}{3a^4c}-\frac{\arctan(ax)}{3a^5c}-\frac{x^2\arctan(ax)}{3a^3c}-\frac{4i\arctan(ax)^2}{3a^5c}\\ &\quad-\frac{x\arctan(ax)^2}{a^4c}+\frac{x^3\arctan(ax)^2}{3a^2c}+\frac{\arctan(ax)^3}{3a^5c}\\ &\quad-\frac{8\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{3a^5c}-\frac{4i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a^5c}\end{aligned}$$

output
```
1/3*x/a^4/c-1/3*arctan(a*x)/a^5/c-1/3*x^2*arctan(a*x)/a^3/c-4/3*I*arctan(a
*x)^2/a^5/c-x*arctan(a*x)^2/a^4/c+1/3*x^3*arctan(a*x)^2/a^2/c+1/3*arctan(a
*x)^3/a^5/c-8/3*arctan(a*x)*ln(2/(1+I*a*x))/a^5/c-4/3*I*polylog(2,1-2/(1+I
*a*x))/a^5/c
```

3.282.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int \frac{x^4\arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{ax+(4i-3ax+a^3x^3)\arctan(ax)^2+\arctan(ax)^3-\arctan(ax)\left(1+a^2x^2+8\log\left(1+e^{2i\arctan(ax)}\right)\right)+4}{3a^5c}$$

input
```
Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
(a*x + (4*I - 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + ArcTan[a*x]^3 - ArcTan[a*x]
*(1 + a^2*x^2 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((
2*I)*ArcTan[a*x])])/(3*a^5*c)
```

3.282.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5451, 27, 5361, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^2}{a^2cx^2+c}dx$$

$$\Big\downarrow \; 5451$$

$$\frac{\int x^2\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int x^2\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 5361$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\int \frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 5451$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{\int \arctan(ax)^2dx}{a^2}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \; 5345$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \; 5361$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^3}{3a^3}}{a^2c}$$

$$\Big\downarrow \text{5455}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5379}$$

3.282. $\int\frac{x^4\arctan(ax)^2}{c+a^2cx^2}dx$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow\ 2849$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow\ 2752$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a
*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1
+ I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2))/3)/(a^2*c
) - (-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x
]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(
1 + I*a*x)])/a)/a))/a^2)/(a^2*c)
```

3.282. $\int \frac{x^4\arctan(ax)^2}{c+a^2cx^2}\,dx$

3.282.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.282.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^2 a^3x^3}{3c}-\frac{\arctan(ax)^2 ax}{c}+\frac{\arctan(ax)^3}{c}-\frac{2\left(\frac{a^2\arctan(ax)x^2}{2}-2\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}-i\left(\ln(ax-i)\ln\right.\right.}{}}{}$
default	$\frac{\frac{\arctan(ax)^2 a^3x^3}{3c}-\frac{\arctan(ax)^2 ax}{c}+\frac{\arctan(ax)^3}{c}-\frac{2\left(\frac{a^2\arctan(ax)x^2}{2}-2\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}-i\left(\ln(ax-i)\ln\right.\right.}{}}{}$
parts	$\frac{x^3\arctan(ax)^2}{3a^2c}-\frac{x\arctan(ax)^2}{a^4c}+\frac{\arctan(ax)^3}{a^5c}-\frac{2\left(\frac{\arctan(ax)^3}{3a^5}+\frac{\frac{a^2\arctan(ax)x^2}{2}-2\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\right.}{}\right.}{}$

input
```
int(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/3/c*arctan(a*x)^2*a^3*x^3-1/c*arctan(a*x)^2*a*x+1/c*arctan(a*x)^3
-2/3/c*(1/2*a^2*arctan(a*x)*x^2-2*arctan(a*x)*ln(a^2*x^2+1)-1/2*a*x+1/2*ar
ctan(a*x)-I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1
/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x
-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)+arctan(a*x)^3))
```

3.282.5 Fricas [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2} \, dx = \int \frac{x^4 \arctan\left(ax\right)^2}{a^2cx^2+c} \, dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^4*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.282.6 Sympy [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2} \, dx = \frac{\int \frac{x^4 \operatorname{atan}^2(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**4*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

3.282.7 Maxima [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2} \, dx = \int \frac{x^4 \arctan\left(ax\right)^2}{a^2cx^2+c} \, dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/48*(4*(432*a^4*integrate(1/48*x^4*arctan(a*x)^2/(a^6*c*x^2 + a^4*c), x)
+ 36*a^4*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x) + 4
8*a^4*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) - 96*a^3
*integrate(1/48*x^3*arctan(a*x)/(a^6*c*x^2 + a^4*c), x) - 144*a^2*integrat
e(1/48*x^2*log(a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) + 288*a*integrate(1/48
*x*arctan(a*x)/(a^6*c*x^2 + a^4*c), x) - arctan(a*x)^3/(a^5*c) - 36*integr
ate(1/48*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x))*a^5*c + 4*(a^3*x^3 -
3*a*x)*arctan(a*x)^2 + 8*arctan(a*x)^3 - (a^3*x^3 - 3*a*x)*log(a^2*x^2 + 1
)^2)/(a^5*c)
```

3.282.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^4 \arctan(ax)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.283 $\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2}\,dx$

3.283.1 Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2}\,dx = -\frac{x\arctan(ax)}{a^3c}+\frac{\arctan(ax)^2}{2a^4c}+\frac{x^2\arctan(ax)^2}{2a^2c}+\frac{i\arctan(ax)^3}{3a^4c}+\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c}+\frac{\log(1+a^2x^2)}{2a^4c}+\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^4c}+\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c}$$

output
```
-x*arctan(a*x)/a^3/c+1/2*arctan(a*x)^2/a^4/c+1/2*x^2*arctan(a*x)^2/a^2/c+1
/3*I*arctan(a*x)^3/a^4/c+arctan(a*x)^2*ln(2/(1+I*a*x))/a^4/c+1/2*ln(a^2*x^
2+1)/a^4/c+I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^4/c+1/2*polylog(3,1-2/
(1+I*a*x))/a^4/c
```

3.283.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{-ax\arctan(ax)+\frac{1}{2}(1+a^2x^2)\arctan(ax)^2-\frac{1}{3}i\arctan(ax)^3+\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)-\log\left(\ldots\right.}{a^4c}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
(-(a*x*ArcTan[a*x]) + ((1 + a^2*x^2)*ArcTan[a*x]^2)/2 - (I/3)*ArcTan[a*x]^
3 + ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - Log[1/Sqrt[1 + a^2*x^2]
] - I*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, -E^((2*I
)*ArcTan[a*x])]/2)/(a^4*c)
```

3.283.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5451, 27, 5361, 5451, 5345, 240, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int \arctan(ax)dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5455}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c}$$

$$\Big\downarrow \text{5529}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

3.283. $\int\frac{x^3\arctan(ax)^2}{c+a^2cx^2}\,dx$

output
```
((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[
1 + a^2*x^2]/(2*a))/a^2))/(a^2*c) - (((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcT
an[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 -
2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)/(a^2*c)
```

3.283.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.283. $\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2} \, dx$

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.283.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.90 (sec) , antiderivative size = 844, normalized size of antiderivative = 4.99

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^2 a^2 x^2}{2c}-\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c}-\frac{-\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+i\arctan(ax)\operatorname{polylog}\left(2,-\frac{(iax+1)^2}{a^2x^2+1}\right)-\frac{\operatorname{polylog}\left(3,-\frac{(}{}\right.}{2}}{}}{}$
default	$\frac{\frac{\arctan(ax)^2 a^2 x^2}{2c}-\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c}-\frac{-\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+i\arctan(ax)\operatorname{polylog}\left(2,-\frac{(iax+1)^2}{a^2x^2+1}\right)-\frac{\operatorname{polylog}\left(3,-\frac{(}{}\right.}{2}}{}}{}$
parts	$\frac{x^2\arctan(ax)^2}{2a^2c}-\frac{\arctan(ax)^2\ln(a^2x^2+1)}{2c\,a^4}-\frac{a\left(-\frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{a^5}+\frac{i\arctan(ax)\operatorname{polylog}\left(2,-\frac{(iax+1)^2}{a^2x^2+1}\right)}{a^5}-\cdots\right.}{}$

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2/c*arctan(a*x)^2*a^2*x^2-1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(
-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)*polylog(2,-(1
+I*a*x)^2/(a^2*x^2+1))-1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/12*I*arct
an(a*x)*(3*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^3-3*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-3*arctan(a*x)*Pi*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)+3*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I/((1+
I*a*x)^2/(a^2*x^2+1)+1)^2)-3*arctan(a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3+6*arctan(a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1))-3*arctan(a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2+3*arctan(a*x)*Pi*csgn(I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-6*arctan(a*x)
*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+
3*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+4*arctan(a*x)^2+12*I*ar
ctan(a*x)*ln(2)+6*I*arctan(a*x)-12-12*I*a*x)+ln((1+I*a*x)^2/(a^2*x^2+1)+1)
))
```

3.283. $\int \frac{x^3\arctan(ax)^2}{c+a^2cx^2}\,dx$

3.283.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)^2}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.283.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^2(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

3.283.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)^2}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.283.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2}\,dx = \int \frac{x^3 \arctan{(ax)}^2}{a^2cx^2 + c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^2}{c\,a^2\,x^2 + c}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.284 $\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2}\,dx$

3.284.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{i\arctan(ax)^2}{a^3c}+\frac{x\arctan(ax)^2}{a^2c}-\frac{\arctan(ax)^3}{3a^3c}+\frac{2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^3c}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}$$

output
```
I*arctan(a*x)^2/a^3/c+x*arctan(a*x)^2/a^2/c-1/3*arctan(a*x)^3/a^3/c+2*arct
an(a*x)*ln(2/(1+I*a*x))/a^3/c+I*polylog(2,1-2/(1+I*a*x))/a^3/c
```

3.284.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{-\frac{1}{3}\arctan(ax)\left((3i-3ax)\arctan(ax)+\arctan(ax)^2-6\log\left(1+e^{2i\arctan(ax)}\right)\right)-i\,\mathrm{PolyLog}\left(2,-e^{2i\arctan}\right.}{a^3c}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
(-1/3*(ArcTan[a*x]*((3*I - 3*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 6*Log[1 +
E^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])])/(a^3*c)
```

3.284.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5451, 27, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int \arctan(ax)^2dx}{a^2c}-\frac{\int \frac{\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \arctan(ax)^2dx}{a^2c}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c}-\frac{\arctan(ax)^3}{3a^3c}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\arctan(ax)^3}{3a^3c}+\frac{x\arctan(ax)^2-2a\left(-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\arctan(ax)^3}{3a^3c}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\arctan(ax)^3}{3a^3c}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\arctan(ax)^3}{3a^3c}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
-1/3*ArcTan[a*x]^3/(a^3*c) + (x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]
^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1
 + I*a*x)])/a)/a))/(a^2*c)
```

3.284.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.284. $\int\frac{x^2\arctan(ax)^2}{c+a^2cx^2}\,dx$

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.284.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(92) = 184$.

Time = 0.52 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^2 ax}{c}-\frac{\arctan(ax)^3}{c}-\frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)\right.}{4}\right.}{}}{a^3}$
default	$\frac{\frac{\arctan(ax)^2 ax}{c}-\frac{\arctan(ax)^3}{c}-\frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)\right.}{4}\right.}{}}{a^3}$
parts	$\frac{x\arctan(ax)^2}{a^2c}-\frac{\arctan(ax)^3}{a^3c}-\frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(\right.\right.}{4}\right.}{}$

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/c*arctan(a*x)^2*a*x-1/c*arctan(a*x)^3-2/c*(1/2*arctan(a*x)*ln(a^2
*x^2+1)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(
-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*
I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)-1/3*arctan(a*x)^3)
)
```

3.284.5 Fricas [F]

$$\int \frac{x^2\arctan(ax)^2}{c+a^2cx^2}\,dx=\int \frac{x^2\arctan\left(ax\right)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.284.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{c + a^2cx^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

3.284.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)^2}{c + a^2cx^2}\,dx = \int \frac{x^2 \arctan(ax)^2}{a^2cx^2 + c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/48*(4*(144*a^2*integrate(1/16*x^2*arctan(a*x)^2/(a^4*c*x^2 + a^2*c), x)
+ 12*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x) + 4
8*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^4*c*x^2 + a^2*c), x) - 96*a*i
ntegrate(1/16*x*arctan(a*x)/(a^4*c*x^2 + a^2*c), x) + arctan(a*x)^3/(a^3*c
) + 12*integrate(1/16*log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x))*a^3*c +
12*a*x*arctan(a*x)^2 - 3*a*x*log(a^2*x^2 + 1)^2 - 8*arctan(a*x)^3)/(a^3*c)
```

3.284.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{c + a^2cx^2}\,dx = \int \frac{x^2 \arctan(ax)^2}{a^2cx^2 + c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.285 $\int \frac{x\arctan(ax)^2}{c+a^2cx^2}\,dx$

3.285.1 Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{x\arctan(ax)^2}{c+a^2cx^2}\,dx = -\frac{i\arctan(ax)^3}{3a^2c}-\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^2c}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^2c}$$

output
```
-1/3*I*arctan(a*x)^3/a^2/c-arctan(a*x)^2*ln(2/(1+I*a*x))/a^2/c-I*arctan(a*
x)*polylog(2,1-2/(1+I*a*x))/a^2/c-1/2*polylog(3,1-2/(1+I*a*x))/a^2/c
```

3.285.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{x\arctan(ax)^2}{c+a^2cx^2}\,dx = -\frac{2i\arctan(ax)^3+6\arctan(ax)^2\log\left(\frac{2i}{i-ax}\right)+6i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right)+3\operatorname{PolyLog}\left(3,\frac{i+ax}{-i+ax}\right)}{6a^2c}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
-1/6*((2*I)*ArcTan[a*x]^3 + 6*ArcTan[a*x]^2*Log[(2*I)/(I - a*x)] + (6*I)*A
rcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)] + 3*PolyLog[3, (I + a*x)/(-I +
 a*x)])/(a^2*c)
```

3.285.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^2}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 5455$$

$$-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{ac}-\frac{i\arctan(ax)^3}{3a^2c}$$

$$\Big\downarrow \; 5379$$

$$-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{ac}-\frac{i\arctan(ax)^3}{3a^2c}$$

$$\Big\downarrow \; 5529$$

$$-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{ac}-\frac{i\arctan(ax)^3}{3a^2c}$$

$$\Big\downarrow \; 7164$$

$$-\frac{i\arctan(ax)^3}{3a^2c}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{ac}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
((-1/3*I)*ArcTan[a*x]^3)/(a^2*c) - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a -
 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1
- 2/(1 + I*a*x)]/(4*a)))/(a*c)
```

3.285.3.1 Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.285.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.35 (sec) , antiderivative size = 756, normalized size of antiderivative = 7.41

method	result
derivativedivides	$\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c} - \frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) - \frac{i\arctan(ax)^3}{3} + \left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\mathrm{csgn}\left(\cdots\right.\right.}{\cdots}$
default	$\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c} - \frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) - \frac{i\arctan(ax)^3}{3} + \left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\mathrm{csgn}\left(\cdots\right.\right.}{\cdots}$
parts	$\frac{\ln\left(a^2x^2+1\right)\arctan(ax)^2}{2a^2c} - \frac{\frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{a} - \frac{i\arctan(ax)^3}{3a} + \left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\cdots\right.}{\cdots}$

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(arctan(a*x)^2*ln((1+I*a*x)/(
a^2*x^2+1)^(1/2))-1/3*I*arctan(a*x)^3+1/4*(-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2
*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+2*I*Pi*csgn(I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+4*ln(2))*arctan(a*x)^2-I*arcta
n(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(1+I*a*x)^2/(a^2
*x^2+1))))
```

3.285.5 Fricas [F]

$$\int \frac{x\arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x\arctan\left(ax\right)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.285.6 Sympy [F]

$$\int \frac{x \arctan(ax)^2}{c + a^2cx^2} \, dx = \frac{\int \frac{x \operatorname{atan}^2(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

3.285.7 Maxima [F]

$$\int \frac{x \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x \arctan(ax)^2}{a^2cx^2 + c} \, dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.285.8 Giac [F]

$$\int \frac{x \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x \arctan(ax)^2}{a^2cx^2 + c} \, dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.286 $\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx$

3.286.1 Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^3}{3ac}$$

output
```
1/3*arctan(a*x)^3/a/c
```

3.286.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^3}{3ac}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^3/(3*a*c)
```

3.286.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \; 5419$$
$$\frac{\arctan(ax)^3}{3ac}$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^3/(3*a*c)
```

3.286.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.286.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\arctan(ax)^3}{3ac}$	15
default	$\frac{\arctan(ax)^3}{3ac}$	15
parallelrisch	$\frac{\arctan(ax)^3}{3ac}$	15
parts	$\frac{\arctan(ax)^3}{3ac}$	15
risch	$\frac{i\ln(iax+1)^3}{24ca}-\frac{i\ln(-iax+1)\ln(iax+1)^2}{8ca}+\frac{i\ln(-iax+1)^2\ln(iax+1)}{8ca}-\frac{i\ln(-iax+1)^3}{24ca}$	94

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(a*x)^3/a/c
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^3}{3\,ac}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/3*arctan(a*x)^3/(a*c)
```

3.286.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\int \frac{\operatorname{atan}^2{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^3}{3\,ac}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3*arctan(a*x)^3/(a*c)
```

3.286.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{\arctan{(ax)}^2}{a^2cx^2+c}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.286.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\mathrm{atan}{(a\,x)}^3}{3\,a\,c}$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^3/(3*a*c)
```

3.287 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx$

3.287.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = -\frac{i\arctan(ax)^3}{3c} + \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{i\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c} + \frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-1/3*I*arctan(a*x)^3/c+arctan(a*x)^2*ln(2-2/(1-I*a*x))/c-I*arctan(a*x)*pol
ylog(2,-1+2/(1-I*a*x))/c+1/2*polylog(3,-1+2/(1-I*a*x))/c
```

3.287.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \frac{i\arctan(ax)^3}{3c} + \frac{\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)}{c} + \frac{i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)}{c} + \frac{\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)}{2c}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

output
```
((I/3)*ArcTan[a*x]^3)/c + (ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])])/
c + (I*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])])/c + PolyLog[3, E^((
-2*I)*ArcTan[a*x])]/(2*c)
```

3.287.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5459}$$

$$\frac{i\int \frac{\arctan(ax)^2}{x(ax+i)}dx}{c}-\frac{i\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{5527}$$

$$\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^3}{3c}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/3*I)*ArcTan[a*x]^3)/c + (I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
 + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog
[3, -1 + 2/(1 - I*a*x)]/(4*a))))/c
```

3.287.3.1 Defintions of rubi rules used

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.287.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.53 (sec) , antiderivative size = 1578, normalized size of antiderivative = 17.34

method	result	size
derivativedivides	Expression too large to display	1578
default	Expression too large to display	1578
parts	Expression too large to display	1989

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*arctan(a*x)^2*ln(a*x)-1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(-arctan(a
*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^
2+1)-1)+1/3*I*arctan(a*x)^3-1/4*(-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(
a^2*x^2+1)+1))^2-2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))^2-2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+2*I*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+2*I*Pi*csgn(((1+I*a*x)^2/(
a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+I*Pi
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)+2*I*Pi-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+...
```

3.287.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

3.287.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**3 + x), x)/c
```

3.287.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x), x)
```

3.287.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)), x)
```

3.288 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)}\,dx$

3.288.1 Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx = -\frac{ia\arctan(ax)^2}{c} - \frac{\arctan(ax)^2}{cx} - \frac{a\arctan(ax)^3}{3c} + \frac{2a\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{ia\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}$$

output
```
-I*a*arctan(a*x)^2/c-arctan(a*x)^2/c/x-1/3*a*arctan(a*x)^3/c+2*a*arctan(a*
x)*ln(2-2/(1-I*a*x))/c-I*a*polylog(2,-1+2/(1-I*a*x))/c
```

3.288.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx = \frac{a\left(-\frac{1}{3}\arctan(ax)\left(\frac{3\arctan(ax)}{ax}+\arctan(ax)(3i+\arctan(ax))-6\log\left(1-e^{2i\arctan(ax)}\right)\right)-i\,\mathrm{PolyLog}\left(2,e^{2\right.}\right.}{c}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

output
```
(a*(-1/3*(ArcTan[a*x]*((3*ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[a
*x]) - 6*Log[1 - E^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, E^((2*I)*ArcTan[a
*x])]))/c
```

3.288.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5453, 27, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c}-\frac{a\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a\arctan(ax)^3}{3c}+\frac{-\frac{\arctan(ax)^2}{x}+2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a\arctan(ax)^3}{3c}+\frac{-\frac{\arctan(ax)^2}{x}+2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{a\arctan(ax)^3}{3c}+\frac{-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-1/3*(a*ArcTan[a*x]^3)/c + (-(ArcTan[a*x]^2/x) + 2*a*((-1/2*I)*ArcTan[a*x]
^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I
*a*x)]/2)))/c
```

3.288.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.288.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(86) = 172$.

Time = 0.50 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.74

method	result
derivativedivides	$a\left(-\frac{\arctan(ax)^3}{c}-\frac{\arctan(ax)^2}{cax}-\frac{2\left(-\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)\ln(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}\right.}{}\right.$
default	$a\left(-\frac{\arctan(ax)^3}{c}-\frac{\arctan(ax)^2}{cax}-\frac{2\left(-\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)\ln(ax)-\frac{i\ln(ax)\ln(iax+1)}{2}\right.}{}\right.$
parts	$-\frac{a\arctan(ax)^3}{c}-\frac{\arctan(ax)^2}{cx}-\frac{2\left(-\frac{a\arctan(ax)^3}{3}-a\left(-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\arctan(ax)\ln(ax)+\frac{i\ln(ax)\ln(iax\right.\right.}{}$

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c*arctan(a*x)^3-1/c*arctan(a*x)^2/a/x-2/c*(-1/3*arctan(a*x)^3+1/2*ar
ctan(a*x)*ln(a^2*x^2+1)-arctan(a*x)*ln(a*x)-1/2*I*ln(a*x)*ln(1+I*a*x)+1/2*
I*ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/2*I*dilog(1-I*a*x)+1/4*I*(ln(
a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/
2*ln(a*x-I)^2)-1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*
x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.288.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)} \, dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2 + c)x^2} \, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

3.288.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)} \, dx = \frac{\int \frac{\operatorname{atan}^2{(ax)}}{a^2x^4+x^2} \, dx}{c}$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**4 + x**2), x)/c
```

3.288.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)} \, dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2 + c)x^2} \, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/48*(8*a*x*arctan(a*x)^3 - 4*(a*arctan(a*x)^3/c + 12*a^2*integrate(1/16*
x^2*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) - 48*a^2*integrate(1/16*x^2
*log(a^2*x^2 + 1)/(a^2*c*x^4 + c*x^2), x) + 96*a*integrate(1/16*x*arctan(a
*x)/(a^2*c*x^4 + c*x^2), x) + 144*integrate(1/16*arctan(a*x)^2/(a^2*c*x^4
+ c*x^2), x) + 12*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x
))*c*x + 12*arctan(a*x)^2 - 3*log(a^2*x^2 + 1)^2)/(c*x)
```

3.288.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\mathrm{atan}(a\,x)^2}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)), x)
```

3.289 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx$

3.289.1 Optimal result

Integrand size = 22, antiderivative size = 178

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx = &-\frac{a\arctan(ax)}{cx}-\frac{a^2\arctan(ax)^2}{2c}-\frac{\arctan(ax)^2}{2cx^2}+\frac{ia^2\arctan(ax)^3}{3c}\\&+\frac{a^2\log(x)}{c}-\frac{a^2\log\left(1+a^2x^2\right)}{2c}-\frac{a^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}\\&+\frac{ia^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}\\&-\frac{a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}\end{aligned}$$

output

```
-a*arctan(a*x)/c/x-1/2*a^2*arctan(a*x)^2/c-1/2*arctan(a*x)^2/c/x^2+1/3*I*a
^2*arctan(a*x)^3/c+a^2*ln(x)/c-1/2*a^2*ln(a^2*x^2+1)/c-a^2*arctan(a*x)^2*l
n(2-2/(1-I*a*x))/c+I*a^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c-1/2*a^2*p
olylog(3,-1+2/(1-I*a*x))/c
```

3.289.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.80

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx = \frac{a^2\left(\frac{i\pi^3}{24}-\frac{\arctan(ax)}{ax}-\frac{(1+a^2x^2)\arctan(ax)^2}{2a^2x^2}-\frac{1}{3}i\arctan(ax)^3-\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)+\log(ax)+\right.}{c}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(a^2*((I/24)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a
^2*x^2) - (I/3)*ArcTan[a*x]^3 - ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x
])] + Log[a*x] + Log[1/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, E^((-
2*I)*ArcTan[a*x])] - PolyLog[3, E^((-2*I)*ArcTan[a*x])]/2))/c
```

3.289.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5453, 27, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3(a^2cx^2+c)}\,dx$$

$$\Big\downarrow \; 5453$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3}dx}{c}-a^2\int \frac{\arctan(ax)^2}{cx(a^2x^2+1)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \; 5361$$

$$\frac{a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a\left(\int \frac{\arctan(ax)}{x^2}dx - a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + a\int \frac{1}{x(a^2x^2+1)}dx - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2 - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{47}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\int \frac{1}{x^2}dx^2 - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{14}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\log\left(x^2\right) - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a\arctan(ax)^2 - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a\arctan(ax)^2 - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c} - \frac{a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx - \frac{1}{3}i\arctan(ax)^3\right)}{c}$$

3.289. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx$

$$\Big\downarrow \textcolor{blue}{5403}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\Big\downarrow \textcolor{blue}{5527}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\Big\downarrow \textcolor{blue}{7164}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(
Log[x^2] - Log[1 + a^2*x^2]))/2))/c - (a^2*((-1/3*I)*ArcTan[a*x]^3 + I*((-
I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*Poly
Log[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a)))))/c
```

3.289.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.289. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.289. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx$

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.289.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 35.01 (sec) , antiderivative size = 1786, normalized size of antiderivative = 10.03

method	result	size
derivativedivides	Expression too large to display	1786
default	Expression too large to display	1786
parts	Expression too large to display	2203

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/c*arctan(a*x)^2/a^2/x^2-1/c*arctan(a*x)^2*ln(a*x)+1/2/c*arctan(a
*x)^2*ln(a^2*x^2+1)-1/c*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-arc
tan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-1/12*arctan(a*x)*(6*I*Pi*arctan(a
*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*a*x-3*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)^3*a*x-6*I*Pi*arctan(a*x)*csgn(((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a*x-12*I*a*x+3*I*Pi*arctan(a*x)*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*a*x-6*I*Pi*arctan(
a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a*x
-3*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)*a*x+6*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*a
*x-6*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x
^2+1)+1))*a*x+3*I*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*Pi*arctan(a*x)*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1))*a*x+6*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a*x+4*I*arctan(a*x)^2*a*x-6*I*csgn(I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))*Pi*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*a
*x-6*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(...
```

3.289.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

3.289.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**5 + x**3), x)/c
```

3.289.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^3), x)
```

3.289.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)), x)
```

3.290 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)}\,dx$

3.290.1 Optimal result

Integrand size = 22, antiderivative size = 166

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)}\,dx = -\frac{a^2}{3cx} - \frac{a^3\arctan(ax)}{3c} - \frac{a\arctan(ax)}{3cx^2} + \frac{4ia^3\arctan(ax)^2}{3c} - \frac{\arctan(ax)^2}{3cx^3} + \frac{a^2\arctan(ax)^2}{cx} + \frac{a^3\arctan(ax)^3}{3c} - \frac{8a^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c} + \frac{4ia^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c}$$

output
```
-1/3*a^2/c/x-1/3*a^3*arctan(a*x)/c-1/3*a*arctan(a*x)/c/x^2+4/3*I*a^3*arcta
n(a*x)^2/c-1/3*arctan(a*x)^2/c/x^3+a^2*arctan(a*x)^2/c/x+1/3*a^3*arctan(a*
x)^3/c-8/3*a^3*arctan(a*x)*ln(2-2/(1-I*a*x))/c+4/3*I*a^3*polylog(2,-1+2/(1
-I*a*x))/c
```

3.290.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)}\,dx$$
$$= \frac{a^3\left(-\frac{1-4\arctan(ax)^2+\frac{(1+a^2x^2)\arctan(ax)^2}{a^2x^2}}{ax} + \arctan(ax)\left(-\frac{1+a^2x^2}{a^2x^2} + \arctan(ax)(4i+\arctan(ax)) - 8\log\left(1-e\right.\right.\right.}{3c}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```

output
```
(a^3*(-((1 - 4*ArcTan[a*x]^2 + ((1 + a^2*x^2)*ArcTan[a*x]^2)/(a^2*x^2))/(a
*x)) + ArcTan[a*x]*(-((1 + a^2*x^2)/(a^2*x^2)) + ArcTan[a*x]*(4*I + ArcTan
[a*x]) - 8*Log[1 - E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*Arc
Tan[a*x])]))/(3*c)
```

3.290.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5453, 27, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4}dx}{c}-a^2\int \frac{\arctan(ax)^2}{cx^2\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}a\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \; 264$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\left(a^2\left(-\int\frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \; 5419$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \; 5459$$

$$\frac{-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)}{c}-\frac{a^2\left(2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \; 5403$$

$$\frac{-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\cdots}{c}$$
$$\frac{a^2\left(2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\cdots}{c}$$

$$\Big\downarrow \; 2897$$

$$\frac{-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\cdots}{c}$$
$$\frac{a^2\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\cdots}{c}$$

3.290. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)}\,dx$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```

output
```
(-1/3*ArcTan[a*x]^2/x^3 + (2*a*(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*Arc
Tan[a*x]))/2 - a^2*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2
/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3)/c - (a^2*(-(ArcTan
[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*A
rcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/c
```

3.290.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.290.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.77

method	result
derivativedivides	$a^3 \left(-\frac{\arctan(ax)^2}{3c\,a^3x^3} + \frac{\arctan(ax)^2}{cax} + \frac{\arctan(ax)^3}{c} - \frac{2\left(\frac{\arctan(ax)}{2a^2x^2} + 4\arctan(ax)\ln(ax) - 2\arctan(ax)\ln(a^2x^2+1)\right.}{}\right.$
default	$a^3 \left(-\frac{\arctan(ax)^2}{3c\,a^3x^3} + \frac{\arctan(ax)^2}{cax} + \frac{\arctan(ax)^3}{c} - \frac{2\left(\frac{\arctan(ax)}{2a^2x^2} + 4\arctan(ax)\ln(ax) - 2\arctan(ax)\ln(a^2x^2+1)\right.}{}\right.$
parts	$\frac{a^3\arctan(ax)^3}{c} - \frac{\arctan(ax)^2}{3c\,x^3} + \frac{a^2\arctan(ax)^2}{cx} - \frac{2\left(a^3\left(\frac{\arctan(ax)}{2a^2x^2} + 4\arctan(ax)\ln(ax) - 2\arctan(ax)\ln(a^2x^2+\right.\right.}{}$

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

3.290. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)}\,dx$

output
```
a^3*(-1/3/c*arctan(a*x)^2/a^3/x^3+1/c*arctan(a*x)^2/a/x+1/c*arctan(a*x)^3-
2/3/c*(1/2*arctan(a*x)/a^2/x^2+4*arctan(a*x)*ln(a*x)-2*arctan(a*x)*ln(a^2*
x^2+1)+1/2*arctan(a*x)+1/2/a/x+2*I*ln(a*x)*ln(1+I*a*x)-2*I*ln(a*x)*ln(1-I*
a*x)+2*I*dilog(1+I*a*x)-2*I*dilog(1-I*a*x)-I*(ln(a*x-I)*ln(a^2*x^2+1)-dilo
g(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+I*(ln(I+a*
x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I
+a*x)^2)+arctan(a*x)^3))
```

3.290.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)
```

3.290.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^6+x^4}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**6 + x**4), x)/c
```

3.290.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4 (c+a^2cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Timed out
```

3.290.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)} \, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)x^4} \, dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)} \, dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^4\,(c\,a^2\,x^2 + c)} \, dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)), x)
```

3.291 $\int \frac{x^3 \arctan(ax)^2}{\left(c+a^2cx^2\right)^2} \, dx$

3.291.1 Optimal result

Integrand size = 22, antiderivative size = 192

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^2}{\left(c+a^2cx^2\right)^2} \, dx = & -\frac{1}{4a^4c^2(1+a^2x^2)} - \frac{x\arctan(ax)}{2a^3c^2(1+a^2x^2)} - \frac{\arctan(ax)^2}{4a^4c^2} \\ & + \frac{\arctan(ax)^2}{2a^4c^2(1+a^2x^2)} - \frac{i\arctan(ax)^3}{3a^4c^2} - \frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c^2} \\ & - \frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^4c^2} - \frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2}\end{aligned}$$

output
```
-1/4/a^4/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)/a^3/c^2/(a^2*x^2+1)-1/4*arctan(
a*x)^2/a^4/c^2+1/2*arctan(a*x)^2/a^4/c^2/(a^2*x^2+1)-1/3*I*arctan(a*x)^3/a
^4/c^2-arctan(a*x)^2*ln(2/(1+I*a*x))/a^4/c^2-I*arctan(a*x)*polylog(2,1-2/(
1+I*a*x))/a^4/c^2-1/2*polylog(3,1-2/(1+I*a*x))/a^4/c^2
```

3.291.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \arctan(ax)^2}{\left(c+a^2cx^2\right)^2} \, dx = \frac{\frac{1}{3}i\arctan(ax)^3 + \frac{1}{8}\left(-1+2\arctan(ax)^2\right)\cos(2\arctan(ax)) - \arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) + i\arctan(}{a^4c^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
((I/3)*ArcTan[a*x]^3 + ((-1 + 2*ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]])/8 - Arc
Tan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, -E^((
2*I)*ArcTan[a*x])] - PolyLog[3, -E^((2*I)*ArcTan[a*x])]/2 - (ArcTan[a*x]*S
in[2*ArcTan[a*x]])/4)/(a^4*c^2)
```

3.291.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5499, 27, 5455, 5379, 5465, 5427, 241, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c^2}-\frac{\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5529}$$

$$-\frac{\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}}{a^2c^2}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
-((-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*Ar
cTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a)/(a^2*c^2)) + (((-1/
3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/
2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 +
I*a*x)]/(4*a)))/a)/(a^2*c^2)
```

3.291. $\int\frac{x^3\arctan(ax)^2}{(c+a^2cx^2)^2}dx$

3.291.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.291. $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx$

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.291.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.50 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.45

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c^2}+\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\frac{i\arctan(ax)^3}{3}-\frac{i\arctan(ax)(ax+i)}{8ax-8i}-\frac{ax+i}{16(ax-i)}+\frac{i\arctan(\cdots}{8a\cdots}}{\cdots}}{\cdots}$
default	$\frac{\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c^2}+\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\frac{i\arctan(ax)^3}{3}-\frac{i\arctan(ax)(ax+i)}{8ax-8i}-\frac{ax+i}{16(ax-i)}+\frac{i\arctan(\cdots}{8a\cdots}}{\cdots}}{\cdots}$
parts	$\frac{\arctan(ax)^2}{2a^4c^2(a^2x^2+1)}+\frac{\arctan(ax)^2\ln(a^2x^2+1)}{2c^2a^4}-\frac{a\left(\frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{a^5}-\frac{i\arctan(ax)^3}{3a^5}-\frac{i\arctan(ax)(ax+i)}{8a^5(ax-i)}-\frac{a\cdots}{16a^5\cdots}\cdots\right.}{\cdots}$

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2/c^2/(a^2*x^2+
1)-1/c^2*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arctan(a*x)^
3-I*arctan(a*x)*(I+a*x)/(8*a*x-8*I)-1/16*(I+a*x)/(a*x-I)+I*arctan(a*x)*(a*
x-I)/(8*a*x+8*I)-1/16*(a*x-I)/(I+a*x)-I*arctan(a*x)*polylog(2,-(1+I*a*x)^2
/(a^2*x^2+1))+1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/4*(I*Pi*csgn(I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-2*I*Pi*csgn(I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+I*Pi*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I/((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+2*I*Pi*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-4*ln(2)-1)*arct
an(a*x)^2))
```

3.291.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.291.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.291.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan{(ax)}^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

3.291.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan{(ax)}^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^2, x)
```

3.292 $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2} dx$

3.292.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{x}{4a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)}{4a^3c^2} - \frac{\arctan(ax)}{2a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^2}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^3}{6a^3c^2}$$

output
```
1/4*x/a^2/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^3/c^2-1/2*arctan(a*x)/a^3/c^2/
(a^2*x^2+1)-1/2*x*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a^3/
c^2
```

3.292.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{3ax + 3(-1+a^2x^2)\arctan(ax) - 6ax\arctan(ax)^2 + 2(1+a^2x^2)\arctan(ax)^3}{12a^3c^2(1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
(3*a*x + 3*(-1 + a^2*x^2)*ArcTan[a*x] - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x
^2)*ArcTan[a*x]^3)/(12*a^3*c^2*(1 + a^2*x^2))
```

3.292.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5471, 27, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5471}$$

$$\frac{\int \frac{x\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{ac^2}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{ac^2}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{ac^2}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}+\frac{\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{ac^2}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^2)/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a^3*c^2)
 + (-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a
*x]/(2*a))/(2*a))/(a*c^2)
```

3.292.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.292. $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2} dx$

3.292.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{2\arctan(ax)^3x^2a^2+3a^2\arctan(ax)x^2-6a\arctan(ax)^2x+2\arctan(ax)^3+3ax-3\arctan(ax)}{12c^2(a^2x^2+1)a^3}$
derivativedivides	$\frac{-\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2a^2x^2+2}-\frac{ax}{4\left(a^2x^2+1\right)}-\frac{\arctan(ax)}{4}}{c^2}}{a^3}$
default	$\frac{-\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2a^2x^2+2}-\frac{ax}{4\left(a^2x^2+1\right)}-\frac{\arctan(ax)}{4}}{c^2}}{a^3}$
parts	$-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2a^3c^2}-\frac{\frac{\arctan(ax)^3}{3a^3}-\frac{-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{a^3}}{c^2}$
risch	$\frac{i\ln(iax+1)^3}{48c^2a^3}-\frac{i\left(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax\right)\ln(iax+1)^2}{16a^3c^2(a^2x^2+1)}+\frac{i\left(a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2+4iax\ln\right.}{16a^3c^2(ax+i)(ax-i}$

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(a*x)^3*x^2*a^2+3*a^2*arctan(a*x)*x^2-6*a*arctan(a*x)^2*x+2*
arctan(a*x)^3+3*a*x-3*arctan(a*x))/c^2/(a^2*x^2+1)/a^3
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.65

$$\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx$$
$$= -\frac{6\,ax\arctan\left(ax\right)^2-2\left(a^2x^2+1\right)\arctan\left(ax\right)^3-3\,ax-3\left(a^2x^2-1\right)\arctan\left(ax\right)}{12\left(a^5c^2x^2+a^3c^2\right)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/12*(6*a*x*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^
2*x^2 - 1)*arctan(a*x))/(a^5*c^2*x^2 + a^3*c^2)
```

3.292.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

$$\int \frac{x^2 \arctan(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = -\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2}-\frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax)^2 + \frac{\left(2\left(a^2x^2+1\right)\arctan(ax)^3+3\,ax+3\left(a^2x^2+1\right)\arctan(ax)\right)a^2}{12\left(a^7c^2x^2+a^5c^2\right)} - \frac{\left(\left(a^2x^2+1\right)\arctan(ax)^2+1\right)a\arctan(ax)}{2\left(a^6c^2x^2+a^4c^2\right)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^2 + 1
/12*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x + 3*(a^2*x^2 + 1)*arctan(a*x))*
a^2/(a^7*c^2*x^2 + a^5*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arct
an(a*x)/(a^6*c^2*x^2 + a^4*c^2)
```

3.292.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = \int \frac{x^2 \arctan(ax)^2}{\left(a^2cx^2+c\right)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{x}{2\,(2\,a^4\,c^2\,x^2+2\,a^2\,c^2)}+\frac{\mathrm{atan}(a\,x)}{4\,a^3\,c^2}+\frac{\mathrm{atan}(a\,x)^3}{6\,a^3\,c^2} - \frac{\mathrm{atan}(a\,x)}{2\,a^5\,c^2\,\left(\frac{1}{a^2}+x^2\right)}-\frac{x\,\mathrm{atan}(a\,x)^2}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
x/(2*(2*a^2*c^2 + 2*a^4*c^2*x^2)) + atan(a*x)/(4*a^3*c^2) + atan(a*x)^3/(6
*a^3*c^2) - atan(a*x)/(2*a^5*c^2*(1/a^2 + x^2)) - (x*atan(a*x)^2)/(2*a^4*c
^2*(1/a^2 + x^2))
```

3.293 $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} dx$

3.293.1 Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{1}{4a^2c^2(1+a^2x^2)} + \frac{x \arctan(ax)}{2ac^2(1+a^2x^2)} + \frac{\arctan(ax)^2}{4a^2c^2} - \frac{\arctan(ax)^2}{2a^2c^2(1+a^2x^2)}$$

output
```
1/4/a^2/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x
)^2/a^2/c^2-1/2*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)
```

3.293.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{1 + 2ax \arctan(ax) + (-1 + a^2x^2) \arctan(ax)^2}{4a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
(1 + 2*a*x*ArcTan[a*x] + (-1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^2*c^2*(1 + a^2
*x^2))
```

3.293.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5465, 27, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{ac^2}-\frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{ac^2}-\frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{ac^2}-\frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^2/(a^2*c^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*A
rcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/(a*c^2)
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.293.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{x^2\arctan(ax)^2a^2-a^2x^2+2x\arctan(ax)a-\arctan(ax)^2}{4c^2(a^2x^2+1)a^2}$
derivativedivides	$\frac{-\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{x\arctan(ax)a}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{c^2}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{x\arctan(ax)a}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{c^2}}{a^2}$
parts	$-\frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}+\frac{\frac{x\arctan(ax)a}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{c^2a^2}$
risch	$-\frac{\left(a^2x^2-1\right)\ln(iax+1)^2}{16a^2c^2(a^2x^2+1)}+\frac{\left(-\ln(-iax+1)+a^2x^2\ln(-iax+1)-2iax\right)\ln(iax+1)}{8(ax+i)a^2c^2(ax-i)}-\frac{-4+a^2x^2\ln(-iax+1)^2-\ln(-iax}{16(ax+i)a^2c^2(}$

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*arctan(a*x)^2*a^2-a^2*x^2+2*x*arctan(a*x)*a-arctan(a*x)^2)/c^2/(a
^2*x^2+1)/a^2
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.53

$$\int \frac{x\arctan(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = \frac{2\,ax\arctan\left(ax\right)+\left(a^2x^2-1\right)\arctan\left(ax\right)^2+1}{4\left(a^4c^2x^2+a^2c^2\right)}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/4*(2*a*x*arctan(a*x) + (a^2*x^2 - 1)*arctan(a*x)^2 + 1)/(a^4*c^2*x^2 + a
^2*c^2)
```

3.293.6 Sympy [F]

$$\int \frac{x\arctan(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{x\operatorname{atan}^2\left(ax\right)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{x\arctan(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = \frac{\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan\left(ax\right)}{2\,ac} - \frac{\left(a^2x^2+1\right)\arctan\left(ax\right)^2-1}{4\left(a^4cx^2+a^2c\right)c} - \frac{\arctan\left(ax\right)^2}{2\left(a^2cx^2+c\right)a^2c}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)/(a*c) - 1/4*((a^2*
x^2 + 1)*arctan(a*x)^2 - 1)/((a^4*c*x^2 + a^2*c)*c) - 1/2*arctan(a*x)^2/((
a^2*c*x^2 + c)*a^2*c)
```

3.293.8 Giac [F]

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx = \int \frac{x \arctan{(ax)}^2}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.293.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx = \frac{a^2\,x^2\,\mathrm{atan}(a\,x)^2+2\,a\,x\,\mathrm{atan}(a\,x)-\mathrm{atan}(a\,x)^2+1}{4\,a^2\,c^2\,\left(a^2\,x^2+1\right)}$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
(2*a*x*atan(a*x) - atan(a*x)^2 + a^2*x^2*atan(a*x)^2 + 1)/(4*a^2*c^2*(a^2*
x^2 + 1))
```

3.294 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx$

3.294.1 Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = -\frac{x}{4c^2(1+a^2x^2)} - \frac{\arctan(ax)}{4ac^2} + \frac{\arctan(ax)}{2ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^2}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^3}{6ac^2}$$

output
```
-1/4*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a/c^2+1/2*arctan(a*x)/a/c^2/(a^2*x^
2+1)+1/2*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a/c^2
```

3.294.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{-3ax+(3-3a^2x^2)\arctan(ax)+6ax\arctan(ax)^2+2(1+a^2x^2)\arctan(ax)^3}{12c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

output
```
(-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x
^2)*ArcTan[a*x]^3)/(12*c^2*(a + a^3*x^2))
```

3.294.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5427, 27, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \textbf{5427}$$

$$-a\int \frac{x\arctan(ax)}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \textbf{5465}$$

$$-\frac{a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \textbf{215}$$

$$-\frac{a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{\arctan(ax)^3}{6ac^2}$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a*c^2) - (a*(-1
/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2
*a))/(2*a)))/c^2
```

3.294.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.294. $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx$

3.294.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{2\arctan(ax)^3x^2a^2-3a^2\arctan(ax)x^2+6a\arctan(ax)^2x+2\arctan(ax)^3-3ax+3\arctan(ax)}{12c^2(a^2x^2+1)a}$
derivativedivides	$\frac{\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{c^2}}{a}$
default	$\frac{\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{c^2}}{a}$
parts	$\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2a\,c^2}-\frac{\frac{\arctan(ax)^3}{3a}+\frac{-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{a}}{c^2}$
risch	$\frac{i\ln(iax+1)^3}{48c^2a}-\frac{i(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax)\ln(iax+1)^2}{16c^2(a^2x^2+1)a}+\frac{i\left(a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2-4iax\ln(\right.}{16c^2(ax+i)(ax-i)a}$

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(a*x)^3*x^2*a^2-3*a^2*arctan(a*x)*x^2+6*a*arctan(a*x)^2*x+2*
arctan(a*x)^3-3*a*x+3*arctan(a*x))/c^2/(a^2*x^2+1)/a
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{6\,ax\arctan(ax)^2+2\,(a^2x^2+1)\arctan(ax)^3-3\,ax-3\,(a^2x^2-1)\arctan(ax)}{12\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/12*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2
*x^2 - 1)*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

3.294.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)^2 + \frac{\left(2\left(a^2x^2+1\right)\arctan(ax)^3-3\,ax-3\left(a^2x^2+1\right)\arctan(ax)\right)a^2}{12\left(a^5c^2x^2+a^3c^2\right)} - \frac{\left(\left(a^2x^2+1\right)\arctan(ax)^2-1\right)a\arctan(ax)}{2\left(a^4c^2x^2+a^2c^2\right)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^2 + 1/12*(2*
(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^
5*c^2*x^2 + a^3*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)
/(a^4*c^2*x^2 + a^2*c^2)
```

3.294.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.294.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\mathrm{atan}(a\,x)}{2\,\left(a^3\,c^2\,x^2+a\,c^2\right)} - \frac{x}{2\,\left(2\,a^2\,c^2\,x^2+2\,c^2\right)} + \frac{x\,\mathrm{atan}(a\,x)^2}{2\,\left(a^2\,c^2\,x^2+c^2\right)} - \frac{\mathrm{atan}(a\,x)}{4\,a\,c^2} + \frac{\mathrm{atan}(a\,x)^3}{6\,a\,c^2}$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)/(2*(a*c^2 + a^3*c^2*x^2)) - x/(2*(2*c^2 + 2*a^2*c^2*x^2)) + (x*a
tan(a*x)^2)/(2*(c^2 + a^2*c^2*x^2)) - atan(a*x)/(4*a*c^2) + atan(a*x)^3/(6
*a*c^2)
```

3.295 $\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx$

3.295.1 Optimal result

Integrand size = 22, antiderivative size = 170

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = -\frac{1}{4c^2\left(1+a^2x^2\right)}-\frac{ax\arctan(ax)}{2c^2\left(1+a^2x^2\right)}-\frac{\arctan(ax)^2}{4c^2}+\frac{\arctan(ax)^2}{2c^2\left(1+a^2x^2\right)}-\frac{i\arctan(ax)^3}{3c^2}+\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}+\frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}$$

output
```
-1/4/c^2/(a^2*x^2+1)-1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^2
/c^2+1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/3*I*arctan(a*x)^3/c^2+arctan(a*x)
^2*ln(2-2/(1-I*a*x))/c^2-I*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c^2+1/2*p
olylog(3,-1+2/(1-I*a*x))/c^2
```

3.295.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{-i\pi^3+8i\arctan(ax)^3-3\cos(2\arctan(ax))+6\arctan(ax)^2\cos(2\arctan(ax))+24\arctan(ax)^2\log\left(1-\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```

output
```
((-I)*Pi^3 + (8*I)*ArcTan[a*x]^3 - 3*Cos[2*ArcTan[a*x]] + 6*ArcTan[a*x]^2*
Cos[2*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (2
4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2
*I)*ArcTan[a*x])] - 6*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/(24*c^2)
```

3.295.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5501, 27, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{cx(a^2x^2+1)}dx}{c}-a^2\int \frac{x\arctan(ax)^2}{c^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$$\Big\downarrow \; 5427$$

$$-\frac{a^2\left(\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$$\Big\downarrow \; 241$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$$\Big\downarrow \; 5527$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(a}{c^2}$$

$$\Big\downarrow \; 7164$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

3.295. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) +
(x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/c^2) + ((-1/3
*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a
*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2
/(1 - I*a*x)]/(4*a))))/c^2
```

3.295.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.295.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.09 (sec) , antiderivative size = 1677, normalized size of antiderivative = 9.86

method	result	size
derivativedivides	Expression too large to display	1677
default	Expression too large to display	1677
parts	Expression too large to display	2098

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*arctan(a*x)^2*ln(a*x)-1/2/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan
(a*x)^2/c^2/(a^2*x^2+1)-1/c^2*(-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2))+1/3*I*arctan(a*x)^3-I*arctan(a*x)*(I+a*x)/(8*a*x-8*I)-1/16*(I+a*x)/(a*
x-I)+I*arctan(a*x)*(a*x-I)/(8*a*x+8*I)-1/16*(a*x-I)/(I+a*x)+arctan(a*x)^2*
ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
+1)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2))+1/4*(I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+2*I*Pi
*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+
I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+2*I*Pi*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a
^2*x^2+1)+1))^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*c
sgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)+2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))^2-2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^3+I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2...
```

3.295.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

3.295.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

3.295.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x), x)
```

3.295.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^2}{x\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^2), x)
```

3.296 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx$

3.296.1 Optimal result

Integrand size = 22, antiderivative size = 177

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx &= \frac{a^2x}{4c^2(1+a^2x^2)}+\frac{a\arctan(ax)}{4c^2}-\frac{a\arctan(ax)}{2c^2(1+a^2x^2)}-\frac{ia\arctan(ax)^2}{c^2}\\ &\quad-\frac{\arctan(ax)^2}{c^2x}-\frac{a^2x\arctan(ax)^2}{2c^2(1+a^2x^2)}-\frac{a\arctan(ax)^3}{2c^2}\\ &\quad+\frac{2a\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{ia\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}\end{aligned}$$

output
```
1/4*a^2*x/c^2/(a^2*x^2+1)+1/4*a*arctan(a*x)/c^2-1/2*a*arctan(a*x)/c^2/(a^2
*x^2+1)-I*a*arctan(a*x)^2/c^2-arctan(a*x)^2/c^2/x-1/2*a^2*x*arctan(a*x)^2/
c^2/(a^2*x^2+1)-1/2*a*arctan(a*x)^3/c^2+2*a*arctan(a*x)*ln(2-2/(1-I*a*x))/
c^2-I*a*polylog(2,-1+2/(1-I*a*x))/c^2
```

3.296.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx = -\frac{4ax\arctan(ax)^3+2ax\arctan(ax)\left(\cos(2\arctan(ax))-8\log\left(1-e^{2i\arctan(ax)}\right)\right)+8iax\,\text{PolyLog}\left(2,e^{2i\cdots}\right.}{8c^2x}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-1/8*(4*a*x*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 8*Log[
1 - E^((2*I)*ArcTan[a*x])]) + (8*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]
- a*x*Sin[2*ArcTan[a*x]] + 2*ArcTan[a*x]^2*(4 + (4*I)*a*x + a*x*Sin[2*ArcT
an[a*x]]))/(c^2*x)
```

3.296.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {5501, 27, 5427, 5453, 5361, 5419, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{cx^2(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}$$

$$\Big\downarrow \text{215}$$

3.296. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx$

$$-\frac{a^2\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+$$
$$\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax}{x}}{c^2}$$

$$\Big\downarrow 216$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+$$
$$\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax}{x}}{c^2}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/
2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*
a))/(2*a))))/c^2) + (-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 + 2*a*((-1/2
*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2
, -1 + 2/(1 - I*a*x)]/2)))/c^2
```

3.296.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.296. $\int\frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx$

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.296. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.296.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.77

method	result
derivativedivides	$a\left(-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2ax}-\frac{-\arctan(ax)^3+\frac{\arctan(ax)}{2a^2x^2+2}+\arctan(ax)\ln(a^2x^2+1)-2\,\mathrm{ar}}{}\right.$
default	$a\left(-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2ax}-\frac{-\arctan(ax)^3+\frac{\arctan(ax)}{2a^2x^2+2}+\arctan(ax)\ln(a^2x^2+1)-2\,\mathrm{ar}}{}\right.$
parts	$-\frac{a^2x\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3a\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2x}-\frac{2\left(-\frac{a\arctan(ax)^3}{2}-\frac{a\left(-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}-\arctan(ax)\ln\left(a^2x^2+1\right)-\right.}{}\right.}{}$

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-3/2*arctan(a*x)^3/c^2-1/c^2*arct
an(a*x)^2/a/x-1/c^2*(-arctan(a*x)^3+1/2*arctan(a*x)/(a^2*x^2+1)+arctan(a*x
)*ln(a^2*x^2+1)-2*arctan(a*x)*ln(a*x)-1/4*a*x/(a^2*x^2+1)-1/4*arctan(a*x)-
I*ln(a*x)*ln(1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*dilog(1-I*a
*x)+1/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2
*I*(I+a*x))-1/2*ln(a*x-I)^2)-1/2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a
*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.296.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

3.296.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^6+2a^2x^4+x^2}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

3.296.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/32*(6*a^3*x^3*arctan2(1, a*x) - 6*a^2*x^2 + 8*(a^3*x^3 + a*x)*arctan(a*
x)^3 + 12*a*x*arctan(a*x) + 4*(3*a^2*x^2 + 2)*arctan(a*x)^2 + 6*a*x*arctan
2(1, a*x) - (3*a^2*x^2 + 2)*log(a^2*x^2 + 1)^2 + 192*(a^6*c^2*x^3 + a^4*c^
2*x)*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^
2), x) - 128*(a^2*c^2*x^3 + c^2*x)*integrate(1/64*(4*(a^2*x^2 + 1)^(7/2)*a
^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(6*arctan(a*x)) - 24*(a^2*x^2 + 1)^3*a^
2*arctan(a*x)*log(a^2*x^2 + 1)*sin(5*arctan(a*x)) + 52*(a^2*x^2 + 1)^(5/2)
*a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(4*arctan(a*x)) - 48*(a^2*x^2 + 1)^2*
a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(3*arctan(a*x)) + 16*(a^2*x^2 + 1)^(3/
2)*a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(2*arctan(a*x)) - (4*(a^2*x^2 + 1)^
(7/2)*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^(7/2)*a^2*log(a^2*x^2 + 1)^2)*cos(
6*arctan(a*x)) + 6*(4*(a^2*x^2 + 1)^3*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^3*
a^2*log(a^2*x^2 + 1)^2)*cos(5*arctan(a*x)) - 13*(4*(a^2*x^2 + 1)^(5/2)*a^2
*arctan(a*x)^2 - (a^2*x^2 + 1)^(5/2)*a^2*log(a^2*x^2 + 1)^2)*cos(4*arctan(
a*x)) + 12*(4*(a^2*x^2 + 1)^2*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^2*a^2*log(
a^2*x^2 + 1)^2)*cos(3*arctan(a*x)) - 4*(4*(a^2*x^2 + 1)^(3/2)*a^2*arctan(a
*x)^2 - (a^2*x^2 + 1)^(3/2)*a^2*log(a^2*x^2 + 1)^2)*cos(2*arctan(a*x)))*sq
rt(a^2*x^2 + 1)/((a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^6*cos(6*arctan(a*x))^2
+ (a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^6*sin(6*arctan(a*x))^2 + 36*(a^2*c^2*x
^2 + c^2)*(a^2*x^2 + 1)^5*cos(5*arctan(a*x))^2 + 36*(a^2*c^2*x^2 + c^2)...
```

3.296.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^2), x)
```

3.297 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx$

3.297.1 Optimal result

Integrand size = 22, antiderivative size = 250

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx &= \frac{a^2}{4c^2(1+a^2x^2)} - \frac{a\arctan(ax)}{c^2x} + \frac{a^3x\arctan(ax)}{2c^2(1+a^2x^2)} - \frac{a^2\arctan(ax)^2}{4c^2} \\ &\quad - \frac{\arctan(ax)^2}{2c^2x^2} - \frac{a^2\arctan(ax)^2}{2c^2(1+a^2x^2)} + \frac{2ia^2\arctan(ax)^3}{3c^2} + \frac{a^2\log(x)}{c^2} \\ &\quad - \frac{a^2\log(1+a^2x^2)}{2c^2} - \frac{2a^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2} \\ &\quad + \frac{2ia^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2} \\ &\quad - \frac{a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^2}\end{aligned}$$

output
```
1/4*a^2/c^2/(a^2*x^2+1)-a*arctan(a*x)/c^2/x+1/2*a^3*x*arctan(a*x)/c^2/(a^2
*x^2+1)-1/4*a^2*arctan(a*x)^2/c^2-1/2*arctan(a*x)^2/c^2/x^2-1/2*a^2*arctan
(a*x)^2/c^2/(a^2*x^2+1)+2/3*I*a^2*arctan(a*x)^3/c^2+a^2*ln(x)/c^2-1/2*a^2*
ln(a^2*x^2+1)/c^2-2*a^2*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2+2*I*a^2*arctan
(a*x)*polylog(2,-1+2/(1-I*a*x))/c^2-a^2*polylog(3,-1+2/(1-I*a*x))/c^2
```

3.297.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.64

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)^2}\,dx$$
$$= \frac{a^2\left(-2i\arctan(ax)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)-\mathrm{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)+\frac{1}{24}\left(2i\pi^3-16i\arctan(ax)^3+\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(a^2*((-2*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - PolyLog[3, E
^((-2*I)*ArcTan[a*x])] + ((2*I)*Pi^3 - (16*I)*ArcTan[a*x]^3 + 3*Cos[2*ArcT
an[a*x]] + 6*ArcTan[a*x]^2*(-2 - 2/(a^2*x^2) - Cos[2*ArcTan[a*x]] - 8*Log[
1 - E^((-2*I)*ArcTan[a*x])]) + 24*Log[a*x] - 12*Log[1 + a^2*x^2] + (6*ArcT
an[a*x]*(-4 + a*x*Sin[2*ArcTan[a*x]]))/(a*x))/24))/c^2
```

3.297.3 Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.46, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5501, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\,(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \; 5501$$
$$\frac{\int \frac{\arctan(ax)^2}{cx^3(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c^2x\,(a^2x^2+1)^2}dx$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$
$$\Big\downarrow \; 5453$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3}dx - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(\int \frac{\arctan(ax)}{x^2}dx - a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) - \frac{\arctan(ax)^2}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + a\int \frac{1}{x(a^2x^2+1)}dx - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2 - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\int \frac{1}{x^2}dx^2 - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\log\left(x^2\right) - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{16}$$

3.297. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{a}{}}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)}{c^2}+\frac{-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)}{c^2}$$

$$\Big\downarrow \text{5403}$$

3.297. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx$

$$-\frac{\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\right.}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)\right.\right.}{c^2}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\text{ar}}{2a}\right.\right.}{c^2}$$

$$\Big\downarrow \text{5527}$$

3.297. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx$

$$-\frac{\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}ia\right.\right.}{c^2}$$

$$\frac{a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{x\,a}{2(}\right.\right.}{c^2}$$

$$\Big\downarrow \textbf{7164}$$

$$-\frac{\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-\right.}{4a}\right.\right.\right.}{c^2}$$

input
```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*((-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2
)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a
*x]^2/(4*a))/a) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(
((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(
1 - I*a*x)]/(4*a)))))/c^2) + (-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x)
 - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((-1/3
*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a
*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2
/(1 - I*a*x)]/(4*a)))))/c^2
```

3.297.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.297. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx$

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.297. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx$

3.297.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 85.27 (sec) , antiderivative size = 4110, normalized size of antiderivative = 16.44

method	result	size
derivativedivides	Expression too large to display	4110
default	Expression too large to display	4110
parts	Expression too large to display	5354

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/c^2*arctan(a*x)^2*ln(a^2*x^2+1)-
1/2/c^2*arctan(a*x)^2/a^2/x^2-2/c^2*arctan(a*x)^2*ln(a*x)-1/c^2*(2*arctan(
a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2
*x^2+1)-1)-1/48/a/x/(I+a*x)*(-24*a^2*arctan(a*x)*x^2-24*I*csgn(I*(1+I*a*x)
^4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^3*Pi*arctan(a*x)^2*a*x-3*a
^3*x^3-48*I*arctan(a*x)*a*x-96*ln(2)*arctan(a*x)^2*a*x-96*ln(2)*arctan(a*x
)^2*a^3*x^3-48*arctan(a*x)+9*a*x-12*a^3*arctan(a*x)^2*x^3-12*a*arctan(a*x)
^2*x+24*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1))*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arc
tan(a*x)^2*a^3*x^3-48*I*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I/((1+I*a
*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1)
)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)-I)*arctan(a*x)^2*a^3*x^3+24*I*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1))*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2*a*x-48*
I*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(
1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*Pi*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1)-I)*arctan(a*x)^2*a*x+32*I*arctan(a*x)^3*a^3*x^3+32*I*arctan(
a*x)^3*a*x+48*I*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)
-I/((1+I*a*x)^2/(a^2*x^2+1)+1))*Pi*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I
*a*x)^2/(a^2*x^2+1)-1/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2*a^3*...
```

3.297.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

3.297.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx=\frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^7+2a^2x^5+x^3}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

3.297.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^3), x)
```

3.297.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^2}{x^3\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^2), x)
```

3.298 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

3.298.1 Optimal result

Integrand size = 22, antiderivative size = 242

$$
\begin{aligned}
\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx = &-\frac{a^2}{3c^2x}-\frac{a^4x}{4c^2(1+a^2x^2)}-\frac{7a^3\arctan(ax)}{12c^2}-\frac{a\arctan(ax)}{3c^2x^2} \\
&+\frac{a^3\arctan(ax)}{2c^2(1+a^2x^2)}+\frac{7ia^3\arctan(ax)^2}{3c^2}-\frac{\arctan(ax)^2}{3c^2x^3} \\
&+\frac{2a^2\arctan(ax)^2}{c^2x}+\frac{a^4x\arctan(ax)^2}{2c^2(1+a^2x^2)}+\frac{5a^3\arctan(ax)^3}{6c^2} \\
&-\frac{14a^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c^2}+\frac{7ia^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^2}
\end{aligned}
$$

output

```
-1/3*a^2/c^2/x-1/4*a^4*x/c^2/(a^2*x^2+1)-7/12*a^3*arctan(a*x)/c^2-1/3*a*ar
ctan(a*x)/c^2/x^2+1/2*a^3*arctan(a*x)/c^2/(a^2*x^2+1)+7/3*I*a^3*arctan(a*x
)^2/c^2-1/3*arctan(a*x)^2/c^2/x^3+2*a^2*arctan(a*x)^2/c^2/x+1/2*a^4*x*arct
an(a*x)^2/c^2/(a^2*x^2+1)+5/6*a^3*arctan(a*x)^3/c^2-14/3*a^3*arctan(a*x)*l
n(2-2/(1-I*a*x))/c^2+7/3*I*a^3*polylog(2,-1+2/(1-I*a*x))/c^2
```

3.298.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx$$
$$= \frac{20a^3x^3\arctan(ax)^3 + 2ax\arctan(ax)\left(-4-4a^2x^2+3a^2x^2\cos(2\arctan(ax)) - 56a^2x^2\log\left(1-e^{2i\arctan(a}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
(20*a^3*x^3*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(-4 - 4*a^2*x^2 + 3*a^2*x^2*
Cos[2*ArcTan[a*x]] - 56*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) + (56*I)*a
^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])] - a^2*x^2*(8 + 3*a*x*Sin[2*ArcTan
[a*x]]) + ArcTan[a*x]^2*(-8 + 48*a^2*x^2 + (56*I)*a^3*x^3 + 6*a^3*x^3*Sin[
2*ArcTan[a*x]]))/(24*c^2*x^3)
```

3.298.3 Rubi [A] (verified)

Time = 3.44 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.69, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.045$, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \; 5501$$
$$\frac{\int \frac{\arctan(ax)^2}{cx^4(a^2x^2+1)}dx}{c} - a^2\int \frac{\arctan(ax)^2}{c^2x^2\left(a^2x^2+1\right)^2}dx$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$
$$\Big\downarrow \; 5453$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5361$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx\right) + \frac{2}{3}a\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{3x^3}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5453$$

$$\frac{-\left(a^2\left(\int \frac{\arctan(ax)^2}{x^2}dx - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)\right) + \frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx - a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) - \frac{\arctan(ax)^2}{3x^3}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5361$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right)\right) + \frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx -}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 264$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right)\right) + \frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) + \frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}\right.\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 216$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right)\right) + \frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) - \frac{\arctan(ax)}{2x^2} + \frac{1}{2}a(-a\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5419$$

$$\frac{-\left(a^2\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{1}{3}a\arctan(ax)^3 - \frac{\arctan(ax)^2}{x}\right)\right) + \frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) - \frac{\arctan(ax)}{2x^2} + \frac{1}{2}a(-a\arctan\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

3.298. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}+\frac{\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\left(a^2\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(}{x}\right.\right.}{}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}+\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-\right.}{}$$

$$\Big\downarrow \text{5501}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)}{c^2}+\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-\right.}{}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)}{c^2}+\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-\right.}{}$$

$$\Big\downarrow \text{5453}$$

$$-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx+\int \frac{\arctan(ax)^2}{x^2}dx\right)}{c^2}+\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-\right.}{}$$

$$\Big\downarrow \text{5361}$$

3.298. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

$$-\frac{a^2\left(-\left(a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int\frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}{c^2}$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-$$

$$\Big\downarrow \text{5419}$$

$$-\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c^2}+$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-$$

$$\Big\downarrow \text{5459}$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-$$

$$\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3\right.}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-$$

$$\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-ia\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2897}$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-$$

$$\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{}{1}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{5465}$$

3.298. $\int\frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-}{a^2\left(-a^2\left(-a\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\right.\right.\right.}\ \ c^2$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-}{a^2\left(-a^2\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\right.\right.\right.\right.}\ \ c^2$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-}{a^2\left(-a^2\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-ia}\right.\right.\right.\right.}\ \ c^2$$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
(-1/3*ArcTan[a*x]^2/x^3 - a^2*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 +
2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] -
 PolyLog[2, -1 + 2/(1 - I*a*x)]/2))) + (2*a*(-1/2*ArcTan[a*x]/x^2 + (a*(-x
^(-1) - a*ArcTan[a*x]))/2 - a^2*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a
*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3)/c^2 -
 (a^2*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 - a^2*((x*ArcTan[a*x]^2)/(
2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2
*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))) + 2*a*((-1/2*I)
*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -
1 + 2/(1 - I*a*x)]/2))))/c^2
```

3.298. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

3.298.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

3.298. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.298. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$

3.298.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.45

method	result
derivativedivides	$a^3\left(-\frac{\arctan(ax)^2}{3c^2a^3x^3}+\frac{2\arctan(ax)^2}{c^2ax}+\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)}{a^2x^2}+14\arctan(ax)\ln(ax)-\frac{3a}{2(\cdots}\cdots}{\cdots}\cdots\right.$
default	$a^3\left(-\frac{\arctan(ax)^2}{3c^2a^3x^3}+\frac{2\arctan(ax)^2}{c^2ax}+\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)}{a^2x^2}+14\arctan(ax)\ln(ax)-\frac{3a}{2(\cdots}\cdots}{\cdots}\cdots\right.$
parts	$\frac{a^4x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5a^3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{3c^2x^3}+\frac{2a^2\arctan(ax)^2}{c^2x}-\frac{2\left(\frac{5a^3\arctan(ax)^3}{2}+\frac{a^3\left(\frac{\arctan(ax)}{a^2x^2}+14a\cdots\right.}{\cdots}\cdots\right.}{\cdots}$

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/c^2*arctan(a*x)^2/a^3/x^3+2/c^2*arctan(a*x)^2/a/x+1/2*a*x*arctan
(a*x)^2/c^2/(a^2*x^2+1)+5/2*arctan(a*x)^3/c^2-1/3/c^2*(arctan(a*x)/a^2/x^2
+14*arctan(a*x)*ln(a*x)-3/2*arctan(a*x)/(a^2*x^2+1)-7*arctan(a*x)*ln(a^2*x
^2+1)+7*I*ln(a*x)*ln(1+I*a*x)-7*I*ln(a*x)*ln(1-I*a*x)+7*I*dilog(1+I*a*x)-7
*I*dilog(1-I*a*x)-7/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(
a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+7/2*I*(ln(I+a*x)*ln(a^2*x^2+1)-
dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)+1/a/x+3/
4*a*x/(a^2*x^2+1)+7/4*arctan(a*x)+5*arctan(a*x)^3))
```

3.298.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

3.298.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^8+2a^2x^6+x^4}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

3.298.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.298.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^2}{x^4\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^2), x)
```

3.299 $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} dx$

3.299.1 Optimal result

Integrand size = 22, antiderivative size = 140

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} dx = -\frac{x^4}{32c^3(1+a^2x^2)^2} + \frac{3}{32a^4c^3(1+a^2x^2)} + \frac{x^3\arctan(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)}{16a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)^2}{32a^4c^3} + \frac{x^4\arctan(ax)^2}{4c^3(1+a^2x^2)^2}$$

output
```
-1/32*x^4/c^3/(a^2*x^2+1)^2+3/32/a^4/c^3/(a^2*x^2+1)+1/8*x^3*arctan(a*x)/a
/c^3/(a^2*x^2+1)^2+3/16*x*arctan(a*x)/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)
^2/a^4/c^3+1/4*x^4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2
```

3.299.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} dx = \frac{4+5a^2x^2+2ax(3+5a^2x^2)\arctan(ax)+(-3-6a^2x^2+5a^4x^4)\arctan(ax)^2}{32a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(4 + 5*a^2*x^2 + 2*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x] + (-3 - 6*a^2*x^2 + 5*a
^4*x^4)*ArcTan[a*x]^2)/(32*a^4*c^3*(1 + a^2*x^2)^2)
```

3.299.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5479, 27, 5473, 5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{1}{2}a\int \frac{x^4 \arctan(ax)}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\int \frac{x^4 \arctan(ax)}{\left(a^2x^2+1\right)^3}dx}{2c^3}$$

$$\Big\downarrow \text{5473}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\left(\frac{3\int \frac{x^2\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{4a^2}-\frac{x^3\arctan(ax)}{4a^2(a^2x^2+1)^2}+\frac{x^4}{16a(a^2x^2+1)^2}\right)}{2c^3}$$

$$\Big\downarrow \text{5469}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\left(\frac{3\left(\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{2a^2}-\frac{x\arctan(ax)}{2a^2\left(a^2x^2+1\right)}-\frac{1}{4a^3\left(a^2x^2+1\right)}\right)}{4a^2}-\frac{x^3\arctan(ax)}{4a^2(a^2x^2+1)^2}+\frac{x^4}{16a(a^2x^2+1)^2}\right)}{2c^3}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\left(-\frac{x^3\arctan(ax)}{4a^2(a^2x^2+1)^2}+\frac{x^4}{16a(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^2}{4a^3}-\frac{x\arctan(ax)}{2a^2\left(a^2x^2+1\right)}-\frac{1}{4a^3\left(a^2x^2+1\right)}\right)}{4a^2}\right)}{2c^3}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

3.299. $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}dx$

output
```
(x^4*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (a*(x^4/(16*a*(1 + a^2*x^2)^
2) - (x^3*ArcTan[a*x])/(4*a^2*(1 + a^2*x^2)^2) + (3*(-1/4*1/(a^3*(1 + a^2*
x^2)) - (x*ArcTan[a*x])/(2*a^2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a^3)))/(4
*a^2)))/(2*c^3)
```

3.299.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.299. $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx$

3.299.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{5a^4\arctan(ax)^2x^4-4a^4x^4+10\arctan(ax)x^3a^3-6x^2\arctan(ax)^2a^2-3a^2x^2+6x\arctan(ax)a-3\arctan(ax)^2}{32c^3(a^2x^2+1)^2a^4}$
derivativedivides	$\frac{-\frac{\arctan(ax)^2}{2c^3\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{-\frac{5\arctan(ax)a^3x^3}{8\left(a^2x^2+1\right)^2}-\frac{3\arctan(ax)ax}{8\left(a^2x^2+1\right)^2}-\frac{5\arctan(ax)^2}{16}-\frac{5}{16\left(a^2x^2+1\right)}+\frac{1}{16\left(a^2x^2+1\right)^2}}{2c^3}}{a^4}$
default	$\frac{-\frac{\arctan(ax)^2}{2c^3\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}-\frac{-\frac{5\arctan(ax)a^3x^3}{8\left(a^2x^2+1\right)^2}-\frac{3\arctan(ax)ax}{8\left(a^2x^2+1\right)^2}-\frac{5\arctan(ax)^2}{16}-\frac{5}{16\left(a^2x^2+1\right)}+\frac{1}{16\left(a^2x^2+1\right)^2}}{2c^3}}{a^4}$
parts	$\frac{\arctan(ax)^2}{4c^3a^4(a^2x^2+1)^2}-\frac{\arctan(ax)^2}{2c^3a^4(a^2x^2+1)}-\frac{-\frac{5\arctan(ax)x^3}{8a\left(a^2x^2+1\right)^2}-\frac{3\arctan(ax)x}{8a^3\left(a^2x^2+1\right)^2}-\frac{5\arctan(ax)^2}{8a^4}+\frac{-\frac{5}{2\left(a^2x^2+1\right)}+\frac{1}{2\left(a^2x^2+1\right)^2}+}{8a^4}}{2c^3}$
risch	$-\frac{(5a^4x^4-6a^2x^2-3)\ln(iax+1)^2}{128a^4c^3(a^2x^2+1)^2}+\frac{(-6a^2x^2\ln(-iax+1)-3\ln(-iax+1)+5x^4\ln(-iax+1)a^4-10ia^3x^3-6iax)\ln(iax}{64a^4(ax+i)^2(ax-i)^2c^3}$

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(5*a^4*arctan(a*x)^2*x^4-4*a^4*x^4+10*arctan(a*x)*x^3*a^3-6*x^2*arcta
n(a*x)^2*a^2-3*a^2*x^2+6*x*arctan(a*x)*a-3*arctan(a*x)^2)/c^3/(a^2*x^2+1)^
2/a^4
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{x^3\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{5\,a^2x^2+\left(5\,a^4x^4-6\,a^2x^2-3\right)\arctan\left(ax\right)^2+2\left(5\,a^3x^3+3\,ax\right)\arctan\left(ax\right)+4}{32\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(5*a^2*x^2 + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^2 + 2*(5*a^3*x^3
 + 3*a*x)*arctan(a*x) + 4)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.299.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.32

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{1}{16}a\left(\frac{5a^2x^3+3x}{a^8c^3x^4+2a^6c^3x^2+a^4c^3}+\frac{5\arctan(ax)}{a^5c^3}\right)\arctan(ax) + \frac{(5a^2x^2-5(a^4x^4+2a^2x^2+1)\arctan(ax)^2+4)a^2}{32(a^{10}c^3x^4+2a^8c^3x^2+a^6c^3)} - \frac{(2a^2x^2+1)\arctan(ax)^2}{4(a^8c^3x^4+2a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/16*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arct
an(a*x)/(a^5*c^3))*arctan(a*x) + 1/32*(5*a^2*x^2 - 5*(a^4*x^4 + 2*a^2*x^2
+ 1)*arctan(a*x)^2 + 4)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) - 1/4
*(2*a^2*x^2 + 1)*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.299.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.299.9 Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{5\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+10\,a^3\,x^3\,\mathrm{atan}(a\,x)-6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+5\,a^2\,x^2+6\,a\,x\,\mathrm{atan}(a\,x)-3\,\mathrm{atan}(a\,x)^2+4}{32\,a^4\,c^3\,(a^2\,x^2+1)^2}$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

output
```
(5*a^2*x^2 - 3*atan(a*x)^2 + 10*a^3*x^3*atan(a*x) + 6*a*x*atan(a*x) - 6*a^
2*x^2*atan(a*x)^2 + 5*a^4*x^4*atan(a*x)^2 + 4)/(32*a^4*c^3*(a^2*x^2 + 1)^2
)
```

3.300 $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx$

3.300.1 Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx = \frac{x}{32a^2c^3(1+a^2x^2)^2} - \frac{x}{64a^2c^3(1+a^2x^2)} - \frac{\arctan(ax)}{64a^3c^3} - \frac{\arctan(ax)}{8a^3c^3(1+a^2x^2)^2} + \frac{\arctan(ax)}{8a^3c^3(1+a^2x^2)} - \frac{x\arctan(ax)^2}{4a^2c^3(1+a^2x^2)^2} + \frac{x\arctan(ax)^2}{8a^2c^3(1+a^2x^2)} + \frac{\arctan(ax)^3}{24a^3c^3}$$

output
```
1/32*x/a^2/c^3/(a^2*x^2+1)^2-1/64*x/a^2/c^3/(a^2*x^2+1)-1/64*arctan(a*x)/a
^3/c^3-1/8*arctan(a*x)/a^3/c^3/(a^2*x^2+1)^2+1/8*arctan(a*x)/a^3/c^3/(a^2*
x^2+1)-1/4*x*arctan(a*x)^2/a^2/c^3/(a^2*x^2+1)^2+1/8*x*arctan(a*x)^2/a^2/c
^3/(a^2*x^2+1)+1/24*arctan(a*x)^3/a^3/c^3
```

3.300.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.52

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx = \frac{3ax - 3a^3x^3 - 3(1-6a^2x^2+a^4x^4)\arctan(ax) + 24ax(-1+a^2x^2)\arctan(ax)^2 + 8(1+a^2x^2)^2\arctan(ax}{192a^3c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(3*a*x - 3*a^3*x^3 - 3*(1 - 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] + 24*a*x*(-1
+ a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(192*a^3*c^3*(
1 + a^2*x^2)^2)
```

3.300.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.72, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5499, 27, 5427, 5435, 215, 215, 216, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^2}{c^3(a^2x^2+1)^3}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^3}-\frac{\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{a^2c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{a^2c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx-\frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}}{a^2c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}}{a^2c^3}$$

$\downarrow$ 215

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}}{a^2c^3}$$

$\downarrow$ 216

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)}{a^2c^3}$$

$\downarrow$ 5427

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\right.}{a^2c^3}$$

$\downarrow$ 5465

$$\frac{-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}\right.\right.}{a^2c^3}$$

$\downarrow$ 215

$$\frac{-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.}{a^2c^3}$$

$\downarrow$ 216

3.300. $\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$

$$\frac{\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-$$

$$\frac{\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}\right.\right.}{a^2c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTa
n[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*
a)))/(a^2*c^3) - (ArcTan[a*x]/(8*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4
*(1 + a^2*x^2)^2) + (-1/4*x/(1 + a^2*x^2)^2 - (3*(x/(2*(1 + a^2*x^2)) + Ar
cTan[a*x]/(2*a)))/4)/8 + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[
a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^
2)) + ArcTan[a*x]/(2*a))/(2*a))))/4)/(a^2*c^3)
```

3.300.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.300. $\int \frac{x^2\arctan(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx$

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.300.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{8a^4\arctan(ax)^3x^4-3\arctan(ax)a^4x^4+24a^3\arctan(ax)^2x^3+16\arctan(ax)^3x^2a^2-3a^3x^3+18a^2\arctan(ax)x^2-24a\,\mathrm{arct}}{192c^3(a^2x^2+1)^2a^3}$
derivativedivides	$\frac{\frac{\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^3}{8c^3}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2(a^2x^2+1)^2}-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{\frac{1}{8}a^3x^3-\frac{1}{8}ax}{2(a^2x^2+1)^2}+\frac{\arctan(ax)}{16}}{4c^3}}{a^3}$
default	$\frac{\frac{\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^3}{8c^3}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2(a^2x^2+1)^2}-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{\frac{1}{8}a^3x^3-\frac{1}{8}ax}{2(a^2x^2+1)^2}+\frac{\arctan(ax)}{16}}{4c^3}}{a^3}$
parts	$\frac{\arctan(ax)^2x^3}{8c^3(a^2x^2+1)^2}-\frac{x\arctan(ax)^2}{8a^2c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^3}{8a^3c^3}-\frac{\frac{\arctan(ax)^3}{3a^3}+\frac{\frac{\arctan(ax)}{2(a^2x^2+1)^2}-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{\frac{1}{8}a^3x^3-\frac{1}{8}ax}{2(a^2x^2+1)^2}+\frac{\arctan(}{16}}{a^3}}{4c^3}$
risch	$\frac{i\ln(iax+1)^3}{192a^3c^3}-\frac{i(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)^2}{64a^3c^3(a^2x^2+1)^2}+\frac{i\left(a^4x^4\ln(-iax\right.}{}$

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*(8*a^4*arctan(a*x)^3*x^4-3*arctan(a*x)*a^4*x^4+24*a^3*arctan(a*x)^2*
x^3+16*arctan(a*x)^3*x^2*a^2-3*a^3*x^3+18*a^2*arctan(a*x)*x^2-24*a*arctan(
a*x)^2*x+8*arctan(a*x)^3+3*a*x-3*arctan(a*x))/c^3/(a^2*x^2+1)^2/a^3
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = -\frac{3\,a^3x^3 - 8\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan\,(ax)^3 - 24\,(a^3x^3 - ax)\arctan\,(ax)^2 - 3\,ax + 3\,(a^4x^4 - 6\,a^2x^2 + 1}{192\,(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 24*(a^3*x^
3 - a*x)*arctan(a*x)^2 - 3*a*x + 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x))/
(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

3.300.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^2{(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.28

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan{(ax)}}{a^3c^3}\right)\arctan{(ax)}^2 - \frac{\left(3\,a^3x^3-8\,(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}^3-3\,ax+3\,(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}\right)a^2}{192\,(a^9c^3x^4+2\,a^7c^3x^2+a^5c^3)} + \frac{\left(a^2x^2-(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}^2\right)a\arctan{(ax)}}{8\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(
a^3*c^3))*arctan(a*x)^2 - 1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*a
rctan(a*x)^3 - 3*a*x + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^9*c
^3*x^4 + 2*a^7*c^3*x^2 + a^5*c^3) + 1/8*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 +
1)*arctan(a*x)^2)*a*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.300.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan{(ax)}^2}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.300.9 Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\frac{x}{8\,a^2}-\frac{x^3}{8}}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3} - \frac{\mathrm{atan}(a\,x)^2\left(\frac{x}{8\,a^4\,c^3}-\frac{x^3}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4} - \frac{\mathrm{atan}(a\,x)}{64\,a^3\,c^3} + \frac{\mathrm{atan}(a\,x)^3}{24\,a^3\,c^3} + \frac{x^2\,\mathrm{atan}(a\,x)}{8\,a^3\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

output
```
(x/(8*a^2) - x^3/8)/(8*c^3 + 16*a^2*c^3*x^2 + 8*a^4*c^3*x^4) - (atan(a*x)^
2*(x/(8*a^4*c^3) - x^3/(8*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4) - atan(a*x)
/(64*a^3*c^3) + atan(a*x)^3/(24*a^3*c^3) + (x^2*atan(a*x))/(8*a^3*c^3*(1/a
^2 + 2*x^2 + a^2*x^4))
```

3.300. $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$

3.301 $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3} dx$

3.301.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3} dx = \frac{1}{32a^2c^3(1+a^2x^2)^2} + \frac{3}{32a^2c^3(1+a^2x^2)} + \frac{x \arctan(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3x \arctan(ax)}{16ac^3(1+a^2x^2)} + \frac{3 \arctan(ax)^2}{32a^2c^3} - \frac{\arctan(ax)^2}{4a^2c^3(1+a^2x^2)^2}$$

output
```
1/32/a^2/c^3/(a^2*x^2+1)^2+3/32/a^2/c^3/(a^2*x^2+1)+1/8*x*arctan(a*x)/a/c^
3/(a^2*x^2+1)^2+3/16*x*arctan(a*x)/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)^2/a^
2/c^3-1/4*arctan(a*x)^2/a^2/c^3/(a^2*x^2+1)^2
```

3.301.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3} dx = \frac{4+3a^2x^2+2ax(5+3a^2x^2)\arctan(ax)+(-5+6a^2x^2+3a^4x^4)\arctan(ax)^2}{32c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + (-5 + 6*a^2*x^2 + 3*a
^4*x^4)*ArcTan[a*x]^2)/(32*c^3*(a + a^3*x^2)^2)
```

3.301.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5465, 27, 5431, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)}{c^3(a^2x^2+1)^3}dx}{2a} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^2/(a^2*c^3*(1 + a^2*x^2)^2) + (1/(16*a*(1 + a^2*x^2)^2) +
 (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcT
an[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)/(2*a*c^3)
```

3.301.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.301. $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3} dx$

3.301.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{3a^4\arctan(ax)^2x^4-4a^4x^4+6\arctan(ax)x^3a^3+6x^2\arctan(ax)^2a^2-5a^2x^2+10x\arctan(ax)a-5\arctan(ax)^2}{32c^3(a^2x^2+1)^2a^2}$
derivativedivides	$\frac{-\frac{\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\frac{\arctan(ax)ax}{4\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)ax}{8\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^2}{16}+\frac{1}{16\left(a^2x^2+1\right)^2}+\frac{3}{16\left(a^2x^2+1\right)}}{2c^3}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\frac{\arctan(ax)ax}{4\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)ax}{8\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^2}{16}+\frac{1}{16\left(a^2x^2+1\right)^2}+\frac{3}{16\left(a^2x^2+1\right)}}{2c^3}}{a^2}$
parts	$-\frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}+\frac{\frac{\arctan(ax)ax}{4\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)ax}{8\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^2}{16}+\frac{1}{16\left(a^2x^2+1\right)^2}+\frac{3}{16\left(a^2x^2+1\right)}}{2c^3a^2}$
risch	$-\frac{(3a^4x^4+6a^2x^2-5)\ln(iax+1)^2}{128a^2c^3(a^2x^2+1)^2}+\frac{(-5\ln(-iax+1)+3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3-10iax)\ln(iax}{64(ax+i)^2c^3(ax-i)^2a^2}$

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(3*a^4*arctan(a*x)^2*x^4-4*a^4*x^4+6*arctan(a*x)*x^3*a^3+6*x^2*arctan
(a*x)^2*a^2-5*a^2*x^2+10*x*arctan(a*x)*a-5*arctan(a*x)^2)/c^3/(a^2*x^2+1)^
2/a^2
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.63

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^2x^2+\left(3\,a^4x^4+6\,a^2x^2-5\right)\arctan\left(ax\right)^2+2\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)+4}{32\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(3*a^2*x^2 + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^2 + 2*(3*a^3*x^3
 + 5*a*x)*arctan(a*x) + 4)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

3.301.6 Sympy [F]

$$\int \frac{x \arctan(ax)^2}{(c + a^2cx^2)^3} dx = \frac{\int \frac{x \operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} dx}{c^3}$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.18

$$\int \frac{x \arctan(ax)^2}{(c + a^2cx^2)^3} dx = \frac{\left(\frac{3a^2x^3+5x}{a^4c^2x^4+2a^2c^2x^2+c^2} + \frac{3\arctan(ax)}{ac^2}\right)\arctan(ax)}{16ac} + \frac{3a^2x^2 - 3(a^4x^4 + 2a^2x^2 + 1)\arctan(ax)^2 + 4}{32(a^6c^2x^4 + 2a^4c^2x^2 + a^2c^2)c} - \frac{\arctan(ax)^2}{4(a^2cx^2 + c)^2a^2c}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/16*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x
)/(a*c^2))*arctan(a*x)/(a*c) + 1/32*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 +
1)*arctan(a*x)^2 + 4)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*c) - 1/4*ar
ctan(a*x)^2/((a^2*c*x^2 + c)^2*a^2*c)
```

3.301.8 Giac [F]

$$\int \frac{x \arctan(ax)^2}{(c + a^2cx^2)^3} dx = \int \frac{x \arctan(ax)^2}{(a^2cx^2 + c)^3} dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.301.9 Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{3\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+6\,a^3\,x^3\,\mathrm{atan}(a\,x)+6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+3\,a^2\,x^2+10\,a\,x\,\mathrm{atan}(a\,x)-5\,\mathrm{atan}(a\,x)^2+4}{32\,a^2\,c^3\,(a^2\,x^2+1)^2}$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

output
```
(3*a^2*x^2 - 5*atan(a*x)^2 + 6*a^3*x^3*atan(a*x) + 10*a*x*atan(a*x) + 6*a^
2*x^2*atan(a*x)^2 + 3*a^4*x^4*atan(a*x)^2 + 4)/(32*a^2*c^3*(a^2*x^2 + 1)^2
)
```

3.302 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$

3.302.1 Optimal result

Integrand size = 19, antiderivative size = 169

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = -\frac{x}{32c^3(1+a^2x^2)^2}-\frac{15x}{64c^3(1+a^2x^2)}-\frac{15\arctan(ax)}{64ac^3}+\frac{\arctan(ax)}{8ac^3(1+a^2x^2)^2}+\frac{3\arctan(ax)}{8ac^3(1+a^2x^2)}+\frac{x\arctan(ax)^2}{4c^3(1+a^2x^2)^2}+\frac{3x\arctan(ax)^2}{8c^3(1+a^2x^2)}+\frac{\arctan(ax)^3}{8ac^3}$$

output
```
-1/32*x/c^3/(a^2*x^2+1)^2-15/64*x/c^3/(a^2*x^2+1)-15/64*arctan(a*x)/a/c^3+
1/8*arctan(a*x)/a/c^3/(a^2*x^2+1)^2+3/8*arctan(a*x)/a/c^3/(a^2*x^2+1)+1/4*
x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^2/c^3/(a^2*x^2+1)+1/8*
arctan(a*x)^3/a/c^3
```

3.302.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{-ax(17+15a^2x^2)+(17-6a^2x^2-15a^4x^4)\arctan(ax)+8ax(5+3a^2x^2)\arctan(ax)^2+8(1+a^2x^2)^2\,\mathrm{arc}}{64ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

output
```
(-(a*x*(17 + 15*a^2*x^2)) + (17 - 6*a^2*x^2 - 15*a^4*x^4)*ArcTan[a*x] + 8*
a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(64*a
*c^3*(1 + a^2*x^2)^2)
```

3.302.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5435, 27, 215, 215, 218, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5435}$$

$$\frac{3\int \frac{\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{4c}-\frac{1}{8}\int \frac{1}{\left(a^2cx^2+c\right)^3}dx+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}-\frac{1}{8}\int \frac{1}{\left(a^2cx^2+c\right)^3}dx+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}+\frac{1}{8}\left(-\frac{3\int \frac{1}{(a^2cx^2+c)^2}dx}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}+\frac{1}{8}\left(-\frac{3\left(\frac{\int \frac{1}{a^2cx^2+c}dx}{2c}+\frac{x}{2c^2(a^2x^2+1)}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 5427$$

$$\frac{3\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 5465$$

$$\frac{3\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 215$$

$$\frac{3\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)$$

3.302. $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

output
```
ArcTan[a*x]/(8*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*
x^2)^2) + (-1/4*x/(c^3*(1 + a^2*x^2)^2) - (3*(x/(2*c^2*(1 + a^2*x^2)) + Ar
cTan[a*x]/(2*a*c^2)))/(4*c))/8 + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) +
 ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1
+ a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(4*c^3)
```

3.302.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.302.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{8a^4 \arctan(ax)^3 x^4 - 15\arctan(ax)a^4x^4 + 24a^3\arctan(ax)^2x^3 + 16\arctan(ax)^3x^2a^2 - 15a^3x^3 - 6a^2\arctan(ax)x^2 + 40a \operatorname{arc}}{64c^3(a^2x^2+1)^2 a}$
derivativedivides	$\frac{\frac{ax\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2} + \frac{3ax\arctan(ax)^2}{8c^3\left(a^2x^2+1\right)} + \frac{3\arctan(ax)^3}{8c^3} - \frac{-\frac{3\arctan(ax)}{2\left(a^2x^2+1\right)} - \frac{\arctan(ax)}{2\left(a^2x^2+1\right)^2} + \frac{\frac{15}{8}a^3x^3 + \frac{17}{8}ax}{2\left(a^2x^2+1\right)^2} + \frac{15\arctan(ax)}{16} + \arctan(ax)^3}{4c^3}}{a}$
default	$\frac{\frac{ax\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2} + \frac{3ax\arctan(ax)^2}{8c^3\left(a^2x^2+1\right)} + \frac{3\arctan(ax)^3}{8c^3} - \frac{-\frac{3\arctan(ax)}{2\left(a^2x^2+1\right)} - \frac{\arctan(ax)}{2\left(a^2x^2+1\right)^2} + \frac{\frac{15}{8}a^3x^3 + \frac{17}{8}ax}{2\left(a^2x^2+1\right)^2} + \frac{15\arctan(ax)}{16} + \arctan(ax)^3}{4c^3}}{a}$
parts	$\frac{x\arctan(ax)^2}{4c^3(a^2x^2+1)^2} + \frac{3x\arctan(ax)^2}{8c^3(a^2x^2+1)} + \frac{3\arctan(ax)^3}{8a\,c^3} - \frac{\frac{-\frac{3\arctan(ax)}{2\left(a^2x^2+1\right)} - \frac{\arctan(ax)}{2\left(a^2x^2+1\right)^2} + \frac{\frac{15}{8}a^3x^3 + \frac{17}{8}ax}{2\left(a^2x^2+1\right)^2} + \frac{15\arctan(ax)}{16}}{a} + \operatorname{ar}}{4c^3}$
risch	$\frac{i\ln(iax+1)^3}{64c^3a} - \frac{i(3x^4\ln(-iax+1)a^4 + 6a^2x^2\ln(-iax+1) - 6ia^3x^3 + 3\ln(-iax+1) - 10iax)\ln(iax+1)^2}{64c^3(a^2x^2+1)^2 a} + \frac{i\left(3a^4x^4\ln(\right.}{}$

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*(8*a^4*arctan(a*x)^3*x^4-15*arctan(a*x)*a^4*x^4+24*a^3*arctan(a*x)^2*
x^3+16*arctan(a*x)^3*x^2*a^2-15*a^3*x^3-6*a^2*arctan(a*x)*x^2+40*a*arctan(
a*x)^2*x+8*arctan(a*x)^3-17*a*x+17*arctan(a*x))/c^3/(a^2*x^2+1)^2/a
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = -\frac{15\,a^3x^3 - 8\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan{(ax)}^3 - 8\,(3\,a^3x^3 + 5\,ax)\arctan{(ax)}^2 + 17\,ax + (15\,a^4x^4 + 6\,a^2}{64\,(a^5c^3x^4 + 2\,a^3c^3x^2 + ac^3)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 8*(3*a^3*x
^3 + 5*a*x)*arctan(a*x)^2 + 17*a*x + (15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(
a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

3.302.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^2{(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.37

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{3\,a^2x^3 + 5\,x}{a^4c^3x^4 + 2\,a^2c^3x^2 + c^3} + \frac{3\,\arctan{(ax)}}{ac^3}\right)\arctan{(ax)}^2 - \frac{\left(15\,a^3x^3 - 8\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan{(ax)}^3 + 17\,ax + 15\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan{(ax)}\right)a^2}{64\,(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3)} + \frac{\left(3\,a^2x^2 - 3\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan{(ax)}^2 + 4\right)a\arctan{(ax)}}{8\,(a^6c^3x^4 + 2\,a^4c^3x^2 + a^2c^3)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)
/(a*c^3))*arctan(a*x)^2 - 1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*a
rctan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^7
*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3) + 1/8*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*
x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a
^2*c^3)
```

3.302.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.302.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(ax)^2}{\left(c+a^2cx^2\right)^3}\,dx=\frac{\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^3\,c^3}+\frac{3\,x^2}{8\,a\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{15\,\mathrm{atan}(a\,x)}{64\,a\,c^3}-\frac{\frac{15\,a^2\,x^3}{8}+\frac{17\,x}{8}}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3}+\frac{\mathrm{atan}(a\,x)^2\,\left(\frac{3\,x^3}{8\,c^3}+\frac{5\,x}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{\mathrm{atan}(a\,x)^3}{8\,a\,c^3}$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^3,x)
```

output
```
(atan(a*x)*(1/(2*a^3*c^3) + (3*x^2)/(8*a*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
- (15*atan(a*x))/(64*a*c^3) - ((17*x)/8 + (15*a^2*x^3)/8)/(8*c^3 + 16*a^2*
c^3*x^2 + 8*a^4*c^3*x^4) + (atan(a*x)^2*((3*x^3)/(8*c^3) + (5*x)/(8*a^2*c^
3)))/(1/a^2 + 2*x^2 + a^2*x^4) + atan(a*x)^3/(8*a*c^3)
```

3.303 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^3} \, dx$

3.303.1 Optimal result

Integrand size = 22, antiderivative size = 236

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^3} \, dx = & -\frac{1}{32c^3(1+a^2x^2)^2} - \frac{11}{32c^3(1+a^2x^2)} - \frac{ax\arctan(ax)}{8c^3(1+a^2x^2)^2} \\ & - \frac{11ax\arctan(ax)}{16c^3(1+a^2x^2)} - \frac{11\arctan(ax)^2}{32c^3} + \frac{\arctan(ax)^2}{4c^3(1+a^2x^2)^2} \\ & + \frac{\arctan(ax)^2}{2c^3(1+a^2x^2)} - \frac{i\arctan(ax)^3}{3c^3} + \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3} \\ & - \frac{i\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3} + \frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
-1/32/c^3/(a^2*x^2+1)^2-11/32/c^3/(a^2*x^2+1)-1/8*a*x*arctan(a*x)/c^3/(a^2
*x^2+1)^2-11/16*a*x*arctan(a*x)/c^3/(a^2*x^2+1)-11/32*arctan(a*x)^2/c^3+1/
4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)^2/c^3/(a^2*x^2+1)-1/3*I*
arctan(a*x)^3/c^3+arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3-I*arctan(a*x)*polylo
g(2,-1+2/(1-I*a*x))/c^3+1/2*polylog(3,-1+2/(1-I*a*x))/c^3
```

3.303.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx$$
$$= \frac{-32i\pi^3 + 256i\arctan(ax)^3 - 144\cos(2\arctan(ax)) + 288\arctan(ax)^2\cos(2\arctan(ax)) - 3\cos(4\arctan}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```

output
```
((-32*I)*Pi^3 + (256*I)*ArcTan[a*x]^3 - 144*Cos[2*ArcTan[a*x]] + 288*ArcTa
n[a*x]^2*Cos[2*ArcTan[a*x]] - 3*Cos[4*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Cos[
4*ArcTan[a*x]] + 768*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (768*
I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 384*PolyLog[3, E^((-2*
I)*ArcTan[a*x])] - 288*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Sin
[4*ArcTan[a*x]])/(768*c^3)
```

3.303.3 Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.39, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5501, 27, 5465, 5431, 5427, 241, 5501, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^3}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2x(a^2x^2+1)^2}dx}{c} - a^2\int \frac{x\arctan(ax)^2}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5431$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5427$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 241$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5459$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3}{c^3}$$

$$\Big\downarrow \; 5403$$

3.303. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$
$$\frac{-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^3}$$

$$\Big\downarrow \; 5465$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$
$$\frac{-a^2\left(\frac{\int\frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\mathrm{arcta}}{c^3}$$

$$\Big\downarrow \; 5427$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$
$$\frac{-a^2\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\right.}{c^3}$$

$$\Big\downarrow \; 241$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$
$$\frac{i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\mathrm{arctan}(}{2a^2(a^2x}\right.}{c^3}$$

$$\Big\downarrow \; 5527$$

3.303. $\int\frac{\arctan(ax)^2}{x(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(\cdots}{2\left(a^2x^2+\cdots\right.}}{\cdots}\right.}{c^3}$$

$$\Big\downarrow \textcolor{blue}{7164}$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{-a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)\cdots\right.}{c^3}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-1/4*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)^2) + (1/(16*a*(1 + a^2*x^2)^
2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*
ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)/(2*a)))/c^3) + (
(-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/
(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4
*a))/a) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*A
rcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*
x)]/(4*a))))/c^3
```

3.303.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.303. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^3}\,dx$

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.303.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.06 (sec) , antiderivative size = 1722, normalized size of antiderivative = 7.30

method	result	size
derivativedivides	Expression too large to display	1722
default	Expression too large to display	1722
parts	Expression too large to display	2148

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*arctan(a*x)^2*ln(a*x)+1/4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-1/2/c^3*ar
ctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2/c^3/(a^2*x^2+1)-1/2/c^3*(-2*ar
ctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+2/3*I*arctan(a*x)^3-3*I*arctan(a*x)*(I+a*x)/(8*a*
x-8*I)+4*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/16*(I+a*x
)/(a*x-I)-3/16*(a*x-I)/(I+a*x)+2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-
1)-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,(1+I*a*x)
/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+3*I*
arctan(a*x)*(a*x-I)/(8*a*x+8*I)-4*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
1/16*(-8*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))+16*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))+16*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))^3-8*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+16*I*Pi*csgn(I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+16*I*Pi*csgn(((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+8*I*Pi*csgn(I/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^2-16*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-16*I*Pi*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^...
```

3.303.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*
x), x)
```

3.303.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

3.303.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x), x)
```

3.303.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^3), x)
```

3.304 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

3.304.1 Optimal result

Integrand size = 22, antiderivative size = 250

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx = \frac{a^2x}{32c^3(1+a^2x^2)^2} + \frac{31a^2x}{64c^3(1+a^2x^2)} + \frac{31a\arctan(ax)}{64c^3} - \frac{a\arctan(ax)}{8c^3(1+a^2x^2)^2} - \frac{7a\arctan(ax)}{8c^3(1+a^2x^2)} - \frac{ia\arctan(ax)^2}{c^3} - \frac{\arctan(ax)^2}{c^3x} - \frac{a^2x\arctan(ax)^2}{4c^3(1+a^2x^2)^2} - \frac{7a^2x\arctan(ax)^2}{8c^3(1+a^2x^2)} - \frac{5a\arctan(ax)^3}{8c^3} + \frac{2a\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3} - \frac{ia\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}$$

output
```
1/32*a^2*x/c^3/(a^2*x^2+1)^2+31/64*a^2*x/c^3/(a^2*x^2+1)+31/64*a*arctan(a*
x)/c^3-1/8*a*arctan(a*x)/c^3/(a^2*x^2+1)^2-7/8*a*arctan(a*x)/c^3/(a^2*x^2+
1)-I*a*arctan(a*x)^2/c^3-arctan(a*x)^2/c^3/x-1/4*a^2*x*arctan(a*x)^2/c^3/(
a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)^2/c^3/(a^2*x^2+1)-5/8*a*arctan(a*x)^3/c
^3+2*a*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3-I*a*polylog(2,-1+2/(1-I*a*x))/c^3
```

3.304.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.56

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx = -\frac{160ax\arctan(ax)^3+4ax\arctan(ax)\left(32\cos(2\arctan(ax))+\cos(4\arctan(ax))-128\log\left(1-e^{2i\arctan(}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
-1/256*(160*a*x*ArcTan[a*x]^3 + 4*a*x*ArcTan[a*x]*(32*Cos[2*ArcTan[a*x]] +
 Cos[4*ArcTan[a*x]] - 128*Log[1 - E^((2*I)*ArcTan[a*x])]) + (256*I)*a*x*Po
lyLog[2, E^((2*I)*ArcTan[a*x])] - a*x*(64*Sin[2*ArcTan[a*x]] + Sin[4*ArcTa
n[a*x]]) + 8*ArcTan[a*x]^2*(32 + (32*I)*a*x + 16*a*x*Sin[2*ArcTan[a*x]] +
a*x*Sin[4*ArcTan[a*x]]))/(c^3*x)
```

3.304.3 Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.60, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.955, Rules used = {5501, 27, 5435, 215, 215, 216, 5427, 5465, 215, 216, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2x^2(a^2x^2+1)^2}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c^3(a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx-\frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 216$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \; 5427$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)\right.\right.}{c^3}$$

$$\Big\downarrow \; 5465$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x\cdots}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\cdots\right.\right.}{c^3}$$

3.304. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{arcta}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{arcta}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{arcta}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{-\left(a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx+\int \frac{\arctan(ax)^2}{x^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{arcta}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{-\left(a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{arcta}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5419}$$

3.304. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^3}-$$

$$\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{arcta}}{2a}\right.\right.\right.}{c^3}$$

$\downarrow$ 5459

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-}{c^3}$$

$\downarrow$ 5403

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)\right.}{c^3}$$

$\downarrow$ 2897

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}\right.\right.\right.}{c^3}$$

$\downarrow$ 5465

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{P}\right.\right.}{c^3}$$

3.304. $\int\frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

$$\Big\downarrow \; 215$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-ia}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 216$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\right.\right.}{c^3}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(ArcTan[a*x]/(8*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*(1 + a^2*
x^2)^2) + (-1/4*x/(1 + a^2*x^2)^2 - (3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/
(2*a))/4)/8 + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*
a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcT
an[a*x]/(2*a))/(2*a))))/4))/c^3) + (-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)
/3 - a^2*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-
1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(
2*a))/(2*a))) + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 -
2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2)))/c^3
```

3.304.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.304. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.304. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.304. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$

3.304.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.48

method	result
derivativedivides	$a\left(-\frac{\arctan(ax)^2}{c^3ax}-\frac{7\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^3}{8c^3}-\frac{-5\arctan(ax)^3-8\arctan(ax)\,\mathrm{l}}{}\right.$
default	$a\left(-\frac{\arctan(ax)^2}{c^3ax}-\frac{7\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^3}{8c^3}-\frac{-5\arctan(ax)^3-8\arctan(ax)\,\mathrm{l}}{}\right.$
parts	$-\frac{7\arctan(ax)^2a^4x^3}{8c^3(a^2x^2+1)^2}-\frac{9a^2x\arctan(ax)^2}{8c^3(a^2x^2+1)^2}-\frac{15a\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{c^3x}-\frac{2\left(-\frac{5a\arctan(ax)^3}{8}-\frac{a\left(8\arctan(a\right.}{}\right.}{}$

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c^3*arctan(a*x)^2/a/x-7/8/c^3*arctan(a*x)^2/(a^2*x^2+1)^2*a^3*x^3-9/
8*a*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-15/8*arctan(a*x)^3/c^3-1/4/c^3*(-5*a
rctan(a*x)^3-8*arctan(a*x)*ln(a*x)+1/2*arctan(a*x)/(a^2*x^2+1)^2+7/2*arcta
n(a*x)/(a^2*x^2+1)+4*arctan(a*x)*ln(a^2*x^2+1)-4*I*ln(a*x)*ln(1+I*a*x)+4*I
*ln(a*x)*ln(1-I*a*x)-4*I*dilog(1+I*a*x)+4*I*dilog(1-I*a*x)+2*I*(ln(a*x-I)*
ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*
x-I)^2)-2*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2
*I*(a*x-I))-1/2*ln(I+a*x)^2)-1/2*(31/8*a^3*x^3+33/8*a*x)/(a^2*x^2+1)^2-31/
16*arctan(a*x)))
```

3.304.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx=\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*
x^2), x)
```

3.304.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^8+3a^4x^6+3a^2x^4+x^2}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c
**3
```

3.304.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.304.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2 c x^2)^3} \, dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^2 \left(c\,a^2\,x^2 + c\right)^3} \, dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^3), x)
```

3.305 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$

3.305.1 Optimal result

Integrand size = 22, antiderivative size = 322

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx &= \frac{a^2}{32c^3(1+a^2x^2)^2}+\frac{19a^2}{32c^3(1+a^2x^2)}-\frac{a\arctan(ax)}{c^3x}+\frac{a^3x\arctan(ax)}{8c^3(1+a^2x^2)^2}\\ &\quad+\frac{19a^3x\arctan(ax)}{16c^3(1+a^2x^2)}+\frac{3a^2\arctan(ax)^2}{32c^3}-\frac{\arctan(ax)^2}{2c^3x^2}\\ &\quad-\frac{a^2\arctan(ax)^2}{4c^3(1+a^2x^2)^2}-\frac{a^2\arctan(ax)^2}{c^3(1+a^2x^2)}+\frac{ia^2\arctan(ax)^3}{c^3}\\ &\quad+\frac{a^2\log(x)}{c^3}-\frac{a^2\log(1+a^2x^2)}{2c^3}-\frac{3a^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\ &\quad+\frac{3ia^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}\\ &\quad-\frac{3a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
1/32*a^2/c^3/(a^2*x^2+1)^2+19/32*a^2/c^3/(a^2*x^2+1)-a*arctan(a*x)/c^3/x+1
/8*a^3*x*arctan(a*x)/c^3/(a^2*x^2+1)^2+19/16*a^3*x*arctan(a*x)/c^3/(a^2*x^
2+1)+3/32*a^2*arctan(a*x)^2/c^3-1/2*arctan(a*x)^2/c^3/x^2-1/4*a^2*arctan(a
*x)^2/c^3/(a^2*x^2+1)^2-a^2*arctan(a*x)^2/c^3/(a^2*x^2+1)+I*a^2*arctan(a*x
)^3/c^3+a^2*ln(x)/c^3-1/2*a^2*ln(a^2*x^2+1)/c^3-3*a^2*arctan(a*x)^2*ln(2-2
/(1-I*a*x))/c^3+3*I*a^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c^3-3/2*a^2*
polylog(3,-1+2/(1-I*a*x))/c^3
```

3.305.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx$$
$$= \frac{a^2\left(\frac{i\pi^3}{8}-\frac{\arctan(ax)}{ax}-\frac{(1+a^2x^2)\arctan(ax)^2}{2a^2x^2}-i\arctan(ax)^3+\frac{5}{16}\cos(2\arctan(ax))-\frac{5}{8}\arctan(ax)^2\cos(2\operatorname{arcta}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(a^2*((I/8)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^
2*x^2) - I*ArcTan[a*x]^3 + (5*Cos[2*ArcTan[a*x]])/16 - (5*ArcTan[a*x]^2*Co
s[2*ArcTan[a*x]])/8 + Cos[4*ArcTan[a*x]]/256 - (ArcTan[a*x]^2*Cos[4*ArcTan
[a*x]])/32 - 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + Log[a*x] +
Log[1/Sqrt[1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a
*x])] - (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 + (5*ArcTan[a*x]*Sin[2*Ar
cTan[a*x]])/8 + (ArcTan[a*x]*Sin[4*ArcTan[a*x]])/64))/c^3
```

3.305.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 689 vs. 2(322) = 644.

Time = 5.62 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.14, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.182, Rules used = {5501, 27, 5501, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5465, 5431, 5427, 241, 5501, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\left(a^2cx^2+c\right)^3}\,dx$$
$$\Big\downarrow \ 5501$$
$$\frac{\int \frac{\arctan(ax)^2}{c^2x^3(a^2x^2+1)^2}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c^3x\left(a^2x^2+1\right)^3}dx$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \ 5501$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 5453$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+\int \frac{\arctan(ax)^2}{x^3}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 5361$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 5453$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 5361$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 243$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(a}{2x^2}}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 47$$

3.305. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 14$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 16$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 5419$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 5459$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}+$$

$$\frac{-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(a\right.}{c^3}$$

$$\Big\downarrow \ 5403$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}+$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c^3}$$

$$\Big\downarrow \ 5465$$

3.305. $\quad \int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^3}dx}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\frac{3}{4}\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$$\Big\downarrow \text{241}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$$\Big\downarrow \text{5501}$$

3.305. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}-}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-\left(a^2\left(\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)\right)-a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-ia}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5459}$$

$$-\left(a^2\left(-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int\frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\,\text{arcta}\right.\right.$$

$$\frac{a^2\left(-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int\frac{\arctan(ax)^2}{x(ax+i)}dx-a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{a}{4a}\right.\right.}{c^3}$$

$$\Big\downarrow \text{5403}$$

$$-\left(a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(-a^2\int\frac{x\,\text{arcta}}{(a^2x}\right.$$

$$\frac{a^2\left(-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}}{}\right.\right.}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$-\left(a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(-a^2\left(\frac{\int\frac{\text{arct}}{(a^2}}{}\right.\right.$$

$$\frac{a^2\left(-a^2\left(\frac{\int\frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

3.305. $\int\frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$

$$-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(-a^2\left(\frac{-\frac{1}{2}a\int\right.\right.$$

$$a^2\left(-a^2\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\right.\right.$$

$$\Big\downarrow \text{241}$$

$$-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(i\left(2ia\int\frac{\text{arc}}{}\right.\right.$$

$$a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\text{ar}}{2a}\right.\right.$$

$$c^3$$

$$\Big\downarrow \text{5527}$$

$$-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,a\right.\right.$$

$$a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\,a}{2(}}{}\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$-a^2\left(-a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}\right.}{4a}\right.\right.\right.$$

$$a^2\left(-a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)-a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{}{16a}}{2a}\right.\right.$$

3.305. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$

input
```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*((-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2
)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a
*x]^2/(4*a))/a) - a^2*(-1/4*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)^2) + (1/(16*a
*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 +
a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)/(
2*a)) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*Arc
Tan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)
]/(4*a)))))/c^3) + (-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*ArcT
an[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((-1/3*I)*ArcTan
[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*A
rcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*
x)]/(4*a)))) - a^2*((-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*
(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)
) + ArcTan[a*x]^2/(4*a))/a) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
 + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog
[3, -1 + 2/(1 - I*a*x)]/(4*a)))))/c^3
```

3.305.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.305. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3} dx$

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.305.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 57.98 (sec) , antiderivative size = 1970, normalized size of antiderivative = 6.12

method	result	size
derivativedivides	Expression too large to display	1970
default	Expression too large to display	1970
parts	Expression too large to display	2425

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-arctan(a*x)^2/c^3/(a^2*x^2+1)-1/4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+3/
2/c^3*arctan(a*x)^2*ln(a^2*x^2+1)-1/2/c^3*arctan(a*x)^2/a^2/x^2-3/c^3*arct
an(a*x)^2*ln(a*x)-1/2/c^3*(3*I*Pi*arctan(a*x)^2-12*I*arctan(a*x)*polylog(2
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))-3/16*arctan(a*x)^2-6*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)
-1)+6*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)^2*ln((
1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*arctan(a*x)^2*ln(2)-1/128*cos(4*arctan(a*x
))+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*ar
ctan(a*x)^2-3/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)*arctan(a*x)^2+12*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*polylog(3
,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln((1+
I*a*x)/(a^2*x^2+1)^(1/2)+1)+arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+ar
ctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+3/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)^2*arctan(a*x)^2-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+3/2*I*Pi*csgn(I*((1+I*a*x)^2/(...
```

3.305.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*
x^3), x)
```

3.305.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx=\frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c
**3
```

3.305.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^3), x)
```

3.305.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^3\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^3), x)
```

3.306 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

3.306.1 Optimal result

Integrand size = 22, antiderivative size = 317

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx &= -\frac{a^2}{3c^3x} - \frac{a^4x}{32c^3(1+a^2x^2)^2} - \frac{47a^4x}{64c^3(1+a^2x^2)} - \frac{205a^3\arctan(ax)}{192c^3} \\ &\quad - \frac{a\arctan(ax)}{3c^3x^2} + \frac{a^3\arctan(ax)}{8c^3(1+a^2x^2)^2} + \frac{11a^3\arctan(ax)}{8c^3(1+a^2x^2)} \\ &\quad + \frac{10ia^3\arctan(ax)^2}{3c^3} - \frac{\arctan(ax)^2}{3c^3x^3} + \frac{3a^2\arctan(ax)^2}{c^3x} \\ &\quad + \frac{a^4x\arctan(ax)^2}{4c^3(1+a^2x^2)^2} + \frac{11a^4x\arctan(ax)^2}{8c^3(1+a^2x^2)} + \frac{35a^3\arctan(ax)^3}{24c^3} \\ &\quad - \frac{20a^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c^3} + \frac{10ia^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^3}\end{aligned}$$

output
```
-1/3*a^2/c^3/x-1/32*a^4*x/c^3/(a^2*x^2+1)^2-47/64*a^4*x/c^3/(a^2*x^2+1)-20
5/192*a^3*arctan(a*x)/c^3-1/3*a*arctan(a*x)/c^3/x^2+1/8*a^3*arctan(a*x)/c^
3/(a^2*x^2+1)^2+11/8*a^3*arctan(a*x)/c^3/(a^2*x^2+1)+10/3*I*a^3*arctan(a*x
)^2/c^3-1/3*arctan(a*x)^2/c^3/x^3+3*a^2*arctan(a*x)^2/c^3/x+1/4*a^4*x*arct
an(a*x)^2/c^3/(a^2*x^2+1)^2+11/8*a^4*x*arctan(a*x)^2/c^3/(a^2*x^2+1)+35/24
*a^3*arctan(a*x)^3/c^3-20/3*a^3*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3+10/3*I*a
^3*polylog(2,-1+2/(1-I*a*x))/c^3
```

3.306.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.60

$$\int \frac{\arctan(ax)^2}{x^4 (c+a^2cx^2)^3}\,dx$$
$$= \frac{a^3\left(-\frac{256(1+a^2x^2)\arctan(ax)}{a^2x^2} - \frac{256(1+a^2x^2)\arctan(ax)^2}{a^3x^3} + 1120\arctan(ax)^3 + \frac{256(-1+10\arctan(ax)^2)}{ax} + 576\arctan(ax\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
(a^3*((-256*(1 + a^2*x^2)*ArcTan[a*x])/(a^2*x^2) - (256*(1 + a^2*x^2)*ArcT
an[a*x]^2)/(a^3*x^3) + 1120*ArcTan[a*x]^3 + (256*(-1 + 10*ArcTan[a*x]^2))/
(a*x) + 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 12*ArcTan[a*x]*Cos[4*ArcTan[a
*x]] - 5120*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + (2560*I)*(ArcTan[
a*x]^2 + PolyLog[2, E^((2*I)*ArcTan[a*x])]) + 288*(-1 + 2*ArcTan[a*x]^2)*S
in[2*ArcTan[a*x]] + 3*(-1 + 8*ArcTan[a*x]^2)*Sin[4*ArcTan[a*x]]))/(768*c^3
)
```

3.306.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4 (a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \text{5501}$$
$$\frac{\int \frac{\arctan(ax)^2}{c^2x^4(a^2x^2+1)^2}dx}{c} - a^2\int \frac{\arctan(ax)^2}{c^3x^2(a^2x^2+1)^3}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^3}dx}{c^3}$$
$$\Big\downarrow \text{5501}$$
$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx - \frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2(a^2x^2+1)}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\text{arc}}{8a(}\right.\right.}{c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+1}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx + \int \frac{\arctan(ax)^2}{x^4}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5361}$$

3.306. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{3x^3}}{c^3}-$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan}{3}}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5419}$$

$$\frac{-\left(a^2\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)\right)-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan}{2x}\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5459}$$

3.306. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx + \frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx - \frac{1}{2}i\arctan(ax)^2\right)\right) - \frac{\arctan(ax)}{2x^2} + \frac{1}{2}a\left(-a\arctan(ax) - \frac{1}{x}\right)\right) - \left(\right.}{c^3}$$

$$\Big\downarrow \color{blue}{5403}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right) - a^2\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{2}i\arctan(ax)^2\right) - \frac{1}{3}\right.$$

$$\Big\downarrow \color{blue}{2897}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx + \frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right) - \frac{1}{2}i\arctan(ax)^2\right.\right.$$

$$\Big\downarrow \color{blue}{5465}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a} - \frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx + \frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right) - \frac{1}{2}i\arctan(ax)^2\right.\right.$$

$$\Big\downarrow \color{blue}{215}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2(a^2x^2+1)}}{2a} - \frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(a}{4(a^2x^2+}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx + \frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right) - \frac{1}{2}i\arctan(ax)^2\right.\right.$$

$$\Big\downarrow \color{blue}{216}$$

3.306. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan}{2(a^2x^2}\right.\right.\right.}{-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right.\right.\right.}$$

c^3

$$\Big\downarrow 5501$$

$$-\frac{a^2\left(-a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{}{4}\right.\right.\right.\right.}{-a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.\right.}$$

c^3

$$\Big\downarrow 5427$$

$$-\frac{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\right.\right.\right.}{-a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\right.\right.\right.\right.\right.}$$

$$\Big\downarrow 5453$$

$$-\frac{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx+\int \frac{\arctan(ax)^2}{x^2}dx-\left(a^2\left(\frac{x\arctan(a}{4(a^2x^2+1}\right.\right.\right.}{-a^2\left(-\left(a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx+\int \frac{\arctan(ax)^2}{x^2}dx\right)+\frac{2}{3}a\left(-\left(a^2\left(\right.\right.\right.}$$

$$\Big\downarrow 5361$$

$$-\frac{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan}{4(a^2x^2}\right.\right.\right.}{-a^2\left(-\left(a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}$$

$$\Big\downarrow 5419$$

3.306. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\right.\right.}{-a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)+\frac{2}{3}}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax\right.}{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-a^2\left(\frac{x\arctan(ax}{4(a^2x^2+1)}\right.\right.}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{}{1-}\right.\right.\right.\right.}{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-i}\right.\right.\right.\right.}$$

$$\Big\downarrow \text{2897}$$

$$\frac{-a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\right.\right.\right.\right.}{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)-a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\text{arc}\right.\right.\right.\right.\right.}$$

$$\Big\downarrow \text{5465}$$

$$\frac{-a^2\left(-a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right.\right.\right.}{a^2\left(-a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)-a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.\right.}$$

3.306. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

3.306.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.306. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.306. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.306.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

method	result
derivativedivides	$a^3\left(-\frac{\arctan(ax)^2}{3c^3a^3x^3}+\frac{3\arctan(ax)^2}{c^3ax}+\frac{11\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^3}{8c^3}-\frac{\frac{4\arctan(a}{a^2x^2}}{}\right.$
default	$a^3\left(-\frac{\arctan(ax)^2}{3c^3a^3x^3}+\frac{3\arctan(ax)^2}{c^3ax}+\frac{11\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^3}{8c^3}-\frac{\frac{4\arctan(a}{a^2x^2}}{}\right.$
parts	$\frac{11\arctan(ax)^2a^6x^3}{8c^3(a^2x^2+1)^2}+\frac{13a^4x\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35a^3\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{3c^3x^3}+\frac{3a^2\arctan(ax)^2}{c^3x}-\frac{2\left(\frac{35a^3\arctan}{8}\right.}{}$

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/c^3*arctan(a*x)^2/a^3/x^3+3/c^3*arctan(a*x)^2/a/x+11/8/c^3*arcta
n(a*x)^2/(a^2*x^2+1)^2*a^3*x^3+13/8*a*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+35
/8*arctan(a*x)^3/c^3-1/12/c^3*(4*arctan(a*x)/a^2/x^2+80*arctan(a*x)*ln(a*x
)-40*arctan(a*x)*ln(a^2*x^2+1)-3/2*arctan(a*x)/(a^2*x^2+1)^2-33/2*arctan(a
*x)/(a^2*x^2+1)+40*I*ln(a*x)*ln(1+I*a*x)-40*I*ln(a*x)*ln(1-I*a*x)+40*I*dil
og(1+I*a*x)-40*I*dilog(1-I*a*x)-20*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I
*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+20*I*(ln(I+a*x)*ln
(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)
^2)+4/a/x+1/2*(141/8*a^3*x^3+147/8*a*x)/(a^2*x^2+1)^2+205/16*arctan(a*x)+3
5*arctan(a*x)^3))
```

3.306.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3
*x^4), x)
```

3.306.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx=\frac{\int \frac{\operatorname{atan}^2\left(ax\right)}{a^6x^{10}+3a^4x^8+3a^2x^6+x^4}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/
c**3
```

3.306.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.306.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^3} \, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)^3 x^4} \, dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^3} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^4 \left(c\,a^2\,x^2 + c\right)^3} \, dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^3), x)
```

3.307 $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

3.307.1 Optimal result

Integrand size = 24, antiderivative size = 385

$$\begin{aligned}\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx &= -\frac{11\sqrt{c+a^2cx^2}}{60a^4}+\frac{\left(c+a^2cx^2\right)^{3/2}}{30a^4c}\\
&\quad+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{12a^3}-\frac{x^3\sqrt{c+a^2cx^2}\arctan(ax)}{10a}\\
&\quad-\frac{2\sqrt{c+a^2cx^2}\arctan(ax)^2}{15a^4}+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{15a^2}\\
&\quad+\frac{1}{5}x^4\sqrt{c+a^2cx^2}\arctan(ax)^2\\
&\quad-\frac{11ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{30a^4\sqrt{c+a^2cx^2}}\\
&\quad+\frac{11ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{c+a^2cx^2}}\\
&\quad-\frac{11ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/30*(a^2*c*x^2+c)^(3/2)/a^4/c-11/30*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+11/60*I*c*pol
ylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^
2+c)^(1/2)-11/60*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2
+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-11/60*(a^2*c*x^2+c)^(1/2)/a^4+1/12*x*arc
tan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3-1/10*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/
a-2/15*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4+1/15*x^2*arctan(a*x)^2*(a^2*c
*x^2+c)^(1/2)/a^2+1/5*x^4*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.307.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx =$$

$$-\frac{(1+a^2x^2)^2\,\sqrt{c\,(1+a^2x^2)}\Big(50-32\arctan(ax)^2+72\cos(2\arctan(ax))+160\arctan(ax)^2\cos(2\arctan(}{}$$

input
```
Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
-1/960*((1 + a^2*x^2)^2*Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*
Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTa
n[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
 - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*Arc
Tan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x
]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*A
rcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x
]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 +
a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*A
rcTan[a*x]]))/a^4
```

3.307.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1189 vs. 2(385) = 770.

Time = 5.68 (sec) , antiderivative size = 1189, normalized size of antiderivative = 3.09, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {5485, 5487, 5465, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5465, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$c\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{5465}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{5425}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+$$

$$a^2c\left(-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{5421}$$

$$a^2c\left(-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$

$$c\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{a\sqrt{a^2cx^2+c}}\right.}{3a^2}\right.$$

$$\Big\downarrow \text{5487}$$

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan}{a}\right.}{}\right.}{3}\right.$$

$$\Big\downarrow \text{241}$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$a^2c\left(-\frac{4\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+x\cdots\right.}{5a}\cdots\right.$$

$$c\left(-\frac{2\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.}{a}\cdots\right.}{\cdots}\right.}{3a^2}\cdots\right.$$

$$\Big\downarrow \ 243$$

$$a^2c\left(-\frac{4\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}+\cdots\right.}{5a}\cdots\right.$$

$$c\left(-\frac{2\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.}{a}\cdots\right.}{\cdots}\right.}{3a^2}\cdots\right.$$

$$\Big\downarrow \ 53$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$a^2c\left(-\frac{4\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{}{a^2\sqrt{c}}\right.}{8a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.}{a}\right.}{}\right.}{3a^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2c\left(-\frac{4\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx}}{4a^2c}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.}{a}\right.}{}\right.}{3a^2}\right.$$

$$\Big\downarrow \text{5425}$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx}}{4a^2c}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)}{}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{5421}$$

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx}}{4a^2c}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\,\mathrm{a}}{}\right.$$

$$\Big\downarrow \text{5465}$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$a^2c\left(-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}d}{4a^2}\right.}{}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\,\mathrm{a}}{}\right.$$

$$\Big\downarrow \color{blue}{5425}$$

$$a^2c\left(-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\,\mathrm{arc}}{\sqrt{a^2}}}{4}\right.}{}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\,\mathrm{a}}{}\right.$$

$$\Big\downarrow \color{blue}{5421}$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2}\cdots}{}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\cdots}{}\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\Big\downarrow \text{5487}$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{x}{\sqrt{a^2cx^2+}}}{2a}\right.}{4a^2}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i}{}\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\downarrow 241$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)}{2a^2c}-\frac{\int\frac{\arctan(ax}{\sqrt{a^2cx^2+}}}{2a^2}\right.}{4a^2}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\,\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i}{}\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5425

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\int\cdots}{2a^2\sqrt{\cdots}}\cdots\right.}{4a^2}\cdots\right.}{5a}\cdots\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\cdots}{\cdots}\cdots\right.}{2a^2\sqrt{a^2cx^2+c}}\cdots\right.}{3a}\cdots\right.$$

$$\Big\downarrow \; 5421$$

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{5a\cdots}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\cdots}{\cdots}\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\cdots\right.$$

input
```
Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

output
```
c*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a
^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[
1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2]))
)/(3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^
2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (
I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^2))
 + a^2*c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/8*((-
2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a
+ (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*
c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(
4*a^2)))/(5*a) - (4*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (
2*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]
/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*...
```

3.307.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.307. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

3.307.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.61

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\left(12a^4\arctan(ax)^2x^4-6\arctan(ax)x^3a^3+4x^2\arctan(ax)^2a^2+2a^2x^2+5x\arctan(ax)a-8\arctan(ax)^2-9\right)}{60a^4}-\frac{11\sqrt{}}{}$

input
```
int(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(12*a^4*arctan(a*x)^2*x^4-6*arctan(a*x)
*x^3*a^3+4*x^2*arctan(a*x)^2*a^2+2*a^2*x^2+5*x*arctan(a*x)*a-8*arctan(a*x)
^2-9)-11/60*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/
(a^2*x^2+1)^(1/2)
```

3.307.5 Fricas [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int\sqrt{a^2cx^2+c}x^3\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2, x)
```

3.307.6 Sympy [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^3\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x**3*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

3.307.7 Maxima [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \sqrt{a^2cx^2+c}x^3\arctan(ax)^2\,dx$$

input
```
integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2, x)
```

3.307.8 Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int x^3\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.308 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

3.308.1 Optimal result

Integrand size = 24, antiderivative size = 436

$$\begin{aligned}\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx &= \frac{x\sqrt{c+a^2cx^2}}{12a^2}+\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{12a^3}\\ &\quad-\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{6a}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a^2}\\ &\quad+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\\ &\quad+\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{4a^3\sqrt{c+a^2cx^2}}\\ &\quad-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^3}\\ &\quad-\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\\ &\quad+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\\ &\quad+\frac{c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\\ &\quad-\frac{c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/6*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3+1/4*I*c*arctan((
1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2
+c)^(1/2)-1/4*I*c*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/4*I*c*arctan(a*x)*polylog(2,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/4*c
*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^
2+c)^(1/2)-1/4*c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2
)/a^3/(a^2*c*x^2+c)^(1/2)+1/12*x*(a^2*c*x^2+c)^(1/2)/a^2+1/12*arctan(a*x)*
(a^2*c*x^2+c)^(1/2)/a^3-1/6*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a+1/8*x*ar
ctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2+1/4*x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1
/2)
```

3.308.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$$
$$=\frac{\sqrt{c+a^2cx^2}\left(8\left(3i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-2\text{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-3i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(8*((3*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 2
*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 3*Pol
yLog[3, (-I)*E^(I*ArcTan[a*x])] - 3*PolyLog[3, I*E^(I*ArcTan[a*x])]) + (1
+ a^2*x^2)^(3/2)*(ArcTan[a*x]*(2 + 6*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]])
 - 3*ArcTan[a*x]^2*(-7*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]) + 2*(a*
x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]))))/(96*a^3*Sqrt[1 + a^2*x^2])
```

3.308.3 Rubi [A] (verified)

Time = 5.47 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.62, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.042, Rules used = {5485, 5487, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 5487, 262, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5485}$$

$$c\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$$\Big\downarrow \text{5425}$$

$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)$$

$$\Big\downarrow 4669$$

$$a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie\right.\right.}{2a^3\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow 3011$$

$$a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arc}\right.\right.}{}\right.$$

$$\Big\downarrow 2720$$

$$a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arct}}\right.\right.\right.}{}\right.$$

$$\Big\downarrow 5465$$

$$a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Po}\right.\right.}{}\right.$$

$$\Big\downarrow 224$$

3.308. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arct\ldots}\right.\right.}{}\right.$$

$$\Big\downarrow 219$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\ldots\right.\right.}{}\right.$$

$$\Big\downarrow 5487$$

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{\ldots}}{2a^2c}\right.}{4a^2}\right.$$
$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\ldots\right.\right.}{}\right.$$

$$\Big\downarrow 262$$

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\ldots\right.}{4a^2}\right.$$
$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\ldots\right.\right.}{}\right.$$

$$\Big\downarrow 224$$

3.308. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$a^2c\left(-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}}{2a}-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}}{2a^2}\right.}{4a^2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$a^2c\left(-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)}{4a^2}-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{x\sqrt{a}}{}}{2a}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \textcolor{blue}{5425}$$

$$a^2c\left(-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)}{4a^2}-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+}}{3a^2c}}{2a}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \textcolor{blue}{5423}$$

3.308. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{}\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \mathrm{a}\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}(-2\right.}{}\right.$$

$$\Big\downarrow \text{3011}$$

3.308. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$a^2c\left(-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}(2(\ldots}{}\right.}{}\right.$$

$\downarrow$ 2720

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$a^2c\left(-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}(2(\ldots}{}\right.}{}\right.$$

$\downarrow$ 5465

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$a^2c\left(-\frac{-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\arctan(ax)\ldots}{a\ldots}\right.}{}\right.$$

$\downarrow$ 224

3.308. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int \frac{}{1-\frac{c}{a^2}}}{}\right.}{3a^2}}{2a}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\text{arc}\right.}{}\right.$$

$$\Big\downarrow 219$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arcta}}{}\right.}{3a^2}}{2a}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\text{arc}\right.}{}\right.$$

$$\Big\downarrow 7143$$

$$c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\right.\right.\right.}{}\right.$$

$$a^2c\left(\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\right.\right.}{}\right.}{}\right.$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

3.308. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

output
```
a^2*c*((x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(4*a^2*c) - ((x^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - A
rcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*a) - (2*((Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^
2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2))/(2*a) - (3*((x*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTan
h[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/a - (Sqrt[1 + a^2*x^2]
*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*
ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x
])])))/(2*a^3*Sqrt[c + a^2*c*x^2])))/(4*a^2)) + c*((x*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - Ar
cTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/a - (Sqrt[1 + a^2*
x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2
*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan
[a*x])])))/(2*a^3*Sqrt[c + a^2*c*x^2]))
```

3.308.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.308. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.308.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.69

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\left(6a^3\arctan(ax)^2x^3-4a^2\arctan(ax)x^2+3a\arctan(ax)^2x+2ax+2\arctan(ax)\right)}{24a^3}-\frac{i\sqrt{c(ax-i)(ax+i)}\,\left(3i\arctan(\right.}{}$

input
```
int(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(6*a^3*arctan(a*x)^2*x^3-4*a^2*arctan(a
*x)*x^2+3*a*arctan(a*x)^2*x+2*a*x+2*arctan(a*x))-1/24*I*(c*(a*x-I)*(I+a*x)
)^(1/2)*(3*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(
a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-8*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2
+1)^(1/2)
```

3.308.5 Fricas [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

3.308.6 Sympy [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^2\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x**2*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

3.308.7 Maxima [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

3.308.8 Giac [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int\sqrt{a^2cx^2+c}\,x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^2\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.309 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

3.309.1 Optimal result

Integrand size = 22, antiderivative size = 279

$$\begin{aligned}\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx &= \frac{\sqrt{c+a^2cx^2}}{3a^2}-\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{3a}\\&+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3a^2c}\\&+\frac{2ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{c+a^2cx^2}}\\&-\frac{ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{c+a^2cx^2}}\\&+\frac{ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a^2/c+2/3*I*c*arctan(a*x)*arctan((1+
I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-1/
3*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/
(a^2*c*x^2+c)^(1/2)+1/3*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+1/3*(a^2*c*x^2+c)^(1/2)/a^2-1/3*x
*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a
```

3.309.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.93

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$$
$$=\frac{(1+a^2x^2)\sqrt{c(1+a^2x^2)}\left(2+4\arctan(ax)^2+2\cos(2\arctan(ax))-\frac{3\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}}-\arctan(\right.}{}$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan
[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] -
ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*
x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*Arc
Tan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1
 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/(12*a^2)
```

3.309.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5465, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int\sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}$$

$$\Big\downarrow \ 5421$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-$$

$$\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2c}}{2}\right.}{3a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a^2*c*
x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-
2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2
, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3*a)
```

3.309.3.1 Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.309.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\left(x^2\arctan(ax)^2a^2-x\arctan(ax)a+\arctan(ax)^2+1\right)}{3a^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan\right.}{}$

input
```
int(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(x^2*arctan(a*x)^2*a^2-x*arctan(a*x)*a+a
rctan(a*x)^2+1)+1/3*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dil
og(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
)))/a^2/(a^2*x^2+1)^(1/2)
```

3.309.5 Fricas [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+cx}\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

3.309.6 Sympy [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x*atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

3.309.7 Maxima [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int\sqrt{a^2cx^2+c}x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

3.309.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.310 $\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

3.310.1 Optimal result

Integrand size = 21, antiderivative size = 340

$$\begin{aligned}\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx &= -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)^2\\
&\quad-\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a\sqrt{c+a^2cx^2}}\\
&\quad+\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a}\\
&\quad+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\
&\quad-\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\
&\quad-\frac{c\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\
&\quad+\frac{c\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a-I*c*arctan((1+I*a*x)/(a
^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+I*c
*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a
/(a^2*c*x^2+c)^(1/2)-I*c*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-c*polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+c*polylog(3,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-arctan(a*x)
*(a^2*c*x^2+c)^(1/2)/a+1/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$$
$$= \frac{\sqrt{c\left(1+a^2x^2\right)}\left(-2\sqrt{1+a^2x^2}\arctan(ax)+ax\sqrt{1+a^2x^2}\arctan(ax)^2-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)\right.}{}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(-2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + a*x*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]^2 - (2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*A
rcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 2*PolyL
og[3, (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*a*S
qrt[1 + a^2*x^2])
```

3.310.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.67, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{5423}$$

$$\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{4669}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(a\right.}{2a\sqrt{a^2cx^2+c}}$$
$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

3.310. $\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$

$$\Big\downarrow \text{3011}$$

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{2a\sqrt{a^2c}}$$

$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}$$

$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[
1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan
[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x
])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(
I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])
```

3.310.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

3.310. $\int \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.310.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.79

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)(x\arctan(ax)a-2)}{2a}+\frac{i\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)^2\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^2\ln\left(1-\frac{i(iax}{\sqrt{a^2}}\right.\right.}{}$

input
```
int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(x*arctan(a*x)*a-2)+1/2*I*(c*(
a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+2*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(
3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/
(a^2*x^2+1)^(1/2)
```

3.310.5 Fricas [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+c}\arctan\left(ax\right)^2\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

3.310.6 Sympy [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

3.310.7 Maxima [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \sqrt{a^2cx^2+c}\arctan\left(ax\right)^2\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

3.310.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.311 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx$

3.311.1 Optimal result

Integrand size = 24, antiderivative size = 439

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = {} & \sqrt{c+a^2cx^2}\arctan(ax)^2 \\ & + \frac{4ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ & - \frac{2c\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & + \frac{2ic\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & - \frac{2ic\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & - \frac{2ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ & + \frac{2ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ & - \frac{2c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & + \frac{2c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
4*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-2*c*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*c*arctan(a*x)*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*c*arcta
n(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2
+c)^(1/2)-2*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c*polylog(3,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*c*polylog(3,(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(a^2
*c*x^2+c)^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(\sqrt{1+a^2x^2}\arctan(ax)^2+\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)-2\arctan(ax)\log\left(1-ie^{i\arctan(ax}\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]^2*Log[
1 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 2*Ar
cTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan
[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I
)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*ArcTan[a*
x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]
```

3.311.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5491}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{4671}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax)\right.}{\sqrt{a^2cx^2+c}}$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(\right.\right.}{\sqrt{a^2cx^2}}$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{2720}$$

3.311. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx$

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\right.}{}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 7143

$$\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i\,\mathrm{arc}}\right.\right.\right.}{\sqrt{a^2cx^2+c}}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```

output
```
a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*
(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyL
og[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1
 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^
2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTa
n[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sq
rt[c + a^2*c*x^2]
```

3.311.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.311. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.311. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.311.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.77

method	result
default	$\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan\left(ax\right)^{2}+\frac{i\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)^{2}\ln\left(\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}+1\right)-i\arctan(ax)^{2}\ln\left(1-\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right.\right.}{}$

input
```
int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2+I*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arc
tan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)^2*ln(1-(1+I*a*x
)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+
1)^(1/2))+2*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*dilog(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.311.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

3.311.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\sqrt{c(a^2x^2+1)}\,\operatorname{atan}^2(ax)}{x}\,dx$$

input
```
integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x, x)
```

3.311.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}{x}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

3.311.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.312 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx$

3.312.1 Optimal result

Integrand size = 24, antiderivative size = 458

$$
\begin{aligned}
\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = &-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x} \\
&-\frac{2iac\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} \\
&-\frac{4ac\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{2iac\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&-\frac{2iac\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{2iac\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
&-\frac{2iac\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
&-\frac{2ac\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{2ac\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-2*I*a*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)-4*a*c*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*a*c*arctan(a*x)*polylog(
2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*
I*a*c*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+2*I*a*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*c*polylog(2,(1+I*a*x)^(1/2)/(
1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*a*c*polylog(3,-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*a*c*po
lylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)-arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x
```

3.312.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx =$$

$$-\frac{a\sqrt{c(1+a^2x^2)}\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)^2}{ax} - 2\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right) - \arctan(ax)^2\log\left(1-ie^{i\arctan(a\right.\right.}}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```

output
```
-((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2*Ar
cTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan
[a*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[1
 + E^(I*ArcTan[a*x])] - (2*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTa
n[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I
*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (
-I)*E^(I*ArcTan[a*x])] - 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*
x^2])
```

3.312.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5423}$$

$$\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4669}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax))-2(i\,\mathrm{arcta}}{\sqrt{a^2c}}$$

$$\Big\downarrow \text{2720}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$

$$\Big\downarrow \color{blue}{5479}$$

$$c\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$

$$\Big\downarrow \color{blue}{5493}$$

$$c\left(\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$

$$\Big\downarrow \color{blue}{5489}$$

$$\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$
$$c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{Poly}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \color{blue}{7143}$$

$$c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{Poly}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.}{\sqrt{a^2cx^2+c}}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```

3.312. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx$

output
```
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(
-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]]))/Sqrt[c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I
*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[
2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*
c*x^2]
```

3.312.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

3.312. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.312. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx$

3.312.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.67

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2}{x}-\frac{a\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^2\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-2i\arctan\right.}{}$

input
```
int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2/x-a*(c*(a*x-I)*(I+a*x))^(1/2)*(ar
ctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(
a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-2*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/
2)+1)-2*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/
2)
```

3.312.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^2}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

3.312.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx=\int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}{x^2}\,dx$$

input
```
integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**2, x)
```

3.312.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

3.312.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

3.313 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx$

3.313.1 Optimal result

Integrand size = 24, antiderivative size = 328

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = & -\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{x}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2}\\
& -\frac{a^2c\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
& -a^2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)\\
& +\frac{ia^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
& -\frac{ia^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
& -\frac{a^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
& +\frac{a^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a^2*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)-a^2*c*arctan(a*x)^2*arct
anh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*a
^2*c*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)-I*a^2*c*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*c*polylog(3,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+a^2*c*polylog(3,(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a*arctan(a*
x)*(a^2*c*x^2+c)^(1/2)/x-1/2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

3.313.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx$$
$$= \frac{a^2\sqrt{c\,(1+a^2x^2)}\left(-4\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)^2\csc^2\left(\frac{1}{2}\arctan(ax)\right)+4\arctan(ax)^2\,(\mathrm{lo}\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```

output
```
(a^2*Sqrt[c*(1 + a^2*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x
]^2*Csc[ArcTan[a*x]/2]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - L
og[1 + E^(I*ArcTan[a*x])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]
*(PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-Po
lyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x
]^2*Sec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 +
a^2*x^2])
```

3.313.3 Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5485, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x^3}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5493}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5491}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 4671}$$

$$c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(a\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3011}$$

$$c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{\sqrt{a^2cx}}$$

$$\Big\downarrow \text{ 2720}$$

$$c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+$$
$$a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.$$

$$\Big\downarrow \text{ 5497}$$

$$c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+$$
$$a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.$$

3.313. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}dx$

$$\Big\downarrow \text{5479}$$

$$c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{243}$$

$$c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{73}$$

$$c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{221}$$

$$c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{5493}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2}}{2cx^2}\right.$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{5491}$$

3.313. $\int\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\text{ar}\right.$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{3042}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+}}{\sqrt{c}}\right.}{\sqrt{c}}\right.\right.$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{4671}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\,\text{arcta}\right.}{2\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\,\text{ar}\right.\right.}{2\sqrt{}}\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

3.313. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-e^{i\right.\right.\right.}}{\sqrt{a^2cx^2+c}}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,\right.\right.\right.}{2\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```

output
```
c*(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[
c]) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])]
+ 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTa
n[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^
(I*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2])) + (a^2*c*Sqrt[1 + a^2*x^2]*(-
2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -
E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*Po
lyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^
2*c*x^2]
```

3.313.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.313. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx$

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.313.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\, \arctan(ax)(2ax+\arctan(ax))}{2x^2} - \frac{a^2\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-\arctan(ax)^2 \ln\left(1-\frac{ia}{\sqrt{a}}\right.\right.}{}$

input
```
int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*a*x+arctan(a*x))/x^2-1/2*a^2
*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1
)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(
2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*
x)/(a^2*x^2+1)^(1/2))+4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^
(1/2)
```

3.313.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^3}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

3.313.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}{x^3}\,dx$$

input
```
integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**3, x)
```

3.313.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^3}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

3.313.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

3.314 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx$

3.314.1 Optimal result

Integrand size = 24, antiderivative size = 275

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = &-\frac{a^2\sqrt{c+a^2cx^2}}{3x}-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{3x^2}\\&-\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3cx^3}\\&-\frac{2a^3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\\&+\frac{ia^3c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\\&-\frac{ia^3c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/c/x^3-2/3*a^3*c*arctan(a*x)*arctanh
((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/
3*I*a^3*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)-1/3*I*a^3*c*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/3*a^2*(a^2*c*x^2+c)^(1/2)/x-1/3*a*a
rctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2
```

3.314.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx =$$
$$-\frac{c\sqrt{1+a^2x^2}\left(-4ia^3x^3\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)+4ia^3x^3\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)+\sqrt{1+a^2x^2}\left(4a^2x^2+\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/12*(c*Sqrt[1 + a^2*x^2]*((-4*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])]
+ (4*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*(4*a^2*x
^2 + 4*(1 + a^2*x^2)*ArcTan[a*x]^2 + ArcTan[a*x]*(a*x*(4 - 3*Sqrt[1 + a^2*
x^2]*Log[1 - E^(I*ArcTan[a*x])] + 3*Sqrt[1 + a^2*x^2]*Log[1 + E^(I*ArcTan[
a*x])]) + (1 + a^2*x^2)*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[
a*x])])*Sin[3*ArcTan[a*x]]))))/(x^3*Sqrt[c + a^2*c*x^2])
```

3.314.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5479, 5481, 242, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x^4}dx$$
$$\Big\downarrow \text{5479}$$
$$\frac{2}{3}a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$
$$\Big\downarrow \text{5481}$$
$$\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$
$$\Big\downarrow \text{242}$$

$$\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5497}$$

$$\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right.$$
$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{242}$$

$$\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2c}}{x}\right.$$
$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5493}$$

$$\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\right.$$
$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5489}$$

$$-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\right.\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

3.314. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx$

output
```
-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(c*x^3) + (2*a*(-((a*Sqrt[c + a
^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(-1/2*(a*Sqrt[c
+ a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^2*S
qrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
+ I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))))/3
```

3.314.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.314.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.71

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,\left(x^2\arctan(ax)^2a^2+a^2x^2+x\arctan(ax)a+\arctan(ax)^2\right)}{3x^3}+\frac{ia^3\sqrt{c(ax-i)(ax+i)}\,\left(i\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+\right.\right.}{}$

input
```
int(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*(a*x-I)*(I+a*x))^(1/2)*(x^2*arctan(a*x)^2*a^2+a^2*x^2+x*arctan(a*x
)*a+arctan(a*x)^2)/x^3+1/3*I*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)*
ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)
^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.314.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

3.314.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2(ax)}{x^4}\,dx$$

input
```
integrate(atan(a*x)**2*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**4, x)
```

3.314.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

3.314.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

3.315 $\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)^2 \, dx$

3.315.1 Optimal result

Integrand size = 24, antiderivative size = 476

$$\int x^3 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^2 \, dx = -\frac{17c\sqrt{c+a^2cx^2}}{280a^4} - \frac{17\left(c+a^2cx^2\right)^{3/2}}{1260a^4} + \frac{\left(c+a^2cx^2\right)^{5/2}}{105a^4c} + \frac{3cx\sqrt{c+a^2cx^2}\arctan(ax)}{56a^3} - \frac{23cx^3\sqrt{c+a^2cx^2}\arctan(ax)}{420a} - \frac{1}{21}acx^5\sqrt{c+a^2cx^2}\arctan(ax) - \frac{2c\sqrt{c+a^2cx^2}\arctan(ax)^2}{35a^4} + \frac{cx^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{35a^2} + \frac{8}{35}cx^4\sqrt{c+a^2c}$$

output
```
-17/1260*(a^2*c*x^2+c)^(3/2)/a^4+1/105*(a^2*c*x^2+c)^(5/2)/a^4/c-17/140*I*
c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
a^4/(a^2*c*x^2+c)^(1/2)+17/280*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-17/280*I*c^2*polylog(2,
I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/
2)-17/280*c*(a^2*c*x^2+c)^(1/2)/a^4+3/56*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/
2)/a^3-23/420*c*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a-1/21*a*c*x^5*arctan(
a*x)*(a^2*c*x^2+c)^(1/2)-2/35*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4+1/35
*c*x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2+8/35*c*x^4*arctan(a*x)^2*(a^2
*c*x^2+c)^(1/2)+1/7*a^2*c*x^6*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.315.2 Mathematica [A] (warning: unable to verify)

Time = 3.92 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.67

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = \frac{c(1+a^2x^2)^2\sqrt{c+a^2cx^2}\left(-168\left(50-32\arctan(ax)^2+72\cos(2\arctan(ax))+\right.\right.}{}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*(-168*(50 - 32*ArcTan[a*x]^2 + 72*C
os[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan
[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
- 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcT
an[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]
*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*Ar
cTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]
]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a
^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*Ar
cTan[a*x]]) + (1 + a^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTa
n[a*x]] - 5376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]]
+ 6720*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*
ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[
a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Co
s[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcT
an[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*
ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log
[1 + I*E^(I*ArcTan[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*
E^(I*ArcTan[a*x])] + 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*...
```

3.315.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3185 vs. 2(476) = 952.

Time = 18.38 (sec) , antiderivative size = 3185, normalized size of antiderivative = 6.69, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5485, 5485, 5487, 5465, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5465, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5465, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int \frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(c\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int}{}\right.\right.$$
$$a^2c\left(a^2c\left(-\frac{2\int \frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4}{}\right.\right.$$

$$\Big\downarrow \text{ 5465}$$

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2\int}{}\right.\right.$$
$$a^2c\left(a^2c\left(-\frac{2\int \frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4}{}\right.\right.$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\right.$$
$$a^2c\left(a^2c\left(-\frac{2\int \frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4}{}\right.\right.$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$\Big\downarrow \text{5421}$$

$$a^2c\left(a^2c\left(-\frac{2\int \frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4}{}\right.\right.$$

$$c\left(a^2c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\Big(}{}\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int \frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\arctan}{5a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int \frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan}{3a^2c}\right.}{}\right.\right.$$

$$\Big\downarrow \text{241}$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\arctan\right.}{5a^2c}\right.\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan\right.}{3a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 243$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx^2}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\arctan\right.}{5a^2c}\right.\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan\right.}{3a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 53$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\frac{1}{a^4\sqrt{a^2cx^2+c}}\right)dx^2}{12a}-\frac{5\int\frac{x^4\arctan}{\sqrt{a^2cx}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4}{}\right.\right.$$

$$\Big\downarrow 2009$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2}{}\right.}{}\right.\right.$$

$$\Big\downarrow 5425$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-4\left(\frac{x^2}{}\right.\right.\right.$$

$$\Big\downarrow \; 5421$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-4\left(\frac{x^2}{}\right.\right.\right.$$

$$\Big\downarrow \; 5465$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(x^2\right.}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(x^2\right.}{}\right.\right.$$

$$\Big\downarrow \text{5421}$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}\cdots}{6a^2}\cdots\right.}{7a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

$\downarrow$ 5487

3.315. $\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int}{\sqrt{a^2c}}\right.}{4a^2}\right.}{5a}\right.\right.$$

↓ 241

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}\cdots}{4a^2c}\cdots\right.}{\cdots}\right.}{7a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{\mathrm{arcta}\cdots}{\sqrt{a^2c\cdots}}}{2\cdots}\right.}{4a^2\cdots}\right.}{5a}\cdots\right.\right.$$

↓ 243

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{\mathrm{arcta}}{\sqrt{a^2c}}}{2}\right.}{4a^2}\right.}{5a}\right.\right.$$

$\downarrow$ 53

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{\mathrm{arcta}}{\sqrt{a^2c}}}{2}\right.}{4a^2}\right.}{5a}\right.\right.$$

↓ 2009

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\operatorname{arc}\cdots}{4a^2c\cdots}\right.}{\cdots}\right.}{7a\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{\operatorname{arcta}\cdots}{\sqrt{a^2c\cdots}}}{2\cdots}\right.}{4a^2\cdots}\right.}{5a\cdots}\right.\right.$$

$\downarrow$ 5425

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2}}{2c}\right.}{4a^2}\right.}{5a}\right.\right.$$

↓ 5421

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2}}{}\right.}{}\right.}{}\right.\right.$$

↓ 5465

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2}}{}\right.}{}\right.}{}\right.\right.$$

$$\Big\downarrow \ 5425$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2}}{}\right.}{}\right.}{}\right.\right.$$

↓ 5421

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2}}{}\right.}{}\right.}{}\right.\right.$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$\downarrow$ 5487

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c} - \frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c} - \frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3} - \frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2} + \frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a} - \frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c} - \frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c} - \frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2} - \frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a} - \frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c} - \frac{\sqrt{a^2x^2}}{}\right.}{}\right.}{}\right.\right.$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$\downarrow$ 241

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}\cdots}{4a^2c\cdots}\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2\cdots}}{\cdots}\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

3.315.

$$\downarrow 5425$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}\cdots}{4a^2c\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$\downarrow$ 5421

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arc}}{4a^2c}\right.}{}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2}}{}\right.}{}\right.}{}\right.\right.$$

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

input
```
Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
c*(c*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c
+ a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sq
rt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Poly
Log[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2
])))/(3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^
2)) + a^2*c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/8*
((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))
/a + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a
^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[
1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2]))
)/(4*a^2)))/(5*a) - (4*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c)
- (2*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a
*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 -...
```

3.315.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.315. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

3.315.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.57

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(360a^6x^6\arctan(ax)^2-120\arctan(ax)a^5x^5+576a^4\arctan(ax)^2x^4+24a^4x^4-138\arctan(ax)x^3a^3+72x^2\arctan(a\right.}{2520a^4}$

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2520*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(360*a^6*x^6*arctan(a*x)^2-120*arct
an(a*x)*a^5*x^5+576*a^4*arctan(a*x)^2*x^4+24*a^4*x^4-138*arctan(a*x)*x^3*a
^3+72*x^2*arctan(a*x)^2*a^2+14*a^2*x^2+135*x*arctan(a*x)*a-144*arctan(a*x)
^2-163)-17/280*c*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(
1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))
/a^4/(a^2*x^2+1)^(1/2)
```

3.315.5 Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^5 + c*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

3.315.6 Sympy [F]

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

3.315.7 Maxima [F]

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2, x)
```

3.315.8 Giac [F(-2)]

Exception generated.

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int x^3\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.316 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

3.316.1 Optimal result

Integrand size = 24, antiderivative size = 531

$$
\begin{aligned}
\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx &= \frac{cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2} \\
&+\frac{31c\sqrt{c+a^2cx^2}\arctan(ax)}{360a^3}-\frac{19cx^2\sqrt{c+a^2cx^2}\arctan(ax)}{180a} \\
&-\frac{1}{15}acx^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)^2}{16a^2} \\
&+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&+\frac{ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{8a^3\sqrt{c+a^2cx^2}}-\frac{41c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{360a^3} \\
&-\frac{ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} \\
&+\frac{ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} \\
&+\frac{c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}-\frac{c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-41/360*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^3+1/8*I*c^2*arc
tan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*
c*x^2+c)^(1/2)-1/8*I*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/8*I*c^2*arctan(a*x)*polyl
og(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1
/2)+1/8*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^
3/(a^2*c*x^2+c)^(1/2)-1/8*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+1/36*c*x*(a^2*c*x^2+c)^(1/2)/a^2+1/
60*c*x^3*(a^2*c*x^2+c)^(1/2)+31/360*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3-
19/180*c*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a-1/15*a*c*x^4*arctan(a*x)*(a
^2*c*x^2+c)^(1/2)+1/16*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2+7/24*c*x^
3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+1/6*a^2*c*x^5*arctan(a*x)^2*(a^2*c*x^2
+c)^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.99

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = \frac{c\sqrt{c+a^2cx^2}\left(960\left(3i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-2\mathrm{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-3\right.\right.}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(960*((3*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
 - 2*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, (-I)*
E^(I*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 3
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 3*PolyLog[3, I*E^(I*ArcTan[a*x])]) +
 32*((-45*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 19*ArcTanh[(a*x)/Sq
rt[1 + a^2*x^2]] + (45*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] -
 (45*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 45*PolyLog[3, (-I)*E
^(I*ArcTan[a*x])] + 45*PolyLog[3, I*E^(I*ArcTan[a*x])]) + 120*(1 + a^2*x^2
)^(3/2)*(ArcTan[a*x]*(2 + 6*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]]) - 3*ArcT
an[a*x]^2*(-7*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]) + 2*(a*x + Sqrt[
1 + a^2*x^2]*Sin[3*ArcTan[a*x]])) + (1 + a^2*x^2)^3*((-56*a*x)/Sqrt[1 + a^
2*x^2] + ArcTan[a*x]*(12/Sqrt[1 + a^2*x^2] + 110*Cos[3*ArcTan[a*x]] - 90*C
os[5*ArcTan[a*x]]) - 108*Sin[3*ArcTan[a*x]] - 52*Sin[5*ArcTan[a*x]] + 15*A
rcTan[a*x]^2*((78*a*x)/Sqrt[1 + a^2*x^2] - 47*Sin[3*ArcTan[a*x]] + 3*Sin[5
*ArcTan[a*x]]))))/(11520*a^3*Sqrt[1 + a^2*x^2])
```

3.316.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{5485}$$

$$c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^2dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^2dx$$

$$\Big\downarrow \text{5485}$$

$$c\left(c\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)+ a^2c\left(a^2c\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(}{\sqrt{a^2cx^2}}}{2a}\right.\right.$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\right.\right.$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2}}{}\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2}}{}\right.\right.$$

$\downarrow$ 3011

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2}}{}\right.\right.$$

$\downarrow$ 2720

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2}}{}\right.\right.$$

$\downarrow$ 5465

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\int}{a}\right.\right.$$

$\downarrow$ 224

3.316. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int \right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int}{}}{a}\right.\right.$$

$$\Big\downarrow 219$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int}{}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\sqrt{a^2x^2+1}(2(i\arctan(ax)}{}\right.\right.$$

$$\Big\downarrow 5487$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\int \frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}\right.}{}\right.\right.$$

$$\Big\downarrow 262$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+}}{2}\right.}{}\right.\right.$$

$$\Big\downarrow 224$$

3.316. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\right.}{}\right.\right.$$

$$\Big\downarrow \; 219$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(x\right.}{}\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(x\right.}{}\right.\right.$$

$$\Big\downarrow \; 224$$

3.316. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\cdots}{\sqrt{\cdots}}}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\cdots}{\cdots}\right.}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \text{ 219}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\operatorname{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\cdots}{\cdots}\right.}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\operatorname{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\cdots}{\cdots}\right.}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \text{ 5423}$$

3.316. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\,\text{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(x\cdots\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\,\text{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(x\cdots\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\,\text{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(x\cdots\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.316. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\operatorname{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\sqrt{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\operatorname{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\sqrt{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \; 5465$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\operatorname{arc}\cdots}{\sqrt{a^2\cdots}}}{5\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int\cdots}{\sqrt{\cdots}}\cdots\right.}{3a^2}}{2a}\cdots\right.\right.$$

$$\Big\downarrow \; 224$$

3.316. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\,\text{arc}}{\sqrt{a^2}}}{5a}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int}{1}\right.}{3a^2}}{2a}\right.\right.$$

$$\Big\downarrow \; 219$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\,\text{arc}}{\sqrt{a^2}}}{5a}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{ar}}{}\right.}{3a^2}}{2a}\right.\right.$$

$$\Big\downarrow \; 5487$$

3.316. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\left(\sqrt{a^2cx\cdots\right.}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\mathrm{ar}\cdots}{\cdots}\right.}{3a^2\cdots}}{2a}\cdots\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\left(\sqrt{a^2cx\cdots\right.}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\mathrm{ar}\cdots}{\cdots}\right.}{3a^2\cdots}}{2a}\cdots\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.316. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

3.316.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.316. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.316. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

3.316.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.64

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\left(120a^5\arctan(ax)^2x^5-48\arctan(ax)a^4x^4+210a^3\arctan(ax)^2x^3+12a^3x^3-76a^2\arctan(ax)x^2+45a\arctan(ax)^2\right.}{720a^3}$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/720*c/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(120*a^5*arctan(a*x)^2*x^5-48*arctan
(a*x)*a^4*x^4+210*a^3*arctan(a*x)^2*x^3+12*a^3*x^3-76*a^2*arctan(a*x)*x^2+
45*a*arctan(a*x)^2*x+20*a*x+62*arctan(a*x))-1/720*I*c*(c*(a*x-I)*(I+a*x))^
(1/2)*(45*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(
a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*polylog(2,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+90*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*polylog(3,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))-164*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a
^2*x^2+1)^(1/2)
```

3.316.5 Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

3.316.6 Sympy [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

3.316.7 Maxima [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int (a^2cx^2+c)^{\frac{3}{2}}x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2, x)
```

3.316.8 Giac [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int (a^2cx^2+c)^{\frac{3}{2}}x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int x^2\,\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}\,dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.317 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

3.317.1 Optimal result

Integrand size = 22, antiderivative size = 334

$$\begin{aligned}\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx &= \frac{3c\sqrt{c+a^2cx^2}}{20a^2}+\frac{(c+a^2cx^2)^{3/2}}{30a^2}\\ &-\frac{3cx\sqrt{c+a^2cx^2}\arctan(ax)}{20a}-\frac{x(c+a^2cx^2)^{3/2}\arctan(ax)}{10a}\\ &+\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{5a^2c}+\frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{10a^2\sqrt{c+a^2cx^2}}\\ &-\frac{3ic^2\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{c+a^2cx^2}}+\frac{3ic^2\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/30*(a^2*c*x^2+c)^(3/2)/a^2-1/10*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a+1/5*
(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/a^2/c+3/10*I*c^2*arctan(a*x)*arctan((1+I
*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-3/2
0*I*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^
2/(a^2*c*x^2+c)^(1/2)+3/20*I*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+3/20*c*(a^2*c*x^2+c)^(1/2)/a
^2-3/20*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a
```

3.317.2 Mathematica [A] (verified)

Time = 3.38 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.80

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = \frac{c(1+a^2x^2)\sqrt{c+a^2cx^2}\left(80\left(2+4\arctan(ax)^2+2\cos(2\arctan(ax))-\frac{3\arctan(a\ldots}{\ldots}\right.\right.}{\ldots}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*(80*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Ar
cTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3
*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
)/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]) - (1 + a^2*x^2)*
(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*A
rcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcT
an[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I
*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*Arc
Tan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x
^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*
ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2,
 I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*
x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]])))/(960*a^2)
```

3.317.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5465, 5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \ 5465$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx}{5a}$$

$$\Big\downarrow \ 5413$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)}{5a}$$

$$\Big\downarrow \ 5413$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)}{5a}$$

$$\Big\downarrow \ 5425$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)}{12a}\right.}{5a}$$

$$\Big\downarrow \ 5421$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\right.\right.}{5a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

3.317. $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

output
```
((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/12*(c + a^2*c*x^2
)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sqrt[c +
a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2
]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*Pol
yLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt
[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/4))/(5*a)
```

3.317.3.1 Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.317.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.71

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\left(12a^4\arctan(ax)^2x^4-6\arctan(ax)x^3a^3+24x^2\arctan(ax)^2a^2+2a^2x^2-15x\arctan(ax)a+12\arctan(ax)^2+11\right)}{60a^2}+$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*c/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(12*a^4*arctan(a*x)^2*x^4-6*arctan(a*
x)*x^3*a^3+24*x^2*arctan(a*x)^2*a^2+2*a^2*x^2-15*x*arctan(a*x)*a+12*arctan
(a*x)^2+11)+3/20*c*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilo
g(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/a^2/(a^2*x^2+1)^(1/2)
```

3.317.5 Fricas [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

3.317.6 Sympy [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

3.317.7 Maxima [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int (a^2cx^2+c)^{\frac{3}{2}}x\arctan{(ax)}^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2, x)
```

3.317.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int x\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.318 $\int (c+a^2cx^2)^{3/2} \arctan(ax)^2 \, dx$

3.318.1 Optimal result

Integrand size = 21, antiderivative size = 438

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^2 \, dx = \frac{1}{12}cx\sqrt{c+a^2cx^2} - \frac{3c\sqrt{c+a^2cx^2}\arctan(ax)}{4a} - \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{6a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^2 + \frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax)^2 - \frac{3ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{4a\sqrt{c+a^2cx^2}} + \frac{5c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a} +$$

output
```
-1/6*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*
x)^2+5/6*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a-3/4*I*c^2*arct
an((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x
^2+c)^(1/2)+3/4*I*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/4*I*c^2*arctan(a*x)*polylog(2,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/4
*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*
x^2+c)^(1/2)+3/4*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/a/(a^2*c*x^2+c)^(1/2)+1/12*c*x*(a^2*c*x^2+c)^(1/2)-3/4*c*arctan(a*x)
*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.318.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = \frac{c\sqrt{c+a^2cx^2}\Big(2ax\sqrt{1+a^2x^2}+2a^3x^3\sqrt{1+a^2x^2}-94\sqrt{1+a^2x^2}\arctan(ax)+2}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(2*a*x*Sqrt[1 + a^2*x^2] + 2*a^3*x^3*Sqrt[1 + a^2*x
^2] - 94*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x] + 69*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 21*a^3*x^3*Sqrt[1 + a^2*
x^2]*ArcTan[a*x]^2 - (72*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 80*A
rcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^
2*x^2*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 6*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[
a*x]] + (72*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*Arc
Tan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 72*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + 72*PolyLog[3, I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]] + 4*a^2
*x^2*Sin[3*ArcTan[a*x]] + 2*a^4*x^4*Sin[3*ArcTan[a*x]] - 3*ArcTan[a*x]^2*S
in[3*ArcTan[a*x]] - 6*a^2*x^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3*a^4*x^4
*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(96*a*Sqrt[1 + a^2*x^2])
```

3.318.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.79, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{219}$$

3.318. $\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

$$\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \; 5425$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right.$$
$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \; 5423$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\right.$$
$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2\cdots}}{a}\right.$$
$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \; 4669$$

3.318. $\int\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \textbf{3011}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{2a}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \textbf{2720}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \textbf{7143}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLo}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

3.318. $\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

output
```
-1/6*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(3/2)*ArcT
an[a*x]^2)/4 + (c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]
*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*S
qrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^
(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c
 + a^2*c*x^2])))/4
```

3.318.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.318. $\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$

3.318.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.69

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\left(6a^3\arctan(ax)^2x^3-4a^2\arctan(ax)x^2+15a\arctan(ax)^2x+2ax-22\arctan(ax)\right)}{24a}+\frac{ic\sqrt{c(ax-i)(ax+i)}\left(9i\,\mathrm{arct}\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*c/a*(c*(a*x-I)*(I+a*x))^(1/2)*(6*a^3*arctan(a*x)^2*x^3-4*a^2*arctan(a
*x)*x^2+15*a*arctan(a*x)^2*x+2*a*x-22*arctan(a*x))+1/24*I*c*(c*(a*x-I)*(I+
a*x))^(1/2)*(9*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*arc
tan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*arctan(a*x)*polylog(2,-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+18*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-40*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(
a^2*x^2+1)^(1/2)
```

3.318.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2, x)
```

3.318.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

3.318.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2, x)
```

3.318.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx = \int \mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.319 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx$

3.319.1 Optimal result

Integrand size = 24, antiderivative size = 530

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx &= \frac{1}{3}c\sqrt{c+a^2cx^2} \\
&- \frac{1}{3}acx\sqrt{c+a^2cx^2}\arctan(ax) + c\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&+ \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2 + \frac{14ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} \\
&- \frac{2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+ \frac{2ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&- \frac{2ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&- \frac{7ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} + \frac{7ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} \\
&- \frac{2c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+ \frac{2c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+14/3*I*c^2*arctan(a*x)*arctan((1+I*a
*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^2*arc
tan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x
^2+c)^(1/2)+2*I*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*c^2*arctan(a*x)*polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7/3*I*c^2*polyl
og(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)+7/3*I*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)-2*c^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*c^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/3*c*(a^2*c*x^2+c)^(1/2)-1/3*a
*c*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.319.2 Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.94

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx = \frac{1}{12}c\sqrt{c+a^2cx^2}\left(\frac{12\left(\sqrt{1+a^2x^2}\arctan(ax)^2+\arctan(ax)^2\log\left(1-e^{i\,\mathrm{arct}}\right.\right.}{}\right.$$

$$+\left(1+a^2x^2\right)\left(2+4\arctan(ax)^2+2\cos(2\arctan(ax))-\frac{3\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}}-\arctan(ax)\cos(3\,a\right.$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*((12*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]
^2*Log[1 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])]
 + 2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I
*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
 - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*Ar
cTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (1 + a
^2*x^2)*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1
 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]
]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x]
)])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcT
an[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2)
 + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[
a*x]*Sin[2*ArcTan[a*x]])))/12
```

3.319.3 Rubi [A] (verified)

Time = 3.24 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5485, 5465, 5413, 5425, 5421, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int \sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+$$
$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\Big\downarrow \; 5485$$

$$c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\Big\downarrow \; 5465$$

$$c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\Big\downarrow \; 5425$$

3.319. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx$

$$c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\downarrow \text{5421}$$

$$c\left(c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\downarrow \text{5493}$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyL}}{}\right.}{a\sqrt{a^2cx^2+}}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

3.319. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx$

$$\Big\downarrow \text{5491}$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\operatorname{arcta}}{a}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\operatorname{arc}}{}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\Big\downarrow \text{4671}$$

3.319. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\downarrow 3011$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arcta\right.\right.}{\sqrt{a^2c}}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\downarrow 2720$$

3.319. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)\,de^{i\arctan(ax)}\right)\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\Big\downarrow \text{ 7143}$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-e\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]
```

3.319. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx$

output
```
a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c +
 a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^
2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*Po
lyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqr
t[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3*a)) + c*(
a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*
(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyL
og[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1
 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^
2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTa
n[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sq
rt[c + a^2*c*x^2])
```

3.319.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.319. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx$

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.319. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.319.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.69

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\left(x^2\arctan(ax)^2a^2-x\arctan(ax)a+4\arctan(ax)^2+1\right)}{3}+\frac{ic\sqrt{c(ax-i)(ax+i)}\left(3i\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*(c*(a*x-I)*(I+a*x))^(1/2)*(x^2*arctan(a*x)^2*a^2-x*arctan(a*x)*a+4*a
rctan(a*x)^2+1)+1/3*I*c*(c*(a*x-I)*(I+a*x))^(1/2)*(3*I*arctan(a*x)^2*ln((1
+I*a*x)/(a^2*x^2+1)^(1/2)+1)-3*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-7*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*I*arctan(a*x)
*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*
polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-7*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*dilog(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.319.5 Fricas [F]

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx=\int \frac{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

3.319.6 Sympy [F]

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^2}{x} \, dx = \int \frac{(c(a^2x^2 + 1))^{\frac{3}{2}} \operatorname{atan}^2(ax)}{x} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x, x)
```

3.319.7 Maxima [F]

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^2}{x} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}} \arctan(ax)^2}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

3.319.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^2}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.320 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx$

3.320.1 Optimal result

Integrand size = 24, antiderivative size = 556

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx &= -ac\sqrt{c+a^2cx^2}\arctan(ax) \\
&- \frac{c\sqrt{c+a^2cx^2}\arctan(ax)^2}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&- \frac{3iac^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} \\
&- \frac{4ac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + ac^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) \\
&+ \frac{3iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&- \frac{3iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+ \frac{2iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
&- \frac{3ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+ \frac{3ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
a*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-3*I*a*c^2*arctan((1+I*a
*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
-4*a*c^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a*c^2*arctan(a*x
)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)
^(1/2)+2*I*a*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*c^2*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a*c^2*polylog(3,-I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*a*c^2*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a*c*a
rctan(a*x)*(a^2*c*x^2+c)^(1/2)-c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x+1/2*a
^2*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.68

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-2ax\sqrt{1+a^2x^2}\arctan(ax)-2\sqrt{1+a^2x^2}\arctan(ax)^2+a\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(-2*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 2*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^2 + a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (2*I)*a
*x*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*a*x*ArcTanh[(a*x)/Sqrt[1 +
a^2*x^2]] + 4*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 2*a*x*ArcTan[a*
x]^2*Log[1 - I*E^(I*ArcTan[a*x])] - 2*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*Arc
Tan[a*x])] - 4*a*x*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)*a*x*Poly
Log[2, -E^(I*ArcTan[a*x])] + (6*I)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])] - (6*I)*a*x*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (4*I
)*a*x*PolyLog[2, E^(I*ArcTan[a*x])] - 6*a*x*PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] + 6*a*x*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*x*Sqrt[1 + a^2*x^2])
```

3.320.3 Rubi [A] (verified)

Time = 4.11 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.96, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5485, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+ c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right. c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+ c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right.$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\right.$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a}}{a}\right.$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{4669}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\,\text{arct}\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\,\text{a}\right.\right.}{2a}\right.$$

$$\Big\downarrow \text{2720}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

3.320. $\int\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx$

$$c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$
$$c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$
$$c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.$$

3.320. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx$

$$\Big\downarrow 2720$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\Big\downarrow 5479$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$c\left(c\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$\Big\downarrow 5493$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.$$

$$\Big\downarrow 5489$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax}\right.\right.}{}\right.$$

$$\Big\downarrow 7143$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyL}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{P}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

3.320. $\int\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
a^2*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (
c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I
*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3
, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*A
rcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c
+ a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*A
rcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyL
og[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan
[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])
```

3.320.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.320. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.320. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.320.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.64

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(a^{2}\arctan(ax)x^{2}-2ax-2\arctan(ax)\right)}{2x}+\frac{ica\sqrt{c(ax-i)(ax+i)}\left(3i\arctan(ax)^{2}\ln\left(1+\frac{i(iax+1)}{\sqrt{a^{2}x^{2}+1}}\right)-3\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(a^2*arctan(a*x)*x^2-2*a*x-2*a
rctan(a*x))/x+1/2*I*c*a*(c*(a*x-I)*(I+a*x))^(1/2)*(3*I*arctan(a*x)^2*ln(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*arctan(a*x)
*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*
polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*dilog((1+I*a*x)/(a^2*x^2+1)^(1/
2)+1)+4*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan((1+I*a*x)/(a^2*x^2+1)^
(1/2)))/(a^2*x^2+1)^(1/2)
```

3.320.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

3.320.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**2, x)
```

3.320.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

3.320.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

3.321 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx$

3.321.1 Optimal result

Integrand size = 24, antiderivative size = 567

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = -\frac{ac\sqrt{c+a^2cx^2}\arctan(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2} + \frac{4ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3a^2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - a^2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{3ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{2ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-a^2*c^(3/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+4*I*a^2*c^2*arctan(a*x)*
arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)-3*a^2*c^2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c^2*arcta
n(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2
+c)^(1/2)-2*I*a^2*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*a^2*c^2*polylog(2,I*(1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a^2*c^2*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*a^2*c
^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^
(1/2)-a*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x+a^2*c*arctan(a*x)^2*(a^2*c*x^2
+c)^(1/2)-1/2*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

3.321.2 Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.80

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \frac{a^2c\sqrt{c+a^2cx^2}\left(-4\arctan(ax)-4\arctan(ax)\cot^2\left(\frac{1}{2}\arctan(ax)\right)+4ax\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
(a^2*c*Sqrt[c + a^2*c*x^2]*(-4*ArcTan[a*x] - 4*ArcTan[a*x]*Cot[ArcTan[a*x]
/2]^2 + 4*a*x*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]^2*Cot[ArcTa
n[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1
 - E^(I*ArcTan[a*x])] - 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*A
rcTan[a*x])] + 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[a*x
])] - 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*C
ot[ArcTan[a*x]/2]*Log[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*Cot[ArcTan[
a*x]/2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (16*I)*Cot[ArcTan[a*x]/2]*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] + (16*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*
ArcTan[a*x])] - (24*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcT
an[a*x])] - 24*Cot[ArcTan[a*x]/2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 24*Cot[
ArcTan[a*x]/2]*PolyLog[3, E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Csc[ArcTan[a*
x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2])
```

3.321.3 Rubi [A] (verified)

Time = 5.75 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.18, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.958, Rules used = {5485, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5465}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5425}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5421}$$

$$c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+ a^2c\left(c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2\right.}{}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{a\sqrt{a^2cx^2}}\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\,\mathrm{ar}}{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{4671}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\,\mathrm{arcta}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

3.321. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\operatorname{arc}\right.\right.}{\sqrt{c}}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5497}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(a\right.\right.}{}\right.$$

$$\Big\downarrow \text{5479}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)\right.$$

$$\Big\downarrow \text{243}$$

3.321. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x^3}\,dx$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{2cx^2}\right.\right.}\right.$$

$$\Big\downarrow 73$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right.\right.}\right.$$

$$\Big\downarrow 221$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{2cx^2}\right.\right.}\right.$$

$$\Big\downarrow 5493$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{c\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2c}}{2cx^2}\right.\right.}\right.$$

$$\Big\downarrow 5491$$

3.321. $\int\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2c}}{\sqrt{}}\right.}{\sqrt{c}}\right.\right.\right.$$

$$\Big\downarrow \text{ 4671}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 3011}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 2720}$$

3.321. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{P}\right.\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
a^2*c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^
2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (
I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt
[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(
I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])
])))/Sqrt[c + a^2*c*x^2]) + c*(c*(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt
[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x
]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[
a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(
I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2])
) + (a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])]
+ 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTa
n[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^
(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])
```

3.321. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx$

3.321.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.321. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.321. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.321.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.73

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(2a^2\arctan(ax)x^2-2ax-\arctan(ax)\right)}{2x^2}-\frac{c\,a^2\sqrt{c(ax-i)(ax+i)}\,\left(3\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-3\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*a^2*arctan(a*x)*x^2-2*a*x-a
rctan(a*x))/x^2-1/2*c*a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(3*arctan(a*x)^2*ln((1
+I*a*x)/(a^2*x^2+1)^(1/2)+1)-3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-6*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*
x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1+I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))+4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*di
log(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+6*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,(1+I*a*x)/(a^2
*x^2+1)^(1/2))-2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+2*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2)+1))/(a^2*x^2+1)^(1/2)
```

3.321.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

3.321.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2{(ax)}}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**3, x)
```

3.321.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

3.321.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

3.322 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx$

3.322.1 Optimal result

Integrand size = 24, antiderivative size = 579

$$\begin{aligned}\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx &= -\frac{a^2c\sqrt{c+a^2cx^2}}{3x}\\
&-\frac{ac\sqrt{c+a^2cx^2}\arctan(ax)}{3x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\\
&-\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3x^3}-\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\\
&-\frac{14a^3c^2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\\
&+\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
&-\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
&+\frac{7ia^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\\
&-\frac{7ia^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\\
&-\frac{2a^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
&+\frac{2a^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3-2*I*a^3*c^2*arctan((1+I*a*x)/(a
^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-14/3*
a^3*c^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)+2*I*a^3*c^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a^3*c^2*arctan(
a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2
+c)^(1/2)+7/3*I*a^3*c^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7/3*I*a^3*c^2*polylog(2,(1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*a^3*c^2*polylog(3,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*a^3
*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2
+c)^(1/2)-1/3*a^2*c*(a^2*c*x^2+c)^(1/2)/x-1/3*a*c*arctan(a*x)*(a^2*c*x^2+c
)^(1/2)/x^2-a^2*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x
```

3.322.2 Mathematica [A] (verified)

Time = 5.01 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.78

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \frac{a^3c^2\sqrt{1+a^2x^2}\left(8i\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-24\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)^2}{ax}-2\right.\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
(a^3*c^2*Sqrt[1 + a^2*x^2]*((8*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - 24*((Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a
*x])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Log[1 +
 I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - (2*I)*P
olyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + (2*I)*Pol
yLog[2, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*Poly
Log[3, I*E^(I*ArcTan[a*x])]) - (2*(1 + a^2*x^2)^(3/2)*(2 + 4*ArcTan[a*x]^2
 - 2*Cos[2*ArcTan[a*x]] + ((4*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])])/(1
 + a^2*x^2)^(3/2) + ArcTan[a*x]*(2*Sin[2*ArcTan[a*x]] + ((Log[1 - E^(I*Arc
Tan[a*x])] - Log[1 + E^(I*ArcTan[a*x])])*(-3*a*x + Sqrt[1 + a^2*x^2]*Sin[3
*ArcTan[a*x]]))/Sqrt[1 + a^2*x^2]))/(a^3*x^3)))/(24*Sqrt[c + a^2*c*x^2])
```

3.322.3 Rubi [A] (verified)

Time = 4.29 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.99, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {5485, 5479, 5481, 242, 5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 5497, 242, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx-\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5481}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{242}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \; 5423$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \; 3042$$

$$a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+$$
$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \; 4669$$

$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\text{lo}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \; 3011$$

$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 5479$$

3.322. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x^4}\,dx$

$$c\left(\frac{2}{3}a\left(-c\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(a}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{2}{3}a\left(-c\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5489}$$

$$c\left(\frac{2}{3}a\left(-c\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5497}$$

$$c\left(\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2-}}{x^2}\right.\right.$$
$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.$$

$$\Big\downarrow \text{242}$$

$$c\left(\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-a\sqrt{}\right.\right.$$
$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5493}$$

3.322. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x^4}\,dx$

$$c\left(\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\right.\right.$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{5489}$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan\right.\right.}{}\right.$$

$$c\left(-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{}}{\sqrt{}}\right.\right.}{2\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{}}{\sqrt{}}\right.\right.}{2\sqrt{a^2cx^2+c}}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(c*x^3) + (2*a*(-((a*Sqrt[c
+ a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(-1/2*(a*Sqrt
[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^
2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[
1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))))/3) + a^2*c*(c*(-(
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*Ar
cTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
]))/Sqrt[c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcT
an[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*
E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2
])
```

3.322. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x^4}\,dx$

3.322.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.322. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx$

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.322. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx$

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.322.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.59

method	result
default	$-\frac{c\sqrt{c(ax-i)(ax+i)}\left(4x^2\arctan(ax)^2a^2+a^2x^2+x\arctan(ax)a+\arctan(ax)^2\right)}{3x^3}-\frac{c\,a^3\sqrt{c(ax-i)(ax+i)}\left(3\arctan(ax)^2\ln\left(1+\frac{i(i\right.\right.}{\sqrt{a^2}}}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(c*(a*x-I)*(I+a*x))^(1/2)*(4*x^2*arctan(a*x)^2*a^2+a^2*x^2+x*arctan
(a*x)*a+arctan(a*x)^2)/x^3-1/3*c*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(3*arctan(a
*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arctan(a*x)
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-7*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-
7*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.322.5 Fricas [F]

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^2}{x^4} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}} \arctan{(ax)}^2}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

3.322.6 Sympy [F]

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^2}{x^4} \, dx = \int \frac{(c(a^2x^2 + 1))^{\frac{3}{2}} \operatorname{atan}^2{(ax)}}{x^4} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**4, x)
```

3.322.7 Maxima [F]

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^2}{x^4} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}} \arctan{(ax)}^2}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

3.322.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

3.323 $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

3.323.1 Optimal result

Integrand size = 24, antiderivative size = 578

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = -\frac{115c^2\sqrt{c+a^2cx^2}}{4032a^4}-\frac{115c(c+a^2cx^2)^{3/2}}{18144a^4} -\frac{23(c+a^2cx^2)^{5/2}}{7560a^4}+\frac{(c+a^2cx^2)^{7/2}}{252a^4c}+\frac{47c^2x\sqrt{c+a^2cx^2}\arctan(ax)}{1344a^3} -\frac{205c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)}{6048a}-\frac{103ac^2x^5\sqrt{c+a^2cx^2}\arctan(ax)}{1512} -\frac{1}{36}a^3c^2x^7\sqrt{c+a^2cx^2}\arctan(ax)-\frac{2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{63a^4}+\frac{c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{63a^2}+\frac{5}{21}c^2x^4\sqrt{c}$$

output
```
-115/18144*c*(a^2*c*x^2+c)^(3/2)/a^4-23/7560*(a^2*c*x^2+c)^(5/2)/a^4+1/252
*(a^2*c*x^2+c)^(7/2)/a^4/c+115/4032*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-115/2016*I*c^3*arc
tan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^
2*c*x^2+c)^(1/2)-115/4032*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-115/4032*c^2*(a^2*c*x^2+c)^(1
/2)/a^4+47/1344*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3-205/6048*c^2*x^3
*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a-103/1512*a*c^2*x^5*arctan(a*x)*(a^2*c*x
^2+c)^(1/2)-1/36*a^3*c^2*x^7*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-2/63*c^2*arct
an(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4+1/63*c^2*x^2*arctan(a*x)^2*(a^2*c*x^2+c)
^(1/2)/a^2+5/21*c^2*x^4*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+19/63*a^2*c^2*x^
6*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+1/9*a^4*c^2*x^8*arctan(a*x)^2*(a^2*c*x
^2+c)^(1/2)
```

3.323.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1320 vs. 2(578) = 1156.

Time = 6.72 (sec) , antiderivative size = 1320, normalized size of antiderivative = 2.28

$$\int x^3 (c + a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
((c + a^2*c*x^2)^(5/2)*(-48384*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*
x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*
ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*
x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*
ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log
[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^
(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x
^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2)
+ 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]) +
576*(1 + a^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] -
5376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*Arc
Tan[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*ArcTan[a*x
]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[a*x]*Cos[3
*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Cos[5*ArcTan
[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*L
og[1 - I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x
])])/Sqrt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(
I*ArcTan[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTa
n[a*x])] + 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x...
```

3.323.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+c\int x^3\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int x^7\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)+$$
$$c\left(a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\Big(a^2c\Big(a^2c\int\frac{x^9\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\Big)+c\Big(a^2c\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}$$
$$c\Big(a^2c\Big(a^2c\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\Big)+c\Big(a^2c\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5487}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{7a^2c}$$
$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{5a^2c}$$

$$\Big\downarrow \text{ 5465}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{7a^2c}$$
$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{5a^2c}$$

$$\Big\downarrow \text{ 5425}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{7a^2c}$$
$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{5a^2c}$$

3.323. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$\Big\downarrow \textbf{5421}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{7a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \textbf{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\frac{x^7}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)}{9a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\arctan}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\arctan}{5a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \textbf{241}$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\frac{x^7}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)}{9a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{ar}}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{ar}}{5a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 243$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx^2}{16a}-\frac{7\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)}{9a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{7}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx^2}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{5}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 53$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{5/2}}{a^6c^3}-\frac{3\left(a^2cx^2+c\right)^{3/2}}{a^6c^2}+\frac{3\sqrt{a^2cx^2+c}}{a^6c}-\frac{1}{a^6\sqrt{a^2cx^2+c}}\right.}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\frac{1}{a^4\sqrt{a^2cx^2+c}}\right)dx^2}{12a}-\frac{5\int\frac{x^4}{\sqrt{}}}{}\right.}{7a}\right.\right.\right.$$

$$\Big\downarrow \; 2009$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(}{\sqrt{a^2cx^2}}}{6a^2}\right.}{7a}\right.\right.\right.$$

$$\Big\downarrow \; 5425$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(}{\sqrt{a^2cx^2}}}{6a^2}\right.}{7a}\right.\right.\right.$$

$$\downarrow \text{5421}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(}{\sqrt{a^2cx^2}}}{6a^2}\right.}{7a}\right.\right.\right.$$

$$\downarrow \text{5465}$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\cdots}{16a}\cdots\right.}{9a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(\cdots}{\sqrt{a^2cx^2\cdots}}}{6a^2}\cdots\right.}{7a}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 5425$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\cdots}{16a}\cdots\right.}{9a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(\cdots}{\sqrt{a^2cx^2\cdots}}}{6a^2}\cdots\right.}{7a}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 5421$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(}{\sqrt{a^2cx^2}}}{6a^2}\right.}{7a}\right.\right.\right.$$

$$\Big\downarrow 5487$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

$\downarrow$ 241

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\sqrt{a^2cx^2+}\right.}{}\right.}{7a}\right.\right.\right.$$

$\downarrow$ 243

3.323. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}\cdots}{16a}\cdots\right.}{9a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+\cdots}}{\cdots}\right.}{\cdots}\cdots\right.}{7a}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 53$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

$\downarrow$ 2009

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\sqrt{a^2cx^2+}\right.}{}\right.}{7a}\right.\right.\right.$$

$\downarrow$ 5425

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\cdots\right.}{\cdots}\right.\right.\right.$$

$$\left(\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+\cdots}}{\cdots}\right.}{\cdots}\right.}{7a}\cdots\right.\right.\right.$$

↓ 5421

3.323. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$\left(c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\sqrt{a^2cx^2+}\right.}{}\right.}{7a}\right.\right.\right.\right.$$

$\downarrow$ 5465

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

↓ 5425

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

↓ 5421

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$\left(\left(\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+}}{}\right.}{}\right.\right.\right.$$

3.323. $\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$\Big\downarrow \text{5487}$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2(a^2cx^2+c)^{7/2}}{7a^8c^4}-\frac{6(a^2cx^2+c)^{5/2}}{5a^8c^3}+\frac{2(a^2cx^2+c)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2(a^2cx^2+c)^{5/2}}{5a^6c^3}-\frac{4(a^2cx^2+c)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+}}{}\right.}{}\right.$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$\downarrow$ 241

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$\left(\left(\left(\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+}}{}\right.}{}\right.}{}\right.\right.\right.$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$\downarrow$ 243

3.323. $\int x^3 (c + a^2 c x^2)^{5/2} \arctan(ax)^2 \, dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+\dots}}{}\right.}{}\right.$$

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

input
```
Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.323.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.323. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.323.4 Maple [A] (verified)

Time = 10.11 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.53

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(20160\arctan(ax)^2a^8x^8-5040\arctan(ax)a^7x^7+54720a^6x^6\arctan(ax)^2+720a^6x^6-12360\arctan(ax)a^5x^5+432\right.}{181}$

input
```
int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/181440*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(20160*arctan(a*x)^2*a^8*x^8-50
40*arctan(a*x)*a^7*x^7+54720*a^6*x^6*arctan(a*x)^2+720*a^6*x^6-12360*arcta
n(a*x)*a^5*x^5+43200*a^4*arctan(a*x)^2*x^4+1608*a^4*x^4-6150*arctan(a*x)*x
^3*a^3+2880*x^2*arctan(a*x)^2*a^2-94*a^2*x^2+6345*x*arctan(a*x)*a-5760*arc
tan(a*x)^2-6157)-115/4032*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)
```

3.323.5 Fricas [F]

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\int(a^2cx^2+c)^{\frac{5}{2}}x^3\arctan(ax)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x)^2, x)
```

3.323.6 Sympy [F]

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)\,dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

3.323.7 Maxima [F]

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \int (a^2cx^2+c)^{\frac{5}{2}}x^3\arctan(ax)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2, x)
```

3.323.8 Giac [F(-2)]

Exception generated.

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \int x^3\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.324 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

3.324.1 Optimal result

Integrand size = 24, antiderivative size = 638

$$
\begin{aligned}
\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx &= \frac{43c^2x\sqrt{c+a^2cx^2}}{4032a^2} \\
&+ \frac{29c^2x^3\sqrt{c+a^2cx^2}}{1680} + \frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2} \\
&+ \frac{1373c^2\sqrt{c+a^2cx^2}\arctan(ax)}{20160a^3} - \frac{737c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)}{10080a} \\
&- \frac{83}{840}ac^2x^4\sqrt{c+a^2cx^2}\arctan(ax) - \frac{1}{28}a^3c^2x^6\sqrt{c+a^2cx^2}\arctan(ax) \\
&+ \frac{5c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2}{128a^2} + \frac{59}{192}c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&+ \frac{17}{48}a^2c^2x^5\sqrt{c+a^2cx^2}\arctan(ax)^2 + \frac{1}{8}a^4c^2x^7\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&+ \frac{5ic^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{64a^3\sqrt{c+a^2cx^2}} - \frac{397c^{5/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{5040a^3} \\
&- \frac{5ic^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}} \\
&+ \frac{5ic^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}} \\
&+ \frac{5c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}} \\
&- \frac{5c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-397/5040*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^3-5/64*I*c^3*
arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^
3/(a^2*c*x^2+c)^(1/2)+5/64*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+5/64*I*c^3*arctan((1
+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+
c)^(1/2)+5/64*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/a^3/(a^2*c*x^2+c)^(1/2)-5/64*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+43/4032*c^2*x*(a^2*c*x^2+c)^
(1/2)/a^2+29/1680*c^2*x^3*(a^2*c*x^2+c)^(1/2)+1/168*a^2*c^2*x^5*(a^2*c*x^2
+c)^(1/2)+1373/20160*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3-737/10080*c^2
*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a-83/840*a*c^2*x^4*arctan(a*x)*(a^2*c
*x^2+c)^(1/2)-1/28*a^3*c^2*x^6*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+5/128*c^2*x
*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2+59/192*c^2*x^3*arctan(a*x)^2*(a^2*c
*x^2+c)^(1/2)+17/48*a^2*c^2*x^5*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+1/8*a^4*
c^2*x^7*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 4.15 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.19

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \frac{c^2\sqrt{c+a^2cx^2}\Big(53760ax(1+a^2x^2)^{3/2}-25088ax(1+a^2x^2)^{5/2}+7006ax(1+a^2}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(53760*a*x*(1 + a^2*x^2)^(3/2) - 25088*a*x*(1 + a
^2*x^2)^(5/2) + 7006*a*x*(1 + a^2*x^2)^(7/2) + 53760*(1 + a^2*x^2)^(3/2)*A
rcTan[a*x] + 5376*(1 + a^2*x^2)^(5/2)*ArcTan[a*x] - 38134*(1 + a^2*x^2)^(7
/2)*ArcTan[a*x] + 564480*a*x*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2 + 524160*a*
x*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^2 + 185325*a*x*(1 + a^2*x^2)^(7/2)*ArcTa
n[a*x]^2 + (201600*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 203264*Arc
Tanh[(a*x)/Sqrt[1 + a^2*x^2]] + 161280*(1 + a^2*x^2)^2*ArcTan[a*x]*Cos[3*A
rcTan[a*x]] + 49280*(1 + a^2*x^2)^3*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 7658*
(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 40320*(1 + a^2*x^2)^3*Arc
Tan[a*x]*Cos[5*ArcTan[a*x]] - 10990*(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[5*ArcT
an[a*x]] + 3150*(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[7*ArcTan[a*x]] - (201600*I
)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (201600*I)*ArcTan[a*x]*
PolyLog[2, I*E^(I*ArcTan[a*x])] + 201600*PolyLog[3, (-I)*E^(I*ArcTan[a*x])
] - 201600*PolyLog[3, I*E^(I*ArcTan[a*x])] + 53760*(1 + a^2*x^2)^2*Sin[3*A
rcTan[a*x]] - 48384*(1 + a^2*x^2)^3*Sin[3*ArcTan[a*x]] + 12246*(1 + a^2*x^
2)^4*Sin[3*ArcTan[a*x]] - 80640*(1 + a^2*x^2)^2*ArcTan[a*x]^2*Sin[3*ArcTan
[a*x]] - 315840*(1 + a^2*x^2)^3*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 93975*(
1 + a^2*x^2)^4*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 23296*(1 + a^2*x^2)^3*Si
n[5*ArcTan[a*x]] + 7678*(1 + a^2*x^2)^4*Sin[5*ArcTan[a*x]] + 20160*(1 + a^
2*x^2)^3*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 41685*(1 + a^2*x^2)^4*ArcTa...
```

3.324.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \ 5485$$

$$c\int x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+a^2c\int x^4\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx$$

$$\Big\downarrow \ 5485$$

$$c\left(c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^2dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)+ a^2c\left(a^2c\int x^6\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)$$

$$\Big\downarrow \ 5485$$

$$c\left(c\left(c\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + a^2c\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right) + a^2c\left(a^2c\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right.\right.$$
$$a^2c\left(a^2c\left(a^2c\int \frac{x^8 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right) + c\left(a^2c\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \; 5487$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 5425$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 5423$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 4669$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 3011$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 5465$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 224$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \; 219$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}\right.\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\frac{\sqrt{a^2cx^2+c}\arctan(a}{6a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\sqrt{a\right.}}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a\right.}}{}\right.\right.\right.$$

$$\Big\downarrow \text{224}$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\sqrt{a\cdots\right.}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a\cdots\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{219}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\sqrt{a\cdots\right.}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a\cdots\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan\cdots}{\sqrt{a^2cx^2\cdots}}}{7a^2}\cdots}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(a\cdots}{\sqrt{a^2cx^2+c\cdots}}}{5a^2}\cdots}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{224}$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan}{\sqrt{a^2cx^2}}}{7a^2}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}}{5a}-\frac{4\int}{}}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{219}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan}{\sqrt{a^2cx^2}}}{7a^2}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3}{}}{}}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{262}$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\frac{x^{\cdots}}{\cdots}}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{\cdots}{\sqrt{a\cdots}}}{2a^2}\right.}{4a^2}\right.}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\frac{x^{\cdots}}{\cdots}}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{219}$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\cdots\right)}{4a^2}\cdots\right)}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\cdots}{\cdots}}{\cdots}}{3a}\cdots\right.\right.\right.$$

$$\Big\downarrow \ \textbf{5425}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\cdots\right)}{4a^2}\cdots\right)}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\cdots}{\cdots}}{\cdots}}{3a}\cdots\right.\right.\right.$$

$$\Big\downarrow \ \textbf{5423}$$

3.324. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\cdots\right)}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int \frac{x^3\cdots}{\cdots}}{\cdots}}{3a}\cdots\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\cdots\right)}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int \frac{x^3\cdots}{\cdots}}{\cdots}}{3a}\cdots\right.\right.\right.$$

$$\Big\downarrow \text{4669}$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}}{\sqrt{a^2cx}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int \frac{x^3}{}}{}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}}{\sqrt{a^2cx}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int \frac{x^3}{}}{}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}}{\sqrt{a^2cx}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x}{}}{}}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{5465}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}}{\sqrt{a^2cx}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x}{}}{}}{3a}\right.\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

3.324.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.324. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.324. $\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^2 \, dx$

3.324.4 Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.59

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(5040\arctan(ax)^2 a^7x^7-1440a^6\arctan(ax)x^6+14280a^5\arctan(ax)^2x^5+240a^5x^5-3984\arctan(ax)a^4x^4+12390\right.}{40320a^3}$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/40320*c^2/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(5040*arctan(a*x)^2*a^7*x^7-1440
*a^6*arctan(a*x)*x^6+14280*a^5*arctan(a*x)^2*x^5+240*a^5*x^5-3984*arctan(a
*x)*a^4*x^4+12390*a^3*arctan(a*x)^2*x^3+696*a^3*x^3-2948*a^2*arctan(a*x)*x
^2+1575*a*arctan(a*x)^2*x+430*a*x+2746*arctan(a*x))-1/40320*I*c^2*(c*(a*x-
I)*(I+a*x))^(1/2)*(1575*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-1575*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*arctan(a*x
)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(2,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-3150*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6352*arctan((1+I*a*x)/(
a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

3.324.5 Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x)^2, x)
```

3.324.6 Sympy [F]

$$\int x^2 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2\,dx = \int x^2 \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^2(ax)\,dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

3.324.7 Maxima [F]

$$\int x^2 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^{\frac{5}{2}} x^2 \arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2, x)
```

3.324.8 Giac [F]

$$\int x^2 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^{\frac{5}{2}} x^2 \arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \int x^2 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2 + c\right)^{5/2} \, dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.325 $\int x(c+a^2cx^2)^{5/2} \arctan(ax)^2\,dx$

3.325.1 Optimal result

Integrand size = 22, antiderivative size = 387

$$\begin{aligned}\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx &= \frac{5c^2\sqrt{c+a^2cx^2}}{56a^2}\\
&+\frac{5c(c+a^2cx^2)^{3/2}}{252a^2}+\frac{(c+a^2cx^2)^{5/2}}{105a^2}-\frac{5c^2x\sqrt{c+a^2cx^2}\arctan(ax)}{56a}\\
&-\frac{5cx(c+a^2cx^2)^{3/2}\arctan(ax)}{84a}-\frac{x(c+a^2cx^2)^{5/2}\arctan(ax)}{21a}\\
&+\frac{(c+a^2cx^2)^{7/2}\arctan(ax)^2}{7a^2c}+\frac{5ic^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{28a^2\sqrt{c+a^2cx^2}}\\
&-\frac{5ic^3\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{c+a^2cx^2}}+\frac{5ic^3\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
5/252*c*(a^2*c*x^2+c)^(3/2)/a^2+1/105*(a^2*c*x^2+c)^(5/2)/a^2-5/84*c*x*(a^
2*c*x^2+c)^(3/2)*arctan(a*x)/a-1/21*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a+1/
7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^2/a^2/c+5/28*I*c^3*arctan(a*x)*arctan((1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-5
/56*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
a^2/(a^2*c*x^2+c)^(1/2)+5/56*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+5/56*c^2*(a^2*c*x^2+c)^(1/
2)/a^2-5/56*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a
```

3.325.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1087 vs. 2(387) = 774.

Time = 7.75 (sec) , antiderivative size = 1087, normalized size of antiderivative = 2.81

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \frac{c^2(1+a^2x^2)\sqrt{c(1+a^2x^2)}\Big(2+4\arctan(ax)^2+2\cos(2\arctan(ax))-\frac{3\arctan(a}{}}{}$$

$$-\frac{c^2(1+a^2x^2)^2\sqrt{c(1+a^2x^2)}\Big(50-32\arctan(ax)^2+72\cos(2\arctan(ax))+160\arctan(ax)^2\cos(2\arctan(}{}$$

$$+\frac{c^2(1+a^2x^2)^3\sqrt{c(1+a^2x^2)}\Big(4116+10944\arctan(ax)^2+6262\cos(2\arctan(ax))-5376\arctan(ax)^2\cos(}{}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
(c^2*(1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Ar
cTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3
*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
)/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/(12*a^2) - (c^2
*(1 + a^2*x^2)^2*Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*A
rcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]]
 - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*A
rcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x
]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1
 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a
*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[
1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1
 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2
)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a
*x]]))/(480*a^2) + (c^2*(1 + a^2*x^2)^3*Sqrt[c*(1 + a^2*x^2)]*(4116 + 1094
4*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] - 5376*ArcTan[a*x]^2*Cos[2*ArcTa
n[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*ArcTan[a*x]^2*Cos[4*ArcTan[a*x...
```

3.325.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5465, 5413, 5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\int \left(a^2cx^2+c\right)^{5/2}\arctan(ax)dx}{7a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\left(\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}\right)}{7a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right. \ldots}{7a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3}}{12a} \ldots\right.\right.}{7a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2c \ldots}{}\right.\right.}{7a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+\right.\right.\right.}{7c}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^2)/(7*a^2*c) - (2*(-1/30*(c + a^2*c*x^2
)^(5/2)/a + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/6 + (5*c*(-1/12*(c + a^2
*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sqr
t[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a
^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a +
(I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (
I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/4))/6))
/(7*a)
```

3.325.3.1 Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.325. $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.325.4 Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.71

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(360a^6x^6\arctan(ax)^2-120\arctan(ax)a^5x^5+1080a^4\arctan(ax)^2x^4+24a^4x^4-390\arctan(ax)x^3a^3+1080x^2\,\text{arct}\right.}{2520a^2}$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2520*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(360*a^6*x^6*arctan(a*x)^2-120*ar
ctan(a*x)*a^5*x^5+1080*a^4*arctan(a*x)^2*x^4+24*a^4*x^4-390*arctan(a*x)*x^
3*a^3+1080*x^2*arctan(a*x)^2*a^2+98*a^2*x^2-495*x*arctan(a*x)*a+360*arctan
(a*x)^2+299)+5/56*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*d
ilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2)))/a^2/(a^2*x^2+1)^(1/2)
```

3.325.5 Fricas [F]

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(
a*x)^2, x)
```

3.325.6 Sympy [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\int x\big(c(a^2x^2+1)\big)^{\frac{5}{2}}\operatorname{atan}^2(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

3.325.7 Maxima [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\int (a^2cx^2+c)^{\frac{5}{2}}x\arctan(ax)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2, x)
```

3.325.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int x\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.326 $\int (c+a^2cx^2)^{5/2} \arctan(ax)^2 \, dx$

3.326.1 Optimal result

Integrand size = 21, antiderivative size = 516

$$\int (c+a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx(c+a^2cx^2)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\arctan(ax)}{8a} - \frac{5c(c+a^2cx^2)^{3/2}\arctan(ax)}{36a} - \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{15a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2 + \frac{5}{24}cx(c+a^2cx^2)^{3/2}\arctan(ax)^2 + \frac{1}{6}x(c+a^2cx^2)^{5/2}\arctan(ax)^2 - \frac{5ic^3\sqrt{1+}}{}$$

output
```
1/60*c*x*(a^2*c*x^2+c)^(3/2)-5/36*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a-1/15
*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x
)^2+1/6*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2+259/360*c^(5/2)*arctanh(a*x*c^
(1/2)/(a^2*c*x^2+c)^(1/2))/a-5/8*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))
*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+5/8*I*c^3*arctan(a*
x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^
2+c)^(1/2)-5/8*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*
(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/8*c^3*polylog(3,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+5/8*c^3*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+17/
180*c^2*x*(a^2*c*x^2+c)^(1/2)-5/8*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a+5/
16*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.326.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.49

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \frac{c^2\sqrt{c+a^2cx^2}\Big(424ax\sqrt{1+a^2x^2}+368a^3x^3\sqrt{1+a^2x^2}-56a^5x^5\sqrt{1+a^2x^2}-11}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(424*a*x*Sqrt[1 + a^2*x^2] + 368*a^3*x^3*Sqrt[1 +
 a^2*x^2] - 56*a^5*x^5*Sqrt[1 + a^2*x^2] - 11028*Sqrt[1 + a^2*x^2]*ArcTan[
a*x] + 504*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 12*a^4*x^4*Sqrt[1 + a^2
*x^2]*ArcTan[a*x] + 11970*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 7380*a^3*x
^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 1170*a^5*x^5*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^2 - (7200*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 8288*ArcTanh[
(a*x)/Sqrt[1 + a^2*x^2]] + 1550*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 3210*a^2*
x^2*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 1770*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan
[a*x]] + 110*a^6*x^6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 90*ArcTan[a*x]*Cos[5
*ArcTan[a*x]] - 270*a^2*x^2*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 270*a^4*x^4*A
rcTan[a*x]*Cos[5*ArcTan[a*x]] - 90*a^6*x^6*ArcTan[a*x]*Cos[5*ArcTan[a*x]]
 + (7200*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (7200*I)*ArcTa
n[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 7200*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + 7200*PolyLog[3, I*E^(I*ArcTan[a*x])] + 372*Sin[3*ArcTan[a*x]] + 6
36*a^2*x^2*Sin[3*ArcTan[a*x]] + 156*a^4*x^4*Sin[3*ArcTan[a*x]] - 108*a^6*x
^6*Sin[3*ArcTan[a*x]] - 1425*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3555*a^2*x
^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 2835*a^4*x^4*ArcTan[a*x]^2*Sin[3*Arc
Tan[a*x]] - 705*a^6*x^6*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 52*Sin[5*ArcTan
[a*x]] - 156*a^2*x^2*Sin[5*ArcTan[a*x]] - 156*a^4*x^4*Sin[5*ArcTan[a*x]] -
 52*a^6*x^6*Sin[5*ArcTan[a*x]] + 45*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + ...
```

3.326.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.94, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {5415, 211, 211, 224, 219, 5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.326. $\int (c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

$$\int \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\int \left(a^2cx^2+c\right)^{3/2}dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{5415}$$

3.326. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx\right.}{6a}\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arc}}{}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{5415}$$

3.326. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2}}{a}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{5425}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}}{\sqrt{a^2cx}}\right.}{a}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{5423}$$

3.326. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+}}{a}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax}{}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{4669}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-\right.}{}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{2720}$$

3.326. $\int\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\,\text{P}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
-1/15*((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(5/2)*Arc
Tan[a*x]^2)/6 + (c*((x*(c + a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*
x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4))
/15 + (5*c*(-1/6*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2
)^(3/2)*ArcTan[a*x]^2)/4 + (c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTan
h[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*
ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*
I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan
[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))
/(2*a*Sqrt[c + a^2*c*x^2])))/4))/6
```

3.326. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

3.326.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.326. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx$

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.326.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.66

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(120a^5\arctan(ax)^2x^5-48\arctan(ax)a^4x^4+390a^3\arctan(ax)^2x^3+12a^3x^3-196a^2\arctan(ax)x^2+495a\arctan(a\right.}{720a}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/720*c^2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(120*a^5*arctan(a*x)^2*x^5-48*arctan
(a*x)*a^4*x^4+390*a^3*arctan(a*x)^2*x^3+12*a^3*x^3-196*a^2*arctan(a*x)*x^2
+495*a*arctan(a*x)^2*x+80*a*x-598*arctan(a*x))+1/720*I*c^2*(c*(a*x-I)*(I+a
*x))^(1/2)*(225*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-225*I*
arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arctan(a*x)*polylog(
2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))+450*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*I*pol
ylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1036*arctan((1+I*a*x)/(a^2*x^2+1)^(1
/2)))/a/(a^2*x^2+1)^(1/2)
```

3.326.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2, x)
```

3.326.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

3.326.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2, x)
```

3.326.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx = \int \mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

3.327 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx$

3.327.1 Optimal result

Integrand size = 24, antiderivative size = 605

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \frac{29}{60}c^2\sqrt{c+a^2cx^2}$$
$$+\frac{1}{30}c\left(c+a^2cx^2\right)^{3/2}-\frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\arctan(ax)$$
$$-\frac{1}{10}acx\left(c+a^2cx^2\right)^{3/2}\arctan(ax)+c^2\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{1}{3}c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2+\frac{1}{5}\left(c+a^2cx^2\right)^{5/}$$

output
```
1/30*c*(a^2*c*x^2+c)^(3/2)-1/10*a*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+1/3*
c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2+
149/30*I*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^3*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*c^3*arctan(a*x)*poly
log(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-
2*I*c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)-149/60*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+149/60*I*c^3*polylog(2,I*(1+
I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^3*
polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)+2*c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*
x^2+c)^(1/2)+29/60*c^2*(a^2*c*x^2+c)^(1/2)-29/60*a*c^2*x*arctan(a*x)*(a^2*
c*x^2+c)^(1/2)+c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.327.2 Mathematica [A] (verified)

Time = 7.17 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.47

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = c^2\sqrt{c(1+a^2x^2)}\left(\arctan(ax)^2 + \frac{\arctan(ax)^2\left(\log\left(1-e^{i\arctan(ax)}\right)-\log\left(1+e^{i\arctan(ax)}\right)\right)}{\sqrt{1+a^2x^2}} - \frac{2\left(\arctan(ax)\left(\log\left(1-ie^{i\arctan(ax)}\right)-\log\left(1+ie^{i\arctan(ax)}\right)\right)+i\left(\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(2\right.\right.\right.}{\sqrt{1+a^2x^2}} + \frac{2i\arctan(ax)\left(\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)\right)}{\sqrt{1+a^2x^2}} + \frac{2\left(-\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)+\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)\right)}{\sqrt{1+a^2x^2}}\right) + \frac{1}{6}c^2(1+a^2x^2)\sqrt{c(1+a^2x^2)}\left(2+4\arctan(ax)^2+2\cos(2\arctan(ax)) - \frac{3\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}} - \arctan(ax)\cos(3\arctan(ax))\log\left(1-ie^{i\arctan(ax)}\right) + \frac{3\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}} + \arctan(ax)\cos(3\arctan(ax))\log\left(1+ie^{i\arctan(ax)}\right) - \frac{4i\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{(1+a^2x^2)^{3/2}} + \frac{4i\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{(1+a^2x^2)^{3/2}} - 2\arctan(ax)\sin(2\arctan(ax))\right) - \frac{1}{960}c^2(1+a^2x^2)^2\sqrt{c(1+a^2x^2)}\left(50-32\arctan(ax)^2+72\cos(2\arctan(\right.$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```

output
```
c^2*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 + (ArcTan[a*x]^2*(Log[1 - E^(I*Ar
cTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (2*(ArcTan[
a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sqrt
[1 + a^2*x^2] + ((2*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcTan[a*x])] - PolyL
og[2, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (2*(-PolyLog[3, -E^(I*ArcTa
n[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]) + (c^2*(1 +
a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]]
 - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan
[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log
[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*
x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2
*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/6 - (c^2*(1 + a^2*x^2)^2*
Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160
*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*
x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*
ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*
x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[
a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I...
```

3.327.3 Rubi [A] (verified)

Time = 5.72 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.36, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5485, 5465, 5413, 5413, 5425, 5421, 5485, 5465, 5413, 5425, 5421, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{ 5465}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx}{5a}\right)+$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/}}{12a}\right.}{5a}\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(\right.}{5a}\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\,\mathrm{ar}\right.}{5a}\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5421}$$

3.327. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}dx$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5465}$$

$$c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int \sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5413}$$

3.327. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

$$c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int\sqrt{a^2c\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 5421}$$

3.327. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

$$c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{P}}{}\right.}{2\sqrt{a^2c}}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5485

$$c\left(c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan}{}\right.}{}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5465

3.327. $\displaystyle\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

$$c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)}{3a^2c}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(\right.}{3a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5421}$$

3.327. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

$$c\left(c\left(c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}(\ldots}{}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+\ldots}}{\sqrt{1-ia\ldots}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \textbf{5493}$$

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{P}\ldots}{}\right.}{a\sqrt{a^2cx^2\ldots}}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+\ldots}}{\sqrt{1-ia\ldots}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \textbf{5491}$$

3.327. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\,\mathrm{a}}{}\right.}{}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2}{}\right.}{}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+}}{\sqrt{1-ia}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \text{4671}$$

3.327. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}(2,}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$\Big\downarrow \; 3011$$

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}(2,}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$\Big\downarrow \; 2720$$

3.327. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

$$c\Bigg(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\Big(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\Big(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)\ \cdots}{\cdots}$$

$$c\Bigg(c\Bigg(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\Bigg(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}(2,\cdots}{a\cdots}\ \cdots}{2\sqrt{a^2cx^2+c}\ \cdots}\ \cdots}{3a\ \cdots}$$

$\big\downarrow$ 7143

$$c\Bigg(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\Big(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\Big(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)\ \cdots}{\cdots}$$

$$c\Bigg(c\Bigg(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\Bigg(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}(2,\cdots}{a\cdots}\ \cdots}{2\sqrt{a^2cx^2+c}\ \cdots}\ \cdots}{3a\ \cdots}$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```

3.327. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x}\,dx$

output
```
a^2*c*(((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/12*(c + a^
2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sq
rt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 +
a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a +
 (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/4))/(5
*a)) + c*(a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/
2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[
1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3
*a)) + c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*S
qrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) -
2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*
x])])))/Sqrt[c + a^2*c*x^2]))
```

3.327.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.327. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx$

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.327. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx$

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.327.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.67

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(12a^4\arctan(ax)^2x^4-6\arctan(ax)x^3a^3+44x^2\arctan(ax)^2a^2+2a^2x^2-35x\arctan(ax)a+92\arctan(ax)^2+31\right)}{60}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(12*a^4*arctan(a*x)^2*x^4-6*arctan(a*x)
*x^3*a^3+44*x^2*arctan(a*x)^2*a^2+2*a^2*x^2-35*x*arctan(a*x)*a+92*arctan(a
*x)^2+31)+1/60*I*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(60*I*arctan(a*x)^2*ln((1+I
*a*x)/(a^2*x^2+1)^(1/2)+1)-60*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-149*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+149*I*arctan(a
*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*arctan(a*x)*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-120*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+120*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*I*polylog(3,(1+I*a*x)
/(a^2*x^2+1)^(1/2))-149*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+149*dilog(1
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.327.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^2}{x} \, dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}} \arctan(ax)^2}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x, x)
```

3.327.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^2}{x} \, dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}^2(ax)}{x} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x, x)
```

3.327.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^2}{x} \, dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}} \arctan(ax)^2}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x, x)
```

3.327.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.328 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx$

3.328.1 Optimal result

Integrand size = 24, antiderivative size = 655

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2} - \frac{7}{4}ac^2\sqrt{c+a^2cx^2}\arctan(ax) - \frac{1}{6}ac(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2 + \frac{1}{4}a^2cx(c+a^2cx^2)^{3/2}\arctan(ax)^2 - \frac{15iac^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{4\sqrt{c+a^2cx^2}} - \frac{4ac^3\sqrt{1+a^2x^2}\,\mathrm{arcta}}{\sqrt{c}}$$

output
```
-1/6*a*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+1/4*a^2*c*x*(a^2*c*x^2+c)^(3/2)*a
rctan(a*x)^2+11/6*a*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-15/4*
I*a*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-4*a*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15/4*I*a*c^3*arctan(a*x)*po
lylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)-15/4*I*a*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*I*a*c^3*polylog(2,-(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*c^3*polylog(2,(1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15/4*
a*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x
^2+c)^(1/2)+15/4*a*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/12*a^2*c^2*x*(a^2*c*x^2+c)^(1/2)-7/4*a*c^2*a
rctan(a*x)*(a^2*c*x^2+c)^(1/2)-c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x+7/8
*a^2*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 626, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(2a^2x^2\sqrt{1+a^2x^2}+2a^4x^4\sqrt{1+a^2x^2}-190ax\sqrt{1+a^2x^2}\text{ ar}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + 2*a^4*x^4*Sqrt[1 +
 a^2*x^2] - 190*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 2*a^3*x^3*Sqrt[1 + a^2
*x^2]*ArcTan[a*x] - 96*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 117*a^2*x^2*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]^2 + 21*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 -
(168*I)*a*x*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 176*a*x*ArcTanh[(a*x
)/Sqrt[1 + a^2*x^2]] + 6*a*x*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^3*x^3*A
rcTan[a*x]*Cos[3*ArcTan[a*x]] + 6*a^5*x^5*ArcTan[a*x]*Cos[3*ArcTan[a*x]] +
 192*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 96*a*x*ArcTan[a*x]^2*Log
[1 - I*E^(I*ArcTan[a*x])] - 96*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x
])] - 192*a*x*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (192*I)*a*x*PolyLog
[2, -E^(I*ArcTan[a*x])] + (360*I)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] - (360*I)*a*x*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (19
2*I)*a*x*PolyLog[2, E^(I*ArcTan[a*x])] - 360*a*x*PolyLog[3, (-I)*E^(I*ArcT
an[a*x])] + 360*a*x*PolyLog[3, I*E^(I*ArcTan[a*x])] + 2*a*x*Sin[3*ArcTan[a
*x]] + 4*a^3*x^3*Sin[3*ArcTan[a*x]] + 2*a^5*x^5*Sin[3*ArcTan[a*x]] - 3*a*x
*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 6*a^3*x^3*ArcTan[a*x]^2*Sin[3*ArcTan[a
*x]] - 3*a^5*x^5*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(96*x*Sqrt[1 + a^2*x^2
])
```

3.328.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2c\right.}{6a}\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+\right.\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{ar}}{}\right.\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{224}$$

3.328. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^2}\,dx$

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2}}{a}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \ 219$$

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \ 5425$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}}{\sqrt{a^2cx}}\right.}{a}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \ 5423$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+}}{a}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \ 3042$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(a}{}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \ 4669$$

3.328. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^2}dx$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx\right)+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5415}$$

$$c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+c\int\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2}}{a}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

3.328. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^2}dx$

$$\Big\downarrow \text{219}$$

$$c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2}}\right.}{a}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)}{}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc}\right.\right.}{}\right.\right.$$
$$c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int a\right.}{2}\right.\right.$$

3.328. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^2}\,dx$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2\right.\right.\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\right.\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.\right.$$

$$c\left(c\left(a^2c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.\right.$$

$$c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan\right.\right.}{}\right.\right.$$

3.328. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^2}dx$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arc\right.\right.}{}\right.\right.$$

$$c\left(c\left(c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{4669}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}\right)\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\left(\right.\right.}{}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{3011}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}\right)\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\left(\right.\right.}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
$Aborted
```

3.328.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.328. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^2}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

3.328. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.328.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.61

method	result
default	$\frac{c^{2}\sqrt{c(ax-i)(ax+i)}\left(6a^{4}\arctan(ax)^{2}x^{4}-4\arctan(ax)x^{3}a^{3}+27x^{2}\arctan(ax)^{2}a^{2}+2a^{2}x^{2}-46x\arctan(ax)a-24\arctan(ax)^{2}\right)}{24x}+$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(6*a^4*arctan(a*x)^2*x^4-4*arctan(a*x)*
x^3*a^3+27*x^2*arctan(a*x)^2*a^2+2*a^2*x^2-46*x*arctan(a*x)*a-24*arctan(a*
x)^2)/x+1/24*I*c^2*a*(c*(a*x-I)*(I+a*x))^(1/2)*(45*I*arctan(a*x)^2*ln(1+I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+48*I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+90*arctan(a*x
)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylog(2,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9
0*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-88*arctan((1+I*a*x)/(a^2*x^2+
1)^(1/2))+48*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+48*dilog((1+I*a*x)/(a^2*
x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.328.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^2}{x^2} \, dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}} \arctan(ax)^2}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x^2, x)
```

3.328.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^2}{x^2} \, dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}^2(ax)}{x^2} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**2, x)
```

3.328.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^2}{x^2} \, dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}} \arctan(ax)^2}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^2, x)
```

3.328.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

3.329 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx$

3.329.1 Optimal result

Integrand size = 24, antiderivative size = 661

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\arctan(ax)}{x}$$
$$-\frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\arctan(ax) + 2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2}$$
$$+\frac{1}{3}a^2c(c+a^2cx^2)^{3/2}\arctan(ax)^2 + \frac{26ia^2c^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}$$
$$-\frac{5a^2c^3\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$-a^2c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{5ia^2c^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{5ia^2c^3\sqrt{1+a^2x^2}}{}$$

output
```
1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2-a^2*c^(5/2)*arctanh((a^2*c*x^2
+c)^(1/2)/c^(1/2))+26/3*I*a^2*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^2*c^3*arctan(a*x)^2*
arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
+5*I*a^2*c^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*I*a^2*c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13/3*I*a^2*c^3*pol
ylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)
^(1/2)+13/3*I*a^2*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^
2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^2*c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*a^2*c^3*polylog(3,(1+I*a*x)
/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+1/3*a^2*c^2*(a^2
*c*x^2+c)^(1/2)-a*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x-1/3*a^3*c^2*x*arct
an(a*x)*(a^2*c*x^2+c)^(1/2)+2*a^2*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)-1/
2*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

3.329. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx$

3.329.2 Mathematica [A] (warning: unable to verify)

Time = 7.59 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.15

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = 2a^2c^2\sqrt{c(1+a^2x^2)}\Bigg(\arctan(ax)^2$$
$$+\frac{\arctan(ax)^2\left(\log\left(1-e^{i\arctan(ax)}\right)-\log\left(1+e^{i\arctan(ax)}\right)\right)}{\sqrt{1+a^2x^2}}$$
$$-\frac{2\left(\arctan(ax)\left(\log\left(1-ie^{i\arctan(ax)}\right)-\log\left(1+ie^{i\arctan(ax)}\right)\right)+i\left(\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\text{PolyLog}\left(2\right.\right.\right.}{\sqrt{1+a^2x^2}}$$
$$+\frac{2i\arctan(ax)\left(\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)\right)}{\sqrt{1+a^2x^2}}$$
$$+\frac{2\left(-\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)+\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)\right)}{\sqrt{1+a^2x^2}}\Bigg)$$
$$+\frac{1}{12}a^2c^2(1+a^2x^2)\sqrt{c(1+a^2x^2)}\Bigg(2+4\arctan(ax)^2+2\cos(2\arctan(ax))$$
$$-\frac{3\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}}-\arctan(ax)\cos(3\arctan(ax))\log\left(1-ie^{i\arctan(ax)}\right)$$
$$+\frac{3\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}}+\arctan(ax)\cos(3\arctan(ax))\log\left(1+ie^{i\arctan(ax)}\right)$$
$$-\frac{4i\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{(1+a^2x^2)^{3/2}}+\frac{4i\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{(1+a^2x^2)^{3/2}}$$
$$-2\arctan(ax)\sin(2\arctan(ax))\Bigg)+\frac{a^2c^2\sqrt{c(1+a^2x^2)}\left(-4\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)^2\csc\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
2*a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 + (ArcTan[a*x]^2*(Log[1 - E
^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (2*(A
rcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) +
 I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]))
)/Sqrt[1 + a^2*x^2] + ((2*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcTan[a*x])] -
 PolyLog[2, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (2*(-PolyLog[3, -E^(I
*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]) + (a^2
*c^2*(1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Ar
cTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3
*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
)/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/12 + (a^2*c^2*S
qrt[c*(1 + a^2*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*Cs
c[ArcTan[a*x]/2]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 +
 E^(I*ArcTan[a*x])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]*(Poly
Log[2, -E^(I*ArcTan[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-PolyLog[
3, -E^(I*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x]^2*Se
c[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 + a^2...
```

3.329.3 Rubi [A] (verified)

Time = 10.82 (sec) , antiderivative size = 1215, normalized size of antiderivative = 1.84, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {5485, 5485, 5465, 5413, 5425, 5421, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{x^3} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \; 5465$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int\sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \; 5413$$

$$a^2c\Bigg(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int\frac{\sqrt{}}{}$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \; 5425$$

$$a^2c\Bigg(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \; 5421$$

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)+$$
$$a^2c\Bigg(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+a^2c\Bigg(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\Bigg(\frac{c\sqrt{a^2x^2+1}\Big(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+}{2\sqrt{}}}{}$$

3.329. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$\Big\downarrow \; 5485$$

$$c\left(a^2c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)\right)+$$
$$a^2c\left(c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\,\mathrm{arc}}{}\right.}{}\right.}{}\right.\right.$$

$$\Big\downarrow \; 5465$$

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+\right.}{3a^2c}\right.\right.$$

$$\Big\downarrow \; 5425$$

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(}{x\sqrt{a^2cx}}\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)}{}\right.\right.$$

$$\Big\downarrow \; 5421$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLo}}{}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$c\left(c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}\right.\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5491

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$a^2c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\right.}{}\right.\right.\right.$$

$$c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\text{ar}}{}\right.\right.$$

$$\Big\downarrow \; 4671$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{5497}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{5479}$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow 243$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow 73$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\ldots\right)}{a}\ldots\right.}{2\sqrt{a^2cx^2+c}}\ldots\right.}{3a}\ldots\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\ldots}{\ldots}\right)}{a}\ldots\right.}{a\sqrt{a^2cx^2+c}}\ldots\right.\right.\right.$$

$$\Big\downarrow \; 221$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\ldots\right)}{a}\ldots\right.}{2\sqrt{a^2cx^2+c}}\ldots\right.}{3a}\ldots\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\ldots}{\ldots}\right)}{a}\ldots\right.}{a\sqrt{a^2cx^2+c}}\ldots\right.\right.\right.$$

$$\Big\downarrow \; 5493$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\cdots\right.}{a}\cdots\right.}{2\sqrt{a^2cx^2+c}}\cdots\right.}{3a}\cdots\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\cdots}{\cdots}\right.}{a}\cdots\right.}{a\sqrt{a^2cx^2+c}}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 5491$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\cdots\right.}{a}\cdots\right.}{2\sqrt{a^2cx^2+c}}\cdots\right.}{3a}\cdots\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\cdots}{\cdots}\right.}{a}\cdots\right.}{a\sqrt{a^2cx^2+c}}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\cdots\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{\cdots}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{4671}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\cdots\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{\cdots}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.329. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{\cdot}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{\cdot}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

3.329. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx$

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
a^2*c*(a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*S
qrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3*a)
) + c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^
2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (
I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt
[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(
I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])
])))/Sqrt[c + a^2*c*x^2])) + c*(a^2*c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a
*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^
(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyL
og[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])
] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]) + c*(c*(-1/2*...
```

3.329. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx$

3.329.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.329. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx$

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.329. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx$

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.329.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.69

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(2a^4\arctan(ax)^2x^4-2\arctan(ax)x^3a^3+14x^2\arctan(ax)^2a^2+2a^2x^2-6x\arctan(ax)a-3\arctan(ax)^2\right)}{6x^2}-\frac{c^2a^2}{}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(2*a^4*arctan(a*x)^2*x^4-2*arctan(a*x)*x
^3*a^3+14*x^2*arctan(a*x)^2*a^2+2*a^2*x^2-6*x*arctan(a*x)*a-3*arctan(a*x)^
2)/x^2-1/6*c^2*a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(15*arctan(a*x)^2*ln((1+I*a*x
)/(a^2*x^2+1)^(1/2)+1)-15*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
30*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*arctan(a*x)*
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+26*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*di
log(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
)+1)+30*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*polylog(3,(1+I*a*x)/(a^
2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.329.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x^3, x)
```

3.329.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**3, x)
```

3.329.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^3, x)
```

3.329.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

3.330 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx$

3.330.1 Optimal result

Integrand size = 24, antiderivative size = 675

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x}$$
$$-a^3c^2\sqrt{c+a^2cx^2}\arctan(ax)-\frac{ac^2\sqrt{c+a^2cx^2}\arctan(ax)}{3x^2}$$
$$-\frac{2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}+\frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2$$
$$-\frac{c(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3x^3}-\frac{5ia^3c^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}}$$
$$-\frac{26a^3c^3\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}$$
$$+a^3c^{5/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)+\frac{5ia^3c^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}-\frac{5ia^3c^3\sqrt{1+a^2x^2}\ldots}{}$$

output
```
-1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3+a^3*c^(5/2)*arctanh(a*x*c^(1/
2)/(a^2*c*x^2+c)^(1/2))-5*I*a^3*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*ar
ctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-26/3*a^3*c^3*arctan(a*x)
*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^
(1/2)+5*I*a^3*c^3*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*I*a^3*c^3*arctan(a*x)*polylog(2,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13/3*I*a^
3*c^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)-13/3*I*a^3*c^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^3*c^3*polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*a^3*c^3*polylog(3,I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/3*a^2
*c^2*(a^2*c*x^2+c)^(1/2)/x-a^3*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/3*a*c
^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x^2-2*a^2*c^2*arctan(a*x)^2*(a^2*c*x^2+
c)^(1/2)/x+1/2*a^4*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
```

3.330.2 Mathematica [A] (warning: unable to verify)

Time = 2.94 (sec) , antiderivative size = 644, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx =$$

$$-\frac{c^3\sqrt{1+a^2x^2}\Big(2(1+a^2x^2)^{3/2}+12a^3x^3\sqrt{1+a^2x^2}\arctan(ax)+24a^2x^2\sqrt{1+a^2x^2}\arctan(ax)^2-6a^4x^4\sqrt{1+}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/12*(c^3*Sqrt[1 + a^2*x^2]*(2*(1 + a^2*x^2)^(3/2) + 12*a^3*x^3*Sqrt[1 +
a^2*x^2]*ArcTan[a*x] + 24*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - 6*a^4*
x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 4*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2
+ (12*I)*a^3*x^3*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - 12*a^3*x^3*ArcT
anh[(a*x)/Sqrt[1 + a^2*x^2]] - 2*(1 + a^2*x^2)^(3/2)*Cos[2*ArcTan[a*x]] -
3*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 51*a^3*x^3*ArcTan[a*x]*Log[
1 - E^(I*ArcTan[a*x])] - 24*a^3*x^3*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*
x])] + 24*a^3*x^3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 3*a*x*ArcTa
n[a*x]*Log[1 + E^(I*ArcTan[a*x])] + 51*a^3*x^3*ArcTan[a*x]*Log[1 + E^(I*Ar
cTan[a*x])] - (52*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])] - (60*I)*a^3*x
^3*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (60*I)*a^3*x^3*ArcTan[
a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + (52*I)*a^3*x^3*PolyLog[2, E^(I*ArcT
an[a*x])] + 60*a^3*x^3*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 60*a^3*x^3*Pol
yLog[3, I*E^(I*ArcTan[a*x])] + 2*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Sin[2*Arc
Tan[a*x]] + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]*Sin
[3*ArcTan[a*x]] - (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x]
)]*Sin[3*ArcTan[a*x]]))/(x^3*Sqrt[c + a^2*c*x^2])
```

3.330.3 Rubi [A] (verified)

Time = 10.47 (sec) , antiderivative size = 1119, normalized size of antiderivative = 1.66, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.208$, Rules used = {5485, 5485, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5481, 242, 5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 5497, 242, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx\right)+c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(a^2c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\right.$$
$$\left.c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)\right)$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(a^2c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\right.$$
$$\left.c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)\right)$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(a^2c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)\right.$$
$$\left.c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)\right.$$
$$\left.c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right.\right.$$
$$\left.c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)\right)$$

$$\Big\downarrow \text{3042}$$

3.330. $\int\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx$

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(a\right.\right.$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$$\Big\downarrow \text{4669}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(a}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(a}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5481}$$

3.330. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^4}\,dx$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2c}}{x^2}\right.\right.\right.$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(a}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{242}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)\right.\right.$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(a}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right.\right.\right.$$
$$a^2c\left(c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{}}{x^2}\right.\right.\right.$$
$$a^2c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax}{x^3\sqrt{a^2cx^2+}}\right.\right.\right.$$
$$a^2c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.330. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^4}\,dx$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+c\left(\frac{2}{3}a\left(-c\int \right.\right.\right.$$
$$a^2c\left(c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+a^2c\left(\frac{c\sqrt{a^2x^2}}{}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.\right.$$
$$c\left(c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.\right.$$
$$c\left(c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\left(\right.\right.}{}\right.\right.$$
$$c\left(c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\left(\right.\right.}{}\right.\right.$$
$$c\left(c\left(c\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{5493}$$

3.330. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^4}\,dx$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan\right.}{}\right.\right.$$

$$\Big\downarrow 5489$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$$\Big\downarrow 5497$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$$\Big\downarrow 242$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$$\Big\downarrow 5493$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$$\Big\downarrow 5489$$

3.330. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^4}\,dx$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan(\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan(\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
a^2*c*(a^2*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]
)/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E
^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - Po
lyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqr
t[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan
[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/
Sqrt[c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a
*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
- PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I
*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]))
+ c*(c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(c*x^3) + (2*a*(-((a*Sq
rt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(-1/2*(a
*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2)
- (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2,
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))))/3) + a^2*c*(
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2...
```

3.330. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx$

3.330.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.330. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx$

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

3.330. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.330. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx$

3.330.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.59

method	result
default	$\frac{c^{2}\sqrt{c(ax-i)(ax+i)}\left(3a^{4}\arctan(ax)^{2}x^{4}-6\arctan(ax)x^{3}a^{3}-14x^{2}\arctan(ax)^{2}a^{2}-2a^{2}x^{2}-2x\arctan(ax)a-2\arctan(ax)^{2}\right)}{6x^{3}}+\frac{ic^{2}a^{3}}{}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(3*a^4*arctan(a*x)^2*x^4-6*arctan(a*x)*x
^3*a^3-14*x^2*arctan(a*x)^2*a^2-2*a^2*x^2-2*x*arctan(a*x)*a-2*arctan(a*x)^
2)/x^3+1/6*I*c^2*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(15*I*arctan(a*x)^2*ln(1+I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+26*I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+30*arctan(a*x
)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*arctan(a*x)*polylog(2,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))+30*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3
0*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*arctan((1+I*a*x)/(a^2*x^2+
1)^(1/2))+26*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))+26*dilog((1+I*a*x)/(a^2*x^
2+1)^(1/2)+1))/(a^2*x^2+1)^(1/2)
```

3.330.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x^4, x)
```

3.330.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**4, x)
```

3.330.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^4, x)
```

3.330.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{x^4}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

3.331 $\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

3.331.1 Optimal result

Integrand size = 24, antiderivative size = 315

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{c+a^2cx^2}}{3a^4c}-\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{3a^3c}\\ &\quad-\frac{2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3a^2c}\\ &\quad-\frac{10i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{c+a^2cx^2}}\\ &\quad+\frac{5i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{c+a^2cx^2}}\\ &\quad-\frac{5i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-10/3*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1
/2)/a^4/(a^2*c*x^2+c)^(1/2)+5/3*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-5/3*I*polylog(2,I*(1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+1/3*(a^
2*c*x^2+c)^(1/2)/a^4/c-1/3*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^3/c-2/3*arc
tan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(
1/2)/a^2/c
```

3.331.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{(1+a^2x^2)\sqrt{c(1+a^2x^2)}\Big(2-2\arctan(ax)^2+2\cos(2\arctan(ax))-6\arctan(ax)^2\cos(2\arctan(ax))+\frac{15}{}}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 - 2*ArcTan[a*x]^2 + 2*Cos[2*ArcTan
[a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + (15*ArcTan[a*x]*Log[1 - I*E^
(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log
[1 - I*E^(I*ArcTan[a*x])] - (15*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/
Sqrt[1 + a^2*x^2] - 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan
[a*x])] + ((20*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2)
- ((20*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[
a*x]*Sin[2*ArcTan[a*x]]))/(12*a^4*c)
```

3.331.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5487, 5465, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \; 5487$$

$$-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5465$$

$$-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5425}$$

$$-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5421}$$

$$-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5487}$$

$$-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5425}$$

3.331. $\int\frac{x^3\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5421$$

$$-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}-\frac{2\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a^2*
c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(
3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x
^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*P
olyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sq
rt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^2)
```

3.331. $\int \frac{x^3\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

3.331.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.331. $\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

3.331.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.65

method	result
default	$\frac{\left(x^{2} \arctan\left(a x\right)^{2} a^{2}-x \arctan\left(a x\right) a-2 \arctan\left(a x\right)^{2}+1\right) \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{3 a^{4} c}-\frac{5 \left(\arctan\left(a x\right) \ln\left(1+\frac{i \left(i a x+1\right)}{\sqrt{a^{2} x^{2}+1}}\right)-\arctan\left(a x\right) \ln\left(1-\frac{i (i}{\sqrt{a}}\right.\right.}{}$

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2*arctan(a*x)^2*a^2-x*arctan(a*x)*a-2*arctan(a*x)^2+1)*(c*(a*x-I)*(
I+a*x))^(1/2)/a^4/c-5/3*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-a
rctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x)
)^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

3.331.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\, dx = \int \frac{x^3 \arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}}\, dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.331.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\, dx = \int \frac{x^3 \operatorname{atan}^2\left(ax\right)}{\sqrt{c\left(a^2x^2+1\right)}}\, dx$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

3.331.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.331.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.332 $\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$

3.332.1 Optimal result

Integrand size = 24, antiderivative size = 344

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx = & -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a^2c} \\ & + \frac{i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^3\sqrt{c+a^2cx^2}} + \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^3\sqrt{c}} \\ & - \frac{i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} \\ & + \frac{i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} \\ & + \frac{\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} \\ & - \frac{\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^3/c^(1/2)+I*arctan((1+I*a*x)/(a
^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-I
*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a
^3/(a^2*c*x^2+c)^(1/2)+I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-polylog(3,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-arctan(a*
x)*(a^2*c*x^2+c)^(1/2)/a^3/c+1/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^2/c
```

3.332.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.51

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(\arctan(ax)(-2+ax\arctan(ax)) + \frac{2\left(i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+\operatorname{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-i\arctan(ax)\right.}{}\right.}{2a^3c}$$

input

```
Integrate[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output

```
(Sqrt[c + a^2*c*x^2]*(ArcTan[a*x]*(-2 + a*x*ArcTan[a*x]) + (2*(I*ArcTan[E^
(I*ArcTan[a*x])]*ArcTan[a*x]^2 + ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcT
an[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, I*E^
(I*ArcTan[a*x])] + PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*
ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]))/(2*a^3*c)
```

3.332.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.70, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5487, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \; 5487$$

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \; 5425$$

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \; 5423$$

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax\right.}{2a^3\sqrt{a^2cx^2+c}}$$
$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(\right.\right.}{2a^3\sqrt{a^2c}}$$
$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$
$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$
$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{224}$$

3.332. $\int \frac{x^2\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\right.}{}\quad -\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}$$

$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 219

$$-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 7143

$$-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-$$
$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{2a^3\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[
c]))/a - (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^
2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*
E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - P
olyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a^3*Sqrt[c + a^2*c*x^2])
```

3.332.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.332. $\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.332.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.80

method	result
default	$\frac{(x\arctan(ax)a-2)\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{2c\,a^3}-\frac{i\left(i\arctan(ax)^2\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^2\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+2\arctan(\right.}{}$

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x*arctan(a*x)*a-2)*arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^3-1/2*I*
(I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^2*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*po
lylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2
)/(a^2*x^2+1)^(1/2)/a^3/c
```

3.332.5 Fricas [F]

$$\int \frac{x^2\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^2\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.332.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

3.332.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.332.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.333 $\int \frac{x \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

3.333.1 Optimal result

Integrand size = 22, antiderivative size = 220

$$\int \frac{x \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{a^2c} + \frac{4i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}}$$

output
```
4*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
a^2/(a^2*c*x^2+c)^(1/2)-2*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*
(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+2*I*polylog(2,I*(1+I*a*x)^(1/2)/
(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(
a^2*c*x^2+c)^(1/2)/a^2/c
```

3.333.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.57

$$
\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx
$$
$$
= \frac{\sqrt{c\,(1+a^2x^2)}\Big(\arctan(ax)^2 - \frac{2(\arctan(ax)(\log(1-ie^{i\arctan(ax)})-\log(1+ie^{i\arctan(ax)}))+i(\text{PolyLog}(2,-ie^{i\arctan(ax)})-\text{PolyLog}(2}{\sqrt{1+a^2x^2}}}{a^2c}
$$

input
```
Integrate[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 - (2*(ArcTan[a*x]*(Log[1 - I*E^(I*Ar
cTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sqrt[1 + a^2*x^2]))/(a^2*c)
```

3.333.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5465, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$
\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx
$$
$$
\Big\downarrow \; 5465
$$
$$
\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}
$$
$$
\Big\downarrow \; 5425
$$
$$
\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}
$$
$$
\Big\downarrow \; 5421
$$

$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I
)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, (
(-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*
x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])
```

3.333.3.1 Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.333. $\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

3.333.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.82

method	result
default	$\frac{\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{a^2c}+\frac{2\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{dilog}\left(\right.\right.}{\sqrt{a^2x^2+1}\,a^2c}$

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/a^2/c+2*(arctan(a*x)*ln(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*d
ilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c
```

3.333.5 Fricas [F]

$$\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.333.6 Sympy [F]

$$\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x\,\mathrm{atan}^2\left(ax\right)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

3.333.7 Maxima [F]

$$\int \frac{x \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.333.8 Giac [F]

$$\int \frac{x \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.334 $\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$

3.334.1 Optimal result

Integrand size = 21, antiderivative size = 256

$$\begin{aligned}\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = & -\frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a\sqrt{c+a^2cx^2}} \\ & + \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} \\ & - \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} \\ & - \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} \\ & + \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a
/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-2*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1
/2)+2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*
x^2+c)^(1/2)
```

3.334.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{2\sqrt{c\left(1+a^2x^2\right)}\left(-i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+i\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\arctan(c\cdots}{ac\sqrt{1+a^2x^2}}$$

input
```
Integrate[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[c*(1 + a^2*x^2)]*((-I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + I
*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*ArcTan[a*x]*PolyLog[2,
 I*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[3, I*
E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2*x^2])
```

3.334.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.56, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \text{ 5425}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 5423}$$
$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 4669}$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax\right.}{a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{3011}$$

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(\right.\right.}{a\sqrt{a^2cx}}$$

$$\Big\downarrow \textbf{2720}$$

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \textbf{7143}$$

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\,\text{PolyLog}\left(2,\right.\right.\right.}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*
ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcT
an[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3,
 I*E^(I*ArcTan[a*x])])))/(a*Sqrt[c + a^2*c*x^2])
```

3.334.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.334.4 Maple [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.334.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.334.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

3.334.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.334.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(1/2), x)
```

3.335 $\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$

3.335.1 Optimal result

Integrand size = 24, antiderivative size = 227

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,(1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*polylog(3,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+2*polylog(3
,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.64

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{1+a^2x^2}\left(\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)-\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)+2i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.}{\sqrt{}}$$

input
```
Integrate[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]
^2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[
a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E
^(I*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)
]
```

3.335.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.56, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \; 5493$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5491$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textcolor{blue}{4671}$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax)\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textcolor{blue}{3011}$$

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(a\right.\right.}{\sqrt{a^2cx^2}}$$

$$\Big\downarrow \textcolor{blue}{2720}$$

$$\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(\right.\right.$$

$$\Big\downarrow \textcolor{blue}{7143}$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\mathrm{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\mathrm{PolyLog}\left(3,-e^{i\arctan(ax)}\right.\right.\right.}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*Arc
Tan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])])
- 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[
a*x])])))/Sqrt[c + a^2*c*x^2]
```

3.335.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.335. $\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.335. $\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$

3.335.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\left(\arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-\arctan(ax)^2 \ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-2i \arctan(ax) \operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+2i \arctan(ax) \operatorname{polylog}\left(\right.}{\sqrt{a^2x^2+1}\, c}$

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^2*ln(1-(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*
(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

3.335.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\, dx = \int \frac{\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}x}\, dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

3.335.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\, dx = \int \frac{\operatorname{atan}^2\left(ax\right)}{x\sqrt{c\left(a^2x^2+1\right)}}\, dx$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

3.335.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.335.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.336 $\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx$

3.336.1 Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{cx} - \frac{4a\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{2ia\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2ia\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-4*a*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)+2*I*a*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a*polylog(2,(1+I*a*x)^(1/2)/(1-I*a
*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^2*(a^2*c*x^2+
c)^(1/2)/c/x
```

3.336.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{a\sqrt{1+a^2x^2}\left(\arctan(ax)\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)}{ax}-2\log\left(1-e^{i\arctan(ax)}\right)+2\log\left(1+e^{i\arctan(ax)}\right)\right)-2i\,\mathrm{PolyL}\right.}{\sqrt{c\left(1+a^2x^2\right)}}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((a*Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x)
 - 2*Log[1 - E^(I*ArcTan[a*x])] + 2*Log[1 + E^(I*ArcTan[a*x])]) - (2*I)*Po
lyLog[2, -E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/Sqrt[
c*(1 + a^2*x^2)])
```

3.336.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{ 5479}$$

$$2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{ 5493}$$

$$\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{ 5489}$$

$$-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\mathrm{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*
ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt
[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]]))/Sqrt[c + a^2*c*x^2]
```

3.336.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.336.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

method	result
default	$-\frac{\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{cx}+\frac{2ia\left(i\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\right.}{\sqrt{a^2x^2+1}\,c}$

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c/x+2*I*a*(I*arctan(a*x)*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+
polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1
/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

3.336.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

3.336.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

3.336.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.336.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.337 $\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$

3.337.1 Optimal result

Integrand size = 24, antiderivative size = 328

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx &= -\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{cx}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{2cx^2}\\
&\quad+\frac{a^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
&\quad-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\\
&\quad-\frac{ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
&\quad+\frac{ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
&\quad+\frac{a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\
&\quad-\frac{a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a^2*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+a^2*arctan(a*x)^2*arctan
h((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*a^2
*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)+I*a^2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+a^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*polylog(3,(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a*arctan(a*x)*(a^2*c*
x^2+c)^(1/2)/c/x-1/2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c/x^2
```

3.337.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{a^2\sqrt{1+a^2x^2}\left(-4\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)^2\csc^2\left(\frac{1}{2}\arctan(ax)\right)-4\arctan(ax)^2\log\left(1\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^2*Sqrt[1 + a^2*x^2]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*
Csc[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcT
an[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Log[Tan[ArcTan[a*x]/2]] - (8*I)*A
rcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (8*I)*ArcTan[a*x]*PolyLog[2, E
^(I*ArcTan[a*x])] + 8*PolyLog[3, -E^(I*ArcTan[a*x])] - 8*PolyLog[3, E^(I*A
rcTan[a*x])] + ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcT
an[a*x]/2]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

3.337.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.67, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{221}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5493}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5491}$$

3.337. $\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(\right.}{2\sqrt{a^2cx^2+c}}$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\,\text{arcta}\right.\right.}{2\sqrt{a^2c}}$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-e\right.\right.\right.}{2\sqrt{a^2cx^2+c}}$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

3.337. $\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$

input
```
Int[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^
2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c])
 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2
*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a
*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*
ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2])
```

3.337.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.337. $\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.337.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.80

method	result
default	$-\frac{(2ax+\arctan(ax))\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{2c\,x^2}+\frac{a^2\left(\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-2i\arctan\right.}{}$

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a*x+arctan(a*x))*arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^2+1/2*a
^2*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^2*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*
arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1
)^(1/2)/c
```

3.337.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx=\int \frac{\arctan{(ax)}^2}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

3.337.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x^3\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

3.337.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.337.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.338 $\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.338.1 Optimal result

Integrand size = 24, antiderivative size = 311

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = &-\frac{a^2\sqrt{c+a^2cx^2}}{3cx}-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{3cx^2}\\&-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3cx}\\&+\frac{10a^3\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\\&-\frac{5ia^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\\&+\frac{5ia^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
10/3*a^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)-5/3*I*a^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5/3*I*a^3*polylog(2,(1+I*a*x)^
(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/3*a^2*(a^2*
c*x^2+c)^(1/2)/c/x-1/3*a*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c/x^2-1/3*arctan(
a*x)^2*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
/c/x
```

3.338.2 Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.73

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{a^3\sqrt{c+a^2cx^2}\left(-20i\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)+\frac{(1+a^2x^2)^{3/2}\left(\arctan(ax)^2(2-6\cos(2\arctan(ax)))+2(-1+\cos(2\arctan(ax)\right.}{}\right.}{12}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^3*Sqrt[c + a^2*c*x^2]*((-20*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a
^2*x^2)^(3/2)*(ArcTan[a*x]^2*(2 - 6*Cos[2*ArcTan[a*x]]) + 2*(-1 + Cos[2*Ar
cTan[a*x]]) + ((20*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)
^(3/2) + ArcTan[a*x]*(-2*Sin[2*ArcTan[a*x]] + (5*(Log[1 - E^(I*ArcTan[a*x]
)] - Log[1 + E^(I*ArcTan[a*x])])*(-3*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[
a*x]]))/Sqrt[1 + a^2*x^2])))/(a^3*x^3)))/(12*c*Sqrt[1 + a^2*x^2])
```

3.338.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5497, 5479, 5493, 5489, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \text{5497}$$
$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$
$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \; 5493$$

$$-\frac{2}{3}a^2\left(\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \; 5489$$

$$\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \; 5497$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-$$
$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \; 242$$

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-$$
$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \; 5493$$

3.338. $\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$

$$\frac{2}{3}a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-$$

$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{ 5489}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{2}{3}a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\text{arct}}{}\right.$$

$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*Arc
Tanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c +
a^2*c*x^2]))/3 + (2*a*(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*Arc
Tanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c
 + a^2*c*x^2])))/3
```

3.338. $\int\frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.338.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.338. $\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.338.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.66

method	result
default	$\frac{\left(2x^2 \arctan(ax)^2 a^2 - a^2x^2 - x \arctan(ax) a - \arctan(ax)^2\right)\sqrt{c(ax-i)(ax+i)}}{3c\,x^3} - \frac{5ia^3\left(i \arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right) - i\arctan(ax)\,\mathrm{l}\right.}{}$

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*x^2*arctan(a*x)^2*a^2-a^2*x^2-x*arctan(a*x)*a-arctan(a*x)^2)*(c*(a*
x-I)*(I+a*x))^(1/2)/c/x^3-5/3*I*a^3*(I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1
)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I
)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

3.338.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)
```

3.338.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2\left(ax\right)}{x^4\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

3.338.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^2}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

3.338.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^2}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.339 $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

3.339.1 Optimal result

Integrand size = 24, antiderivative size = 305

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2}{a^4c\sqrt{c+a^2cx^2}} - \frac{2x\arctan(ax)}{a^3c\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)^2}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{a^4c^2} + \frac{4i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{c+a^2cx^2}}$$

output
```
-2/a^4/c/(a^2*c*x^2+c)^(1/2)-2*x*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)+arc
tan(a*x)^2/a^4/c/(a^2*c*x^2+c)^(1/2)+4*I*arctan(a*x)*arctan((1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(1/2)-2*I*polylog
(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+
c)^(1/2)+2*I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2
)/a^4/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^4/c^2
```

3.339.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.69

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\Big(-2+3\arctan(ax)^2-2\cos(2\arctan(ax))+\arctan(ax)^2\cos(2\arctan(ax)$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(-2 + 3*ArcTan[a*x]^2 - 2*Cos[2*ArcTan[a*x]] + ArcT
an[a*x]^2*Cos[2*ArcTan[a*x]] - (4*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])]
)/Sqrt[1 + a^2*x^2] + (4*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1
+ a^2*x^2] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
+ ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 2*ArcTan[a*x
]*Sin[2*ArcTan[a*x]]))/(2*a^4*c^2)
```

3.339.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5499, 5465, 5425, 5421, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \; 5499$$

$$\frac{\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \; 5425$$

$$\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \textcolor{blue}{5421}$$

$$-\frac{\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \textcolor{blue}{5429}$$

$$-\frac{\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a)/a^2) + ((Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*
x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1
 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
- I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2]))/(a^2*c)
```

3.339.3.1 Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

3.339. $\int \frac{x^3\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.339.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.96

method	result
default	$\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^4c^2} - \frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)}{2(a^2x^2+1)a^4c^2} + \text{arcta}$

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/
(a^2*x^2+1)/a^4/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2
-2-2*I*arctan(a*x))/(a^2*x^2+1)/a^4/c^2+arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^
(1/2)/a^4/c^2+2*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*
x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/
(a^2*x^2+1)^(1/2)/a^4/c^2
```

3.339.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

3.339.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.339.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

3.339.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.340 $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} dx$

3.340.1 Optimal result

Integrand size = 24, antiderivative size = 349

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} dx &= \frac{2x}{a^2c\sqrt{c+a^2cx^2}} - \frac{2\arctan(ax)}{a^3c\sqrt{c+a^2cx^2}} \\ &- \frac{x\arctan(ax)^2}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^3c\sqrt{c+a^2cx^2}} \\ &+ \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\ &- \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\ &- \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2*x/a^2/c/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)-x*ar
ctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(1/2)-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/
2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a
*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2
*c*x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-2*polylog(3,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+2*polylog(3,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.65

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx =$$

$$-\frac{\sqrt{1+a^2x^2}\left(-\frac{2ax}{\sqrt{1+a^2x^2}}+\frac{2\arctan(ax)}{\sqrt{1+a^2x^2}}+\frac{ax\arctan(ax)^2}{\sqrt{1+a^2x^2}}-\arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)+\arctan(ax)^2\log\left(1\right.\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*((-2*a*x)/Sqrt[1 + a^2*x^2] + (2*ArcTan[a*x])/Sqrt[1
+ a^2*x^2] + (a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - ArcTan[a*x]^2*Log[1 -
 I*E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (2*I)
*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*PolyL
og[3, I*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

3.340.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5499, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$\downarrow$ 3042

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$\downarrow$ 4669

$$-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+$$
$$\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax\right.}{a^3c\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3011

$$-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+$$
$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(\right.\right.}{a^3c\sqrt{a^2c}}$$

$\downarrow$ 2720

$$-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+$$
$$\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.$$

$\downarrow$ 5433

$$-\frac{-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}}{a^2}+$$
$$\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.$$

$\downarrow$ 208

$$-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+$$
$$\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.$$

$\downarrow$ 7143

3.340. $\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

$$-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.}{a^3c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^
2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))/a^2) + (Sqrt[1 + a^2*x^2]
*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*
ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x
])])))/(a^3*c*Sqrt[c + a^2*c*x^2])
```

3.340.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.340. $\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.340. $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

3.340.4 Maple [F]

$$\int \frac{x^2 \arctan(ax)^2}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

3.340.5 Fricas [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

3.340.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.340.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

3.340.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.341 $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} dx$

3.341.1 Optimal result

Integrand size = 22, antiderivative size = 78

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} dx = \frac{2}{a^2c\sqrt{c+a^2cx^2}} + \frac{2x \arctan(ax)}{ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^2}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
2/a^2/c/(a^2*c*x^2+c)^(1/2)+2*x*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)-arctan
(a*x)^2/a^2/c/(a^2*c*x^2+c)^(1/2)
```

3.341.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} dx = \frac{\sqrt{c+a^2cx^2}(2+2ax \arctan(ax) - \arctan(ax)^2)}{a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(2 + 2*a*x*ArcTan[a*x] - ArcTan[a*x]^2))/(a^2*c^2*(1
+ a^2*x^2))
```

3.341.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5465, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{5429}$$
$$\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x
^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a
```

3.341.3.1 Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.341.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result	siz
default	$-\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)}{2(a^2x^2+1)a^2c^2}$	11

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)
/(a^2*x^2+1)/a^2/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^
2-2-2*I*arctan(a*x))/(a^2*x^2+1)/a^2/c^2
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}\left(2\,ax\arctan{(ax)}-\arctan{(ax)}^2+2\right)}{a^4c^2x^2+a^2c^2}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(2*a*x*arctan(a*x) - arctan(a*x)^2 + 2)/(a^4*c^2*x^2 +
 a^2*c^2)
```

3.341.6 Sympy [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x\operatorname{atan}^2{(ax)}}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \sqrt{c}\left(\frac{2\,x\arctan\left(ax\right)}{\sqrt{a^2x^2+1}ac^2} - \frac{\arctan\left(ax\right)^2}{\sqrt{a^2x^2+1}a^2c^2} + \frac{2}{\sqrt{a^2x^2+1}a^2c^2}\right)$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(c)*(2*x*arctan(a*x)/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^2/(sqrt(a
^2*x^2 + 1)*a^2*c^2) + 2/(sqrt(a^2*x^2 + 1)*a^2*c^2))
```

3.341.8 Giac [F]

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\arctan\left(ax\right)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.342 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

3.342.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{2x}{c\sqrt{c+a^2cx^2}} + \frac{2\arctan(ax)}{ac\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}$$

output
```
-2*x/c/(a^2*c*x^2+c)^(1/2)+2*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(
a*x)^2/c/(a^2*c*x^2+c)^(1/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-2ax+2\arctan(ax)+ax\arctan(ax)^2)}{c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x + 2*ArcTan[a*x] + a*x*ArcTan[a*x]^2))/(c^2*(a
 + a^3*x^2))
```

3.342.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \ 5433$$

$$-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 208$$

$$\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
 + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])
```

3.342.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

3.342.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a\,c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)}{2(a^2x^2+1)a\,c^2}$	114

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a
^2*x^2+1)/a/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I
*arctan(a*x))/(a^2*x^2+1)/a/c^2
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}\left(ax\arctan\left(ax\right)^2-2\,ax+2\,\arctan\left(ax\right)\right)}{a^3c^2x^2+ac^2}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^2 - 2*a*x + 2*arctan(a*x))/(a^3*c^2*x
^2 + a*c^2)
```

3.342.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan{(ax)}^2}{\sqrt{a^2cx^2+c}c} - \frac{2\,(ax-\arctan{(ax)})}{\sqrt{a^2x^2+1}ac^{\frac{3}{2}}}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*c) - 2*(a*x - arctan(a*x))/(sqrt(a^2*
x^2 + 1)*a*c^(3/2))
```

3.342.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(3/2), x)
```

3.343 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx$

3.343.1 Optimal result

Integrand size = 24, antiderivative size = 310

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx &= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\arctan(ax)}{c\sqrt{c+a^2cx^2}}\\
&+\frac{\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&+\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&-\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&-\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/c/(a^2*c*x^2+c)^(1/2)-2*a*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+arctan(a*
x)^2/c/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-2*I*ar
ctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*
c*x^2+c)^(1/2)-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/c/(a^2*c*x^2+c)^(1/2)+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/c/(a^2*c*x^2+c)^(1/2)
```

3.343.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.66

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{2}{\sqrt{1+a^2x^2}} - \frac{2ax\arctan(ax)}{\sqrt{1+a^2x^2}} + \frac{\arctan(ax)^2}{\sqrt{1+a^2x^2}} + \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2/Sqrt[1 + a^2*x^2] - (2*a*x*ArcTan[a*x])/Sqrt[1 + a^
2*x^2] + ArcTan[a*x]^2/Sqrt[1 + a^2*x^2] + ArcTan[a*x]^2*Log[1 - E^(I*ArcT
an[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*P
olyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a
*x])] - 2*PolyLog[3, -E^(I*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]
))/(c*Sqrt[c*(1 + a^2*x^2)])
```

3.343.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.69, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5501, 5465, 5429, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \, 5501$$

$$\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \, 5465$$

$$\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \, 5429$$

$$\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 5493$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 5491$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 4671$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right) + \frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax)\right.}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3011$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right) + \frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(a\right.\right.}{c\sqrt{a^2cx^2}}$$

$$\Big\downarrow \; 2720$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right) + \frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(\right.\right.}{}$$

$$\Big\downarrow \; 7143$$

3.343. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right.\right.\right.}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c +
a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^
2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTa
n[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/(c
*Sqrt[c + a^2*c*x^2])
```

3.343.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.343. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx$

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.343. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx$

3.343.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.99

method	result
default	$\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)}{2(a^2x^2+1)c^2}-\frac{\left(\mathrm{arc}\right.}{}$

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/
(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2
*I*arctan(a*x))/(a^2*x^2+1)/c^2-(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)+1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*pol
ylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(
a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1
+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^
2
```

3.343.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 +
c^2*x), x)
```

3.343.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.343.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.343.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{{\mathrm{atan}\left(a\,x\right)}^2}{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.344 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{3/2}}\,dx$

3.344.1 Optimal result

Integrand size = 24, antiderivative size = 293

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{3/2}}\,dx &= \frac{2a^2x}{c\sqrt{c+a^2cx^2}} - \frac{2a\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\arctan(ax)^2}{c\sqrt{c+a^2cx^2}} \\ &\quad - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{c^2x} - \frac{4a\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} \\ &\quad + \frac{2ia\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} - \frac{2ia\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2*a^2*x/c/(a^2*c*x^2+c)^(1/2)-2*a*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-a^2*x*
arctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)-4*a*arctan(a*x)*arctanh((1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+2*I*a*polylog(2
,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)
-2*I*a*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2
*c*x^2+c)^(1/2)-arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^2/x
```

3.344.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.77

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \frac{a\Big(4ax - 4\arctan(ax) - 2ax\arctan(ax)^2 - \frac{1}{2}ax\arctan(ax)^2\csc^2\left(\frac{1}{2}\arctan(ax)\right) +$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a*(4*a*x - 4*ArcTan[a*x] - 2*a*x*ArcTan[a*x]^2 - (a*x*ArcTan[a*x]^2*Csc[A
rcTan[a*x]/2]^2)/2 + 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a
*x])] - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)
*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*Sqrt[1 + a^2*x^2
]*PolyLog[2, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^2*Sin[ArcTa
n[a*x]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

3.344.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5501, 5433, 208, 5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5433}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \ 5479$$

$$\frac{2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \ 5493$$

$$\frac{\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \ 5489$$

$$-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)+\frac{-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))) + (-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTa
nh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^
2*c*x^2])/c
```

3.344.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.344.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)a}{2(a^2x^2+1)c^2}-$ ar

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)
/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*
I*arctan(a*x))*a/(a^2*x^2+1)/c^2-arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c
^2/x+2*I*a*(I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*
ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2
+1)^(1/2)/c^2
```

3.344.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 +
c^2*x^2), x)
```

3.344.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^2\left(ax\right)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.344.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.344.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.345 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$

3.345.1 Optimal result

Integrand size = 24, antiderivative size = 422

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\arctan(ax)}{c\sqrt{c+a^2cx^2}} \\ &- \frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{c^2x}-\frac{a^2\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{2c^2x^2} \\ &+ \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}} \\ &- \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+ \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+ \frac{3a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}-\frac{3a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a^2*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+2*a^2/c/(a^2*c*x^2+c)^(1
/2)+2*a^3*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-a^2*arctan(a*x)^2/c/(a^2*c*x
^2+c)^(1/2)+3*a^2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-3*I*a^2*arctan(a*x)*polylog(2,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+3*I*a^2*arcta
n(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x
^2+c)^(1/2)+3*a^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2
)/c/(a^2*c*x^2+c)^(1/2)-3*a^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-a*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x
-1/2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^2/x^2
```

3.345.2 Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{x^3 (c+a^2cx^2)^{3/2}}\,dx = \frac{a^2\left(16+16ax\arctan(ax)-8\arctan(ax)^2-2ax\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)-\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a^2*(16 + 16*a*x*ArcTan[a*x] - 8*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]*Csc[Ar
cTan[a*x]/2]^2 - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - 12
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 12*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Sqrt[1 + a^2*x^2]*Lo
g[Tan[ArcTan[a*x]/2]] - (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E
^(I*ArcTan[a*x])] + (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*A
rcTan[a*x])] + 24*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] - 24*Sq
rt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^2*Sec[ArcTan[a*x]/2]^2 - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Tan[ArcTan[a*
x]/2]))/(8*c*Sqrt[c + a^2*c*x^2])
```

3.345.3 Rubi [A] (verified)

Time = 4.26 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.05, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5501, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 5501, 5465, 5429, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \ 5501$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \ 5497$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx + a\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c} - a^2\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \ 5479$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx + a\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right) - \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c} - a^2\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \ 243$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx + a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2 - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right) - \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c} - a^2\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \ 73$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx + a\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac} - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right) - \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c} - a^2\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \ 221$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{5493}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{5491}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}$$

$$a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{4671}$$

$$-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx+$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax)-2\arctan(ax)^2\text{arctanh}\left(e^{i \text{arc}}\right.\right.}{2\sqrt{a^2cx^2+c}}}{c}$$

$$\Big\downarrow \textbf{3011}$$

3.345. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$

$$-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)-i\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 2720}$$

$$-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5501}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx\right)+$$
$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5465}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5429}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5493}$$

3.345. $\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx$

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a\right.}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{5491}$$

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a\right.}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{3042}$$

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right.\right.$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a\right.}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{4671}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a\right.}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\,a\right.}{}\right.$$

$$\Big\downarrow \textbf{3011}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i a\right.}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

3.345. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$

$$\Big\downarrow \; 2720$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\,a}\right.\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{Po}\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$c$$

$$a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\right.}{}\right.$$

input
```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x
^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] +
2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[
a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I
*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2]))/c - a^2*(-(a^2*(-(ArcTan[a*x]^2
/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTa
n[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]
^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a
*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I
*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/(c*Sqrt[c + a^2*c*x^2]))
```

3.345. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$

3.345.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.345. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.345. $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.345.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^2\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)a^2}{2(a^2x^2+1)c^2}$

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(
1/2)/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^
2-2-2*I*arctan(a*x))*a^2/(a^2*x^2+1)/c^2-1/2*(2*a*x+arctan(a*x))*arctan(a*
x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x^2+1/2*a^2*(3*arctan(a*x)^2*ln((1+I*a*x)
/(a^2*x^2+1)^(1/2)+1)-3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

3.345.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 +
c^2*x^3), x)
```

3.345.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^2\left(ax\right)}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.345.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

3.345.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2 c x^2)^{3/2}} \, dx = \int \frac{\arctan(ax)^2}{(a^2 c x^2 + c)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2 c x^2)^{3/2}} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^3 \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.346 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx$

3.346.1 Optimal result

Integrand size = 24, antiderivative size = 397

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx &= -\frac{2a^4x}{c\sqrt{c+a^2cx^2}}-\frac{a^2\sqrt{c+a^2cx^2}}{3c^2x}+\frac{2a^3\arctan(ax)}{c\sqrt{c+a^2cx^2}}\\ &-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{3c^2x^2}+\frac{a^4x\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{3c^2x^3}\\ &+\frac{5a^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3c^2x}+\frac{22a^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{c+a^2cx^2}}\\ &-\frac{11ia^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{c+a^2cx^2}}+\frac{11ia^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2*a^4*x/c/(a^2*c*x^2+c)^(1/2)+2*a^3*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+a^4
*x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)+22/3*a^3*arctan(a*x)*arctanh((1+I*a
*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-11/3*I*
a^3*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c
*x^2+c)^(1/2)+11/3*I*a^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x
^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/3*a^2*(a^2*c*x^2+c)^(1/2)/c^2/x-1/3*a*
arctan(a*x)*(a^2*c*x^2+c)^(1/2)/c^2/x^2-1/3*arctan(a*x)^2*(a^2*c*x^2+c)^(1
/2)/c^2/x^3+5/3*a^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^2/x
```

3.346.2 Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.68

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx = \frac{a^3\sqrt{1+a^2x^2}\left(-88i\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)+\frac{(1+a^2x^2)^{3/2}\left(-22+28\cos(2\arctan(ax))-6\cos\right.}{}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*Sqrt[1 + a^2*x^2]*((-88*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a^2
*x^2)^(3/2)*(-22 + 28*Cos[2*ArcTan[a*x]] - 6*Cos[4*ArcTan[a*x]] + ArcTan[a
*x]^2*(25 - 36*Cos[2*ArcTan[a*x]] + 3*Cos[4*ArcTan[a*x]]) + ((88*I)*a^3*x^
3*PolyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ArcTan[a*x]*((66*a*
x*(-Log[1 - E^(I*ArcTan[a*x])] + Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2
*x^2] + 8*Sin[2*ArcTan[a*x]] + 22*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^
(I*ArcTan[a*x])])*Sin[3*ArcTan[a*x]] - 6*Sin[4*ArcTan[a*x]])))/(a^3*x^3)))
/(24*c*Sqrt[c + a^2*c*x^2])
```

3.346.3 Rubi [A] (verified)

Time = 3.86 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5501, 5497, 5479, 5493, 5489, 5497, 242, 5493, 5489, 5501, 5433, 208, 5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{-\frac{2}{3}a^2\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \ 5479$$

$$\frac{-\frac{2}{3}a^2\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \ 5493$$

$$\frac{-\frac{2}{3}a^2\left(\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \ 5489$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+\frac{\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{Poly}}{\sqrt{a^2cx^2+c}}\right.}{c}$$

$$\Big\downarrow \ 5497$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+\frac{\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\,\text{ar}\right.}{}\right.}{c}$$

$$\Big\downarrow \ 242$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+\frac{\frac{2}{3}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{a}\right.}{}\right.}{c}$$

$$\Big\downarrow \ 5493$$

3.346. $\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx$

$$-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+$$

$$\frac{\frac{2}{3}a\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan\right.}{}\right.}{c}$$

$$\Big\downarrow \text{5489}$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)+}{}$$

$$\Big\downarrow \text{5501}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx\right)+$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)+}{}$$

$$\Big\downarrow \text{5433}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)\right)+$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)+}{}$$

$$\Big\downarrow \text{208}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)\right)+$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)+}{}$$

$$\Big\downarrow \text{5479}$$

3.346. $\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx$

$$-a^2\left(\frac{2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)+$$

$$\Big\downarrow \text{5493}$$

$$-a^2\left(\frac{\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)+$$

$$\Big\downarrow \text{5489}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)+$$
$$a^2\left(-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)+\frac{-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctan}\right.}{}}{c}\right.$$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

3.346. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx$

output
```
-(a^2*(-(a^2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c
 + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))) + (-((Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x
]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x
]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt
[c + a^2*c*x^2])/c)) + (-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^3) -
 (2*a^2*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*
x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[
2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]))/3 + (2*a*(-1/2*(a*Sqrt[c + a^2*c*x^2]
)/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^2*Sqrt[1 + a^2*
x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[
2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2])))/3)/c
```

3.346.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.346. $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx$

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.346.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.80

method	result
default	$\frac{a^{3}\left(\arctan\left(ax\right)^{2}-2+2i\arctan\left(ax\right)\right)\left(ax-i\right)\sqrt{c\left(ax-i\right)\left(ax+i\right)}}{2\left(a^{2}x^{2}+1\right)c^{2}}+\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\left(ax+i\right)\left(\arctan\left(ax\right)^{2}-2-2i\arctan\left(ax\right)\right)a^{3}}{2\left(a^{2}x^{2}+1\right)c^{2}}+\frac{(5}{}$

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2
)/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2
*I*arctan(a*x))*a^3/(a^2*x^2+1)/c^2+1/3*(5*x^2*arctan(a*x)^2*a^2-a^2*x^2-x
*arctan(a*x)*a-arctan(a*x)^2)*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x^3-11/3*I*a^3
*(I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*ln(1-(1+I*
a*x)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,
(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/
c^2
```

3.346.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{(a^2cx^2 + c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 +
c^2*x^4), x)
```

3.346.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^2\left(ax\right)}{x^4 \left(c\left(a^2x^2 + 1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.346.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2 + c)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^4), x)
```

3.346.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2 + c)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^4\,{(c\,a^2\,x^2 + c)}^{3/2}} \, dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(3/2)), x)
```

3.347 $\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

3.347.1 Optimal result

Integrand size = 24, antiderivative size = 400

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{2}{27a^6c(c+a^2cx^2)^{3/2}} - \frac{32}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{2x^3\arctan(ax)}{9a^3c(c+a^2cx^2)^{3/2}} - \frac{10x\arctan(ax)}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2\arctan(ax)^2}{3a^4c(c+a^2cx^2)^{3/2}} + \frac{5\arctan(ax)^2}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{a^6c^3} + \frac{4i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{c+a^2cx^2}}$$

output
```
2/27/a^6/c/(a^2*c*x^2+c)^(3/2)-2/9*x^3*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(3/
2)+1/3*x^2*arctan(a*x)^2/a^4/c/(a^2*c*x^2+c)^(3/2)-32/9/a^6/c^2/(a^2*c*x^2
+c)^(1/2)-10/3*x*arctan(a*x)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+5/3*arctan(a*x)^2
/a^6/c^2/(a^2*c*x^2+c)^(1/2)+4*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a
*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)-2*I*polylog(2,-I*
(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(
1/2)+2*I*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^
6/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^6/c^3
```

3.347.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.57

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{8(-95+\cos(2\arctan(ax)))-9(1+a^2x^2)\arctan(ax)^2(-45-20\cos(2\arctan(ax))}{}$$

input
```
Integrate[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(8*(-95 + Cos[2*ArcTan[a*x]]) - 9*(1 + a^2*x^2)*ArcTan[a*x]^2*(-45 - 20*Co
s[2*ArcTan[a*x]] + Cos[4*ArcTan[a*x]]) - (432*I)*Sqrt[1 + a^2*x^2]*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] + (432*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, I*E^(I*
ArcTan[a*x])] + 6*ArcTan[a*x]*(-124*a*x - 72*Sqrt[1 + a^2*x^2]*Log[1 - I*E
^(I*ArcTan[a*x])] + 72*Sqrt[1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] + (1
 + a^2*x^2)*Sin[4*ArcTan[a*x]]))/(216*a^6*c^2*Sqrt[c + a^2*c*x^2])
```

3.347.3 Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5499, 5475, 243, 53, 2009, 5465, 5429, 5499, 5465, 5425, 5421, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^2}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5475}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}}-\frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

3.347. $\displaystyle\int \frac{x^5\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 5425$$

$$\frac{\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 5421$$

$$-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\frac{-\frac{\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2\ldots\right.}{}\right.}{a\sqrt{a^2cx^2+c}}}{a^2c}}{a^2c}$$

$$\Big\downarrow \; 5429$$

$$-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\frac{-\frac{\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\ldots\right.}{a\sqrt{a^2cx^2+c}}}{a^2c}}{a^2c}$$

input
```
Int[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

3.347. $\int \frac{x^5\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

output
```
-(((-2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/
9 + (2*x^3*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2
)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^
2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c +
 a^2*c*x^2])))/a))/(3*a^2*c))/a^2) + (-((-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a
^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c
+ a^2*c*x^2])))/a)/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (
2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*
PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2
]))/(a^2*c))/(a^2*c)
```

3.347.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.347. $\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx$

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.347.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.14

method	result
default	$\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216\left(a^2x^2+1\right)^2a^6c^3}+\frac{7\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-}}{8c^3a^6\left(a^2x^2+1\right)}$

input
```
int(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*
(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^6/c^3+7/8*(arctan(a*x)^2-2+2*I*a
rctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^6/(a^2*x^2+1)-7/8*(c
*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^
6/(a^2*x^2+1)-1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x
+1)*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/c^3/a^6/(a^4*x^4+2*a^2*x^2+1)+arc
tan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/a^6/c^3+2*(arctan(a*x)*ln(1+I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*di
log(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^3/a^6
```

3.347.5 Fricas [F]

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

3.347.6 Sympy [F]

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**5*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.347.7 Maxima [F]

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

3.347.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^5*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.348 $\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx$

3.348.1 Optimal result

Integrand size = 24, antiderivative size = 444

$$
\begin{aligned}
\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx &= \frac{2x^3}{27a^2c(c+a^2cx^2)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} \\
&- \frac{2x^2\arctan(ax)}{9a^3c(c+a^2cx^2)^{3/2}} - \frac{22\arctan(ax)}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\arctan(ax)^2}{3a^2c(c+a^2cx^2)^{3/2}} \\
&- \frac{x\arctan(ax)^2}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^5c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\
&- \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\
&- \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
2/27*x^3/a^2/c/(a^2*c*x^2+c)^(3/2)-2/9*x^2*arctan(a*x)/a^3/c/(a^2*c*x^2+c)
^(3/2)-1/3*x^3*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)+22/9*x/a^4/c^2/(a^2
*c*x^2+c)^(1/2)-22/9*arctan(a*x)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)
^2/a^4/c^2/(a^2*c*x^2+c)^(1/2)-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arc
tan(a*x)^2*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*p
olylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*
x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-2*polylog(3,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+2*polylog(3,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)
```

3.348.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.54

$$\int \frac{x^4 \arctan(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{c\left(1+a^2x^2\right)}\left(-\frac{270\arctan(ax)}{\sqrt{1+a^2x^2}}-\frac{135ax\left(-2+\arctan(ax)^2\right)}{\sqrt{1+a^2x^2}}+6\arctan(ax)\cos(3\arctan(ax))\right.}{}$$

input
```
Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*((-270*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (135*a*x*(-
2 + ArcTan[a*x]^2))/Sqrt[1 + a^2*x^2] + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] +
 108*ArcTan[a*x]^2*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a
*x])]) + (216*I)*ArcTan[a*x]*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog
[2, I*E^(I*ArcTan[a*x])]) - 216*(PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - Poly
Log[3, I*E^(I*ArcTan[a*x])]) + (-2 + 9*ArcTan[a*x]^2)*Sin[3*ArcTan[a*x]]))
/(108*a^5*c^3*Sqrt[1 + a^2*x^2])
```

3.348.3 Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.88, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5499, 5479, 5473, 5465, 208, 5499, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^2}{(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \; 5499$$
$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$
$$\Big\downarrow \; 5479$$
$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} - \frac{2}{3}a\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$
$$\Big\downarrow \; 5473$$
$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} - \frac{2}{3}a\left(\frac{2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$
$$\Big\downarrow \; 5465$$
$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} - \frac{2}{3}a\left(\frac{2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$
$$\Big\downarrow \; 208$$
$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} - \frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$
$$\Big\downarrow \; 5499$$
$$\frac{\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c} - \frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} - \frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

3.348. $\int \frac{x^4\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \text{5425}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arcta\right.}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{PolyL}\right.\right.}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

3.348. $\displaystyle\int \frac{x^4\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \; 2720$$

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+$$
$$\frac{-\frac{\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\operatorname{arct}\right.}{a^3c}}{a^2c}$$

$$\Big\downarrow \; 5433$$

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+$$
$$-\frac{-2\int\frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}}{a^2}+\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{ar}}\right.\right.\right.$$

$$\Big\downarrow \; 208$$

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+$$
$$-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.$$

$$\Big\downarrow \; 7143$$

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+$$
$$\frac{-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\right.\right.}{a^3c\sqrt{a^2cx}}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

3.348. $\int\frac{x^4\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

output
```
-(((x^3*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (2*a*(x^3/(9*a*c*(c +
 a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (
2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))
/(3*a^2*c)))/3)/a^2) + (-(((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x]
)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))/a
^2) + (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 +
 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(
I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - Poly
Log[3, I*E^(I*ArcTan[a*x])])))/(a^3*c*Sqrt[c + a^2*c*x^2]))/(a^2*c)
```

3.348.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.348. $\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.348. $\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.348.4 Maple [F]

$$\int \frac{x^4 \arctan(ax)^2}{(a^2 c\, x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

3.348.5 Fricas [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} dx = \int \frac{x^4 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}} dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

3.348.6 Sympy [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.348.7 Maxima [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

3.348.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.349 $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

3.349.1 Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{2}{27a^4c(c+a^2cx^2)^{3/2}} + \frac{14}{9a^4c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\arctan(ax)}{9ac(c+a^2cx^2)^{3/2}} + \frac{4x\arctan(ax)}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)^2}{3a^2c(c+a^2cx^2)^{3/2}} - \frac{2\arctan(ax)^2}{3a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/27/a^4/c/(a^2*c*x^2+c)^(3/2)+2/9*x^3*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2
)-1/3*x^2*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)+14/9/a^4/c^2/(a^2*c*x^2+
c)^(1/2)+4/3*x*arctan(a*x)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-2/3*arctan(a*x)^2/a
^4/c^2/(a^2*c*x^2+c)^(1/2)
```

3.349.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(40+42a^2x^2+6ax(6+7a^2x^2)\arctan(ax)-9(2+3a^2x^2)\arctan(ax)^2)}{27a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(40 + 42*a^2*x^2 + 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]
- 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2))/(27*a^4*c^3*(1 + a^2*x^2)^2)
```

3.349.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5475, 243, 53, 2009, 5465, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \left(\frac{1}{a^2c\,(a^2cx^2+c)^{3/2}}-\frac{1}{a^2\,(a^2cx^2+c)^{5/2}}\right)dx^2-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$-\frac{x^2\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/9 +
 (2*x^3*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2)/(
3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c
*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^
2*c*x^2])))/a))/(3*a^2*c)
```

3.349.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.349. $\int \frac{x^3\arctan(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

3.349.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^4c^3}-\frac{3\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(}}{8c^3a^4(a^2x^2+1)}$

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)
*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(arctan(a*x)^2-2+2*I*
arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(
c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a
^4/(a^2*x^2+1)+1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*
x+1)*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.53

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\left(42\,a^2x^2 - 9\,(3\,a^2x^2+2)\arctan\left(ax\right)^2 + 6\,(7\,a^3x^3+6\,ax)\arctan\left(ax\right) + 4}{27\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*sqrt(a^2*c*x^2 + c)*(42*a^2*x^2 - 9*(3*a^2*x^2 + 2)*arctan(a*x)^2 + 6
*(7*a^3*x^3 + 6*a*x)*arctan(a*x) + 40)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*
c^3)
```

3.349.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\mathrm{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.349.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

3.349.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.350 $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

3.350.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{2x^3}{27c\,(c+a^2cx^2)^{3/2}} - \frac{4x}{9a^2c^2\sqrt{c+a^2cx^2}} + \frac{2x^2\arctan(ax)}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{4\arctan(ax)}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)^2}{3c\,(c+a^2cx^2)^{3/2}}$$

output
```
-2/27*x^3/c/(a^2*c*x^2+c)^(3/2)+2/9*x^2*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2
)+1/3*x^3*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-4/9*x/a^2/c^2/(a^2*c*x^2+c)^
(1/2)+4/9*arctan(a*x)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

3.350.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(-2ax(6+7a^2x^2)+6(2+3a^2x^2)\arctan(ax)+9a^3x^3\arctan(ax)^2\right)}{27a^3c^3\,(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(6 + 7*a^2*x^2) + 6*(2 + 3*a^2*x^2)*ArcTan[a*
x] + 9*a^3*x^3*ArcTan[a*x]^2))/(27*a^3*c^3*(1 + a^2*x^2)^2)
```

3.350.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5479, 5473, 5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\int \frac{x^3 \arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5473}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\left(\frac{2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\left(\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (2*a*(x^3/(9*a*c*(c + a^
2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(
x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))/(3
*a^2*c)))/3
```

3.350.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.350. $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx$

3.350.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.96

method	result
default	$\frac{\left(6 i \arctan (a x)+9 \arctan (a x)^{2}-2\right)\left(a^{3} x^{3}-3 i a^{2} x^{2}-3 a x+i\right) \sqrt{c(a x-i)(a x+i)}}{216\left(a^{2} x^{2}+1\right)^{2} a^{3} c^{3}}+\frac{\left(\arctan (a x)^{2}-2+2 i \arctan (a x)\right)(a x-i) \sqrt{c(a x-i)(}}{8 c^{3} a^{3}\left(a^{2} x^{2}+1\right)}$

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c
*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^3/c^3+1/8*(arctan(a*x)^2-2+2*I*arc
tan(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^3/(a^2*x^2+1)+1/8*(c*(a*
x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^3/(a^2
*x^2+1)+1/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(I+a*x))^(1/
2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a^3/c^3
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\left(9\,a^3x^3\arctan\left(ax\right)^2 - 14\,a^3x^3 - 12\,ax + 6\left(3\,a^2x^2+2\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}}{27\left(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3\right)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*arctan(a*x)^2 - 14*a^3*x^3 - 12*a*x + 6*(3*a^2*x^2 + 2)*ar
ctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

3.350.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{x}{\sqrt{a^2cx^2+c}\,a^2c^2} - \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right)\arctan(ax)^2 - \frac{2\,(7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan(ax))a}{27\,(a^6c^2x^2+a^4c^2)\sqrt{a^2x^2+1}\sqrt{c}}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*ar
ctan(a*x)^2 - 2/27*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*a/(
(a^6*c^2*x^2 + a^4*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

3.350.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)^2}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.351 $\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

3.351.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{2}{27a^2c(c+a^2cx^2)^{3/2}} + \frac{4}{9a^2c^2\sqrt{c+a^2cx^2}} + \frac{2x\arctan(ax)}{9ac(c+a^2cx^2)^{3/2}} + \frac{4x\arctan(ax)}{9ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^2}{3a^2c(c+a^2cx^2)^{3/2}}$$

output
```
2/27/a^2/c/(a^2*c*x^2+c)^(3/2)+2/9*x*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)-1
/3*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)+4/9/a^2/c^2/(a^2*c*x^2+c)^(1/2)
+4/9*x*arctan(a*x)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.351.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(2(7+6a^2x^2)+6ax(3+2a^2x^2)\arctan(ax)-9\arctan(ax)^2)}{27c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(2*(7 + 6*a^2*x^2) + 6*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x
] - 9*ArcTan[a*x]^2))/(27*c^3*(a + a^3*x^2)^2)
```

3.351.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5465, 5431, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{\left(a^2cx^2+c\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{2 \int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5431}$$

$$\frac{2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5429}$$

$$\frac{2\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^2/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(9*a*c*(c + a^2*c
*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(a*c*Sq
rt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c)))/(3*
a)
```

3.351.3.1 Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.351.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.01

method	result
default	$\frac{\left(6 i \arctan \left(a x \right)+9 \arctan \left(a x \right)^{2}-2\right) \left(i a^{3} x^{3}+3 a^{2} x^{2}-3 i a x -1\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{216 \left(a^{2} x^{2}+1\right)^{2} a^{2} c^{3}}-\frac{\left(\arctan \left(a x \right)^{2}-2+2 i \arctan \left(a x \right)\right) \left(i a x +1\right) \sqrt{c (ax-}}{8 c^{3} a^{2} \left(a^{2} x^{2}+1\right)}$

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*
(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arctan(a*x)^2-2+2*I*a
rctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c
*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^
2/(a^2*x^2+1)-1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x
+1)*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/c^3/a^2/(a^4*x^4+2*a^2*x^2+1)
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.60

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\left(12\,a^2x^2+6\,(2\,a^3x^3+3\,ax)\arctan{(ax)}-9\,\arctan{(ax)}^2+14\right)}{27\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*sqrt(a^2*c*x^2 + c)*(12*a^2*x^2 + 6*(2*a^3*x^3 + 3*a*x)*arctan(a*x) -
 9*arctan(a*x)^2 + 14)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

3.351.6 Sympy [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}^2\,(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.351.7 Maxima [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

3.351.8 Giac [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^2}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

3.352 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

3.352.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{2x}{27c(c+a^2cx^2)^{3/2}} - \frac{40x}{27c^2\sqrt{c+a^2cx^2}} + \frac{2\arctan(ax)}{9ac(c+a^2cx^2)^{3/2}} + \frac{4\arctan(ax)}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)^2}{3c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/27*x/c/(a^2*c*x^2+c)^(3/2)+2/9*arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)+1/3*
x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-40/27*x/c^2/(a^2*c*x^2+c)^(1/2)+4/3*
arctan(a*x)/a/c^2/(a^2*c*x^2+c)^(1/2)+2/3*x*arctan(a*x)^2/c^2/(a^2*c*x^2+c
)^(1/2)
```

3.352.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-2ax(21+20a^2x^2)+6(7+6a^2x^2)\arctan(ax)+9ax(3+2a^2x^2)\arctan}{27ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[
a*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2))/(27*a*c^3*(1 + a^2*x^2)^2)
```

3.352.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5435, 209, 208, 5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5435}$$

$$\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c}-\frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c}-\frac{2}{9}\left(\frac{2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-2*(x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2])))/9
 + (2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2)/(3*c*
(c + a^2*c*x^2)^(3/2)) + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*
x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))
)/(3*c)
```

3.352.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

3.352. $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$

3.352.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\left(6i\arctan(ax)+9\arctan(ax)^{2}-2\right)\left(a^{3}x^{3}-3ia^{2}x^{2}-3ax+i\right)\sqrt{c(ax-i)(ax+i)}}{216(a^{2}x^{2}+1)^{2}c^{3}a}+\frac{3\left(\arctan(ax)^{2}-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(ax-}}{8a\,c^{3}(a^{2}x^{2}+1)}$

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(
c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3/a+3/8*(arctan(a*x)^2-2+2*I*arct
an(a*x))*(a*x-I)*(c*(a*x-I)*(I+a*x))^(1/2)/a/c^3/(a^2*x^2+1)+3/8*(c*(a*x-I
)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^2-2-2*I*arctan(a*x))/a/c^3/(a^2*x^2+
1)-1/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(I+a*x))^(1/2)*(a
^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\left(40\,a^3x^3-9\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)^2+42\,ax-6\left(6\,a^2x^2+7\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}}{27\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 42*a*x - 6*(6*a^
2*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 +
 a*c^3)
```

3.352.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2x}{\sqrt{a^2cx^2+c}c^2} + \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}c}\right)\arctan(ax)^2 - \frac{2(20a^3x^3+21ax-3(6a^2x^2+7)\arctan(ax))a}{27(a^4c^2x^2+a^2c^2)\sqrt{a^2x^2+1}\sqrt{c}}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan(a
*x)^2 - 2/27*(20*a^3*x^3 + 21*a*x - 3*(6*a^2*x^2 + 7)*arctan(a*x))*a/((a^4
*c^2*x^2 + a^2*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

3.352.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(5/2), x)
```

3.353 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

3.353.1 Optimal result

Integrand size = 24, antiderivative size = 389

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx = &-\frac{2}{27c(c+a^2cx^2)^{3/2}}-\frac{22}{9c^2\sqrt{c+a^2cx^2}}\\&-\frac{2ax\arctan(ax)}{9c(c+a^2cx^2)^{3/2}}-\frac{22ax\arctan(ax)}{9c^2\sqrt{c+a^2cx^2}}+\frac{\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}}\\&+\frac{\arctan(ax)^2}{c^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\&+\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\&-\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\&-\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/27/c/(a^2*c*x^2+c)^(3/2)-2/9*a*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1/3*
arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-22/9/c^2/(a^2*c*x^2+c)^(1/2)-22/9*a*x*
arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(1/2)-
2*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2
/(a^2*c*x^2+c)^(1/2)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2*I*arctan(a*x)*polylog(2,(1+
I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2*poly
log(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1
/2)+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*
x^2+c)^(1/2)
```

3.353.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\left(1+a^2x^2\right)^{3/2}\left(-\frac{270}{\sqrt{1+a^2x^2}}-\frac{270ax\arctan(ax)}{\sqrt{1+a^2x^2}}+\frac{135\arctan(ax)^2}{\sqrt{1+a^2x^2}}-2\cos(3\arctan(ax))+9\,a$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(-270/Sqrt[1 + a^2*x^2] - (270*a*x*ArcTan[a*x])/Sqrt[
1 + a^2*x^2] + (135*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 2*Cos[3*ArcTan[a*x]
] + 9*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]] + 108*ArcTan[a*x]^2*Log[1 - E^(I*Ar
cTan[a*x])] - 108*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (216*I)*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (216*I)*ArcTan[a*x]*PolyLog[2, E^(
I*ArcTan[a*x])] - 216*PolyLog[3, -E^(I*ArcTan[a*x])] + 216*PolyLog[3, E^(I
*ArcTan[a*x])] - 6*ArcTan[a*x]*Sin[3*ArcTan[a*x]]))/(108*c*(c*(1 + a^2*x^2
))^(3/2))
```

3.353.3 Rubi [A] (verified)

Time = 2.61 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5501, 5465, 5431, 5429, 5501, 5465, 5429, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \; 5465$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{3a} - \frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5431$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c} + \frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}} + \frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a} - \frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5429$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a} - \frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5501$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a} - \frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5465$$

3.353. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5429$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5493$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5491$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

3.353. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{4671}$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)+\frac{-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log(\right.}{c\sqrt{a^2cx^2+c}}}{c}$$

$$\Big\downarrow \text{3011}$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)+\frac{-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax}\right.\right.\right.}{}}{c}$$

$$\Big\downarrow \text{2720}$$

3.353. $\displaystyle \int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)+\frac{-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,\ldots\right.\right.}{}}{}$$

$$\Big\downarrow \text{7143}$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)+\frac{-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e\ldots\right.\right.\right.}{}}{c}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*(-1/3*ArcTan[a*x]^2/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(9*a*c*(c
+ a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/
(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c
)))/(3*a))) + (-(a^2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1
/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a))
 + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*
ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])
]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcT
an[a*x])])))/(c*Sqrt[c + a^2*c*x^2]))/c
```

3.353. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

3.353.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.353. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.353.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2c^3}+\frac{5\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(}}{8c^3(a^2x^2+1)}$

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

3.353. $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$

output
```
-1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)
*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)^2-2+2*I*arct
an(a*x))*(1+I*a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I
)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/(a^2*x^2+
1)+1/216*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-6*I*a
rctan(a*x)+9*arctan(a*x)^2-2)/c^3/(a^4*x^4+2*a^2*x^2+1)-(arctan(a*x)^2*ln(
(1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*
x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1
/2)/(a^2*x^2+1)^(1/2)/c^3
```

3.353.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 +
3*a^2*c^3*x^3 + c^3*x), x)
```

3.353.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.353.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.353.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\mathrm{atan}(a\,x)^2}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.354 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx$

3.354.1 Optimal result

Integrand size = 24, antiderivative size = 381

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx &= \frac{2a^2x}{27c(c+a^2cx^2)^{3/2}}+\frac{94a^2x}{27c^2\sqrt{c+a^2cx^2}}\\
&-\frac{2a\arctan(ax)}{9c(c+a^2cx^2)^{3/2}}-\frac{10a\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\arctan(ax)^2}{3c^2\sqrt{c+a^2cx^2}}\\
&-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{c^3x}-\frac{4a\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&+\frac{2ia\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}-\frac{2ia\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2/27*a^2*x/c/(a^2*c*x^2+c)^(3/2)-2/9*a*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-1
/3*a^2*x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)+94/27*a^2*x/c^2/(a^2*c*x^2+c)
^(1/2)-10/3*a*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-5/3*a^2*x*arctan(a*x)^2/
c^2/(a^2*c*x^2+c)^(1/2)-4*a*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+2*I*a*polylog(2,-(1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2*I*a*p
olylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2
+c)^(1/2)-arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c^3/x
```

3.354.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{a\left(-378ax+378\arctan(ax)+189ax\arctan(ax)^2+6\sqrt{1+a^2x^2}\arctan(ax)\cos(3\arctan(ax))+27ax\,\mathrm{arct}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-1/108*(a*(-378*a*x + 378*ArcTan[a*x] + 189*a*x*ArcTan[a*x]^2 + 6*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^2*Csc[ArcTan
[a*x]/2]^2 - 216*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]
+ 216*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - (216*I)*S
qrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] + (216*I)*Sqrt[1 + a^2*x^2
]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]] +
 9*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(c^2*Sqrt[c + a^2*c*x^2])
```

3.354.3 Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5501, 5435, 209, 208, 5433, 208, 5501, 5433, 208, 5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} - \frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 209$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} - \frac{2}{9}\left(\frac{2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c} + \frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right) + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 208$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)\right)$$

$$\Big\downarrow \; 5433$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\Bigg(\frac{2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} - \frac{2}{9}\Bigg(\frac{2x}{3c^2\sqrt{a^2cx}}$$

$$\Big\downarrow \; 208$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\Bigg(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\Bigg(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} +$$

$$\Big\downarrow \; 5501$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\Bigg(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\Bigg(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} +$$

$$\Big\downarrow \; 5433$$

3.354. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\left(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{}{3}\right.\right.$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\left(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{}{3}\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$\frac{\frac{2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\left(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{}{3}\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$\frac{\frac{\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\left(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{}{3}\right.\right.$$

$$\Big\downarrow \text{5489}$$

$$-a^2\left(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\right.\right.$$
$$\frac{-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)+\frac{-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\right.\right.}{\sqrt{a^2cx^2+c}}}{c}}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

3.354. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx$

output
```
-(a^2*((-2*(x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^
2])))/9 + (2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2
)/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*Ar
cTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c
*x^2])))/(3*c))) + (-(a^2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x]
)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))
+ (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(
-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]]))/Sqrt[c + a^2*c*x^2])/c)/c
```

3.354.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

3.354. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx$

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.354.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\left(54\arctan(ax)\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)a^5x^5-54\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^5x^5+108i\operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^3x^3-54i\operatorname{polylog}\left(2,-\right.\right.}{}$

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

3.354. $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx$

output
```
-1/27*(54*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a^5*x^5-54*arctan(
a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^5*x^5+108*I*polylog(2,(1+I*a*x)/(
a^2*x^2+1)^(1/2))*a^3*x^3-54*I*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x
+72*arctan(a*x)^2*(a^2*x^2+1)^(1/2)*a^4*x^4-94*(a^2*x^2+1)^(1/2)*a^4*x^4+9
0*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^3+108*arctan(a*x)*ln((1+I*a*x)/(a^2*
x^2+1)^(1/2)+1)*a^3*x^3-108*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))*
a^3*x^3+54*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^5*x^5-108*I*polylog(
2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^3*x^3+108*arctan(a*x)^2*(a^2*x^2+1)^(1/2
)*a^2*x^2-96*a^2*x^2*(a^2*x^2+1)^(1/2)+96*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*
x+54*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)*a*x-54*arctan(a*x)*ln(1
-(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-54*I*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))*a^5*x^5+54*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x+27*arctan(a*x
)^2*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/x/c^3/(
a^4*x^4+2*a^2*x^2+1)
```

3.354.5 Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 +
3*a^2*c^3*x^4 + c^3*x^2), x)
```

3.354.6 Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\operatorname{atan}^2\left(ax\right)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.354.7 Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2 c x^2)^{5/2}} \, dx = \int \frac{\arctan(ax)^2}{(a^2 c x^2 + c)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

3.354.8 Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2 c x^2)^{5/2}} \, dx = \int \frac{\arctan(ax)^2}{(a^2 c x^2 + c)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2 c x^2)^{5/2}} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,(c\,a^2\,x^2 + c)^{5/2}} \, dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.355 $\int x^m (c+a^2cx^2)^2 \arctan(ax)^2 \, dx$

3.355.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2 \, dx = \operatorname{Int}\left(x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

3.355.2 Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2 \, dx = \int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2 \, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2, x]
```

3.355.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \; 5560$$

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.355.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.355.4 Maple [N/A] (verified)

Not integrable

Time = 1.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

3.355.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^2, x)
```

3.355.6 Sympy [N/A]

Not integrable

Time = 14.90 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = c^2\left(\int x^m \operatorname{atan}^2(ax)\,dx + \int 2a^2x^2x^m \operatorname{atan}^2(ax)\,dx + \int a^4x^4x^m \operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
c**2*(Integral(x**m*atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m*atan(a*x)
**2, x) + Integral(a**4*x**4*x**m*atan(a*x)**2, x))
```

3.355.7 Maxima [N/A]

Not integrable

Time = 9.32 (sec) , antiderivative size = 841, normalized size of antiderivative = 38.23

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/16*(4*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*
a^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*
x)^2 - ((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a
^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*log(a^2*x^
2 + 1)^2 + 16*(m^3 + 9*m^2 + 23*m + 15)*integrate(1/16*(12*((a^6*c^2*m^3 +
 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^3 + 3*(a^4*c^2*m^3
 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^2*m^2 + 23*c^2*m +
 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*c^2)*x^2 + 15*c^2)
*x^m*arctan(a*x)^2 + ((a^6*c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6
*c^2)*x^6 + c^2*m^3 + 3*(a^4*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a
^4*c^2)*x^4 + 9*c^2*m^2 + 23*c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a
^2*c^2*m + 15*a^2*c^2)*x^2 + 15*c^2)*x^m*log(a^2*x^2 + 1)^2 - 8*((a^5*c^2*
m^2 + 4*a^5*c^2*m + 3*a^5*c^2)*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*
c^2)*x^3 + (a*c^2*m^2 + 8*a*c^2*m + 15*a*c^2)*x)*x^m*arctan(a*x) + 4*((a^6
*c^2*m^2 + 4*a^6*c^2*m + 3*a^6*c^2)*x^6 + 2*(a^4*c^2*m^2 + 6*a^4*c^2*m + 5
*a^4*c^2)*x^4 + (a^2*c^2*m^2 + 8*a^2*c^2*m + 15*a^2*c^2)*x^2)*x^m*log(a^2*
x^2 + 1))/(m^3 + (a^2*m^3 + 9*a^2*m^2 + 23*a^2*m + 15*a^2)*x^2 + 9*m^2 + 2
3*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

3.355.8 Giac [N/A]

Not integrable

Time = 110.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x^m (c + a^2cx^2)^2 \arctan(ax)^2 \, dx = \int \left(a^2cx^2 + c\right)^2 x^m \arctan\left(ax\right)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.355.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m (c + a^2 c x^2)^2 \arctan(ax)^2 \, dx = \int x^m \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2 + c\right)^2 dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

3.356 $\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx$

3.356.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx=\mathrm{Int}\left(x^m(c+a^2cx^2)\arctan(ax)^2,x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

3.356.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx=\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2, x]
```

3.356.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right) \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.356.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.356.4 Maple [N/A] (verified)

Not integrable

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2 c\, x^2+c\right) \arctan \left(a x \right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

3.356.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \left(c+a^2 c x^2\right) \arctan(ax)^2 \, dx = \int \left(a^2 c x^2 + c\right) x^m \arctan\left(ax\right)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

3.356.6 Sympy [N/A]

Not integrable

Time = 6.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^m \left(c+a^2 c x^2\right) \arctan(ax)^2 \, dx = c\left(\int x^m \operatorname{atan}^2\left(ax\right) dx + \int a^2 x^2 x^m \operatorname{atan}^2\left(ax\right) dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
c*(Integral(x**m*atan(a*x)**2, x) + Integral(a**2*x**2*x**m*atan(a*x)**2,
x))
```

3.356.7 Maxima [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 19.30

$$\int x^m \left(c+a^2 c x^2\right) \arctan(ax)^2 \, dx = \int \left(a^2 c x^2 + c\right) x^m \arctan\left(ax\right)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/16*(4*((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)^2 - ((a^2*
c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*log(a^2*x^2 + 1)^2 + 16*(m^2 + 4*m +
 3)*integrate(1/16*(12*((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*
(a^2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arctan(a*x)^2 + (
(a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*(a^2*c*m^2 + 4*a^2*c*m +
 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*log(a^2*x^2 + 1)^2 - 8*((a^3*c*m + a^3*c)
*x^3 + (a*c*m + 3*a*c)*x)*x^m*arctan(a*x) + 4*((a^4*c*m + a^4*c)*x^4 + (a^
2*c*m + 3*a^2*c)*x^2)*x^m*log(a^2*x^2 + 1))/((a^2*m^2 + 4*a^2*m + 3*a^2)*x
^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

3.356.8 Giac [N/A]

Not integrable

Time = 107.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx=\int (a^2cx^2+c)x^m\arctan(ax)^2\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.356.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx=\int x^m\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2), x)
```

3.357 $\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2} \, dx$

3.357.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2} \, dx = \text{Int}\left(\frac{x^m \arctan(ax)^2}{c+a^2cx^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

3.357.2 Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2} \, dx = \int \frac{x^m \arctan(ax)^2}{c+a^2cx^2} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2), x]
```

3.357.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5560\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^2}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.357.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.357.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan\left(ax\right)^2}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

3.357.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan\left(ax\right)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.357.6 Sympy [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

3.357.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan\left(ax\right)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

3.357.8 Giac [N/A]

Not integrable

Time = 99.65 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan{(ax)}^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.357.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^m \,\mathrm{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

3.358 $\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx$

3.358.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

3.358.2 Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2, x]
```

3.358.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.358.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.358.4 Maple [N/A] (verified)

Not integrable

Time = 2.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan\left(ax\right)^2}{\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

3.358.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.358.6 Sympy [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.358.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

3.358.8 Giac [N/A]

Not integrable

Time = 116.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.358.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^2, x)
```

3.359 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$

3.359.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2,x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

3.359.2 Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2, x]
```

3.359.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.359.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.359.4 Maple [N/A] (verified)

Not integrable

Time = 1.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

3.359.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2cx^2)^{3/2} \arctan(ax)^2 \, dx = \int (a^2cx^2 + c)^{\frac{3}{2}} x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

3.359.6 Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2cx^2)^{3/2} \arctan(ax)^2 \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Timed out
```

3.359.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2cx^2)^{3/2} \arctan(ax)^2 \, dx = \int (a^2cx^2 + c)^{\frac{3}{2}} x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

3.359.8 Giac [F(-2)]

Exception generated.

$$\int x^m (c+a^2cx^2)^{3/2} \arctan(ax)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.359.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^{3/2} \arctan(ax)^2 \, dx = \int x^m \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^{3/2} \, dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

3.360 $\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx$

3.360.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx = \text{Int}\left(x^m \sqrt{c+a^2cx^2} \arctan(ax)^2, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

3.360.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx = \int x^m \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2, x]
```

3.360.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int x^m \arctan(ax)^2 \sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.360.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.360.4 Maple [N/A] (verified)

Not integrable

Time = 1.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

3.360.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int\sqrt{a^2cx^2+c}x^m\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

3.360.6 Sympy [N/A]

Not integrable

Time = 29.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^m\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

3.360.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int\sqrt{a^2cx^2+c}x^m\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

3.360.8 Giac [F(-2)]

Exception generated.

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.360.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx = \int x^m \operatorname{atan}(a\,x)^2 \sqrt{c\,a^2\,x^2+c} \, dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

3.361 $\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$

3.361.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx = \text{Int}\left(\frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.361.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2], x]
```

3.361.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{a^2cx^2+c}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^2}{\sqrt{a^2cx^2+c}} dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.361.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.361.4 Maple [N/A] (verified)

Not integrable

Time = 0.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(a x\right)^2}{\sqrt{a^2 c\, x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.361.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.361.6 Sympy [N/A]

Not integrable

Time = 18.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}^2{(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

3.361.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

3.361.8 Giac [N/A]

Not integrable

Time = 41.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.361.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\,\mathrm{atan}{(a\,x)}^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

3.362 $\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$

3.362.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

3.362.2 Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2), x]
```

3.362.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.362.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.362.4 Maple [N/A] (verified)

Not integrable

Time = 1.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^2}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

3.362.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

3.362.6 Sympy [N/A]

Not integrable

Time = 20.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}^2{(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.362.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

3.362.8 Giac [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.362.9 Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

3.363 $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

3.363.1 Optimal result

Integrand size = 20, antiderivative size = 219

$$\begin{aligned}\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx &= \frac{cx}{15a^3}-\frac{cx^3}{60a}-\frac{c\arctan(ax)}{15a^4}-\frac{cx^2\arctan(ax)}{60a^2}\\&\quad+\frac{1}{20}cx^4\arctan(ax)+\frac{7ic\arctan(ax)^2}{30a^4}+\frac{cx\arctan(ax)^2}{4a^3}\\&\quad-\frac{cx^3\arctan(ax)^2}{12a}-\frac{1}{10}acx^5\arctan(ax)^2-\frac{c\arctan(ax)^3}{12a^4}\\&\quad+\frac{1}{4}cx^4\arctan(ax)^3+\frac{1}{6}a^2cx^6\arctan(ax)^3\\&\quad+\frac{7c\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a^4}+\frac{7ic\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{30a^4}\end{aligned}$$

output
```
1/15*c*x/a^3-1/60*c*x^3/a-1/15*c*arctan(a*x)/a^4-1/60*c*x^2*arctan(a*x)/a^
2+1/20*c*x^4*arctan(a*x)+7/30*I*c*arctan(a*x)^2/a^4+1/4*c*x*arctan(a*x)^2/
a^3-1/12*c*x^3*arctan(a*x)^2/a-1/10*a*c*x^5*arctan(a*x)^2-1/12*c*arctan(a*
x)^3/a^4+1/4*c*x^4*arctan(a*x)^3+1/6*a^2*c*x^6*arctan(a*x)^3+7/15*c*arctan
(a*x)*ln(2/(1+I*a*x))/a^4+7/30*I*c*polylog(2,1-2/(1+I*a*x))/a^4
```

3.363.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.62

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$=\frac{c(4ax-a^3x^3-(14i-15ax+5a^3x^3+6a^5x^5)\arctan(ax)^2+5(-1+3a^4x^4+2a^6x^6)\arctan(ax)^3+\text{arcta}}{60a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(4*a*x - a^3*x^3 - (14*I - 15*a*x + 5*a^3*x^3 + 6*a^5*x^5)*ArcTan[a*x]^
2 + 5*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^3 + ArcTan[a*x]*(-4 - a^2*x
^2 + 3*a^4*x^4 + 28*Log[1 + E^((2*I)*ArcTan[a*x])]) - (14*I)*PolyLog[2, -E
^((2*I)*ArcTan[a*x])]))/(60*a^4)
```

3.363.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 711 vs. 2(219) = 438.

Time = 4.44 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.25, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {5485, 5361, 5451, 5361, 5451, 5345, 5361, 254, 262, 216, 2009, 5419, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)^3\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5\arctan(ax)^3dx+c\int x^3\arctan(ax)^3dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\int\frac{x^6\arctan(ax)^2}{a^2x^2+1}dx\right)+c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\int\frac{x^4\arctan(ax)^2}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5451}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\int x^2\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\int x^4\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x^4\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\int\frac{x^5\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^4\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{5451}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int\arctan(ax)^2dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}d}{a^2}}{a^2}\right.\right.$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\int x^3\arctan(ax)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)^2dx}{a^2}-\int\frac{x^2 a}{}}{a^2}\right.\right.$$

$$\Big\downarrow \text{5345}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right.\right.$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\int x^3\arctan(ax)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)^2dx}{a^2}-\int\frac{x^2 a}{}}{a^2}\right.\right.$$

$$\Big\downarrow \text{5361}$$

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2-2a\cdots}{a^2}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\frac{x^4}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(ax)\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 254$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2-2a\cdots}{a^2}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}\cdots\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(a\cdots}{\cdots}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}\cdots\right.\right.$$

$$\Big\downarrow \; 216$$

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2}{\cdots}\right.\right.$$

$$\Big\downarrow 2009$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2}{\cdots}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{3}x^3\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow 5419$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2}{\cdots}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{3}x^3\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow 5451$$

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\Bigg(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\Bigg(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\bigg(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\bigg)}{a^2}-\frac{x\arctan(ax)^2}{}$$

$$a^2c\Bigg(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\Bigg(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\bigg(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(a}{a^2x^2+1}}{a^2}}{a^2}}{a^2}$$

$\Big\downarrow$ 5345

$$c\Bigg(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\Bigg(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\bigg(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\bigg)}{a^2}-\frac{x\arctan(ax)^2}{}$$

$$a^2c\Bigg(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\Bigg(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\bigg(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(a}{a^2x^2+1}}{a^2}}{a^2}}{a^2}$$

$\Big\downarrow$ 5361

$$c\Bigg(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\Bigg(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\bigg(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\bigg)}{a^2}-\frac{x\arctan(ax)^2}{}$$

$$a^2c\Bigg(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\Bigg(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\bigg(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}}{a^2}}{a^2}}{a^2}$$

$\Big\downarrow$ 262

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2\cdots}{\cdots}\right.\right.\cdots$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{\cdots}{a^2\cdots}}{\cdots}\right.\cdots}{a^2}\cdots}{a\cdots}\right.\cdots}{a^2}\cdots\right.\right.\cdots$$

$$\Big\downarrow \ 216$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2\cdots}{\cdots}\right.\right.\cdots$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan\cdots}{\cdots}\right.\cdots}{a^2}\cdots}{a^2\cdots}\right.\cdots}{a^2}\cdots\right.\right.\cdots$$

$$\Big\downarrow \ 5419$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2\cdots}{\cdots}\right.\right.\cdots$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan\cdots}{\cdots}\right.\cdots}{a^2}\cdots}{a^2\cdots}\right.\cdots}{a^2}\cdots\right.\right.\cdots$$

$$\Big\downarrow \ 5455$$

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2}-\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\mathrm{arcta}}{}\right.}{a^2}\right.}{a^2}\right.\right.$$

$$\Big\downarrow \; 5379$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}}{a}}{a^2}\right.}{a^2}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\mathrm{arcta}\right.}{a^2}\right.}{a^2}\right.\right.$$

$$\Big\downarrow \; 2849$$

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{x^3}{3a^2}-\frac{x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(}{a^3}\right.}{a^2}\right.}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a}}{a^2}\right.}{a^2}\right.\right.$$

$$\Big\downarrow \; 2752$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a^2}\right.}{a^2}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan}{}\right.}{a^2}\right.}{a^2}\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

output
```
c*((x^4*ArcTan[a*x]^3)/4 - (3*a*(((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2*ArcT
an[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]
^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1
 + I*a*x)])/a)/a)/a^2))/3)/a^2 - (-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*x]^
2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/
a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2))/4) + a^2*c*((x
^6*ArcTan[a*x]^3)/6 - (a*(((x^5*ArcTan[a*x]^2)/5 - (2*a*(((x^4*ArcTan[a*x]
)/4 - (a*(-(x/a^4) + x^3/(3*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*ArcTa
n[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^
2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1
+ I*a*x)])/a)/a)/a^2)/a^2))/5)/a^2 - (((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2
*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan
[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 -
 2/(1 + I*a*x)])/a)/a)/a^2))/3)/a^2 - (-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[
a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*
x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2)/a^2))/2)
```

3.363.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

3.363. $\int x^3(c + a^2cx^2)\arctan(ax)^3\,dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.363. $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.363.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{\frac{c\arctan(ax)^3a^6x^6}{6}+\frac{c\arctan(ax)^3a^4x^4}{4}-\frac{c\left(\frac{2a^5\arctan(ax)^2x^5}{5}+\frac{a^3\arctan(ax)^2x^3}{3}-a\arctan(ax)^2x+\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)a^4x\cdots}{5}\cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{c\arctan(ax)^3a^6x^6}{6}+\frac{c\arctan(ax)^3a^4x^4}{4}-\frac{c\left(\frac{2a^5\arctan(ax)^2x^5}{5}+\frac{a^3\arctan(ax)^2x^3}{3}-a\arctan(ax)^2x+\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)a^4x\cdots}{5}\cdots\right)}{\cdots}}{\cdots}$
parts	$\frac{a^2cx^6\arctan(ax)^3}{6}+\frac{cx^4\arctan(ax)^3}{4}-\frac{c\left(\frac{2a\arctan(ax)^2x^5}{5}+\frac{\arctan(ax)^2x^3}{3a}-\frac{\arctan(ax)^2x}{a^3}+\frac{\arctan(ax)^3}{a^4}-\frac{2\left(\frac{3\arctan\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/6*c*arctan(a*x)^3*a^6*x^6+1/4*c*arctan(a*x)^3*a^4*x^4-1/4*c*(2/5*
a^5*arctan(a*x)^2*x^5+1/3*a^3*arctan(a*x)^2*x^3-a*arctan(a*x)^2*x+1/3*arct
an(a*x)^3-1/5*arctan(a*x)*a^4*x^4+1/15*a^2*arctan(a*x)*x^2+14/15*arctan(a*
x)*ln(a^2*x^2+1)+1/15*a^3*x^3-4/15*a*x+4/15*arctan(a*x)+7/15*I*(ln(a*x-I)*
ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*
x-I)^2)-7/15*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(
1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.363.5 Fricas [F]

$$\int x^3 \left(c+a^2 c x^2\right) \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right) x^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^5 + c*x^3)*arctan(a*x)^3, x)
```

3.363.6 Sympy [F]

$$\int x^3 \left(c+a^2 c x^2\right) \arctan(ax)^3 \, dx = c \left(\int x^3 \operatorname{atan}^3\left(ax\right) dx + \int a^2 x^5 \operatorname{atan}^3\left(ax\right) dx\right)$$

input
```
integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x**3*atan(a*x)**3, x) + Integral(a**2*x**5*atan(a*x)**3, x))
```

3.363.7 Maxima [F]

$$\int x^3 \left(c+a^2 c x^2\right) \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right) x^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/960*(20*(23040*a^7*c*integrate(1/960*x^7*arctan(a*x)^3/(a^5*x^2 + a^3),
x) - 5760*a^6*c*integrate(1/960*x^6*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 14
40*a^6*c*integrate(1/960*x^6*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 1152
*a^6*c*integrate(1/960*x^6*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 46080*a^
5*c*integrate(1/960*x^5*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 2304*a^5*c*int
egrate(1/960*x^5*arctan(a*x)/(a^5*x^2 + a^3), x) - 8640*a^4*c*integrate(1/
960*x^4*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 2160*a^4*c*integrate(1/960*x^4
*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 960*a^4*c*integrate(1/960*x^4*lo
g(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 23040*a^3*c*integrate(1/960*x^3*arcta
n(a*x)^3/(a^5*x^2 + a^3), x) + 1920*a^3*c*integrate(1/960*x^3*arctan(a*x)/
(a^5*x^2 + a^3), x) + 2880*a^2*c*integrate(1/960*x^2*log(a^2*x^2 + 1)/(a^5
*x^2 + a^3), x) - 5760*a*c*integrate(1/960*x*arctan(a*x)/(a^5*x^2 + a^3),
x) + 720*c*integrate(1/960*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + c*arct
an(a*x)^3/a^4)*a^4 + 40*(2*a^6*c*x^6 + 3*a^4*c*x^4 - c)*arctan(a*x)^3 - 4*
(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x)*arctan(a*x)^2 + (6*a^5*c*x^5 + 5*a^
3*c*x^3 - 15*a*c*x)*log(a^2*x^2 + 1)^2)/a^4
```

3.363.8 Giac [F]

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx=\int x^3\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.364 $\int x^2(c + a^2cx^2) \arctan(ax)^3 \, dx$

3.364.1 Optimal result

Integrand size = 20, antiderivative size = 211

$$\begin{aligned}\int x^2(c + a^2cx^2) \arctan(ax)^3 \, dx = & -\frac{cx^2}{20a} + \frac{cx \arctan(ax)}{10a^2} + \frac{1}{10}cx^3 \arctan(ax) \\ & - \frac{c \arctan(ax)^2}{20a^3} - \frac{cx^2 \arctan(ax)^2}{5a} - \frac{3}{20}acx^4 \arctan(ax)^2 \\ & - \frac{2ic \arctan(ax)^3}{15a^3} + \frac{1}{3}cx^3 \arctan(ax)^3 \\ & + \frac{1}{5}a^2cx^5 \arctan(ax)^3 - \frac{2c \arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{5a^3} \\ & - \frac{2ic \arctan(ax) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{5a^3} \\ & - \frac{c \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+iax}\right)}{5a^3}\end{aligned}$$

output
```
-1/20*c*x^2/a+1/10*c*x*arctan(a*x)/a^2+1/10*c*x^3*arctan(a*x)-1/20*c*arcta
n(a*x)^2/a^3-1/5*c*x^2*arctan(a*x)^2/a-3/20*a*c*x^4*arctan(a*x)^2-2/15*I*c
*arctan(a*x)^3/a^3+1/3*c*x^3*arctan(a*x)^3+1/5*a^2*c*x^5*arctan(a*x)^3-2/5
*c*arctan(a*x)^2*ln(2/(1+I*a*x))/a^3-2/5*I*c*arctan(a*x)*polylog(2,1-2/(1+
I*a*x))/a^3-1/5*c*polylog(3,1-2/(1+I*a*x))/a^3
```

3.364.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.81

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$=\frac{c(-3-3a^2x^2+6ax\arctan(ax)+6a^3x^3\arctan(ax)-3\arctan(ax)^2-12a^2x^2\arctan(ax)^2-9a^4x^4\arctan$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(-3 - 3*a^2*x^2 + 6*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 3*ArcTan[
a*x]^2 - 12*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 + (8*I)*ArcTan
[a*x]^3 + 20*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^5*ArcTan[a*x]^3 - 24*ArcTan[
a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(
(2*I)*ArcTan[a*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(60*a^3)
```

3.364.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 503 vs. 2(211) = 422.

Time = 3.73 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.38, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5485, 5361, 5451, 5361, 5451, 5345, 240, 5361, 243, 49, 2009, 5419, 5451, 5345, 240, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)^3\left(a^2cx^2+c\right)\,dx$$
$$\Big\downarrow \text{5485}$$
$$a^2c\int x^4\arctan(ax)^3dx+c\int x^2\arctan(ax)^3dx$$
$$\Big\downarrow \text{5361}$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\int\frac{x^5\arctan(ax)^2}{a^2x^2+1}dx\right)+c\left(\frac{1}{3}x^3\arctan(ax)^3-a\int\frac{x^3\arctan(ax)^2}{a^2x^2+1}dx\right)$$
$$\Big\downarrow \text{5451}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\int x^3\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\int\frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{5451}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan}{a^2}}{}}{a^2}\right.\right.$$

$$\Big\downarrow \text{5345}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan}{a^2}}{}}{a^2}\right.\right.$$

$$\Big\downarrow \text{240}$$

3.364. $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)^2dx}{a^2}-\int\frac{x\,\mathrm{arct}\cdots}{a^2\cdots}}{a^2}\cdots\right.\right.$$

$$\Big\downarrow\ 5361$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow\ 243$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(a\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow\ 49$$

3.364. $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right)dx^2}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\cdots\right.\right.$$

$$\Big\downarrow \; 2009$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 5419$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\cdots}{\cdots}\right.\right.$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \; 5451$$

3.364. $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$

$$a^2c\Bigg(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\Bigg(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\Bigg(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}}{a^2}$$

$$c\Bigg(\frac{1}{3}x^3\arctan(ax)^3-a\Bigg(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\Bigg(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\Bigg)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\Bigg)\Bigg)\Bigg)$$

$$\Big\downarrow 5345$$

$$a^2c\Bigg(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\Bigg(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\Bigg(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-}{a^2}}{a^2}$$

$$c\Bigg(\frac{1}{3}x^3\arctan(ax)^3-a\Bigg(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\Bigg(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\Bigg)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\Bigg)\Bigg)\Bigg)$$

$$\Big\downarrow 240$$

$$a^2c\Bigg(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\Bigg(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\Bigg(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-}{a^2}}{a^2}$$

$$c\Bigg(\frac{1}{3}x^3\arctan(ax)^3-a\Bigg(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\Bigg(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\Bigg)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\Bigg)\Bigg)\Bigg)$$

$$\Big\downarrow 5419$$

3.364. $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\text{arc}\cdots}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$\Big\downarrow \; 5455$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(a\cdots}{3a^2}\cdots}{a^2}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\text{arc}\cdots}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$\Big\downarrow \; 5379$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\text{ar}\cdots}{\cdots}}{a}\cdots}{\cdots}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\text{arc}\cdots}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$\Big\downarrow \; 5529$$

3.364. $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\right.}{}}{}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\text{arc}}{a^2}\right.}{a^2}\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\,\text{lo}}{a}}{}}{}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\text{arc}}{a^2}\right.}{a^2}\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

3.364. $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$

output
```
c*((x^3*ArcTan[a*x]^3)/3 - a*(((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]
^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((-1/3*I)*A
rcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*A
rcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)
]/(4*a)))/a)/a^2)) + a^2*c*((x^5*ArcTan[a*x]^3)/5 - (3*a*(((x^4*ArcTan[a*x
]^2)/4 - (a*(((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6
)/a^2 - (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))
/a^2)/a^2))/2)/a^2 - (((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2/a^3 +
 (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((-1/3*I)*ArcTan[a*
x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*
x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a))
)/a)/a^2)/a^2))/5)
```

3.364.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.364. $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.364.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 46.49 (sec) , antiderivative size = 900, normalized size of antiderivative = 4.27

method	result	size
derivativedivides	Expression too large to display	900
default	Expression too large to display	900
parts	Expression too large to display	902

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*c*arctan(a*x)^3*a^5*x^5+1/3*c*arctan(a*x)^3*a^3*x^3-1/5*c*(3/4*
a^4*arctan(a*x)^2*x^4+x^2*arctan(a*x)^2*a^2-arctan(a*x)^2*ln(a^2*x^2+1)+2*
arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/12*I*(-6*Pi*arctan(a*x)^2*
csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+12*Pi*arctan(a*x)^2*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))-6*Pi*arctan(
a*x)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)+1))^2+6*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1))-12*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+6*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1))^3+6*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn
(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)-6*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-6*Pi*arctan
(a*x)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+6*Pi*arctan(a*x)^2*csgn(I*(1+I*a*x)^2/
(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+8*arctan(a*x)^3+3*I*a^2*x^2-6
*I*arctan(a*x)*a*x+3*I*arctan(a*x)^2-6*I*arctan(a*x)*a^3*x^3+24*I*ln(2)*ar
ctan(a*x)^2+3*I)-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+polyl
og(3,-(1+I*a*x)^2/(a^2*x^2+1))))
```

3.364.5 Fricas [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^3, x)
```

3.364.6 Sympy [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx = c\left(\int x^2\,\mathrm{atan}^3(ax)\,dx+\int a^2x^4\,\mathrm{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x**2*atan(a*x)**3, x) + Integral(a**2*x**4*atan(a*x)**3, x))
```

3.364.7 Maxima [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/120*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^3 - 1/160*(3*a^2*c*x^5 + 5*c*x^3
)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/160*(140*(a^4*c*x^6 + 2*a^2
*c*x^4 + c*x^2)*arctan(a*x)^3 - 4*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x)^2
+ 4*(3*a^4*c*x^6 + 5*a^2*c*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (3*a^3*c*x^
5 + 5*a*c*x^3 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*arctan(a*x))*log(a^2*
x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

3.364.8 Giac [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx=\int x^2\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.365 $\int x(c+a^2cx^2)\arctan(ax)^3\,dx$

3.365.1 Optimal result

Integrand size = 18, antiderivative size = 160

$$\begin{aligned}\int x(c+a^2cx^2)\arctan(ax)^3\,dx =& -\frac{cx}{4a}+\frac{c(1+a^2x^2)\arctan(ax)}{4a^2}\\ &-\frac{ic\arctan(ax)^2}{2a^2}-\frac{cx\arctan(ax)^2}{2a}\\ &-\frac{cx(1+a^2x^2)\arctan(ax)^2}{4a}+\frac{c(1+a^2x^2)^2\arctan(ax)^3}{4a^2}\\ &-\frac{c\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^2}-\frac{ic\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2}\end{aligned}$$

output
```
-1/4*c*x/a+1/4*c*(a^2*x^2+1)*arctan(a*x)/a^2-1/2*I*c*arctan(a*x)^2/a^2-1/2
*c*x*arctan(a*x)^2/a-1/4*c*x*(a^2*x^2+1)*arctan(a*x)^2/a+1/4*c*(a^2*x^2+1)
^2*arctan(a*x)^3/a^2-c*arctan(a*x)*ln(2/(1+I*a*x))/a^2-1/2*I*c*polylog(2,1
-2/(1+I*a*x))/a^2
```

3.365.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.63

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$=\frac{c\Big(-ax-(-2i+3ax+a^3x^3)\arctan(ax)^2+(1+a^2x^2)^2\arctan(ax)^3+\arctan(ax)\left(1+a^2x^2-4\log\left(1+\right.\right.}{4a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(-(a*x) - (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + (1 + a^2*x^2)^2*ArcT
an[a*x]^3 + ArcTan[a*x]*(1 + a^2*x^2 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) +
 (2*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(4*a^2)
```

3.365.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5465, 27, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3 \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^3}{4a^2}-\frac{3\int c\left(a^2x^2+1\right)\arctan(ax)^2dx}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^3}{4a^2}-\frac{3c\int \left(a^2x^2+1\right)\arctan(ax)^2dx}{4a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)}{4a}$$

$$\Big\downarrow \text{24}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)}{4a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)}{4a}$$

$$\Big\downarrow \textbf{5455}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)}{4a}$$

$$\Big\downarrow \textbf{5379}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a}{}\right.}{4a}$$

$$\Big\downarrow \textbf{2849}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2\right.}{4a}$$

$$\Big\downarrow \textbf{2752}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\right.}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(4*a^2) - (3*c*(x/3 - ((1 + a^2*x^2)*Arc
Tan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a*x]^2
- 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a
+ ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)))/3))/(4*a)
```

3.365. $\int x(c+a^2cx^2)\arctan(ax)^3\,dx$

3.365.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

3.365. $\int x(c+a^2cx^2)\arctan(ax)^3\,dx$

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.365.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.48

method	result
parts	$\frac{c\arctan(ax)^3 a^2 x^4}{4}+\frac{c\arctan(ax)^3 x^2}{2}+\frac{c\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{a^3\arctan(ax)^2x^3}{3}+a\arctan(ax)^2x-\frac{a^2\arctan(ax)x^2}{3}-\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{c\arctan(ax)^3a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^3}{2}+\frac{c\arctan(ax)^3}{4}-\frac{3c\left(\frac{a^3\arctan(ax)^2x^3}{3}+a\arctan(ax)^2x-\frac{a^2\arctan(ax)x^2}{3}-\frac{2\arctan(ax)\ln\cdots}{3}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{c\arctan(ax)^3a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^3}{2}+\frac{c\arctan(ax)^3}{4}-\frac{3c\left(\frac{a^3\arctan(ax)^2x^3}{3}+a\arctan(ax)^2x-\frac{a^2\arctan(ax)x^2}{3}-\frac{2\arctan(ax)\ln\cdots}{3}\cdots\right.}{\cdots}}{\cdots}$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*c*arctan(a*x)^3*a^2*x^4+1/2*c*arctan(a*x)^3*x^2+1/4*c*arctan(a*x)^3/a^
2-3/4/a^2*c*(1/3*a^3*arctan(a*x)^2*x^3+a*arctan(a*x)^2*x-1/3*a^2*arctan(a*
x)*x^2-2/3*arctan(a*x)*ln(a^2*x^2+1)+1/3*a*x-1/3*arctan(a*x)-1/3*I*(ln(a*x
-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*l
n(a*x-I)^2)+1/3*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*
ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2))
```

3.365.5 Fricas [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*arctan(a*x)^3, x)
```

3.365.6 Sympy [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx=c\left(\int x\,\mathrm{atan}^3(ax)\,dx+\int a^2x^3\,\mathrm{atan}^3(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x*atan(a*x)**3, x) + Integral(a**2*x**3*atan(a*x)**3, x))
```

3.365.7 Maxima [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/64*(8*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 4*(512*a^5*c*integra
te(1/64*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) - 192*a^4*c*integrate(1/64*x^4
*arctan(a*x)^2/(a^3*x^2 + a), x) - 48*a^4*c*integrate(1/64*x^4*log(a^2*x^2
 + 1)^2/(a^3*x^2 + a), x) - 64*a^4*c*integrate(1/64*x^4*log(a^2*x^2 + 1)/(
a^3*x^2 + a), x) + 1024*a^3*c*integrate(1/64*x^3*arctan(a*x)^3/(a^3*x^2 +
a), x) + 128*a^3*c*integrate(1/64*x^3*arctan(a*x)/(a^3*x^2 + a), x) - 384*
a^2*c*integrate(1/64*x^2*arctan(a*x)^2/(a^3*x^2 + a), x) - 96*a^2*c*integr
ate(1/64*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 192*a^2*c*integrate(1/
64*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 512*a*c*integrate(1/64*x*arcta
n(a*x)^3/(a^3*x^2 + a), x) + 384*a*c*integrate(1/64*x*arctan(a*x)/(a^3*x^2
 + a), x) - c*arctan(a*x)^3/a^2 - 48*c*integrate(1/64*log(a^2*x^2 + 1)^2/(
a^3*x^2 + a), x))*a^2 - 4*(a^3*c*x^3 + 3*a*c*x)*arctan(a*x)^2 + (a^3*c*x^3
 + 3*a*c*x)*log(a^2*x^2 + 1)^2)/a^2
```

3.365.8 Giac [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx=\int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.366 $\int (c+a^2cx^2) \arctan(ax)^3 \, dx$

3.366.1 Optimal result

Integrand size = 17, antiderivative size = 172

$$\begin{aligned}\int (c+a^2cx^2) \arctan(ax)^3 \, dx = {} & cx\arctan(ax) - \frac{c(1+a^2x^2)\arctan(ax)^2}{2a} + \frac{2ic\arctan(ax)^3}{3a} \\ & + \frac{2}{3}cx\arctan(ax)^3 + \frac{1}{3}cx\left(1+a^2x^2\right)\arctan(ax)^3 \\ & + \frac{2c\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} - \frac{c\log\left(1+a^2x^2\right)}{2a} \\ & + \frac{2ic\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a} \\ & + \frac{c\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{a}\end{aligned}$$

output
```
c*x*arctan(a*x)-1/2*c*(a^2*x^2+1)*arctan(a*x)^2/a+2/3*I*c*arctan(a*x)^3/a+
2/3*c*x*arctan(a*x)^3+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^3+2*c*arctan(a*x)^2*
ln(2/(1+I*a*x))/a-1/2*c*ln(a^2*x^2+1)/a+2*I*c*arctan(a*x)*polylog(2,1-2/(1
+I*a*x))/a+c*polylog(3,1-2/(1+I*a*x))/a
```

3.366.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int (c + a^2cx^2) \arctan(ax)^3 \, dx$$
$$= \frac{c(6ax \arctan(ax) - 3\arctan(ax)^2 - 3a^2x^2 \arctan(ax)^2 - 4i \arctan(ax)^3 + 6ax \arctan(ax)^3 + 2a^3x^3 \text{arcta}}{}$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(6*a*x*ArcTan[a*x] - 3*ArcTan[a*x]^2 - 3*a^2*x^2*ArcTan[a*x]^2 - (4*I)*
ArcTan[a*x]^3 + 6*a*x*ArcTan[a*x]^3 + 2*a^3*x^3*ArcTan[a*x]^3 + 12*ArcTan[
a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 3*Log[1 + a^2*x^2] - (12*I)*ArcTan
[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 6*PolyLog[3, -E^((2*I)*ArcTan[a
*x])]))/(6*a)
```

3.366.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5415, 5345, 240, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2 + c\right) \, dx$$

$$\Big\downarrow \; 5415$$

$$c\int \arctan(ax)dx + \frac{2}{3}c\int \arctan(ax)^3dx + \frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3 - \frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}$$

$$\Big\downarrow \; 5345$$

$$c\left(x\arctan(ax) - a\int \frac{x}{a^2x^2+1}dx\right) + \frac{2}{3}c\left(x\arctan(ax)^3 - 3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right) + \frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3 - \frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}$$

$$\Big\downarrow \; 240$$

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c(a^2x^2+1)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c(a^2x^2+1)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c(a^2x^2+1)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{5529}$$

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\right.\right.$$
$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c(a^2x^2+1)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \text{7164}$$

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\right.}{4a}\right.}{a}\right.\right.$$
$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c(a^2x^2+1)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

3.366. $\int\left(c+a^2cx^2\right)\arctan(ax)^3\,dx$

output
```
-1/2*(c*(1 + a^2*x^2)*ArcTan[a*x]^2)/a + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^3)
/3 + c*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)) + (2*c*(x*ArcTan[a*x]^3 -
3*a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a
- 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1
 - 2/(1 + I*a*x)]/(4*a)))/a)))/3
```

3.366.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.366.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.82 (sec) , antiderivative size = 860, normalized size of antiderivative = 5.00

method	result
parts	$\frac{c \arctan(ax)^3 a^2 x^3}{3} + cx \arctan(ax)^3 - c \left(\frac{a \arctan(ax)^2 x^2}{2} + \frac{\arctan(ax)^2 \ln(a^2x^2+1)}{a} - \frac{2 \arctan(ax)^2 \ln}{} \right.$
derivativedivides	$\frac{\frac{c \arctan(ax)^3 a^3 x^3}{3} + c \arctan(ax)^3 ax - c \left(\frac{x^2 \arctan(ax)^2 a^2}{2} + \arctan(ax)^2 \ln(a^2x^2+1) - 2\arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) + 2i a \right.}{}$
default	$\frac{\frac{c \arctan(ax)^3 a^3 x^3}{3} + c \arctan(ax)^3 ax - c \left(\frac{x^2 \arctan(ax)^2 a^2}{2} + \arctan(ax)^2 \ln(a^2x^2+1) - 2\arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) + 2i a \right.}{}$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*arctan(a*x)^3*a^2*x^3+c*x*arctan(a*x)^3-c*(1/2*a*arctan(a*x)^2*x^2+1
/a*arctan(a*x)^2*ln(a^2*x^2+1)-1/a*(2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+
1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+polylog(3,-(
1+I*a*x)^2/(a^2*x^2+1))-1/6*I*arctan(a*x)*(-3*arctan(a*x)*Pi*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)+3*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)+3*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^3-3*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+3*arctan(a*x
)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-6*arctan(a*x)*Pi*csgn(I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+3*arctan(a*x)*Pi*csgn(
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-3*arctan(
a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+6*arctan(a*x)*Pi*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))-3*arcta
n(a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1))^2+4*arctan(a*x)^2+12*I*arctan(a*x)*ln(2)-3*I*arctan(a*x)+6+6*I*a
*x)+ln((1+I*a*x)^2/(a^2*x^2+1)+1)))
```

3.366.5 Fricas [F]

$$\int (c+a^2cx^2)\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

3.366.6 Sympy [F]

$$\int (c+a^2cx^2)\arctan(ax)^3\,dx=c\left(\int a^2x^2\operatorname{atan}^3(ax)\,dx+\int \operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**3, x) + Integral(atan(a*x)**3, x))
```

3.366.7 Maxima [F]

$$\int (c + a^2cx^2) \arctan(ax)^3 \, dx = \int (a^2cx^2 + c) \arctan(ax)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
28*a^4*c*integrate(1/32*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 3*a^4*c*inte
grate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*
integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 4*a^3*
c*integrate(1/32*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + a^3*c*integrate(1/3
2*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/24*(a^2*c*x^3 + 3*c*x)*arct
an(a*x)^3 + 7/32*c*arctan(a*x)^4/a + 56*a^2*c*integrate(1/32*x^2*arctan(a*
x)^3/(a^2*x^2 + 1), x) + 6*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^
2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a
^2*x^2 + 1)/(a^2*x^2 + 1), x) - 1/32*(a^2*c*x^3 + 3*c*x)*arctan(a*x)*log(a
^2*x^2 + 1)^2 - 12*a*c*integrate(1/32*x*arctan(a*x)^2/(a^2*x^2 + 1), x) +
3*a*c*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3*c*integrat
e(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

3.366.8 Giac [F]

$$\int (c + a^2cx^2) \arctan(ax)^3 \, dx = \int (a^2cx^2 + c) \arctan(ax)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int (c+a^2cx^2) \arctan(ax)^3 \, dx = \int \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right) \, dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.367 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx$

3.367.1 Optimal result

Integrand size = 20, antiderivative size = 276

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = &-\frac{3}{2}ic\arctan(ax)^2-\frac{3}{2}acx\arctan(ax)^2 \\
&+\frac{1}{2}c\arctan(ax)^3+\frac{1}{2}a^2cx^2\arctan(ax)^3 \\
&+2c\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) \\
&-3c\arctan(ax)\log\left(\frac{2}{1+iax}\right) \\
&-\frac{3}{2}ic\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
&-\frac{3}{2}ic\arctan(ax)^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
&+\frac{3}{2}ic\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) \\
&-\frac{3}{2}c\arctan(ax)\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+iax}\right) \\
&+\frac{3}{2}c\arctan(ax)\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1+iax}\right) \\
&+\frac{3}{4}ic\,\mathrm{PolyLog}\left(4,1-\frac{2}{1+iax}\right) \\
&-\frac{3}{4}ic\,\mathrm{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)
\end{aligned}
$$

output
```
-3/2*I*c*arctan(a*x)^2-3/2*a*c*x*arctan(a*x)^2+1/2*c*arctan(a*x)^3+1/2*a^2
*c*x^2*arctan(a*x)^3-2*c*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-3*c*arctan(
a*x)*ln(2/(1+I*a*x))-3/2*I*c*polylog(2,1-2/(1+I*a*x))-3/2*I*c*arctan(a*x)^
2*polylog(2,1-2/(1+I*a*x))+3/2*I*c*arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))
-3/2*c*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+3/2*c*arctan(a*x)*polylog(3,-1
+2/(1+I*a*x))+3/4*I*c*polylog(4,1-2/(1+I*a*x))-3/4*I*c*polylog(4,-1+2/(1+I
*a*x))
```

3.367.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = {} & \frac{1}{2}c(1+a^2x^2)\arctan(ax)^3 - \frac{3}{2}c\big(-i\arctan(ax)^2 \\
& + ax\arctan(ax)^2 + 2\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right) \\
& - i\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)\big) - \frac{1}{64}ic\big(\pi^4 - 32\arctan(ax)^4 \\
& + 64i\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right) \\
& - 64i\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right) \\
& - 96\arctan(ax)^2\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) \\
& - 96\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) \\
& + 96i\arctan(ax)\,\mathrm{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) \\
& - 96i\arctan(ax)\,\mathrm{PolyLog}\left(3,-e^{2i\arctan(ax)}\right) \\
& + 48\,\mathrm{PolyLog}\left(4,e^{-2i\arctan(ax)}\right) \\
& + 48\,\mathrm{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\big)
\end{aligned}
$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]
```

output
```
(c*(1 + a^2*x^2)*ArcTan[a*x]^3)/2 - (3*c*((-I)*ArcTan[a*x]^2 + a*x*ArcTan[
a*x]^2 + 2*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - I*PolyLog[2, -E^((
2*I)*ArcTan[a*x])]))/2 - (I/64)*c*(Pi^4 - 32*ArcTan[a*x]^4 + (64*I)*ArcTan
[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - (64*I)*ArcTan[a*x]^3*Log[1 + E^(
(2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])]
- 96*ArcTan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]
*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (96*I)*ArcTan[a*x]*PolyLog[3, -E^((2
*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4,
-E^((2*I)*ArcTan[a*x])])
```

3.367.3 Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5485, 5357, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\arctan(ax)^3dx+c\int \frac{\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{5357}$$

$$a^2c\int x\arctan(ax)^3dx+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\int \frac{x^2\arctan(ax)^2}{a^2x^2+1}dx\right)+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5451}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{\int \arctan(ax)^2dx}{a^2}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \textbf{5419}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^3}{3a^3}\right)\right)+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int\frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \textbf{5455}$$

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int\frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)+a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)\right)$$

$$\Big\downarrow \textbf{5379}$$

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int\frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)+a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right.}{a^2}\right.\right.$$

$$\Big\downarrow \textbf{2849}$$

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int\frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)+a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan}{2}\right.}{a^2}\right.\right.$$

$$\Big\downarrow \textbf{2752}$$

3.367. $\int\frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx$

$$c\Bigg(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int\frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\Bigg)+$$

$$a^2c\Bigg(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\Bigg(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\Bigg(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\big(2,1-}{2a}}{a}}{a^2}$$

$$\Big\downarrow \ 5523$$

$$c\Bigg(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\Bigg(\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right.}{a^2x^2+1}$$

$$a^2c\Bigg(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\Bigg(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\Bigg(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\big(2,1-}{2a}}{a}}{a^2}$$

$$\Big\downarrow \ 5529$$

$$c\Bigg(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\Bigg(\frac{1}{2}\Bigg(\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\int\frac{\arctan(ax)\,\text{PolyLo}}{a^2x^2}$$

$$a^2c\Bigg(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\Bigg(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\Bigg(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\big(2,1-}{2a}}{a}}{a^2}$$

$$\Big\downarrow \ 5533$$

$$c\Bigg(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\Bigg(\frac{1}{2}\Bigg(\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\Bigg(\frac{i\arctan(ax)\,\text{PolyLo}}{2a}$$

$$a^2c\Bigg(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\Bigg(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\Bigg(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\big(2,1-}{2a}}{a}}{a^2}$$

$$\Big\downarrow \ 7164$$

3.367. $\int\frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a}}{a}\right)}{a^2}\right.\right.$$

$$c\left(2\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\ldots}{2a}\right.\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]
```

output
```
a^2*c*((x^2*ArcTan[a*x]^3)/2 - (3*a*(-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*
x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)
])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2))/2) + c*(2*ArcTan
[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 6*a*((((I/2)*ArcTan[a*x]^2*PolyLog[2,
 1 - 2/(1 + I*a*x)])/a - I*(((I/2)*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x
)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)]/(4*a)))/2 + (((-1/2*I)*ArcTan[a*x]^2
*PolyLog[2, -1 + 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTan[a*x]*PolyLog[3, -1 +
 2/(1 + I*a*x)])/a + PolyLog[4, -1 + 2/(1 + I*a*x)]/(4*a)))/2))
```

3.367.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.367. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx$

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.367. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.367. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx$

3.367.4 Maple [A] (verified)

Time = 19.54 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{c\arctan(ax)^2(-i\arctan(ax)+x\arctan(ax)a-3)(ax+i)}{2}+\frac{3ic\arctan(ax)^2\operatorname{polylog}\left(2,-\frac{(iax+1)^2}{a^2x^2+1}\right)}{2}-3c\arctan($
default	$\frac{c\arctan(ax)^2(-i\arctan(ax)+x\arctan(ax)a-3)(ax+i)}{2}+\frac{3ic\arctan(ax)^2\operatorname{polylog}\left(2,-\frac{(iax+1)^2}{a^2x^2+1}\right)}{2}-3c\arctan($

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*arctan(a*x)^2*(-I*arctan(a*x)+x*arctan(a*x)*a-3)*(I+a*x)+3/2*I*c*arc
tan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-3*c*arctan(a*x)*ln((1+I*a*x
)^2/(a^2*x^2+1)+1)+3*I*c*arctan(a*x)^2+c*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x
^2+1)^(1/2)+1)-3*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
+6*c*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c*polylog(4,(
1+I*a*x)/(a^2*x^2+1)^(1/2))-c*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+
6*I*c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*c*arctan(a*x)*polylog(3,
-(1+I*a*x)^2/(a^2*x^2+1))+3/2*I*c*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+c*ar
ctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/4*I*c*polylog(4,-(1+I*a*x)
^2/(a^2*x^2+1))+6*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I
*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

3.367.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

3.367.6 Sympy [F]

$$\int \frac{(c + a^2cx^2) \arctan(ax)^3}{x} \, dx = c\left(\int \frac{\operatorname{atan}^3(ax)}{x} \, dx + \int a^2x \operatorname{atan}^3(ax) \, dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x,x)
```

output
```
c*(Integral(atan(a*x)**3/x, x) + Integral(a**2*x*atan(a*x)**3, x))
```

3.367.7 Maxima [F]

$$\int \frac{(c + a^2cx^2) \arctan(ax)^3}{x} \, dx = \int \frac{(a^2cx^2 + c) \arctan(ax)^3}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
1/16*a^2*c*x^2*arctan(a*x)^3 - 3/64*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1)
^2 + integrate(1/64*(12*a^4*c*x^4*arctan(a*x)*log(a^2*x^2 + 1) - 12*a^3*c*
x^3*arctan(a*x)^2 + 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a^
3*c*x^3 + 2*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)
/(a^2*x^3 + x), x)
```

3.367.8 Giac [F]

$$\int \frac{(c + a^2cx^2) \arctan(ax)^3}{x} \, dx = \int \frac{(a^2cx^2 + c) \arctan(ax)^3}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x, x)
```

3.368 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx$

3.368.1 Optimal result

Integrand size = 20, antiderivative size = 169

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = & -\frac{c\arctan(ax)^3}{x}+a^2cx\arctan(ax)^3\\ & +3ac\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)\\ & +3ac\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\\ & -3iac\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\ & +3iac\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\ & +\frac{3}{2}ac\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)\\ & +\frac{3}{2}ac\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-c*arctan(a*x)^3/x+a^2*c*x*arctan(a*x)^3+3*a*c*arctan(a*x)^2*ln(2/(1+I*a*x
))+3*a*c*arctan(a*x)^2*ln(2-2/(1-I*a*x))-3*I*a*c*arctan(a*x)*polylog(2,-1+
2/(1-I*a*x))+3*I*a*c*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+3/2*a*c*polylog(
3,-1+2/(1-I*a*x))+3/2*a*c*polylog(3,1-2/(1+I*a*x))
```

3.368.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = -iac\arctan(ax)^3 + a^2cx\arctan(ax)^3 + 3ac\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) - 3iac\arctan(ax)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + ac\left(-\frac{i\pi^3}{8} + i\arctan(ax)^3 - \frac{\arctan(ax)^3}{ax} + 3\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) + 3i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + \frac{3}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)\right) + \frac{3}{2}ac\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
(-I)*a*c*ArcTan[a*x]^3 + a^2*c*x*ArcTan[a*x]^3 + 3*a*c*ArcTan[a*x]^2*Log[1
 + E^((2*I)*ArcTan[a*x])] - (3*I)*a*c*ArcTan[a*x]*PolyLog[2, -E^((2*I)*Arc
Tan[a*x])] + a*c*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) +
3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLo
g[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2) +
 (3*a*c*PolyLog[3, -E^((2*I)*ArcTan[a*x])])/2
```

3.368.3 Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5345, 5361, 5455, 5379, 5459, 5403, 5527, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\,(a^2cx^2+c)}{x^2}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \arctan(ax)^3dx+c\int \frac{\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \ 5345$$

$$a^2c\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+c\int \frac{\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \ 5361$$

$$a^2c\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)$$

$$\Big\downarrow \ 5455$$

$$c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)$$

$$\Big\downarrow \ 5379$$

$$c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)$$

$$\Big\downarrow \ 5459$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+ c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)$$

$$\Big\downarrow \ 5403$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+ c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan\right.\right.$$

$$\Big\downarrow \ 5527$$

3.368. $\int \frac{\left(c+a^2cx^2\right)\arctan(ax)^3}{x^2}\,dx$

$$c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\operatorname{arc}\right.\right.\right.$$
$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)$$

$$\Big\downarrow \; 5529$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\operatorname{arc}\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\operatorname{PolyLog}(3,1-}{4a}\right.}{a}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
c*(-(ArcTan[a*x]^3/x) + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^
2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(
1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))) + a^2*c*(x*ArcTa
n[a*x]^3 - 3*a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 +
I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - P
olyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a))
```

3.368. $\int\frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}dx$

3.368.3.1 Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.368. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.368.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.21 (sec) , antiderivative size = 1653, normalized size of antiderivative = 9.78

method	result	size
parts	Expression too large to display	1653
derivativedivides	Expression too large to display	1654
default	Expression too large to display	1654

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x*arctan(a*x)^3-c*arctan(a*x)^3/x-3*c*(a*arctan(a*x)^2*ln(a^2*x^2+1)
-a*arctan(a*x)^2*ln(a*x)-2*a*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
)-1/3*I*arctan(a*x)^3+1/4*(I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi
*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)^3+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((
1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))+I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))^3-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*
a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))^2+I*Pi+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+...
```

3.368.5 Fricas [F]

$$\int \frac{(c + a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2 + c)\arctan{(ax)}^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

3.368.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = c\left(\int a^2\operatorname{atan}^3(ax)\,dx + \int \frac{\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x))
```

3.368.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/64*(8*(a^2*c*x^2 - c)*arctan(a*x)^3 - 6*(a^2*c*x^2 - c)*arctan(a*x)*log(
a^2*x^2 + 1)^2 + (28*a*c*arctan(a*x)^4 + 1792*a^4*c*integrate(1/32*x^4*arc
tan(a*x)^3/(a^2*x^4 + x^2), x) + 192*a^4*c*integrate(1/32*x^4*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 768*a^4*c*integrate(1/32*x^4*arct
an(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 768*a^3*c*integrate(1/32*x^
3*arctan(a*x)^2/(a^2*x^4 + x^2), x) + a*c*log(a^2*x^2 + 1)^3 + 384*a^2*c*i
ntegrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 768
*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x)
 + 768*a*c*integrate(1/32*x*arctan(a*x)^2/(a^2*x^4 + x^2), x) - 192*a*c*in
tegrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 1792*c*integrate(1
/32*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 192*c*integrate(1/32*arctan(a*x)*l
og(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

3.368.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^2, x)
```

3.369 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx$

3.369.1 Optimal result

Integrand size = 20, antiderivative size = 310

$$\begin{aligned}
\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = &-\frac{3}{2}ia^2c\arctan(ax)^2-\frac{3ac\arctan(ax)^2}{2x}-\frac{1}{2}a^2c\arctan(ax)^3\\
&-\frac{c\arctan(ax)^3}{2x^2}+2a^2c\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)\\
&+3a^2c\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\
&-\frac{3}{2}ia^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\
&-\frac{3}{2}ia^2c\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}ia^2c\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\
&-\frac{3}{2}a^2c\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}a^2c\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\\
&+\frac{3}{4}ia^2c\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right)\\
&-\frac{3}{4}ia^2c\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)
\end{aligned}$$

output
```
-3/2*I*a^2*c*arctan(a*x)^2-3/2*a*c*arctan(a*x)^2/x-1/2*a^2*c*arctan(a*x)^3
-1/2*c*arctan(a*x)^3/x^2-2*a^2*c*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))+3*a
^2*c*arctan(a*x)*ln(2-2/(1-I*a*x))-3/2*I*a^2*c*polylog(2,-1+2/(1-I*a*x))-3
/2*I*a^2*c*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))+3/2*I*a^2*c*arctan(a*x)^
2*polylog(2,-1+2/(1+I*a*x))-3/2*a^2*c*arctan(a*x)*polylog(3,1-2/(1+I*a*x))
+3/2*a^2*c*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/4*I*a^2*c*polylog(4,1-2
/(1+I*a*x))-3/4*I*a^2*c*polylog(4,-1+2/(1+I*a*x))
```

3.369.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx &= \frac{1}{2}a^2c\arctan(ax)^3+\frac{c(-1-a^2x^2)\arctan(ax)^3}{2x^2}\\ &\quad+\frac{3}{2}a^2c\Bigg(-\frac{1}{3}\arctan(ax)\Bigg(\frac{3\arctan(ax)}{ax}\\ &\quad+\arctan(ax)(3i+\arctan(ax))-6\log\left(1-e^{2i\arctan(ax)}\right)\Bigg)\\ &\quad-i\operatorname{PolyLog}\left(2,e^{2i\arctan(ax)}\right)\Bigg)-\frac{1}{64}ia^2c\big(\pi^4\\ &\quad-32\arctan(ax)^4+64i\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right)\\ &\quad-64i\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right)\\ &\quad-96\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)\\ &\quad-96\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)\\ &\quad+96i\arctan(ax)\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)\\ &\quad-96i\arctan(ax)\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\\ &\quad+48\operatorname{PolyLog}\left(4,e^{-2i\arctan(ax)}\right)\\ &\quad+48\operatorname{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\big)\end{aligned}$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c*ArcTan[a*x]^3)/2 + (c*(-1 - a^2*x^2)*ArcTan[a*x]^3)/(2*x^2) + (3*a^
2*c*(-1/3*(ArcTan[a*x]*((3*ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[
a*x]) - 6*Log[1 - E^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, E^((2*I)*ArcTan[
a*x])]))/2 - (I/64)*a^2*c*(Pi^4 - 32*ArcTan[a*x]^4 + (64*I)*ArcTan[a*x]^3*
Log[1 - E^((-2*I)*ArcTan[a*x])] - (64*I)*ArcTan[a*x]^3*Log[1 + E^((2*I)*Ar
cTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - 96*Arc
Tan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]*PolyLog
[3, E^((-2*I)*ArcTan[a*x])] - (96*I)*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcT
an[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4, -E^((2*I
)*ArcTan[a*x])])
```

3.369.3 Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5485, 5357, 5361, 5453, 5361, 5419, 5459, 5403, 2897, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^3}{x}dx+c\int \frac{\arctan(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{5357}$$

$$c\int \frac{\arctan(ax)^3}{x^3}dx+ a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{2x^2}\right)+ a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5453}$$

$$c\left(\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) - \frac{\arctan(ax)^3}{2x^2}\right) + a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right) - \frac{\arctan(ax)^3}{2x^2}\right) + a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5419}$$

$$c\left(\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx - \frac{1}{3}a\arctan(ax)^3 - \frac{\arctan(ax)^2}{x}\right) - \frac{\arctan(ax)^3}{2x^2}\right) + a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5459}$$

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right) + c\left(-\frac{\arctan(ax)^3}{2x^2} + \frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx - \frac{1}{2}i\arctan(ax)^2\right) - \frac{1}{3}a\arctan(ax)^3 - \frac{\arctan(ax)^2}{x}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right) + c\left(-\frac{\arctan(ax)^3}{2x^2} + \frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right)$$

$$\Big\downarrow \text{2897}$$

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right) + c\left(-\frac{\arctan(ax)^3}{2x^2} + \frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right) - \frac{1}{2}i\arctan(ax\right.\right.$$

$$\Big\downarrow \text{5523}$$

3.369. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx$

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax}\right.}{a^2x^2+1}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax\right.\right.\right.$$

$\Big\downarrow$ 5529

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\int\frac{\arctan(ax)\text{Poly}}{a^2}\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax\right.\right.\right.$$

$\Big\downarrow$ 5533

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\left(\frac{i\arctan(ax)\text{Poly}}{}\right.\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax\right.\right.\right.$$

$\Big\downarrow$ 7164

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\left(\frac{i\arctan(ax)\text{Poly}}{}\right.\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
c*(-1/2*ArcTan[a*x]^3/x^2 + (3*a*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3
 + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)
] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2) + a^2*c*(2*ArcTan[a*x]^3*ArcTa
nh[1 - 2/(1 + I*a*x)] - 6*a*((((I/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I
*a*x)])/a - I*(((I/2)*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyL
og[4, 1 - 2/(1 + I*a*x)]/(4*a)))/2 + (((-1/2*I)*ArcTan[a*x]^2*PolyLog[2, -
1 + 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x
)])/a + PolyLog[4, -1 + 2/(1 + I*a*x)]/(4*a)))/2))
```

3.369. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx$

3.369.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.369. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.369. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx$

3.369.4 Maple [A] (verified)

Time = 45.81 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.69

method	result
derivativedivides	$a^2\left(-\frac{c\arctan(ax)^2(-i\arctan(ax)-3iax+x\arctan(ax)a)(ax+i)}{2a^2x^2}-3ic\arctan(ax)^2+6c\arctan(ax)\,\mathrm{po}\right.$
default	$a^2\left(-\frac{c\arctan(ax)^2(-i\arctan(ax)-3iax+x\arctan(ax)a)(ax+i)}{2a^2x^2}-3ic\arctan(ax)^2+6c\arctan(ax)\,\mathrm{po}\right.$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*c*arctan(a*x)^2*(-I*arctan(a*x)-3*I*a*x+x*arctan(a*x)*a)*(I+a*x)
/a^2/x^2-3*I*c*arctan(a*x)^2+6*c*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))+6*I*c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c*arctan(a*x)*l
n((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-3*I*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(
a^2*x^2+1)^(1/2))+6*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3
*I*c*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+c*arctan(a*x)^3*ln((1+I*a*x)/(
a^2*x^2+1)^(1/2)+1)-3*I*c*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*c*ar
ctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+3/2*I*c*arctan(a*x)^2*polylo
g(2,-(1+I*a*x)^2/(a^2*x^2+1))+c*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-3/4*I*c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+3*c*arctan(a*x)*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))-c*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+6*I*c*polylog(4,
(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

3.369.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx=\int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

3.369.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = c\left(\int \frac{\operatorname{atan}^3(ax)}{x^3}\,dx + \int \frac{a^2\operatorname{atan}^3(ax)}{x}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x**3,x)
```

output
```
c*(Integral(atan(a*x)**3/x**3, x) + Integral(a**2*atan(a*x)**3/x, x))
```

3.369.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
-1/64*(4*c*arctan(a*x)^3 - 3*c*arctan(a*x)*log(a^2*x^2 + 1)^2 - 64*x^2*int
egrate(-1/64*(12*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1) - 12*a*c*x*arctan(
a*x)^2 - 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a*c*x - 2*(a^
4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^5 + x^3
), x))/x^2
```

3.369.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^3, x)
```

3.370 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx$

3.370.1 Optimal result

Integrand size = 20, antiderivative size = 189

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx &= -\frac{a^2c\arctan(ax)}{x}-\frac{1}{2}a^3c\arctan(ax)^2-\frac{ac\arctan(ax)^2}{2x^2}\\ &\quad-\frac{2}{3}ia^3c\arctan(ax)^3-\frac{c\arctan(ax)^3}{3x^3}\\ &\quad-\frac{a^2c\arctan(ax)^3}{x}+a^3c\log(x)-\frac{1}{2}a^3c\log\left(1+a^2x^2\right)\\ &\quad+2a^3c\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\\ &\quad-2ia^3c\arctan(ax)\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\ &\quad+a^3c\,\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)\end{aligned}$$

output
```
-a^2*c*arctan(a*x)/x-1/2*a^3*c*arctan(a*x)^2-1/2*a*c*arctan(a*x)^2/x^2-2/3
*I*a^3*c*arctan(a*x)^3-1/3*c*arctan(a*x)^3/x^3-a^2*c*arctan(a*x)^3/x+a^3*c
*ln(x)-1/2*a^3*c*ln(a^2*x^2+1)+2*a^3*c*arctan(a*x)^2*ln(2-2/(1-I*a*x))-2*I
*a^3*c*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+a^3*c*polylog(3,-1+2/(1-I*a*x
))
```

3.370.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \frac{1}{12}c\left(-ia^3\pi^3 - \frac{12a^2\arctan(ax)}{x} - 6a^3\arctan(ax)^2 - \frac{6a\arctan(ax)^2}{x^2} + 8ia^3\arctan(ax)^3 - \frac{4\arctan(ax)^3}{x^3} - \frac{12a^2\arctan(ax)^3}{x} + 24a^3\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) + 12a^3\log(ax) - 6a^3\log\left(1+a^2x^2\right) + 24ia^3\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + 12a^3\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
(c*((-I)*a^3*Pi^3 - (12*a^2*ArcTan[a*x])/x - 6*a^3*ArcTan[a*x]^2 - (6*a*Ar
cTan[a*x]^2)/x^2 + (8*I)*a^3*ArcTan[a*x]^3 - (4*ArcTan[a*x]^3)/x^3 - (12*a
^2*ArcTan[a*x]^3)/x + 24*a^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])]
 + 12*a^3*Log[a*x] - 6*a^3*Log[1 + a^2*x^2] + (24*I)*a^3*ArcTan[a*x]*PolyL
og[2, E^((-2*I)*ArcTan[a*x])] + 12*a^3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])
)/12
```

3.370.3 Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.61, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5485, 5361, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)}{x^4}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^3}{x^2}dx + c\int \frac{\arctan(ax)^3}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+c\left(a\int \frac{\arctan(ax)^2}{x^3\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{5453}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(\int \frac{\arctan(ax)^2}{x^3}dx-a^2\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\int \frac{\arctan(ax)}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{5453}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{x}\right)-\right.\right.$$

$$\Big\downarrow \text{47}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\right.\right.\right.$$

$$\Big\downarrow \text{14}$$

3.370. $\int \frac{\left(c+a^2cx^2\right)\arctan(ax)^3}{x^4}\,dx$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+$$
$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax}{x}\right.\right.\right.$$

$$\Big\downarrow \ 16$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+$$
$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\right.\right.$$

$$\Big\downarrow \ 5419$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+$$
$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(a}{2x^2}\right.\right.$$

$$\Big\downarrow \ 5459$$

$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\,a\right.\right.\right.$$
$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)$$

$$\Big\downarrow \ 5403$$

$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\mathrm{arct}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}\right.\right.\right.\right.$$

$$\Big\downarrow \ 5527$$

$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\,a\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)\right.\right.\right.\right.\right.$$

$$\Big\downarrow \ 7164$$

3.370. $\int \frac{\left(c+a^2cx^2\right)\arctan(ax)^3}{x^4}\,dx$

$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\operatorname{arcta}\right.\right.\right.\right.\right.$$

$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(a\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
a^2*c*(-(ArcTan[a*x]^3/x) + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a
*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 +
 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))) + c*(-1/3*Ar
cTan[a*x]^3/x^3 + a*(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*Arc
Tan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((-1/3*I)*ArcTa
n[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*
ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a
*x)]/(4*a))))))
```

3.370.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.370. $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx$

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.370.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 38.36 (sec) , antiderivative size = 1807, normalized size of antiderivative = 9.56

method	result	size
derivativedivides	Expression too large to display	1807
default	Expression too large to display	1807
parts	Expression too large to display	1809

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-c*arctan(a*x)^3/a/x-1/3*c*arctan(a*x)^3/a^3/x^3-c*(arctan(a*x)^2*ln(
a^2*x^2+1)+1/2*arctan(a*x)^2/a^2/x^2-2*arctan(a*x)^2*ln(a*x)-2*arctan(a*x)
^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2
+1)-1)+1/6*arctan(a*x)*(-6*I*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*Pi*arctan
(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*a*x-3*I*Pi*arctan(a*x)*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*a*x+3*I*P
i*arctan(a*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*a*x-3*I*Pi*arctan(a*x)*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))*a*x+6*I*a*x+6*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1))*a*x+6*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*a*x+3*I*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*Pi*arctan(a*x)*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))*a*x-6*I*Pi*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a*x-6*I*Pi*arctan(a*x)*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a
^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*a*x-6*I*Pi*arctan(a*x)*csgn(((1+
I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*a*x+3*I*Pi*arctan(a
*x)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I/(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*a*x+3*I*P...
```

3.370.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

3.370.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = c\left(\int \frac{\operatorname{atan}^3(ax)}{x^4}\,dx + \int \frac{a^2\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x**4,x)
```

output
```
c*(Integral(atan(a*x)**3/x**4, x) + Integral(a**2*atan(a*x)**3/x**2, x))
```

3.370.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/96*(3*(7*a^3*c*arctan(a*x)^4 + 96*a^4*c*integrate(1/32*x^4*arctan(a*x)*l
og(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 384*a^4*c*integrate(1/32*x^4*arcta
n(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 384*a^3*c*integrate(1/32*x^3
*arctan(a*x)^2/(a^2*x^6 + x^4), x) - 96*a^3*c*integrate(1/32*x^3*log(a^2*x
^2 + 1)^2/(a^2*x^6 + x^4), x) + 1792*a^2*c*integrate(1/32*x^2*arctan(a*x)^
3/(a^2*x^6 + x^4), x) + 192*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x
^2 + 1)^2/(a^2*x^6 + x^4), x) - 128*a^2*c*integrate(1/32*x^2*arctan(a*x)*l
og(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 128*a*c*integrate(1/32*x*arctan(a*x)
^2/(a^2*x^6 + x^4), x) - 32*a*c*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x
^6 + x^4), x) + 896*c*integrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 9
6*c*integrate(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3
 - 4*(3*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(3*a^2*c*x^2 + c)*arctan(a*x)*log
(a^2*x^2 + 1)^2)/x^3
```

3.370.8 Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan\left(ax\right)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^4, x)
```

3.371 $\int x^3(c+a^2cx^2)^2 \arctan(ax)^3\,dx$

3.371.1 Optimal result

Integrand size = 22, antiderivative size = 313

$$\begin{aligned}\int x^3(c+a^2cx^2)^2 \arctan(ax)^3\,dx &= \frac{c^2x}{21a^3}-\frac{c^2x^3}{168a}-\frac{1}{280}ac^2x^5-\frac{c^2\arctan(ax)}{21a^4}\\
&-\frac{5c^2x^2\arctan(ax)}{168a^2}+\frac{1}{28}c^2x^4\arctan(ax)\\
&+\frac{1}{56}a^2c^2x^6\arctan(ax)+\frac{2ic^2\arctan(ax)^2}{21a^4}\\
&+\frac{c^2x\arctan(ax)^2}{8a^3}-\frac{c^2x^3\arctan(ax)^2}{24a}\\
&-\frac{1}{8}ac^2x^5\arctan(ax)^2-\frac{3}{56}a^3c^2x^7\arctan(ax)^2\\
&-\frac{c^2\arctan(ax)^3}{24a^4}+\frac{1}{4}c^2x^4\arctan(ax)^3\\
&+\frac{1}{3}a^2c^2x^6\arctan(ax)^3+\frac{1}{8}a^4c^2x^8\arctan(ax)^3\\
&+\frac{4c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{21a^4}\\
&+\frac{2ic^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{21a^4}\end{aligned}$$

output
```
1/21*c^2*x/a^3-1/168*c^2*x^3/a-1/280*a*c^2*x^5-1/21*c^2*arctan(a*x)/a^4-5/
168*c^2*x^2*arctan(a*x)/a^2+1/28*c^2*x^4*arctan(a*x)+1/56*a^2*c^2*x^6*arct
an(a*x)+2/21*I*c^2*arctan(a*x)^2/a^4+1/8*c^2*x*arctan(a*x)^2/a^3-1/24*c^2*
x^3*arctan(a*x)^2/a-1/8*a*c^2*x^5*arctan(a*x)^2-3/56*a^3*c^2*x^7*arctan(a*
x)^2-1/24*c^2*arctan(a*x)^3/a^4+1/4*c^2*x^4*arctan(a*x)^3+1/3*a^2*c^2*x^6*
arctan(a*x)^3+1/8*a^4*c^2*x^8*arctan(a*x)^3+4/21*c^2*arctan(a*x)*ln(2/(1+I
*a*x))/a^4+2/21*I*c^2*polylog(2,1-2/(1+I*a*x))/a^4
```

3.371.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.53

$$\int x^3 \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx$$
$$= \frac{c^2 \Big(40ax - 5a^3x^3 - 3a^5x^5 - 5(16i - 21ax + 7a^3x^3 + 21a^5x^5 + 9a^7x^7) \arctan(ax)^2 + 35(1+a^2x^2)^3 (-1 +$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(40*a*x - 5*a^3*x^3 - 3*a^5*x^5 - 5*(16*I - 21*a*x + 7*a^3*x^3 + 21*a
^5*x^5 + 9*a^7*x^7)*ArcTan[a*x]^2 + 35*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*Ar
cTan[a*x]^3 + 5*ArcTan[a*x]*(-8 - 5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 + 32*L
og[1 + E^((2*I)*ArcTan[a*x])]) - (80*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]
))/(840*a^4)
```

3.371.3 Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \left(a^2cx^2+c\right)^2 \, dx$$
$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^4c^2x^7\arctan(ax)^3+2a^2c^2x^5\arctan(ax)^3+c^2x^3\arctan(ax)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}a^4c^2x^8\arctan(ax)^3-\frac{c^2\arctan(ax)^3}{24a^4}+\frac{2ic^2\arctan(ax)^2}{21a^4}-\frac{c^2\arctan(ax)}{21a^4}+\frac{4c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{21a^4}+\frac{2ic^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{21a^4}-\frac{3}{56}a^3c^2x^7\arctan(ax)^2+\frac{c^2x\arctan(ax)^2}{8a^3}+\frac{c^2x}{21a^3}+\frac{1}{3}a^2c^2x^6\arctan(ax)^3+\frac{1}{56}a^2c^2x^6\arctan(ax)-\frac{5c^2x^2\arctan(ax)}{168a^2}-\frac{1}{8}ac^2x^5\arctan(ax)^2+\frac{1}{4}c^2x^4\arctan(ax)^3+\frac{1}{28}c^2x^4\arctan(ax)-\frac{c^2x^3\arctan(ax)^2}{24a}-\frac{1}{280}ac^2x^5-\frac{c^2x^3}{168a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*x)/(21*a^3) - (c^2*x^3)/(168*a) - (a*c^2*x^5)/280 - (c^2*ArcTan[a*x])
/(21*a^4) - (5*c^2*x^2*ArcTan[a*x])/(168*a^2) + (c^2*x^4*ArcTan[a*x])/28 +
 (a^2*c^2*x^6*ArcTan[a*x])/56 + (((2*I)/21)*c^2*ArcTan[a*x]^2)/a^4 + (c^2*
x*ArcTan[a*x]^2)/(8*a^3) - (c^2*x^3*ArcTan[a*x]^2)/(24*a) - (a*c^2*x^5*Arc
Tan[a*x]^2)/8 - (3*a^3*c^2*x^7*ArcTan[a*x]^2)/56 - (c^2*ArcTan[a*x]^3)/(24
*a^4) + (c^2*x^4*ArcTan[a*x]^3)/4 + (a^2*c^2*x^6*ArcTan[a*x]^3)/3 + (a^4*c
^2*x^8*ArcTan[a*x]^3)/8 + (4*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(21*a^4)
+ (((2*I)/21)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

3.371.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.371. $\int x^3(c+a^2cx^2)^2\arctan(ax)^3\,dx$

3.371.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{c^2\arctan(ax)^3a^8x^8}{8}+\frac{c^2\arctan(ax)^3a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^3}{4}-\frac{c^2\arctan(ax)^3}{24}-\frac{c^2\left(\frac{3\arctan(ax)^2a^7x^7}{7}+a^5\arctan(ax)^2x^5+\frac{a^3}{}\right.}{}$
default	$\frac{c^2\arctan(ax)^3a^8x^8}{8}+\frac{c^2\arctan(ax)^3a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^3}{4}-\frac{c^2\arctan(ax)^3}{24}-\frac{c^2\left(\frac{3\arctan(ax)^2a^7x^7}{7}+a^5\arctan(ax)^2x^5+\frac{a^3}{}\right.}{}$
parts	$\frac{a^4c^2x^8\arctan(ax)^3}{8}+\frac{a^2c^2x^6\arctan(ax)^3}{3}+\frac{c^2x^4\arctan(ax)^3}{4}-\frac{c^2\left(\frac{3a^3\arctan(ax)^2x^7}{7}+a\arctan(ax)^2x^5+\frac{\arctan(}{3a}\right.}{}$

input
```
int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*c^2*arctan(a*x)^3*a^8*x^8+1/3*c^2*arctan(a*x)^3*a^6*x^6+1/4*a^4
*c^2*x^4*arctan(a*x)^3-1/24*c^2*arctan(a*x)^3-1/8*c^2*(3/7*arctan(a*x)^2*a
^7*x^7+a^5*arctan(a*x)^2*x^5+1/3*a^3*arctan(a*x)^2*x^3-a*arctan(a*x)^2*x-1
/7*a^6*arctan(a*x)*x^6-2/7*arctan(a*x)*a^4*x^4+5/21*a^2*arctan(a*x)*x^2+16
/21*arctan(a*x)*ln(a^2*x^2+1)+1/35*a^5*x^5+1/21*a^3*x^3-8/21*a*x+8/21*arct
an(a*x)+8/21*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln
(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)-8/21*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/
2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.371.5 Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3, x)
```

3.371.6 Sympy [F]

$$\int x^3(c+a^2cx^2)^2\arctan(ax)^3\,dx = c^2\left(\int x^3\operatorname{atan}^3(ax)\,dx + \int 2a^2x^5\operatorname{atan}^3(ax)\,dx + \int a^4x^7\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**3*atan(a*x)**3, x) + Integral(2*a**2*x**5*atan(a*x)**3,
x) + Integral(a**4*x**7*atan(a*x)**3, x))
```

3.371.7 Maxima [F]

$$\int x^3(c+a^2cx^2)^2\arctan(ax)^3\,dx = \int (a^2cx^2+c)^2x^3\arctan(ax)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2688*(28*(129024*a^9*c^2*integrate(1/2688*x^9*arctan(a*x)^3/(a^5*x^2 + a
^3), x) - 24192*a^8*c^2*integrate(1/2688*x^8*arctan(a*x)^2/(a^5*x^2 + a^3)
, x) - 6048*a^8*c^2*integrate(1/2688*x^8*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3
), x) - 3456*a^8*c^2*integrate(1/2688*x^8*log(a^2*x^2 + 1)/(a^5*x^2 + a^3)
, x) + 387072*a^7*c^2*integrate(1/2688*x^7*arctan(a*x)^3/(a^5*x^2 + a^3),
x) + 6912*a^7*c^2*integrate(1/2688*x^7*arctan(a*x)/(a^5*x^2 + a^3), x) - 6
4512*a^6*c^2*integrate(1/2688*x^6*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 1612
8*a^6*c^2*integrate(1/2688*x^6*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 80
64*a^6*c^2*integrate(1/2688*x^6*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 387
072*a^5*c^2*integrate(1/2688*x^5*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 16128
*a^5*c^2*integrate(1/2688*x^5*arctan(a*x)/(a^5*x^2 + a^3), x) - 48384*a^4*
c^2*integrate(1/2688*x^4*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 12096*a^4*c^2
*integrate(1/2688*x^4*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 2688*a^4*c^
2*integrate(1/2688*x^4*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 129024*a^3*c
^2*integrate(1/2688*x^3*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 5376*a^3*c^2*i
ntegrate(1/2688*x^3*arctan(a*x)/(a^5*x^2 + a^3), x) + 8064*a^2*c^2*integra
te(1/2688*x^2*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) - 16128*a*c^2*integrate
(1/2688*x*arctan(a*x)/(a^5*x^2 + a^3), x) + 2016*c^2*integrate(1/2688*log(
a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + c^2*arctan(a*x)^3/a^4)*a^4 + 56*(3*a^
8*c^2*x^8 + 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x)^3 - 4*(9*a...
```

3.371.8 Giac [F]

$$\int x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^2 x^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = \int x^3 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^2 dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.372 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^3 \, dx$

3.372.1 Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int x^2(c+a^2cx^2)^2 \arctan(ax)^3 \, dx = &-\frac{11c^2x^2}{420a}-\frac{1}{140}ac^2x^4-\frac{c^2x\arctan(ax)}{70a^2}\\
&+\frac{17}{210}c^2x^3\arctan(ax)+\frac{1}{35}a^2c^2x^5\arctan(ax)\\
&+\frac{c^2\arctan(ax)^2}{140a^3}-\frac{4c^2x^2\arctan(ax)^2}{35a}\\
&-\frac{27}{140}ac^2x^4\arctan(ax)^2-\frac{1}{14}a^3c^2x^6\arctan(ax)^2\\
&-\frac{8ic^2\arctan(ax)^3}{105a^3}+\frac{1}{3}c^2x^3\arctan(ax)^3\\
&+\frac{2}{5}a^2c^2x^5\arctan(ax)^3+\frac{1}{7}a^4c^2x^7\arctan(ax)^3\\
&-\frac{8c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{35a^3}+\frac{c^2\log(1+a^2x^2)}{30a^3}\\
&-\frac{8ic^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^3}\\
&-\frac{4c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{35a^3}\end{aligned}$$

output
```
-11/420*c^2*x^2/a-1/140*a*c^2*x^4-1/70*c^2*x*arctan(a*x)/a^2+17/210*c^2*x^
3*arctan(a*x)+1/35*a^2*c^2*x^5*arctan(a*x)+1/140*c^2*arctan(a*x)^2/a^3-4/3
5*c^2*x^2*arctan(a*x)^2/a-27/140*a*c^2*x^4*arctan(a*x)^2-1/14*a^3*c^2*x^6*
arctan(a*x)^2-8/35*I*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^3+1/3*c^2*
x^3*arctan(a*x)^3+2/5*a^2*c^2*x^5*arctan(a*x)^3+1/7*a^4*c^2*x^7*arctan(a*x
)^3-8/35*c^2*arctan(a*x)^2*ln(2/(1+I*a*x))/a^3+1/30*c^2*ln(a^2*x^2+1)/a^3-
8/105*I*c^2*arctan(a*x)^3/a^3-4/35*c^2*polylog(3,1-2/(1+I*a*x))/a^3
```

3.372.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.73

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^3\,dx$$
$$= \frac{c^2(-8-11a^2x^2-3a^4x^4-6ax\arctan(ax)+34a^3x^3\arctan(ax)+12a^5x^5\arctan(ax)+3\arctan(ax)^2-4}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(-8 - 11*a^2*x^2 - 3*a^4*x^4 - 6*a*x*ArcTan[a*x] + 34*a^3*x^3*ArcTan[
a*x] + 12*a^5*x^5*ArcTan[a*x] + 3*ArcTan[a*x]^2 - 48*a^2*x^2*ArcTan[a*x]^2
 - 81*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 + (32*I)*ArcTan[a*x
]^3 + 140*a^3*x^3*ArcTan[a*x]^3 + 168*a^5*x^5*ArcTan[a*x]^3 + 60*a^7*x^7*A
rcTan[a*x]^3 - 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 14*Log[1
+ a^2*x^2] + (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 48*Po
lyLog[3, -E^((2*I)*ArcTan[a*x])]))/(420*a^3)
```

3.372.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)^3\left(a^2cx^2+c\right)^2\,dx$$
$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^6\arctan(ax)^3+2a^2c^2x^4\arctan(ax)^3+c^2x^2\arctan(ax)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}a^4c^2x^7\arctan(ax)^3-\frac{8ic^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{35a^3}-\frac{1}{14}a^3c^2x^6\arctan(ax)^2-\frac{8ic^2\arctan(ax)^3}{105a^3}+\frac{c^2\arctan(ax)^2}{140a^3}-\frac{8c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{35a^3}-\frac{4c^2\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{35a^3}+\frac{2}{5}a^2c^2x^5\arctan(ax)^3+\frac{1}{35}a^2c^2x^5\arctan(ax)-\frac{c^2x\arctan(ax)}{70a^2}+\frac{c^2\log\left(a^2x^2+1\right)}{30a^3}-\frac{27}{140}ac^2x^4\arctan(ax)^2+\frac{1}{3}c^2x^3\arctan(ax)^3+\frac{17}{210}c^2x^3\arctan(ax)-\frac{4c^2x^2\arctan(ax)^2}{35a}-\frac{1}{140}ac^2x^4-\frac{11c^2x^2}{420a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(-11*c^2*x^2)/(420*a) - (a*c^2*x^4)/140 - (c^2*x*ArcTan[a*x])/(70*a^2) + (
17*c^2*x^3*ArcTan[a*x])/210 + (a^2*c^2*x^5*ArcTan[a*x])/35 + (c^2*ArcTan[a
*x]^2)/(140*a^3) - (4*c^2*x^2*ArcTan[a*x]^2)/(35*a) - (27*a*c^2*x^4*ArcTan
[a*x]^2)/140 - (a^3*c^2*x^6*ArcTan[a*x]^2)/14 - (((8*I)/105)*c^2*ArcTan[a*
x]^3)/a^3 + (c^2*x^3*ArcTan[a*x]^3)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^3)/5 +
(a^4*c^2*x^7*ArcTan[a*x]^3)/7 - (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(
35*a^3) + (c^2*Log[1 + a^2*x^2])/(30*a^3) - (((8*I)/35)*c^2*ArcTan[a*x]*Po
lyLog[2, 1 - 2/(1 + I*a*x)])/a^3 - (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(
35*a^3)
```

3.372.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.372.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 72.00 (sec) , antiderivative size = 1256, normalized size of antiderivative = 3.91

method	result	size
derivativedivides	Expression too large to display	1256
default	Expression too large to display	1256
parts	Expression too large to display	1256

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/7*c^2*arctan(a*x)^3*a^7*x^7+2/5*c^2*arctan(a*x)^3*a^5*x^5+1/3*c^2
*arctan(a*x)^3*a^3*x^3-1/35*c^2*(-2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-1/4*arctan(a*x)^2+4*x^2*arctan(a*x)
^2*a^2+8*arctan(a*x)^2*ln(2)+1/4*(I+a*x)^4+5/2*a^6*x^6*arctan(a*x)^2+27/4*
a^4*arctan(a*x)^2*x^4+4*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+7/3*ln((1+I*a*
x)^2/(a^2*x^2+1)+1)-4*arctan(a*x)^2*ln(a^2*x^2+1)-5*I*arctan(a*x)*(a*x-I)^
4-8*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+5*arctan(a*x)*(a*x-I
)^4*(I+a*x)-5*arctan(a*x)*(a*x-I)*(I+a*x)^4+10*arctan(a*x)*(a*x-I)^2*(I+a*
x)^3-2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
3*arctan(a*x)^2+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)
^2-2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2-30*I*arctan(a*x)
*(a*x-I)^2*(I+a*x)^2+20*I*arctan(a*x)*(a*x-I)*(I+a*x)^3-7/12*(I+a*x)^2-8/3
*I*arctan(a*x)^3-5/6*I*(I+a*x)-I*(I+a*x)^3+43/6*arctan(a*x)*(a*x-I)^3-arct
an(a*x)*(a*x-I)^5+20*I*arctan(a*x)*(a*x-I)^3*(I+a*x)-3*I*arctan(a*x)*(a*x-
I)*(I+a*x)+3/2*I*arctan(a*x)*(a*x-I)^2-10*arctan(a*x)*(a*x-I)^3*(I+a*x)^2+
43/2*arctan(a*x)*(a*x-I)*(I+a*x)^2-43/2*arctan(a*x)*(a*x-I)^2*(I+a*x)+8*ar
ctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan(a*x)*(a*x-I)-2*I*Pi*c
sgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arcta
n(a*x)^2+2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^...
```

3.372.5 Fricas [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^3\,dx = \int (a^2cx^2+c)^2x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3, x)
```

3.372.6 Sympy [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^3\,dx = c^2\left(\int x^2\operatorname{atan}^3(ax)\,dx + \int 2a^2x^4\operatorname{atan}^3(ax)\,dx + \int a^4x^6\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**3, x) + Integral(2*a**2*x**4*atan(a*x)**3,
x) + Integral(a**4*x**6*atan(a*x)**3, x))
```

3.372.7 Maxima [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^3\,dx = \int (a^2cx^2+c)^2x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/840*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^3 - 1/112
0*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)*log(a^2*x^2 +
 1)^2 + integrate(1/1120*(980*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4
 + c^2*x^2)*arctan(a*x)^3 - 4*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*
x^3)*arctan(a*x)^2 + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*
arctan(a*x)*log(a^2*x^2 + 1) + (15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2
*x^3 + 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*arctan(
a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

3.372.8 Giac [F]

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx=\int x^2\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.373 $\int x(c+a^2cx^2)^2 \arctan(ax)^3 \, dx$

3.373.1 Optimal result

Integrand size = 20, antiderivative size = 242

$$\begin{aligned}\int x(c+a^2cx^2)^2 \arctan(ax)^3 \, dx = & -\frac{11c^2x}{60a} - \frac{1}{60}ac^2x^3 + \frac{2c^2(1+a^2x^2)\arctan(ax)}{15a^2} \\ & + \frac{c^2(1+a^2x^2)^2 \arctan(ax)}{20a^2} - \frac{4ic^2\arctan(ax)^2}{15a^2} \\ & - \frac{4c^2x\arctan(ax)^2}{15a} - \frac{2c^2x(1+a^2x^2)\arctan(ax)^2}{15a} \\ & - \frac{c^2x(1+a^2x^2)^2\arctan(ax)^2}{10a} \\ & + \frac{c^2(1+a^2x^2)^3\arctan(ax)^3}{6a^2} - \frac{8c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a^2} \\ & - \frac{4ic^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^2}\end{aligned}$$

output
```
-11/60*c^2*x/a-1/60*a*c^2*x^3+2/15*c^2*(a^2*x^2+1)*arctan(a*x)/a^2+1/20*c^
2*(a^2*x^2+1)^2*arctan(a*x)/a^2-4/15*I*c^2*arctan(a*x)^2/a^2-4/15*c^2*x*ar
ctan(a*x)^2/a-2/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^2/a-1/10*c^2*x*(a^2*x^2+1
)^2*arctan(a*x)^2/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^3/a^2-8/15*c^2*arcta
n(a*x)*ln(2/(1+I*a*x))/a^2-4/15*I*c^2*polylog(2,1-2/(1+I*a*x))/a^2
```

3.373.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.54

$$\int x(c+a^2cx^2)^2\arctan(ax)^3\,dx$$
$$=\frac{c^2\Big(-ax(11+a^2x^2)-2(-8i+15ax+10a^3x^3+3a^5x^5)\arctan(ax)^2+10(1+a^2x^2)^3\arctan(ax)^3+\text{arcta}}{60a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(-(a*x*(11 + a^2*x^2)) - 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*A
rcTan[a*x]^2 + 10*(1 + a^2*x^2)^3*ArcTan[a*x]^3 + ArcTan[a*x]*(11 + 14*a^2
*x^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (16*I)*PolyLog[2,
-E^((2*I)*ArcTan[a*x])]))/(60*a^2)
```

3.373.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5465, 27, 5415, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^3\left(a^2cx^2+c\right)^2\,dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^2dx}{2a}$$
$$\Big\downarrow \text{27}$$
$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx}{2a}$$
$$\Big\downarrow \text{5415}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-$$
$$\frac{c^2\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}\int\left(a^2x^2+1\right)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}\right)}{2a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-$$
$$\frac{c^2\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}+x\right)\right)}{2a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-$$
$$\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2\right.}{2a}$$

$$\Big\downarrow \text{24}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-$$
$$\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\right.}{2a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-$$
$$\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2x^2+\right.\right.}{2a}$$

$$\Big\downarrow \text{5455}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-$$
$$\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\right.\right.}{2a}$$

$$\Big\downarrow \text{5379}$$

3.373. $\int x\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx$

$$\frac{c^2(a^2x^2+1)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-}{2a}$$

$$\Big\downarrow \textbf{2849}$$

$$\frac{c^2(a^2x^2+1)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(a}{2a}$$

$$\Big\downarrow \textbf{2752}$$

$$\frac{c^2(a^2x^2+1)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax}{2a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^3)/(6*a^2) - (c^2*((x + (a^2*x^3)/3)/10 -
 ((1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) + (x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/
5 + (4*(x/3 - ((1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[
a*x]^2)/3 + (2*(x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((Ar
cTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)
/a)))/3))/5))/(2*a)
```

3.373.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.373. $\int x(c+a^2cx^2)^2\arctan(ax)^3\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.373. $\int x(c+a^2cx^2)^2 \arctan(ax)^3 \, dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.373.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.22

method	result
parts	$\frac{c^2\arctan(ax)^3a^4x^6}{6}+\frac{c^2\arctan(ax)^3a^2x^4}{2}+\frac{c^2\arctan(ax)^3x^2}{2}+\frac{c^2\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{a^5\arctan(ax)^2x^5}{5}+\frac{2a^3\operatorname{arc}\ldots\right.}{\ldots}$
derivativedivides	$\frac{\frac{c^2\arctan(ax)^3a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^3}{2}+\frac{a^2c^2x^2\arctan(ax)^3}{2}+\frac{c^2\arctan(ax)^3}{6}-\frac{c^2\left(\frac{a^5\arctan(ax)^2x^5}{5}+\frac{2a^3\arctan(ax)^2x^3}{3}+a\ldots\right.}{\ldots}}{\ldots}$
default	$\frac{\frac{c^2\arctan(ax)^3a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^3}{2}+\frac{a^2c^2x^2\arctan(ax)^3}{2}+\frac{c^2\arctan(ax)^3}{6}-\frac{c^2\left(\frac{a^5\arctan(ax)^2x^5}{5}+\frac{2a^3\arctan(ax)^2x^3}{3}+a\ldots\right.}{\ldots}}{\ldots}$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*arctan(a*x)^3*a^4*x^6+1/2*c^2*arctan(a*x)^3*a^2*x^4+1/2*c^2*arctan
(a*x)^3*x^2+1/6*c^2*arctan(a*x)^3/a^2-1/2/a^2*c^2*(1/5*a^5*arctan(a*x)^2*x
^5+2/3*a^3*arctan(a*x)^2*x^3+a*arctan(a*x)^2*x-1/10*arctan(a*x)*a^4*x^4-7/
15*a^2*arctan(a*x)*x^2-8/15*arctan(a*x)*ln(a^2*x^2+1)+1/30*a^3*x^3+11/30*a
*x-11/30*arctan(a*x)-4/15*I*(ln(a*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))
-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+4/15*I*(ln(I+a*x)*ln(a^2*x^
2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2))
```

3.373.5 Fricas [F]

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3, x)
```

3.373.6 Sympy [F]

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx=c^2\left(\int x\operatorname{atan}^3\left(ax\right)dx+\int 2a^2x^3\operatorname{atan}^3\left(ax\right)dx\right.$$
$$\left.+\int a^4x^5\operatorname{atan}^3\left(ax\right)dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x*atan(a*x)**3, x) + Integral(2*a**2*x**3*atan(a*x)**3, x)
+ Integral(a**4*x**5*atan(a*x)**3, x))
```

3.373.7 Maxima [F]

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/480*(40*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x)^
3 + 20*(5760*a^7*c^2*integrate(1/480*x^7*arctan(a*x)^3/(a^3*x^2 + a), x) -
 1440*a^6*c^2*integrate(1/480*x^6*arctan(a*x)^2/(a^3*x^2 + a), x) - 360*a^
6*c^2*integrate(1/480*x^6*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 288*a^6*c
^2*integrate(1/480*x^6*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 17280*a^5*c^2*
integrate(1/480*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) + 576*a^5*c^2*integrat
e(1/480*x^5*arctan(a*x)/(a^3*x^2 + a), x) - 4320*a^4*c^2*integrate(1/480*x
^4*arctan(a*x)^2/(a^3*x^2 + a), x) - 1080*a^4*c^2*integrate(1/480*x^4*log(
a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 960*a^4*c^2*integrate(1/480*x^4*log(a^2
*x^2 + 1)/(a^3*x^2 + a), x) + 17280*a^3*c^2*integrate(1/480*x^3*arctan(a*x
)^3/(a^3*x^2 + a), x) + 1920*a^3*c^2*integrate(1/480*x^3*arctan(a*x)/(a^3*
x^2 + a), x) - 4320*a^2*c^2*integrate(1/480*x^2*arctan(a*x)^2/(a^3*x^2 + a
), x) - 1080*a^2*c^2*integrate(1/480*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a),
 x) - 1440*a^2*c^2*integrate(1/480*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x)
+ 5760*a*c^2*integrate(1/480*x*arctan(a*x)^3/(a^3*x^2 + a), x) + 2880*a*c^
2*integrate(1/480*x*arctan(a*x)/(a^3*x^2 + a), x) - c^2*arctan(a*x)^3/a^2
- 360*c^2*integrate(1/480*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x))*a^2 - 4*(3
*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x)^2 + (3*a^5*c^2*x^5
 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a^2*x^2 + 1)^2)/a^2
```

3.373.8 Giac [F]

$$\int x(c+a^2cx^2)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^2\arctan(ax)^3\,dx=\int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.374 $\int (c+a^2cx^2)^2 \arctan(ax)^3\,dx$

3.374.1 Optimal result

Integrand size = 19, antiderivative size = 289

$$\begin{aligned}\int (c+a^2cx^2)^2 \arctan(ax)^3\,dx = &-\frac{c^2(1+a^2x^2)}{20a}+c^2x\arctan(ax)\\&+\frac{1}{10}c^2x(1+a^2x^2)\arctan(ax)-\frac{2c^2(1+a^2x^2)\arctan(ax)^2}{5a}\\&-\frac{3c^2(1+a^2x^2)^2\arctan(ax)^2}{20a}+\frac{8ic^2\arctan(ax)^3}{15a}\\&+\frac{8}{15}c^2x\arctan(ax)^3+\frac{4}{15}c^2x(1+a^2x^2)\arctan(ax)^3\\&+\frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax)^3\\&+\frac{8c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a}-\frac{c^2\log(1+a^2x^2)}{2a}\\&+\frac{8ic^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a}\\&+\frac{4c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{5a}\end{aligned}$$

output
```
-1/20*c^2*(a^2*x^2+1)/a+c^2*x*arctan(a*x)+1/10*c^2*x*(a^2*x^2+1)*arctan(a*
x)-2/5*c^2*(a^2*x^2+1)*arctan(a*x)^2/a-3/20*c^2*(a^2*x^2+1)^2*arctan(a*x)^
2/a+8/15*I*c^2*arctan(a*x)^3/a+8/15*c^2*x*arctan(a*x)^3+4/15*c^2*x*(a^2*x^
2+1)*arctan(a*x)^3+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)^3+8/5*c^2*arctan(a*
x)^2*ln(2/(1+I*a*x))/a-1/2*c^2*ln(a^2*x^2+1)/a+8/5*I*c^2*arctan(a*x)*polyl
og(2,1-2/(1+I*a*x))/a+4/5*c^2*polylog(3,1-2/(1+I*a*x))/a
```

3.374.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.67

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3\,dx$$
$$= \frac{c^2\left(-3-3a^2x^2+66ax\arctan(ax)+6a^3x^3\arctan(ax)-33\arctan(ax)^2-42a^2x^2\arctan(ax)^2-9a^4x^4\text{ar}\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(-3 - 3*a^2*x^2 + 66*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 33*Arc
Tan[a*x]^2 - 42*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 - (32*I)*A
rcTan[a*x]^3 + 60*a*x*ArcTan[a*x]^3 + 40*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^
5*ArcTan[a*x]^3 + 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 30*Log
[1 + a^2*x^2] - (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 48
*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(60*a)
```

3.374.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {5415, 27, 5413, 5345, 240, 5415, 5345, 240, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3\left(a^2cx^2+c\right)^2dx$$
$$\Big\downarrow \text{5415}$$
$$\frac{3}{10}c\int c\left(a^2x^2+1\right)\arctan(ax)dx+\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$
$$\Big\downarrow \text{27}$$
$$\frac{3}{10}c^2\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{3}{10}c^2\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{3}{10}c^2\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$

$$\Big\downarrow \text{240}$$

$$\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{5415}$$

$$\frac{4}{5}c^2\left(\int \arctan(ax)dx+\frac{2}{3}\int \arctan(ax)^3dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{5345}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)-a\int \frac{x}{a^2x^2+1}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\text{a}}{2a}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{240}$$

3.374. $\int \left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}-\frac{\log\left(\right.}{}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\mathrm{arc}}{2a}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2\right.\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{5529}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right.}{a}\right.\right.\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{7164}$$

3.374. $\int\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\mathrm{PolyLog}}{}\right.}{a}\right.\right.\right.$$

$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(-3*c^2*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(20*a) + (c^2*x*(1 + a^2*x^2)^2*Arc
Tan[a*x]^3)/5 + (3*c^2*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x
])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/10 + (4*c^2*(x*Arc
Tan[a*x] - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a) + (x*(1 + a^2*x^2)*ArcTan[a
*x]^3)/3 - Log[1 + a^2*x^2]/(2*a) + (2*(x*ArcTan[a*x]^3 - 3*a*(((-1/3*I)*A
rcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*A
rcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)
]/(4*a)))/a)))/3))/5
```

3.374.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5413

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.374. $\int (c + a^2cx^2)^2 \arctan(ax)^3 \, dx$

3.374.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.29 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.30

method	result	size
derivativedivides	Expression too large to display	953
default	Expression too large to display	953
parts	Expression too large to display	955

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*c^2*arctan(a*x)^3*a^5*x^5+2/3*c^2*arctan(a*x)^3*a^3*x^3+c^2*arcta
n(a*x)^3*a*x-1/5*c^2*(3/4*a^4*arctan(a*x)^2*x^4+7/2*x^2*arctan(a*x)^2*a^2+
4*arctan(a*x)^2*ln(a^2*x^2+1)-8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2))+1/12*I*(24*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-48*arcta
n(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+24*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))^2-24*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+24*arcta
n(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+24*a
rctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)^3-24*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-24*arctan(a*x)^2*Pi*
csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)+48*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)^2-24*arctan(a*x)^2*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3+32*arctan(a*x)^3-3*I*a^2*x^2+66*I*arctan(a*x)*a*x-33*I*arctan(a*
x)^2+60*arctan(a*x)+6*I*arctan(a*x)*a^3*x^3+96*I*arctan(a*x)^2*ln(2)-3*I)-
5*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+8*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^
2*x^2+1))-4*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))))
```

3.374.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3, x)
```

3.374.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = c^2 \left(\int 2a^2x^2 \operatorname{atan}^3\left(ax\right) dx + \int a^4x^4 \operatorname{atan}^3\left(ax\right) dx + \int \operatorname{atan}^3\left(ax\right) dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**3, x) + Integral(a**4*x**4*atan(a*x)
**3, x) + Integral(atan(a*x)**3, x))
```

3.374.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
140*a^6*c^2*integrate(1/160*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 15*a^6*c
^2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
12*a^6*c^2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1),
 x) - 12*a^5*c^2*integrate(1/160*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a
^5*c^2*integrate(1/160*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^4*
c^2*integrate(1/160*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 45*a^4*c^2*integ
rate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c
^2*integrate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40
*a^3*c^2*integrate(1/160*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + 10*a^3*c^2*
integrate(1/160*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*c^2*arctan
(a*x)^4/a + 420*a^2*c^2*integrate(1/160*x^2*arctan(a*x)^3/(a^2*x^2 + 1), x
) + 45*a^2*c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2
 + 1), x) + 60*a^2*c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a
^2*x^2 + 1), x) + 1/120*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan
(a*x)^3 - 60*a*c^2*integrate(1/160*x*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*
a*c^2*integrate(1/160*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/160*(3*a^
4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 15
*c^2*integrate(1/160*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

3.374.8 Giac [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3\,dx = \int \mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.375 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx$

3.375.1 Optimal result

Integrand size = 22, antiderivative size = 370

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x}\,dx =& -\frac{1}{4}ac^2x+\frac{1}{4}c^2\arctan(ax)+\frac{1}{4}a^2c^2x^2\arctan(ax)\\
&-2ic^2\arctan(ax)^2-\frac{9}{4}ac^2x\arctan(ax)^2\\
&-\frac{1}{4}a^3c^2x^3\arctan(ax)^2+\frac{3}{4}c^2\arctan(ax)^3\\
&+a^2c^2x^2\arctan(ax)^3+\frac{1}{4}a^4c^2x^4\arctan(ax)^3\\
&+2c^2\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)\\
&-4c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\
&-2ic^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\
&-\frac{3}{2}ic^2\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}ic^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\
&-\frac{3}{2}c^2\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}c^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\\
&+\frac{3}{4}ic^2\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right)\\
&-\frac{3}{4}ic^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-1/4*a*c^2*x+1/4*c^2*arctan(a*x)+1/4*a^2*c^2*x^2*arctan(a*x)-3/4*I*c^2*pol
ylog(4,-1+2/(1+I*a*x))-9/4*a*c^2*x*arctan(a*x)^2-1/4*a^3*c^2*x^3*arctan(a*
x)^2+3/4*c^2*arctan(a*x)^3+a^2*c^2*x^2*arctan(a*x)^3+1/4*a^4*c^2*x^4*arcta
n(a*x)^3-2*c^2*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-4*c^2*arctan(a*x)*ln(
2/(1+I*a*x))-2*I*c^2*arctan(a*x)^2+3/4*I*c^2*polylog(4,1-2/(1+I*a*x))-3/2*
I*c^2*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))-3/2*c^2*arctan(a*x)*polylog(3
,1-2/(1+I*a*x))+3/2*c^2*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/2*I*c^2*ar
ctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))-2*I*c^2*polylog(2,1-2/(1+I*a*x))
```

3.375.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.82

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x}\,dx = \frac{1}{64}c^2\left(-i\pi^4-16ax+16\arctan(ax)+16a^2x^2\arctan(ax)\right.$$
$$+128i\arctan(ax)^2-144ax\arctan(ax)^2$$
$$-16a^3x^3\arctan(ax)^2+48\arctan(ax)^3$$
$$+64a^2x^2\arctan(ax)^3+16a^4x^4\arctan(ax)^3$$
$$+32i\arctan(ax)^4+64\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right)$$
$$-256\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right)$$
$$-64\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right)$$
$$+96i\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)$$
$$+32i\left(4+3\arctan(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)$$
$$+96\arctan(ax)\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)$$
$$-96\arctan(ax)\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)$$
$$-48i\operatorname{PolyLog}\left(4,e^{-2i\arctan(ax)}\right)$$
$$\left.-48i\operatorname{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```

output
```
(c^2*((-I)*Pi^4 - 16*a*x + 16*ArcTan[a*x] + 16*a^2*x^2*ArcTan[a*x] + (128*
I)*ArcTan[a*x]^2 - 144*a*x*ArcTan[a*x]^2 - 16*a^3*x^3*ArcTan[a*x]^2 + 48*A
rcTan[a*x]^3 + 64*a^2*x^2*ArcTan[a*x]^3 + 16*a^4*x^4*ArcTan[a*x]^3 + (32*I
)*ArcTan[a*x]^4 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - 256*A
rcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 64*ArcTan[a*x]^3*Log[1 + E^((2
*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])
] + (32*I)*(4 + 3*ArcTan[a*x]^2)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 96*A
rcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3,
-E^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (48*
I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]))/64
```

3.375.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^2}{x} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^3\arctan(ax)^3+2a^2c^2x\arctan(ax)^3+\frac{c^2\arctan(ax)^3}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^4c^2x^4\arctan(ax)^3-\frac{1}{4}a^3c^2x^3\arctan(ax)^2+a^2c^2x^2\arctan(ax)^3+\frac{1}{4}a^2c^2x^2\arctan(ax)+2c^2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}ic^2\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{2}ic^2\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{3}{2}c^2\arctan(ax)\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{3}{2}c^2\arctan(ax)\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)-\frac{9}{4}ac^2x\arctan(ax)^2+\frac{3}{4}c^2\arctan(ax)^3-2ic^2\arctan(ax)^2+\frac{1}{4}c^2\arctan(ax)-4c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)-2ic^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{4}ic^2\text{PolyLog}\left(4,1-\frac{2}{iax+1}\right)-\frac{3}{4}ic^2\text{PolyLog}\left(4,\frac{2}{iax+1}-1\right)-\frac{1}{4}ac^2x$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```

output
```
-1/4*(a*c^2*x) + (c^2*ArcTan[a*x])/4 + (a^2*c^2*x^2*ArcTan[a*x])/4 - (2*I)
*c^2*ArcTan[a*x]^2 - (9*a*c^2*x*ArcTan[a*x]^2)/4 - (a^3*c^2*x^3*ArcTan[a*x
]^2)/4 + (3*c^2*ArcTan[a*x]^3)/4 + a^2*c^2*x^2*ArcTan[a*x]^3 + (a^4*c^2*x^
4*ArcTan[a*x]^3)/4 + 2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 4*c^
2*ArcTan[a*x]*Log[2/(1 + I*a*x)] - (2*I)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)]
 - ((3*I)/2)*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*c
^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^2*ArcTan[a*x]*PolyL
og[3, 1 - 2/(1 + I*a*x)])/2 + (3*c^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*
a*x)])/2 + ((3*I)/4)*c^2*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^2*Pol
yLog[4, -1 + 2/(1 + I*a*x)]
```

3.375. $\int \frac{\left(c+a^2cx^2\right)^2\arctan(ax)^3}{x}\,dx$

3.375.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.375.4 Maple [A] (verified)

Time = 24.32 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{c^2\left(-3i\arctan(ax)^3+3\arctan(ax)^3ax-i\arctan(ax)^3a^2x^2+\arctan(ax)^3a^3x^3-8\arctan(ax)^2+i\arctan(ax)^2ax-x^2 \text{arct}\right.}{4}$
default	$\frac{c^2\left(-3i\arctan(ax)^3+3\arctan(ax)^3ax-i\arctan(ax)^3a^2x^2+\arctan(ax)^3a^3x^3-8\arctan(ax)^2+i\arctan(ax)^2ax-x^2 \text{arct}\right.}{4}$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*(-3*I*arctan(a*x)^3+3*arctan(a*x)^3*a*x-I*arctan(a*x)^3*a^2*x^2+ar
ctan(a*x)^3*a^3*x^3-8*arctan(a*x)^2+I*arctan(a*x)^2*a*x-x^2*arctan(a*x)^2*
a^2-I*arctan(a*x)+x*arctan(a*x)*a-1)*(I+a*x)+c^2*arctan(a*x)^3*ln((1+I*a*x
)/(a^2*x^2+1)^(1/2)+1)+2*I*c^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+6*c^2*a
rctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^2*polylog(4,(1+I*
a*x)/(a^2*x^2+1)^(1/2))-c^2*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+4*
I*c^2*arctan(a*x)^2-3/2*c^2*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1)
)-3*I*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+c^2*arctan(
a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^2*arctan(a*x)^2*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^
2+1)^(1/2))-3/4*I*c^2*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+3/2*I*c^2*arctan
(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-4*c^2*arctan(a*x)*ln((1+I*a*x)
^2/(a^2*x^2+1)+1)+6*I*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

3.375.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x, x)
```

3.375.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx = c^2 \left(\int \frac{\operatorname{atan}^3(ax)}{x} \, dx + \int 2a^2x \operatorname{atan}^3(ax) \, dx + \int a^4x^3 \operatorname{atan}^3(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x,x)
```

output
```
c**2*(Integral(atan(a*x)**3/x, x) + Integral(2*a**2*x*atan(a*x)**3, x) + I
ntegral(a**4*x**3*atan(a*x)**3, x))
```

3.375.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
1/32*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^3 - 3/128*(a^4*c^2*x^4 + 4*
a^2*c^2*x^2)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/128*(112*(a^6*c^
2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x)^3 - 12*(a^5*c^2*x
^5 + 4*a^3*c^2*x^3)*arctan(a*x)^2 + 12*(a^6*c^2*x^6 + 4*a^4*c^2*x^4)*arcta
n(a*x)*log(a^2*x^2 + 1) + 3*(a^5*c^2*x^5 + 4*a^3*c^2*x^3 + 4*(a^6*c^2*x^6
+ 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a
^2*x^3 + x), x)
```

3.375.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
Timed out
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x} \, dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x, x)
```

3.376 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} \, dx$

3.376.1 Optimal result

Integrand size = 22, antiderivative size = 284

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} \, dx &= a^2c^2x\arctan(ax) - \frac{1}{2}ac^2\arctan(ax)^2 - \frac{1}{2}a^3c^2x^2\arctan(ax)^2 \\ &+ \frac{2}{3}iac^2\arctan(ax)^3 - \frac{c^2\arctan(ax)^3}{x} + 2a^2c^2x\arctan(ax)^3 \\ &+ \frac{1}{3}a^4c^2x^3\arctan(ax)^3 + 5ac^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) \\ &- \frac{1}{2}ac^2\log\left(1+a^2x^2\right) + 3ac^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\ &- 3iac^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ &+ 5iac^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\ &+ \frac{3}{2}ac^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) \\ &+ \frac{5}{2}ac^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
a^2*c^2*x*arctan(a*x)-1/2*a*c^2*arctan(a*x)^2-1/2*a^3*c^2*x^2*arctan(a*x)^
2+2/3*I*a*c^2*arctan(a*x)^3-c^2*arctan(a*x)^3/x+2*a^2*c^2*x*arctan(a*x)^3+
1/3*a^4*c^2*x^3*arctan(a*x)^3+5*a*c^2*arctan(a*x)^2*ln(2/(1+I*a*x))-1/2*a*
c^2*ln(a^2*x^2+1)+3*a*c^2*arctan(a*x)^2*ln(2-2/(1-I*a*x))-3*I*a*c^2*arctan
(a*x)*polylog(2,-1+2/(1-I*a*x))+5*I*a*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a
*x))+3/2*a*c^2*polylog(3,-1+2/(1-I*a*x))+5/2*a*c^2*polylog(3,1-2/(1+I*a*x)
)
```

3.376.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.87

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2}\,dx$$
$$= \frac{c^2\left(-3ia\pi^3x + 24a^2x^2\arctan(ax) - 12ax\arctan(ax)^2 - 12a^3x^3\arctan(ax)^2 - 24\arctan(ax)^3 - 16iax\operatorname{arc}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```

output
```
(c^2*((-3*I)*a*Pi^3*x + 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2 - 12
*a^3*x^3*ArcTan[a*x]^2 - 24*ArcTan[a*x]^3 - (16*I)*a*x*ArcTan[a*x]^3 + 48*
a^2*x^2*ArcTan[a*x]^3 + 8*a^4*x^4*ArcTan[a*x]^3 + 72*a*x*ArcTan[a*x]^2*Log
[1 - E^((-2*I)*ArcTan[a*x])] + 120*a*x*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcT
an[a*x])] - 12*a*x*Log[1 + a^2*x^2] + (72*I)*a*x*ArcTan[a*x]*PolyLog[2, E^
((-2*I)*ArcTan[a*x])] - (120*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTa
n[a*x])] + 36*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, -
E^((2*I)*ArcTan[a*x])]))/(24*x)
```

3.376.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)^2}{x^2}\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^2\arctan(ax)^3+2a^2c^2\arctan(ax)^3+\frac{c^2\arctan(ax)^3}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^4c^2x^3\arctan(ax)^3-\frac{1}{2}a^3c^2x^2\arctan(ax)^2+2a^2c^2x\arctan(ax)^3+a^2c^2x\arctan(ax)-\frac{1}{2}ac^2\log\left(a^2x^2+1\right)-3iac^2\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+5iac^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{2}{3}iac^2\arctan(ax)^3-\frac{1}{2}ac^2\arctan(ax)^2-\frac{c^2\arctan(ax)^3}{x}+5ac^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ac^2\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+\frac{5}{2}ac^2\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```

output
```
a^2*c^2*x*ArcTan[a*x] - (a*c^2*ArcTan[a*x]^2)/2 - (a^3*c^2*x^2*ArcTan[a*x]
^2)/2 + ((2*I)/3)*a*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/x + 2*a^2*c^2*
x*ArcTan[a*x]^3 + (a^4*c^2*x^3*ArcTan[a*x]^3)/3 + 5*a*c^2*ArcTan[a*x]^2*Lo
g[2/(1 + I*a*x)] - (a*c^2*Log[1 + a^2*x^2])/2 + 3*a*c^2*ArcTan[a*x]^2*Log[
2 - 2/(1 - I*a*x)] - (3*I)*a*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)
] + (5*I)*a*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^2*PolyL
og[3, -1 + 2/(1 - I*a*x)])/2 + (5*a*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

3.376.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.376.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 76.25 (sec) , antiderivative size = 1793, normalized size of antiderivative = 6.31

method	result	size
derivativedivides	Expression too large to display	1793
default	Expression too large to display	1793
parts	Expression too large to display	1794

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*c^2*arctan(a*x)^3*a^3*x^3+2*c^2*arctan(a*x)^3*a*x-c^2*arctan(a*x)^3
/a/x-c^2*(1/2*x^2*arctan(a*x)^2*a^2-3*arctan(a*x)^2*ln(a*x)+4*arctan(a*x)^
2*ln(a^2*x^2+1)-8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/6*I*arct
an(a*x)*(-9*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a
*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2
*x^2+1)+1))*Pi+9*arctan(a*x)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi-12*arctan(a*x)
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+24*arctan(a*x)*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))-12*arctan(
a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)+1))^2-12*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)^2*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+12*arctan(a*x)*Pi*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+12*arctan(a*x)*P
i*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-12*arcta
n(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+12*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1))^3-24*arctan(a*x)*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1))^2+12*arctan(a*x)*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+9*arctan(a*x)*csgn(I/((1+I*a*x)^2...
```

3.376.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^2}\,dx=\int \frac{(a^2cx^2+c)^2\arctan{(ax)}^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^2, x)
```

3.376.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^2}\,dx=c^2\left(\int 2a^2\operatorname{atan}^3{(ax)}\,dx+\int \frac{\operatorname{atan}^3{(ax)}}{x^2}\,dx\right.$$
$$\left.+\int a^4x^2\operatorname{atan}^3{(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) +
Integral(a**4*x**2*atan(a*x)**3, x))
```

3.376.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^2}\,dx=\int \frac{(a^2cx^2+c)^2\arctan{(ax)}^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/96*(4*(a^4*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)^3 - 3*(a^4*c^2*x
^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 3*(896*a^6*c^
2*integrate(1/32*x^6*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 96*a^6*c^2*integr
ate(1/32*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 128*a^6*
c^2*integrate(1/32*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) -
128*a^5*c^2*integrate(1/32*x^5*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 32*a^5*
c^2*integrate(1/32*x^5*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 21*a*c^2*a
rctan(a*x)^4 + 2688*a^4*c^2*integrate(1/32*x^4*arctan(a*x)^3/(a^2*x^4 + x^
2), x) + 288*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^
2*x^4 + x^2), x) + 768*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2
+ 1)/(a^2*x^4 + x^2), x) - 768*a^3*c^2*integrate(1/32*x^3*arctan(a*x)^2/(a
^2*x^4 + x^2), x) + a*c^2*log(a^2*x^2 + 1)^3 + 288*a^2*c^2*integrate(1/32*
x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 384*a^2*c^2*integ
rate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 384*a*c^2
*integrate(1/32*x*arctan(a*x)^2/(a^2*x^4 + x^2), x) - 96*a*c^2*integrate(1
/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 896*c^2*integrate(1/32*arct
an(a*x)^3/(a^2*x^4 + x^2), x) + 96*c^2*integrate(1/32*arctan(a*x)*log(a^2*
x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

3.376.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x^2} dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^2, x)
```

3.377 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3} dx$

3.377.1 Optimal result

Integrand size = 22, antiderivative size = 399

$$\begin{aligned}
\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3}\,dx = &-3ia^2c^2\arctan(ax)^2-\frac{3ac^2\arctan(ax)^2}{2x} \\
&-\frac{3}{2}a^3c^2x\arctan(ax)^2-\frac{c^2\arctan(ax)^3}{2x^2} \\
&+\frac{1}{2}a^4c^2x^2\arctan(ax)^3 \\
&+4a^2c^2\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) \\
&-3a^2c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) \\
&+3a^2c^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) \\
&-\frac{3}{2}ia^2c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\
&-\frac{3}{2}ia^2c^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
&-3ia^2c^2\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
&+3ia^2c^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) \\
&-3a^2c^2\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) \\
&+3a^2c^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right) \\
&+\frac{3}{2}ia^2c^2\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right) \\
&-\frac{3}{2}ia^2c^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)
\end{aligned}$$

output
```
-3*I*a^2*c^2*arctan(a*x)^2-3/2*a*c^2*arctan(a*x)^2/x-3/2*a^3*c^2*x*arctan(
a*x)^2-1/2*c^2*arctan(a*x)^3/x^2+1/2*a^4*c^2*x^2*arctan(a*x)^3-4*a^2*c^2*a
rctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-3*a^2*c^2*arctan(a*x)*ln(2/(1+I*a*x))
+3*a^2*c^2*arctan(a*x)*ln(2-2/(1-I*a*x))+3/2*I*a^2*c^2*polylog(4,1-2/(1+I*
a*x))-3/2*I*a^2*c^2*polylog(4,-1+2/(1+I*a*x))-3/2*I*a^2*c^2*polylog(2,-1+2
/(1-I*a*x))-3*I*a^2*c^2*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))-3*a^2*c^2*a
rctan(a*x)*polylog(3,1-2/(1+I*a*x))+3*a^2*c^2*arctan(a*x)*polylog(3,-1+2/(
1+I*a*x))-3/2*I*a^2*c^2*polylog(2,1-2/(1+I*a*x))+3*I*a^2*c^2*arctan(a*x)^2
*polylog(2,-1+2/(1+I*a*x))
```

3.377.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.76

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx = \frac{1}{32}a^2c^2\Bigg(&-i\pi^4-\frac{48\arctan(ax)^2}{ax}-48ax\arctan(ax)^2 \\ &-\frac{16\arctan(ax)^3}{a^2x^2}+16a^2x^2\arctan(ax)^3+32i\arctan(ax)^4 \\ &+64\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right) \\ &+96\arctan(ax)\log\left(1-e^{2i\arctan(ax)}\right) \\ &-96\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right) \\ &-64\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right) \\ &+96i\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) \\ &+48i\left(1+2\arctan(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) \\ &-48i\operatorname{PolyLog}\left(2,e^{2i\arctan(ax)}\right) \\ &+96\arctan(ax)\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) \\ &-96\arctan(ax)\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right) \\ &-48i\operatorname{PolyLog}\left(4,e^{-2i\arctan(ax)}\right) \\ &-48i\operatorname{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\Bigg)\end{aligned}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c^2*((-I)*Pi^4 - (48*ArcTan[a*x]^2)/(a*x) - 48*a*x*ArcTan[a*x]^2 - (1
6*ArcTan[a*x]^3)/(a^2*x^2) + 16*a^2*x^2*ArcTan[a*x]^3 + (32*I)*ArcTan[a*x]
^4 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log
[1 - E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])
] - 64*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]^2
*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (48*I)*(1 + 2*ArcTan[a*x]^2)*PolyLog
[2, -E^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 9
6*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[
3, -E^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (
48*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]))/32
```

3.377.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)^2}{x^3}dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^4c^2x\arctan(ax)^3+\frac{2a^2c^2\arctan(ax)^3}{x}+\frac{c^2\arctan(ax)^3}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}a^4c^2x^2\arctan(ax)^3-\frac{3}{2}a^3c^2x\arctan(ax)^2+4a^2c^2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-3ia^2c^2\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+3ia^2c^2\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-3a^2c^2\arctan(ax)\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+3a^2c^2\arctan(ax)\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)-3ia^2c^2\arctan(ax)^2-3a^2c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)+3a^2c^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{2}ia^2c^2\text{PolyLog}\left(4,1-\frac{2}{iax+1}\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(4,\frac{2}{iax+1}-1\right)-\frac{c^2\arctan(ax)^3}{2x^2}-\frac{3ac^2\arctan(ax)^2}{2x}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```

output
```
(-3*I)*a^2*c^2*ArcTan[a*x]^2 - (3*a*c^2*ArcTan[a*x]^2)/(2*x) - (3*a^3*c^2*
x*ArcTan[a*x]^2)/2 - (c^2*ArcTan[a*x]^3)/(2*x^2) + (a^4*c^2*x^2*ArcTan[a*x
]^3)/2 + 4*a^2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 3*a^2*c^2*Ar
cTan[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)
] - ((3*I)/2)*a^2*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^2*P
olyLog[2, 1 - 2/(1 + I*a*x)] - (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, 1 -
2/(1 + I*a*x)] + (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)
] - 3*a^2*c^2*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan
[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c^2*PolyLog[4, 1 - 2/
(1 + I*a*x)] - ((3*I)/2)*a^2*c^2*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

3.377.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.377.4 Maple [A] (verified)

Time = 32.79 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.56

method	result
derivativedivides	$a^2\left(\frac{c^2\arctan(ax)^2(a^2\arctan(ax)x^2-\arctan(ax)-3ax)(ax-i)(ax+i)}{2a^2x^2}+2c^2\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.$
default	$a^2\left(\frac{c^2\arctan(ax)^2(a^2\arctan(ax)x^2-\arctan(ax)-3ax)(ax-i)(ax+i)}{2a^2x^2}+2c^2\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*c^2*arctan(a*x)^2*(a^2*arctan(a*x)*x^2-arctan(a*x)-3*a*x)*(a*x-I)
*(I+a*x)/a^2/x^2+2*c^2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*
I*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^2*arctan(a*x)*ln((1+I*a*x
)/(a^2*x^2+1)^(1/2)+1)+3*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x
^2+1))+12*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*I*c^2
*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))-3*c^2*arctan(a*x)*ln((1+I*a*x)^2/(a^2
*x^2+1)+1)-6*I*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*
c^2*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+12*I*c^2*polylog(4,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))+2*c^2*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)+1)-3*I*c^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*c^2*arctan(a*x)*p
olylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*c^2*arctan(a*x)^2*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-3*I*c^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*c^2*arctan(a*x)^3*l
n((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I*c^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
)
```

3.377.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx=\int \frac{(a^2cx^2+c)^2\arctan{(ax)}^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^3, x)
```

3.377.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx=c^2\left(\int \frac{\operatorname{atan}^3{(ax)}}{x^3}\,dx+\int \frac{2a^2\operatorname{atan}^3{(ax)}}{x}\,dx+\int a^4x\operatorname{atan}^3{(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**3,x)
```

output
```
c**2*(Integral(atan(a*x)**3/x**3, x) + Integral(2*a**2*atan(a*x)**3/x, x)
+ Integral(a**4*x*atan(a*x)**3, x))
```

3.377.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan{(ax)}^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
1/64*(12*a^4*c^2*x^2*integrate(4*x*arctan(a*x)^3 + x*arctan(a*x)*log(a^2*x
^2 + 1)^2, x) + 8*a^3*c^2*x^2*integrate(-1/8*(24*(a^2*x^2 + 1)*a*x*arctan(
a*x)^3 - 18*(a^2*x^2 + 1)*a*x*arctan(a*x)*log(a^2*x^2 + 1)^2 + 36*(a^2*x^2
 + 1)*arctan(a*x)^2*log(a^2*x^2 + 1) - 3*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^3
- sqrt(a^2*x^2 + 1)*(12*sqrt(a^2*x^2 + 1)*arctan(a*x)^2*log(a^2*x^2 + 1) -
 sqrt(a^2*x^2 + 1)*log(a^2*x^2 + 1)^3 - (12*(a^2*x^2 + 1)^2*arctan(a*x)^2*
log(a^2*x^2 + 1) - (a^2*x^2 + 1)^2*log(a^2*x^2 + 1)^3)*cos(3*arctan(a*x))
+ 3*(12*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^2*log(a^2*x^2 + 1) - (a^2*x^2 + 1)
^(3/2)*log(a^2*x^2 + 1)^3)*cos(2*arctan(a*x)) - 2*(4*(a^2*x^2 + 1)^2*arcta
n(a*x)^3 - 3*(a^2*x^2 + 1)^2*arctan(a*x)*log(a^2*x^2 + 1)^2)*sin(3*arctan(
a*x)) + 6*(4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^3 - 3*(a^2*x^2 + 1)^(3/2)*arc
tan(a*x)*log(a^2*x^2 + 1)^2)*sin(2*arctan(a*x))))/((a^2*x^2 + 1)^4*cos(3*a
rctan(a*x))^2 + (a^2*x^2 + 1)^4*sin(3*arctan(a*x))^2 - 6*(a^2*x^2 + 1)^(7/
2)*sin(3*arctan(a*x))*sin(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^3*cos(2*arctan(
a*x))^2 + 9*(a^2*x^2 + 1)^3*sin(2*arctan(a*x))^2 + a^2*x^2 + 6*(a^2*x^2 +
1)^2*cos(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^2 - 2*(3*(a^2*x^2 + 1)^(7/2)*cos
(2*arctan(a*x)) + (a^2*x^2 + 1)^(5/2))*cos(3*arctan(a*x)) + 6*((a^2*x^2 +
1)^2*a*x*sin(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*a*x*sin(2*arctan(a*x))
 + (a^2*x^2 + 1)^2*cos(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*cos(2*arctan
(a*x)) - sqrt(a^2*x^2 + 1))*sqrt(a^2*x^2 + 1) + 1), x) - 12*a^3*c^2*x^2...
```

3.377.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^3, x)
```

3.378 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx$

3.378.1 Optimal result

Integrand size = 22, antiderivative size = 311

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx = & -\frac{a^2c^2\arctan(ax)}{x} - \frac{1}{2}a^3c^2\arctan(ax)^2 - \frac{ac^2\arctan(ax)^2}{2x^2} \\ & - \frac{2}{3}ia^3c^2\arctan(ax)^3 - \frac{c^2\arctan(ax)^3}{3x^3} \\ & - \frac{2a^2c^2\arctan(ax)^3}{x} + a^4c^2x\arctan(ax)^3 + a^3c^2\log(x) \\ & + 3a^3c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) - \frac{1}{2}a^3c^2\log\left(1+a^2x^2\right) \\ & + 5a^3c^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\ & - 5ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ & + 3ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\ & + \frac{5}{2}a^3c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) \\ & + \frac{3}{2}a^3c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-a^2*c^2*arctan(a*x)/x-1/2*a^3*c^2*arctan(a*x)^2-1/2*a*c^2*arctan(a*x)^2/x
^2-2/3*I*a^3*c^2*arctan(a*x)^3-1/3*c^2*arctan(a*x)^3/x^3-2*a^2*c^2*arctan(
a*x)^3/x+a^4*c^2*x*arctan(a*x)^3+a^3*c^2*ln(x)+3*a^3*c^2*arctan(a*x)^2*ln(
2/(1+I*a*x))-1/2*a^3*c^2*ln(a^2*x^2+1)+5*a^3*c^2*arctan(a*x)^2*ln(2-2/(1-I
*a*x))-5*I*a^3*c^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+3*I*a^3*c^2*arcta
n(a*x)*polylog(2,1-2/(1+I*a*x))+5/2*a^3*c^2*polylog(3,-1+2/(1-I*a*x))+3/2*
a^3*c^2*polylog(3,1-2/(1+I*a*x))
```

3.378.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} dx$$
$$= \frac{c^2(-5ia^3\pi^3x^3 - 24a^2x^2\arctan(ax) - 12ax\arctan(ax)^2 - 12a^3x^3\arctan(ax)^2 - 8\arctan(ax)^3 - 48a^2x^2 \text{a}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```

output
```
(c^2*((-5*I)*a^3*Pi^3*x^3 - 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2
- 12*a^3*x^3*ArcTan[a*x]^2 - 8*ArcTan[a*x]^3 - 48*a^2*x^2*ArcTan[a*x]^3 +
(16*I)*a^3*x^3*ArcTan[a*x]^3 + 24*a^4*x^4*ArcTan[a*x]^3 + 120*a^3*x^3*ArcT
an[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 72*a^3*x^3*ArcTan[a*x]^2*Log[1
 + E^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*Log[a*x] - 12*a^3*x^3*Log[1 + a^2*x
^2] + (120*I)*a^3*x^3*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (72
*I)*a^3*x^3*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 60*a^3*x^3*Po
lyLog[3, E^((-2*I)*ArcTan[a*x])] + 36*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[
a*x])]))/(24*x^3)
```

3.378.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 (a^2cx^2+c)^2}{x^4} dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^4c^2\arctan(ax)^3 + \frac{2a^2c^2\arctan(ax)^3}{x^2} + \frac{c^2\arctan(ax)^3}{x^4}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$a^4c^2x\arctan(ax)^3 - 5ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right) + 3ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right) - \frac{2}{3}ia^3c^2\arctan(ax)^3 - \frac{1}{2}a^3c^2\arctan(ax)^2 + 3a^3c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) + 5a^3c^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) + \frac{5}{2}a^3c^2\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right) + \frac{3}{2}a^3c^2\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right) + a^3c^2\log(x) - \frac{2a^2c^2\arctan(ax)^3}{x} - \frac{a^2c^2\arctan(ax)}{x} - \frac{1}{2}a^3c^2\log\left(a^2x^2+1\right) - \frac{c^2\arctan(ax)^3}{3x^3} - \frac{ac^2\arctan(ax)^2}{2x^2}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```

output
```
-((a^2*c^2*ArcTan[a*x])/x) - (a^3*c^2*ArcTan[a*x]^2)/2 - (a*c^2*ArcTan[a*x
]^2)/(2*x^2) - ((2*I)/3)*a^3*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/(3*x^
3) - (2*a^2*c^2*ArcTan[a*x]^3)/x + a^4*c^2*x*ArcTan[a*x]^3 + a^3*c^2*Log[x
] + 3*a^3*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)] - (a^3*c^2*Log[1 + a^2*x^2]
)/2 + 5*a^3*c^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (5*I)*a^3*c^2*ArcTa
n[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a^3*c^2*ArcTan[a*x]*PolyLog[
2, 1 - 2/(1 + I*a*x)] + (5*a^3*c^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*
a^3*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

3.378.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.378. $\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^4} dx$

3.378.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 103.36 (sec) , antiderivative size = 1883, normalized size of antiderivative = 6.05

method	result	size
derivativedivides	Expression too large to display	1883
default	Expression too large to display	1883
parts	Expression too large to display	1884

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(c^2*arctan(a*x)^3*a*x-1/3*c^2*arctan(a*x)^3/a^3/x^3-2*c^2*arctan(a*x)
^3/a/x-c^2*(1/2*arctan(a*x)^2/a^2/x^2-5*arctan(a*x)^2*ln(a*x)+4*arctan(a*x
)^2*ln(a^2*x^2+1)-8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/6*arct
an(a*x)*(12*I*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))*Pi*arctan(a*x)*a*x+12*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I/(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)*Pi*arctan(a*x)*a*x+15*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-
1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*a
rctan(a*x)*a*x+12*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3*Pi*arctan(a*x)*a*x+6*I*a*x-12*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^3*Pi*arctan(a*x)*a*x-12*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)*a*x+15*I*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2
+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)*a*x-15*I*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*Pi*arctan(a*x)*a*x-1
5*I*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*Pi*arc
tan(a*x)*a*x+15*I*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)*a*x-12*I*csgn
(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)^2*Pi*arctan(a*x)*a*x+16*I*arctan(a*x)^2*a*x-15*I*...
```

3.378.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^4, x)
```

3.378.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx = c^2 \left(\int a^4 \operatorname{atan}^3(ax) \, dx + \int \frac{\operatorname{atan}^3(ax)}{x^4} \, dx + \int \frac{2a^2 \operatorname{atan}^3(ax)}{x^2} \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**4,x)
```

output
```
c**2*(Integral(a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) + In
tegral(2*a**2*atan(a*x)**3/x**2, x))
```

3.378.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/192*(3*(42*a^3*c^2*arctan(a*x)^4 + 1792*a^6*c^2*integrate(1/32*x^6*arcta
n(a*x)^3/(a^2*x^6 + x^4), x) + 192*a^6*c^2*integrate(1/32*x^6*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 768*a^6*c^2*integrate(1/32*x^6*ar
ctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 768*a^5*c^2*integrate(1/3
2*x^5*arctan(a*x)^2/(a^2*x^6 + x^4), x) + a^3*c^2*log(a^2*x^2 + 1)^3 + 576
*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4)
, x) - 1536*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x
^6 + x^4), x) + 1536*a^3*c^2*integrate(1/32*x^3*arctan(a*x)^2/(a^2*x^6 + x
^4), x) - 384*a^3*c^2*integrate(1/32*x^3*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4
), x) + 5376*a^2*c^2*integrate(1/32*x^2*arctan(a*x)^3/(a^2*x^6 + x^4), x)
+ 576*a^2*c^2*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 +
 x^4), x) - 256*a^2*c^2*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a
^2*x^6 + x^4), x) + 256*a*c^2*integrate(1/32*x*arctan(a*x)^2/(a^2*x^6 + x^
4), x) - 64*a*c^2*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x)
+ 1792*c^2*integrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 192*c^2*inte
grate(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 + 8*(3*
a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x)^3 - 6*(3*a^4*c^2*x^4 - 6*a^
2*c^2*x^2 - c^2)*arctan(a*x)*log(a^2*x^2 + 1)^2)/x^3
```

3.378.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^4, x)
```

3.379 $\int x^3(c+a^2cx^2)^3 \arctan(ax)^3 \, dx$

3.379.1 Optimal result

Integrand size = 22, antiderivative size = 381

$$\begin{aligned}\int x^3\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx &= \frac{389c^3x}{12600a^3}-\frac{17c^3x^3}{9450a}-\frac{1}{252}ac^3x^5-\frac{1}{840}a^3c^3x^7\\
&-\frac{389c^3\arctan(ax)}{12600a^4}-\frac{107c^3x^2\arctan(ax)}{4200a^2}\\
&+\frac{53c^3x^4\arctan(ax)}{2100}+\frac{71a^2c^3x^6\arctan(ax)}{2520}\\
&+\frac{1}{120}a^4c^3x^8\arctan(ax)+\frac{26ic^3\arctan(ax)^2}{525a^4}\\
&+\frac{3c^3x\arctan(ax)^2}{40a^3}-\frac{c^3x^3\arctan(ax)^2}{40a}\\
&-\frac{27}{200}ac^3x^5\arctan(ax)^2-\frac{33}{280}a^3c^3x^7\arctan(ax)^2\\
&-\frac{1}{30}a^5c^3x^9\arctan(ax)^2-\frac{c^3\arctan(ax)^3}{40a^4}\\
&+\frac{1}{4}c^3x^4\arctan(ax)^3+\frac{1}{2}a^2c^3x^6\arctan(ax)^3\\
&+\frac{3}{8}a^4c^3x^8\arctan(ax)^3+\frac{1}{10}a^6c^3x^{10}\arctan(ax)^3\\
&+\frac{52c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{525a^4}\\
&+\frac{26ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{525a^4}\end{aligned}$$

output
```
389/12600*c^3*x/a^3-17/9450*c^3*x^3/a-1/252*a*c^3*x^5-1/840*a^3*c^3*x^7-38
9/12600*c^3*arctan(a*x)/a^4-107/4200*c^3*x^2*arctan(a*x)/a^2+53/2100*c^3*x
^4*arctan(a*x)+71/2520*a^2*c^3*x^6*arctan(a*x)+1/120*a^4*c^3*x^8*arctan(a*
x)+26/525*I*c^3*polylog(2,1-2/(1+I*a*x))/a^4+3/40*c^3*x*arctan(a*x)^2/a^3-
1/40*c^3*x^3*arctan(a*x)^2/a-27/200*a*c^3*x^5*arctan(a*x)^2-33/280*a^3*c^3
*x^7*arctan(a*x)^2-1/30*a^5*c^3*x^9*arctan(a*x)^2-1/40*c^3*arctan(a*x)^3/a
^4+1/4*c^3*x^4*arctan(a*x)^3+1/2*a^2*c^3*x^6*arctan(a*x)^3+3/8*a^4*c^3*x^8
*arctan(a*x)^3+1/10*a^6*c^3*x^10*arctan(a*x)^3+52/525*c^3*arctan(a*x)*ln(2
/(1+I*a*x))/a^4+26/525*I*c^3*arctan(a*x)^2/a^4
```

3.379.2 Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.50

$$\int x^3 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx$$
$$= \frac{c^3 \Big(-ax(-1167 + 68a^2x^2 + 150a^4x^4 + 45a^6x^6) - 9(208i - 315ax + 105a^3x^3 + 567a^5x^5 + 495a^7x^7 + 140a}{}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-(a*x*(-1167 + 68*a^2*x^2 + 150*a^4*x^4 + 45*a^6*x^6)) - 9*(208*I -
315*a*x + 105*a^3*x^3 + 567*a^5*x^5 + 495*a^7*x^7 + 140*a^9*x^9)*ArcTan[a*
x]^2 + 945*(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^3 + 3*ArcTan[a*x]*
(-389 - 321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 + 1248*Log[1
 + E^((2*I)*ArcTan[a*x])]) - (1872*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))
/(37800*a^4)
```

3.379.3 Rubi [A] (verified)

Time = 3.48 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \left(a^2 c x^2 + c\right)^3 \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^9\arctan(ax)^3+3a^4c^3x^7\arctan(ax)^3+3a^2c^3x^5\arctan(ax)^3+c^3x^3\arctan(ax)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}a^6c^3x^{10}\arctan(ax)^3-\frac{1}{30}a^5c^3x^9\arctan(ax)^2+\frac{3}{8}a^4c^3x^8\arctan(ax)^3+\frac{1}{120}a^4c^3x^8\arctan(ax)-\frac{c^3\arctan(ax)^3}{40a^4}+\frac{26ic^3\arctan(ax)^2}{525a^4}-\frac{389c^3\arctan(ax)}{12600a^4}+\frac{52c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{525a^4}+\frac{26ic^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{525a^4}-\frac{33}{280}a^3c^3x^7\arctan(ax)^2+\frac{3c^3x\arctan(ax)^2}{40a^3}-\frac{1}{840}a^3c^3x^7+\frac{389c^3x}{12600a^3}+\frac{1}{2}a^2c^3x^6\arctan(ax)^3+\frac{71a^2c^3x^6\arctan(ax)}{2520}-\frac{107c^3x^2\arctan(ax)}{4200a^2}-\frac{27}{200}ac^3x^5\arctan(ax)^2+\frac{1}{4}c^3x^4\arctan(ax)^3+\frac{53c^3x^4\arctan(ax)}{2100}-\frac{c^3x^3\arctan(ax)^2}{40a}-\frac{1}{252}ac^3x^5-\frac{17c^3x^3}{9450a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(389*c^3*x)/(12600*a^3) - (17*c^3*x^3)/(9450*a) - (a*c^3*x^5)/252 - (a^3*c
^3*x^7)/840 - (389*c^3*ArcTan[a*x])/(12600*a^4) - (107*c^3*x^2*ArcTan[a*x]
)/(4200*a^2) + (53*c^3*x^4*ArcTan[a*x])/2100 + (71*a^2*c^3*x^6*ArcTan[a*x]
)/2520 + (a^4*c^3*x^8*ArcTan[a*x])/120 + (((26*I)/525)*c^3*ArcTan[a*x]^2)/
a^4 + (3*c^3*x*ArcTan[a*x]^2)/(40*a^3) - (c^3*x^3*ArcTan[a*x]^2)/(40*a) -
(27*a*c^3*x^5*ArcTan[a*x]^2)/200 - (33*a^3*c^3*x^7*ArcTan[a*x]^2)/280 - (a
^5*c^3*x^9*ArcTan[a*x]^2)/30 - (c^3*ArcTan[a*x]^3)/(40*a^4) + (c^3*x^4*Arc
Tan[a*x]^3)/4 + (a^2*c^3*x^6*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]
^3)/8 + (a^6*c^3*x^10*ArcTan[a*x]^3)/10 + (52*c^3*ArcTan[a*x]*Log[2/(1 + I
*a*x)])/(525*a^4) + (((26*I)/525)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

3.379.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.379.4 Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{c^3\arctan(ax)^3a^{10}x^{10}}{10}+\frac{3c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{a^4c^3x^4\arctan(ax)^3}{4}-\frac{c^3\arctan(ax)^3}{40}-\frac{3c^3\left(\frac{4\arctan(ax)^2a^9}{9}\right.}{}$
default	$\frac{c^3\arctan(ax)^3a^{10}x^{10}}{10}+\frac{3c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{a^4c^3x^4\arctan(ax)^3}{4}-\frac{c^3\arctan(ax)^3}{40}-\frac{3c^3\left(\frac{4\arctan(ax)^2a^9}{9}\right.}{}$
parts	$\frac{a^6c^3x^{10}\arctan(ax)^3}{10}+\frac{3a^4c^3x^8\arctan(ax)^3}{8}+\frac{a^2c^3x^6\arctan(ax)^3}{2}+\frac{c^3x^4\arctan(ax)^3}{4}-\frac{3c^3\left(\frac{4a^5\arctan(ax)^2x^9}{9}\right.}{}$

input
```
int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/10*c^3*arctan(a*x)^3*a^10*x^10+3/8*c^3*arctan(a*x)^3*a^8*x^8+1/2*
a^6*c^3*x^6*arctan(a*x)^3+1/4*a^4*c^3*x^4*arctan(a*x)^3-1/40*c^3*arctan(a*
x)^3-3/40*c^3*(4/9*arctan(a*x)^2*a^9*x^9+11/7*arctan(a*x)^2*a^7*x^7+9/5*a^
5*arctan(a*x)^2*x^5+1/3*a^3*arctan(a*x)^2*x^3-a*arctan(a*x)^2*x-1/9*arctan
(a*x)*a^8*x^8-71/189*a^6*arctan(a*x)*x^6-106/315*arctan(a*x)*a^4*x^4+107/3
15*a^2*arctan(a*x)*x^2+208/315*arctan(a*x)*ln(a^2*x^2+1)+1/63*a^7*x^7+10/1
89*a^5*x^5+68/2835*a^3*x^3-389/945*a*x+389/945*arctan(a*x)+104/315*I*(ln(a
*x-I)*ln(a^2*x^2+1)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2
*ln(a*x-I)^2)-104/315*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I
+a*x)*ln(1/2*I*(a*x-I))-1/2*ln(I+a*x)^2)))
```

3.379.5 Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^3x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(a*
x)^3, x)
```

3.379.6 Sympy [F]

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^3\,dx = c^3\left(\int x^3\operatorname{atan}^3(ax)\,dx + \int 3a^2x^5\operatorname{atan}^3(ax)\,dx + \int 3a^4x^7\operatorname{atan}^3(ax)\,dx + \int a^6x^9\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(x**3*atan(a*x)**3, x) + Integral(3*a**2*x**5*atan(a*x)**3,
x) + Integral(3*a**4*x**7*atan(a*x)**3, x) + Integral(a**6*x**9*atan(a*x)*
*3, x))
```

3.379.7 Maxima [F]

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^3\,dx = \int (a^2cx^2+c)^3x^3\arctan(ax)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/67200*(420*(5376000*a^11*c^3*integrate(1/67200*x^11*arctan(a*x)^3/(a^5*x
^2 + a^3), x) - 806400*a^10*c^3*integrate(1/67200*x^10*arctan(a*x)^2/(a^5*
x^2 + a^3), x) - 201600*a^10*c^3*integrate(1/67200*x^10*log(a^2*x^2 + 1)^2
/(a^5*x^2 + a^3), x) - 89600*a^10*c^3*integrate(1/67200*x^10*log(a^2*x^2 +
 1)/(a^5*x^2 + a^3), x) + 21504000*a^9*c^3*integrate(1/67200*x^9*arctan(a*
x)^3/(a^5*x^2 + a^3), x) + 179200*a^9*c^3*integrate(1/67200*x^9*arctan(a*x
)/(a^5*x^2 + a^3), x) - 3024000*a^8*c^3*integrate(1/67200*x^8*arctan(a*x)^
2/(a^5*x^2 + a^3), x) - 756000*a^8*c^3*integrate(1/67200*x^8*log(a^2*x^2 +
 1)^2/(a^5*x^2 + a^3), x) - 316800*a^8*c^3*integrate(1/67200*x^8*log(a^2*x
^2 + 1)/(a^5*x^2 + a^3), x) + 32256000*a^7*c^3*integrate(1/67200*x^7*arcta
n(a*x)^3/(a^5*x^2 + a^3), x) + 633600*a^7*c^3*integrate(1/67200*x^7*arctan
(a*x)/(a^5*x^2 + a^3), x) - 4032000*a^6*c^3*integrate(1/67200*x^6*arctan(a
*x)^2/(a^5*x^2 + a^3), x) - 1008000*a^6*c^3*integrate(1/67200*x^6*log(a^2*
x^2 + 1)^2/(a^5*x^2 + a^3), x) - 362880*a^6*c^3*integrate(1/67200*x^6*log(
a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 21504000*a^5*c^3*integrate(1/67200*x^5*
arctan(a*x)^3/(a^5*x^2 + a^3), x) + 725760*a^5*c^3*integrate(1/67200*x^5*a
rctan(a*x)/(a^5*x^2 + a^3), x) - 2016000*a^4*c^3*integrate(1/67200*x^4*arc
tan(a*x)^2/(a^5*x^2 + a^3), x) - 504000*a^4*c^3*integrate(1/67200*x^4*log(
a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 67200*a^4*c^3*integrate(1/67200*x^4*l
og(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 5376000*a^3*c^3*integrate(1/67200...
```

3.379.8 Giac [F]

$$\int x^3 \left(c+a^2cx^2\right)^3 \arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^3 x^3 \arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int x^3\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.380 $\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx$

3.380.1 Optimal result

Integrand size = 22, antiderivative size = 389

$$
\begin{aligned}
\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx = & -\frac{107 c^3 x^2}{7560 a} - \frac{11 a c^3 x^4}{1260} - \frac{1}{504} a^3 c^3 x^6 \\
& - \frac{47 c^3 x \arctan(ax)}{1260 a^2} + \frac{239 c^3 x^3 \arctan(ax)}{3780} \\
& + \frac{59 a^2 c^3 x^5 \arctan(ax)}{1260} + \frac{1}{84} a^4 c^3 x^7 \arctan(ax) \\
& + \frac{47 c^3 \arctan(ax)^2}{2520 a^3} - \frac{8 c^3 x^2 \arctan(ax)^2}{105 a} \\
& - \frac{89}{420} a c^3 x^4 \arctan(ax)^2 - \frac{10}{63} a^3 c^3 x^6 \arctan(ax)^2 \\
& - \frac{1}{24} a^5 c^3 x^8 \arctan(ax)^2 - \frac{16 i c^3 \arctan(ax)^3}{315 a^3} \\
& + \frac{1}{3} c^3 x^3 \arctan(ax)^3 + \frac{3}{5} a^2 c^3 x^5 \arctan(ax)^3 \\
& + \frac{3}{7} a^4 c^3 x^7 \arctan(ax)^3 + \frac{1}{9} a^6 c^3 x^9 \arctan(ax)^3 \\
& - \frac{16 c^3 \arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{105 a^3} + \frac{31 c^3 \log(1 + a^2 x^2)}{945 a^3} \\
& - \frac{16 i c^3 \arctan(ax) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{105 a^3} \\
& - \frac{8 c^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+iax}\right)}{105 a^3}
\end{aligned}
$$

output
```
-107/7560*c^3*x^2/a-11/1260*a*c^3*x^4-1/504*a^3*c^3*x^6-47/1260*c^3*x*arct
an(a*x)/a^2+239/3780*c^3*x^3*arctan(a*x)+59/1260*a^2*c^3*x^5*arctan(a*x)+1
/84*a^4*c^3*x^7*arctan(a*x)+47/2520*c^3*arctan(a*x)^2/a^3-8/105*c^3*x^2*ar
ctan(a*x)^2/a-89/420*a*c^3*x^4*arctan(a*x)^2-10/63*a^3*c^3*x^6*arctan(a*x)
^2-1/24*a^5*c^3*x^8*arctan(a*x)^2-16/105*I*c^3*arctan(a*x)*polylog(2,1-2/(
1+I*a*x))/a^3+1/3*c^3*x^3*arctan(a*x)^3+3/5*a^2*c^3*x^5*arctan(a*x)^3+3/7*
a^4*c^3*x^7*arctan(a*x)^3+1/9*a^6*c^3*x^9*arctan(a*x)^3-16/105*c^3*arctan(
a*x)^2*ln(2/(1+I*a*x))/a^3+31/945*c^3*ln(a^2*x^2+1)/a^3-16/315*I*c^3*arcta
n(a*x)^3/a^3-8/105*c^3*polylog(3,1-2/(1+I*a*x))/a^3
```

3.380.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.72

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$$
$$=\frac{c^3\left(-56-107a^2x^2-66a^4x^4-15a^6x^6-282ax\arctan(ax)+478a^3x^3\arctan(ax)+354a^5x^5\arctan(ax)+\right.}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-56 - 107*a^2*x^2 - 66*a^4*x^4 - 15*a^6*x^6 - 282*a*x*ArcTan[a*x] +
478*a^3*x^3*ArcTan[a*x] + 354*a^5*x^5*ArcTan[a*x] + 90*a^7*x^7*ArcTan[a*x]
 + 141*ArcTan[a*x]^2 - 576*a^2*x^2*ArcTan[a*x]^2 - 1602*a^4*x^4*ArcTan[a*x
]^2 - 1200*a^6*x^6*ArcTan[a*x]^2 - 315*a^8*x^8*ArcTan[a*x]^2 + (384*I)*Arc
Tan[a*x]^3 + 2520*a^3*x^3*ArcTan[a*x]^3 + 4536*a^5*x^5*ArcTan[a*x]^3 + 324
0*a^7*x^7*ArcTan[a*x]^3 + 840*a^9*x^9*ArcTan[a*x]^3 - 1152*ArcTan[a*x]^2*L
og[1 + E^((2*I)*ArcTan[a*x])] + 248*Log[1 + a^2*x^2] + (1152*I)*ArcTan[a*x
]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 576*PolyLog[3, -E^((2*I)*ArcTan[a*x
])]))/(7560*a^3)
```

3.380.3 Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^8 \arctan(ax)^3 + 3a^4c^3x^6 \arctan(ax)^3 + 3a^2c^3x^4 \arctan(ax)^3 + c^3x^2 \arctan(ax)^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}a^6c^3x^9 \arctan(ax)^3 - \frac{1}{24}a^5c^3x^8 \arctan(ax)^2 + \frac{3}{7}a^4c^3x^7 \arctan(ax)^3 + \frac{1}{84}a^4c^3x^7 \arctan(ax) - \frac{16ic^3 \arctan(ax) \text{PolyLog}\left(2, 1-\frac{2}{iax+1}\right)}{105a^3} - \frac{10}{63}a^3c^3x^6 \arctan(ax)^2 - \frac{16ic^3 \arctan(ax)^3}{315a^3} + \frac{47c^3 \arctan(ax)^2}{2520a^3} - \frac{16c^3 \arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{105a^3} - \frac{8c^3 \text{PolyLog}\left(3, 1-\frac{2}{iax+1}\right)}{105a^3} - \frac{1}{504}a^3c^3x^6 + \frac{3}{5}a^2c^3x^5 \arctan(ax)^3 + \frac{59a^2c^3x^5 \arctan(ax)}{1260} - \frac{47c^3x \arctan(ax)}{1260a^2} + \frac{31c^3 \log\left(a^2x^2+1\right)}{945a^3} - \frac{89}{420}ac^3x^4 \arctan(ax)^2 + \frac{1}{3}c^3x^3 \arctan(ax)^3 + \frac{239c^3x^3 \arctan(ax)}{3780} - \frac{8c^3x^2 \arctan(ax)^2}{105a} - \frac{11ac^3x^4}{1260} - \frac{107c^3x^2}{7560a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(-107*c^3*x^2)/(7560*a) - (11*a*c^3*x^4)/1260 - (a^3*c^3*x^6)/504 - (47*c^
3*x*ArcTan[a*x])/(1260*a^2) + (239*c^3*x^3*ArcTan[a*x])/3780 + (59*a^2*c^3
*x^5*ArcTan[a*x])/1260 + (a^4*c^3*x^7*ArcTan[a*x])/84 + (47*c^3*ArcTan[a*x
]^2)/(2520*a^3) - (8*c^3*x^2*ArcTan[a*x]^2)/(105*a) - (89*a*c^3*x^4*ArcTan
[a*x]^2)/420 - (10*a^3*c^3*x^6*ArcTan[a*x]^2)/63 - (a^5*c^3*x^8*ArcTan[a*x
]^2)/24 - (((16*I)/315)*c^3*ArcTan[a*x]^3)/a^3 + (c^3*x^3*ArcTan[a*x]^3)/3
 + (3*a^2*c^3*x^5*ArcTan[a*x]^3)/5 + (3*a^4*c^3*x^7*ArcTan[a*x]^3)/7 + (a^
6*c^3*x^9*ArcTan[a*x]^3)/9 - (16*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(10
5*a^3) + (31*c^3*Log[1 + a^2*x^2])/(945*a^3) - (((16*I)/105)*c^3*ArcTan[a*
x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3 - (8*c^3*PolyLog[3, 1 - 2/(1 + I*a*x
)])/(105*a^3)
```

3.380.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.380.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 121.68 (sec) , antiderivative size = 1576, normalized size of antiderivative = 4.05

method	result	size
derivativedivides	Expression too large to display	1576
default	Expression too large to display	1576
parts	Expression too large to display	1576

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/9*c^3*arctan(a*x)^3*a^9*x^9+3/7*c^3*arctan(a*x)^3*a^7*x^7+3/5*c^3
*arctan(a*x)^3*a^5*x^5+1/3*c^3*arctan(a*x)^3*a^3*x^3-1/105*c^3*(-4*I*Pi*cs
gn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-47/2
4*arctan(a*x)^2+35/8*arctan(a*x)^2*a^8*x^8+8*x^2*arctan(a*x)^2*a^2+175/4*a
rctan(a*x)*(a*x-I)^4*(I+a*x)^3-35/4*I*arctan(a*x)*(a*x-I)^6-105/4*arctan(a
*x)*(a*x-I)^5*(I+a*x)^2+35/4*arctan(a*x)*(a*x-I)^6*(I+a*x)+115/6*I*arctan(
a*x)*(a*x-I)^4+3*I*arctan(a*x)*(a*x-I)^2-16*I*arctan(a*x)*polylog(2,-(1+I*
a*x)^2/(a^2*x^2+1))-175/4*arctan(a*x)*(a*x-I)^3*(I+a*x)^4+16*arctan(a*x)^2
*ln(2)+4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-230/3*
I*arctan(a*x)*(a*x-I)^3*(I+a*x)+115*I*arctan(a*x)*(a*x-I)^2*(I+a*x)^2+105/
2*I*arctan(a*x)*(a*x-I)^5*(I+a*x)-525/4*I*arctan(a*x)*(a*x-I)^4*(I+a*x)^2-
4*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arc
tan(a*x)^2-4*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2-525/4*I*
arctan(a*x)*(a*x-I)^2*(I+a*x)^4-6*I*arctan(a*x)*(a*x-I)*(I+a*x)+105/2*I*ar
ctan(a*x)*(a*x-I)*(I+a*x)^5-230/3*I*arctan(a*x)*(a*x-I)*(I+a*x)^3+175*I*ar
ctan(a*x)*(a*x-I)^3*(I+a*x)^3-53/24*(I+a*x)^4+50/3*a^6*x^6*arctan(a*x)^2+5
/24*(I+a*x)^6+4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn
(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arcta...
```

3.380.5 Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*
x)^3, x)
```

3.380.6 Sympy [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^3\,dx = c^3\left(\int x^2\operatorname{atan}^3(ax)\,dx+\int 3a^2x^4\operatorname{atan}^3(ax)\,dx+\int 3a^4x^6\operatorname{atan}^3(ax)\,dx+\int a^6x^8\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(x**2*atan(a*x)**3, x) + Integral(3*a**2*x**4*atan(a*x)**3,
x) + Integral(3*a**4*x**6*atan(a*x)**3, x) + Integral(a**6*x**8*atan(a*x)*
*3, x))
```

3.380.7 Maxima [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^3\,dx = \int (a^2cx^2+c)^3x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2520*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*
arctan(a*x)^3 - 1/3360*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5
 + 105*c^3*x^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/3360*(2940*(a
^8*c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arc
tan(a*x)^3 - 4*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a
*c^3*x^3)*arctan(a*x)^2 + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 + 189*a^4*c
^3*x^6 + 105*a^2*c^3*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (35*a^7*c^3*x^9 +
 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a*c^3*x^3 + 315*(a^8*c^3*x^10 + 4
*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x))*log(a
^2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

3.380.8 Giac [F]

$$\int x^2 \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^3 x^2 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx = \int x^2 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^3 dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.381 $\int x(c+a^2cx^2)^3 \arctan(ax)^3 \, dx$

3.381.1 Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}\int x(c+a^2cx^2)^3 \arctan(ax)^3 \, dx = & -\frac{19c^3x}{140a} - \frac{19}{840}ac^3x^3 - \frac{1}{280}a^3c^3x^5 \\ & + \frac{3c^3(1+a^2x^2)\arctan(ax)}{35a^2} + \frac{9c^3(1+a^2x^2)^2\arctan(ax)}{280a^2} \\ & + \frac{c^3(1+a^2x^2)^3\arctan(ax)}{56a^2} - \frac{6ic^3\arctan(ax)^2}{35a^2} \\ & - \frac{6c^3x\arctan(ax)^2}{35a} - \frac{3c^3x(1+a^2x^2)\arctan(ax)^2}{35a} \\ & - \frac{9c^3x(1+a^2x^2)^2\arctan(ax)^2}{140a} \\ & - \frac{3c^3x(1+a^2x^2)^3\arctan(ax)^2}{56a} \\ & + \frac{c^3(1+a^2x^2)^4\arctan(ax)^3}{8a^2} \\ & - \frac{12c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{35a^2} \\ & - \frac{6ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^2}\end{aligned}$$

output
```
-19/140*c^3*x/a-19/840*a*c^3*x^3-1/280*a^3*c^3*x^5+3/35*c^3*(a^2*x^2+1)*ar
ctan(a*x)/a^2+9/280*c^3*(a^2*x^2+1)^2*arctan(a*x)/a^2+1/56*c^3*(a^2*x^2+1)
^3*arctan(a*x)/a^2-6/35*I*c^3*arctan(a*x)^2/a^2-6/35*c^3*x*arctan(a*x)^2/a
-3/35*c^3*x*(a^2*x^2+1)*arctan(a*x)^2/a-9/140*c^3*x*(a^2*x^2+1)^2*arctan(a
*x)^2/a-3/56*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^2/a+1/8*c^3*(a^2*x^2+1)^4*arc
tan(a*x)^3/a^2-12/35*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a^2-6/35*I*c^3*polylo
g(2,1-2/(1+I*a*x))/a^2
```

3.381.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.51

$$\int x(c+a^2cx^2)^3 \arctan(ax)^3\,dx$$
$$= \frac{c^3\Big(-ax(114+19a^2x^2+3a^4x^4)-9(-16i+35ax+35a^3x^3+21a^5x^5+5a^7x^7)\arctan(ax)^2+105(1+a^2x^2}{}$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4)) - 9*(-16*I + 35*a*x + 35*a^3*x
^3 + 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 + 105*(1 + a^2*x^2)^4*ArcTan[a*
x]^3 + 3*ArcTan[a*x]*(38 + 57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 96*Log[1
+ E^((2*I)*ArcTan[a*x])]) + (144*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(
840*a^2)
```

3.381.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5465, 27, 5415, 210, 2009, 5415, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^3\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \; 5465$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3\int c^3\left(a^2x^2+1\right)^3\arctan(ax)^2dx}{8a}$$

$$\Big\downarrow 27$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\int\left(a^2x^2+1\right)^3\arctan(ax)^2dx}{8a}$$

$$\Big\downarrow 5415$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}\int\left(a^2x^2+1\right)^2dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^3\arctan(ax)}{21a}\right)}{8a}$$

$$\Big\downarrow 210$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}\int\left(a^4x^4+2a^2x^2+1\right)dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^3\arctan(}{21a}\right.}{8a}$$

$$\Big\downarrow 2009$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}\left(\frac{a^4x^5}{5}+\frac{2a^2x^3}{3}+x\right)\right)}{8a}$$

$$\Big\downarrow 5415$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}\int\left(a^2x^2+1\right)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}\right)+\right.}{8a}$$

$$\Big\downarrow 2009$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}+x\right)\right)+\frac{1}{7}x(\right.}{8a}$$

$$\Big\downarrow 5415$$

3.381. $\int x\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(\right.\right.\right.$$

$$\Big\downarrow \text{24}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2\right.\right.$$

$$\Big\downarrow \text{5345}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2\right.\right.\right.$$

$$\Big\downarrow \text{5455}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(a\right.}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{5379}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(a\right.\right.\right.$$

$$\Big\downarrow \text{2849}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{arct}\right.\right.\right.$$

$$\Big\downarrow \text{2752}$$

3.381. $\int x\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2} -$$

$$\frac{3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^3)/(8*a^2) - (3*c^3*((x + (2*a^2*x^3)/3 +
 (a^4*x^5)/5)/21 - ((1 + a^2*x^2)^3*ArcTan[a*x])/(21*a) + (x*(1 + a^2*x^2)
^3*ArcTan[a*x]^2)/7 + (6*((x + (a^2*x^3)/3)/10 - ((1 + a^2*x^2)^2*ArcTan[a
*x])/(10*a) + (x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/5 + (4*(x/3 - ((1 + a^2*x^
2)*ArcTan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a
*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x
)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)))/3))/5))/7))/(8*a)
```

3.381.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.381. $\int x(c+a^2cx^2)^3 \arctan(ax)^3\,dx$

3.381.4 Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.12

method	result
parts	$\frac{c^3\arctan(ax)^3a^6x^8}{8}+\frac{c^3\arctan(ax)^3a^4x^6}{2}+\frac{3c^3\arctan(ax)^3a^2x^4}{4}+\frac{c^3\arctan(ax)^3x^2}{2}+\frac{c^3\arctan(ax)^3}{8a^2}-\frac{3c^3}{}$
derivativedivides	$\frac{\frac{c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{3a^4c^3x^4\arctan(ax)^3}{4}+\frac{a^2c^3x^2\arctan(ax)^3}{2}+\frac{c^3\arctan(ax)^3}{8}-\frac{3c^3\left(\frac{\arctan(ax)^2a^7x^7}{7}\right.}{}}{}$
default	$\frac{\frac{c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{3a^4c^3x^4\arctan(ax)^3}{4}+\frac{a^2c^3x^2\arctan(ax)^3}{2}+\frac{c^3\arctan(ax)^3}{8}-\frac{3c^3\left(\frac{\arctan(ax)^2a^7x^7}{7}\right.}{}}{}$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*arctan(a*x)^3*a^6*x^8+1/2*c^3*arctan(a*x)^3*a^4*x^6+3/4*c^3*arctan
(a*x)^3*a^2*x^4+1/2*c^3*arctan(a*x)^3*x^2+1/8*c^3*arctan(a*x)^3/a^2-3/8/a^
2*c^3*(1/7*arctan(a*x)^2*a^7*x^7+3/5*a^5*arctan(a*x)^2*x^5+a^3*arctan(a*x)
^2*x^3+a*arctan(a*x)^2*x-1/21*a^6*arctan(a*x)*x^6-8/35*arctan(a*x)*a^4*x^4
-19/35*a^2*arctan(a*x)*x^2-16/35*arctan(a*x)*ln(a^2*x^2+1)+1/105*a^5*x^5+1
9/315*a^3*x^3+38/105*a*x-38/105*arctan(a*x)-8/35*I*(ln(a*x-I)*ln(a^2*x^2+1
)-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x))-1/2*ln(a*x-I)^2)+8/35
*I*(ln(I+a*x)*ln(a^2*x^2+1)-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I
))-1/2*ln(I+a*x)^2))
```

3.381.5 Fricas [F]

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*x)
^3, x)
```

3.381.6 Sympy [F]

$$\int x(c+a^2cx^2)^3 \arctan(ax)^3 \,dx = c^3\left(\int x \operatorname{atan}^3(ax)\,dx + \int 3a^2x^3 \operatorname{atan}^3(ax)\,dx + \int 3a^4x^5 \operatorname{atan}^3(ax)\,dx + \int a^6x^7 \operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(x*atan(a*x)**3, x) + Integral(3*a**2*x**3*atan(a*x)**3, x)
+ Integral(3*a**4*x**5*atan(a*x)**3, x) + Integral(a**6*x**7*atan(a*x)**3,
 x))
```

3.381.7 Maxima [F]

$$\int x(c+a^2cx^2)^3 \arctan(ax)^3 \,dx = \int (a^2cx^2+c)^3 x \arctan(ax)^3 \,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/4480*(280*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 +
 c^3)*arctan(a*x)^3 + 140*(71680*a^9*c^3*integrate(1/4480*x^9*arctan(a*x)^
3/(a^3*x^2 + a), x) - 13440*a^8*c^3*integrate(1/4480*x^8*arctan(a*x)^2/(a^
3*x^2 + a), x) - 3360*a^8*c^3*integrate(1/4480*x^8*log(a^2*x^2 + 1)^2/(a^3
*x^2 + a), x) - 1920*a^8*c^3*integrate(1/4480*x^8*log(a^2*x^2 + 1)/(a^3*x^
2 + a), x) + 286720*a^7*c^3*integrate(1/4480*x^7*arctan(a*x)^3/(a^3*x^2 +
a), x) + 3840*a^7*c^3*integrate(1/4480*x^7*arctan(a*x)/(a^3*x^2 + a), x) -
 53760*a^6*c^3*integrate(1/4480*x^6*arctan(a*x)^2/(a^3*x^2 + a), x) - 1344
0*a^6*c^3*integrate(1/4480*x^6*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 8064
*a^6*c^3*integrate(1/4480*x^6*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 430080*
a^5*c^3*integrate(1/4480*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) + 16128*a^5*c
^3*integrate(1/4480*x^5*arctan(a*x)/(a^3*x^2 + a), x) - 80640*a^4*c^3*inte
grate(1/4480*x^4*arctan(a*x)^2/(a^3*x^2 + a), x) - 20160*a^4*c^3*integrate
(1/4480*x^4*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 13440*a^4*c^3*integrate
(1/4480*x^4*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 286720*a^3*c^3*integrate(
1/4480*x^3*arctan(a*x)^3/(a^3*x^2 + a), x) + 26880*a^3*c^3*integrate(1/448
0*x^3*arctan(a*x)/(a^3*x^2 + a), x) - 53760*a^2*c^3*integrate(1/4480*x^2*a
rctan(a*x)^2/(a^3*x^2 + a), x) - 13440*a^2*c^3*integrate(1/4480*x^2*log(a^
2*x^2 + 1)^2/(a^3*x^2 + a), x) - 13440*a^2*c^3*integrate(1/4480*x^2*log(a^
2*x^2 + 1)/(a^3*x^2 + a), x) + 71680*a*c^3*integrate(1/4480*x*arctan(a*...
```

3.381.8 Giac [F]

$$\int x(c+a^2cx^2)^3\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.382 $\int (c + a^2cx^2)^3 \arctan(ax)^3 \, dx$

3.382.1 Optimal result

Integrand size = 19, antiderivative size = 388

$$\begin{aligned}
\int (c + a^2cx^2)^3 \arctan(ax)^3 \, dx = & -\frac{13c^3(1+a^2x^2)}{210a} - \frac{c^3(1+a^2x^2)^2}{140a} \\
& + \frac{14}{15}c^3x \arctan(ax) + \frac{13}{105}c^3x(1+a^2x^2) \arctan(ax) \\
& + \frac{1}{35}c^3x(1+a^2x^2)^2 \arctan(ax) \\
& - \frac{12c^3(1+a^2x^2)\arctan(ax)^2}{35a} \\
& - \frac{9c^3(1+a^2x^2)^2\arctan(ax)^2}{70a} - \frac{c^3(1+a^2x^2)^3\arctan(ax)^2}{14a} \\
& + \frac{16ic^3\arctan(ax)^3}{35a} + \frac{16}{35}c^3x\arctan(ax)^3 \\
& + \frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^3 \\
& + \frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^3 \\
& + \frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^3 \\
& + \frac{48c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{35a} - \frac{7c^3\log(1+a^2x^2)}{15a} \\
& + \frac{48ic^3\arctan(ax)\operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{35a} \\
& + \frac{24c^3\operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{35a}
\end{aligned}$$

output
```
-13/210*c^3*(a^2*x^2+1)/a-1/140*c^3*(a^2*x^2+1)^2/a+14/15*c^3*x*arctan(a*x
)+13/105*c^3*x*(a^2*x^2+1)*arctan(a*x)+1/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x
)-12/35*c^3*(a^2*x^2+1)*arctan(a*x)^2/a-9/70*c^3*(a^2*x^2+1)^2*arctan(a*x)
^2/a-1/14*c^3*(a^2*x^2+1)^3*arctan(a*x)^2/a+48/35*I*c^3*arctan(a*x)*polylo
g(2,1-2/(1+I*a*x))/a+16/35*c^3*x*arctan(a*x)^3+8/35*c^3*x*(a^2*x^2+1)*arct
an(a*x)^3+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^3+1/7*c^3*x*(a^2*x^2+1)^3*a
rctan(a*x)^3+48/35*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))/a-7/15*c^3*ln(a^2*x^2
+1)/a+16/35*I*c^3*arctan(a*x)^3/a+24/35*c^3*polylog(3,1-2/(1+I*a*x))/a
```

3.382.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.63

$$\int (c+a^2cx^2)^3 \arctan(ax)^3 \, dx$$
$$= \frac{c^3(-29-32a^2x^2-3a^4x^4+456ax\arctan(ax)+76a^3x^3\arctan(ax)+12a^5x^5\arctan(ax)-228\arctan(ax$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-29 - 32*a^2*x^2 - 3*a^4*x^4 + 456*a*x*ArcTan[a*x] + 76*a^3*x^3*ArcT
an[a*x] + 12*a^5*x^5*ArcTan[a*x] - 228*ArcTan[a*x]^2 - 342*a^2*x^2*ArcTan[
a*x]^2 - 144*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 - (192*I)*Ar
cTan[a*x]^3 + 420*a*x*ArcTan[a*x]^3 + 420*a^3*x^3*ArcTan[a*x]^3 + 252*a^5*
x^5*ArcTan[a*x]^3 + 60*a^7*x^7*ArcTan[a*x]^3 + 576*ArcTan[a*x]^2*Log[1 + E
^((2*I)*ArcTan[a*x])] - 196*Log[1 + a^2*x^2] - (576*I)*ArcTan[a*x]*PolyLog
[2, -E^((2*I)*ArcTan[a*x])] + 288*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(42
0*a)
```

3.382.3 Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.22, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {5415, 27, 5413, 5413, 5345, 240, 5415, 5413, 5345, 240, 5415, 5345, 240, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \; 5415$$

$$\frac{1}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}$$

$$\Big\downarrow \; 5413$$

$$\frac{1}{7}c^3\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}$$

$$\Big\downarrow \; 5413$$

$$\frac{1}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right.$$
$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}$$

$$\Big\downarrow \; 5345$$

$$\frac{1}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(\right.$$
$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}$$

$$\Big\downarrow \; 240$$

$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \; 5415$$

3.382. $\int \left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$

$$\frac{6}{7}c^3\left(\frac{3}{10}\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3\left(a^2x^2+1\right.}{}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{5413}$$

$$\frac{6}{7}c^3\left(\frac{3}{10}\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}x\left(a^2\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{5345}$$

$$\frac{6}{7}c^3\left(\frac{3}{10}\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}\int \left(a^2x^2+1\right)\text{arctan}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{240}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}\left(\frac{1}{3}x\left(a^2x^2+\right.\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{5415}$$

3.382. $\int \left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\int \arctan(ax)dx+\frac{2}{3}\int \arctan(ax)^3dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}\right)+\frac{1}{5}x(a\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{5345}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)-a\int \frac{x}{a^2x^2+1}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right.}{}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{240}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}-\frac{\text{lo}}{}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{5455}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)}{2}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \text{5379}$$

3.382. $\int \left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x(a^2$$

$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$

$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \; 5529$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax}\right.}{2a}\right.}{a}\right.\right.\right.\right.$$

$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$

$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

$$\Big\downarrow \; 7164$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{Pol}}{}\right.}{a}\right.\right.\right.\right.$$

$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$

$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right.\right.$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

3.382. $\int\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$

output
```
-1/14*(c^3*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/a + (c^3*x*(1 + a^2*x^2)^3*ArcTa
n[a*x]^3)/7 + (c^3*(-1/20*(1 + a^2*x^2)^2/a + (x*(1 + a^2*x^2)^2*ArcTan[a*
x])/5 + (4*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x
*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/5))/7 + (6*c^3*((-3*(1 + a^2*x
^2)^2*ArcTan[a*x]^2)/(20*a) + (x*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/5 + (3*(-1
/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] -
 Log[1 + a^2*x^2]/(2*a)))/3))/10 + (4*(x*ArcTan[a*x] - ((1 + a^2*x^2)*ArcT
an[a*x]^2)/(2*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^3)/3 - Log[1 + a^2*x^2]/(2
*a) + (2*(x*ArcTan[a*x]^3 - 3*a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a
*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1
 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)))/3))/5))/7
```

3.382.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.382.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 86.79 (sec) , antiderivative size = 1267, normalized size of antiderivative = 3.27

method	result	size
derivativedivides	Expression too large to display	1267
default	Expression too large to display	1267
parts	Expression too large to display	1268

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/7*c^3*arctan(a*x)^3*a^7*x^7+3/5*c^3*arctan(a*x)^3*a^5*x^5+c^3*arcta
n(a*x)^3*a^3*x^3+c^3*arctan(a*x)^3*a*x-3/35*c^3*(4*I*Pi*csgn(I/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+19/3*arctan(a*x)^2+1
9/2*x^2*arctan(a*x)^2*a^2-16*arctan(a*x)^2*ln(2)+1/12*(I+a*x)^4+5/6*a^6*x^
6*arctan(a*x)^2-11/3*arctan(a*x)*(a*x-I)^2*(I+a*x)+5/3*arctan(a*x)*(a*x-I)
^4*(I+a*x)+10/3*arctan(a*x)*(a*x-I)^2*(I+a*x)^3+11/3*arctan(a*x)*(a*x-I)*(
I+a*x)^2-10/3*arctan(a*x)*(a*x-I)^3*(I+a*x)^2-5/3*arctan(a*x)*(a*x-I)*(I+a
*x)^4+4*a^4*arctan(a*x)^2*x^4+11/9*arctan(a*x)*(a*x-I)^3-8*arctan(a*x)*(a*
x-I)-1/3*arctan(a*x)*(a*x-I)^5-8*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-98/9*
ln((1+I*a*x)^2/(a^2*x^2+1)+1)+8*arctan(a*x)^2*ln(a^2*x^2+1)+7/18*(I+a*x)^2
-5/3*I*arctan(a*x)*(a*x-I)^4-4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
3*arctan(a*x)^2-10*I*arctan(a*x)*(a*x-I)^2*(I+a*x)^2+20/3*I*arctan(a*x)*(a
*x-I)^3*(I+a*x)+20/3*I*arctan(a*x)*(a*x-I)*(I+a*x)^3+6*I*arctan(a*x)*(a*x-
I)*(I+a*x)+4*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^3*arctan(a*x)^2+4*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)
^2-8*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1))^2*arctan(a*x)^2+8*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1...
```

3.382.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3\,dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
, x)
```

3.382.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3\,dx = c^3\left(\int 3a^2x^2\operatorname{atan}^3(ax)\,dx + \int 3a^4x^4\operatorname{atan}^3(ax)\,dx + \int a^6x^6\operatorname{atan}^3(ax)\,dx + \int \operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**3, x) + Integral(3*a**4*x**4*atan(a*
x)**3, x) + Integral(a**6*x**6*atan(a*x)**3, x) + Integral(atan(a*x)**3, x
))
```

3.382.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3\,dx = \int \left(a^2cx^2+c\right)^3 \arctan(ax)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
980*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)^3/(a^2*x^2 + 1), x) + 105*a^8
*c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
 + 60*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 +
 1), x) - 60*a^7*c^3*integrate(1/1120*x^7*arctan(a*x)^2/(a^2*x^2 + 1), x)
+ 15*a^7*c^3*integrate(1/1120*x^7*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3
920*a^6*c^3*integrate(1/1120*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 420*a^6
*c^3*integrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
 + 252*a^6*c^3*integrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2
+ 1), x) - 252*a^5*c^3*integrate(1/1120*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x
) + 63*a^5*c^3*integrate(1/1120*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
 5880*a^4*c^3*integrate(1/1120*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 630*a
^4*c^3*integrate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1),
x) + 420*a^4*c^3*integrate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^
2 + 1), x) - 420*a^3*c^3*integrate(1/1120*x^3*arctan(a*x)^2/(a^2*x^2 + 1),
 x) + 105*a^3*c^3*integrate(1/1120*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 7/32*c^3*arctan(a*x)^4/a + 3920*a^2*c^3*integrate(1/1120*x^2*arctan(a*
x)^3/(a^2*x^2 + 1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)*log(
a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan
(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 420*a*c^3*integrate(1/1120*x*ar
ctan(a*x)^2/(a^2*x^2 + 1), x) + 105*a*c^3*integrate(1/1120*x*log(a^2*x^...
```

3.382.8 Giac [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx = \int \mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3 \, dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

3.383 $\int \frac{\left(c+a^2cx^2\right)^3\arctan(ax)^3}{x}\,dx$

3.383.1 Optimal result

Integrand size = 22, antiderivative size = 447

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx = & -\frac{13}{30}ac^3x-\frac{1}{60}a^3c^3x^3+\frac{13}{30}c^3\arctan(ax)\\
&+\frac{29}{60}a^2c^3x^2\arctan(ax)+\frac{1}{20}a^4c^3x^4\arctan(ax)\\
&-\frac{34}{15}ic^3\arctan(ax)^2-\frac{11}{4}ac^3x\arctan(ax)^2\\
&-\frac{7}{12}a^3c^3x^3\arctan(ax)^2-\frac{1}{10}a^5c^3x^5\arctan(ax)^2\\
&+\frac{11}{12}c^3\arctan(ax)^3+\frac{3}{2}a^2c^3x^2\arctan(ax)^3\\
&+\frac{3}{4}a^4c^3x^4\arctan(ax)^3+\frac{1}{6}a^6c^3x^6\arctan(ax)^3\\
&+2c^3\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)\\
&-\frac{68}{15}c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\
&-\frac{34}{15}ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\
&-\frac{3}{2}ic^3\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}ic^3\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\
&-\frac{3}{2}c^3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}c^3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\\
&+\frac{3}{4}ic^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right)\\
&-\frac{3}{4}ic^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-13/30*a*c^3*x-1/60*a^3*c^3*x^3+13/30*c^3*arctan(a*x)+29/60*a^2*c^3*x^2*ar
ctan(a*x)+1/20*a^4*c^3*x^4*arctan(a*x)-34/15*I*c^3*arctan(a*x)^2-11/4*a*c^
3*x*arctan(a*x)^2-7/12*a^3*c^3*x^3*arctan(a*x)^2-1/10*a^5*c^3*x^5*arctan(a
*x)^2+11/12*c^3*arctan(a*x)^3+3/2*a^2*c^3*x^2*arctan(a*x)^3+3/4*a^4*c^3*x^
4*arctan(a*x)^3+1/6*a^6*c^3*x^6*arctan(a*x)^3-2*c^3*arctan(a*x)^3*arctanh(
-1+2/(1+I*a*x))-68/15*c^3*arctan(a*x)*ln(2/(1+I*a*x))+3/2*I*c^3*arctan(a*x
)^2*polylog(2,-1+2/(1+I*a*x))-3/2*I*c^3*arctan(a*x)^2*polylog(2,1-2/(1+I*a
*x))-3/4*I*c^3*polylog(4,-1+2/(1+I*a*x))-3/2*c^3*arctan(a*x)*polylog(3,1-2
/(1+I*a*x))+3/2*c^3*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/4*I*c^3*polylo
g(4,1-2/(1+I*a*x))-34/15*I*c^3*polylog(2,1-2/(1+I*a*x))
```

3.383.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx = \frac{1}{960}c^3\big(&-15i\pi^4-416ax-16a^3x^3+416\arctan(ax)\\
&+464a^2x^2\arctan(ax)+48a^4x^4\arctan(ax)\\
&+2176i\arctan(ax)^2-2640ax\arctan(ax)^2\\
&-560a^3x^3\arctan(ax)^2-96a^5x^5\arctan(ax)^2\\
&+880\arctan(ax)^3+1440a^2x^2\arctan(ax)^3\\
&+720a^4x^4\arctan(ax)^3+160a^6x^6\arctan(ax)^3\\
&+480i\arctan(ax)^4+960\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right)\\
&-4352\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right)\\
&-960\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right)\\
&+1440i\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)\\
&+32i\left(68+45\arctan(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)\\
&+1440\arctan(ax)\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)\\
&-1440\arctan(ax)\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\\
&-720i\operatorname{PolyLog}\left(4,e^{-2i\arctan(ax)}\right)\\
&-720i\operatorname{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\big)\end{aligned}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```

output
```
(c^3*((-15*I)*Pi^4 - 416*a*x - 16*a^3*x^3 + 416*ArcTan[a*x] + 464*a^2*x^2*
ArcTan[a*x] + 48*a^4*x^4*ArcTan[a*x] + (2176*I)*ArcTan[a*x]^2 - 2640*a*x*A
rcTan[a*x]^2 - 560*a^3*x^3*ArcTan[a*x]^2 - 96*a^5*x^5*ArcTan[a*x]^2 + 880*
ArcTan[a*x]^3 + 1440*a^2*x^2*ArcTan[a*x]^3 + 720*a^4*x^4*ArcTan[a*x]^3 + 1
60*a^6*x^6*ArcTan[a*x]^3 + (480*I)*ArcTan[a*x]^4 + 960*ArcTan[a*x]^3*Log[1
 - E^((-2*I)*ArcTan[a*x])] - 4352*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x]
)] - 960*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (1440*I)*ArcTan[a*
x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*(68 + 45*ArcTan[a*x]^2)*P
olyLog[2, -E^((2*I)*ArcTan[a*x])] + 1440*ArcTan[a*x]*PolyLog[3, E^((-2*I)*
ArcTan[a*x])] - 1440*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (720
*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (720*I)*PolyLog[4, -E^((2*I)*ArcT
an[a*x])]))/960
```

3.383.3 Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}{x}\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^5\arctan(ax)^3+3a^4c^3x^3\arctan(ax)^3+3a^2c^3x\arctan(ax)^3+\frac{c^3\arctan(ax)^3}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a^6c^3x^6\arctan(ax)^3 - \frac{1}{10}a^5c^3x^5\arctan(ax)^2 + \frac{3}{4}a^4c^3x^4\arctan(ax)^3 + \frac{1}{20}a^4c^3x^4\arctan(ax) - \frac{7}{12}a^3c^3x^3\arctan(ax)^2 - \frac{1}{60}a^3c^3x^3 + \frac{3}{2}a^2c^3x^2\arctan(ax)^3 + \frac{29}{60}a^2c^3x^2\arctan(ax) + 2c^3\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - \frac{3}{2}ic^3\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right) + \frac{3}{2}ic^3\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right) - \frac{3}{2}c^3\arctan(ax)\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right) + \frac{3}{2}c^3\arctan(ax)\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right) - \frac{11}{4}ac^3x\arctan(ax)^2 + \frac{11}{12}c^3\arctan(ax)^3 - \frac{34}{15}ic^3\arctan(ax)^2 + \frac{13}{30}c^3\arctan(ax) - \frac{68}{15}c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right) - \frac{34}{15}ic^3\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right) + \frac{3}{4}ic^3\text{PolyLog}\left(4,1-\frac{2}{iax+1}\right) - \frac{3}{4}ic^3\text{PolyLog}\left(4,\frac{2}{iax+1}-1\right) - \frac{13}{30}ac^3x$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```

output
```
(-13*a*c^3*x)/30 - (a^3*c^3*x^3)/60 + (13*c^3*ArcTan[a*x])/30 + (29*a^2*c^
3*x^2*ArcTan[a*x])/60 + (a^4*c^3*x^4*ArcTan[a*x])/20 - ((34*I)/15)*c^3*Arc
Tan[a*x]^2 - (11*a*c^3*x*ArcTan[a*x]^2)/4 - (7*a^3*c^3*x^3*ArcTan[a*x]^2)/
12 - (a^5*c^3*x^5*ArcTan[a*x]^2)/10 + (11*c^3*ArcTan[a*x]^3)/12 + (3*a^2*c
^3*x^2*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^3)/4 + (a^6*c^3*x^6*A
rcTan[a*x]^3)/6 + 2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - (68*c^3
*ArcTan[a*x]*Log[2/(1 + I*a*x)])/15 - ((34*I)/15)*c^3*PolyLog[2, 1 - 2/(1
+ I*a*x)] - ((3*I)/2)*c^3*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((
3*I)/2)*c^3*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^3*ArcTan[a
*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*c^3*ArcTan[a*x]*PolyLog[3, -1 +
2/(1 + I*a*x)])/2 + ((3*I)/4)*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4
)*c^3*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

3.383.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.383. $\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx$

3.383.4 Maple [A] (verified)

Time = 56.76 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{c^3\left(-136\arctan(ax)^2-55i\arctan(ax)^3+iax+55\arctan(ax)^3ax-29x^2\arctan(ax)^2a^2-26i\arctan(ax)-3i\arctan(ax)a^2\right.}{}$
default	$\frac{c^3\left(-136\arctan(ax)^2-55i\arctan(ax)^3+iax+55\arctan(ax)^3ax-29x^2\arctan(ax)^2a^2-26i\arctan(ax)-3i\arctan(ax)a^2\right.}{}$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*c^3*(-136*arctan(a*x)^2-55*I*arctan(a*x)^3+I*a*x+55*arctan(a*x)^3*a*x
-29*x^2*arctan(a*x)^2*a^2-26*I*arctan(a*x)-3*I*arctan(a*x)*a^2*x^2+35*arct
an(a*x)^3*a^3*x^3-6*a^4*arctan(a*x)^2*x^4-35*I*arctan(a*x)^3*a^2*x^2+10*ar
ctan(a*x)^3*a^5*x^5-25+29*I*arctan(a*x)^2*a*x-10*I*arctan(a*x)^3*a^4*x^4+2
6*x*arctan(a*x)*a-a^2*x^2+6*I*arctan(a*x)^2*a^3*x^3+3*arctan(a*x)*x^3*a^3)
*(I+a*x)+3/2*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-68/15
*c^3*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+6*I*c^3*polylog(4,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))+c^3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+3
4/15*I*c^3*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+6*c^3*arctan(a*x)*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(
a^2*x^2+1)^(1/2))-c^3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+68/15*I*
c^3*arctan(a*x)^2-3/2*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-
3/4*I*c^3*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+c^3*arctan(a*x)^3*ln(1-(1+I*
a*x)/(a^2*x^2+1)^(1/2))-3*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))+6*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c
^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))
```

3.383.5 Fricas [F]

$$\int\frac{\left(c+a^2cx^2\right)^3\arctan(ax)^3}{x}\,dx=\int\frac{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x, x)
```

3.383.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx = c^3\left(\int \frac{\operatorname{atan}^3(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}^3(ax)\,dx + \int 3a^4x^3\operatorname{atan}^3(ax)\,dx + \int a^6x^5\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x,x)
```

output
```
c**3*(Integral(atan(a*x)**3/x, x) + Integral(3*a**2*x*atan(a*x)**3, x) + I
ntegral(3*a**4*x**3*atan(a*x)**3, x) + Integral(a**6*x**5*atan(a*x)**3, x)
)
```

3.383.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
1/96*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)^3 - 1/12
8*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)*log(a^2*x^2
 + 1)^2 + integrate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^
4 + 4*a^2*c^3*x^2 + c^3)*arctan(a*x)^3 - 4*(2*a^7*c^3*x^7 + 9*a^5*c^3*x^5
+ 18*a^3*c^3*x^3)*arctan(a*x)^2 + 4*(2*a^8*c^3*x^8 + 9*a^6*c^3*x^6 + 18*a^
4*c^3*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (2*a^7*c^3*x^7 + 9*a^5*c^3*x^5 +
 18*a^3*c^3*x^3 + 12*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*
c^3*x^2 + c^3)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^3 + x), x)
```

3.383.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x} \, dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
Timed out
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x} \, dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}{x} \, dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x, x)
```

3.384 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} dx$

3.384.1 Optimal result

Integrand size = 22, antiderivative size = 354

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} dx = & -\frac{1}{20}a^3c^3x^2 + \frac{21}{10}a^2c^3x\arctan(ax) + \frac{1}{10}a^4c^3x^3\arctan(ax) \\ & - \frac{21}{20}ac^3\arctan(ax)^2 - \frac{6}{5}a^3c^3x^2\arctan(ax)^2 \\ & - \frac{3}{20}a^5c^3x^4\arctan(ax)^2 + \frac{6}{5}iac^3\arctan(ax)^3 \\ & - \frac{c^3\arctan(ax)^3}{x} + 3a^2c^3x\arctan(ax)^3 + a^4c^3x^3\arctan(ax)^3 \\ & + \frac{1}{5}a^6c^3x^5\arctan(ax)^3 + \frac{33}{5}ac^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) \\ & - ac^3\log\left(1+a^2x^2\right) + 3ac^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\ & - 3iac^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ & + \frac{33}{5}iac^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\ & + \frac{3}{2}ac^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) \\ & + \frac{33}{10}ac^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-1/20*a^3*c^3*x^2+21/10*a^2*c^3*x*arctan(a*x)+1/10*a^4*c^3*x^3*arctan(a*x)
-21/20*a*c^3*arctan(a*x)^2-6/5*a^3*c^3*x^2*arctan(a*x)^2-3/20*a^5*c^3*x^4*
arctan(a*x)^2-3*I*a*c^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))-c^3*arctan(a
*x)^3/x+3*a^2*c^3*x*arctan(a*x)^3+a^4*c^3*x^3*arctan(a*x)^3+1/5*a^6*c^3*x^
5*arctan(a*x)^3+33/5*a*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))-a*c^3*ln(a^2*x^2+
1)+3*a*c^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))+33/5*I*a*c^3*arctan(a*x)*polylo
g(2,1-2/(1+I*a*x))+6/5*I*a*c^3*arctan(a*x)^3+3/2*a*c^3*polylog(3,-1+2/(1-I
*a*x))+33/10*a*c^3*polylog(3,1-2/(1+I*a*x))
```

3.384.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.84

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} dx = \frac{c^3(-2ax - 5ia\pi^3x - 2a^3x^3 + 84a^2x^2\arctan(ax) + 4a^4x^4\arctan(ax) - 42ax\arctan(ax)^2 - 48a^3x^3\arctan$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```

output
```
(c^3*(-2*a*x - (5*I)*a*Pi^3*x - 2*a^3*x^3 + 84*a^2*x^2*ArcTan[a*x] + 4*a^4
*x^4*ArcTan[a*x] - 42*a*x*ArcTan[a*x]^2 - 48*a^3*x^3*ArcTan[a*x]^2 - 6*a^5
*x^5*ArcTan[a*x]^2 - 40*ArcTan[a*x]^3 - (48*I)*a*x*ArcTan[a*x]^3 + 120*a^2
*x^2*ArcTan[a*x]^3 + 40*a^4*x^4*ArcTan[a*x]^3 + 8*a^6*x^6*ArcTan[a*x]^3 +
120*a*x*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 264*a*x*ArcTan[a*x
]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 40*a*x*Log[1 + a^2*x^2] + (120*I)*a*x
*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (264*I)*a*x*ArcTan[a*x]*
PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*
x])] + 132*a*x*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(40*x)
```

3.384.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}{x^2} \, dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^6c^3x^4\arctan(ax)^3+3a^4c^3x^2\arctan(ax)^3+3a^2c^3\arctan(ax)^3+\frac{c^3\arctan(ax)^3}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{5}a^6c^3x^5\arctan(ax)^3-\frac{3}{20}a^5c^3x^4\arctan(ax)^2+a^4c^3x^3\arctan(ax)^3+\frac{1}{10}a^4c^3x^3\arctan(ax)-\frac{6}{5}a^3c^3x^2\arctan(ax)^2-\frac{1}{20}a^3c^3x^2+3a^2c^3x\arctan(ax)^3+\frac{21}{10}a^2c^3x\arctan(ax)-ac^3\log\left(a^2x^2+1\right)-3iac^3\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{33}{5}iac^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{6}{5}iac^3\arctan(ax)^3-\frac{21}{20}ac^3\arctan(ax)^2-\frac{c^3\arctan(ax)^3}{x}+\frac{33}{5}ac^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ac^3\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+\frac{33}{10}ac^3\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```

output
```
-1/20*(a^3*c^3*x^2) + (21*a^2*c^3*x*ArcTan[a*x])/10 + (a^4*c^3*x^3*ArcTan[
a*x])/10 - (21*a*c^3*ArcTan[a*x]^2)/20 - (6*a^3*c^3*x^2*ArcTan[a*x]^2)/5 -
 (3*a^5*c^3*x^4*ArcTan[a*x]^2)/20 + ((6*I)/5)*a*c^3*ArcTan[a*x]^3 - (c^3*A
rcTan[a*x]^3)/x + 3*a^2*c^3*x*ArcTan[a*x]^3 + a^4*c^3*x^3*ArcTan[a*x]^3 +
(a^6*c^3*x^5*ArcTan[a*x]^3)/5 + (33*a*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)]
)/5 - a*c^3*Log[1 + a^2*x^2] + 3*a*c^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)
] - (3*I)*a*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + ((33*I)/5)*a*
c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^3*PolyLog[3, -1 + 2
/(1 - I*a*x)])/2 + (33*a*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/10
```

3.384. $\int \frac{\left(c+a^2cx^2\right)^3\arctan(ax)^3}{x^2}\,dx$

3.384.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.384.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 163.40 (sec) , antiderivative size = 1894, normalized size of antiderivative = 5.35

method	result	size
derivativedivides	Expression too large to display	1894
default	Expression too large to display	1894
parts	Expression too large to display	1897

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*c^3*arctan(a*x)^3*a^5*x^5+c^3*arctan(a*x)^3*a^3*x^3+3*c^3*arctan(a*
x)^3*a*x-c^3*arctan(a*x)^3/a/x-3/5*c^3*(1/4*a^4*arctan(a*x)^2*x^4+2*x^2*ar
ctan(a*x)^2*a^2+8*arctan(a*x)^2*ln(a^2*x^2+1)-5*arctan(a*x)^2*ln(a*x)-16*a
rctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+10*I*arctan(a*x)*polylog(2,(1
+I*a*x)/(a^2*x^2+1)^(1/2))-10/3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+5*arctan(a*x
)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-5*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)
^(1/2)+1)-1/12*I*(I-64*arctan(a*x)^3+I*a^2*x^2-40*arctan(a*x)-48*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*arctan(a*x)^2
*Pi+48*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2*Pi+48*csgn(I*(1+I*a*x)^2/
(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^2*arctan(a*x)^2*Pi-96*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2*Pi+48*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2*Pi+96*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)
^2*Pi-48*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(
a*x)^2*Pi+30*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))
*arctan(a*x)^2*Pi+30*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2...
```

3.384.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x^2, x)
```

3.384.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2}\,dx = c^3\left(\int 3a^2 \operatorname{atan}^3(ax)\,dx + \int \frac{\operatorname{atan}^3(ax)}{x^2}\,dx + \int 3a^4x^2 \operatorname{atan}^3(ax)\,dx + \int a^6x^4 \operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) +
Integral(3*a**4*x**2*atan(a*x)**3, x) + Integral(a**6*x**4*atan(a*x)**3, x
))
```

3.384.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/320*(8*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x
)^3 - 6*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x)
*log(a^2*x^2 + 1)^2 + 5*(8960*a^8*c^3*integrate(1/160*x^8*arctan(a*x)^3/(a
^2*x^4 + x^2), x) + 960*a^8*c^3*integrate(1/160*x^8*arctan(a*x)*log(a^2*x^
2 + 1)^2/(a^2*x^4 + x^2), x) + 768*a^8*c^3*integrate(1/160*x^8*arctan(a*x)
*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 768*a^7*c^3*integrate(1/160*x^7*ar
ctan(a*x)^2/(a^2*x^4 + x^2), x) + 192*a^7*c^3*integrate(1/160*x^7*log(a^2*
x^2 + 1)^2/(a^2*x^4 + x^2), x) + 35840*a^6*c^3*integrate(1/160*x^6*arctan(
a*x)^3/(a^2*x^4 + x^2), x) + 3840*a^6*c^3*integrate(1/160*x^6*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 3840*a^6*c^3*integrate(1/160*x^6*
arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 3840*a^5*c^3*integrate(
1/160*x^5*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 960*a^5*c^3*integrate(1/160*
x^5*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 56*a*c^3*arctan(a*x)^4 + 5376
0*a^4*c^3*integrate(1/160*x^4*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 5760*a^4
*c^3*integrate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x
) + 11520*a^4*c^3*integrate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^
4 + x^2), x) - 11520*a^3*c^3*integrate(1/160*x^3*arctan(a*x)^2/(a^2*x^4 +
x^2), x) + 3*a*c^3*log(a^2*x^2 + 1)^3 + 3840*a^2*c^3*integrate(1/160*x^2*a
rctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 3840*a^2*c^3*integrate
(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 3840*a*c^...
```

3.384.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} \, dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} \, dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}{x^2} \, dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^2, x)
```

3.385 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3} dx$

3.385.1 Optimal result

Integrand size = 22, antiderivative size = 503

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3}\,dx = & -\frac{1}{4}a^3c^3x + \frac{1}{4}a^2c^3\arctan(ax) + \frac{1}{4}a^4c^3x^2\arctan(ax) \\
& - 5ia^2c^3\arctan(ax)^2 - \frac{3ac^3\arctan(ax)^2}{2x} \\
& - \frac{15}{4}a^3c^3x\arctan(ax)^2 - \frac{1}{4}a^5c^3x^3\arctan(ax)^2 \\
& + \frac{3}{4}a^2c^3\arctan(ax)^3 - \frac{c^3\arctan(ax)^3}{2x^2} \\
& + \frac{3}{2}a^4c^3x^2\arctan(ax)^3 + \frac{1}{4}a^6c^3x^4\arctan(ax)^3 \\
& + 6a^2c^3\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) \\
& - 7a^2c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right) \\
& + 3a^2c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) \\
& - \frac{3}{2}ia^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\
& - \frac{7}{2}ia^2c^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
& - \frac{9}{2}ia^2c^3\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
& + \frac{9}{2}ia^2c^3\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) \\
& - \frac{9}{2}a^2c^3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) \\
& + \frac{9}{2}a^2c^3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right) \\
& + \frac{9}{4}ia^2c^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right) \\
& - \frac{9}{4}ia^2c^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)
\end{aligned}
$$

output
```
-1/4*a^3*c^3*x+1/4*a^2*c^3*arctan(a*x)+1/4*a^4*c^3*x^2*arctan(a*x)-9/4*I*a
^2*c^3*polylog(4,-1+2/(1+I*a*x))-3/2*a*c^3*arctan(a*x)^2/x-15/4*a^3*c^3*x*
arctan(a*x)^2-1/4*a^5*c^3*x^3*arctan(a*x)^2+3/4*a^2*c^3*arctan(a*x)^3-1/2*
c^3*arctan(a*x)^3/x^2+3/2*a^4*c^3*x^2*arctan(a*x)^3+1/4*a^6*c^3*x^4*arctan
(a*x)^3-6*a^2*c^3*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-7*a^2*c^3*arctan(a
*x)*ln(2/(1+I*a*x))+3*a^2*c^3*arctan(a*x)*ln(2-2/(1-I*a*x))+9/4*I*a^2*c^3*
polylog(4,1-2/(1+I*a*x))-3/2*I*a^2*c^3*polylog(2,-1+2/(1-I*a*x))-5*I*a^2*c
^3*arctan(a*x)^2-7/2*I*a^2*c^3*polylog(2,1-2/(1+I*a*x))-9/2*a^2*c^3*arctan
(a*x)*polylog(3,1-2/(1+I*a*x))+9/2*a^2*c^3*arctan(a*x)*polylog(3,-1+2/(1+I
*a*x))-9/2*I*a^2*c^3*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))+9/2*I*a^2*c^3*
arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))
```

3.385.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx$$
$$= \frac{c^3\left(-3ia^2\pi^4x^2-16a^3x^3+16a^2x^2\arctan(ax)+16a^4x^4\arctan(ax)-96ax\arctan(ax)^2+128ia^2x^2\arctan\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```

output
```
(c^3*((-3*I)*a^2*Pi^4*x^2 - 16*a^3*x^3 + 16*a^2*x^2*ArcTan[a*x] + 16*a^4*x
^4*ArcTan[a*x] - 96*a*x*ArcTan[a*x]^2 + (128*I)*a^2*x^2*ArcTan[a*x]^2 - 24
0*a^3*x^3*ArcTan[a*x]^2 - 16*a^5*x^5*ArcTan[a*x]^2 - 32*ArcTan[a*x]^3 + 48
*a^2*x^2*ArcTan[a*x]^3 + 96*a^4*x^4*ArcTan[a*x]^3 + 16*a^6*x^6*ArcTan[a*x]
^3 + (96*I)*a^2*x^2*ArcTan[a*x]^4 + 192*a^2*x^2*ArcTan[a*x]^3*Log[1 - E^((
-2*I)*ArcTan[a*x])] + 192*a^2*x^2*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x]
)] - 448*a^2*x^2*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 192*a^2*x^2*
ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (288*I)*a^2*x^2*ArcTan[a*x]
^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*a^2*x^2*(7 + 9*ArcTan[a*x]^
2)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - (96*I)*a^2*x^2*PolyLog[2, E^((2*I)
*ArcTan[a*x])] + 288*a^2*x^2*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])
] - 288*a^2*x^2*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (144*I)*a
^2*x^2*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (144*I)*a^2*x^2*PolyLog[4, -E^
((2*I)*ArcTan[a*x])]))/(64*x^2)
```

3.385.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}{x^3}dx$$

$$\Big\downarrow \ 5483$$

$$\int \left(a^6c^3x^3\arctan(ax)^3+3a^4c^3x\arctan(ax)^3+\frac{3a^2c^3\arctan(ax)^3}{x}+\frac{c^3\arctan(ax)^3}{x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}a^6c^3x^4\arctan(ax)^3-\frac{1}{4}a^5c^3x^3\arctan(ax)^2+\frac{3}{2}a^4c^3x^2\arctan(ax)^3+\frac{1}{4}a^4c^3x^2\arctan(ax)-\frac{15}{4}a^3c^3x\arctan(ax)^2-\frac{1}{4}a^3c^3x+6a^2c^3\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-\frac{9}{2}ia^2c^3\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{9}{2}ia^2c^3\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{9}{2}a^2c^3\arctan(ax)\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{9}{2}a^2c^3\arctan(ax)\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)+\frac{3}{4}a^2c^3\arctan(ax)^3-5ia^2c^3\arctan(ax)^2+\frac{1}{4}a^2c^3\arctan(ax)-7a^2c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)+3a^2c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{3}{2}ia^2c^3\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-\frac{7}{2}ia^2c^3\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{9}{4}ia^2c^3\text{PolyLog}\left(4,1-\frac{2}{iax+1}\right)-\frac{9}{4}ia^2c^3\text{PolyLog}\left(4,\frac{2}{iax+1}-1\right)-\frac{c^3\arctan(ax)^3}{2x^2}-\frac{3ac^3\arctan(ax)^2}{2x}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```

output
```
-1/4*(a^3*c^3*x) + (a^2*c^3*ArcTan[a*x])/4 + (a^4*c^3*x^2*ArcTan[a*x])/4 -
 (5*I)*a^2*c^3*ArcTan[a*x]^2 - (3*a*c^3*ArcTan[a*x]^2)/(2*x) - (15*a^3*c^3
*x*ArcTan[a*x]^2)/4 - (a^5*c^3*x^3*ArcTan[a*x]^2)/4 + (3*a^2*c^3*ArcTan[a*
x]^3)/4 - (c^3*ArcTan[a*x]^3)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^3)/2 +
(a^6*c^3*x^4*ArcTan[a*x]^3)/4 + 6*a^2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 +
 I*a*x)] - 7*a^2*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^3*ArcTan[a*x
]*Log[2 - 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^3*PolyLog[2, -1 + 2/(1 - I*a*x)
] - ((7*I)/2)*a^2*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((9*I)/2)*a^2*c^3*Ar
cTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((9*I)/2)*a^2*c^3*ArcTan[a*x]^
2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, 1 - 2
/(1 + I*a*x)])/2 + (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/
2 + ((9*I)/4)*a^2*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((9*I)/4)*a^2*c^3*Po
lyLog[4, -1 + 2/(1 + I*a*x)]
```

3.385.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.385.4 Maple [A] (verified)

Time = 77.00 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.52

method	result
derivativedivides	$a^2\left(\frac{c^3\left(2i\arctan(ax)^3+6i\arctan(ax)^2ax-2\arctan(ax)^3ax-14x^2\arctan(ax)^2a^2-5i\arctan(ax)^3a^2x^2+i\arctan(ax)^2\right.}{}\right.$
default	$a^2\left(\frac{c^3\left(2i\arctan(ax)^3+6i\arctan(ax)^2ax-2\arctan(ax)^3ax-14x^2\arctan(ax)^2a^2-5i\arctan(ax)^3a^2x^2+i\arctan(ax)^2\right.}{}\right.$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/4*c^3*(2*I*arctan(a*x)^3+6*I*arctan(a*x)^2*a*x-2*arctan(a*x)^3*a*x-
14*x^2*arctan(a*x)^2*a^2-5*I*arctan(a*x)^3*a^2*x^2+I*arctan(a*x)^2*a^3*x^3
+5*arctan(a*x)^3*a^3*x^3-a^4*arctan(a*x)^2*x^4-I*arctan(a*x)^3*a^4*x^4+arc
tan(a*x)^3*a^5*x^5-a^2*x^2-I*arctan(a*x)*a^2*x^2+arctan(a*x)*x^3*a^3)*(I+a
*x)/a^2/x^2-9*I*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-7
*c^3*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3*I*c^3*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))+3*c^3*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+7/
2*I*c^3*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+3*c^3*arctan(a*x)^3*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2)+1)+18*I*c^3*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1
8*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^3*polylog(
2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^
2+1)^(1/2))+18*I*c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-9/2*c^3*arctan
(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+4*I*c^3*arctan(a*x)^2+18*c^3*arc
tan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*c^3*arctan(a*x)^2*poly
log(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*c^3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^
2*x^2+1)+1)-9/4*I*c^3*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+3*c^3*arctan(a*x
)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+9/2*I*c^3*arctan(a*x)^2*polylog(2,-(1+
I*a*x)^2/(a^2*x^2+1)))
```

3.385.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^3\arctan{(ax)}^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x^3, x)
```

3.385.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx = c^3\left(\int \frac{\operatorname{atan}^3{(ax)}}{x^3}\,dx + \int \frac{3a^2\operatorname{atan}^3{(ax)}}{x}\,dx + \int 3a^4x\operatorname{atan}^3{(ax)}\,dx + \int a^6x^3\operatorname{atan}^3{(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**3,x)
```

output
```
c**3*(Integral(atan(a*x)**3/x**3, x) + Integral(3*a**2*atan(a*x)**3/x, x)
+ Integral(3*a**4*x*atan(a*x)**3, x) + Integral(a**6*x**3*atan(a*x)**3, x)
)
```

3.385.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
1/128*(4*(a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^3 - 3*(a^6*c^3*
x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 128*x^2*inte
grate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3 - 12*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x)*ar
ctan(a*x)^2 + 12*(a^8*c^3*x^8 + 6*a^6*c^3*x^6 - 2*a^2*c^3*x^2)*arctan(a*x)
*log(a^2*x^2 + 1) + 3*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x + 4*(a^8*c^
3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan(a*x))*
log(a^2*x^2 + 1)^2)/(a^2*x^5 + x^3), x))/x^2
```

3.385.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3} \, dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^3 \arctan(ax)^3}{x^3} \, dx = \int \frac{\operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3}{x^3} \, dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^3, x)
```

3.386 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4} dx$

3.386.1 Optimal result

Integrand size = 22, antiderivative size = 336

$$\begin{aligned}
\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4} dx = &-\frac{a^2c^3\arctan(ax)}{x} + a^4c^3x\arctan(ax) - a^3c^3\arctan(ax)^2 \\
&- \frac{ac^3\arctan(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2\arctan(ax)^2 \\
&- \frac{c^3\arctan(ax)^3}{3x^3} - \frac{3a^2c^3\arctan(ax)^3}{x} \\
&+ 3a^4c^3x\arctan(ax)^3 + \frac{1}{3}a^6c^3x^3\arctan(ax)^3 + a^3c^3\log(x) \\
&+ 8a^3c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) - a^3c^3\log\left(1+a^2x^2\right) \\
&+ 8a^3c^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\
&- 8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\
&+ 8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
&+ 4a^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) \\
&+ 4a^3c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)
\end{aligned}$$

output
```
-a^2*c^3*arctan(a*x)/x+a^4*c^3*x*arctan(a*x)-a^3*c^3*arctan(a*x)^2-1/2*a*c
^3*arctan(a*x)^2/x^2-1/2*a^5*c^3*x^2*arctan(a*x)^2-1/3*c^3*arctan(a*x)^3/x
^3-3*a^2*c^3*arctan(a*x)^3/x+3*a^4*c^3*x*arctan(a*x)^3+1/3*a^6*c^3*x^3*arc
tan(a*x)^3+a^3*c^3*ln(x)+8*a^3*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))-a^3*c^3*l
n(a^2*x^2+1)+8*a^3*c^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))+8*I*a^3*c^3*arctan(
a*x)*polylog(2,1-2/(1+I*a*x))-8*I*a^3*c^3*arctan(a*x)*polylog(2,-1+2/(1-I*
a*x))+4*a^3*c^3*polylog(3,-1+2/(1-I*a*x))+4*a^3*c^3*polylog(3,1-2/(1+I*a*x
))
```

3.386.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^4}\,dx = \frac{c^3(-2ia^3\pi^3x^3-6a^2x^2\arctan(ax)+6a^4x^4\arctan(ax)-3ax\arctan(ax)^2-6a^3x^3\arctan(ax)^2-3a^5x^5\,\text{ar}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```

output
```
(c^3*((-2*I)*a^3*Pi^3*x^3 - 6*a^2*x^2*ArcTan[a*x] + 6*a^4*x^4*ArcTan[a*x]
- 3*a*x*ArcTan[a*x]^2 - 6*a^3*x^3*ArcTan[a*x]^2 - 3*a^5*x^5*ArcTan[a*x]^2
- 2*ArcTan[a*x]^3 - 18*a^2*x^2*ArcTan[a*x]^3 + 18*a^4*x^4*ArcTan[a*x]^3 +
2*a^6*x^6*ArcTan[a*x]^3 + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTa
n[a*x])] + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 6*a^3
*x^3*Log[a*x] - 6*a^3*x^3*Log[1 + a^2*x^2] + (48*I)*a^3*x^3*ArcTan[a*x]*Po
lyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*a^3*x^3*ArcTan[a*x]*PolyLog[2, -
E^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 2
4*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(6*x^3)
```

3.386.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}{x^4} \, dx$$

$$\Big\downarrow \; 5483$$

$$\int \left(a^6c^3x^2\arctan(ax)^3+3a^4c^3\arctan(ax)^3+\frac{3a^2c^3\arctan(ax)^3}{x^2}+\frac{c^3\arctan(ax)^3}{x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}a^6c^3x^3\arctan(ax)^3-\frac{1}{2}a^5c^3x^2\arctan(ax)^2+3a^4c^3x\arctan(ax)^3+a^4c^3x\arctan(ax)-8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)-a^3c^3\arctan(ax)^2+8a^3c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)+8a^3c^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)+4a^3c^3\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+4a^3c^3\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+a^3c^3\log(x)-\frac{3a^2c^3\arctan(ax)^3}{x}-\frac{a^2c^3\arctan(ax)}{x}-a^3c^3\log\left(a^2x^2+1\right)-\frac{c^3\arctan(ax)^3}{3x^3}-\frac{ac^3\arctan(ax)^2}{2x^2}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```

output
```
-((a^2*c^3*ArcTan[a*x])/x) + a^4*c^3*x*ArcTan[a*x] - a^3*c^3*ArcTan[a*x]^2
 - (a*c^3*ArcTan[a*x]^2)/(2*x^2) - (a^5*c^3*x^2*ArcTan[a*x]^2)/2 - (c^3*Ar
cTan[a*x]^3)/(3*x^3) - (3*a^2*c^3*ArcTan[a*x]^3)/x + 3*a^4*c^3*x*ArcTan[a*
x]^3 + (a^6*c^3*x^3*ArcTan[a*x]^3)/3 + a^3*c^3*Log[x] + 8*a^3*c^3*ArcTan[a
*x]^2*Log[2/(1 + I*a*x)] - a^3*c^3*Log[1 + a^2*x^2] + 8*a^3*c^3*ArcTan[a*x
]^2*Log[2 - 2/(1 - I*a*x)] - (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(
1 - I*a*x)] + (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + 4*
a^3*c^3*PolyLog[3, -1 + 2/(1 - I*a*x)] + 4*a^3*c^3*PolyLog[3, 1 - 2/(1 + I
*a*x)]
```

3.386.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

3.386. $\int \frac{\left(c+a^2cx^2\right)^3\arctan(ax)^3}{x^4}\,dx$

3.386.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 138.35 (sec) , antiderivative size = 1948, normalized size of antiderivative = 5.80

method	result	size
parts	Expression too large to display	1948
derivativedivides	Expression too large to display	1949
default	Expression too large to display	1949

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^6*c^3*x^3*arctan(a*x)^3+3*a^4*c^3*x*arctan(a*x)^3-3*a^2*c^3*arctan(a
*x)^3/x-1/3*c^3*arctan(a*x)^3/x^3-c^3*(1/2*a^5*arctan(a*x)^2*x^2+1/2*a*arc
tan(a*x)^2/x^2-8*a^3*arctan(a*x)^2*ln(a*x)+8*a^3*arctan(a*x)^2*ln(a^2*x^2+
1)-a^3*(16*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-8*arctan(a*x)^2*l
n((1+I*a*x)^2/(a^2*x^2+1)-1)+8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
)+1)-16*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+16*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+8*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-16*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+16*polylog(
3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-8*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2
*x^2+1))+4*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/3*arctan(a*x)*(3-12*I*arc
tan(a*x)*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^
3*a*x-12*I*arctan(a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^3*a*x+12*I*arctan(a*x)*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a*x+12*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*Pi*arctan(a*x)*a*x-12*I*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)^3*Pi*arctan(a*x)*a*x+12*I*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
))^3*Pi*arctan(a*x)*a*x+6*I*a*x-3*a^2*x^2+3*x*arctan(a*x)*a-12*I*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)^2*Pi*arctan(a*x)*a*x-12*I*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*Pi*arc...
```

3.386.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x^4, x)
```

3.386.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4}\,dx = c^3\left(\int 3a^4 \operatorname{atan}^3(ax)\,dx + \int \frac{\operatorname{atan}^3(ax)}{x^4}\,dx + \int \frac{3a^2 \operatorname{atan}^3(ax)}{x^2}\,dx + \int a^6x^2 \operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**4,x)
```

output
```
c**3*(Integral(3*a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) +
Integral(3*a**2*atan(a*x)**3/x**2, x) + Integral(a**6*x**2*atan(a*x)**3, x
))
```

3.386.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/192*(3*(1792*a^8*c^3*integrate(1/32*x^8*arctan(a*x)^3/(a^2*x^6 + x^4), x
) + 192*a^8*c^3*integrate(1/32*x^8*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6
 + x^4), x) + 256*a^8*c^3*integrate(1/32*x^8*arctan(a*x)*log(a^2*x^2 + 1)/
(a^2*x^6 + x^4), x) - 256*a^7*c^3*integrate(1/32*x^7*arctan(a*x)^2/(a^2*x^
6 + x^4), x) + 64*a^7*c^3*integrate(1/32*x^7*log(a^2*x^2 + 1)^2/(a^2*x^6 +
 x^4), x) + 84*a^3*c^3*arctan(a*x)^4 + 7168*a^6*c^3*integrate(1/32*x^6*arc
tan(a*x)^3/(a^2*x^6 + x^4), x) + 768*a^6*c^3*integrate(1/32*x^6*arctan(a*x
)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 2304*a^6*c^3*integrate(1/32*x^6
*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 2304*a^5*c^3*integrate
(1/32*x^5*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 3*a^3*c^3*log(a^2*x^2 + 1)^3
 + 1152*a^4*c^3*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6
 + x^4), x) - 2304*a^4*c^3*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)
/(a^2*x^6 + x^4), x) + 2304*a^3*c^3*integrate(1/32*x^3*arctan(a*x)^2/(a^2*
x^6 + x^4), x) - 576*a^3*c^3*integrate(1/32*x^3*log(a^2*x^2 + 1)^2/(a^2*x^
6 + x^4), x) + 7168*a^2*c^3*integrate(1/32*x^2*arctan(a*x)^3/(a^2*x^6 + x^
4), x) + 768*a^2*c^3*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^
2*x^6 + x^4), x) - 256*a^2*c^3*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2
+ 1)/(a^2*x^6 + x^4), x) + 256*a*c^3*integrate(1/32*x*arctan(a*x)^2/(a^2*x
^6 + x^4), x) - 64*a*c^3*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^
4), x) + 1792*c^3*integrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 19...
```

3.386.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)^3}{x^4} dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4} \, dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}{x^4} \, dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^4, x)
```

3.387 $\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} \, dx$

3.387.1 Optimal result

Integrand size = 22, antiderivative size = 217

$$\begin{aligned}\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} \, dx = {} & \frac{x \arctan(ax)}{a^4c} - \frac{\arctan(ax)^2}{2a^5c} - \frac{x^2 \arctan(ax)^2}{2a^3c} \\ & - \frac{4i \arctan(ax)^3}{3a^5c} - \frac{x \arctan(ax)^3}{a^4c} + \frac{x^3 \arctan(ax)^3}{3a^2c} \\ & + \frac{\arctan(ax)^4}{4a^5c} - \frac{4 \arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{a^5c} - \frac{\log(1+a^2x^2)}{2a^5c} \\ & - \frac{4i \arctan(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^5c} - \frac{2 \operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{a^5c}\end{aligned}$$

output
```
x*arctan(a*x)/a^4/c-1/2*arctan(a*x)^2/a^5/c-1/2*x^2*arctan(a*x)^2/a^3/c-4/
3*I*arctan(a*x)^3/a^5/c-x*arctan(a*x)^3/a^4/c+1/3*x^3*arctan(a*x)^3/a^2/c+
1/4*arctan(a*x)^4/a^5/c-4*arctan(a*x)^2*ln(2/(1+I*a*x))/a^5/c-1/2*ln(a^2*x
^2+1)/a^5/c-4*I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^5/c-2*polylog(3,1-2
/(1+I*a*x))/a^5/c
```

3.387.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.71

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} dx$$
$$= \frac{12ax\arctan(ax) - 6\arctan(ax)^2 - 6a^2x^2\arctan(ax)^2 + 16i\arctan(ax)^3 - 12ax\arctan(ax)^3 + 4a^3x^3\arctan}{}$$

input
```
Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
(12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 6*a^2*x^2*ArcTan[a*x]^2 + (16*I)*A
rcTan[a*x]^3 - 12*a*x*ArcTan[a*x]^3 + 4*a^3*x^3*ArcTan[a*x]^3 + 3*ArcTan[a
*x]^4 - 48*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 6*Log[1 + a^2*x^
2] + (48*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*PolyLog[3,
 -E^((2*I)*ArcTan[a*x])])/(12*a^5*c)
```

3.387.3 Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.53, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5451, 27, 5361, 5451, 5345, 5361, 5419, 5451, 5345, 240, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^3}{a^2cx^2+c} dx$$
$$\Big\downarrow \text{ 5451}$$
$$\frac{\int x^2\arctan(ax)^3dx}{a^2c} - \frac{\int \frac{x^2\arctan(ax)^3}{c(a^2x^2+1)}dx}{a^2}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int x^2\arctan(ax)^3dx}{a^2c} - \frac{\int \frac{x^2\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \text{ 5361}$$
$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\int \frac{x^3\arctan(ax)^2}{a^2x^2+1}dx}{a^2c} - \frac{\int \frac{x^2\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{\int\arctan(ax)^3dx}{a^2}-\frac{\int\frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \text{5345}$$

3.387. $\displaystyle\int\frac{x^4\arctan(ax)^3}{c+a^2cx^2}dx$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \; 240$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \; 5419$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \; 5455$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^4}{4a^3}+\frac{x\arctan(ax)^3-3a\left(-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow \; 5379$$

3.387. $\int \frac{x^4\arctan(ax)^3}{c+a^2cx^2}\,dx$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2} - \frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}\cdots}{a}\cdots}{a^2}\cdots\right.}{a^2c}$$

$$\frac{-\frac{\arctan(ax)^4}{4a^3}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow \; 5529$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2} - \frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}\cdots\right.}{a}\cdots}{\cdots}\cdots\right.}{a^2c}$$

$$\frac{-\frac{\arctan(ax)^4}{4a^3}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^3}{3a^2}\cdots\right.}{a^2}}{a^2c}$$

$$\Big\downarrow \; 7164$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2} - \frac{-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan\cdots}{\cdots}\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{a^2c}$$

$$\frac{-\frac{\arctan(ax)^4}{4a^3}+\frac{x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}\right)}{a^2}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

3.387. $\int \frac{x^4\arctan(ax)^3}{c+a^2cx^2}\,dx$

output
```
((x^3*ArcTan[a*x]^3)/3 - a*(((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2
/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((-1/3*I)*Arc
Tan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*Arc
Tan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/
(4*a))/a)/a^2))/(a^2*c) - (-1/4*ArcTan[a*x]^4/a^3 + (x*ArcTan[a*x]^3 - 3*
a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a -
2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 -
 2/(1 + I*a*x)]/(4*a)))/a))/a^2)/(a^2*c)
```

3.387.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

3.387. $\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} \, dx$

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.387.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.16 (sec) , antiderivative size = 888, normalized size of antiderivative = 4.09

method	result	size
derivativedivides	Expression too large to display	888
default	Expression too large to display	888
parts	Expression too large to display	898

input
```
int(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/3/c*arctan(a*x)^3*a^3*x^3-1/c*arctan(a*x)^3*a*x+1/c*arctan(a*x)^4
-1/c*(1/2*x^2*arctan(a*x)^2*a^2-2*arctan(a*x)^2*ln(a^2*x^2+1)+4*arctan(a*x
)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/6*I*arctan(a*x)*(6*arctan(a*x)*Pi*cs
gn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-12*arc
tan(a*x)*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^
2+1))^2+6*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+6*arctan(a*x)*P
i*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-6*arctan(a*x)*
Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))-6*arctan(a*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2+12*arctan(a*x)*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))-6*arctan(a
*x)*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-6*arctan(a*x)*Pi*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I/((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)+6*arctan(a*x)*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^3+8*arctan(a*x)^2+24*I*arctan(a*x)*ln(2)+3*I*arctan
(a*x)-6-6*I*a*x)-ln((1+I*a*x)^2/(a^2*x^2+1)+1)-4*I*arctan(a*x)*polylog(2,-
(1+I*a*x)^2/(a^2*x^2+1))+2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+3/4*arctan(
a*x)^4))
```

3.387.5 Fricas [F]

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^4 \arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^4*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.387.6 Sympy [F]

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^4 \operatorname{atan}^3{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**4*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

3.387.7 Maxima [F]

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^4 \arctan{(ax)}^3}{a^2cx^2+c} \, dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3072*(48*(7168*a^4*integrate(1/128*x^4*arctan(a*x)^3/(a^6*c*x^2 + a^4*c)
, x) + 768*a^4*integrate(1/128*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^6*c*x
^2 + a^4*c), x) + 1024*a^4*integrate(1/128*x^4*arctan(a*x)*log(a^2*x^2 + 1
)/(a^6*c*x^2 + a^4*c), x) - 1024*a^3*integrate(1/128*x^3*arctan(a*x)^2/(a^
6*c*x^2 + a^4*c), x) + 256*a^3*integrate(1/128*x^3*log(a^2*x^2 + 1)^2/(a^6
*c*x^2 + a^4*c), x) - 3072*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2
 + 1)/(a^6*c*x^2 + a^4*c), x) + 768*a*integrate(1/128*x*arctan(a*x)^2*log(
a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) + 192*a*integrate(1/128*x*log(a^2*x^2
 + 1)^3/(a^6*c*x^2 + a^4*c), x) + 3072*a*integrate(1/128*x*arctan(a*x)^2/(
a^6*c*x^2 + a^4*c), x) - 768*a*integrate(1/128*x*log(a^2*x^2 + 1)^2/(a^6*c
*x^2 + a^4*c), x) - 3*arctan(a*x)^4/(a^5*c) - 384*integrate(1/128*arctan(a
*x)*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x))*a^5*c + 128*(a^3*x^3 - 3*a
*x)*arctan(a*x)^3 + 240*arctan(a*x)^4 - 9*log(a^2*x^2 + 1)^4 - 24*(4*(a^3*
x^3 - 3*a*x)*arctan(a*x) + 3*arctan(a*x)^2)*log(a^2*x^2 + 1)^2)/(a^5*c)
```

3.387.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^4 \arctan{(ax)}^3}{a^2cx^2+c} \, dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^4 \operatorname{atan}(a\,x)^3}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.388 $\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} dx$

3.388.1 Optimal result

Integrand size = 22, antiderivative size = 260

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} dx = &-\frac{3i \arctan(ax)^2}{2a^4c} - \frac{3x \arctan(ax)^2}{2a^3c} + \frac{\arctan(ax)^3}{2a^4c} + \frac{x^2 \arctan(ax)^3}{2a^2c} \\ &+ \frac{i \arctan(ax)^4}{4a^4c} - \frac{3 \arctan(ax) \log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{\arctan(ax)^3 \log\left(\frac{2}{1+iax}\right)}{a^4c} \\ &- \frac{3i \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{2a^4c} + \frac{3i \arctan(ax)^2 \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{2a^4c} \\ &+ \frac{3 \arctan(ax) \operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^4c} - \frac{3i \operatorname{PolyLog}\left(4, 1-\frac{2}{1+iax}\right)}{4a^4c}\end{aligned}$$

output
```
-3/2*I*arctan(a*x)^2/a^4/c-3/2*x*arctan(a*x)^2/a^3/c+1/2*arctan(a*x)^3/a^4
/c+1/2*x^2*arctan(a*x)^3/a^2/c+1/4*I*arctan(a*x)^4/a^4/c-3*arctan(a*x)*ln(
2/(1+I*a*x))/a^4/c+arctan(a*x)^3*ln(2/(1+I*a*x))/a^4/c-3/2*I*polylog(2,1-2
/(1+I*a*x))/a^4/c+3/2*I*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))/a^4/c+3/2*a
rctan(a*x)*polylog(3,1-2/(1+I*a*x))/a^4/c-3/4*I*polylog(4,1-2/(1+I*a*x))/a
^4/c
```

3.388.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.62

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx$$
$$= \frac{6i \arctan(ax)^2 - 6ax \arctan(ax)^2 + 2(1 + a^2x^2) \arctan(ax)^3 - i \arctan(ax)^4 - 12 \arctan(ax) \log\left(1 + e^{2i\cdots\right.}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((6*I)*ArcTan[a*x]^2 - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^2)*ArcTan[a*x]^3
 - I*ArcTan[a*x]^4 - 12*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] + 4*Arc
Tan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] - (6*I)*(-1 + ArcTan[a*x]^2)*Pol
yLog[2, -E^((2*I)*ArcTan[a*x])] + 6*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTa
n[a*x])] + (3*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])])/(4*a^4*c)
```

3.388.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5451, 27, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{a^2cx^2 + c} \, dx$$
$$\Big\downarrow \; 5451$$
$$\frac{\int x \arctan(ax)^3 dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^3}{c(a^2x^2+1)} dx}{a^2}$$
$$\Big\downarrow \; 27$$
$$\frac{\int x \arctan(ax)^3 dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^3}{a^2x^2+1} dx}{a^2c}$$
$$\Big\downarrow \; 5361$$
$$\frac{\frac{1}{2}x^2 \arctan(ax)^3 - \frac{3}{2}a \int \frac{x^2 \arctan(ax)^2}{a^2x^2+1} dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^3}{a^2x^2+1} dx}{a^2c}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{\int\arctan(ax)^2dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^3}{3a^3}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5455}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\int\frac{\arctan(ax)^3}{i-ax}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$$\Big\downarrow \text{2849}$$

3.388. $\int\frac{x^3\arctan(ax)^3}{c+a^2cx^2}\,dx$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$$\Big\downarrow \text{5529}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$$\Big\downarrow \text{5533}$$

3.388. $\int\frac{x^3\arctan(ax)^3}{c+a^2cx^2}\,dx$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(}{4a^2}}{a^2c}$$

$$\Big\downarrow \; 7164$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{i\arctan(ax)^4}{4a^2}-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{iax+1}\right)}{4a}\right)-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((x^2*ArcTan[a*x]^3)/2 - (3*a*(-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*x]^2 -
 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a +
 ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2))/2)/(a^2*c) - (((-1/4*I
)*ArcTan[a*x]^4)/a^2 - ((ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/a - 3*(((-1/2*I
)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTan[a*x]*P
olyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)]/(4*a))))/a
)/(a^2*c)
```

3.388.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

3.388. $\int \frac{x^3\arctan(ax)^3}{c+a^2cx^2}\,dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.388. $\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} \, dx$

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.388.4 Maple [A] (verified)

Time = 39.60 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{i\arctan(ax)^4}{4c}+\frac{\arctan(ax)^2(-i\arctan(ax)+x\arctan(ax)a-3)(ax+i)}{2c}+\frac{\arctan(ax)^3\ln\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)}{c}-\frac{3i\arctan(ax)^2\operatorname{polylog}(2,}{2c}$
default	$-\frac{i\arctan(ax)^4}{4c}+\frac{\arctan(ax)^2(-i\arctan(ax)+x\arctan(ax)a-3)(ax+i)}{2c}+\frac{\arctan(ax)^3\ln\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)}{c}-\frac{3i\arctan(ax)^2\operatorname{polylog}(2,}{2c}$

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/4*I*arctan(a*x)^4/c+1/2/c*arctan(a*x)^2*(-I*arctan(a*x)+x*arctan
(a*x)*a-3)*(I+a*x)+1/c*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3/2*I/c
*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+3/2/c*arctan(a*x)*polyl
og(3,-(1+I*a*x)^2/(a^2*x^2+1))+3/4*I/c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))
+3*I/c*arctan(a*x)^2-3/c*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I/c
*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1)))
```

3.388.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.388.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

3.388.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.388.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan{(ax)}^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^3}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.389 $\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2}\,dx$

3.389.1 Optimal result

Integrand size = 22, antiderivative size = 130

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2}\,dx = {} & \frac{i \arctan(ax)^3}{a^3c}+\frac{x \arctan(ax)^3}{a^2c} \\ & -\frac{\arctan(ax)^4}{4a^3c}+\frac{3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^3c} \\ & +\frac{3i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^3c}+\frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^3c}\end{aligned}$$

output
```
I*arctan(a*x)^3/a^3/c+x*arctan(a*x)^3/a^2/c-1/4*arctan(a*x)^4/a^3/c+3*arct
an(a*x)^2*ln(2/(1+I*a*x))/a^3/c+3*I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a
^3/c+3/2*polylog(3,1-2/(1+I*a*x))/a^3/c
```

3.389.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2}\,dx$$
$$= \frac{-\frac{1}{4}\arctan(ax)^2\left((4i-4ax)\arctan(ax)+\arctan(ax)^2-12\log\left(1+e^{2i\arctan(ax)}\right)\right)-3i\arctan(ax)\operatorname{PolyLo}}{a^3c}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
(-1/4*(ArcTan[a*x]^2*((4*I - 4*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 12*Log[1
 + E^((2*I)*ArcTan[a*x])])) - (3*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTa
n[a*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[a*x])])/2)/(a^3*c)
```

3.389.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5451, 27, 5345, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)^3}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int \arctan(ax)^3dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \arctan(ax)^3dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}-\frac{\arctan(ax)^4}{4a^3c}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2c}$$

$$\Big\downarrow \textcolor{blue}{5529}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2c}$$

$$\Big\downarrow \textcolor{blue}{7164}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}\right)}{a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
-1/4*ArcTan[a*x]^4/(a^3*c) + (x*ArcTan[a*x]^3 - 3*a*(((-1/3*I)*ArcTan[a*x]
^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]
*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/
a))/(a^2*c)
```

3.389.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.389. $\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2} \, dx$

3.389.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.14 (sec) , antiderivative size = 785, normalized size of antiderivative = 6.04

method	result
derivativedivides	$\frac{\frac{\arctan(ax)^3 ax}{c} - \frac{\arctan(ax)^4}{c} - \frac{3\left(\frac{\arctan(ax)^2 \ln\left(a^2x^2+1\right)}{2} - \arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) + \frac{i\arctan(ax)^3}{3} - \frac{\left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}\right.}\right.\right.}{}\right.}{}}{}$
default	$\frac{\frac{\arctan(ax)^3 ax}{c} - \frac{\arctan(ax)^4}{c} - \frac{3\left(\frac{\arctan(ax)^2 \ln\left(a^2x^2+1\right)}{2} - \arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) + \frac{i\arctan(ax)^3}{3} - \frac{\left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}\right.}\right.\right.}{}\right.}{}}{}$
parts	$\frac{x\arctan(ax)^3}{a^2c} - \frac{\arctan(ax)^4}{a^3c} - \frac{3\left(\frac{\frac{\arctan(ax)^2 \ln\left(a^2x^2+1\right)}{2} - \arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) + \frac{i\arctan(ax)^3}{3} - \frac{\left(-i\pi\ \mathrm{csgn}\left(\left(\right.\right.\right.}{}}{}\right.}{}$

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/c*arctan(a*x)^3*a*x-1/c*arctan(a*x)^4-3/c*(1/2*arctan(a*x)^2*ln(a
^2*x^2+1)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/3*I*arctan(a*x)^
3-1/4*(-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*P
i*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))+2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^3+4*ln(2))*arctan(a*x)^2+I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x
^2+1))-1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/4*arctan(a*x)^4))
```

3.389.5 Fricas [F]

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^2 \arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.389.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^3{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

3.389.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/1024*(16*(7168*a^2*integrate(1/128*x^2*arctan(a*x)^3/(a^4*c*x^2 + a^2*c)
, x) + 768*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c*x
^2 + a^2*c), x) + 3072*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1
)/(a^4*c*x^2 + a^2*c), x) - 768*a*integrate(1/128*x*arctan(a*x)^2*log(a^2*
x^2 + 1)/(a^4*c*x^2 + a^2*c), x) - 192*a*integrate(1/128*x*log(a^2*x^2 + 1
)^3/(a^4*c*x^2 + a^2*c), x) - 3072*a*integrate(1/128*x*arctan(a*x)^2/(a^4*
c*x^2 + a^2*c), x) + 768*a*integrate(1/128*x*log(a^2*x^2 + 1)^2/(a^4*c*x^2
 + a^2*c), x) + 3*arctan(a*x)^4/(a^3*c) + 384*integrate(1/128*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x))*a^3*c + 128*a*x*arctan(a*x)^3
- 80*arctan(a*x)^4 + 3*log(a^2*x^2 + 1)^4 - 24*(4*a*x*arctan(a*x) - arctan
(a*x)^2)*log(a^2*x^2 + 1)^2)/(a^3*c)
```

3.389.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^3}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.390 $\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx$

3.390.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = -\frac{i\arctan(ax)^4}{4a^2c} - \frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c} - \frac{3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^2c} + \frac{3i\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^2c}$$

output
```
-1/4*I*arctan(a*x)^4/a^2/c-arctan(a*x)^3*ln(2/(1+I*a*x))/a^2/c-3/2*I*arcta
n(a*x)^2*polylog(2,1-2/(1+I*a*x))/a^2/c-3/2*arctan(a*x)*polylog(3,1-2/(1+I
*a*x))/a^2/c+3/4*I*polylog(4,1-2/(1+I*a*x))/a^2/c
```

3.390.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = -\frac{i\left(\arctan(ax)^4 - 4i\arctan(ax)^3\log\left(\frac{2i}{i-ax}\right) + 6\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right) - 6i\arctan(ax)\operatorname{PolyLog}\right.}{4a^2c}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((-1/4*I)*(ArcTan[a*x]^4 - (4*I)*ArcTan[a*x]^3*Log[(2*I)/(I - a*x)] + 6*Ar
cTan[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)] - (6*I)*ArcTan[a*x]*PolyLog[3
, (I + a*x)/(-I + a*x)] - 3*PolyLog[4, (I + a*x)/(-I + a*x)]))/(a^2*c)
```

3.390.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5455, 5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^3}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \; 5455$$

$$-\frac{\int \frac{\arctan(ax)^3}{i-ax}dx}{ac}-\frac{i \arctan(ax)^4}{4a^2c}$$

$$\Big\downarrow \; 5379$$

$$-\frac{\frac{\arctan(ax)^3 \log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2 \log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{ac}-\frac{i \arctan(ax)^4}{4a^2c}$$

$$\Big\downarrow \; 5529$$

$$-\frac{\frac{\arctan(ax)^3 \log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\int \frac{\arctan(ax) \operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i \arctan(ax)^2 \operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{ac}-\frac{i \arctan(ax)^4}{4a^2c}$$

$$\Big\downarrow \; 5533$$

$$-\frac{\frac{\arctan(ax)^3 \log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i \arctan(ax) \operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)-\frac{i \arctan(ax)^2 \operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{ac}-\frac{i \arctan(ax)^4}{4a^2c}$$

$$\Big\downarrow \; 7164$$

$$
-\frac{i\arctan(ax)^4}{4a^2c}-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{iax+1}\right)}{4a}\right)-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{ac}
$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((-1/4*I)*ArcTan[a*x]^4)/(a^2*c) - ((ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/a -
 3*(((-1/2*I)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + I*(((I/2)*A
rcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)
]/(4*a))))/(a*c)
```

3.390.3.1 Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.390.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.79 (sec) , antiderivative size = 789, normalized size of antiderivative = 5.72

method	result
derivativedivides	$\frac{\ln\left(a^2x^2+1\right)\arctan(ax)^3}{2c} - \frac{3\left(\frac{2\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{3} - \frac{i\arctan(ax)^4}{6} + \frac{\left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\mathrm{cs}\right.}{}\right.}{}$
default	$\frac{\ln\left(a^2x^2+1\right)\arctan(ax)^3}{2c} - \frac{3\left(\frac{2\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{3} - \frac{i\arctan(ax)^4}{6} + \frac{\left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\mathrm{cs}\right.}{}\right.}{}$
parts	$\frac{\ln(a^2x^2+1)\arctan(ax)^3}{2a^2c} - \frac{3\left(\frac{2\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{3a} - \frac{i\arctan(ax)^4}{6a} + \frac{\left(-i\pi\ \mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1}{a^2x^2+}\right.\right.}{}\right.}{}$

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2/c*ln(a^2*x^2+1)*arctan(a*x)^3-3/2/c*(2/3*arctan(a*x)^3*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2))-1/6*I*arctan(a*x)^4+1/6*(-I*Pi*csgn(I/((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*
Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+2*I*Pi*csgn(I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+4*ln(2))*arctan(a*x)^3-I
*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+arctan(a*x)*polylog(3,-
(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))))
```

3.390.5 Fricas [F]

$$\int \frac{x \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x \arctan{(ax)}^3}{a^2cx^2+c} \, dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.390.6 Sympy [F]

$$\int \frac{x \arctan(ax)^3}{c+a^2cx^2} \, dx = \frac{\int \frac{x \operatorname{atan}^3{(ax)}}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

3.390.7 Maxima [F]

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x\arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.390.8 Giac [F]

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x\arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.391 $\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx$

3.391.1 Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\arctan(ax)^4}{4ac}$$

output
```
1/4*arctan(a*x)^4/a/c
```

3.391.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\arctan(ax)^4}{4ac}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^4/(4*a*c)
```

3.391.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \; 5419$$
$$\frac{\arctan(ax)^4}{4ac}$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^4/(4*a*c)
```

3.391.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.391.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\arctan(ax)^4}{4ac}$
default	$\frac{\arctan(ax)^4}{4ac}$
parallelrisch	$\frac{\arctan(ax)^4}{4ac}$
parts	$\frac{\arctan(ax)^4}{4ac}$
risch	$\frac{\ln(iax+1)^4}{64ca}-\frac{\ln(-iax+1)\ln(iax+1)^3}{16ca}+\frac{3\ln(-iax+1)^2\ln(iax+1)^2}{32ca}-\frac{\ln(-iax+1)^3\ln(iax+1)}{16ca}+\frac{\ln(-iax+1)^4}{64ca}$

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(a*x)^4/a/c
```

3.391.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^4}{4\,ac}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/4*arctan(a*x)^4/(a*c)
```

3.391.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{\operatorname{atan}^3{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

3.391.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^4}{4\,ac}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/4*arctan(a*x)^4/(a*c)
```

3.391.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{c + a^2cx^2}\,dx = \int \frac{\arctan{(ax)}^3}{a^2cx^2 + c}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.391.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c + a^2cx^2}\,dx = \frac{\mathrm{atan}(a\,x)^4}{4\,a\,c}$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^4/(4*a*c)
```

3.392 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)}\,dx$

3.392.1 Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)}\,dx=-\frac{i\arctan(ax)^4}{4c}+\frac{\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}+\frac{3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}+\frac{3i\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}$$

output
```
-1/4*I*arctan(a*x)^4/c+arctan(a*x)^3*ln(2-2/(1-I*a*x))/c-3/2*I*arctan(a*x)
^2*polylog(2,-1+2/(1-I*a*x))/c+3/2*arctan(a*x)*polylog(3,-1+2/(1-I*a*x))/c
+3/4*I*polylog(4,-1+2/(1-I*a*x))/c
```

3.392.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)}\,dx=-\frac{i\left(\pi^4-16\arctan(ax)^4+64i\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right)-96\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(\right.\right.}{64c}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/64*I)*(Pi^4 - 16*ArcTan[a*x]^4 + (64*I)*ArcTan[a*x]^3*Log[1 - E^((-2*
I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (
96*I)*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-
2*I)*ArcTan[a*x])]))/c
```

3.392.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5459, 5403, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5459}$$

$$\frac{i\int \frac{\arctan(ax)^3}{x(ax+i)}dx}{c}-\frac{i\arctan(ax)^4}{4c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^4}{4c}$$

$$\Big\downarrow \text{5527}$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int \frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^4}{4c}$$

$$\Big\downarrow \text{5531}$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(\ldots}{c}$$
$$\frac{i\arctan(ax)^4}{4c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{\operatorname{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(ax)^3\log\right.}{c}$$

$$\frac{i\arctan(ax)^4}{4c}$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/4*I)*ArcTan[a*x]^4)/c + (I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)]
 + (3*I)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((
-1/2*I)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/
(1 - I*a*x)]/(4*a)))))/c
```

3.392.3.1 Defintions of rubi rules used

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.392.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 68.93 (sec) , antiderivative size = 1640, normalized size of antiderivative = 13.23

method	result	size
derivativedivides	Expression too large to display	1640
default	Expression too large to display	1640
parts	Expression too large to display	2057

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*arctan(a*x)^3*ln(a*x)-1/2/c*ln(a^2*x^2+1)*arctan(a*x)^3-3/2/c*(-2/3*ar
ctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/6*I*arctan(a*x)^4-1/6*(-I*Pi
*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-I*P
i*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-2*I*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*
a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-2*I*Pi*csgn(((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-2*I*Pi*csgn(I/((1+I*a*x)
^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))^2+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^3+2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*c
sgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+2*I*Pi-2
*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^2+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^...
```

3.392.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

3.392.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**3 + x), x)/c
```

3.392.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x), x)
```

3.392.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)), x)
```

3.393 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx$

3.393.1 Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = -\frac{ia\arctan(ax)^3}{c} - \frac{\arctan(ax)^3}{cx} - \frac{a\arctan(ax)^4}{4c} + \frac{3a\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3ia\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c} + \frac{3a\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-I*a*arctan(a*x)^3/c-arctan(a*x)^3/c/x-1/4*a*arctan(a*x)^4/c+3*a*arctan(a*
x)^2*ln(2-2/(1-I*a*x))/c-3*I*a*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c+3/2
*a*polylog(3,-1+2/(1-I*a*x))/c
```

3.393.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \frac{a\left(-\frac{i\pi^3}{8}+i\arctan(ax)^3-\frac{\arctan(ax)^3}{ax}-\frac{1}{4}\arctan(ax)^4+3\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)+3i\arctan(a\right.}{c}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]
```

output
```
(a*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - ArcTan[a*x]^4/
4 + 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*Po
lyLog[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/
2))/c
```

3.393.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5453, 27, 5361, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2 (a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c (a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{x}}{c}-\frac{a\arctan(ax)^4}{4c}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a\arctan(ax)^4}{4c}+\frac{-\frac{\arctan(ax)^3}{x}+3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a\arctan(ax)^4}{4c}+\frac{-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\Big\downarrow \text{5527}$$

$$-\frac{a\arctan(ax)^4}{4c}+\frac{-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)\right.}{c}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{a\arctan(ax)^4}{4c}+\frac{-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\right.}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-1/4*(a*ArcTan[a*x]^4)/c + (-(ArcTan[a*x]^3/x) + 3*a*((-1/3*I)*ArcTan[a*x]
^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan
[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(
4*a)))))/c
```

3.393.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.393. $\int\frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}dx$

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.393. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx$

3.393.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.13 (sec) , antiderivative size = 1609, normalized size of antiderivative = 13.19

method	result	size
parts	Expression too large to display	1609
derivativedivides	Expression too large to display	1611
default	Expression too large to display	1611

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-a*arctan(a*x)^4/c-arctan(a*x)^3/c/x-3/c*(-1/4*a*arctan(a*x)^4-a*(arctan(a
*x)^2*ln(a*x)-1/2*arctan(a*x)^2*ln(a^2*x^2+1)+arctan(a*x)^2*ln((1+I*a*x)/(
a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-1/3*I*arctan
(a*x)^3+1/4*(-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1))-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-2*I
*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-2*I*Pi
*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((
1+I*a*x)^2/(a^2*x^2+1)+1))^2+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^3+2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*
a*x)^2/(a^2*x^2+1)+1))^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I
/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+I*Pi*csgn(I*(1+I*a*x)^2/
(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)+2*I*Pi-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*cs...
```

3.393.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)} \, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^2} \, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

3.393.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)} \, dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^4+x^2} \, dx}{c}$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**4 + x**2), x)/c
```

3.393.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)} \, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^2} \, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/1024*(80*a*x*arctan(a*x)^4 - 3*a*x*log(a^2*x^2 + 1)^4 - (48*a*arctan(a*
x)^4/c - 12288*a^3*integrate(1/128*x^3*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^2
*c*x^4 + c*x^2), x) - 3*a*log(a^2*x^2 + 1)^4/c + 6144*a^2*integrate(1/128*
x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) - 49152*a^2*int
egrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*c*x^4 + c*x^2), x) + 49
152*a*integrate(1/128*x*arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x) - 12288*a*in
tegrate(1/128*x*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) + 114688*integr
ate(1/128*arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x) + 12288*integrate(1/128*ar
ctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x))*c*x + 128*arctan(a*x
)^3 - 24*(a*x*arctan(a*x)^2 + 4*arctan(a*x))*log(a^2*x^2 + 1)^2)/(c*x)
```

3.393.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)), x)
```

3.394 $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx$

3.394.1 Optimal result

Integrand size = 22, antiderivative size = 262

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx =& -\frac{3ia^2\arctan(ax)^2}{2c}-\frac{3a\arctan(ax)^2}{2cx}-\frac{a^2\arctan(ax)^3}{2c}\\&-\frac{\arctan(ax)^3}{2cx^2}+\frac{ia^2\arctan(ax)^4}{4c}+\frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}\\&-\frac{a^2\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}\\&+\frac{3ia^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}\\&-\frac{3a^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}\\&-\frac{3ia^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}\end{aligned}$$

output
```
-3/2*I*a^2*arctan(a*x)^2/c-3/2*a*arctan(a*x)^2/c/x-1/2*a^2*arctan(a*x)^3/c
-1/2*arctan(a*x)^3/c/x^2+1/4*I*a^2*arctan(a*x)^4/c+3*a^2*arctan(a*x)*ln(2-
2/(1-I*a*x))/c-a^2*arctan(a*x)^3*ln(2-2/(1-I*a*x))/c-3/2*I*a^2*polylog(2,-
1+2/(1-I*a*x))/c+3/2*I*a^2*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c-3/2*a
^2*arctan(a*x)*polylog(3,-1+2/(1-I*a*x))/c-3/4*I*a^2*polylog(4,-1+2/(1-I*a
*x))/c
```

3.394.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx = \frac{ia^2\left(\pi^4 - 96\arctan(ax)^2 + \frac{96i\arctan(ax)^2}{ax} + \frac{32i(1+a^2x^2)\arctan(ax)^3}{a^2x^2} - 16\arctan(ax)^4 + 64i\arctan(ax)^3\log(1-\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

output
```
((I/64)*a^2*(Pi^4 - 96*ArcTan[a*x]^2 + ((96*I)*ArcTan[a*x]^2)/(a*x) + ((32
*I)*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - 16*ArcTan[a*x]^4 + (64*I)*Arc
Tan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - (192*I)*ArcTan[a*x]*Log[1 - E
^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])
] - 96*PolyLog[2, E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]*PolyLog[3, E
^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])]))/c
```

3.394.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5453, 27, 5361, 5453, 5361, 5419, 5459, 5403, 2897, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^3(a^2cx^2+c)}\,dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3}dx}{c} - a^2\int \frac{\arctan(ax)^3}{cx(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3}dx}{c} - \frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax\right.}{c}$$
$$\frac{a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)}{c}$$

$$\Big\downarrow \text{2897}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\operatorname{arct}\right.}{c}$$
$$\frac{a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)}{c}$$

$$\Big\downarrow \text{5527}$$

3.394. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arct\right.$$
$$\frac{\qquad}{c}$$
$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\right.}{c}$$

$$\Big\downarrow \text{5531}$$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arct\right.$$
$$\frac{\qquad}{c}$$
$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arct\right.\right.}{c}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arct\right.$$
$$\frac{\qquad}{c}$$
$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{\text{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(ax)\right.\right.}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(-1/2*ArcTan[a*x]^3/x^2 + (3*a*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 +
 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)]
- PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2)/c - (a^2*((-1/4*I)*ArcTan[a*x]^4
 + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3*I)*a*(((I/2)*ArcTan[a
*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*I)*ArcTan[a*x]*PolyLog
[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 - I*a*x)]/(4*a))))))/c
```

3.394. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx$

3.394.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.394. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx$

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.394.4 Maple [A] (verified)

Time = 91.86 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.68

method	result
derivativedivides	$a^2\left(-\frac{6i\,\mathrm{polylog}\left(4,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{\arctan(ax)^2(-i\arctan(ax)-3iax+x\arctan(ax)a)(ax+i)}{2c\,a^2x^2}+\frac{3i\arctan(ax)^2\,\mathrm{poly}}{}\right.$
default	$a^2\left(-\frac{6i\,\mathrm{polylog}\left(4,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}-\frac{\arctan(ax)^2(-i\arctan(ax)-3iax+x\arctan(ax)a)(ax+i)}{2c\,a^2x^2}+\frac{3i\arctan(ax)^2\,\mathrm{poly}}{}\right.$

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-6*I/c*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2/c*arctan(a*x)^2*(-I
*arctan(a*x)-3*I*a*x+x*arctan(a*x)*a)*(I+a*x)/a^2/x^2+3*I/c*arctan(a*x)^2*
polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/c*arctan(a*x)^3*ln(1-(1+I*a*x)/(
a^2*x^2+1)^(1/2))-3*I/c*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6/c*arctan
(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I/c*arctan(a*x)^2*polylog(2
,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/c*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)+1)-6*I/c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6/c*arctan(a*x)*poly
log(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c*polylog(2,(1+I*a*x)/(a^2*x^2+1)^
(1/2))+1/4*I/c*arctan(a*x)^4+3/c*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-3*I/c*arctan(a*x)^2+3/c*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)
)
```

3.394.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

3.394.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**5 + x**3), x)/c
```

3.394.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^3), x)
```

3.394.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)), x)
```

3.395 $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx$

3.395.1 Optimal result

Integrand size = 22, antiderivative size = 227

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx= & -\frac{a^2\arctan(ax)}{cx}-\frac{a^3\arctan(ax)^2}{2c}-\frac{a\arctan(ax)^2}{2cx^2}+\frac{4ia^3\arctan(ax)^3}{3c}\\ & -\frac{\arctan(ax)^3}{3cx^3}+\frac{a^2\arctan(ax)^3}{cx}+\frac{a^3\arctan(ax)^4}{4c}+\frac{a^3\log(x)}{c}\\ & -\frac{a^3\log\left(1+a^2x^2\right)}{2c}-\frac{4a^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}\\ & +\frac{4ia^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}\\ & -\frac{2a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c}\end{aligned}$$

output
```
-a^2*arctan(a*x)/c/x-1/2*a^3*arctan(a*x)^2/c-1/2*a*arctan(a*x)^2/c/x^2+4/3
*I*a^3*arctan(a*x)^3/c-1/3*arctan(a*x)^3/c/x^3+a^2*arctan(a*x)^3/c/x+1/4*a
^3*arctan(a*x)^4/c+a^3*ln(x)/c-1/2*a^3*ln(a^2*x^2+1)/c-4*a^3*arctan(a*x)^2
*ln(2-2/(1-I*a*x))/c+4*I*a^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c-2*a^3
*polylog(3,-1+2/(1-I*a*x))/c
```

3.395.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \frac{a^3\left(\frac{1}{12}\left(2i\pi^3-\frac{12\arctan(ax)}{ax}+\left(-16i-\frac{4}{a^3x^3}+\frac{12}{ax}\right)\arctan(ax)^3+3\arctan(ax)^4+\arctan(ax)^2\left(-6-\frac{6}{a^2x^2}-\right.\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```

output
```
(a^3*(((2*I)*Pi^3 - (12*ArcTan[a*x])/(a*x) + (-16*I - 4/(a^3*x^3) + 12/(a*
x))*ArcTan[a*x]^3 + 3*ArcTan[a*x]^4 + ArcTan[a*x]^2*(-6 - 6/(a^2*x^2) - 48
*Log[1 - E^((-2*I)*ArcTan[a*x])]) + 12*Log[a*x] - 6*Log[1 + a^2*x^2])/12 -
 (4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - 2*PolyLog[3, E^((-
2*I)*ArcTan[a*x])]))/c
```

3.395.3 Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {5453, 27, 5361, 5453, 5361, 5419, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^4(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4}dx}{c}-a^2\int \frac{\arctan(ax)^3}{cx^2(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \; 5453$$

$$\frac{a\left(\int \frac{\arctan(ax)^2}{x^3}dx-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \; 5361$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \; 5419$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \; 5453$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \; 5361$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \; 243$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \; 47$$

3.395. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{14}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan}{3x}}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2\right.\right.}{c}$$

$$\frac{a^2\left(3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)\right.}{c}$$

$$\frac{a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\right.}{c}$$

3.395. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx$

$$\Big\downarrow \text{5527}$$

$$-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2\right.\right.\right.\right.\right.$$

$$\frac{a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\right.\right.}{c}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right.\right.\right.\right.\right.$$

$$c$$

$$\frac{a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(\right.\right.}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```

output
```
-((a^2*(-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 + 3*a*((-1/3*I)*ArcTan[a*
x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcT
an[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]
/(4*a))))))/c) + (-1/3*ArcTan[a*x]^3/x^3 + a*(-1/2*ArcTan[a*x]^2/x^2 + a*(
-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))
/2) - a^2*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I
*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - P
olyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))))/c
```

3.395.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.395. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx$

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.395.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 77.33 (sec) , antiderivative size = 1831, normalized size of antiderivative = 8.07

method	result	size
derivativedivides	Expression too large to display	1831
default	Expression too large to display	1831
parts	Expression too large to display	1930

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/c*arctan(a*x)^4-1/3/c*arctan(a*x)^3/a^3/x^3+1/c*arctan(a*x)^3/a/x-1
/c*(-2*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2/a^2/x^2+4*arctan(a*x)
^2*ln(a*x)+4*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)^2
*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+1/6*arctan(a*x)*(6*I*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*
Pi*arctan(a*x)*a*x+6*I*a*x+12*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^3*Pi*arctan(a*x)*a*x+12*I*csgn(I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*Pi*arctan(a*x)*a*x+6*I*csgn(I/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)^2*Pi*arctan(a*x)*a*x+12*I*csgn(((1+I*a*x)^2/(a^2*x^2+1)
-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*Pi*arctan(a*x)*a*x-6*I*csgn(I/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*Pi*arctan(a*x)*a*x+12*I*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*Pi*arctan(a*x)*a*x-12*I*csgn((
(1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)*a
*x-12*I*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^2*Pi*arctan(a*x)*a*x-8*I*arctan(a*x)^2*a*x-12*I*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*Pi*arctan(a*x)*a*x+12*I*csgn(I*((1+...
```

3.395.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan{(ax)}^3}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)
```

3.395.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^6+x^4}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**6 + x**4), x)/c
```

3.395.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Timed out
```

3.395.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \int \frac{\operatorname{atan}(a\,x)^3}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)), x)
```

3.396 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} dx$

3.396.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} dx &= \frac{3x}{8a^3c^2(1+a^2x^2)} + \frac{3\arctan(ax)}{8a^4c^2} - \frac{3\arctan(ax)}{4a^4c^2(1+a^2x^2)} \\ &\quad - \frac{3x\arctan(ax)^2}{4a^3c^2(1+a^2x^2)} - \frac{\arctan(ax)^3}{4a^4c^2} + \frac{\arctan(ax)^3}{2a^4c^2(1+a^2x^2)} - \frac{i\arctan(ax)^4}{4a^4c^2} \\ &\quad - \frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c^2} - \frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2} \\ &\quad - \frac{3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2} + \frac{3i\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^4c^2}\end{aligned}$$

output
```
3/8*x/a^3/c^2/(a^2*x^2+1)+3/8*arctan(a*x)/a^4/c^2-3/4*arctan(a*x)/a^4/c^2/
(a^2*x^2+1)-3/4*x*arctan(a*x)^2/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^3/a^4/
c^2+1/2*arctan(a*x)^3/a^4/c^2/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/a^4/c^2-arct
an(a*x)^3*ln(2/(1+I*a*x))/a^4/c^2-3/2*I*arctan(a*x)^2*polylog(2,1-2/(1+I*a
*x))/a^4/c^2-3/2*arctan(a*x)*polylog(3,1-2/(1+I*a*x))/a^4/c^2+3/4*I*polylo
g(4,1-2/(1+I*a*x))/a^4/c^2
```

3.396.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} \, dx$$
$$= \frac{4i \arctan(ax)^4 - 6 \arctan(ax) \cos(2 \arctan(ax)) + 4 \arctan(ax)^3 \cos(2 \arctan(ax)) - 16 \arctan(ax)^3 \log (}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
((4*I)*ArcTan[a*x]^4 - 6*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 4*ArcTan[a*x]^3*
Cos[2*ArcTan[a*x]] - 16*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (24
*I)*ArcTan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*ArcTan[a*x]*Poly
Log[3, -E^((2*I)*ArcTan[a*x])] - (12*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]
 + 3*Sin[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]])/(16*a^4*c^2)
```

3.396.3 Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5499, 27, 5455, 5379, 5465, 5427, 5465, 215, 216, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \ 5499$$

$$\frac{\int \frac{x \arctan(ax)^3}{c(a^2x^2+1)}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^3}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x \arctan(ax)^3}{a^2x^2+1}dx}{a^2c^2} - \frac{\int \frac{x \arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \ 5455$$

$$-\frac{\int \frac{x \arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^2} + \frac{-\frac{\int \frac{\arctan(ax)^3}{i-ax}dx}{a} - \frac{i \arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5379$$

$$-\frac{\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5465$$

$$-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5427$$

$$-\frac{\frac{3\left(-a\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5465$$

$$-\frac{\frac{3\left(-a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 215$$

$$-\frac{\frac{3\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

3.396. $\int \frac{x^3\arctan(ax)^3}{\left(c+a^2cx^2\right)^2}\,dx$

$$\Big\downarrow \text{5529}$$

$$\frac{-\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5533}$$

$$\frac{-\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{7164}$$

$$\frac{-\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^4}{4a^2}-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{iax+1}\right)}{4a}\right)-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}}{a^2c^2}$$

input

```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output

```
-((-1/2*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*(1 +
a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2))
+ (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(2*a))/(a^2*c^2)) + (
((-1/4*I)*ArcTan[a*x]^4)/a^2 - ((ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/a - 3*(
((-1/2*I)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTa
n[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)]/(4
*a))))/a)/(a^2*c^2)
```

3.396. $\int \frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$

3.396.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.396. $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.396.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.99 (sec) , antiderivative size = 936, normalized size of antiderivative = 3.47

method	result	size
derivativedivides	Expression too large to display	936
default	Expression too large to display	936
parts	Expression too large to display	971

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2/c^2*arctan(a*x)^3*ln(a^2*x^2+1)+1/2*arctan(a*x)^3/c^2/(a^2*x^2+
1)-3/2/c^2*(2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/6*I*arctan
(a*x)^4-I*arctan(a*x)^2*(I+a*x)/(8*a*x-8*I)-1/8*arctan(a*x)*(I+a*x)/(a*x-I
)+I*(I+a*x)/(16*a*x-16*I)+I*arctan(a*x)^2*(a*x-I)/(8*a*x+8*I)-1/8*arctan(a
*x)*(a*x-I)/(I+a*x)-I*(a*x-I)/(16*a*x+16*I)-I*arctan(a*x)^2*polylog(2,-(1+
I*a*x)^2/(a^2*x^2+1))+arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*
I*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+1/6*(I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^3-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+2*I*P
i*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+I*
Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1))-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+4*ln(2)+1)*arctan(a*x)^3))
```

3.396.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.396.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.396.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2+c)^2} dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.396.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2+c)^2} dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \int \frac{x^3 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^2} dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^2, x)
```

3.397 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx$

3.397.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{3}{8a^3c^2(1+a^2x^2)} + \frac{3x\arctan(ax)}{4a^2c^2(1+a^2x^2)} + \frac{3\arctan(ax)^2}{8a^3c^2} - \frac{3\arctan(ax)^2}{4a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^3}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^4}{8a^3c^2}$$

output
```
3/8/a^3/c^2/(a^2*x^2+1)+3/4*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+3/8*arctan(a
*x)^2/a^3/c^2-3/4*arctan(a*x)^2/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)^3/a^
2/c^2/(a^2*x^2+1)+1/8*arctan(a*x)^4/a^3/c^2
```

3.397.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.55

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{3+6ax\arctan(ax)+3(-1+a^2x^2)\arctan(ax)^2-4ax\arctan(ax)^3+(1+a^2x^2)\arctan(ax)^4}{8a^3c^2(1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
(3 + 6*a*x*ArcTan[a*x] + 3*(-1 + a^2*x^2)*ArcTan[a*x]^2 - 4*a*x*ArcTan[a*x
]^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*a^3*c^2*(1 + a^2*x^2))
```

3.397.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5471, 27, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{5471}$$

$$\frac{3\int \frac{x\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{2a}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{2ac^2}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2ac^2}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2ac^2}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}+\frac{3\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2ac^2}$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^3)/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a^3*c^2)
 + (3*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (
x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/(2*a*c^2)
```

3.397.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.397. $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$

3.397.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{\arctan(ax)^4 x^2 a^2+3x^2 \arctan(ax)^2 a^2-4\arctan(ax)^3 ax-3a^2x^2+\arctan(ax)^4+6x\arctan(ax)a-3\arctan(ax)^2}{8c^2(a^2x^2+1)a^3}$
derivativedivides	$\frac{-\frac{\arctan(ax)^3 ax}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2a^2x^2+2}-\frac{\arctan(ax)ax}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{4}-\frac{1}{4\left(a^2x^2+1\right)}\right)}{2c^2}}{a^3}$
default	$\frac{-\frac{\arctan(ax)^3 ax}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2a^2x^2+2}-\frac{\arctan(ax)ax}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{4}-\frac{1}{4\left(a^2x^2+1\right)}\right)}{2c^2}}{a^3}$
parts	$-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{2a^3c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4a^3}-\frac{-\frac{\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{x\arctan(ax)a}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{a^3}\right)}{2c^2}$
risch	$\frac{\ln(iax+1)^4}{128c^2a^3}-\frac{\left(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax\right)\ln(iax+1)^3}{32a^3c^2(a^2x^2+1)}+\frac{3\left(a^2x^2\ln(-iax+1)^2+4iax\ln(-iax+1)-2a^2x\right.}{64a^3c^2(ax+i)(ax}$

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(arctan(a*x)^4*x^2*a^2+3*x^2*arctan(a*x)^2*a^2-4*arctan(a*x)^3*a*x-3*a
^2*x^2+arctan(a*x)^4+6*x*arctan(a*x)*a-3*arctan(a*x)^2)/c^2/(a^2*x^2+1)/a^
3
```

3.397.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56

$$\int \frac{x^2\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = -\frac{4\,ax\arctan(ax)^3-(a^2x^2+1)\arctan(ax)^4-6\,ax\arctan(ax)-3\,(a^2x^2-1)\arctan(ax)^2-3}{8\,(a^5c^2x^2+a^3c^2)}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*a*x*arctan(a*x)^3 - (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x
) - 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^5*c^2*x^2 + a^3*c^2)
```

3.397.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = -\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2} - \frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax)^3 - \frac{3\left((a^2x^2+1)\arctan(ax)^2+1\right)a\arctan(ax)^2}{4\,(a^6c^2x^2+a^4c^2)} - \frac{1}{8}\left(\frac{\left((a^2x^2+1)\arctan(ax)^4+3\,(a^2x^2+1)\arctan(ax)^2-3\right)a^2}{a^8c^2x^2+a^6c^2} - \frac{2\left(2\,(a^2x^2+1)\arctan(ax)^3+3\,ax+\right.}{a^7c^2x}\right.$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^3 - 3
/4*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arctan(a*x)^2/(a^6*c^2*x^2 + a^4*c^
2) - 1/8*(((a^2*x^2 + 1)*arctan(a*x)^4 + 3*(a^2*x^2 + 1)*arctan(a*x)^2 - 3
)*a^2/(a^8*c^2*x^2 + a^6*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x +
 3*(a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^2*x^2 + a^5*c^2))*a
```

3.397.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.397.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} \, dx = \frac{3}{2\,a^2\,\left(4\,a^3\,c^2\,x^2+4\,a\,c^2\right)}+\mathrm{atan}(a\,x)^2\,\left(\frac{3}{8\,a^3\,c^2}-\frac{3}{4\,a^5\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right)+\frac{\mathrm{atan}(a\,x)^4}{8\,a^3\,c^2}+\frac{3\,x\,\mathrm{atan}(a\,x)}{4\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}-\frac{x\,\mathrm{atan}(a\,x)^3}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
3/(2*a^2*(4*a*c^2 + 4*a^3*c^2*x^2)) + atan(a*x)^2*(3/(8*a^3*c^2) - 3/(4*a^
5*c^2*(1/a^2 + x^2))) + atan(a*x)^4/(8*a^3*c^2) + (3*x*atan(a*x))/(4*a^4*c
^2*(1/a^2 + x^2)) - (x*atan(a*x)^3)/(2*a^4*c^2*(1/a^2 + x^2))
```

3.398 $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^2} dx$

3.398.1 Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^2} dx = -\frac{3x}{8ac^2(1+a^2x^2)} - \frac{3\arctan(ax)}{8a^2c^2} + \frac{3\arctan(ax)}{4a^2c^2(1+a^2x^2)} + \frac{3x\arctan(ax)^2}{4ac^2(1+a^2x^2)} + \frac{\arctan(ax)^3}{4a^2c^2} - \frac{\arctan(ax)^3}{2a^2c^2(1+a^2x^2)}$$

output
```
-3/8*x/a/c^2/(a^2*x^2+1)-3/8*arctan(a*x)/a^2/c^2+3/4*arctan(a*x)/a^2/c^2/(
a^2*x^2+1)+3/4*x*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^3/a^2/c^2
-1/2*arctan(a*x)^3/a^2/c^2/(a^2*x^2+1)
```

3.398.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{-3ax + (3-3a^2x^2)\arctan(ax) + 6ax\arctan(ax)^2 + 2(-1+a^2x^2)\arctan(ax)^3}{8a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
(-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(-1 + a^2*
x^2)*ArcTan[a*x]^3)/(8*a^2*c^2*(1 + a^2*x^2))
```

3.398.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5465, 27, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^3}{\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\int \frac{\arctan(ax)^2}{c^2\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-a\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^3/(a^2*c^2*(1 + a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*(1 +
 a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2))
 + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(2*a*c^2)
```

3.398.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.398. $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^2} dx$

3.398.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

method	result
parallelrisch	$\frac{2\arctan(ax)^3x^2a^2-3a^2\arctan(ax)x^2+6a\arctan(ax)^2x-2\arctan(ax)^3-3ax+3\arctan(ax)}{8c^2(a^2x^2+1)a^2}$
derivativedivides	$\frac{-\frac{\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\frac{3a\arctan(ax)^2x}{2(2a^2x^2+2)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2(2a^2x^2+2)}-\frac{3ax}{8(a^2x^2+1)}-\frac{3\arctan(ax)}{8}}{c^2}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\frac{3a\arctan(ax)^2x}{2(2a^2x^2+2)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2(2a^2x^2+2)}-\frac{3ax}{8(a^2x^2+1)}-\frac{3\arctan(ax)}{8}}{c^2}}{a^2}$
parts	$-\frac{\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}+\frac{\frac{3a\arctan(ax)^2x}{2(2a^2x^2+2)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2(2a^2x^2+2)}-\frac{3ax}{8(a^2x^2+1)}-\frac{3\arctan(ax)}{8}}{a^2c^2}$
risch	$\frac{i(a^2x^2-1)\ln(iax+1)^3}{32a^2c^2(a^2x^2+1)}-\frac{3i(-\ln(-iax+1)+a^2x^2\ln(-iax+1)-2iax)\ln(iax+1)^2}{32(ax+i)a^2c^2(ax-i)}+\frac{3i\left(-4+a^2x^2\ln(-iax+1)^2-\ln(\right.}{32(ax}$

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*arctan(a*x)^3*x^2*a^2-3*a^2*arctan(a*x)*x^2+6*a*arctan(a*x)^2*x-2*a
rctan(a*x)^3-3*a*x+3*arctan(a*x))/c^2/(a^2*x^2+1)/a^2
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.52

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{6\,ax\arctan(ax)^2+2\,(a^2x^2-1)\arctan(ax)^3-3\,ax-3\,(a^2x^2-1)\arctan(ax)}{8\,(a^4c^2x^2+a^2c^2)}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 - 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*
x^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

3.398.6 Sympy [F]

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx &= \frac{3\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan(ax)^2}{4\,ac} \\ &\quad + \frac{\frac{\left(2\,(a^2x^2+1)\arctan(ax)^3-3\,ax-3\,(a^2x^2+1)\arctan(ax)\right)a^2}{a^5cx^2+a^3c}-\frac{6\left((a^2x^2+1)\arctan(ax)^2-1\right)a\arctan(ax)}{a^4cx^2+a^2c}}{8\,ac} \\ &\quad - \frac{\arctan(ax)^3}{2\,(a^2cx^2+c)a^2c}\end{aligned}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
3/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)^2/(a*c) + 1/8*((2*
(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^
5*c*x^2 + a^3*c) - 6*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)/(a^4*
c*x^2 + a^2*c))/(a*c) - 1/2*arctan(a*x)^3/((a^2*c*x^2 + c)*a^2*c)
```

3.398.8 Giac [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x\arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.398.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \mathrm{atan}(a\,x)^3\left(\frac{1}{4\,a^2\,c^2}-\frac{1}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right)-\frac{3\,x}{2\,\left(4\,a^3\,c^2\,x^2+4\,a\,c^2\right)} - \frac{3\,\mathrm{atan}(a\,x)}{8\,a^2\,c^2}+\frac{3\,\mathrm{atan}(a\,x)}{4\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}+\frac{3\,x\,\mathrm{atan}(a\,x)^2}{4\,a^3\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)^3*(1/(4*a^2*c^2) - 1/(2*a^4*c^2*(1/a^2 + x^2))) - (3*x)/(2*(4*a*
c^2 + 4*a^3*c^2*x^2)) - (3*atan(a*x))/(8*a^2*c^2) + (3*atan(a*x))/(4*a^4*c
^2*(1/a^2 + x^2)) + (3*x*atan(a*x)^2)/(4*a^3*c^2*(1/a^2 + x^2))
```

3.399 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2} dx$

3.399.1 Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2} dx = -\frac{3}{8ac^2(1+a^2x^2)} - \frac{3x\arctan(ax)}{4c^2(1+a^2x^2)} - \frac{3\arctan(ax)^2}{8ac^2} + \frac{3\arctan(ax)^2}{4ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^3}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^4}{8ac^2}$$

output
```
-3/8/a/c^2/(a^2*x^2+1)-3/4*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/8*arctan(a*x)^2
/a/c^2+3/4*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)^3/c^2/(a^2*x^
2+1)+1/8*arctan(a*x)^4/a/c^2
```

3.399.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{-3-6ax\arctan(ax)+(3-3a^2x^2)\arctan(ax)^2+4ax\arctan(ax)^3+(1+a^2x^2)\arctan(ax)^4}{8c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
```

output
```
(-3 - 6*a*x*ArcTan[a*x] + (3 - 3*a^2*x^2)*ArcTan[a*x]^2 + 4*a*x*ArcTan[a*x
]^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*c^2*(a + a^3*x^2))
```

3.399.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5427, 27, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \textbf{5427}$$

$$-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{3a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{2c^2}+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \textbf{5465}$$

$$-\frac{3a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2c^2}+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \textbf{5427}$$

$$-\frac{3a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2c^2}+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \textbf{241}$$

$$\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}-\frac{3a\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2c^2}+\frac{\arctan(ax)^4}{8ac^2}$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a*c^2) - (3*a*(
-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTa
n[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/(2*c^2)
```

3.399.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.399. $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$

3.399.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{\arctan(ax)^4 x^2 a^2 - 3x^2 \arctan(ax)^2 a^2 + 4\arctan(ax)^3 ax + 3a^2x^2 + \arctan(ax)^4 - 6x\arctan(ax)a + 3\arctan(ax)^2}{8c^2(a^2x^2+1)a}$
derivativedivides	$\frac{\frac{\arctan(ax)^3 ax}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)^4}{2c^2} - \frac{3\left(\frac{\arctan(ax)^4}{4} - \frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{x\arctan(ax)a}{2a^2x^2+2} + \frac{\arctan(ax)^2}{4} + \frac{1}{4a^2x^2+4}\right)}{2c^2}}{a}$
default	$\frac{\frac{\arctan(ax)^3 ax}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)^4}{2c^2} - \frac{3\left(\frac{\arctan(ax)^4}{4} - \frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{x\arctan(ax)a}{2a^2x^2+2} + \frac{\arctan(ax)^2}{4} + \frac{1}{4a^2x^2+4}\right)}{2c^2}}{a}$
parts	$\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)^4}{2a\,c^2} - \frac{3\left(\frac{\arctan(ax)^4}{4a} + \frac{-\frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{x\arctan(ax)a}{2a^2x^2+2} + \frac{\arctan(ax)^2}{4} + \frac{1}{4a^2x^2+4}}{a}\right)}{2c^2}$
risch	$\frac{\ln(iax+1)^4}{128c^2a} - \frac{(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax)\ln(iax+1)^3}{32c^2(a^2x^2+1)a} + \frac{3\left(a^2x^2\ln(-iax+1)^2+2a^2x^2+\ln(-iax+1)^2-4\right.}{64c^2(ax+i)(ax-}$

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(arctan(a*x)^4*x^2*a^2-3*x^2*arctan(a*x)^2*a^2+4*arctan(a*x)^3*a*x+3*a
^2*x^2+arctan(a*x)^4-6*x*arctan(a*x)*a+3*arctan(a*x)^2)/c^2/(a^2*x^2+1)/a
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{4\,ax\arctan(ax)^3 + (a^2x^2+1)\arctan(ax)^4 - 6\,ax\arctan(ax) - 3\,(a^2x^2-1)\arctan(ax)^2 - 3}{8\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*x*arctan(a*x)^3 + (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x)
 - 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^3*c^2*x^2 + a*c^2)
```

3.399.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.65

$$\begin{aligned}\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx &= \frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)^3 \\ &\quad - \frac{3\left((a^2x^2+1)\arctan(ax)^2-1\right)a\arctan(ax)^2}{4\left(a^4c^2x^2+a^2c^2\right)} \\ &\quad - \frac{1}{8}\left(\frac{\left((a^2x^2+1)\arctan(ax)^4-3\left(a^2x^2+1\right)\arctan(ax)^2+3\right)a^2}{a^6c^2x^2+a^4c^2} - \frac{2\left(2\left(a^2x^2+1\right)\arctan(ax)^3-3\,ax-\right.}{a^5c^2x}\right.\end{aligned}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^3 - 3/4*((a^
2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)^2/(a^4*c^2*x^2 + a^2*c^2) - 1/
8*(((a^2*x^2 + 1)*arctan(a*x)^4 - 3*(a^2*x^2 + 1)*arctan(a*x)^2 + 3)*a^2/(
a^6*c^2*x^2 + a^4*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2
*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^5*c^2*x^2 + a^3*c^2))*a
```

3.399.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.399.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\mathrm{atan}(a\,x)^4}{8\,a\,c^2} - \mathrm{atan}(a\,x)^2\left(\frac{3}{8\,a\,c^2} - \frac{3}{4\,a^3\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right) - \frac{3}{2\,a\,\left(4\,a^2\,c^2\,x^2+4\,c^2\right)} - \frac{3\,x\,\mathrm{atan}(a\,x)}{4\,a^2\,c^2\,\left(\frac{1}{a^2}+x^2\right)} + \frac{x\,\mathrm{atan}(a\,x)^3}{2\,a^2\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)^4/(8*a*c^2) - atan(a*x)^2*(3/(8*a*c^2) - 3/(4*a^3*c^2*(1/a^2 + x
^2))) - 3/(2*a*(4*c^2 + 4*a^2*c^2*x^2)) - (3*x*atan(a*x))/(4*a^2*c^2*(1/a^
2 + x^2)) + (x*atan(a*x)^3)/(2*a^2*c^2*(1/a^2 + x^2))
```

3.400 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx$

3.400.1 Optimal result

Integrand size = 22, antiderivative size = 240

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx &= \frac{3ax}{8c^2(1+a^2x^2)}+\frac{3\arctan(ax)}{8c^2}-\frac{3\arctan(ax)}{4c^2(1+a^2x^2)}\\ &\quad-\frac{3ax\arctan(ax)^2}{4c^2(1+a^2x^2)}-\frac{\arctan(ax)^3}{4c^2}+\frac{\arctan(ax)^3}{2c^2(1+a^2x^2)}\\ &\quad-\frac{i\arctan(ax)^4}{4c^2}+\frac{\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^2}\\ &\quad-\frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}\\ &\quad+\frac{3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}+\frac{3i\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^2}\end{aligned}$$

output
```
3/8*a*x/c^2/(a^2*x^2+1)+3/8*arctan(a*x)/c^2-3/4*arctan(a*x)/c^2/(a^2*x^2+1
)-3/4*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^3/c^2+1/2*arctan(a
*x)^3/c^2/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c^2+arctan(a*x)^3*ln(2-2/(1-I*a*
x))/c^2-3/2*I*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^2+3/2*arctan(a*x)*
polylog(3,-1+2/(1-I*a*x))/c^2+3/4*I*polylog(4,-1+2/(1-I*a*x))/c^2
```

3.400.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx$$
$$= \frac{-i\pi^4+16i\arctan(ax)^4-24\arctan(ax)\cos(2\arctan(ax))+16\arctan(ax)^3\cos(2\arctan(ax))+64\arctan}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```

output
```
((-I)*Pi^4 + (16*I)*ArcTan[a*x]^4 - 24*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 16
*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*Arc
Tan[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 96*
ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*
I)*ArcTan[a*x])] + 12*Sin[2*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[2*ArcTan[a
*x]])/(64*c^2)
```

3.400.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5501, 27, 5459, 5403, 5465, 5427, 5465, 215, 216, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{5501}$$
$$\frac{\int \frac{\arctan(ax)^3}{cx(a^2x^2+1)}dx}{c}-a^2\int \frac{x\arctan(ax)^3}{c^2\left(a^2x^2+1\right)^2}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}$$
$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{ 5403}$$

$$-\frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{ 5465}$$

$$-\frac{a^2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{ 5427}$$

$$-\frac{a^2\left(\frac{3\left(-a\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{ 5465}$$

$$-\frac{a^2\left(\frac{3\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{ 215}$$

3.400. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx$

$$-\frac{a^2\left(\frac{3\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{5527}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\,\text{a}}{c^2}$$

$$\Big\downarrow \text{5531}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(a}{c^2}$$

$$\Big\downarrow \text{7164}$$

3.400. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{\operatorname{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(ax)^3\log\right.}{c^2}$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-1/2*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*
(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x
^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(2*a)))/c^2) + (
(-1/4*I)*ArcTan[a*x]^4 + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3
*I)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*
I)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 -
I*a*x)]/(4*a)))))/c^2
```

3.400.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

3.400. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx$

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.400.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 84.45 (sec) , antiderivative size = 1787, normalized size of antiderivative = 7.45

method	result	size
derivativedivides	Expression too large to display	1787
default	Expression too large to display	1787
parts	Expression too large to display	2218

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*arctan(a*x)^3*ln(a*x)+1/2*arctan(a*x)^3/c^2/(a^2*x^2+1)-1/2/c^2*arct
an(a*x)^3*ln(a^2*x^2+1)-3/2/c^2*(-2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+
1)^(1/2))+2*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*ar
ctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/8*arctan(a*x)*(I+a*x)
/(a*x-I)-I*arctan(a*x)^2*(I+a*x)/(8*a*x-8*I)-I*(a*x-I)/(16*a*x+16*I)-1/8*a
rctan(a*x)*(a*x-I)/(I+a*x)+1/6*I*arctan(a*x)^4+2/3*arctan(a*x)^3*ln((1+I*a
*x)^2/(a^2*x^2+1)-1)-2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-4
*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*polylog(3,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))+I*(I+a*x)/(16*a*x-16*I)-2/3*arctan(a*x)^3*ln(1-(1+I*
a*x)/(a^2*x^2+1)^(1/2))-4*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arct
an(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)^2*(a*x-I)/(8*
a*x+8*I)-1/6*(2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1))^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-I*Pi*csgn(I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-2*I*Pi*csgn(((1+I*a
*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-2*I*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))^2+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^...
```

3.400.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

3.400.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx=\frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

3.400.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x), x)
```

3.400.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^2), x)
```

3.401 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx$

3.401.1 Optimal result

Integrand size = 22, antiderivative size = 234

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx = {} & \frac{3a}{8c^2(1+a^2x^2)}+\frac{3a^2x\arctan(ax)}{4c^2(1+a^2x^2)}+\frac{3a\arctan(ax)^2}{8c^2} \\ & -\frac{3a\arctan(ax)^2}{4c^2(1+a^2x^2)}-\frac{ia\arctan(ax)^3}{c^2}-\frac{\arctan(ax)^3}{c^2x}-\frac{a^2x\arctan(ax)^3}{2c^2(1+a^2x^2)} \\ & -\frac{3a\arctan(ax)^4}{8c^2}+\frac{3a\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2} \\ & -\frac{3ia\arctan(ax)\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2} \\ & +\frac{3a\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}\end{aligned}$$

output
```
3/8*a/c^2/(a^2*x^2+1)+3/4*a^2*x*arctan(a*x)/c^2/(a^2*x^2+1)+3/8*a*arctan(a
*x)^2/c^2-3/4*a*arctan(a*x)^2/c^2/(a^2*x^2+1)-I*a*arctan(a*x)^3/c^2-arctan
(a*x)^3/c^2/x-1/2*a^2*x*arctan(a*x)^3/c^2/(a^2*x^2+1)-3/8*a*arctan(a*x)^4/
c^2+3*a*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2-3*I*a*arctan(a*x)*polylog(2,-1
+2/(1-I*a*x))/c^2+3/2*a*polylog(3,-1+2/(1-I*a*x))/c^2
```

3.401.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = \frac{a\left(-2i\pi^3+16i\arctan(ax)^3-\frac{16\arctan(ax)^3}{ax}-6\arctan(ax)^4+3\cos(2\arctan(ax))-6\arctan(ax)^2\cos(2\operatorname{arc}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
(a*((-2*I)*Pi^3 + (16*I)*ArcTan[a*x]^3 - (16*ArcTan[a*x]^3)/(a*x) - 6*ArcT
an[a*x]^4 + 3*Cos[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 48
*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLo
g[2, E^((-2*I)*ArcTan[a*x])] + 24*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 6*A
rcTan[a*x]*Sin[2*ArcTan[a*x]] - 4*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]]))/(16*c
^2)
```

3.401.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5501, 27, 5427, 5453, 5361, 5419, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\left(a^2cx^2+c\right)^2}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{cx^2(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \; 5453$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \; 5361$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx-\frac{\arctan(ax)^3}{x}}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \; 5419$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \; 5459$$

$$-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+\frac{3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}}{c^2}$$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+$$
$$\frac{3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{arc}}{c^2}$$

$$\Big\downarrow \; 5465$$

$$-\frac{a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+$$
$$\frac{3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{arc}}{c^2}$$

3.401. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx$

$$\Big\downarrow \text{5427}$$

$$-\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+\frac{3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{arct}}{c^2}$$

$$\Big\downarrow \text{241}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+\frac{3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{arct}}{c^2}$$

$$\Big\downarrow \text{5527}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+\frac{3a\left(i\left(2ia\left(\frac{i\arctan(ax)\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\text{arcta}}{c^2}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+\frac{3a\left(i\left(2ia\left(\frac{i\arctan(ax)\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

3.401. $\int\frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}dx$

output
```
-((a^2*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(
-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTa
n[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2))/c^2) + (-(ArcTan[
a*x]^3/x) - (a*ArcTan[a*x]^4)/4 + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*Ar
cTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2
, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a)))))/c^2
```

3.401.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.401. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx$

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.401. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx$

3.401.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 77.72 (sec) , antiderivative size = 1731, normalized size of antiderivative = 7.40

method	result	size
derivativedivides	Expression too large to display	1731
default	Expression too large to display	1731
parts	Expression too large to display	1735

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c^2*arctan(a*x)^3/a/x-1/2/c^2*arctan(a*x)^3*a*x/(a^2*x^2+1)-3/2/c^2*
arctan(a*x)^4-3/2/c^2*(-3/4*arctan(a*x)^4-2*arctan(a*x)^2*ln(a*x)+1/2*arct
an(a*x)^2/(a^2*x^2+1)+arctan(a*x)^2*ln(a^2*x^2+1)-2*arctan(a*x)^2*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2/3*I*arctan(a*x)^3+1/16*(I+a*x)/(a*x-I)+I*arctan(a*x)*(I+a*x)/(8*a*x
-8*I)+1/16*(a*x-I)/(I+a*x)+2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2
*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)*(a*x-I)/(8*
a*x+8*I)-4*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1-(1
+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1
)^(1/2))-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*(2*I*Pi*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^2+4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^
2*x^2+1)+1))^3-2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)+1)^2)-4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^3-4*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-
4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+4*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+...
```

3.401.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

3.401.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^3\left(ax\right)}{a^4x^6+2a^2x^4+x^2}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

3.401.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2048*(240*(a^3*x^3 + a*x)*arctan(a*x)^4 - 9*(a^3*x^3 + a*x)*log(a^2*x^2
 + 1)^4 + 128*(3*a^2*x^2 + 2)*arctan(a*x)^3 - 24*(3*(a^3*x^3 + a*x)*arctan
(a*x)^2 + 4*(3*a^2*x^2 + 2)*arctan(a*x))*log(a^2*x^2 + 1)^2 - 4*(a^2*c^2*x
^3 + c^2*x)*(72*a^5*(a^2/(a^8*c^2*x^2 + a^6*c^2) + log(a^2*x^2 + 1)/(a^6*c
^2*x^2 + a^4*c^2)) - 18432*a^5*integrate(1/256*x^5*arctan(a*x)^2*log(a^2*x
^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 4608*a^5*integrate(1
/256*x^5*log(a^2*x^2 + 1)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) +
36864*a^4*integrate(1/256*x^4*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 +
 c^2*x^2), x) + 9216*a^4*integrate(1/256*x^4*arctan(a*x)*log(a^2*x^2 + 1)^
2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 73728*a^4*integrate(1/256*
x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2),
x) + 9*a^3*log(a^2*x^2 + 1)^3/(a^4*c^2*x^2 + a^2*c^2) + 27*(2*a^4*(a^2/(a^
10*c^2*x^2 + a^8*c^2) + log(a^2*x^2 + 1)/(a^8*c^2*x^2 + a^6*c^2)) + a^2*lo
g(a^2*x^2 + 1)^2/(a^6*c^2*x^2 + a^4*c^2))*a^3 - 18432*a^3*integrate(1/256*
x^3*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)
, x) + 73728*a^3*integrate(1/256*x^3*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^
2*x^4 + c^2*x^2), x) + 36*a^3*log(a^2*x^2 + 1)^2/(a^4*c^2*x^2 + a^2*c^2) +
 36864*a^2*integrate(1/256*x^2*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4
+ c^2*x^2), x) + 9216*a^2*integrate(1/256*x^2*arctan(a*x)*log(a^2*x^2 + 1)
^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 49152*a^2*integrate(1/...
```

3.401.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^3}{x^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^2), x)
```

3.402 $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

3.402.1 Optimal result

Integrand size = 22, antiderivative size = 374

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx =& -\frac{3a^3x}{8c^2(1+a^2x^2)}-\frac{3a^2\arctan(ax)}{8c^2}+\frac{3a^2\arctan(ax)}{4c^2(1+a^2x^2)}\\&-\frac{3ia^2\arctan(ax)^2}{2c^2}-\frac{3a\arctan(ax)^2}{2c^2x}+\frac{3a^3x\arctan(ax)^2}{4c^2(1+a^2x^2)}\\&-\frac{a^2\arctan(ax)^3}{4c^2}-\frac{\arctan(ax)^3}{2c^2x^2}-\frac{a^2\arctan(ax)^3}{2c^2(1+a^2x^2)}\\&+\frac{ia^2\arctan(ax)^4}{2c^2}+\frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}\\&-\frac{2a^2\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^2}-\frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}\\&+\frac{3ia^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}\\&-\frac{3a^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^2}\\&-\frac{3ia^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{2c^2}\end{aligned}$$

output
```
-3/8*a^3*x/c^2/(a^2*x^2+1)-3/8*a^2*arctan(a*x)/c^2+3/4*a^2*arctan(a*x)/c^2
/(a^2*x^2+1)-3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^2-3/2*a*arctan(a*x)^2/c
^2/x+3/4*a^3*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/4*a^2*arctan(a*x)^3/c^2-1/2
*arctan(a*x)^3/c^2/x^2-1/2*a^2*arctan(a*x)^3/c^2/(a^2*x^2+1)+1/2*I*a^2*arc
tan(a*x)^4/c^2+3*a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c^2-2*a^2*arctan(a*x)^3
*ln(2-2/(1-I*a*x))/c^2+3*I*a^2*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^2
-3/2*I*a^2*polylog(4,-1+2/(1-I*a*x))/c^2-3*a^2*arctan(a*x)*polylog(3,-1+2/
(1-I*a*x))/c^2-3/2*I*a^2*arctan(a*x)^2/c^2
```

3.402.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$$
$$= \frac{a^2\left(i\pi^4 - 48i\arctan(ax)^2 - \frac{48\arctan(ax)^2}{ax} - \frac{16(1+a^2x^2)\arctan(ax)^3}{a^2x^2} - 16i\arctan(ax)^4 + 12\arctan(ax)\cos(2\operatorname{arc}$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(a^2*(I*Pi^4 - (48*I)*ArcTan[a*x]^2 - (48*ArcTan[a*x]^2)/(a*x) - (16*(1 +
a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (16*I)*ArcTan[a*x]^4 + 12*ArcTan[a*x]*
Cos[2*ArcTan[a*x]] - 8*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 64*ArcTan[a*x]^3
*Log[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[
a*x])] - (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*
PolyLog[2, E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, E^((-2*I)*Ar
cTan[a*x])] + (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - 6*Sin[2*ArcTan[a
*x]] + 12*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]]))/(32*c^2)
```

3.402.3 Rubi [A] (verified)

Time = 4.18 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.42, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 5419, 5459, 5403, 2897, 5501, 5459, 5403, 5465, 5427, 5465, 215, 216, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

$$\int \frac{\arctan(ax)^3}{x^3\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{cx^3(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c^2x\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-a^2\left(i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax}{x}}{c^2}$$

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}dx$

$$\Big\downarrow \textcolor{blue}{5403}$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}+\frac{\frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right.}{c^2}$$

$$\Big\downarrow \textcolor{blue}{2897}$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\Big\downarrow \textcolor{blue}{5501}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx\right)}{c^2}+\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\Big\downarrow \textcolor{blue}{5459}$$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)}{c^2}$$

$$\Big\downarrow \textcolor{blue}{5403}$$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)}{c^2}$$

$$\Big\downarrow \textcolor{blue}{5465}$$

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{a^2\left(-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\right.}$$

$$c^2$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{a^2\left(-a^2\left(\frac{3\left(-a\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan\right.\right.}$$

$$c^2$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{a^2\left(-a^2\left(\frac{3\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right.}{a^2x^2+1}\right.\right.}$$

$$c^2$$

$$\Big\downarrow \text{ 215}$$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{a^2\left(-a^2\left(\frac{3\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int \frac{\arctan(ax)^2\log}{a^2x^2}\right.\right.}$$

$$c^2$$

$$\Big\downarrow \text{ 216}$$

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\frac{a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}\right.\right.}{2a}\right.\right.}{c^2}$$

$\downarrow$ 5527

$$\frac{-a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int \frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)\right.}{}$$

$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int \frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\right.}{c^2}$$

$\downarrow$ 5531

$$\frac{-a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan\right.\right.}{}$$

$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan\right.\right.}{}$$

$\downarrow$ 7164

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

$$\frac{-a^2\Bigg(i\Bigg(3ia\Bigg(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\Bigg(\frac{\operatorname{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\Bigg)\Bigg)\Bigg)-i\arctan(ax}{a^2\Bigg(-a^2\Bigg(\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\Bigg)+i\Bigg(3ia\Bigg(\frac{i\arctan(ax)^2\operatorname{PolyLo}}{2a}}$$

input
```
Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*((-1/4*I)*ArcTan[a*x]^4 - a^2*(-1/2*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2
)) + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1
/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2
*a))/(2*a))))/(2*a)) + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3*I
)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*I)
*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 - I*
a*x)]/(4*a))))))/c^2) + (-1/2*ArcTan[a*x]^3/x^2 + (3*a*(-(ArcTan[a*x]^2/x)
 - (a*ArcTan[a*x]^3)/3 + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]
*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2 - a^2*((-
1/4*I)*ArcTan[a*x]^4 + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3*I
)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*I)
*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 - I*
a*x)]/(4*a))))))/c^2
```

3.402.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.402. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$

3.402.4 Maple [A] (verified)

Time = 96.56 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.42

method	result
derivativedivides	$a^2\left(\frac{6i\arctan(ax)^2\operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^2}+\frac{\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{32c^2(ax+i)}+\frac{\left(-6i\arctan\right.}{}\right.$
default	$a^2\left(\frac{6i\arctan(ax)^2\operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^2}+\frac{\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{32c^2(ax+i)}+\frac{\left(-6i\arctan\right.}{}\right.$

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(6*I/c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/32*(6*
I*arctan(a*x)^2+4*arctan(a*x)^3-3*I-6*arctan(a*x))*(a*x-I)/c^2/(I+a*x)+1/3
2*(-6*I*arctan(a*x)^2+4*arctan(a*x)^3+3*I-6*arctan(a*x))*(I+a*x)/c^2/(a*x-
I)-1/2/c^2*arctan(a*x)^2*(-I*arctan(a*x)-3*I*a*x+x*arctan(a*x)*a)*(I+a*x)/
a^2/x^2-3*I/c^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/c^2*arctan(a*x)*l
n(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c^2*arctan(a*x)^2+3/c^2*arctan(a*x)*l
n((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-3*I/c^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-12*I/c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2/c^2*arctan(a*x)^3
*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+1/2*I/c^2*arctan(a*x)^4-12/c^2*arctan(a
*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I/c^2*arctan(a*x)^2*polylog(
2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2/c^2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^
2+1)^(1/2))-12*I/c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12/c^2*arctan
(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

3.402.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

3.402.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3 \left(c + a^2cx^2\right)^2} \, dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^7 + 2a^2x^5 + x^3} \, dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

3.402.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3 \left(c + a^2cx^2\right)^2} \, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^3), x)
```

3.402.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3 \left(c + a^2cx^2\right)^2} \, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^3}{x^3\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^2), x)
```

3.403 $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

3.403.1 Optimal result

Integrand size = 22, antiderivative size = 332

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx = & -\frac{3a^3}{8c^2(1+a^2x^2)}-\frac{a^2\arctan(ax)}{c^2x}-\frac{3a^4x\arctan(ax)}{4c^2(1+a^2x^2)} \\ & -\frac{7a^3\arctan(ax)^2}{8c^2}-\frac{a\arctan(ax)^2}{2c^2x^2}+\frac{3a^3\arctan(ax)^2}{4c^2(1+a^2x^2)} \\ & +\frac{7ia^3\arctan(ax)^3}{3c^2}-\frac{\arctan(ax)^3}{3c^2x^3}+\frac{2a^2\arctan(ax)^3}{c^2x} \\ & +\frac{a^4x\arctan(ax)^3}{2c^2(1+a^2x^2)}+\frac{5a^3\arctan(ax)^4}{8c^2}+\frac{a^3\log(x)}{c^2} \\ & -\frac{a^3\log(1+a^2x^2)}{2c^2}-\frac{7a^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2} \\ & +\frac{7ia^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2} \\ & -\frac{7a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}\end{aligned}$$

output
```
-3/8*a^3/c^2/(a^2*x^2+1)-a^2*arctan(a*x)/c^2/x-3/4*a^4*x*arctan(a*x)/c^2/(
a^2*x^2+1)-7/8*a^3*arctan(a*x)^2/c^2-1/2*a*arctan(a*x)^2/c^2/x^2+3/4*a^3*a
rctan(a*x)^2/c^2/(a^2*x^2+1)+7/3*I*a^3*arctan(a*x)^3/c^2-1/3*arctan(a*x)^3
/c^2/x^3+2*a^2*arctan(a*x)^3/c^2/x+1/2*a^4*x*arctan(a*x)^3/c^2/(a^2*x^2+1)
+5/8*a^3*arctan(a*x)^4/c^2+a^3*ln(x)/c^2-1/2*a^3*ln(a^2*x^2+1)/c^2-7*a^3*a
rctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2+7*I*a^3*arctan(a*x)*polylog(2,-1+2/(1-I
*a*x))/c^2-7/2*a^3*polylog(3,-1+2/(1-I*a*x))/c^2
```

3.403.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.59

$$\int \frac{\arctan(ax)^3}{x^4 (c+a^2cx^2)^2} \, dx$$
$$= \frac{a^3\left(-7i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)+\frac{1}{48}\left(14i\pi^3+30\arctan(ax)^4-9\cos(2\arctan(ax))+6\operatorname{arct}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
(a^3*((-7*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + ((14*I)*Pi^3
 + 30*ArcTan[a*x]^4 - 9*Cos[2*ArcTan[a*x]] + 6*ArcTan[a*x]^2*(-4 - 4/(a^2*
x^2) + 3*Cos[2*ArcTan[a*x]] - 56*Log[1 - E^((-2*I)*ArcTan[a*x])]) + 48*Log
[a*x] - 24*Log[1 + a^2*x^2] - 168*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 4*A
rcTan[a*x]^3*(-28*I - 4/(a^3*x^3) + 24/(a*x) + 3*Sin[2*ArcTan[a*x]]) - (6*
ArcTan[a*x]*(8 + 3*a*x*Sin[2*ArcTan[a*x]]))/(a*x))/48))/c^2
```

3.403.3 Rubi [A] (verified)

Time = 6.08 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.75, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.273, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 5419, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^4 (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{cx^4(a^2x^2+1)}dx}{c} - a^2 \int \frac{\arctan(ax)^3}{c^2x^2 (a^2x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx}{c^2} - \frac{a^2 \int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx + a\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{3x^3}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{-a^2\left(\int \frac{\arctan(ax)^3}{x^2}dx - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx\right) + a\left(\int \frac{\arctan(ax)^2}{x^3}dx - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) - \frac{\arctan(ax)^3}{3x^3}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{2x^2}\right) - a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx - \frac{\arctan(ax)^3}{x}\right.}{c^2} \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{1}{4}a\arctan(ax)^4 - \frac{\arctan(ax)^3}{x}\right) + a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{2x^2}\right.}{c^2} \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{1}{4}a\arctan(ax)^4 - \frac{\arctan(ax)^3}{x}\right) + a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(\int \frac{\arctan(ax)}{x^2}dx - a^2\int \frac{\arctan}{a^2x^2}\right.\right.}{c^2} \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{1}{4}a\arctan(ax)^4 - \frac{\arctan(ax)^3}{x}\right) + a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + a\right.\right.}{c^2} \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

3.403. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

$$\Big\downarrow \text{243}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\right.\right.}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\right.\right.}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\right.\right.}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\right.\right.}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+\right.\right.\right.\right.}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}+$$
$$\frac{a\left(-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\right.}{c^2}$$

3.403. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}+$$

$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4}{}$$

$$\Big\downarrow \; 5501$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx\right)}{c^2}+$$

$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4}{}$$

$$\Big\downarrow \; 5427$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)\right)}{c^2}+$$

$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4}{}$$

$$\Big\downarrow \; 5453$$

$$-\frac{a^2\left(-\left(a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)\right)-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx+\int \frac{\arctan(ax)^3}{x^2}dx\right)}{c^2}+$$

$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4}{}$$

$$\Big\downarrow \; 5361$$

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx-\frac{\arctan(ax)^3}{x}\right)}{c^2}$$

$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4}{}$$

$$\Big\downarrow \; 5419$$

3.403. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c^2}+$$

$$-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4$$

$$\Big\downarrow \; 5459$$

$$-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4$$

$$\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\int\frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(}{c^2}$$

$$\Big\downarrow \; 5403$$

$$-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4$$

$$\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\right.\right.\right.}{c^2}$$

$$\Big\downarrow \; 5465$$

$$-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4$$

$$\frac{a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{\int\frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx\right.\right.\right.}{c^2}$$

$$\Big\downarrow \; 5427$$

$$-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4$$

$$\frac{a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int\frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\right.\right.\right.}{c^2}$$

3.403. $\int\frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

$$\Big\downarrow \; 241$$

$$\frac{-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4}{a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}\right.}$$

$$c^2$$

$$\Big\downarrow \; 5527$$

$$\frac{-a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}\right.}{a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\right.}$$

$$\Big\downarrow \; 7164$$

$$\frac{a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\operatorname{arcta}\right.\right.\right.}{a^2\left(-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\left(\frac{i\arctan(a}{}\right.\right.\right.\right.}$$

input
```
Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

3.403. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

output
```
-((a^2*(-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 - a^2*((x*ArcTan[a*x]^3)/
(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1
 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2))
+ ArcTan[a*x]^2/(4*a))/a))/2) + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcT
an[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2,
-1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))))/c^2) +
(-1/3*ArcTan[a*x]^3/x^3 - a^2*(-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 +
3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
 + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog
[3, -1 + 2/(1 - I*a*x)]/(4*a))))) + a*(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTa
n[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a
^2*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
+ (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[
3, -1 + 2/(1 - I*a*x)]/(4*a))))))/c^2
```

3.403.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.403. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.403.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 233.77 (sec) , antiderivative size = 4175, normalized size of antiderivative = 12.58

method	result	size
derivativedivides	Expression too large to display	4175
default	Expression too large to display	4175
parts	Expression too large to display	4182

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/c^2*arctan(a*x)^3/a^3/x^3+2/c^2*arctan(a*x)^3/a/x+1/2/c^2*arctan
(a*x)^3*a*x/(a^2*x^2+1)+5/2/c^2*arctan(a*x)^4-1/2/c^2*(arctan(a*x)^2/a^2/x
^2+14*arctan(a*x)^2*ln(a*x)-3/2*arctan(a*x)^2/(a^2*x^2+1)-7*arctan(a*x)^2*
ln(a^2*x^2+1)+14*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-14*arctan(a
*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-1/48/a/x/(I+a*x)*(-168*a^2*arctan(a*x)
*x^2-168*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*a^3*x^3-336*I*arct
an(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*csgn(I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*a^3*x^3-168*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*a^3*x^3+168*
I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))^2*a^3*x^3+336*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1)-I)*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*a^3*x^3+336*I*arctan(a*x)^2*Pi*csgn(I/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1
)-I/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a^3*x^3+336*I*arctan(a*x)^2*Pi*csgn(I/(
(1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-I/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(1/((1+I*a*x)^2/(a^2*x^2+1)+1)*(1+I*a*x)^2/(a^2*x^2+1)-1/((1+I
*a*x)^2/(a^2*x^2+1)+1))^2*a^3*x^3+336*I*arctan(a*x)^2*Pi*csgn(I*(1+I*a*x)^
4/(a^2*x^2+1)^2+2*I*(1+I*a*x)^2/(a^2*x^2+1)+I)^2*csgn(I*(1+I*a*x)^2/(a^...
```

3.403.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c+a^2cx^2)^2}\,dx = \int \frac{\arctan{(ax)}^3}{(a^2cx^2+c)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

3.403.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4 \left(c + a^2cx^2\right)^2} \, dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^8 + 2a^2x^6 + x^4} \, dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

3.403.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^4 \left(c + a^2cx^2\right)^2} \, dx = \int \frac{\arctan(ax)^3}{\left(a^2cx^2 + c\right)^2 x^4} \, dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/6144*(1200*(a^5*x^5 + a^3*x^3)*arctan(a*x)^4 - 45*(a^5*x^5 + a^3*x^3)*lo
g(a^2*x^2 + 1)^4 + 128*(15*a^4*x^4 + 10*a^2*x^2 - 2)*arctan(a*x)^3 - 24*(1
5*(a^5*x^5 + a^3*x^3)*arctan(a*x)^2 + 4*(15*a^4*x^4 + 10*a^2*x^2 - 2)*arct
an(a*x))*log(a^2*x^2 + 1)^2 - 12*(a^2*c^2*x^5 + c^2*x^3)*(120*a^7*(a^2/(a^
8*c^2*x^2 + a^6*c^2) + log(a^2*x^2 + 1)/(a^6*c^2*x^2 + a^4*c^2)) - 30720*a
^7*integrate(1/256*x^7*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^4*c^2*x^8 + 2*a^2
*c^2*x^6 + c^2*x^4), x) - 7680*a^7*integrate(1/256*x^7*log(a^2*x^2 + 1)^3/
(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 61440*a^6*integrate(1/256*x^
6*arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15360*a^6*in
tegrate(1/256*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*a^2*c^2*
x^6 + c^2*x^4), x) - 122880*a^6*integrate(1/256*x^6*arctan(a*x)*log(a^2*x^
2 + 1)/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15*a^5*log(a^2*x^2 +
1)^3/(a^4*c^2*x^2 + a^2*c^2) + 45*(2*a^4*(a^2/(a^10*c^2*x^2 + a^8*c^2) + l
og(a^2*x^2 + 1)/(a^8*c^2*x^2 + a^6*c^2)) + a^2*log(a^2*x^2 + 1)^2/(a^6*c^2
*x^2 + a^4*c^2))*a^5 - 30720*a^5*integrate(1/256*x^5*arctan(a*x)^2*log(a^2
*x^2 + 1)/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 122880*a^5*integra
te(1/256*x^5*arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 6
0*a^5*log(a^2*x^2 + 1)^2/(a^4*c^2*x^2 + a^2*c^2) + 61440*a^4*integrate(1/2
56*x^4*arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15360*a
^4*integrate(1/256*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*...
```

3.403.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^3}{x^4\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^2), x)
```

3.404 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx$

3.404.1 Optimal result

Integrand size = 22, antiderivative size = 212

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx = &-\frac{3x^3}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256a^3c^3(1+a^2x^2)} - \frac{27\arctan(ax)}{256a^4c^3} \\ &- \frac{3x^4\arctan(ax)}{32c^3(1+a^2x^2)^2} + \frac{9\arctan(ax)}{32a^4c^3(1+a^2x^2)} + \frac{3x^3\arctan(ax)^2}{16ac^3(1+a^2x^2)^2} \\ &+ \frac{9x\arctan(ax)^2}{32a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)^3}{32a^4c^3} + \frac{x^4\arctan(ax)^3}{4c^3(1+a^2x^2)^2}\end{aligned}$$

output
```
-3/128*x^3/a/c^3/(a^2*x^2+1)^2-45/256*x/a^3/c^3/(a^2*x^2+1)-27/256*arctan(
a*x)/a^4/c^3-3/32*x^4*arctan(a*x)/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)/a^4/c
^3/(a^2*x^2+1)+3/16*x^3*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/32*x*arctan(a*
x)^2/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)^3/a^4/c^3+1/4*x^4*arctan(a*x)^3/
c^3/(a^2*x^2+1)^2
```

3.404.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx$$
$$= \frac{-3ax(15+17a^2x^2) + (45+18a^2x^2-51a^4x^4)\arctan(ax) + 24ax(3+5a^2x^2)\arctan(ax)^2 + 8(-3-6a^2x}{256a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(-3*a*x*(15 + 17*a^2*x^2) + (45 + 18*a^2*x^2 - 51*a^4*x^4)*ArcTan[a*x] + 2
4*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x]^2 + 8*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTa
n[a*x]^3)/(256*a^4*c^3*(1 + a^2*x^2)^2)
```

3.404.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5479, 27, 5475, 252, 252, 216, 5471, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4 \arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3}{4}a\int \frac{x^4 \arctan(ax)^2}{c^3(a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4 \arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3a\int \frac{x^4 \arctan(ax)^2}{(a^2x^2+1)^3}dx}{4c^3}$$

$$\Big\downarrow \text{5475}$$

$$\frac{x^4 \arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3a\left(\frac{3\int \frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}-\frac{1}{8}\int \frac{x^4}{(a^2x^2+1)^3}dx+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{4c^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4 \arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3a\left(\frac{3\int \frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\int \frac{x^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}\right)+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{4c^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\int\frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\int\frac{1}{a^2x^2+1}dx}{2a^2}-\frac{x}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}\right)+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{4c^3}$$

$$\Big\downarrow \; 216$$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\int\frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\arctan(ax)}{2a^3}-\frac{x}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}\right)\right)}{4c^3}$$

$$\Big\downarrow \; 5471$$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\left(\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}+\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\arctan(ax)}{2a^3}-\frac{x}{2a^2\left(a^2x^2\right.}\right.}{4a^2}\right.\right.}{4c^3}$$

$$\Big\downarrow \; 5465$$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\left(\frac{\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a}+\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\arctan(}{2a^3}\right.}{}\right.\right.}{4c^3}$$

$$\Big\downarrow \; 215$$

3.404. $\int \frac{x^3\arctan(ax)^3}{\left(c+a^2cx^2\right)^3}\,dx$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\left(\frac{\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a}+\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\right.\right.}{4c^3}$$

$\downarrow$ 216

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}+\frac{\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a}\right)}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3(}{}\right.\right.}{4c^3}$$

input
```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) - (3*a*((x^4*ArcTan[a*x])/(8*a
*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^2)/(4*a^2*(1 + a^2*x^2)^2) + (x^3/(4*
a^2*(1 + a^2*x^2)^2) - (3*(-1/2*x/(a^2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a^3
)))/(4*a^2))/8 + (3*(-1/2*(x*ArcTan[a*x]^2)/(a^2*(1 + a^2*x^2)) + ArcTan[a
*x]^3/(6*a^3) + (-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2
)) + ArcTan[a*x]/(2*a))/(2*a))/a))/(4*a^2)))/(4*c^3)
```

3.404. $\int \frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

3.404.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.404. $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.404.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{40a^4\arctan(ax)^3x^4-51\arctan(ax)a^4x^4+120a^3\arctan(ax)^2x^3-48\arctan(ax)^3x^2a^2-51a^3x^3+18a^2\arctan(ax)x^2+72a\cdots}{256c^3(a^2x^2+1)^2a^4}$
derivativedivides	$\frac{-\frac{\arctan(ax)^3}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3\left(-\frac{5\arctan(ax)^2a^3x^3}{8(a^2x^2+1)^2}-\frac{3\arctan(ax)^2ax}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^3}{24}-\frac{5\arctan(ax)}{8(a^2x^2+1)}+\frac{\arctan(ax)}{8(a^2x^2+1)^2}+\frac{\frac{17}{8}a\cdots}{8(a\cdots}\cdots\right.}{4c^3}}{a^4}$
default	$\frac{-\frac{\arctan(ax)^3}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3\left(-\frac{5\arctan(ax)^2a^3x^3}{8(a^2x^2+1)^2}-\frac{3\arctan(ax)^2ax}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^3}{24}-\frac{5\arctan(ax)}{8(a^2x^2+1)}+\frac{\arctan(ax)}{8(a^2x^2+1)^2}+\frac{\frac{17}{8}a\cdots}{8(a\cdots}\cdots\right.}{4c^3}}{a^4}$
parts	$\frac{\arctan(ax)^3}{4c^3a^4(a^2x^2+1)^2}-\frac{\arctan(ax)^3}{2c^3a^4(a^2x^2+1)}-\frac{3\left(-\frac{5\arctan(ax)^2x^3}{8a(a^2x^2+1)^2}-\frac{3\arctan(ax)^2x}{8a^3(a^2x^2+1)^2}-\frac{5\arctan(ax)^3}{8a^4}+\frac{-\frac{5\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(a\cdots}{2(a^2x^2+1\cdots}}{\cdots}\cdots\right.}{4c^3}$
risch	$\frac{i(5a^4x^4-6a^2x^2-3)\ln(iax+1)^3}{256a^4c^3(a^2x^2+1)^2}-\frac{3i(-6a^2x^2\ln(-iax+1)-3\ln(-iax+1)+5x^4\ln(-iax+1)a^4-10ia^3x^3-6iax)\ln(iax\cdots}{256a^4(ax+i)^2(ax-i)^2c^3}$

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/256*(40*a^4*arctan(a*x)^3*x^4-51*arctan(a*x)*a^4*x^4+120*a^3*arctan(a*x)
^2*x^3-48*arctan(a*x)^3*x^2*a^2-51*a^3*x^3+18*a^2*arctan(a*x)*x^2+72*a*arc
tan(a*x)^2*x-24*arctan(a*x)^3-45*a*x+45*arctan(a*x))/c^3/(a^2*x^2+1)^2/a^4
```

3.404.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.55

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{51\,a^3x^3 - 8\,(5\,a^4x^4 - 6\,a^2x^2 - 3)\arctan{(ax)}^3 - 24\,(5\,a^3x^3 + 3\,ax)\arctan{(ax)}^2 + 45\,ax + 3\,(17\,a^4x^4 -}{256\,(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/256*(51*a^3*x^3 - 8*(5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^3 - 24*(5*a
^3*x^3 + 3*a*x)*arctan(a*x)^2 + 45*a*x + 3*(17*a^4*x^4 - 6*a^2*x^2 - 15)*a
rctan(a*x))/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.404.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^3{(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

3.404.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.36

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{3}{32}\,a\left(\frac{5\,a^2x^3+3\,x}{a^8c^3x^4+2\,a^6c^3x^2+a^4c^3}+\frac{5\,\arctan\left(ax\right)}{a^5c^3}\right)\arctan\left(ax\right)^2$$
$$-\frac{(2\,a^2x^2+1)\arctan\left(ax\right)^3}{4\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$
$$-\frac{1}{256}\left(\frac{(51\,a^3x^3-40\,(a^4x^4+2\,a^2x^2+1)\arctan\left(ax\right)^3+45\,ax+51\,(a^4x^4+2\,a^2x^2+1)\arctan\left(ax\right))a^2}{a^{11}c^3x^4+2\,a^9c^3x^2+a^7c^3}\right.$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
3/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arct
an(a*x)/(a^5*c^3))*arctan(a*x)^2 - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)^3/(a^8*
c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/256*((51*a^3*x^3 - 40*(a^4*x^4 + 2*
a^2*x^2 + 1)*arctan(a*x)^3 + 45*a*x + 51*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(
a*x))*a^2/(a^11*c^3*x^4 + 2*a^9*c^3*x^2 + a^7*c^3) - 24*(5*a^2*x^2 - 5*(a^
4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^10*c^3*x^4 + 2*
a^8*c^3*x^2 + a^6*c^3))*a
```

3.404.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \arctan\left(ax\right)^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.404.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\mathrm{atan}(a\,x)^2\,\left(\frac{9\,x}{32\,a^5\,c^3}+\frac{15\,x^3}{32\,a^3\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4} - \mathrm{atan}(a\,x)^3\,\left(\frac{\frac{1}{4\,a^6\,c^3}+\frac{x^2}{2\,a^4\,c^3}}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{5}{32\,a^4\,c^3}\right) - \frac{\frac{51\,a^2\,x^3}{8}+\frac{45\,x}{8}}{32\,a^7\,c^3\,x^4+64\,a^5\,c^3\,x^2+32\,a^3\,c^3} - \frac{51\,\mathrm{atan}(a\,x)}{256\,a^4\,c^3}+\frac{\mathrm{atan}(a\,x)\,\left(\frac{3}{8\,a^6\,c^3}+\frac{15\,x^2}{32\,a^4\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

output
```
(atan(a*x)^2*((9*x)/(32*a^5*c^3) + (15*x^3)/(32*a^3*c^3)))/(1/a^2 + 2*x^2
+ a^2*x^4) - atan(a*x)^3*((1/(4*a^6*c^3) + x^2/(2*a^4*c^3))/(1/a^2 + 2*x^2
 + a^2*x^4) - 5/(32*a^4*c^3)) - ((45*x)/8 + (51*a^2*x^3)/8)/(32*a^3*c^3 +
64*a^5*c^3*x^2 + 32*a^7*c^3*x^4) - (51*atan(a*x))/(256*a^4*c^3) + (atan(a*
x)*(3/(8*a^6*c^3) + (15*x^2)/(32*a^4*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
```

3.405 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

3.405.1 Optimal result

Integrand size = 22, antiderivative size = 237

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx &= \frac{3}{128a^3c^3(1+a^2x^2)^2}-\frac{3}{128a^3c^3(1+a^2x^2)}\\ &\quad+\frac{3x\arctan(ax)}{32a^2c^3(1+a^2x^2)^2}-\frac{3x\arctan(ax)}{64a^2c^3(1+a^2x^2)}\\ &\quad-\frac{3\arctan(ax)^2}{128a^3c^3}-\frac{3\arctan(ax)^2}{16a^3c^3(1+a^2x^2)^2}+\frac{3\arctan(ax)^2}{16a^3c^3(1+a^2x^2)}\\ &\quad-\frac{x\arctan(ax)^3}{4a^2c^3(1+a^2x^2)^2}+\frac{x\arctan(ax)^3}{8a^2c^3(1+a^2x^2)}+\frac{\arctan(ax)^4}{32a^3c^3}\end{aligned}$$

output
```
3/128/a^3/c^3/(a^2*x^2+1)^2-3/128/a^3/c^3/(a^2*x^2+1)+3/32*x*arctan(a*x)/a
^2/c^3/(a^2*x^2+1)^2-3/64*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)-3/128*arctan(a
*x)^2/a^3/c^3-3/16*arctan(a*x)^2/a^3/c^3/(a^2*x^2+1)^2+3/16*arctan(a*x)^2/
a^3/c^3/(a^2*x^2+1)-1/4*x*arctan(a*x)^3/a^2/c^3/(a^2*x^2+1)^2+1/8*x*arctan
(a*x)^3/a^2/c^3/(a^2*x^2+1)+1/32*arctan(a*x)^4/a^3/c^3
```

3.405.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{-3a^2x^2 + (6ax - 6a^3x^3)\arctan(ax) - 3(1-6a^2x^2+a^4x^4)\arctan(ax)^2 + 16ax(-1+a^2x^2)\arctan(ax)^3 +}{128a^3c^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(-3*a^2*x^2 + (6*a*x - 6*a^3*x^3)*ArcTan[a*x] - 3*(1 - 6*a^2*x^2 + a^4*x^4
)*ArcTan[a*x]^2 + 16*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^3 + 4*(1 + a^2*x^2)^2*
ArcTan[a*x]^4)/(128*a^3*c^3*(1 + a^2*x^2)^2)
```

3.405.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.73, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5499, 27, 5427, 5435, 5427, 5431, 5427, 241, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \ 5499$$
$$\frac{\int \frac{\arctan(ax)^3}{c^2(a^2x^2+1)^2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^3}{c^3(a^2x^2+1)^3}dx}{a^2}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^3} - \frac{\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx}{a^2c^3}$$
$$\Big\downarrow \ 5427$$
$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}}{a^2c^3} - \frac{\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx}{a^2c^3}$$
$$\Big\downarrow \ 5435$$

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}}{a^2c^3}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}}{a^2c^3}$$

$$\Big\downarrow \text{ 5431}$$

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan}{4(a^2x^2}}{a^2c^3}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x}{2}\right.}{a^2c^3}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}\right.\right.}{a^2c^3}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}\right.}{a^2c^3}$$

$$\Big\downarrow \text{ 5427}$$

3.405. $\int \frac{x^2\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

$$\frac{-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-$$

$$\frac{\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arc}{16a(}}{a^2c^3}$$

$$\Big\downarrow 241$$

$$\frac{\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-$$

$$\frac{\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\Bigg(\frac{x\arctan}{2(a^2x}}{a^2c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*Ar
cTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])
/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2)/(a^2*c^3) - ((3*ArcTan[a*
x]^2)/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(4*(1 + a^2*x^2)^2) - (3*
(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4
*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a
)))/4))/8 + (3*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a)
- (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) +
(x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2))/4)/(a^2*c
^3)
```

3.405.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.405. $\int \frac{x^2\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.405. $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

3.405.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{4a^4x^4\arctan(ax)^4-3a^4\arctan(ax)^2x^4+16\arctan(ax)^3a^3x^3+8\arctan(ax)^4x^2a^2-6\arctan(ax)x^3a^3+18x^2\arctan(ax)^2}{128c^3(a^2x^2+1)^2a^3}$
derivativedivides	$\frac{\frac{\arctan(ax)^3a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{\arctan(ax)^3ax}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^4}{8c^3}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{\arctan(ax)ax}{8(a^2x^2+1)^2}\right.}{8c^3}}{a^3}$
default	$\frac{\frac{\arctan(ax)^3a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{\arctan(ax)^3ax}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^4}{8c^3}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{\arctan(ax)ax}{8(a^2x^2+1)^2}\right.}{8c^3}}{a^3}$
parts	$\frac{\arctan(ax)^3x^3}{8c^3(a^2x^2+1)^2}-\frac{x\arctan(ax)^3}{8a^2c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^4}{8a^3c^3}-\frac{3\left(\frac{\arctan(ax)^4}{4a^3}+\frac{\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-}{}\right.}{8c^3}$
risch	$\frac{\ln(iax+1)^4}{512a^3c^3}-\frac{(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)^3}{128a^3c^3(a^2x^2+1)^2}+\frac{3\left(2a^4x^4\ln(-iax-\right.}{}$

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128*(4*a^4*x^4*arctan(a*x)^4-3*a^4*arctan(a*x)^2*x^4+16*arctan(a*x)^3*a^
3*x^3+8*arctan(a*x)^4*x^2*a^2-6*arctan(a*x)*x^3*a^3+18*x^2*arctan(a*x)^2*a
^2-16*arctan(a*x)^3*a*x-3*a^2*x^2+4*arctan(a*x)^4+6*x*arctan(a*x)*a-3*arct
an(a*x)^2)/c^3/(a^2*x^2+1)^2/a^3
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

$$\int \frac{x^2\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$$
$$=\frac{4\,(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}^4-3\,a^2x^2+16\,(a^3x^3-ax)\arctan{(ax)}^3-3\,(a^4x^4-6\,a^2x^2+1)\arctan{(}}{128\,(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3)}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/128*(4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 3*a^2*x^2 + 16*(a^3*x^3
 - a*x)*arctan(a*x)^3 - 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x)^2 - 6*(a^3
*x^3 - a*x)*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

3.405.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx &= \frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan(ax)}{a^3c^3}\right)\arctan(ax)^3 \\ &\quad+\frac{3\left(a^2x^2-(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2\right)a\arctan(ax)^2}{16\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)} \\ &\quad-\frac{1}{128}\left(\frac{\left(4\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^4+3\,a^2x^2-3\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2\right)a^2}{a^{10}c^3x^4+2\,a^8c^3x^2+a^6c^3}+\frac{2\,(3\,a^3x^3\right.\end{aligned}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(
a^3*c^3))*arctan(a*x)^3 + 3/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan
(a*x)^2)*a*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/128*(
(4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 3*a^2*x^2 - 3*(a^4*x^4 + 2*a^
2*x^2 + 1)*arctan(a*x)^2)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) + 2
*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x + 3*(a^4*x
^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^9*c^3*x^4 + 2*a^7*c^3*x^
2 + a^5*c^3))*a
```

3.405.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{(a^2cx^2+c)^3} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.405.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx = \frac{\mathrm{atan}(a\,x)\,\left(\frac{3\,x}{64\,a^4\,c^3}-\frac{3\,x^3}{64\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{3\,x^2}{2\,\left(64\,a^5\,c^3\,x^4+128\,a^3\,c^3\,x^2+64\,a\,c^3\right)}-\mathrm{atan}(a\,x)^2\left(\frac{3}{128\,a^3\,c^3}-\frac{3\,x^2}{16\,a^3\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}\right)-\frac{\mathrm{atan}(a\,x)^3\,\left(\frac{x}{8\,a^4\,c^3}-\frac{x^3}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{\mathrm{atan}(a\,x)^4}{32\,a^3\,c^3}$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

output
```
(atan(a*x)*((3*x)/(64*a^4*c^3) - (3*x^3)/(64*a^2*c^3)))/(1/a^2 + 2*x^2 + a
^2*x^4) - (3*x^2)/(2*(64*a*c^3 + 128*a^3*c^3*x^2 + 64*a^5*c^3*x^4)) - atan
(a*x)^2*(3/(128*a^3*c^3) - (3*x^2)/(16*a^3*c^3*(1/a^2 + 2*x^2 + a^2*x^4)))
 - (atan(a*x)^3*(x/(8*a^4*c^3) - x^3/(8*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^
4) + atan(a*x)^4/(32*a^3*c^3)
```

3.406 $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

3.406.1 Optimal result

Integrand size = 20, antiderivative size = 208

$$\begin{aligned}\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = & -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256ac^3(1+a^2x^2)} - \frac{45\arctan(ax)}{256a^2c^3} \\ & + \frac{3\arctan(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9\arctan(ax)}{32a^2c^3(1+a^2x^2)} + \frac{3x\arctan(ax)^2}{16ac^3(1+a^2x^2)^2} \\ & + \frac{9x\arctan(ax)^2}{32ac^3(1+a^2x^2)} + \frac{3\arctan(ax)^3}{32a^2c^3} - \frac{\arctan(ax)^3}{4a^2c^3(1+a^2x^2)^2}\end{aligned}$$

output
```
-3/128*x/a/c^3/(a^2*x^2+1)^2-45/256*x/a/c^3/(a^2*x^2+1)-45/256*arctan(a*x)
/a^2/c^3+3/32*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)/a^2/c^3/(
a^2*x^2+1)+3/16*x*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/32*x*arctan(a*x)^2/a
/c^3/(a^2*x^2+1)+3/32*arctan(a*x)^3/a^2/c^3-1/4*arctan(a*x)^3/a^2/c^3/(a^2
*x^2+1)^2
```

3.406.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.50

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{-3ax(17+15a^2x^2) - 3(-17+6a^2x^2+15a^4x^4)\arctan(ax) + 24ax(5+3a^2x^2)\arctan(ax)^2 + 8(-5+6a}{256c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(-3*a*x*(17 + 15*a^2*x^2) - 3*(-17 + 6*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] +
 24*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*Arc
Tan[a*x]^3)/(256*c^3*(a + a^3*x^2)^2)
```

3.406.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5465, 27, 5435, 215, 215, 216, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^3}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\int \frac{\arctan(ax)^2}{c^3\left(a^2x^2+1\right)^3}dx}{4a}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^3}dx}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5435}$$

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx-\frac{1}{8}\int \frac{1}{\left(a^2x^2+1\right)^3}dx+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{\left(a^2x^2+1\right)^2}dx-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow 216$$

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow 5427$$

$$\frac{3\Big(\frac{3}{4}\Big(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\Big)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\Big(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\Big)\cdots}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow 5465$$

$$\frac{3\Bigg(\frac{3}{4}\Bigg(-a\Bigg(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\Bigg)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\Bigg)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\Big(\frac{x}{2(a^2x^2\cdots}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow 215$$

$$\frac{3\Bigg(\frac{3}{4}\Bigg(-a\Bigg(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\Bigg)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\Bigg)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\cdots}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow 216$$

$$\frac{3\Bigg(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\Big(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\Big)-\frac{x}{4(a^2x^2+1)^2}\Big)+\frac{3}{4}\Bigg(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\Bigg(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\cdots}{2a}\cdots}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

3.406. $\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}dx$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^3/(a^2*c^3*(1 + a^2*x^2)^2) + (3*(ArcTan[a*x]/(8*a*(1 + a
^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*(1 + a^2*x^2)^2) + (-1/4*x/(1 + a^2*x^2)
^2 - (3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/8 + (3*((x*ArcTan[a*
x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(
1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/4))/(4*
a*c^3)
```

3.406.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.406.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{24a^4\arctan(ax)^3x^4-45\arctan(ax)a^4x^4+72a^3\arctan(ax)^2x^3+48\arctan(ax)^3x^2a^2-45a^3x^3-18a^2\arctan(ax)x^2+120a}{256c^3(a^2x^2+1)^2a^2}$
derivativedivides	$\frac{-\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9a\arctan(ax)^2x}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3\left(\frac{15}{8}a^3x^3+\frac{17}{8}ax\right)}{32(a^2x^2+1)^2}-45\,\mathrm{arc}}{c^3}}{a^2}$
default	$\frac{-\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9a\arctan(ax)^2x}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3\left(\frac{15}{8}a^3x^3+\frac{17}{8}ax\right)}{32(a^2x^2+1)^2}-45\,\mathrm{arc}}{c^3}}{a^2}$
parts	$-\frac{\arctan(ax)^3}{4a^2c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9a\arctan(ax)^2x}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3\left(\frac{15}{8}a^3x^3+\frac{17}{8}ax\right)}{32(a^2x^2+1)^2}}{a^2c^3}$
risch	$\frac{i(3a^4x^4+6a^2x^2-5)\ln(iax+1)^3}{256a^2c^3(a^2x^2+1)^2}-\frac{3i(-5\ln(-iax+1)+3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3-10iax)\ln(iax}{256(ax+i)^2c^3(ax-i)^2a^2}$

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/256*(24*a^4*arctan(a*x)^3*x^4-45*arctan(a*x)*a^4*x^4+72*a^3*arctan(a*x)^
2*x^3+48*arctan(a*x)^3*x^2*a^2-45*a^3*x^3-18*a^2*arctan(a*x)*x^2+120*a*arc
tan(a*x)^2*x-40*arctan(a*x)^3-51*a*x+51*arctan(a*x))/c^3/(a^2*x^2+1)^2/a^2
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{45\,a^3x^3 - 8\,(3\,a^4x^4 + 6\,a^2x^2 - 5)\arctan(ax)^3 - 24\,(3\,a^3x^3 + 5\,ax)\arctan(ax)^2 + 51\,ax + 3\,(15\,a^4x^4 +}{256\,(a^6c^3x^4 + 2\,a^4c^3x^2 + a^2c^3)}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/256*(45*a^3*x^3 - 8*(3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^3 - 24*(3*a
^3*x^3 + 5*a*x)*arctan(a*x)^2 + 51*a*x + 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*a
rctan(a*x))/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

3.406.6 Sympy [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x\,\mathrm{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.31

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{3\left(\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2} + \frac{3\,\arctan(ax)}{ac^2}\right)\arctan(ax)^2}{32\,ac} - \frac{3\left(\frac{\left(15\,a^3x^3-8\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^3+17\,ax+15\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)\right)a^2}{a^7c^2x^4+2\,a^5c^2x^2+a^3c^2} - \frac{8\left(3\,a^2x^2-3\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2\right.}{a^6c^2x^4+2\,a^4c^2x^2+a^2c^2}\right.}{256\,ac} - \frac{\arctan(ax)^3}{4\,(a^2cx^2+c)^2a^2c}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
3/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x
)/(a*c^2))*arctan(a*x)^2/(a*c) - 3/256*((15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x
^2 + 1)*arctan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))
*a^2/(a^7*c^2*x^4 + 2*a^5*c^2*x^2 + a^3*c^2) - 8*(3*a^2*x^2 - 3*(a^4*x^4 +
 2*a^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^2*x^4 + 2*a^4*c^2*
x^2 + a^2*c^2))/(a*c) - 1/4*arctan(a*x)^3/((a^2*c*x^2 + c)^2*a^2*c)
```

3.406.8 Giac [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{x\arctan{(ax)}^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.406.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.91

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \mathrm{atan}(a\,x)^3\left(\frac{3}{32\,a^2\,c^3}-\frac{1}{4\,a^4\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}\right) - \frac{\frac{45\,a^2\,x^3}{8}+\frac{51\,x}{8}}{32\,a^5\,c^3\,x^4+64\,a^3\,c^3\,x^2+32\,a\,c^3} + \frac{\mathrm{atan}(a\,x)^2\,\left(\frac{15\,x}{32\,a^3\,c^3}+\frac{9\,x^3}{32\,a\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4} - \frac{45\,\mathrm{atan}(a\,x)}{256\,a^2\,c^3} + \frac{\mathrm{atan}(a\,x)\,\left(\frac{3}{8\,a^4\,c^3}+\frac{9\,x^2}{32\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^3*(3/(32*a^2*c^3) - 1/(4*a^4*c^3*(1/a^2 + 2*x^2 + a^2*x^4))) - (
(51*x)/8 + (45*a^2*x^3)/8)/(32*a*c^3 + 64*a^3*c^3*x^2 + 32*a^5*c^3*x^4) +
(atan(a*x)^2*((15*x)/(32*a^3*c^3) + (9*x^3)/(32*a*c^3)))/(1/a^2 + 2*x^2 +
a^2*x^4) - (45*atan(a*x))/(256*a^2*c^3) + (atan(a*x)*(3/(8*a^4*c^3) + (9*x
^2)/(32*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
```

3.406. $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3} dx$

3.407 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

3.407.1 Optimal result

Integrand size = 19, antiderivative size = 225

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{3}{128ac^3(1+a^2x^2)^2}-\frac{45}{128ac^3(1+a^2x^2)}-\frac{3x\arctan(ax)}{32c^3(1+a^2x^2)^2} -\frac{45x\arctan(ax)}{64c^3(1+a^2x^2)}-\frac{45\arctan(ax)^2}{128ac^3}+\frac{3\arctan(ax)^2}{16ac^3(1+a^2x^2)^2} +\frac{9\arctan(ax)^2}{16ac^3(1+a^2x^2)}+\frac{x\arctan(ax)^3}{4c^3(1+a^2x^2)^2}+\frac{3x\arctan(ax)^3}{8c^3(1+a^2x^2)}+\frac{3\arctan(ax)^4}{32ac^3}$$

output
```
-3/128/a/c^3/(a^2*x^2+1)^2-45/128/a/c^3/(a^2*x^2+1)-3/32*x*arctan(a*x)/c^3
/(a^2*x^2+1)^2-45/64*x*arctan(a*x)/c^3/(a^2*x^2+1)-45/128*arctan(a*x)^2/a/
c^3+3/16*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/16*arctan(a*x)^2/a/c^3/(a^2*x
^2+1)+1/4*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^3/c^3/(a^2*x
^2+1)+3/32*arctan(a*x)^4/a/c^3
```

3.407.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.51

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{48+45a^2x^2+6ax(17+15a^2x^2)\arctan(ax)+3(-17+6a^2x^2+15a^4x^4)\arctan(ax)^2-16ax(5+3a^2x}{128ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```

output
```
-1/128*(48 + 45*a^2*x^2 + 6*a*x*(17 + 15*a^2*x^2)*ArcTan[a*x] + 3*(-17 + 6
*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x]^2 - 16*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^
3 - 12*(1 + a^2*x^2)^2*ArcTan[a*x]^4)/(a*c^3*(1 + a^2*x^2)^2)
```

3.407.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5435, 27, 5427, 5431, 5427, 241, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{3}{8}\int \frac{\arctan(ax)}{c^3\left(a^2x^2+1\right)^3}dx+\frac{3\int \frac{\arctan(ax)^3}{c^2\left(a^2x^2+1\right)^2}dx}{4c}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^3}dx}{8c^3}+\frac{3\int \frac{\arctan(ax)^3}{\left(a^2x^2+1\right)^2}dx}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^3}dx}{8c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^3}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5431}$$

$$-\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{1}{16a\left(a^2x^2+1\right)^2}\right)}{8c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^3}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16ac^3(a^2x^2+1)^2}-\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16ac^3(a^2x^2+1)^2}-\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16ac^3(a^2x^2+1)^2}-\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16ac^3(a^2x^2+1)^2}-\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}+\frac{3\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```

3.407. $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

output
```
(3*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(4*c^3*(1
 + a^2*x^2)^2) - (3*(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^
2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2))
+ ArcTan[a*x]^2/(4*a)))/4))/(8*c^3) + (3*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^
2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) +
 (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^
2/(4*a))/a))/2))/(4*c^3)
```

3.407.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

3.407. $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3} dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.407.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{12a^4x^4\arctan(ax)^4-45a^4\arctan(ax)^2x^4+48\arctan(ax)^3a^3x^3+48a^4x^4+24\arctan(ax)^4x^2a^2-90\arctan(ax)x^3a^3-18a}{128c^3(a^2x^2+1)^2a}$
derivativedivides	$\frac{\frac{\arctan(ax)^3ax}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^3ax}{8c^3\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^4}{8c^3}-\frac{3\left(-\frac{3\arctan(ax)^2}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{2\left(a^2x^2+1\right)^2}+\frac{15\arctan(ax)a^3x^3}{8\left(a^2x^2+1\right)^2}+\frac{17\arctan(ax)ax}{8\left(a^2x^2+1\right)^2}+15a\right.}{8c^3}}{a}$
default	$\frac{\frac{\arctan(ax)^3ax}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^3ax}{8c^3\left(a^2x^2+1\right)}+\frac{3\arctan(ax)^4}{8c^3}-\frac{3\left(-\frac{3\arctan(ax)^2}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{2\left(a^2x^2+1\right)^2}+\frac{15\arctan(ax)a^3x^3}{8\left(a^2x^2+1\right)^2}+\frac{17\arctan(ax)ax}{8\left(a^2x^2+1\right)^2}+15a\right.}{8c^3}}{a}$
parts	$\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3x\arctan(ax)^3}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^4}{8a\,c^3}-\frac{3\left(\frac{-\frac{3\arctan(ax)^2}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{2\left(a^2x^2+1\right)^2}+\frac{15\arctan(ax)a^3x^3}{8\left(a^2x^2+1\right)^2}+\frac{17\arctan(}{8\left(a^2x^2+\right.}}{a}\right.}{}$
risch	$\frac{3\ln(iax+1)^4}{512c^3a}-\frac{(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax)\ln(iax+1)^3}{128c^3(a^2x^2+1)^2a}+\frac{3\left(6a^4x^4\ln(\right.}{}$

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128*(12*a^4*x^4*arctan(a*x)^4-45*a^4*arctan(a*x)^2*x^4+48*arctan(a*x)^3*
a^3*x^3+48*a^4*x^4+24*arctan(a*x)^4*x^2*a^2-90*arctan(a*x)*x^3*a^3-18*x^2*
arctan(a*x)^2*a^2+80*arctan(a*x)^3*a*x+51*a^2*x^2+12*arctan(a*x)^4-102*x*a
rctan(a*x)*a+51*arctan(a*x)^2)/c^3/(a^2*x^2+1)^2/a
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{12\,(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^4-45\,a^2x^2+16\,(3\,a^3x^3+5\,ax)\arctan\,(ax)^3-3\,(15\,a^4x^4+6\,a^2x^2-17}{128\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/128*(12*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 45*a^2*x^2 + 16*(3*a^3
*x^3 + 5*a*x)*arctan(a*x)^3 - 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(a*x)^
2 - 6*(15*a^3*x^3 + 17*a*x)*arctan(a*x) - 48)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2
 + a*c^3)
```

3.407.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.49

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan\,(ax)}{ac^3}\right)\arctan\,(ax)^3$$
$$+\frac{3\left(3\,a^2x^2-3\,(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^2+4\right)a\arctan\,(ax)^2}{16\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$
$$-\frac{3}{128}\left(\frac{\left(4\,(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^4+15\,a^2x^2-15\,(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^2+16\right)a^2}{a^8c^3x^4+2\,a^6c^3x^2+a^4c^3}+\frac{2}{}\right.$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)
/(a*c^3))*arctan(a*x)^3 + 3/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*ar
ctan(a*x)^2 + 4)*a*arctan(a*x)^2/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) -
 3/128*((4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 15*a^2*x^2 - 15*(a^4*
x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 16)*a^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2
+ a^4*c^3) + 2*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 + 1
7*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^3*x
^4 + 2*a^5*c^3*x^2 + a^3*c^3))*a
```

3.407.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx=\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.407.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx=\operatorname{atan}(a\,x)^2\left(\frac{\frac{3}{4\,a^3\,c^3}+\frac{9\,x^2}{16\,a\,c^3}}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{45}{128\,a\,c^3}\right)-\frac{\frac{45\,a\,x^2}{2}+\frac{24}{a}}{64\,a^4\,c^3\,x^4+128\,a^2\,c^3\,x^2+64\,c^3}-\frac{\operatorname{atan}(a\,x)\,\left(\frac{45\,x^3}{64\,c^3}+\frac{51\,x}{64\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{\operatorname{atan}(a\,x)^3\,\left(\frac{3\,x^3}{8\,c^3}+\frac{5\,x}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{3\,\operatorname{atan}(a\,x)^4}{32\,a\,c^3}$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^2*((3/(4*a^3*c^3) + (9*x^2)/(16*a*c^3))/(1/a^2 + 2*x^2 + a^2*x^4
) - 45/(128*a*c^3)) - ((45*a*x^2)/2 + 24/a)/(64*c^3 + 128*a^2*c^3*x^2 + 64
*a^4*c^3*x^4) - (atan(a*x)*((45*x^3)/(64*c^3) + (51*x)/(64*a^2*c^3)))/(1/a
^2 + 2*x^2 + a^2*x^4) + (atan(a*x)^3*((3*x^3)/(8*c^3) + (5*x)/(8*a^2*c^3))
)/(1/a^2 + 2*x^2 + a^2*x^4) + (3*atan(a*x)^4)/(32*a*c^3)
```

3.407. $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$

3.408 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$

3.408.1 Optimal result

Integrand size = 22, antiderivative size = 332

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx = {} & \frac{3ax}{128c^3(1+a^2x^2)^2}+\frac{141ax}{256c^3(1+a^2x^2)}+\frac{141\arctan(ax)}{256c^3}\\ & -\frac{3\arctan(ax)}{32c^3(1+a^2x^2)^2}-\frac{33\arctan(ax)}{32c^3(1+a^2x^2)}-\frac{3ax\arctan(ax)^2}{16c^3(1+a^2x^2)^2}\\ & -\frac{33ax\arctan(ax)^2}{32c^3(1+a^2x^2)}-\frac{11\arctan(ax)^3}{32c^3}+\frac{\arctan(ax)^3}{4c^3(1+a^2x^2)^2}\\ & +\frac{\arctan(ax)^3}{2c^3(1+a^2x^2)}-\frac{i\arctan(ax)^4}{4c^3}+\frac{\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\ & -\frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}\\ & +\frac{3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{3i\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}\end{aligned}$$

output
```
3/128*a*x/c^3/(a^2*x^2+1)^2+141/256*a*x/c^3/(a^2*x^2+1)+141/256*arctan(a*x
)/c^3-3/32*arctan(a*x)/c^3/(a^2*x^2+1)^2-33/32*arctan(a*x)/c^3/(a^2*x^2+1)
-3/16*a*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-33/32*a*x*arctan(a*x)^2/c^3/(a^2
*x^2+1)-11/32*arctan(a*x)^3/c^3+1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+1/2*ar
ctan(a*x)^3/c^3/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c^3+arctan(a*x)^3*ln(2-2/(
1-I*a*x))/c^3-3/2*I*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^3+3/2*arctan
(a*x)*polylog(3,-1+2/(1-I*a*x))/c^3+3/4*I*polylog(4,-1+2/(1-I*a*x))/c^3
```

3.408.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$$
$$= \frac{-16i\pi^4 + 256i\arctan(ax)^4 - 576\arctan(ax)\cos(2\arctan(ax)) + 384\arctan(ax)^3\cos(2\arctan(ax)) - 12}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```

output
```
((-16*I)*Pi^4 + (256*I)*ArcTan[a*x]^4 - 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]]
 + 384*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Cos[4*ArcTan[a*x]
] + 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + 1024*ArcTan[a*x]^3*Log[1 - E^((-
2*I)*ArcTan[a*x])] + (1536*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*
x])] + 1536*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (768*I)*PolyL
og[4, E^((-2*I)*ArcTan[a*x])] + 288*Sin[2*ArcTan[a*x]] - 576*ArcTan[a*x]^2
*Sin[2*ArcTan[a*x]] + 3*Sin[4*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[4*ArcTan
[a*x]])/(1024*c^3)
```

3.408.3 Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.55, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5501, 27, 5465, 5435, 215, 215, 216, 5427, 5465, 215, 216, 5501, 5459, 5403, 5465, 5427, 5465, 215, 216, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \text{ 5501}$$
$$\frac{\int \frac{\arctan(ax)^3}{c^2x(a^2x^2+1)^2}dx}{c} - a^2\int \frac{x\arctan(ax)^3}{c^3(a^2x^2+1)^3}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^3}dx}{4a}-\frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx-\frac{1}{8}\int \frac{1}{\left(a^2x^2+1\right)^3}dx+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a}-\frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{\left(a^2x^2+1\right)^2}dx-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a}-\frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a}-\frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)\right)}{4a}-\frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

3.408. $\displaystyle\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)\right)}{4a}\right.}{c^3}$$

$$\Big\downarrow \; 5465$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right.\right.}{4a}\right.}{c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\right.\right.}{4a}\right.}{c^3}$$

$$\Big\downarrow \; 216$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(}{2a^2\left(a^2x\right.}\right.\right.}{4a}\right.}{c^3}$$

$$\Big\downarrow \; 5501$$

3.408. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx - a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(}{2a^2(a^2x}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}}{2a^2(a}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx + i\int \frac{\arctan(ax)^3}{x(ax+i)}dx - \frac{1}{4}i\arctan(ax)^4}{c^3}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}}{2a^2(a}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx + i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}}{2a^2(a}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{2a} - \frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right) + i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\,\text{ar}}{c^3}$$

3.408. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$

$$\Big\downarrow \text{5427}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\cdots\right)\right)\cdots}{4a}\cdots\right)}{c^3}$$

$$-\frac{a^2\left(\frac{3\left(-a\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\cdots\right)}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\cdots\right)\right)\cdots}{4a}\cdots\right)}{c^3}$$

$$-\frac{a^2\left(\frac{3\left(-a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx\cdots\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\cdots\right)\right)\cdots}{4a}\cdots\right)}{c^3}$$

$$-\frac{a^2\left(\frac{3\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\cdots\right)}{a^2x^2+1}\cdots\right)}{c^3}$$

$$\Big\downarrow \text{216}$$

3.408. $\int\frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}\cdots}{2a^2\left(a\cdots\right.}\right.\right.\right.}{4a}\cdots}{c^3}\cdots$$

$$\frac{i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}\cdots}{2a^2\cdots}\right.\right.}{2a}\cdots\right.}{c^3}\cdots$$

$$\Big\downarrow \; 5527$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}\cdots}{2a^2\left(a\cdots\right.}\right.\right.\right.}{4a}\cdots}{c^3}\cdots$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\cdots}{c^3}\cdots$$

$$\Big\downarrow \; 5531$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}\cdots}{2a^2\left(a\cdots\right.}\right.\right.\right.}{4a}\cdots}{c^3}\cdots$$

$$i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(\cdots\right.$$

$$\Big\downarrow \; 7164$$

3.408. $\displaystyle\int\frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}}{2a^2\left(a\right.}\right.\right.\right.}{4a}\right.}{c^3}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\right.}{2a}\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-1/4*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2)^2) + (3*(ArcTan[a*x]/(8*a*(1
 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*(1 + a^2*x^2)^2) + (-1/4*x/(1 + a^2*
x^2)^2 - (3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/8 + (3*((x*ArcTa
n[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a
^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/4))
/(4*a)))/c^3) + ((-1/4*I)*ArcTan[a*x]^4 - a^2*(-1/2*ArcTan[a*x]^3/(a^2*(1
+ a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a
) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTa
n[a*x]/(2*a))/(2*a))))/(2*a)) + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x
)] + (3*I)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(
((-1/2*I)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 +
2/(1 - I*a*x)]/(4*a)))))/c^3
```

3.408.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.408. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.408.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 93.26 (sec) , antiderivative size = 1841, normalized size of antiderivative = 5.55

method	result	size
derivativedivides	Expression too large to display	1841
default	Expression too large to display	1841
parts	Expression too large to display	2278

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*arctan(a*x)^3*ln(a*x)+1/2*arctan(a*x)^3/c^3/(a^2*x^2+1)-1/2/c^3*arct
an(a*x)^3*ln(a^2*x^2+1)+1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2-3/4/c^3*(-4/3*
arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*(I+a*x)/(16*a*x-16*I)-3/
8*arctan(a*x)*(a*x-I)/(I+a*x)-3*I*(a*x-I)/(16*a*x+16*I)+4*I*arctan(a*x)^2*
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/3*I*arctan(a*x)^4-3/8*arctan(a*x)
*(I+a*x)/(a*x-I)+3*I*arctan(a*x)^2*(a*x-I)/(8*a*x+8*I)-8*I*polylog(4,(1+I*
a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*(I+a*x)/(8*a*x-8*I)+4/3*arctan(a
*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-4/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^
2+1)^(1/2)+1)-8*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-8*I*po
lylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4/3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^
2*x^2+1)^(1/2))+4*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
8*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/24*(-8*I*Pi*csgn(I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+16*I*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+16*I*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-8*I*Pi*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1))^3+16*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))^2+16*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^3+8*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-16*I*Pi*...
```

3.408.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*
x), x)
```

3.408.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

3.408.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x), x)
```

3.408.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\mathrm{atan}(a\,x)^3}{x\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^3), x)
```

3.409 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

3.409.1 Optimal result

Integrand size = 22, antiderivative size = 332

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx =& \frac{3a}{128c^3(1+a^2x^2)^2}+\frac{93a}{128c^3(1+a^2x^2)}+\frac{3a^2x\arctan(ax)}{32c^3(1+a^2x^2)^2}\\ &+\frac{93a^2x\arctan(ax)}{64c^3(1+a^2x^2)}+\frac{93a\arctan(ax)^2}{128c^3}\\ &-\frac{3a\arctan(ax)^2}{16c^3(1+a^2x^2)^2}-\frac{21a\arctan(ax)^2}{16c^3(1+a^2x^2)}-\frac{ia\arctan(ax)^3}{c^3}\\ &-\frac{\arctan(ax)^3}{c^3x}-\frac{a^2x\arctan(ax)^3}{4c^3(1+a^2x^2)^2}-\frac{7a^2x\arctan(ax)^3}{8c^3(1+a^2x^2)}\\ &-\frac{15a\arctan(ax)^4}{32c^3}+\frac{3a\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\ &-\frac{3ia\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}\\ &+\frac{3a\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
3/128*a/c^3/(a^2*x^2+1)^2+93/128*a/c^3/(a^2*x^2+1)+3/32*a^2*x*arctan(a*x)/
c^3/(a^2*x^2+1)^2+93/64*a^2*x*arctan(a*x)/c^3/(a^2*x^2+1)+93/128*a*arctan(
a*x)^2/c^3-3/16*a*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-21/16*a*arctan(a*x)^2/c^
3/(a^2*x^2+1)-I*a*arctan(a*x)^3/c^3-arctan(a*x)^3/c^3/x-1/4*a^2*x*arctan(a
*x)^3/c^3/(a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)^3/c^3/(a^2*x^2+1)-15/32*a*ar
ctan(a*x)^4/c^3+3*a*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3-3*I*a*arctan(a*x)*
polylog(2,-1+2/(1-I*a*x))/c^3+3/2*a*polylog(3,-1+2/(1-I*a*x))/c^3
```

3.409.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx$$
$$= \frac{a\left(-\frac{i\pi^3}{8}+i\arctan(ax)^3-\frac{\arctan(ax)^3}{ax}-\frac{ax\arctan(ax)^3}{1+a^2x^2}-\frac{15}{32}\arctan(ax)^4+\frac{3}{8}\cos(2\arctan(ax))-\frac{3}{4}\arctan(ax)\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
(a*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - (a*x*ArcTan[a*
x]^3)/(1 + a^2*x^2) - (15*ArcTan[a*x]^4)/32 + (3*Cos[2*ArcTan[a*x]])/8 - (
3*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/4 + (3*Cos[4*ArcTan[a*x]])/1024 - (3*A
rcTan[a*x]^2*Cos[4*ArcTan[a*x]])/128 + 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*A
rcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (3*P
olyLog[3, E^((-2*I)*ArcTan[a*x])])/2 + (3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/
4 + (3*ArcTan[a*x]*Sin[4*ArcTan[a*x]])/256 - (ArcTan[a*x]^3*Sin[4*ArcTan[a
*x]])/32))/c^3
```

3.409.3 Rubi [A] (verified)

Time = 4.12 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.62, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5501, 27, 5435, 5427, 5431, 5427, 241, 5465, 5427, 241, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\left(a^2cx^2+c\right)^3}\,dx$$
$$\Big\downarrow \text{5501}$$
$$\frac{\int \frac{\arctan(ax)^3}{c^2x^2\left(a^2x^2+1\right)^2}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c^3\left(a^2x^2+1\right)^3}dx$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(-\frac{3}{8}\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\,\mathrm{ar}}{4(}\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(-\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right.\right.}{c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(}{2(a^2x^2+}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(}{4(a^2x^2+}\right.\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

3.409. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+}{c^3}$$

$$\Big\downarrow \; 241$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\right.\right.}{c^3}$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\right.\right.}{c^3}$$

$$\Big\downarrow \; 5427$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\right.\right.}{c^3}$$

$$\Big\downarrow \; 5453$$

$$\frac{-\left(a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)\right)-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx+\int \frac{\arctan(ax)^3}{x^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\right.\right.}{c^3}$$

$$\Big\downarrow \; 5361$$

3.409. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx-\frac{\arctan(ax)^3}{x}}{c^3}-$$

$$\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\cdots\right.}{c^3}$$

$$\Big\downarrow \text{5419}$$

$$\frac{-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}}{c^3}-$$

$$\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\cdots\right.}{c^3}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\cdots\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4\cdots}{c^3}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\cdots\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\cdots\right.\right.}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\cdots\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\cdots\right.\right.}{c^3}$$

3.409. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

$$\Big\downarrow \text{5427}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int\frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int\frac{\text{arct}}{}\right.\right.}{c^3}$$

$$\Big\downarrow \text{241}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$\frac{3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\right.}{c^3}$$

$$\Big\downarrow \text{5527}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\text{arcta}\right.$$

$$\Big\downarrow \text{7164}$$

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{Po}}{}\right.\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```

3.409. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

output
```
-((a^2*((3*ArcTan[a*x]^2)/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(4*(1
 + a^2*x^2)^2) - (3*(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^
2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2))
+ ArcTan[a*x]^2/(4*a)))/4))/8 + (3*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) +
ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4
*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a
))/a))/2))/4))/c^3) + (-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 - a^2*((x*
ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan
[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*
(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2) + 3*a*((-1/3*I)*ArcTan[a*x]^3
 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a
*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*
a)))))/c^3
```

3.409.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.409. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.409. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.409.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 82.41 (sec) , antiderivative size = 1799, normalized size of antiderivative = 5.42

method	result	size
derivativedivides	Expression too large to display	1799
default	Expression too large to display	1799
parts	Expression too large to display	1802

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-7/8/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a^3*x^3-9/8/c^3*arctan(a*x)^3/(a^2
*x^2+1)^2*a*x-15/8/c^3*arctan(a*x)^4-1/c^3*arctan(a*x)^3/a/x-3/8/c^3*(-15/
4*arctan(a*x)^4+1/2*arctan(a*x)^2/(a^2*x^2+1)^2+4*arctan(a*x)^2*ln(a^2*x^2
+1)+7/2*arctan(a*x)^2/(a^2*x^2+1)-8*arctan(a*x)^2*ln(a*x)-8*arctan(a*x)^2*
ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+16*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*
x^2+1)^(1/2))+8/3*I*arctan(a*x)^3-I*arctan(a*x)*(a*x-I)/(I+a*x)+1/2*(I+a*x
)/(a*x-I)+1/2*(a*x-I)/(I+a*x)+8*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1
)-8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+16*I*arctan(a*x)*polyl
og(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-16*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))-8*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)*(I+a*x
)/(a*x-I)-16*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/16*(32*I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-32*I*P
i*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-64
*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-32*I
*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+32*I*Pi*csgn
(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)^2+64*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^3-32*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3-64*I*Pi*csgn(I
/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*...
```

3.409.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx=\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*
x^2), x)
```

3.409.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^8+3a^4x^6+3a^2x^4+x^2}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c
**3
```

3.409.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16384*(2400*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^4 - 90*(a^5*x^5 + 2
*a^3*x^3 + a*x)*log(a^2*x^2 + 1)^4 + 256*(15*a^4*x^4 + 25*a^2*x^2 + 8)*arc
tan(a*x)^3 - 48*(15*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^2 + 4*(15*a^4*
x^4 + 25*a^2*x^2 + 8)*arctan(a*x))*log(a^2*x^2 + 1)^2 - (a^4*c^3*x^5 + 2*a
^2*c^3*x^3 + c^3*x)*(360*((8*a^2*x^2 + 7)*a^2/(a^12*c^3*x^4 + 2*a^10*c^3*x
^2 + a^8*c^3) + 2*(4*a^2*x^2 + 3)*log(a^2*x^2 + 1)/(a^10*c^3*x^4 + 2*a^8*c
^3*x^2 + a^6*c^3))*a^7 - 2949120*a^7*integrate(1/1024*x^7*arctan(a*x)^2*lo
g(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x)
 - 737280*a^7*integrate(1/1024*x^7*log(a^2*x^2 + 1)^3/(a^6*c^3*x^8 + 3*a^4
*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 360*(2*a^2*x^2 + 1)*a^5*log(a^2*
x^2 + 1)^3/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5898240*a^6*integrate
(1/1024*x^6*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c
^3*x^2), x) + 1474560*a^6*integrate(1/1024*x^6*arctan(a*x)*log(a^2*x^2 + 1
)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) - 11796480
*a^6*integrate(1/1024*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^
4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 720*(2*a^2*x^2 + 1)*a^5*log(a^2
*x^2 + 1)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 270*(((16*a^2*x^2 +
15)*a^2/(a^14*c^3*x^4 + 2*a^12*c^3*x^2 + a^10*c^3) + 2*(8*a^2*x^2 + 7)*log
(a^2*x^2 + 1)/(a^12*c^3*x^4 + 2*a^10*c^3*x^2 + a^8*c^3))*a^4 + 2*(4*a^2*x^
2 + 3)*a^2*log(a^2*x^2 + 1)^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3))...
```

3.409.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\mathrm{atan}(a\,x)^3}{x^2\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^3), x)
```

3.410 $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

3.410.1 Optimal result

Integrand size = 22, antiderivative size = 478

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx = & -\frac{3a^3x}{128c^3\left(1+a^2x^2\right)^2}-\frac{237a^3x}{256c^3\left(1+a^2x^2\right)}-\frac{237a^2\arctan(ax)}{256c^3} \\ & +\frac{3a^2\arctan(ax)}{32c^3\left(1+a^2x^2\right)^2}+\frac{57a^2\arctan(ax)}{32c^3\left(1+a^2x^2\right)}-\frac{3ia^2\arctan(ax)^2}{2c^3} \\ & -\frac{3a\arctan(ax)^2}{2c^3x}+\frac{3a^3x\arctan(ax)^2}{16c^3\left(1+a^2x^2\right)^2}+\frac{57a^3x\arctan(ax)^2}{32c^3\left(1+a^2x^2\right)} \\ & +\frac{3a^2\arctan(ax)^3}{32c^3}-\frac{\arctan(ax)^3}{2c^3x^2}-\frac{a^2\arctan(ax)^3}{4c^3\left(1+a^2x^2\right)^2}-\frac{a^2\arctan(ax)^3}{c^3\left(1+a^2x^2\right)} \\ & +\frac{3ia^2\arctan(ax)^4}{4c^3}+\frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3} \\ & -\frac{3a^2\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3} \\ & +\frac{9ia^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3} \\ & -\frac{9a^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3} \\ & -\frac{9ia^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}\end{aligned}$$

output
```
-3/128*a^3*x/c^3/(a^2*x^2+1)^2-237/256*a^3*x/c^3/(a^2*x^2+1)-237/256*a^2*a
rctan(a*x)/c^3+3/32*a^2*arctan(a*x)/c^3/(a^2*x^2+1)^2+57/32*a^2*arctan(a*x
)/c^3/(a^2*x^2+1)-3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^3-3/2*a*arctan(a*x
)^2/c^3/x+3/16*a^3*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+57/32*a^3*x*arctan(a*
x)^2/c^3/(a^2*x^2+1)+3/32*a^2*arctan(a*x)^3/c^3-1/2*arctan(a*x)^3/c^3/x^2-
1/4*a^2*arctan(a*x)^3/c^3/(a^2*x^2+1)^2-a^2*arctan(a*x)^3/c^3/(a^2*x^2+1)+
9/2*I*a^2*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^3+3*a^2*arctan(a*x)*ln
(2-2/(1-I*a*x))/c^3-3*a^2*arctan(a*x)^3*ln(2-2/(1-I*a*x))/c^3+3/4*I*a^2*ar
ctan(a*x)^4/c^3-9/4*I*a^2*polylog(4,-1+2/(1-I*a*x))/c^3-9/2*a^2*arctan(a*x
)*polylog(3,-1+2/(1-I*a*x))/c^3-3/2*I*a^2*arctan(a*x)^2/c^3
```

3.410.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)^3}{x^3 (c+a^2cx^2)^3} \, dx$$
$$= \frac{a^2 \left(48i\pi^4 - 1536i \arctan(ax)^2 - \frac{1536 \arctan(ax)^2}{ax} - \frac{512(1+a^2x^2)\arctan(ax)^3}{a^2x^2} - 768i \arctan(ax)^4 + 960 \arctan(ax\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(a^2*((48*I)*Pi^4 - (1536*I)*ArcTan[a*x]^2 - (1536*ArcTan[a*x]^2)/(a*x) -
(512*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (768*I)*ArcTan[a*x]^4 + 960*
ArcTan[a*x]*Cos[2*ArcTan[a*x]] - 640*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 12
*ArcTan[a*x]*Cos[4*ArcTan[a*x]] - 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 30
72*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 3072*ArcTan[a*x]*Log[1
- E^((2*I)*ArcTan[a*x])] - (4608*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*Arc
Tan[a*x])] - (1536*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] - 4608*ArcTan[a*x]
*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + (2304*I)*PolyLog[4, E^((-2*I)*ArcTan
[a*x])] - 480*Sin[2*ArcTan[a*x]] + 960*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] -
3*Sin[4*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/(1024*c^3)
```

3.410.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^3 \left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{c^2x^3(a^2x^2+1)^2}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c^3x\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx+\int \frac{\arctan(ax)^3}{x^3}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \; 5419$$

$$\frac{\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \; 5459$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}+$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\left(i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\,\mathrm{arc}\right.}{c^3}$$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}+$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx+\frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\,\mathrm{arc}\right.$$

$$\Big\downarrow \; 2897$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}+$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)-$$

$$\Big\downarrow \; 5465$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{4a}-\frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)-$$

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

$$\Big\downarrow \text{5435}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx - \frac{1}{8}\int \frac{1}{\left(a^2x^2+1\right)^3}dx + \frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2} + \frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a} - \frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right) -}{}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{\left(a^2x^2+1\right)^2}dx - \frac{x}{4\left(a^2x^2+1\right)^2}\right) + \frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2} + \frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a} - \frac{\arctan(ax}{4a^2(a^2x^2+}\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right) -}{}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2\left(a^2x^2+1\right)}\right) - \frac{x}{4\left(a^2x^2+1\right)^2}\right) + \frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2} + \frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a}\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right) -}{}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx + \frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2} + \frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4\left(a^2x^2+1\right)^2}\right)\right)}{4a} - \frac{}{4}\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right) -}{}$$

$$\Big\downarrow \text{5427}$$

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\right.\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan}{8a\left(a^2x\right.}\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{}{2(}\right.\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)$$

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

$$\Big\downarrow \; 5501$$

$$-\frac{a^2\Big(-a^2\int\frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+\int\frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\Big(a^2\Big(\frac{3\Big(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\Big(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\Big)-\frac{x}{4(a^2x^2+}}{c^3}}{-a^2\Big(\int\frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-a^2\int\frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx\Big)-a^2\Big(i\Big(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\Big(2-\frac{2}{1-iax}}$$

$$\Big\downarrow \; 5459$$

$$\frac{-a^2\Big(-a^2\int\frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\int\frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\Big)-a^2\Big(i\Big(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan}{a^2\Big(-a^2\int\frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\int\frac{\arctan(ax)^3}{x(ax+i)}dx-a^2\Big(\frac{3\Big(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\Big(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\Big)-\frac{x}{4(a^2x^2+1)^2}}{c^3}}$$

$$\Big\downarrow \; 5403$$

$$\frac{-a^2\Big(i\Big(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\Big)-\frac{1}{4}i\arctan(ax)^4\Big)-a^2\Big(-a^2\int\frac{x\arctan(}{(a^2x^2+1}}{a^2\Big(-a^2\int\frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\Big(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\Big)-a^2\Big(\frac{3\Big(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}}{}}$$

$$\Big\downarrow \; 5465$$

3.410. $\displaystyle\int\frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)-a^2\left(-a^2\left(\frac{3\int \frac{\arctan}{(a^2x^2}}{2a}\right.\right.}{a^2\left(-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\right.}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.}{a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{x}{4\left(a^2x^2+1\right)^2}-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)\right)\right)+\frac{3}{4}\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan}{2\left(a^2x^2\right.}\right.}{4a}\right.\right.}$$

$$\Big\downarrow \text{5465}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.}{a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{x}{4\left(a^2x^2+1\right)^2}-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)\right)\right)+\frac{3}{4}\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan}{2\left(a^2x^2\right.}\right.}{4a}\right.\right.}$$

$$\Big\downarrow \text{215}$$

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{x}{4\left(a^2x^2+1\right)^2}-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)\right)\right)+\frac{3}{4}\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan}{2\left(a^2x^2\right.}\right.}{4a}\right.\right.$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)-a^2\left(3\right.\right.$$

$$\Big\downarrow \text{5527}$$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)-a^2\left(3\right.\right.$$

$$\Big\downarrow \text{5531}$$

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)-a^2\left(3\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

3.410.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

3.410. $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx$

3.410.4 Maple [A] (verified)

Time = 95.91 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.21

method	result
derivativedivides	$a^2\left(\frac{3i\arctan(ax)^4}{4c^3}+\frac{5\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{64c^3(ax+i)}+\frac{5\left(-6i\arctan(ax)^2+4\arctan(\right.}{64c^3(}\right.$
default	$a^2\left(\frac{3i\arctan(ax)^4}{4c^3}+\frac{5\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{64c^3(ax+i)}+\frac{5\left(-6i\arctan(ax)^2+4\arctan(\right.}{64c^3(}\right.$

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(3/4*I/c^3*arctan(a*x)^4+5/64*(6*I*arctan(a*x)^2+4*arctan(a*x)^3-3*I-6
*arctan(a*x))*(a*x-I)/c^3/(I+a*x)+5/64*(-6*I*arctan(a*x)^2+4*arctan(a*x)^3
+3*I-6*arctan(a*x))*(I+a*x)/c^3/(a*x-I)-1/2/c^3*arctan(a*x)^2*(-I*arctan(a
*x)-3*I*a*x+x*arctan(a*x)*a)*(I+a*x)/a^2/x^2+9*I/c^3*arctan(a*x)^2*polylog
(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x
^2+1)^(1/2))+9*I/c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-
18/c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c^3*arctan(a
*x)^2-3/c^3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-18*I/c^3*polyl
og(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18/c^3*arctan(a*x)*polylog(3,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))-3*I/c^3*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c^3
*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/c^3*arctan(a*x)*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2)+1)-18*I/c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/c^3*a
rctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/256*arctan(a*x)*(8*arctan(a
*x)^2-3)/c^3*cos(4*arctan(a*x))+3/1024*(8*arctan(a*x)^2-1)/c^3*sin(4*arcta
n(a*x)))
```

3.410.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*
x^3), x)
```

3.410.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c
**3
```

3.410.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^3), x)
```

3.410.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3 (c + a^2cx^2)^3} \, dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^3\,(c\,a^2\,x^2 + c)^3} \, dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^3), x)
```

3.411 $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

3.411.1 Optimal result

Integrand size = 22, antiderivative size = 432

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx = &-\frac{3a^3}{128c^3(1+a^2x^2)^2}-\frac{141a^3}{128c^3(1+a^2x^2)}-\frac{a^2\arctan(ax)}{c^3x}\\
&-\frac{3a^4x\arctan(ax)}{32c^3(1+a^2x^2)^2}-\frac{141a^4x\arctan(ax)}{64c^3(1+a^2x^2)}-\frac{205a^3\arctan(ax)^2}{128c^3}\\
&-\frac{a\arctan(ax)^2}{2c^3x^2}+\frac{3a^3\arctan(ax)^2}{16c^3(1+a^2x^2)^2}+\frac{33a^3\arctan(ax)^2}{16c^3(1+a^2x^2)}\\
&+\frac{10ia^3\arctan(ax)^3}{3c^3}-\frac{\arctan(ax)^3}{3c^3x^3}+\frac{3a^2\arctan(ax)^3}{c^3x}\\
&+\frac{a^4x\arctan(ax)^3}{4c^3(1+a^2x^2)^2}+\frac{11a^4x\arctan(ax)^3}{8c^3(1+a^2x^2)}+\frac{35a^3\arctan(ax)^4}{32c^3}\\
&+\frac{a^3\log(x)}{c^3}-\frac{a^3\log(1+a^2x^2)}{2c^3}-\frac{10a^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\
&+\frac{10ia^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}\\
&-\frac{5a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^3}\end{aligned}$$

output
```
-3/128*a^3/c^3/(a^2*x^2+1)^2-141/128*a^3/c^3/(a^2*x^2+1)-a^2*arctan(a*x)/c
^3/x-3/32*a^4*x*arctan(a*x)/c^3/(a^2*x^2+1)^2-141/64*a^4*x*arctan(a*x)/c^3
/(a^2*x^2+1)-205/128*a^3*arctan(a*x)^2/c^3-1/2*a*arctan(a*x)^2/c^3/x^2+3/1
6*a^3*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+33/16*a^3*arctan(a*x)^2/c^3/(a^2*x^2
+1)+10/3*I*a^3*arctan(a*x)^3/c^3-1/3*arctan(a*x)^3/c^3/x^3+3*a^2*arctan(a*
x)^3/c^3/x+1/4*a^4*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+11/8*a^4*x*arctan(a*x
)^3/c^3/(a^2*x^2+1)+35/32*a^3*arctan(a*x)^4/c^3+a^3*ln(x)/c^3-1/2*a^3*ln(a
^2*x^2+1)/c^3-10*a^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3+10*I*a^3*arctan(a
*x)*polylog(2,-1+2/(1-I*a*x))/c^3-5*a^3*polylog(3,-1+2/(1-I*a*x))/c^3
```

3.411.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)^3}\,dx$$
$$= \frac{a^3\left(\frac{5i\pi^3}{12}-\frac{\arctan(ax)}{ax}-\frac{1}{2}\arctan(ax)^2-\frac{\arctan(ax)^2}{2a^2x^2}-\frac{10}{3}i\arctan(ax)^3-\frac{\arctan(ax)^3}{3a^3x^3}+\frac{3\arctan(ax)^3}{ax}+\frac{35}{32}\arctan(\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
(a^3*(((5*I)/12)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^
2/(2*a^2*x^2) - ((10*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + (3*
ArcTan[a*x]^3)/(a*x) + (35*ArcTan[a*x]^4)/32 - (9*Cos[2*ArcTan[a*x]])/16 +
 (9*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/8 - (3*Cos[4*ArcTan[a*x]])/1024 + (3
*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]])/128 - 10*ArcTan[a*x]^2*Log[1 - E^((-2*I
)*ArcTan[a*x])] + Log[a*x] - Log[1 + a^2*x^2]/2 - (10*I)*ArcTan[a*x]*PolyL
og[2, E^((-2*I)*ArcTan[a*x])] - 5*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (9*
ArcTan[a*x]*Sin[2*ArcTan[a*x]])/8 + (3*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]])/4
 - (3*ArcTan[a*x]*Sin[4*ArcTan[a*x]])/256 + (ArcTan[a*x]^3*Sin[4*ArcTan[a*
x]])/32))/c^3
```

3.411.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^4\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{c^2x^4(a^2x^2+1)^2}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c^3x^2\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\right.}{c^3}$$

$$\Big\downarrow \text{ 5431}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3}-$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(-\frac{3}{8}\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax}{2(a^2x^2+1}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(-\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 241

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5453

$$\frac{a^2\left(-\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx + \int \frac{\arctan(ax)^3}{x^4}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5361

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx + a\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{3x^3}}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5453

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\int \frac{\arctan(ax)^3}{x^2}dx - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx\right) + a\left(\int \frac{\arctan(ax)^2}{x^3}dx - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) - \frac{\arctan(a}{3x^3}}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5361

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{2x^2}\right) - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - a^2\int \frac{\text{arct}}{a^2}\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5419

3.411. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 5453$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan}{a^2x^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 5361$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 243$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 47$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 14$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 16$$

3.411. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5419

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\frac{1}{2}a(\mathrm{lo}\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5459

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx+a\left(-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\,\mathrm{arct}\right.\right.}{}$$

$\downarrow$ 5403

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)\right.\right.}{}$$

$\downarrow$ 5465

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)\right.\right.}{}$$

$\downarrow$ 5427

3.411. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan}{8}\right.\right.\right.}{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)\right.\right.}$$

$\downarrow$ 241

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right.\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)\right.\right.$$

$\downarrow$ 5501

$$-\frac{a^2\left(-a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx+\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{}{4}\right.\right.\right.\right.\right.}{-a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx\right)-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-i}\right.\right.\right.\right.}$$

$\downarrow$ 5427

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}\right.\right.\right.}{-a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)\right)-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{}{1-}\right.}{a^2x^2+1}\right.\right.\right.}$$

$\downarrow$ 5453

3.411. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)-a^2\int\frac{\arctan(ax)^3}{a^2x^2+1}dx+\int\frac{\arctan(ax)^3}{x^2}dx-\left(a^2\left(\frac{x\arctan}{4(a^2x^2}\right.\right.\right.}{-a^2\left(-\left(a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)\right)-a^2\int\frac{\arctan(ax)^3}{a^2x^2+1}dx+\int\frac{\arctan(ax)^3}{x^2}dx\right)-a^2\left(3a\left(i\right.\right.}$$

$$\Big\downarrow \text{5361}$$

$$-\frac{\arctan(ax)^3}{3x^3}-a^2\left(-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{\arctan(ax)^4}{8a}+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)+3a\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int\frac{\mathrm{arc}}{}\right.$$

$$\overline{a^2\left(-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{\arctan(ax)^2}{4a}+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}\right)+\frac{}{16a}\right.\right.\right.}$$

$$\Big\downarrow \text{5419}$$

$$-\frac{\arctan(ax)^3}{3x^3}-a^2\left(-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{\arctan(ax)^4}{8a}+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)+3a\int\frac{\mathrm{arcta}}{x(a^2}\right.$$

$$\overline{a^2\left(-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{\arctan(ax)^2}{4a}+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\right.\right.\right.\right.}$$

input
```
Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

3.411.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.411. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.411. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.411. $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$

3.411.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 136.48 (sec) , antiderivative size = 2062, normalized size of antiderivative = 4.77

method	result	size
derivativedivides	Expression too large to display	2062
default	Expression too large to display	2062
parts	Expression too large to display	2069

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/c^3*arctan(a*x)^3/a^3/x^3+3/c^3*arctan(a*x)^3/a/x+11/8/c^3*arcta
n(a*x)^3/(a^2*x^2+1)^2*a^3*x^3+13/8/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a*x+35
/8/c^3*arctan(a*x)^4-1/8/c^3*(105/4*arctan(a*x)^4+205/16*arctan(a*x)^2+80*
arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+80*arctan(a*x)^2*ln((1+I*a
*x)/(a^2*x^2+1)^(1/2)+1)+80*arctan(a*x)^2*ln(2)+3/128*cos(4*arctan(a*x))+1
60*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+160*polylog(3,(1+I*a*x)/(a^2*x^
2+1)^(1/2))-9/4*(I+a*x)/(a*x-I)-9/4*(a*x-I)/(I+a*x)-3/2*arctan(a*x)^2/(a^2
*x^2+1)^2-8*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-8*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)+1)-33/2*arctan(a*x)^2/(a^2*x^2+1)-40*arctan(a*x)^2*ln(a^2*x^2+1)+3/32
*arctan(a*x)*sin(4*arctan(a*x))-80/3*I*arctan(a*x)^3+80*arctan(a*x)^2*ln(a
*x)-80*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-160*I*arctan(a*x)*polyl
og(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+40*I*Pi*arctan(a*x)^2+4*arctan(a*x)^2/a
^2/x^2-160*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+80*arctan(
a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-40*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+4*arctan(a*
x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+40*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1
)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-20*I*Pi*csgn(I/((1+I*a*x)^2/(
a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+40*I*Pi*csgn(I*((1+I...
```

3.411.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2 cx^2)^3} \, dx = \int \frac{\arctan(ax)^3}{(a^2 cx^2 + c)^3 x^4} \, dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3
*x^4), x)
```

3.411.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2 cx^2)^3} \, dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6 x^{10} + 3a^4 x^8 + 3a^2 x^6 + x^4} \, dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/
c**3
```

3.411.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2 cx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.411.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c+a^2cx^2)^3} \, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3 x^4} \, dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4 (c+a^2cx^2)^3} \, dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^4\,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^3), x)
```

3.412 $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

3.412.1 Optimal result

Integrand size = 24, antiderivative size = 523

$$\begin{aligned}\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = &-\frac{x\sqrt{c+a^2cx^2}}{20a^3}-\frac{9\sqrt{c+a^2cx^2}\arctan(ax)}{20a^4}\\&+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{10a^2}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a^3}\\&-\frac{3x^3\sqrt{c+a^2cx^2}\arctan(ax)^2}{20a}\\&-\frac{11ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{20a^4\sqrt{c+a^2cx^2}}\\&-\frac{2\sqrt{c+a^2cx^2}\arctan(ax)^3}{15a^4}+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{15a^2}\\&+\frac{1}{5}x^4\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{2a^4}\\&+\frac{11ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}\\&-\frac{11ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}\\&-\frac{11c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}\\&+\frac{11c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/2*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4-11/20*I*c*arctan(
(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^
2+c)^(1/2)+11/20*I*c*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-11/20*I*c*arctan(a*x)*polylog(2
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-
11/20*c*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a
^2*c*x^2+c)^(1/2)+11/20*c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^
2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-1/20*x*(a^2*c*x^2+c)^(1/2)/a^3-9/20*arc
tan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4+1/10*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/
a^2+1/8*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-3/20*x^3*arctan(a*x)^2*(a^
2*c*x^2+c)^(1/2)/a-2/15*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^4+1/15*x^2*arc
tan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2+1/5*x^4*arctan(a*x)^3*(a^2*c*x^2+c)^(1/
2)
```

3.412.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.50

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$$

$$= \frac{\sqrt{c+a^2cx^2}\left(\frac{48\left(-11i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+10\text{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+11i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-11i\arctan\right.}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((48*((-11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
 + 10*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (11*I)*ArcTan[a*x]*PolyLog[2, (-I
)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])]
- 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a*x]
)]))/Sqrt[1 + a^2*x^2] - (1 + a^2*x^2)^2*((48*a*x)/(1 + a^2*x^2)^2 + 32*Ar
cTan[a*x]^3*(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*Arc
Tan[a*x]] + 11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] -
 33*Sin[4*ArcTan[a*x]]))))/(960*a^4)
```

3.412.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{x^5 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{ 5465}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{ 5423}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2}}{3a^2c}\right.$$
$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$\Big\downarrow$ 3042

$$c\left(-\frac{\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)}{3a}\right.$$
$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$\Big\downarrow$ 4669

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$
$$c\left(-\frac{\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log(\right.}{a^2\sqrt{a^2cx^2+c}}\right.}{3a^2}\right.$$

$\Big\downarrow$ 3011

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$
$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{a^2\sqrt{}}\right.}{3a^2}\right.$$

$\Big\downarrow$ 2720

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$
$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.}{3}\right.$$

$\Big\downarrow$ 5487

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(\right.\right.}{}\right.}{3}\right.$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(\right.\right.}{}\right.}{3}\right.$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(\right.\right.}{}\right.}{3}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(\right.\right.}{}\right.}{3}\right.$$

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$\downarrow$ 4669

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.}{3}\right.$$

$\downarrow$ 3011

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.}{3}\right.$$

$\downarrow$ 2720

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.}{3}\right.$$

$\downarrow$ 5465

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\arc}{3a^2c}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(a}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \textbf{224}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\arc}{3a^2c}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}}{}\right.$$

$$\Big\downarrow \textbf{219}$$

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,\mathrm{arc}}{3a^2c}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e}{}}{}\right.$$

$$\Big\downarrow \; 5425$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,\mathrm{arc}}{3a^2c}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e}{}}{}\right.$$

$$\Big\downarrow \; 5423$$

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,\mathrm{arc}}{3a^2c}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e}{}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,\mathrm{arc}}{3a^2c}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e}{}}{}\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,\mathrm{arc}}{3a^2c}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e}{}}{}\right.$$

$$\Big\downarrow \text{3011}$$

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\begin{array}{l}\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\arc\ldots}{3a^2c}\right.}{\ldots}\\ c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e\ldots}{\ldots}}{\ldots}\right.\end{array}\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(\begin{array}{l}\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\arc\ldots}{3a^2c}\right.}{\ldots}\\ c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e\ldots}{\ldots}}{\ldots}\right.\end{array}\right.$$

$$\Big\downarrow \; 5487$$

$$c\left(\begin{array}{l}\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\sqrt{\ldots}}{\ldots}\right.}{\ldots}\right.}{5a}\\ c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e\ldots}{\ldots}}{\ldots}\right.\end{array}\right.$$

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$\Big\downarrow \; 262$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}-\frac{2\int\frac{x\arctan(ax}{\sqrt{a^2cx^2+c}}}{3a^2}}{2a}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e}{}}{}\right.$$

$$\Big\downarrow \; 224$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}-\frac{2\int}{}}{2a}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e}{}}{}\right.$$

$$\Big\downarrow \; 219$$

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x \cdots}{\cdots}}{\cdots}}{2a}\cdots\right)}{5a}\cdots\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e\cdots}{\cdots}}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x \cdots}{\cdots}}{\cdots}}{2a}\cdots\right)}{5a}\cdots\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan(e\cdots}{\cdots}}{\cdots}\cdots\right.$$

input
```
Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

3.412.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.412. $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

3.412.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.80

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\left(24a^4\arctan(ax)^3x^4-18a^3\arctan(ax)^2x^3+8\arctan(ax)^3x^2a^2+12a^2\arctan(ax)x^2+15a\arctan(ax)^2x-16\arctan\right.}{120a^4}$

input
```
int(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(24*a^4*arctan(a*x)^3*x^4-18*a^3*arcta
n(a*x)^2*x^3+8*arctan(a*x)^3*x^2*a^2+12*a^2*arctan(a*x)*x^2+15*a*arctan(a*
x)^2*x-16*arctan(a*x)^3-6*a*x-54*arctan(a*x))+11/120*(c*(a*x-I)*(I+a*x))^(
1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-11/120*(c*(a*x-I)*(I+a*
x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-I/a^4*(c*(a*x-I)*(I+
a*x))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

3.412.5 Fricas [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int\sqrt{a^2cx^2+c}\,x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

3.412.6 Sympy [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x^3\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate(x**3*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

3.412.7 Maxima [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int\sqrt{a^2cx^2+c}\,x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

3.412.8 Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int x^3\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.413 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

3.413.1 Optimal result

Integrand size = 24, antiderivative size = 747

$$
\begin{aligned}
\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx =& -\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{4a^2} \\
&+\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a^3}-\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{4a} \\
&+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^3}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\arctan(ax)^3 \\
&+\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{4a^3\sqrt{c+a^2cx^2}} \\
&+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}} \\
&-\frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} \\
&+\frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} \\
&-\frac{ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}} \\
&+\frac{ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}} \\
&+\frac{3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}} \\
&-\frac{3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}} \\
&+\frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}} \\
&-\frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
1/4*I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2
)/a^3/(a^2*c*x^2+c)^(1/2)+I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/4*I*c*polylog(4,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/8*I*
c*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/a^3/(a^2*c*x^2+c)^(1/2)+1/2*I*c*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/8*I*c*arctan(a*x)^2*polylo
g(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1
/2)+3/4*c*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/4*c*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-1/2*I*c*polylog
(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)
^(1/2)-3/4*I*c*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/
a^3/(a^2*c*x^2+c)^(1/2)-1/4*(a^2*c*x^2+c)^(1/2)/a^3+1/4*x*arctan(a*x)*(a^2
*c*x^2+c)^(1/2)/a^2+1/8*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-1/4*x^2*arct
an(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+1/8*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^
2+1/4*x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

3.413.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1844 vs. 2(747) = 1494.

Time = 12.23 (sec) , antiderivative size = 1844, normalized size of antiderivative = 2.47

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \text{Too large to display}$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqr
t[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 +
I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2
, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(
-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[a*x
])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]
))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2
- ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2
- ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - Ar
cTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/
2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLo
g[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 +
 (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[
1 + E^(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*
x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2
 + (-1/2*Pi + ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan
[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - Ar
cTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2
 + (-1/2*Pi + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a
*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/...
```

3.413.3 Rubi [A] (verified)

Time = 9.63 (sec) , antiderivative size = 1306, normalized size of antiderivative = 1.75, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5485, 5487, 5425, 5423, 3042, 4669, 3011, 5465, 5425, 5421, 5487, 5425, 5423, 3042, 4669, 3011, 5465, 5425, 5421, 5487, 241, 5425, 5421, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5485}$$

$$c\int \frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+$$
$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$$\Big\downarrow \text{5425}$$

$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+$$
$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1\right.\right.}{2a^3\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie\right.\right.\right.}{}\right.$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$

$$c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \operatorname{arcta}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$

$$c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2\right.\right.}{}\right.$$

$$\Big\downarrow \text{5421}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$\Big\downarrow \text{5487}$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$a^2c\left(-\frac{3\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\right.}{4a^2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(-\frac{3\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax}{\sqrt{a^2x^2+}}}{2a^2\sqrt{a^2cx^2+c}}\right.}{4a^2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$\Big\downarrow \text{5423}$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+}}{2a^3\sqrt{}}\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int\arctan(a}{}\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$\Big\downarrow \text{4669}$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(-3\int\arctan\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{5465}$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\begin{array}{l} \frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}\right.}{4a} \\ \frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a} \end{array}\right.$$

$\big\downarrow$ 5425

$$c\left(\begin{array}{l} \frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan}{\sqrt{a^2cx}}}{3a}\right.}{4a} \\ \frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a} \end{array}\right.$$

$\big\downarrow$ 5421

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\cdots\right.}{a\cdots}\right.}{3a^2\cdots}\right.}{4a\cdots}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{a\sqrt{a^2cx^2+c}}\cdots\right.}{2a}\cdots\right.$$

$\downarrow$ 5487

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{a}\right.}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\downarrow$ 241

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{a}\right.}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i}{}\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\downarrow$ 5425

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{a}\right.}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$$\Big\downarrow \ 5421$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\right.}{2a^2}\right.}{3a}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$$\Big\downarrow \; 7163$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\cdots\right)}{2a^2\cdots}\right)}{3a}\cdots\right)}{\cdots}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right)}{a\sqrt{a^2cx^2+c}}\cdots\right)}{2a}\cdots\right.$$

$\big\downarrow$ 2720

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\cdots\right.}{2a^2\cdots}\right.}{3a\cdots}\right.}{\cdots}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$$\Big\downarrow \; 7143$$

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\cdots\right.}{2a^2\cdots}\right.}{3a}\right.}{\cdots}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

output
```
c*((x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(2*a^2*c) - (3*((Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*Arc
Tan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*
x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(2*a) - (Sqrt[1 + a^2*x^2]*((-2*I)*ArcT
an[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2
, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[
a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(2*a^3*Sqrt[c + a^2*c*x^2]))
+ a^2*c*((x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(4*a^2*c) - (3*((x^2*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a^2*c*x^2]/(a
^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]
*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*Poly
Log[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[
1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(3*a) - (2
*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2
*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2,
 ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^2)))/(4*a) -...
```

3.413.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.413. $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.413.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.62

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\left(2\arctan(ax)^3a^3x^3-2x^2\arctan(ax)^2a^2+\arctan(ax)^3ax+2x\arctan(ax)a+\arctan(ax)^2-2\right)}{8a^3}+\frac{\sqrt{c(ax-i)(ax+i)}}{}$

input
```
int(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*x^2*arctan(a*
x)^2*a^2+arctan(a*x)^3*a*x+2*x*arctan(a*x)*a+arctan(a*x)^2-2)+1/8*(c*(a*x-
I)*(I+a*x))^(1/2)*(arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arcta
n(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+4*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan
(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+6*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*dilog(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog
(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

3.413.5 Fricas [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int\sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

3.413.6 Sympy [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x^2\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate(x**2*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

3.413.7 Maxima [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int \sqrt{a^2cx^2+c}x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

3.413.8 Giac [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int \sqrt{a^2cx^2+c}x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int x^2\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.414 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

3.414.1 Optimal result

Integrand size = 22, antiderivative size = 373

$$\begin{aligned}\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx &= \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^2}-\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a}\\&+\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^2\sqrt{c+a^2cx^2}}\\&+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^2}\\&-\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\&+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\&+\frac{c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\&-\frac{c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/a^2/c-arctanh(a*x*c^(1/2)/(a^2*c*x^2
+c)^(1/2))*c^(1/2)/a^2+I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)
^2*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-I*c*arctan(a*x)*polylog(2,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+I*c
*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^
2/(a^2*c*x^2+c)^(1/2)+c*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)*(a^2*c*x^2+c)^(1/
2)/a^2-1/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a
```

3.414.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.55

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \frac{\sqrt{c+a^2cx^2}\left(\frac{12\left(i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-\operatorname{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)+i\arctan(ax)\operatorname{PolyL}\right.}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((12*(I*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - Arc
Tanh[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[
a*x])] + I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + PolyLog[3, (-I)*E
^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] +
(1 + a^2*x^2)*ArcTan[a*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*
ArcTan[a*x]*Sin[2*ArcTan[a*x]])))/(12*a^2)
```

3.414.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5465, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3 \sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}$$
$$\Big\downarrow \text{5415}$$
$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}}{a}$$
$$\Big\downarrow \text{224}$$
$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}}{a}$$
$$\Big\downarrow \text{219}$$
$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}}{a}$$
$$\Big\downarrow \text{5425}$$
$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}}{a}$$
$$\Big\downarrow \text{5423}$$
$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}}{a}$$
$$\Big\downarrow \text{3042}$$

3.414. $\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\cdot}{\sqrt{a^2\cdots}}\right.}{a}}{a}$$

$$\Big\downarrow \; 4669$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(a\cdots\right.}{2a\sqrt{a^2cx^2+c}}}{a}$$

$$\Big\downarrow \; 3011$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-i\int\cdots\right.\right.}{2a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 2720$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\,\text{arc}\cdots}\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 7143$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,ie^{i\,\text{arcta}\cdots}\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}}{a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

3.414. $\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

output
```
((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*a^2*c) - (-((Sqrt[c + a^2*c*x^2]*
ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*Ar
cTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a
*Sqrt[c + a^2*c*x^2]))/a
```

3.414.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.414. $\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

3.414.4 Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(2x^2\arctan(ax)^2a^2-3x\arctan(ax)a+2\arctan(ax)^2+6\right)}{6a^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,\left(i\arctan(ax)^3-3\arcta\right.}{}$

input
```
int(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*x^2*arctan(a*x)^2*a^2-3*x
*arctan(a*x)*a+2*arctan(a*x)^2+6)-1/6*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(
a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)
*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+1/6*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arcta
n(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*
x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+2*I/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*arct
an((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

3.414.5 Fricas [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int\sqrt{a^2cx^2+cx}\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

3.414.6 Sympy [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate(x*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

3.414.7 Maxima [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int \sqrt{a^2cx^2+cx}\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

3.414.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int x\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.415 $\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

3.415.1 Optimal result

Integrand size = 21, antiderivative size = 626

$$
\begin{aligned}
\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx =& -\frac{3\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)^3 \\
&-\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a\sqrt{c+a^2cx^2}} \\
&-\frac{6ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} \\
&+\frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{2a\sqrt{c+a^2cx^2}} \\
&-\frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\,\mathrm{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{2a\sqrt{c+a^2cx^2}} \\
&+\frac{3ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} \\
&-\frac{3ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} \\
&-\frac{3c\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} \\
&+\frac{3c\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} \\
&-\frac{3ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} \\
&+\frac{3ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a
/(a^2*c*x^2+c)^(1/2)-6*I*c*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+3/2*I*c*arctan(a*x)^2*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
-3/2*I*c*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1
)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+3*I*c*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*I*c*polylog(2,I*(1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*c*arcta
n(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*
c*x^2+c)^(1/2)+3*c*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*I*c*polylog(4,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+3*I*c*polylog(4,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/2*arcta
n(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+1/2*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

3.415.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.41

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx =$$
$$-\frac{i\sqrt{c\left(1+a^2x^2\right)}\left(12\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)-3i\sqrt{1+a^2x^2}\arctan(ax)^2+iax\sqrt{1+a^2x^2}\arctan(\right.}{}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
((-1/2*I)*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]
- (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^
(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] +
(6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I
*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2])
```

3.415.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5415, 5425, 5421, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5415}$$

$$3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5423}$$

$$\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{4669}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(\right.}{2a\sqrt{a^2cx^2+c}}$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{3011}$$

$$c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{7163}$$

$$c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int \text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\,\text{arctan}\right.\right.\right.$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{2720}$$

3.415. $\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax}\right.\right.\right.}{}$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+$$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\left(3\right.\right.\right.\right.}{}$$

$$\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
(-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1
- I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqr
t[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*
ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2
*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)
*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])]
 - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*
E^(I*ArcTan[a*x])]))))/(2*a*Sqrt[c + a^2*c*x^2])
```

3.415.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.415. $\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

3.415. $\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.415.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.67

method	result
default	$\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2(x\arctan(ax)a-3)}{2a} - \frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^3\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^3\ln\left(1-\frac{i(iax+}{\sqrt{a^2x^2}}\right.\right.}{}$

input
```
int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(x*arctan(a*x)*a-3)-1/2*(c*(
a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-a
rctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*ar
ctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+6*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))-6*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*d
ilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

3.415.5 Fricas [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{a^2cx^2+c}\arctan\left(ax\right)^3\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

3.415.6 Sympy [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^3\left(ax\right)dx$$

input
```
integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

3.415.7 Maxima [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{a^2cx^2+c}\arctan\left(ax\right)^3\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

3.415.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.415.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int \mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.416 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$

3.416.1 Optimal result

Integrand size = 24, antiderivative size = 600

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \frac{6ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\arctan(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\arctan(ax)^3\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
6*I*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)-2*c*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*c*arctan(a*x)^2*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*c*arcta
n(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*
x^2+c)^(1/2)+6*I*c*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*c*arctan(a*x)^2*polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*c*arctan(a*x)
*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)+6*c*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*
c*x^2+c)^(1/2)-6*c*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)+6*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*c*polylog(4,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*c*polylog(4,(1+I*
a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^
3*(a^2*c*x^2+c)^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(-i\pi^4+8\sqrt{1+a^2x^2}\arctan(ax)^3+2i\arctan(ax)^4+8\arctan(ax)^3\log\left(1-e^{-i\arctan(ax)}\right)-24\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((-I)*Pi^4 + 8*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (2*I
)*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 24*ArcTa
n[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 24*ArcTan[a*x]^2*Log[1 + I*E^(I*Ar
cTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a
*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, -
E^(I*ArcTan[a*x])] - (48*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
 + (48*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*Pol
yLog[3, E^((-I)*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x
])] + 48*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 48*PolyLog[3, I*E^(I*ArcTan[
a*x])] - (48*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I
*ArcTan[a*x])]))/(8*Sqrt[1 + a^2*x^2])
```

3.416.3 Rubi [A] (verified)

Time = 2.71 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.60, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {5485, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5493, 5491, 3042, 4671, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x}dx$$

$$\Big\downarrow \textbf{5485}$$

$$a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \textbf{5465}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \textbf{5425}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \textbf{5423}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+$$
$$c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \textbf{4669}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\right.}{a^2\sqrt{a^2cx^2-}}\right.$$
$$c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+$$

$$\downarrow 3011$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.$$
$$c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+$$

$$\downarrow 2720$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$
$$c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+$$

$$\downarrow 5493$$

$$\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$

$$\downarrow 5491$$

$$\frac{c\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$

$$\downarrow 3042$$

$$\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$

$$\downarrow 4671$$

$$\frac{c\sqrt{a^2x^2+1}\left(-3\int\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arctan(a\right.}{\sqrt{a^2cx^2+c}}$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$

3.416. $\int\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$

$$\Big\downarrow \text{3011}$$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.}$$

$$\Big\downarrow \text{7143}$$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.}$$

$$\Big\downarrow \text{7163}$$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan(\right.\right.\right.}{a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.}$$

$$\Big\downarrow \text{2720}$$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.}{a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.}$$

$$\Big\downarrow \text{7143}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-\right.\right.\right.\right.}{a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

3.416. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$

output
```
a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*
((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*A
rcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x]
)])))/(a^2*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*
ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*
x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4,
 -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])]
- (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*
ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]
```

3.416.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.416. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.416. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.416.4 Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.76

method	result
default	$\sqrt{c\,(ax-i)\,(ax+i)}\,\arctan\,(ax)^3-\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.}{}$

input
```
int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^3-(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(
a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2
*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3
*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*ln
(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*a
rctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^
2+1)^(1/2))+6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.416.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

3.416.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}{x}\,dx$$

input
```
integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x, x)
```

3.416.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

3.416.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + a^2cx^2}\arctan(ax)^3}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c + a^2cx^2}\arctan(ax)^3}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2 + c}}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.417 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx$

3.417.1 Optimal result

Integrand size = 24, antiderivative size = 622

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6iac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3iac\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3iac\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6iac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-2*I*a*c*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)-6*a*c*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a*c*arctan(a*x)*polylog(2,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a*
c*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-3*I*a*c*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c*arctan(a*x)*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*
a*c*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c
)^(1/2)-6*a*c*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a*c*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a*c*polylog(3,(1+
I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c*po
lylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)+6*I*a*c*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)-arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x
```

3.417.2 Mathematica [A] (warning: unable to verify)

Time = 2.67 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx$$
$$= \frac{a\sqrt{c+a^2cx^2}\csc\left(\frac{1}{2}\arctan(ax)\right)\left(-7ia\pi^4x-8ia\pi^3x\arctan(ax)+24ia\pi^2x\arctan(ax)^2-32ia\pi x\arctan(a\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

output
```
(a*Sqrt[c + a^2*c*x^2]*Csc[ArcTan[a*x]/2]*((-7*I)*a*Pi^4*x - (8*I)*a*Pi^3*
x*ArcTan[a*x] + (24*I)*a*Pi^2*x*ArcTan[a*x]^2 - (32*I)*a*Pi*x*ArcTan[a*x]^
3 - 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (16*I)*a*x*ArcTan[a*x]^4 + 48*a*P
i^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] - 96*a*Pi*x*ArcTan[a*x]^2*L
og[1 - I/E^(I*ArcTan[a*x])] - 8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])] + 64
*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 192*a*x*ArcTan[a*x]^2*Lo
g[1 - E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])] - 48*a*
Pi^2*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*a*Pi*x*ArcTan[a*x]^2*
Log[1 + I*E^(I*ArcTan[a*x])] - 64*a*x*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[
a*x])] - 192*a*x*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log
[Tan[(Pi + 2*ArcTan[a*x])/4]] + (192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/
E^(I*ArcTan[a*x])] + (48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*
ArcTan[a*x])] + (384*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (
48*I)*a*Pi^2*x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*a*Pi*x*ArcTan[
a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (192*I)*a*x*ArcTan[a*x]^2*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - (384*I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*Arc
Tan[a*x])] + 384*a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] - 192*
a*Pi*x*PolyLog[3, I/E^(I*ArcTan[a*x])] - 384*a*x*PolyLog[3, -E^(I*ArcTan[a
*x])] + 192*a*Pi*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 384*a*x*ArcTan[a*x
]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 384*a*x*PolyLog[3, E^(I*ArcTan[a...
```

3.417.3 Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.58, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5485, 5425, 5423, 3042, 4669, 3011, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \; 5423$$

$$\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \; 3042$$

$$c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 4669$$

$$c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3011$$

$$c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}$$

$$\Big\downarrow \; 5479$$

$$c\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}$$

$$\Big\downarrow \; 5493$$

$$c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}$$

$$\Big\downarrow \; 5491$$

$$c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}$$

$$\Big\downarrow \; 3042$$

3.417. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\underline{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}$$

$$\Big\downarrow \text{4671}$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\underline{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}$$

$$\Big\downarrow \text{3011}$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan}\right.\right.\right.}{}\right.$$
$$\underline{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}$$

$$\Big\downarrow \text{2720}$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$
$$\underline{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}$$

$$\Big\downarrow \text{7143}$$

$$\underline{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}$$
$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{7163}$$

$$\underline{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\arcta\right.\right.\right.}$$
$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

3.417. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx$

$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.\right.}{}$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\Big\downarrow 7143$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\right.\right.\right.}{}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

output
```
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(
-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2,
-E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*P
olyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a
^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcT
an[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*
((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)*E^(
I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (
2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*E^(I
*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]
```

3.417.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.417. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx$

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.417. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx$

3.417.4 Maple [A] (verified)

Time = 5.53 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.75

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^3}{x}+\frac{ia\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)^3\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^3\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+3i\,\mathrm{arc}\right.}{}$

input
```
int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^3/x+I*a*(c*(a*x-I)*(I+a*x))^(1/2)*(
I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)+1)-3*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^
2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arct
an(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,(1
+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I
*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.417.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x^2}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

3.417.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3(ax)}{x^2}\,dx$$

input
```
integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**2, x)
```

3.417.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

3.417.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

3.418 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx$

3.418.1 Optimal result

Integrand size = 24, antiderivative size = 602

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = -\frac{3a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{2x^2}$$
$$- \frac{a^2c\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$- \frac{6a^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$
$$+ \frac{3ia^2c\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}}$$
$$- \frac{3ia^2c\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}}$$
$$+ \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$
$$- \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$
$$- \frac{3a^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$+ \frac{3a^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$- \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$+ \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-a^2*c*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-6*a^2*c*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3/2*I*a^2*c*arctan(a*x)^2*p
olylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)-3/2*I*a^2*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c*polylog(2,-(1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c*polylog(2,(1+I
*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a^2*c
*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)+3*a^2*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c*polylog(4,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c*polylog(4,
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*a*a
rctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x-1/2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x
^2
```

3.418.2 Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx$$
$$= \frac{a^2\sqrt{c(1+a^2x^2)}\left(-i\pi^4+2i\arctan(ax)^4-12\arctan(ax)^2\cot\left(\frac{1}{2}\arctan(ax)\right)-2\arctan(ax)^3\csc^2\left(\frac{1}{2}\operatorname{arcta}\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*Sqrt[c*(1 + a^2*x^2)]*((-I)*Pi^4 + (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*
x]^2*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 + 8*ArcTan[
a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[
a*x])] - 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1
 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x
])] + (24*I)*(2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*P
olyLog[2, E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*
x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4, E
^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*
x]^3*Sec[ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt
[1 + a^2*x^2])
```

3.418.3 Rubi [A] (verified)

Time = 3.46 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {5485, 5493, 5491, 3042, 4671, 3011, 5497, 5479, 5493, 5489, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5493}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5491}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{4671}$$

$$c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arctan\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$

$$\Big\downarrow \text{5497}$$

$$c\left(\frac{3}{2}a\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}\right)+$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right)\right.}{}$$

$$\Big\downarrow \text{5479}$$

$$c\left(\frac{3}{2}a\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}\right)+$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right)\right.}{}$$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{3}{2}a\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{c}}{2cx^2}\right.$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right)\right.}{}$$

$$\Big\downarrow \text{5489}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right)\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+ia}}{\sqrt{1-ia}}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right)\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right)\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-\right.}{}\right.\right.$$

3.418. $\int\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx$

$$\Big\downarrow \; 4671$$

$$\frac{a^2c\sqrt{a^2x^2+1}\Big(3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\Big(-3\int\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\operatorname{arc}}{2\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \; 3011$$

$$\frac{a^2c\sqrt{a^2x^2+1}\Big(3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\Big(3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan}{}\right.$$

$$\Big\downarrow \; 7163$$

$$\frac{a^2c\sqrt{a^2x^2+1}\Big(3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\big(i\int\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\operatorname{arcta}}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\Big(3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\big(i\int\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\operatorname{a}}{}\right.$$

$$\Big\downarrow \; 2720$$

$$\frac{a^2c\sqrt{a^2x^2+1}\Big(3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\big(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(a}}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\Big(3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\big(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\operatorname{arcta}}}{}\right.$$

$$\Big\downarrow \; 7143$$

$$\frac{a^2c\sqrt{a^2x^2+1}\Big(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\big(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\big(\operatorname{PolyLog}\left(4,\right.}{}$$

$$c\left(\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```

3.418. $\int\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx$

output
```
(a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*
(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*
PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*A
rcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyL
og[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*
c*x^2] + c*(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^2) + (3*a*(-((Sq
rt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))
/Sqrt[c + a^2*c*x^2]))/2 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTan
h[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] -
 (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I
*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I
)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan
[a*x])]))))/(2*Sqrt[c + a^2*c*x^2]))
```

3.418.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.418. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx$

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.418. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.418.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.67

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2(3ax+\arctan(ax))}{2x^2}+\frac{ia^2\sqrt{c(ax-i)(ax+i)}\,\left(i\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\arctan(ax)^3\ln\left(1-\right.\right.}{}$

input
```
int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(3*a*x+arctan(a*x))/x^2+1/2*I
*a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)+1)-I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*
polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*I
*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*ln(1-
(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+
1)^(1/2))+6*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(
1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.418.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x^3}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

3.418.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}{x^3}\,dx$$

input
```
integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**3, x)
```

3.418.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x^3}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

3.418.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

3.419 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx$

3.419.1 Optimal result

Integrand size = 24, antiderivative size = 361

$$
\begin{aligned}
\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx =& -\frac{a^2\sqrt{c+a^2cx^2}\arctan(ax)}{x} - \frac{a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2} \\
& - \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{3cx^3} \\
& - \frac{a^3c\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& - a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) \\
& + \frac{ia^3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{ia^3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{a^3c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& + \frac{a^3c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/c/x^3-a^3*arctanh((a^2*c*x^2+c)^(1/
2)/c^(1/2))*c^(1/2)-a^3*c*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+I*a^3*c*arctan(a*x)*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-I*a^3*c*a
rctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c
*x^2+c)^(1/2)-a^3*c*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)+a^3*c*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x-1/2
*a*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/x^2
```

3.419.2 Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx$$
$$= \frac{a^3c\sqrt{1+a^2x^2}\left(-12\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)-2\arctan(ax)^3\cot\left(\frac{1}{2}\arctan(ax)\right)-3\arctan(ax)^2\csc\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```

output
```
(a^3*c*Sqrt[1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*
x]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcT
an[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*L
og[1 - E^(I*ArcTan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] +
24*Log[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*
x])] - (24*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E
^(I*ArcTan[a*x])] + 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec
[ArcTan[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2
]^4)/(a^3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[A
rcTan[a*x]/2]))/(24*Sqrt[c*(1 + a^2*x^2)])
```

3.419.3 Rubi [A] (verified)

Time = 3.17 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5479, 5485, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5485}$$

$$a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5493}$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5491}$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\Big\downarrow$ 3011

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 2720

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\mathrm{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 5497

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(a\right.\right.}{}\right.$$

$\Big\downarrow$ 5479

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)\right.$$

$\Big\downarrow$ 243

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{2cx^2}\right.\right.$$

$\Big\downarrow$ 73

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right.\right.$$

3.419. $\int\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx$

$$\Big\downarrow 221$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{2cx^2}\right.\right.$$

$$\Big\downarrow 5493$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2c}}{2cx^2}\right.\right.$$

$$\Big\downarrow 5491$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\right.\right.$$

$$\Big\downarrow 3042$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2c}}{\sqrt{}}\right)}{\sqrt{c}}\right.\right.\right.$$

$$\Big\downarrow 4671$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow 3011$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow 2720$$

3.419. $\int\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}dx$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```

output
```
-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(c*x^3) + a*(c*(-1/2*(Sqrt[c +
a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) - (a^2*Sqrt[1
+ a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]
*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*A
rcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])]))
)/(2*Sqrt[c + a^2*c*x^2])) + (a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*Ar
cTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])]
 - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcT
an[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])
```

3.419.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.419. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.419. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.419.4 Maple [A] (verified)

Time = 3.85 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\sqrt{c(ax-i)(ax+i)}\, \arctan(ax)\left(2x^2\arctan(ax)^2a^2+6a^2x^2+3x\arctan(ax)a+2\arctan(ax)^2\right)}{6x^3}-\frac{a^3\sqrt{c(ax-i)(ax+i)}\,\left(\arctan(ax)^2\right.}{}$

3.419. $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx$

input
```
int(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*x^2*arctan(a*x)^2*a^2+6*a^2*
x^2+3*x*arctan(a*x)*a+2*arctan(a*x)^2)/x^3-1/2*a^3*(c*(a*x-I)*(I+a*x))^(1/
2)*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^2*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(
a^2*x^2+1)^(1/2)-2*a^3*(c*(a*x-I)*(I+a*x))^(1/2)*arctanh((1+I*a*x)/(a^2*x^
2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

3.419.5 Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x^4}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

3.419.6 Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}{x^4}\,dx$$

input
```
integrate(atan(a*x)**3*(a**2*c*x**2+c)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**4, x)
```

3.419.7 Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x^4}\,dx$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

3.419.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.419.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

3.420 $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

3.420.1 Optimal result

Integrand size = 24, antiderivative size = 652

$$
\begin{aligned}
\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx &= \frac{cx\sqrt{c+a^2cx^2}}{420a^3}-\frac{cx^3\sqrt{c+a^2cx^2}}{140a} \\
&-\frac{163c\sqrt{c+a^2cx^2}\arctan(ax)}{840a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\arctan(ax)}{60a^2} \\
&+\frac{1}{35}cx^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{9cx\sqrt{c+a^2cx^2}\arctan(ax)^2}{112a^3} \\
&-\frac{23cx^3\sqrt{c+a^2cx^2}\arctan(ax)^2}{280a}-\frac{1}{14}acx^5\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&-\frac{51ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{280a^4\sqrt{c+a^2cx^2}} \\
&-\frac{2c\sqrt{c+a^2cx^2}\arctan(ax)^3}{35a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{35a^2} \\
&+\frac{8}{35}cx^4\sqrt{c+a^2cx^2}\arctan(ax)^3 \\
&+\frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{23c^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{120a^4} \\
&+\frac{51ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}} \\
&-\frac{51ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}} \\
&-\frac{51c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}} \\
&+\frac{51c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
23/120*c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4+51/280*I*c^2*a
rctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4
/(a^2*c*x^2+c)^(1/2)-51/280*I*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arct
an(a*x)^2*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-51/280*I*c^2*arctan(a*
x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x
^2+c)^(1/2)-51/280*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+
1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+51/280*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+1/420*c*x*(a^2*c*x^2+
c)^(1/2)/a^3-1/140*c*x^3*(a^2*c*x^2+c)^(1/2)/a-163/840*c*arctan(a*x)*(a^2*
c*x^2+c)^(1/2)/a^4+1/60*c*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+1/35*c*x
^4*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+9/112*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(
1/2)/a^3-23/280*c*x^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-1/14*a*c*x^5*arc
tan(a*x)^2*(a^2*c*x^2+c)^(1/2)-2/35*c*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^
4+1/35*c*x^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2+8/35*c*x^4*arctan(a*x)^
3*(a^2*c*x^2+c)^(1/2)+1/7*a^2*c*x^6*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

3.420.2 Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \frac{c\sqrt{c+a^2cx^2}\left(64\left(309i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-259\text{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)\right.\right.}{}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(64*((309*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^
2 - 259*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[a*x]*PolyLog[2,
(-I)*E^(I*ArcTan[a*x])] + (309*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x
])] + 309*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 309*PolyLog[3, I*E^(I*ArcTa
n[a*x])]) + 2688*((-11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 10*Arc
Tanh[(a*x)/Sqrt[1 + a^2*x^2]] + (11*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 11*Poly
Log[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a*x])]) - 56*
(1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*(-1 + 5*C
os[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] + 11*Cos[4*
ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*ArcTan[a*x]
])) + (1 + a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*ArcTan[a*x]] +
35*Cos[4*ArcTan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*ArcTan[a*x]] + 30
9*Cos[4*ArcTan[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a*x]] + 88*Sin[
4*ArcTan[a*x]] + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(211*Sin[2*ArcTa
n[a*x]] - 60*Sin[4*ArcTan[a*x]] + 103*Sin[6*ArcTan[a*x]]))))/(53760*a^4*Sq
rt[1 + a^2*x^2])
```

3.420.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)^3dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(c\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\right.\right.$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$\downarrow 5465$$

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int\right.\right.$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$\downarrow 5425$$

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\right.$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$\downarrow 5423$$

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2}}{3a^2c}\right.\right.$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$\downarrow 3042$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(\right.\right.$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\right.}{}\right.\right.$$

$$\Big\downarrow \text{5487}$$

3.420. $\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\right.\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 4669$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\right.\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 3011$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\right.\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

3.420. $\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{5a}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}}{3a}\right.}{}\right.\right.$$

$$\Big\downarrow \; 5465$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{5a}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+}}{3a}\right.}{}\right.\right.$$

$$\Big\downarrow \; 224$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a\ldots}{5a\ldots}\ldots\right.}{\ldots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+\ldots}}{3a\ldots}\ldots\right.}{\ldots}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a\ldots}{5a\ldots}\ldots\right.}{\ldots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+\ldots}}{3a\ldots}\ldots\right.}{\ldots}\right.\right.$$

$$\Big\downarrow \text{5425}$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a}{5a}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+}}{3a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a}{5a}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+}}{3a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\cdots\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{5a\cdots}\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+\cdots}}{3a\cdots}\cdots\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{5a}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+}}{3a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{5a}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+}}{3a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 5487}$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\ldots\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\ldots\right.}{5a}\right.\right.$$

$$\Big\downarrow 262$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\ldots}{3a}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}-\ldots}{2a}\right.}{5a}\right.\right.$$

$$\Big\downarrow 224$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3 a}{\sqrt{a}}}{}}{3a}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}-}{2a}\right.}{5a}\right.\right.$$

$$\Big\downarrow \; 219$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3 a}{\sqrt{a}}}{}}{3a}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2}{}}{2a}\right.}{5a}\right.\right.$$

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

input
```
Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.420.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.420. $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.420. $\int x^3(c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx$

3.420.4 Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.72

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(240\arctan(ax)^3a^6x^6-120a^5\arctan(ax)^2x^5+384a^4\arctan(ax)^3x^4+48\arctan(ax)a^4x^4-138a^3\arctan(ax)^2x^3+\right.}{1680a^4}$

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1680*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(240*arctan(a*x)^3*a^6*x^6-120*a^5*
arctan(a*x)^2*x^5+384*a^4*arctan(a*x)^3*x^4+48*arctan(a*x)*a^4*x^4-138*a^3
*arctan(a*x)^2*x^3+48*arctan(a*x)^3*x^2*a^2-12*a^3*x^3+28*a^2*arctan(a*x)*
x^2+135*a*arctan(a*x)^2*x-96*arctan(a*x)^3+4*a*x-326*arctan(a*x))+17/560*c
*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-1
7/560*c*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1
/2)-23/60*I*c/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(
1/2))/(a^2*x^2+1)^(1/2)
```

3.420.5 Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^5 + c*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

3.420.6 Sympy [F]

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\int x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3(ax)\,dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

3.420.7 Maxima [F]

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan(ax)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3, x)
```

3.420.8 Giac [F(-2)]

Exception generated.

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int x^3 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^{3/2} \, dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.421 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

3.421.1 Optimal result

Integrand size = 24, antiderivative size = 882

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=-\frac{c\sqrt{c+a^2cx^2}}{30a^3}-\frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3}$$
$$+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)}{12a^2}+\frac{1}{20}cx^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{31c\sqrt{c+a^2cx^2}\arctan(ax)^2}{240a^3}$$
$$-\frac{19cx^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{120a}-\frac{1}{10}acx^4\sqrt{c+a^2cx^2}\arctan(ax)^2$$
$$+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)^3}{16a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\arctan(ax)^3$$
$$+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{8a^3\sqrt{c+a^2cx^2}}$$
$$+\frac{41ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^3\sqrt{c+a^2cx^2}}$$
$$-\frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{16a^3\sqrt{c+a^2cx^2}}$$
$$+\frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)^2\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{16a^3\sqrt{c+a^2cx^2}}$$
$$-\frac{41ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{120a^3\sqrt{c+a^2cx^2}}+\frac{41ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{120a^3\sqrt{c+a^2cx^2}}$$
$$+\frac{3c^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}$$
$$-\frac{3c^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}$$
$$+\frac{3ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}-\frac{3ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}$$

output
```
-1/60*(a^2*c*x^2+c)^(3/2)/a^3+1/8*I*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)
)*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/8*I*c^2*polylo
g(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1
/2)+3/16*I*c^2*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+41/120*I*c^2*polylog(2,I*(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/8*I*c^2*
polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+
c)^(1/2)-3/16*I*c^2*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3/8*c^2*arctan(a*x)*polylog(3,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-
3/8*c^2*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/a^3/(a^2*c*x^2+c)^(1/2)+41/60*I*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-41/120*I*c^2*
polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c
*x^2+c)^(1/2)-1/30*c*(a^2*c*x^2+c)^(1/2)/a^3+1/12*c*x*arctan(a*x)*(a^2*c*x
^2+c)^(1/2)/a^2+1/20*c*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+31/240*c*arctan
(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-19/120*c*x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(
1/2)/a-1/10*a*c*x^4*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)+1/16*c*x*arctan(a*x)
^3*(a^2*c*x^2+c)^(1/2)/a^2+7/24*c*x^3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)+1/
6*a^2*c*x^5*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

3.421.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4015 vs. $2(882) = 1764$.

Time = 18.47 (sec) , antiderivative size = 4015, normalized size of antiderivative = 4.55

$$\int x^2 (c + a^2 cx^2)^{3/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
(c*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (
Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1
 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLo
g[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)
]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[
a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a
*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi
/2 - ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi
/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 -
 ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(
Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + Pol
yLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^
4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*L
og[1 + E^(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan
[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (P
i/2 + (-1/2*Pi + ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + Arc
Tan[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 -
 ArcTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (P
i/2 + (-1/2*Pi + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTa
n[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x...
```

3.421.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^3dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^3dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(c\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+ a^2c\left(a^2c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3 \text{ar}}{\sqrt{c}}\right.\right.$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right.\right.$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5}{}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5}{}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5}{}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a}}{}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5}{}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a}}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$

$$\Big\downarrow 5465$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\right.}{2a}\right.\right.$$

$$\Big\downarrow 5425$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\right.}{2}\right.\right.$$

$$\Big\downarrow 5421$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\sqrt{a^2x^2+1}(3(i\arctan(\right.\right.$$

$$\Big\downarrow 5487$$

3.421. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{arct}\cdots}{4a^2c}\cdots\right.}{\cdots}\cdots\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\cdots}{2a\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \textcolor{blue}{5425}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{arct}\cdots}{4a^2c}\cdots\right.}{\cdots}\cdots\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\cdots}{2a\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \textcolor{blue}{5423}$$

3.421. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{arct}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{arct}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{4669}$$

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\operatorname{arct}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a}\right.}{}\right.\right.$$

$$\Big\downarrow \; 3011$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\operatorname{arct}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a}\right.}{}\right.\right.$$

$$\Big\downarrow \; 5465$$

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{arct}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}}{3a}\right.}{4a}\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{arct}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2}{}}{}\right.}{4a}\right.\right.$$

$\downarrow$ 5421

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax}}{\sqrt{1-i}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{4a}\right.\right.$$

↓ 5487

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\cdots}{2a}\cdots\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax\cdots}}{\sqrt{1-i\cdots}}\right)}{a}\cdots\right.}{\cdots}\right.}{3a^2}\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 241

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}\,dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx}{4a^2}\right)}{5a}-\cdots}{2a}\cdots\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax\cdots}}{\sqrt{1-i\cdots}}\right)}{a}\cdots\right.}{\cdots}\right)}{3a^2}\cdots\right.}{\cdots}\cdots\right.\right.$$

↓ 243

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax-}}{\sqrt{1-i}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{}\right.\right.$$

$\downarrow$ 53

3.421. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c} - \frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c} - \frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c} - \frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c} - \frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a} - \frac{3\int \frac{x^2 \ldots}{\sqrt{\ldots}}}{\ldots}\ldots\right.}{5a}\ldots}{2a}\ldots\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c} - \frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c} - \frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax\ldots}}{\sqrt{1-i\ldots}}\right)}{a}\ldots\right.}{\ldots}\ldots\right.}{3a^2\ldots}\ldots\right.}{\ldots}\ldots\right.\right.$$

↓ 2009

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan}{\sqrt{a^2cx}}}{4a^2}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax-}}{\sqrt{1-i}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{}\right.\right.$$

↓ 5425

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arcta}{\sqrt{a^2cx}}}{4a^2}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax}}{\sqrt{1-i}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{}\right.\right.$$

↓ 5421

3.421. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan}{\sqrt{a^2cx}}}{4a^2}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+}}{\sqrt{1-iax}}\right.}{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

$\Big\downarrow$ 5423

3.421. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan}{\sqrt{a^2cx}}}{4a^2}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+}}{\sqrt{1-iax}}\right.}{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 3042

3.421. $\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\operatorname{arcta}\cdots}{\sqrt{a^2cx\cdots}}}{4a^2\cdots}\cdots\right.}{5a}\cdots}{2a}\cdots\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+\cdots}}{\sqrt{1-iax\cdots}}\right.}{a}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.\right.$$

↓ 4669

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan}{\sqrt{a^2cx}}}{4a^2}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+}}{\sqrt{1-iax}}\right)}{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

3.421.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.421. $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.421. $\int x^2(c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx$

3.421.4 Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.58

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(40\arctan(ax)^3a^5x^5-24a^4\arctan(ax)^2x^4+70\arctan(ax)^3a^3x^3+12\arctan(ax)x^3a^3-38x^2\arctan(ax)^2a^2+15\,\mathrm{ar}\right.}{240a^3}$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/240*c/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*a^4*arc
tan(a*x)^2*x^4+70*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*x^3*a^3-38*x^2*arct
an(a*x)^2*a^2+15*arctan(a*x)^3*a*x-4*a^2*x^2+20*x*arctan(a*x)*a+31*arctan(
a*x)^2-12)+1/240*c*(c*(a*x-I)*(I+a*x))^(1/2)*(15*arctan(a*x)^3*ln(1+I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-45*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+45*I*arct
an(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+82*arctan(a*x)*ln(1+I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-82*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*
x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(4,-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-90*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-82*I*dilog(
1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+82*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
)))/a^3/(a^2*x^2+1)^(1/2)
```

3.421.5 Fricas [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \int (a^2cx^2+c)^{\frac{3}{2}}x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

3.421.6 Sympy [F]

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int x^2 \left(c(a^2x^2+1)\right)^{\frac{3}{2}} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

3.421.7 Maxima [F]

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3, x)
```

3.421.8 Giac [F]

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \int x^2\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.422 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

3.422.1 Optimal result

Integrand size = 22, antiderivative size = 477

$$
\begin{aligned}
\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx &= -\frac{cx\sqrt{c+a^2cx^2}}{20a} \\
&+\frac{9c\sqrt{c+a^2cx^2}\arctan(ax)}{20a^2}+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{10a^2} \\
&-\frac{9cx\sqrt{c+a^2cx^2}\arctan(ax)^2}{40a}-\frac{3x(c+a^2cx^2)^{3/2}\arctan(ax)^2}{20a} \\
&+\frac{9ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{20a^2\sqrt{c+a^2cx^2}}+\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{5a^2c} \\
&-\frac{c^{3/2}\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{2a^2}-\frac{9ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}} \\
&+\frac{9ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}} \\
&+\frac{9c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}} \\
&-\frac{9c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
1/10*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^2-3/20*x*(a^2*c*x^2+c)^(3/2)*arctan
(a*x)^2/a+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/a^2/c-1/2*c^(3/2)*arctanh(
a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^2+9/20*I*c^2*arctan((1+I*a*x)/(a^2*x^2+
1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-9/20*I*c
^2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/a^2/(a^2*c*x^2+c)^(1/2)+9/20*I*c^2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+9/20*c^2*polylog(
3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2
)-9/20*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/
(a^2*c*x^2+c)^(1/2)-1/20*c*x*(a^2*c*x^2+c)^(1/2)/a+9/20*c*arctan(a*x)*(a^2
*c*x^2+c)^(1/2)/a^2-9/40*c*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a
```

3.422.2 Mathematica [A] (verified)

Time = 2.70 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \frac{c\sqrt{c+a^2cx^2}\Big(960\Big(i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-\operatorname{arctanh}\Big(\frac{ax}{\sqrt{1+a^2x^2}}\Big)-i\operatorname{ar}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(960*(I*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - A
rcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])] + I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + PolyLog[3, (-I)
*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])]) + 48*((-11*I)*ArcTa
n[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 10*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] +
 (11*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x
]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
+ 11*PolyLog[3, I*E^(I*ArcTan[a*x])]) + 80*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]
*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*Sin[2*ArcTan[
a*x]]) - (1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*
(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] +
11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*Ar
cTan[a*x]])))/(960*a^2*Sqrt[1 + a^2*x^2])
```

3.422.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.80, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5465, 5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \ 5465$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx}{5a}$$

$$\Big\downarrow \ 5415$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}\right)}{5a}$$

$$\Big\downarrow \ 211$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\text{ar}\ldots$$

$$\Big\downarrow \ 224$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\ldots\right.\right.}{5a}$$

$$\Big\downarrow \ 219$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\ldots}{\sqrt{a^2c\ldots}}\right.}{2a}\ldots\right.\right.}{5a}$$

$$\Big\downarrow \; 5415$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\arctan(ax)^2\left(a\right.}{5a}$$

$$\Big\downarrow \; 224$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\,\mathrm{arcta}\right.}{5a}$$

$$\Big\downarrow \; 219$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\frac{1}{4}x\,\mathrm{arctan}\right.}{5a}$$

$$\Big\downarrow \; 5425$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\frac{1}{4}x\,\mathrm{ar}\right.}{5a}$$

$$\Big\downarrow \; 5423$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a}{\sqrt{a^2}}\right.}{a}\right.\right.}{5}$$

$$\Big\downarrow \; 3042$$

3.422. $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arcta}}{}\right.\right.$$

$\Big\downarrow$ 4669

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan\left(e^{i\arctan(ax)}\right)\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$\Big\downarrow$ 3011

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan}\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$\Big\downarrow$ 2720

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLo}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$\Big\downarrow$ 7143

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

3.422. $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

output
```
((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/(5*a^2*c) - (3*(-1/6*((c + a^2*c*x^2
)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 + (c*((
x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x
^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a
^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*Arc
Tan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog
[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a
*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])))/4))
/(5*a)
```

3.422.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.422. $\int x(c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx$

3.422.4 Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.88

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(8a^4\arctan(ax)^3x^4-6a^3\arctan(ax)^2x^3+16\arctan(ax)^3x^2a^2+4a^2\arctan(ax)x^2-15a\arctan(ax)^2x+8\arctan(a\right.}{40a^2}$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/40*c/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(8*a^4*arctan(a*x)^3*x^4-6*a^3*arctan
(a*x)^2*x^3+16*arctan(a*x)^3*x^2*a^2+4*a^2*arctan(a*x)*x^2-15*a*arctan(a*x
)^2*x+8*arctan(a*x)^3-2*a*x+22*arctan(a*x))+3/40*c*(c*(a*x-I)*(I+a*x))^(1/
2)*(-I*arctan(a*x)^3+3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6
*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+3/40*c*(c*(a*x-I)*(I+a*x
))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+I*c/a^2*(c*(a*x-I)*(I
+a*x))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

3.422.5 Fricas [F]

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

3.422.6 Sympy [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

3.422.7 Maxima [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3, x)
```

3.422.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.423 $\int (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx$

3.423.1 Optimal result

Integrand size = 21, antiderivative size = 760

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = -\frac{c\sqrt{c+a^2cx^2}}{4a} + \frac{1}{4}cx\sqrt{c+a^2cx^2}\arctan(ax)$$
$$-\frac{9c\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a} - \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{4a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^3$$
$$+\frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax)^3 - \frac{3ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{4a\sqrt{c+a^2cx^2}} - \frac{5ic^2\sqrt{1+a^2x^2}\arctan(ax)}{a\sqrt{c+a^2c}}$$

output
```
-1/4*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(
a*x)^3-5*I*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^
2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+9/4*I*c^2*polylog(4,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-5/2*I*c^2*polylog(2,I*(1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-9/4
*I*c^2*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*
c*x^2+c)^(1/2)-9/8*I*c^2*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+5/2*I*c^2*polylog(2,-I*(1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-9/4*c^
2*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/
a/(a^2*c*x^2+c)^(1/2)+9/4*c^2*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3/4*I*c^2*arctan((1+I*a*x
)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
+9/8*I*c^2*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^
2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-1/4*c*(a^2*c*x^2+c)^(1/2)/a+1/4*c*x*arcta
n(a*x)*(a^2*c*x^2+c)^(1/2)-9/8*c*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+3/8*c
*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

3.423.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2105 vs. 2(760) = 1520.

Time = 12.73 (sec) , antiderivative size = 2105, normalized size of antiderivative = 2.77

$$\int \left(c + a^2cx^2\right)^{3/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
((-1/2*I)*c*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2
)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*
E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
+ (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^
(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2
]) + (c*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]
) + (Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] -
Log[1 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - P
olyLog[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2
*x^2)]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - Ar
cTan[a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - Arc
Tan[a*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(
I*(Pi/2 - ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(
I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(P
i/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E
^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))]
+ PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a
*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a...
```

3.423.3 Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 652, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5415, 5413, 5415, 5425, 5421, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{2}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+ \frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{2}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^3dx+\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{2}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{3}{4}c\left(3c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2c}}{2a}\right.$$
$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{}}{\sqrt{}}\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c}\right.$$
$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{5423}$$

3.423. $\int\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{\sqrt{a^2cx^2}}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{4669}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int\arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int\arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\operatorname{ar}\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{3011}$$

3.423. $\int\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow 7163$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\,a\right.\right.\right.}{}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow 2720$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arct}\right.\right.\right.}{}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow 7143$$

3.423. $\int\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2c\right.$$

$$\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}(3(i\,\mathrm{ar}\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
-1/4*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/a + (x*(c + a^2*c*x^2)^(3/2)*Ar
cTan[a*x]^3)/4 + (c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 -
 I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*S
qrt[c + a^2*c*x^2])))/2 + (3*c*((-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*
a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-
2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2
, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((
-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I
*ArcTan[a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2
*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^
(I*ArcTan[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(2*a*Sqrt[c + a^2*c
*x^2])))/4
```

3.423.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.423. $\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

3.423. $\int (c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.423.4 Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.61

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\left(2\arctan(ax)^3a^3x^3-2x^2\arctan(ax)^2a^2+5\arctan(ax)^3ax+2x\arctan(ax)a-11\arctan(ax)^2-2\right)}{8a}-\frac{c\sqrt{c(ax-i)(}}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c/a*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*x^2*arctan(a*
x)^2*a^2+5*arctan(a*x)^3*a*x+2*x*arctan(a*x)*a-11*arctan(a*x)^2-2)-1/8*c*(
c*(a*x-I)*(I+a*x))^(1/2)*(3*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-3*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*arctan(a*x)^2*
polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+9*I*arctan(a*x)^2*polylog(2,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+20*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+18*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*arctan(a*x
)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*arctan(a*x)*polylog(3,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+18*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*po
lylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+20*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

3.423.5 Fricas [F]

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2cx^2+c)^{\frac{3}{2}} \arctan(ax)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3, x)
```

3.423.6 Sympy [F]

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \int (c(a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

3.423.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3, x)
```

3.423.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int \mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.424 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx$

3.424.1 Optimal result

Integrand size = 24, antiderivative size = 726

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = c\sqrt{c+a^2cx^2}\arctan(ax) - \frac{1}{2}acx\sqrt{c+a^2cx^2}\arctan(ax)^2$$
$$+ \frac{7ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} + c\sqrt{c+a^2cx^2}\arctan(ax)^3$$
$$+ \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3 - \frac{2c^2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$-c^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) + \frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{7ic^2\sqrt{1+a^2x^2}\operatorname{arctan}}{}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3-c^(3/2)*arctanh(a*x*c^(1/2)/(a^2*c*x
^2+c)^(1/2))+3*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-2*c^2*arctan(a*x)^3*arctanh((1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7*I*c^2*arctan
(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x
^2+c)^(1/2)+6*I*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+7*I*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(
a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+7*I*c^2*arctan(a*x)*polylog(2
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*c^
2*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)+7*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7*c^2*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*c^2*arctan(a*x)*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*c^2*poly
log(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-
3*I*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)+c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-1/2*a*c*x*arcta
n(a*x)^2*(a^2*c*x^2+c)^(1/2)+c*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)
```

3.424.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.76

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-3i\pi^4+\frac{24\arctan(ax)}{\sqrt{1+a^2x^2}}+\frac{24a^2x^2\arctan(ax)}{\sqrt{1+a^2x^2}}-\frac{12ax\arctan(ax)^2}{\sqrt{1+a^2x^2}}-12a\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*((-3*I)*Pi^4 + (24*ArcTan[a*x])/Sqrt[1 + a^2*x^2] +
 (24*a^2*x^2*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (12*a*x*ArcTan[a*x]^2)/Sqrt[
1 + a^2*x^2] - (12*a^3*x^3*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (24*I)*ArcTa
n[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + (32*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2]
+ (40*a^2*x^2*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (8*a^4*x^4*ArcTan[a*x]^3)
/Sqrt[1 + a^2*x^2] + (6*I)*ArcTan[a*x]^4 - 24*ArcTanh[(a*x)/Sqrt[1 + a^2*x
^2]] + 24*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 72*ArcTan[a*x]^2*L
og[1 - I*E^(I*ArcTan[a*x])] + 72*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])
] - 24*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*Pol
yLog[2, E^((-I)*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcT
an[a*x])] - (168*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (168*
I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 144*ArcTan[a*x]*PolyLog[3
, E^((-I)*ArcTan[a*x])] - 144*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] +
 168*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 168*PolyLog[3, I*E^(I*ArcTan[a*x
])] - (144*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (144*I)*PolyLog[4, -E^(I*
ArcTan[a*x])]))/(24*Sqrt[1 + a^2*x^2])
```

3.424.3 Rubi [A] (verified)

Time = 5.84 (sec) , antiderivative size = 629, normalized size of antiderivative = 0.87, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {5485, 5465, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5485, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5493, 5491, 3042, 4671, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{}}{a}}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\downarrow 224$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\downarrow 219$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arcta}}{}}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\downarrow 5425$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{a}}{}}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\downarrow 5423$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(a}{}}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\downarrow 3042$$

3.424. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x}dx$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{4669}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)\right.}{2a\sqrt{a^2cx^2+c}}}{}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int\frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$\Big\downarrow \text{5465}$$

3.424. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x}\,dx$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$\Big\downarrow 5425$$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$\Big\downarrow 5423$$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$\Big\downarrow 3042$$

$$c\left(c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan\right.}{a^2\sqrt{a^2cx^2+c}}\right.$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$\Big\downarrow 4669$$

3.424. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x}\,dx$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arc\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arcta}}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5491}$$

3.424. $\displaystyle\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x}\,dx$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{Pol}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\operatorname{arct}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{4671}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\operatorname{arctar}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

3.424. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x}\,dx$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\right.\right.}{2a\sqrt{a^2cx^2}}\right.$$

$$\Big\downarrow 7163$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\arcta\right.\right.\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\right.\right.}{2a\sqrt{a^2cx^2}}\right.$$

$$\Big\downarrow 2720$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\right.\right.}{2a\sqrt{a^2cx^2}}\right.$$

$$\Big\downarrow 7143$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\right.\right.}{2a\sqrt{a^2cx^2}}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4\right.\right.\right.\right.}{}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

3.424. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x}\,dx$

output
```
a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*a^2*c) - (-((Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c
]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-
2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2,
 (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])
))/(2*a*Sqrt[c + a^2*c*x^2]))/a) + c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLo
g[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[
a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2])) + (
c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*Ar
cTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyL
og[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan
[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3,
 E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2
])
```

3.424.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.424. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.424. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.424. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx$

3.424.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.70

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(2x^2\arctan(ax)^2a^2-3x\arctan(ax)a+8\arctan(ax)^2+6\right)}{6}-\frac{c\sqrt{c(ax-i)(ax+i)}\left(2\arctan(ax)^3\ln\left(\right.\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*x^2*arctan(a*x)^2*a^2-3*x*a
rctan(a*x)*a+8*arctan(a*x)^2+6)-1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*(2*arctan(
a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
+6*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*arctan(a*x)^2*
ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))+14*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1
4*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*arctan(a*x)*po
lylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*arctan(a*x)*polylog(3,(1+I*a*x)/(
a^2*x^2+1)^(1/2))+12*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*polylo
g(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-1
4*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*polylog(3,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.424.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

3.424.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x, x)
```

3.424.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

3.424.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x}\,dx=\int \frac{\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.425 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx$

3.425.1 Optimal result

Integrand size = 24, antiderivative size = 901

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = -\frac{3}{2}ac\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)^3}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\arctan(ax)^3 - \frac{3iac^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6iac^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ac^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{9iac^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} - \frac{9iac^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} - \frac{6iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{9ac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{9ac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{9iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{9iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
3*I*a*c^2*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)-9/2*I*a*c^2*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*c^2*arctan(a*x)^2*a
rctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-
3*I*a*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)+6*I*a*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c^2*arctan(a*x)*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
-6*I*a*c^2*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)
^(1/2)/(a^2*c*x^2+c)^(1/2)-9*I*a*c^2*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a*c^2*polylog(2,I*(1+I*a*x)
^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*c^2*poly
log(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-
9*a*c^2*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)+9*a*c^2*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a*c^2*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+9*I*a*c^2*po
lylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)+9/2*I*a*c^2*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*a*c*arctan(a*x)^2*(a^2*c*x^2+c)^...
```

3.425.2 Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 1387, normalized size of antiderivative = 1.54

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
(a*c*Sqrt[c + a^2*c*x^2]*((-7*I)*Pi^4*Sqrt[1 + a^2*x^2] - (8*I)*Pi^3*Sqrt[
1 + a^2*x^2]*ArcTan[a*x] - (384*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*
x])]*ArcTan[a*x] - 96*ArcTan[a*x]^2 - 96*a^2*x^2*ArcTan[a*x]^2 + (24*I)*Pi
^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (64*ArcTan[a*x]^3)/(a*x) - 32*a*x*Arc
Tan[a*x]^3 + 32*a^3*x^3*ArcTan[a*x]^3 - (32*I)*Pi*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^3 - (64*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3
 + (16*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^4 + 48*Pi^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] - 96*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
^2*Log[1 - I/E^(I*ArcTan[a*x])] - 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[1 + I/E^(I*
ArcTan[a*x])] + 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a
*x])] + 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 8
*Pi^3*Sqrt[1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] - 48*Pi^2*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*Pi*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 64*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] - 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*L
og[1 + E^(I*ArcTan[a*x])] + 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[2*Sqrt[1 + a^2*x^
2]*Sin[(Pi + 2*ArcTan[a*x])/4]^2] + (192*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^
2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] + (48*I)*Pi*Sqrt[1 + a^2*x^2]*(Pi - 4
*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] + (384*I)*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (192*I)*Sqrt[1 + a^2*x^2]*...
```

3.425.3 Rubi [A] (verified)

Time = 5.79 (sec) , antiderivative size = 768, normalized size of antiderivative = 0.85, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.042$, Rules used = {5485, 5415, 5425, 5421, 5423, 3042, 4669, 3011, 5485, 5425, 5423, 3042, 4669, 3011, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{x^2} \, dx$$

$$\Big\downarrow \ 5485$$

$$a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx + c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \ 5415$$

$$a^2c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right)+ c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \; 5425$$

$$a^2c\left(\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2}}{2a}\right. \; c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \; 5421$$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+ a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \; 5423$$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+ a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \; 3042$$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+ a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{\sqrt{a^2cx}}\right.$$

$$\Big\downarrow \; 4669$$

3.425. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\,\mathrm{ar}\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

3.425. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}dx$

$$\Big\downarrow \text{3042}$$

$$c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$
$$c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$
$$c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5479}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$
$$c\left(c\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5493}$$

3.425. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^2}\,dx$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\right.\right.}{}\right.\right.$$

$$\Big\downarrow 5491$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\right.\right.}{}\right.$$

$$\Big\downarrow 3042$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\,\mathrm{ar}\right.\right.}{}\right.$$

$$\Big\downarrow 4671$$

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(a\right.}{\sqrt{a^2cx^2}}\right.\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow 3011$$

3.425. $\int\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-e^{i\arc\right.\right.\right.}{}\right.\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\downarrow \text{2720}$$

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLo}\right.\right.}{}\right.\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\downarrow \text{7143}$$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\downarrow \text{7163}$$

$$c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(3\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\downarrow \text{2720}$$

3.425. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\mathrm{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\mathrm{PolyLog}\left(3\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\mathrm{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2\right.\right.\right.}{}\right.\right.$$

$$a^2c\left(\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\left(3\left(i\,\mathrm{ar}\right.\right.}{}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
a^2*c*((-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/
a))/Sqrt[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[
a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[
4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan
[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLo
g[4, I*E^(I*ArcTan[a*x])]))))/(2*a*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*
x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan
[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^
(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]) +
 (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 +
3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan
[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x
])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*A
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x]
)]))))/Sqrt[c + a^2*c*x^2])
```

3.425. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^2}\,dx$

3.425.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

3.425. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.425. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.425.4 Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.67

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2\left(a^2\arctan(ax)x^2-3ax-2\arctan(ax)\right)}{2x}-\frac{3ica\sqrt{c(ax-i)(ax+i)}\left(-2i\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(a^2*arctan(a*x)*x^2-3*a*x-2
*arctan(a*x))/x-3/2*I*c*a*(c*(a*x-I)*(I+a*x))^(1/2)*(-2*I*arctan(a*x)^2*ln
((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-4*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3
*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+4*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)^3*ln(1-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))-2*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)
*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan(a*x)*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*
I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(2,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(
2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)
```

3.425.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

3.425.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**2, x)
```

3.425.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

3.425.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

3.426 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx$

3.426.1 Optimal result

Integrand size = 24, antiderivative size = 919

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = & -\frac{3ac\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x} \\
& + \frac{6ia^2c^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} \\
& + a^2c\sqrt{c+a^2cx^2}\arctan(ax)^3 - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)^3}{2x^2} \\
& - \frac{3a^2c^2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{6a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
& + \frac{9ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} \\
& - \frac{6ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& + \frac{6ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{9ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} \\
& + \frac{3ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{3ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{9a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& + \frac{6a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{6a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& + \frac{9a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& - \frac{9ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& + \frac{9ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-9/2*I*a^2*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*a^2*c^2*arctan(a*x)^3*arctanh((1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a^2*c^2*arctan(
a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2
+c)^(1/2)+6*I*a^2*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+9*I*a^2*c^2*polylog(4,(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a^2*c^2*arctan(a*x)
*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)
^(1/2)-9*I*a^2*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+6*I*a^2*c^2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c^2*polylog(2,
(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-9*a
^2*c^2*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+6*a^2*c^2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a^2*c^2*polylog(3,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+9*a^2*c^2*arctan(a*x
)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)+9/2*I*a^2*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c^2*polylog(2,-(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*a*c*arctan...
```

3.426.2 Mathematica [A] (verified)

Time = 6.82 (sec) , antiderivative size = 691, normalized size of antiderivative = 0.75

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \frac{a^2c\sqrt{c+a^2cx^2}\left(-12\arctan(ax)^2-3i\pi^4\cot\left(\frac{1}{2}\arctan(ax)\right)+6i\arctan(a\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c*Sqrt[c + a^2*c*x^2]*(-12*ArcTan[a*x]^2 - (3*I)*Pi^4*Cot[ArcTan[a*x]
/2] + (6*I)*ArcTan[a*x]^4*Cot[ArcTan[a*x]/2] - 12*ArcTan[a*x]^2*Cot[ArcTan
[a*x]/2]^2 + 8*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 2*ArcTan[a*x]^3*Co
t[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 24*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2
]*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1
- E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*
ArcTan[a*x])] + 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[
a*x])] - 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] - 24
*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + (72*I)*ArcT
an[a*x]^2*Cot[ArcTan[a*x]/2]*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*(2
+ 3*ArcTan[a*x]^2)*Cot[ArcTan[a*x]/2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (96
*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (9
6*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (48*
I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])] + 144*ArcTan[a*x]*Cot[
ArcTan[a*x]/2]*PolyLog[3, E^((-I)*ArcTan[a*x])] - 144*ArcTan[a*x]*Cot[ArcT
an[a*x]/2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 96*Cot[ArcTan[a*x]/2]*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])] - 96*Cot[ArcTan[a*x]/2]*PolyLog[3, I*E^(I*ArcTa
n[a*x])] - (144*I)*Cot[ArcTan[a*x]/2]*PolyLog[4, E^((-I)*ArcTan[a*x])] - (
144*I)*Cot[ArcTan[a*x]/2]*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3
*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(16*Sqrt[1 ...
```

3.426.3 Rubi [A] (verified)

Time = 8.37 (sec) , antiderivative size = 925, normalized size of antiderivative = 1.01, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5485, 5485, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5493, 5491, 3042, 4671, 3011, 5497, 5479, 5493, 5489, 5491, 3042, 4671, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \; 5465$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \; 5425$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \; 5423$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right) c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \; 3042$$

$$a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\operatorname{arct}}{a^2\sqrt{a^2cx^2+c}}\right.\right. c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \; 4669$$

$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+ a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\,a\right.}{}\right.\right.$$

$$\Big\downarrow \; 3011$$

3.426. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx$

$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 5493$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\,\mathrm{ar}}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 5491$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{I}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\,\mathrm{ar}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 4671$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\,\mathrm{arct}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$c\left(c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\,\mathrm{lo}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \; 3011$$

3.426. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^3}\,dx$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$
$$c\left(c\int\frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5497}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$
$$c\left(c\left(\frac{3}{2}a\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(a\right.\right.}{}\right.$$

$$\Big\downarrow \text{5479}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$
$$c\left(c\left(\frac{3}{2}a\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$
$$c\left(c\left(\frac{3}{2}a\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3}{2c}\right.\right.$$

$$\Big\downarrow \text{5489}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax\right.\right.}{}\right.$$

$$\Big\downarrow \text{5491}$$

3.426. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^3}\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax\right.\right.}{}\right.$$

$\downarrow$ 3042

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax\right.\right.}{}\right.$$

$\downarrow$ 4671

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax\right.\right.}{}\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax\right.\right.}{}\right.$$

$\downarrow$ 7143

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax\right.\right.}{}\right.$$

$\downarrow$ 7163

3.426. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4\right.\right.\right.\right.}{}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]
```

3.426. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^3}\,dx$

output
```
a^2*c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^
2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) -
 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcT
an[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[
a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*Ar
cTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + Pol
yLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[
a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4
, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]) + c*((a^2*c*Sqrt[1 + a^2*x^2
]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLo
g[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan
[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2,
 E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])
] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2] + c*(-1/2*(Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^2) + (3*a*(-((Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])
] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]))/
2 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])]...
```

3.426.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.426. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx$

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.426. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.426. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.426.4 Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 592, normalized size of antiderivative = 0.64

method	result
default	$\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2\left(2a^2\arctan(ax)x^2-3ax-\arctan(ax)\right)}{2x^2}-\frac{3c\,a^2\sqrt{c(ax-i)(ax+i)}\,\left(\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-a\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)^2*(2*a^2*arctan(a*x)*x^2-3*a*x
-arctan(a*x))/x^2-3/2*c*a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(arctan(a*x)^3*ln((1
+I*a*x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2
))+2*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(2,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arct
an(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*arctan(a*x)*polylog(3,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-4*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan
(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,-I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))+4*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^
(1/2)
```

3.426.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

3.426.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3{(ax)}}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**3, x)
```

3.426.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

3.426.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.426.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

3.427 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx$

3.427.1 Optimal result

Integrand size = 24, antiderivative size = 788

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = -\frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)}{x}$$
$$-\frac{ac\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}$$
$$-\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{3x^3}-\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{\sqrt{c+a^2cx^2}}$$
$$-\frac{7a^3c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$-a^3c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)+\frac{7ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}+\frac{3ia^3c^2\sqrt{1+a^2x^2}}{}$$

output
```
-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3-a^3*c^(3/2)*arctanh((a^2*c*x^2+
c)^(1/2)/c^(1/2))+3*I*a^3*c^2*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7*a^3*c^2*arctan(a*x)^2*
arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
+7*I*a^3*c^2*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)-7*I*a^3*c^2*arctan(a*x)*polylog(2,(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a^3*c^2*polylo
g(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6
*I*a^3*c^2*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^
2*c*x^2+c)^(1/2)-7*a^3*c^2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^
2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a^3*c^2*arctan(a*x)*polylog(3,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a^3*c^2*arcta
n(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x
^2+c)^(1/2)+7*a^3*c^2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/(a^2*c*x^2+c)^(1/2)-2*I*a^3*c^2*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*a
rctan(a*x)^3*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^3*c^2*arctan(a*x)
^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c
)^(1/2)-a^2*c*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/x-1/2*a*c*arctan(a*x)^2*(a^2
*c*x^2+c)^(1/2)/x^2-a^2*c*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/x
```

3.427.2 Mathematica [A] (warning: unable to verify)

Time = 9.61 (sec) , antiderivative size = 1508, normalized size of antiderivative = 1.91

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
(a^3*c*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1
+ a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi
^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*
ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2
*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^
2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a
^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (6
4*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192
*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi
^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan
[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[
a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*
x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x
]^2*Log[1 + E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(P
i + 2*ArcTan[a*x])/4]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*Pol
yLog[2, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi -
4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*
I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((4
8*I)*a*Pi^2*x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((19
2*I)*a*Pi*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^...
```

3.427.3 Rubi [A] (verified)

Time = 7.70 (sec) , antiderivative size = 755, normalized size of antiderivative = 0.96, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5485, 5479, 5485, 5425, 5423, 3042, 4669, 3011, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+ c\left(a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+ c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{4669}$$

$$c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+ a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

3.427. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5493}$$

$$c\left(a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-i\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5491}$$

$$c\left(a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+\right.}{3cx^3}\right.$$
$$a^2c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\right.\right.}{}\right.$$

$$\Big\downarrow \text{4671}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan\right.}{\sqrt{a^2c}}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.427. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^4}\,dx$

$$a^2c\Bigg(c\Bigg(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\right.}$$

$$c\Bigg(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\Bigg(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(\right.}$$

$$\Big\downarrow \text{2720}$$

$$a^2c\Bigg(c\Bigg(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}$$

$$c\Bigg(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\Bigg(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(\right.}$$

$$\Big\downarrow \text{5497}$$

$$a^2c\Bigg(c\Bigg(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}$$

$$c\Bigg(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\Bigg(c\Bigg(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{2cx^2}$$

$$\Big\downarrow \text{5479}$$

$$a^2c\Bigg(c\Bigg(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}$$

$$c\Bigg(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\Bigg(c\Bigg(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\Bigg(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2}}{cx}$$

$$\Big\downarrow \text{243}$$

$$a^2c\Bigg(c\Bigg(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}$$

$$c\Bigg(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\Bigg(c\Bigg(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\Bigg(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2}}{cx}$$

$$\Big\downarrow \text{73}$$

3.427. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^4}\,dx$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2}}{cx}\right.\right.\right.\right.$$

$$\Big\downarrow \text{221}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\sqrt{a}\right.}{\sqrt{c}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctan}}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2c}}{cx}\right.\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Poly}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\operatorname{arcta}\right.\right.\right.\right.$$

$$\Big\downarrow \text{4671}$$

3.427. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^4}dx$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(\right.\right.}{}\right.$$
$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(a}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 3011

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(\right.\right.}{}\right.$$
$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(a}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 2720

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(\right.\right.}{}\right.$$
$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(a}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 7143

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.}{}\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyL}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 7163

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-\right.\right.\right.}{}\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyL}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 2720

3.427. $\int\frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}(3\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}(3,\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow 7143$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyL}\right.\right.}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(c*x^3) + a*(c*(-1/2*(Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) - (a^2*Sqrt
[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(
I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])
])))/(2*Sqrt[c + a^2*c*x^2])) + (a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2
*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x
])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*A
rcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])) + a^
2*c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x
^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLo
g[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a
*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[
c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]
*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (
2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I
)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])
] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I
*E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2])
```

3.427. $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^4}\,dx$

3.427.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.427. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx$

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.427. $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.427.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 557, normalized size of antiderivative = 0.71

method	result
default	$-\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(8x^2\arctan(ax)^2a^2+6a^2x^2+3x\arctan(ax)a+2\arctan(ax)^2\right)}{6x^3}-\frac{ic\,a^3\sqrt{c(ax-i)(ax+i)}\,\left(2i\arctan\right.}{}$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*c*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(8*x^2*arctan(a*x)^2*a^2+6*a^
2*x^2+3*x*arctan(a*x)*a+2*arctan(a*x)^2)/x^3-1/2*I*c*a^3*(c*(a*x-I)*(I+a*x
))^(1/2)*(2*I*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*arcta
n(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*I*arctan(a*x)^2*ln((1+I
*a*x)/(a^2*x^2+1)^(1/2)+1)-2*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*arctan(
a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))-14*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-14*arctan(a*x
)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*arctan(a*x)*polylog(2,(1+I*a*
x)/(a^2*x^2+1)^(1/2))+7*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+
2*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+14*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)
^(1/2))-2*I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*polylog(4
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
)))/(a^2*x^2+1)^(1/2)
```

3.427.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

3.427.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**4, x)
```

3.427.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

3.427.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

3.428 $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

3.428.1 Optimal result

Integrand size = 24, antiderivative size = 798

$$\begin{aligned}
\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx &= \frac{85c^2x\sqrt{c+a^2cx^2}}{12096a^3} \\
&- \frac{c^2x^3\sqrt{c+a^2cx^2}}{240a} - \frac{1}{504}ac^2x^5\sqrt{c+a^2cx^2} \\
&- \frac{6157c^2\sqrt{c+a^2cx^2}\arctan(ax)}{60480a^4} - \frac{47c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)}{30240a^2} \\
&+ \frac{67c^2x^4\sqrt{c+a^2cx^2}\arctan(ax)}{2520} + \frac{1}{84}a^2c^2x^6\sqrt{c+a^2cx^2}\arctan(ax) \\
&+ \frac{47c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2}{896a^3} - \frac{205c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)^2}{4032a} \\
&- \frac{103ac^2x^5\sqrt{c+a^2cx^2}\arctan(ax)^2}{1008} - \frac{1}{24}a^3c^2x^7\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&- \frac{115ic^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{1344a^4\sqrt{c+a^2cx^2}} \\
&- \frac{2c^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{63a^4} + \frac{c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{63a^2} \\
&+ \frac{5}{21}c^2x^4\sqrt{c+a^2cx^2}\arctan(ax)^3 + \frac{19}{63}a^2c^2x^6\sqrt{c+a^2cx^2}\arctan(ax)^3 \\
&+ \frac{1}{9}a^4c^2x^8\sqrt{c+a^2cx^2}\arctan(ax)^3 + \frac{1433c^{5/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{15120a^4} \\
&+ \frac{115ic^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{1344a^4\sqrt{c+a^2cx^2}} \\
&- \frac{115ic^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{1344a^4\sqrt{c+a^2cx^2}} \\
&- \frac{115c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{1344a^4\sqrt{c+a^2cx^2}} \\
&+ \frac{115c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{1344a^4\sqrt{c+a^2cx^2}}
\end{aligned}$$

output
```
1433/15120*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4+115/1344*I
*c^3*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/a^4/(a^2*c*x^2+c)^(1/2)-115/1344*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/
2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-115/1344*I*c^3
*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^
4/(a^2*c*x^2+c)^(1/2)-115/1344*c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+115/1344*c^3*polylog(3,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+85/1209
6*c^2*x*(a^2*c*x^2+c)^(1/2)/a^3-1/240*c^2*x^3*(a^2*c*x^2+c)^(1/2)/a-1/504*
a*c^2*x^5*(a^2*c*x^2+c)^(1/2)-6157/60480*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1/
2)/a^4-47/30240*c^2*x^2*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^2+67/2520*c^2*x^
4*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/84*a^2*c^2*x^6*arctan(a*x)*(a^2*c*x^2+
c)^(1/2)+47/896*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3-205/4032*c^2*x
^3*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-103/1008*a*c^2*x^5*arctan(a*x)^2*(a
^2*c*x^2+c)^(1/2)-1/24*a^3*c^2*x^7*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)-2/63*
c^2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^4+1/63*c^2*x^2*arctan(a*x)^3*(a^2*
c*x^2+c)^(1/2)/a^2+5/21*c^2*x^4*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)+19/63*a^
2*c^2*x^6*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)+1/9*a^4*c^2*x^8*arctan(a*x)^3*
(a^2*c*x^2+c)^(1/2)
```

3.428.2 Mathematica [A] (verified)

Time = 5.37 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.07

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(774144\left(-11i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+10\operatorname{arctanh}\left(\frac{a}{\sqrt{1+}}\right.\right.\right.}{}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(774144*((-11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan
[a*x]^2 + 10*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (11*I)*ArcTan[a*x]*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[
a*x])] - 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcT
an[a*x])]) + 256*((-16407*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 127
88*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (16407*I)*ArcTan[a*x]*PolyLog[2, (-I
)*E^(I*ArcTan[a*x])] - (16407*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x]
)] - 16407*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 16407*PolyLog[3, I*E^(I*Ar
cTan[a*x])]) - 16128*(1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*Ar
cTan[a*x]^3*(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*Arc
Tan[a*x]] + 11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] -
 33*Sin[4*ArcTan[a*x]])) + 576*(64*((309*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcT
an[a*x]^2 - 259*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[a*x]*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] + (309*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*Ar
cTan[a*x])] + 309*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 309*PolyLog[3, I*E^
(I*ArcTan[a*x])]) + (1 + a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*A
rcTan[a*x]] + 35*Cos[4*ArcTan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*Arc
Tan[a*x]] + 309*Cos[4*ArcTan[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a
*x]] + 88*Sin[4*ArcTan[a*x]] + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(2
11*Sin[2*ArcTan[a*x]] - 60*Sin[4*ArcTan[a*x]] + 103*Sin[6*ArcTan[a*x]])...
```

3.428.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^5\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+c\int x^3\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int x^7\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^3dx\right)+$$
$$c\left(a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)^3dx\right)$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\left(a^2c\int \frac{x^9\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$c\left(a^2c\left(a^2c\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right.\right.$$

$$\Big\downarrow \textbf{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int \frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \textbf{5465}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int \frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \textbf{5425}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int \frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \textbf{5423}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int \frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$\Big\downarrow \text{3042}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int\frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int\frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int\frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int\frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int\frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int\frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\int\frac{x^8\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{8\int\frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x}{7a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5487}$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 5425$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 5423$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$\downarrow$ 2720

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\arctan}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\arctan}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$\downarrow$ 224

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}\cdots}{7a^2c\cdots}\right.\cdots}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2\cdots}}{\cdots}\right.\cdots}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 219$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}\cdots}{7a^2c\cdots}\right.\cdots}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2\cdots}}{\cdots}\right.\cdots}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 5425$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2-}}{}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 5423$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2-}}{}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}\cdots}{7a^2c\cdots}\right.}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2\cdots}\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}\cdots}{7a^2c\cdots}\right.}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2\cdots}\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}}{3a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{arct}}{7a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 5487$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\cdots}{3a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}\cdots\right.}{7a}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\cdots}{4a}\cdots}{3a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\cdots}{3a}\cdots\right.}{7a}\cdots\right.\right.\right.$$

$$\Big\downarrow \; 224$$

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^8}{9a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\frac{6\int\frac{x^5\,\mathrm{ar}}{\sqrt{a}}}{}}{4a}}{3a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int}{}}{3a}\right.}{7a}\right.\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.428.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.428. $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.428.4 Maple [A] (verified)

Time = 19.27 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.66

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(13440\arctan(ax)^3a^8x^8-5040\arctan(ax)^2a^7x^7+36480\arctan(ax)^3a^6x^6+1440a^6\arctan(ax)x^6-12360a^5\operatorname{arcta}\right.}{}$

input
```
int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/120960*c^2/a^4*(c*(a*x-I)*(I+a*x))^(1/2)*(13440*arctan(a*x)^3*a^8*x^8-50
40*arctan(a*x)^2*a^7*x^7+36480*arctan(a*x)^3*a^6*x^6+1440*a^6*arctan(a*x)*
x^6-12360*a^5*arctan(a*x)^2*x^5+28800*a^4*arctan(a*x)^3*x^4-240*a^5*x^5+32
16*arctan(a*x)*a^4*x^4-6150*a^3*arctan(a*x)^2*x^3+1920*arctan(a*x)^3*x^2*a
^2-504*a^3*x^3-188*a^2*arctan(a*x)*x^2+6345*a*arctan(a*x)^2*x-3840*arctan(
a*x)^3+850*a*x-12314*arctan(a*x))+115/8064*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(
I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*ar
ctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-115/8064*c^2*(c*(a*x-I)*(I+a*
x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-1433/7560*I*c^2/a^4*
(c*(a*x-I)*(I+a*x))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^
(1/2)
```

3.428.5 Fricas [F]

$$\int x^3 (c + a^2 c x^2)^{5/2} \arctan(ax)^3 \, dx = \int (a^2 c x^2 + c)^{\frac{5}{2}} x^3 \arctan(ax)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x)^3, x)
```

3.428.6 Sympy [F]

$$\int x^3 (c + a^2 c x^2)^{5/2} \arctan(ax)^3 \, dx = \int x^3 \left(c \left(a^2 x^2 + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

3.428.7 Maxima [F]

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^3, x)
```

3.428.8 Giac [F(-2)]

Exception generated.

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.428.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int x^3\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.429 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

3.429.1 Optimal result

Integrand size = 24, antiderivative size = 1019

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx=\frac{13c^2\sqrt{c+a^2cx^2}}{6720a^3}$$

$$-\frac{3c(c+a^2cx^2)^{3/2}}{560a^3}-\frac{(c+a^2cx^2)^{5/2}}{280a^3}+\frac{43c^2x\sqrt{c+a^2cx^2}\arctan(ax)}{1344a^2}$$

$$+\frac{29}{560}c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1}{56}a^2c^2x^5\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1373c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{13440a^3}-\frac{737c^2}{}$$

output
```
-3/560*c*(a^2*c*x^2+c)^(3/2)/a^3-1/280*(a^2*c*x^2+c)^(5/2)/a^3+15/64*I*c^3
*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^
2+c)^(1/2)+5/64*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a
^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-15/64*I*c^3*polylog(4,I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+397/840*I*c^3
*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3
/(a^2*c*x^2+c)^(1/2)+15/128*I*c^3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-397/1680*I*c^3*po
lylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x
^2+c)^(1/2)+15/64*c^3*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-15/64*c^3*arctan(a*x)*polylog(
3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)
+397/1680*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(
1/2)/a^3/(a^2*c*x^2+c)^(1/2)-15/128*I*c^3*arctan(a*x)^2*polylog(2,-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+13/6720*
c^2*(a^2*c*x^2+c)^(1/2)/a^3+43/1344*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)/
a^2+29/560*c^2*x^3*arctan(a*x)*(a^2*c*x^2+c)^(1/2)+1/56*a^2*c^2*x^5*arctan
(a*x)*(a^2*c*x^2+c)^(1/2)+1373/13440*c^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)
/a^3-737/6720*c^2*x^2*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a-83/560*a*c^2*x^4
*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)-3/56*a^3*c^2*x^6*arctan(a*x)^2*(a^2*...
```

3.429.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6517 vs. 2(1019) = 2038.

Time = 24.69 (sec) , antiderivative size = 6517, normalized size of antiderivative = 6.40

$$\int x^2 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
Result too large to show
```

3.429.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5485}$$

$$c\int x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+a^2c\int x^4\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx$$

$$\Big\downarrow \text{5485}$$

$$c\left(c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^3dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^3dx\right)+ a^2c\left(a^2c\int x^6\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^3dx\right)$$

$$\Big\downarrow \text{5485}$$

$$c\left(c\left(c\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(a^2c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right.\right.$$
$$a^2c\left(a^2c\left(a^2c\int \frac{x^8\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int \frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int \frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\Bigg)a^2+c\Bigg(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}$$

$$\Big\downarrow \ 3042$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\Bigg)a^2+c\Bigg(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}$$

$$\Big\downarrow \ 4669$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\Bigg)a^2+c\Bigg(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}$$

$$\Big\downarrow \ 3011$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\Bigg)a^2+c\Bigg(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}$$

$$\Big\downarrow \ 5465$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\Bigg)a^2+c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}$$

$$c\Bigg(c\Bigg(c\Bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\Bigg)a^2+c\Bigg(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}$$

$$\Big\downarrow \ 5425$$

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\downarrow \text{5423}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\downarrow \text{3042}$$

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.429. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow 5465$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow 5425$$

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2cx\right.}{}\right.\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \; 5421$$

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a}\right.}{}\right.\right.\right.$$

$\downarrow$ 5487

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}d}{6a^2}\right.}{7a}\right.}{8a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}}{2a}\right.\right.\right.$$

$\downarrow$ 241

3.429. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}d}{6a^2}\right.}{7a}\right.}{8a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}}{2a}\right.\right.\right.$$

↓ 243

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx^2}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.}{8a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}}{2a}\right.\right.\right.$$

$$\Big\downarrow 53$$

3.429. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\cdots\right.}{12a}\cdots\right.}{7a}\cdots\right.}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a}-\frac{3\int\cdots}{\cdots}\right.}{5a}\cdots}{2a\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 2009$$

3.429. $\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\cdots}{12a}\cdots}{7a}\cdots}{\cdots}\cdots\right.\right.\right.$$

$$\left(\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\cdots}{\sqrt{\cdots}}}{\cdots}\cdots\right.}{5a}\cdots}{2a}\cdots\right.\right.\right.$$

$\downarrow$ 5425

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2}{\sqrt{}}}{}\right.}{5a}}{2a}\right.\right.\right.$$

$$\Big\downarrow \ 5421$$

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\cdots}{12a}\cdots\right.}{7a}\cdots\right.}{\cdots}\right.\right.\right.$$

$$\left(\left(c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\cdots}{\sqrt{\cdots}}\cdots}{\cdots}\right.}{5a}\cdots}{2a}\right.\right.\right.\right.\right.$$

↓ 5423

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2}{\sqrt{}}}{}\right.}{5a}}{2a}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2}{\sqrt{}}}{}\right.}{5a}}{2a}\right.\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

3.429.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.429. $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

3.429.4 Maple [A] (verified)

Time = 12.63 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.56

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(1680\arctan(ax)^3a^7x^7-720a^6x^6\arctan(ax)^2+4760\arctan(ax)^3a^5x^5+240\arctan(ax)a^5x^5-1992a^4\arctan(ax)\right.}{}$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13440*c^2/a^3*(c*(a*x-I)*(I+a*x))^(1/2)*(1680*arctan(a*x)^3*a^7*x^7-720*
a^6*x^6*arctan(a*x)^2+4760*arctan(a*x)^3*a^5*x^5+240*arctan(a*x)*a^5*x^5-1
992*a^4*arctan(a*x)^2*x^4+4130*arctan(a*x)^3*a^3*x^3-48*a^4*x^4+696*arctan
(a*x)*x^3*a^3-1474*x^2*arctan(a*x)^2*a^2+525*arctan(a*x)^3*a*x-168*a^2*x^2
+430*x*arctan(a*x)*a+1373*arctan(a*x)^2-94)+1/13440*c^2*(c*(a*x-I)*(I+a*x)
)^(1/2)*(525*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-525*arctan(
a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1575*I*arctan(a*x)^2*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+1575*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+3176*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3
150*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3176*arctan(a*x)
*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(3,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+3150*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150
*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3176*I*dilog(1+I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+3176*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^
2+1)^(1/2)
```

3.429.5 Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x)^3, x)
```

3.429.6 Sympy [F]

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^3 \, dx = \int x^2 \left(c \left(a^2 x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

3.429.7 Maxima [F]

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^3 \, dx = \int \left(a^2 c x^2+c\right)^{\frac{5}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3, x)
```

3.429.8 Giac [F]

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^3 \, dx = \int \left(a^2 c x^2+c\right)^{\frac{5}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.429.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^3 \, dx = \int x^2 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^{5/2} dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.430 $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

3.430.1 Optimal result

Integrand size = 22, antiderivative size = 561

$$\begin{aligned}
\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx =& -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx(c+a^2cx^2)^{3/2}}{140a} \\
&+\frac{15c^2\sqrt{c+a^2cx^2}\arctan(ax)}{56a^2}+\frac{5c(c+a^2cx^2)^{3/2}\arctan(ax)}{84a^2} \\
&+\frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{35a^2}-\frac{15c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2}{112a} \\
&-\frac{5cx(c+a^2cx^2)^{3/2}\arctan(ax)^2}{56a}-\frac{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}{14a} \\
&+\frac{15ic^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{56a^2\sqrt{c+a^2cx^2}} \\
&+\frac{(c+a^2cx^2)^{7/2}\arctan(ax)^3}{7a^2c}-\frac{37c^{5/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{120a^2} \\
&-\frac{15ic^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}} \\
&+\frac{15ic^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}} \\
&+\frac{15c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}} \\
&-\frac{15c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

output
```
-1/140*c*x*(a^2*c*x^2+c)^(3/2)/a+5/84*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^
2+1/35*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a^2-5/56*c*x*(a^2*c*x^2+c)^(3/2)*ar
ctan(a*x)^2/a-1/14*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/a+1/7*(a^2*c*x^2+c)
^(7/2)*arctan(a*x)^3/a^2/c-37/120*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c
)^(1/2))/a^2+15/56*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2
*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-15/56*I*c^3*arctan(a*x)*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/
2)+15/56*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+15/56*c^3*polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)-15/56*c^3*polylog(
3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)
-17/420*c^2*x*(a^2*c*x^2+c)^(1/2)/a+15/56*c^2*arctan(a*x)*(a^2*c*x^2+c)^(1
/2)/a^2-15/112*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a
```

3.430.2 Mathematica [A] (verified)

Time = 4.53 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.28

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(64\left(309i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-259\operatorname{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)\right.\right.}{}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(64*((309*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
]^2 - 259*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[a*x]*PolyLog[2
, (-I)*E^(I*ArcTan[a*x])] + (309*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a
*x])] + 309*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 309*PolyLog[3, I*E^(I*Arc
Tan[a*x])]) + 53760*(I*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - ArcTanh[(
a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
 + I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + PolyLog[3, (-I)*E^(I*Ar
cTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])]) + 4480*(1 + a^2*x^2)^(3/2)*
ArcTan[a*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*Si
n[2*ArcTan[a*x]]) - 112*(48*((11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^
2 - 10*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] - (11*I)*ArcTan[a*x]*PolyLog[2, (-
I)*E^(I*ArcTan[a*x])] + (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])]
 + 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 11*PolyLog[3, I*E^(I*ArcTan[a*x
])]) + (1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*(-
1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] + 11
*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*ArcT
an[a*x]]))) + (1 + a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*ArcTan[
a*x]] + 35*Cos[4*ArcTan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*ArcTan[a*
x]] + 309*Cos[4*ArcTan[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a*x]] +
 88*Sin[4*ArcTan[a*x]] + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(211*...
```

3.430.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.93, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {5465, 5415, 211, 211, 224, 219, 5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2dx}{7a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\int\left(a^2cx^2+c\right)^{3/2}dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)(a^2cx^2}{15a}\right.}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+\right.\right.}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\,\mathrm{ar}\right.}{7a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right.\right.}{7a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\,\mathrm{arcta}}{}\right.\right.\right.}{7a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)}{6a}\right.\right.$$

$$\Big\downarrow \text{211}$$

3.430. $\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}\right.\right.$$

$$\Big\downarrow \ 224$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx\right.\right.\right.$$

$$\Big\downarrow \ 219$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\right.}{2c}\right.\right.\right.$$

$$\Big\downarrow \ 5415$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\arctan(ax\right.\right.$$

$$\Big\downarrow \ 224$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\,a\right.\right.$$

$$\Big\downarrow \ 219$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\frac{1}{4}x\,\text{ar}\right.\right.$$

3.430. $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$\Big\downarrow \ 5425$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\right.\right.$$

$$\Big\downarrow \ 5423$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}}{}\right.\right.\right.$$

$$\Big\downarrow \ 3042$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\right.\right.\right.$$

$$\Big\downarrow \ 4669$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan\left(e^{i\arctan(ax)}\right)\ldots\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \ 3011$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\,\text{PolyLog}\left(2,ie^{i}\ldots\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \ 2720$$

3.430. $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{P}\right.\right.}{2a\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$\Big\downarrow \textbf{7143}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\operatorname{Po}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^3)/(7*a^2*c) - (3*(-1/15*((c + a^2*c*x^
2)^(5/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/6 + (c*(
(x*(c + a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*A
rcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4))/15 + (5*c*(-1/6*((c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^
2)/4 + (c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a)
 + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x
)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTa
n[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E
^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c
*x^2])))/4))/6))/(7*a)
```

3.430.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

3.430. $\int x(c + a^2cx^2)^{5/2} \arctan(ax)^3 \, dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.430.4 Maple [A] (verified)

Time = 8.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.85

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(240\arctan(ax)^3a^6x^6-120a^5\arctan(ax)^2x^5+720a^4\arctan(ax)^3x^4+48\arctan(ax)a^4x^4-390a^3\arctan(ax)^2x^3+\right.}{1680a^2}$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1680*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(240*arctan(a*x)^3*a^6*x^6-120*a^
5*arctan(a*x)^2*x^5+720*a^4*arctan(a*x)^3*x^4+48*arctan(a*x)*a^4*x^4-390*a
^3*arctan(a*x)^2*x^3+720*arctan(a*x)^3*x^2*a^2-12*a^3*x^3+196*a^2*arctan(a
*x)*x^2-495*a*arctan(a*x)^2*x+240*arctan(a*x)^3-80*a*x+598*arctan(a*x))-5/
112*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^
(1/2)+5/112*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2
*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x
^2+1)^(1/2)+37/60*I*c^2/a^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan((1+I*a*x)/(a^
2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

3.430.5 Fricas [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx=\int (a^2cx^2+c)^{\frac{5}{2}}x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(
a*x)^3, x)
```

3.430.6 Sympy [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx=\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

3.430.7 Maxima [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx=\int (a^2cx^2+c)^{\frac{5}{2}}x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3, x)
```

3.430.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.430.9 Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx = \int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.431 $\int (c+a^2cx^2)^{5/2} \arctan(ax)^3 \, dx$

3.431.1 Optimal result

Integrand size = 21, antiderivative size = 870

$$\int (c+a^2cx^2)^{5/2} \arctan(ax)^3 \, dx = -\frac{17c^2\sqrt{c+a^2cx^2}}{60a}$$

$$-\frac{c(c+a^2cx^2)^{3/2}}{60a} + \frac{17}{60}c^2x\sqrt{c+a^2cx^2}\arctan(ax)$$

$$+\frac{1}{20}cx\left(c+a^2cx^2\right)^{3/2}\arctan(ax) - \frac{15c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{16a} - \frac{5c(c+a^2cx^2)^{3/2}\arctan(ax)^2}{24a} - \frac{(c+a^2cx^2)^{5}}{}$$

output
```
-1/60*c*(a^2*c*x^2+c)^(3/2)/a+1/20*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-5/2
4*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a-1/10*(a^2*c*x^2+c)^(5/2)*arctan(a*
x)^2/a+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3+1/6*x*(a^2*c*x^2+c)^(5/2
)*arctan(a*x)^3-5/8*I*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^
3*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-15/8*I*c^3*polylog(4,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+15/16*I*c^3*a
rctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a
/(a^2*c*x^2+c)^(1/2)-15/16*I*c^3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-259/60*I*c^3*arctan(
a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^
2+c)^(1/2)-259/120*I*c^3*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2
*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-15/8*c^3*arctan(a*x)*polylog(3,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+15/8*c^3*
arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(
a^2*c*x^2+c)^(1/2)+259/120*I*c^3*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+15/8*I*c^3*polylog(4,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-17/60*c^2*(
a^2*c*x^2+c)^(1/2)/a+17/60*c^2*x*arctan(a*x)*(a^2*c*x^2+c)^(1/2)-15/16*c^2
*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/a+5/16*c^2*x*arctan(a*x)^3*(a^2*c*x^2+c
)^(1/2)
```

3.431.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4281 vs. $2(870) = 1740$.

Time = 18.94 (sec) , antiderivative size = 4281, normalized size of antiderivative = 4.92

$$\int (c + a^2cx^2)^{5/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
((-1/2*I)*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a
*x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]
^2)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2,
I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])
] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*
E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x
^2]) + (2*c^2*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^
2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x
])] - Log[1 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1
 + a^2*x^2)]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/
2 - ArcTan[a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2
 - ArcTan[a*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[
2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1
 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2
*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLo
g[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*
x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - Ar
cTan[a*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - Ar...
```

3.431.3 Rubi [A] (verified)

Time = 3.64 (sec) , antiderivative size = 954, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5415, 5413, 5413, 5415, 5413, 5415, 5425, 5421, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2} \, dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{5}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+$$
$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2\right.$$
$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2\right.$$
$$\frac{5}{6}c\left(\frac{1}{2}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\mathrm{arcta}}{}\right.$$
$$\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2\right.$$
$$\frac{5}{6}c\left(\frac{1}{2}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+\right.$$
$$\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2\right.$$
$$\frac{5}{6}c\left(\frac{1}{2}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\mathrm{ar}}{\sqrt{}}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5425}$$

3.431. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}\right.$$

$$\frac{5}{6}c\left(\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$

$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(a\right.}{}}{}\right.\right.$$

$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$

$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(a\right.}{}}{}\right.\right.$$

$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.431. $\int\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$

$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(a\right.}{}}{}\right.\right.$$

$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$

$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(a\right.}{}}{}\right.\right.$$

$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$

$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(a\right.}{}}{}\right.\right.$$

$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$\Big\downarrow \text{7163}$$

3.431. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax}{\right.}}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$\Big\downarrow$ 2720

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax}{\right.}}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$\Big\downarrow$ 7143

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax}{\right.}}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

3.431. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx$

output
```
-1/10*((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/a + (x*(c + a^2*c*x^2)^(5/2)*A
rcTan[a*x]^3)/6 + (c*(-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(
3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sq
rt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a
))/(2*Sqrt[c + a^2*c*x^2])))/4))/5 + (5*c*(-1/4*((c + a^2*c*x^2)^(3/2)*Arc
Tan[a*x]^2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/4 + (c*(-1/2*Sqrt[
c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2
*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I
*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/2 + (3*c*
((-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sq
rt[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]
*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (
2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I
)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*...
```

3.431.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.431. $\int (c + a^2cx^2)^{5/2} \arctan(ax)^3 \, dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.431.4 Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.60

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(40\arctan(ax)^3a^5x^5-24a^4\arctan(ax)^2x^4+130\arctan(ax)^3a^3x^3+12\arctan(ax)x^3a^3-98x^2\arctan(ax)^2a^2+16\right.}{240a}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/240*c^2/a*(c*(a*x-I)*(I+a*x))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*a^4*arc
tan(a*x)^2*x^4+130*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*x^3*a^3-98*x^2*arc
tan(a*x)^2*a^2+165*arctan(a*x)^3*a*x-4*a^2*x^2+80*x*arctan(a*x)*a-299*arct
an(a*x)^2-72)-1/240*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(75*arctan(a*x)^3*ln(1+I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-75*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))-225*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+225
*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+518*arctan(a*x)*
ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arctan(a*x)*polylog(3,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-518*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45
0*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*I*polylog(4,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-518*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+518*I*dilog(1-I*(1+I*a*x)/(
a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

3.431.5 Fricas [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3, x)
```

3.431.6 Sympy [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

3.431.7 Maxima [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3, x)
```

3.431.8 Giac [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx = \int \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

3.432 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx$

3.432.1 Optimal result

Integrand size = 24, antiderivative size = 845

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2} + \frac{29}{20}c^2\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{10}c(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{29}{40}ac^2x\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{3}{20}acx(c+a^2cx^2)^{3/2}\arctan(ax)^2 + \frac{149ic^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{20\sqrt{c+a^2cx^2}} + c^2\sqrt{c+a^2cx^2}\,\text{arctar}$$

output
```
1/10*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-3/20*a*c*x*(a^2*c*x^2+c)^(3/2)*arct
an(a*x)^2+1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3+1/5*(a^2*c*x^2+c)^(5/2)*
arctan(a*x)^3-3/2*c^(5/2)*arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))-6*I*c^3
*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)-2*c^3*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*c^3*arctan(a*x)^2*p
olylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/
2)-149/20*I*c^3*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*c^3*arctan(a*x)*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+149/20*
c^3*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2
+c)^(1/2)-149/20*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)+6*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+149/20*I*c^3*arctan((1+I*a*
x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+
149/20*I*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1/20*a*c^2*x*(a^2*c*x^2+c)^(1/2)+29/20*c^2*a
rctan(a*x)*(a^2*c*x^2+c)^(1/2)-29/40*a*c^2*x*arctan(a*x)^2*(a^2*c*x^2+c...
```

3.432.2 Mathematica [A] (verified)

Time = 5.86 (sec) , antiderivative size = 723, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\Big(-120i\pi^4+960(1+a^2x^2)^{3/2}\arctan(ax)-150(1+a^2x^2)^5}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*((-120*I)*Pi^4 + 960*(1 + a^2*x^2)^(3/2)*ArcTan[a
*x] - 150*(1 + a^2*x^2)^(5/2)*ArcTan[a*x] + (1392*I)*ArcTan[E^(I*ArcTan[a*
x])]*ArcTan[a*x]^2 + 960*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + 640*(1 + a^2*x^
2)^(3/2)*ArcTan[a*x]^3 + 32*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3 + (240*I)*Ar
cTan[a*x]^4 - 1440*ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + 960*(1 + a^2*x^2)^(3
/2)*ArcTan[a*x]*Cos[2*ArcTan[a*x]] - 216*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]*C
os[2*ArcTan[a*x]] - 160*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3*Cos[2*ArcTan[a*x
]] - 66*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 960*ArcTan[a*
x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 2880*ArcTan[a*x]^2*Log[1 - I*E^(I*Arc
Tan[a*x])] + 2880*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 960*ArcTan[
a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (2880*I)*ArcTan[a*x]^2*PolyLog[2, E^((
-I)*ArcTan[a*x])] + (2880*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])]
- (7152*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (7152*I)*ArcTa
n[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 5760*ArcTan[a*x]*PolyLog[3, E^((-
I)*ArcTan[a*x])] - 5760*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 7152*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 7152*PolyLog[3, I*E^(I*ArcTan[a*x])]
- (5760*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (5760*I)*PolyLog[4, -E^(I*Ar
cTan[a*x])] - 12*(1 + a^2*x^2)^(5/2)*Sin[2*ArcTan[a*x]] - 480*(1 + a^2*x^2
)^(3/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 6*(1 + a^2*x^2)^(5/2)*ArcTan[a*
x]^2*Sin[2*ArcTan[a*x]] - 6*(1 + a^2*x^2)^(5/2)*Sin[4*ArcTan[a*x]] + 33...
```

3.432.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{x} \, dx$$

$$\Big\downarrow \ 5485$$

$$a^2c\int x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \ 5465$$

$$a^2c\left(\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx}{5a}\right)+ c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \ 5415$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2\right.\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \; 211$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \; 224$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx\right.\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \; 219$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(a\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \; 5415$$

3.432. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arct\cdots}{\cdots}\right.\right.}{\cdots}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+\cdots}\right.\right.}{\cdots}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\cdots\right.\right.}{\cdots}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+\cdots}}{a}\right.\right.}{\cdots}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{5423}$$

3.432. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \; 3042$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \; 4669$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\,a}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$$\Big\downarrow \; 3011$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\,\mathrm{arc}\right.\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow \; 2720$$

3.432. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5465}$$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5415}$$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(a}{}}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{224}$$

3.432. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\right.}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arc\right.\right.\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow 219$$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\mathrm{ar}}{}}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arc\right.\right.\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow 5425$$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arc\right.\right.\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow 5423$$

3.432. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\text{arcta}}{}}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arc}}\right.\right.\right.}{}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\,\text{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

3.432. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arc\right.\right.}\right.\right.}{}\right.\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right.\right.\right.}{}\right.\right.\right.$$

$\big\downarrow$ 2720

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arc\right.\right.}\right.\right.}{}\right.\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax}\right.\right.\right.}{}\right.\right.\right.$$

$\big\downarrow$ 5485

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+c\sqrt{}}{}\right.\right.$$

$\big\downarrow$ 5465

3.432. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{}}{}}{}\right.\right.$$

$\Big\downarrow$ 5425

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{}}{}}{}\right.\right.$$

$\Big\downarrow$ 5423

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{}}{}}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

output
```
$Aborted
```

3.432. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$

3.432.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.432. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx$

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

3.432. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx$

3.432.4 Maple [A] (verified)

Time = 7.49 (sec) , antiderivative size = 562, normalized size of antiderivative = 0.67

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(24a^4\arctan(ax)^3x^4-18a^3\arctan(ax)^2x^3+88\arctan(ax)^3x^2a^2+12a^2\arctan(ax)x^2-105a\arctan(ax)^2x+184 a\right.}{120}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/120*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(24*a^4*arctan(a*x)^3*x^4-18*a^3*arcta
n(a*x)^2*x^3+88*arctan(a*x)^3*x^2*a^2+12*a^2*arctan(a*x)*x^2-105*a*arctan(
a*x)^2*x+184*arctan(a*x)^3-6*a*x+186*arctan(a*x))-1/40*c^2*(c*(a*x-I)*(I+a
*x))^(1/2)*(40*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-40*arctan(a
*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*I*arctan(a*x)^2*polylog(2,-(1+
I*a*x)/(a^2*x^2+1)^(1/2))+120*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2
+1)^(1/2))-149*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+149*arcta
n(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+298*I*arctan(a*x)*polylog(2,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-298*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+240*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-24
0*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+240*I*polylog(4,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-240*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*
I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-298*polylog(3,-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+298*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.432.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x, x)
```

3.432.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x, x)
```

3.432.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x, x)
```

3.432.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.433 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx$

3.433.1 Optimal result

Integrand size = 24, antiderivative size = 1027

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = -\frac{1}{4}ac^2\sqrt{c+a^2cx^2} + \frac{1}{4}a^2c^2x\sqrt{c+a^2cx^2}\arctan(ax) - \frac{21}{8}ac^2\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{1}{4}ac(c + a^2cx^2)^{3/2}\arctan(ax)^2 - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\arctan(ax)^3 + \frac{1}{4}a^2cx(c+a^2cx^2)^{3/2}\arctan(ax)^3 - \frac{15iac^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{4\sqrt{c+a^2cx^2}} - \frac{11iac^3\sqrt{1+a^2x^2}\,\mathrm{arc}}{\sqrt{c}}$$

output
```
-1/4*a*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+1/4*a^2*c*x*(a^2*c*x^2+c)^(3/2)
*arctan(a*x)^3-45/8*I*a*c^3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-11/2*I*a*c^3*polylog(2,I*(1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*c
^3*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)-11*I*a*c^3*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+11/2*I*a*c^3*polylog(2,-I*(1+
I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*a*
c^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)+45/4*I*a*c^3*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*
(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15/4*I*a*c^3*arctan((1+I*a*x)/(a^2*x
^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+45/8*I*a*
c^3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/(a^2*c*x^2+c)^(1/2)-6*a*c^3*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a
^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-45/4*a*c^3*arctan(a*x)*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+45/4*a*c^
3*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)+6*a*c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*c^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-45/4*I*a*c^3*pol...
```

3.433.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3267 vs. $2(1027) = 2054$.

Time = 15.01 (sec) , antiderivative size = 3267, normalized size of antiderivative = 3.18

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
((-I)*a*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2
)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*
E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
+ (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^
(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] +
 (a*c^2*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1
 + a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*P
i^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x
*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^
2*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a
^2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 +
a^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (
64*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (19
2*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*P
i^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan
[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a
*x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan...
```

3.433.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5415}$$

$$a^2c\left(\frac{1}{2}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\text{arcta}}{}\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5413}$$

$$a^2c\left(\frac{1}{2}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^3dx+\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(\frac{1}{2}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(3c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int\frac{a}{\sqrt{}}\right.\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLo}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLo}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.433. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}\right.}{\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\,\text{a}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx\right)+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\,\text{a}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5415}$$

$$c\left(a^2c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right)-\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\,\text{a}\right.\right.}{}\right.\right.$$

3.433. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

$$\Big\downarrow \text{5425}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int\frac{\text{arcta}}{\sqrt{a^2}}}{2\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.433. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 4669

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 3011

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 5485

3.433. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\big\downarrow$ 5425

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\big\downarrow$ 5423

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\Big(\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\big\downarrow$ 3042

3.433. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{5479}$$

3.433. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5493

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5491

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 3042

3.433. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+c}\cdots}\right.\right.$$

$$\Big\downarrow \color{blue}{4671}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+c}\cdots}\right.\right.$$

$$\Big\downarrow \color{blue}{3011}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+c}\cdots}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
$Aborted
```

3.433. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^2}\,dx$

3.433.3.1 Defintions of rubi rules used

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

3.433. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx$

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.433. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.433.4 Maple [A] (verified)

Time = 7.82 (sec) , antiderivative size = 655, normalized size of antiderivative = 0.64

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(2a^4\arctan(ax)^3x^4-2a^3\arctan(ax)^2x^3+9\arctan(ax)^3x^2a^2+2a^2\arctan(ax)x^2-23a\arctan(ax)^2x-8\arctan(a\right.}{8x}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*(2*a^4*arctan(a*x)^3*x^4-2*a^3*arctan(a*
x)^2*x^3+9*arctan(a*x)^3*x^2*a^2+2*a^2*arctan(a*x)*x^2-23*a*arctan(a*x)^2*
x-8*arctan(a*x)^3-2*a*x)/x+1/8*I*c^2*a*(c*(a*x-I)*(I+a*x))^(1/2)*(24*I*arc
tan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-90*I*arctan(a*x)*polylog(3,I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))+48*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
48*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*arctan(a*x)^2*polylog(2,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+45*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+15*I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*a
rctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*arctan(a*x)*polylog(
3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+44*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-48*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+48*arct
an(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-24*I*arctan(a*x)^2*ln(1-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-44*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-44*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*polylog(4,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+44*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*polylo
g(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.433.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x^2, x)
```

3.433.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**2, x)
```

3.433.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^2, x)
```

3.433.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

3.434 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx$

3.434.1 Optimal result

Integrand size = 24, antiderivative size = 1043

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)$$
$$-\frac{3ac^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x}-\frac{1}{2}a^3c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2$$
$$+\frac{13ia^2c^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}}$$
$$+2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)^3-\frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{2x^2}$$
$$+\frac{1}{3}a^2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3-\frac{5a^2c^3\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$-\frac{6a^2c^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$
$$-a^2c^{5/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)+\frac{15ia^2c^3\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}}-\frac{13ia^2c^3\sqrt{1+a^2}}{}$$

output
```
1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3-a^2*c^(5/2)*arctanh(a*x*c^(1/2
)/(a^2*c*x^2+c)^(1/2))+13*I*a^2*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*ar
ctan(a*x)^2*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^2*c^3*arctan(a*x)^3*
arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
-6*a^2*c^3*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1
)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c^3*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c^3*polylog(2,-(1+I
*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13*I*a^
2*c^3*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)-13*I*a^2*c^3*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15*I*a^2*c^3*polylog
(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15/
2*I*a^2*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+
1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15*a^2*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13*a^2*c^3*polylog
(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-1
3*a^2*c^3*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*
c*x^2+c)^(1/2)+15*a^2*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15/2*I*a^2*c^3*arctan(a*x)^2*poly
log(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/...
```

3.434.2 Mathematica [A] (verified)

Time = 9.02 (sec) , antiderivative size = 934, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c^2*Sqrt[c + a^2*c*x^2]*(-36*ArcTan[a*x]^2 - (15*I)*Pi^4*Cot[ArcTan[a
*x]/2] + (48*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]
 + (30*I)*ArcTan[a*x]^4*Cot[ArcTan[a*x]/2] - 48*ArcTanh[(a*x)/Sqrt[1 + a^2
*x^2]]*Cot[ArcTan[a*x]/2] - 36*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]^2 + 12*a*x
*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 12*a^3*x^3*ArcTan[a*x]*Csc[ArcTan[a*x]
/2]^2 + 56*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 + 8*a^3*x^3*ArcTan[a*x]^
3*Csc[ArcTan[a*x]/2]^2 + 12*a*x*ArcTan[a*x]*Cos[2*ArcTan[a*x]]*Csc[ArcTan[
a*x]/2]^2 + 12*a^3*x^3*ArcTan[a*x]*Cos[2*ArcTan[a*x]]*Csc[ArcTan[a*x]/2]^2
 - 6*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 120*ArcTan[a*
x]^3*Cot[ArcTan[a*x]/2]*Log[1 - E^((-I)*ArcTan[a*x])] + 144*ArcTan[a*x]*Co
t[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 288*ArcTan[a*x]^2*Cot[ArcTan
[a*x]/2]*Log[1 - I*E^(I*ArcTan[a*x])] + 288*ArcTan[a*x]^2*Cot[ArcTan[a*x]/
2]*Log[1 + I*E^(I*ArcTan[a*x])] - 144*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1
 + E^(I*ArcTan[a*x])] - 120*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*
ArcTan[a*x])] + (360*I)*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*PolyLog[2, E^((-I
)*ArcTan[a*x])] + (72*I)*(2 + 5*ArcTan[a*x]^2)*Cot[ArcTan[a*x]/2]*PolyLog[
2, -E^(I*ArcTan[a*x])] - (624*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2,
 (-I)*E^(I*ArcTan[a*x])] + (624*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[
2, I*E^(I*ArcTan[a*x])] - (144*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTa
n[a*x])] + 720*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[3, E^((-I)*ArcTan...
```

3.434.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+ c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan}{}}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}}{}}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{5423}$$

3.434. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^3}dx$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\text{arc}}{a}\right.\right.$$

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2}\ -}{a}\right.\right.$$

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$$\Big\downarrow \text{4669}$$

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)+$$

$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)+$$

$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)+$$

$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(}\right.\right.\right.}{}\right.\right.$$

3.434. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^3}\,dx$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\left(a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)\right)+$$
$$a^2c\left(c\left(a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}(2(i\arctan(a$$

$$\Big\downarrow \text{5465}$$

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+\right.}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan}{x\sqrt{a^2c}}\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\arctan(ax\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}d\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}d\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.434. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^3}\,dx$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d a}{a^2\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d a}{a^2\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.\right.$$

$$c\left(c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{}}{}}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\right.\right.\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{}}{}}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\right.\right.\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{5493}$$

3.434. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 5491$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 4671$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \; 3011$$

3.434. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\left(2\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \ 5497$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\left(2\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \ 5479$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\left(2\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \ 5493$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\left(2\cdots\right.\right.\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \ 5489$$

3.434. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^3}\,dx$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{}}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\left(\right.\right.\right.}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{}}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\left(\right.\right.\right.}{}\right.\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
$Aborted
```

3.434.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.434. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^3}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

3.434. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.434. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.434.4 Maple [A] (verified)

Time = 7.42 (sec) , antiderivative size = 660, normalized size of antiderivative = 0.63

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\Big(2a^4\arctan(ax)^2x^4-3\arctan(ax)x^3a^3+14x^2\arctan(ax)^2a^2+6a^2x^2-9x\arctan(ax)a-3\arctan(ax)^2}{6x^2}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(2*a^4*arctan(a*x)^2*x^4-3*a
rctan(a*x)*x^3*a^3+14*x^2*arctan(a*x)^2*a^2+6*a^2*x^2-9*x*arctan(a*x)*a-3*
arctan(a*x)^2)/x^2-1/2*c^2*a^2*(c*(a*x-I)*(I+a*x))^(1/2)*(5*arctan(a*x)^3*
ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-5*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+
1)^(1/2))+6*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-13*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+13*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*arctan(a*x)*po
lylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I*polylog(4,(1+I*a*x)/(a^2*x^2+
1)^(1/2))-30*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a
*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+30*arctan(a*x)*polylog(3,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*I*ar
ctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*polylog(4,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-26*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-15*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arcta
n((1+I*a*x)/(a^2*x^2+1)^(1/2))-26*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+26*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

3.434.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x^3, x)
```

3.434.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**3, x)
```

3.434.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^3, x)
```

3.434.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

3.435 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx$

3.435.1 Optimal result

Integrand size = 24, antiderivative size = 1061

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = -\frac{a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)}{x} - \frac{3}{2}a^3c^2\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{ac^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2} - \frac{2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{x} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\arctan(ax)^3 - \frac{c(c+a^2cx^2)^{3/2}\arctan(ax)^3}{3x^3} - \frac{5ia^3c^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6ia^3c^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{13a^3c^3\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - a^3c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{13ia^3c^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{15ia^3c^3\sqrt{1+a^2}}{}$$

output
```
-1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3-a^3*c^(5/2)*arctanh((a^2*c*x^
2+c)^(1/2)/c^(1/2))+15/2*I*a^3*c^3*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15*I*a^3*c^3*polylo
g(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-
13*a^3*c^3*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^
(1/2)/(a^2*c*x^2+c)^(1/2)-15/2*I*a^3*c^3*arctan(a*x)^2*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*I*a^3*c^3*po
lylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c
)^(1/2)+13*I*a^3*c^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+15*I*a^3*c^3*polylog(4,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13*I*a^3*c^3*arctan
(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+
c)^(1/2)-5*I*a^3*c^3*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-13*a^3*c^3*polylog(3,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-15*a^3*c^3*arctan(a*x)*p
olylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)+15*a^3*c^3*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+13*a^3*c^3*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a^3*c^3*arctan(a*x)*arc
tan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1...
```

3.435.2 Mathematica [A] (warning: unable to verify)

Time = 10.26 (sec) , antiderivative size = 1771, normalized size of antiderivative = 1.67

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
((-1/2*I)*a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcT
an[a*x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[
a*x]^2)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog
[2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a
*x])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (
-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*
x^2] + (a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x
)/Sqrt[1 + a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((2
4*I)*a*Pi^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I
)*a*Pi*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqr
t[1 + a^2*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sq
rt[1 + a^2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/S
qrt[1 + a^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*
x^2] + (64*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^
2] + (192*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
+ (8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2
*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*
x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*
ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*...
```

3.435.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{x^4}\,dx$$

$$\Big\downarrow \; 5485$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^4}dx$$

$$\Big\downarrow \; 5485$$

$$a^2c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx\right)+ c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)$$

$$\Big\downarrow \; 5415$$

$$a^2c\left(a^2c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right)\right.$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(a^2c\left(\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)}{2}\right.\right.$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)$$

$$\Big\downarrow \text{5421}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}\right.}{\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{}}{}\right.\right.$$

$$\Big\downarrow \text{4669}$$

3.435. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$

$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$

$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\left(a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+$$

$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\right.\right.$$

$$a^2c\left(c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5425}$$

3.435. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$a^2c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}(3(i\arctan(ax)^2\operatorname{PolyLog}(2,-\right.\right.$$

$$\Big\downarrow \; 5423$$

$$c\left(a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$a^2c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}(3(i\operatorname{arct}}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+c\left(a\left(a^2c\int \frac{\operatorname{ar}}{x\sqrt{}}\right.\right.\right.$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+a^2c\left(\frac{c\sqrt{a^2x^2-}}{}\right.\right.$$

$$\Big\downarrow \; 4669$$

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}(3(i\arctan(ax)^2\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-2i\int \arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})\,d}{}\right.\right.$$

$$c\left(c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \; 3011$$

3.435. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-i\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow 5479$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(c\left(3a\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3\right.}{}\right.\right.$$

$$\Big\downarrow 5493$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\mathrm{arcta}\right.}{}\right.\right.$$

$$\Big\downarrow 5491$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(\right.\right.}{}\right.\right.$$

$$\Big\downarrow 3042$$

3.435. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}(-}{}\right.\right.\right.$$

$$\Big\downarrow 4671$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow 3011$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow 2720$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow 5497$$

3.435. $\int\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow 5479$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow 243$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow 73$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow 221$$

3.435. $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x^4}\,dx$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\mathrm{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 5493$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\mathrm{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 5491$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\mathrm{arcta}\right.\right.}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
$Aborted
```

3.435. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx$

3.435.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.435. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx$

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.435. $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx$

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.435.4 Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 694, normalized size of antiderivative = 0.65

method	result
default	$\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(3a^4\arctan(ax)^2x^4-9\arctan(ax)x^3a^3-14x^2\arctan(ax)^2a^2-6a^2x^2-3x\arctan(ax)a-2\arctan(ax)^2\right.}{6x^3}$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(I+a*x))^(1/2)*arctan(a*x)*(3*a^4*arctan(a*x)^2*x^4-9*a
rctan(a*x)*x^3*a^3-14*x^2*arctan(a*x)^2*a^2-6*a^2*x^2-3*x*arctan(a*x)*a-2*
arctan(a*x)^2)/x^3+1/2*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)*(5*arct
an(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-5*arctan(a*x)^3*ln(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-13*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)
+13*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*arctan(a*x)*polyl
og(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+6*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*arctan(a*x)*pol
ylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*arctan(a*x)*polylog(3,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*
I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(2,-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-2*ln((1+
I*a*x)/(a^2*x^2+1)^(1/2)+1)-26*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*
polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(2,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-15*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I
*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*arctan(a*x)*polylog(2,-(1+
I*a*x)/(a^2*x^2+1)^(1/2)))*c^2*a^3
```

3.435.5 Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x^4, x)
```

3.435.6 Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3{(ax)}}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**4, x)
```

3.435.7 Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^4, x)
```

3.435.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

3.436 $\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

3.436.1 Optimal result

Integrand size = 24, antiderivative size = 408

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^4c}-\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a^3c}\\&\quad-\frac{5i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^4\sqrt{c+a^2cx^2}}\\&\quad-\frac{2\sqrt{c+a^2cx^2}\arctan(ax)^3}{3a^4c}\\&\quad+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{3a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^4\sqrt{c}}\\&\quad+\frac{5i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}}\\&\quad-\frac{5i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}}\\&\quad-\frac{5\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}}\\&\quad+\frac{5\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-arctanh(a*x*c^(1/2)/(a^2*c*x^2+c)^(1/2))/a^4/c^(1/2)-5*I*arctan((1+I*a*x)
/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2
)+5*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1
/2)/a^4/(a^2*c*x^2+c)^(1/2)-5*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)-5*polylog(3,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2)+5*polylog
(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/(a^2*c*x^2+c)^(1/2
)+arctan(a*x)*(a^2*c*x^2+c)^(1/2)/a^4/c-1/2*x*arctan(a*x)^2*(a^2*c*x^2+c)^
(1/2)/a^3/c-2/3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x
)^3*(a^2*c*x^2+c)^(1/2)/a^2/c
```

3.436.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.54

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(\frac{12\left(-5i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-\operatorname{arctanh}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+5i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-5i\arctan(ax)\,\mathrm{P}\right.}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((12*((-5*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
- ArcTanh[(a*x)/Sqrt[1 + a^2*x^2]] + (5*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])] - (5*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 5*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])] + 5*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqr
t[1 + a^2*x^2] - (1 + a^2*x^2)*ArcTan[a*x]*(-6 + 2*ArcTan[a*x]^2 + 6*(-1 +
 ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]] + 3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])))/(
12*a^4*c)
```

3.436.3 Rubi [A] (verified)

Time = 3.21 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {5487, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5487, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \color{blue}{5487}$$

$$-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \color{blue}{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \color{blue}{5425}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \color{blue}{5423}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \color{blue}{3042}$$

$$-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \color{blue}{4669}$$

$$-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a} - \frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2\right.}{a^2\sqrt{a^2cx^2+c}}}{3a^2}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 3011$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(\right.\right.}{a^2\sqrt{a^2c}}}{3a^2}$$

$$\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a} + \frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 2720$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.\right.}{}}{3a^2}$$

$$\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a} + \frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5487$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.\right.}{}}{3a^2}$$

$$\frac{-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a} - \frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2} + \frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a} + \frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5425$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.\right.}{}}{3a^2}$$

$$\frac{-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a} - \frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}} + \frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a} + \frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5423$$

3.436. $\int \frac{x^3\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan(\right.}{2a^3\sqrt{a^2cx^2+c}}}{a}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{2a^3\sqrt{a^2cx^2+c}}}{a}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{2720}$$

3.436. $\int\frac{x^3\arctan(ax)^3}{\sqrt{c+a^2cx^2}}dx$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan\right.\right.}{2a^3\sqrt{}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5465$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\right.\right.}}{}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 224$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan\right.\right.}}{}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.}{3a^2}$$

$$-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\right.\right.\right.}}{2a^3\sqrt{a^2cx^2+c}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 7143$$

3.436. $\int\frac{x^3\arctan(ax)^3}{\sqrt{c+a^2cx^2}}dx$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie\right.\right.\right.}{a^2\sqrt{a^2cx^2+c}}}{3a^2}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}-$$

$$\frac{-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(\right.\right.\right.}{}}{a}$$

input
```
Int[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) -
ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/a - (Sqrt[1 + a^
2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) -
 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcT
an[a*x])])))/(2*a^3*Sqrt[c + a^2*c*x^2]))/a - (2*((Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x]
)]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - P
olyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*Ar
cTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2])
))/(3*a^2)
```

3.436.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.436. $\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.436.4 Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(2x^2\arctan(ax)^2a^2-3x\arctan(ax)a-4\arctan(ax)^2+6\right)\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{6c\,a^4}+\frac{5\left(i\arctan(ax)^3-3\arctan(ax)^2\ln\left(1+\frac{i(}{\sqrt{c}}\right.\right.}{}$

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^2*arctan(a*x)^2*a^2-3*x*arctan(a*x)*a-4*arctan(a*x)^2+6)*arctan(a
*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^4+5/6*(I*arctan(a*x)^3-3*arctan(a*x)^2*l
n(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)
*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c-5/6*(I*arctan(a*x)^3-3*arctan(a*x)
^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-
I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c+2*I*arctan((1+I*a*x)/(a^2*x^2+1)
^(1/2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

3.436.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.436.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^3 \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}} \, dx$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

3.436.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.436.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.437 $\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

3.437.1 Optimal result

Integrand size = 24, antiderivative size = 625

$$\begin{aligned}\int \frac{x^2\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = &-\frac{3\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a^3c}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^3}{2a^2c}\\
&+\frac{i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a^3\sqrt{c+a^2cx^2}}\\
&-\frac{6i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}}\\
&-\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{2a^3\sqrt{c+a^2cx^2}}\\
&+\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{2a^3\sqrt{c+a^2cx^2}}\\
&+\frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}}\\
&-\frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}}\\
&+\frac{3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}}\\
&-\frac{3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}}\\
&+\frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}}\\
&-\frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a^3/
(a^2*c*x^2+c)^(1/2)-6*I*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2)
)*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/2*I*arctan(a*x)^2*polylog(2,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+
3/2*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3*I*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3*I*polylog(2,I*(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3*arctan(a
*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c
*x^2+c)^(1/2)-3*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)+3*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3*I*polylog(4,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/(a^2*c*x^2+c)^(1/2)-3/2*arctan
(a*x)^2*(a^2*c*x^2+c)^(1/2)/a^3/c+1/2*x*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/
a^2/c
```

3.437.2 Mathematica [A] (verified)

Time = 5.66 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.30

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c\,(1+a^2x^2)}\left(\frac{7i\pi^4}{32}+\frac{1}{4}i\pi^3\arctan(ax)-6\arctan(ax)^2-\frac{3}{4}i\pi^2\arctan(ax)^2+i\pi\arctan(ax)^3-\frac{1}{2}i\arctan(\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(((7*I)/32)*Pi^4 + (I/4)*Pi^3*ArcTan[a*x] - 6*ArcTa
n[a*x]^2 - ((3*I)/4)*Pi^2*ArcTan[a*x]^2 + I*Pi*ArcTan[a*x]^3 - (I/2)*ArcTa
n[a*x]^4 - (3*Pi^2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/2 + 3*Pi*ArcT
an[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])] + (Pi^3*Log[1 + I/E^(I*ArcTan[a*x])
])/4 - 2*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 12*ArcTan[a*x]*Log[1
 - I*E^(I*ArcTan[a*x])] - (Pi^3*Log[1 + I*E^(I*ArcTan[a*x])])/4 - 12*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + (3*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*A
rcTan[a*x])])/2 - 3*Pi*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcT
an[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] - (Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x]
)/4]])/4 - (6*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - ((3*I)
/2)*Pi*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] + (12*I)*PolyL
og[2, (-I)*E^(I*ArcTan[a*x])] - ((3*I)/2)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])] + (6*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (6*I)*
ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (12*I)*PolyLog[2, I*E^(
I*ArcTan[a*x])] - 12*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 6*Pi
*PolyLog[3, I/E^(I*ArcTan[a*x])] - 6*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
 + 12*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (12*I)*PolyLog[4, (
-I)/E^(I*ArcTan[a*x])] + (12*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + ArcTa
n[a*x]^3/(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2 - (6*ArcTan[a*x]^2*Si
n[ArcTan[a*x]/2])/(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]) - ArcTan[a*...
```

3.437.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.66, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5487, 5425, 5423, 3042, 4669, 3011, 5465, 5425, 5421, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5425}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5423}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(\right.}{2a^3\sqrt{a^2cx^2+c}}$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5425}$$

3.437. $\int \frac{x^2\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5421}$$

$$-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}$$

$$\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\operatorname{arcta}\right.\right.\right.}{}$$

$$\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.\right.}{}$$

$$\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{7143}$$

3.437. $\int \frac{x^2\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}-$$

$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\left(3,\right.\right.\right.\right.$$

input
```
Int[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(2*a^2*c) - (3*((Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan
[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]]
)/a))/(a*Sqrt[c + a^2*c*x^2])))/(2*a) - (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[
E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])
] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I
*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x
])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(2*a^3*Sqrt[c + a^2*c*x^2])
```

3.437.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.437. $\int \frac{x^2\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.437. $\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.437.4 Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.68

method	result
default	$\frac{(x\arctan(ax)a-3)\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{2c\,a^3}+\frac{\left(\arctan(ax)^3\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^3\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-3i\arctan(a\ldots\right.}{}$

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x*arctan(a*x)*a-3)*arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c/a^3+1/2*
(arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(3,-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(4
,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+6*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*dilog(1-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

3.437.5 Fricas [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.437.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

3.437.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.437.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan{(ax)}^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.438 $\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

3.438.1 Optimal result

Integrand size = 22, antiderivative size = 283

$$\begin{aligned}\int \frac{x\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx &= \frac{6i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^2\sqrt{c+a^2cx^2}}\\
&+\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{a^2c}\\
&-\frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\
&+\frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\
&+\frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\
&-\frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
6*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^
2/(a^2*c*x^2+c)^(1/2)-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+6*I*arctan(a*x)*polylog(2,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^2+c)^(1/2)+6
*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^2/(a^2*c*x^
2+c)^(1/2)-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^
2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^2/c
```

3.438.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.59

$$\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c(1+a^2x^2)}\left(\arctan(ax)^3 - \frac{3(\arctan(ax)^2(\log(1-ie^{i\arctan(ax)})-\log(1+ie^{i\arctan(ax)}))+2i\arctan(ax)(\operatorname{PolyLog}(2,-ie^{i\arctan(ax)}}{\sqrt{1+a^2x^2}}\right.}{a^2c}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^3 - (3*(ArcTan[a*x]^2*(Log[1 - I*E^(I*
ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (2*I)*ArcTan[a*x]*(PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) - 2*PolyLog
[3, (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 +
 a^2*x^2]))/(a^2*c)
```

3.438.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.61, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5465, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}$$
$$\Big\downarrow \text{5425}$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{5423}$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2 d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 4669$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-$$
$$\frac{3\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(a\right.}{a^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3011$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-$$
$$\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{a^2\sqrt{a^2c}}$$

$$\Big\downarrow \; 2720$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-$$
$$3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.$$

$$\Big\downarrow \; 7143$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-$$
$$\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\right.\right.\right.}{a^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)
*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I
)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a
*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(
a^2*Sqrt[c + a^2*c*x^2])
```

3.438. $\int \frac{x\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

3.438.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.438. $\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.438.4 Maple [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{a^2 c\, x^2 + c}}\,dx$$

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.438.5 Fricas [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c + a^2 c x^2}}\,dx = \int \frac{x \arctan(ax)^3}{\sqrt{a^2 c x^2 + c}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.438.6 Sympy [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2 + 1)}}\,dx$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

3.438.7 Maxima [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2 + c}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.438.8 Giac [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2 + c}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.439 $\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

3.439.1 Optimal result

Integrand size = 21, antiderivative size = 368

$$\begin{aligned}\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx =& -\frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a\sqrt{c+a^2cx^2}}\\ &+\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &-\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &-\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &+\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &-\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &+\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a
/(a^2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-6*
arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/
(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a/(a^2*c*x^2+c)^(1/2)
```

3.439.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.52

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{c\left(1+a^2x^2\right)}\left(2\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3-3\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)+3\arctan\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

output
```
((-I)*Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3
*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*PolyLo
g[2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4,
(-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1
+ a^2*x^2])
```

3.439.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5425, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \; 5425$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5423$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 4669$$

$$\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(\right.}{a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3011$$

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\Big\downarrow \; 7163$$

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan\right.\right.\right.}{}$$

$$\Big\downarrow \; 2720$$

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.}{}$$

$$\Big\downarrow \; 7143$$

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\left(3,\right.\right.\right.\right.}{}$$

input
```
Int[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

3.439. $\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

output
```
(Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*
ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])]))
 - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan
[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])])))
)/(a*Sqrt[c + a^2*c*x^2])
```

3.439.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

3.439. $\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.439.4 Maple [F]

$$\int \frac{\arctan\left(a x\right)^{3}}{\sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.439.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^{3}}{\sqrt{a^{2}cx^{2}+c}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.439.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

3.439.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.439.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

3.440 $\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$

3.440.1 Optimal result

Integrand size = 24, antiderivative size = 327

$$
\begin{aligned}
\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = &-\frac{2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&-\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&-\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&-\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-2*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a
^2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2,(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)
*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(
1/2)+6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2
)/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2
+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)
```

3.440.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.64

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx =$$
$$-\frac{i\sqrt{1+a^2x^2}\left(\pi^4-2\arctan(ax)^4+8i\arctan(ax)^3\log\left(1-e^{-i\arctan(ax)}\right)-8i\arctan(ax)^3\log\left(1+e^{i\arctan}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((-1/8*I)*Sqrt[1 + a^2*x^2]*(Pi^4 - 2*ArcTan[a*x]^4 + (8*I)*ArcTan[a*x]^3*
Log[1 - E^((-I)*ArcTan[a*x])] - (8*I)*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*
x])] - 24*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - 24*ArcTan[a*x]^
2*PolyLog[2, -E^(I*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLog[3, E^((-I)*A
rcTan[a*x])] - (48*I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 48*Poly
Log[4, E^((-I)*ArcTan[a*x])] + 48*PolyLog[4, -E^(I*ArcTan[a*x])]))/Sqrt[c*
(1 + a^2*x^2)]
```

3.440.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.54, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5493, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$\Big\downarrow$ 5493

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5491

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 4671

$$\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax\right.}{\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3011

$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-\right.$$

$\Big\downarrow$ 7163

$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan(a\right.\right.\right.$$

$\Big\downarrow$ 2720

$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}-\right.\right.\right.$$

$\Big\downarrow$ 7143

$$\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\text{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-e^{\right.\right.\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```

3.440. $\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}dx$

output
```
(Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*Arc
Tan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLo
g[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[
a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3,
E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]
```

3.440.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.440. $\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.440.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.80

method	result
default	$\frac{i\left(i\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+3\arctan(ax)^2\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-3\arctan(ax)^2\operatorname{polylog}\right.}{}$

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(I*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)^3*ln(1-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arct
an(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,
(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*p
olylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+
1)^(1/2)/c
```

3.440.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}\,x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

3.440.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

3.440.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}\,x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x), x)
```

3.440.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.441 $\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$

3.441.1 Optimal result

Integrand size = 24, antiderivative size = 260

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{cx} - \frac{6a\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-6*a*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/
(a^2*c*x^2+c)^(1/2)+6*I*a*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*I*a*arctan(a*x)*polylog(2,(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-6*a*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+6*a
*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1
/2)-arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c/x
```

3.441.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx =$$
$$-\frac{a\sqrt{1+a^2x^2}\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)^3}{ax}-3\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)+3\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((a*Sqrt[1 + a^2*x^2]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3)/(a*x) - 3*ArcTan
[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a
*x])] - (6*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*
x]*PolyLog[2, E^(I*ArcTan[a*x])] + 6*PolyLog[3, -E^(I*ArcTan[a*x])] - 6*Po
lyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)])
```

3.441.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.60, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \text{ 5479}$$
$$3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$
$$\Big\downarrow \text{ 5493}$$
$$\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$
$$\Big\downarrow \text{ 5491}$$
$$\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+$$
$$\frac{3a\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(a$$
$$\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+$$
$$\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{\sqrt{a^2cx^2}}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+$$
$$\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+$$
$$\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i a}\right.\right.\right.}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*
ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^
(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*Poly
Log[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*
c*x^2]
```

3.441. $\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$

3.441.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

3.441. $\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.441.4 Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\arctan(ax)^3\sqrt{c(ax-i)(ax+i)}}{cx}-\frac{3a\left(\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-2i\arctan(ax)\operatorname{polylog}\left(2,-\right.\right.}{}$

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(a*x)^3*(c*(a*x-I)*(I+a*x))^(1/2)/c/x-3*a*(arctan(a*x)^2*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*
I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^
2*x^2+1)^(1/2)/c
```

3.441.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

3.441.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3{(ax)}}{x^2\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

3.441.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

3.441.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(a\,x)^3}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.442 $\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$

3.442.1 Optimal result

Integrand size = 24, antiderivative size = 597

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx &= -\frac{3a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{2cx^2} \\ &+ \frac{a^2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{6a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} \\ &+ \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} \\ &+ \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
a^2*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(
a^2*c*x^2+c)^(1/2)-6*a^2*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*I*a^2*arctan(a*x)^2*polylog(
2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3/2*
I*a^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*polylog(2,(1+I*a*x)^(1/2)/(
1-I*a*x)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+3*a^2*arctan(a*x)*po
lylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2
)-3*a^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/(a^2*c*x^2+c)^(1/2)+3*I*a^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3*I*a^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)
^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-3/2*a*arctan(a*x)^2*(a^2*c*x
^2+c)^(1/2)/c/x-1/2*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c/x^2
```

3.442.2 Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.58

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{a^2\sqrt{1+a^2x^2}\left(i\pi^4 - 2i\arctan(ax)^4 - 12\arctan(ax)^2\cot\left(\frac{1}{2}\arctan(ax)\right) - 2\arctan(ax)^3\csc^2\left(\frac{1}{2}\arctan(ax\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^2*Sqrt[1 + a^2*x^2]*(I*Pi^4 - (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x]^2*Co
t[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 8*ArcTan[a*x]^3*
Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]
- 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + 8*ArcTan[a*x]^3*Log[1 + E^(I
*ArcTan[a*x])] - (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - (
24*I)*(-2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog
[2, E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] +
 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + (48*I)*PolyLog[4, E^((-I)
*ArcTan[a*x])] + (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*S
ec[ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt[c*(1
+ a^2*x^2)])
```

3.442.3 Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.63, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5497, 5479, 5493, 5489, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5479}$$

$$\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5493}$$

$$\frac{3}{2}a\left(\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5489}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5491}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$
$$\Big\downarrow \textbf{3042}$$
$$-\frac{a^2\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$
$$\Big\downarrow \textbf{4671}$$
$$-\frac{a^2\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\,\text{arcta}\right.}{2\sqrt{a^2cx^2+c}}$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$
$$\Big\downarrow \textbf{3011}$$
$$-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax\right.\right.}{}$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$
$$\Big\downarrow \textbf{7163}$$

3.442. $\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\operatorname{arct}\right.\right.\right.}{}$$

$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \; 2720$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.\right.}{}$$

$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \; 7143$$

$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-\right.\right.\right.\right.}{}$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

input
```
Int[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^2) + (3*a*(-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTa
nh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^
2*c*x^2]))/2 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTa
n[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)
*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x]
)])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcT
an[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/
(2*Sqrt[c + a^2*c*x^2])
```

3.442. $\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$

3.442.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

3.442. $\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.442.4 Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.69

method	result
default	$-\frac{(3ax+\arctan(ax))\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{2c\,x^2}-\frac{ia^2\left(i\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+1\right)-i\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+3\,\text{ar}\right.}{}$

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*a*x+arctan(a*x))*arctan(a*x)^2*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^2-1/2
*I*a^2*(I*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)^3*
ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I
*arctan(a*x)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)+6*I*arctan(a*x)*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(2,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/
2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

3.442.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

3.442.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3{(ax)}}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

3.442.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

3.442.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.443 $\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.443.1 Optimal result

Integrand size = 24, antiderivative size = 396

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = &-\frac{a^2\sqrt{c+a^2cx^2}\arctan(ax)}{cx}-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2cx^2}\\&-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{3cx}\\&+\frac{5a^3\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&-\frac{a^3\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\\&-\frac{5ia^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&+\frac{5ia^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&+\frac{5a^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&-\frac{5a^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a^3*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+5*a^3*arctan(a*x)^2*arct
anh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*I
*a^3*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/(a^2*c*x^2+c)^(1/2)+5*I*a^3*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(
1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)+5*a^3*polylog(3,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-5*a^3*polylog(3,(1+I
*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/(a^2*c*x^2+c)^(1/2)-a^2*arctan(
a*x)*(a^2*c*x^2+c)^(1/2)/c/x-1/2*a*arctan(a*x)^2*(a^2*c*x^2+c)^(1/2)/c/x^2
-1/3*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^3*(a^2*c*
x^2+c)^(1/2)/c/x
```

3.443.2 Mathematica [A] (verified)

Time = 4.92 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{a^3\sqrt{c\left(1+a^2x^2\right)}\left(-12\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)+10\arctan(ax)^3\cot\left(\frac{1}{2}\arctan(ax)\right)-3\arctan(ax)^2\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^3*Sqrt[c*(1 + a^2*x^2)]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] + 10*ArcTan
[a*x]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*A
rcTan[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) - 60*ArcTan[a*x]^
2*Log[1 - E^(I*ArcTan[a*x])] + 60*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])]
 + 24*Log[Tan[ArcTan[a*x]/2]] - (120*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTa
n[a*x])] + (120*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 120*PolyLog
[3, -E^(I*ArcTan[a*x])] - 120*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x
]^2*Sec[ArcTan[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan
[a*x]/2]^4)/(a^3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] + 10*ArcTan[a*x]
^3*Tan[ArcTan[a*x]/2]))/(24*c*Sqrt[1 + a^2*x^2])
```

3.443.3 Rubi [A] (verified)

Time = 3.40 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5497, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5493}$$

$$-\frac{2}{3}a^2\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5491}$$

$$-\frac{2}{3}a^2\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}a^2\left(\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan\right.}{\sqrt{a^2cx}}\right.$$

$$a\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 3011$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\right.\right.\right.}$$

$$a\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 2720$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.}{}\right.$$

$$a\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 5497$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.}{}\right.$$

$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)-$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 5479$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.}{}\right.$$

$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)-$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 243$$

3.443. $\int\frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyI}\right.\right.}{}\right.$$

$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 73$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyI}\right.\right.}{}\right.$$

$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 221$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyI}\right.\right.}{}\right.$$

$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 5493$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyI}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2}}{2cx^2}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 5491$$

3.443. $\int\frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\text{ar}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+}}{\sqrt{c}}\right.}{\sqrt{c}}\right.\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\text{arcta}\right.}{2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{3011}$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\,\text{ar}\right.\right.}{2\sqrt{}}\right.$$

$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLo}\right.\right.}{}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{2720}$$

3.443. $\int\frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax}\right.\right.}{}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,\right.\right.\right.}{2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*A
rcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])
] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*Arc
Tan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]))/3 + a*
(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x
^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] +
2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[
a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I
*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2]))
```

3.443. $\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.443.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.443. $\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.443.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.82

method	result
default	$\frac{\left(4x^2\arctan(ax)^2a^2-6a^2x^2-3x\arctan(ax)a-2\arctan(ax)^2\right)\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{6c\,x^3}-\frac{2a^3\operatorname{arctanh}\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\sqrt{c(ax-i)(}}{\sqrt{a^2x^2+1}\,c}$

3.443. $\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*x^2*arctan(a*x)^2*a^2-6*a^2*x^2-3*x*arctan(a*x)*a-2*arctan(a*x)^2)*
arctan(a*x)*(c*(a*x-I)*(I+a*x))^(1/2)/c/x^3-2*a^3*arctanh((1+I*a*x)/(a^2*x
^2+1)^(1/2))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c+5/2*a^3*(arctan
(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*
I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*
(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

3.443.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)
```

3.443.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3{(ax)}}{x^4\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

3.443.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

3.443.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.444 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$

3.444.1 Optimal result

Integrand size = 24, antiderivative size = 403

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx = {} & \frac{6x}{a^3c\sqrt{c+a^2cx^2}} - \frac{6\arctan(ax)}{a^4c\sqrt{c+a^2cx^2}} - \frac{3x\arctan(ax)^2}{a^3c\sqrt{c+a^2cx^2}} \\ & + \frac{6i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^4c\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)^3}{a^4c\sqrt{c+a^2cx^2}} \\ & + \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{a^4c^2} - \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} \\ & + \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} \\ & + \frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} - \frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
6*x/a^3/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(1/2)-3*x*
arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3/a^4/c/(a^2*c*x^2+c)^
(1/2)+6*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1
/2)/a^4/c/(a^2*c*x^2+c)^(1/2)-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(1/2)+6*I*arctan(a*x)*
polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^
2+c)^(1/2)+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a
^4/c/(a^2*c*x^2+c)^(1/2)-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x
^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^
4/c^2
```

3.444.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.76

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(\frac{6ax}{\sqrt{1+a^2x^2}} - 3\sqrt{1+a^2x^2}\arctan(ax) - \frac{3ax\arctan(ax)^2}{\sqrt{1+a^2x^2}} + \frac{3}{2}\sqrt{1+a^2x^2}\arctan(\right.}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((6*a*x)/Sqrt[1 + a^2*x^2] - 3*Sqrt[1 + a^2*x^2]*ArcTan
[a*x] - (3*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (3*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]^3)/2 - 3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + (Sqrt
[1 + a^2*x^2]*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]])/2 - 3*ArcTan[a*x]^2*Log[1
- I*E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (6
*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*Pol
yLog[2, I*E^(I*ArcTan[a*x])] + 6*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 6*Po
lyLog[3, I*E^(I*ArcTan[a*x])]))/(a^4*c*Sqrt[c*(1 + a^2*x^2)])
```

3.444.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.72, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5499, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c} - \frac{\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c} - \frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c} - \frac{\frac{3\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \; 5423$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \; 4669$$

$$-\frac{\frac{3\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\,\mathrm{ar}\ldots\right.}{a^2\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \; 3011$$

$$-\frac{\frac{3\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{Po}\ldots\right.\right.}{a^2\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \; 2720$$

$$-\frac{\frac{3\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\,\mathrm{a}\ldots\right.\right.}{\ldots}}{a^2c}$$

$$\Big\downarrow \; 5433$$

3.444. $\int\frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

$$\frac{-\frac{\frac{3\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})de^{i\arctan(ax)})-2(i a}{}}{a^2c}$$

$$\Big\downarrow \; 208$$

$$\frac{-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})de^{i\arctan(ax)})-2(i a}{}}{a^2c}$$

$$\Big\downarrow \; 7143$$

$$\frac{-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\operatorname{PolyLog}(3,-ie^{i\arctan(ax)}))-2(i\arctan(ax)\operatorname{PolyLog}(2,ie^{i\arctan}}{a^2\sqrt{a^2cx^2+c}}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a
^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)
/(c*Sqrt[c + a^2*c*x^2])))/a)/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/
(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*
x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-
I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])]
- PolyLog[3, I*E^(I*ArcTan[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2]))/(a^2*c)
```

3.444. $\int \frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

3.444.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.444. $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.444.4 Maple [F]

$$\int \frac{x^3 \arctan(ax)^3}{(a^2 c\, x^2 + c)^{\frac{3}{2}}} dx$$

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

3.444.5 Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

3.444.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.444.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

3.444.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.444.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.445 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

3.445.1 Optimal result

Integrand size = 24, antiderivative size = 495

$$\begin{aligned}
\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{6}{a^3c\sqrt{c+a^2cx^2}} + \frac{6x\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\arctan(ax)^2}{a^3c\sqrt{c+a^2cx^2}} \\
&- \frac{x\arctan(ax)^3}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a^3c\sqrt{c+a^2cx^2}} \\
&+ \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&- \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&- \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&+ \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&- \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}
\end{aligned}$$

output
```
6/a^3/c/(a^2*c*x^2+c)^(1/2)+6*x*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)-3*ar
ctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^
(1/2)-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1
/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x
)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*
c*x^2+c)^(1/2)-6*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)*polylog(3,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-6*I*poly
log(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c
)^(1/2)+6*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^3
/c/(a^2*c*x^2+c)^(1/2)
```

3.445.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.29

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx =$$
$$-\frac{\sqrt{1+a^2x^2}\left(7i\pi^4 - \frac{384}{\sqrt{1+a^2x^2}} + 8i\pi^3\arctan(ax) - \frac{384ax\arctan(ax)}{\sqrt{1+a^2x^2}} - 24i\pi^2\arctan(ax)^2 + \frac{192\arctan(ax)^2}{\sqrt{1+a^2x^2}} + 32i\pi\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-1/64*(Sqrt[1 + a^2*x^2]*((7*I)*Pi^4 - 384/Sqrt[1 + a^2*x^2] + (8*I)*Pi^3*
ArcTan[a*x] - (384*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (24*I)*Pi^2*ArcTan
[a*x]^2 + (192*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (32*I)*Pi*ArcTan[a*x]^3
+ (64*a*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (16*I)*ArcTan[a*x]^4 - 48*Pi^
2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 96*Pi*ArcTan[a*x]^2*Log[1 - I
/E^(I*ArcTan[a*x])] + 8*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] - 64*ArcTan[a*x]
^3*Log[1 + I/E^(I*ArcTan[a*x])] - 8*Pi^3*Log[1 + I*E^(I*ArcTan[a*x])] + 48
*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 96*Pi*ArcTan[a*x]^2*Log[1
 + I*E^(I*ArcTan[a*x])] + 64*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] -
8*Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]] - (192*I)*ArcTan[a*x]^2*PolyLog[2,
 (-I)/E^(I*ArcTan[a*x])] - (48*I)*Pi*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(
I*ArcTan[a*x])] - (48*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (192*I)
*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*ArcTan[a*x]^2
*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 384*ArcTan[a*x]*PolyLog[3, (-I)/E^(I
*ArcTan[a*x])] + 192*Pi*PolyLog[3, I/E^(I*ArcTan[a*x])] - 192*Pi*PolyLog[3
, (-I)*E^(I*ArcTan[a*x])] + 384*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] + (384*I)*PolyLog[4, (-I)/E^(I*ArcTan[a*x])] + (384*I)*PolyLog[4, (-I
)*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)])
```

3.445.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.63, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5499, 5425, 5423, 3042, 4669, 3011, 5433, 5429, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3 d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \; 4669$$

$$-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(\right.}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3011$$

$$-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\Big\downarrow \; 5433$$

$$-\frac{-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\Big\downarrow \; 5429$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\Big\downarrow \; 7163$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan\right.\right.\right.}{}$$

$$\Big\downarrow \; 2720$$

3.445. $\int \frac{x^2\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+$$
$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+$$
$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\left(3,\right.\right.\right.\right.$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*Sqrt
[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqr
t[c + a^2*c*x^2])))/a^2) + (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a
*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])
] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4
, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[
a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog
[4, I*E^(I*ArcTan[a*x])]))))/(a^3*c*Sqrt[c + a^2*c*x^2])
```

3.445.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

3.445. $\int \frac{x^2\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.445. $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.445.4 Maple [F]

$$\int \frac{x^2 \arctan (ax)^3}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

3.445.5 Fricas [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan (ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

3.445.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c + a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}^3(ax)}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.445.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c + a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

3.445.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c + a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.446 $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx$

3.446.1 Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx = -\frac{6x}{ac\sqrt{c+a^2cx^2}} + \frac{6\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}} + \frac{3x\arctan(ax)^2}{ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^3}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
-6*x/a/c/(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)+3*x*a
rctan(a*x)^2/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^(1/
2)
```

3.446.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.57

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{c+a^2cx^2}(-6ax+6\arctan(ax)+3ax\arctan(ax)^2-\arctan(ax)^3)}{a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-6*a*x + 6*ArcTan[a*x] + 3*a*x*ArcTan[a*x]^2 - ArcTa
n[a*x]^3))/(a^2*c^2*(1 + a^2*x^2))
```

3.446.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5465, 5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^3}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \textbf{5465}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{5433}$$

$$\frac{3\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{208}$$

$$\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2*
c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c
*Sqrt[c + a^2*c*x^2])))/a
```

3.446.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.446.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

method	result
default	$-\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^3-6\arctan\right.}{2(a^2x^2+1)a^2c^2}$

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x
-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/a^2/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*
x-1)*(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/a^2/c
^2
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\left(3\,ax\arctan\left(ax\right)^2-\arctan\left(ax\right)^3-6\,ax+6\,\arctan\left(ax\right)\right)}{a^4c^2x^2+a^2c^2}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(3*a*x*arctan(a*x)^2 - arctan(a*x)^3 - 6*a*x + 6*arcta
n(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

3.446.6 Sympy [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}^3\,(ax)}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \sqrt{c}\left(\frac{3\,x\arctan{(ax)}^2}{\sqrt{a^2x^2+1}ac^2} - \frac{\arctan{(ax)}^3}{\sqrt{a^2x^2+1}a^2c^2} - \frac{6\left(\frac{x}{\sqrt{a^2x^2+1}} - \frac{\arctan(ax)}{\sqrt{a^2x^2+1}a}\right)}{ac^2}\right)$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(c)*(3*x*arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^3/(sqrt
(a^2*x^2 + 1)*a^2*c^2) - 6*(x/sqrt(a^2*x^2 + 1) - arctan(a*x)/(sqrt(a^2*x^
2 + 1)*a))/(a*c^2))
```

3.446.8 Giac [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.446.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^3}{{(c\,a^2\,x^2+c)}^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.447 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

3.447.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=-\frac{6}{ac\sqrt{c+a^2cx^2}}-\frac{6x\arctan(ax)}{c\sqrt{c+a^2cx^2}}+\frac{3\arctan(ax)^2}{ac\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^3}{c\sqrt{c+a^2cx^2}}$$

output
```
-6/a/c/(a^2*c*x^2+c)^(1/2)-6*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+3*arctan(
a*x)^2/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2)
```

3.447.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{c+a^2cx^2}(-6-6ax\arctan(ax)+3\arctan(ax)^2+ax\arctan(ax)^3)}{c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-6 - 6*a*x*ArcTan[a*x] + 3*ArcTan[a*x]^2 + a*x*ArcTa
n[a*x]^3))/(c^2*(a + a^3*x^2))
```

3.447.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5433, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5433}$$

$$-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5429}$$

$$\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*Sqrt[c
+ a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c
 + a^2*c*x^2]))
```

3.447.3.1 Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

3.447.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

method	result
default	$\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a\,c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^3-6\arctan(ax)\cdot\right.}{2(a^2x^2+1)a\,c^2}$

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)
*(I+a*x))^(1/2)/(a^2*x^2+1)/a/c^2+1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(a
rctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/a/c^2
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}\left(ax\arctan(ax)^3-6\,ax\arctan(ax)+3\arctan(ax)^2-6\right)}{a^3c^2x^2+ac^2}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^3 - 6*a*x*arctan(a*x) + 3*arctan(a*x)
^2 - 6)/(a^3*c^2*x^2 + a*c^2)
```

3.447.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan{(ax)}^3}{\sqrt{a^2cx^2+cc}} - \frac{3\,a\left(\frac{2\,x\arctan(ax)}{\sqrt{a^2x^2+1}ac} - \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}a^2c} + \frac{2}{\sqrt{a^2x^2+1}a^2c}\right)}{\sqrt{c}}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
x*arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*c) - 3*a*(2*x*arctan(a*x)/(sqrt(a^2*x
^2 + 1)*a*c) - arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a^2*c) + 2/(sqrt(a^2*x^2 +
 1)*a^2*c))/sqrt(c)
```

3.447.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.447.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

3.448 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{3/2}}\,dx$

3.448.1 Optimal result

Integrand size = 24, antiderivative size = 443

$$
\begin{aligned}
\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^{3/2}}\,dx &= \frac{6ax}{c\sqrt{c+a^2cx^2}}-\frac{6\arctan(ax)}{c\sqrt{c+a^2cx^2}}-\frac{3ax\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}\\
&+\frac{\arctan(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&+\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&-\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&-\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&+\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\
&-\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}+\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
6*a*x/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-3*a*x*arct
an(a*x)^2/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2)-2*arct
an(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*
x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)*
polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^
(1/2)+6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/
2)/c/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*
x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+6*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/
2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)
```

3.448.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-i\pi^4+\frac{48ax}{\sqrt{1+a^2x^2}}-\frac{48\arctan(ax)}{\sqrt{1+a^2x^2}}-\frac{24ax\arctan(ax)^2}{\sqrt{1+a^2x^2}}+\frac{8\arctan(ax)^3}{\sqrt{1+a^2x^2}}+2i\arctan(\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-I)*Pi^4 + (48*a*x)/Sqrt[1 + a^2*x^2] - (48*ArcTan[a*
x])/Sqrt[1 + a^2*x^2] - (24*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (8*ArcT
an[a*x]^3)/Sqrt[1 + a^2*x^2] + (2*I)*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1
 - E^((-I)*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (2
4*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2
*PolyLog[2, -E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan
[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4
, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])]))/(8*c*Sqr
t[c*(1 + a^2*x^2)])
```

3.448.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.66, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5501, 5465, 5433, 208, 5493, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5491}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{4671}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax\right.}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-\right.}{}$$

$$\Big\downarrow \text{7163}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan(a\right.\right.\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}-\right.\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-e\right.\right.\right.\right.}{}$$

3.448. $\displaystyle\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{3/2}}\,dx$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c
 + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x
]^2)/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*
ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*
x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4,
 -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])]
- (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*
ArcTan[a*x])]))))/(c*Sqrt[c + a^2*c*x^2])
```

3.448.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.448. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{3/2}}\,dx$

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.448. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{3/2}}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.448.4 Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^3-6\arctan(a\right.}{2(a^2x^2+1)c^2}$

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-
I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*
(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^2+I*(I*a
rctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)^3*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x
)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog
(4,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/
2)/c^2
```

3.448.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 +
c^2*x), x)
```

3.448.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.448.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x), x)
```

3.448.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.449 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx$

3.449.1 Optimal result

Integrand size = 24, antiderivative size = 377

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx &= \frac{6a}{c\sqrt{c+a^2cx^2}}+\frac{6a^2x\arctan(ax)}{c\sqrt{c+a^2cx^2}}\\ &-\frac{3a\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{c^2x}\\ &-\frac{6a\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\ &+\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\ &-\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\\ &-\frac{6a\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}+\frac{6a\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
6*a/c/(a^2*c*x^2+c)^(1/2)+6*a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-3*a*ar
ctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)-a^2*x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2
)-6*a*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)
/c/(a^2*c*x^2+c)^(1/2)+6*I*a*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^
(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-6*I*a*arctan(a*x)*polylog(2
,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-6*a*
polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^
(1/2)+6*a*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c/(a^2*
c*x^2+c)^(1/2)-arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/c^2/x
```

3.449.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.80

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{a\Big(12+12ax\arctan(ax)-6\arctan(ax)^2-2ax\arctan(ax)^3-\frac{1}{2}ax\arctan(ax)^3\operatorname{cs}}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a*(12 + 12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]^3 - (a*x
*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2)/2 + 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2
*Log[1 - E^(I*ArcTan[a*x])] - 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^
(I*ArcTan[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*A
rcTan[a*x])] - (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan
[a*x])] - 12*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 12*Sqrt[1
+ a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^3*
Sin[ArcTan[a*x]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

3.449.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5501, 5433, 5429, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\left(a^2cx^2+c\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(-6\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5479}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5491}$$

$$\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{4671}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right) + \frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx} + \frac{3a\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax)-2\,a}{\sqrt{a^2cx^2+c}}}{c}$$

$$\Big\downarrow \text{3011}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right) + \frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx} + \frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\right.}{\sqrt{a^2cx^2+}}}{c}$$

$$\Big\downarrow \text{2720}$$

3.449. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+$$
$$\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2(}{}}{c}$$

$$\Big\downarrow \text{ 7143}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+$$
$$\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i\arc}\right.\right.\right.}{\sqrt{a^2cx^2+c}}}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*
Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c
*Sqrt[c + a^2*c*x^2])))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) +
 (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(
I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x
])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*Ar
cTan[a*x])])))/Sqrt[c + a^2*c*x^2])/c
```

3.449.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.449. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx$

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.449. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.449.4 Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^3-6\arctan(\right.}{2(a^2x^2+1)c^2}$

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x
-I)*(I+a*x))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(
arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/(a^2*x^2+1)/c^2-arcta
n(a*x)^3*(c*(a*x-I)*(I+a*x))^(1/2)/c^2/x-3*a*(arctan(a*x)^2*ln((1+I*a*x)/(
a^2*x^2+1)^(1/2)+1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*ar
ctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(
2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2
*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^
2+1)^(1/2)/c^2
```

3.449.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 +
c^2*x^2), x)
```

3.449.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{x^2(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.449.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

3.449.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,(c\,a^2\,x^2 + c)^{3/2}} \, dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.450 $\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

3.450.1 Optimal result

Integrand size = 24, antiderivative size = 534

$$
\begin{aligned}
\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{2x^3}{27a^3c\,(c+a^2cx^2)^{3/2}}+\frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} \\
&- \frac{2x^2\arctan(ax)}{9a^4c\,(c+a^2cx^2)^{3/2}}-\frac{94\arctan(ax)}{9a^6c^2\sqrt{c+a^2cx^2}}-\frac{x^3\arctan(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}} \\
&- \frac{5x\arctan(ax)^2}{a^5c^2\sqrt{c+a^2cx^2}}+\frac{6i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^6c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{x^2\arctan(ax)^3}{3a^4c\,(c+a^2cx^2)^{3/2}}+\frac{5\arctan(ax)^3}{3a^6c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{a^6c^3} \\
&- \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}}-\frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
2/27*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)-2/9*x^2*arctan(a*x)/a^4/c/(a^2*c*x^2+c)
^(3/2)-1/3*x^3*arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(3/2)+1/3*x^2*arctan(a*x)
^3/a^4/c/(a^2*c*x^2+c)^(3/2)+94/9*x/a^5/c^2/(a^2*c*x^2+c)^(1/2)-94/9*arcta
n(a*x)/a^6/c^2/(a^2*c*x^2+c)^(1/2)-5*x*arctan(a*x)^2/a^5/c^2/(a^2*c*x^2+c)
^(1/2)+5/3*arctan(a*x)^3/a^6/c^2/(a^2*c*x^2+c)^(1/2)+6*I*arctan((1+I*a*x)/
(a^2*x^2+1)^(1/2))*arctan(a*x)^2*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(
1/2)-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)
^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+6*polylog(
3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^
(1/2)-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^6/c^2
/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3*(a^2*c*x^2+c)^(1/2)/a^6/c^3
```

3.450.2 Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.69

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$

$$-\frac{(1+a^2x^2)^2 \Big(1134\arctan(ax) - 405\arctan(ax)^3 + 1128\arctan(ax)\cos(2\arctan(ax)) - 180\arctan(ax)^3 \mathrm{c}$$

input
```
Integrate[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/216*((1 + a^2*x^2)^2*(1134*ArcTan[a*x] - 405*ArcTan[a*x]^3 + 1128*ArcTa
n[a*x]*Cos[2*ArcTan[a*x]] - 180*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 6*ArcTa
n[a*x]*Cos[4*ArcTan[a*x]] + 9*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + (648*ArcT
an[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (648*ArcTan[a*
x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((1296*I)*ArcTan[a*
x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((1296*I)*ArcTa
n[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (1296*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (1296*PolyLog[3, I*E^(I*Ar
cTan[a*x])])/Sqrt[1 + a^2*x^2] - 1132*Sin[2*ArcTan[a*x]] + 558*ArcTan[a*x]
^2*Sin[2*ArcTan[a*x]] + 2*Sin[4*ArcTan[a*x]] - 9*ArcTan[a*x]^2*Sin[4*ArcTa
n[a*x]]))/(a^6*c*(c*(1 + a^2*x^2))^(3/2))
```

3.450.3 Rubi [A] (verified)

Time = 4.19 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.14, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {5499, 5475, 5465, 5433, 208, 5473, 5465, 208, 5499, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5475}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x \arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{2}{3}\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2 \arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3 \arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{2}{3}\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2 \arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3 \arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\left(\frac{3\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x \arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{2}{3}\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2 \arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3 \arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{2}{3}\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5473}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{2}{3}\left(\frac{2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c} - \frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right) - \frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\cdots\right)}{a}\cdots\right)}{3a^2c}\cdots}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{2}{3}\left(\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right) - \frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{\cdots}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots}{a^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}} - \frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \cdots\right)}{a^2}$$

$$\Big\downarrow \text{5499}$$

3.450. $\int \frac{x^5\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{\frac{\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\right.}{a^2}$$

$$\Big\downarrow \ 5465$$

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\right.}{a^2}$$

$$\Big\downarrow \ 5425$$

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\right.}{a^2}$$

$$\Big\downarrow \ 5423$$

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\right.}{a^2}$$

$$\Big\downarrow \ 3042$$

3.450. $\int \frac{x^5\arctan(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\right.}{a^2}$$

$\downarrow$ 4669

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$\frac{-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i}\right.\right.}{a^2\sqrt{a^2cx^2+c}}}{a^2c}}{a^2c}$$

$\downarrow$ 3011

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$\frac{-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}}{a^2c}$$

$\downarrow$ 2720

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie\right.\right.\right.}{}}{}$$

$\downarrow$ 5433

3.450. $\int \frac{x^5\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\left(-2\int\frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(}\right.\right.\right.}{}}{}$$

$$\Big\downarrow \; 208$$

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-}\right.\right.}{}}{}$$

$$\Big\downarrow \; 7143$$

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{Poly}\right.\right.}{}}{a^2c}$$

input
```
Int[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

3.450. $\int \frac{x^5\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

output
```
-(((x^3*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^3)
/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*(x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) -
 (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a
^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))/(3*a^2*c)))/3 + (2*
(-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2
*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(
c*Sqrt[c + a^2*c*x^2])))/a))/(3*a^2*c))/a^2) + (-((-(ArcTan[a*x]^3/(a^2*c*
Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x
])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))
/a)/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*
x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2
*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan
[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2]))/(a^2*c))/(a^2*c)
```

3.450.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.450. $\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx$

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

3.450. $\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.450.4 Maple [F]

$$\int \frac{x^5 \arctan\left(ax\right)^3}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

3.450.5 Fricas [F]

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

3.450.6 Sympy [F]

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**5*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.450.7 Maxima [F]

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.450.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^5*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.451 $\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

3.451.1 Optimal result

Integrand size = 24, antiderivative size = 622

$$\begin{aligned}\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx &= -\frac{2}{27a^5c\,(c+a^2cx^2)^{3/2}}+\frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} \\ &+\frac{2x^3\arctan(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{22x\arctan(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}-\frac{x^2\arctan(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}} \\ &-\frac{11\arctan(ax)^2}{3a^5c^2\sqrt{c+a^2cx^2}}-\frac{x^3\arctan(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{x\arctan(ax)^3}{a^4c^2\sqrt{c+a^2cx^2}} \\ &-\frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a^5c^2\sqrt{c+a^2cx^2}} \\ &+\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\ &-\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\,\mathrm{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\ &-\frac{6\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\ &+\frac{6\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\ &-\frac{6i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}}+\frac{6i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/27/a^5/c/(a^2*c*x^2+c)^(3/2)+2/9*x^3*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3
/2)-1/3*x^2*arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(3/2)-1/3*x^3*arctan(a*x)^3/
a^2/c/(a^2*c*x^2+c)^(3/2)+68/9/a^5/c^2/(a^2*c*x^2+c)^(1/2)+22/3*x*arctan(a
*x)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-11/3*arctan(a*x)^2/a^5/c^2/(a^2*c*x^2+c)^(
1/2)-x*arctan(a*x)^3/a^4/c^2/(a^2*c*x^2+c)^(1/2)-2*I*arctan((1+I*a*x)/(a^2
*x^2+1)^(1/2))*arctan(a*x)^3*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)
+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(
1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-3*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-6*arctan(a
*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a
^2*c*x^2+c)^(1/2)+6*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+6*I*polylog
(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^
(1/2)
```

3.451.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.11

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{\sqrt{c(1+a^2x^2)}\left(189i\pi^4 - \frac{12960}{\sqrt{1+a^2x^2}} + 216i\pi^3\arctan(ax) - \frac{12960ax\arctan(ax)}{\sqrt{1+a^2x^2}} - 648i\pi^2\arctan(ax)^2 + \frac{6480\arctan}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/1728*(Sqrt[c*(1 + a^2*x^2)]*((189*I)*Pi^4 - 12960/Sqrt[1 + a^2*x^2] + (
216*I)*Pi^3*ArcTan[a*x] - (12960*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (648
*I)*Pi^2*ArcTan[a*x]^2 + (6480*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (864*I)*
Pi*ArcTan[a*x]^3 + (2160*a*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (432*I)*Ar
cTan[a*x]^4 + 32*Cos[3*ArcTan[a*x]] - 144*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]]
 - 1296*Pi^2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 2592*Pi*ArcTan[a*x
]^2*Log[1 - I/E^(I*ArcTan[a*x])] + 216*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] -
 1728*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] - 216*Pi^3*Log[1 + I*E^(I
*ArcTan[a*x])] + 1296*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 2592
*Pi*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 1728*ArcTan[a*x]^3*Log[1
+ I*E^(I*ArcTan[a*x])] - 216*Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]] - (5184
*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - (1296*I)*Pi*(Pi - 4
*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] - (1296*I)*Pi^2*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] + (5184*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - (5184*I)*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 10
368*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 5184*Pi*PolyLog[3, I/
E^(I*ArcTan[a*x])] - 5184*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 10368*Ar
cTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (10368*I)*PolyLog[4, (-I)/E
^(I*ArcTan[a*x])] + (10368*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + 96*ArcT
an[a*x]*Sin[3*ArcTan[a*x]] - 144*ArcTan[a*x]^3*Sin[3*ArcTan[a*x]]))/(a^...
```

3.451.3 Rubi [A] (verified)

Time = 3.66 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.91, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5499, 5479, 5475, 243, 53, 2009, 5465, 5429, 5499, 5425, 5423, 3042, 4669, 3011, 5433, 5429, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5479}$$

$$\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \textcolor{blue}{5475}$$

$$\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c} - \frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \textcolor{blue}{243}$$

$$\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c} - \frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2 - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \textcolor{blue}{53}$$

$$\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c} - \frac{1}{9}\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}} - \frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2 - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c} - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}} - \frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)\right)}{a^2}$$

$$\Big\downarrow \textcolor{blue}{5465}$$

$$\frac{\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(\frac{2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}} - \cdots 3a^4c(a^2\cdots\right.\right.}{a^2}$$

3.451. $\int \frac{x^4\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \text{5429}$$

$$\frac{\frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - }{}$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c} -$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}} - \frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c} -$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3 d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}} - \frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c} -$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$\Big\downarrow \text{3042}$$

3.451. $\int \frac{x^4\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$\downarrow$ 4669

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2}}\right.\right.}{a^2}$$

$$\frac{-\frac{\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan\right.}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$\downarrow$ 3011

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2}}\right.\right.}{a^2}$$

$$\frac{-\frac{\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-3\left(i\arctan\right.\right.}{a^3c}}{a^2c}$$

$\downarrow$ 5433

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2}}\right.\right.}{a^2}$$

$$-\frac{-6\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i}\right.\right.\right.}{}$$

$\downarrow$ 5429

3.451. $$\int \frac{x^4\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$$

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2\cdots}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\cdots\right.\right.}{\cdots}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2\cdots}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\,\mathrm{a}\cdots}\right.\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2\cdots}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{Poly}\cdots\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2\cdots}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\,\mathrm{arcta}\cdots}\right.\right.\right.\right.}{\cdots}$$

3.451. $\int \frac{x^4\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

input
```
Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-(((x^3*ArcTan[a*x]^3)/(3*c*(c + a^2*c*x^2)^(3/2)) - a*((-2/(3*a^4*c*(c +
a^2*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/9 + (2*x^3*ArcTan[a*x
])/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2)/(3*a^2*c*(c + a^2*c
*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a
*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a))/(3
*a^2*c)))/a^2) + (-(((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcT
an[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*A
rcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a^2) + (Sqrt[1 + a^2*x^2]*((-2*I)*Ar
cTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog
[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTa
n[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(a^3*c*Sqrt[c + a^2*c*x^2])
)/(a^2*c)
```

3.451.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.451. $\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

3.451. $\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.451. $\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

3.451.4 Maple [F]

$$\int \frac{x^4 \arctan(ax)^3}{(a^2 c\, x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

3.451.5 Fricas [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c + a^2cx^2)^{5/2}} \, dx = \int \frac{x^4 \arctan(ax)^3}{(a^2cx^2 + c)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

3.451.6 Sympy [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c + a^2cx^2)^{5/2}} \, dx = \int \frac{x^4 \operatorname{atan}^3(ax)}{(c(a^2x^2 + 1))^{\frac{5}{2}}} \, dx$$

input
```
integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.451.7 Maxima [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.451.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.452 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

3.452.1 Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx &= -\frac{2x^3}{27ac\,(c+a^2cx^2)^{3/2}} - \frac{40x}{9a^3c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{2x^2\arctan(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{40\arctan(ax)}{9a^4c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} \\ &+ \frac{2x\arctan(ax)^2}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\arctan(ax)^3}{3a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/27*x^3/a/c/(a^2*c*x^2+c)^(3/2)+2/9*x^2*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^
(3/2)+1/3*x^3*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)-1/3*x^2*arctan(a*x)^3/
a^2/c/(a^2*c*x^2+c)^(3/2)-40/9*x/a^3/c^2/(a^2*c*x^2+c)^(1/2)+40/9*arctan(a
*x)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+2*x*arctan(a*x)^2/a^3/c^2/(a^2*c*x^2+c)^(1
/2)-2/3*arctan(a*x)^3/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

3.452.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.44

$$\int \frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-2ax(60+61a^2x^2)+6(20+21a^2x^2)\arctan(ax)+9ax(6+7a^2x^2)\operatorname{arct}}{27a^4c^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(60 + 61*a^2*x^2) + 6*(20 + 21*a^2*x^2)*ArcTa
n[a*x] + 9*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]^2 - 9*(2 + 3*a^2*x^2)*ArcTan[a*
x]^3))/(27*a^4*c^3*(1 + a^2*x^2)^2)
```

3.452.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5475, 5465, 5433, 208, 5473, 5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\arctan(ax)^3}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{2\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{2}{3}\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{2}{3}\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(\frac{3\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{2}{3}\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{2}{3}\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5473}$$

$$-\frac{2}{3}\left(\frac{2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}\right)-\frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2}{3}\left(\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}\right)-\frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{208}$$

3.452. $\int \frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

$$-\frac{x^2\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^3)/(3
*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*(x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x
^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a^2*
c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))/(3*a^2*c)))/3 + (2*(-(
ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2*c*
x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*S
qrt[c + a^2*c*x^2])))/a))/(3*a^2*c)
```

3.452.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.452. $\int \frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

3.452.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.30 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^4c^3}-\frac{3\left(\arctan(ax)^3-6\arctan(ax)+\right.}{8}$

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*
a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(ar
ctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(I+a
*x))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(ar
ctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^4/(a^2*x^2+1)+1/216
*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*
x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{\left(122\,a^3x^3+9\,(3\,a^2x^2+2)\arctan{(ax)}^3-9\,(7\,a^3x^3+6\,ax)\arctan{(ax)}^2+120\,ax-6\,(21\,a^2x^2+20)\arctan\right.}{27\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*(122*a^3*x^3 + 9*(3*a^2*x^2 + 2)*arctan(a*x)^3 - 9*(7*a^3*x^3 + 6*a*
x)*arctan(a*x)^2 + 120*a*x - 6*(21*a^2*x^2 + 20)*arctan(a*x))*sqrt(a^2*c*x
^2 + c)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

3.452.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{atan}^3{(ax)}}{\left(c\,(a^2x^2+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.452.7 Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.452.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^3}{(c + a^2cx^2)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{(c + a^2cx^2)^{5/2}} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2 + c)^{5/2}} \, dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.453 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

3.453.1 Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{2}{27a^3c\,(c+a^2cx^2)^{3/2}} - \frac{14}{9a^3c^2\sqrt{c+a^2cx^2}} - \frac{2x^3\arctan(ax)}{9c\,(c+a^2cx^2)^{3/2}} - \frac{4x\arctan(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x^2\arctan(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} + \frac{2\arctan(ax)^2}{3a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)^3}{3c\,(c+a^2cx^2)^{3/2}}$$

output
```
2/27/a^3/c/(a^2*c*x^2+c)^(3/2)-2/9*x^3*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1
/3*x^2*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x^3*arctan(a*x)^3/c/(a^2*
c*x^2+c)^(3/2)-14/9/a^3/c^2/(a^2*c*x^2+c)^(1/2)-4/3*x*arctan(a*x)/a^2/c^2/
(a^2*c*x^2+c)^(1/2)+2/3*arctan(a*x)^2/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

3.453.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.48

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-40-42a^2x^2-6ax(6+7a^2x^2)\arctan(ax)+9(2+3a^2x^2)\arctan(ax)^2}{27a^3c^3\,(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-40 - 42*a^2*x^2 - 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]
 + 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2 + 9*a^3*x^3*ArcTan[a*x]^3))/(27*a^3*c^3
*(1 + a^2*x^2)^2)
```

3.453.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5479, 5475, 243, 53, 2009, 5465, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}-a\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{x^3\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{x^3\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{53}$$

$$\frac{x^3\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \left(\frac{1}{a^2c\,(a^2cx^2+c)^{3/2}}-\frac{1}{a^2\,(a^2cx^2+c)^{5/2}}\right)dx^2-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(}{9ac\,(a^2cx^2+}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c\,(a^2cx^2+c)^{3/2}}\right)\right)$$

$$\Big\downarrow \ 5465$$

$$\frac{x^3 \arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}} - a\left(\frac{2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}} - \frac{}{3a^4c}\right.\right.$$

$$\Big\downarrow \ 5429$$

$$\frac{x^3 \arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}} - a\left(-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+}}\right.\right.$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^3)/(3*c*(c + a^2*c*x^2)^(3/2)) - a*((-2/(3*a^4*c*(c + a^2
*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/9 + (2*x^3*ArcTan[a*x])/
(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2)/(3*a^2*c*(c + a^2*c*x^
2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*
Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a))/(3*a^
2*c))
```

3.453.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.453. $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.453.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.79 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.55

method	result
default	$\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^3c^3}+\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\,\mathrm{arc}\right.}{8c^3a^3}$

3.453. $\int \frac{x^2\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*a
^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^3/c^3+1/8*(arcta
n(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^
(1/2)/c^3/a^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*
x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^3/(a^2*x^2+1)+1/216*(-9*I*
arctan(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)
*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a^3/c^3
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.53

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\left(9\,a^3x^3 \arctan\left(ax\right)^3 - 42\,a^2x^2 + 9\left(3\,a^2x^2+2\right)\arctan\left(ax\right)^2 - 6\left(7\,a^3x^3+6\,ax\right)\text{arcta}}{27\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*arctan(a*x)^3 - 42*a^2*x^2 + 9*(3*a^2*x^2 + 2)*arctan(a*x)
^2 - 6*(7*a^3*x^3 + 6*a*x)*arctan(a*x) - 40)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*
x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

3.453.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.453.7 Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.453.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.454 $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

3.454.1 Optimal result

Integrand size = 22, antiderivative size = 199

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{2x}{27ac\,(c+a^2cx^2)^{3/2}} - \frac{40x}{27ac^2\sqrt{c+a^2cx^2}} + \frac{2\arctan(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{4\arctan(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)^2}{3ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}}$$

output
```
-2/27*x/a/c/(a^2*c*x^2+c)^(3/2)+2/9*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)+
1/3*x*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)-1/3*arctan(a*x)^3/a^2/c/(a^2*c
*x^2+c)^(3/2)-40/27*x/a/c^2/(a^2*c*x^2+c)^(1/2)+4/3*arctan(a*x)/a^2/c^2/(a
^2*c*x^2+c)^(1/2)+2/3*x*arctan(a*x)^2/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.454.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-2ax(21+20a^2x^2)+6(7+6a^2x^2)\arctan(ax)+9ax(3+2a^2x^2)\arctan}{27c^3\,(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[
a*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2 - 9*ArcTan[a*x]^3))/(27*c^3*(a
+ a^3*x^2)^2)
```

3.454.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5465, 5435, 209, 208, 5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^3}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{2}{9}\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{\frac{2\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \textbf{208}$$

$$\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^3/(a^2*c*(c + a^2*c*x^2)^(3/2)) + ((-2*(x/(3*c*(c + a^2*c
*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2])))/9 + (2*ArcTan[a*x])/(9*
a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2))
 + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))/(3*c))/a
```

3.454.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

3.454. $\int \frac{x\arctan(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.454.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.57

method	result
default	$\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216\left(a^2x^2+1\right)^2a^2c^3}-\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\,\mathrm{ar}\right.}{8c^3}$

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*a
^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arc
tan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(I+a*
x))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arc
tan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^2/(a^2*x^2+1)-1/216*
(c*(a*x-I)*(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x
)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))/c^3/a^2/(a^4*x^4+2*a^2*x^2+1)
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.52

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{\left(40\,a^3x^3-9\,(2\,a^3x^3+3\,ax)\arctan\left(ax\right)^2+9\,\arctan\left(ax\right)^3+42\,ax-6\,(6\,a^2x^2+7)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+}}{27\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 9*arctan(a*x)^3
+ 42*a*x - 6*(6*a^2*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4
 + 2*a^4*c^3*x^2 + a^2*c^3)
```

3.454.6 Sympy [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\operatorname{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.454.7 Maxima [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan\left(ax\right)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.454.8 Giac [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

3.455 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

3.455.1 Optimal result

Integrand size = 21, antiderivative size = 215

$$\begin{aligned}\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx &= -\frac{2}{27ac\,(c+a^2cx^2)^{3/2}}-\frac{40}{9ac^2\sqrt{c+a^2cx^2}}\\ &\quad-\frac{2x\arctan(ax)}{9c\,(c+a^2cx^2)^{3/2}}-\frac{40x\arctan(ax)}{9c^2\sqrt{c+a^2cx^2}}+\frac{\arctan(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}}\\ &\quad+\frac{2\arctan(ax)^2}{ac^2\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^3}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\arctan(ax)^3}{3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/27/a/c/(a^2*c*x^2+c)^(3/2)-2/9*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)+1/3*
arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x*arctan(a*x)^3/c/(a^2*c*x^2+c)^
(3/2)-40/9/a/c^2/(a^2*c*x^2+c)^(1/2)-40/9*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^
(1/2)+2*arctan(a*x)^2/a/c^2/(a^2*c*x^2+c)^(1/2)+2/3*x*arctan(a*x)^3/c^2/(a
^2*c*x^2+c)^(1/2)
```

3.455.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.48

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(-2(61+60a^2x^2)-6ax(21+20a^2x^2)\arctan(ax)+9(7+6a^2x^2)\arctan(\right.}{27ac^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*(61 + 60*a^2*x^2) - 6*a*x*(21 + 20*a^2*x^2)*ArcTa
n[a*x] + 9*(7 + 6*a^2*x^2)*ArcTan[a*x]^2 + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*
x]^3))/(27*a*c^3*(1 + a^2*x^2)^2)
```

3.455.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5435, 5431, 5429, 5433, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \; 5435$$

$$-\frac{2}{3}\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5431$$

$$-\frac{2}{3}\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5429$$

$$\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output
```
ArcTan[a*x]^2/(3*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^3)/(3*c*(c +
a^2*c*x^2)^(3/2)) + (2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*A
rcTan[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (
x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]))))/(3*c) - (2*(1/(9*a*c*(c + a^2*c*
x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(a*c*Sqr
t[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c)))/3
```

3.455.3.1 Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.455. $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

3.455.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.17 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\left(9 i \arctan(a x)^{2}+9 \arctan(a x)^{3}-2 i-6 \arctan(a x)\right)\left(a^{3} x^{3}-3 i a^{2} x^{2}-3 a x+i\right) \sqrt{c(a x-i)(a x+i)}}{216\left(a^{2} x^{2}+1\right)^{2} c^{3} a}+\frac{3\left(\arctan(a x)^{3}-6 \arctan(a x)+3\right.}{8 a}$

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*
a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3/a+3/8*(arctan
(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^(
1/2)/a/c^3/(a^2*x^2+1)+3/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^
3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/a/c^3/(a^2*x^2+1)-1/216*(-9*I*arcta
n(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)*(a^3
*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/c^3/a
```

3.455.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{\sqrt{a^2cx^2+c}\left(120\,a^2x^2-9\,(2\,a^3x^3+3\,ax)\arctan{(ax)}^3-9\,(6\,a^2x^2+7)\arctan{(ax)}^2+6\,(20\,a^3x^3+21\,ax)\,\mathrm{a}\right.}{27\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*sqrt(a^2*c*x^2 + c)*(120*a^2*x^2 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)
^3 - 9*(6*a^2*x^2 + 7)*arctan(a*x)^2 + 6*(20*a^3*x^3 + 21*a*x)*arctan(a*x)
 + 122)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

3.455.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^3{(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.455.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

3.455.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

3.456 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

3.456.1 Optimal result

Integrand size = 24, antiderivative size = 553

$$\begin{aligned}
\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx &= \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{202ax}{27c^2\sqrt{c+a^2cx^2}}-\frac{2\arctan(ax)}{9c\left(c+a^2cx^2\right)^{3/2}} \\
&- \frac{22\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{ax\arctan(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}}-\frac{11ax\arctan(ax)^2}{3c^2\sqrt{c+a^2cx^2}}+\frac{\arctan(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}} \\
&+ \frac{\arctan(ax)^3}{c^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\
&- \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\
&- \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\
&- \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}+\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}
\end{aligned}$$

output
```
2/27*a*x/c/(a^2*c*x^2+c)^(3/2)-2/9*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-1/3*a
*x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)+1/3*arctan(a*x)^3/c/(a^2*c*x^2+c)^(
3/2)+202/27*a*x/c^2/(a^2*c*x^2+c)^(1/2)-22/3*arctan(a*x)/c^2/(a^2*c*x^2+c)
^(1/2)-11/3*a*x*arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3/c^2/(a
^2*c*x^2+c)^(1/2)-2*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^
2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+3*I*arctan(a*x)^2*polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-3*I*arcta
n(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2
*c*x^2+c)^(1/2)-6*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2
*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-6*I*polylog(4,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*I
*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c
)^(1/2)
```

3.456.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\left(1+a^2x^2\right)^{3/2}\left(-27i\pi^4+\frac{1620ax}{\sqrt{1+a^2x^2}}-\frac{1620\arctan(ax)}{\sqrt{1+a^2x^2}}-\frac{810ax\arctan(ax)^2}{\sqrt{1+a^2x^2}}+\frac{270\arctan(ax)^3}{\sqrt{1+a^2x^2}}+\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
((1 + a^2*x^2)^(3/2)*((-27*I)*Pi^4 + (1620*a*x)/Sqrt[1 + a^2*x^2] - (1620*
ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (810*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2]
 + (270*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (54*I)*ArcTan[a*x]^4 - 12*ArcTa
n[a*x]*Cos[3*ArcTan[a*x]] + 18*ArcTan[a*x]^3*Cos[3*ArcTan[a*x]] + 216*ArcT
an[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 216*ArcTan[a*x]^3*Log[1 + E^(I*A
rcTan[a*x])] + (648*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (6
48*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] + 1296*ArcTan[a*x]*Poly
Log[3, E^((-I)*ArcTan[a*x])] - 1296*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*
x])] - (1296*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (1296*I)*PolyLog[4, -E^
(I*ArcTan[a*x])] + 4*Sin[3*ArcTan[a*x]] - 18*ArcTan[a*x]^2*Sin[3*ArcTan[a*
x]]))/(216*c*(c*(1 + a^2*x^2))^(3/2))
```

3.456.3 Rubi [A] (verified)

Time = 3.37 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.95, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {5501, 5465, 5435, 209, 208, 5433, 208, 5501, 5465, 5433, 208, 5493, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \ 5501$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\int \frac{x\arctan(ax)^3}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \ 5465$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \ 5435$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \ 209$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{2}{9}\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)}\right)$$

$$\Big\downarrow \ 208$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{\frac{2\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right.}{a}\right.$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \text{5465}$$

3.456. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \text{5433}$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \text{5493}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \text{5491}$$

3.456. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3a^2}\right.$$

$$\Big\downarrow \; 4671$$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3}\right.$$

$$\frac{-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int\right.}{}}{c}$$

$$\Big\downarrow \; 3011$$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3}\right.$$

$$\frac{-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(\right.\right.}{}}{}$$

$$\Big\downarrow \; 7163$$

3.456. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\cdots}{3\cdots}\right.$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyL}\cdots\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \; 2720$$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\cdots}{3\cdots}\right.$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\,\mathrm{arct}}\cdots\right.\right.\right.}{\cdots}$$

$$\Big\downarrow \; 7143$$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\cdots}{3\cdots}\right.$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)\cdots\right.\right.}{\cdots}$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

3.456. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

output
```
-(a^2*(-1/3*ArcTan[a*x]^3/(a^2*c*(c + a^2*c*x^2)^(3/2)) + ((-2*(x/(3*c*(c
+ a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2])))/9 + (2*ArcTan[a*
x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)
^(3/2)) + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c
 + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))/(3*c))/a)) +
(-(a^2*(-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[
c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*
x]^2)/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3
*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a
*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4
, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])]
 - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I
*ArcTan[a*x])]))))/(c*Sqrt[c + a^2*c*x^2]))/c
```

3.456.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.456. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

3.456. $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.456.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.01

method	result
default	$-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2c^3}+\frac{5\left(\arctan(ax)^3-6\arctan(ax)+\right.}{}$

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*
a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan
(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(I+a*x))
^(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I*a*x-1)*(arctan(a*x
)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/(a^2*x^2+1)+1/216*(c*(a*x-I)*
(I+a*x))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x)^2+9*arcta
n(a*x)^3+2*I-6*arctan(a*x))/c^3/(a^4*x^4+2*a^2*x^2+1)+I*(I*arctan(a*x)^3*l
n((1+I*a*x)/(a^2*x^2+1)^(1/2)+1)-I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(
a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(1+I*a*x)/(
a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

3.456.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 +
3*a^2*c^3*x^3 + c^3*x), x)
```

3.456.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\operatorname{atan}^3\left(ax\right)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.456.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x), x)
```

3.456.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.457 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx$

3.457.1 Optimal result

Integrand size = 24, antiderivative size = 493

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx &= \frac{2a}{27c(c+a^2cx^2)^{3/2}}+\frac{94a}{9c^2\sqrt{c+a^2cx^2}} \\ &+\frac{2a^2x\arctan(ax)}{9c(c+a^2cx^2)^{3/2}}+\frac{94a^2x\arctan(ax)}{9c^2\sqrt{c+a^2cx^2}}-\frac{a\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}} \\ &-\frac{5a\arctan(ax)^2}{c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)^3}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\arctan(ax)^3}{3c^2\sqrt{c+a^2cx^2}} \\ &-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{c^3x}-\frac{6a\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\ &+\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\ &-\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\ &-\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}+\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2/27*a/c/(a^2*c*x^2+c)^(3/2)+2/9*a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-1
/3*a*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-1/3*a^2*x*arctan(a*x)^3/c/(a^2*c*
x^2+c)^(3/2)+94/9*a/c^2/(a^2*c*x^2+c)^(1/2)+94/9*a^2*x*arctan(a*x)/c^2/(a^
2*c*x^2+c)^(1/2)-5*a*arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(1/2)-5/3*a^2*x*arcta
n(a*x)^3/c^2/(a^2*c*x^2+c)^(1/2)-6*a*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*I*a*arctan(a*x)*
polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c
)^(1/2)-6*I*a*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+
1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-6*a*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+6*a*polylog(3,(1+I*a*x)/(a^2*
x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^3*(a^2
*c*x^2+c)^(1/2)/c^3/x
```

3.457.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^{5/2}} \, dx =$$
$$-\frac{a\left(-1134 - 1134ax\arctan(ax) + 567\arctan(ax)^2 + 189ax\arctan(ax)^3 - 2\sqrt{1 + a^2x^2}\cos(3\arctan(ax)) + \right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-1/108*(a*(-1134 - 1134*a*x*ArcTan[a*x] + 567*ArcTan[a*x]^2 + 189*a*x*ArcT
an[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]] + 9*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^2*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^
2 - 324*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 324*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] - (648*I)*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (648*I)*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 648*Sqrt[1 + a^2*x^2]
*PolyLog[3, -E^(I*ArcTan[a*x])] - 648*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*Ar
cTan[a*x])] - 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Sin[3*ArcTan[a*x]] + 9*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]^3*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^3*Tan[ArcTan[a*x]/2]))/(c^2*Sqrt[c + a^2*c*x^2])
```

3.457.3 Rubi [A] (verified)

Time = 3.99 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {5501, 5435, 5431, 5429, 5433, 5429, 5501, 5433, 5429, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(-\frac{2}{3}\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 5431}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(-\frac{2}{3}\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}\right.$$

$$\Big\downarrow \text{ 5429}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}\right.\right.$$

$$\Big\downarrow \text{ 5433}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\left(-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}} + \frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}} - \frac{2}{3}\left(\frac{x\arcta}{3c\left(a^2cx^2\right.}\right.\right.$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}} + \frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}} + \frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

$$\Big\downarrow \text{5433}$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}} + \frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

$$\Big\downarrow \text{5429}$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}} + \frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$\frac{\frac{3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}} + \frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

3.457. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \; 5493$$

$$\frac{\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{c}-$$

$$a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\Bigg(\right.$$

$$\Big\downarrow \; 5491$$

$$\frac{\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+}}\right.\right.}{c}$$

$$a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\Bigg(\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2c}}\right.\right.}{c}$$

$$a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\Bigg(\right.$$

$$\Big\downarrow \; 4671$$

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$\frac{-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i}\right.\right.}{}}{c}$$

$$\Big\downarrow \; 3011$$

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,\right.\right.\right.}{}}{}$$

3.457. $\displaystyle\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx$

$\downarrow$ 2720

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$\frac{-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}(2,\right.\right.}{}}{}$$

$\downarrow$ 7143

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$\frac{-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\right.}{}}{}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*(ArcTan[a*x]^2/(3*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^3)/(3*
c*(c + a^2*c*x^2)^(3/2)) + (2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2])
 + (x*ArcTan[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^
2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]))))/(3*c) - (2*(1/(9*a*c*(c +
 a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(
a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c)
))/3)) + (-(a^2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a
*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTa
n[a*x])/(c*Sqrt[c + a^2*c*x^2])))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*
x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*
ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[
3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])/c)/c
```

3.457. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx$

3.457.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

3.457. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx$

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.457. $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.457.4 Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.07

method	result
default	$\frac{a\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c(ax-i)(ax+i)}}{216\left(a^2x^2+1\right)^2c^3} - \frac{7a\left(\arctan(ax)^3-6\arctan(ax)+3\right.}{8c}$

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*a*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I
*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^2/c^3-7/8*a*(arcta
n(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(I+a*x))^
(1/2)/c^3/(a^2*x^2+1)-7/8*(c*(a*x-I)*(I+a*x))^(1/2)*(I+a*x)*(arctan(a*x)^3
-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/c^3/(a^2*x^2+1)+1/216*(c*(a*x-I)*(
I+a*x))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*(-9*I*arctan(a*x)^2+9*arctan(a
*x)^3+2*I-6*arctan(a*x))*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)^3*(c*(a*x
-I)*(I+a*x))^(1/2)/c^3/x-3*a*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
+1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2
*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

3.457.5 Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 +
3*a^2*c^3*x^4 + c^3*x^2), x)
```

3.457.6 Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.457.7 Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

3.457.8 Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.458 $\int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx$

3.458.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \operatorname{Int}\left(x^m (c+a^2cx^2)^2 \arctan(ax)^3, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

3.458.2 Mathematica [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3, x]
```

3.458.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \; 5560$$

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.458.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.458.4 Maple [N/A] (verified)

Not integrable

Time = 3.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^3 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

3.458.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^3, x)
```

3.458.6 Sympy [N/A]

Not integrable

Time = 28.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = c^2 \left(\int x^m \operatorname{atan}^3\left(ax\right) dx + \int 2a^2x^2x^m \operatorname{atan}^3\left(ax\right) dx + \int a^4x^4x^m \operatorname{atan}^3\left(ax\right) dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**m*atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m*atan(a*x)
**3, x) + Integral(a**4*x**4*x**m*atan(a*x)**3, x))
```

3.458.7 Maxima [N/A]

Not integrable

Time = 13.16 (sec) , antiderivative size = 952, normalized size of antiderivative = 43.27

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(4*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*
a^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*
x)^3 - 3*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6
*a^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a
*x)*log(a^2*x^2 + 1)^2 + 32*(m^3 + 9*m^2 + 23*m + 15)*integrate(1/32*(28*(
(a^6*c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^3 +
3*(a^4*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^2*m^
2 + 23*c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*c^2)
*x^2 + 15*c^2)*x^m*arctan(a*x)^3 - 12*((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*
c^2)*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*
a*c^2*m + 15*a*c^2)*x)*x^m*arctan(a*x)^2 + 12*((a^6*c^2*m^2 + 4*a^6*c^2*m
+ 3*a^6*c^2)*x^6 + 2*(a^4*c^2*m^2 + 6*a^4*c^2*m + 5*a^4*c^2)*x^4 + (a^2*c^
2*m^2 + 8*a^2*c^2*m + 15*a^2*c^2)*x^2)*x^m*arctan(a*x)*log(a^2*x^2 + 1) +
3*(((a^6*c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^
3 + 3*(a^4*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^
2*m^2 + 23*c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*
c^2)*x^2 + 15*c^2)*x^m*arctan(a*x) + ((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*c
^2)*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*a
*c^2*m + 15*a*c^2)*x)*x^m)*log(a^2*x^2 + 1)^2)/(m^3 + (a^2*m^3 + 9*a^2*m^2
 + 23*a^2*m + 15*a^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m...
```

3.458.8 Giac [N/A]

Not integrable

Time = 149.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x^m(c+a^2cx^2)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x^m\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.458.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m \left(c + a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = \int x^m \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^2 dx$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

3.459 $\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx$

3.459.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx=\operatorname{Int}\left(x^m(c+a^2cx^2)\arctan(ax)^3,x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

3.459.2 Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx=\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3, x]
```

3.459.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right) \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right) dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.459.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.459.4 Maple [N/A] (verified)

Not integrable

Time = 3.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2 c\,x^2+c\right) \arctan\left(a x\right)^3 dx$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

3.459.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)x^m\arctan(ax)^3\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

3.459.6 Sympy [N/A]

Not integrable

Time = 12.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx = c\left(\int x^m\operatorname{atan}^3(ax)\,dx+\int a^2x^2x^m\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x**m*atan(a*x)**3, x) + Integral(a**2*x**2*x**m*atan(a*x)**3,
x))
```

3.459.7 Maxima [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 435, normalized size of antiderivative = 21.75

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)x^m\arctan(ax)^3\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(4*((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)^3 - 3*((a^
2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)*log(a^2*x^2 + 1)^2 + 3
2*(m^2 + 4*m + 3)*integrate(1/32*(28*((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^
4 + c*m^2 + 2*(a^2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arc
tan(a*x)^3 - 12*((a^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m*arctan(a*x
)^2 + 12*((a^4*c*m + a^4*c)*x^4 + (a^2*c*m + 3*a^2*c)*x^2)*x^m*arctan(a*x)
*log(a^2*x^2 + 1) + 3*(((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*
(a^2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arctan(a*x) + ((a
^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m)*log(a^2*x^2 + 1)^2)/((a^2*m^
2 + 4*a^2*m + 3*a^2)*x^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

3.459.8 Giac [N/A]

Not integrable

Time = 145.87 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int x^m \left(c+a^2 c x^2\right) \arctan (a x)^3 \, dx=\int \left(a^2 c x^2+c\right) x^m \arctan \left(a x\right)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.459.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \left(c+a^2 c x^2\right) \arctan (a x)^3 \, dx=\int x^m\,{\mathrm{atan}\left(a\,x\right)}^3\,\left(c\,a^2\,x^2+c\right) \,d x$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2), x)
```

3.460 $\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2} \, dx$

3.460.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2} \, dx = \text{Int}\left(\frac{x^m \arctan(ax)^3}{c+a^2cx^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

3.460.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^m \arctan(ax)^3}{c+a^2cx^2} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2), x]
```

3.460.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^3}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.460.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.460.4 Maple [N/A] (verified)

Not integrable

Time = 1.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan\left(ax\right)^3}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

3.460.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.460.6 Sympy [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^3{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

3.460.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

3.460.8 Giac [N/A]

Not integrable

Time = 136.66 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m \arctan(ax)^3}{c + a^2cx^2}\, dx = \int \frac{x^m \arctan{(ax)}^3}{a^2cx^2 + c}\, dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.460.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c + a^2cx^2}\, dx = \int \frac{x^m \,\mathrm{atan}(a\,x)^3}{c\,a^2\,x^2 + c}\, dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

3.461 $\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$

3.461.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

3.461.2 Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2, x]
```

3.461.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^2} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^2} dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.461.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.461.4 Maple [N/A] (verified)

Not integrable

Time = 4.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan\left(ax\right)^3}{\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

3.461.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.461.6 Sympy [N/A]

Not integrable

Time = 4.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^3{(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.461.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

3.461.8 Giac [N/A]

Not integrable

Time = 159.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.461.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^2, x)
```

3.462 $\int x^m (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx$

3.462.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \operatorname{Int}\left(x^m (c+a^2cx^2)^{3/2} \arctan(ax)^3, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

3.462.2 Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \int x^m (c+a^2cx^2)^{3/2} \arctan(ax)^3 \, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3, x]
```

3.462.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.462.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.462.4 Maple [N/A] (verified)

Not integrable

Time = 4.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{3}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

3.462.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2 c x^2 + c)^{\frac{3}{2}} x^m \arctan(ax)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

3.462.6 Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Timed out
```

3.462.7 Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2 c x^2 + c)^{\frac{3}{2}} x^m \arctan(ax)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

3.462.8 Giac [F(-2)]

Exception generated.

$$\int x^m (c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.462.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \int x^m \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

3.463 $\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^3 \, dx$

3.463.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^3 \, dx = \text{Int}\left(x^m \sqrt{c+a^2 c x^2} \arctan(ax)^3, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

3.463.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^3 \, dx = \int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^3 \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3, x]
```

3.463.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^m \arctan(ax)^3 \sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.463.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.463.4 Maple [N/A] (verified)

Not integrable

Time = 3.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \arctan(ax)^3 \sqrt{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x)
```

3.463.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \int \sqrt{a^2cx^2+c}x^m \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

3.463.6 Sympy [N/A]

Not integrable

Time = 71.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \int x^m \sqrt{c\left(a^2x^2+1\right)} \operatorname{atan}^3\left(ax\right) dx$$

input
```
integrate(x**m*atan(a*x)**3*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

3.463.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \int \sqrt{a^2cx^2+c}x^m \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

3.463.8 Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^3*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.463.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int x^m\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

3.464 $\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$

3.464.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.464.2 Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2], x]
```

3.464.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.464.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.464.4 Maple [N/A] (verified)

Not integrable

Time = 3.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^3}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.464.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.464.6 Sympy [N/A]

Not integrable

Time = 49.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

3.464.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

3.464.8 Giac [N/A]

Not integrable

Time = 59.76 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan\left(ax\right)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.464.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

3.465 $\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$

3.465.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

3.465.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2), x]
```

3.465.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.465.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.465.4 Maple [N/A] (verified)

Not integrable

Time = 3.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^3}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

3.465.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

3.465.6 Sympy [N/A]

Not integrable

Time = 59.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.465.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

3.465.8 Giac [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.465.9 Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

3.466 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx$

3.466.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

3.466.2 Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

3.466.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.466.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.466.4 Maple [N/A] (verified)

Not integrable

Time = 13.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

3.466.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)/arctan(a*x), x)
```

3.466.6 Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = c\left(\int \frac{x}{\operatorname{atan}(ax)}\,dx + \int \frac{a^2x^3}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
c*(Integral(x/atan(a*x), x) + Integral(a**2*x**3/atan(a*x), x))
```

3.466.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*x/arctan(a*x), x)
```

3.466.8 Giac [N/A]

Not integrable

Time = 30.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.466.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x), x)
```

3.467 $\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx$

3.467.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/arctan(a*x),x)
```

3.467.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \int \frac{c+a^2cx^2}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x], x]
```

3.467.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{a^2cx^2+c}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.467.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.467.4 Maple [N/A] (verified)

Not integrable

Time = 13.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x),x)
```

3.467.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx=\int \frac{a^2cx^2+c}{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arctan(a*x), x)
```

3.467.6 Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx=c\left(\int \frac{a^2x^2}{\operatorname{atan}\left(ax\right)}\,dx+\int \frac{1}{\operatorname{atan}\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x),x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x), x) + Integral(1/atan(a*x), x))
```

3.467.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx=\int \frac{a^2cx^2+c}{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)/arctan(a*x), x)
```

3.467.8 Giac [N/A]

Not integrable

Time = 28.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.18

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.467.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x), x)
```

3.468 $\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx$

3.468.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/x/arctan(a*x),x)
```

3.468.2 Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]), x]
```

3.468.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5560\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{a^2cx^2+c}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.468.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.468.4 Maple [N/A] (verified)

Not integrable

Time = 8.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x),x)
```

3.468.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

3.468.6 Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = c\left(\int \frac{1}{x\,\mathrm{atan}(ax)}\,dx + \int \frac{a^2x}{\mathrm{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x),x)
```

output
```
c*(Integral(1/(x*atan(a*x)), x) + Integral(a**2*x/atan(a*x), x))
```

3.468.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

3.468.8 Giac [N/A]

Not integrable

Time = 30.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx=\int \frac{a^2cx^2+c}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.468.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx=\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)), x)
```

3.469 $\int \frac{x\left(c+a^2cx^2\right)^2}{\arctan(ax)}\,dx$

3.469.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.469.2 Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

3.469.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.469.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.469.4 Maple [N/A] (verified)

Not integrable

Time = 13.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.469.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x), x)
```

3.469.6 Sympy [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
c**2*(Integral(x/atan(a*x), x) + Integral(2*a**2*x**3/atan(a*x), x) + Inte
gral(a**4*x**5/atan(a*x), x))
```

3.469.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/arctan(a*x), x)
```

3.469.8 Giac [N/A]

Not integrable

Time = 36.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.469.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x), x)
```

3.470 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx$

3.470.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.470.2 Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x], x]
```

3.470.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.470.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.470.4 Maple [N/A] (verified)

Not integrable

Time = 28.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.470.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^2}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x), x)
```

3.470.6 Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx=c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}(ax)}\,dx+\int \frac{a^4x^4}{\operatorname{atan}(ax)}\,dx+\int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x), x) + Integral(a**4*x**4/atan(a*x), x
) + Integral(1/atan(a*x), x))
```

3.470.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^2}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2/arctan(a*x), x)
```

3.470.8 Giac [N/A]

Not integrable

Time = 33.65 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.470.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x), x)
```

3.471 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx$

3.471.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

3.471.2 Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]), x]
```

3.471.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.471.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.471.4 Maple [N/A] (verified)

Not integrable

Time = 43.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

3.471.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)), x)
```

3.471.6 Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = c^2\left(\int \frac{1}{x\,\mathrm{atan}\,(ax)}\,dx + \int \frac{2a^2x}{\mathrm{atan}\,(ax)}\,dx + \int \frac{a^4x^3}{\mathrm{atan}\,(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x),x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)), x) + Integral(2*a**2*x/atan(a*x), x) + Int
egral(a**4*x**3/atan(a*x), x))
```

3.471.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)), x)
```

3.471.8 Giac [N/A]

Not integrable

Time = 35.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.471.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)), x)
```

3.472 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx$

3.472.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.472.2 Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

3.472.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^3}{\arctan(ax)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^3}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.472.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.472.4 Maple [N/A] (verified)

Not integrable

Time = 71.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.472.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)
, x)
```

3.472.6 Sympy [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
c**3*(Integral(x/atan(a*x), x) + Integral(3*a**2*x**3/atan(a*x), x) + Inte
gral(3*a**4*x**5/atan(a*x), x) + Integral(a**6*x**7/atan(a*x), x))
```

3.472.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/arctan(a*x), x)
```

3.472.8 Giac [N/A]

Not integrable

Time = 40.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.472.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x), x)
```

3.473 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx$

3.473.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.473.2 Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x], x]
```

3.473.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.473.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.473.4 Maple [N/A] (verified)

Not integrable

Time = 59.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\,(ax)}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.473.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x),
x)
```

3.473.6 Sympy [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}(ax)}\,dx + \int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x), x) + Integral(3*a**4*x**4/atan(a*x),
 x) + Integral(a**6*x**6/atan(a*x), x) + Integral(1/atan(a*x), x))
```

3.473.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3/arctan(a*x), x)
```

3.473.8 Giac [N/A]

Not integrable

Time = 39.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.473.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x), x)
```

3.474 $\int \frac{(c+a^2cx^2)^3}{x \arctan(ax)} dx$

3.474.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^3}{x \arctan(ax)} dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^3}{x \arctan(ax)}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

3.474.2 Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x \arctan(ax)} dx = \int \frac{(c+a^2cx^2)^3}{x \arctan(ax)} dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]), x]
```

3.474.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.474.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.474.4 Maple [N/A] (verified)

Not integrable

Time = 160.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

3.474.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x
)), x)
```

3.474.6 Sympy [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = c^3\left(\int \frac{1}{x\operatorname{atan}{(ax)}}\,dx + \int \frac{3a^2x}{\operatorname{atan}{(ax)}}\,dx + \int \frac{3a^4x^3}{\operatorname{atan}{(ax)}}\,dx + \int \frac{a^6x^5}{\operatorname{atan}{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x),x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)), x) + Integral(3*a**2*x/atan(a*x), x) + Int
egral(3*a**4*x**3/atan(a*x), x) + Integral(a**6*x**5/atan(a*x), x))
```

3.474.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)), x)
```

3.474.8 Giac [N/A]

Not integrable

Time = 42.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.474.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{x\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)), x)
```

3.475 $\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx$

3.475.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^2}{(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Unintegrable(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.475.2 Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

3.475.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^2}{\arctan(ax)\,(a^2cx^2+c)}dx$$

input
```
Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.475.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.475.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2c\,x^2+c)\arctan\,(ax)}dx$$

input
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.475.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.475.6 Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x^2}{a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c}$$

input
```
integrate(x**2/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(x**2/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

3.475.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.475.8 Giac [N/A]

Not integrable

Time = 19.57 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.475.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)), x)
```

3.476 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx$

3.476.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \text{Int}\left(\frac{x}{(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Unintegrable(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.476.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

3.476.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.476.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.476.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a^{2} c\,x^{2}+c\right)\arctan\left(a x\right)}dx$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.476.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.476.6 Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x}{a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

3.476.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.476.8 Giac [N/A]

Not integrable

Time = 14.97 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.476.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)), x)
```

3.477 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx$

3.477.1 Optimal result

Integrand size = 19, antiderivative size = 12

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\arctan(ax))}{ac}$$

output
```
ln(arctan(a*x))/a/c
```

3.477.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\arctan(ax))}{ac}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Log[ArcTan[a*x]]/(a*c)
```

3.477.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5417}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{5417}$$
$$\frac{\log(\arctan(ax))}{ac}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Log[ArcTan[a*x]]/(a*c)
```

3.477.3.1 Defintions of rubi rules used

rule 5417
```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.477.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\ln(\arctan(ax))}{ac}$	13
default	$\frac{\ln(\arctan(ax))}{ac}$	13
parallelrisch	$\frac{\ln(\arctan(ax))}{ac}$	13
risch	$\frac{\ln(\ln(iax+1)-\ln(-iax+1))}{ca}$	28

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(arctan(a*x))/a/c
```

3.477.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\arctan(ax))}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
log(arctan(a*x))/(a*c)
```

3.477.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\operatorname{atan}(ax))}{ac}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
log(atan(a*x))/(a*c)
```

3.477.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(2|\arctan(ax)|)}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
log(2*abs(arctan(a*x)))/(a*c)
```

3.477.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)\arctan{(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.477.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\ln{(\mathrm{atan}(a\,x))}}{a\,c}$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
log(atan(a*x))/(a*c)
```

3.478 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)}\,dx$

3.478.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{1}{x\,(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.478.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)}\,dx=\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

3.478.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{1}{x \arctan(ax)\left(a^2cx^2+c\right)}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.478.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.478.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right) \arctan\left(a x\right)}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.478.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

3.478.6 Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{1}{a^2x^3\operatorname{atan}(ax)+x\operatorname{atan}(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x) + x*atan(a*x)), x)/c
```

3.478.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

3.478.8 Giac [N/A]

Not integrable

Time = 15.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan{(ax)}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.478.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)), x)
```

3.479 $\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)}\,dx$

3.479.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{1}{x^2(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.479.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

3.479.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x^2 \arctan(ax)\left(a^2cx^2+c\right)}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.479.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.479.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)\arctan\left(ax\right)}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.479.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)), x)
```

3.479.6 Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)} \, dx = \frac{\int \frac{1}{a^2 x^4 \operatorname{atan}(ax) + x^2 \operatorname{atan}(ax)} \, dx}{c}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(1/(a**2*x**4*atan(a*x) + x**2*atan(a*x)), x)/c
```

3.479.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)), x)
```

3.479.8 Giac [N/A]

Not integrable

Time = 20.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)x^2\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.479.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\arctan(ax)}\,dx=\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)), x)
```

3.480 $\int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx$

3.480.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{x^4}{(c+a^2cx^2)^2\arctan(ax)},x\right)$$

output
```
Unintegrable(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.480.2 Mathematica [N/A]

Not integrable

Time = 3.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx=\int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

3.480.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.480.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.480.4 Maple [N/A] (verified)

Not integrable

Time = 9.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.480.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^4/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

3.480.6 Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{x^4}{a^4x^4\operatorname{atan}(ax)+2a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**4/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

3.480.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.480.8 Giac [N/A]

Not integrable

Time = 49.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^4}{(a^2cx^2+c)^2 \arctan{(ax)}} \, dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.480.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^4}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^2} \, dx$$

input
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.481 $\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx$

3.481.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\arctan(ax)},x\right)$$

output
```
Unintegrable(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.481.2 Mathematica [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

3.481.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.481.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.481.4 Maple [N/A] (verified)

Not integrable

Time = 2.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.481.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

3.481.6 Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{\int \frac{x^3}{a^4x^4\operatorname{atan}(ax)+2a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

3.481.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.481.8 Giac [N/A]

Not integrable

Time = 47.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan{(ax)}} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.481.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.482 $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx$

3.482.1 Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = -\frac{\text{CosIntegral}(2\arctan(ax))}{2a^3c^2} + \frac{\log(\arctan(ax))}{2a^3c^2}$$

output
```
-1/2*Ci(2*arctan(a*x))/a^3/c^2+1/2*ln(arctan(a*x))/a^3/c^2
```

3.482.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{-\text{CosIntegral}(2\arctan(ax)) + \log(\arctan(ax))}{2a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(-CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a^3*c^2)
```

3.482.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^3c^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]]/2)/(a^3*c^2)
```

3.482.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.482.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\ln(\arctan(ax))-\mathrm{Ci}(2\arctan(ax))}{2a^3c^2}$	24
default	$\frac{\ln(\arctan(ax))-\mathrm{Ci}(2\arctan(ax))}{2a^3c^2}$	24

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(ln(arctan(a*x))-Ci(2*arctan(a*x)))/c^2
```

3.482.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{2\,\log\left(\arctan\left(ax\right)\right)-\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^3c^2}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
1/4*(2*log(arctan(a*x)) - log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
 1)) - log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^3*c^2)
```

3.482.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{x^2}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

3.482.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.482.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^2}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.483 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} \, dx$

3.483.1 Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\mathrm{Si}(2\arctan(ax))}{2a^2c^2}$$

output
```
1/2*Si(2*arctan(a*x))/a^2/c^2
```

3.483.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\mathrm{Si}(2\arctan(ax))}{2a^2c^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

3.483.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{\text{Si}(2\arctan(ax))}{2a^2c^2}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

3.483.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.483.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\mathrm{Si}(2\arctan(ax))}{2a^2c^2}$	16
default	$\frac{\mathrm{Si}(2\arctan(ax))}{2a^2c^2}$	16

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*Si(2*arctan(a*x))/a^2/c^2
```

3.483.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.94

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{i\ \mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - i\ \mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^2c^2}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
1/4*(I*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - I*log_integr
al(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^2*c^2)
```

3.483.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{\int \frac{x}{a^4x^4\operatorname{atan}(ax)+2a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c
**2
```

3.483.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.483.8 Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.484 $\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx$

3.484.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\text{CosIntegral}(2\arctan(ax))}{2ac^2} + \frac{\log(\arctan(ax))}{2ac^2}$$

output
```
1/2*Ci(2*arctan(a*x))/a/c^2+1/2*ln(arctan(a*x))/a/c^2
```

3.484.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\text{CosIntegral}(2\arctan(ax)) + \log(\arctan(ax))}{2ac^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a*c^2)
```

3.484.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{5439}$$
$$\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{ac^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{ac^2}$$
$$\Big\downarrow \text{3793}$$
$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{ac^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/(a*c^2)
```

3.484.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_^2))^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.484.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{\ln(\arctan(ax))+\mathrm{Ci}(2\arctan(ax))}{2a\,c^2}$	22
default	$\frac{\ln(\arctan(ax))+\mathrm{Ci}(2\arctan(ax))}{2a\,c^2}$	22

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(ln(arctan(a*x))+Ci(2*arctan(a*x)))/c^2
```

3.484.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{2\log(\arctan(ax)) + \mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) + \mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,ac^2}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
1/4*(2*log(arctan(a*x)) + log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
 1)) + log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a*c^2)
```

3.484.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c
**2
```

3.484.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.484.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.484.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.485 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)} dx$

3.485.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)} dx = \text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)}, x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.485.2 Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)} dx = \int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)} dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

3.485.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^2} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^2} dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.485.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.485.4 Maple [N/A] (verified)

Not integrable

Time = 3.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.485.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2x\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)), x)
```

3.485.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{\int \frac{1}{a^4x^5\operatorname{atan}(ax)+2a^2x^3\operatorname{atan}(ax)+x\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x) + 2*a**2*x**3*atan(a*x) + x*atan(a*x)), x)
/c**2
```

3.485.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2x\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)), x)
```

3.485.8 Giac [N/A]

Not integrable

Time = 36.89 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.485.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.486 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)} dx$

3.486.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)} dx = \text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)}, x\right)$$

output
```
Unintegrable(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.486.2 Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)} dx = \int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)} dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

3.486.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^2} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^2}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.486.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.486.4 Maple [N/A] (verified)

Not integrable

Time = 4.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.486.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)), x)
```

3.486.6 Sympy [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\int \frac{1}{a^4x^6 \operatorname{atan}(ax) + 2a^2x^4 \operatorname{atan}(ax) + x^2 \operatorname{atan}(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x) + 2*a**2*x**4*atan(a*x) + x**2*atan(a*x)),
 x)/c**2
```

3.486.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)), x)
```

3.486.8 Giac [N/A]

Not integrable

Time = 38.58 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.486.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)\,(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.487 $\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx$

3.487.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x^6}{(c+a^2cx^2)^3\arctan(ax)},x\right)$$

output
```
Unintegrable(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.487.2 Mathematica [N/A]

Not integrable

Time = 6.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx$$

input
```
Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

3.487.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\arctan(ax)\,(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^6}{\arctan(ax)\,(a^2cx^2+c)^3}dx$$

input
```
Int[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.487.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.487.4 Maple [N/A] (verified)

Not integrable

Time = 8.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)}dx$$

input
```
int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.487.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^6}{\left(c+a^2cx^2\right)^3 \arctan(ax)}\,dx = \int \frac{x^6}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)}\,dx$$

input
```
integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^6/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x)
```

3.487.6 Sympy [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x^6}{\left(c+a^2cx^2\right)^3 \arctan(ax)}\,dx = \frac{\int \frac{x^6}{a^6x^6\,\mathrm{atan}\,(ax)+3a^4x^4\,\mathrm{atan}\,(ax)+3a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c^3}$$

input
```
integrate(x**6/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**6/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

3.487.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^6}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^6/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.487.8 Giac [N/A]

Not integrable

Time = 72.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^6}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.487.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^6}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^6/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^6/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.488 $\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$

3.488.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \text{Int}\left(\frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)}, x\right)$$

output
```
Unintegrable(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.488.2 Mathematica [N/A]

Not integrable

Time = 6.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

3.488.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.488.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.488.4 Maple [N/A] (verified)

Not integrable

Time = 8.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.488.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\int \frac{x^5}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x)
```

3.488.6 Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{\int \frac{x^5}{a^6x^6\operatorname{atan}\left(ax\right)+3a^4x^4\operatorname{atan}\left(ax\right)+3a^2x^2\operatorname{atan}\left(ax\right)+\operatorname{atan}\left(ax\right)}\,dx}{c^3}$$

input
```
integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**5/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

3.488.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{(a^2cx^2+c)^3 \arctan(ax)} \, dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.488.8 Giac [N/A]

Not integrable

Time = 73.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{(a^2cx^2+c)^3 \arctan(ax)} \, dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.488.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.489 $\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx$

3.489.1 Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = -\frac{\text{CosIntegral}(2\arctan(ax))}{2a^5c^3} + \frac{\text{CosIntegral}(4\arctan(ax))}{8a^5c^3} + \frac{3\log(\arctan(ax))}{8a^5c^3}$$

output
```
-1/2*Ci(2*arctan(a*x))/a^5/c^3+1/8*Ci(4*arctan(a*x))/a^5/c^3+3/8*ln(arctan
(a*x))/a^5/c^3
```

3.489.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{-4\,\text{CosIntegral}(2\arctan(ax)) + \text{CosIntegral}(4\arctan(ax)) + 3\log(\arctan(ax))}{8a^5c^3}$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan
[a*x]])/(8*a^5*c^3)
```

3.489.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \; 5505$$

$$\frac{\int \frac{a^4x^4}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin(\arctan(ax))^4}{\arctan(ax)}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \; 3793$$

$$\frac{\int \left(-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^5c^3}$$

input
```
Int[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-1/2*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]]/8 + (3*Log[A
rcTan[a*x]])/8)/(a^5*c^3)
```

3.489.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.489.4 Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{-3\ln(\arctan(ax))+4\operatorname{Ci}(2\arctan(ax))-\operatorname{Ci}(4\arctan(ax))}{8a^5c^3}$	35
default	$-\frac{-3\ln(\arctan(ax))+4\operatorname{Ci}(2\arctan(ax))-\operatorname{Ci}(4\arctan(ax))}{8a^5c^3}$	35

input
```
int(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^5*(-3*ln(arctan(a*x))+4*Ci(2*arctan(a*x))-Ci(4*arctan(a*x)))/c^3
```

3.489.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.48

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx$$
$$= \frac{6 \log(\arctan(ax)) + \text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + \text{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) -}{16\,a^5c^3}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a
^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 4*log_integ
ral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - 4*log_integral(-(a^2*x^2 - 2
*I*a*x - 1)/(a^2*x^2 + 1)))/(a^5*c^3)
```

3.489.6 Sympy [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x^4}{a^6x^6 \operatorname{atan}(ax)+3a^4x^4 \operatorname{atan}(ax)+3a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**4/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

3.489.7 Maxima [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.489.8 Giac [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^4}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.490 $\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx$

3.490.1 Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\mathrm{Si}(2\arctan(ax))}{4a^4c^3} - \frac{\mathrm{Si}(4\arctan(ax))}{8a^4c^3}$$

output
```
1/4*Si(2*arctan(a*x))/a^4/c^3-1/8*Si(4*arctan(a*x))/a^4/c^3
```

3.490.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = -\frac{-2\mathrm{Si}(2\arctan(ax)) + \mathrm{Si}(4\arctan(ax))}{8a^4c^3}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
-1/8*(-2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(a^4*c^3
)
```

3.490.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^3x^3}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}-\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\text{Si}(2\arctan(ax))-\frac{1}{8}\text{Si}(4\arctan(ax))}{a^4c^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(SinIntegral[2*ArcTan[a*x]]/4 - SinIntegral[4*ArcTan[a*x]]/8)/(a^4*c^3)
```

3.490.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.490.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{\mathrm{Si}(4\arctan(ax))-2\ \mathrm{Si}(2\arctan(ax))}{8a^4c^3}$	26
default	$-\frac{\mathrm{Si}(4\arctan(ax))-2\ \mathrm{Si}(2\arctan(ax))}{8a^4c^3}$	26

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^4*(Si(4*arctan(a*x))-2*Si(2*arctan(a*x)))/c^3
```

3.490.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.89

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{-i\ \mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+i\ \mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\ \mathrm{log\_integra}}{16\,a^4c^3}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(-I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a
^4*x^4 + 2*a^2*x^2 + 1)) + I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x
^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2
+ 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/
(a^2*x^2 + 1)))/(a^4*c^3)
```

3.490.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\int \frac{x^3}{a^6x^6\,\mathrm{atan}\,(ax)+3a^4x^4\,\mathrm{atan}\,(ax)+3a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

3.490.7 Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3\arctan\,(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.490.8 Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3\arctan\,(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^3}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.491 $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$

3.491.1 Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = -\frac{\text{CosIntegral}(4\arctan(ax))}{8a^3c^3} + \frac{\log(\arctan(ax))}{8a^3c^3}$$

output
```
-1/8*Ci(4*arctan(a*x))/a^3/c^3+1/8*ln(arctan(a*x))/a^3/c^3
```

3.491.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{-\text{CosIntegral}(4\arctan(ax)) + \log(\arctan(ax))}{8a^3c^3}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]])/(8*a^3*c^3)
```

3.491.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))}{a^3c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-1/8*CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]]/8)/(a^3*c^3)
```

3.491.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.491.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\ln(\arctan(ax))-\mathrm{Ci}(4\arctan(ax))}{8a^3c^3}$	24
default	$\frac{\ln(\arctan(ax))-\mathrm{Ci}(4\arctan(ax))}{8a^3c^3}$	24

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3*(ln(arctan(a*x))-Ci(4*arctan(a*x)))/c^3
```

3.491.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.64

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{2\,\log(\arctan(ax)) - \log\_integral\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) - \log\_integral\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)}{16\,a^3c^3}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(2*log(arctan(a*x)) - log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - log_integral((a^4*x^4 - 4*I*a
^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/(a^3*c^3)
```

3.491.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\int \frac{x^2}{a^6x^6\,\mathrm{atan}\,(ax)+3a^4x^4\,\mathrm{atan}\,(ax)+3a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

3.491.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3\arctan{(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.491.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3\arctan{(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^2}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.492 $\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$

3.492.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{\text{Si}(2\arctan(ax))}{4a^2c^3} + \frac{\text{Si}(4\arctan(ax))}{8a^2c^3}$$

output
```
1/4*Si(2*arctan(a*x))/a^2/c^3+1/8*Si(4*arctan(a*x))/a^2/c^3
```

3.492.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{2\text{Si}(2\arctan(ax)) + \text{Si}(4\arctan(ax))}{8a^2c^3}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(8*a^2*c^3)
```

3.492.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \; 5505$$

$$\frac{\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \; 4906$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))}{a^2c^3}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8)/(a^2*c^3)
```

3.492.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.492.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{\mathrm{Si}(4\arctan(ax))+2\,\mathrm{Si}(2\arctan(ax))}{8a^2c^3}$	26
default	$\frac{\mathrm{Si}(4\arctan(ax))+2\,\mathrm{Si}(2\arctan(ax))}{8a^2c^3}$	26

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2*(Si(4*arctan(a*x))+2*Si(2*arctan(a*x)))/c^3
```

3.492.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.89

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{i\ \mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-i\ \mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\ \mathrm{log\_integral}\left(\right.}{16\,a^2c^3}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^
4*x^4 + 2*a^2*x^2 + 1)) - I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^
2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2 +
 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(
a^2*x^2 + 1)))/(a^2*c^3)
```

3.492.6 Sympy [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)}\,dx = \frac{\int \frac{x}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan
(a*x) + atan(a*x)), x)/c**3
```

3.492.7 Maxima [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)}\,dx = \int \frac{x}{\left(a^2cx^2+c\right)^3 \arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.492.8 Giac [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)}\,dx = \int \frac{x}{\left(a^2cx^2+c\right)^3 \arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.493 $\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx$

3.493.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{\text{CosIntegral}(2\arctan(ax))}{2ac^3}+\frac{\text{CosIntegral}(4\arctan(ax))}{8ac^3}+\frac{3\log(\arctan(ax))}{8ac^3}$$

output
```
1/2*Ci(2*arctan(a*x))/a/c^3+1/8*Ci(4*arctan(a*x))/a/c^3+3/8*ln(arctan(a*x)
)/a/c^3
```

3.493.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{4\,\text{CosIntegral}(2\arctan(ax))+\text{CosIntegral}(4\arctan(ax))+3\log(\arctan(ax))}{8ac^3}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan[
a*x]])/(8*a*c^3)
```

3.493.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(CosIntegral[2*ArcTan[a*x]]/2 + CosIntegral[4*ArcTan[a*x]]/8 + (3*Log[ArcT
an[a*x]])/8)/(a*c^3)
```

3.493.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.493.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{3\ln(\arctan(ax))+4\ \mathrm{Ci}(2\arctan(ax))+\mathrm{Ci}(4\arctan(ax))}{8a\,c^{3}}$	33
default	$\frac{3\ln(\arctan(ax))+4\ \mathrm{Ci}(2\arctan(ax))+\mathrm{Ci}(4\arctan(ax))}{8a\,c^{3}}$	33

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(3*ln(arctan(a*x))+4*Ci(2*arctan(a*x))+Ci(4*arctan(a*x)))/c^3
```

3.493.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.48

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)}\,dx$$
$$= \frac{6\log(\arctan(ax)) + \mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + \mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)}{16\,ac^3}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a
^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*log_integ
ral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + 4*log_integral(-(a^2*x^2 - 2
*I*a*x - 1)/(a^2*x^2 + 1)))/(a*c^3)
```

3.493.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\int \frac{1}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan
(a*x) + atan(a*x)), x)/c**3
```

3.493.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.493.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)^3\arctan{(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)}\,dx=\int \frac{1}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.494 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)} dx$

3.494.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x (c+a^2cx^2)^3 \arctan(ax)} dx = \operatorname{Int}\left(\frac{1}{x (c+a^2cx^2)^3 \arctan(ax)}, x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.494.2 Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x (c+a^2cx^2)^3 \arctan(ax)} dx = \int \frac{1}{x (c+a^2cx^2)^3 \arctan(ax)} dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

3.494.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^3} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^3}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.494.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.494.4 Maple [N/A] (verified)

Not integrable

Time = 4.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.494.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a
*x)), x)
```

3.494.6 Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{\int \frac{1}{a^6x^7\operatorname{atan}\left(ax\right)+3a^4x^5\operatorname{atan}\left(ax\right)+3a^2x^3\operatorname{atan}\left(ax\right)+x\operatorname{atan}\left(ax\right)}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x) + 3*a**4*x**5*atan(a*x) + 3*a**2*x**3*atan
(a*x) + x*atan(a*x)), x)/c**3
```

3.494.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)), x)
```

3.494.8 Giac [N/A]

Not integrable

Time = 49.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.494.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.495 $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)}\,dx$

3.495.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.495.2 Mathematica [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

3.495.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2 c x^2+c\right)^3} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{x^2 \arctan(ax) \left(a^2 c x^2+c\right)^3}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.495.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.495.4 Maple [N/A] (verified)

Not integrable

Time = 4.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.495.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan
(a*x)), x)
```

3.495.6 Sympy [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \frac{\int \frac{1}{a^6x^8 \operatorname{atan}(ax) + 3a^4x^6 \operatorname{atan}(ax) + 3a^2x^4 \operatorname{atan}(ax) + x^2 \operatorname{atan}(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x) + 3*a**4*x**6*atan(a*x) + 3*a**2*x**4*atan
(a*x) + x**2*atan(a*x)), x)/c**3
```

3.495.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)), x)
```

3.495.8 Giac [N/A]

Not integrable

Time = 53.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.495.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x) \, (c\,a^2\,x^2 + c)^3} \, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.496 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)} \, dx$

3.496.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)} \, dx = \text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

3.496.2 Mathematica [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)} \, dx = \int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)} \, dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

3.496.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.496.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.496.4 Maple [N/A] (verified)

Not integrable

Time = 5.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan(ax)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

3.496.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+cx}}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

3.496.6 Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

3.496.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+cx}}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

3.496.8 Giac [N/A]

Not integrable

Time = 33.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.496.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x), x)
```

3.497 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$

3.497.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

3.497.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x], x]
```

3.497.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.497.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.497.4 Maple [N/A] (verified)

Not integrable

Time = 5.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

3.497.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

3.497.6 Sympy [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

3.497.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

3.497.8 Giac [N/A]

Not integrable

Time = 32.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.497.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x), x)
```

3.498 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx$

3.498.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

3.498.2 Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]), x]
```

3.498.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.498.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.498.4 Maple [N/A] (verified)

Not integrable

Time = 14.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a^{2} c \,x^{2}+c}}{x \arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

3.498.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

3.498.6 Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)), x)
```

3.498.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

3.498.8 Giac [N/A]

Not integrable

Time = 34.76 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.498.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)), x)
```

3.499 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)} dx$

3.499.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)} dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}, x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.499.2 Mathematica [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)} dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

3.499.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.499.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.499.4 Maple [N/A] (verified)

Not integrable

Time = 19.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.499.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

3.499.6 Sympy [N/A]

Not integrable

Time = 4.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

3.499.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x), x)
```

3.499.8 Giac [N/A]

Not integrable

Time = 47.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.499.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x), x)
```

3.500 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$

3.500.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.500.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x], x]
```

3.500.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.500.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.500.4 Maple [N/A] (verified)

Not integrable

Time = 18.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.500.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

3.500.6 Sympy [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

3.500.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

3.500.8 Giac [N/A]

Not integrable

Time = 43.80 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.500.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x), x)
```

3.501 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx$

3.501.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

3.501.2 Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]), x]
```

3.501.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.501.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.501.4 Maple [N/A] (verified)

Not integrable

Time = 25.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

3.501.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

3.501.6 Sympy [N/A]

Not integrable

Time = 6.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\,\mathrm{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)), x)
```

3.501.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

3.501.8 Giac [N/A]

Not integrable

Time = 46.64 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.501.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)), x)
```

3.502 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$

3.502.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.502.2 Mathematica [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

3.502.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.502.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.502.4 Maple [N/A] (verified)

Not integrable

Time = 16.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.502.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(
a*x), x)
```

3.502.6 Sympy [N/A]

Not integrable

Time = 15.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x), x)
```

3.502.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x), x)
```

3.502.8 Giac [N/A]

Not integrable

Time = 56.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.502.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x), x)
```

3.503 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$

3.503.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.503.2 Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x], x]
```

3.503.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.503.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.503.4 Maple [N/A] (verified)

Not integrable

Time = 15.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.503.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*
x), x)
```

3.503.6 Sympy [N/A]

Not integrable

Time = 11.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x), x)
```

3.503.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x), x)
```

3.503.8 Giac [N/A]

Not integrable

Time = 54.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.503.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x), x)
```

3.504 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx$

3.504.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

3.504.2 Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]), x]
```

3.504.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.504.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.504.4 Maple [N/A] (verified)

Not integrable

Time = 98.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

3.504.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan
(a*x)), x)
```

3.504.6 Sympy [N/A]

Not integrable

Time = 15.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)), x)
```

3.504.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)), x)
```

3.504.8 Giac [N/A]

Not integrable

Time = 56.62 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.504.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)), x)
```

3.505 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$

3.505.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.505.2 Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

3.505.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.505.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.505.4 Maple [N/A] (verified)

Not integrable

Time = 1.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\arctan\left(ax\right)\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.505.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

input
```
integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.505.6 Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x}{\sqrt{c(a^2x^2+1)}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

3.505.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

input
```
integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.505.8 Giac [N/A]

Not integrable

Time = 35.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.505.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.506 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$

3.506.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Unintegrable(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.506.2 Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

3.506.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.506.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.506.4 Maple [N/A] (verified)

Not integrable

Time = 3.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\arctan\left(ax\right)\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.506.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

input
```
integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.506.6 Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(1/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

3.506.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

input
```
integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.506.8 Giac [N/A]

Not integrable

Time = 33.86 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(1/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.506.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{1}{\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.507 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$

3.507.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.507.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

3.507.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x \arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.507.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.507.4 Maple [N/A] (verified)

Not integrable

Time = 4.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \arctan\left(a x \right) \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.507.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+cx}\arctan{(ax)}}\,dx$$

input
```
integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

3.507.6 Sympy [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{x\sqrt{c\,(a^2x^2+1)}\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(1/x/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

3.507.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+cx}\arctan{(ax)}}\,dx$$

input
```
integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

3.507.8 Giac [N/A]

Not integrable

Time = 34.33 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+cx}\arctan{(ax)}}\,dx$$

input
```
integrate(1/x/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.507.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.508 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$

3.508.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.508.2 Mathematica [N/A]

Not integrable

Time = 5.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

3.508.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.508.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.508.4 Maple [N/A] (verified)

Not integrable

Time = 1.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.508.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)), x)
```

3.508.6 Sympy [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

3.508.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.508.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.508.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.509 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$

3.509.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.509.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \$\mathrm{Aborted}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.509.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.509.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.509.4 Maple [N/A] (verified)

Not integrable

Time = 1.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.509.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)), x)
```

3.509.6 Sympy [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

3.509.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.509.8 Giac [N/A]

Not integrable

Time = 30.79 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.509.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.510 $\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$

3.510.1 Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

3.510.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{a^2c\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a^2*c*Sqrt[c*(1 + a^2*x^2)])
```

3.510.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5506, 5505, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

3.510.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.510.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

method	result	size
default	$-\frac{\operatorname{csgn}\left(\arctan\left(a x\right)\right) \pi \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{2 \sqrt{a^{2} x^{2}+1}\, a^{2} c^{2}}+\frac{\operatorname{Si}\left(\arctan\left(a x\right)\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{\sqrt{a^{2} x^{2}+1}\, a^{2} c^{2}}$	82

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(arctan(a*x))*Pi*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/
c^2+Si(arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c^2
```

3.510.5 Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan
(a*x)), x)
```

3.510.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

3.510.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.510.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}\,(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.511 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$

3.511.1 Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{ac\sqrt{c+a^2cx^2}}$$

output
```
Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

3.511.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{ac\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a*c*Sqrt[c*(1 + a^2*x^2)])
```

3.511.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5440, 5439, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \ 5440$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \ 5439$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \ 3042$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \ 3783$$
$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

3.511.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.511.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.95 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.49

method	result
default	$-\frac{i\,\mathrm{csgn}(\arctan(ax))\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^2}+\frac{i\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^2}+\frac{\mathrm{Ci}(\arctan(ax))\sqrt{c(ax-i)}}{\sqrt{a^2x^2+1}\,a\,c^2}$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(I+a*x))^(1/2)/
(a^2*x^2+1)^(1/2)/a/c^2+1/2*I*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(I+a*x))^(
1/2)/(a^2*x^2+1)^(1/2)/a/c^2+Ci(arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^
2*x^2+1)^(1/2)/a/c^2
```

3.511.5 Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a
*x)), x)
```

3.511.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

3.511.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.511.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.512 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$

3.512.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.512.2 Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

3.512.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.512.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.512.4 Maple [N/A] (verified)

Not integrable

Time = 2.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.512.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan
(a*x)), x)
```

3.512.6 Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

3.512.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)), x)
```

3.512.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.512.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.513 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$

3.513.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.513.2 Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

3.513.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{3/2}} \, dx$$
$$\Big\downarrow \, 5560$$
$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.513.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.513.4 Maple [N/A] (verified)

Not integrable

Time = 2.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.513.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arct
an(a*x)), x)
```

3.513.6 Sympy [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

3.513.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)), x)
```

3.513.8 Giac [N/A]

Not integrable

Time = 32.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)}\, dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)}\, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.513.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)}\, dx = \int \frac{1}{x^2\, \mathrm{atan}(a\, x)\, (c\, a^2\, x^2 + c)^{3/2}}\, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.514 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.514.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.514.2 Mathematica [N/A]

Not integrable

Time = 6.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

3.514.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.514.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.514.4 Maple [N/A] (verified)

Not integrable

Time = 6.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.514.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

3.514.6 Sympy [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.514.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.514.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.514.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{\mathrm{atan}\,(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.515 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.515.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\text{Int}\left(\frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.515.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\$\text{Aborted}$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.515.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.515.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.515.4 Maple [N/A] (verified)

Not integrable

Time = 12.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.515.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

3.515.6 Sympy [N/A]

Not integrable

Time = 5.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^4}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.515.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.515.8 Giac [N/A]

Not integrable

Time = 75.96 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.515.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^4}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.516 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.516.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
3/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-1/4*Si(3
*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

3.516.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{(1+a^2x^2)^{3/2}\,(3\mathrm{Si}(\arctan(ax)) - \mathrm{Si}(3\arctan(ax)))}{4a^4c\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(3*SinIntegral[ArcTan[a*x]] - SinIntegral[3*ArcTan[a*
x]]))/(4*a^4*c*(c*(1 + a^2*x^2))^(3/2))
```

3.516.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^3}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{Si}(\arctan(ax))-\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*SinIntegral[ArcTan[a*x]])/4 - SinIntegral[3*ArcTan[
a*x]]/4))/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```

3.516.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.516.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.44

method	result	size
default	$-\frac{\operatorname{csgn}(\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^4c^3}-\frac{\operatorname{Si}(3\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^4c^3}+\frac{3\operatorname{Si}(\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^4c^3}$	125

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*csgn(arctan(a*x))*Pi*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/
c^3-1/4*Si(3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/
c^3+3/4*Si(arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c^
3
```

3.516.5 Fricas [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

3.516.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^3}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.516.7 Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.516.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.517 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.517.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/4*Ci(3
*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

3.517.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}(\mathrm{CosIntegral}(\arctan(ax)) - \mathrm{CosIntegral}(3\arctan(ax)))}{4a^3c^2\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x]])
)/(4*a^3*c^2*Sqrt[c*(1 + a^2*x^2)])
```

3.517.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]
]/4))/(a^3*c^2*Sqrt[c + a^2*c*x^2])
```

3.517.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.517.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.89 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\operatorname{Ci}(3\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^3c^3}+\frac{\operatorname{Ci}(\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,a^3c^3}$	84

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Ci(3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c^3
+1/4*Ci(arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c^3
```

3.517.5 Fricas [F]

$$\int \frac{x^2}{(c + a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{x^2}{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

3.517.6 Sympy [F]

$$\int \frac{x^2}{(c + a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{x^2}{(c(a^2x^2 + 1))^{\frac{5}{2}} \operatorname{atan}(ax)} \, dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.517.7 Maxima [F]

$$\int \frac{x^2}{(c + a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{x^2}{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.517.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.518 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.518.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
1/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/4*Si(3
*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

3.518.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{(1+a^2x^2)^{3/2}\,(\mathrm{Si}(\arctan(ax))+\mathrm{Si}(3\arctan(ax)))}{4a^2c\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(SinIntegral[ArcTan[a*x]] + SinIntegral[3*ArcTan[a*x]
]))/(4*a^2*c*(c*(1 + a^2*x^2))^(3/2))
```

3.518.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\,(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \text{5506}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{5505}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{4906}$$
$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4 + SinIntegral[3*ArcTan[a*x]
]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2])
```

3.518.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.518.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.44

method	result	size
default	$-\frac{\operatorname{csgn}\left(\arctan\left(a x\right)\right) \pi \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{2} c^{3}}+\frac{\operatorname{Si}\left(3 \arctan\left(a x\right)\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{2} c^{3}}+\frac{\operatorname{Si}\left(\arctan\left(a x\right)\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{2} c^{3}}$	125

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*csgn(arctan(a*x))*Pi*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/
c^3+1/4*Si(3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/
c^3+1/4*Si(arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c^
3
```

3.518.5 Fricas [F]

$$\int \frac{x}{(c + a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{x}{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)} \, dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x
^2 + c^3)*arctan(a*x)), x)
```

3.518.6 Sympy [F]

$$\int \frac{x}{(c + a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{x}{(c (a^2x^2 + 1))^{\frac{5}{2}} \operatorname{atan}(ax)} \, dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.518.7 Maxima [F]

$$\int \frac{x}{(c + a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{x}{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)} \, dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.518.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.518.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.519 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.519.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}}$$

output
```
3/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/4*Ci(3*a
rctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.519.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{(1+a^2x^2)^{3/2}\,(3\,\text{CosIntegral}(\arctan(ax))+\text{CosIntegral}(3\arctan(ax)))}{4ac\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(3*CosIntegral[ArcTan[a*x]] + CosIntegral[3*ArcTan[a*
x]]))/(4*a*c*(c*(1 + a^2*x^2))^(3/2))
```

3.519.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*CosIntegral[ArcTan[a*x]])/4 + CosIntegral[3*ArcTan[
a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2])
```

3.519.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.519.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.06

method	result
default	$-\frac{i\,\mathrm{csgn}(\arctan(ax))\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,c^3a}+\frac{i\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,c^3a}+\frac{\mathrm{Ci}(3\arctan(ax))\sqrt{c(ax-}}{4\sqrt{a^2x^2+1}\,c^3a}$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(I+a*x))^(1/2)/
(a^2*x^2+1)^(1/2)/c^3/a+1/2*I*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(I+a*x))^(
1/2)/(a^2*x^2+1)^(1/2)/c^3/a+1/4*Ci(3*arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/
2)/(a^2*x^2+1)^(1/2)/c^3/a+3/4*Ci(arctan(a*x))*(c*(a*x-I)*(I+a*x))^(1/2)/(
a^2*x^2+1)^(1/2)/c^3/a
```

3.519.5 Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3)*arctan(a*x)), x)
```

3.519.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.519.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.519.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.520 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.520.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.520.2 Mathematica [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

3.520.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.520.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.520.4 Maple [N/A] (verified)

Not integrable

Time = 2.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.520.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
 + c^3*x)*arctan(a*x)), x)
```

3.520.6 Sympy [N/A]

Not integrable

Time = 9.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.520.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)), x)
```

3.520.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.520.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.521 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$

3.521.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.521.2 Mathematica [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

3.521.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.521.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.521.4 Maple [N/A] (verified)

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.521.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
 + c^3*x^2)*arctan(a*x)), x)
```

3.521.6 Sympy [N/A]

Not integrable

Time = 14.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{x^2 (c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}(ax)} \, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

3.521.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)), x)
```

3.521.8 Giac [N/A]

Not integrable

Time = 61.45 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.521.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)\, (c\,a^2\,x^2+c)^{5/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.522 $\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)} \, dx$

3.522.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)} \, dx = \operatorname{Int}\left(\frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.522.2 Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)} \, dx = \int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

3.522.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.522.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.522.4 Maple [N/A] (verified)

Not integrable

Time = 4.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)^3}{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.522.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)} \, dx = \int \frac{(a^2 c x^2 + c)^3 x^m}{\arctan(ax)} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x), x)
```

3.522.6 Sympy [N/A]

Not integrable

Time = 12.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)} \, dx = c^3 \left(\int \frac{x^m}{\operatorname{atan}(ax)} \, dx + \int \frac{3a^2 x^2 x^m}{\operatorname{atan}(ax)} \, dx + \int \frac{3a^4 x^4 x^m}{\operatorname{atan}(ax)} \, dx + \int \frac{a^6 x^6 x^m}{\operatorname{atan}(ax)} \, dx \right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
c**3*(Integral(x**m/atan(a*x), x) + Integral(3*a**2*x**2*x**m/atan(a*x), x
) + Integral(3*a**4*x**4*x**m/atan(a*x), x) + Integral(a**6*x**6*x**m/atan
(a*x), x))
```

3.522.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x), x)
```

3.522.8 Giac [N/A]

Not integrable

Time = 134.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.522.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x), x)
```

3.523 $\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx$

3.523.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.523.2 Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

3.523.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2 c x^2 + c)^2}{\arctan(ax)} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m (a^2 c x^2 + c)^2}{\arctan(ax)} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.523.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.523.4 Maple [N/A] (verified)

Not integrable

Time = 3.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2 c\, x^2 + c)^2}{\arctan(ax)} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.523.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x), x)
```

3.523.6 Sympy [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx=c^2\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx+\int \frac{2a^2x^2x^m}{\operatorname{atan}(ax)}\,dx+\int \frac{a^4x^4x^m}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
c**2*(Integral(x**m/atan(a*x), x) + Integral(2*a**2*x**2*x**m/atan(a*x), x
) + Integral(a**4*x**4*x**m/atan(a*x), x))
```

3.523.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x), x)
```

3.523.8 Giac [N/A]

Not integrable

Time = 114.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.523.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}(a\,x)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x), x)
```

3.524 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx$

3.524.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

3.524.2 Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

3.524.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.524.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.524.4 Maple [N/A] (verified)

Not integrable

Time = 3.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)}{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

3.524.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

3.524.6 Sympy [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=c\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx+\int \frac{a^2x^2x^m}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
c*(Integral(x**m/atan(a*x), x) + Integral(a**2*x**2*x**m/atan(a*x), x))
```

3.524.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

3.524.8 Giac [N/A]

Not integrable

Time = 91.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.524.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x), x)
```

3.525 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx$

3.525.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^m}{(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.525.2 Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

3.525.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5560\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.525.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.525.4 Maple [N/A] (verified)

Not integrable

Time = 2.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

3.525.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.525.6 Sympy [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x^m}{a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

3.525.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.525.8 Giac [N/A]

Not integrable

Time = 62.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.525.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)), x)
```

3.526 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)} \, dx$

3.526.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}, x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.526.2 Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

3.526.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.526.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.526.4 Maple [N/A] (verified)

Not integrable

Time = 5.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

3.526.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

3.526.6 Sympy [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{x^m}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**m/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

3.526.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

3.526.8 Giac [N/A]

Not integrable

Time = 111.85 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.526.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^m}{\mathrm{atan}(a\,x)\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

3.527 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx$

3.527.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.527.2 Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

3.527.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.527.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.527.4 Maple [N/A] (verified)

Not integrable

Time = 3.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

3.527.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x)
```

3.527.6 Sympy [N/A]

Not integrable

Time = 9.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\int \frac{x^m}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**m/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

3.527.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

3.527.8 Giac [N/A]

Not integrable

Time = 157.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.527.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

3.528 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} \, dx$

3.528.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} \, dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.528.2 Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} \, dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

3.528.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.528.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.528.4 Maple [N/A] (verified)

Not integrable

Time = 6.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.528.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x), x)
```

3.528.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Timed out
```

3.528.7 Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x), x)
```

3.528.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.528.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x), x)
```

3.529 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$

3.529.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.529.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

3.529.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a^2cx^2+c)^{3/2}}{\arctan(ax)}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m(a^2cx^2+c)^{3/2}}{\arctan(ax)}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.529.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.529.4 Maple [N/A] (verified)

Not integrable

Time = 4.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.529.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

3.529.6 Sympy [N/A]

Not integrable

Time = 120.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x^m(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**m*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

3.529.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

3.529.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.529.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x), x)
```

3.530 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$

3.530.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

3.530.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

3.530.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.530.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.530.4 Maple [N/A] (verified)

Not integrable

Time = 4.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

3.530.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

3.530.6 Sympy [N/A]

Not integrable

Time = 6.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x^m\sqrt{c(a^2x^2+1)}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

3.530.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

3.530.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.530.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x), x)
```

3.531 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$

3.531.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.531.2 Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

3.531.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.531.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.531.4 Maple [N/A] (verified)

Not integrable

Time = 4.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\arctan\left(ax\right)\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

3.531.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

input
```
integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.531.6 Sympy [N/A]

Not integrable

Time = 8.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x^m}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**m/atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

3.531.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

input
```
integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

3.531.8 Giac [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.531.9 Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

3.532 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$

3.532.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.532.2 Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

3.532.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.532.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.532.4 Maple [N/A] (verified)

Not integrable

Time = 4.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

3.532.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)), x)
```

3.532.6 Sympy [N/A]

Not integrable

Time = 39.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

3.532.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

3.532.8 Giac [N/A]

Not integrable

Time = 6.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx=\int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.532.9 Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx=\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

3.533 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx$

3.533.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.533.2 Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

3.533.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.533.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.533.4 Maple [N/A] (verified)

Not integrable

Time = 6.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

3.533.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

3.533.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Timed out
```

3.533.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

3.533.8 Giac [N/A]

Not integrable

Time = 12.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.533.9 Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

3.534 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2} dx$

3.534.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2} dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^2}, x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.534.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2} dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^2} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

3.534.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.534.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.534.4 Maple [N/A] (verified)

Not integrable

Time = 10.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.534.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)/arctan(a*x)^2, x)
```

3.534.6 Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = c\left(\int \frac{x}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
c*(Integral(x/atan(a*x)**2, x) + Integral(a**2*x**3/atan(a*x)**2, x))
```

3.534.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^4*c*x^5 + 2*a^2*c*x^3 + c*x - arctan(a*x)*integrate((5*a^4*c*x^4 + 6*a
^2*c*x^2 + c)/arctan(a*x), x))/(a*arctan(a*x))
```

3.534.8 Giac [N/A]

Not integrable

Time = 57.99 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.534.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^2, x)
```

3.535 $\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx$

3.535.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.535.2 Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2, x]
```

3.535.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^2}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{a^2cx^2+c}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.535.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.535.4 Maple [N/A] (verified)

Not integrable

Time = 19.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^{2} c \,x^{2}+c}{\arctan \left(a x \right)^{2}}d x$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.535.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

3.535.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}^2\left(ax\right)}\,dx + \int \frac{1}{\operatorname{atan}^2\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**2, x) + Integral(atan(a*x)**(-2), x))
```

3.535.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^4*c*x^4 + 2*a^2*c*x^2 - a*arctan(a*x)*integrate(4*(a^3*c*x^3 + a*c*x)/
arctan(a*x), x) + c)/(a*arctan(a*x))
```

3.535.8 Giac [N/A]

Not integrable

Time = 55.51 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.18

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.535.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^2, x)
```

3.536 $\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx$

3.536.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

3.536.2 Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2), x]
```

3.536.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{a^2cx^2+c}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.536.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.536.4 Maple [N/A] (verified)

Not integrable

Time = 29.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

3.536.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

3.536.6 Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = c\left(\int \frac{1}{x\,\mathrm{atan}^2\left(ax\right)}\,dx + \int \frac{a^2x}{\mathrm{atan}^2\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**2,x)
```

output
```
c*(Integral(1/(x*atan(a*x)**2), x) + Integral(a**2*x/atan(a*x)**2, x))
```

3.536.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^4*c*x^4 + 2*a^2*c*x^2 - x*arctan(a*x)*integrate((3*a^4*c*x^4 + 2*a^2*c
*x^2 - c)/(x^2*arctan(a*x)), x) + c)/(a*x*arctan(a*x))
```

3.536.8 Giac [N/A]

Not integrable

Time = 57.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.536.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^2), x)
```

3.537 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2} dx$

3.537.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2} dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}, x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.537.2 Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2} dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

3.537.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.537.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.537.4 Maple [N/A] (verified)

Not integrable

Time = 22.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.537.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^2, x)
```

3.537.6 Sympy [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^2(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
c**2*(Integral(x/atan(a*x)**2, x) + Integral(2*a**2*x**3/atan(a*x)**2, x)
+ Integral(a**4*x**5/atan(a*x)**2, x))
```

3.537.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.05

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^6*c^2*x^7 + 3*a^4*c^2*x^5 + 3*a^2*c^2*x^3 + c^2*x - arctan(a*x)*integr
ate((7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x), x)
)/(a*arctan(a*x))
```

3.537.8 Giac [N/A]

Not integrable

Time = 66.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.537.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{x\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^2, x)
```

3.538 $\int \frac{\left(c+a^2cx^2\right)^2}{\arctan(ax)^2}\,dx$

3.538.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\left(c+a^2cx^2\right)^2}{\arctan(ax)^2}\,dx=\text{Int}\left(\frac{\left(c+a^2cx^2\right)^2}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.538.2 Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\left(c+a^2cx^2\right)^2}{\arctan(ax)^2}\,dx=\int \frac{\left(c+a^2cx^2\right)^2}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2, x]
```

3.538.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.538.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.538.4 Maple [N/A] (verified)

Not integrable

Time = 56.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.538.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^2, x)
```

3.538.6 Sympy [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{1}{\operatorname{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**2, x) + Integral(a**4*x**4/atan(a*x)
**2, x) + Integral(atan(a*x)**(-2), x))
```

3.538.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - a*arctan(a*x)*integrate(6*
(a^5*c^2*x^5 + 2*a^3*c^2*x^3 + a*c^2*x)/arctan(a*x), x) + c^2)/(a*arctan(a
*x))
```

3.538.8 Giac [N/A]

Not integrable

Time = 64.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.538.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^2, x)
```

3.539 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx$

3.539.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

3.539.2 Mathematica [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2), x]
```

3.539.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.539.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.539.4 Maple [N/A] (verified)

Not integrable

Time = 135.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

3.539.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^2), x)
```

3.539.6 Sympy [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = c^2\left(\int \frac{1}{x\,\mathrm{atan}^2(ax)}\,dx + \int \frac{2a^2x}{\mathrm{atan}^2(ax)}\,dx + \int \frac{a^4x^3}{\mathrm{atan}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**2,x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**2), x) + Integral(2*a**2*x/atan(a*x)**2, x)
 + Integral(a**4*x**3/atan(a*x)**2, x))
```

3.539.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - x*arctan(a*x)*integrate((5
*a^6*c^2*x^6 + 9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)/(x^2*arctan(a*x)), x)
+ c^2)/(a*x*arctan(a*x))
```

3.539.8 Giac [N/A]

Not integrable

Time = 64.50 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.539.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^2), x)
```

3.540 $\int \frac{x\left(c+a^2cx^2\right)^3}{\arctan(ax)^2}\,dx$

3.540.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.540.2 Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

3.540.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^2}dx$$
$$\Big\downarrow \, 5560$$
$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.540.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.540.4 Maple [N/A] (verified)

Not integrable

Time = 95.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.540.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)
^2, x)
```

3.540.6 Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
c**3*(Integral(x/atan(a*x)**2, x) + Integral(3*a**2*x**3/atan(a*x)**2, x)
+ Integral(3*a**4*x**5/atan(a*x)**2, x) + Integral(a**6*x**7/atan(a*x)**2,
 x))
```

3.540.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 6.15

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^8*c^3*x^9 + 4*a^6*c^3*x^7 + 6*a^4*c^3*x^5 + 4*a^2*c^3*x^3 + c^3*x - ar
ctan(a*x)*integrate((9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 + 12*
a^2*c^3*x^2 + c^3)/arctan(a*x), x))/(a*arctan(a*x))
```

3.540.8 Giac [N/A]

Not integrable

Time = 70.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.540.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^2, x)
```

3.541 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx$

3.541.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.541.2 Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2, x]
```

3.541.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.541.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.541.4 Maple [N/A] (verified)

Not integrable

Time = 75.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.541.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^2
, x)
```

3.541.6 Sympy [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{1}{\operatorname{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**2, x) + Integral(3*a**4*x**4/atan(a*
x)**2, x) + Integral(a**6*x**6/atan(a*x)**2, x) + Integral(atan(a*x)**(-2)
, x))
```

3.541.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 6.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - a*ar
ctan(a*x)*integrate(8*(a^7*c^3*x^7 + 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 + a*c^3
*x)/arctan(a*x), x))/(a*arctan(a*x))
```

3.541.8 Giac [N/A]

Not integrable

Time = 68.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.541.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\mathrm{atan}(a\,x)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^2, x)
```

3.542 $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx$

3.542.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{x\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

3.542.2 Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2), x]
```

3.542.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^2}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.542.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.542.4 Maple [N/A] (verified)

Not integrable

Time = 135.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

3.542.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x
)^2), x)
```

3.542.6 Sympy [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = c^3\left(\int \frac{1}{x\operatorname{atan}^2{(ax)}}\,dx + \int \frac{3a^2x}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{3a^4x^3}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{a^6x^5}{\operatorname{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**2,x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**2), x) + Integral(3*a**2*x/atan(a*x)**2, x)
 + Integral(3*a**4*x**3/atan(a*x)**2, x) + Integral(a**6*x**5/atan(a*x)**2
, x))
```

3.542.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.91

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - x*ar
ctan(a*x)*integrate((7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a^4*c^3*x^4 + 4*a
^2*c^3*x^2 - c^3)/(x^2*arctan(a*x)), x))/(a*x*arctan(a*x))
```

3.542.8 Giac [N/A]

Not integrable

Time = 70.33 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.542.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{x\,\mathrm{atan}\,{(a\,x)}^2}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^2), x)
```

3.543 $\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.543.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x^3}{ac\arctan(ax)}+\frac{3\mathrm{Int}\left(\frac{x^2}{\arctan(ax)},x\right)}{ac}$$

output
```
-x^3/a/c/arctan(a*x)+3*Unintegrable(x^2/arctan(a*x),x)/a/c
```

3.543.2 Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.543.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2\,(a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5461}$$
$$\frac{3\int \frac{x^2}{\arctan(ax)}dx}{ac}-\frac{x^3}{ac\arctan(ax)}$$
$$\Big\downarrow \text{5377}$$
$$\frac{3\int \frac{x^2}{\arctan(ax)}dx}{ac}-\frac{x^3}{ac\arctan(ax)}$$

input
```
Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.543.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.543.4 Maple [N/A] (verified)

Not integrable

Time = 21.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^2c\,x^2+c)\arctan\left(ax\right)^2}dx$$

input
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.543.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.543.6 Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x^3}{a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c}$$

input
```
integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x**3/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

3.543.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^3 - 3*arctan(a*x)*integrate(x^2/arctan(a*x), x))/(a*c*arctan(a*x))
```

3.543.8 Giac [N/A]

Not integrable

Time = 43.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.543.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.544 $\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.544.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x^2}{ac\arctan(ax)} + \frac{2\mathrm{Int}\left(\frac{x}{\arctan(ax)},x\right)}{ac}$$

output
```
-x^2/a/c/arctan(a*x)+2*Unintegrable(x/arctan(a*x),x)/a/c
```

3.544.2 Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.544.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2(a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5461}$$
$$\frac{2\int \frac{x}{\arctan(ax)}dx}{ac}-\frac{x^2}{ac\arctan(ax)}$$
$$\Big\downarrow \text{5377}$$
$$\frac{2\int \frac{x}{\arctan(ax)}dx}{ac}-\frac{x^2}{ac\arctan(ax)}$$

input
```
Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.544.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.544.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a^2 c\, x^2+c\right) \arctan \left(a x\right)^2}dx$$

input
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.544.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\, dx = \int \frac{x^2}{\left(a^2cx^2+c\right) \arctan \left(ax\right)^2}\, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.544.6 Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\, dx = \frac{\int \frac{x^2}{a^2x^2\operatorname{atan}^2\left(ax\right)+\operatorname{atan}^2\left(ax\right)}\, dx}{c}$$

input
```
integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x**2/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

3.544.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^2 - 2*arctan(a*x)*integrate(x/arctan(a*x), x))/(a*c*arctan(a*x))
```

3.544.8 Giac [N/A]

Not integrable

Time = 42.68 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.544.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.545 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.545.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x}{ac\arctan(ax)} + \frac{\mathrm{Int}\left(\frac{1}{\arctan(ax)},x\right)}{ac}$$

output
```
-x/a/c/arctan(a*x)+Unintegrable(1/arctan(a*x),x)/a/c
```

3.545.2 Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.545.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 (a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{\int \frac{1}{\arctan(ax)}dx}{ac}-\frac{x}{ac\arctan(ax)}$$
$$\Big\downarrow \text{5353}$$
$$\frac{\int \frac{1}{\arctan(ax)}dx}{ac}-\frac{x}{ac\arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.545.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.545.4 Maple [N/A] (verified)

Not integrable

Time = 0.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a^2c\,x^2+c)\arctan\left(ax\right)^2}dx$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.545.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.545.6 Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x}{a^2x^2\operatorname{atan}^2\left(ax\right)+\operatorname{atan}^2\left(ax\right)}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

3.545.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
(arctan(a*x)*integrate(1/arctan(a*x), x) - x)/(a*c*arctan(a*x))
```

3.545.8 Giac [N/A]

Not integrable

Time = 33.98 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.545.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.546 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.546.1 Optimal result

Integrand size = 19, antiderivative size = 14

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan(ax)}$$

output
```
-1/a/c/arctan(a*x)
```

3.546.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan(ax)}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*ArcTan[a*x]))
```

3.546.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5419}$$
$$-\frac{1}{ac\arctan(ax)}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*ArcTan[a*x]))
```

3.546.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.546.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$-\frac{1}{ac\arctan(ax)}$	15
default	$-\frac{1}{ac\arctan(ax)}$	15
parallelrisch	$-\frac{1}{ac\arctan(ax)}$	15
risch	$\frac{2i}{ac(\ln(-iax+1)-\ln(iax+1))}$	31

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a/c/arctan(a*x)
```

3.546.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan{(ax)}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/(a*c*arctan(a*x))
```

3.546.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\,\mathrm{atan}{(ax)}}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
-1/(a*c*atan(a*x))
```

3.546.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan{(ax)}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/(a*c*arctan(a*x))
```

3.546.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.546.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{a\,c\,\mathrm{atan}\left(a\,x\right)}$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
-1/(a*c*atan(a*x))
```

3.547 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.547.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{acx\arctan(ax)} - \frac{\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)},x\right)}{ac}$$

output
```
-1/a/c/x/arctan(a*x)-Unintegrable(1/x^2/arctan(a*x),x)/a/c
```

3.547.2 Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.547.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{\int \frac{1}{x^2 \arctan(ax)}dx}{ac} - \frac{1}{acx \arctan(ax)}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{\int \frac{1}{x^2 \arctan(ax)}dx}{ac} - \frac{1}{acx \arctan(ax)}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.547.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.547.4 Maple [N/A] (verified)

Not integrable

Time = 1.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.547.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=\int \frac{1}{{\left(a^{2} c x^{2} + c\right)} x \arctan\left(a x\right)^{2}}\,{d x}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)
```

3.547.6 Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=\frac{\int \frac{1}{a^{2} x^{3} \operatorname{atan}^{2}{\left(a x \right)} + x \operatorname{atan}^{2}{\left(a x \right)}}\, dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**2 + x*atan(a*x)**2), x)/c
```

3.547.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan(ax)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x*arctan(a*x)*integrate(1/(x^2*arctan(a*x)), x) + 1)/(a*c*x*arctan(a*x))
```

3.547.8 Giac [N/A]

Not integrable

Time = 33.95 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan(ax)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.547.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.548 $\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.548.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{acx^2\arctan(ax)} - \frac{2\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)},x\right)}{ac}$$

output
```
-1/a/c/x^2/arctan(a*x)-2*Unintegrable(1/x^3/arctan(a*x),x)/a/c
```

3.548.2 Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.548.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{ 5461}$$
$$-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{ac} - \frac{1}{acx^2 \arctan(ax)}$$
$$\Big\downarrow \text{ 5377}$$
$$-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{ac} - \frac{1}{acx^2 \arctan(ax)}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.548.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.548.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a^2 c x^2 + c) \arctan(ax)^2} dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.548.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)^2} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^2), x)
```

3.548.6 Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} dx = \frac{\int \frac{1}{a^2 x^4 \operatorname{atan}^2(ax) + x^2 \operatorname{atan}^2(ax)} dx}{c}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**4*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c
```

3.548.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(2*x^2*arctan(a*x)*integrate(1/(x^3*arctan(a*x)), x) + 1)/(a*c*x^2*arctan
(a*x))
```

3.548.8 Giac [N/A]

Not integrable

Time = 42.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.548.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^2 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.549 $\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.549.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{acx^3\arctan(ax)} - \frac{3\mathrm{Int}\left(\frac{1}{x^4\arctan(ax)},x\right)}{ac}$$

output
```
-1/a/c/x^3/arctan(a*x)-3*Unintegrable(1/x^4/arctan(a*x),x)/a/c
```

3.549.2 Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.549.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{ac} - \frac{1}{acx^3 \arctan(ax)}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{ac} - \frac{1}{acx^3 \arctan(ax)}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.549.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.549.4 Maple [N/A] (verified)

Not integrable

Time = 27.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.549.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3 (c+a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)x^3 \arctan\left(ax\right)^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^2), x)
```

3.549.6 Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (c+a^2cx^2) \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^{2} x^{5} \operatorname{atan}^{2}{\left(a x \right)} + x^{3} \operatorname{atan}^{2}{\left(a x \right)}}\, dx}{c}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**5*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c
```

3.549.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^3 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(3*x^3*arctan(a*x)*integrate(1/(x^4*arctan(a*x)), x) + 1)/(a*c*x^3*arctan
(a*x))
```

3.549.8 Giac [N/A]

Not integrable

Time = 43.58 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^3 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.549.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^2 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.550 $\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.550.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{acx^4\arctan(ax)} - \frac{4\mathrm{Int}\left(\frac{1}{x^5\arctan(ax)},x\right)}{ac}$$

output
```
-1/a/c/x^4/arctan(a*x)-4*Unintegrable(1/x^5/arctan(a*x),x)/a/c
```

3.550.2 Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.550.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{ac}-\frac{1}{acx^4 \arctan(ax)}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{ac}-\frac{1}{acx^4 \arctan(ax)}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.550.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.550.4 Maple [N/A] (verified)

Not integrable

Time = 24.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(a^2 c\, x^2+c\right) \arctan \left(a x\right)^2}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.550.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right) \arctan(a x)^2} \, dx=\int \frac{1}{(a^2 c x^2+c) x^4 \arctan \left(a x\right)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^2), x)
```

3.550.6 Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right) \arctan(a x)^2} \, dx=\frac{\int \frac{1}{a^{2} x^{6} \operatorname{atan}^{2}{\left(a x \right)} + x^{4} \operatorname{atan}^{2}{\left(a x \right)}}\, dx}{c}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c
```

3.550.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)x^4 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(4*x^4*arctan(a*x)*integrate(1/(x^5*arctan(a*x)), x) + 1)/(a*c*x^4*arctan
(a*x))
```

3.550.8 Giac [N/A]

Not integrable

Time = 42.95 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)x^4 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.550.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{x^4 \operatorname{atan}{(a\,x)}^2 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.551 $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} dx$

3.551.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = -\frac{x}{a^3c^2 \arctan(ax)} + \frac{x}{a^3c^2(1+a^2x^2)\arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{a^4c^2} + \frac{\text{Int}\left(\frac{1}{\arctan(ax)}, x\right)}{a^3c^2}$$

output
```
-x/a^3/c^2/arctan(a*x)+x/a^3/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*arctan(a*x))
/a^4/c^2+Unintegrable(1/arctan(a*x),x)/a^3/c^2
```

3.551.2 Mathematica [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = \int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

3.551.3 Rubi [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5499, 27, 5457, 5353, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c(a^2x^2+1)\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)\arctan(ax)^2}dx}{a^2c^2}-\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5457}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5503}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 2009$$

3.551. $\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

$$-\frac{\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}}{}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.551.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.551. $\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.551.4 Maple [N/A] (verified)

Not integrable

Time = 8.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^2c\,x^2+c)^2\arctan(ax)^2}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.551.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

3.551.6 Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}^2{(ax)}+2a^2x^2 \operatorname{atan}^2{(ax)}+\operatorname{atan}^2{(ax)}} \, dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*
x)**2), x)/c**2
```

3.551.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.68

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^3 - (a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^4 + 3*x^2)/((a^
5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x))/((a^3*c^2*x^2 + a*c^2
)*arctan(a*x))
```

3.551.8 Giac [N/A]

Not integrable

Time = 120.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.551.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.552 $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} dx$

3.552.1 Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = -\frac{x^2}{ac^2(1+a^2x^2)\arctan(ax)} + \frac{\text{Si}(2\arctan(ax))}{a^3c^2}$$

output
```
-x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/a^3/c^2
```

3.552.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = \frac{-\frac{a^2x^2}{(1+a^2x^2)\arctan(ax)} + \text{Si}(2\arctan(ax))}{a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
(-((a^2*x^2)/((1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]])/(a
^3*c^2)
```

3.552.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5477, 27, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2 (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \; 5477$$

$$\frac{2\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)}dx}{a} - \frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx}{ac^2} - \frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \; 5505$$

$$\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^3c^2} - \frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \; 4906$$

$$\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^3c^2} - \frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^3c^2} - \frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^3c^2} - \frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \; 3780$$

$$\frac{\text{Si}(2\arctan(ax))}{a^3c^2} - \frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]]/(a^3
*c^2)
```

3.552.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.552. $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$

3.552.4 Maple [A] (verified)

Time = 6.99 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{2a^3c^2\arctan(ax)}$	37
default	$\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{2a^3c^2\arctan(ax)}$	37

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))-1)/arctan(
a*x)
```

3.552.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.86

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx =$$
$$-\frac{2\,a^2x^2-(i\,a^2x^2+i)\arctan(ax)\log\_integral\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-(-i\,a^2x^2-i)\arctan(ax)\log\_integral}{2\,(a^5c^2x^2+a^3c^2)\arctan(ax)}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*x^2 - (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I
*a*x - 1)/(a^2*x^2 + 1)) - (-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2
*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/((a^5*c^2*x^2 + a^3*c^2)*arctan(a*x))
```

3.552.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{x^2}{a^4x^4 \operatorname{atan}^2(ax)+2a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)} \, dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*
x)**2), x)/c**2
```

3.552.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
(4*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate(1/2*x/((a^5*c^2*x^4 + 2*a^3
*c^2*x^2 + a*c^2)*arctan(a*x)), x) - x^2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*
x))
```

3.552.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.553 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$

3.553.1 Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = -\frac{x}{ac^2(1+a^2x^2)\arctan(ax)} + \frac{\text{CosIntegral}(2\arctan(ax))}{a^2c^2}$$

output
```
-x/a/c^2/(a^2*x^2+1)/arctan(a*x)+Ci(2*arctan(a*x))/a^2/c^2
```

3.553.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \frac{-\frac{ax}{(1+a^2x^2)\arctan(ax)} + \text{CosIntegral}(2\arctan(ax))}{a^2c^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
(-((a*x)/((1 + a^2*x^2)*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]])/(a^2*c
^2)
```

3.553.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. $2(41) = 82$.

Time = 0.78 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\arctan(ax)}dx}{a} - a\int \frac{x^2}{c^2(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{ac^2} - \frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2} - \frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2} + \frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2c^2} - \frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2} + \frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2c^2} - \frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2} + \frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)} + \frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2c^2} - \frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax)) + \frac{1}{2}\log(\arctan(ax))}{a^2c^2} - \frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{ 5505}$$

$$-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]]
+ Log[ArcTan[a*x]]/2)/(a^2*c^2) + (CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcT
an[a*x]]/2)/(a^2*c^2)
```

3.553.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.553. $\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.553.4 Maple [A] (verified)

Time = 8.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)-\sin(2\arctan(ax))}{2a^2c^2\arctan(ax)}$	38
default	$\frac{2\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)-\sin(2\arctan(ax))}{2a^2c^2\arctan(ax)}$	38

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/c^2*(2*Ci(2*arctan(a*x))*arctan(a*x)-sin(2*arctan(a*x)))/arctan(a*
x)
```

3.553.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.80

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx$$
$$= \frac{(a^2x^2+1)\arctan(ax)\,\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(a^2x^2+1)\arctan(ax)\,\text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right.}{2\,(a^4c^2x^2+a^2c^2)\arctan(ax)}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*
x^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1
)/(a^2*x^2 + 1)) - 2*a*x)/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x))
```

3.553.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{x}{a^4x^4\,\text{atan}^2(ax)+2a^2x^2\,\text{atan}^2(ax)+\text{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)*
*2), x)/c**2
```

3.553.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)^2\arctan(ax)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^2 - 1)/((a^5*c^2*x^4
+ 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x) + x)/((a^3*c^2*x^2 + a*c^2)*arct
an(a*x))
```

3.553.8 Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x}{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.554 $\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^2} dx$

3.554.1 Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = -\frac{1}{ac^2 (1+a^2x^2) \arctan(ax)} - \frac{\text{Si}(2\arctan(ax))}{ac^2}$$

output
```
-1/a/c^2/(a^2*x^2+1)/arctan(a*x)-Si(2*arctan(a*x))/a/c^2
```

3.554.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = -\frac{\frac{1}{\arctan(ax)+a^2x^2\arctan(ax)} + \text{Si}(2\arctan(ax))}{ac^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(((ArcTan[a*x] + a^2*x^2*ArcTan[a*x])^(-1) + SinIntegral[2*ArcTan[a*x]])/
(a*c^2))
```

3.554.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5437, 27, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$-2a\int \frac{x}{c^2 (a^2x^2+1)^2 \arctan(ax)}dx-\frac{1}{ac^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\int \frac{x}{(a^2x^2+1)^2 \arctan(ax)}dx}{c^2}-\frac{1}{ac^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\int \frac{ax}{(a^2x^2+1) \arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{1}{ac^2 (a^2x^2+1) \arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^2
)
```

3.554.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.554. $\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$

3.554.4 Maple [A] (verified)

Time = 7.73 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{2a\,c^{2}\arctan(ax)}$	37
default	$-\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{2a\,c^{2}\arctan(ax)}$	37

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))+1)/arctan(a
*x)
```

3.554.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.73

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx$$
$$= \frac{(-i\,a^2x^2-i)\arctan{(ax)}\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(i\,a^2x^2+i)\arctan{(ax)}\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,a}{a^2x^2+1}\right.}{2\left(a^3c^2x^2+ac^2\right)\arctan{(ax)}}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*((-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a
^2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*
x - 1)/(a^2*x^2 + 1)) - 2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x))
```

3.554.6 Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{1}{a^4x^4\,\mathrm{atan}^2\,(ax)+2a^2x^2\,\mathrm{atan}^2\,(ax)+\mathrm{atan}^2\,(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)*
*2), x)/c**2
```

3.554.7 Maxima [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(4*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)*integrate(1/2*x/((a^4*c^2*x^4 + 2*
a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x
))
```

3.554.8 Giac [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx=\int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.555 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$

3.555.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = -\frac{1}{ac^2x\arctan(ax)} + \frac{ax}{c^2(1+a^2x^2)\arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{c^2} - \frac{\text{Int}\left(\frac{1}{x^2\arctan(ax)}, x\right)}{ac^2}$$

output
```
-1/a/c^2/x/arctan(a*x)+a*x/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*arctan(a*x))/c
^2-Unintegrable(1/x^2/arctan(a*x),x)/a/c^2
```

3.555.2 Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

3.555.3 Rubi [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2 \left(a^2 c x^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{cx(a^2x^2+1)\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5461}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 3793$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 5505$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

3.555. $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

$$\Big\downarrow \textcolor{blue}{3793}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.555.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.555. $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.555. $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

3.555.4 Maple [N/A] (verified)

Not integrable

Time = 12.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)^{2}\arctan\left(ax\right)^{2}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.555.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\left(c+a^{2}cx^{2}\right)^{2}\arctan(ax)^{2}}\,dx=\int \frac{1}{\left(a^{2}cx^{2}+c\right)^{2}x\arctan\left(ax\right)^{2}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^2), x)
```

3.555.6 Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(c+a^{2}cx^{2}\right)^{2}\arctan(ax)^{2}}\,dx=\frac{\int \frac{1}{a^{4}x^{5}\operatorname{atan}^{2}\left(ax\right)+2a^{2}x^{3}\operatorname{atan}^{2}\left(ax\right)+x\operatorname{atan}^{2}\left(ax\right)}\,dx}{c^{2}}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**2 + 2*a**2*x**3*atan(a*x)**2 + x*atan(a*x
)**2), x)/c**2
```

3.555.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.64

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)^2x\arctan(ax)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^3 + a*c^2*x)*arctan(a*x)*integrate((3*a^2*x^2 + 1)/((a^5*c^2*
x^6 + 2*a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^3 + a*c
^2*x)*arctan(a*x))
```

3.555.8 Giac [N/A]

Not integrable

Time = 84.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)^2x\arctan(ax)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.555.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.556 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$

3.556.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = -\frac{1}{ac^2x^2 \arctan(ax)} + \frac{a}{c^2 (1+a^2x^2) \arctan(ax)} + \frac{a\text{Si}(2\arctan(ax))}{c^2} - \frac{2\text{Int}\left(\frac{1}{x^3 \arctan(ax)}, x\right)}{ac^2}$$

output
```
-1/a/c^2/x^2/arctan(a*x)+a/c^2/(a^2*x^2+1)/arctan(a*x)+a*Si(2*arctan(a*x))
/c^2-2*Unintegrable(1/x^3/arctan(a*x),x)/a/c^2
```

3.556.2 Mathematica [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

3.556.3 Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 27, 5437, 5461, 5377, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^2(a^2x^2+1)\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^2 (a^2x^2+1)^2 \arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx}{c^2} - \frac{a^2 \int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx}{c^2} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^2} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^2} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^2} - \frac{a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^2} - \frac{a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 3780$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)}{c^2}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.556.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.556. $\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.556.4 Maple [N/A] (verified)

Not integrable

Time = 6.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^2}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.556.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^2x^2\arctan(ax)^2}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2), x)
```

3.556.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \frac{\int \frac{1}{a^4x^6\operatorname{atan}^2(ax)+2a^2x^4\operatorname{atan}^2(ax)+x^2\operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**2 + 2*a**2*x**4*atan(a*x)**2 + x**2*atan(
a*x)**2), x)/c**2
```

3.556.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.86

$$\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^2x^2\arctan(ax)^2}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)*integrate(2*(2*a^2*x^2 + 1)/((a^5*
c^2*x^7 + 2*a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)), x) + 1)/((a^3*c^2*x^4 +
 a*c^2*x^2)*arctan(a*x))
```

3.556.8 Giac [N/A]

Not integrable

Time = 88.95 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx = \int \frac{1}{\left(a^2cx^2+c\right)^2 x^2 \arctan\left(ax\right)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.556.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x^2 \operatorname{atan}\left(a\,x\right)^2 \left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.557 $\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

3.557.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^2}\,dx = &-\frac{1}{ac^2x^3\arctan(ax)}+\frac{a}{c^2x\arctan(ax)}\\&-\frac{a^3x}{c^2(1+a^2x^2)\arctan(ax)}\\&+\frac{a^2\,\mathrm{CosIntegral}(2\arctan(ax))}{c^2}\\&-\frac{3\mathrm{Int}\left(\frac{1}{x^4\arctan(ax)},x\right)}{ac^2}+\frac{a\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)},x\right)}{c^2}\end{aligned}$$

output
```
-1/a/c^2/x^3/arctan(a*x)+a/c^2/x/arctan(a*x)-a^3*x/c^2/(a^2*x^2+1)/arctan(
a*x)+a^2*Ci(2*arctan(a*x))/c^2-3*Unintegrable(1/x^4/arctan(a*x),x)/a/c^2+a
*Unintegrable(1/x^2/arctan(a*x),x)/c^2
```

3.557.2 Mathematica [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

3.557.3 Rubi [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5461, 5377, 5501, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^3(a^2x^2+1)\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)\arctan(ax)^2}dx}{c^2}-\frac{a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

$\Big\downarrow$ 5377

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

$\Big\downarrow$ 5503

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

$\Big\downarrow$ 5439

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

$\Big\downarrow$ 3793

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

3.557. $\displaystyle\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(a}}{a}}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-a^2\left(-\left(a^2\left(-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{}{x^2}}{}}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan}}{a}}{c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-a^2\left(-\left(a^2\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{}{a(a}\right.\right.\right.}{c^2}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.557. $\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

3.557.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.557. $\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.557.4 Maple [N/A] (verified)

Not integrable

Time = 27.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.557.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)^2} \, dx=\int { \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{2} x^{3} \arctan \left(a x\right)^{2}} \,d x }$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^2), x)
```

3.557.6 Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{1}{a^4x^7 \operatorname{atan}^2(ax)+2a^2x^5 \operatorname{atan}^2(ax)+x^3 \operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**2 + 2*a**2*x**5*atan(a*x)**2 + x**3*atan(
a*x)**2), x)/c**2
```

3.557.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.82

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)*integrate((5*a^2*x^2 + 3)/((a^5*c^
2*x^8 + 2*a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)), x) + 1)/((a^3*c^2*x^5 + a
*c^2*x^3)*arctan(a*x))
```

3.557.8 Giac [N/A]

Not integrable

Time = 89.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.557.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.558 $\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

3.558.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx &= -\frac{1}{ac^2x^4\arctan(ax)}+\frac{a}{c^2x^2\arctan(ax)} \\ &\quad -\frac{a^3}{c^2\left(1+a^2x^2\right)\arctan(ax)}-\frac{a^3\mathrm{Si}(2\arctan(ax))}{c^2} \\ &\quad -\frac{4\mathrm{Int}\left(\frac{1}{x^5\arctan(ax)},x\right)}{ac^2}+\frac{2a\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)},x\right)}{c^2}\end{aligned}$$

output
```
-1/a/c^2/x^4/arctan(a*x)+a/c^2/x^2/arctan(a*x)-a^3/c^2/(a^2*x^2+1)/arctan(
a*x)-a^3*Si(2*arctan(a*x))/c^2-4*Unintegrable(1/x^5/arctan(a*x),x)/a/c^2+2
*a*Unintegrable(1/x^3/arctan(a*x),x)/c^2
```

3.558.2 Mathematica [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

3.558.3 Rubi [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 27, 5461, 5377, 5501, 5437, 5461, 5377, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 \left(a^2cx^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^4(a^2x^2+1)\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^2x^2\left(a^2x^2+1\right)^2\arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^2}dx}{c^2} - \frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{1}{ax^4\arctan(ax)}}{c^2} - \frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{1}{ax^4\arctan(ax)}}{c^2} - \frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{1}{ax^4\arctan(ax)}}{c^2} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx - a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)}{c^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{1}{ax^4\arctan(ax)}}{c^2} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4 \arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2 \arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{a}-\frac{1}{ax^2 \arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 5377$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4 \arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2 \arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{a}-\frac{1}{ax^2 \arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4 \arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{a}-\frac{1}{ax^2 \arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 4906$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4 \arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{a}-\frac{1}{ax^2 \arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4 \arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{a}-\frac{1}{ax^2 \arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4 \arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{a}-\frac{1}{ax^2 \arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \; 3780$$

3.558. $\int \frac{1}{x^4(c+a^2cx^2)^2 \arctan(ax)^2}\,dx$

$$\frac{-\frac{4\int\frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{2\int\frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\mathrm{Si}(2\arctan(ax))}{a}\right)\right)-\frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.558.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

3.558. $\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^2}\,dx$

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.558.4 Maple [N/A] (verified)

Not integrable

Time = 21.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a^2 c x^2 + c)^2 \arctan (ax)^2} dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.558.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^2), x)
```

3.558.6 Sympy [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^4x^8 \operatorname{atan}^2(ax) + 2a^2x^6 \operatorname{atan}^2(ax) + x^4 \operatorname{atan}^2(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**2 + 2*a**2*x**6*atan(a*x)**2 + x**4*atan(
a*x)**2), x)/c**2
```

3.558.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.86

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)*integrate(2*(3*a^2*x^2 + 2)/((a^5*
c^2*x^9 + 2*a^3*c^2*x^7 + a*c^2*x^5)*arctan(a*x)), x) + 1)/((a^3*c^2*x^6 +
 a*c^2*x^4)*arctan(a*x))
```

3.558.8 Giac [N/A]

Not integrable

Time = 89.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.558.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^2 \,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.559 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx$

3.559.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{x}{a^3c^3(1+a^2x^2)^2\arctan(ax)} - \frac{x}{a^3c^3(1+a^2x^2)\arctan(ax)} + \frac{\text{CosIntegral}(2\arctan(ax))}{2a^4c^3} - \frac{\text{CosIntegral}(4\arctan(ax))}{2a^4c^3}$$

output
```
x/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)-x/a^3/c^3/(a^2*x^2+1)/arctan(a*x)+1/2*
Ci(2*arctan(a*x))/a^4/c^3-1/2*Ci(4*arctan(a*x))/a^4/c^3
```

3.559.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{-2a^3x^3 + (1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral}(2\arctan(ax)) - (1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral(4 arcta}}{2a^4c^3(1+a^2x^2)^2\arctan(ax)}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-2*a^3*x^3 + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] - (1
+ a^2*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*a^4*c^3*(1 + a^2*x
^2)^2*ArcTan[a*x])
```

3.559.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. $2(86) = 172$.

Time = 1.53 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5499, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x}{c^3(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^3}-\frac{\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{5439}$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \ 3793$$

$$\frac{-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-3a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-3a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \ 5505$$

$$\frac{-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a\left(a^2x^2+1\right)^2\arctan(}}{a^2c^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a\left(a^2x^2+1\right)^2\arctan(}}{a^2c^3}$$

$$\Big\downarrow \ 3793$$

$$\frac{-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a\left(a^2x^2+1\right)^2\arctan(}}{a^2c^3}$$

$$\Big\downarrow \ 2009$$

3.559. $\int \frac{x^3}{\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx$

$$\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(}}{a^2c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+}}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x}}{a^2c^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-((-(x/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a*
x]] + Log[ArcTan[a*x]]/8))/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + CosIntegr
al[4*ArcTan[a*x]]/8 + (3*Log[ArcTan[a*x]])/8)/a^2)/(a^2*c^3)) + (-(x/(a*(1
 + a^2*x^2)*ArcTan[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[
a*x]]/2)/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/a^2)/(a
^2*c^3)
```

3.559.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.559. $\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.559. $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx$

3.559.4 Maple [A] (verified)

Time = 9.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)-4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)-\sin(4\arctan(ax))+2\sin(2\arctan(ax))}{8a^4c^3\arctan(ax)}$	60
default	$-\frac{4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)-4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)-\sin(4\arctan(ax))+2\sin(2\arctan(ax))}{8a^4c^3\arctan(ax)}$	60

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^4/c^3*(4*Ci(4*arctan(a*x))*arctan(a*x)-4*Ci(2*arctan(a*x))*arctan(a
*x)-sin(4*arctan(a*x))+2*sin(2*arctan(a*x)))/arctan(a*x)
```

3.559.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.40

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx =$$
$$-\frac{4\,a^3x^3 + (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + (a^4x^4 + 2\,a^2x^2 + 1)}{}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a^3*x^3 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*
x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) +
(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3
- 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (a^4*x^4 + 2*a^2*x
^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) -
 (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x -
1)/(a^2*x^2 + 1)))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x))
```

3.559.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \frac{\int \frac{x^3}{a^6x^6 \operatorname{atan}^2(ax)+3a^4x^4 \operatorname{atan}^2(ax)+3a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)} dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*
x**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

3.559.7 Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^2} dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^3 + (a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((a^2*x
^4 - 3*x^2)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(
a*x)), x))/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

3.559.8 Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^2} dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.560 $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx$

3.560.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \frac{1}{a^3c^3(1+a^2x^2)^2 \arctan(ax)} - \frac{1}{a^3c^3(1+a^2x^2)\arctan(ax)} + \frac{\operatorname{Si}(4\arctan(ax))}{2a^3c^3}$$

output
```
1/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)-1/a^3/c^3/(a^2*x^2+1)/arctan(a*x)+1/2*
Si(4*arctan(a*x))/a^3/c^3
```

3.560.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \frac{-2a^2x^2 + (1+a^2x^2)^2 \arctan(ax)\operatorname{Si}(4\arctan(ax))}{2a^3c^3(1+a^2x^2)^2 \arctan(ax)}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-2*a^2*x^2 + (1 + a^2*x^2)^2*ArcTan[a*x]*SinIntegral[4*ArcTan[a*x]])/(2*a
^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])
```

3.560.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5499, 27, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2 (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^3}-\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2c^3}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}}{a^2c^3}-\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-(1/(a*(1 + a^2*x^2)*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/a)/(a^2*c
^3) - (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[a*x
]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/(a^2*c^3)
```

3.560.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.560. $\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.560. $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx$

3.560.4 Maple [A] (verified)

Time = 8.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\frac{4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{8a^3c^3\arctan(ax)}$	37
default	$\frac{4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{8a^3c^3\arctan(ax)}$	37

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3/c^3*(4*Si(4*arctan(a*x))*arctan(a*x)+cos(4*arctan(a*x))-1)/arctan(
a*x)
```

3.560.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.93

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx =$$
$$-\frac{4\,a^2x^2-(i\,a^4x^4+2i\,a^2x^2+i)\arctan(ax)\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-(-i\,a^4x^4-2i\,a^2x^2}{4\,(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3)\arctan(ax)}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a^2*x^2 - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((
a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)
) - (-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I
*a^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/((a^7*c^3*
x^4 + 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x))
```

3.560.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{\int \frac{x^2}{a^6x^6 \operatorname{atan}^2(ax)+3a^4x^4 \operatorname{atan}^2(ax)+3a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*
x**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

3.560.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate(2*(a^2*x^3 -
 x)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x
) + x^2)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

3.560.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x^2}{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.561 $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

3.561.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = -\frac{x}{ac^3(1+a^2x^2)^2\arctan(ax)}+\frac{\text{CosIntegral}(2\arctan(ax))}{2a^2c^3}+\frac{\text{CosIntegral}(4\arctan(ax))}{2a^2c^3}$$

output
```
-x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)+1/2*Ci(2*arctan(a*x))/a^2/c^3+1/2*Ci(4*
arctan(a*x))/a^2/c^3
```

3.561.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{-2ax+(1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral}(2\arctan(ax))+(1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral}(4\arctan$$
$$2c^3(a+a^3x^2)^2\arctan(ax)$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-2*a*x + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] + (1 + a^
2*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*c^3*(a + a^3*x^2)^2*Ar
cTan[a*x])
```

3.561.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \textcolor{blue}{5503}$$

$$\frac{\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{c^3(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{ac^3}-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \textcolor{blue}{5439}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \textcolor{blue}{3793}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2c^3}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{3\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2c^3}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2c^3}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-(x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a
*x]] + Log[ArcTan[a*x]]/8))/(a^2*c^3) + (CosIntegral[2*ArcTan[a*x]]/2 + Co
sIntegral[4*ArcTan[a*x]]/8 + (3*Log[ArcTan[a*x]])/8)/(a^2*c^3)
```

3.561.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.561. $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.561.4 Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)+4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)-\sin(4\arctan(ax))-2\sin(2\arctan(ax))}{8a^2c^3\arctan(ax)}$	60
default	$\frac{4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)+4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)-\sin(4\arctan(ax))-2\sin(2\arctan(ax))}{8a^2c^3\arctan(ax)}$	60

3.561. $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2/c^3*(4*Ci(4*arctan(a*x))*arctan(a*x)+4*Ci(2*arctan(a*x))*arctan(a*
x)-sin(4*arctan(a*x))-2*sin(2*arctan(a*x)))/arctan(a*x)
```

3.561.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 4.69

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{(a^4x^4+2\,a^2x^2+1)\arctan(ax)\,\text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+(a^4x^4+2\,a^2x^2+1)\arctan(ax)}{}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/4*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 + 4*I*a^3
*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*
a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2 +
 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*arcta
n(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*x^4 + 2
*a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 +
 1)) - 4*a*x)/((a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x))
```

3.561.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{\int \frac{x}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**
2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

3.561.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x}{(a^2cx^2+c)^3 \arctan(ax)^2} \, dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((3*a^2*x^2 -
 1)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x
) + x)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

3.561.8 Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x}{(a^2cx^2+c)^3 \arctan(ax)^2} \, dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.562 $\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx$

3.562.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx = -\frac{1}{ac^3\left(1+a^2x^2\right)^2\arctan(ax)} - \frac{\operatorname{Si}(2\arctan(ax))}{ac^3} - \frac{\operatorname{Si}(4\arctan(ax))}{2ac^3}$$

output
```
-1/a/c^3/(a^2*x^2+1)^2/arctan(a*x)-Si(2*arctan(a*x))/a/c^3-1/2*Si(4*arctan
(a*x))/a/c^3
```

3.562.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx = -\frac{\frac{1}{\left(1+a^2x^2\right)^2\arctan(ax)} + \operatorname{Si}(2\arctan(ax)) + \frac{1}{2}\operatorname{Si}(4\arctan(ax))}{ac^3}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-((1/((1 + a^2*x^2)^2*ArcTan[a*x]) + SinIntegral[2*ArcTan[a*x]] + SinInteg
ral[4*ArcTan[a*x]]/2)/(a*c^3))
```

3.562.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5437, 27, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5437}$$

$$-4a\int \frac{x}{c^3 (a^2x^2+1)^3 \arctan(ax)}dx-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4a\int \frac{x}{(a^2x^2+1)^3 \arctan(ax)}dx}{c^3}-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{4\int \frac{ax}{(a^2x^2+1)^2 \arctan(ax)}d\arctan(ax)}{ac^3}-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{ac^3}-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[a*x]]/
4 + SinIntegral[4*ArcTan[a*x]]/8))/(a*c^3)
```

3.562.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.562.4 Maple [A] (verified)

Time = 8.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+4\cos(2\arctan(ax))+\cos(4\arctan(ax))+3}{8a\,c^3\arctan(ax)}$	59
default	$-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+4\cos(2\arctan(ax))+\cos(4\arctan(ax))+3}{8a\,c^3\arctan(ax)}$	59

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)+4*Si(4*arctan(a*x))*arctan(a*x
)+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)
```

3.562.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.95

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{(-i\,a^4x^4 - 2i\,a^2x^2 - i)\arctan(ax)\operatorname{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + (i\,a^4x^4 + 2i\,a^2x^2 + i)\operatorname{arctan}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/4*((-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 + 4*
I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^4*x
^4 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*
a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 2*(I*a^4*x^4 + 2*I*a^2
*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1))
 - 2*(-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2
*I*a*x - 1)/(a^2*x^2 + 1)) - 4)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arc
tan(a*x))
```

3.562.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{\int \frac{1}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**
2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

3.562.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)^3\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(8*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)*integrate(1/2*x/((
a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)), x) + 1)/(
(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

3.562.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)^3\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.562.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx=\int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.563 $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

3.563.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx =& -\frac{1}{ac^3x\arctan(ax)}+\frac{ax}{c^3(1+a^2x^2)^2\arctan(ax)}\\ &+\frac{ax}{c^3(1+a^2x^2)\arctan(ax)}-\frac{3\,\mathrm{CosIntegral}(2\arctan(ax))}{2c^3}\\ &-\frac{\mathrm{CosIntegral}(4\arctan(ax))}{2c^3}-\frac{\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)},x\right)}{ac^3}\end{aligned}$$

output
```
-1/a/c^3/x/arctan(a*x)+a*x/c^3/(a^2*x^2+1)^2/arctan(a*x)+a*x/c^3/(a^2*x^2+
1)/arctan(a*x)-3/2*Ci(2*arctan(a*x))/c^3-1/2*Ci(4*arctan(a*x))/c^3-Uninteg
rable(1/x^2/arctan(a*x),x)/a/c^3
```

3.563.2 Mathematica [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

3.563.3 Rubi [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5501, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^2\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x\left(a^2x^2+1\right)\arctan(ax)^2}dx-a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\left(a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^3}-\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{x}{a\left(a^2x^2+1\right)^2\arctan(ax)}\right)}{c^3}$$

3.563. $\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}dx$

$$\downarrow \text{5439}$$

$$-\frac{\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{}{ax\arctan}}{c^3}$$
$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\downarrow \text{3042}$$

$$-\frac{\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{}{ax\arctan}}{c^3}$$
$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\downarrow \text{3793}$$

$$-\frac{\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}}{c^3}$$
$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\downarrow \text{2009}$$

$$-\frac{\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}}{c^3}$$
$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(}\right.}{c^3}$$

$$\downarrow \text{5505}$$

3.563. $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

$$\frac{-\left(a^2\left(-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}}{a}}{c^3}$$

$$\frac{a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2 a}\right.}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}}{a}}{c^3}$$

$$\frac{a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2 a}\right.}{c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\left(a^2\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-}{c^3}$$

$$\frac{a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2 a}\right.}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+}\right.\right.}{c^3}$$

$$\frac{a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2 a}\right.}{c^3}$$

$$\Big\downarrow \text{4906}$$

3.563. $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

$$\frac{-\frac{\int \frac{1}{x^2 \arctan(ax)}dx}{a} - \left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{}{a(a^2x^2+}\right.\right.}{c^3}$$

$$\frac{a^2\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2} + \frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2} - \frac{}{a(a}\right.}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{1}{x^2 \arctan(ax)}dx}{a} - \left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{}{a(a^2x^2+}\right.\right.}{c^3}$$

$$\frac{a^2\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2} + \frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2} - \frac{}{a}\right.}{c^3}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.563.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.563. $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^2} dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.563. $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.563.4 Maple [N/A] (verified)

Not integrable

Time = 8.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3} \arctan\left(a x\right)^{2}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.563.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^{2} c x^{2} + c\right)^{3} x \arctan\left(a x\right)^{2}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a
*x)^2), x)
```

3.563.6 Sympy [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{\int \frac{1}{a^6x^7\operatorname{atan}^2(ax)+3a^4x^5\operatorname{atan}^2(ax)+3a^2x^3\operatorname{atan}^2(ax)+x\operatorname{atan}^2(ax)}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**2 + 3*a**4*x**5*atan(a*x)**2 + 3*a**2*x**
3*atan(a*x)**2 + x*atan(a*x)**2), x)/c**3
```

3.563.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.14

$$\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^3x\arctan(ax)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x)*integrate((5*a^2*x^2
 + 1)/((a^7*c^3*x^8 + 3*a^5*c^3*x^6 + 3*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*
x)), x) + 1)/((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))
```

3.563.8 Giac [N/A]

Not integrable

Time = 122.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^3x\arctan(ax)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.563.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.564 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx$

3.564.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx = -\frac{1}{ac^3x^2\arctan(ax)}+\frac{a}{c^3\left(1+a^2x^2\right)^2\arctan(ax)}+\frac{a}{c^3\left(1+a^2x^2\right)\arctan(ax)}+\frac{2a\mathrm{Si}(2\arctan(ax))}{c^3}+\frac{a\mathrm{Si}(4\arctan(ax))}{2c^3}-\frac{2\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)},x\right)}{ac^3}$$

output
```
-1/a/c^3/x^2/arctan(a*x)+a/c^3/(a^2*x^2+1)^2/arctan(a*x)+a/c^3/(a^2*x^2+1)
/arctan(a*x)+2*a*Si(2*arctan(a*x))/c^3+1/2*a*Si(4*arctan(a*x))/c^3-2*Unint
egrable(1/x^3/arctan(a*x),x)/a/c^3
```

3.564.2 Mathematica [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

3.564.3 Rubi [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5437, 5501, 5437, 5461, 5377, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \color{blue}{5501}$$

$$\frac{\int \frac{1}{c^2x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^3 (a^2x^2+1)^3 \arctan(ax)^2} dx$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2 \int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \color{blue}{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \color{blue}{5501}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx - a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \color{blue}{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^3} - \frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \color{blue}{5461}$$

$$\frac{-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{4\int \frac{ax}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}\right)}{c^3}$$

3.564. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}\right)}{c^3}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)\right)-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}\right)}{c^3}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.564.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.564. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.564. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

3.564.4 Maple [N/A] (verified)

Not integrable

Time = 10.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.564.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(a x)^2} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{3} x^{2} \arctan \left (a x\right)^{2}} \,d x }$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan
(a*x)^2), x)
```

3.564.6 Sympy [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(a x)^2} \, dx=\frac {\int \frac {1}{a^{6} x^{8} \operatorname {atan}^{2}{\left (a x \right)} + 3 a^{4} x^{6} \operatorname {atan}^{2}{\left (a x \right)} + 3 a^{2} x^{4} \operatorname {atan}^{2}{\left (a x \right)} + x^{2} \operatorname {atan}^{2}{\left (a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**2 + 3*a**4*x**6*atan(a*x)**2 + 3*a**2*x**
4*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c**3
```

3.564.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{1}{(a^2cx^2+c)^3 x^2 \arctan{(ax)}^2} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x)*integrate(2*(3*a^2
*x^2 + 1)/((a^7*c^3*x^9 + 3*a^5*c^3*x^7 + 3*a^3*c^3*x^5 + a*c^3*x^3)*arcta
n(a*x)), x) + 1)/((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x))
```

3.564.8 Giac [N/A]

Not integrable

Time = 128.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{1}{(a^2cx^2+c)^3 x^2 \arctan{(ax)}^2} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.564.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^2 \,(c\,a^2\,x^2+c)^3} dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.565 $\int \frac{1}{x^3(c+a^2cx^2)^3 \arctan(ax)^2} dx$

3.565.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} dx = & -\frac{1}{ac^3x^3 \arctan(ax)} + \frac{2a}{c^3x \arctan(ax)} \\ & - \frac{a^3x}{c^3 (1+a^2x^2)^2 \arctan(ax)} - \frac{2a^3x}{c^3 (1+a^2x^2) \arctan(ax)} \\ & + \frac{5a^2 \operatorname{CosIntegral}(2\arctan(ax))}{2c^3} \\ & + \frac{a^2 \operatorname{CosIntegral}(4\arctan(ax))}{2c^3} \\ & - \frac{3\operatorname{Int}\left(\frac{1}{x^4 \arctan(ax)}, x\right)}{ac^3} + \frac{2a\operatorname{Int}\left(\frac{1}{x^2 \arctan(ax)}, x\right)}{c^3}\end{aligned}$$

output
```
-1/a/c^3/x^3/arctan(a*x)+2*a/c^3/x/arctan(a*x)-a^3*x/c^3/(a^2*x^2+1)^2/arc
tan(a*x)-2*a^3*x/c^3/(a^2*x^2+1)/arctan(a*x)+5/2*a^2*Ci(2*arctan(a*x))/c^3
+1/2*a^2*Ci(4*arctan(a*x))/c^3-3*Unintegrable(1/x^4/arctan(a*x),x)/a/c^3+2
*a*Unintegrable(1/x^2/arctan(a*x),x)/c^3
```

3.565.2 Mathematica [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

3.565.3 Rubi [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5501, 5461, 5377, 5501, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{c^2x^3(a^2x^2+1)^2 \arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^3x(a^2x^2+1)^3 \arctan(ax)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^3} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^3 \arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1) \arctan(ax)^2}dx - a^2 \int \frac{1}{x(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^3} - \frac{a^2 \left(\int \frac{1}{x(a^2x^2+1)^2 \arctan(ax)^2}dx - a^2 \int \frac{x}{(a^2x^2+1)^3 \arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^3}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx-a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^3}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx-a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\left(a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx+\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5503}$$

3.565. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

$$-\frac{\left(a^2\left(-\left(a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{}{ax\arctan}\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)\right)-a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}\right.}{c^3}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)\right)-a^2\left(-a\int \frac{}{(a^2x^2}\right.}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-a^2\left(-3a\int \frac{}{(a^2x^2+}\right.}{c^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{}{x^2\mathrm{ar}}}{}\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-a^2\left(-3a\right.}{}$$

$$\Big\downarrow \text{2009}$$

3.565. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{}{x^2\text{arc}}}{}\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1)}\right.\right.\right.$$

$$\Big\downarrow \textbf{5505}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a}\right.\right.\right.$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int}{x^2}\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a}\right.\right.\right.$$

$$\Big\downarrow \textbf{3793}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(a}\right.\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a}\right.\right.\right.$$

$$\Big\downarrow \textbf{2009}$$

3.565. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

$$-\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\right.\right.\right.\right.$$
$$c^3$$
$$a^2\left(-a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x\right.\right.$$

$$\Big\downarrow \text{4906}$$

$$-\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\right.\right.\right.\right.$$
$$c^3$$
$$a^2\left(-a^2\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\right.\right.\right.\right.$$
$$c^3$$
$$a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan}{a^2}\right.\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.565.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.565. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.565. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.565.4 Maple [N/A] (verified)

Not integrable

Time = 20.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^2}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.565.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^3x^3\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan
(a*x)^2), x)
```

3.565.6 Sympy [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{\int \frac{1}{a^6x^9\,\mathrm{atan}^2\,(ax)+3a^4x^7\,\mathrm{atan}^2\,(ax)+3a^2x^5\,\mathrm{atan}^2\,(ax)+x^3\,\mathrm{atan}^2\,(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**2 + 3*a**4*x**7*atan(a*x)**2 + 3*a**2*x**
5*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c**3
```

3.565.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.32

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^3x^3\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x)*integrate((7*a^2*x
^2 + 3)/((a^7*c^3*x^10 + 3*a^5*c^3*x^8 + 3*a^3*c^3*x^6 + a*c^3*x^4)*arctan
(a*x)), x) + 1)/((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x))
```

3.565.8 Giac [N/A]

Not integrable

Time = 127.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^3x^3\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.565.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.566 $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

3.566.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^2}\,dx =& -\frac{1}{ac^3x^4\arctan(ax)}+\frac{2a}{c^3x^2\arctan(ax)}\\ &-\frac{a^3}{c^3(1+a^2x^2)^2\arctan(ax)}-\frac{2a^3}{c^3(1+a^2x^2)\arctan(ax)}\\ &-\frac{3a^3\mathrm{Si}(2\arctan(ax))}{c^3}-\frac{a^3\mathrm{Si}(4\arctan(ax))}{2c^3}\\ &-\frac{4\mathrm{Int}\left(\frac{1}{x^5\arctan(ax)},x\right)}{ac^3}+\frac{4a\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)},x\right)}{c^3}\end{aligned}$$

output
```
-1/a/c^3/x^4/arctan(a*x)+2*a/c^3/x^2/arctan(a*x)-a^3/c^3/(a^2*x^2+1)^2/arc
tan(a*x)-2*a^3/c^3/(a^2*x^2+1)/arctan(a*x)-3*a^3*Si(2*arctan(a*x))/c^3-1/2
*a^3*Si(4*arctan(a*x))/c^3-4*Unintegrable(1/x^5/arctan(a*x),x)/a/c^3+4*a*U
nintegrable(1/x^3/arctan(a*x),x)/c^3
```

3.566.2 Mathematica [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

3.566.3 Rubi [N/A]

Not integrable

Time = 2.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5501, 5437, 5461, 5377, 5501, 5437, 5461, 5377, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{c^2x^4(a^2x^2+1)^2\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^3x^2 (a^2x^2+1)^3 \arctan(ax)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2 \int \frac{1}{x^2(a^2x^2+1)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^2}dx - a^2 \int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx - a^2 \int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 5437}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^2}dx-a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx-a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5461$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^3}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx-a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5377$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^3}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx-a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5501$$

$$\frac{-\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx-a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)+\int \frac{1}{x^2(a^2x^2+1)\arctan(ax\cdots}\cdots\right)}{c^3}$$

$$\Big\downarrow \ 5437$$

$$\frac{-\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\cdots}}{c^3}$$
$$\frac{a^2\left(-\left(a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(\cdots}\cdots\right)\right)}{c^3}$$

$$\Big\downarrow \ 5461$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\,\mathrm{arcta}\cdots}}{a}}{c^3}$$
$$\frac{a^2\left(-\left(a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(\cdots}\cdots\right)\right)}{c^3}$$

$$\Big\downarrow \ 5377$$

3.566. $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

$$\frac{-\left(a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\cdots}}{a}\cdots}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan\cdots}\right.\cdots}{c^3}$$

$$\Big\downarrow \; 5505$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int\cdots}{}\cdots}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)-a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{}{a(a^2x\cdots}\right.\cdots}{c^3}$$

$$\Big\downarrow \; 4906$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{}{x^5\,\mathrm{arc}\cdots}}{\cdots}\cdots}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}\right.\cdots}{c^3}$$

$$\Big\downarrow \; 27$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\,\mathrm{arcta}\cdots}}{a}\cdots}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\cdots\right.}{c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\,\mathrm{arcta}\cdots}}{a}\cdots}{c^3}$$

$$\frac{a^2\left(-a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\mathrm{S}\cdots\right.}{}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 3042$$

3.566. $\displaystyle\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\arctan}}{a}}{c^3}$$

$$\frac{a^2\left(-a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}S\right.}{}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\left(a^2\left(-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)\right)-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{}{ax}}{c^3}$$

$$\frac{a^2\left(-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)\right)-a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax\right.}{}\right.\right.}{c^3}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.566.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.566. $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^2}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.566. $\int \frac{1}{x^4 (c+a^2 c x^2)^3 \arctan (a x)^2} \, dx$

3.566.4 Maple [N/A] (verified)

Not integrable

Time = 41.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.566.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^3 \arctan(a x)^2} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{3} x^{4} \arctan \left (a x\right)^{2}} \,d x }$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arcta
n(a*x)^2), x)
```

3.566.6 Sympy [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^3 \arctan(a x)^2} \, dx=\frac {\int \frac {1}{a^{6} x^{10} \operatorname {atan}^{2}{\left (a x \right)} + 3 a^{4} x^{8} \operatorname {atan}^{2}{\left (a x \right)} + 3 a^{2} x^{6} \operatorname {atan}^{2}{\left (a x \right)} + x^{4} \operatorname {atan}^{2}{\left (a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**2 + 3*a**4*x**8*atan(a*x)**2 + 3*a**2*x*
*6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c**3
```

3.566.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x)*integrate(4*(2*a^2
*x^2 + 1)/((a^7*c^3*x^11 + 3*a^5*c^3*x^9 + 3*a^3*c^3*x^7 + a*c^3*x^5)*arct
an(a*x)), x) + 1)/((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x))
```

3.566.8 Giac [N/A]

Not integrable

Time = 129.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.566.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^2 \,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.567 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$

3.567.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

3.567.2 Mathematica [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

3.567.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.567.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.567.4 Maple [N/A] (verified)

Not integrable

Time = 26.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{a^{2} c\,x^{2}+c}}{\arctan \left(a x \right)^{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

3.567.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

3.567.6 Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

3.567.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

3.567.8 Giac [N/A]

Not integrable

Time = 61.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.567.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2, x)
```

3.568 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$

3.568.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

3.568.2 Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2, x]
```

3.568.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.568.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.568.4 Maple [N/A] (verified)

Not integrable

Time = 19.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

3.568.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

3.568.6 Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

3.568.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

3.568.8 Giac [N/A]

Not integrable

Time = 59.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.568.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^2, x)
```

3.569 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx$

3.569.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx=\mathrm{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

3.569.2 Mathematica [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2), x]
```

3.569.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5560\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^2}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.569.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.569.4 Maple [N/A] (verified)

Not integrable

Time = 60.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a^{2} c \,x^{2}+c}}{x \arctan \left(a x \right)^{2}}d x$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

3.569.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

3.569.6 Sympy [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**2), x)
```

3.569.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

3.569.8 Giac [N/A]

Not integrable

Time = 62.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.569.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^2), x)
```

3.570 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx$

3.570.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.570.2 Mathematica [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

3.570.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.570.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.570.4 Maple [N/A] (verified)

Not integrable

Time = 34.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.570.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

3.570.6 Sympy [N/A]

Not integrable

Time = 5.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

3.570.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^2, x)
```

3.570.8 Giac [N/A]

Not integrable

Time = 76.91 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.570.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2, x)
```

3.571 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx$

3.571.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.571.2 Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2, x]
```

3.571.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.571.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.571.4 Maple [N/A] (verified)

Not integrable

Time = 16.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)^{2}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.571.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

3.571.6 Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

3.571.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

3.571.8 Giac [N/A]

Not integrable

Time = 73.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.571.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^2, x)
```

3.572 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx$

3.572.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

3.572.2 Mathematica [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2), x]
```

3.572.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.572.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.572.4 Maple [N/A] (verified)

Not integrable

Time = 122.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)^2}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

3.572.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

3.572.6 Sympy [N/A]

Not integrable

Time = 7.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\,\mathrm{atan}^2(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**2), x)
```

3.572.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

3.572.8 Giac [N/A]

Not integrable

Time = 77.60 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.572.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^2), x)
```

3.573 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$

3.573.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.573.2 Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

3.573.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.573.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.573.4 Maple [N/A] (verified)

Not integrable

Time = 62.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.573.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(
a*x)^2, x)
```

3.573.6 Sympy [N/A]

Not integrable

Time = 29.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**2, x)
```

3.573.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^2, x)
```

3.573.8 Giac [N/A]

Not integrable

Time = 89.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.573.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2, x)
```

3.574 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$

3.574.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.574.2 Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2, x]
```

3.574.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.574.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.574.4 Maple [N/A] (verified)

Not integrable

Time = 37.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.574.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*
x)^2, x)
```

3.574.6 Sympy [N/A]

Not integrable

Time = 14.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**2, x)
```

3.574.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^2, x)
```

3.574.8 Giac [N/A]

Not integrable

Time = 85.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.574.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^2, x)
```

3.575 $\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx$

3.575.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

3.575.2 Mathematica [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2), x]
```

3.575.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.575.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.575.4 Maple [N/A] (verified)

Not integrable

Time = 118.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)^2}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

3.575.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan
(a*x)^2), x)
```

3.575.6 Sympy [N/A]

Not integrable

Time = 20.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\,\mathrm{atan}^2(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**2), x)
```

3.575.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^2), x)
```

3.575.8 Giac [N/A]

Not integrable

Time = 90.58 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.575.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^2), x)
```

3.576 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$

3.576.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.576.2 Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

3.576.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.576.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.576.4 Maple [N/A] (verified)

Not integrable

Time = 3.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\arctan\left(ax\right)^{2}\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.576.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.576.6 Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

3.576.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.576.8 Giac [N/A]

Not integrable

Time = 69.46 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.576.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.577 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$

3.577.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)$$

output
```
Unintegrable(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.577.2 Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

3.577.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.577.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.577.4 Maple [N/A] (verified)

Not integrable

Time = 6.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\arctan\left(ax\right)^{2} \sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.577.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.577.6 Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{1}{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(1/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

3.577.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.577.8 Giac [N/A]

Not integrable

Time = 66.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.577.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.578 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$

3.578.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = -\frac{\sqrt{c+a^2cx^2}}{acx\arctan(ax)} - \frac{\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{a}$$

output
```
-(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)-Unintegrable(1/x^2/arctan(a*x)/(a^2
*c*x^2+c)^(1/2),x)/a
```

3.578.2 Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

3.578.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5477, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2 \sqrt{a^2cx^2+c}} \, dx$$
$$\Big\downarrow \text{5477}$$
$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}$$
$$\Big\downarrow \text{5560}$$
$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.578.3.1 Defintions of rubi rules used

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.578.4 Maple [N/A] (verified)

Not integrable

Time = 7.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \arctan (ax)^2 \sqrt{a^2 c\, x^2 + c}}dx$$

input
```
int(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.578.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{c + a^2cx^2}\arctan(ax)^2}\, dx = \int \frac{1}{\sqrt{a^2cx^2 + c}x \arctan (ax)^2}\, dx$$

input
```
integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)
```

3.578.6 Sympy [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{x\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^2\,(ax)}\,dx$$

input
```
integrate(1/x/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

3.578.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan\,(ax)^2}\,dx$$

input
```
integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

3.578.8 Giac [N/A]

Not integrable

Time = 65.52 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan\,(ax)^2}\,dx$$

input
```
integrate(1/x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.578.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.579 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.579.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{x}{a^3c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{a^4c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^2c}$$

output
```
x/a^3/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/
a^4/c/(a^2*c*x^2+c)^(1/2)+Unintegrable(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)
,x)/a^2/c
```

3.579.2 Mathematica [N/A]

Not integrable

Time = 9.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

3.579.3 Rubi [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5499, 5477, 5440, 5439, 3042, 3783, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.579.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.579. $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.579.4 Maple [N/A] (verified)

Not integrable

Time = 3.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.579.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^2), x)
```

3.579.6 Sympy [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.579.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.579.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.579.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.580 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$

3.580.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \frac{1}{a^3c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{a^3c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^2c}$$

output
```
1/a^3/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/
a^3/c/(a^2*c*x^2+c)^(1/2)+Unintegrable(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)
,x)/a^2/c
```

3.580.2 Mathematica [N/A]

Not integrable

Time = 6.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

3.580.3 Rubi [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5499, 5437, 5506, 5505, 3042, 3780, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5506}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.580.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.580. $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.580.4 Maple [N/A] (verified)

Not integrable

Time = 2.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^2}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.580.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^2), x)
```

3.580.6 Sympy [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.580.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.580.8 Giac [N/A]

Not integrable

Time = 60.96 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.580.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.581 $\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.581.1 Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = -\frac{x}{ac\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
-x/a/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a
^2/c/(a^2*c*x^2+c)^(1/2)
```

3.581.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{-ax+\sqrt{1+a^2x^2}\arctan(ax)\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{c+a^2cx^2}\arctan(ax)}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
(-(a*x) + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*CosIntegral[ArcTan[a*x]])/(a^2*c*S
qrt[c + a^2*c*x^2]*ArcTan[a*x])
```

3.581.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5477, 5440, 5439, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
-(x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegra
l[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

3.581.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.581.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\left(\arctan(ax)\operatorname{Ei}_1(-i\arctan(ax))a^2x^2+\arctan(ax)\operatorname{Ei}_1(i\arctan(ax))a^2x^2+2\sqrt{a^2x^2+1}\,ax+\operatorname{Ei}_1(-i\arctan(ax))\arctan(ax)+\operatorname{Ei}_1(i\,a\right.}{2(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)a^2c^2}$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+arctan(a*x)*Ei(1,I*arctan(a
*x))*a^2*x^2+2*(a^2*x^2+1)^(1/2)*a*x+Ei(1,-I*arctan(a*x))*arctan(a*x)+Ei(1
,I*arctan(a*x))*arctan(a*x))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a
rctan(a*x)/a^2/c^2
```

3.581.5 Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan
(a*x)^2), x)
```

3.581.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.581.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.581.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.581.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x}{\mathrm{atan}\,(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.582 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.582.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = -\frac{1}{ac\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{ac\sqrt{c+a^2cx^2}}$$

output
```
-1/a/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a
/c/(a^2*c*x^2+c)^(1/2)
```

3.582.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = -\frac{1+\sqrt{1+a^2x^2}\arctan(ax)\mathrm{Si}(\arctan(ax))}{ac\sqrt{c+a^2cx^2}\arctan(ax)}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
-((1 + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*SinIntegral[ArcTan[a*x]])/(a*c*Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]))
```

3.582.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5437, 5506, 5505, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegra
l[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

3.582.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.582.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.79 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.86

method	result
default	$-\frac{i\left(\arctan(ax)\operatorname{Ei}_1(-i\arctan(ax))a^2x^2-\arctan(ax)\operatorname{Ei}_1(i\arctan(ax))a^2x^2+\operatorname{Ei}_1(-i\arctan(ax))\arctan(ax)-\operatorname{Ei}_1(i\arctan(ax))\operatorname{arct}\right.}{2(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)a\,c^2}$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2-arctan(a*x)*Ei(1,I*arctan
(a*x))*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(a*x)-Ei(1,I*arctan(a*x))*arctan
(a*x)-2*I*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a
rctan(a*x)/a/c^2
```

3.582.5 Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a
*x)^2), x)
```

3.582.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.582.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.582.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.582.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.583 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.583.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \frac{ax}{c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^2x\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{c\sqrt{c+a^2cx^2}} - \frac{\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{ac}$$

output
```
a*x/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/
(a^2*c*x^2+c)^(1/2)-(a^2*c*x^2+c)^(1/2)/a/c^2/x/arctan(a*x)-Unintegrable(1
/x^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)/a/c
```

3.583.2 Mathematica [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

3.583.3 Rubi [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 5477, 5440, 5439, 3042, 3783, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5440}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5439}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-$$
$$a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3783}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-$$
$$a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5560}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-$$
$$a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.583.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.583. $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.583.4 Maple [N/A] (verified)

Not integrable

Time = 3.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{2}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.583.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan
(a*x)^2), x)
```

3.583.6 Sympy [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.583.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2), x)
```

3.583.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.583.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.584 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.584.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \frac{a}{c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{a\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c}$$

output
```
a/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+a*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/
(a^2*c*x^2+c)^(1/2)+Unintegrable(1/x^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x
)/c
```

3.584.2 Mathematica [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

3.584.3 Rubi [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5437, 5506, 5505, 3042, 3780, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5506}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.584.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.584.4 Maple [N/A] (verified)

Not integrable

Time = 3.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.584.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arct
an(a*x)^2), x)
```

3.584.6 Sympy [N/A]

Not integrable

Time = 6.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^2 (c (a^2x^2 + 1))^{\frac{3}{2}} \operatorname{atan}^2{(ax)}} \, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.584.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2), x)
```

3.584.8 Giac [N/A]

Not integrable

Time = 62.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.584.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^2 \,\mathrm{atan}\left(a\,x\right)^2 \left(c\,a^2\,x^2+c\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.585 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.585.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = -\frac{a^3x}{c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{a\sqrt{c+a^2cx^2}}{c^2x\arctan(ax)} + \frac{a^2\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c} + \frac{a\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{c}$$

output
```
-a^3*x/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+a^2*Ci(arctan(a*x))*(a^2*x^2+1)^(
1/2)/c/(a^2*c*x^2+c)^(1/2)+a*(a^2*c*x^2+c)^(1/2)/c^2/x/arctan(a*x)+Uninteg
rable(1/x^3/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c+a*Unintegrable(1/x^2/ar
ctan(a*x)/(a^2*c*x^2+c)^(1/2),x)/c
```

3.585.2 Mathematica [N/A]

Not integrable

Time = 4.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

3.585.3 Rubi [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5501, 5477, 5440, 5439, 3042, 3783, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx\right)$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5440}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-$$
$$a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-$$
$$a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2\right.\right.$$

$$\Big\downarrow \text{3783}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-$$
$$a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+\right.\right.$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-$$
$$a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.585. $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.585.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.585. $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.585.4 Maple [N/A] (verified)

Not integrable

Time = 20.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{2}}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.585.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} x^{3} \arctan \left(a x\right)^{2}} \,d x$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arct
an(a*x)^2), x)
```

3.585.6 Sympy [N/A]

Not integrable

Time = 8.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.585.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2), x)
```

3.585.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.585.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^{3/2}} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.586 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.586.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4\,(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = -\frac{a^3}{c\sqrt{c+a^2cx^2}\arctan(ax)}-\frac{a^3\sqrt{1+a^2x^2}\mathrm{Si}(\arctan(ax))}{c\sqrt{c+a^2cx^2}}+\frac{\mathrm{Int}\left(\frac{1}{x^4\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c}-\frac{a^2\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c}$$

output
```
-a^3/c/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-a^3*Si(arctan(a*x))*(a^2*x^2+1)^(1/
2)/c/(a^2*c*x^2+c)^(1/2)+Unintegrable(1/x^4/arctan(a*x)^2/(a^2*c*x^2+c)^(1
/2),x)/c-a^2*Unintegrable(1/x^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c
```

3.586.2 Mathematica [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4\,(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{x^4\,(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

3.586.3 Rubi [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5501, 5437, 5506, 5505, 3042, 3780, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{x^2 (a^2cx^2+c)^{3/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5506}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5505}$$

$$a^2\left(\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \ 3042$$

$$a^2\left(\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \ 3780$$

$$a^2\left(\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \ 5560$$

$$a^2\left(\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.586.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.586. $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.586. $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.586.4 Maple [N/A] (verified)

Not integrable

Time = 22.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.586.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{\frac {3}{2}} x^{4} \arctan \left (a x\right)^{2}} \,d x }$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arct
an(a*x)^2), x)
```

3.586.6 Sympy [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx=\int \frac {1}{x^{4} \left (c \left (a^{2} x^{2} + 1\right)\right)^{\frac {3}{2}} \operatorname {atan}^{2}{\left (a x \right)}}\, dx$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**4*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.586.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}} x^4 \arctan\left(ax\right)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^2), x)
```

3.586.8 Giac [N/A]

Not integrable

Time = 65.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}} x^4 \arctan\left(ax\right)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.586.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.587 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.587.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{7\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4a^6c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4a^6c^2\sqrt{c+a^2cx^2}} + \frac{\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^4c^2}$$

output
```
x^3/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+x/a^5/c^2/arctan(a*x)/(a^2*c*x^2
+c)^(1/2)-7/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2
)+3/4*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a^2*c*x^2+c)^(1/2)+Unin
tegrable(x/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/a^4/c^2
```

3.587.2 Mathematica [N/A]

Not integrable

Time = 11.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

3.587.3 Rubi [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5499, 5477, 5499, 5477, 5440, 5439, 3042, 3783, 5506, 5505, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x^3}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5439$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 3783$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5506$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5505$$

3.587. $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 4906$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5560$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

input
```
Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.587.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.587. $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.587. $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.587.4 Maple [N/A] (verified)

Not integrable

Time = 18.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^2}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.587.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

3.587.6 Sympy [N/A]

Not integrable

Time = 7.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.587.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.587.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.587.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^5/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^5/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.588 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.588.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx =& -\frac{1}{a^5c\,(c+a^2cx^2)^{3/2}\arctan(ax)} \\ &+ \frac{2}{a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{5\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4a^5c^2\sqrt{c+a^2cx^2}} \\ &- \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4a^5c^2\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^4c^2}\end{aligned}$$

output
```
-1/a^5/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+2/a^5/c^2/arctan(a*x)/(a^2*c*x^2+
c)^(1/2)+5/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)
-3/4*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+Unint
egrable(1/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/a^4/c^2
```

3.588.2 Mathematica [N/A]

Not integrable

Time = 11.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

3.588.3 Rubi [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5499, 5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}} dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2c} - \frac{\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}}{a^2c} - \frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2c} - \frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{a^2}}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \text{5506}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \ 5505$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)}}{a^2}}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \ 3780$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \ 4906$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \ 2009$$

3.588. $\displaystyle\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.588.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.588. $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.588. $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.588.4 Maple [N/A] (verified)

Not integrable

Time = 17.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^2}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.588.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{\left(a^2cx^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

3.588.6 Sympy [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.588.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.588.8 Giac [N/A]

Not integrable

Time = 167.80 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.588.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^4/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.589 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx$

3.589.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = -\frac{x^3}{ac\left(c+a^2cx^2\right)^{3/2}\arctan(ax)} + \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
-x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+3/4*Ci(arctan(a*x))*(a^2*x^2+1)^(
1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-3/4*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a
^4/c^2/(a^2*c*x^2+c)^(1/2)
```

3.589.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \frac{-\frac{4a^3cx^3}{(1+a^2x^2)\arctan(ax)} + 3c\sqrt{1+a^2x^2}(\text{CosIntegral}(\arctan(ax)) - \text{CosIntegra}}{4a^4c^3\sqrt{c+a^2cx^2}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
((-4*a^3*c*x^3)/((1 + a^2*x^2)*ArcTan[a*x]) + 3*c*Sqrt[1 + a^2*x^2]*(CosIn
tegral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x]]))/(4*a^4*c^3*Sqrt[c + a^2
*c*x^2])
```

3.589.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5477, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2 (a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{3\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
-(x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (3*Sqrt[1 + a^2*x^2]*(Cos
Integral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]]/4))/(a^4*c^2*Sqrt[c +
 a^2*c*x^2])
```

3.589.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.589.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.63 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.62

method	result
default	$-\frac{\left(3\arctan(ax)\,\mathrm{Ei}_1(-i\arctan(ax))a^4x^4+3\arctan(ax)\,\mathrm{Ei}_1(i\arctan(ax))a^4x^4-3\arctan(ax)\,\mathrm{Ei}_1(3i\arctan(ax))a^4x^4-3\arctan(ax)\,\mathrm{E}\right.}{}$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(3*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^4*x^4+3*arctan(a*x)*Ei(1,I*arct
an(a*x))*a^4*x^4-3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4-3*arctan(a*x)
*Ei(1,-3*I*arctan(a*x))*a^4*x^4+8*(a^2*x^2+1)^(1/2)*a^3*x^3+6*arctan(a*x)*
Ei(1,-I*arctan(a*x))*a^2*x^2+6*arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2-6*a
rctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2-6*arctan(a*x)*Ei(1,-3*I*arctan(a*
x))*a^2*x^2+3*Ei(1,-I*arctan(a*x))*arctan(a*x)+3*Ei(1,I*arctan(a*x))*arcta
n(a*x)-3*Ei(1,3*I*arctan(a*x))*arctan(a*x)-3*Ei(1,-3*I*arctan(a*x))*arctan
(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a^4/c^3/(a^
4*x^4+2*a^2*x^2+1)
```

3.589.5 Fricas [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

3.589.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.589.7 Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.589.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.589.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.590 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.590.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{1}{a^3c(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-1/a^3/c^2/arctan(a*x)/(a^2*c*x^2+c
)^(1/2)-1/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+
3/4*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

3.590.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{4a^2x^2 + (1+a^2x^2)^{3/2}\arctan(ax)\mathrm{Si}(\arctan(ax)) - 3(1+a^2x^2)^{3/2}\arctan(ax)\mathrm{Si}(3\arctan(ax))}{4a^3c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax)}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
-1/4*(4*a^2*x^2 + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[ArcTan[a*x]]
 - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[3*ArcTan[a*x]])/(a^3*c^2*
(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

3.590.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \; 5499$$

$$\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \; 5437$$

$$\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5506$$

$$\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

$$\Big\downarrow \color{blue}{3780}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\operatorname{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

$$\Big\downarrow \color{blue}{4906}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\operatorname{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\operatorname{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\operatorname{Si}(\arctan(ax))+\frac{1}{4}\operatorname{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
(-(1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegr
al[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2]))/(a^2*c) - (-(1/(a*c*(c + a^2*c
*x^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]
/4 + SinIntegral[3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/a^2
```

3.590. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.590.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.590. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.590.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.75 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.18

method	result
default	$-\frac{i\left(\arctan(ax)\operatorname{Ei}_1(-i\arctan(ax))a^4x^4-3\arctan(ax)\operatorname{Ei}_1(-3i\arctan(ax))a^4x^4-\arctan(ax)\operatorname{Ei}_1(i\arctan(ax))a^4x^4+3\arctan(ax)\operatorname{E}}{}$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*(arctan(a*x)*Ei(1,-I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*Ei(1,-3*I*a
rctan(a*x))*a^4*x^4-arctan(a*x)*Ei(1,I*arctan(a*x))*a^4*x^4+3*arctan(a*x)*
Ei(1,3*I*arctan(a*x))*a^4*x^4+2*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2-6
*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2-2*arctan(a*x)*Ei(1,I*arctan(a*
x))*a^2*x^2+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2-8*I*(a^2*x^2+1)^(1
/2)*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(a*x)-3*Ei(1,-3*I*arctan(a*x))*arct
an(a*x)-Ei(1,I*arctan(a*x))*arctan(a*x)+3*Ei(1,3*I*arctan(a*x))*arctan(a*x
))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a^3/c^3/(a^4*x^
4+2*a^2*x^2+1)
```

3.590.5 Fricas [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

3.590.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.590.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.590.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.590.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.591 $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2} \, dx$

3.591.1 Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2} \, dx = -\frac{x}{ac\left(c+a^2cx^2\right)^{3/2} \arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
-x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+1/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/
2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+3/4*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2
/c^2/(a^2*c*x^2+c)^(1/2)
```

3.591.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2} \, dx = \frac{-4ax+\left(1+a^2x^2\right)^{3/2}\arctan(ax)\,\mathrm{CosIntegral}(\arctan(ax))+3(1+a^2x^2}{4a^2c^2\left(1+a^2x^2\right)\sqrt{c+a^2cx^2}\,\arctan}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
(-4*a*x + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[ArcTan[a*x]] + 3*(1
+ a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[3*ArcTan[a*x]])/(4*a^2*c^2*(1 + a
^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

3.591.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 (a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx}{a} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\operatorname{CosIntegral}(\arctan(ax))-\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

3.591. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

output
```
-(x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*Sqrt[1 + a^2*x^2]*(CosIn
tegral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a
^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*((3*CosIntegral[ArcTan[a*x]])/4 + CosInteg
ral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2])
```

3.591.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.591. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.591.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.59

method	result
default	$-\frac{\left(3\arctan(ax)\operatorname{Ei}_1(3i\arctan(ax))a^4x^4+\arctan(ax)\operatorname{Ei}_1(i\arctan(ax))a^4x^4+3\arctan(ax)\operatorname{Ei}_1(-3i\arctan(ax))a^4x^4+\arctan(ax)\operatorname{Ei}_1\right.}{}$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4+arctan(a*x)*Ei(1,I*arcta
n(a*x))*a^4*x^4+3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+arctan(a*x)*E
i(1,-I*arctan(a*x))*a^4*x^4+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2+2*
arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2+6*arctan(a*x)*Ei(1,-3*I*arctan(a*x
))*a^2*x^2+2*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+8*(a^2*x^2+1)^(1/2)*
a*x+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+Ei(1,I*arctan(a*x))*arctan(a*x)+3*
Ei(1,-3*I*arctan(a*x))*arctan(a*x)+Ei(1,-I*arctan(a*x))*arctan(a*x))/(a^2*
x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a^2/c^3/(a^4*x^4+2*a^2*
x^2+1)
```

3.591.5 Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x
^2 + c^3)*arctan(a*x)^2), x)
```

3.591.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.591.7 Maxima [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.591.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.592 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.592.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}}$$

output
```
-1/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-3/4*Si(arctan(a*x))*(a^2*x^2+1)^(1/
2)/a/c^2/(a^2*c*x^2+c)^(1/2)-3/4*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2
/(a^2*c*x^2+c)^(1/2)
```

3.592.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{-\frac{4}{\arctan(ax)} - 3(1+a^2x^2)^{3/2}\left(\mathrm{Si}(\arctan(ax)) + \mathrm{Si}(3\arctan(ax))\right)}{4ac\,(c+a^2cx^2)^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
(-4/ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*(SinIntegral[ArcTan[a*x]] + SinInt
egral[3*ArcTan[a*x]]))/(4*a*c*(c + a^2*c*x^2)^(3/2))
```

3.592.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5437, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIn
tegral[ArcTan[a*x]]/4 + SinIntegral[3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2
*c*x^2])
```

3.592.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.592.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.79 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.65

method	result
default	$i\left(3\arctan(ax)\operatorname{Ei}_1(i\arctan(ax))a^4x^4+3\arctan(ax)\operatorname{Ei}_1(3i\arctan(ax))a^4x^4-3\arctan(ax)\operatorname{Ei}_1(-i\arctan(ax))a^4x^4-3\arctan(ax)\operatorname{Ei}\right.$

3.592. $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*I*(3*arctan(a*x)*Ei(1,I*arctan(a*x))*a^4*x^4+3*arctan(a*x)*Ei(1,3*I*ar
ctan(a*x))*a^4*x^4-3*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^4*x^4-3*arctan(a*x
)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+6*arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2
+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2-6*arctan(a*x)*Ei(1,-I*arctan(
a*x))*a^2*x^2-6*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2+3*Ei(1,I*arctan
(a*x))*arctan(a*x)+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)-3*Ei(1,-I*arctan(a*
x))*arctan(a*x)-3*Ei(1,-3*I*arctan(a*x))*arctan(a*x)+8*I*(a^2*x^2+1)^(1/2)
)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)/a/c^3/(a^4*x^4+2
*a^2*x^2+1)
```

3.592.5 Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3)*arctan(a*x)^2), x)
```

3.592.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{(c\left(a^2x^2+1\right))^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.592.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.592.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.593 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.593.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx &= \frac{ax}{c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\\ &+\frac{ax}{c^2\sqrt{c+a^2cx^2}\arctan(ax)}-\frac{\sqrt{c+a^2cx^2}}{ac^3x\arctan(ax)}-\frac{5\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}}\\ &-\frac{3\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}}-\frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{ac^2}\end{aligned}$$

output
```
a*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+a*x/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1
/2)-5/4*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-3/4*Ci(3
*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-(a^2*c*x^2+c)^(1/2
)/a/c^3/x/arctan(a*x)-Unintegrable(1/x^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2),x
)/a/c^2
```

3.593.2 Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

3.593.3 Rubi [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5501, 5477, 5440, 5439, 3042, 3783, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5439}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{3783}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5503}$$

3.593. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\left(\frac{\int\frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx}{a}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5440$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5439$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\Bigg(\frac{\sqrt{a^2x^2+1}\int\frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+\right.}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\Bigg(\frac{\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3}}$$

$$\Big\downarrow \; 3793$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\Bigg(\frac{\sqrt{a^2x^2+1}\int\left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{}{ac\,\text{ar}}$$

$$\Big\downarrow \; 2009$$

3.593. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{5506}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\,a\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{4906}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))\right.}{a^2c^2\sqrt{a^2cx^2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan\right.}{a^2c^2\sqrt{}}\right.$$

$$\Big\downarrow \text{5560}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan\right.}{a^2c^2\sqrt{}}\right.$$

3.593. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.593.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.593. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.593. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.593.4 Maple [N/A] (verified)

Not integrable

Time = 4.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{2}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.593.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^{2} c x^{2}+c\right)^{\frac{5}{2}} x \arctan\left(a x\right)^{2}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
 + c^3*x)*arctan(a*x)^2), x)
```

3.593.6 Sympy [N/A]

Not integrable

Time = 14.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.593.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2), x)
```

3.593.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.593.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.594 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.594.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{a}{c(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{7a\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{3a\sqrt{1+a^2x^2}\operatorname{Si}(3\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c^2}$$

output
```
a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+a/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2)+
7/4*a*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+3/4*a*Si(3
*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+Unintegrable(1/x^2
/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c^2
```

3.594.2 Mathematica [N/A]

Not integrable

Time = 5.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

3.594.3 Rubi [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5437, 5501, 5437, 5506, 5505, 3042, 3780, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{c}-a^2\left(-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{c}-a^2\left(-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5506$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5505$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 3780$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 4906$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 2009$$

3.594. $\displaystyle \int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax)) + \frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5560$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax)) + \frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.594.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.594. $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.594. $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.594.4 Maple [N/A] (verified)

Not integrable

Time = 4.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.594.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(a x)^2} \, dx=\int \frac{1}{\left(a^{2} c x^{2} + c\right)^{\frac{5}{2}} x^{2} \arctan \left(a x\right)^{2}} \,d x$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
 + c^3*x^2)*arctan(a*x)^2), x)
```

3.594.6 Sympy [N/A]

Not integrable

Time = 18.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(a x)^2} \, dx=\int \frac{1}{x^{2} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}^{2}{\left(a x \right)}}\, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.594.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^2} dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{5}{2}} x^2 \arctan{(ax)}^2} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2), x)
```

3.594.8 Giac [N/A]

Not integrable

Time = 123.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^2} dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{5}{2}} x^2 \arctan{(ax)}^2} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.594.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^2} dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^2 \,(c\,a^2\,x^2 + c)^{5/2}} dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.595 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.595.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{a^3x}{c(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{2a^3x}{c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{2a\sqrt{c+a^2cx^2}}{c^3x\arctan(ax)} + \frac{9a^2\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{3a^2\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{\text{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c^2} + \frac{2a\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{c^2}$$

output
```
-a^3*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-2*a^3*x/c^2/arctan(a*x)/(a^2*c*x^
2+c)^(1/2)+9/4*a^2*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/
2)+3/4*a^2*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+2*a
*(a^2*c*x^2+c)^(1/2)/c^3/x/arctan(a*x)+Unintegrable(1/x^3/arctan(a*x)^2/(a
^2*c*x^2+c)^(1/2),x)/c^2+2*a*Unintegrable(1/x^2/arctan(a*x)/(a^2*c*x^2+c)^
(1/2),x)/c^2
```

3.595.2 Mathematica [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2}\,dx = \int \frac{1}{x^3 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

3.595.3 Rubi [N/A]

Not integrable

Time = 8.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 5501, 5501, 5477, 5440, 5439, 3042, 3783, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{1}{x^3 \left(a^2cx^2+c\right)^{3/2} \arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{x \left(a^2cx^2+c\right)^{5/2} \arctan(ax)^2}dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\frac{\int \frac{1}{x^3 \sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{x \left(a^2cx^2+c\right)^{3/2} \arctan(ax)^2}dx}{c} - a^2 \left(\frac{\int \frac{1}{x \left(a^2cx^2+c\right)^{3/2} \arctan(ax)^2}dx}{c} - a^2 \int \frac{x}{\left(a^2cx^2+c\right)^{5/2} \arctan(ax)^2}dx\right)$$

$$\Big\downarrow \; 5501$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx\right)}{c} - a^2\left(\frac{\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx\right)$$

$$\Big\downarrow \; 5477$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\int \frac{}{(a^2cx^2+c)}\right.$$

$$\Big\downarrow \; 5440$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}} - \frac{}{ac\arctan(}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\int \frac{}{(a^2c}\right.$$

$$\Big\downarrow \; 5439$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{}{ac\,\mathrm{ar}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\int\right.$$

3.595. $\displaystyle\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\downarrow 3042$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{}{ac\,\mathrm{arc}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int\right.$$

$$\downarrow 3783$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int\frac{}{(a^2cx^2}\right.$$

$$\downarrow 5503$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\int\frac{}{(a^2cx^2+}}{}\right.\right.$$

$$\downarrow 5440$$

3.595. $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow 5439$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow 3042$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow 3793$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+\cdots}}{\cdots}\right.\right.$$

3.595. $\displaystyle\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-2a\int \frac{\ldots}{(\ldots}\right.\right.$$

$$\Big\downarrow \; 5506$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{2a\sqrt{a^2\ldots}}{\ldots}\right.\right.$$

$$\Big\downarrow \; 5505$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{2\sqrt{a^2x\ldots}}{\ldots}\right.\right.$$

$$\Big\downarrow \; 4906$$

3.595. $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{\right.}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{2\sqrt{a^2x}}{}\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{\right.}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{2\sqrt{a^2x}}{}\right.\right.$$

$\Big\downarrow$ 5560

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{\right.}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{2\sqrt{a^2x}}{}\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.595. $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.595.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.595. $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.595. $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.595.4 Maple [N/A] (verified)

Not integrable

Time = 21.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.595.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{\frac {5}{2}} x^{3} \arctan \left (a x\right)^{2}} \,d x }$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5
 + c^3*x^3)*arctan(a*x)^2), x)
```

3.595.6 Sympy [N/A]

Not integrable

Time = 24.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac {1}{x^{3} \left (c \left (a^{2} x^{2} + 1\right)\right)^{\frac {5}{2}} \operatorname {atan}^{2}{\left (a x \right)}}\, dx$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.595.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^3 \arctan{(ax)}^2} dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2), x)
```

3.595.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.595.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} dx = \int \frac{1}{x^3 \operatorname{atan}{(a\,x)}^2 \,{\left(c\,a^2\,x^2+c\right)}^{5/2}} dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.596 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.596.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{a^3}{c(c+a^2cx^2)^{3/2}\arctan(ax)}-\frac{2a^3}{c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{11a^3\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} - \frac{3a^3\sqrt{1+a^2x^2}\operatorname{Si}(3\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{\operatorname{Int}\left(\frac{1}{x^4\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c^2} - \frac{2a^2\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c^2}$$

output
```
-a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-2*a^3/c^2/arctan(a*x)/(a^2*c*x^2+c)
^(1/2)-11/4*a^3*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-
3/4*a^3*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+Uninte
grable(1/x^4/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c^2-2*a^2*Unintegrable(1
/x^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/c^2
```

3.596.2 Mathematica [N/A]

Not integrable

Time = 7.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

3.596.3 Rubi [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 5501, 5437, 5501, 5437, 5506, 5505, 3042, 3780, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4 (a^2cx^2+c)^{3/2} \arctan(ax)^2} dx}{c} - a^2 \int \frac{1}{x^2 (a^2cx^2+c)^{5/2} \arctan(ax)^2} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^4 \sqrt{a^2cx^2+c} \arctan(ax)^2} dx}{c} - a^2 \int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \arctan(ax)^2} dx}{c} - a^2 \left(\frac{\int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \arctan(ax)^2} dx}{c} - a^2 \int \frac{1}{(a^2cx^2+c)^{5/2} \arctan(ax)^2} dx \right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} -$$
$$a^2\left(\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2\left(-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}\right)\right)$$

$$\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx\right)}{c} -$$
$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2\left(-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{}{ac\arctan}\right.\right.$$

$$\downarrow \text{5437}$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c} -$$
$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-3a\int \frac{x}{(a^2cx^2+c)^{5/}}\right.\right.$$

$$\downarrow \text{5506}$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+}}\right.\right.}{c}$$
$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{3a\sqrt{a^2x^2+1}}{c}\right.\right.$$

$$\downarrow \text{5505}$$

3.596. $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2c\cdots}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c\cdots}}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \; 3780$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c}-$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\cdots}}{ac^2\sqrt{a^2cx\cdots}}\right.\right.$$

$$\Big\downarrow \; 4906$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c}-$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{a\cdots}{4\sqrt{a^2x^2+1}\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2009$$

3.596. $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c} - a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(\right.}{ac^2\sqrt{a^2}}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{5560}$$

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c} - a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(\right.}{ac^2\sqrt{a^2}}\right.\right.$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.596.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.596. $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.596. $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.596.4 Maple [N/A] (verified)

Not integrable

Time = 33.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.596.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^{5/2} \arctan(a x)^2} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{\frac {5}{2}} x^{4} \arctan \left (a x\right)^{2}} \,d x }$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^
6 + c^3*x^4)*arctan(a*x)^2), x)
```

3.596.6 Sympy [N/A]

Not integrable

Time = 31.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^{5/2} \arctan(a x)^2} \, dx=\int \frac {1}{x^{4} \left (c \left (a^{2} x^{2} + 1\right)\right)^{\frac {5}{2}} \operatorname {atan}^{2}{\left (a x \right)}}\, dx$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**4*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

3.596.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^4 \arctan{(ax)}^2}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^2), x)
```

3.596.8 Giac [N/A]

Not integrable

Time = 126.88 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^4 \arctan{(ax)}^2}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.596.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2}\,dx = \int \frac{1}{x^4 \operatorname{atan}{(a\,x)}^2 \left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.597 $\int \frac{\sqrt{fx}}{\left(d+c^2dx^2\right)^2(a+b\arctan(cx))^2}\,dx$

3.597.1 Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\arctan(cx))^2}\,dx=\operatorname{Int}\left(\frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\arctan(cx))^2},x\right)$$

output
```
Unintegrable((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

3.597.2 Mathematica [N/A]

Not integrable

Time = 55.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\arctan(cx))^2}\,dx=\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\arctan(cx))^2}\,dx$$

input
```
Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
```

output
```
Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2), x]
```

3.597.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{fx}}{(c^2dx^2+d)^2\,(a+b\arctan(cx))^2}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{fx}}{(c^2dx^2+d)^2\,(a+b\arctan(cx))^2}dx$$

input
```
Int[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
```

output
```
$Aborted
```

3.597.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.597.4 Maple [N/A] (verified)

Not integrable

Time = 5.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{fx}}{\left(c^{2} d\,x^{2}+d\right)^{2} \left(a +b \arctan\left(c x \right)\right)^{2}}d x$$

input
```
int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

output
```
int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

3.597.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.17

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2} \, dx = \int \frac{\sqrt{fx}}{(c^2dx^2+d)^2(b\arctan{(cx)}+a)^2} \, dx$$

input
```
integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="fr
icas")
```

output
```
integral(sqrt(f*x)/(a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c
^4*d^2*x^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^4*d^2*x
^4 + 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x)), x)
```

3.597.6 Sympy [N/A]

Not integrable

Time = 108.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2} \, dx = \frac{\int \frac{\sqrt{fx}}{a^2c^4x^4+2a^2c^2x^2+a^2+2abc^4x^4\operatorname{atan}{(cx)}+4abc^2x^2\operatorname{atan}{(cx)}+2ab\operatorname{atan}{(cx)}+b^2c^4x^4\operatorname{atan}^2{(cx)}+2b^2c^2x^2\operatorname{atan}^2{(cx)}+b^2\operatorname{atan}^2{(cx)}}\, dx}{d^2}$$

input
```
integrate((f*x)**(1/2)/(c**2*d*x**2+d)**2/(a+b*atan(c*x))**2,x)
```

output
```
Integral(sqrt(f*x)/(a**2*c**4*x**4 + 2*a**2*c**2*x**2 + a**2 + 2*a*b*c**4*
x**4*atan(c*x) + 4*a*b*c**2*x**2*atan(c*x) + 2*a*b*atan(c*x) + b**2*c**4*x
**4*atan(c*x)**2 + 2*b**2*c**2*x**2*atan(c*x)**2 + b**2*atan(c*x)**2), x)/
d**2
```

3.597.7 Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 371, normalized size of antiderivative = 12.37

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\arctan(cx))^2}\,dx=\int \frac{\sqrt{fx}}{(c^2dx^2+d)^2(b\arctan{(cx)}+a)^2}\,dx$$

input
```
integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="ma
xima")
```

output
```
1/2*(2*(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x
)^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x))*sqrt(f)*integrate(1/4*(a*
c^2*x^2 + 4*b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)*sqrt(x)/(a^3*c^4*d^2*
x^4 + 2*a^3*c^2*d^2*x^2 + a^3*d^2 + (b^3*c^4*d^2*x^4 + 2*b^3*c^2*d^2*x^2 +
 b^3*d^2)*arctan(c*x)^3 + 3*(a*b^2*c^4*d^2*x^4 + 2*a*b^2*c^2*d^2*x^2 + a*b
^2*d^2)*arctan(c*x)^2 + 3*(a^2*b*c^4*d^2*x^4 + 2*a^2*b*c^2*d^2*x^2 + a^2*b
*d^2)*arctan(c*x)), x) + sqrt(f)*x^(3/2))/(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^
2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arc
tan(c*x))
```

3.597.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\arctan(cx))^2}\,dx=\text{Timed out}$$

input
```
integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="gi
ac")
```

output
```
Timed out
```

3.597.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2\,(a+b\arctan(cx))^2}\,dx = \int \frac{\sqrt{f\,x}}{(a+b\,\mathrm{atan}\,(c\,x))^2\,(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((f*x)^(1/2)/((a + b*atan(c*x))^2*(d + c^2*d*x^2)^2),x)
```

output
```
int((f*x)^(1/2)/((a + b*atan(c*x))^2*(d + c^2*d*x^2)^2), x)
```

3.598 $\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx$

3.598.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.598.2 Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

3.598.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.598.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.598.4 Maple [F(-1)]

Timed out.

hanged

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.598.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{\left(a^2cx^2+c\right)^3x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^2, x)
```

3.598.6 Sympy [N/A]

Not integrable

Time = 14.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\int \frac{x^m}{\operatorname{atan}^2\left(ax\right)}\,dx + \int \frac{3a^2x^2x^m}{\operatorname{atan}^2\left(ax\right)}\,dx + \int \frac{3a^4x^4x^m}{\operatorname{atan}^2\left(ax\right)}\,dx + \int \frac{a^6x^6x^m}{\operatorname{atan}^2\left(ax\right)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
c**3*(Integral(x**m/atan(a*x)**2, x) + Integral(3*a**2*x**2*x**m/atan(a*x)
**2, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**2, x) + Integral(a**6*x**6*
x**m/atan(a*x)**2, x))
```

3.598.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 8.05

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{\left(a^2cx^2+c\right)^3x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*x^m
- arctan(a*x)*integrate(((a^8*c^3*m + 8*a^8*c^3)*x^8 + 4*(a^6*c^3*m + 6*a^
6*c^3)*x^6 + 6*(a^4*c^3*m + 4*a^4*c^3)*x^4 + c^3*m + 4*(a^2*c^3*m + 2*a^2*
c^3)*x^2)*x^m/(x*arctan(a*x)), x))/(a*arctan(a*x))
```

3.598.8 Giac [N/A]

Not integrable

Time = 174.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.598.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^2, x)
```

3.599 $\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx$

3.599.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.599.2 Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

3.599.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.599.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.599.4 Maple [N/A] (verified)

Not integrable

Time = 7.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2 c\,x^2+c)^2}{\arctan\left(a x\right)^2}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.599.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^2, x)
```

3.599.6 Sympy [N/A]

Not integrable

Time = 7.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\int \frac{x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
c**2*(Integral(x**m/atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m/atan(a*x)
**2, x) + Integral(a**4*x**4*x**m/atan(a*x)**2, x))
```

3.599.7 Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.55

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*x^m - arctan(a*x)*in
tegrate(((a^6*c^2*m + 6*a^6*c^2)*x^6 + 3*(a^4*c^2*m + 4*a^4*c^2)*x^4 + c^2
*m + 3*(a^2*c^2*m + 2*a^2*c^2)*x^2)*x^m/(x*arctan(a*x)), x))/(a*arctan(a*x
))
```

3.599.8 Giac [N/A]

Not integrable

Time = 154.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m (c + a^2 c x^2)^2}{\arctan(ax)^2} \, dx = \int \frac{(a^2 c x^2 + c)^2 x^m}{\arctan(ax)^2} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.599.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m (c + a^2 c x^2)^2}{\arctan(ax)^2} \, dx = \int \frac{x^m \left(c\,a^2\,x^2 + c\right)^2}{\mathrm{atan}\left(a\,x\right)^2} \, dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^2, x)
```

3.600 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx$

3.600.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.600.2 Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

3.600.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^2}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.600.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.600.4 Maple [N/A] (verified)

Not integrable

Time = 5.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)}{\arctan\left(ax\right)^2}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.600.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

3.600.6 Sympy [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = c\left(\int \frac{x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
c*(Integral(x**m/atan(a*x)**2, x) + Integral(a**2*x**2*x**m/atan(a*x)**2,
x))
```

3.600.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.75

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^4*c*x^4 + 2*a^2*c*x^2 + c)*x^m - arctan(a*x)*integrate(((a^4*c*m + 4*
a^4*c)*x^4 + 2*(a^2*c*m + 2*a^2*c)*x^2 + c*m)*x^m/(x*arctan(a*x)), x))/(a*
arctan(a*x))
```

3.600.8 Giac [N/A]

Not integrable

Time = 133.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.600.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^2, x)
```

3.601 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx$

3.601.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x^m}{ac\arctan(ax)} + \frac{m\mathrm{Int}\left(\frac{x^{-1+m}}{\arctan(ax)},x\right)}{ac}$$

output
```
-x^m/a/c/arctan(a*x)+m*Unintegrable(x^(-1+m)/arctan(a*x),x)/a/c
```

3.601.2 Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

3.601.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{5461}$$
$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)}dx}{ac}-\frac{x^m}{ac\arctan(ax)}$$
$$\Big\downarrow \text{5377}$$
$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)}dx}{ac}-\frac{x^m}{ac\arctan(ax)}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.601.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.601.4 Maple [N/A] (verified)

Not integrable

Time = 3.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^{m}}{\left(a^{2} c\,x^{2}+c \right) \arctan \left(a x \right)^{2}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

3.601.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx=\int \frac{x^{m}}{{\left(a^{2} c x^{2} + c\right)} \arctan\left(a x\right)^{2}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.601.6 Sympy [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx=\frac{\int \frac{x^{m}}{a^{2} x^{2} \operatorname{atan}^{2}{\left(a x \right)} + \operatorname{atan}^{2}{\left(a x \right)}}\, dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

3.601.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
(m*arctan(a*x)*integrate(x^m/(x*arctan(a*x)), x) - x^m)/(a*c*arctan(a*x))
```

3.601.8 Giac [N/A]

Not integrable

Time = 112.91 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.601.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}{(a\,x)}^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

3.602 $\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx$

3.602.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx = \text{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.602.2 Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx = \int \frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

3.602.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^2} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^2} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.602.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.602.4 Maple [N/A] (verified)

Not integrable

Time = 7.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^2}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

3.602.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

3.602.6 Sympy [N/A]

Not integrable

Time = 10.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x**m/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*
x)**2), x)/c**2
```

3.602.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 5.05

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
((a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate(((a^2*m - 2*a^2)*x^2 + m)*x^m
/((a^5*c^2*x^5 + 2*a^3*c^2*x^3 + a*c^2*x)*arctan(a*x)), x) - x^m)/((a^3*c^
2*x^2 + a*c^2)*arctan(a*x))
```

3.602.8 Giac [N/A]

Not integrable

Time = 230.71 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.602.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2} \, dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

3.603 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} dx$

3.603.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2}, x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.603.2 Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

3.603.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^3} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.603.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.603.4 Maple [N/A] (verified)

Not integrable

Time = 6.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^2}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

3.603.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^2), x)
```

3.603.6 Sympy [N/A]

Not integrable

Time = 32.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{a^6x^6\operatorname{atan}^2{(ax)}+3a^4x^4\operatorname{atan}^2{(ax)}+3a^2x^2\operatorname{atan}^2{(ax)}+\operatorname{atan}^2{(ax)}}\,dx}{c^3}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x**m/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*
x**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

3.603.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.55

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate(((a^2*m - 4*a
^2)*x^2 + m)*x^m/((a^7*c^3*x^7 + 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 + a*c^3*x)*
arctan(a*x)), x) - x^m)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)
)
```

3.603.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.603.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

3.604 $\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^2} \, dx$

3.604.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^2} \, dx = \text{Int}\left(\frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^2}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.604.2 Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^2} \, dx = \int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^2} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

3.604.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan (a x)^2} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan (a x)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.604.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.604.4 Maple [N/A] (verified)

Not integrable

Time = 10.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\arctan \left(a x \right)^{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.604.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^2, x)
```

3.604.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.604.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^2, x)
```

3.604.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.604.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2, x)
```

3.605 $\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^2}\,dx$

3.605.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.605.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

3.605.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{3/2}}{\arctan(ax)^2} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)^{3/2}}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.605.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.605.4 Maple [N/A] (verified)

Not integrable

Time = 10.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2 c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.605.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

3.605.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Timed out
```

3.605.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

3.605.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.605.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2, x)
```

3.606 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$

3.606.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

3.606.2 Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

3.606.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.606.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.606.4 Maple [N/A] (verified)

Not integrable

Time = 6.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

3.606.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

3.606.6 Sympy [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

3.606.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

3.606.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.606.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2, x)
```

3.607 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$

3.607.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.607.2 Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

3.607.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.607.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.607.4 Maple [N/A] (verified)

Not integrable

Time = 6.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\arctan\left(ax\right)^{2} \sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

3.607.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.607.6 Sympy [N/A]

Not integrable

Time = 16.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x**m/atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

3.607.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

3.607.8 Giac [N/A]

Not integrable

Time = 7.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.607.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.608 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$

3.608.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.608.2 Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

3.608.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^{3/2}} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^{3/2}} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.608.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.608.4 Maple [N/A] (verified)

Not integrable

Time = 6.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^2} dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

3.608.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^2), x)
```

3.608.6 Sympy [N/A]

Not integrable

Time = 67.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

3.608.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

3.608.8 Giac [N/A]

Not integrable

Time = 15.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.608.9 Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.609 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$

3.609.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.609.2 Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

3.609.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.609.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.609.4 Maple [N/A] (verified)

Not integrable

Time = 15.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

3.609.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

3.609.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Timed out
```

3.609.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

3.609.8 Giac [N/A]

Not integrable

Time = 30.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.609.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.610 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx$

3.610.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.610.2 Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

3.610.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^3}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.610.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.610.4 Maple [N/A] (verified)

Not integrable

Time = 31.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.610.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)/arctan(a*x)^3, x)
```

3.610.6 Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = c\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
c*(Integral(x/atan(a*x)**3, x) + Integral(a**2*x**3/atan(a*x)**3, x))
```

3.610.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*a^2*arctan(a*x)^2*integrate((15*a^4*c*x^
5 + 22*a^2*c*x^3 + 7*c*x)/arctan(a*x), x) + a*c*x + (5*a^6*c*x^6 + 11*a^4*
c*x^4 + 7*a^2*c*x^2 + c)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

3.610.8 Giac [N/A]

Not integrable

Time = 77.72 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.610.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^3, x)
```

3.611 $\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx$

3.611.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx=\mathrm{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.611.2 Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx=\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3, x]
```

3.611.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^3}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{a^2cx^2+c}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.611.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.611.4 Maple [N/A] (verified)

Not integrable

Time = 49.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.611.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx=\int \frac{a^2cx^2+c}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

3.611.6 Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx=c\left(\int \frac{a^2x^2}{\operatorname{atan}^3{(ax)}}\,dx+\int \frac{1}{\operatorname{atan}^3{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**3, x) + Integral(atan(a*x)**(-3), x))
```

3.611.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.71

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx=\int \frac{a^2cx^2+c}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^4*c*x^4 + 2*a^2*c*x^2 - 2*a*arctan(a*x)^2*integrate(2*(5*a^4*c*x^4
 + 6*a^2*c*x^2 + c)/arctan(a*x), x) + 4*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x)*
arctan(a*x) + c)/(a*arctan(a*x)^2)
```

3.611.8 Giac [N/A]

Not integrable

Time = 74.78 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.18

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.611.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = \int \frac{c\,a^2\,x^2+c}{\mathrm{atan}{(a\,x)}^3}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^3, x)
```

3.612 $\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx$

3.612.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx=\operatorname{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

3.612.2 Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx=\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3), x]
```

3.612.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^3}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a^2cx^2+c}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.612.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.612.4 Maple [N/A] (verified)

Not integrable

Time = 78.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

3.612.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

3.612.6 Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = c\left(\int \frac{1}{x\,\mathrm{atan}^3\left(ax\right)}\,dx + \int \frac{a^2x}{\mathrm{atan}^3\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**3,x)
```

output
```
c*(Integral(1/(x*atan(a*x)**3), x) + Integral(a**2*x/atan(a*x)**3, x))
```

3.612.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 5.70

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*x^2*arctan(a*x)^2*integrate((6*a^6*c*x^6
 + 5*a^4*c*x^4 + c)/(x^3*arctan(a*x)), x) + a*c*x + (3*a^6*c*x^6 + 5*a^4*c
*x^4 + a^2*c*x^2 - c)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

3.612.8 Giac [N/A]

Not integrable

Time = 79.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.612.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^3), x)
```

3.613 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$

3.613.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.613.2 Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

3.613.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.613.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.613.4 Maple [N/A] (verified)

Not integrable

Time = 145.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.613.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^3, x)
```

3.613.6 Sympy [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
c**2*(Integral(x/atan(a*x)**3, x) + Integral(2*a**2*x**3/atan(a*x)**3, x)
+ Integral(a**4*x**5/atan(a*x)**3, x))
```

3.613.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 164, normalized size of antiderivative = 8.20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*a^2*arctan(a*x)^2*in
tegrate(2*(14*a^6*c^2*x^7 + 33*a^4*c^2*x^5 + 24*a^2*c^2*x^3 + 5*c^2*x)/arc
tan(a*x), x) + a*c^2*x + (7*a^8*c^2*x^8 + 22*a^6*c^2*x^6 + 24*a^4*c^2*x^4
+ 10*a^2*c^2*x^2 + c^2)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

3.613.8 Giac [N/A]

Not integrable

Time = 87.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.613.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^3, x)
```

3.614 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3} \, dx$

3.614.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)^3}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.614.2 Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3} \, dx = \int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3, x]
```

3.614.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^3}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.614.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.614.4 Maple [N/A] (verified)

Not integrable

Time = 127.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.614.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^2}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^3, x)
```

3.614.6 Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx=c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^3{(ax)}}\,dx+\int \frac{a^4x^4}{\operatorname{atan}^3{(ax)}}\,dx+\int \frac{1}{\operatorname{atan}^3{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**3, x) + Integral(a**4*x**4/atan(a*x)
**3, x) + Integral(atan(a*x)**(-3), x))
```

3.614.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 7.79

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^2}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - 2*a*arctan(a*x)^2*inte
grate(3*(7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x)
, x) + c^2 + 6*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*arc
tan(a*x))/(a*arctan(a*x)^2)
```

3.614.8 Giac [N/A]

Not integrable

Time = 86.59 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.614.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\mathrm{atan}(a\,x)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^3, x)
```

3.615 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx$

3.615.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

3.615.2 Mathematica [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3), x]
```

3.615.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.615.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.615.4 Maple [N/A] (verified)

Not integrable

Time = 182.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

3.615.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^3), x)
```

3.615.6 Sympy [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = c^2\left(\int \frac{1}{x\,\mathrm{atan}^3{(ax)}}\,dx + \int \frac{2a^2x}{\mathrm{atan}^3{(ax)}}\,dx + \int \frac{a^4x^3}{\mathrm{atan}^3{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**3,x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**3), x) + Integral(2*a**2*x/atan(a*x)**3, x)
 + Integral(a**4*x**3/atan(a*x)**3, x))
```

3.615.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 7.64

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*x^2*arctan(a*x)^2*in
tegrate((15*a^8*c^2*x^8 + 28*a^6*c^2*x^6 + 12*a^4*c^2*x^4 + c^2)/(x^3*arct
an(a*x)), x) + a*c^2*x + (5*a^8*c^2*x^8 + 14*a^6*c^2*x^6 + 12*a^4*c^2*x^4
+ 2*a^2*c^2*x^2 - c^2)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

3.615.8 Giac [N/A]

Not integrable

Time = 88.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.615.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^3), x)
```

3.616 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$

3.616.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.616.2 Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

3.616.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.616.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.616.4 Maple [F(-1)]

Timed out.

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.616.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{\left(a^2cx^2+c\right)^3 x}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)
^3, x)
```

3.616.6 Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
c**3*(Integral(x/atan(a*x)**3, x) + Integral(3*a**2*x**3/atan(a*x)**3, x)
+ Integral(3*a**4*x**5/atan(a*x)**3, x) + Integral(a**6*x**7/atan(a*x)**3,
 x))
```

3.616.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 196, normalized size of antiderivative = 9.80

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{\left(a^2cx^2+c\right)^3 x}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*
x - 2*a^2*arctan(a*x)^2*integrate((45*a^8*c^3*x^9 + 148*a^6*c^3*x^7 + 174*
a^4*c^3*x^5 + 84*a^2*c^3*x^3 + 13*c^3*x)/arctan(a*x), x) + (9*a^10*c^3*x^1
0 + 37*a^8*c^3*x^8 + 58*a^6*c^3*x^6 + 42*a^4*c^3*x^4 + 13*a^2*c^3*x^2 + c^
3)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

3.616.8 Giac [N/A]

Not integrable

Time = 97.51 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^3x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.616.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx=\int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^3, x)
```

3.617 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$

3.617.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.617.2 Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3, x]
```

3.617.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.617.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.617.4 Maple [N/A] (verified)

Not integrable

Time = 154.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan(ax)^3}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.617.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^3
, x)
```

3.617.6 Sympy [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^3(ax)}\,dx + \int \frac{1}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**3, x) + Integral(3*a**4*x**4/atan(a*
x)**3, x) + Integral(a**6*x**6/atan(a*x)**3, x) + Integral(atan(a*x)**(-3)
, x))
```

3.617.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 9.53

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 - 2*a*ar
ctan(a*x)^2*integrate(4*(9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 +
 12*a^2*c^3*x^2 + c^3)/arctan(a*x), x) + c^3 + 8*(a^9*c^3*x^9 + 4*a^7*c^3*
x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))/(a*arctan(a*x)
^2)
```

3.617.8 Giac [N/A]

Not integrable

Time = 94.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.617.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^3, x)
```

3.617. $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$

3.618 $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx$

3.618.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{x\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

3.618.2 Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3), x]
```

3.618.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.618.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.618.4 Maple [N/A] (verified)

Not integrable

Time = 177.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\,(ax)^3}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

3.618.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^3} \, dx = \int \frac{(a^2cx^2 + c)^3}{x \arctan{(ax)}^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x
)^3), x)
```

3.618.6 Sympy [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^3} \, dx = c^3 \left(\int \frac{1}{x \operatorname{atan}^3{(ax)}} \, dx + \int \frac{3a^2x}{\operatorname{atan}^3{(ax)}} \, dx + \int \frac{3a^4x^3}{\operatorname{atan}^3{(ax)}} \, dx + \int \frac{a^6x^5}{\operatorname{atan}^3{(ax)}} \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**3,x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**3), x) + Integral(3*a**2*x/atan(a*x)**3, x)
 + Integral(3*a**4*x**3/atan(a*x)**3, x) + Integral(a**6*x**5/atan(a*x)**3
, x))
```

3.618.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 201, normalized size of antiderivative = 9.14

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*
x - 2*x^2*arctan(a*x)^2*integrate((28*a^10*c^3*x^10 + 81*a^8*c^3*x^8 + 76*
a^6*c^3*x^6 + 22*a^4*c^3*x^4 + c^3)/(x^3*arctan(a*x)), x) + (7*a^10*c^3*x^
10 + 27*a^8*c^3*x^8 + 38*a^6*c^3*x^6 + 22*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^
3)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

3.618.8 Giac [N/A]

Not integrable

Time = 97.83 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.618.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^3), x)
```

3.618. $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx$

3.619 $\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.619.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x^3}{2ac\arctan(ax)^2}+\frac{3\mathrm{Int}\left(\frac{x^2}{\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2*x^3/a/c/arctan(a*x)^2+3/2*Unintegrable(x^2/arctan(a*x)^2,x)/a/c
```

3.619.2 Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.619.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5461}$$

$$\frac{3\int \frac{x^2}{\arctan(ax)^2}dx}{2ac}-\frac{x^3}{2ac\arctan(ax)^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{3\int \frac{x^2}{\arctan(ax)^2}dx}{2ac}-\frac{x^3}{2ac\arctan(ax)^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.619.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.619.4 Maple [N/A] (verified)

Not integrable

Time = 31.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a^2 c\,x^2+c\right) \arctan\left(a x\right)^3}dx$$

input
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.619.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{\left(a^2cx^2+c\right) \arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.619.6 Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^3}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx}{c}$$

input
```
integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

3.619.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^3 - 2*arctan(a*x)^2*integrate(3*(2*a^2*x^3 + x)/arctan(a*x), x)
+ 3*(a^2*x^4 + x^2)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

3.619.8 Giac [N/A]

Not integrable

Time = 62.67 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.619.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.620 $\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.620.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x^2}{2ac\arctan(ax)^2} + \frac{\mathrm{Int}\left(\frac{x}{\arctan(ax)^2},x\right)}{ac}$$

output
```
-1/2*x^2/a/c/arctan(a*x)^2+Unintegrable(x/arctan(a*x)^2,x)/a/c
```

3.620.2 Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.620.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3 (a^2cx^2+c)} dx$$
$$\Big\downarrow \text{5461}$$
$$\frac{\int \frac{x}{\arctan(ax)^2}dx}{ac} - \frac{x^2}{2ac\arctan(ax)^2}$$
$$\Big\downarrow \text{5377}$$
$$\frac{\int \frac{x}{\arctan(ax)^2}dx}{ac} - \frac{x^2}{2ac\arctan(ax)^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.620.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.620.4 Maple [N/A] (verified)

Not integrable

Time = 6.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2c\,x^2+c)\arctan(ax)^3}dx$$

input
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.620.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.620.6 Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^2}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c}$$

input
```
integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x**2/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

3.620.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^2 - 2*arctan(a*x)^2*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) +
2*(a^2*x^3 + x)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

3.620.8 Giac [N/A]

Not integrable

Time = 61.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.620.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.621 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.621.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x}{2ac\arctan(ax)^2} + \frac{\operatorname{Int}\left(\frac{1}{\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2*x/a/c/arctan(a*x)^2+1/2*Unintegrable(1/arctan(a*x)^2,x)/a/c
```

3.621.2 Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.621.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3\,(a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{\int \frac{1}{\arctan(ax)^2}dx}{2ac}-\frac{x}{2ac\arctan(ax)^2}$$
$$\Big\downarrow \text{5353}$$
$$\frac{\int \frac{1}{\arctan(ax)^2}dx}{2ac}-\frac{x}{2ac\arctan(ax)^2}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.621.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.621.4 Maple [N/A] (verified)

Not integrable

Time = 8.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{3}}d x$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.621.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx=\int \frac{x}{{\left(a^{2} c x^{2} + c\right)} \arctan \left(a x\right)^{3}} \,d x$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.621.6 Sympy [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx=\frac{\int \frac{x}{a^{2} x^{2} \operatorname{atan}^{3}{\left(a x \right)} + \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

3.621.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*a^2*arctan(a*x)^2*integrate(x/arctan(a*x), x) - a*x - (a^2*x^2 + 1)
*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

3.621.8 Giac [N/A]

Not integrable

Time = 48.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.621.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.622 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.622.1 Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2ac\arctan(ax)^2}$$

output
```
-1/2/a/c/arctan(a*x)^2
```

3.622.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2ac\arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*ArcTan[a*x]^2)
```

3.622.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \ 5419$$
$$-\frac{1}{2ac\arctan(ax)^2}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*ArcTan[a*x]^2)
```

3.622.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.622.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{1}{2ac\arctan(ax)^2}$	15
default	$-\frac{1}{2ac\arctan(ax)^2}$	15
parallelrisch	$-\frac{1}{2ac\arctan(ax)^2}$	15
risch	$\frac{2}{ac(\ln(-iax+1)-\ln(iax+1))^2}$	30

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c/arctan(a*x)^2
```

3.622.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2\,ac\arctan{(ax)}^2}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2/(a*c*arctan(a*x)^2)
```

3.622.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2ac\operatorname{atan}^2{(ax)}}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
-1/(2*a*c*atan(a*x)**2)
```

3.622.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2\,ac\arctan{(ax)}^2}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2/(a*c*arctan(a*x)^2)
```

3.622.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.622.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2\,a\,c\,\mathrm{atan}\left(a\,x\right)^2}$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
-1/(2*a*c*atan(a*x)^2)
```

3.623 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.623.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^3}\,dx=-\frac{1}{2acx\arctan(ax)^2}-\frac{\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2/a/c/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan(a*x)^2,x)/a/c
```

3.623.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^3}\,dx=\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.623.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^3 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2ac}-\frac{1}{2acx\arctan(ax)^2}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2ac}-\frac{1}{2acx\arctan(ax)^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.623.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.623.4 Maple [N/A] (verified)

Not integrable

Time = 1.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^2c\,x^2+c)\arctan\left(ax\right)^3}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.623.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

3.623.6 Sympy [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{1}{a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**3 + x*atan(a*x)**3), x)/c
```

3.623.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{1}{x (c + a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)x \arctan(ax)^3} \, dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^2*arctan(a*x)^2*integrate(1/(x^3*arctan(a*x)), x) - a*x + (a^2*x^
2 + 1)*arctan(a*x))/(a^2*c*x^2*arctan(a*x)^2)
```

3.623.8 Giac [N/A]

Not integrable

Time = 48.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x (c + a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)x \arctan(ax)^3} \, dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.623.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x (c + a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{x \operatorname{atan}(a x)^3 (c a^2 x^2 + c)} \, dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.624 $\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.624.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=-\frac{1}{2acx^2\arctan(ax)^2}-\frac{\operatorname{Int}\left(\frac{1}{x^3\arctan(ax)^2},x\right)}{ac}$$

output
```
-1/2/a/c/x^2/arctan(a*x)^2-Unintegrable(1/x^3/arctan(a*x)^2,x)/a/c
```

3.624.2 Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.624.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 (a^2cx^2 + c)} dx$$

$$\Big\downarrow \text{5461}$$

$$-\frac{\int \frac{1}{x^3 \arctan(ax)^2} dx}{ac} - \frac{1}{2acx^2 \arctan(ax)^2}$$

$$\Big\downarrow \text{5377}$$

$$-\frac{\int \frac{1}{x^3 \arctan(ax)^2} dx}{ac} - \frac{1}{2acx^2 \arctan(ax)^2}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.624.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.624.4 Maple [N/A] (verified)

Not integrable

Time = 10.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.624.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)^3} \, dx = \int { \frac{1}{{\left(a^{2} c x^{2} + c\right)} x^{2} \arctan\left(a x\right)^{3}} \,d x }$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

3.624.6 Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^{2} x^{4} \operatorname{atan}^{3}{\left(a x \right)} + x^{2} \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**4*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c
```

3.624.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)x^2 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^3*arctan(a*x)^2*integrate((a^2*x^2 + 3)/(x^4*arctan(a*x)), x) - a
*x + 2*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^3*arctan(a*x)^2)
```

3.624.8 Giac [N/A]

Not integrable

Time = 60.78 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)x^2 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.624.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^3 \, (c\,a^2\,x^2+c)} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.625 $\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.625.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=-\frac{1}{2acx^3\arctan(ax)^2}-\frac{3\text{Int}\left(\frac{1}{x^4\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2/a/c/x^3/arctan(a*x)^2-3/2*Unintegrable(1/x^4/arctan(a*x)^2,x)/a/c
```

3.625.2 Mathematica [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.625.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2ac}-\frac{1}{2acx^3\arctan(ax)^2}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2ac}-\frac{1}{2acx^3\arctan(ax)^2}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.625.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.625.4 Maple [N/A] (verified)

Not integrable

Time = 79.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(a^2 c\,x^2+c\right) \arctan\left(a x\right)^3}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.625.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\int \frac{1}{(a^2cx^2+c)x^3\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

3.625.6 Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\frac{\int \frac{1}{a^2x^5\operatorname{atan}^3\left(ax\right)+x^3\operatorname{atan}^3\left(ax\right)}\,dx}{c}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**5*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c
```

3.625.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.18

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^4*arctan(a*x)^2*integrate(3*(a^2*x^2 + 2)/(x^5*arctan(a*x)), x) -
 a*x + 3*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^4*arctan(a*x)^2)
```

3.625.8 Giac [N/A]

Not integrable

Time = 62.51 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.625.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \int \frac{1}{x^3 \operatorname{atan}{(a\,x)}^3 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.626 $\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.626.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx = -\frac{1}{2acx^4\arctan(ax)^2} - \frac{2\mathrm{Int}\left(\frac{1}{x^5\arctan(ax)^2},x\right)}{ac}$$

output
```
-1/2/a/c/x^4/arctan(a*x)^2-2*Unintegrable(1/x^5/arctan(a*x)^2,x)/a/c
```

3.626.2 Mathematica [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx = \int \frac{1}{x^4\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.626.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^3 \left(a^2cx^2+c\right)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{ac}-\frac{1}{2acx^4\arctan(ax)^2}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{ac}-\frac{1}{2acx^4\arctan(ax)^2}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.626.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.626.4 Maple [N/A] (verified)

Not integrable

Time = 49.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(a^2 c\, x^2+c\right) \arctan\left(a x\right)^3}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.626.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right) \arctan(a x)^3} \, dx=\int \frac{1}{(a^2 c x^2 + c) x^4 \arctan\left(a x\right)^3} \,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^3), x)
```

3.626.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right) \arctan(a x)^3} \, dx=\frac{\int \frac{1}{a^{2} x^{6} \operatorname{atan}^{3}{\left(a x \right)} + x^{4} \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c
```

3.626.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.23

$$\int \frac{1}{x^4 (c + a^2 cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)x^4 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^5*arctan(a*x)^2*integrate(2*(3*a^2*x^2 + 5)/(x^6*arctan(a*x)), x)
 - a*x + 4*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^5*arctan(a*x)^2)
```

3.626.8 Giac [N/A]

Not integrable

Time = 61.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4 (c + a^2 cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)x^4 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.626.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c + a^2 cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{x^4 \operatorname{atan}{(a\,x)}^3 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.627 $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$

3.627.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = -\frac{x}{2a^3c^2\arctan(ax)^2} + \frac{x}{2a^3c^2(1+a^2x^2)\arctan(ax)^2} + \frac{1-a^2x^2}{2a^4c^2(1+a^2x^2)\arctan(ax)} + \frac{\mathrm{Si}(2\arctan(ax))}{a^4c^2} + \frac{\mathrm{Int}\left(\frac{1}{\arctan(ax)^2}, x\right)}{2a^3c^2}$$

output
```
-1/2*x/a^3/c^2/arctan(a*x)^2+1/2*x/a^3/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(
-a^2*x^2+1)/a^4/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/a^4/c^2+1/2*
Unintegrable(1/arctan(a*x)^2,x)/a^3/c^2
```

3.627.2 Mathematica [N/A]

Not integrable

Time = 6.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

3.627.3 Rubi [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5499, 27, 5457, 5353, 5467, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c(a^2x^2+1)\arctan(ax)^3}dx}{a^2c} - \frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)\arctan(ax)^3}dx}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^2}$$

$$\Big\downarrow \text{5457}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a} - \frac{x}{2a\arctan(ax)^2}}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^2}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a} - \frac{x}{2a\arctan(ax)^2}}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^2}$$

$$\Big\downarrow \text{5467}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a} - \frac{x}{2a\arctan(ax)^2}}{a^2c^2} - \frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 4906$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3780$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.627. $\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

3.627.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.627. $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.627.4 Maple [N/A] (verified)

Not integrable

Time = 15.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^2 c\, x^2+c)^2 \arctan (a x)^3} dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.627.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3}\, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan{(ax)}^3}\, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

3.627.6 Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}^3(ax)+2a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} \, dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*
x)**3), x)/c**2
```

3.627.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 6.23

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^3 - 2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^4*x^5 +
 2*a^2*x^3 + 3*x)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x
) + (a^2*x^4 + 3*x^2)*arctan(a*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

3.627.8 Giac [N/A]

Not integrable

Time = 173.51 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.627.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3 \left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.628 $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx$

3.628.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = -\frac{x^2}{2ac^2(1+a^2x^2)\arctan(ax)^2} - \frac{x}{a^2c^2(1+a^2x^2)\arctan(ax)} + \frac{\text{CosIntegral}(2\arctan(ax))}{a^3c^2}$$

output
```
-1/2*x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)^2-x/a^2/c^2/(a^2*x^2+1)/arctan(a*x)
+Ci(2*arctan(a*x))/a^3/c^2
```

3.628.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{-\frac{ax(ax+2\arctan(ax))}{(1+a^2x^2)\arctan(ax)^2} + 2\,\text{CosIntegral}(2\arctan(ax))}{2a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
(-((a*x*(a*x + 2*ArcTan[a*x]))/((1 + a^2*x^2)*ArcTan[a*x]^2)) + 2*CosInteg
ral[2*ArcTan[a*x]])/(2*a^3*c^2)
```

3.628.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5477, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \ 5477$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \ 5503$$

$$\frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^{2}\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \ 5439$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \ 3793$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

3.628. $\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

output
```
-1/2*x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) + (-(x/(a*(1 + a^2*x^2)*ArcTa
n[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]]/2)/a^2 + (C
osIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/a^2)/(a*c^2)
```

3.628.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.628. $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.628.4 Maple [A] (verified)

Time = 9.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{4\ \mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{4a^3c^2\arctan(ax)^2}$	52
default	$\frac{4\ \mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{4a^3c^2\arctan(ax)^2}$	52

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^3/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan
(a*x)+cos(2*arctan(a*x))-1)/arctan(a*x)^2
```

3.628.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx =$$
$$-\frac{a^2x^2-(a^2x^2+1)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-(a^2x^2+1)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\right.}{2\left(a^5c^2x^2+a^3c^2\right)\arctan(ax)^2}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2 - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a
*x - 1)/(a^2*x^2 + 1)) - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^
2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) + 2*a*x*arctan(a*x))/((a^5*c^2*x^2 + a^3*c
^2)*arctan(a*x)^2)
```

3.628.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{x^2}{a^4x^4 \operatorname{atan}^3(ax)+2a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} \, dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*
x)**3), x)/c**2
```

3.628.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^
6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x) + a*x^2 + 2*x*arctan
(a*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

3.628.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.629 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$

3.629.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = -\frac{x}{2ac^2(1+a^2x^2)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\arctan(ax)} - \frac{\mathrm{Si}(2\arctan(ax))}{a^2c^2}$$

output
```
-1/2*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(a^2*x^2-1)/a^2/c^2/(a^2*x^2+1)
/arctan(a*x)-Si(2*arctan(a*x))/a^2/c^2
```

3.629.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{-ax+(-1+a^2x^2)\arctan(ax)-2(1+a^2x^2)\arctan(ax)^2\mathrm{Si}(2\arctan(ax))}{2a^2c^2(1+a^2x^2)\arctan(ax)^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
(-(a*x) + (-1 + a^2*x^2)*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*SinIn
tegral[2*ArcTan[a*x]])/(2*a^2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)
```

3.629.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5467, 27, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3 \left(a^2cx^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{5467}$$

$$-2\int \frac{x}{c^2 \left(a^2x^2+1\right)^2 \arctan(ax)}dx-\frac{x}{2ac^2 \left(a^2x^2+1\right) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 \left(a^2x^2+1\right) \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{x}{\left(a^2x^2+1\right)^2 \arctan(ax)}dx}{c^2}-\frac{x}{2ac^2 \left(a^2x^2+1\right) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 \left(a^2x^2+1\right) \arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\int \frac{ax}{\left(a^2x^2+1\right) \arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2 \left(a^2x^2+1\right) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 \left(a^2x^2+1\right) \arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2 \left(a^2x^2+1\right) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 \left(a^2x^2+1\right) \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2 \left(a^2x^2+1\right) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 \left(a^2x^2+1\right) \arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2 \left(a^2x^2+1\right) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 \left(a^2x^2+1\right) \arctan(ax)}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\text{Si}(2\arctan(ax))}{a^2c^2}-\frac{x}{2ac^2 \left(a^2x^2+1\right) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 \left(a^2x^2+1\right) \arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

3.629. $\int \frac{x}{\left(c+a^2cx^2\right)^2 \arctan(ax)^3} \, dx$

output
```
-1/2*x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (1 - a^2*x^2)/(2*a^2*c^2*(1 +
 a^2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[a*x]]/(a^2*c^2)
```

3.629.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.629. $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$

3.629.4 Maple [A] (verified)

Time = 8.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$-\frac{4\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2+2\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{4a^2c^2\arctan(ax)^2}$	51
default	$-\frac{4\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2+2\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{4a^2c^2\arctan(ax)^2}$	51

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/c^2*(4*Si(2*arctan(a*x))*arctan(a*x)^2+2*cos(2*arctan(a*x))*arcta
n(a*x)+sin(2*arctan(a*x)))/arctan(a*x)^2
```

3.629.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.67

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx = \frac{\left(-i\,a^2x^2-i\right)\arctan\left(ax\right)^2\text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+\left(i\,a^2x^2+i\right)\arctan\left(ax\right)^2\text{log\_integral}\left(-\frac{a^2x^2-2}{a^2x^2}\right.}{2\left(a^4c^2x^2+a^2c^2\right)\arctan\left(ax\right)^2}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
1/2*((-I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/
(a^2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*
I*a*x - 1)/(a^2*x^2 + 1)) - a*x + (a^2*x^2 - 1)*arctan(a*x))/((a^4*c^2*x^2
 + a^2*c^2)*arctan(a*x)^2)
```

3.629.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \frac{\int \frac{x}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)*
*3), x)/c**2
```

3.629.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(8*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate(1/2*x/((a^4*c^2*x^
4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + a*x - (a^2*x^2 - 1)*arctan(a*x
))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

3.629.8 Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.630 $\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

3.630.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = -\frac{1}{2ac^2(1+a^2x^2)\arctan(ax)^2} + \frac{x}{c^2(1+a^2x^2)\arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{ac^2}$$

output
```
-1/2/a/c^2/(a^2*x^2+1)/arctan(a*x)^2+x/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*ar
ctan(a*x))/a/c^2
```

3.630.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \frac{-1+2ax\arctan(ax)-2(1+a^2x^2)\arctan(ax)^2\,\text{CosIntegral}(2\arctan(ax))}{2c^2(a+a^3x^2)\arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
(-1 + 2*a*x*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*CosIntegral[2*ArcT
an[a*x]])/(2*c^2*(a + a^3*x^2)*ArcTan[a*x]^2)
```

3.630.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5437, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$-a\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}dx-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 5505$$

$$-\frac{a\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 3793$$

$$-\frac{a\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

3.630. $\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (a*(-(x/(a*(1 + a^2*x^2)*ArcT
an[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]]/2)/a^2 + (
CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/a^2))/c^2
```

3.630.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.630.4 Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{4a\,c^2\arctan(ax)^2}$	52
default	$-\frac{4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{4a\,c^2\arctan(ax)^2}$	52

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan(
a*x)+cos(2*arctan(a*x))+1)/arctan(a*x)^2
```

3.630.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx =$$

$$-\frac{(a^2x^2+1)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(a^2x^2+1)\arctan(ax)^2\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,a}{a^2x^2+}\right.}{2\,(a^3c^2x^2+ac^2)\arctan(ax)^2}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a
^2*x^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*
x - 1)/(a^2*x^2 + 1)) - 2*a*x*arctan(a*x) + 1)/((a^3*c^2*x^2 + a*c^2)*arct
an(a*x)^2)
```

3.630.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \frac{\int \frac{1}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)*
*3), x)/c**2
```

3.630.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^4*c
^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 2*a*x*arctan(a*x) - 1)/((
a^3*c^2*x^2 + a*c^2)*arctan(a*x)^2)
```

3.630.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.631 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$

3.631.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = -\frac{1}{2ac^2x \arctan(ax)^2}+\frac{ax}{2c^2 (1+a^2x^2) \arctan(ax)^2} + \frac{1-a^2x^2}{2c^2 (1+a^2x^2) \arctan(ax)} + \frac{\operatorname{Si}(2\arctan(ax))}{c^2}-\frac{\operatorname{Int}\left(\frac{1}{x^2 \arctan(ax)^2},x\right)}{2ac^2}$$

output
```
-1/2/a/c^2/x/arctan(a*x)^2+1/2*a*x/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(-a^2
*x^2+1)/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/c^2-1/2*Unintegrable
(1/x^2/arctan(a*x)^2,x)/a/c^2
```

3.631.2 Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{x (c+a^2cx^2)^2 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

3.631.3 Rubi [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 27, 5461, 5377, 5467, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^3\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \textbf{5501}$$

$$\frac{\int \frac{1}{cx(a^2x^2+1)\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^3}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \textbf{5461}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \textbf{5377}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \textbf{5467}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \textbf{5505}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \textbf{4906}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \textbf{3780}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{\mathrm{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.631. $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

3.631.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.631.4 Maple [N/A] (verified)

Not integrable

Time = 9.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2}c\,x^{2}+c\right)^{2}\arctan\left(ax\right)^{3}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.631.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3), x)
```

3.631.6 Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\frac{\int \frac{1}{a^4x^5\operatorname{atan}^3(ax)+2a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**3 + 2*a**2*x**3*atan(a*x)**3 + x*atan(a*x
)**3), x)/c**2
```

3.631.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 6.41

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)^2*integrate((3*a^4*x^4 + 2*
a^2*x^2 + 1)/((a^6*c^2*x^7 + 2*a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)), x)
 - a*x + (3*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(
a*x)^2)
```

3.631.8 Giac [N/A]

Not integrable

Time = 121.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.631.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\,(c+a^2cx^2)^2\arctan(ax)^3}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.632 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^3} dx$

3.632.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^3} dx = & -\frac{1}{2ac^2x^2 \arctan(ax)^2} + \frac{a}{2c^2 (1 + a^2x^2) \arctan(ax)^2} \\ & - \frac{a^2x}{c^2 (1 + a^2x^2) \arctan(ax)} \\ & + \frac{a \operatorname{CosIntegral}(2 \arctan(ax))}{c^2} - \frac{\operatorname{Int}\left(\frac{1}{x^3 \arctan(ax)^2}, x\right)}{ac^2}\end{aligned}$$

output
```
-1/2/a/c^2/x^2/arctan(a*x)^2+1/2*a/c^2/(a^2*x^2+1)/arctan(a*x)^2-a^2*x/c^2
/(a^2*x^2+1)/arctan(a*x)+a*Ci(2*arctan(a*x))/c^2-Unintegrable(1/x^3/arctan
(a*x)^2,x)/a/c^2
```

3.632.2 Mathematica [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^3} dx = \int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^3} dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

3.632.3 Rubi [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5437, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 \left(a^2 c x^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c x^2 (a^2x^2+1) \arctan(ax)^3}dx}{c}-a^2 \int \frac{1}{c^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^2 (a^2x^2+1) \arctan(ax)^3}dx}{c^2}-\frac{a^2 \int \frac{1}{\left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2 (a^2x^2+1) \arctan(ax)^3}dx}{c^2}-\frac{a^2 \left(-a \int \frac{x}{\left(a^2x^2+1\right)^2 \arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2 \left(-a \int \frac{x}{\left(a^2x^2+1\right)^2 \arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2 \left(-a \int \frac{x}{\left(a^2x^2+1\right)^2 \arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2 \left(-a \left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2 \arctan(ax)}dx}{a}-a \int \frac{x^2}{\left(a^2x^2+1\right)^2 \arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\text{arcta}}\right.}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\text{arcta}}\right.}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-a^2\left(-a\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2+1}\right.}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-a^2\left(-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\text{arct}}\right.}{c^2}$$

3.632. $\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

$$\Big\downarrow \text{3793}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-a\left(-\frac{\int\left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\right.}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\right.}{c^2}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.632.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.632. $\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.632. $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$

3.632.4 Maple [N/A] (verified)

Not integrable

Time = 9.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.632.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^2 \arctan(a x)^3} \, dx=\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{2} x^{2} \arctan \left(a x\right)^{3}} \,d x$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3), x)
```

3.632.6 Sympy [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^2 \arctan(a x)^3} \, dx=\frac{\int \frac{1}{a^{4} x^{6} \operatorname{atan}^{3}{\left(a x \right)} + 2 a^{2} x^{4} \operatorname{atan}^{3}{\left(a x \right)} + x^{2} \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**3 + 2*a**2*x**4*atan(a*x)**3 + x**2*atan(
a*x)**3), x)/c**2
```

3.632.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)^2*integrate((6*a^4*x^4 + 7*
a^2*x^2 + 3)/((a^6*c^2*x^8 + 2*a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)), x)
 - a*x + 2*(2*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^5 + a^2*c^2*x^3)*arcta
n(a*x)^2)
```

3.632.8 Giac [N/A]

Not integrable

Time = 128.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.632.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^3 \,(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.633 $\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx$

3.633.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx = &-\frac{1}{2ac^2x^3\arctan(ax)^2}+\frac{a}{2c^2x\arctan(ax)^2}\\&-\frac{a^3x}{2c^2\left(1+a^2x^2\right)\arctan(ax)^2}\\&-\frac{a^2(1-a^2x^2)}{2c^2\left(1+a^2x^2\right)\arctan(ax)}-\frac{a^2\mathrm{Si}(2\arctan(ax))}{c^2}\\&-\frac{3\mathrm{Int}\left(\frac{1}{x^4\arctan(ax)^2},x\right)}{2ac^2}+\frac{a\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)^2},x\right)}{2c^2}\end{aligned}$$

output
```
-1/2/a/c^2/x^3/arctan(a*x)^2+1/2*a/c^2/x/arctan(a*x)^2-1/2*a^3*x/c^2/(a^2*
x^2+1)/arctan(a*x)^2-1/2*a^2*(-a^2*x^2+1)/c^2/(a^2*x^2+1)/arctan(a*x)-a^2*
Si(2*arctan(a*x))/c^2-3/2*Unintegrable(1/x^4/arctan(a*x)^2,x)/a/c^2+1/2*a*
Unintegrable(1/x^2/arctan(a*x)^2,x)/c^2
```

3.633.2 Mathematica [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} dx = \int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

3.633.3 Rubi [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5461, 5377, 5501, 5461, 5377, 5467, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^3(a^2x^2+1)\arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{c^2x (a^2x^2+1)^2 \arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)\arctan(ax)^3}dx}{c^2} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^2 \arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2a} - \frac{1}{2ax^3\arctan(ax)^2}}{c^2} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^2 \arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \ 5501$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)}{c^2}$$

$$\Big\downarrow \ 5461$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \ 5377$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \ 5467$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan}\right.}{c^2}$$

$$\Big\downarrow \ 5505$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2a}\right.}{c^2}$$

$$\Big\downarrow \ 4906$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\arctan}\right.}{c^2}$$

3.633. $\displaystyle\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\,\mathrm{arcta}}\right.}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\,\mathrm{arcta}}\right.}{c^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\mathrm{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{1}{2ax\arctan(ax)^2}\right)}{c^2}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.633.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.633. $\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.633. $\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

3.633.4 Maple [N/A] (verified)

Not integrable

Time = 81.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.633.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^2 \arctan(a x)^3} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{2} x^{3} \arctan \left (a x\right)^{3}} \,d x }$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3), x)
```

3.633.6 Sympy [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^2 \arctan(a x)^3} \, dx=\frac {\int \frac {1}{a^{4} x^{7} \operatorname {atan}^{3}{\left (a x \right)} + 2 a^{2} x^{5} \operatorname {atan}^{3}{\left (a x \right)} + x^{3} \operatorname {atan}^{3}{\left (a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**3 + 2*a**2*x**5*atan(a*x)**3 + x**3*atan(
a*x)**3), x)/c**2
```

3.633.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)^2*integrate(2*(5*a^4*x^4 +
7*a^2*x^2 + 3)/((a^6*c^2*x^9 + 2*a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)),
x) - a*x + (5*a^2*x^2 + 3)*arctan(a*x))/((a^4*c^2*x^6 + a^2*c^2*x^4)*arcta
n(a*x)^2)
```

3.633.8 Giac [N/A]

Not integrable

Time = 125.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.633.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{x^3 \operatorname{atan}{(a\,x)}^3 \, (c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.634 $\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

3.634.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx = -\frac{1}{2ac^2x^4\arctan(ax)^2}+\frac{a}{2c^2x^2\arctan(ax)^2}-\frac{a^3}{2c^2\left(1+a^2x^2\right)\arctan(ax)^2}+\frac{a^4x}{c^2\left(1+a^2x^2\right)\arctan(ax)}-\frac{a^3\,\mathrm{CosIntegral}(2\arctan(ax))}{c^2}-\frac{2\mathrm{Int}\left(\frac{1}{x^5\arctan(ax)^2},x\right)}{ac^2}+\frac{a\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)^2},x\right)}{c^2}$$

output
```
-1/2/a/c^2/x^4/arctan(a*x)^2+1/2*a/c^2/x^2/arctan(a*x)^2-1/2*a^3/c^2/(a^2*
x^2+1)/arctan(a*x)^2+a^4*x/c^2/(a^2*x^2+1)/arctan(a*x)-a^3*Ci(2*arctan(a*x
))/c^2-2*Unintegrable(1/x^5/arctan(a*x)^2,x)/a/c^2+a*Unintegrable(1/x^3/ar
ctan(a*x)^2,x)/c^2
```

3.634.2 Mathematica [N/A]

Not integrable

Time = 5.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3} \, dx = \int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

3.634.3 Rubi [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 27, 5461, 5377, 5501, 5437, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^3 \left(a^2 c x^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c x^4 \left(a^2 x^2+1\right) \arctan(ax)^3}dx}{c}-a^2 \int \frac{1}{c^2 x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^4 \left(a^2 x^2+1\right) \arctan(ax)^3}dx}{c^2}-\frac{a^2 \int \frac{1}{x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{2 \int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2 a x^4 \arctan(ax)^2}}{c^2}-\frac{a^2 \int \frac{1}{x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \ 5501$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)}{c^2}$$

$$\Big\downarrow \ 5437$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx-a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)}{c^2}$$

$$\Big\downarrow \ 5461$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \ 5377$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \ 5503$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)}{c^2}$$

$$\Big\downarrow \ 5439$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan}\right.\right.\right.}{c^2}$$

3.634. $\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)-\frac{1}{2a\left(a^2x^2+1\right)\arctan}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \textbf{3793}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)-\frac{}{2a(a^2x^2}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)-\frac{}{2a(a^2x^2}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \textbf{5505}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-a\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)-\right.\right.\right.}{c^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)-\frac{}{2a(a^2}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \textbf{3793}$$

3.634. $\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx$

$$\frac{-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\left(a^2\left(-a\left(-\frac{\int\left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(\right.\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-$$

$$\frac{a^2\left(-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}\right.\right.\right.\right.}{c^2}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.634.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.634. $\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.634. $\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^3}\,dx$

3.634.4 Maple [N/A] (verified)

Not integrable

Time = 32.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.634.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x^4\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^3), x)
```

3.634.6 Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\frac{\int \frac{1}{a^4x^8\operatorname{atan}^3\left(ax\right)+2a^2x^6\operatorname{atan}^3\left(ax\right)+x^4\operatorname{atan}^3\left(ax\right)}\,dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**3 + 2*a**2*x**6*atan(a*x)**3 + x**4*atan(
a*x)**3), x)/c**2
```

3.634.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^2 x^4 \arctan(ax)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)^2*integrate((15*a^4*x^4 + 2
3*a^2*x^2 + 10)/((a^6*c^2*x^10 + 2*a^4*c^2*x^8 + a^2*c^2*x^6)*arctan(a*x))
, x) - a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x))/((a^4*c^2*x^7 + a^2*c^2*x^5)*a
rctan(a*x)^2)
```

3.634.8 Giac [N/A]

Not integrable

Time = 127.47 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^2 x^4 \arctan(ax)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.634.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^3}\,dx = \int \frac{1}{x^4 \mathrm{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.635 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} dx$

3.635.1 Optimal result

Integrand size = 22, antiderivative size = 177

$$\begin{aligned}\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} dx &= \frac{x}{2a^3c^3(1+a^2x^2)^2 \arctan(ax)^2} \\ &\quad - \frac{x}{2a^3c^3(1+a^2x^2)\arctan(ax)^2} \\ &\quad + \frac{2}{a^4c^3(1+a^2x^2)^2\arctan(ax)} \\ &\quad - \frac{3}{2a^4c^3(1+a^2x^2)\arctan(ax)} \\ &\quad - \frac{1-a^2x^2}{2a^4c^3(1+a^2x^2)\arctan(ax)} \\ &\quad - \frac{\mathrm{Si}(2\arctan(ax))}{2a^4c^3} + \frac{\mathrm{Si}(4\arctan(ax))}{a^4c^3}\end{aligned}$$

output
```
1/2*x/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-1/2*x/a^3/c^3/(a^2*x^2+1)/arctan
(a*x)^2+2/a^4/c^3/(a^2*x^2+1)^2/arctan(a*x)-3/2/a^4/c^3/(a^2*x^2+1)/arctan
(a*x)+1/2*(a^2*x^2-1)/a^4/c^3/(a^2*x^2+1)/arctan(a*x)-1/2*Si(2*arctan(a*x)
)/a^4/c^3+Si(4*arctan(a*x))/a^4/c^3
```

3.635.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.41

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{\frac{a^2x^2(-ax+(-3+a^2x^2)\arctan(ax))}{(1+a^2x^2)^2\arctan(ax)^2} - \text{Si}(2\arctan(ax)) + 2\text{Si}(4\arctan(ax))}{2a^4c^3}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
((a^2*x^2*(-(a*x) + (-3 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a
*x]^2) - SinIntegral[2*ArcTan[a*x]] + 2*SinIntegral[4*ArcTan[a*x]])/(2*a^4
*c^3)
```

3.635.3 Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.54, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5499, 27, 5467, 5503, 5437, 5499, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2 \arctan(ax)^3} dx}{a^2c} - \frac{\int \frac{x}{c^3(a^2x^2+1)^3 \arctan(ax)^3} dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2 \arctan(ax)^3} dx}{a^2c^3} - \frac{\int \frac{x}{(a^2x^2+1)^3 \arctan(ax)^3} dx}{a^2c^3}$$

$$\Big\downarrow \text{5467}$$

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2 \arctan(ax)} dx - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{\int \frac{x}{(a^2x^2+1)^3 \arctan(ax)^3} dx}{a^2c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2a} - \frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$$\Big\downarrow \; 5437$$

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{-\frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx + \frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx - \frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a} - \frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$$\Big\downarrow \; 5499$$

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{a^2} - \frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{a^2}\right) + \frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx - \frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a} - \frac{x}{2a(a^2x^2+1)^2 \cdots}}{a^2c^3}$$

$$\Big\downarrow \; 5437$$

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx - \frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a} - \frac{3}{2}a\left(\frac{-2a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx - \frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2} - \frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan \cdots}}{\cdots}\right.}{a^2c^3 \cdots}$$

$$\Big\downarrow \; 5505$$

$$\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2} - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a} - \frac{3}{2}a\left(\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2} - \frac{-\frac{4\int \frac{\cdots}{\left(a^2 \cdots\right.}}{\cdots}}{\cdots}\right.}{a^2c^3 \cdots}$$

$$\Big\downarrow \; 4906$$

3.635. $\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{2\int\frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\right.$$
$$\overline{\qquad\qquad a^2c^3}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{\int\frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\right.$$
$$\overline{\qquad\qquad a^2c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$-\frac{3}{2}a\left(\frac{-\frac{\int\frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-}{a\left(a^2\right.}$$
$$\overline{\qquad\qquad a^2c^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$-\frac{3}{2}a\left(\frac{-\frac{\int\frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-}{a\left(a^2\right.}$$
$$\overline{\qquad\qquad a^2c^3}$$

$$\Big\downarrow \; 3780$$

$$\frac{-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$-\frac{3}{2}a\left(\frac{-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan}}{}$$
$$\overline{\qquad\qquad a^2c^3}$$

3.635. $\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
(-1/2*x/(a*(1 + a^2*x^2)*ArcTan[a*x]^2) - (1 - a^2*x^2)/(2*a^2*(1 + a^2*x^
2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[a*x]]/a^2)/(a^2*c^3) - (-1/2*x/(a*(
1 + a^2*x^2)^2*ArcTan[a*x]^2) - (3*a*((-(1/(a*(1 + a^2*x^2)*ArcTan[a*x]))
- SinIntegral[2*ArcTan[a*x]]/a)/a^2 - (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])
) - (4*(SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/a
^2))/2 + (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[
a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/(2*a))/(a^2*c^3)
```

3.635.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.635.4 Maple [A] (verified)

Time = 10.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{16\,\mathrm{Si}(4\arctan(ax))\arctan(ax)^2-8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2+4\cos(4\arctan(ax))\arctan(ax)-4\cos(2\arctan(ax))\,a}{16a^4c^3\arctan(ax)^2}$
default	$\frac{16\,\mathrm{Si}(4\arctan(ax))\arctan(ax)^2-8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2+4\cos(4\arctan(ax))\arctan(ax)-4\cos(2\arctan(ax))\,a}{16a^4c^3\arctan(ax)^2}$

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/a^4/c^3*(16*Si(4*arctan(a*x))*arctan(a*x)^2-8*Si(2*arctan(a*x))*arcta
n(a*x)^2+4*cos(4*arctan(a*x))*arctan(a*x)-4*cos(2*arctan(a*x))*arctan(a*x)
-2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^2
```

3.635.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx =$$
$$-\frac{2\,a^3x^3+2\left(-i\,a^4x^4-2i\,a^2x^2-i\right)\arctan\left(ax\right)^2\log\_integral\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2\left(i\,a^4x^4+2i\right.}{}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^3*x^3 + 2*(-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integ
ral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2
 + 1)) + 2*(I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral((a^4*x
^4 - 4*I*a^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (
-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a
*x - 1)/(a^2*x^2 + 1)) - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_i
ntegral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*(a^4*x^4 - 3*a^2*x^2)*
arctan(a*x))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x)^2)
```

3.635.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \frac{\int \frac{x^3}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*
x**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

3.635.7 Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^3 + 2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*inte
grate((5*a^2*x^3 - 3*x)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^
2*c^3)*arctan(a*x)), x) - (a^2*x^4 - 3*x^2)*arctan(a*x))/((a^6*c^3*x^4 + 2
*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

3.635.8 Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{x^3}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.636 $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$

3.636.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\begin{aligned}\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = {} & \frac{1}{2a^3c^3(1+a^2x^2)^2 \arctan(ax)^2} \\ & - \frac{1}{2a^3c^3(1+a^2x^2) \arctan(ax)^2} \\ & - \frac{2x}{a^2c^3(1+a^2x^2)^2 \arctan(ax)} \\ & + \frac{x}{a^2c^3(1+a^2x^2) \arctan(ax)} + \frac{\text{CosIntegral}(4\arctan(ax))}{a^3c^3}\end{aligned}$$

output
```
1/2/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-1/2/a^3/c^3/(a^2*x^2+1)/arctan(a*x
)^2-2*x/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)+x/a^2/c^3/(a^2*x^2+1)/arctan(a*x
)+Ci(4*arctan(a*x))/a^3/c^3
```

3.636.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{\frac{ax(-ax+2(-1+a^2x^2)\arctan(ax))}{(1+a^2x^2)^2 \arctan(ax)^2} + 2\,\text{CosIntegral}(4\arctan(ax))}{2a^3c^3}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
((a*x*(-(a*x) + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a*x
]^2) + 2*CosIntegral[4*ArcTan[a*x]])/(2*a^3*c^3)
```

3.636.3 Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5499, 27, 5437, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^3}-\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^3}dx}{a^2c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$$\Big\downarrow \text{5439}$$

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax\ldots}}{a^2c^3}$$

$$\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{\ldots}{2\ldots}}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax\ldots}}{a^2c^3}$$

$$\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\operatorname{arcta}\ldots}\right.}{a^2c^3}$$

$$\Big\downarrow \text{5505}$$

3.636. $\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$\frac{-a\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1)^2}\right.}{a^2c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(a}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1)^2}\right.}{a^2c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-a\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1)^2}\right.}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1)^2}\right.}{a^2c^3}$$

$$\Big\downarrow \text{4906}$$

3.636. $\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a}$$
$$\overline{\qquad a^2c^3 \qquad}$$

$$-2a\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a}\right.$$
$$\overline{\qquad a^2c^3 \qquad}$$

$$\Big\downarrow \text{2009}$$

$$-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a}$$
$$\overline{\qquad a^2c^3 \qquad}$$

$$-2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\right.$$
$$\overline{\qquad a^2c^3 \qquad}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-((-1/2*1/(a*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 2*a*(-(x/(a*(1 + a^2*x^2)^2*
ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]]/8))
/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + CosIntegral[4*ArcTan[a*x]]/8 + (3*L
og[ArcTan[a*x]])/8)/a^2))/(a^2*c^3)) + (-1/2*1/(a*(1 + a^2*x^2)*ArcTan[a*x
]^2) - a*(-(x/(a*(1 + a^2*x^2)*ArcTan[a*x])) - (-1/2*CosIntegral[2*ArcTan[
a*x]] + Log[ArcTan[a*x]]/2)/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcT
an[a*x]]/2)/a^2))/(a^2*c^3)
```

3.636.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.636. $\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.636. $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.636.4 Maple [A] (verified)

Time = 10.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

method	result	size
derivativedivides	$\frac{16\ \mathrm{Ci}(4\arctan(ax))\arctan(ax)^{2}-4\sin(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{16a^{3}c^{3}\arctan(ax)^{2}}$	52
default	$\frac{16\ \mathrm{Ci}(4\arctan(ax))\arctan(ax)^{2}-4\sin(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{16a^{3}c^{3}\arctan(ax)^{2}}$	52

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/a^3/c^3*(16*Ci(4*arctan(a*x))*arctan(a*x)^2-4*sin(4*arctan(a*x))*arct
an(a*x)+cos(4*arctan(a*x))-1)/arctan(a*x)^2
```

3.636.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = -\frac{a^2x^2-(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}^2\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-(a^4x^4+2\,a^2x^2+1)\,a}{2\,(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3)\,\mathrm{arc}}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*
x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) -
(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^
3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 2*(a^3*x^3 - a*x
)*arctan(a*x))/((a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x)^2)
```

3.636.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = \frac{\int \frac{x^2}{a^6x^6 \operatorname{atan}^3(ax)+3a^4x^4 \operatorname{atan}^3(ax)+3a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*
x**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

3.636.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)^3} dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((a^
4*x^4 - 6*a^2*x^2 + 1)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^2
*c^3)*arctan(a*x)), x) - a*x^2 + 2*(a^2*x^3 - x)*arctan(a*x))/((a^6*c^3*x^
4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

3.636.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)^3} dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^3} \, dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3 \left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.637 $\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx$

3.637.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = -\frac{x}{2ac^3(1+a^2x^2)^2\arctan(ax)^2} - \frac{2}{a^2c^3(1+a^2x^2)^2\arctan(ax)} + \frac{3}{2a^2c^3(1+a^2x^2)\arctan(ax)} - \frac{\mathrm{Si}(2\arctan(ax))}{2a^2c^3} - \frac{\mathrm{Si}(4\arctan(ax))}{a^2c^3}$$

output
```
-1/2*x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-2/a^2/c^3/(a^2*x^2+1)^2/arctan(a*
x)+3/2/a^2/c^3/(a^2*x^2+1)/arctan(a*x)-1/2*Si(2*arctan(a*x))/a^2/c^3-Si(4*
arctan(a*x))/a^2/c^3
```

3.637.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = -\frac{ax+\arctan(ax)-3a^2x^2\arctan(ax)+(1+a^2x^2)^2\arctan(ax)^2\mathrm{Si}(2\arctan(ax))+2(1+a^2x^2)^2\arctan(}{2a^2c^3(1+a^2x^2)^2\arctan(ax)^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-1/2*(a*x + ArcTan[a*x] - 3*a^2*x^2*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a
*x]^2*SinIntegral[2*ArcTan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*SinInte
gral[4*ArcTan[a*x]])/(a^2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

3.637.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.72, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5503, 27, 5437, 5499, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{\int \frac{1}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2a}-\frac{3}{2}a\int \frac{x^2}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}dx-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2ac^3}-\frac{3a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{3a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2c^3}+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 5499}$$

$$-\frac{3a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{a^2}-\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{a^2}\right)}{2c^3}+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 5437}$$

$$\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{3a\left(\frac{-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2}-\frac{-4a\int\frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2}\right)}{2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{-\frac{4\int \frac{ax}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{3a\left(\frac{-\frac{2\int\frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2}-\frac{-\frac{4\int\frac{ax}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2}\right)}{2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{3a\left(\frac{-\frac{2\int\frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2}-\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2}\right)}{2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{3a\left(\frac{-\frac{\int\frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2}-\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2}\right)}{2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 2009}$$

3.637. $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}dx$

$$-\frac{3a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)}{2c^3}+\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{3a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)}{2c^3}+\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \textbf{3780}$$

$$-\frac{3a\left(\frac{-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)}{2c^3}+\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-1/2*x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - (3*a*((-(1/(a*(1 + a^2*x^2)
*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/a)/a^2 - (-(1/(a*(1 + a^2*x^2)
^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan
[a*x]]/8))/a)/a^2))/(2*c^3) + (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(
SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/(2*a*c^3)
```

3.637. $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

3.637.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.637. $\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.637.4 Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{16\ \mathrm{Si}(4\arctan(ax))\arctan(ax)^2+8\ \mathrm{Si}(2\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)+4\cos(4\arctan(ax)}{16a^2c^3\arctan(ax)^2}$
default	$-\frac{16\ \mathrm{Si}(4\arctan(ax))\arctan(ax)^2+8\ \mathrm{Si}(2\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)+4\cos(4\arctan(ax)}{16a^2c^3\arctan(ax)^2}$

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16/a^2/c^3*(16*Si(4*arctan(a*x))*arctan(a*x)^2+8*Si(2*arctan(a*x))*arct
an(a*x)^2+4*cos(2*arctan(a*x))*arctan(a*x)+4*cos(4*arctan(a*x))*arctan(a*x
)+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^2
```

3.637.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.81

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx =$$
$$-\frac{2\left(i\,a^4x^4+2i\,a^2x^2+i\right)\arctan\left(ax\right)^2\log\_integral\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2\left(-i\,a^4x^4-2i\,a^2x^2-i\right)}{}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral((a^4*x^4
+ 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*(-
I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (-I*a^4*x^4 -
 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2
*x^2 + 1)) - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^
2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) + 2*a*x - 2*(3*a^2*x^2 - 1)*arctan(a*x
))/((a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

3.637.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \frac{\int \frac{x}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**
2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

3.637.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^3\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((3*
a^2*x^3 - 5*x)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan
(a*x)), x) - a*x + (3*a^2*x^2 - 1)*arctan(a*x))/((a^6*c^3*x^4 + 2*a^4*c^3*
x^2 + a^2*c^3)*arctan(a*x)^2)
```

3.637.8 Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x}{(a^2cx^2+c)^3 \arctan(ax)^3} \, dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.637.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x}{\operatorname{atan}(a\,x)^3 \,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.638 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$

3.638.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = -\frac{1}{2ac^3(1+a^2x^2)^2 \arctan(ax)^2} + \frac{2x}{c^3(1+a^2x^2)^2 \arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{ac^3} - \frac{\text{CosIntegral}(4\arctan(ax))}{ac^3}$$

output
```
-1/2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+2*x/c^3/(a^2*x^2+1)^2/arctan(a*x)-C
i(2*arctan(a*x))/a/c^3-Ci(4*arctan(a*x))/a/c^3
```

3.638.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = -\frac{1-4ax\arctan(ax)+2(1+a^2x^2)^2\arctan(ax)^2\,\text{CosIntegral}(2\arctan(ax))+2(1+a^2x^2)^2\arctan(ax)^2\,\text{C}}{2ac^3(1+a^2x^2)^2\arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-1/2*(1 - 4*a*x*ArcTan[a*x] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[
2*ArcTan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[4*ArcTan[a*x]
])/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

3.638.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {5437, 27, 5503, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5437}$$

$$-2a\int \frac{x}{c^3 (a^2x^2+1)^3 \arctan(ax)^2}dx-\frac{1}{2ac^3 (a^2x^2+1)^2 \arctan(ax)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\int \frac{x}{(a^2x^2+1)^3 \arctan(ax)^2}dx}{c^3}-\frac{1}{2ac^3 (a^2x^2+1)^2 \arctan(ax)^2}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{2a\left(\frac{\int \frac{1}{(a^2x^2+1)^3 \arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2 \arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3 (a^2x^2+1)^2 \arctan(ax)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2 \arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2 \arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3 (a^2x^2+1)^2 \arctan(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2 \arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3 (a^2x^2+1)^2 \arctan(ax)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\arcta}\right.}{c^3}$$
$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 5505$$

$$-\frac{2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1)^2}\right.}{c^3}$$
$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 4906$$

$$-\frac{2a\left(-\frac{3\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\right.}{c^3}$$
$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\right.}{c^3}$$
$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

3.638. $\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

output
```
-1/2*1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - (2*a*(-(x/(a*(1 + a^2*x^2)^
2*ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]]/8
))/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + CosIntegral[4*ArcTan[a*x]]/8 + (3
*Log[ArcTan[a*x]])/8)/a^2))/c^3
```

3.638.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.638. $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.638.4 Maple [A] (verified)

Time = 11.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{16\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)^2+16\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-4\sin(4\arctan(ax))\arctan(ax)-8\sin(2\arctan(a}{16a\,c^3\arctan(ax)^2}$
default	$-\frac{16\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)^2+16\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-4\sin(4\arctan(ax))\arctan(ax)-8\sin(2\arctan(a}{16a\,c^3\arctan(ax)^2}$

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16/a/c^3*(16*Ci(4*arctan(a*x))*arctan(a*x)^2+16*Ci(2*arctan(a*x))*arcta
n(a*x)^2-4*sin(4*arctan(a*x))*arctan(a*x)-8*sin(2*arctan(a*x))*arctan(a*x)
+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^2
```

3.638.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.67

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx =$$

$$-\frac{(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2\,\text{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+(a^4x^4+2\,a^2x^2+1)\arctan(}{}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 + 4*I*
a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 +
 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*
x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*
arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*
x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(
a^2*x^2 + 1)) - 4*a*x*arctan(a*x) + 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c
^3)*arctan(a*x)^2)
```

3.638.6 Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx = \frac{\int \frac{1}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**
2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

3.638.7 Maxima [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)^2*integrate(2*(3*
a^2*x^2 - 1)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x) + 4*a*x*arctan(a*x) - 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*a
rctan(a*x)^2)
```

3.638.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.639 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3} dx$

3.639.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$
\begin{aligned}
\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3} dx = &-\frac{1}{2ac^3x\arctan(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\arctan(ax)^2} \\
&+ \frac{ax}{2c^3(1+a^2x^2)\arctan(ax)^2} \\
&+ \frac{2}{c^3(1+a^2x^2)^2\arctan(ax)} - \frac{3}{2c^3(1+a^2x^2)\arctan(ax)} \\
&+ \frac{1-a^2x^2}{2c^3(1+a^2x^2)\arctan(ax)} + \frac{3\operatorname{Si}(2\arctan(ax))}{2c^3} \\
&+ \frac{\operatorname{Si}(4\arctan(ax))}{c^3} - \frac{\operatorname{Int}\left(\frac{1}{x^2\arctan(ax)^2}, x\right)}{2ac^3}
\end{aligned}
$$

output
```
-1/2/a/c^3/x/arctan(a*x)^2+1/2*a*x/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+1/2*a*x
/c^3/(a^2*x^2+1)/arctan(a*x)^2+2/c^3/(a^2*x^2+1)^2/arctan(a*x)-3/2/c^3/(a^
2*x^2+1)/arctan(a*x)+1/2*(-a^2*x^2+1)/c^3/(a^2*x^2+1)/arctan(a*x)+3/2*Si(2
*arctan(a*x))/c^3+Si(4*arctan(a*x))/c^3-1/2*Unintegrable(1/x^2/arctan(a*x)
^2,x)/a/c^3
```

3.639.2 Mathematica [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

3.639.3 Rubi [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 27, 5501, 5461, 5377, 5467, 5503, 5437, 5499, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^3\left(a^2cx^2+c\right)^3}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x\left(a^2x^2+1\right)\arctan(ax)^3}dx-a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \; 5377$$

$$\frac{a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \; 5467$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}$$
$$\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \; 5503$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}$$
$$\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2a}-\frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5437$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}$$
$$\frac{a^2\left(-\frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5499$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}$$
$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{a^2}-\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{a^2}\right)+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5437$$

3.639. $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$\frac{-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}$$

$$\frac{a^2\left(\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-2a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3}}{}\right.\right.}{c^3}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{-\left(a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\arctan}}{c^3}$$

$$\frac{a^2\left(\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{4\int}{}}{}\right.\right.}{c^3}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(}}{c^3}$$

$$\frac{a^2\left(\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax}}{a^2}\right.\right.}{c^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax}}{c^3}$$

$$\frac{a^2\left(\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}\right.\right.}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

3.639. $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax}}{c^3}$$

$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\mathrm{Si}(2\arctan(ax))+\frac{1}{8}\mathrm{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+ -}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax}}{c^3}$$

$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\mathrm{Si}(2\arctan(ax))+\frac{1}{8}\mathrm{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+ -}{c^3}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\mathrm{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{1}{2ax\arctan(ax)^2}}{c^3}-$$

$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}-\frac{\mathrm{Si}(2\arctan(ax))}{a}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\mathrm{Si}(2\arctan(ax))+\frac{1}{8}\mathrm{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-\frac{1}{a\left(a^2x^2+1\right)^2}}{}}{c^3}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.639. $\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx$

3.639.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.639. $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.639. $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$

3.639.4 Maple [N/A] (verified)

Not integrable

Time = 5.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3} \arctan\left(a x\right)^{3}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.639.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^{2} c x^{2} + c\right)^{3} x \arctan\left(a x\right)^{3}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a
*x)^3), x)
```

3.639.6 Sympy [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\frac{\int \frac{1}{a^{6} x^{7} \operatorname{atan}^{3}{\left(a x \right)} + 3 a^{4} x^{5} \operatorname{atan}^{3}{\left(a x \right)} + 3 a^{2} x^{3} \operatorname{atan}^{3}{\left(a x \right)} + x \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**3 + 3*a**4*x**5*atan(a*x)**3 + 3*a**2*x**
3*atan(a*x)**3 + x*atan(a*x)**3), x)/c**3
```

3.639.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.91

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^3 x \arctan(ax)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2*integrate
((10*a^4*x^4 + 3*a^2*x^2 + 1)/((a^8*c^3*x^9 + 3*a^6*c^3*x^7 + 3*a^4*c^3*x^
5 + a^2*c^3*x^3)*arctan(a*x)), x) - a*x + (5*a^2*x^2 + 1)*arctan(a*x))/((a
^6*c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2)
```

3.639.8 Giac [N/A]

Not integrable

Time = 180.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^3 x \arctan(ax)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.639.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.640 $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

3.640.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx = &-\frac{1}{2ac^3x^2\arctan(ax)^2}+\frac{a}{2c^3(1+a^2x^2)^2\arctan(ax)^2}\\&+\frac{a}{2c^3(1+a^2x^2)\arctan(ax)^2}\\&-\frac{2a^2x}{c^3(1+a^2x^2)^2\arctan(ax)}-\frac{a^2x}{c^3(1+a^2x^2)\arctan(ax)}\\&+\frac{2a\,\mathrm{CosIntegral}(2\arctan(ax))}{c^3}\\&+\frac{a\,\mathrm{CosIntegral}(4\arctan(ax))}{c^3}-\frac{\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)^2},x\right)}{ac^3}\end{aligned}$$

output
```
-1/2/a/c^3/x^2/arctan(a*x)^2+1/2*a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+1/2*a/c
^3/(a^2*x^2+1)/arctan(a*x)^2-2*a^2*x/c^3/(a^2*x^2+1)^2/arctan(a*x)-a^2*x/c
^3/(a^2*x^2+1)/arctan(a*x)+2*a*Ci(2*arctan(a*x))/c^3+a*Ci(4*arctan(a*x))/c
^3-Unintegrable(1/x^3/arctan(a*x)^2,x)/a/c^3
```

3.640.2 Mathematica [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

3.640.3 Rubi [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 27, 5437, 5501, 5437, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\arctan(ax)^3\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x^2\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c}-a^2\int \frac{1}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^2\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\left(-2a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx-\frac{1}{2a\left(a^2x^2+1\right)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx - a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^3} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right) - \frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a} - \frac{1}{2ax^2\arctan(ax)^2}}{c^3} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right) - \frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a} - \frac{1}{2ax^2\arctan(ax)^2}}{c^3} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\left(a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a} - a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{a(a^2x^2+1)\arctan(ax)}\right) - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right) - \cdots}{c^3}$$

$$\frac{a^2\left(-2a\left(\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{a} - 3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right) - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5439}$$

3.640. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$\frac{-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)}\right.}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)}\right.}{c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\ldots}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)\right.}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\ldots}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\ldots}\right.\right.}{c^3}$$

$$\Big\downarrow \text{5505}$$

3.640. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$-\frac{\left(a^2\left(-a\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x}\right.\right.}{c^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\left(a^2\left(-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2+1}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x}\right.\right.}{c^3}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\left(a^2\left(-a\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x}\right.\right.}{c^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{}{a}\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x}\right.\right.}{c^3}$$

$$\Big\downarrow \; 4906$$

3.640. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a} - \left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{}{a}\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\right.\right.}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a} - \left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{}{a}\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax\right.\right.}{a^2}}{c^3}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.640.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.640. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.640. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.640.4 Maple [N/A] (verified)

Not integrable

Time = 7.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.640.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan
(a*x)^3), x)
```

3.640.6 Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^6 x^8 \operatorname{atan}^3(ax) + 3 a^4 x^6 \operatorname{atan}^3(ax) + 3 a^2 x^4 \operatorname{atan}^3(ax) + x^2 \operatorname{atan}^3(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**3 + 3*a**4*x**6*atan(a*x)**3 + 3*a**2*x**
4*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c**3
```

3.640.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 7.95

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2*integrate
((15*a^4*x^4 + 10*a^2*x^2 + 3)/((a^8*c^3*x^10 + 3*a^6*c^3*x^8 + 3*a^4*c^3*
x^6 + a^2*c^3*x^4)*arctan(a*x)), x) - a*x + 2*(3*a^2*x^2 + 1)*arctan(a*x))
/((a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2)
```

3.640.8 Giac [N/A]

Not integrable

Time = 187.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.640.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^3 \,(c\,a^2\,x^2 + c)^3} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.641 $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

3.641.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^3}\,dx = &-\frac{1}{2ac^3x^3\arctan(ax)^2}+\frac{a}{c^3x\arctan(ax)^2}\\ &-\frac{a^3x}{2c^3(1+a^2x^2)^2\arctan(ax)^2}\\ &-\frac{a^3x}{c^3(1+a^2x^2)\arctan(ax)^2}-\frac{2a^2}{c^3(1+a^2x^2)^2\arctan(ax)}\\ &+\frac{3a^2}{2c^3(1+a^2x^2)\arctan(ax)}-\frac{a^2(1-a^2x^2)}{c^3(1+a^2x^2)\arctan(ax)}\\ &-\frac{5a^2\mathrm{Si}(2\arctan(ax))}{2c^3}-\frac{a^2\mathrm{Si}(4\arctan(ax))}{c^3}\\ &-\frac{3\mathrm{Int}\left(\frac{1}{x^4\arctan(ax)^2},x\right)}{2ac^3}+\frac{a\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)^2},x\right)}{c^3}\end{aligned}$$

output
```
-1/2/a/c^3/x^3/arctan(a*x)^2+a/c^3/x/arctan(a*x)^2-1/2*a^3*x/c^3/(a^2*x^2+
1)^2/arctan(a*x)^2-a^3*x/c^3/(a^2*x^2+1)/arctan(a*x)^2-2*a^2/c^3/(a^2*x^2+
1)^2/arctan(a*x)+3/2*a^2/c^3/(a^2*x^2+1)/arctan(a*x)-a^2*(-a^2*x^2+1)/c^3/
(a^2*x^2+1)/arctan(a*x)-5/2*a^2*Si(2*arctan(a*x))/c^3-a^2*Si(4*arctan(a*x)
)/c^3-3/2*Unintegrable(1/x^4/arctan(a*x)^2,x)/a/c^3+a*Unintegrable(1/x^2/a
rctan(a*x)^2,x)/c^3
```

3.641.2 Mathematica [N/A]

Not integrable

Time = 3.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

3.641.3 Rubi [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5501, 5461, 5377, 5501, 5461, 5377, 5467, 5503, 5437, 5499, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{1}{c^2x^3(a^2x^2+1)^2 \arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{c^3x (a^2x^2+1)^3 \arctan(ax)^3}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)^2 \arctan(ax)^3}dx}{c^3} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^3 \arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1) \arctan(ax)^3}dx - a^2 \int \frac{1}{x(a^2x^2+1)^2 \arctan(ax)^3}dx}{c^3} - \frac{a^2 \left(\int \frac{1}{x(a^2x^2+1)^2 \arctan(ax)^3}dx - a^2 \int \frac{x}{(a^2x^2+1)^3 \arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\left(a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx+\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5467}$$

3.641. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$-\Bigg(a^2\Bigg(-\Bigg(a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\Bigg)\Bigg)-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\ \mathrm{ar}\cdots}$$
$$\overline{\qquad\qquad\qquad\qquad c^3\qquad\qquad}$$

$$a^2\Bigg(a^2\Bigg(-\int\frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\Bigg)-a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\Bigg)\cdots$$
$$\overline{\qquad\qquad\qquad\qquad c^3\qquad\qquad}$$

$$\Big\downarrow \; 5503$$

$$-\Bigg(a^2\Bigg(-\Bigg(a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\Bigg)\Bigg)-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\ \mathrm{ar}\cdots}$$
$$\overline{\qquad\qquad\qquad\qquad c^3\qquad\qquad}$$

$$a^2\Bigg(-\Bigg(a^2\Bigg(\frac{\int\frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{2a}-\frac{3}{2}a\int\frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}\Bigg)\Bigg)\Bigg)-a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\mathrm{arc}\cdots}$$
$$\overline{\qquad\qquad\qquad\qquad c^3\qquad\qquad}$$

$$\Big\downarrow \; 5437$$

$$-\Bigg(a^2\Bigg(-\Bigg(a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\Bigg)\Bigg)-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\ \mathrm{ar}\cdots}$$
$$\overline{\qquad\qquad\qquad\qquad c^3\qquad\qquad}$$

$$a^2\Bigg(-\Bigg(a^2\Bigg(-\frac{3}{2}a\int\frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx+\frac{-4a\int\frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2a}-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}\Bigg)\Bigg)\cdots$$

$$\Big\downarrow \; 5499$$

$$-\Bigg(a^2\Bigg(-\Bigg(a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\Bigg)\Bigg)-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\ \mathrm{ar}\cdots}$$
$$\overline{\qquad\qquad\qquad\qquad c^3\qquad\qquad}$$

$$a^2\Bigg(-\Bigg(a^2\Bigg(-\frac{3}{2}a\Bigg(\frac{\int\frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2}-\frac{\int\frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2}\Bigg)+\frac{-4a\int\frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2a}\cdots$$

$$\Big\downarrow \; 5437$$

$$-\Bigg(a^2\Bigg(-\Bigg(a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\Bigg)\Bigg)-\frac{\int\frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\ \mathrm{ar}\cdots}$$
$$\overline{\qquad\qquad\qquad\qquad c^3\qquad\qquad}$$

$$a^2\Bigg(-\Bigg(a^2\Bigg(-2\int\frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\Bigg)\Bigg)-a^2\Bigg(\frac{-4a\int\frac{x}{(a^2x^2+1)^3\arctan(ax)}\cdots}{2\cdots}$$

3.641. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$\downarrow$ 5505

$$-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}$$

$$c^3$$

$$a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\operatorname{arc}}}{}\right.\right.$$

$\downarrow$ 4906

$$-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2a}$$

$$c^3$$

$$a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\right.}{}\right.\right.$$

$\downarrow$ 27

$$-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}$$

$$c^3$$

$$a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\sin\right.}{}\right.\right.$$

$\downarrow$ 2009

$$-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}$$

$$c^3$$

$$a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right.\right.\right.$$

$\downarrow$ 3042

3.641. $\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}\right.\right.}{c^3}$$

$$\frac{a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right.\right.\right.}{}$$

$$\Big\downarrow \; 3780$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{1}{2ax\arctan(ax)^2}\right)\right.}{c^3}$$

$$\frac{a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{}{a\left(a^2x^2\right.}}{}\right.\right.\right.}{}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.641.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.641. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

3.641. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.641.4 Maple [N/A] (verified)

Not integrable

Time = 84.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \left(a^2 c\, x^2+c\right)^3 \arctan \left(a x\right)^3}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.641.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^3 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan
(a*x)^3), x)
```

3.641.6 Sympy [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^6 x^9 \operatorname{atan}^3(ax) + 3a^4 x^7 \operatorname{atan}^3(ax) + 3a^2 x^5 \operatorname{atan}^3(ax) + x^3 \operatorname{atan}^3(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**3 + 3*a**4*x**7*atan(a*x)**3 + 3*a**2*x**
5*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c**3
```

3.641.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.91

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^3 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2*integrate
((21*a^4*x^4 + 19*a^2*x^2 + 6)/((a^8*c^3*x^11 + 3*a^6*c^3*x^9 + 3*a^4*c^3*
x^7 + a^2*c^3*x^5)*arctan(a*x)), x) - a*x + (7*a^2*x^2 + 3)*arctan(a*x))/(
(a^6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2)
```

3.641.8 Giac [N/A]

Not integrable

Time = 184.69 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.641.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^3 \operatorname{atan}{(a\,x)}^3 \,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.642 $\int \frac{1}{x^4(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$

3.642.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$
\begin{aligned}
\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = & -\frac{1}{2ac^3x^4\arctan(ax)^2} + \frac{a}{c^3x^2\arctan(ax)^2} \\
& - \frac{a^3}{2c^3(1+a^2x^2)^2\arctan(ax)^2} \\
& - \frac{a^3}{c^3(1+a^2x^2)\arctan(ax)^2} \\
& + \frac{2a^4x}{c^3(1+a^2x^2)^2\arctan(ax)} + \frac{2a^4x}{c^3(1+a^2x^2)\arctan(ax)} \\
& - \frac{3a^3\,\mathrm{CosIntegral}(2\arctan(ax))}{c^3} \\
& - \frac{a^3\,\mathrm{CosIntegral}(4\arctan(ax))}{c^3} \\
& - \frac{2\mathrm{Int}\left(\frac{1}{x^5\arctan(ax)^2}, x\right)}{ac^3} + \frac{2a\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)^2}, x\right)}{c^3}
\end{aligned}
$$

output
```
-1/2/a/c^3/x^4/arctan(a*x)^2+a/c^3/x^2/arctan(a*x)^2-1/2*a^3/c^3/(a^2*x^2+
1)^2/arctan(a*x)^2-a^3/c^3/(a^2*x^2+1)/arctan(a*x)^2+2*a^4*x/c^3/(a^2*x^2+
1)^2/arctan(a*x)+2*a^4*x/c^3/(a^2*x^2+1)/arctan(a*x)-3*a^3*Ci(2*arctan(a*x
))/c^3-a^3*Ci(4*arctan(a*x))/c^3-2*Unintegrable(1/x^5/arctan(a*x)^2,x)/a/c
^3+2*a*Unintegrable(1/x^3/arctan(a*x)^2,x)/c^3
```

3.642.2 Mathematica [N/A]

Not integrable

Time = 8.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

3.642.3 Rubi [N/A]

Not integrable

Time = 4.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 27, 5501, 5437, 5461, 5377, 5501, 5437, 5461, 5377, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^3 (a^2cx^2 + c)^3} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{c^2x^4(a^2x^2+1)^2 \arctan(ax)^3} dx}{c} - a^2 \int \frac{1}{c^3x^2 (a^2x^2 + 1)^3 \arctan(ax)^3} dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)^2 \arctan(ax)^3} dx}{c^3} - \frac{a^2 \int \frac{1}{x^2(a^2x^2+1)^3 \arctan(ax)^3} dx}{c^3}$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1) \arctan(ax)^3} dx - a^2 \int \frac{1}{x^2(a^2x^2+1)^2 \arctan(ax)^3} dx}{c^3} - \frac{a^2 \left(\int \frac{1}{x^2(a^2x^2+1)^2 \arctan(ax)^3} dx - a^2 \int \frac{1}{(a^2x^2+1)^3 \arctan(ax)^3} dx\right)}{c^3}$$

$$\Big\downarrow \ 5437$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^3}dx - a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5461$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx\right) - \frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a} - \frac{1}{2ax^4\arctan(ax)^2}}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5377$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx\right) - \frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a} - \frac{1}{2ax^4\arctan(ax)^2}}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5501$$

$$\frac{-\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx - a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)\right) - \frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a} - \frac{1}{2ax^4\arctan(ax)^2}}{c^3} - \frac{a^2\left(a^2\left(-\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx\right) - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right) + \int \frac{1}{x^2(a^2x^2+1)\,\mathrm{arctan}\cdots}\cdots\right.}{c^3}$$

$$\Big\downarrow \ 5437$$

$$\frac{-\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)\right) - \frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a} - \cdots}{c^3}$$

$$- \frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right) - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\,\mathrm{arc}\cdots}\right.\right.}{c^3}$$

$$\Big\downarrow \ 5461$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right) - \frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a} - \frac{1}{2ax^2\arctan(ax)^2}\right)\right) - \frac{2\int \frac{1}{x^5\,\mathrm{a}\cdots}}{\cdots}}{c^3}$$

$$- \frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right) - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\,\mathrm{arc}\cdots}\right.\right.}{c^3}$$

3.642. $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$\big\downarrow$ 5377

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}\right)\right)-\frac{2\int \frac{}{x^5 a}}{}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)-a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\,\mathrm{arc}}\right.\right.}{c^3}$$

$\big\downarrow$ 5503

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right.\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\left(\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right.\right.\right.}{}$$

$\big\downarrow$ 5439

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\,\mathrm{ar}}\right.\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{}{2a(a^2x^2+1)^{}}\right.\right.\right.}{}$$

$\big\downarrow$ 3042

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\,\mathrm{a}}\right.\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\,\mathrm{arctar}}\right.\right.\right.}{}$$

$\big\downarrow$ 3793

3.642. $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a}\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2}\right.\right.\right.$$

$$\downarrow \text{2009}$$

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(}\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a}\right.\right.\right.\right.$$

$$\downarrow \text{5505}$$

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-\frac{\int\frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-2a\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\right.\right.\right.\right.$$

$$\downarrow \text{3042}$$

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-\frac{\int\frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2}\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-2a\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\right.\right.\right.\right.$$

$$\downarrow \text{3793}$$

3.642. $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\left(-\frac{\int\left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\operatorname{arc}}\right.\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-2a\left(-\frac{3\int\frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2\left(-\frac{\int\frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-a^2\left(-2a\left(-\frac{3\int\frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\right.\right.\right.$$

$$\Big\downarrow \text{4906}$$

$$\frac{-\left(a^2\left(-\frac{\int\frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-a^2\left(-2a\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\operatorname{arcta}}{a^2}\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2\left(-\frac{\int\frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\frac{\int\frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}}{a^2}\right.\right.\right.\right.$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.642. $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

3.642.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.642. $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.642.4 Maple [N/A] (verified)

Not integrable

Time = 89.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(a^2 c\,x^2+c\right)^3 \arctan\left(ax\right)^3}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.642.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^4 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arcta
n(a*x)^3), x)
```

3.642.6 Sympy [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^6x^{10}\operatorname{atan}^3(ax) + 3a^4x^8\operatorname{atan}^3(ax) + 3a^2x^6\operatorname{atan}^3(ax) + x^4\operatorname{atan}^3(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**3 + 3*a**4*x**8*atan(a*x)**3 + 3*a**2*x*
*6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c**3
```

3.642.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 176, normalized size of antiderivative = 8.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^4 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2*integrate
(2*(14*a^4*x^4 + 15*a^2*x^2 + 5)/((a^8*c^3*x^12 + 3*a^6*c^3*x^10 + 3*a^4*c
^3*x^8 + a^2*c^3*x^6)*arctan(a*x)), x) - a*x + 4*(2*a^2*x^2 + 1)*arctan(a*
x))/((a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2)
```

3.642.8 Giac [N/A]

Not integrable

Time = 185.35 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.642.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^3 \,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.643 $\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx$

3.643.1 Optimal result

Integrand size = 38, antiderivative size = 16

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx=-\frac{x^3}{2a\arctan(ax)^2}$$

output
```
-1/2*x^3/a/arctan(a*x)^2
```

3.643.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx=-\frac{x^3}{2a\arctan(ax)^2}$$

input
```
Integrate[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),
x]
```

output
```
-1/2*x^3/(a*ArcTan[a*x]^2)
```

3.643.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^3}{(a^2x^2+1)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{x^3}{2a\arctan(ax)^2}$$

input
```
Int[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),x]
```

output
```
-1/2*x^3/(a*ArcTan[a*x]^2)
```

3.643.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.643.4 Maple [A] (verified)

Time = 114.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
parallelrisch	$-\frac{x^3}{2a\arctan(ax)^2}$
risch	$\frac{2x^3}{a(\ln(-iax+1)-\ln(iax+1))^2}$
default	$-\frac{x^3}{2a\arctan(ax)^2}-\frac{x^2}{2a^2\arctan(ax)^3}-\frac{1}{2a^4\arctan(ax)^3}-\frac{3x}{2a^3\arctan(ax)^4}-\frac{3i}{2a^4\arctan(ax)^4}+\frac{(-i\arctan(ax)+x\,\text{ar}}{2\,\text{arctan}}$
parts	$-\frac{x^3}{2a\arctan(ax)^2}-\frac{x^2}{2a^2\arctan(ax)^3}-\frac{1}{2a^4\arctan(ax)^3}-\frac{3x}{2a^3\arctan(ax)^4}-\frac{3i}{2a^4\arctan(ax)^4}+\frac{(-i\arctan(ax)+x\,\text{ar}}{2\,\text{arctan}}$

input
```
int(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x,method=_RETURN
VERBOSE)
```

output
```
-1/2*x^3/a/arctan(a*x)^2
```

3.643.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx=-\frac{x^3}{2\,a\arctan\left(ax\right)^2}$$

input
```
integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorit
hm="fricas")
```

output
```
-1/2*x^3/(a*arctan(a*x)^2)
```

3.643.6 Sympy [F]

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx= -\frac{\int\left(-\frac{2ax^3}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\right)dx+\int\frac{3x^2\operatorname{atan}(ax)}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx+\int\frac{3a^2x^4\operatorname{atan}(ax)}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{2a}$$

input
```
integrate(x**3/(a**2*x**2+1)/atan(a*x)**3-3/2*x**2/a/atan(a*x)**2,x)
```

output
```
-(Integral(-2*a*x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integra
l(3*x**2*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integral(
3*a**2*x**4*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x))/(2*a)
```

3.643.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx=-\frac{x^3}{2\,a\arctan\left(ax\right)^2}$$

input
```
integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorit
hm="maxima")
```

output
```
-1/2*x^3/(a*arctan(a*x)^2)
```

3.643.8 Giac [F]

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx$$
$$=\int \frac{x^3}{(a^2x^2+1)\arctan\left(ax\right)^3}-\frac{3\,x^2}{2\,a\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorit
hm="giac")
```

output
```
sage0*x
```

3.643.9 Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx=-\frac{x^3}{2\,a\,\mathrm{atan}\left(a\,x\right)^2}$$

input
```
int(x^3/(atan(a*x)^3*(a^2*x^2 + 1)) - (3*x^2)/(2*a*atan(a*x)^2),x)
```

output
```
-x^3/(2*a*atan(a*x)^2)
```

3.644 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$

3.644.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

3.644.2 Mathematica [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

3.644.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.644.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.644.4 Maple [N/A] (verified)

Not integrable

Time = 33.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

3.644.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

3.644.6 Sympy [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

3.644.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

3.644.8 Giac [N/A]

Not integrable

Time = 78.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.644.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3, x)
```

3.645 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$

3.645.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx=\operatorname{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

3.645.2 Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3, x]
```

3.645.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^3} dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.645.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.645.4 Maple [N/A] (verified)

Not integrable

Time = 32.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan \left(a x \right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

3.645.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

3.645.6 Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

3.645.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

3.645.8 Giac [N/A]

Not integrable

Time = 77.58 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.645.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^3, x)
```

3.646 $\int \frac{\sqrt{c+a^2cx^2}}{x \arctan(ax)^3} \, dx$

3.646.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{c+a^2cx^2}}{x \arctan(ax)^3} \, dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x \arctan(ax)^3}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

3.646.2 Mathematica [N/A]

Not integrable

Time = 3.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x \arctan(ax)^3} \, dx = \int \frac{\sqrt{c+a^2cx^2}}{x \arctan(ax)^3} \, dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3), x]
```

3.646.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^3}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.646.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.646.4 Maple [N/A] (verified)

Not integrable

Time = 86.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

3.646.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

3.646.6 Sympy [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\,\mathrm{atan}^3\,(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**3), x)
```

3.646.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

3.646.8 Giac [N/A]

Not integrable

Time = 85.33 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.646.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^3), x)
```

3.647 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$

3.647.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}, x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.647.2 Mathematica [N/A]

Not integrable

Time = 6.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

3.647.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^3} dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^3} dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.647.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.647.4 Maple [N/A] (verified)

Not integrable

Time = 50.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^3} dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.647.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} \, dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^3} \, dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

3.647.6 Sympy [N/A]

Not integrable

Time = 6.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} \, dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^3(ax)} \, dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

3.647.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} \, dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^3} \, dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^3, x)
```

3.647.8 Giac [N/A]

Not integrable

Time = 97.47 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.647.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3, x)
```

3.648 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx$

3.648.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.648.2 Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3, x]
```

3.648.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.648.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.648.4 Maple [N/A] (verified)

Not integrable

Time = 66.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^3}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.648.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

3.648.6 Sympy [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

3.648.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

3.648.8 Giac [N/A]

Not integrable

Time = 94.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.648.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^3, x)
```

3.649 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx$

3.649.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

3.649.2 Mathematica [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3), x]
```

3.649.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.649.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.649.4 Maple [N/A] (verified)

Not integrable

Time = 85.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{x \arctan \left(a x \right)^{3}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

3.649.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

3.649.6 Sympy [N/A]

Not integrable

Time = 9.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\,\mathrm{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**3), x)
```

3.649.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

3.649.8 Giac [N/A]

Not integrable

Time = 100.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.649.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx=\int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^3), x)
```

3.650 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$

3.650.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.650.2 Mathematica [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

3.650.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.650.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.650.4 Maple [N/A] (verified)

Not integrable

Time = 108.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.650.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(
a*x)^3, x)
```

3.650.6 Sympy [N/A]

Not integrable

Time = 25.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**3, x)
```

3.650.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^3, x)
```

3.650.8 Giac [N/A]

Not integrable

Time = 111.69 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.650.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x\,{(c\,a^2\,x^2+c)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3, x)
```

3.651 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3} dx$

3.651.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3} dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.651.2 Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3} dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3} dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3, x]
```

3.651.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.651.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.651.4 Maple [N/A] (verified)

Not integrable

Time = 93.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^3}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.651.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*
x)^3, x)
```

3.651.6 Sympy [N/A]

Not integrable

Time = 14.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**3, x)
```

3.651.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^3, x)
```

3.651.8 Giac [N/A]

Not integrable

Time = 108.49 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.651.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^3, x)
```

3.652 $\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx$

3.652.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

3.652.2 Mathematica [N/A]

Not integrable

Time = 2.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3), x]
```

3.652.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.652.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.652.4 Maple [N/A] (verified)

Not integrable

Time = 177.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

3.652.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan
(a*x)^3), x)
```

3.652.6 Sympy [N/A]

Not integrable

Time = 19.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\,\mathrm{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**3), x)
```

3.652.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^3), x)
```

3.652.8 Giac [N/A]

Not integrable

Time = 112.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.652.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^3), x)
```

3.653 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$

3.653.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.653.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

3.653.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.653.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.653.4 Maple [N/A] (verified)

Not integrable

Time = 3.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\arctan\left(ax\right)^{3}\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.653.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.653.6 Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(x/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

3.653.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.653.8 Giac [N/A]

Not integrable

Time = 90.53 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.653.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.654 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$

3.654.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Unintegrable(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.654.2 Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

3.654.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.654.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.654.4 Maple [N/A] (verified)

Not integrable

Time = 7.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\arctan\left(ax\right)^{3}\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.654.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.654.6 Sympy [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(1/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

3.654.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.654.8 Giac [N/A]

Not integrable

Time = 86.55 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.654.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.655 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$

3.655.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = -\frac{\sqrt{c+a^2cx^2}}{2acx\arctan(ax)^2} - \frac{\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2a}$$

output
```
-1/2*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan
(a*x)^2/(a^2*c*x^2+c)^(1/2),x)/a
```

3.655.2 Mathematica [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

3.655.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5477, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^3 \sqrt{a^2cx^2+c}} \, dx$$
$$\Big\downarrow \text{5477}$$
$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}$$
$$\Big\downarrow \text{5560}$$
$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.655.3.1 Defintions of rubi rules used

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.655.4 Maple [N/A] (verified)

Not integrable

Time = 8.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \arctan\left(ax\right)^{3} \sqrt{a^{2} c\, x^{2}+c}}dx$$

input
```
int(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.655.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{a^{2} c x^{2} + c} x \arctan\left(a x\right)^{3}}\,dx$$

input
```
integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

3.655.6 Sympy [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

3.655.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

3.655.8 Giac [N/A]

Not integrable

Time = 88.76 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.655.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.656 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$

3.656.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Unintegrable(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.656.2 Mathematica [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

3.656.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{1}{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.656.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.656.4 Maple [N/A] (verified)

Not integrable

Time = 8.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \arctan(ax)^3\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.656.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

3.656.6 Sympy [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^2\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(1/x**2/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

3.656.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

3.656.8 Giac [N/A]

Not integrable

Time = 95.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.656.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.657 $\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$

3.657.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Unintegrable(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.657.2 Mathematica [N/A]

Not integrable

Time = 5.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

3.657.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x^3 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.657.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.657.4 Maple [N/A] (verified)

Not integrable

Time = 30.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 \arctan\left(ax\right)^3 \sqrt{a^2 c\,x^2+c}}dx$$

input
```
int(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.657.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^3\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

3.657.6 Sympy [N/A]

Not integrable

Time = 6.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^3\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^3\,(ax)}\,dx$$

input
```
integrate(1/x**3/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

3.657.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^3\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

3.657.8 Giac [N/A]

Not integrable

Time = 94.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^3\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^3/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.657.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.658 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2} \arctan(ax)^3} \, dx$

3.658.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2} \arctan(ax)^3} \, dx = \frac{x}{2a^3c\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\mathrm{Si}(\arctan(ax))}{2a^4c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^2c}$$

output
```
1/2*x/a^3/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2/a^4/c/arctan(a*x)/(a^2*c
*x^2+c)^(1/2)+1/2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^2+c)^(1
/2)+Unintegrable(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^2/c
```

3.658.2 Mathematica [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2} \arctan(ax)^3} \, dx = \int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2} \arctan(ax)^3} \, dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

3.658.3 Rubi [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5499, 5477, 5437, 5506, 5505, 3042, 3780, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5506}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \textbf{3780}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \textbf{5560}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.658.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

3.658. $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.658. $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.658.4 Maple [N/A] (verified)

Not integrable

Time = 3.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^3}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.658.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^3), x)
```

3.658.6 Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.658.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.658.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.658.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.659 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.659.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{1}{2a^3c\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{2a^3c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^2c}$$

output
```
1/2/a^3/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*x/a^2/c/arctan(a*x)/(a^2*c
*x^2+c)^(1/2)+1/2*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1
/2)+Unintegrable(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^2/c
```

3.659.2 Mathematica [N/A]

Not integrable

Time = 5.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

3.659.3 Rubi [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5499, 5437, 5477, 5440, 5439, 3042, 3783, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5499}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{ 5437}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{ 5477}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$\big\downarrow$ 3783

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$\big\downarrow$ 5560

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.659.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.659. $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.659. $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.659.4 Maple [N/A] (verified)

Not integrable

Time = 2.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^3}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.659.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^3), x)
```

3.659.6 Sympy [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.659.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.659.8 Giac [N/A]

Not integrable

Time = 79.78 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.659.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.660 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx$

3.660.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{x}{2ac\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{1}{2a^2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{2a^2c\sqrt{c+a^2cx^2}}$$

output
```
-1/2*x/a/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2/a^2/c/arctan(a*x)/(a^2*c*
x^2+c)^(1/2)-1/2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/
2)
```

3.660.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{ax+\arctan(ax)+\sqrt{1+a^2x^2}\arctan(ax)^2\mathrm{Si}(\arctan(ax))}{2a^2c\sqrt{c+a^2cx^2}\arctan(ax)^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*(a*x + ArcTan[a*x] + Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*SinIntegral[ArcT
an[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

3.660.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5477, 5437, 5506, 5505, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

3.660. $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3} \, dx$

output
```
-1/2*x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (-(1/(a*c*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a*c*Sqr
t[c + a^2*c*x^2]))/(2*a)
```

3.660.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.660. $\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.660.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.50

method	result
default	$-\frac{i\left(\arctan(ax)^2 \operatorname{Ei}_1(-i\arctan(ax))a^2x^2-\arctan(ax)^2 \operatorname{Ei}_1(i\arctan(ax))a^2x^2+\operatorname{Ei}_1(-i\arctan(ax))\arctan(ax)^2-\operatorname{Ei}_1(i\arctan(ax))\, a\right.}{4\left(a^2x^2+1\right)^{\frac{3}{2}}\arctan(ax)^2a^2c^2}$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2-arctan(a*x)^2*Ei(1,I*ar
ctan(a*x))*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-Ei(1,I*arctan(a*x))*
arctan(a*x)^2-2*I*(a^2*x^2+1)^(1/2)*a*x-2*I*(a^2*x^2+1)^(1/2)*arctan(a*x))
*(c*(a*x-I)*(I+a*x))^(1/2)/(a^2*x^2+1)^(3/2)/arctan(a*x)^2/a^2/c^2
```

3.660.5 Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan
(a*x)^3), x)
```

3.660.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{x}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.660.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.660.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x}{\mathrm{atan}\,(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.661 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.661.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{1}{2ac\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{2ac\sqrt{c+a^2cx^2}}$$

output
```
-1/2/a/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2*x/c/arctan(a*x)/(a^2*c*x^2+
c)^(1/2)-1/2*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

3.661.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{-1+ax\arctan(ax)-\sqrt{1+a^2x^2}\arctan(ax)^2\,\mathrm{CosIntegral}(\arctan(ax))}{2ac\sqrt{c+a^2cx^2}\arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
(-1 + a*x*ArcTan[a*x] - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*CosIntegral[ArcTan
[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

3.661.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5437, 5477, 5440, 5439, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5477}$$

$$-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - (a*(-(x/(a*c*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a^2*
c*Sqrt[c + a^2*c*x^2])))/2
```

3.661.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] && !(IntegerQ[q] || GtQ[d, 0])
```

3.661. $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.661.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

method	result
default	$\frac{\left(\arctan(ax)^2 \operatorname{Ei}_1(i\arctan(ax))a^2x^2+\arctan(ax)^2 \operatorname{Ei}_1(-i\arctan(ax))a^2x^2+2\arctan(ax)\sqrt{a^2x^2+1}\,ax+\operatorname{Ei}_1(i\arctan(ax))\arctan(ax)\right.}{4(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)^2 a\,c^2}$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+arctan(a*x)^2*Ei(1,-I*arcta
n(a*x))*a^2*x^2+2*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*ar
ctan(a*x)^2+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-2*(a^2*x^2+1)^(1/2))/(a^2*x
^2+1)^(3/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a/c^2
```

3.661.5 Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a
*x)^3), x)
```

3.661.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.661.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.661.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.662 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.662.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{ax}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^2x\arctan(ax)^2} + \frac{1}{2c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}} - \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2ac}$$

output
```
1/2*a*x/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2/c/arctan(a*x)/(a^2*c*x^2+c
)^(1/2)+1/2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*(a
^2*c*x^2+c)^(1/2)/a/c^2/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan(a*x)
^2/(a^2*c*x^2+c)^(1/2),x)/a/c
```

3.662.2 Mathematica [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

3.662.3 Rubi [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5477, 5437, 5506, 5505, 3042, 3780, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5506}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \textbf{3780}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \textbf{5560}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.662. $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.662.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.662. $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.662.4 Maple [N/A] (verified)

Not integrable

Time = 4.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.662.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^{2} c x^{2} + c\right)^{\frac{3}{2}} x \arctan\left(a x\right)^{3}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan
(a*x)^3), x)
```

3.662.6 Sympy [N/A]

Not integrable

Time = 7.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.662.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x\arctan(ax)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3), x)
```

3.662.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.662.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.663 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.663.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx &= \frac{a}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2}\\ &\quad-\frac{a^2x}{2c\sqrt{c+a^2cx^2}\arctan(ax)}+\frac{a\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}}\\ &\quad+\frac{\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c}\end{aligned}$$

output
```
1/2*a/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*a^2*x/c/arctan(a*x)/(a^2*c*x
^2+c)^(1/2)+1/2*a*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+
Unintegrable(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c
```

3.663.2 Mathematica [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

3.663.3 Rubi [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5437, 5477, 5440, 5439, 3042, 3783, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5440}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3783}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.663.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.663. $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.663. $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.663.4 Maple [N/A] (verified)

Not integrable

Time = 4.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.663.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(a x)^3} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{\frac {3}{2}} x^{2} \arctan \left (a x\right)^{3}} \,d x }$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arct
an(a*x)^3), x)
```

3.663.6 Sympy [N/A]

Not integrable

Time = 8.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(a x)^3} \, dx=\int \frac {1}{x^{2} \left (c \left (a^{2} x^{2} + 1\right)\right)^{\frac {3}{2}} \operatorname {atan}^{3}{\left (a x \right)}}\, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.663.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3} dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan{(ax)}^3} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3), x)
```

3.663.8 Giac [N/A]

Not integrable

Time = 82.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3} dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan{(ax)}^3} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.663.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3} dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^3 \left(c\,a^2\,x^2 + c\right)^{3/2}} dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.664 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.664.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx &= -\frac{a^3x}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2} \\ &+ \frac{a\sqrt{c+a^2cx^2}}{2c^2x\arctan(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{a^2\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}} \\ &+ \frac{\operatorname{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c} + \frac{a\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2c}\end{aligned}$$

output

```
-1/2*a^3*x/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*a^2/c/arctan(a*x)/(a^2*
c*x^2+c)^(1/2)-1/2*a^2*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(
1/2)+1/2*a*(a^2*c*x^2+c)^(1/2)/c^2/x/arctan(a*x)^2+Unintegrable(1/x^3/arct
an(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c+1/2*a*Unintegrable(1/x^2/arctan(a*x)^2/
(a^2*c*x^2+c)^(1/2),x)/c
```

3.664.2 Mathematica [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3}\,dx = \int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

3.664.3 Rubi [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5501, 5477, 5437, 5506, 5505, 3042, 3780, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{x (a^2cx^2+c)^{3/2} \arctan(ax)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2 \left(\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2 \int \frac{x}{(a^2cx^2+c)^{3/2} \arctan(ax)^3}dx \right)$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2ac\arc}\right.\right.$$

$$\Big\downarrow \text{5506}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2a}\right.\right.$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2a}\right.\right.$$

$$\Big\downarrow \text{3780}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\mathrm{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2ac\arctan(a}\right.\right.$$

3.664. $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

$$\Big\downarrow \text{ 5560}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2ac\arctan(a}\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.664.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.664.4 Maple [N/A] (verified)

Not integrable

Time = 36.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 \left(a^2 c\, x^2 + c\right)^{\frac{3}{2}} \arctan\left(a x\right)^3} dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.664.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x^3\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arct
an(a*x)^3), x)
```

3.664.6 Sympy [N/A]

Not integrable

Time = 14.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{x^3(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.664.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x^3\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3), x)
```

3.664.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.664.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^{3/2}} \, dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.665 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.665.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx &= -\frac{a^3}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2} \\ &+ \frac{a^4x}{2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{a^3\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}} \\ &+ \frac{\mathrm{Int}\left(\frac{1}{x^4\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c} - \frac{a^2\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c}\end{aligned}$$

output
```
-1/2*a^3/c/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2*a^4*x/c/arctan(a*x)/(a^2*
c*x^2+c)^(1/2)-1/2*a^3*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)^(
1/2)+Unintegrable(1/x^4/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c-a^2*Uninteg
rable(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c
```

3.665.2 Mathematica [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

3.665.3 Rubi [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 5501, 5437, 5477, 5440, 5439, 3042, 3783, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^3 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\int \frac{1}{x^2 (a^2cx^2+c)^{3/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{}{2ac\arctan(}\right.\right.$$

$$\Big\downarrow \text{5440}$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\right.\right.$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$\Big\downarrow \text{5439}$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$\Big\downarrow \text{3783}$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\right.\right.$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$\Big\downarrow \text{5560}$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\right.\right.$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.665. $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.665.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

3.665. $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.665.4 Maple [N/A] (verified)

Not integrable

Time = 31.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{3}}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.665.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx=\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} x^{4} \arctan \left(a x\right)^{3}} \,d x$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arct
an(a*x)^3), x)
```

3.665.6 Sympy [N/A]

Not integrable

Time = 22.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^3}\,dx = \int \frac{1}{x^4 \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**4*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.665.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^3}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}} x^4 \arctan(ax)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^3), x)
```

3.665.8 Giac [N/A]

Not integrable

Time = 82.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^3}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}} x^4 \arctan(ax)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.665.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.666 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.666.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx &= \frac{x^3}{2a^3c(c+a^2cx^2)^{3/2}\arctan(ax)^2} \\ &+ \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{3}{2a^6c(c+a^2cx^2)^{3/2}\arctan(ax)} \\ &+ \frac{2}{a^6c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{7\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{8a^6c^2\sqrt{c+a^2cx^2}} \\ &- \frac{9\sqrt{1+a^2x^2}\operatorname{Si}(3\arctan(ax))}{8a^6c^2\sqrt{c+a^2cx^2}} + \frac{\operatorname{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^4c^2}\end{aligned}$$

output
```
1/2*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2/a^6/c/(a^2*c*x^2+c)^(3
/2)/arctan(a*x)+1/2*x/a^5/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+2/a^6/c^2/
arctan(a*x)/(a^2*c*x^2+c)^(1/2)+7/8*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/
c^2/(a^2*c*x^2+c)^(1/2)-9/8*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^6/c^2/(a
^2*c*x^2+c)^(1/2)+Unintegrable(x/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^4/
c^2
```

3.666.2 Mathematica [N/A]

Not integrable

Time = 7.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

3.666.3 Rubi [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5499, 5477, 5499, 5437, 5477, 5437, 5506, 5505, 3042, 3780, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x^3}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ \textbf{5437}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ \textbf{5477}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ \textbf{5437}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ \textbf{5506}$$

3.666. $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\cdots}{a^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\cdots}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\cdots}{a^2}$$

$$\Big\downarrow \; 3780$$

3.666. $\displaystyle\int \frac{x^5}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{\cdots}{2ac\arctan\cdots}}{a^2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3\cdots}}}{a^2}\cdots\right)}{2a}\cdots}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{\cdots}{2ac\arctan\cdots}}{a^2}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{\cdots}{2ac\arctan\cdots}}{a^2}$$

3.666. $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

input
```
Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.666.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.666. $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.666.4 Maple [N/A] (verified)

Not integrable

Time = 19.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a^2 c\, x^2 + c)^{\frac{5}{2}} \arctan(ax)^3} dx$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.666.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

3.666.6 Sympy [N/A]

Not integrable

Time = 12.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.666.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.666.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.666.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^5/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^5/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.667 $\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

3.667.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx &= -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\arctan(ax)^2} \\ &+ \frac{1}{a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\arctan(ax)} \\ &- \frac{x}{a^4c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{5\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{8a^5c^2\sqrt{c+a^2cx^2}} \\ &- \frac{9\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{8a^5c^2\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^4c^2}\end{aligned}$$

output
```
-1/2/a^5/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2*x/a^4/c/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)+1/a^5/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-x/a^4/c^2/arcta
n(a*x)/(a^2*c*x^2+c)^(1/2)+5/8*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(
a^2*c*x^2+c)^(1/2)-9/8*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^5/c^2/(a^2*c*
x^2+c)^(1/2)+Unintegrable(1/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/a^4/c^2
```

3.667.2 Mathematica [N/A]

Not integrable

Time = 7.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

3.667.3 Rubi [N/A]

Not integrable

Time = 4.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5499, 5499, 5437, 5477, 5440, 5439, 3042, 3783, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2}}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx}{a^2}}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a\cdots\right.}}{a^2}}{a^2}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{\cdots}{2ac\,\mathrm{arcta}\cdots}}{a^2}}{a^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{\cdots}{2a\cdots}}{a^2}}{a^2}$$

$$\Big\downarrow \text{3042}$$

3.667. $\displaystyle\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx - \frac{}{2ac}}{a^2}}{a^2}$$

$$\Big\downarrow \; 3783$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2}}{a^2}}{a^2}$$

$$\Big\downarrow \; 5503$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a} - 2a\int \frac{}{\left(a^2cx^2+c\right.}\right.}{a^2}}{a^2}$$

$$\Big\downarrow \; 5440$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}} - 2a\int\right.}{a^2}}{a^2}$$

$$\Big\downarrow \; 5439$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}}{a^2}$$

$$\Big\downarrow \; 3042$$

3.667. $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} -$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - \cdots\right.}{a^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} -$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)} + \frac{3}{4\sqrt{a^2x^2+1}\,\text{arct}\cdots}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} -$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx + \frac{\sqrt{a^2x^2+1}\cdots}{\cdots}\right.}{a^2}$$

$$\Big\downarrow \text{5506}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} -$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}} + \cdots\right.}{a^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} -$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan\cdots}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$$\Big\downarrow \text{4906}$$

3.667. $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\,\text{ar}}{4\,\text{arc}}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosI}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$$\Big\downarrow \; 5560$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosI}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.667.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.667. $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.667. $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.667. $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.667.4 Maple [N/A] (verified)

Not integrable

Time = 18.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(a^2 c\, x^2 + c)^{\frac{5}{2}} \arctan(ax)^3} dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.667.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\, dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}} \arctan(ax)^3}\, dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

3.667.6 Sympy [N/A]

Not integrable

Time = 12.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\, dx = \int \frac{x^4}{(c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}^3(ax)}\, dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.667.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.667.8 Giac [N/A]

Not integrable

Time = 227.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.667.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^4/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.668 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.668.1 Optimal result

Integrand size = 24, antiderivative size = 180

$$\begin{aligned}\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx &= -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\arctan(ax)^2}\\ &\quad+\frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\arctan(ax)}-\frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\arctan(ax)}\\ &\quad-\frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{8a^4c^2\sqrt{c+a^2cx^2}}+\frac{9\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{8a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/2*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2/a^4/c/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)-3/2/a^4/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-3/8*Si(arctan(a
*x))*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+9/8*Si(3*arctan(a*x))*(
a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

3.668.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{-4a^2x^2(ax+3\arctan(ax))-3(1+a^2x^2)^{3/2}\arctan(ax)^2\mathrm{Si}(\arctan(ax))}{8a^4c^2\,(1+a^2x^2)\,\sqrt{c+a^2cx^2}\,\mathrm{arcta}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-4*a^2*x^2*(a*x + 3*ArcTan[a*x]) - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Si
nIntegral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*SinIntegral[3
*ArcTan[a*x]])/(8*a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

3.668.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5477, 5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{3\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5499}$$

$$\frac{3\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$\frac{3\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{3\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}$$

$$\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 5505$$

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2\right.}}{a^2}\right.}{2a}$$

$$\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3}}}{a^2}\right.}{2a}$$

$$\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 3780$$

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\operatorname{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-$$

$$\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 4906$$

3.668. $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\operatorname{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+\right.}}{a^2}\right.}{2a}$$

$$\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\operatorname{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\operatorname{Si}(\arctan(ax))+\frac{1}{4}\operatorname{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-$$

$$\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (3*((-(1/(a*c*Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/
(a*c*Sqrt[c + a^2*c*x^2]))/(a^2*c) - (-(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTa
n[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4 + SinIntegral[
3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/a^2))/(2*a)
```

3.668.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.668. $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.668. $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.668.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 24.65 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.99

method	result
default	$-\frac{i\left(9\arctan(ax)^2 \operatorname{Ei}_1(3i\arctan(ax))a^4x^4-3\arctan(ax)^2 \operatorname{Ei}_1(i\arctan(ax))a^4x^4+3\arctan(ax)^2 \operatorname{Ei}_1(-i\arctan(ax))a^4x^4-9\arctan(\right.}{}$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4-3*arctan(a*x)^2*Ei(
1,I*arctan(a*x))*a^4*x^4+3*arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^4*x^4-9*ar
ctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4+18*arctan(a*x)^2*Ei(1,3*I*arcta
n(a*x))*a^2*x^2-6*arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+6*arctan(a*x)^
2*Ei(1,-I*arctan(a*x))*a^2*x^2-18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2
*x^2-8*I*(a^2*x^2+1)^(1/2)*a^3*x^3-24*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*
x^2+9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^2-3*Ei(1,I*arctan(a*x))*arctan(a*x
)^2+3*Ei(1,-I*arctan(a*x))*arctan(a*x)^2-9*Ei(1,-3*I*arctan(a*x))*arctan(a
*x)^2)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a^4/c^3/(
a^4*x^4+2*a^2*x^2+1)
```

3.668.5 Fricas [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

3.668.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^3}{(c\,(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.668.7 Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.668.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.669 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.669.1 Optimal result

Integrand size = 24, antiderivative size = 209

$$\begin{aligned}\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx &= \frac{1}{2a^3c\,(c+a^2cx^2)^{3/2}\arctan(ax)^2} \\ &- \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{3x}{2a^2c\,(c+a^2cx^2)^{3/2}\arctan(ax)} \\ &+ \frac{x}{2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{8a^3c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{9\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{8a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/2/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2*x/a^2/c/(a^2*c*x^2+c)^(3/2
)/arctan(a*x)-1/2/a^3/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)+1/2*x/a^2/c^2/
arctan(a*x)/(a^2*c*x^2+c)^(1/2)-1/8*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/
c^2/(a^2*c*x^2+c)^(1/2)+9/8*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a^3/c^2/(a
^2*c*x^2+c)^(1/2)
```

3.669.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{4ax(-ax+(-2+a^2x^2)\arctan(ax))-(1+a^2x^2)^{3/2}\arctan(ax)^2\,\mathrm{CosInt}}{8a^3c^2(1+a^2x^2)\sqrt{c}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(4*a*x*(-(a*x) + (-2 + a^2*x^2)*ArcTan[a*x]) - (1 + a^2*x^2)^(3/2)*ArcTan[
a*x]^2*CosIntegral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosI
ntegral[3*ArcTan[a*x]])/(8*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)
```

3.669.3 Rubi [A] (verified)

Time = 3.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5499, 5437, 5477, 5440, 5439, 3042, 3783, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-\frac{1}{2}a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5440}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{3783}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

3.669. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\Big\downarrow \; 5440$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2(a^2cx}}{a^2}$$

$$\Big\downarrow \; 5439$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{}{2ac\arctan(}}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2}}{a^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)(a^2cx^2+c}\right.}{a^2}$$

$$\Big\downarrow \; 5506$$

3.669. $\int\frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int\frac{x^2}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)(a^2 \cdots}\right.}{a^2}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{\cdots}{ac\arctan \cdots}\right.}{a^2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(\cdots\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan \cdots\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-1/2*1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - (a*(-(x/(a*c*Sqrt[c + a^
2*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a^2
*c*Sqrt[c + a^2*c*x^2])))/2)/(a^2*c) - (-1/2*1/(a*c*(c + a^2*c*x^2)^(3/2)*
ArcTan[a*x]^2) - (3*a*(-(x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*S
qrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]]/
4))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*((3*CosIntegral[Arc
Tan[a*x]])/4 + CosIntegral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2]
)))/2)/a^2
```

3.669. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.669.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.669. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.669. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.669.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 20.79 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\left(9 \arctan(ax)^2 \operatorname{Ei}_1(-3i \arctan(ax)) a^4 x^4 + 9 \arctan(ax)^2 \operatorname{Ei}_1(3i \arctan(ax)) a^4 x^4 - \arctan(ax)^2 \operatorname{Ei}_1(i \arctan(ax)) a^4 x^4 - \arctan(ax)\right.}{}$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4+9*arctan(a*x)^2*Ei(1
,3*I*arctan(a*x))*a^4*x^4-arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^4*x^4-arctan
(a*x)^2*Ei(1,-I*arctan(a*x))*a^4*x^4-8*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x
^3+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^2+18*arctan(a*x)^2*Ei(1,3
*I*arctan(a*x))*a^2*x^2-2*arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2-2*arct
an(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+8*a^2*x^2*(a^2*x^2+1)^(1/2)+16*arct
an(a*x)*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2+9*Ei(
1,3*I*arctan(a*x))*arctan(a*x)^2-Ei(1,I*arctan(a*x))*arctan(a*x)^2-Ei(1,-I
*arctan(a*x))*arctan(a*x)^2)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a
rctan(a*x)^2/a^3/c^3/(a^4*x^4+2*a^2*x^2+1)
```

3.669.5 Fricas [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

3.669.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.669.7 Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.669.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.670 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.670.1 Optimal result

Integrand size = 22, antiderivative size = 175

$$\begin{aligned}\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx =& -\frac{x}{2ac(c+a^2cx^2)^{3/2}\arctan(ax)^2}\\ &-\frac{3}{2a^2c(c+a^2cx^2)^{3/2}\arctan(ax)}+\frac{1}{a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)}\\ &-\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{8a^2c^2\sqrt{c+a^2cx^2}}-\frac{9\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{8a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/2*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2/a^2/c/(a^2*c*x^2+c)^(3/2)
/arctan(a*x)+1/a^2/c^2/arctan(a*x)/(a^2*c*x^2+c)^(1/2)-1/8*Si(arctan(a*x))
*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-9/8*Si(3*arctan(a*x))*(a^2*
x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

3.670.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.67

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{-4ax-4\arctan(ax)+8a^2x^2\arctan(ax)-(1+a^2x^2)^{3/2}\arctan(ax)^2\mathrm{Si}(}{8a^2c^2(1+a^2x^2)\sqrt{c+a^2cx}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-4*a*x - 4*ArcTan[a*x] + 8*a^2*x^2*ArcTan[a*x] - (1 + a^2*x^2)^(3/2)*ArcT
an[a*x]^2*SinIntegral[ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*S
inIntegral[3*ArcTan[a*x]])/(8*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)
```

3.670.3 Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.68, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5503, 5437, 5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{2a}-a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$-a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx+\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{2a}-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5499}$$

$$-a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{a^2}\right)+\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{2a}-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-$$

$$a\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)$$

$$\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-$$

$$a\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx}}{a^2}\right.$$

$$\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-$$

$$a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac\arc}}{a^2}\right.$$

$$\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-$$

$$a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac\arctan(}}{a^2}\right.$$

$$\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3780}$$

3.670. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}+\frac{-\frac{3\sqrt{a^2x^2+1}\int\frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}\right)-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}+\frac{-\frac{3\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}\right)-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}+\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}\right)-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - a*((-(1/(a*c*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a*
c*Sqrt[c + a^2*c*x^2]))/(a^2*c) - (-(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a
*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4 + SinIntegral[3*A
rcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/a^2) + (-(1/(a*c*(c + a^2*c*x
^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4
 + SinIntegral[3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/(2*a)
```

3.670. $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

3.670.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.670. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.670.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.43 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.12

method	result
default	$-\frac{i\left(\arctan(ax)^2 \mathrm{Ei}_1(-i\arctan(ax))a^4x^4-\arctan(ax)^2 \mathrm{Ei}_1(i\arctan(ax))a^4x^4+9\arctan(ax)^2 \mathrm{Ei}_1(-3i\arctan(ax))a^4x^4-9\arctan(a\right.}{}$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*(arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^4*x^4-arctan(a*x)^2*Ei(1,I*a
rctan(a*x))*a^4*x^4+9*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4-9*arcta
n(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4+2*arctan(a*x)^2*Ei(1,-I*arctan(a*x)
)*a^2*x^2-2*arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+18*arctan(a*x)^2*Ei(
1,-3*I*arctan(a*x))*a^2*x^2-18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^2*x^2
+16*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+Ei(1,-I*arctan(a*x))*arctan(a*
x)^2-Ei(1,I*arctan(a*x))*arctan(a*x)^2+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x
)^2-9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^2-8*I*(a^2*x^2+1)^(1/2)*a*x-8*I*ar
ctan(a*x)*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/a
rctan(a*x)^2/a^2/c^3/(a^4*x^4+2*a^2*x^2+1)
```

3.670.5 Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x
^2 + c^3)*arctan(a*x)^3), x)
```

3.670.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.670.7 Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.670.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.670.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x}{\mathrm{atan}\,(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.671 $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

3.671.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx = -\frac{1}{2ac\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2} + \frac{3x}{2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)} - \frac{3\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{8ac^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{8ac^2\sqrt{c+a^2cx^2}}$$

output
```
-1/2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2*x/c/(a^2*c*x^2+c)^(3/2)/arc
tan(a*x)-3/8*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-9
/8*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.671.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx = \frac{-4+12ax\arctan(ax)-3(1+a^2x^2)^{3/2}\arctan(ax)^2\,\mathrm{CosIntegral}(\arctan(}{8c^2\left(a+a^3x^2\right)\sqrt{c+a^2cx}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-4 + 12*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral
[ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral[3*ArcTan[a
*x]])/(8*c^2*(a + a^3*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

3.671.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5437, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \ 5437$$

$$-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 5503$$

$$-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 5440$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 5439$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2\right.}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\,(a^2cx^2+c)}\right.$$

$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{}{ac}\right.$$

$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}a\left(-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax)\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int\frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

3.671. $\int\frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax\right.}{a^2c^2\sqrt{a^2c\right.}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\text{arct}\right.}{a^2c\right.}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - (3*a*(-(x/(a*c*(c + a^2
*c*x^2)^(3/2)*ArcTan[a*x])) - (2*Sqrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x
]]/4 - CosIntegral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqr
t[1 + a^2*x^2]*((3*CosIntegral[ArcTan[a*x]])/4 + CosIntegral[3*ArcTan[a*x]
]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2])))/2
```

3.671.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.671. $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.671. $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.671.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 18.91 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.39

method	result
default	$\frac{\left(9\arctan(ax)^2\operatorname{Ei}_1(3i\arctan(ax))a^4x^4+3\arctan(ax)^2\operatorname{Ei}_1(-i\arctan(ax))a^4x^4+3\arctan(ax)^2\operatorname{Ei}_1(i\arctan(ax))a^4x^4+9\arctan(ax)^2\right.}{}$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4+3*arctan(a*x)^2*Ei(1,-
I*arctan(a*x))*a^4*x^4+3*arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^4*x^4+9*arcta
n(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4+18*arctan(a*x)^2*Ei(1,3*I*arctan(a
*x))*a^2*x^2+6*arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+6*arctan(a*x)^2*
Ei(1,I*arctan(a*x))*a^2*x^2+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^
2+24*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+9*Ei(1,3*I*arctan(a*x))*arctan(a*x)
^2+3*Ei(1,-I*arctan(a*x))*arctan(a*x)^2+3*Ei(1,I*arctan(a*x))*arctan(a*x)^
2+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-8*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^
(1/2)*(c*(a*x-I)*(I+a*x))^(1/2)/arctan(a*x)^2/a/c^3/(a^4*x^4+2*a^2*x^2+1)
```

3.671.5 Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3)*arctan(a*x)^3), x)
```

3.671.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.671.7 Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.671.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{\mathrm{atan}\,(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.672 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.672.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx &= \frac{ax}{2c(c+a^2cx^2)^{3/2}\arctan(ax)^2} \\ &+ \frac{ax}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\arctan(ax)^2} + \frac{3}{2c(c+a^2cx^2)^{3/2}\arctan(ax)} \\ &- \frac{1}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{5\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{9\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}} - \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2ac^2}\end{aligned}$$

output
```
1/2*a*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2/c/(a^2*c*x^2+c)^(3/2)/arct
an(a*x)+1/2*a*x/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2/c^2/arctan(a*x)/
(a^2*c*x^2+c)^(1/2)+5/8*Si(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c
)^(1/2)+9/8*Si(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-1/
2*(a^2*c*x^2+c)^(1/2)/a/c^3/x/arctan(a*x)^2-1/2*Unintegrable(1/x^2/arctan(
a*x)^2/(a^2*c*x^2+c)^(1/2),x)/a/c^2
```

3.672.2 Mathematica [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

3.672.3 Rubi [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5501, 5501, 5477, 5437, 5503, 5437, 5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \textbf{5437}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right.}{c}$$
$$a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \textbf{5503}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right.}{c}$$
$$a^2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{2a}-a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \textbf{5437}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right.}{c}$$
$$a^2\left(-a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx+\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-\frac{}{2ac\arctan(a}\right.$$

$$\Big\downarrow \textbf{5499}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right.}{c}$$
$$a^2\left(-a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}\right)+\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a}}{2a}\right.$$

3.672. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\Big\downarrow \text{5437}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2} - a^2\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right.$$
$$\overline{\qquad c \qquad}$$
$$a^2\left(\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a} - a\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2-}}}{a^2c}\right.\right.$$

$$\Big\downarrow \text{5506}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2} - a^2\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a}}\right.$$
$$\overline{\qquad c \qquad}$$
$$a^2\left(-\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a} - a\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{}{ac\arctan(}}{a^2c}\right.\right.$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{x}{2ac\arctan(ax)}\right.$$
$$\overline{\qquad c \qquad}$$
$$a^2\left(-\frac{\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a} - a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}}{a^2c}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2}}\right.$$
$$\overline{\qquad c \qquad}$$
$$a^2\left(-\frac{\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a} - a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{}{ac}}{a^2c}\right.\right.$$

3.672. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\Big\downarrow \text{3780}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\Bigg(-a\Bigg(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2\cdots}}{a^2}\cdots$$

$$\Big\downarrow \text{4906}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\Bigg(-a\Bigg(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\cdots}{a^2}\cdots$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\Bigg(-a\Bigg(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(\cdots}}{a^2}\cdots$$

$$\Big\downarrow \text{5560}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\Bigg(-a\Bigg(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(\cdots}}{a^2}\cdots$$

3.672. $\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.672.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.672. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.672. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.672.4 Maple [N/A] (verified)

Not integrable

Time = 14.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{3}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.672.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} x \arctan\left(a x\right)^{3}}\,{d x}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
 + c^3*x)*arctan(a*x)^3), x)
```

3.672.6 Sympy [N/A]

Not integrable

Time = 21.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.672.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3), x)
```

3.672.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.672.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.673 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

3.673.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx &= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\\ &+\frac{a}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}-\frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\\ &-\frac{a^2x}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)}+\frac{7a\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}}\\ &+\frac{9a\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}}+\frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c^2}\end{aligned}$$

output
```
1/2*a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2*a^2*x/c/(a^2*c*x^2+c)^(3/2)/
arctan(a*x)+1/2*a/c^2/arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-1/2*a^2*x/c^2/arct
an(a*x)/(a^2*c*x^2+c)^(1/2)+7/8*a*Ci(arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a
^2*c*x^2+c)^(1/2)+9/8*a*Ci(3*arctan(a*x))*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2
+c)^(1/2)+Unintegrable(1/x^2/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)/c^2
```

3.673.2 Mathematica [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3}\,dx = \int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

3.673.3 Rubi [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5501, 5437, 5501, 5437, 5477, 5440, 5439, 3042, 3783, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 (a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{c} - a^2\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5440}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5439}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

3.673. $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2}}\right.}{c}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$\downarrow$ 3783

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$\downarrow$ 5503

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx}{a} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right) - \frac{}{2ac}\right.$$

$\downarrow$ 5440

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/}}\right.\right.$$

$\downarrow$ 5439

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)(a^2}\right.\right.$$

$\downarrow$ 3042

3.673. $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)\,(a^2cx\right.\right.$$

$$\Big\downarrow \; 3793$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)} + \frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx -\right.\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax)) + \frac{1}{4}\text{CosIntegral}(3\arctan\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \; 5506$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax)) + \frac{1}{4}\text{CosIntegral}(3\arc\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \; 5505$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} -$$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax)) + \frac{1}{4}\text{CosInte\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \; 4906$$

3.673. $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} - $$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)} - \frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\arctan\right.}{a^2c^2\sqrt{}}\right.\right.$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} - $$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\text{CosIntegral}(\arctan(ax)) - \frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\right.}{}\right.\right.$$

$$\Big\downarrow \color{blue}{5560}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c} - $$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\text{CosIntegral}(\arctan(ax)) - \frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\right.}{}\right.\right.$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.673.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.673. $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.673. $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.673. $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.673.4 Maple [N/A] (verified)

Not integrable

Time = 13.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.673.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right)}^{\frac {5}{2}} x^{2} \arctan \left (a x\right)^{3}} \,d x }$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
 + c^3*x^2)*arctan(a*x)^3), x)
```

3.673.6 Sympy [N/A]

Not integrable

Time = 29.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx=\int \frac {1}{x^{2} \left (c \left (a^{2} x^{2} + 1\right)\right)^{\frac {5}{2}} \operatorname {atan}^{3}{\left (a x \right)}}\, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

3.673.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3), x)
```

3.673.8 Giac [N/A]

Not integrable

Time = 176.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.673.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^3 \, (c\,a^2\,x^2+c)^{5/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.674 $\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3} dx$

3.674.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3} dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.674.2 Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3} dx = \int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3} dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

3.674.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^3} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.674.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.674.4 Maple [N/A] (verified)

Not integrable

Time = 10.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)^3}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.674.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^3, x)
```

3.674.6 Sympy [N/A]

Not integrable

Time = 17.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^4x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^6x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
c**3*(Integral(x**m/atan(a*x)**3, x) + Integral(3*a**2*x**2*x**m/atan(a*x)
**3, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**3, x) + Integral(a**6*x**6*
x**m/atan(a*x)**3, x))
```

3.674.7 Maxima [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 399, normalized size of antiderivative = 18.14

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^10*c^3*m^2 + 17*a^10*c^3*m + 72*a^10*c^
3)*x^10 + (5*a^8*c^3*m^2 + 67*a^8*c^3*m + 224*a^8*c^3)*x^8 + 2*(5*a^6*c^3*
m^2 + 49*a^6*c^3*m + 120*a^6*c^3)*x^6 + c^3*m^2 + 2*(5*a^4*c^3*m^2 + 31*a^
4*c^3*m + 48*a^4*c^3)*x^4 - c^3*m + (5*a^2*c^3*m^2 + 13*a^2*c^3*m + 8*a^2*
c^3)*x^2)*x^m/(x^2*arctan(a*x)), x) - ((a^10*c^3*m + 8*a^10*c^3)*x^10 + (5
*a^8*c^3*m + 32*a^8*c^3)*x^8 + 2*(5*a^6*c^3*m + 24*a^6*c^3)*x^6 + 2*(5*a^4
*c^3*m + 16*a^4*c^3)*x^4 + c^3*m + (5*a^2*c^3*m + 8*a^2*c^3)*x^2)*x^m*arct
an(a*x) - (a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a
*c^3*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

3.674.8 Giac [N/A]

Not integrable

Time = 218.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.674.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^3, x)
```

3.675 $\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$

3.675.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.675.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

3.675.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^3} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.675.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.675.4 Maple [N/A] (verified)

Not integrable

Time = 9.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \left(a^2 c\,x^2+c\right)^2}{\arctan\left(a x\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.675.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^3, x)
```

3.675.6 Sympy [N/A]

Not integrable

Time = 8.87 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = c^2\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**m/atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m/atan(a*x)
**3, x) + Integral(a**4*x**4*x**m/atan(a*x)**3, x))
```

3.675.7 Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 333, normalized size of antiderivative = 15.14

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^8*c^2*m^2 + 13*a^8*c^2*m + 42*a^8*c^2)*
x^8 + 2*(2*a^6*c^2*m^2 + 19*a^6*c^2*m + 45*a^6*c^2)*x^6 + 6*(a^4*c^2*m^2 +
 6*a^4*c^2*m + 9*a^4*c^2)*x^4 + c^2*m^2 - c^2*m + 2*(2*a^2*c^2*m^2 + 5*a^2
*c^2*m + 3*a^2*c^2)*x^2)*x^m/(x^2*arctan(a*x)), x) - ((a^8*c^2*m + 6*a^8*c
^2)*x^8 + 2*(2*a^6*c^2*m + 9*a^6*c^2)*x^6 + 6*(a^4*c^2*m + 3*a^4*c^2)*x^4
+ c^2*m + 2*(2*a^2*c^2*m + 3*a^2*c^2)*x^2)*x^m*arctan(a*x) - (a^7*c^2*x^7
+ 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

3.675.8 Giac [N/A]

Not integrable

Time = 193.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3} \, dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan{(ax)}^3} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.675.9 Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3} \, dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^3} \, dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^3, x)
```

3.676 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx$

3.676.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.676.2 Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

3.676.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.676.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.676.4 Maple [N/A] (verified)

Not integrable

Time = 10.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.676.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

3.676.6 Sympy [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx=c\left(\int \frac{x^m}{\operatorname{atan}^3{(ax)}}\,dx+\int \frac{a^2x^2x^m}{\operatorname{atan}^3{(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
c*(Integral(x**m/atan(a*x)**3, x) + Integral(a**2*x**2*x**m/atan(a*x)**3,
x))
```

3.676.7 Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 221, normalized size of antiderivative = 11.05

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^6*c*m^2 + 9*a^6*c*m + 20*a^6*c)*x^6 + (
3*a^4*c*m^2 + 17*a^4*c*m + 24*a^4*c)*x^4 + c*m^2 + (3*a^2*c*m^2 + 7*a^2*c*
m + 4*a^2*c)*x^2 - c*m)*x^m/(x^2*arctan(a*x)), x) - ((a^6*c*m + 4*a^6*c)*x
^6 + (3*a^4*c*m + 8*a^4*c)*x^4 + (3*a^2*c*m + 4*a^2*c)*x^2 + c*m)*x^m*arct
an(a*x) - (a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

3.676.8 Giac [N/A]

Not integrable

Time = 167.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.676.9 Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^3, x)
```

3.677 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx$

3.677.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x^m}{2ac\arctan(ax)^2} + \frac{m\mathrm{Int}\left(\frac{x^{-1+m}}{\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2*x^m/a/c/arctan(a*x)^2+1/2*m*Unintegrable(x^(-1+m)/arctan(a*x)^2,x)/a/
c
```

3.677.2 Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

3.677.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)^2}dx}{2ac} - \frac{x^m}{2ac\arctan(ax)^2}$$
$$\Big\downarrow \text{5377}$$
$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)^2}dx}{2ac} - \frac{x^m}{2ac\arctan(ax)^2}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.677.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.677.4 Maple [N/A] (verified)

Not integrable

Time = 4.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan\left(ax\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

3.677.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.677.6 Sympy [N/A]

Not integrable

Time = 6.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{a^2x^2\,\mathrm{atan}^3\,(ax)+\mathrm{atan}^3\,(ax)}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

3.677.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.14

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^2*m^2 + a^2*m)*x^2 + m^2 - m)*x^m/(x^2*
arctan(a*x)), x) - a*x*x^m - (a^2*m*x^2 + m)*x^m*arctan(a*x))/(a^2*c*x*arc
tan(a*x)^2)
```

3.677.8 Giac [N/A]

Not integrable

Time = 155.56 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.677.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

3.678 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} dx$

3.678.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3}, x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.678.2 Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

3.678.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)^2} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)^2} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.678.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.678.4 Maple [N/A] (verified)

Not integrable

Time = 11.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

3.678.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

3.678.6 Sympy [N/A]

Not integrable

Time = 18.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x**m/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*
x)**3), x)/c**2
```

3.678.7 Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 8.68

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*x)^2*integrate(1/2*((a^4*m^2 - 3
*a^4*m + 2*a^4)*x^4 + 2*(a^2*m^2 - 2*a^2*m - a^2)*x^2 + m^2 - m)*x^m/((a^6
*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)), x) - a*x*x^m - ((a^2
*m - 2*a^2)*x^2 + m)*x^m*arctan(a*x))/((a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*
x)^2)
```

3.678.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\text{Timed out}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.678.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

3.679 $\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$

3.679.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.679.2 Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

3.679.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^3} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.679.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.679.4 Maple [N/A] (verified)

Not integrable

Time = 7.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

3.679.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^3), x)
```

3.679.6 Sympy [N/A]

Not integrable

Time = 52.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x**m/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*
x**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

3.679.7 Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 224, normalized size of antiderivative = 10.18

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2*integrate(1
/2*((a^4*m^2 - 7*a^4*m + 12*a^4)*x^4 + 2*(a^2*m^2 - 4*a^2*m - 2*a^2)*x^2 +
 m^2 - m)*x^m/((a^8*c^3*x^8 + 3*a^6*c^3*x^6 + 3*a^4*c^3*x^4 + a^2*c^3*x^2)
*arctan(a*x)), x) - a*x*x^m - ((a^2*m - 4*a^2)*x^2 + m)*x^m*arctan(a*x))/(
(a^6*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2)
```

3.679.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.679.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\,(a\,x)^3\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

3.680 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$

3.680.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.680.2 Mathematica [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

3.680.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.680.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.680.4 Maple [N/A] (verified)

Not integrable

Time = 11.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.680.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^3, x)
```

3.680.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.680.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^3, x)
```

3.680.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.680.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3, x)
```

3.681 $\int \frac{x^m (c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$

3.681.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \text{Int}\left(\frac{x^m (c+a^2cx^2)^{3/2}}{\arctan(ax)^3}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.681.2 Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \int \frac{x^m (c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

3.681.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.681.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.681.4 Maple [N/A] (verified)

Not integrable

Time = 13.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.681.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

3.681.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Timed out
```

3.681.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

3.681.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.681.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3, x)
```

3.682 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$

3.682.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

3.682.2 Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

3.682.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

3.682.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.682.4 Maple [N/A] (verified)

Not integrable

Time = 12.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

3.682.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

3.682.6 Sympy [N/A]

Not integrable

Time = 24.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

3.682.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

3.682.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.682.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3, x)
```

3.683 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$

3.683.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.683.2 Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

3.683.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.683.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.683.4 Maple [N/A] (verified)

Not integrable

Time = 13.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\arctan\left(ax\right)^3\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

3.683.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.683.6 Sympy [N/A]

Not integrable

Time = 37.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(x**m/atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

3.683.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

3.683.8 Giac [N/A]

Not integrable

Time = 9.66 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m/arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.683.9 Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.684 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$

3.684.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.684.2 Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

3.684.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.684.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.684.4 Maple [N/A] (verified)

Not integrable

Time = 12.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

3.684.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^3), x)
```

3.684.6 Sympy [N/A]

Not integrable

Time = 122.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

3.684.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

3.684.8 Giac [N/A]

Not integrable

Time = 20.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.684.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

3.685 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$

3.685.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.685.2 Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

3.685.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

3.685.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.685.4 Maple [N/A] (verified)

Not integrable

Time = 15.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

3.685.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

3.685.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Timed out
```

3.685.7 Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

3.685.8 Giac [N/A]

Not integrable

Time = 41.72 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.685.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

3.686 $\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$

3.686.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Int}\left(x^m(c+a^2cx^2)\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

3.686.2 Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

3.686.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)}\left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.686.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.686.4 Maple [N/A] (verified)

Not integrable

Time = 9.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \sqrt{\arctan \left(a x \right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

3.686.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)x^m\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

3.686.6 Sympy [N/A]

Not integrable

Time = 34.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = c\left(\int x^m\sqrt{\operatorname{atan}(ax)}\,dx + \int a^2x^2x^m\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x**m*sqrt(atan(a*x)), x) + Integral(a**2*x**2*x**m*sqrt(atan(a
*x)), x))
```

3.686.7 Maxima [F(-2)]

Exception generated.

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.686.8 Giac [N/A]

Not integrable

Time = 52.96 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx=\int (a^2cx^2+c)x^m\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.686.9 Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx=\int x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

3.687 $\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$

3.687.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \frac{c(1+a^2x^2)^2\sqrt{\arctan(ax)}}{4a^2} - \frac{\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)}{8a}$$

output
```
1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a^2-1/8*Unintegrable((a^2*c*x^2+c)/a
rctan(a*x)^(1/2),x)/a
```

3.687.2 Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

3.687.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)\,dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{c\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{4a^2}-\frac{\int \frac{c\left(a^2x^2+1\right)}{\sqrt{\arctan(ax)}}dx}{8a}$$
$$\Big\downarrow \text{27}$$
$$\frac{c\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{4a^2}-\frac{c\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx}{8a}$$
$$\Big\downarrow \text{5560}$$
$$\frac{c\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{4a^2}-\frac{c\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.687.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.687.4 Maple [N/A] (verified)

Not integrable

Time = 4.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a^{2} c\,x^{2}+c\right) \sqrt{\arctan \left(a x\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

3.687.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.687.6 Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = c\left(\int x\sqrt{\operatorname{atan}(ax)}\,dx + \int a^2x^3\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x*sqrt(atan(a*x)), x) + Integral(a**2*x**3*sqrt(atan(a*x)), x)
)
```

3.687.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.687.8 Giac [N/A]

Not integrable

Time = 52.40 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)x\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.687.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx=\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

3.688 $\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$

3.688.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx=\text{Int}\left((c+a^2cx^2)\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

3.688.2 Mathematica [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx=\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

3.688.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)dx$$

input
```
Int[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.688.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.688.4 Maple [N/A] (verified)

Not integrable

Time = 3.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a^{2} c \,x^{2}+c \right) \sqrt{\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

3.688.5 Fricas [F(-2)]

Exception generated.

$$\int (c + a^2cx^2) \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.688.6 Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int (c + a^2cx^2) \sqrt{\arctan(ax)}\, dx = c\left(\int a^2x^2\sqrt{\operatorname{atan}(ax)}\, dx + \int \sqrt{\operatorname{atan}(ax)}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

output
```
c*(Integral(a**2*x**2*sqrt(atan(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

3.688.7 Maxima [F(-2)]

Exception generated.

$$\int (c + a^2cx^2) \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.688.8 Giac [N/A]

Not integrable

Time = 52.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)\sqrt{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.688.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

3.689 $\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx$

3.689.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

3.689.2 Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x, x]
```

3.689.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)}{x}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)}{x}dx$$

input
```
Int[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
$Aborted
```

3.689.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.689.4 Maple [N/A] (verified)

Not integrable

Time = 5.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^{2} c\,x^{2}+c \right) \sqrt{\arctan \left(a x \right)}}{x}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

3.689.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.689.6 Sympy [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = c\left(\int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx + \int a^2x\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(1/2)/x,x)
```

output
```
c*(Integral(sqrt(atan(a*x))/x, x) + Integral(a**2*x*sqrt(atan(a*x)), x))
```

3.689.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.689.8 Giac [N/A]

Not integrable

Time = 233.46 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c + a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(a^2cx^2 + c)\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.689.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2 + c)}{x}\,dx$$

input
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2))/x, x)
```

3.690 $\int x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}\, dx$

3.690.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}\, dx = \operatorname{Int}\left(x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

3.690.2 Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}\, dx = \int x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}\, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

3.690.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \; 5560$$

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.690.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.690.4 Maple [N/A] (verified)

Not integrable

Time = 9.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2c\,x^2+c\right)^2 \sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

3.690.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\arctan(a x)}\, dx = \int \left(a^2 c x^2 + c\right)^2 x^m \sqrt{\arctan\left(a x\right)}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*sqrt(arctan(a*x)), x)
```

3.690.6 Sympy [N/A]

Not integrable

Time = 120.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\arctan(a x)}\, dx = c^2 \left(\int x^m \sqrt{\operatorname{atan}\left(a x\right)}\, dx + \int 2 a^2 x^2 x^m \sqrt{\operatorname{atan}\left(a x\right)}\, dx + \int a^4 x^4 x^m \sqrt{\operatorname{atan}\left(a x\right)}\, dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x**m*sqrt(atan(a*x)), x) + Integral(2*a**2*x**2*x**m*sqrt(a
tan(a*x)), x) + Integral(a**4*x**4*x**m*sqrt(atan(a*x)), x))
```

3.690.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^2 \sqrt{\arctan(a x)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.690.8 Giac [N/A]

Not integrable

Time = 53.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^m (c + a^2cx^2)^2 \sqrt{\arctan(ax)}\, dx = \int (a^2cx^2 + c)^2 x^m \sqrt{\arctan(ax)}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.690.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2cx^2)^2 \sqrt{\arctan(ax)}\, dx = \int x^m \sqrt{\operatorname{atan}(a\,x)}\, \left(c\,a^2\,x^2 + c\right)^2 dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

3.691 $\int x(c+a^2cx^2)^2 \sqrt{\arctan(ax)}\, dx$

3.691.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^2 \sqrt{\arctan(ax)}\, dx = \frac{c^2(1+a^2x^2)^3 \sqrt{\arctan(ax)}}{6a^2} - \frac{\text{Int}\left(\frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}, x\right)}{12a}$$

output
```
1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(1/2)/a^2-1/12*Unintegrable((a^2*c*x^2+c
)^2/arctan(a*x)^(1/2),x)/a
```

3.691.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^2 \sqrt{\arctan(ax)}\, dx = \int x(c+a^2cx^2)^2 \sqrt{\arctan(ax)}\, dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

3.691.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{6a^2}-\frac{\int \frac{c^2\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx}{12a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{6a^2}-\frac{c^2\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx}{12a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{6a^2}-\frac{c^2\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx}{12a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.691.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.691.4 Maple [N/A] (verified)

Not integrable

Time = 4.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c\,x^{2}+c\right)^{2} \sqrt{\arctan\left(a x\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

3.691.5 Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.691.6 Sympy [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int x(c+a^2cx^2)^2\sqrt{\arctan(ax)}\,dx = c^2\left(\int x\sqrt{\operatorname{atan}(ax)}\,dx + \int 2a^2x^3\sqrt{\operatorname{atan}(ax)}\,dx + \int a^4x^5\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x*sqrt(atan(a*x)), x) + Integral(2*a**2*x**3*sqrt(atan(a*x)
), x) + Integral(a**4*x**5*sqrt(atan(a*x)), x))
```

3.691.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.691.8 Giac [N/A]

Not integrable

Time = 52.39 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x(c+a^2cx^2)^2\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)^2x\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.691.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^2\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\,(a\,x)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

3.692 $\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx$

3.692.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx = \mathrm{Int}\left((c+a^2cx^2)^2 \sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

3.692.2 Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

3.692.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2 dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2 dx$$

input
```
Int[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.692.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.692.4 Maple [N/A] (verified)

Not integrable

Time = 3.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^2c\,x^2+c\right)^2 \sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

3.692.5 Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.692.6 Sympy [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx = c^2\left(\int 2a^2x^2\sqrt{\operatorname{atan}(ax)}\,dx + \int a^4x^4\sqrt{\operatorname{atan}(ax)}\,dx + \int \sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2*sqrt(atan(a*x)), x) + Integral(a**4*x**4*sqrt(a
tan(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

3.692.7 Maxima [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.692.8 Giac [N/A]

Not integrable

Time = 52.87 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\, dx = \int \left(a^2cx^2+c\right)^2 \sqrt{\arctan\left(ax\right)}\, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.692.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\, dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2 \,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

3.693 $\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx$

3.693.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

3.693.2 Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x, x]
```

3.693.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}(a^2cx^2+c)^2}{x}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{\arctan(ax)}(a^2cx^2+c)^2}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
$Aborted
```

3.693.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.693.4 Maple [N/A] (verified)

Not integrable

Time = 4.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^2\sqrt{\arctan{(ax)}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

3.693.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.693.6 Sympy [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = c^2\left(\int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx + \int 2a^2x\sqrt{\operatorname{atan}(ax)}\,dx + \int a^4x^3\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2)/x,x)
```

output
```
c**2*(Integral(sqrt(atan(a*x))/x, x) + Integral(2*a**2*x*sqrt(atan(a*x)),
x) + Integral(a**4*x**3*sqrt(atan(a*x)), x))
```

3.693.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \sqrt{\arctan(ax)}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.693.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \sqrt{\arctan(ax)}}{x} \, dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

3.693.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \sqrt{\arctan(ax)}}{x} \, dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2}{x} \,d x$$

input
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^2)/x, x)
```

3.694 $\int x^m(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$

3.694.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \text{Int}\left(x^m \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

3.694.2 Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \int x^m \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

3.694.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3 dx$$
$$\Big\downarrow \text{5560}$$
$$\int x^m \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.694.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.694.4 Maple [N/A] (verified)

Not integrable

Time = 10.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2c\,x^2+c\right)^3 \sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

3.694.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^m \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \int \left(a^2cx^2+c\right)^3 x^m \sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*sqrt(arct
an(a*x)), x)
```

3.694.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.694.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.694.8 Giac [N/A]

Not integrable

Time = 52.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^m (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\, dx = \int (a^2cx^2+c)^3 x^m \sqrt{\arctan(ax)}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.694.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\, dx = \int x^m \sqrt{\mathrm{atan}(a\,x)}\, (c\,a^2\,x^2+c)^3\, dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

3.695 $\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$

3.695.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \frac{c^3(1+a^2x^2)^4 \sqrt{\arctan(ax)}}{8a^2} - \frac{\text{Int}\left(\frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}},x\right)}{16a}$$

output
```
1/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^(1/2)/a^2-1/16*Unintegrable((a^2*c*x^2+c
)^3/arctan(a*x)^(1/2),x)/a
```

3.695.2 Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

3.695.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \textcolor{blue}{5465}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\sqrt{\arctan(ax)}}{8a^2}-\frac{\int \frac{c^3\left(a^2x^2+1\right)^3}{\sqrt{\arctan(ax)}}dx}{16a}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\sqrt{\arctan(ax)}}{8a^2}-\frac{c^3\int \frac{\left(a^2x^2+1\right)^3}{\sqrt{\arctan(ax)}}dx}{16a}$$

$$\Big\downarrow \textcolor{blue}{5560}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\sqrt{\arctan(ax)}}{8a^2}-\frac{c^3\int \frac{\left(a^2x^2+1\right)^3}{\sqrt{\arctan(ax)}}dx}{16a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.695.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.695.4 Maple [N/A] (verified)

Not integrable

Time = 4.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c \,x^{2}+c \right)^{3} \sqrt{\arctan \left(a x \right)}d x$$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

3.695.5 Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.695.6 Sympy [N/A]

Not integrable

Time = 6.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int x(c+a^2cx^2)^3\sqrt{\arctan(ax)}\,dx = c^3\left(\int x\sqrt{\operatorname{atan}(ax)}\,dx+\int 3a^2x^3\sqrt{\operatorname{atan}(ax)}\,dx+\int 3a^4x^5\sqrt{\operatorname{atan}(ax)}\,dx+\int a^6x^7\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(x*sqrt(atan(a*x)), x) + Integral(3*a**2*x**3*sqrt(atan(a*x)
), x) + Integral(3*a**4*x**5*sqrt(atan(a*x)), x) + Integral(a**6*x**7*sqrt
(atan(a*x)), x))
```

3.695.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.695.8 Giac [N/A]

Not integrable

Time = 56.69 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x(c+a^2cx^2)^3\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)^3x\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.695.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^3\sqrt{\arctan(ax)}\,dx=\int x\,\sqrt{\mathrm{atan}\,(a\,x)}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

3.696 $\int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$

3.696.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \text{Int}\Big((c+a^2cx^2)^3 \sqrt{\arctan(ax)}, x\Big)$$

output
```
Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

3.696.2 Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

3.696.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \; 5560$$

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3 dx$$

input
```
Int[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.696.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.696.4 Maple [N/A] (verified)

Not integrable

Time = 3.64 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^2c\,x^2+c\right)^3 \sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

3.696.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.696.6 Sympy [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = c^3\left(\int 3a^2x^2\sqrt{\operatorname{atan}(ax)}\,dx+\int 3a^4x^4\sqrt{\operatorname{atan}(ax)}\,dx + \int a^6x^6\sqrt{\operatorname{atan}(ax)}\,dx+\int \sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2*sqrt(atan(a*x)), x) + Integral(3*a**4*x**4*sqrt
(atan(a*x)), x) + Integral(a**6*x**6*sqrt(atan(a*x)), x) + Integral(sqrt(a
tan(a*x)), x))
```

3.696.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.696.8 Giac [N/A]

Not integrable

Time = 53.33 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \int \left(a^2cx^2+c\right)^3 \sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.696.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

3.697 $\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx$

3.697.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

3.697.2 Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx=\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x, x]
```

3.697.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}{x}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
$Aborted
```

3.697.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.697.4 Maple [N/A] (verified)

Not integrable

Time = 5.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}}{x}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

3.697.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.697.6 Sympy [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx = c^3\left(\int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx + \int 3a^2x\sqrt{\operatorname{atan}(ax)}\,dx + \int 3a^4x^3\sqrt{\operatorname{atan}(ax)}\,dx + \int a^6x^5\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2)/x,x)
```

output
```
c**3*(Integral(sqrt(atan(a*x))/x, x) + Integral(3*a**2*x*sqrt(atan(a*x)),
x) + Integral(3*a**4*x**3*sqrt(atan(a*x)), x) + Integral(a**6*x**5*sqrt(at
an(a*x)), x))
```

3.697.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}{x} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.697.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}{x} dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

3.697.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}{x} dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}{x} \,d x$$

input
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^3)/x, x)
```

3.698 $\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx$

3.698.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \text{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

3.698.2 Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

3.698.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \sqrt{\arctan(ax)}}{a^2cx^2+c}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.698.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.698.4 Maple [N/A] (verified)

Not integrable

Time = 6.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan \left(a x \right)}}{a^2 c \, x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

3.698.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \int \frac{x^m \sqrt{\arctan{(ax)}}}{a^2cx^2+c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

3.698.6 Sympy [N/A]

Not integrable

Time = 4.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \frac{\int \frac{x^m \sqrt{\operatorname{atan}{(ax)}}}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.698.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.698.8 Giac [N/A]

Not integrable

Time = 51.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{a^2cx^2+c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.698.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \int \frac{x^m \sqrt{\mathrm{atan}(a\,x)}}{c\,a^2\,x^2+c} \, dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.699 $\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$

3.699.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{2x\arctan(ax)^{3/2}}{3a^3c}+\frac{\operatorname{Int}\left(x\sqrt{\arctan(ax)},x\right)}{a^2c}+\frac{2\operatorname{Int}\left(\arctan(ax)^{3/2},x\right)}{3a^3c}$$

output
```
-2/3*x*arctan(a*x)^(3/2)/a^3/c+2/3*Unintegrable(arctan(a*x)^(3/2),x)/a^3/c
+Unintegrable(x*arctan(a*x)^(1/2),x)/a^2/c
```

3.699.2 Mathematica [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

3.699.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5451, 27, 5377, 5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{x\sqrt{\arctan(ax)}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{x\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{x\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5457}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\frac{2x\arctan(ax)^{3/2}}{3a}-\frac{2\int \arctan(ax)^{3/2}dx}{3a}}{a^2c}$$

$$\Big\downarrow \text{ 5353}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\frac{2x\arctan(ax)^{3/2}}{3a}-\frac{2\int \arctan(ax)^{3/2}dx}{3a}}{a^2c}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.699.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.699.4 Maple [N/A] (verified)

Not integrable

Time = 3.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \sqrt{\arctan (ax)}}{a^2 c\, x^2 + c} dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

3.699.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.699.6 Sympy [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.699.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.699.8 Giac [N/A]

Not integrable

Time = 51.88 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.699.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.700 $\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$

3.700.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{2\arctan(ax)^{3/2}}{3a^3c} + \frac{\operatorname{Int}\left(\sqrt{\arctan(ax)},x\right)}{a^2c}$$

output
```
-2/3*arctan(a*x)^(3/2)/a^3/c+Unintegrable(arctan(a*x)^(1/2),x)/a^2/c
```

3.700.2 Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

3.700.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5451, 27, 5353, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \ 5451$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c} - \frac{\int \frac{\sqrt{\arctan(ax)}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c} - \frac{\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 5353$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c} - \frac{\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 5419$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c} - \frac{2\arctan(ax)^{3/2}}{3a^3c}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.700.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.700.4 Maple [N/A] (verified)

Not integrable

Time = 3.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \sqrt{\arctan (ax)}}{a^2 c\, x^2 + c} dx$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

3.700.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.700.6 Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.700.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.700.8 Giac [N/A]

Not integrable

Time = 52.62 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.700.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.701 $\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$

3.701.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{2x\arctan(ax)^{3/2}}{3ac}-\frac{2\mathrm{Int}\left(\arctan(ax)^{3/2},x\right)}{3ac}$$

output
```
2/3*x*arctan(a*x)^(3/2)/a/c-2/3*Unintegrable(arctan(a*x)^(3/2),x)/a/c
```

3.701.2 Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

3.701.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{2x\arctan(ax)^{3/2}}{3ac}-\frac{2\int \arctan(ax)^{3/2}dx}{3ac}$$
$$\Big\downarrow \text{5353}$$
$$\frac{2x\arctan(ax)^{3/2}}{3ac}-\frac{2\int \arctan(ax)^{3/2}dx}{3ac}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.701.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.701.4 Maple [N/A] (verified)

Not integrable

Time = 3.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{\arctan\left(ax\right)}}{a^{2}c\,x^{2}+c}dx$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

3.701.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.701.6 Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{x\sqrt{\operatorname{atan}\left(ax\right)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.701.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.701.8 Giac [N/A]

Not integrable

Time = 48.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.701.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

3.702 $\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$

3.702.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{2\arctan(ax)^{3/2}}{3ac}$$

output
```
2/3*arctan(a*x)^(3/2)/a/c
```

3.702.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{2\arctan(ax)^{3/2}}{3ac}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(3/2))/(3*a*c)
```

3.702.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 5419$$

$$\frac{2\arctan(ax)^{3/2}}{3ac}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(3/2))/(3*a*c)
```

3.702.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.702.4 Maple [A] (verified)

Time = 3.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\arctan(ax)^{\frac{3}{2}}}{3ac}$	15

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(a*x)^(3/2)/a/c
```

3.702.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{2\arctan{(ax)}^{\frac{3}{2}}}{3\,ac}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
2/3*arctan(a*x)^(3/2)/(a*c)
```

3.702.6 Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}{(ax)}}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

3.702.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.702.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\arctan(ax)}}{c + a^2cx^2}\,dx = \frac{2\arctan{(ax)}^{\frac{3}{2}}}{3\,ac}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/3*arctan(a*x)^(3/2)/(a*c)
```

3.702.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\arctan(ax)}}{c + a^2cx^2}\,dx = \frac{2\,\mathrm{atan}(a\,x)^{3/2}}{3\,a\,c}$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2),x)
```

output
```
(2*atan(a*x)^(3/2))/(3*a*c)
```

3.703 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)}\,dx$

3.703.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = -\frac{2i\arctan(ax)^{3/2}}{3c}+\frac{i\mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{x(i+ax)},x\right)}{c}$$

output
```
-2/3*I*arctan(a*x)^(3/2)/c+I*Unintegrable(arctan(a*x)^(1/2)/x/(I+a*x),x)/c
```

3.703.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)), x]
```

3.703.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5459, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)}\,dx$$
$$\Big\downarrow \; 5459$$
$$\frac{i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{3/2}}{3c}$$
$$\Big\downarrow \; 5560$$
$$\frac{i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{3/2}}{3c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.703.3.1 Defintions of rubi rules used

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.703.4 Maple [N/A] (verified)

Not integrable

Time = 3.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan (ax)}}{x\,(a^2c\,x^2+c)}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

3.703.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.703.6 Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**3 + x), x)/c
```

3.703.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.703.8 Giac [N/A]

Not integrable

Time = 24.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.703.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)), x)
```

3.704 $\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx$

3.704.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = -\frac{2a\arctan(ax)^{3/2}}{3c} + \frac{\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^2},x\right)}{c}$$

output
```
-2/3*a*arctan(a*x)^(3/2)/c+Unintegrable(arctan(a*x)^(1/2)/x^2,x)/c
```

3.704.2 Mathematica [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)), x]
```

3.704.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(a^2cx^2+c)}dx$$
$$\Big\downarrow \ 5453$$
$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c}-a^2\int \frac{\sqrt{\arctan(ax)}}{c(a^2x^2+1)}dx$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c}-\frac{a^2\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \ 5377$$
$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c}-\frac{a^2\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \ 5419$$
$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c}-\frac{2a\arctan(ax)^{3/2}}{3c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.704.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.704.4 Maple [N/A] (verified)

Not integrable

Time = 4.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(a^2cx^2+c)}dx$$

input
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

3.704.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.704.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^4+x^2}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**4 + x**2), x)/c
```

3.704.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.704.8 Giac [N/A]

Not integrable

Time = 29.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.704.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)), x)
```

3.705 $\int \frac{\sqrt{\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx$

3.705.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\,(c+a^2cx^2)}\,dx = \frac{2ia^2\arctan(ax)^{3/2}}{3c} + \frac{\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^3},x\right)}{c} - \frac{ia^2\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x(i+ax)},x\right)}{c}$$

output
```
2/3*I*a^2*arctan(a*x)^(3/2)/c+Unintegrable(arctan(a*x)^(1/2)/x^3,x)/c-I*a^
2*Unintegrable(arctan(a*x)^(1/2)/x/(I+a*x),x)/c
```

3.705.2 Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^3\,(c+a^2cx^2)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)), x]
```

3.705.3 Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5459, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c}-a^2\int \frac{\sqrt{\arctan(ax)}}{cx\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c}-\frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c}-\frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx-\frac{2}{3}i\arctan(ax)^{3/2}\right)}{c}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx-\frac{2}{3}i\arctan(ax)^{3/2}\right)}{c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.705.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.705.4 Maple [N/A] (verified)

Not integrable

Time = 3.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x^{3}\left(a^{2} c\,x^{2}+c\right)}dx$$

input
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

3.705.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.705.6 Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}{\left(ax \right)}}}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**5 + x**3), x)/c
```

3.705.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.705.8 Giac [N/A]

Not integrable

Time = 28.66 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx = \int \frac{\sqrt{\arctan{(ax)}}}{(a^2 c x^2 + c) x^3} \, dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.705.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^3 (c + a^2 c x^2)} \, dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^3\,\left(c\,a^2\,x^2+c\right)} \, dx$$

input
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)), x)
```

3.706 $\int \frac{\sqrt{\arctan(ax)}}{x^4(c+a^2cx^2)}\,dx$

3.706.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x^4(c+a^2cx^2)}\,dx = \frac{2a^3\arctan(ax)^{3/2}}{3c} + \frac{\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^4},x\right)}{c} - \frac{a^2\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^2},x\right)}{c}$$

output
```
2/3*a^3*arctan(a*x)^(3/2)/c+Unintegrable(arctan(a*x)^(1/2)/x^4,x)/c-a^2*Un
integrable(arctan(a*x)^(1/2)/x^2,x)/c
```

3.706.2 Mathematica [N/A]

Not integrable

Time = 5.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^4(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^4(c+a^2cx^2)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)), x]
```

3.706.3 Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5453, 5377, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \textbf{5453}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c}-a^2\int \frac{\sqrt{\arctan(ax)}}{cx^2\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c}-\frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \textbf{5377}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c}-\frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \textbf{5453}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\sqrt{\arctan(ax)}}{x^2}dx-a^2\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \textbf{5377}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\sqrt{\arctan(ax)}}{x^2}dx-a^2\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \textbf{5419}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\sqrt{\arctan(ax)}}{x^2}dx-\frac{2}{3}a\arctan(ax)^{3/2}\right)}{c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.706. $\int \frac{\sqrt{\arctan(ax)}}{x^4(c+a^2cx^2)}dx$

3.706.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.706.4 Maple [N/A] (verified)

Not integrable

Time = 4.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x^{4}\left(a^{2}c\,x^{2}+c\right)}dx$$

input
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

3.706.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (c + a^2cx^2)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.706.6 Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (c + a^2cx^2)} \, dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^6+x^4} \, dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**6 + x**4), x)/c
```

3.706.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (c + a^2cx^2)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.706.8 Giac [N/A]

Not integrable

Time = 28.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)x^4}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.706.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)), x)
```

3.707 $\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

3.707.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Int}\left(\frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

3.707.2 Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

3.707.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^2} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.707.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.707.4 Maple [N/A] (verified)

Not integrable

Time = 13.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan \left(a x \right)}}{\left(a^2 c \,x^2+c \right)^2}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

3.707.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.707.6 Sympy [N/A]

Not integrable

Time = 29.84 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.707.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.707.8 Giac [N/A]

Not integrable

Time = 51.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.707.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\,\sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.708 $\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} dx$

3.708.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} dx = \text{Int}\left(\frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Unintegrable(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

3.708.2 Mathematica [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} dx = \int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

3.708.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.708.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.708.4 Maple [N/A] (verified)

Not integrable

Time = 12.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

3.708.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.708.6 Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.708.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.708.8 Giac [N/A]

Not integrable

Time = 61.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.708.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.709 $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

3.709.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = -\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{3/2}}{3a^3c^2} + \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}$$

output
```
1/3*arctan(a*x)^(3/2)/a^3/c^2+1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*P
i^(1/2)/a^3/c^2-1/2*x*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

3.709.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\arctan(ax)}\left(-\frac{3ax}{1+a^2x^2}+2\arctan(ax)\right)+3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{24a^3c^2}$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*((-3*a*x)/(1 + a^2*x^2) + 2*ArcTan[a*x]) + 3*Sqrt[Pi]
*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(24*a^3*c^2)
```

3.709.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5471, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5471}$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4ac^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 3832}$$

$$\frac{\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2\,(a^2x^2+1)}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*Sqrt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*
a^3*c^2) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^3*c^2)
```

3.709.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.709. $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.709.4 Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{3\sqrt{\arctan(ax)}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+8\arctan(ax)^{2}-6\sin(2\arctan(ax))\arctan(ax)}{24c^{2}a^{3}\sqrt{\arctan(ax)}}$	60

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/c^2/a^3*(3*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi
^(1/2))+8*arctan(a*x)^2-6*sin(2*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2
)
```

3.709.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.709.6 Sympy [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.709.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.709.8 Giac [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^2 \sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.710 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

3.710.1 Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\sqrt{\arctan(ax)}}{4a^2c^2} - \frac{\sqrt{\arctan(ax)}}{2a^2c^2(1+a^2x^2)} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}$$

output
```
1/8*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2+1/4*arctan(a*x
)^(1/2)/a^2/c^2-1/2*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

3.710.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) + \frac{\frac{16(-1+a^2x^2)\arctan(ax)}{1+a^2x^2} - i\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) + i\sqrt{2}\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},2i\arctan(\right.}{\sqrt{\arctan(ax)}}}{64a^2c^2}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((16*(-1 + a^2*x^2)
*ArcTan[a*x])/(1 + a^2*x^2) - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
(-2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTa
n[a*x]])/Sqrt[ArcTan[a*x]])/(64*a^2*c^2)
```

3.710.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5465, 27, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4ac^2}-\frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$
$$\Big\downarrow \text{5439}$$
$$\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2c^2}-\frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2c^2}-\frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$
$$\Big\downarrow \text{3793}$$
$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2c^2}-\frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$
$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2c^2}-\frac{\sqrt{\arctan(ax)}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*Sqrt[ArcTan[a*x]]/(a^2*c^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqr
t[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2*c^2)
```

3.710.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.710.4 Maple [A] (verified)

Time = 5.68 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{-2\sqrt{\arctan\left(a x\right)}\, \sqrt{\pi}\, \cos\left(2 \arctan\left(a x\right)\right)+\pi \,\mathrm{FresnelC}\left(\frac{2 \sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)}{8 c^{2} a^{2} \sqrt{\pi}}$	45

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^2/a^2/Pi^(1/2)*(-2*arctan(a*x)^(1/2)*Pi^(1/2)*cos(2*arctan(a*x))+Pi*
FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

3.710.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.710.6 Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.710.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.710.8 Giac [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

3.711 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$

3.711.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{x\sqrt{\arctan(ax)}}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{3/2}}{3ac^2} - \frac{\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8ac^2}$$

output
```
1/3*arctan(a*x)^(3/2)/a/c^2-1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^
(1/2)/a/c^2+1/2*x*arctan(a*x)^(1/2)/c^2/(a^2*x^2+1)
```

3.711.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\arctan(ax)}\left(\frac{3x}{1+a^2x^2} + \frac{2\arctan(ax)}{a}\right) - \frac{3\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a}}{24c^2}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*((3*x)/(1 + a^2*x^2) + (2*ArcTan[a*x])/a) - (3*Sqrt[P
i]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/a)/(24*c^2)
```

3.711.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5427, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-\frac{1}{4}a\int \frac{x}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4c^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{x\sqrt{\arctan(ax)}}{2c^2\left(a^2x^2+1\right)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8ac^2}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

output
```
(x*Sqrt[ArcTan[a*x]])/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a*c^2)
- (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a*c^2)
```

3.711.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.711.4 Maple [A] (verified)

Time = 6.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{8\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}+6\sin(2\arctan(ax))\sqrt{\arctan(ax)}\,\sqrt{\pi}-3\pi\ \mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{24c^2a\sqrt{\pi}}$	57

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/c^2/a*(8*arctan(a*x)^(3/2)*Pi^(1/2)+6*sin(2*arctan(a*x))*arctan(a*x)^
(1/2)*Pi^(1/2)-3*Pi*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

3.711.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.711.6 Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.711.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.711.8 Giac [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{\sqrt{\mathrm{atan}\,(a\,x)}}{{(c\,a^2\,x^2+c)}^2}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^2, x)
```

3.712 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^2}\,dx$

3.712.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx=\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

3.712.2 Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2), x]
```

3.712.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
$Aborted
```

3.712.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.712.4 Maple [N/A] (verified)

Not integrable

Time = 4.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

3.712.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.712.6 Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

3.712.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.712.8 Giac [N/A]

Not integrable

Time = 30.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.712.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^2), x)
```

3.713 $\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx$

3.713.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx = \operatorname{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

3.713.2 Mathematica [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

3.713.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.713.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.713.4 Maple [N/A] (verified)

Not integrable

Time = 9.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^3}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

3.713.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3), x)
```

3.713.6 Sympy [N/A]

Not integrable

Time = 114.91 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^m\sqrt{\operatorname{atan}{(ax)}}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

3.713.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.713.8 Giac [N/A]

Not integrable

Time = 52.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.713.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.714 $\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

3.714.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Int}\left(\frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3},x\right)$$

output
```
Unintegrable(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

3.714.2 Mathematica [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$

input
```
Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

3.714.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^5\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.714.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.714.4 Maple [N/A] (verified)

Not integrable

Time = 12.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^3}dx$$

input
```
int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

3.714.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.714.6 Sympy [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^5\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**5*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**5*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

3.714.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.714.8 Giac [N/A]

Not integrable

Time = 69.47 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.714.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x^5*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^5*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.715 $\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

3.715.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx=\frac{\arctan(ax)^{3/2}}{4a^5c^3}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^5c^3}+\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3}-\frac{\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{4a^5c^3}+\frac{\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{32a^5c^3}$$

output
```
1/4*arctan(a*x)^(3/2)/a^5/c^3-1/128*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c^3+1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2
))*Pi^(1/2)/a^5/c^3-1/4*sin(2*arctan(a*x))*arctan(a*x)^(1/2)/a^5/c^3+1/32*
sin(4*arctan(a*x))*arctan(a*x)^(1/2)/a^5/c^3
```

3.715.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.30

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx=\frac{-\frac{96ax\arctan(ax)}{(1+a^2x^2)^2}-\frac{160a^3x^3\arctan(ax)}{(1+a^2x^2)^2}+64\arctan(ax)^2-8\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)-8\sqrt{2}\sqrt{i}}{256a^5c^3\sqrt{}}$$

input
```
Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
((-96*a*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (160*a^3*x^3*ArcTan[a*x])/(1 + a^
2*x^2)^2 + 64*ArcTan[a*x]^2 - 8*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
(-2*I)*ArcTan[a*x]] - 8*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTa
n[a*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*A
rcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(256*a^5*c^3*Sqrt[ArcTan[a*x]])
```

3.715.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^4x^4\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^4d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(-\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(2a}{a^5c^3}$$

input
```
Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/64 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 - (Sqrt[ArcT
an[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32
)/(a^5*c^3)
```

3.715.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.715.4 Maple [A] (verified)

Time = 6.96 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

method	result
default	$\frac{-\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+16\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\arctan(ax)^2-32\sin(2\arctan(}{128c^3a^5\sqrt{\arctan(ax)}}$

input
```
int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128/c^3/a^5*(-FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arc
tan(a*x)^(1/2)*Pi^(1/2)+16*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*
x)^(1/2)/Pi^(1/2))+32*arctan(a*x)^2-32*sin(2*arctan(a*x))*arctan(a*x)+4*si
n(4*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)
```

3.715.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.715.6 Sympy [F]

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^4\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**4*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

3.715.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.715.8 Giac [F]

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.715.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^4\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.716 $\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx$

3.716.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx = -\frac{3\sqrt{\arctan(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\arctan(ax)}}{4c^3(1+a^2x^2)^2} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^4c^3} + \frac{\sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16a^4c^3}$$

output
```
-1/128*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4
/c^3+1/16*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3-3/32*arc
tan(a*x)^(1/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(1/2)/c^3/(a^2*x^2+1)^2
```

3.716.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.95

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx$$

$$= \frac{-10\sqrt{2\pi} \operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + 80\sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) + \frac{\frac{64\left(-3-6a^2x^2+5a^4x^4\right)\arctan(ax)}{\left(1+a^2x^2\right)^2} - 12i\sqrt{2}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(-10*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 80*Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*
ArcTan[a*x])/(1 + a^2*x^2)^2 - (12*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma
[1/2, (-2*I)*ArcTan[a*x]] + (12*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(2*I)*ArcTan[a*x]] + (3*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan
[a*x]] - (3*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[Arc
Tan[a*x]])/(2048*a^4*c^3)
```

3.716.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5479, 27, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3(a^2x^2+1)^2}-\frac{1}{8}a\int \frac{x^4}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3(a^2x^2+1)^2}-\frac{a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{8c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3(a^2x^2+1)^2}-\frac{\int \frac{a^4x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3(a^2x^2+1)^2}-\frac{\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{\int\left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{8a^4c^3}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*Sqrt[ArcTan[a*x]])/(4*c^3*(1 + a^2*x^2)^2) - ((3*Sqrt[ArcTan[a*x]])/4
 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 - (Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(8*a^4*c^3)
```

3.716.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

3.716. $\int \frac{x^3\sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.716.4 Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+16\cos(2\arctan(ax))\arctan(ax)-8\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\arctan(ax)}\,\sqrt{\pi}}{128c^3a^4\sqrt{\arctan(ax)}}$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/128/c^3/a^4/arctan(a*x)^(1/2)*(FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^
(1/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+16*cos(2*arctan(a*x))*arctan(a*x
)-8*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)^(1/2)*Pi^(1/2)-4*co
s(4*arctan(a*x))*arctan(a*x))
```

3.716.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.716.6 Sympy [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

3.716.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.716.8 Giac [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.716.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.717 $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

3.717.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\arctan(ax)^{3/2}}{12a^3c^3} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^3c^3} - \frac{\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{32a^3c^3}$$

output
```
1/12*arctan(a*x)^(3/2)/a^3/c^3+1/128*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3-1/32*sin(4*arctan(a*x))*arctan(a*x)^(1/
2)/a^3/c^3
```

3.717.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{32\arctan(ax)\left(3ax(-1+a^2x^2)+2(1+a^2x^2)^2\arctan(ax)\right) - 3(1+a^2x^2)^2\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-4i\arct\right.}{768a^3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(32*ArcTan[a*x]*(3*a*x*(-1 + a^2*x^2) + 2*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3
*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - 3
*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(768*a
^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

3.717.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{1}{8}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\arctan(ax)^{3/2}-\frac{1}{32}\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{a^3c^3}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(3/2)/12 + (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]])/64 - (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32)/(a^3*c^3)
```

3.717.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.717.4 Maple [A] (verified)

Time = 5.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{3\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+32\arctan(ax)^{2}-12\sin(4\arctan(ax))\arctan(ax)}{384c^{3}a^{3}\sqrt{\arctan(ax)}}$	66

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/384/c^3/a^3*(3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*ar
ctan(a*x)^(1/2)*Pi^(1/2)+32*arctan(a*x)^2-12*sin(4*arctan(a*x))*arctan(a*x
))/arctan(a*x)^(1/2)
```

3.717.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.717.6 Sympy [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

3.717.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.717.8 Giac [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.717.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.718 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

3.718.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{3\sqrt{\arctan(ax)}}{32a^2c^3} - \frac{\sqrt{\arctan(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^2c^3} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16a^2c^3}$$

output
```
1/128*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/
c^3+1/16*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3+3/32*arct
an(a*x)^(1/2)/a^2/c^3-1/4*arctan(a*x)^(1/2)/a^2/c^3/(a^2*x^2+1)^2
```

3.718.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.95

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$

$$= \frac{-6\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + 48\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) + \frac{\frac{64(-5+6a^2x^2+3a^4x^4)\arctan(ax)}{(1+a^2x^2)^2} - 20i\sqrt{2}\sqrt{}}{}}{}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(-6*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 48*Sqrt[Pi]*Fres
nelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*A
rcTan[a*x])/(1 + a^2*x^2)^2 - (20*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[
1/2, (-2*I)*ArcTan[a*x]] + (20*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (
2*I)*ArcTan[a*x]] - (11*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan
[a*x]] + (11*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[Ar
cTan[a*x]])/(2048*a^2*c^3)
```

3.718.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5465, 27, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{1}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{8a}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{8ac^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*Sqrt[ArcTan[a*x]]/(a^2*c^3*(1 + a^2*x^2)^2) + ((3*Sqrt[ArcTan[a*x]])/
4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*Fr
esnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(8*a^2*c^3)
```

3.718.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.718. $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.718.4 Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
default	$-\frac{-\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+16\cos(2\arctan(ax))\arctan(ax)-8\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\arctan(ax)}\,\sqrt{}}{128c^3a^2\sqrt{\arctan(ax)}}$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/128/c^3/a^2/arctan(a*x)^(1/2)*(-FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+16*cos(2*arctan(a*x))*arctan(a*
x)-8*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)^(1/2)*Pi^(1/2)+4*c
os(4*arctan(a*x))*arctan(a*x))
```

3.718.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.718.6 Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

3.718.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.718.8 Giac [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.718.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

3.719 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$

3.719.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\arctan(ax)^{3/2}}{4ac^3} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64ac^3} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8ac^3} + \frac{\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{4ac^3} + \frac{\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{32ac^3}$$

output
```
1/4*arctan(a*x)^(3/2)/a/c^3-1/128*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^
(1/2))*2^(1/2)*Pi^(1/2)/a/c^3-1/8*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*P
i^(1/2)/a/c^3+1/4*sin(2*arctan(a*x))*arctan(a*x)^(1/2)/a/c^3+1/32*sin(4*ar
ctan(a*x))*arctan(a*x)^(1/2)/a/c^3
```

3.719.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{-16\sqrt{\arctan(ax)}\left(\frac{ax(5+3a^2x^2)}{(1+a^2x^2)^2}+2\arctan(ax)\right)+\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+16\sqrt{\pi}\,\text{FresnelS}}{128ac^3}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

output
```
-1/128*(-16*Sqrt[ArcTan[a*x]]*((a*x*(5 + 3*a^2*x^2))/(1 + a^2*x^2)^2 + 2*A
rcTan[a*x]) + Sqrt[2*Pi]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 16*Sqr
t[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a*c^3)
```

3.719.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \ 5439$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \ 3793$$

$$\frac{\int \left(\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin(2a}{ac^3}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/64 - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 + (Sqrt[ArcT
an[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32
)/(a*c^3)
```

3.719.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.719.4 Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

method	result
default	$\frac{-\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+32\arctan(ax)^2+32\sin(2\arctan(ax))\arctan(ax)+4\sin(4\arctan(ax))\arctan(ax}{128c^3a\sqrt{\arctan(ax)}}$

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128/c^3/a/arctan(a*x)^(1/2)*(-FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+32*arctan(a*x)^2+32*sin(2*arctan(a
*x))*arctan(a*x)+4*sin(4*arctan(a*x))*arctan(a*x)-16*arctan(a*x)^(1/2)*Pi^
(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

3.719.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c + a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.719.6 Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c + a^2cx^2)^3} \, dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c
**3
```

3.719.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c + a^2cx^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.719.8 Giac [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx=\int \frac{\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.719.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^3, x)
```

3.720 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^3}\,dx$

3.720.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx=\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

3.720.2 Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3), x]
```

3.720.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

3.720.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.720.4 Maple [N/A] (verified)

Not integrable

Time = 4.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^2c\,x^2+c\right)^3}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

3.720.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.720.6 Sympy [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c
**3
```

3.720.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.720.8 Giac [N/A]

Not integrable

Time = 34.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.720.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^3), x)
```

3.721 $\int x^m \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$

3.721.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Int}\left(x^m \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.721.2 Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x^m \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

3.721.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \textcolor{blue}{5560}$$

$$\int x^m \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.721.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.721.4 Maple [N/A] (verified)

Not integrable

Time = 12.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.721.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{c+a^2cx^2} \sqrt{\arctan(ax)}\, dx = \int \sqrt{a^2cx^2+c}\, x^m \sqrt{\arctan(ax)}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

3.721.6 Sympy [N/A]

Not integrable

Time = 63.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \sqrt{\arctan(ax)}\, dx = \int x^m \sqrt{c\,(a^2x^2+1)} \sqrt{\operatorname{atan}(ax)}\, dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

3.721.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \sqrt{c+a^2cx^2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.721.8 Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.721.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.722 $\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$

3.722.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\text{Int}\left(x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.722.2 Mathematica [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

3.722.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \textcolor{blue}{5560}$$

$$\int x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.722.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.722.4 Maple [N/A] (verified)

Not integrable

Time = 14.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \sqrt{a^{2} c\,x^{2}+c}\,\sqrt{\arctan\left(a x \right)}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.722.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.722.6 Sympy [N/A]

Not integrable

Time = 5.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x^{2}\sqrt{c\left(a^{2}x^{2}+1\right)}\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

3.722.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.722.8 Giac [N/A]

Not integrable

Time = 77.99 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\int \sqrt{a^2cx^2+c}x^2\sqrt{\arctan{(ax)}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.722.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\int x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.723 $\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$

3.723.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\frac{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}{3a^2c}-\frac{\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)}{6a}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a^2/c-1/6*Unintegrable((a^2*c*x^
2+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

3.723.2 Mathematica [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

3.723.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}{3a^2c}-\frac{\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx}{6a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}{3a^2c}-\frac{\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx}{6a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.723.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.723.4 Maple [N/A] (verified)

Not integrable

Time = 9.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.723.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.723.6 Sympy [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

3.723.7 Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.723.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.723.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.724 $\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$

3.724.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \mathrm{Int}\left(\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.724.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

3.724.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.724.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.724.4 Maple [N/A] (verified)

Not integrable

Time = 12.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.724.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.724.6 Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

3.724.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.724.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.724.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.725 $\int x^m(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$

3.725.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

3.725.2 Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

3.725.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2 c x^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \sqrt{\arctan(ax)} \left(a^2 c x^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.725.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.725.4 Maple [N/A] (verified)

Not integrable

Time = 16.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

3.725.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan(ax)}\, dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m \sqrt{\arctan(ax)}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*sqrt(arctan(a*x)), x)
```

3.725.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.725.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.725.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.725.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \sqrt{\arctan(ax)}\,dx = \int x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.726 $\int x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$

3.726.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

3.726.2 Mathematica [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

3.726.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{\arctan(ax)} \left(a^2 c x^2+c\right)^{3/2} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int x^2 \sqrt{\arctan(ax)} \left(a^2 c x^2+c\right)^{3/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.726.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.726.4 Maple [N/A] (verified)

Not integrable

Time = 16.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

3.726.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \sqrt{\arctan(ax)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.726.6 Sympy [N/A]

Not integrable

Time = 129.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 c x^2)^{3/2} \sqrt{\arctan(ax)} \, dx = \int x^2 (c (a^2 x^2 + 1))^{\frac{3}{2}} \sqrt{\operatorname{atan}(ax)} \, dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

3.726.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \sqrt{\arctan(ax)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.726.8 Giac [N/A]

Not integrable

Time = 73.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)^{\frac{3}{2}}x^2\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.726.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \int x^2\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^{3/2}\,dx$$

input
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.727 $\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$

3.727.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}{5a^2c}-\frac{\operatorname{Int}\left(\frac{\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\arctan(ax)}},x\right)}{10a}$$

output
```
1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2)/a^2/c-1/10*Unintegrable((a^2*c*x
^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a
```

3.727.2 Mathematica [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

3.727.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx}{10a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx}{10a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.727.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.727.4 Maple [N/A] (verified)

Not integrable

Time = 13.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}d x$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

3.727.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.727.6 Sympy [N/A]

Not integrable

Time = 60.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \int x \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

3.727.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.727.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.727.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.728 $\int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$

3.728.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Int}\left((c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

3.728.2 Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

3.728.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.728.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.728.4 Maple [N/A] (verified)

Not integrable

Time = 13.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

3.728.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.728.6 Sympy [N/A]

Not integrable

Time = 32.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

3.728.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.728.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.728.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.729 $\int x^m (c+a^2cx^2)^{5/2} \sqrt{\arctan(ax)}\,dx$

3.729.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx = \text{Int}\left(x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

3.729.2 Mathematica [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx = \int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

3.729.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^{5/2} dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.729.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.729.4 Maple [N/A] (verified)

Not integrable

Time = 22.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \sqrt{\arctan\left(a x\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

3.729.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx = \int \left(a^2 c x^2+c\right)^{\frac{5}{2}} x^m \sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*sqrt(
arctan(a*x)), x)
```

3.729.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.729.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.729.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.729.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}\,dx = \int x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2} \,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.730 $\int x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$

3.730.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\mathrm{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

3.730.2 Mathematica [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

3.730.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{\arctan(ax)} \left(a^2 c x^2+c\right)^{5/2} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \sqrt{\arctan(ax)} \left(a^2 c x^2+c\right)^{5/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.730.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.730.4 Maple [N/A] (verified)

Not integrable

Time = 14.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

3.730.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2cx^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.730.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2cx^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.730.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2cx^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.730.8 Giac [N/A]

Not integrable

Time = 73.54 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \int (a^2 c x^2 + c)^{\frac{5}{2}} x^2 \sqrt{\arctan(ax)}\, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.730.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \int x^2 \sqrt{\mathrm{atan}(a\,x)}\, (c\,a^2\,x^2 + c)^{5/2}\, dx$$

input
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.731 $\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$

3.731.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx=\frac{(c+a^2cx^2)^{7/2}\sqrt{\arctan(ax)}}{7a^2c}-\frac{\operatorname{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}},x\right)}{14a}$$

output
```
1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(1/2)/a^2/c-1/14*Unintegrable((a^2*c*x
^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a
```

3.731.2 Mathematica [N/A]

Not integrable

Time = 4.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx=\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

3.731.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx}{14a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx}{14a}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.731.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.731.4 Maple [N/A] (verified)

Not integrable

Time = 14.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

3.731.5 Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.731.6 Sympy [F(-1)]

Timed out.

$$\int x\left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.731.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.731.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.731.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.732 $\int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$

3.732.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Int}\Big((c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)},x\Big)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

3.732.2 Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

3.732.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.732.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.732.4 Maple [N/A] (verified)

Not integrable

Time = 13.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

3.732.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.732.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.732.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.732.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.732.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\int \sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.733 $\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}} \, dx$

3.733.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}} \, dx = \operatorname{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.733.2 Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}} \, dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

3.733.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}} dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.733.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.733.4 Maple [N/A] (verified)

Not integrable

Time = 16.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \sqrt{\arctan\left(a x\right)}}{\sqrt{a^2 c\, x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.733.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

3.733.6 Sympy [N/A]

Not integrable

Time = 32.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^m\sqrt{\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.733.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.733.8 Giac [N/A]

Not integrable

Time = 51.72 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.733.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.734 $\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

3.734.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3a^2c}+\frac{\operatorname{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{3a^3}-\frac{\operatorname{Int}\left(\frac{x^2}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{6a}$$

output
```
-2/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^4/c+1/3*x^2*(a^2*c*x^2+c)^(1/
2)*arctan(a*x)^(1/2)/a^2/c+1/3*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a
*x)^(1/2),x)/a^3-1/6*Unintegrable(x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2
),x)/a
```

3.734.2 Mathematica [N/A]

Not integrable

Time = 4.89 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

3.734.3 Rubi [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5487, 5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{6a}-\frac{2\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{6a}+\frac{x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{6a}+\frac{x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3a^2c}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.734.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.734.4 Maple [N/A] (verified)

Not integrable

Time = 14.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{\sqrt{a^2 c x^2 + c}} dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.734.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.734.6 Sympy [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.734.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.734.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.734.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.735 $\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

3.735.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{2a^2c} - \frac{\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{4a} - \frac{\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{2a^2}$$

output
```
1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2/c-1/4*Unintegrable(x/(a^2*
c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-1/2*Unintegrable(arctan(a*x)^(1/2)/(
a^2*c*x^2+c)^(1/2),x)/a^2
```

3.735.2 Mathematica [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

3.735.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5487, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.735.3.1 Defintions of rubi rules used

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.735.4 Maple [N/A] (verified)

Not integrable

Time = 13.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2\sqrt{\arctan\left(ax\right)}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.735.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.735.6 Sympy [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.735.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.735.8 Giac [N/A]

Not integrable

Time = 182.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.735.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.736 $\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

3.736.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{a^2c}-\frac{\operatorname{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{2a}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2/c-1/2*Unintegrable(1/(a^2*c*x^2+
c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

3.736.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

3.736.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c} - \frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c} - \frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.736.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.736.4 Maple [N/A] (verified)

Not integrable

Time = 7.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{\arctan{(ax)}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.736.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.736.6 Sympy [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\sqrt{\operatorname{atan}{(ax)}}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.736.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.736.8 Giac [N/A]

Not integrable

Time = 191.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.736.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.737 $\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$

3.737.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.737.2 Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x]
```

3.737.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.737.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.737.4 Maple [N/A] (verified)

Not integrable

Time = 7.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{\sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.737.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.737.6 Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

3.737.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.737.8 Giac [N/A]

Not integrable

Time = 169.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan{(ax)}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.737.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.738 $\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$

3.738.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.738.2 Mathematica [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x]
```

3.738.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.738.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.738.4 Maple [N/A] (verified)

Not integrable

Time = 14.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan \left(a x \right)}}{x \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.738.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.738.6 Sympy [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

3.738.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.738.8 Giac [N/A]

Not integrable

Time = 168.35 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan{(ax)}}}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.738.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.739 $\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$

3.739.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{cx} + \frac{1}{2}a\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x+1/2*a*Unintegrable(1/x/(a^2*c*x
^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.739.2 Mathematica [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

3.739.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5479, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{5560}$$

$$\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.739.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.739.4 Maple [N/A] (verified)

Not integrable

Time = 14.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{x^{2} \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

3.739.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.739.6 Sympy [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

3.739.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.739.8 Giac [N/A]

Not integrable

Time = 184.65 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}\,x^2}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.739.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.740 $\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$

3.740.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{2cx^2}+\frac{1}{4}a\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{1}{2}a^2\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^2+1/4*a*Unintegrable(1/x^2/
(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)-1/2*a^2*Unintegrable(arctan(a*x)^
(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.740.2 Mathematica [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

3.740.3 Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5497, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5497}$$
$$-\frac{1}{2}a^2\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}$$
$$\Big\downarrow \text{5560}$$
$$-\frac{1}{2}a^2\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.740.3.1 Defintions of rubi rules used

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.740.4 Maple [N/A] (verified)

Not integrable

Time = 14.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{x^{3} \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

3.740.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.740.6 Sympy [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

3.740.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.740.8 Giac [N/A]

Not integrable

Time = 185.55 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+cx^3}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.740.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.741 $\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.741.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3cx} + \frac{1}{6}a\mathrm{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{1}{3}a^3\mathrm{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^3+2/3*a^2*(a^2*c*x^2+c)^(1/
2)*arctan(a*x)^(1/2)/c/x+1/6*a*Unintegrable(1/x^3/(a^2*c*x^2+c)^(1/2)/arct
an(a*x)^(1/2),x)-1/3*a^3*Unintegrable(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^
(1/2),x)
```

3.741.2 Mathematica [N/A]

Not integrable

Time = 16.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

3.741.3 Rubi [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5497, 5479, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\int \frac{1}{x^3\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{6}a\int \frac{1}{x^3\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2}{3}a^2\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{6}a\int \frac{1}{x^3\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.741.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.741.4 Maple [N/A] (verified)

Not integrable

Time = 14.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x^4\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

3.741.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.741.6 Sympy [N/A]

Not integrable

Time = 19.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^4\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

3.741.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.741.8 Giac [N/A]

Not integrable

Time = 186.46 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.741.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.742 $\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$

3.742.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \text{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.742.2 Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

3.742.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.742.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.742.4 Maple [N/A] (verified)

Not integrable

Time = 12.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m\sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.742.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^
2*x^2 + c^2), x)
```

3.742.6 Sympy [N/A]

Not integrable

Time = 102.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.742.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.742.8 Giac [N/A]

Not integrable

Time = 52.40 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.742.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.743 $\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

3.743.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.743.2 Mathematica [N/A]

Not integrable

Time = 7.73 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

3.743.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.743.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.743.4 Maple [N/A] (verified)

Not integrable

Time = 7.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3\sqrt{\arctan\left(a x\right)}}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.743.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.743.6 Sympy [N/A]

Not integrable

Time = 9.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.743.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.743.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.743.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.744 $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

3.744.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.744.2 Mathematica [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

3.744.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.744.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.744.4 Maple [N/A] (verified)

Not integrable

Time = 7.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \sqrt{\arctan \left(a x \right)}}{\left(a^2 c \,x^2+c \right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.744.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.744.6 Sympy [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.744.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.744.8 Giac [N/A]

Not integrable

Time = 67.52 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.744.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.745 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

3.745.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
1/2*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^
(1/2)
```

3.745.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{-4\arctan(ax) - i\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right) + i\sqrt{1+a^2x^2}\sqrt{i}}{4a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(-4*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
I)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*Arc
Tan[a*x]])/(4*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.745.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5465, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{2a}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{2ac\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(Sqrt[ArcTan[a*x]]/(a^2*c*Sqrt[c + a^2*c*x^2])) + (Sqrt[Pi/2]*Sqrt[1 + a^
2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

3.745.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.745. $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

3.745.4 Maple [F]

$$\int \frac{x\sqrt{\arctan\left(a x\right)}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.745.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.745.6 Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\sqrt{\operatorname{atan}{\left(a x \right)}}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.745.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.745.8 Giac [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.745.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.746 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

3.746.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\sqrt{\arctan(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

output
```
-1/2*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/
2)
```

3.746.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{4ax\arctan(ax) + \sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2}, -i\arctan(ax)\right) + \sqrt{1+a^2x^2}\sqrt{i \text{ ar}}}{4ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(4*a*x*ArcTan[a*x] + Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (
-I)*ArcTan[a*x]] + Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcT
an[a*x]])/(4*a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.746.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5440, 5439, 3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - Sqrt[Pi/2]
*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(a*c*Sqrt[c + a^2*c*x^2])
```

3.746.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.746. $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.746.4 Maple [F]

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}d x$$

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.746.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.746.6 Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}\left(a x\right)}}{\left(c\left(a^{2} x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.746.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.746.8 Giac [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.746.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.747 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{3/2}}\,dx$

3.747.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

3.747.2 Mathematica [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)), x]
```

3.747.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.747.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.747.4 Maple [N/A] (verified)

Not integrable

Time = 11.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan \left(a x \right)}}{x \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

3.747.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.747.6 Sympy [N/A]

Not integrable

Time = 8.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.747.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.747.8 Giac [N/A]

Not integrable

Time = 35.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.747.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.748 $\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)^{3/2}}\,dx$

3.748.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

3.748.2 Mathematica [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

3.748.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(a^2cx^2+c\right)^{3/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.748.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.748.4 Maple [N/A] (verified)

Not integrable

Time = 11.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

3.748.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.748.6 Sympy [N/A]

Not integrable

Time = 18.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}}\, dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^2 (c (a^2x^2 + 1))^{\frac{3}{2}}}\, dx$$

input
```
integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.748.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.748.8 Giac [N/A]

Not integrable

Time = 40.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c+a^2cx^2)^{3/2}}\, dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\, dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.748.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c+a^2cx^2)^{3/2}}\, dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\, dx$$

input
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.749 $\int \frac{x^m \sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

3.749.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.749.2 Mathematica [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{x^m \sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

3.749.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.749.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.749.4 Maple [N/A] (verified)

Not integrable

Time = 33.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.749.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

3.749.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.749.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.749.8 Giac [N/A]

Not integrable

Time = 51.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{x^m\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.749.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.750 $\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

3.750.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \mathrm{Int}\left(\frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.750.2 Mathematica [N/A]

Not integrable

Time = 7.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

3.750.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.750.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.750.4 Maple [N/A] (verified)

Not integrable

Time = 30.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^4\sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.750.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.750.6 Sympy [N/A]

Not integrable

Time = 50.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\sqrt{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.750.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.750.8 Giac [N/A]

Not integrable

Time = 83.62 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.750.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.751 $\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

3.751.1 Optimal result

Integrand size = 26, antiderivative size = 215

$$\begin{aligned}\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx &= -\frac{3\sqrt{\arctan(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\cos(3\arctan(ax))}{12a^4c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^4c^2\sqrt{c+a^2cx^2}} \\ &- \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/72*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x
^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+3/8*FresnelC(2^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1
/2)-3/4*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+1/12*cos(3*arctan(a*
x))*(a^2*x^2+1)^(1/2)*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

3.751.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.51

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{-96\arctan(ax)-144a^2x^2\arctan(ax)-27i(1+a^2x^2)^{3/2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\right.}{}$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-96*ArcTan[a*x] - 144*a^2*x^2*ArcTan[a*x] - (27*I)*(1 + a^2*x^2)^(3/2)*Sq
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + (27*I)*(1 + a^2*x^2)^(
3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]
*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3
+ 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - I*Sqr
t[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] - I*a^2
*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]]
)/(144*a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.751.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \; 5506$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5505$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^3d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3793$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(3\arctan(ax))\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}+\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{arc}\right.}{a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-3*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[A
rcTan[a*x]]*Cos[3*ArcTan[a*x]])/12 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqr
t[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/1
2))/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```

3.751.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.751. $\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.751.4 Maple [F]

$$\int \frac{x^{3} \sqrt{\arctan \left(a x \right)}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.751.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c + a^2cx^2)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.751.6 Sympy [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.751.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.751.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.751.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.752 $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

3.752.1 Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3\sqrt{\arctan(ax)}}{3c(c+a^2cx^2)^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/72*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/8*FresnelS(2^(1/2)/Pi^(1/2)*arcta
n(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/
2)+1/3*x^3*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)
```

3.752.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{24a^3x^3\sqrt{\arctan(ax)} - 9\sqrt{2\pi}(1+a^2x^2)^{3/2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + \sqrt{6\pi}(1-}{72a^3c^2(1+a^2x^2)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(24*a^3*x^3*Sqrt[ArcTan[a*x]] - 9*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[
Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[Sq
rt[6/Pi]*Sqrt[ArcTan[a*x]]])/(72*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]
)
```

3.752.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5479, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c(a^2cx^2+c)^{3/2}} - \frac{1}{6}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c(a^2cx^2+c)^{3/2}} - \frac{a\sqrt{a^2x^2+1}\int \frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{6c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c(a^2cx^2+c)^{3/2}} - \frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*Sqrt[ArcTan[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[1 + a^2*x^2]*(
(3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*Fres
nelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(6*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

3.752.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.752. $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.752.4 Maple [F]

$$\int \frac{x^{2} \sqrt{\arctan \left(a x \right)}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.752.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.752.6 Sympy [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.752.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.752.8 Giac [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.752.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.753 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

3.753.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\begin{aligned}\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx &= -\frac{\sqrt{\arctan(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} \\ &+ \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^2c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/72*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/8*FresnelC(2^(1/2)/Pi^(1/2)*arcta
n(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/
2)-1/3*arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(3/2)
```

3.753.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{-48\arctan(ax) - i(1+a^2x^2)^{3/2}\left(9\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right) - 9\sqrt{i\arct}\right.}{144}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-48*ArcTan[a*x] - I*(1 + a^2*x^2)^(3/2)*(9*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-I)*ArcTan[a*x]] - 9*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] +
Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(144*a^2*c*(c + a^2*c*x^2)^(3/
2)*Sqrt[ArcTan[a*x]])
```

3.753.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5465, 5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{6a} - \frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{6ac^2\sqrt{a^2cx^2+c}} - \frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*Sqrt[ArcTan[a*x]]/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (Sqrt[1 + a^2*x^2]*
((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]*Fre
snelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(6*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

3.753.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.753. $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.753.4 Maple [F]

$$\int \frac{x\sqrt{\arctan\left(ax\right)}}{\left(a^{2} c \,x^{2}+c\right)^{\frac{5}{2}}}d x$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.753.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.753.6 Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.753.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.753.8 Giac [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.753.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\sqrt{\operatorname{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.754 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

3.754.1 Optimal result

Integrand size $=23$, antiderivative size $=213$

$$\begin{aligned}\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{3x\sqrt{\arctan(ax)}}{4c^2\sqrt{c+a^2cx^2}} \\ &\quad - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4ac^2\sqrt{c+a^2cx^2}} \\ &\quad - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12ac^2\sqrt{c+a^2cx^2}} \\ &\quad + \frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\sin(3\arctan(ax))}{12ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/72*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x
^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-3/8*FresnelS(2^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+
3/4*x*arctan(a*x)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+1/12*sin(3*arctan(a*x))*(a
^2*x^2+1)^(1/2)*arctan(a*x)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.754.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{48x(3+2a^2x^2)\sqrt{\arctan(ax)} - \frac{4\sqrt{6\pi}(1+a^2x^2)^{3/2}\left(3\sqrt{3}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\cdots}\right.\right.}{a}}{\cdots}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(48*x*(3 + 2*a^2*x^2)*Sqrt[ArcTan[a*x]] - (4*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2
)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6/Pi]*
Sqrt[ArcTan[a*x]]]))/a + (3*(1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*
Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a
*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sq
rt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(a*Sqrt[ArcTan[a*x]]))/
(144*c*(c + a^2*c*x^2)^(3/2))
```

3.754.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \; 5440$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5439$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{4}\sqrt{\arctan(ax)}\cos(3\arctan(ax))+\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{arct}}\right.}{ac^2\sqrt{a^2cx^2+c}}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*a*x*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) - (3*S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelS
[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/12 + (Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]]
)/12))/(a*c^2*Sqrt[c + a^2*c*x^2])
```

3.754.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.754. $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.754.4 Maple [F]

$$\int \frac{\sqrt{\arctan (ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.754.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.754.6 Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.754.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.754.8 Giac [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.754.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.755 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx$

3.755.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)^{5/2}}\,dx = \mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

3.755.2 Mathematica [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)), x]
```

3.755.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.755.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.755.4 Maple [N/A] (verified)

Not integrable

Time = 14.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

3.755.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.755.6 Sympy [N/A]

Not integrable

Time = 61.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x\,(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.755.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.755.8 Giac [N/A]

Not integrable

Time = 45.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.755.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.756 $\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx$

3.756.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\operatorname{Int}\left(x^m(c+a^2cx^2)\arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

3.756.2 Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

3.756.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.756.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.756.4 Maple [N/A] (verified)

Not integrable

Time = 8.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^m \left(a^2 c\, x^2+c\right) \arctan\left(a x\right)^{\frac{3}{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

3.756.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)x^m\arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

3.756.6 Sympy [F(-1)]

Timed out.

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.756.7 Maxima [F(-2)]

Exception generated.

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.756.8 Giac [N/A]

Not integrable

Time = 51.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int (a^2cx^2+c)x^m\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.756.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int x^m\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.757 $\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx$

3.757.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^2(c+a^2cx^2)\arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

3.757.2 Mathematica [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

3.757.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right) \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right) dx$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.757.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.757.4 Maple [N/A] (verified)

Not integrable

Time = 4.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

3.757.5 Fricas [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.757.6 Sympy [N/A]

Not integrable

Time = 8.87 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = c\left(\int x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^2x^4\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(a**2*x**4*atan(a*x)**(3/2
), x))
```

3.757.7 Maxima [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.757.8 Giac [N/A]

Not integrable

Time = 78.97 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)x^2\arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.757.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \int x^2\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.758 $\int x(c + a^2cx^2) \arctan(ax)^{3/2} dx$

3.758.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}\,dx=\frac{c(1+a^2x^2)^2\arctan(ax)^{3/2}}{4a^2}-\frac{3\mathrm{Int}\left(\left(c+a^2cx^2\right)\sqrt{\arctan(ax)},x\right)}{8a}$$

output
```
1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(3/2)/a^2-3/8*Unintegrable((a^2*c*x^2+c)*a
rctan(a*x)^(1/2),x)/a
```

3.758.2 Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}\,dx=\int x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

3.758.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right) dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c(a^2x^2+1)^2 \arctan(ax)^{3/2}}{4a^2}-\frac{3\int c(a^2x^2+1)\sqrt{\arctan(ax)}dx}{8a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c(a^2x^2+1)^2 \arctan(ax)^{3/2}}{4a^2}-\frac{3c\int (a^2x^2+1)\sqrt{\arctan(ax)}dx}{8a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{c(a^2x^2+1)^2 \arctan(ax)^{3/2}}{4a^2}-\frac{3c\int (a^2x^2+1)\sqrt{\arctan(ax)}dx}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.758.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.758.4 Maple [N/A] (verified)

Not integrable

Time = 3.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

3.758.5 Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.758.6 Sympy [N/A]

Not integrable

Time = 6.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = c\left(\int x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x*atan(a*x)**(3/2), x) + Integral(a**2*x**3*atan(a*x)**(3/2),
x))
```

3.758.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.758.8 Giac [N/A]

Not integrable

Time = 78.99 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)x\arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.758.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.759 $\int (c+a^2cx^2) \arctan(ax)^{3/2} \, dx$

3.759.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (c+a^2cx^2) \arctan(ax)^{3/2} \, dx = -\frac{c(1+a^2x^2)\sqrt{\arctan(ax)}}{4a} + \frac{1}{3}cx(1+a^2x^2)\arctan(ax)^{3/2} + \frac{1}{8}c\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right) + \frac{2}{3}c\text{Int}\left(\arctan(ax)^{3/2},x\right)$$

output
```
1/3*c*x*(a^2*x^2+1)*arctan(a*x)^(3/2)-1/4*c*(a^2*x^2+1)*arctan(a*x)^(1/2)/
a+2/3*c*Unintegrable(arctan(a*x)^(3/2),x)+1/8*c*Unintegrable(1/arctan(a*x)
^(1/2),x)
```

3.759.2 Mathematica [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+a^2cx^2) \arctan(ax)^{3/2} \, dx = \int (c+a^2cx^2) \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

3.759.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{8}c\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}c\int \arctan(ax)^{3/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{3/2}-\frac{c\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{1}{8}c\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}c\int \arctan(ax)^{3/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{3/2}-\frac{c\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.759.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

3.759.4 Maple [N/A] (verified)

Not integrable

Time = 3.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

3.759.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right) \arctan(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.759.6 Sympy [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int (c+a^2cx^2) \arctan(ax)^{3/2} \, dx = c\left(\int a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int \operatorname{atan}^{\frac{3}{2}}(ax) \, dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2), x)
)
```

3.759.7 Maxima [F(-2)]

Exception generated.

$$\int (c+a^2cx^2) \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.759.8 Giac [N/A]

Not integrable

Time = 77.83 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int (c+a^2cx^2) \arctan(ax)^{3/2} \, dx = \int (a^2cx^2 + c) \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.759.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c + a^2cx^2) \arctan(ax)^{3/2} \, dx = \int \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

3.760 $\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx$

3.760.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

3.760.2 Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x, x]
```

3.760.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2} (a^2cx^2+c)}{x} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{3/2} (a^2cx^2+c)}{x}dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

3.760.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.760.4 Maple [N/A] (verified)

Not integrable

Time = 4.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a^2c\,x^2+c)\arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

3.760.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.760.6 Sympy [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = c\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx + \int a^2x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x,x)
```

output
```
c*(Integral(atan(a*x)**(3/2)/x, x) + Integral(a**2*x*atan(a*x)**(3/2), x))
```

3.760.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.760.8 Giac [N/A]

Not integrable

Time = 194.85 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.760.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x, x)
```

3.761 $\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx$

3.761.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

3.761.2 Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2, x]
```

3.761.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)}{x^2} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
$Aborted
```

3.761.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.761.4 Maple [N/A] (verified)

Not integrable

Time = 5.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}}{x^{2}}d x$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

3.761.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.761.6 Sympy [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = c\left(\int a^2\,\mathrm{atan}^{\frac{3}{2}}(ax)\,dx + \int \frac{\mathrm{atan}^{\frac{3}{2}}(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**2, x)
)
```

3.761.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.761.8 Giac [N/A]

Not integrable

Time = 192.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.761.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x^2, x)
```

3.762 $\int x^m(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$

3.762.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \text{Int}\left(x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

3.762.2 Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

3.762.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.762.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.762.4 Maple [N/A] (verified)

Not integrable

Time = 10.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c \,x^2+c \right)^2 \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

3.762.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x^m \left(c+a^2 c x^2\right)^2 \arctan(ax)^{3/2}\,dx=\int \left(a^2 c x^2+c\right)^2 x^m \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(3/2), x)
```

3.762.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^2 \arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.762.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^2 \arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.762.8 Giac [N/A]

Not integrable

Time = 51.64 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.762.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.763 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$

3.763.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

3.763.2 Mathematica [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \int x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

3.763.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.763.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.763.4 Maple [N/A] (verified)

Not integrable

Time = 4.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2 \left(a^2 c\, x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

3.763.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.763.6 Sympy [N/A]

Not integrable

Time = 18.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = c^2 \left(\int x^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int 2a^2 x^4 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int a^4 x^6 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx \right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(2*a**2*x**4*atan(a*x)*
*(3/2), x) + Integral(a**4*x**6*atan(a*x)**(3/2), x))
```

3.763.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.763.8 Giac [N/A]

Not integrable

Time = 78.60 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c)^2 x^2 \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.763.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^2 dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.764 $\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$

3.764.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \frac{c^2(1+a^2x^2)^3 \arctan(ax)^{3/2}}{6a^2} - \frac{\operatorname{Int}\left((c+a^2cx^2)^2 \sqrt{\arctan(ax)}, x\right)}{4a}$$

output
```
1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a^2-1/4*Unintegrable((a^2*c*x^2+c)
^2*arctan(a*x)^(1/2),x)/a
```

3.764.2 Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

3.764.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \; 5465$$

$$\frac{c^2 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}{6a^2}-\frac{\int c^2 \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}dx}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{c^2 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}{6a^2}-\frac{c^2 \int \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}dx}{4a}$$

$$\Big\downarrow \; 5560$$

$$\frac{c^2 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}{6a^2}-\frac{c^2 \int \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}dx}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.764.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.764.4 Maple [N/A] (verified)

Not integrable

Time = 4.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

3.764.5 Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.764.6 Sympy [N/A]

Not integrable

Time = 13.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int x(c+a^2cx^2)^2\arctan(ax)^{3/2}\,dx = c^2\left(\int x\,\mathrm{atan}^{\frac{3}{2}}(ax)\,dx + \int 2a^2x^3\,\mathrm{atan}^{\frac{3}{2}}(ax)\,dx + \int a^4x^5\,\mathrm{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x*atan(a*x)**(3/2), x) + Integral(2*a**2*x**3*atan(a*x)**(3
/2), x) + Integral(a**4*x**5*atan(a*x)**(3/2), x))
```

3.764.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.764.8 Giac [N/A]

Not integrable

Time = 78.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x(c+a^2cx^2)^2\arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)^2x\arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.764.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^2\arctan(ax)^{3/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.765 $\int (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$

3.765.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\begin{aligned}\int (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = {} & -\frac{c^2(1+a^2x^2)\sqrt{\arctan(ax)}}{5a} - \frac{3c^2(1+a^2x^2)^2\sqrt{\arctan(ax)}}{40a} \\ & + \frac{4}{15}c^2x(1+a^2x^2)\arctan(ax)^{3/2} + \frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax)^{3/2} \\ & + \frac{1}{10}c^2\mathrm{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right) + \frac{3}{80}c\,\mathrm{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right) + \frac{8}{15}c^2\mathrm{Int}\left(\arctan(ax)^{3/2},x\right)\end{aligned}$$

output
```
4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^(3/2)+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*
x)^(3/2)-1/5*c^2*(a^2*x^2+1)*arctan(a*x)^(1/2)/a-3/40*c^2*(a^2*x^2+1)^2*ar
ctan(a*x)^(1/2)/a+8/15*c^2*Unintegrable(arctan(a*x)^(3/2),x)+1/10*c^2*Unin
tegrable(1/arctan(a*x)^(1/2),x)+3/80*c*Unintegrable((a^2*c*x^2+c)/arctan(a
*x)^(1/2),x)
```

3.765.2 Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx=\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

3.765.3 Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 27, 5415, 5353, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2dx$$

$$\Big\downarrow \text{ 5415}$$

$$\frac{3}{80}c\int \frac{c\left(a^2x^2+1\right)}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)^{3/2}dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}-\frac{3c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^{3/2}dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}-\frac{3c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \text{ 5415}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+ \frac{4}{5}c^2\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{\arctan(ax)}}{4a}\right)+ \frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{3/2}-\frac{3c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \textbf{5353}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+ \frac{4}{5}c^2\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{\arctan(ax)}}{4a}\right)+ \frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{3/2}-\frac{3c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \textbf{5560}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+ \frac{4}{5}c^2\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{\arctan(ax)}}{4a}\right)+ \frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{3/2}-\frac{3c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}{40a}$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.765.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.765. $\int (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.765.4 Maple [N/A] (verified)

Not integrable

Time = 3.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

3.765.5 Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.765.6 Sympy [N/A]

Not integrable

Time = 9.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = c^2\left(\int 2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**(3/2), x) + Integral(a**4*x**4*atan(
a*x)**(3/2), x) + Integral(atan(a*x)**(3/2), x))
```

3.765.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.765.8 Giac [N/A]

Not integrable

Time = 78.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \int \left(a^2cx^2+c\right)^2 \arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.765.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2)^2 \arctan(ax)^{3/2} \, dx = \int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^2 \, dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

3.766 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx$

3.766.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

3.766.2 Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x, x]
```

3.766.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2}{x} \, dx$$
$$\Big\downarrow \, 5560$$
$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2}{x} dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

3.766.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.766.4 Maple [N/A] (verified)

Not integrable

Time = 5.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

3.766.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.766.6 Sympy [N/A]

Not integrable

Time = 9.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}\,dx = c^2\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx + \int 2a^2x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^4x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x,x)
```

output
```
c**2*(Integral(atan(a*x)**(3/2)/x, x) + Integral(2*a**2*x*atan(a*x)**(3/2)
, x) + Integral(a**4*x**3*atan(a*x)**(3/2), x))
```

3.766.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.766.8 Giac [N/A]

Not integrable

Time = 260.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.766.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}{x} \, dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x, x)
```

3.766. $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx$

3.767 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx$

3.767.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

3.767.2 Mathematica [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2, x]
```

3.767.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$
$$\Big\downarrow \, 5560$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
$Aborted
```

3.767.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.767.4 Maple [N/A] (verified)

Not integrable

Time = 5.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

3.767.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.767.6 Sympy [N/A]

Not integrable

Time = 7.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}\,dx = c^2\left(\int 2a^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2}\,dx + \int a^4x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**
2, x) + Integral(a**4*x**2*atan(a*x)**(3/2), x))
```

3.767.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.767.8 Giac [N/A]

Not integrable

Time = 253.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx = \int \frac{(a^2cx^2+c)^2 \arctan{(ax)}^{\frac{3}{2}}}{x^2} dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.767.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}{x^2} dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x^2, x)
```

3.768 $\int x^m(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$

3.768.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \text{Int}\left(x^m(c+a^2cx^2)^3 \arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

3.768.2 Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int x^m(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

3.768.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{5560\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 \, dx$$
$$\Big\downarrow \; 5560$$
$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.768.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.768.4 Maple [N/A] (verified)

Not integrable

Time = 9.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c\, x^2+c\right)^3 \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

3.768.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^m (c + a^2cx^2)^3 \arctan(ax)^{3/2} \, dx = \int (a^2cx^2 + c)^3 x^m \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*
x)^(3/2), x)
```

3.768.6 Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2cx^2)^3 \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.768.7 Maxima [F(-2)]

Exception generated.

$$\int x^m (c + a^2cx^2)^3 \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.768.8 Giac [N/A]

Not integrable

Time = 52.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^m (c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)^3 x^m \arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.768.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.769 $\int x^2(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$

3.769.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx = \operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

3.769.2 Mathematica [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx = \int x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

3.769.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 \, dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.769.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.769.4 Maple [N/A] (verified)

Not integrable

Time = 4.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

3.769.5 Fricas [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.769.6 Sympy [N/A]

Not integrable

Time = 37.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx=c^3\left(\int x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^2x^4\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^4x^6\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^6x^8\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(3*a**2*x**4*atan(a*x)*
*(3/2), x) + Integral(3*a**4*x**6*atan(a*x)**(3/2), x) + Integral(a**6*x**
8*atan(a*x)**(3/2), x))
```

3.769.7 Maxima [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.769.8 Giac [N/A]

Not integrable

Time = 77.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)^3 x^2 \arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.769.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int x^2 \operatorname{atan}(a\,x)^{3/2}\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.770 $\int x(c + a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$

3.770.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c + a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \frac{c^3(1 + a^2x^2)^4 \arctan(ax)^{3/2}}{8a^2} - \frac{3\mathrm{Int}\left((c + a^2cx^2)^3 \sqrt{\arctan(ax)}, x\right)}{16a}$$

output
```
1/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^(3/2)/a^2-3/16*Unintegrable((a^2*c*x^2+c
)^3*arctan(a*x)^(1/2),x)/a
```

3.770.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c + a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int x(c + a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

3.770.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^3\left(a^2x^2+1\right)^4 \arctan(ax)^{3/2}}{8a^2}-\frac{3\int c^3\left(a^2x^2+1\right)^3 \sqrt{\arctan(ax)}dx}{16a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\left(a^2x^2+1\right)^4 \arctan(ax)^{3/2}}{8a^2}-\frac{3c^3\int \left(a^2x^2+1\right)^3 \sqrt{\arctan(ax)}dx}{16a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{c^3\left(a^2x^2+1\right)^4 \arctan(ax)^{3/2}}{8a^2}-\frac{3c^3\int \left(a^2x^2+1\right)^3 \sqrt{\arctan(ax)}dx}{16a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.770.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.770.4 Maple [N/A] (verified)

Not integrable

Time = 4.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

3.770.5 Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.770.6 Sympy [N/A]

Not integrable

Time = 29.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = c^3\left(\int x \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 3a^2x^3 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 3a^4x^5 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^6x^7 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(x*atan(a*x)**(3/2), x) + Integral(3*a**2*x**3*atan(a*x)**(3
/2), x) + Integral(3*a**4*x**5*atan(a*x)**(3/2), x) + Integral(a**6*x**7*a
tan(a*x)**(3/2), x))
```

3.770.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.770.8 Giac [N/A]

Not integrable

Time = 77.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)^3 x \arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.770.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.771 $\int (c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$

3.771.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = -\frac{6c^3(1+a^2x^2)\sqrt{\arctan(ax)}}{35a} - \frac{9c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}}{140a} - \frac{c^3(1+a^2x^2)^3\sqrt{\arctan(ax)}}{28a} + \frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^{3/2} + \frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^{3/2} + \frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^{3/2} + \frac{3}{35}c^3\mathrm{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right) + \frac{9}{280}c^2\mathrm{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right) + \frac{1}{56}c\mathrm{Int}\left(\frac{(c\cdots}{\sqrt{\mathrm{a}\cdots}}\right.$$

output
```
8/35*c^3*x*(a^2*x^2+1)*arctan(a*x)^(3/2)+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a
*x)^(3/2)+1/7*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^(3/2)-6/35*c^3*(a^2*x^2+1)*a
rctan(a*x)^(1/2)/a-9/140*c^3*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a-1/28*c^3*(a
^2*x^2+1)^3*arctan(a*x)^(1/2)/a+16/35*c^3*Unintegrable(arctan(a*x)^(3/2),x
)+3/35*c^3*Unintegrable(1/arctan(a*x)^(1/2),x)+9/280*c^2*Unintegrable((a^2
*c*x^2+c)/arctan(a*x)^(1/2),x)+1/56*c*Unintegrable((a^2*c*x^2+c)^2/arctan(
a*x)^(1/2),x)
```

3.771.2 Mathematica [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx = \int \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

3.771.3 Rubi [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5415, 27, 5415, 5415, 5353, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{56}c\int \frac{c^2\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{28a}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}c^3\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{28a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{56}c^3\int \frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\int (a^2x^2+1)\arctan(ax)^{3/2}dx+\frac{1}{5}x(a^2x^2+1)^2\arctan(ax)^{3/2}-\frac{3(a^2x^2+1)^2\sqrt{}}{40a}\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^{3/2}-\frac{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}{28a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{56}c^3\int \frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\right.\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^{3/2}-\frac{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}{28a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{1}{56}c^3\int \frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\right.\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^{3/2}-\frac{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}{28a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{1}{56}c^3\int \frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\right.\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^{3/2}-\frac{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}{28a}$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.771. $\int (c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx$

3.771.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.771.4 Maple [N/A] (verified)

Not integrable

Time = 3.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

3.771.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.771.6 Sympy [N/A]

Not integrable

Time = 18.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=c^3\left(\int 3a^2x^2\,\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^4x^4\,\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^6x^6\,\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**(3/2), x) + Integral(3*a**4*x**4*ata
n(a*x)**(3/2), x) + Integral(a**6*x**6*atan(a*x)**(3/2), x) + Integral(ata
n(a*x)**(3/2), x))
```

3.771.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.771.8 Giac [N/A]

Not integrable

Time = 76.85 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2} \, dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.771.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2} \, dx = \int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3 \, dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

3.772 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx$

3.772.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

3.772.2 Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x, x]
```

3.772.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}{x}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

3.772.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.772.4 Maple [N/A] (verified)

Not integrable

Time = 5.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

3.772.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.772.6 Sympy [N/A]

Not integrable

Time = 17.76 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x}\,dx = c^3\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 3a^4x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^6x^5\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x,x)
```

output
```
c**3*(Integral(atan(a*x)**(3/2)/x, x) + Integral(3*a**2*x*atan(a*x)**(3/2)
, x) + Integral(3*a**4*x**3*atan(a*x)**(3/2), x) + Integral(a**6*x**5*atan
(a*x)**(3/2), x))
```

3.772.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.772.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

3.772.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2 + c)^3}{x} \, dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x, x)
```

3.773 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx$

3.773.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx = \text{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

3.773.2 Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2, x]
```

3.773.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
$Aborted
```

3.773.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.773.4 Maple [N/A] (verified)

Not integrable

Time = 4.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

3.773.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.773.6 Sympy [N/A]

Not integrable

Time = 15.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx = c^3 \left(\int 3a^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2} \, dx + \int 3a^4x^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int a^6x^4 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**
2, x) + Integral(3*a**4*x**2*atan(a*x)**(3/2), x) + Integral(a**6*x**4*ata
n(a*x)**(3/2), x))
```

3.773.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.773.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.773.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}{x^2}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x^2, x)
```

3.774 $\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2} dx$

3.774.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

3.774.2 Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2} dx = \int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

3.774.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m \arctan(ax)^{3/2}}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.774.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.774.4 Maple [N/A] (verified)

Not integrable

Time = 5.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{a^2 c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

3.774.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan{(ax)}^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

3.774.6 Sympy [N/A]

Not integrable

Time = 71.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^{\frac{3}{2}}{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

3.774.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.774.8 Giac [N/A]

Not integrable

Time = 52.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.774.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.775 $\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx$

3.775.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx = -\frac{2x \arctan(ax)^{5/2}}{5a^3c} + \frac{\operatorname{Int}\left(x \arctan(ax)^{3/2}, x\right)}{a^2c} + \frac{2\operatorname{Int}\left(\arctan(ax)^{5/2}, x\right)}{5a^3c}$$

output
```
-2/5*x*arctan(a*x)^(5/2)/a^3/c+Unintegrable(x*arctan(a*x)^(3/2),x)/a^2/c+2
/5*Unintegrable(arctan(a*x)^(5/2),x)/a^3/c
```

3.775.2 Mathematica [N/A]

Not integrable

Time = 3.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx = \int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

3.775.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5451, 27, 5377, 5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \textbf{5451}$$

$$\frac{\int x \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{3/2}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int x \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \textbf{5377}$$

$$\frac{\int x \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \textbf{5457}$$

$$\frac{\int x \arctan(ax)^{3/2}dx}{a^2c} - \frac{\frac{2x \arctan(ax)^{5/2}}{5a} - \frac{2\int \arctan(ax)^{5/2}dx}{5a}}{a^2c}$$

$$\Big\downarrow \textbf{5353}$$

$$\frac{\int x \arctan(ax)^{3/2}dx}{a^2c} - \frac{\frac{2x \arctan(ax)^{5/2}}{5a} - \frac{2\int \arctan(ax)^{5/2}dx}{5a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.775.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.775.4 Maple [N/A] (verified)

Not integrable

Time = 3.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan\left(ax\right)^{\frac{3}{2}}}{a^2 c\, x^2 + c} dx$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

3.775.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c + a^2cx^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.775.6 Sympy [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c + a^2cx^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

3.775.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c + a^2cx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.775.8 Giac [N/A]

Not integrable

Time = 78.81 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^3 \arctan{(ax)}^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.775.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.776 $\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} dx$

3.776.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} dx = -\frac{2\arctan(ax)^{5/2}}{5a^3c} + \frac{\mathrm{Int}\left(\arctan(ax)^{3/2}, x\right)}{a^2c}$$

output
```
-2/5*arctan(a*x)^(5/2)/a^3/c+Unintegrable(arctan(a*x)^(3/2),x)/a^2/c
```

3.776.2 Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} dx = \int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

3.776.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5451, 27, 5353, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{5451}$$
$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^{3/2}}{c(a^2x^2+1)}dx}{a^2}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \text{5353}$$
$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \text{5419}$$
$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{2\arctan(ax)^{5/2}}{5a^3c}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.776.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.776.4 Maple [N/A] (verified)

Not integrable

Time = 3.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{2} \arctan \left(a x \right)^{\frac{3}{2}}}{a^{2} c \,x^{2}+c}d x$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

3.776.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c + a^2 c x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.776.6 Sympy [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c + a^2 c x^2} \, dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

3.776.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c + a^2 c x^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.776.8 Giac [N/A]

Not integrable

Time = 78.43 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c + a^2cx^2} \, dx = \int \frac{x^2 \arctan{(ax)}^{\frac{3}{2}}}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.776.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c + a^2cx^2} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{3/2}}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.777 $\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2} dx$

3.777.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2} dx = \frac{2x \arctan(ax)^{5/2}}{5ac} - \frac{2\text{Int}\left(\arctan(ax)^{5/2}, x\right)}{5ac}$$

output
```
2/5*x*arctan(a*x)^(5/2)/a/c-2/5*Unintegrable(arctan(a*x)^(5/2),x)/a/c
```

3.777.2 Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2} dx = \int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

3.777.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{3/2}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \; 5457$$
$$\frac{2x\arctan(ax)^{5/2}}{5ac}-\frac{2\int \arctan(ax)^{5/2}dx}{5ac}$$
$$\Big\downarrow \; 5353$$
$$\frac{2x\arctan(ax)^{5/2}}{5ac}-\frac{2\int \arctan(ax)^{5/2}dx}{5ac}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.777.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.777.4 Maple [N/A] (verified)

Not integrable

Time = 4.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan{(ax)}^{\frac{3}{2}}}{a^2 c\, x^2 + c} dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

3.777.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{c + a^2 c x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.777.6 Sympy [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan(ax)^{3/2}}{c + a^2 c x^2} dx = \frac{\int \frac{x \operatorname{atan}^{\frac{3}{2}}{(ax)}}{a^2 x^2 + 1} dx}{c}$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

3.777.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{c + a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.777.8 Giac [N/A]

Not integrable

Time = 74.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x \arctan(ax)^{3/2}}{c + a^2cx^2}\,dx = \int \frac{x \arctan\left(ax\right)^{\frac{3}{2}}}{a^2cx^2 + c}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.777.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^{3/2}}{c + a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{c\,a^2\,x^2 + c}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

3.778 $\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2} \, dx$

3.778.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2} \, dx = \frac{2\arctan(ax)^{5/2}}{5ac}$$

output
```
2/5*arctan(a*x)^(5/2)/a/c
```

3.778.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2} \, dx = \frac{2\arctan(ax)^{5/2}}{5ac}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(5/2))/(5*a*c)
```

3.778.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 5419}$$
$$\frac{2\arctan(ax)^{5/2}}{5ac}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(5/2))/(5*a*c)
```

3.778.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.778.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\arctan(ax)^{\frac{5}{2}}}{5ac}$	15

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/5*arctan(a*x)^(5/2)/a/c
```

3.778.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{2\arctan(ax)^{\frac{5}{2}}}{5\,ac}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
2/5*arctan(a*x)^(5/2)/(a*c)
```

3.778.6 Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

3.778.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.778.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2\arctan\left(ax\right)^{\frac{5}{2}}}{5\,ac}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/5*arctan(a*x)^(5/2)/(a*c)
```

3.778.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2\,\mathrm{atan}(a\,x)^{5/2}}{5\,a\,c}$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2),x)
```

output
```
(2*atan(a*x)^(5/2))/(5*a*c)
```

3.779 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx$

3.779.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx = -\frac{2i\arctan(ax)^{5/2}}{5c} + \frac{i\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x(i+ax)},x\right)}{c}$$

output
```
-2/5*I*arctan(a*x)^(5/2)/c+I*Unintegrable(arctan(a*x)^(3/2)/x/(I+a*x),x)/c
```

3.779.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)), x]
```

3.779.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5459, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)}\,dx$$
$$\Big\downarrow \text{5459}$$
$$\frac{i\int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{5/2}}{5c}$$
$$\Big\downarrow \text{5560}$$
$$\frac{i\int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{5/2}}{5c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.779.3.1 Defintions of rubi rules used

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.779.4 Maple [N/A] (verified)

Not integrable

Time = 4.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^{2} c\,x^{2}+c\right)}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

3.779.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.779.6 Sympy [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}{a^{2} x^{3} + x}\, dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**3 + x), x)/c
```

3.779.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.779.8 Giac [N/A]

Not integrable

Time = 72.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.779.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)), x)
```

3.780 $\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx$

3.780.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx=-\frac{2a\arctan(ax)^{5/2}}{5c}+\frac{\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2},x\right)}{c}$$

output
```
-2/5*a*arctan(a*x)^(5/2)/c+Unintegrable(arctan(a*x)^(3/2)/x^2,x)/c
```

3.780.2 Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)), x]
```

3.780.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)^{3/2}}{c\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-\frac{2a\arctan(ax)^{5/2}}{5c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.780.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.780.4 Maple [N/A] (verified)

Not integrable

Time = 3.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x^2 (a^2 c x^2 + c)} dx$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

3.780.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.780.6 Sympy [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^4+x^2}\,dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**4 + x**2), x)/c
```

3.780.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.780.8 Giac [N/A]

Not integrable

Time = 73.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.780.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)), x)
```

3.781 $\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)} dx$

3.781.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)} dx = \frac{2ia^2\arctan(ax)^{5/2}}{5c} + \frac{\operatorname{Int}\left(\frac{\arctan(ax)^{3/2}}{x^3},x\right)}{c} - \frac{ia^2\operatorname{Int}\left(\frac{\arctan(ax)^{3/2}}{x(i+ax)},x\right)}{c}$$

output
```
2/5*I*a^2*arctan(a*x)^(5/2)/c+Unintegrable(arctan(a*x)^(3/2)/x^3,x)/c-I*a^
2*Unintegrable(arctan(a*x)^(3/2)/x/(I+a*x),x)/c
```

3.781.2 Mathematica [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)} dx = \int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)} dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)), x]
```

3.781.3 Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5459, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c}-a^2\int \frac{\arctan(ax)^{3/2}}{cx\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx-\frac{2}{5}i\arctan(ax)^{5/2}\right)}{c}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx-\frac{2}{5}i\arctan(ax)^{5/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.781.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.781.4 Maple [N/A] (verified)

Not integrable

Time = 3.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{x^3 \left(a^2 c\,x^2+c\right)}dx$$

input
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

3.781.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.781.6 Sympy [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**5 + x**3), x)/c
```

3.781.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.781.8 Giac [N/A]

Not integrable

Time = 73.64 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.781.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)), x)
```

3.782 $\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx$

3.782.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx = \frac{2a^3\arctan(ax)^{5/2}}{5c} + \frac{\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^4},x\right)}{c} - \frac{a^2\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2},x\right)}{c}$$

output
```
2/5*a^3*arctan(a*x)^(5/2)/c+Unintegrable(arctan(a*x)^(3/2)/x^4,x)/c-a^2*Un
integrable(arctan(a*x)^(3/2)/x^2,x)/c
```

3.782.2 Mathematica [N/A]

Not integrable

Time = 4.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)), x]
```

3.782.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5453, 5377, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^4\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-a^2\int \frac{\arctan(ax)^{3/2}}{cx^2\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{3/2}}{x^2}dx-a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{3/2}}{x^2}dx-a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{3/2}}{x^2}dx-\frac{2}{5}a\arctan(ax)^{5/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.782. $\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx$

3.782.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.782.4 Maple [N/A] (verified)

Not integrable

Time = 4.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{x^4 (a^2 c x^2 + c)} dx$$

input
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

3.782.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4 (c + a^2cx^2)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.782.6 Sympy [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x^4 (c + a^2cx^2)} \, dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^6+x^4} \, dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**6 + x**4), x)/c
```

3.782.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4 (c + a^2cx^2)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.782.8 Giac [N/A]

Not integrable

Time = 72.60 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.782.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx = \int \frac{\operatorname{atan}(a\,x)^{3/2}}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)), x)
```

3.783 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$

3.783.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

3.783.2 Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

3.783.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.783.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.783.4 Maple [N/A] (verified)

Not integrable

Time = 15.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

3.783.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} \, dx = \int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.783.6 Sympy [N/A]

Not integrable

Time = 92.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^m \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} \, dx = \frac{\int \frac{x^m \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.783.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.783.8 Giac [N/A]

Not integrable

Time = 51.94 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.783.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.784 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$

3.784.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \mathrm{Int}\left(\frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Unintegrable(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

3.784.2 Mathematica [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

3.784.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^3 \arctan(ax)^{3/2}}{\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.784.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.784.4 Maple [N/A] (verified)

Not integrable

Time = 14.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{3} \arctan \left(a x \right)^{\frac{3}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{2}}d x$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

3.784.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.784.6 Sympy [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.784.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.784.8 Giac [N/A]

Not integrable

Time = 87.40 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.784.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.785 $\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$

3.785.1 Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \frac{3\sqrt{\arctan(ax)}}{16a^3c^2} - \frac{3\sqrt{\arctan(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{5/2}}{5a^3c^2} + \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^3c^2}$$

output
```
-1/2*x*arctan(a*x)^(3/2)/a^2/c^2/(a^2*x^2+1)+1/5*arctan(a*x)^(5/2)/a^3/c^2
+3/32*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2+3/16*arctan(
a*x)^(1/2)/a^3/c^2-3/8*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)
```

3.785.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.47

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \frac{\frac{16\sqrt{\arctan(ax)}(15(-1+a^2x^2)-40ax\arctan(ax)+16(1+a^2x^2)\arctan(ax)^2)}{1+a^2x^2} + 60\left(-2\sqrt{\arctan(ax)} + \right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
((16*Sqrt[ArcTan[a*x]]*(15*(-1 + a^2*x^2) - 40*a*x*ArcTan[a*x] + 16*(1 + a
^2*x^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) + 60*(-2*Sqrt[ArcTan[a*x]] + Sqrt[Pi
]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + (15*(8*ArcTan[a*x] - I*Sqrt[
2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[
I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(1280*a^
3*c^2)
```

3.785.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5471, 27, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5471}$$

$$\frac{3\int \frac{x\sqrt{\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5439}$$

$$\frac{3\left(\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan(ax)^{5/2}}{5a^3c^2}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^(3/2))/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*
a^3*c^2) + (3*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a
*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4
*a*c^2)
```

3.785.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.785.4 Maple [A] (verified)

Time = 7.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

method	result
default	$\frac{32\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}-40\arctan(ax)^{\frac{3}{2}}\sin(2\arctan(ax))\sqrt{\pi}+15\pi\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-30\sqrt{\arctan(ax)}\,\sqrt{\pi}\ \cos(2\arctan(ax))}{160c^2a^3\sqrt{\pi}}$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/160/c^2/a^3/Pi^(1/2)*(32*arctan(a*x)^(5/2)*Pi^(1/2)-40*arctan(a*x)^(3/2)
*sin(2*arctan(a*x))*Pi^(1/2)+15*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-
30*arctan(a*x)^(1/2)*Pi^(1/2)*cos(2*arctan(a*x)))
```

3.785.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c + a^2 cx^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.785.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c + a^2 cx^2)^2} \, dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.785.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c + a^2 cx^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.785.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.785.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.786 $\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$

3.786.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \frac{3x\sqrt{\arctan(ax)}}{8ac^2(1+a^2x^2)} + \frac{\arctan(ax)^{3/2}}{4a^2c^2} - \frac{\arctan(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

output
```
1/4*arctan(a*x)^(3/2)/a^2/c^2-1/2*arctan(a*x)^(3/2)/a^2/c^2/(a^2*x^2+1)-3/
32*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2+3/8*x*arctan(a*
x)^(1/2)/a/c^2/(a^2*x^2+1)
```

3.786.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \frac{\frac{4\sqrt{\arctan(ax)}(3ax+2(-1+a^2x^2)\arctan(ax))}{1+a^2x^2} - 3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
((4*Sqrt[ArcTan[a*x]]*(3*a*x + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/(1 + a^2*x^2
) - 3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^2*c^2)
```

3.786.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5465, 27, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \textit{5465}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \textit{27}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \textit{5427}$$

$$\frac{3\left(-\frac{1}{4}a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \textit{5505}$$

$$\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \textit{4906}$$

$$\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \textit{27}$$

$$\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \textit{3042}$$

$$\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 3786$$

$$\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 3832$$

$$\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^(3/2)/(a^2*c^2*(1 + a^2*x^2)) + (3*((x*Sqrt[ArcTan[a*x]])
/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (Sqrt[Pi]*FresnelS[(2*Sqrt[
ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a*c^2)
```

3.786.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.786. $\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.786.4 Maple [A] (verified)

Time = 6.78 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{8 \arctan(ax)^{\frac{3}{2}} \sqrt{\pi}\, \cos(2 \arctan(ax)) - 6 \sqrt{\arctan(ax)}\, \sqrt{\pi}\, \sin(2 \arctan(ax)) + 3\pi \,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32 c^{2} a^{2} \sqrt{\pi}}$	64

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32/c^2/a^2/Pi^(1/2)*(8*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))-6*
arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))+3*Pi*FresnelS(2*arctan(a*x)^
(1/2)/Pi^(1/2)))
```

3.786. $\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\, dx$

3.786.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.786.6 Sympy [F]

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.786.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.786.8 Giac [F]

$$\int \frac{x\arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx = \int \frac{x\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.786.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

3.787 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$

3.787.1 Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = -\frac{3\sqrt{\arctan(ax)}}{16ac^2} + \frac{3\sqrt{\arctan(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^{3/2}}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{5/2}}{5ac^2} - \frac{3\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32ac^2}$$

output
```
1/2*x*arctan(a*x)^(3/2)/c^2/(a^2*x^2+1)+1/5*arctan(a*x)^(5/2)/a/c^2-3/32*F
resnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2-3/16*arctan(a*x)^(1/2
)/a/c^2+3/8*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)
```

3.787.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.50

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\frac{16\sqrt{\arctan(ax)}\left(15-15a^2x^2+40ax\arctan(ax)+16(1+a^2x^2)\arctan(ax)^2\right)}{1+a^2x^2} + 60\left(2\sqrt{\arctan(ax)} - \sqrt{\pi}\,\mathrm{Fr}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```

output
```
((16*Sqrt[ArcTan[a*x]]*(15 - 15*a^2*x^2 + 40*a*x*ArcTan[a*x] + 16*(1 + a^2
*x^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) + 60*(2*Sqrt[ArcTan[a*x]] - Sqrt[Pi]*F
resnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) - (15*(8*ArcTan[a*x] - I*Sqrt[2]*
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*A
rcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(1280*a*c^2
)
```

3.787.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5427, 27, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{3a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3a\left(\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3a\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^(3/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a*c^2)
- (3*a*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] +
(Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4*c^2)
```

3.787.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.787.4 Maple [A] (verified)

Time = 7.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

method	result
default	$\frac{32\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}+40\arctan(ax)^{\frac{3}{2}}\sin(2\arctan(ax))\sqrt{\pi}+30\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\cos(2\arctan(ax))-15\pi\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{160c^2a\sqrt{\pi}}$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/160/c^2/a*(32*arctan(a*x)^(5/2)*Pi^(1/2)+40*arctan(a*x)^(3/2)*sin(2*arct
an(a*x))*Pi^(1/2)+30*arctan(a*x)^(1/2)*Pi^(1/2)*cos(2*arctan(a*x))-15*Pi*F
resnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

3.787.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.787.6 Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.787.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.787.8 Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.787.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^2, x)
```

3.788 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^2}\,dx$

3.788.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

3.788.2 Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2), x]
```

3.788.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^2}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
$Aborted
```

3.788.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.788.4 Maple [N/A] (verified)

Not integrable

Time = 4.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^{2}c\,x^{2}+c\right)^{2}}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

3.788.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.788.6 Sympy [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

3.788.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.788.8 Giac [N/A]

Not integrable

Time = 82.79 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.788.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\mathrm{atan}\left(a\,x\right)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^2), x)
```

3.789 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

3.789.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

3.789.2 Mathematica [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

3.789.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^3} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^3} dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.789.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.789.4 Maple [N/A] (verified)

Not integrable

Time = 13.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^3} dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

3.789.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3), x)
```

3.789.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Timed out
```

3.789.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.789.8 Giac [N/A]

Not integrable

Time = 57.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.789.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.790 $\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

3.790.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \text{Int}\left(\frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Unintegrable(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

3.790.2 Mathematica [N/A]

Not integrable

Time = 5.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

3.790.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.790.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.790.4 Maple [N/A] (verified)

Not integrable

Time = 14.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5 \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2 c\,x^2+c\right)^3}dx$$

input
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

3.790.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.790.6 Sympy [N/A]

Not integrable

Time = 9.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^5 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**5*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.790.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.790.8 Giac [N/A]

Not integrable

Time = 93.60 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5 \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.790.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.791 $\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

3.791.1 Optimal result

Integrand size = 24, antiderivative size = 230

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \frac{27\sqrt{\arctan(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\arctan(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x\arctan(ax)^{3/2}}{8a^4c^3(1+a^2x^2)} + \frac{3\arctan(ax)^{5/2}}{20a^5c^3} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^5c^3} + \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^5c^3}$$

output

```
-1/4*x^3*arctan(a*x)^(3/2)/a^2/c^3/(a^2*x^2+1)^2-3/8*x*arctan(a*x)^(3/2)/a
^4/c^3/(a^2*x^2+1)+3/20*arctan(a*x)^(5/2)/a^5/c^3-3/1024*FresnelC(2*2^(1/2
)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c^3+3/32*FresnelC(2*arc
tan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^5/c^3+27/256*arctan(a*x)^(1/2)/a^5/c^3
+3/32*x^4*arctan(a*x)^(1/2)/a/c^3/(a^2*x^2+1)^2-9/32*arctan(a*x)^(1/2)/a^5
/c^3/(a^2*x^2+1)
```

3.791.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.54

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\frac{64\sqrt{\arctan(ax)}\left(15(-15-6a^2x^2+17a^4x^4)-160ax(3+5a^2x^2)\arctan(ax)+192(1+a^2x^2)^2\arctan(ax)^2\right)}{(1+a^2x^2)^2} - 510}{}$$

input

```
Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output

```
((64*Sqrt[ArcTan[a*x]]*(15*(-15 - 6*a^2*x^2 + 17*a^4*x^4) - 160*a*x*(3 + 5
*a^2*x^2)*ArcTan[a*x] + 192*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^
2 - 510*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + 90*Sqrt
[ArcTan[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] +
 Gamma[1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (225*(24*ArcTan[a*x]
 - (4*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (
4*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-
I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gam
ma[1/2, (4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^5*c^3)
```

3.791.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5475, 27, 5471, 5465, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5475}$$

$$-\frac{3}{64}\int \frac{x^4}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx + \frac{3\int \frac{x^2\arctan(ax)^{3/2}}{c^2(a^2x^2+1)^2}dx}{4a^2c} + \frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2} - \frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3\int \frac{x^2\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4a^2c^3}-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 5471$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 5465$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 5439$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 3042$$

3.791. $\int \frac{x^4\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{3\int \frac{a^4x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \text{3042}$$

3.791. $\int \frac{x^4\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}dx$

$$-\frac{3\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \textbf{3793}$$

$$-\frac{3\int\left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

input
```
Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(3*x^4*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^(3
/2))/(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/
2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcTan[a*x]])/Sqrt[Pi]])/2))/(64*a^5*c^3) + (3*(-1/2*(x*ArcTan[a*x]^(3/2
))/(a^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a^3) + (3*(-1/2*Sqrt[ArcTan[
a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4*a)))/(4*a^2*c^3)
```

3.791. $\int \frac{x^4\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

3.791.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.791. $\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.791.4 Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.57

method	result
default	$\frac{-15\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+768\arctan(ax)^3+160\arctan(ax)^2\sin(4\arctan(ax))-1280\arctan(ax)^2\sin(2}{5120c^3a^5\sqrt{a}}$

input
```
int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5120/c^3/a^5*(-15*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)
*arctan(a*x)^(1/2)*Pi^(1/2)+768*arctan(a*x)^3+160*arctan(a*x)^2*sin(4*arct
an(a*x))-1280*arctan(a*x)^2*sin(2*arctan(a*x))+480*FresnelC(2*arctan(a*x)^
(1/2)/Pi^(1/2))*arctan(a*x)^(1/2)*Pi^(1/2)-960*cos(2*arctan(a*x))*arctan(a
*x)+60*cos(4*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)
```

3.791.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.791.6 Sympy [F]

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^4 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**4*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.791.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.791.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^4 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.791.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.792 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

3.792.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx &= -\frac{3\arctan(ax)^{3/2}}{32a^4c^3}+\frac{x^4\arctan(ax)^{3/2}}{4c^3(1+a^2x^2)^2}\\ &+\frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^4c^3}-\frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{64a^4c^3}\\ &+\frac{3\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{32a^4c^3}-\frac{3\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{256a^4c^3}\end{aligned}$$

output
```
-3/32*arctan(a*x)^(3/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(3/2)/c^3/(a^2*x^2+1)^
2+3/1024*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a
^4/c^3-3/64*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3+3/32*s
in(2*arctan(a*x))*arctan(a*x)^(1/2)/a^4/c^3-3/256*sin(4*arctan(a*x))*arcta
n(a*x)^(1/2)/a^4/c^3
```

3.792.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \sqrt{\arctan(ax)}\left(\frac{3x(3+5a^2x^2)}{64a^3c^3(1+a^2x^2)^2} + \frac{(-3-6a^2x^2+5a^4x^4)\arctan(ax)}{32a^4c^3(1+a^2x^2)^2}\right) - \frac{9\left(-2\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) - \sqrt{-i\arctan(a\right.}{4096a^4c^3\sqrt{\arctan(ax)}} - \frac{15\left(-2\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) + \sqrt{-i\arctan(\right.}{4096a^4c^3\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Sqrt[ArcTan[a*x]]*((3*x*(3 + 5*a^2*x^2))/(64*a^3*c^3*(1 + a^2*x^2)^2) + ((
-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c^3*(1 + a^2*x^2)^2)) - (
9*(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sq
rt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan
[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*
I)*ArcTan[a*x]]))/(4096*a^4*c^3*Sqrt[ArcTan[a*x]]) - (15*(-2*Sqrt[2]*Sqrt[
(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[2]*Sqrt[I*ArcTan
[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (
-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(
4096*a^4*c^3*Sqrt[ArcTan[a*x]])
```

3.792.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5479, 27, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \textcolor{blue}{5479}$$

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 \left(a^2x^2+1\right)^2}-\frac{3}{8}a\int \frac{x^4\sqrt{\arctan(ax)}}{c^3 \left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 \left(a^2x^2+1\right)^2}-\frac{3a\int \frac{x^4\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^3}dx}{8c^3}$$

$$\Big\downarrow \textcolor{blue}{5505}$$

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 \left(a^2x^2+1\right)^2}-\frac{3\int \frac{a^4x^4\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 \left(a^2x^2+1\right)^2}-\frac{3\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^4d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \textcolor{blue}{3793}$$

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 \left(a^2x^2+1\right)^2}-\frac{3\int \left(-\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 \left(a^2x^2+1\right)^2}-\frac{3\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(\right.}{8a^4c^3}$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x]^(3/2))/(4*c^3*(1 + a^2*x^2)^2) - (3*(ArcTan[a*x]^(3/2)/4
- (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/64 + (Sqrt[Pi]*Fre
snelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 - (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan
[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32))/(8*a^4*c^3)
```

3.792. $\int \frac{x^3\arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx$

3.792.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.792.4 Maple [A] (verified)

Time = 7.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

method	result
default	$-\frac{-3\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+128\arctan(ax)^2\cos(2\arctan(ax))-32\arctan(ax)^2\cos(4\arctan(ax))+48\sqrt{}}{1024c^3a^4\sqrt{\arctan(ax)}}$

3.792. $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/1024/c^3/a^4*(-3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)
*arctan(a*x)^(1/2)*Pi^(1/2)+128*arctan(a*x)^2*cos(2*arctan(a*x))-32*arctan
(a*x)^2*cos(4*arctan(a*x))+48*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan
(a*x)^(1/2)/Pi^(1/2))-96*sin(2*arctan(a*x))*arctan(a*x)+12*sin(4*arctan(a*
x))*arctan(a*x))/arctan(a*x)^(1/2)
```

3.792.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.792.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.792.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.792.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.792.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.793 $\int \frac{x^2 \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^3} dx$

3.793.1 Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \frac{\arctan(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\arctan(ax)}\cos(4\arctan(ax))}{256a^3c^3} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^3c^3} - \frac{\arctan(ax)^{3/2}\sin(4\arctan(ax))}{32a^3c^3}$$

output
```
1/20*arctan(a*x)^(5/2)/a^3/c^3-1/32*arctan(a*x)^(3/2)*sin(4*arctan(a*x))/a
^3/c^3+3/1024*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)/a^3/c^3-3/256*cos(4*arctan(a*x))*arctan(a*x)^(1/2)/a^3/c^3
```

3.793.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.27

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \frac{\frac{64\sqrt{\arctan(ax)}\left(-15(1-6a^2x^2+a^4x^4)+160ax(-1+a^2x^2)\arctan(ax)+64(1+a^2x^2)^2\arctan(ax)^2\right)}{(1+a^2x^2)^2} + 30\left(12\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
((64*Sqrt[ArcTan[a*x]]*(-15*(1 - 6*a^2*x^2 + a^4*x^4) + 160*a*x*(-1 + a^2*
x^2)*ArcTan[a*x] + 64*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^2 + 30
*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) - 90*Sqrt[ArcTan
[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] + Gamma[
1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (15*(24*ArcTan[a*x] - (4*I)
*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (4*I)*Sqr
t[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-I)*ArcTa
n[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^3*c^3)
```

3.793.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2\arctan(ax)^{3/2}}{(a^2x^2+1)^2}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{1}{8}\arctan(ax)^{3/2}-\frac{1}{8}\arctan(ax)^{3/2}\cos(4\arctan(ax))\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{512}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{20}\arctan(ax)^{5/2}-\frac{1}{32}\arctan(ax)^{3/2}\sin(4\arctan(ax))-\frac{3}{256}\sqrt{\arctan(ax}}{a^3c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

3.793. $\int \frac{x^2\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

output
```
(ArcTan[a*x]^(5/2)/20 - (3*Sqrt[ArcTan[a*x]]*Cos[4*ArcTan[a*x]])/256 + (3*
Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/512 - (ArcTan[a*x]^(3
/2)*Sin[4*ArcTan[a*x]])/32)/(a^3*c^3)
```

3.793.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.793.4 Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.75

method	result
default	$\frac{15\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}-160\arctan(ax)^2\sin(4\arctan(ax))+256\arctan(ax)^3-60\cos(4\arctan(ax))\,\mathrm{arct}}{5120c^3a^3\sqrt{\arctan(ax)}}$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5120/c^3/a^3*(15*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*
arctan(a*x)^(1/2)*Pi^(1/2)-160*arctan(a*x)^2*sin(4*arctan(a*x))+256*arctan
(a*x)^3-60*cos(4*arctan(a*x))*arctan(a*x))/arctan(a*x)^(1/2)
```

3.793.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.793.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.793.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.793.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.793.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.794 $\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$

3.794.1 Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \frac{3\arctan(ax)^{3/2}}{32a^2c^3} - \frac{\arctan(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^2c^3} - \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{64a^2c^3} + \frac{3\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{32a^2c^3} + \frac{3\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{256a^2c^3}$$

output
```
3/32*arctan(a*x)^(3/2)/a^2/c^3-1/4*arctan(a*x)^(3/2)/a^2/c^3/(a^2*x^2+1)^2
-3/1024*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^
2/c^3-3/64*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3+3/32*si
n(2*arctan(a*x))*arctan(a*x)^(1/2)/a^2/c^3+3/256*sin(4*arctan(a*x))*arctan
(a*x)^(1/2)/a^2/c^3
```

3.794.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.07

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{480ax\arctan(ax)+288a^3x^3\arctan(ax)-320\arctan(ax)^2+384a^2x^2\arctan(ax)^2 \cdots}{}$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(480*a*x*ArcTan[a*x] + 288*a^3*x^3*ArcTan[a*x] - 320*ArcTan[a*x]^2 + 384*a
^2*x^2*ArcTan[a*x]^2 + 192*a^4*x^4*ArcTan[a*x]^2 + 24*Sqrt[2]*(1 + a^2*x^2
)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 24*Sqrt[2]*(1
+ a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + 3*Sqrt[(-
I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 6*a^2*x^2*Sqrt[(-I)*ArcTa
n[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 3*a^4*x^4*Sqrt[(-I)*ArcTan[a*x]]*
Gamma[1/2, (-4*I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]] + 6*a^2*x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]] +
 3*a^4*x^4*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(2048*c^3*(a
 + a^3*x^2)^2*Sqrt[ArcTan[a*x]])
```

3.794.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5465, 27, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{c^3(a^2x^2+1)^3}dx}{8a}-\frac{\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{8ac^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \ 5439$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{8a^2c^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4d\arctan(ax)}{8a^2c^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \ 3793$$

$$\frac{3\int \left(\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{8a^2c^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{3\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin(}{8a^2c^3}$$
$$\frac{\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^(3/2)/(a^2*c^3*(1 + a^2*x^2)^2) + (3*(ArcTan[a*x]^(3/2)/4
 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/64 - (Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 + (Sqrt[ArcTan[a*x]]*Sin[2*ArcTa
n[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32))/(8*a^2*c^3)
```

3.794. $\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

3.794.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.794.4 Maple [A] (verified)

Time = 7.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

method	result
default	$-\frac{3\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+128\arctan(ax)^2\cos(2\arctan(ax))+48\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(a}}{\sqrt{\pi}}\right.}{1024c^3a^2\sqrt{\arctan(ax)}}$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/1024/c^3/a^2*(3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*
arctan(a*x)^(1/2)*Pi^(1/2)+128*arctan(a*x)^2*cos(2*arctan(a*x))+48*arctan(
a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))+32*arctan(a*x)^
2*cos(4*arctan(a*x))-96*sin(2*arctan(a*x))*arctan(a*x)-12*sin(4*arctan(a*x
))*arctan(a*x))/arctan(a*x)^(1/2)
```

3.794.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.794.6 Sympy [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\, dx = \frac{\int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\, dx}{c^3}$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(a**6*x***6 + 3*a**4*x***4 + 3*a**2*x***2 + 1), x
)/c**3
```

3.794.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.794.8 Giac [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^3} \, dx = \int \frac{x \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^3} \, dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.794.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^3} \, dx = \int \frac{x \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2 + c)^3} \, dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

3.795 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

3.795.1 Optimal result

Integrand size = 21, antiderivative size = 219

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = -\frac{45\sqrt{\arctan(ax)}}{256ac^3}+\frac{3\sqrt{\arctan(ax)}}{32ac^3(1+a^2x^2)^2}+\frac{9\sqrt{\arctan(ax)}}{32ac^3(1+a^2x^2)}+\frac{x\arctan(ax)^{3/2}}{4c^3(1+a^2x^2)^2}+\frac{3x\arctan(ax)^{3/2}}{8c^3(1+a^2x^2)}+\frac{3\arctan(ax)^{5/2}}{20ac^3}-\frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512ac^3}-\frac{3\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32ac^3}$$

output
```
1/4*x*arctan(a*x)^(3/2)/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^(3/2)/c^3/(a^2
*x^2+1)+3/20*arctan(a*x)^(5/2)/a/c^3-3/1024*FresnelC(2*2^(1/2)/Pi^(1/2)*ar
ctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a/c^3-3/32*FresnelC(2*arctan(a*x)^(1/2)/
Pi^(1/2))*Pi^(1/2)/a/c^3-45/256*arctan(a*x)^(1/2)/a/c^3+3/32*arctan(a*x)^(
1/2)/a/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)^(1/2)/a/c^3/(a^2*x^2+1)
```

3.795.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.62

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\frac{64\sqrt{\arctan(ax)}\left(-15(-17+6a^2x^2+15a^4x^4)+160ax(5+3a^2x^2)\arctan(ax)+192(1+a^2x^2)^2\arctan(ax)^2\right)}{(1+a^2x^2)^2}+450\Big(}{}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```

output
```
((64*Sqrt[ArcTan[a*x]]*(-15*(-17 + 6*a^2*x^2 + 15*a^4*x^4) + 160*a*x*(5 +
3*a^2*x^2)*ArcTan[a*x] + 192*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)
^2 + 450*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[Arc
Tan[a*x]]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + 90*Sqr
t[ArcTan[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]]
+ Gamma[1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) - (255*(24*ArcTan[a*x
] - (4*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] +
(4*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(
-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Ga
mma[1/2, (4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a*c^3)
```

3.795.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5435, 27, 5427, 5439, 3042, 3793, 2009, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{3}{64}\int \frac{1}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3\int \frac{\arctan(ax)^{3/2}}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\arctan(ax)^{3/2}}{4c^3(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\int \frac{\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64ac^3}+\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}-\frac{3\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}-\frac{3\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{5465}$$

3.795. $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{5439}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{2009}$$

3.795. $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```

output
```
(3*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^(3/2))/(
4*c^3*(1 + a^2*x^2)^2) - (3*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*Fresnel
C[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a
*x]])/Sqrt[Pi]])/2))/(64*a*c^3) + (3*((x*ArcTan[a*x]^(3/2))/(2*(1 + a^2*x^
2)) + ArcTan[a*x]^(5/2)/(5*a) - (3*a*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2
*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqr
t[Pi]])/2)/(4*a^2)))/4))/(4*c^3)
```

3.795.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.795.4 Maple [A] (verified)

Time = 7.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.60

method	result
default	$\frac{768\arctan(ax)^3+1280\arctan(ax)^2\sin(2\arctan(ax))+160\arctan(ax)^2\sin(4\arctan(ax))-15\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\mathrm{arcta}}}{5120c^3a\sqrt{\mathrm{arc}}}$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

3.795. $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$

output
```
1/5120/c^3/a/arctan(a*x)^(1/2)*(768*arctan(a*x)^3+1280*arctan(a*x)^2*sin(2
*arctan(a*x))+160*arctan(a*x)^2*sin(4*arctan(a*x))-15*FresnelC(2*2^(1/2)/P
i^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+960*cos(2*ar
ctan(a*x))*arctan(a*x)-480*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a
*x)^(1/2)*Pi^(1/2)+60*cos(4*arctan(a*x))*arctan(a*x))
```

3.795.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.795.6 Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

3.795.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.795.8 Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.795.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^3, x)
```

3.796 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^3}\,dx$

3.796.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^3}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^3},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

3.796.2 Mathematica [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^3}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3), x]
```

3.796.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

3.796.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.796.4 Maple [N/A] (verified)

Not integrable

Time = 4.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^{2}c\,x^{2}+c\right)^{3}}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

3.796.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.796.6 Sympy [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/
c**3
```

3.796.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.796.8 Giac [N/A]

Not integrable

Time = 87.54 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^3}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.796.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^3), x)
```

3.797 $\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{3/2} \, dx$

3.797.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{3/2} \, dx = \text{Int}\left(x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{3/2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.797.2 Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{3/2} \, dx = \int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

3.797.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int x^m \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.797.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.797.4 Maple [N/A] (verified)

Not integrable

Time = 16.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \arctan(ax)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.797.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int \sqrt{a^2cx^2+c}x^m\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

3.797.6 Sympy [F(-1)]

Timed out.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.797.7 Maxima [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.797.8 Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.797.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.798 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$

3.798.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.798.2 Mathematica [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

3.798.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.798.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.798.4 Maple [N/A] (verified)

Not integrable

Time = 13.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \arctan \left(a x \right)^{\frac{3}{2}} \sqrt{a^{2} c \,x^{2}+c}d x$$

input
```
int(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.798.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{c + a^2 c x^2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.798.6 Sympy [N/A]

Not integrable

Time = 92.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2 \sqrt{c + a^2 c x^2} \arctan(ax)^{3/2} \, dx = \int x^2 \sqrt{c \left(a^2 x^2 + 1\right)} \operatorname{atan}^{\frac{3}{2}}(ax) \, dx$$

input
```
integrate(x**2*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

3.798.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \sqrt{c + a^2 c x^2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.798.8 Giac [N/A]

Not integrable

Time = 88.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan{(ax)}^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.798.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.799 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$

3.799.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{3a^2c}-\frac{\mathrm{Int}\left(\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)},x\right)}{2a}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a^2/c-1/2*Unintegrable((a^2*c*x^
2+c)^(1/2)*arctan(a*x)^(1/2),x)/a
```

3.799.2 Mathematica [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

3.799.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx}{2a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx}{2a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.799.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.799.4 Maple [N/A] (verified)

Not integrable

Time = 6.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x \arctan(ax)^{\frac{3}{2}} \sqrt{a^2c\,x^2+c}dx$$

input
```
int(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.799.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.799.6 Sympy [N/A]

Not integrable

Time = 53.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int x\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(x*atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

3.799.7 Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.799.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.799.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.800 $\int \sqrt{c+a^2cx^2} \arctan(ax)^{3/2} \, dx$

3.800.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sqrt{c+a^2cx^2} \arctan(ax)^{3/2} \, dx = -\frac{3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4a} + \frac{1}{2}x\sqrt{c+a^2cx^2} \arctan(ax)^{3/2} + \frac{3}{8}c\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) + \frac{1}{2}c\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
1/2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)-3/4*(a^2*c*x^2+c)^(1/2)*arctan
(a*x)^(1/2)/a+1/2*c*Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)+
3/8*c*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.800.2 Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{c+a^2cx^2} \arctan(ax)^{3/2} \, dx = \int \sqrt{c+a^2cx^2} \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

3.800.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.800.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.800.4 Maple [N/A] (verified)

Not integrable

Time = 13.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \arctan\left(ax\right)^{\frac{3}{2}} \sqrt{a^{2} c \,x^{2}+c}dx$$

input
```
int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.800.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.800.6 Sympy [N/A]

Not integrable

Time = 26.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^{\frac{3}{2}}\left(ax\right)dx$$

input
```
integrate(atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

3.800.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.800.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.800.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int \mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.801 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx$

3.801.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \mathrm{Int}\left(\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

3.801.2 Mathematica [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x, x]
```

3.801.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{x}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{x}dx$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

3.801.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.801.4 Maple [N/A] (verified)

Not integrable

Time = 13.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}\sqrt{a^{2}c\,x^{2}+c}}{x}dx$$

input
```
int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

output
```
int(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

3.801.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + a^2 c x^2} \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.801.6 Sympy [N/A]

Not integrable

Time = 19.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c + a^2 c x^2} \arctan(ax)^{3/2}}{x} \, dx = \int \frac{\sqrt{c (a^2 x^2 + 1)} \operatorname{atan}^{\frac{3}{2}}(ax)}{x} \, dx$$

input
```
integrate(atan(a*x)**(3/2)*(a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)/x, x)
```

3.801.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + a^2 c x^2} \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.801.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.801.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.802 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$

3.802.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

3.802.2 Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

3.802.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.802.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.802.4 Maple [N/A] (verified)

Not integrable

Time = 20.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

3.802.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}\,dx=\int \left(a^2 c x^2+c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(3/2), x)
```

3.802.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.802.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.802.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.802.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.803 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$

3.803.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx = \mathrm{Int}\left(x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

3.803.2 Mathematica [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx = \int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

3.803.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.803.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.803.4 Maple [N/A] (verified)

Not integrable

Time = 15.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

3.803.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.803.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.803.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.803.8 Giac [N/A]

Not integrable

Time = 89.43 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.803.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.804 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$

3.804.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{5a^2c}-\frac{3\mathrm{Int}\left(\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)},x\right)}{10a}$$

output
```
1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/a^2/c-3/10*Unintegrable((a^2*c*x
^2+c)^(3/2)*arctan(a*x)^(1/2),x)/a
```

3.804.2 Mathematica [N/A]

Not integrable

Time = 3.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

3.804.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{3/2} \sqrt{\arctan(ax)}dx}{10a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{3/2} \sqrt{\arctan(ax)}dx}{10a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.804.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.804.4 Maple [N/A] (verified)

Not integrable

Time = 15.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

3.804.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.804.6 Sympy [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.804.7 Maxima [F(-2)]

Exception generated.

$$\int x(c + a^2cx^2)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.804.8 Giac [F(-2)]

Exception generated.

$$\int x(c + a^2cx^2)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.804.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c + a^2cx^2)^{3/2} \arctan(ax)^{3/2} \, dx = \int x \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.805 $\int \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx$

3.805.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = -\frac{9c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{16a} - \frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}{8a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^{3/2} + \frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2} + \frac{9}{32}c^2\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}, x\right) + \cdot$$

output
```
1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)+3/8*c*x*arctan(a*x)^(3/2)*(a^2
*c*x^2+c)^(1/2)-1/8*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a-9/16*c*(a^2*c*
x^2+c)^(1/2)*arctan(a*x)^(1/2)/a+3/8*c^2*Unintegrable(arctan(a*x)^(3/2)/(a
^2*c*x^2+c)^(1/2),x)+9/32*c^2*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*
x)^(1/2),x)+1/16*c*Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.805.2 Mathematica [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \int \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

3.805.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{8a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}}{4a}\right.$$
$$\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{8a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}}{4a}\right.$$
$$\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{8a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.805.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.805.4 Maple [N/A] (verified)

Not integrable

Time = 15.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

3.805.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.805.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.805.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.805.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.805.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx = \int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.806 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx$

3.806.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

3.806.2 Mathematica [N/A]

Not integrable

Time = 3.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x, x]
```

3.806.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{x}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

3.806.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.806.4 Maple [N/A] (verified)

Not integrable

Time = 16.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}{x}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

3.806.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.806.6 Sympy [N/A]

Not integrable

Time = 141.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2)/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)/x, x)
```

3.806.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.806.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.806.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.807 $\int x^m (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2} \, dx$

3.807.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Int}\left(x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

3.807.2 Mathematica [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2} \, dx = \int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

3.807.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.807.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.807.4 Maple [N/A] (verified)

Not integrable

Time = 21.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

3.807.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{3/2}\, dx = \int \left(a^2 c x^2 + c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)^{\frac{3}{2}}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arcta
n(a*x)^(3/2), x)
```

3.807.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{3/2}\, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.807.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{3/2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.807.8 Giac [F(-2)]

Exception generated.

$$\int x^m (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.807.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2} dx = \int x^m \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{5/2} dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.808 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx$

3.808.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

3.808.2 Mathematica [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

3.808.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2 c x^2 + c\right)^{5/2} dx$$
$$\Big\downarrow \ 5560$$
$$\int x^2 \arctan(ax)^{3/2} \left(a^2 c x^2 + c\right)^{5/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.808.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.808.4 Maple [N/A] (verified)

Not integrable

Time = 14.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2 \left(a^2 c\,x^2 + c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

3.808.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.808.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.808.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.808.8 Giac [N/A]

Not integrable

Time = 89.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.808.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{5/2} dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.809 $\int x(c+a^2cx^2)^{5/2} \arctan(ax)^{3/2} \, dx$

3.809.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2} \, dx = \frac{\left(c+a^2cx^2\right)^{7/2} \arctan(ax)^{3/2}}{7a^2c} - \frac{3\text{Int}\left(\left(c+a^2cx^2\right)^{5/2} \sqrt{\arctan(ax)}, x\right)}{14a}$$

output
```
1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(3/2)/a^2/c-3/14*Unintegrable((a^2*c*x
^2+c)^(5/2)*arctan(a*x)^(1/2),x)/a
```

3.809.2 Mathematica [N/A]

Not integrable

Time = 6.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2} \, dx = \int x\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

3.809.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} \, dx$$

$$\Big\downarrow \; 5465$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}dx}{14a}$$

$$\Big\downarrow \; 5560$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}dx}{14a}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.809.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.809.4 Maple [N/A] (verified)

Not integrable

Time = 13.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

3.809.5 Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.809.6 Sympy [F(-1)]

Timed out.

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.809.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.809.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.809.9 Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.810 $\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx$

3.810.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx = -\frac{15c^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{32a} - \frac{5c\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}{48a} - \frac{\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}{20a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2} + \frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2} + \frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2} + \frac{15}{64}c^3$$

output
```
5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)+1/6*x*(a^2*c*x^2+c)^(5/2)*a
rctan(a*x)^(3/2)+5/16*c^2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)-5/48*c*(
a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a-1/20*(a^2*c*x^2+c)^(5/2)*arctan(a*x
)^(1/2)/a-15/32*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a+5/16*c^3*Unint
egrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)+1/40*c*Unintegrable((a^2*
c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)+15/64*c^3*Unintegrable(1/(a^2*c*x^2+c)
^(1/2)/arctan(a*x)^(1/2),x)+5/96*c^2*Unintegrable((a^2*c*x^2+c)^(1/2)/arct
an(a*x)^(1/2),x)
```

3.810.2 Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

3.810.3 Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5415, 5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}dx$$

$$\Big\downarrow \ 5415$$

$$\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}dx+\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

$$\Big\downarrow \ 5415$$

$$\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\left(\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(}}{}\right.$$
$$\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

$$\Big\downarrow \ 5415$$

$$\frac{1}{40}c\int\frac{(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{5}{6}c\left(\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{1}{40}c\int\frac{(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{5}{6}c\left(\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.810.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.810.4 Maple [N/A] (verified)

Not integrable

Time = 13.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

3.810.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.810.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.810.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.810.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.810.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx = \int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.811 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx$

3.811.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

3.811.2 Mathematica [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x, x]
```

3.811.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{x}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

3.811.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.811.4 Maple [N/A] (verified)

Not integrable

Time = 15.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}{x}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

3.811.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.811.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}}{x} dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2)/x,x)
```

output
```
Timed out
```

3.811.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}}{x} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.811.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.811.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.812 $\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.812.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.812.2 Mathematica [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

3.812.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.812.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.812.4 Maple [N/A] (verified)

Not integrable

Time = 17.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.812.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

3.812.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.812.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.812.8 Giac [N/A]

Not integrable

Time = 52.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.812.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.813 $\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}} dx$

3.813.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}} dx = -\frac{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4a^3c} - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3a^2c} + \frac{\mathrm{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{8a^2} + \frac{5\mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{4a^3}$$

output
```
-2/3*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)^(3/2)
*(a^2*c*x^2+c)^(1/2)/a^2/c-1/4*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^3
/c+1/8*Unintegrable(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a^2+5/4*Uni
ntegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)/a^3
```

3.813.2 Mathematica [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}} dx = \int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}} dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

3.813.3 Rubi [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5487, 5465, 5487, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{2\int \frac{x\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}-\frac{\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5487}$$

$$-\frac{-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}}{2a}-\frac{2\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}}{2a}-\frac{2\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.813. $\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}dx$

3.813.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.813.4 Maple [N/A] (verified)

Not integrable

Time = 14.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arctan (ax)^{\frac{3}{2}}}{\sqrt{a^2 c\, x^2 + c}}dx$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.813.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.813.6 Sympy [N/A]

Not integrable

Time = 128.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c(a^2x^2 + 1)}} \, dx$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.813.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.813.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.813.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.814 $\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.814.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4a^3c} + \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{2a^2c} + \frac{3\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{8a^2} - \frac{\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{2a^2}$$

output
```
1/2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^2/c-3/4*(a^2*c*x^2+c)^(1/2)*
arctan(a*x)^(1/2)/a^3/c-1/2*Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(
1/2),x)/a^2+3/8*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a^
2
```

3.814.2 Mathematica [N/A]

Not integrable

Time = 3.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

3.814.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5487, 5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{3\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.814.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.814.4 Maple [N/A] (verified)

Not integrable

Time = 13.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2 c\,x^2 + c}}dx$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.814.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.814.6 Sympy [N/A]

Not integrable

Time = 62.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.814.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.814.8 Giac [N/A]

Not integrable

Time = 151.58 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.814.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2 + c}}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.815 $\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.815.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{a^2c} - \frac{3\mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{2a}$$

output
```
arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^2/c-3/2*Unintegrable(arctan(a*x)^(
1/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

3.815.2 Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

3.815.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.815.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.815.4 Maple [N/A] (verified)

Not integrable

Time = 7.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x \arctan (ax)^{\frac{3}{2}}}{\sqrt{a^{2} c \,x^{2}+c}}dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.815.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.815.6 Sympy [N/A]

Not integrable

Time = 36.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}{\sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.815.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.815.8 Giac [N/A]

Not integrable

Time = 151.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.815.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.816 $\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.816.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.816.2 Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]
```

3.816.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.816.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.816.4 Maple [N/A] (verified)

Not integrable

Time = 9.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.816.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.816.6 Sympy [N/A]

Not integrable

Time = 12.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.816.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.816.8 Giac [N/A]

Not integrable

Time = 143.93 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.816.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.817 $\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$

3.817.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.817.2 Mathematica [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

3.817.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.817.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.817.4 Maple [N/A] (verified)

Not integrable

Time = 13.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.817.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.817.6 Sympy [N/A]

Not integrable

Time = 13.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

3.817.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.817.8 Giac [N/A]

Not integrable

Time = 145.72 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.817.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.818 $\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$

3.818.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{cx} + \frac{3}{2}a\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x+3/2*a*Unintegrable(arctan(a*x)^
(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.818.2 Mathematica [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

3.818.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5479, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5479}$$
$$\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}$$
$$\Big\downarrow \text{5560}$$
$$\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.818.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.818.4 Maple [N/A] (verified)

Not integrable

Time = 15.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(a x\right)^{\frac{3}{2}}}{x^{2} \sqrt{a^{2} c\, x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

3.818.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.818.6 Sympy [N/A]

Not integrable

Time = 18.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

3.818.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.818.8 Giac [N/A]

Not integrable

Time = 147.54 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.818.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.819 $\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$

3.819.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{3a\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4cx} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{2cx^2} + \frac{3}{8}a^2\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{1}{2}a^2\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-1/2*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x^2-3/4*a*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^(1/2)/c/x-1/2*a^2*Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2
+c)^(1/2),x)+3/8*a^2*Unintegrable(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2
),x)
```

3.819.2 Mathematica [N/A]

Not integrable

Time = 7.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

3.819.3 Rubi [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5497, 5479, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.819.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.819.4 Maple [N/A] (verified)

Not integrable

Time = 15.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x^3\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

3.819.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.819.6 Sympy [N/A]

Not integrable

Time = 33.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

3.819.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.819.8 Giac [N/A]

Not integrable

Time = 149.98 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.819.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.820 $\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.820.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = & -\frac{a\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4cx^2} \\ & -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3cx} \\ & +\frac{1}{8}a^2\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)-\frac{5}{4}a^3\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)\end{aligned}$$

output
```
-1/3*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^(3/2)
*(a^2*c*x^2+c)^(1/2)/c/x-1/4*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^2
+1/8*a^2*Unintegrable(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)-5/4*a
^3*Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.820.2 Mathematica [N/A]

Not integrable

Time = 18.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

3.820.3 Rubi [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5497, 5479, 5497, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{2}a\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\left(\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{2}a\left(-\frac{1}{2}a^2\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2}{3}a^2\left(\frac{3}{2}a\int\frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{1}{2}a\left(-\frac{1}{2}a^2\int\frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}\right)-$$
$$\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.820.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.820. $\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.820.4 Maple [N/A] (verified)

Not integrable

Time = 15.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x\right)^{\frac{3}{2}}}{x^{4} \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

3.820.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.820.6 Sympy [N/A]

Not integrable

Time = 58.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{x^{4} \sqrt{c \left(a^{2} x^{2} + 1\right)}}\, dx$$

input
```
integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

3.820.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.820.8 Giac [N/A]

Not integrable

Time = 147.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.820.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.821 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} dx$

3.821.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.821.2 Mathematica [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

3.821.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.821.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.821.4 Maple [N/A] (verified)

Not integrable

Time = 12.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan \left(a x \right)^{\frac{3}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.821.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^
2*x^2 + c^2), x)
```

3.821.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.821.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.821.8 Giac [N/A]

Not integrable

Time = 52.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.821.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.822 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.822.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.822.2 Mathematica [N/A]

Not integrable

Time = 6.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

3.822.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \, 5560$$

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}} dx$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.822.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.822.4 Maple [N/A] (verified)

Not integrable

Time = 8.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arctan \left(a x \right)^{\frac{3}{2}}}{\left(a^2 c \,x^2+c \right)^{\frac{3}{2}}}d x$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.822.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.822.6 Sympy [N/A]

Not integrable

Time = 154.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.822.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.822.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.822.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.823 $\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.823.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.823.2 Mathematica [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

3.823.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.823.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.823.4 Maple [N/A] (verified)

Not integrable

Time = 7.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.823.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.823.6 Sympy [N/A]

Not integrable

Time = 76.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.823.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.823.8 Giac [N/A]

Not integrable

Time = 85.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.823.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \,\mathrm{atan}(a\,x)^{3/2}}{{(c\,a^2\,x^2+c)}^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.824 $\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$

3.824.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx = \frac{3x\sqrt{\arctan(ax)}}{2ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

output
```
-arctan(a*x)^(3/2)/a^2/c/(a^2*c*x^2+c)^(1/2)-3/4*FresnelS(2^(1/2)/Pi^(1/2)
*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)
^(1/2)+3/2*x*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

3.824.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx = \frac{4(3ax - 2\arctan(ax))\arctan(ax) + 3\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2}, -i\arctan(ax\right.}{8a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(4*(3*a*x - 2*ArcTan[a*x])*ArcTan[a*x] + 3*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*Arc
Tan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan
[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(8*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan
[a*x]])
```

3.824.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5465, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{2ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3832}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^(3/2)/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*Sqrt[1 + a^2*x^2]*((
a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*
Sqrt[ArcTan[a*x]]]))/(2*a^2*c*Sqrt[c + a^2*c*x^2])
```

3.824.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.824.4 Maple [F]

$$\int \frac{x\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.824.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.824.6 Sympy [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^{3/2}}\,dx = \int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.824.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.824.8 Giac [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.824.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.825 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.825.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}$$

output
```
x*arctan(a*x)^(3/2)/c/(a^2*c*x^2+c)^(1/2)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*ar
ctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2
)+3/2*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

3.825.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\sqrt{\arctan(ax)}(3+2ax\arctan(ax))-3\sqrt{2\pi}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[ArcTan[a*x]]*(3 + 2*a*x*ArcTan[a*x]) - 3*Sqrt[2*Pi]*Sqrt[1 + a^2*x
^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a*c*Sqrt[c + a^2*c*x^2])
```

3.825.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5433, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \text{ 5433}$$
$$-\frac{3}{4}\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 5440}$$
$$-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 5439}$$
$$-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 3042}$$
$$-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 3785}$$
$$-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 3833}$$
$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/
(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

3.825.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.825. $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.825.4 Maple [F]

$$\int \frac{\arctan{(ax)}^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.825.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.825.6 Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}{(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.825.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.825.8 Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.825.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.826 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{3/2}}\,dx$

3.826.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

3.826.2 Mathematica [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

3.826.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.826.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.826.4 Maple [N/A] (verified)

Not integrable

Time = 12.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

3.826.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.826.6 Sympy [N/A]

Not integrable

Time = 34.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.826.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.826.8 Giac [N/A]

Not integrable

Time = 80.86 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.826.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.827 $\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{3/2}}\,dx$

3.827.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

3.827.2 Mathematica [N/A]

Not integrable

Time = 6.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

3.827.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{3/2}}{x^2 (a^2cx^2+c)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.827.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.827.4 Maple [N/A] (verified)

Not integrable

Time = 10.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x \right)^{\frac{3}{2}}}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

3.827.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.827.6 Sympy [N/A]

Not integrable

Time = 57.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.827.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.827.8 Giac [N/A]

Not integrable

Time = 79.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.827.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

3.828 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.828.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.828.2 Mathematica [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

3.828.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.828.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.828.4 Maple [N/A] (verified)

Not integrable

Time = 24.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.828.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

3.828.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.828.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.828.8 Giac [N/A]

Not integrable

Time = 52.79 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.828.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.829 $\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.829.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.829.2 Mathematica [N/A]

Not integrable

Time = 10.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

3.829.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} dx$$

input
```
Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.829.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.829.4 Maple [N/A] (verified)

Not integrable

Time = 27.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^5 \arctan \left(a x \right)^{\frac{3}{2}}}{\left(a^2 c \,x^2+c \right)^{\frac{5}{2}}}d x$$

input
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.829.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.829.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.829.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.829.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.829.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,dx$$

input
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.830 $\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.830.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \operatorname{Int}\left(\frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.830.2 Mathematica [N/A]

Not integrable

Time = 7.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

3.830.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^4 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} dx$$

input
```
Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.830.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.830.4 Maple [N/A] (verified)

Not integrable

Time = 20.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^4 \arctan (ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.830.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.830.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.830.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.830.8 Giac [N/A]

Not integrable

Time = 96.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.830.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.831 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.831.1 Optimal result

Integrand size = 26, antiderivative size = 263

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{x^3\sqrt{\arctan(ax)}}{6ac(c+a^2cx^2)^{3/2}} + \frac{x\sqrt{\arctan(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)^{3/2}}{3a^2c(c+a^2cx^2)^{3/2}} \\ &\quad - \frac{2\arctan(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^4c^2\sqrt{c+a^2cx^2}} \\ &\quad + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/3*x^2*arctan(a*x)^(3/2)/a^2/c/(a^2*c*x^2+c)^(3/2)-2/3*arctan(a*x)^(3/2)
/a^4/c^2/(a^2*c*x^2+c)^(1/2)+1/144*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-9/16*
FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^
(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+1/6*x^3*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2
+c)^(3/2)+x*arctan(a*x)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

3.831.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{24\arctan(ax)\left(ax(6+7a^2x^2)-2(2+3a^2x^2)\arctan(ax)\right)-7\sqrt{6\pi}(1+a^2x^2)^{3/2}\sqrt{}}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(24*ArcTan[a*x]*(a*x*(6 + 7*a^2*x^2) - 2*(2 + 3*a^2*x^2)*ArcTan[a*x]) - 7*
Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1
+ a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] +
3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTa
n[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3
*I)*ArcTan[a*x]])))/(144*a^4*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

3.831.3 Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5475, 5465, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{2\int \frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^{3/2}}dx}{2ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{25}$$

3.831. $\int \frac{x^3\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow 3786$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow 3832$$

$$-\frac{1}{12}\int\frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow 5506$$

$$-\frac{\sqrt{a^2x^2+1}\int\frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

3.831. $\int \frac{x^3\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \; 5505$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12a^4c^2\sqrt{a^2cx^2+c}}$$

3.831. $\int \frac{x^3\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*Sqrt[ArcTan[a*x]])/(6*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^(
3/2))/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^(3/2)/(a^2*c*Sqr
t[c + a^2*c*x^2])) + (3*Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1
+ a^2*x^2] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^2*c*
Sqrt[c + a^2*c*x^2])))/(3*a^2*c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*Fresn
elS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqr
t[ArcTan[a*x]]])/2))/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

3.831.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.831. $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.831. $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.831.4 Maple [F]

$$\int \frac{x^{3} \arctan \left(a x \right)^{\frac{3}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.831.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.831.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^{3} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.831.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.831.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.831.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}}\, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2 + c)^{5/2}}\, dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.832 $\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$

3.832.1 Optimal result

Integrand size = 26, antiderivative size = 247

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx &= \frac{3\sqrt{\arctan(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3 \arctan(ax)^{3/2}}{3c(c+a^2cx^2)^{3/2}} \\ &- \frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\cos(3\arctan(ax))}{24a^3c^2\sqrt{c+a^2cx^2}} \\ &- \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^3c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/3*x^3*arctan(a*x)^(3/2)/c/(a^2*c*x^2+c)^(3/2)+1/144*FresnelC(6^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c
*x^2+c)^(1/2)-3/16*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi
^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+3/8*arctan(a*x)^(1/2)
/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/24*cos(3*arctan(a*x))*(a^2*x^2+1)^(1/2)*arc
tan(a*x)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

3.832.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.37

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{96\arctan(ax)+144a^2x^2\arctan(ax)+96a^3x^3\arctan(ax)^2+27i(1+a^2x^2)^{3/2}\sqrt{-}}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(96*ArcTan[a*x] + 144*a^2*x^2*ArcTan[a*x] + 96*a^3*x^3*ArcTan[a*x]^2 + (27
*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]
] - (27*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x
]] - I*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan
[a*x]] - I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (
-3*I)*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gam
ma[1/2, (3*I)*ArcTan[a*x]])/(288*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]
*Sqrt[ArcTan[a*x]])
```

3.832.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5479, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \; 5479$$

$$\frac{x^3 \arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{1}{2}a\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \; 5506$$

$$\frac{x^3 \arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{a\sqrt{a^2x^2+1}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{2c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\frac{a^3x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\sqrt{\arctan(ax)}\sin(\arctan(ax))^3d\arctan(ax)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(3\arctan(ax))\right)d\arctan(ax)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}-$$
$$\frac{\sqrt{a^2x^2+1}\left(-\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}+\frac{3}{4}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\mathrm{arc}}\right.}{2a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^(3/2))/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[1 + a^2*x^2]*(
(-3*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[ArcTan[a*x]]*Cos[3*Ar
cTan[a*x]])/12 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 -
 (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/12))/(2*a^3*c^2*Sqrt[
c + a^2*c*x^2])
```

3.832.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.832. $\int \frac{x^2\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.832.4 Maple [F]

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2 c\, x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.832.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.832.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.832.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.832.8 Giac [F]

$$\int \frac{x^2\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.833 $\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.833.1 Optimal result

Integrand size = 24, antiderivative size = 248

$$\begin{aligned}\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{3x\sqrt{\arctan(ax)}}{8ac^2\sqrt{c+a^2cx^2}}-\frac{\arctan(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}}\\&\quad-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^2c^2\sqrt{c+a^2cx^2}}\\&\quad-\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24a^2c^2\sqrt{c+a^2cx^2}}\\&\quad+\frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\sin(3\arctan(ax))}{24a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/3*arctan(a*x)^(3/2)/a^2/c/(a^2*c*x^2+c)^(3/2)-1/144*FresnelS(6^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*
c*x^2+c)^(1/2)-3/16*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*P
i^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+3/8*x*arctan(a*x)^(1
/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/24*sin(3*arctan(a*x))*(a^2*x^2+1)^(1/2)*ar
ctan(a*x)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

3.833.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.05

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{48(3ax+2a^3x^3-2\arctan(ax))\arctan(ax)-4\sqrt{6\pi}(1+a^2x^2)^{3/2}\sqrt{\arctan(ax)}\Big(3\sqrt{}$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(48*(3*a*x + 2*a^3*x^3 - 2*ArcTan[a*x])*ArcTan[a*x] - 4*Sqrt[6*Pi]*(1 + a^
2*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[
a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1 + a^2*x^2)^(3/2)*(
3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
 (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]]))
)/(288*a^2*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

3.833.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5465, 5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx}{2a}-\frac{\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{2ac^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5439$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{2a^2c^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{2a^2c^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{4}\sqrt{\arctan(ax)}\cos(3\arctan(ax))+\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}\right)d\arctan(ax)}{2a^2c^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{arct}}\right.}{2a^2c^2\sqrt{a^2cx^2+c}}$$

$$\frac{\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^(3/2)/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (Sqrt[1 + a^2*x^2]*
((3*a*x*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) - (3*Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[Ar
cTan[a*x]]])/12 + (Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]])/12))/(2*a^2*c^2*S
qrt[c + a^2*c*x^2])
```

3.833.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.833. $\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.833.4 Maple [F]

$$\int \frac{x \arctan (ax)^{\frac{3}{2}}}{(a^2 c\, x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.833.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.833.6 Sympy [F]

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.833.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.833.8 Giac [F]

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.833.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.834 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.834.1 Optimal result

Integrand size = 23, antiderivative size = 252

$$\begin{aligned}\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{\sqrt{\arctan(ax)}}{6ac\,(c+a^2cx^2)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{ac^2\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}}\\&+\frac{2x\arctan(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}}-\frac{9\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8ac^2\sqrt{c+a^2cx^2}}\\&-\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/3*x*arctan(a*x)^(3/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan(a*x)^(3/2)/c^2/
(a^2*c*x^2+c)^(1/2)-1/144*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(
1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-9/16*FresnelC(2^
(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2
/(a^2*c*x^2+c)^(1/2)+1/6*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(3/2)+arctan(
a*x)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.834.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.37

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{336\arctan(ax)+288a^2x^2\arctan(ax)+288ax\arctan(ax)^2+192a^3x^3\arctan(ax)^2+}{}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(336*ArcTan[a*x] + 288*a^2*x^2*ArcTan[a*x] + 288*a*x*ArcTan[a*x]^2 + 192*a
^3*x^3*ArcTan[a*x]^2 + (81*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*G
amma[1/2, (-I)*ArcTan[a*x]] - (81*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x
]]*Gamma[1/2, I*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]
]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)
*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - I*Sqrt[3 + 3*a^2*x^2]*Sqrt[
I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] - I*a^2*x^2*Sqrt[3 + 3*a^2*x^
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(288*c^2*(a + a^3*x^
2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.834.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5435, 5433, 5440, 5439, 3042, 3785, 3793, 2009, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{1}{12}\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$-\frac{1}{12}\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\left(-\frac{3}{4}\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5440$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5439$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3785$$

3.834. $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]
```

3.834. $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

output
```
Sqrt[ArcTan[a*x]]/(6*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^(3/2))/(3
*c*(c + a^2*c*x^2)^(3/2)) + (2*((3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sq
rt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^
2*c*x^2])))/(3*c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*
Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]
)/2))/(12*a*c^2*Sqrt[c + a^2*c*x^2])
```

3.834.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

3.834. $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.834.4 Maple [F]

$$\int \frac{\arctan \left(a x \right)^{\frac{3}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.834.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.834.6 Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.834.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.834.8 Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.835 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}} dx$

3.835.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}} dx = \operatorname{Int}\left(\frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}}, x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

3.835.2 Mathematica [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}} dx = \int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}} dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

3.835.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\,(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\arctan(ax)^{3/2}}{x\,(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.835.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.835.4 Maple [N/A] (verified)

Not integrable

Time = 6.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(a x \right)^{\frac{3}{2}}}{x \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

3.835.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.835.6 Sympy [N/A]

Not integrable

Time = 109.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.835.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.835.8 Giac [N/A]

Not integrable

Time = 90.94 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.835.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.836 $\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx$

3.836.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

3.836.2 Mathematica [N/A]

Not integrable

Time = 7.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x]
```

3.836.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 \left(a^2cx^2+c\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}}{x^2 \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.836.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.836.4 Maple [N/A] (verified)

Not integrable

Time = 6.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

3.836.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.836.6 Sympy [N/A]

Not integrable

Time = 176.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

3.836.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.836.8 Giac [N/A]

Not integrable

Time = 87.68 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.836.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

3.837 $\int x^m(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$

3.837.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2}\,dx=\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

3.837.2 Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

3.837.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.837.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.837.4 Maple [N/A] (verified)

Not integrable

Time = 9.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

3.837.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2} \, dx = \int (a^2 c x^2 + c) x^m \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

3.837.6 Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.837.7 Maxima [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.837.8 Giac [N/A]

Not integrable

Time = 52.89 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x^m (c + a^2cx^2) \arctan(ax)^{5/2} \, dx = \int (a^2cx^2 + c)x^m \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.837.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2cx^2) \arctan(ax)^{5/2} \, dx = \int x^m \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.838 $\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$

3.838.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\mathrm{Int}\left(x^2(c+a^2cx^2)\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

3.838.2 Mathematica [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

3.838.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right) \, dx$$
$$\Big\downarrow \; 5560$$
$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right) dx$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.838.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.838.4 Maple [N/A] (verified)

Not integrable

Time = 3.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

3.838.5 Fricas [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.838.6 Sympy [N/A]

Not integrable

Time = 41.87 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = c\left(\int x^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int a^2x^4\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
c*(Integral(x**2*atan(a*x)**(5/2), x) + Integral(a**2*x**4*atan(a*x)**(5/2
), x))
```

3.838.7 Maxima [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.838.8 Giac [N/A]

Not integrable

Time = 108.42 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int (a^2cx^2+c)x^2\arctan(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.838.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.839 $\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$

3.839.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \frac{5c(1+a^2x^2)\sqrt{\arctan(ax)}}{32a^2} - \frac{5cx(1+a^2x^2)\arctan(ax)^{3/2}}{24a} + \frac{c(1+a^2x^2)^2\arctan(ax)^{5/2}}{4a^2} - \frac{5c\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{64a} - \frac{5c\text{Int}\left(\arctan(ax)^{3/2},x\right)}{12a}$$

output
```
-5/24*c*x*(a^2*x^2+1)*arctan(a*x)^(3/2)/a+1/4*c*(a^2*x^2+1)^2*arctan(a*x)^
(5/2)/a^2+5/32*c*(a^2*x^2+1)*arctan(a*x)^(1/2)/a^2-5/12*c*Unintegrable(arc
tan(a*x)^(3/2),x)/a-5/64*c*Unintegrable(1/arctan(a*x)^(1/2),x)/a
```

3.839.2 Mathematica [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

3.839.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5415, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{5/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5\int c\left(a^2x^2+1\right) \arctan(ax)^{3/2}dx}{8a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5c\int \left(a^2x^2+1\right) \arctan(ax)^{3/2}dx}{8a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5c\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right) \arctan(ax)^{3/2}-\frac{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}\right)}{8a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5c\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right) \arctan(ax)^{3/2}-\frac{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}\right)}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.839.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.839.4 Maple [N/A] (verified)

Not integrable

Time = 1.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a^{2} c\,x^{2}+c\right) \arctan\left(a x\right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

3.839.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.839.6 Sympy [N/A]

Not integrable

Time = 26.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = c\left(\int x\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx + \int a^2x^3\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
c*(Integral(x*atan(a*x)**(5/2), x) + Integral(a**2*x**3*atan(a*x)**(5/2),
x))
```

3.839.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.839.8 Giac [N/A]

Not integrable

Time = 111.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int x(c + a^2cx^2) \arctan(ax)^{5/2} dx = \int (a^2cx^2 + c)x \arctan(ax)^{\frac{5}{2}} dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.839.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c + a^2cx^2) \arctan(ax)^{5/2} dx = \int x \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right) dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.840 $\int (c+a^2cx^2) \arctan(ax)^{5/2} dx$

3.840.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (c+a^2cx^2) \arctan(ax)^{5/2} dx = -\frac{5c(1+a^2x^2)\arctan(ax)^{3/2}}{12a} + \frac{1}{3}cx(1+a^2x^2)\arctan(ax)^{5/2} + \frac{5}{8}c\text{Int}\left(\sqrt{\arctan(ax)},x\right) + \frac{2}{3}c\text{Int}\left(\arctan(ax)^{5/2},x\right)$$

output
```
-5/12*c*(a^2*x^2+1)*arctan(a*x)^(3/2)/a+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^(5
/2)+2/3*c*Unintegrable(arctan(a*x)^(5/2),x)+5/8*c*Unintegrable(arctan(a*x)
^(1/2),x)
```

3.840.2 Mathematica [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+a^2cx^2) \arctan(ax)^{5/2} dx = \int (c+a^2cx^2) \arctan(ax)^{5/2} dx$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

3.840.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{8}c\int \sqrt{\arctan(ax)}dx+\frac{2}{3}c\int \arctan(ax)^{5/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5c\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{5}{8}c\int \sqrt{\arctan(ax)}dx+\frac{2}{3}c\int \arctan(ax)^{5/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5c\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.840.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

3.840.4 Maple [N/A] (verified)

Not integrable

Time = 1.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

3.840.5 Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2) \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.840.6 Sympy [N/A]

Not integrable

Time = 21.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int (c+a^2cx^2)\arctan(ax)^{5/2}\,dx = c\left(\int a^2x^2\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx+\int \mathrm{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2), x)
)
```

3.840.7 Maxima [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.840.8 Giac [N/A]

Not integrable

Time = 108.65 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int (c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \int (a^2cx^2+c)\arctan(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.840.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c + a^2cx^2) \arctan(ax)^{5/2} \, dx = \int \mathrm{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

3.841 $\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx$

3.841.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

3.841.2 Mathematica [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x, x]
```

3.841.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2} (a^2cx^2 + c)}{x} dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\arctan(ax)^{5/2} (a^2cx^2 + c)}{x} dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

3.841.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.841.4 Maple [N/A] (verified)

Not integrable

Time = 2.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a^2c\,x^2 + c) \arctan(ax)^{\frac{5}{2}}}{x} dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

3.841.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.841.6 Sympy [N/A]

Not integrable

Time = 16.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = c\left(\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x}\,dx + \int a^2x\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x,x)
```

output
```
c*(Integral(atan(a*x)**(5/2)/x, x) + Integral(a**2*x*atan(a*x)**(5/2), x))
```

3.841.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.841.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

3.841.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2 + c)}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x, x)
```

3.842 $\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx$

3.842.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

3.842.2 Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2, x]
```

3.842.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}{x^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
$Aborted
```

3.842.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.842.4 Maple [N/A] (verified)

Not integrable

Time = 2.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

3.842.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.842.6 Sympy [N/A]

Not integrable

Time = 19.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = c\left(\int a^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**2, x)
)
```

3.842.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.842.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.842.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x^2, x)
```

3.843 $\int x^m(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$

3.843.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Int}\left(x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

3.843.2 Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

3.843.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \, 5560$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.843.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.843.4 Maple [N/A] (verified)

Not integrable

Time = 8.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

3.843.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(5/2), x)
```

3.843.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.843.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.843.8 Giac [N/A]

Not integrable

Time = 53.60 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^{5/2}\, dx = \int (a^2cx^2+c)^2 x^m \arctan(ax)^{\frac{5}{2}}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.843.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^{5/2}\, dx = \int x^m \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^2 dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.844 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$

3.844.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \mathrm{Int}\left(x^2(c+a^2cx^2)^2 \arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

3.844.2 Mathematica [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \int x^2(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

3.844.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.844.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.844.4 Maple [N/A] (verified)

Not integrable

Time = 4.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

3.844.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.844.6 Sympy [N/A]

Not integrable

Time = 80.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{5/2} \, dx = c^2 \left(\int x^2 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int 2a^2 x^4 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int a^4 x^6 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx \right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**(5/2), x) + Integral(2*a**2*x**4*atan(a*x)*
*(5/2), x) + Integral(a**4*x**6*atan(a*x)**(5/2), x))
```

3.844.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.844.8 Giac [N/A]

Not integrable

Time = 112.39 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{5/2} \, dx = \int (a^2 c x^2 + c)^2 x^2 \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.844.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{5/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right)^2 dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.845 $\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$

3.845.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \frac{c^2(1+a^2x^2)\sqrt{\arctan(ax)}}{12a^2} + \frac{c^2(1+a^2x^2)^2\sqrt{\arctan(ax)}}{32a^2} - \frac{c^2x(1+a^2x^2)\arctan(ax)^{3/2}}{9a} - \frac{c^2x(1+a^2x^2)^2\arctan(ax)^{3/2}}{12a} + \frac{c^2(1+a^2x^2)^3\arctan(ax)^{5/2}}{6a^2} - \frac{c^2\mathrm{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{24a} - \frac{c\,\mathrm{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)}{64a} - \frac{2c^2\mathrm{Int}\left(\arctan(ax)^{3/2},x\right)}{9a}$$

output

```
-1/9*c^2*x*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-1/12*c^2*x*(a^2*x^2+1)^2*arctan
(a*x)^(3/2)/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(5/2)/a^2+1/12*c^2*(a^2*x^
2+1)*arctan(a*x)^(1/2)/a^2+1/32*c^2*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a^2-2/
9*c^2*Unintegrable(arctan(a*x)^(3/2),x)/a-1/24*c^2*Unintegrable(1/arctan(a
*x)^(1/2),x)/a-1/64*c*Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)/a
```

3.845.2 Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}\,dx=\int x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

3.845.3 Rubi [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5415, 5415, 5353, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \ 5465$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-\frac{5\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx}{12a}$$

$$\Big\downarrow \ 27$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-\frac{5c^2\int \left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx}{12a}$$

$$\Big\downarrow \ 5415$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-$$
$$\frac{5c^2\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^{3/2}dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}-\frac{3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{40a}\right)}{12a}$$

$$\Big\downarrow \ 5415$$

$$\frac{c^2(a^2x^2+1)^3\arctan(ax)^{5/2}}{6a^2}-\frac{5c^2\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{a}}{4a}\right.\right.}{12a}$$

$$\Big\downarrow \; 5353$$

$$\frac{c^2(a^2x^2+1)^3\arctan(ax)^{5/2}}{6a^2}-\frac{5c^2\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{a}}{4a}\right.\right.}{12a}$$

$$\Big\downarrow \; 5560$$

$$\frac{c^2(a^2x^2+1)^3\arctan(ax)^{5/2}}{6a^2}-\frac{5c^2\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{a}}{4a}\right.\right.}{12a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.845.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

3.845. $\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.845.4 Maple [N/A] (verified)

Not integrable

Time = 1.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

3.845.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.845.6 Sympy [N/A]

Not integrable

Time = 51.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx = c^2\left(\int x\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int 2a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^4x^5\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(x*atan(a*x)**(5/2), x) + Integral(2*a**2*x**3*atan(a*x)**(5
/2), x) + Integral(a**4*x**5*atan(a*x)**(5/2), x))
```

3.845.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.845.8 Giac [N/A]

Not integrable

Time = 109.50 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx = \int (a^2cx^2+c)^2x\arctan(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.845.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.846 $\int (c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$

3.846.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = -\frac{c^2(1+a^2x^2)\arctan(ax)^{3/2}}{3a} - \frac{c^2(1+a^2x^2)^2\arctan(ax)^{3/2}}{8a} + \frac{4}{15}c^2x(1+a^2x^2)\arctan(ax)^{5/2} + \frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax)^{5/2} + \frac{1}{2}c^2\mathrm{Int}\left(\sqrt{\arctan(ax)},x\right) + \frac{3}{16}c\mathrm{Int}\left((c+a^2\right.$$

output
```
-1/3*c^2*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-1/8*c^2*(a^2*x^2+1)^2*arctan(a*x)
^(3/2)/a+4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^(5/2)+1/5*c^2*x*(a^2*x^2+1)^2*
arctan(a*x)^(5/2)+8/15*c^2*Unintegrable(arctan(a*x)^(5/2),x)+1/2*c^2*Unint
egrable(arctan(a*x)^(1/2),x)+3/16*c*Unintegrable((a^2*c*x^2+c)*arctan(a*x)
^(1/2),x)
```

3.846.2 Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \int (c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

3.846.3 Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 27, 5415, 5353, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{16}c\int c\left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)^{5/2}dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{16}c^2\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^{5/2}dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{16}c^2\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}c^2\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{3}{16}c^2\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}c^2\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{3}{16}c^2\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+$$
$$\frac{4}{5}c^2\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}\right)+$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.846.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.846.4 Maple [N/A] (verified)

Not integrable

Time = 2.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

3.846.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.846.6 Sympy [N/A]

Not integrable

Time = 40.92 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = c^{2} \left(\int 2 a^{2} x^{2} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int a^{4} x^{4} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**(5/2), x) + Integral(a**4*x**4*atan(
a*x)**(5/2), x) + Integral(atan(a*x)**(5/2), x))
```

3.846.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.846.8 Giac [N/A]

Not integrable

Time = 111.91 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2} \, dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.846.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2} \, dx = \int \mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^2 \, dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

3.847 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx$

3.847.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

3.847.2 Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x, x]
```

3.847.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}{x}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

3.847.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.847.4 Maple [N/A] (verified)

Not integrable

Time = 2.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}{x}d x$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

3.847.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.847.6 Sympy [N/A]

Not integrable

Time = 29.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c + a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = c^2 \left(\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x} \, dx + \int 2a^2x \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int a^4x^3 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x,x)
```

output
```
c**2*(Integral(atan(a*x)**(5/2)/x, x) + Integral(2*a**2*x*atan(a*x)**(5/2)
, x) + Integral(a**4*x**3*atan(a*x)**(5/2), x))
```

3.847.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^2 \arctan(ax)^{5/2}}{x} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.847.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^2 \arctan(ax)^{5/2}}{x} dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

3.847.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^2 \arctan(ax)^{5/2}}{x} dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2 + c)^2}{x} dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x, x)
```

3.848 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} dx$

3.848.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

3.848.2 Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2, x]
```

3.848.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
$Aborted
```

3.848.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.848.4 Maple [N/A] (verified)

Not integrable

Time = 2.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

3.848.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.848.6 Sympy [N/A]

Not integrable

Time = 31.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c + a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} dx = c^2 \left(\int 2a^2 \operatorname{atan}^{\frac{5}{2}}(ax)\, dx + \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^2} dx + \int a^4x^2 \operatorname{atan}^{\frac{5}{2}}(ax)\, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**
2, x) + Integral(a**4*x**2*atan(a*x)**(5/2), x))
```

3.848.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.848.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.848.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}{x^2} \,d x$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x^2, x)
```

3.849 $\int x^m(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx$

3.849.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \mathrm{Int}\left(x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

3.849.2 Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \int x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

3.849.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.849.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.849.4 Maple [N/A] (verified)

Not integrable

Time = 10.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c\,x^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

3.849.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^m \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2} \, dx = \int \left(a^2 c x^2 + c\right)^3 x^m \arctan\left(ax\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*
x)^(5/2), x)
```

3.849.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.849.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.849.8 Giac [N/A]

Not integrable

Time = 51.06 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2} \, dx = \int \left(a^2cx^2+c\right)^3 x^m \arctan\left(ax\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.849.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2} \, dx = \int x^m \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^3 dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.850 $\int x^2(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx$

3.850.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \text{Int}\left(x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

3.850.2 Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \int x^2\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

3.850.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 dx$$
$$\Big\downarrow \; 5560$$
$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.850.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.850.4 Maple [N/A] (verified)

Not integrable

Time = 4.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

3.850.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.850.6 Sympy [N/A]

Not integrable

Time = 149.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2} \, dx = c^3 \left(\int x^2 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int 3a^2 x^4 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int 3a^4 x^6 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int a^6 x^8 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx \right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(x**2*atan(a*x)**(5/2), x) + Integral(3*a**2*x**4*atan(a*x)*
*(5/2), x) + Integral(3*a**4*x**6*atan(a*x)**(5/2), x) + Integral(a**6*x**
8*atan(a*x)**(5/2), x))
```

3.850.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.850.8 Giac [N/A]

Not integrable

Time = 108.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\int \left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.850.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.851 $\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx$

3.851.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx &= \frac{3c^3(1+a^2x^2)\sqrt{\arctan(ax)}}{56a^2} \\ &+ \frac{9c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}}{448a^2} + \frac{5c^3(1+a^2x^2)^3\sqrt{\arctan(ax)}}{448a^2} \\ &- \frac{c^3x(1+a^2x^2)\arctan(ax)^{3/2}}{14a} - \frac{3c^3x(1+a^2x^2)^2\arctan(ax)^{3/2}}{56a} \\ &- \frac{5c^3x(1+a^2x^2)^3\arctan(ax)^{3/2}}{112a} + \frac{c^3(1+a^2x^2)^4\arctan(ax)^{5/2}}{8a^2} \\ &- \frac{3c^3\mathrm{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{112a} - \frac{9c^2\mathrm{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)}{896a} \\ &- \frac{5c\mathrm{Int}\left(\frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)}{896a} - \frac{c^3\mathrm{Int}\left(\arctan(ax)^{3/2},x\right)}{7a}\end{aligned}$$

output
```
-1/14*c^3*x*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-3/56*c^3*x*(a^2*x^2+1)^2*arcta
n(a*x)^(3/2)/a-5/112*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a+1/8*c^3*(a^2*
x^2+1)^4*arctan(a*x)^(5/2)/a^2+3/56*c^3*(a^2*x^2+1)*arctan(a*x)^(1/2)/a^2+
9/448*c^3*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a^2+5/448*c^3*(a^2*x^2+1)^3*arct
an(a*x)^(1/2)/a^2-1/7*c^3*Unintegrable(arctan(a*x)^(3/2),x)/a-3/112*c^3*Un
integrable(1/arctan(a*x)^(1/2),x)/a-9/896*c^2*Unintegrable((a^2*c*x^2+c)/a
rctan(a*x)^(1/2),x)/a-5/896*c*Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(1/
2),x)/a
```

3.851.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx=\int x(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

3.851.3 Rubi [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 27, 5415, 5415, 5415, 5353, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-\frac{5\int c^3(a^2x^2+1)^3\arctan(ax)^{3/2}dx}{16a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-\frac{5c^3\int(a^2x^2+1)^3\arctan(ax)^{3/2}dx}{16a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-\frac{5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\int(a^2x^2+1)^2\arctan(ax)^{3/2}dx+\frac{1}{7}x(a^2x^2+1)^3\arctan(ax)^{3/2}-\frac{(a^2x^2+1)^3\sqrt{\arctan(ax)}}{28a}\right)}{16a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$\frac{5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\int(a^2x^2+1)\arctan(ax)^{3/2}dx+\frac{1}{5}x(a^2x^2+1)^2\arctan(ax)^{3/2}\right.\right.}{16a}$$

$$\Big\downarrow \; 5415$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\text{arc}\right.\right.\right.$$

$$\Big\downarrow \; 5353$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\text{arc}\right.\right.\right.$$

$$\Big\downarrow \; 5560$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\text{arc}\right.\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.851.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.851. $\int x(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx$

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.851.4 Maple [N/A] (verified)

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

3.851.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.851.6 Sympy [N/A]

Not integrable

Time = 96.84 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int x(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx = c^3\left(\int x\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx + \int 3a^2x^3\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx + \int 3a^4x^5\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx + \int a^6x^7\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(x*atan(a*x)**(5/2), x) + Integral(3*a**2*x**3*atan(a*x)**(5
/2), x) + Integral(3*a**4*x**5*atan(a*x)**(5/2), x) + Integral(a**6*x**7*a
tan(a*x)**(5/2), x))
```

3.851.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.851.8 Giac [N/A]

Not integrable

Time = 104.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx = \int (a^2cx^2+c)^3 x \arctan(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.851.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{5/2}\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.852 $\int (c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx$

3.852.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx = -\frac{2c^3(1+a^2x^2)\arctan(ax)^{3/2}}{7a} - \frac{3c^3(1+a^2x^2)^2\arctan(ax)^{3/2}}{28a} - \frac{5c^3(1+a^2x^2)^3\arctan(ax)^{3/2}}{84a} + \frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^{5/2} + \frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^{5/2} + \frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^{5/2} + \frac{3}{7}c^3\mathrm{Int}\Big($$

output
```
-2/7*c^3*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-3/28*c^3*(a^2*x^2+1)^2*arctan(a*x
)^(3/2)/a-5/84*c^3*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a+8/35*c^3*x*(a^2*x^2+1
)*arctan(a*x)^(5/2)+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^(5/2)+1/7*c^3*x*(
a^2*x^2+1)^3*arctan(a*x)^(5/2)+16/35*c^3*Unintegrable(arctan(a*x)^(5/2),x)
+3/7*c^3*Unintegrable(arctan(a*x)^(1/2),x)+9/56*c^2*Unintegrable((a^2*c*x^
2+c)*arctan(a*x)^(1/2),x)+5/56*c*Unintegrable((a^2*c*x^2+c)^2*arctan(a*x)^
(1/2),x)
```

3.852.2 Mathematica [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx=\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

3.852.3 Rubi [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 27, 5415, 5415, 5353, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{56}c\int c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^{5/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^{5/2}dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{(a^2x\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.852. $\int \left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx$

3.852.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.852.4 Maple [N/A] (verified)

Not integrable

Time = 2.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

3.852.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.852.6 Sympy [N/A]

Not integrable

Time = 85.96 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = c^3\left(\int 3a^2x^2\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx + \int 3a^4x^4\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx + \int a^6x^6\,\mathrm{atan}^{\frac{5}{2}}(ax)\,dx + \int \mathrm{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**(5/2), x) + Integral(3*a**4*x**4*ata
n(a*x)**(5/2), x) + Integral(a**6*x**6*atan(a*x)**(5/2), x) + Integral(ata
n(a*x)**(5/2), x))
```

3.852.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.852.8 Giac [N/A]

Not integrable

Time = 108.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2} \, dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.852.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2} \, dx = \int \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^3 \, dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

3.853 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx$

3.853.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

3.853.2 Mathematica [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x, x]
```

3.853.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x}dx$$
$$\Big\downarrow \ \color{blue}{5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

3.853.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.853.4 Maple [N/A] (verified)

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}}{x}d x$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

3.853.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.853.6 Sympy [N/A]

Not integrable

Time = 57.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x}\,dx = c^3\left(\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int 3a^4x^3\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^6x^5\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x,x)
```

output
```
c**3*(Integral(atan(a*x)**(5/2)/x, x) + Integral(3*a**2*x*atan(a*x)**(5/2)
, x) + Integral(3*a**4*x**3*atan(a*x)**(5/2), x) + Integral(a**6*x**5*atan
(a*x)**(5/2), x))
```

3.853.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.853.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

3.853.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}{x} \,d x$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x, x)
```

3.854 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx$

3.854.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

3.854.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2, x]
```

3.854.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
$Aborted
```

3.854.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.854.4 Maple [N/A] (verified)

Not integrable

Time = 2.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

3.854.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.854.6 Sympy [N/A]

Not integrable

Time = 61.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = c^3\left(\int 3a^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^2}\,dx + \int 3a^4x^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^6x^4\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**
2, x) + Integral(3*a**4*x**2*atan(a*x)**(5/2), x) + Integral(a**6*x**4*ata
n(a*x)**(5/2), x))
```

3.854.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.854.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.854.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3}{x^2}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x^2, x)
```

3.855 $\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx$

3.855.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

3.855.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx = \int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

3.855.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.855.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.855.4 Maple [N/A] (verified)

Not integrable

Time = 5.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan \left(a x\right)^{\frac{5}{2}}}{a^2 c \,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

3.855.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

3.855.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Timed out
```

3.855.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.855.8 Giac [N/A]

Not integrable

Time = 52.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \int \frac{x^m \arctan{(ax)}^{\frac{5}{2}}}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.855.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.856 $\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx$

3.856.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = -\frac{2x\arctan(ax)^{7/2}}{7a^3c} + \frac{\operatorname{Int}\left(x\arctan(ax)^{5/2},x\right)}{a^2c} + \frac{2\operatorname{Int}\left(\arctan(ax)^{7/2},x\right)}{7a^3c}$$

output
```
-2/7*x*arctan(a*x)^(7/2)/a^3/c+Unintegrable(x*arctan(a*x)^(5/2),x)/a^2/c+2
/7*Unintegrable(arctan(a*x)^(7/2),x)/a^3/c
```

3.856.2 Mathematica [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

3.856.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5451, 27, 5377, 5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{5/2}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5457}$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\frac{2x \arctan(ax)^{7/2}}{7a} - \frac{2\int \arctan(ax)^{7/2}dx}{7a}}{a^2c}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\frac{2x \arctan(ax)^{7/2}}{7a} - \frac{2\int \arctan(ax)^{7/2}dx}{7a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.856.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.856.4 Maple [N/A] (verified)

Not integrable

Time = 2.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{3} \arctan \left(a x \right)^{\frac{5}{2}}}{a^{2} c \,x^{2}+c}d x$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

3.856.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.856.6 Sympy [N/A]

Not integrable

Time = 9.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

3.856.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.856.8 Giac [N/A]

Not integrable

Time = 108.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx=\int \frac{x^3 \arctan{(ax)}^{\frac{5}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.856.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx=\int \frac{x^3\,{\mathrm{atan}\left(a\,x\right)}^{5/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.857 $\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx$

3.857.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx = -\frac{2\arctan(ax)^{7/2}}{7a^3c} + \frac{\text{Int}\left(\arctan(ax)^{5/2}, x\right)}{a^2c}$$

output
```
-2/7*arctan(a*x)^(7/2)/a^3/c+Unintegrable(arctan(a*x)^(5/2),x)/a^2/c
```

3.857.2 Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx = \int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

3.857.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5451, 27, 5353, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{a^2cx^2+c}dx$$
$$\Big\downarrow \ 5451$$
$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^{5/2}}{c(a^2x^2+1)}dx}{a^2}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \ 5353$$
$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \ 5419$$
$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{2\arctan(ax)^{7/2}}{7a^3c}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.857.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.857.4 Maple [N/A] (verified)

Not integrable

Time = 1.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{a^2c\,x^2+c}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

3.857.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.857.6 Sympy [N/A]

Not integrable

Time = 7.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

3.857.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.857.8 Giac [N/A]

Not integrable

Time = 109.59 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^2 \arctan{(ax)}^{\frac{5}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.857.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^2 \,\mathrm{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.858 $\int \frac{x\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx$

3.858.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx=\frac{2x\arctan(ax)^{7/2}}{7ac}-\frac{2\mathrm{Int}\left(\arctan(ax)^{7/2},x\right)}{7ac}$$

output
```
2/7*x*arctan(a*x)^(7/2)/a/c-2/7*Unintegrable(arctan(a*x)^(7/2),x)/a/c
```

3.858.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx=\int \frac{x\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

3.858.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{2x \arctan(ax)^{7/2}}{7ac}-\frac{2\int \arctan(ax)^{7/2}dx}{7ac}$$
$$\Big\downarrow \text{5353}$$
$$\frac{2x \arctan(ax)^{7/2}}{7ac}-\frac{2\int \arctan(ax)^{7/2}dx}{7ac}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.858.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.858.4 Maple [N/A] (verified)

Not integrable

Time = 1.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan\left(ax\right)^{\frac{5}{2}}}{a^{2} c\,x^{2}+c}dx$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

3.858.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.858.6 Sympy [N/A]

Not integrable

Time = 5.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}{a^{2} x^{2} + 1}\, dx}{c}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

3.858.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.858.8 Giac [N/A]

Not integrable

Time = 107.69 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x\arctan\left(ax\right)^{\frac{5}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.858.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

3.859 $\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx$

3.859.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\arctan(ax)^{7/2}}{7ac}$$

output
```
2/7*arctan(a*x)^(7/2)/a/c
```

3.859.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\arctan(ax)^{7/2}}{7ac}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(7/2))/(7*a*c)
```

3.859.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{2\arctan(ax)^{7/2}}{7ac}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(7/2))/(7*a*c)
```

3.859.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.859.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2\arctan(ax)^{\frac{7}{2}}}{7ac}$	15

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/7*arctan(a*x)^(7/2)/a/c
```

3.859.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\arctan{(ax)}^{\frac{7}{2}}}{7\,ac}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
2/7*arctan(a*x)^(7/2)/(a*c)
```

3.859.6 Sympy [A] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \begin{cases} \frac{2\operatorname{atan}^{\frac{7}{2}}{(ax)}}{7ac} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((2*atan(a*x)**(7/2)/(7*a*c), Ne(a, 0)), (0, True))
```

3.859.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.859.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\arctan(ax)^{\frac{7}{2}}}{7\,ac}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/7*arctan(a*x)^(7/2)/(a*c)
```

3.859.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\,\mathrm{atan}(a\,x)^{7/2}}{7\,a\,c}$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2),x)
```

output
```
(2*atan(a*x)^(7/2))/(7*a*c)
```

3.860 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx$

3.860.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx=-\frac{2i\arctan(ax)^{7/2}}{7c}+\frac{i\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x(i+ax)},x\right)}{c}$$

output
```
-2/7*I*arctan(a*x)^(7/2)/c+I*Unintegrable(arctan(a*x)^(5/2)/x/(I+a*x),x)/c
```

3.860.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx=\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)), x]
```

3.860.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5459, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5459}$$

$$\frac{i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{7/2}}{7c}$$

$$\Big\downarrow \text{5560}$$

$$\frac{i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{7/2}}{7c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.860.3.1 Defintions of rubi rules used

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.860.4 Maple [N/A] (verified)

Not integrable

Time = 1.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan (ax)^{\frac{5}{2}}}{x\,(a^2c\,x^2+c)}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

3.860.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.860.6 Sympy [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**3 + x), x)/c
```

3.860.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.860.8 Giac [N/A]

Not integrable

Time = 103.47 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.860.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)), x)
```

3.861 $\int \frac{\arctan(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx$

3.861.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx=-\frac{2a\arctan(ax)^{7/2}}{7c}+\frac{\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x^2},x\right)}{c}$$

output
```
-2/7*a*arctan(a*x)^(7/2)/c+Unintegrable(arctan(a*x)^(5/2)/x^2,x)/c
```

3.861.2 Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)), x]
```

3.861.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^2 (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \; 5453$$
$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - a^2 \int \frac{\arctan(ax)^{5/2}}{c(a^2x^2+1)}dx$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \; 5377$$
$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{c}$$
$$\Big\downarrow \; 5419$$
$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - \frac{2a \arctan(ax)^{7/2}}{7c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.861.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.861.4 Maple [N/A] (verified)

Not integrable

Time = 1.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x^{2}\left(a^{2}c\,x^{2}+c\right)}dx$$

input
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

3.861.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.861.6 Sympy [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^4+x^2}\,dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**4 + x**2), x)/c
```

3.861.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.861.8 Giac [N/A]

Not integrable

Time = 107.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x^2 (c + a^2cx^2)}\,dx = \int \frac{\arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2 + c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.861.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^2 (c + a^2cx^2)}\,dx = \int \frac{{\mathrm{atan}\left(a\,x\right)}^{5/2}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)), x)
```

3.862 $\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx$

3.862.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \frac{2ia^2\arctan(ax)^{7/2}}{7c} + \frac{\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x^3},x\right)}{c} - \frac{ia^2\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x(i+ax)},x\right)}{c}$$

output
```
2/7*I*a^2*arctan(a*x)^(7/2)/c+Unintegrable(arctan(a*x)^(5/2)/x^3,x)/c-I*a^
2*Unintegrable(arctan(a*x)^(5/2)/x/(I+a*x),x)/c
```

3.862.2 Mathematica [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)), x]
```

3.862.3 Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5459, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-a^2\int \frac{\arctan(ax)^{5/2}}{cx\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{5/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{5/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx-\frac{2}{7}i\arctan(ax)^{7/2}\right)}{c}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx-\frac{2}{7}i\arctan(ax)^{7/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.862.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.862.4 Maple [N/A] (verified)

Not integrable

Time = 1.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan (ax)^{\frac{5}{2}}}{x^{3} (a^{2}c\,x^{2}+c)}dx$$

input
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

3.862.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3 (c + a^2cx^2)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.862.6 Sympy [N/A]

Not integrable

Time = 5.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{x^3 (c + a^2cx^2)} \, dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^5 + x^3} \, dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**5 + x**3), x)/c
```

3.862.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3 (c + a^2cx^2)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.862.8 Giac [N/A]

Not integrable

Time = 107.95 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x^3 (c+a^2cx^2)} \, dx = \int \frac{\arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)x^3} \, dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.862.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^3 (c+a^2cx^2)} \, dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^3\,\left(c\,a^2\,x^2+c\right)} \, dx$$

input
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)), x)
```

3.863 $\int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx$

3.863.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx = \frac{2a^3\arctan(ax)^{7/2}}{7c} + \frac{\mathrm{Int}\left(\frac{\arctan(ax)^{5/2}}{x^4},x\right)}{c} - \frac{a^2\mathrm{Int}\left(\frac{\arctan(ax)^{5/2}}{x^2},x\right)}{c}$$

output
```
2/7*a^3*arctan(a*x)^(7/2)/c+Unintegrable(arctan(a*x)^(5/2)/x^4,x)/c-a^2*Un
integrable(arctan(a*x)^(5/2)/x^2,x)/c
```

3.863.2 Mathematica [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)), x]
```

3.863.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5453, 27, 5377, 5453, 5377, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^4 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c} - a^2 \int \frac{\arctan(ax)^{5/2}}{cx^2 (a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^{5/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^{5/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c} - \frac{a^2 \left(\int \frac{\arctan(ax)^{5/2}}{x^2}dx - a^2 \int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c} - \frac{a^2 \left(\int \frac{\arctan(ax)^{5/2}}{x^2}dx - a^2 \int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c} - \frac{a^2 \left(\int \frac{\arctan(ax)^{5/2}}{x^2}dx - \frac{2}{7}a\arctan(ax)^{7/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

3.863. $\int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)} \, dx$

3.863.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.863.4 Maple [N/A] (verified)

Not integrable

Time = 4.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan (ax)^{\frac{5}{2}}}{x^4 (a^2 c\, x^2 + c)} dx$$

input
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

3.863.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.863.6 Sympy [N/A]

Not integrable

Time = 7.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^6+x^4}\,dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**6 + x**4), x)/c
```

3.863.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.863.8 Giac [N/A]

Not integrable

Time = 109.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x^4 (c + a^2cx^2)} \, dx = \int \frac{\arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2 + c)x^4} \, dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.863.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^4 (c + a^2cx^2)} \, dx = \int \frac{\mathrm{atan}{(a\,x)}^{5/2}}{x^4\,\left(c\,a^2\,x^2 + c\right)} \, dx$$

input
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)), x)
```

3.864 $\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

3.864.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

3.864.2 Mathematica [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

3.864.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.864.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.864.4 Maple [N/A] (verified)

Not integrable

Time = 9.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

3.864.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

3.864.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Timed out
```

3.864.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.864.8 Giac [N/A]

Not integrable

Time = 52.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.864.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.865 $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx$

3.865.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \text{Int}\left(\frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Unintegrable(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

3.865.2 Mathematica [N/A]

Not integrable

Time = 3.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

3.865.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^2} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^2} dx$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

3.865.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.865.4 Maple [N/A] (verified)

Not integrable

Time = 7.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2 c\, x^2+c\right)^2}dx$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

3.865.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.865.6 Sympy [N/A]

Not integrable

Time = 15.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.865.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.865.8 Giac [N/A]

Not integrable

Time = 118.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^3 \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.865.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.866 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

3.866.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{15x\sqrt{\arctan(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\arctan(ax)^{3/2}}{16a^3c^2} - \frac{5\arctan(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{7/2}}{7a^3c^2} - \frac{15\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128a^3c^2}$$

output
```
5/16*arctan(a*x)^(3/2)/a^3/c^2-5/8*arctan(a*x)^(3/2)/a^3/c^2/(a^2*x^2+1)-1
/2*x*arctan(a*x)^(5/2)/a^2/c^2/(a^2*x^2+1)+1/7*arctan(a*x)^(7/2)/a^3/c^2-1
5/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2+15/32*x*arct
an(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

3.866.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\arctan(ax)}(105ax + 70(-1+a^2x^2)\arctan(ax) - 112ax\arctan(ax)^2 + 32(1+}{896a^3c^2(1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*(105*a*x + 70*(-1 + a^2*x^2)*ArcTan[a*x] - 112*a*x*Ar
cTan[a*x]^2 + 32*(1 + a^2*x^2)*ArcTan[a*x]^3) - 105*Sqrt[Pi]*(1 + a^2*x^2)
*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(896*a^3*c^2*(1 + a^2*x^2))
```

3.866.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5471, 27, 5465, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)^{5/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5471}$$

$$\frac{5\int \frac{x\arctan(ax)^{3/2}}{c^2(a^2x^2+1)^2}dx}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{5\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5505}$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{4906}$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3786}$$

$$\frac{5\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3832}$$

3.866. $\int \frac{x^2\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

$$\frac{\arctan(ax)^{7/2}}{7a^3c^2}+\frac{5\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^(5/2))/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*
a^3*c^2) + (5*(-1/2*ArcTan[a*x]^(3/2)/(a^2*(1 + a^2*x^2)) + (3*((x*Sqrt[Ar
cTan[a*x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (Sqrt[Pi]*Fresne
lS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a)))/(4*a*c^2)
```

3.866.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.866. $\int \frac{x^2\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.866.4 Maple [A] (verified)

Time = 25.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

method	result
default	$\frac{128\arctan(ax)^{\frac{7}{2}}\sqrt{\pi}-224\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\,\sin(2\arctan(ax))-280\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}\,\cos(2\arctan(ax))+210\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\sin(2\,\mathrm{arc}}{896c^2a^3\sqrt{\pi}}$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/896/c^2/a^3*(128*arctan(a*x)^(7/2)*Pi^(1/2)-224*arctan(a*x)^(5/2)*Pi^(1/
2)*sin(2*arctan(a*x))-280*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))+21
0*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))-105*Pi*FresnelS(2*arctan(a
*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

3.866.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.866.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.866.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.866.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.867 $\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx$

3.867.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = -\frac{15\sqrt{\arctan(ax)}}{64a^2c^2} + \frac{15\sqrt{\arctan(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\arctan(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\arctan(ax)^{5/2}}{4a^2c^2} - \frac{\arctan(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} - \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128a^2c^2}$$

output
```
5/8*x*arctan(a*x)^(3/2)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^(5/2)/a^2/c^2-1/
2*arctan(a*x)^(5/2)/a^2/c^2/(a^2*x^2+1)-15/128*FresnelC(2*arctan(a*x)^(1/2
)/Pi^(1/2))*Pi^(1/2)/a^2/c^2-15/64*arctan(a*x)^(1/2)/a^2/c^2+15/32*arctan(
a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

3.867.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.50

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \frac{240\arctan(ax) - 240a^2x^2\arctan(ax) + 640ax\arctan(ax)^2 - 256\arctan(ax)^3 + 25}{}$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
(240*ArcTan[a*x] - 240*a^2*x^2*ArcTan[a*x] + 640*a*x*ArcTan[a*x]^2 - 256*A
rcTan[a*x]^3 + 256*a^2*x^2*ArcTan[a*x]^3 - 60*Sqrt[Pi]*(1 + a^2*x^2)*Sqrt[
ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (15*I)*Sqrt[2]*(1
+ a^2*x^2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - (15*I)*
Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - (15*I)*Sqrt[2]
*a^2*x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(1024*a^2*c^2*
(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])
```

3.867.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5465, 27, 5427, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{c^2\left(a^2x^2+1\right)^2}dx}{4a}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{\left(a^2x^2+1\right)^2}dx}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{5\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^{3/2}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^{3/2}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3793}$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^(5/2)/(a^2*c^2*(1 + a^2*x^2)) + (5*((x*ArcTan[a*x]^(3/2))
/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a) - (3*a*(-1/2*Sqrt[ArcTan[a*x]
]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[Ar
cTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/4))/(4*a*c^2)
```

3.867. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

3.867.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.867. $\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

3.867.4 Maple [A] (verified)

Time = 25.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

method	result
default	$-\frac{32 \arctan(ax)^{\frac{5}{2}} \cos(2 \arctan(ax))\sqrt{\pi}-40 \arctan(ax)^{\frac{3}{2}} \sin(2 \arctan(ax))\sqrt{\pi}-30\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\cos(2 \arctan(ax))+15\pi\ \mathrm{FresnelC}}{128c^2a^2\sqrt{\pi}}$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/128/c^2/a^2/Pi^(1/2)*(32*arctan(a*x)^(5/2)*cos(2*arctan(a*x))*Pi^(1/2)-
40*arctan(a*x)^(3/2)*sin(2*arctan(a*x))*Pi^(1/2)-30*arctan(a*x)^(1/2)*Pi^(
1/2)*cos(2*arctan(a*x))+15*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

3.867.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.867.6 Sympy [F]

$$\int \frac{x\arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.867.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.867.8 Giac [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

3.868 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

3.868.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = -\frac{15x\sqrt{\arctan(ax)}}{32c^2(1+a^2x^2)} - \frac{5\arctan(ax)^{3/2}}{16ac^2} + \frac{5\arctan(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{7/2}}{7ac^2} + \frac{15\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128ac^2}$$

output
```
-5/16*arctan(a*x)^(3/2)/a/c^2+5/8*arctan(a*x)^(3/2)/a/c^2/(a^2*x^2+1)+1/2*
x*arctan(a*x)^(5/2)/c^2/(a^2*x^2+1)+1/7*arctan(a*x)^(7/2)/a/c^2+15/128*Fre
snelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2-15/32*x*arctan(a*x)^(1/
2)/c^2/(a^2*x^2+1)
```

3.868.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\arctan(ax)}(-105ax - 70(-1+a^2x^2)\arctan(ax) + 112ax\arctan(ax)^2 + 32(1+a}{896c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*(-105*a*x - 70*(-1 + a^2*x^2)*ArcTan[a*x] + 112*a*x*A
rcTan[a*x]^2 + 32*(1 + a^2*x^2)*ArcTan[a*x]^3) + 105*Sqrt[Pi]*(1 + a^2*x^2
)*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(896*c^2*(a + a^3*x^2))
```

3.868.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5427, 27, 5465, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{c^2\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5a\int \frac{x\arctan(ax)^{3/2}}{\left(a^2x^2+1\right)^2}dx}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{5a\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{5a\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{4906}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{3786}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{3832}$$

3.868. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

$$-\frac{5a\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\,(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^(5/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a*c^2)
- (5*a*(-1/2*ArcTan[a*x]^(3/2)/(a^2*(1 + a^2*x^2)) + (3*((x*Sqrt[ArcTan[a*
x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (Sqrt[Pi]*FresnelS[(2*S
qrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a)))/(4*c^2)
```

3.868.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.868. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.868.4 Maple [A] (verified)

Time = 25.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

method	result
default	$\frac{128\arctan(ax)^{\frac{7}{2}}\sqrt{\pi}+224\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\,\sin(2\arctan(ax))+280\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}\,\cos(2\arctan(ax))+105\pi\ \mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{896c^2a\sqrt{\pi}}$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/896/c^2/a/Pi^(1/2)*(128*arctan(a*x)^(7/2)*Pi^(1/2)+224*arctan(a*x)^(5/2)
*Pi^(1/2)*sin(2*arctan(a*x))+280*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a
*x))+105*Pi*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))-210*arctan(a*x)^(1/2)*P
i^(1/2)*sin(2*arctan(a*x)))
```

3.868.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.868.6 Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

3.868.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.868.8 Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^2, x)
```

3.869 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^2}\,dx$

3.869.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2},x\right)$$

output
```
Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

3.869.2 Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2), x]
```

3.869.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{5/2}}{x(a^2cx^2+c)^2}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
$Aborted
```

3.869.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.869.4 Maple [N/A] (verified)

Not integrable

Time = 3.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^{2}c\,x^{2}+c\right)^{2}}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

3.869.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.869.6 Sympy [N/A]

Not integrable

Time = 5.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

3.869.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.869.8 Giac [N/A]

Not integrable

Time = 116.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^2}\,dx=\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.869.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^2), x)
```

3.870 $\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

3.870.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

3.870.2 Mathematica [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

3.870.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.870.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.870.4 Maple [N/A] (verified)

Not integrable

Time = 7.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^3}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

3.870.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3), x)
```

3.870.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Timed out
```

3.870.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.870.8 Giac [N/A]

Not integrable

Time = 54.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.870.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.871 $\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$

3.871.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \operatorname{Int}\left(\frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Unintegrable(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

3.871.2 Mathematica [N/A]

Not integrable

Time = 6.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$$

input
```
Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

3.871.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^5 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

3.871.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.871.4 Maple [N/A] (verified)

Not integrable

Time = 8.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5 \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^3}dx$$

input
```
int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

3.871.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.871.6 Sympy [N/A]

Not integrable

Time = 31.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^5 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**5*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**5*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.871.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.871.8 Giac [N/A]

Not integrable

Time = 128.67 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.871.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^5*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^5*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.872 $\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

3.872.1 Optimal result

Integrand size = 24, antiderivative size = 310

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \frac{45x\sqrt{\arctan(ax)}}{128a^4c^3(1+a^2x^2)} + \frac{45\arctan(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} - \frac{15\arctan(ax)^{3/2}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x\arctan(ax)^{5/2}}{8a^4c^3(1+a^2x^2)} + \frac{3\arctan(ax)^{7/2}}{28a^5c^3} + \frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^5c^3} - \frac{15\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128a^5c^3} + \frac{15\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{256a^5c^3} - \frac{15\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{2048a^5c^3}$$

output

```
45/256*arctan(a*x)^(3/2)/a^5/c^3+5/32*x^4*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2
+1)^2-15/32*arctan(a*x)^(3/2)/a^5/c^3/(a^2*x^2+1)-1/4*x^3*arctan(a*x)^(5/2
)/a^2/c^3/(a^2*x^2+1)^2-3/8*x*arctan(a*x)^(5/2)/a^4/c^3/(a^2*x^2+1)+3/28*a
rctan(a*x)^(7/2)/a^5/c^3+15/8192*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))*2^(1/2)*Pi^(1/2)/a^5/c^3-15/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2
))*Pi^(1/2)/a^5/c^3+45/128*x*arctan(a*x)^(1/2)/a^4/c^3/(a^2*x^2+1)+15/256*
sin(2*arctan(a*x))*arctan(a*x)^(1/2)/a^5/c^3-15/2048*sin(4*arctan(a*x))*ar
ctan(a*x)^(1/2)/a^5/c^3
```

3.872.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.93

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \frac{50400ax\arctan(ax) + 57120a^3x^3\arctan(ax) - 33600\arctan(ax)^2 - 13440a^2x^2 \mathrm{arc}}{}$$

input
```
Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(50400*a*x*ArcTan[a*x] + 57120*a^3*x^3*ArcTan[a*x] - 33600*ArcTan[a*x]^2 -
 13440*a^2*x^2*ArcTan[a*x]^2 + 38080*a^4*x^4*ArcTan[a*x]^2 - 43008*a*x*Arc
Tan[a*x]^3 - 71680*a^3*x^3*ArcTan[a*x]^3 + 12288*(1 + a^2*x^2)^2*ArcTan[a*
x]^4 + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*
I)*ArcTan[a*x]] + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1
/2, (2*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[
1/2, (-4*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1
/2, (4*I)*ArcTan[a*x]])/(114688*a^5*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

3.872.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5475, 27, 5471, 5465, 5427, 5505, 3042, 3793, 2009, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} dx$$

$$\Big\downarrow \color{blue}{5475}$$

$$-\frac{15}{64}\int \frac{x^4\sqrt{\arctan(ax)}}{c^3(a^2x^2+1)^3}dx + \frac{3\int \frac{x^2\arctan(ax)^{5/2}}{c^2(a^2x^2+1)^2}dx}{4a^2c} + \frac{5x^4\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2} - \frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{3\int \frac{x^2\arctan(ax)^{5/2}}{(a^2x^2+1)^2}dx}{4a^2c^3} - \frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3} + \frac{5x^4\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2} - \frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \; 5471$$

$$-\frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(\frac{5\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 5465$$

$$-\frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(\frac{5\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 5427$$

$$-\frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(\frac{5\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 5505$$

3.872. $\int \frac{x^4\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-\frac{15\int \frac{a^4x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{64a^5c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{15\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^4d\arctan(ax)}{64a^5c^3}+\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{15\int \left(-\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{64a^5c^3}+\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

3.872. $\int \frac{x^4\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}dx$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

3.872. $\int \frac{x^4\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3786}$$

3.872. $\int \frac{x^4\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\,\text{s}\right.}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+$$

$$\frac{3\left(\frac{\arctan(ax)^{7/2}}{7a^3}+\frac{5\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}-\frac{\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

input
```
Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(5*x^4*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^(5
/2))/(4*a^2*c^3*(1 + a^2*x^2)^2) + (3*(-1/2*(x*ArcTan[a*x]^(5/2))/(a^2*(1
+ a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a^3) + (5*(-1/2*ArcTan[a*x]^(3/2)/(a^2*
(1 + a^2*x^2)) + (3*((x*Sqrt[ArcTan[a*x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]
^(3/2)/(3*a) - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))
/(4*a)))/(4*a)))/(4*a^2*c^3) - (15*(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*Fres
nelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/64 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]])/8 - (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[
ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32))/(64*a^5*c^3)
```

3.872. $\int \frac{x^4\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

3.872.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.872. $\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.872.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.54

$$\frac{6144\arctan(ax)^{\frac{7}{2}}\sqrt{\pi}-14336\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\sin(2\arctan(ax))+1792\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\sin(4\arctan(a}{}$$

input
```
int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
1/57344/c^3/a^5*(6144*arctan(a*x)^(7/2)*Pi^(1/2)-14336*arctan(a*x)^(5/2)*P
i^(1/2)*sin(2*arctan(a*x))+1792*arctan(a*x)^(5/2)*Pi^(1/2)*sin(4*arctan(a*
x))-17920*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))+1120*arctan(a*x)^(
3/2)*Pi^(1/2)*cos(4*arctan(a*x))+13440*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*ar
ctan(a*x))-420*arctan(a*x)^(1/2)*Pi^(1/2)*sin(4*arctan(a*x))+105*Pi*2^(1/2
)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-6720*Pi*FresnelS(2*arctan
(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

3.872.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.872.6 Sympy [F]

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^4 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**4*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.872.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c + a^2cx^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.872.8 Giac [F]

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c + a^2cx^2)^3} \, dx = \int \frac{x^4 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2 + c)^3} \, dx$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.872.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c + a^2cx^2)^3} \, dx = \int \frac{x^4 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2 + c)^3} \, dx$$

input
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.873 $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx$

3.873.1 Optimal result

Integrand size = 24, antiderivative size = 256

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = -\frac{135\sqrt{\arctan(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\arctan(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\arctan(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\arctan(ax)^{3/2}}{64a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)^{5/2}}{32a^4c^3} + \frac{x^4\arctan(ax)^{5/2}}{4c^3(1+a^2x^2)^2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^4c^3} - \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{256a^4c^3}$$

output
```
5/32*x^3*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2+15/64*x*arctan(a*x)^(3/2)/a
^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)^(5/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(5/2
)/c^3/(a^2*x^2+1)^2+15/8192*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))
*2^(1/2)*Pi^(1/2)/a^4/c^3-15/256*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi
^(1/2)/a^4/c^3-135/2048*arctan(a*x)^(1/2)/a^4/c^3-15/256*x^4*arctan(a*x)^(
1/2)/c^3/(a^2*x^2+1)^2+45/256*arctan(a*x)^(1/2)/a^4/c^3/(a^2*x^2+1)
```

3.873.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.40

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{510\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + \frac{14400 \arctan(ax)+5760a^2x^2 \arctan(ax)-16320a^4x^4 \text{ar}}{}}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(510*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (14400*ArcTan[a
*x] + 5760*a^2*x^2*ArcTan[a*x] - 16320*a^4*x^4*ArcTan[a*x] + 30720*a*x*Arc
Tan[a*x]^2 + 51200*a^3*x^3*ArcTan[a*x]^2 - 12288*ArcTan[a*x]^3 - 24576*a^2
*x^2*ArcTan[a*x]^3 + 20480*a^4*x^4*ArcTan[a*x]^3 - 4080*Sqrt[Pi]*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (900*I
)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[
a*x]] - (900*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*
I)*ArcTan[a*x]] + (135*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-4*I)*ArcTan[a*x]] - (135*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[
1/2, (4*I)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^4*
c^3)
```

3.873.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {5479, 27, 5475, 5471, 5465, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \ 5479$$

$$\frac{x^4 \arctan(ax)^{5/2}}{4c^3(a^2x^2+1)^2} - \frac{5}{8}a\int \frac{x^4 \arctan(ax)^{3/2}}{c^3(a^2x^2+1)^3}dx$$

$$\Big\downarrow \ 27$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\int\frac{x^4\arctan(ax)^{3/2}}{(a^2x^2+1)^3}dx}{8c^3}$$

$$\Big\downarrow \; 5475$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(\frac{3\int\frac{x^2\arctan(ax)^{3/2}}{\left(a^2x^2+1\right)^2}dx}{4a^2}-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}\right)}{8c^3}$$

$$\Big\downarrow \; 5471$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3\left(\frac{3\int\frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}\cdots\right)}{8c^3}$$

$$\Big\downarrow \; 5465$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3\left(\frac{3\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{3x^4\sqrt{\arctan\cdots}}{32a(a^2x^2\cdots}\cdots\right)}{8c^3}$$

$$\Big\downarrow \; 5439$$

3.873. $\int \frac{x^3\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-5a\left(-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3\left(\frac{3\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{3x^4}{32}\cdots\right)}{8c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-5a\left(-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3\left(\frac{3\left(\frac{\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{3x^4\sqrt{\cdots}}{32a\cdots}\right)}{8c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-5a\left(-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3\left(\frac{3\left(\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/\cdots}}{2a^2\left(a^2x^2+1\right)}\cdots\right)}{4a^2}\cdots\right)}{8c^3}$$

$$\Big\downarrow \text{2009}$$

3.873. $\displaystyle\int\frac{x^3\arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\text{arct}}}{4a^2}\right.}{4a}\right.}{4a^2}\right.}{8c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3\int\frac{a^4x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+}{4a^2}\right.}{4a}\right.}{4a^2}\right.}{8c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3\int\frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\text{arctan}}}{4a^2}\right.}{4a}\right.}{4a^2}\right.}{8c^3}$$

$$\Big\downarrow \text{3793}$$

3.873. $\int\frac{x^3\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3\int\left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\cdots\right)}{\cdots}\right)}{8c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\cdots\right)}{8c^3}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a^2*x^2)^2) - (5*a*((3*x^4*Sqrt[ArcTan
[a*x]])/(32*a*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^(3/2))/(4*a^2*(1 + a^2*x
^2)^2) - (3*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*S
qrt[ArcTan[a*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])
/2))/(64*a^5) + (3*(-1/2*(x*ArcTan[a*x]^(3/2))/(a^2*(1 + a^2*x^2)) + ArcTa
n[a*x]^(5/2)/(5*a^3) + (3*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (S
qrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/
(4*a^2)))/(4*a))/(4*a^2)))/(8*c^3)
```

3.873. $\int \frac{x^3\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

3.873.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.873. $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.873.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.61

$$\frac{-1024 \arctan(ax)^{\frac{5}{2}} \cos(2\arctan(ax))\sqrt{\pi} + 256 \arctan(ax)^{\frac{5}{2}} \cos(4\arctan(ax))\sqrt{\pi} + 1280 \arctan(ax)^{\frac{3}{2}} \mathrm{si}}{}$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
1/8192/c^3/a^4*(-1024*arctan(a*x)^(5/2)*cos(2*arctan(a*x))*Pi^(1/2)+256*ar
ctan(a*x)^(5/2)*cos(4*arctan(a*x))*Pi^(1/2)+1280*arctan(a*x)^(3/2)*sin(2*a
rctan(a*x))*Pi^(1/2)-160*arctan(a*x)^(3/2)*Pi^(1/2)*sin(4*arctan(a*x))+15*
Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+960*arctan(a*x)^
(1/2)*Pi^(1/2)*cos(2*arctan(a*x))-60*cos(4*arctan(a*x))*arctan(a*x)^(1/2)*
Pi^(1/2)-480*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

3.873.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.873.6 Sympy [F]

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.873.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.873.8 Giac [F]

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.873.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.874 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

3.874.1 Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \frac{\arctan(ax)^{7/2}}{28a^3c^3} - \frac{5\arctan(ax)^{3/2}\cos(4\arctan(ax))}{256a^3c^3} - \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^3c^3} + \frac{15\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{2048a^3c^3} - \frac{\arctan(ax)^{5/2}\sin(4\arctan(ax))}{32a^3c^3}$$

output
```
1/28*arctan(a*x)^(7/2)/a^3/c^3-5/256*arctan(a*x)^(3/2)*cos(4*arctan(a*x))/
a^3/c^3-1/32*arctan(a*x)^(5/2)*sin(4*arctan(a*x))/a^3/c^3-15/8192*FresnelS
(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3+15/2048*si
n(4*arctan(a*x))*arctan(a*x)^(1/2)/a^3/c^3
```

3.874.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.39

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \frac{32\arctan(ax)\left(-105ax(-1+a^2x^2) - 70(1-6a^2x^2+a^4x^4)\arctan(ax) + 448ax(\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(32*ArcTan[a*x]*(-105*a*x*(-1 + a^2*x^2) - 70*(1 - 6*a^2*x^2 + a^4*x^4)*Ar
cTan[a*x] + 448*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^2 + 128*(1 + a^2*x^2)^2*Arc
Tan[a*x]^3) + 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)
*ArcTan[a*x]] + 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*A
rcTan[a*x]])/(114688*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

3.874.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2\arctan(ax)^{5/2}}{(a^2x^2+1)^2}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{1}{8}\arctan(ax)^{5/2}-\frac{1}{8}\arctan(ax)^{5/2}\cos(4\arctan(ax))\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{15\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096}+\frac{1}{28}\arctan(ax)^{7/2}-\frac{1}{32}\arctan(ax)^{5/2}\sin(4\arctan(ax))+\frac{15\sqrt{\arctan(ax)}\sin(4\arctan(}{2048}}{a^3c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(7/2)/28 - (5*ArcTan[a*x]^(3/2)*Cos[4*ArcTan[a*x]])/256 - (15
*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4096 + (15*Sqrt[ArcT
an[a*x]]*Sin[4*ArcTan[a*x]])/2048 - (ArcTan[a*x]^(5/2)*Sin[4*ArcTan[a*x]])
/32)/(a^3*c^3)
```

3.874.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.874.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$-\frac{-2048\arctan(ax)^4+1792\arctan(ax)^3\sin(4\arctan(ax))+1120\arctan(ax)^2\cos(4\arctan(ax))+105\ \mathrm{F}}{57344c^3a^3\sqrt{\arctan(ax}}$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
-1/57344/c^3/a^3/arctan(a*x)^(1/2)*(-2048*arctan(a*x)^4+1792*arctan(a*x)^3
*sin(4*arctan(a*x))+1120*arctan(a*x)^2*cos(4*arctan(a*x))+105*FresnelS(2*2
^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)-420*
sin(4*arctan(a*x))*arctan(a*x))
```

3.874.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.874.6 Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

3.874.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.874.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.874.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.875 $\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$

3.875.1 Optimal result

Integrand size = 22, antiderivative size = 254

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = -\frac{225\sqrt{\arctan(ax)}}{2048a^2c^3} + \frac{15\sqrt{\arctan(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\arctan(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\arctan(ax)^{3/2}}{64ac^3(1+a^2x^2)} + \frac{3\arctan(ax)^{5/2}}{32a^2c^3} - \frac{\arctan(ax)^{5/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^2c^3} - \frac{15\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{256a^2c^3}$$

output
```
5/32*x*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2+15/64*x*arctan(a*x)^(3/2)/a/c
^3/(a^2*x^2+1)+3/32*arctan(a*x)^(5/2)/a^2/c^3-1/4*arctan(a*x)^(5/2)/a^2/c^
3/(a^2*x^2+1)^2-15/8192*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(
1/2)*Pi^(1/2)/a^2/c^3-15/256*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/
2)/a^2/c^3-225/2048*arctan(a*x)^(1/2)/a^2/c^3+15/256*arctan(a*x)^(1/2)/a^2
/c^3/(a^2*x^2+1)^2+45/256*arctan(a*x)^(1/2)/a^2/c^3/(a^2*x^2+1)
```

3.875.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.41

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{450\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + \frac{16320\arctan(ax)-5760a^2x^2\arctan(ax)-14400a^4x^4\operatorname{arc}}{}}{}$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(450*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (16320*ArcTan[a
*x] - 5760*a^2*x^2*ArcTan[a*x] - 14400*a^4*x^4*ArcTan[a*x] + 51200*a*x*Arc
Tan[a*x]^2 + 30720*a^3*x^3*ArcTan[a*x]^2 - 20480*ArcTan[a*x]^3 + 24576*a^2
*x^2*ArcTan[a*x]^3 + 12288*a^4*x^4*ArcTan[a*x]^3 - 3600*Sqrt[Pi]*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (1020*
I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan
[a*x]] - (1020*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (
2*I)*ArcTan[a*x]] + (345*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-4*I)*ArcTan[a*x]] - (345*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamm
a[1/2, (4*I)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^
2*c^3)
```

3.875.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5465, 27, 5435, 5427, 5439, 3042, 3793, 2009, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \ 5465$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{c^3(a^2x^2+1)^3}dx}{8a} - \frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2x^2+1)^3}dx}{8ac^3}-\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5435}$$

$$\frac{5\left(-\frac{3}{64}\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3}{4}\int \frac{\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}\right)}{8ac^3}-\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{5\left(-\frac{3}{64}\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\arc}}{32a(a^2}}{8ac^3}$$
$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{5\left(-\frac{3\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64a}+\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\right.}{8ac^3}$$
$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)-\frac{3\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64a}+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\ar}}{32a(a}\right.}{8ac^3}$$
$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)-\frac{3\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(a}{64a}\right.}{8ac^3}$$
$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

3.875. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}dx$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\cdots}\right.\right.}{\cdots}\cdots\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}\cdots\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 5439$$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\text{arct}\cdots}}{32a(a^2\cdots}\cdots\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\text{arcta}\cdots}}{32a(a^2x\cdots}\cdots\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3\cdots}}{4(a^2x^2+1)^2}\cdots\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \; 2009$$

3.875. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^(5/2)/(a^2*c^3*(1 + a^2*x^2)^2) + (5*((3*Sqrt[ArcTan[a*x]
])/(32*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^(3/2))/(4*(1 + a^2*x^2)^2) - (3
*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[
a*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/(64*a)
 + (3*((x*ArcTan[a*x]^(3/2))/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a) -
 (3*a*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (
Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/4))/4))/(8
*a*c^3)
```

3.875.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.875. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.875.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.62

$$-\frac{1024 \arctan (ax)^{\frac{5}{2}} \cos (2 \arctan (ax)) \sqrt{\pi}+256 \arctan (ax)^{\frac{5}{2}} \cos (4 \arctan (ax)) \sqrt{\pi}-1280 \arctan (ax)^{\frac{3}{2}} \mathrm{si}}{}$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
-1/8192/c^3/a^2/Pi^(1/2)*(1024*arctan(a*x)^(5/2)*cos(2*arctan(a*x))*Pi^(1/
2)+256*arctan(a*x)^(5/2)*cos(4*arctan(a*x))*Pi^(1/2)-1280*arctan(a*x)^(3/2
)*sin(2*arctan(a*x))*Pi^(1/2)+15*Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*ar
ctan(a*x)^(1/2))-160*arctan(a*x)^(3/2)*Pi^(1/2)*sin(4*arctan(a*x))-960*arc
tan(a*x)^(1/2)*Pi^(1/2)*cos(2*arctan(a*x))+480*Pi*FresnelC(2*arctan(a*x)^(
1/2)/Pi^(1/2))-60*cos(4*arctan(a*x))*arctan(a*x)^(1/2)*Pi^(1/2))
```

3.875.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.875.6 Sympy [F]

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x
)/c**3
```

3.875.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.875.8 Giac [F]

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

3.876 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

3.876.1 Optimal result

Integrand size = 21, antiderivative size = 296

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = -\frac{45x\sqrt{\arctan(ax)}}{128c^3(1+a^2x^2)} - \frac{75\arctan(ax)^{3/2}}{256ac^3} + \frac{5\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)} + \frac{x\arctan(ax)^{5/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)^{5/2}}{8c^3(1+a^2x^2)} + \frac{3\arctan(ax)^{7/2}}{28ac^3} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096ac^3} + \frac{15\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128ac^3} - \frac{15\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{256ac^3} - \frac{15\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{2048ac^3}$$

output
```
-75/256*arctan(a*x)^(3/2)/a/c^3+5/32*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2
+15/32*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)+1/4*x*arctan(a*x)^(5/2)/c^3/(a^
2*x^2+1)^2+3/8*x*arctan(a*x)^(5/2)/c^3/(a^2*x^2+1)+3/28*arctan(a*x)^(7/2)/
a/c^3+15/8192*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1
/2)/a/c^3+15/128*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^3-45/
128*x*arctan(a*x)^(1/2)/c^3/(a^2*x^2+1)-15/256*sin(2*arctan(a*x))*arctan(a
*x)^(1/2)/a/c^3-15/2048*sin(4*arctan(a*x))*arctan(a*x)^(1/2)/a/c^3
```

3.876.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.97

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx =$$
$$-\frac{57120ax\arctan(ax)+50400a^3x^3\arctan(ax)-38080\arctan(ax)^2+13440a^2x^2\arctan(ax)^2+33600a^4x^4}{}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```

output
```
-1/114688*(57120*a*x*ArcTan[a*x] + 50400*a^3*x^3*ArcTan[a*x] - 38080*ArcTa
n[a*x]^2 + 13440*a^2*x^2*ArcTan[a*x]^2 + 33600*a^4*x^4*ArcTan[a*x]^2 - 716
80*a*x*ArcTan[a*x]^3 - 43008*a^3*x^3*ArcTan[a*x]^3 - 12288*(1 + a^2*x^2)^2
*ArcTan[a*x]^4 + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma
[1/2, (-2*I)*ArcTan[a*x]] + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x
]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*
x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x
]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]
)
```

3.876.3 Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5435, 27, 5427, 5439, 3042, 3793, 2009, 5465, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^3}dx$$
$$\Big\downarrow \text{5435}$$
$$-\frac{15}{64}\int \frac{\sqrt{\arctan(ax)}}{c^3(a^2x^2+1)^3}dx+\frac{3\int \frac{\arctan(ax)^{5/2}}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\arctan(ax)^{5/2}}{4c^3(a^2x^2+1)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}$$
$$\Big\downarrow \text{27}$$

$$-\frac{15\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\int \frac{\arctan(ax)^{5/2}}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{15\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{15\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{64ac^3}+\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}-\frac{15\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}-\frac{15\int \left(\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64ac^3}$$

$$\Big\downarrow \text{5465}$$

3.876. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}dx$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin}{64ac^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin}{64ac^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin}{64ac^3}$$

$$\Big\downarrow \text{4906}$$

3.876. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int\frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64ac^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int\frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64ac^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int\frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64ac^3}$$

$$\Big\downarrow \text{3786}$$

3.876. $\int\frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64ac^3}$$

$$\Big\downarrow \ 3832$$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}-\frac{\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{64ac^3}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```

output
```
(5*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^(5/2))/(
4*c^3*(1 + a^2*x^2)^2) + (3*((x*ArcTan[a*x]^(5/2))/(2*(1 + a^2*x^2)) + Arc
Tan[a*x]^(7/2)/(7*a) - (5*a*(-1/2*ArcTan[a*x]^(3/2)/(a^2*(1 + a^2*x^2)) +
(3*((x*Sqrt[ArcTan[a*x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (S
qrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a)))/4))/(4*c
^3) - (15*(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[Ar
cTan[a*x]]])/64 - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 +
(Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan
[a*x]])/32))/(64*a*c^3)
```

3.876. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

3.876.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.876. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.876.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.57

$$\frac{6144 \arctan(ax)^{\frac{7}{2}} \sqrt{\pi} + 14336 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi} \sin(2\arctan(ax)) + 1792 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi} \sin(4\arctan(a}{}$$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
1/57344/c^3/a/Pi^(1/2)*(6144*arctan(a*x)^(7/2)*Pi^(1/2)+14336*arctan(a*x)^
(5/2)*Pi^(1/2)*sin(2*arctan(a*x))+1792*arctan(a*x)^(5/2)*Pi^(1/2)*sin(4*ar
ctan(a*x))+105*Pi*2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+1
7920*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))+1120*arctan(a*x)^(3/2)*
Pi^(1/2)*cos(4*arctan(a*x))+6720*Pi*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))
-13440*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))-420*arctan(a*x)^(1/2)
*Pi^(1/2)*sin(4*arctan(a*x)))
```

3.876.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.876.6 Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

3.876.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.876.8 Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^3, x)
```

3.877 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3} dx$

3.877.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3} dx = \text{Int}\left(\frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3}, x\right)$$

output
```
Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

3.877.2 Mathematica [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3} dx = \int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3} dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3), x]
```

3.877.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{5/2}}{x\,(a^2cx^2+c)^3}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

3.877.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.877.4 Maple [N/A] (verified)

Not integrable

Time = 3.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^{2}c\,x^{2}+c\right)^{3}}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

3.877.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.877.6 Sympy [N/A]

Not integrable

Time = 10.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/
c**3
```

3.877.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.877.8 Giac [N/A]

Not integrable

Time = 125.50 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.877.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\mathrm{atan}\left(a\,x\right)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^3), x)
```

3.878 $\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$

3.878.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.878.2 Mathematica [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

3.878.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.878.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.878.4 Maple [N/A] (verified)

Not integrable

Time = 11.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \arctan(ax)^{\frac{5}{2}}\sqrt{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.878.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{5/2}\, dx = \int \sqrt{a^2cx^2+c}x^m \arctan{(ax)}^{\frac{5}{2}}\, dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

3.878.6 Sympy [F(-1)]

Timed out.

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{5/2}\, dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.878.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{5/2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.878.8 Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.878.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\int x^m\,\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.879 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$

3.879.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \operatorname{Int}\left(x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.879.2 Mathematica [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

3.879.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^2 \arctan(ax)^{5/2} \sqrt{a^2cx^2+c}dx$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.879.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.879.4 Maple [N/A] (verified)

Not integrable

Time = 5.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2 \arctan(ax)^{\frac{5}{2}} \sqrt{a^2c\,x^2+c}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.879.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.879.6 Sympy [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.879.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.879.8 Giac [N/A]

Not integrable

Time = 123.40 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int \sqrt{a^2cx^2+c}x^2\arctan(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.879.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int x^2\,\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.880 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$

3.880.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \frac{5\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{8a^2} - \frac{5x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{12a} + \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{3a^2c} - \frac{5c\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{16a} - \frac{5c\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{12a}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/a^2/c-5/12*x*arctan(a*x)^(3/2)*(
a^2*c*x^2+c)^(1/2)/a+5/8*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2-5/12*c*
Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a-5/16*c*Unintegrabl
e(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

3.880.2 Mathematica [N/A]

Not integrable

Time = 4.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

3.880.3 Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \; 5465$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{5\int \sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx}{6a}$$

$$\Big\downarrow \; 5415$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{5\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}\right)}{6a}$$

$$\Big\downarrow \; 5560$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{5\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}\right)}{6a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.880.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.880.4 Maple [N/A] (verified)

Not integrable

Time = 3.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x \arctan(ax)^{\frac{5}{2}} \sqrt{a^2 c\, x^2 + c}dx$$

input
```
int(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.880.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.880.6 Sympy [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x*atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.880.7 Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.880.8 Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.880.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.881 $\int \sqrt{c+a^2cx^2} \arctan(ax)^{5/2} \, dx$

3.881.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sqrt{c+a^2cx^2} \arctan(ax)^{5/2} \, dx = -\frac{5\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{4a} + \frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2} + \frac{15}{8}c\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right) + \frac{1}{2}c\text{Int}\left(\frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
-5/4*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+1/2*x*arctan(a*x)^(5/2)*(a^2*
c*x^2+c)^(1/2)+1/2*c*Unintegrable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
+15/8*c*Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.881.2 Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{c+a^2cx^2} \arctan(ax)^{5/2} \, dx = \int \sqrt{c+a^2cx^2} \arctan(ax)^{5/2} \, dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

3.881.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{4a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{4a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.881.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.881.4 Maple [N/A] (verified)

Not integrable

Time = 3.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \arctan\left(ax\right)^{\frac{5}{2}} \sqrt{a^{2} c\,x^{2}+c}dx$$

input
```
int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x)
```

3.881.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.881.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{c + a^2cx^2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.881.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c + a^2cx^2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.881.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{c + a^2cx^2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.881.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int \mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

3.882 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx$

3.882.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Unintegrable(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

3.882.2 Mathematica [N/A]

Not integrable

Time = 3.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x, x]
```

3.882.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{x}\,dx$$
$$\Big\downarrow \textbf{5560}$$
$$\int \frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{x}dx$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

3.882.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.882.4 Maple [N/A] (verified)

Not integrable

Time = 3.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x \right)^{\frac{5}{2}} \sqrt{a^{2} c \,x^{2}+c}}{x}d x$$

input
```
int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

output
```
int(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x)
```

3.882.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.882.6 Sympy [N/A]

Not integrable

Time = 168.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^{\frac{5}{2}}\,(ax)}{x}\,dx$$

input
```
integrate(atan(a*x)**(5/2)*(a**2*c*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)/x, x)
```

3.882.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.882.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.882.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2))/x, x)
```

3.883 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$

3.883.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

3.883.2 Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx=\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

3.883.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} \, dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.883.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.883.4 Maple [N/A] (verified)

Not integrable

Time = 9.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

3.883.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(5/2), x)
```

3.883.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.883.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.883.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.883.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)^{5/2} \, dx = \int x^m \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.884 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$

3.884.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

3.884.2 Mathematica [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

3.884.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.884.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.884.4 Maple [N/A] (verified)

Not integrable

Time = 4.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

3.884.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.884.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.884.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.884.8 Giac [N/A]

Not integrable

Time = 122.43 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{5/2} \, dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^2 \arctan\left(ax\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.884.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{5/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^{3/2} \, dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.885 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$

3.885.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \frac{9c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{32a^2} + \frac{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}{16a^2} - \frac{3cx\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{16a} - \frac{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{8a} + \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{5a^2c} - \frac{9c^2\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{64a} - \frac{c\mathrm{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)}{32a} - \frac{3c^2\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{16a}$$

output
```
-1/8*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a+1/5*(a^2*c*x^2+c)^(5/2)*arc
tan(a*x)^(5/2)/a^2/c-3/16*c*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+1/16
*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a^2+9/32*c*(a^2*c*x^2+c)^(1/2)*arct
an(a*x)^(1/2)/a^2-3/16*c^2*Unintegrable(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1
/2),x)/a-9/64*c^2*Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/
a-1/32*c*Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

3.885.2 Mathematica [N/A]

Not integrable

Time = 3.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx$$

input

```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output

```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

3.885.3 Rubi [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5415, 5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}dx}{2a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/}}{8a}}{2a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\Big(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\text{arc}}}{}}{2a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$

$$\frac{\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\mathrm{arc}}}{}\right.}{2a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.885.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.885. $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$

3.885.4 Maple [N/A] (verified)

Not integrable

Time = 2.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

3.885.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.885.6 Sympy [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.885.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.885.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.885.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.886 $\int (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}\,dx$

3.886.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}\,dx = -\frac{15c\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{16a} - \frac{5(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{24a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^{5/2} + \frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2} + \frac{45}{32}c^2\mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right) + \frac{5}{16}c\mathrm{Int}\Big(\sqrt{c}$$

output
```
-5/24*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a+1/4*x*(a^2*c*x^2+c)^(3/2)*ar
ctan(a*x)^(5/2)-15/16*c*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+3/8*c*x*ar
ctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)+3/8*c^2*Unintegrable(arctan(a*x)^(5/2)
/(a^2*c*x^2+c)^(1/2),x)+45/32*c^2*Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^
2+c)^(1/2),x)+5/16*c*Unintegrable((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

3.886.2 Mathematica [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}\,dx = \int (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

3.886.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{5/2}dx+\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}-\frac{5\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{24a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2+}}{4a}\right.$$
$$\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}-\frac{5\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{24a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2+}}{4a}\right.$$
$$\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}-\frac{5\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{24a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.886.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.886.4 Maple [N/A] (verified)

Not integrable

Time = 2.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^{\frac{5}{2}} dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

3.886.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.886.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.886.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.886.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.886.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\, dx = \int \mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,d x$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

3.887 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx$

3.887.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

3.887.2 Mathematica [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x, x]
```

3.887.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{x}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

3.887.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.887.4 Maple [N/A] (verified)

Not integrable

Time = 3.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

3.887.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.887.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2)/x,x)
```

output
```
Timed out
```

3.887.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.887.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.887.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2))/x, x)
```

3.888 $\int x^m(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$

3.888.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \operatorname{Int}\left(x^m(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

3.888.2 Mathematica [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \int x^m(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

3.888.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.888.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.888.4 Maple [N/A] (verified)

Not integrable

Time = 12.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

3.888.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}\, dx = \int \left(a^2 c x^2+c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)^{\frac{5}{2}}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arcta
n(a*x)^(5/2), x)
```

3.888.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}\, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.888.7 Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.888.8 Giac [F(-2)]

Exception generated.

$$\int x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.888.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}\, dx = \int x^m \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right)^{5/2} dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.889 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$

3.889.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2},x\right)$$

output
```
Unintegrable(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

3.889.2 Mathematica [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

3.889.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.889.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.889.4 Maple [N/A] (verified)

Not integrable

Time = 5.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

3.889.5 Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.889.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.889.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.889.8 Giac [N/A]

Not integrable

Time = 124.48 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx=\int (a^2cx^2+c)^{\frac{5}{2}}x^2\arctan(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.889.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.890 $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$

3.890.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx &= \frac{75c^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{448a^2}\\ &+\frac{25c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}{672a^2}+\frac{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}{56a^2}\\ &-\frac{25c^2x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{224a}-\frac{25cx(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{336a}\\ &-\frac{5x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{84a}+\frac{(c+a^2cx^2)^{7/2}\arctan(ax)^{5/2}}{7a^2c}\\ &-\frac{75c^3\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{896a}-\frac{25c^2\mathrm{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)}{1344a}\\ &-\frac{c\mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}},x\right)}{112a}-\frac{25c^3\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{224a}\end{aligned}$$

output
```
-25/336*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a-5/84*x*(a^2*c*x^2+c)^(
5/2)*arctan(a*x)^(3/2)/a+1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(5/2)/a^2/c-2
5/224*c^2*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a+25/672*c*(a^2*c*x^2+c)
^(3/2)*arctan(a*x)^(1/2)/a^2+1/56*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2)/a^
2+75/448*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2-25/224*c^3*Unintegr
able(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a-1/112*c*Unintegrable((a^2*
c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-75/896*c^3*Unintegrable(1/(a^2*c*x^2
+c)^(1/2)/arctan(a*x)^(1/2),x)/a-25/1344*c^2*Unintegrable((a^2*c*x^2+c)^(1
/2)/arctan(a*x)^(1/2),x)/a
```

3.890.2 Mathematica [N/A]

Not integrable

Time = 5.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

3.890.3 Rubi [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5415, 5415, 5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{5\int\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}dx}{14a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{5\left(\frac{1}{40}c\int\frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}dx+\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}(a^2c}{20a}\right.}{14a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{5\left(\frac{1}{40}c\int\frac{(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\left(\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+\right.\right.\right.}{14a}$$

$\Big\downarrow$ 5415

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$5\left(\frac{1}{40}c\int\frac{(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\left(\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\,\mathrm{arct}\right.\right.\right.$$

$\Big\downarrow$ 5560

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$5\left(\frac{1}{40}c\int\frac{(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\left(\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\,\mathrm{arct}\right.\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.890.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.890. $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.890.4 Maple [N/A] (verified)

Not integrable

Time = 3.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

3.890.5 Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.890.6 Sympy [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.890.7 Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.890.8 Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.890.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.891 $\int (c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$

3.891.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=-\frac{25c^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{32a}$$
$$-\frac{25c(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{144a}-\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{12a}$$
$$+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}+\frac{5}{24}cx\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}+\frac{1}{6}x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}+\frac{75}{64}c^3$$

output
```
-25/144*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a-1/12*(a^2*c*x^2+c)^(5/2)
*arctan(a*x)^(3/2)/a+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)+1/6*x*
(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)-25/32*c^2*arctan(a*x)^(3/2)*(a^2*c*x
^2+c)^(1/2)/a+5/16*c^2*x*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)+5/16*c^3*Un
integrable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)+1/8*c*Unintegrable((a^
2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)+75/64*c^3*Unintegrable(arctan(a*x)^(
1/2)/(a^2*c*x^2+c)^(1/2),x)+25/96*c^2*Unintegrable((a^2*c*x^2+c)^(1/2)*arc
tan(a*x)^(1/2),x)
```

3.891.2 Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

3.891.3 Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5415, 5415, 5415, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{5/2}dx+\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+\frac{5}{6}c\left(\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{5/2}dx+\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}-\right.$$
$$\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+$$
$$\frac{5}{6}c\left(\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^5\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+$$
$$\frac{5}{6}c\left(\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^5\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.891.3.1 Defintions of rubi rules used

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.891. $\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx$

3.891.4 Maple [N/A] (verified)

Not integrable

Time = 3.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

3.891.5 Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.891.6 Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.891.7 Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.891.8 Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.891.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx = \int \mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

3.892 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx$

3.892.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

3.892.2 Mathematica [N/A]

Not integrable

Time = 3.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x, x]
```

3.892.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{x}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

3.892.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.892.4 Maple [N/A] (verified)

Not integrable

Time = 4.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}{x}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

3.892.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.892.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2)/x,x)
```

output
```
Timed out
```

3.892.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.892.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.892.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2))/x, x)
```

3.893 $\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.893.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}, x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.893.2 Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

3.893.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.893.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.893.4 Maple [N/A] (verified)

Not integrable

Time = 13.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.893.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

3.893.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.893.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.893.8 Giac [N/A]

Not integrable

Time = 53.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.893.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.894 $\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.894.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx &= \frac{5\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{8a^4c} - \frac{5x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{12a^3c} \\ &\quad - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3a^2c} \\ &\quad - \frac{5\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{16a^3} + \frac{25\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{12a^3}\end{aligned}$$

output
```
-5/12*x*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^3/c-2/3*arctan(a*x)^(5/2)*
(a^2*c*x^2+c)^(1/2)/a^4/c+1/3*x^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a^
2/c+5/8*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^4/c+25/12*Unintegrable(arc
tan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a^3-5/16*Unintegrable(1/(a^2*c*x^2+c
)^(1/2)/arctan(a*x)^(1/2),x)/a^3
```

3.894.2 Mathematica [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

3.894.3 Rubi [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5487, 5465, 5487, 5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{5\int \frac{x^2\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{2\int \frac{x\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}-\frac{5\int \frac{x^2\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{6a}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5487}$$

$$-\frac{5\left(-\frac{3\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}\right)}{6a}-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{5\left(-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}\right)}{6a}-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{5\left(-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}\right)}{6a}-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.894.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

3.894. $\int \frac{x^3\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.894.4 Maple [N/A] (verified)

Not integrable

Time = 6.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2 c x^2 + c}} dx$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.894.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c + a^2 c x^2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.894.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.894.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.894.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.894.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.895 $\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.895.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{5\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{4a^3c} + \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{2a^2c} + \frac{15\mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{8a^2} - \frac{\mathrm{Int}\left(\frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)}{2a^2}$$

output
```
-5/4*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/a^3/c+1/2*x*arctan(a*x)^(5/2)*(
a^2*c*x^2+c)^(1/2)/a^2/c-1/2*Unintegrable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^
(1/2),x)/a^2+15/8*Unintegrable(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)/a^
2
```

3.895.2 Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

3.895.3 Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5487, 5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{5\int \frac{x\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{5\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{5\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.895.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.895.4 Maple [N/A] (verified)

Not integrable

Time = 4.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2 c\,x^2+c}}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.895.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.895.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.895.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.895.8 Giac [N/A]

Not integrable

Time = 227.56 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.895.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2 + c}} \, dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.896 $\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$

3.896.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx = \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{a^2c} - \frac{5\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}, x\right)}{2a}$$

output
```
arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/a^2/c-5/2*Unintegrable(arctan(a*x)^(
3/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

3.896.2 Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx = \int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}} dx$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

3.896.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5465, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.896.3.1 Defintions of rubi rules used

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.896.4 Maple [N/A] (verified)

Not integrable

Time = 4.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2 c x^2 + c}} dx$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.896.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c + a^2 cx^2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.896.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c + a^2 cx^2}} dx = \text{Timed out}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.896.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.896.8 Giac [N/A]

Not integrable

Time = 224.85 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.896.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

3.897 $\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$

3.897.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\text{Int}\left(\frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.897.2 Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x]
```

3.897.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

3.897.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.897.4 Maple [N/A] (verified)

Not integrable

Time = 5.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.897.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.897.6 Sympy [N/A]

Not integrable

Time = 120.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

3.897.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.897.8 Giac [N/A]

Not integrable

Time = 213.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.897.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

3.898 $\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$

3.898.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \operatorname{Int}\left(\frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.898.2 Mathematica [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

3.898.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.898.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.898.4 Maple [N/A] (verified)

Not integrable

Time = 5.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(a x \right)^{\frac{5}{2}}}{x \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.898.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.898.6 Sympy [N/A]

Not integrable

Time = 141.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

3.898.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.898.8 Giac [N/A]

Not integrable

Time = 215.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.898.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

3.899 $\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$

3.899.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{cx} + \frac{5}{2}a\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/c/x+5/2*a*Unintegrable(arctan(a*x)^
(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

3.899.2 Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

3.899.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5479, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5479}$$
$$\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}$$
$$\Big\downarrow \text{5560}$$
$$\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.899.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.899.4 Maple [N/A] (verified)

Not integrable

Time = 5.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x\right)^{\frac{5}{2}}}{x^{2} \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

3.899.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.899.6 Sympy [N/A]

Not integrable

Time = 171.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^2\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

3.899.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.899.8 Giac [N/A]

Not integrable

Time = 223.40 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.899.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

3.900 $\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$

3.900.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{5a\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{4cx} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{2cx^2} + \frac{15}{8}a^2\mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right) - \frac{1}{2}a^2\mathrm{Int}\left(\frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-5/4*a*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x-1/2*arctan(a*x)^(5/2)*(a^
2*c*x^2+c)^(1/2)/c/x^2-1/2*a^2*Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2
+c)^(1/2),x)+15/8*a^2*Unintegrable(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2)
,x)
```

3.900.2 Mathematica [N/A]

Not integrable

Time = 8.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

3.900.3 Rubi [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5497, 5479, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{5}{4}a\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5479}$$

$$\frac{5}{4}a\left(\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{5}{4}a\left(\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.900.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.900.4 Maple [N/A] (verified)

Not integrable

Time = 5.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x^3\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

3.900.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.900.6 Sympy [N/A]

Not integrable

Time = 169.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

3.900.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.900.8 Giac [N/A]

Not integrable

Time = 224.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.900.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

3.901 $\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

3.901.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx &= -\frac{5a^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{8cx}-\frac{5a\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{12cx^2}\\&\quad-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3cx}\\&\quad+\frac{5}{16}a^3\mathrm{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)-\frac{25}{12}a^3\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)\end{aligned}$$

output
```
-5/12*a*arctan(a*x)^(3/2)*(a^2*c*x^2+c)^(1/2)/c/x^2-1/3*arctan(a*x)^(5/2)*
(a^2*c*x^2+c)^(1/2)/c/x^3+2/3*a^2*arctan(a*x)^(5/2)*(a^2*c*x^2+c)^(1/2)/c/
x-5/8*a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x-25/12*a^3*Unintegrable
(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)+5/16*a^3*Unintegrable(1/x/(a^2
*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.901.2 Mathematica [N/A]

Not integrable

Time = 17.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx=\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

3.901.3 Rubi [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5497, 5479, 5497, 5479, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{a^2cx^2+c}}dx+\frac{5}{6}a\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{5}{6}a\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{5}{6}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{5}{6}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan}{}\right.$$
$$\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow 5560$$

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{5}{6}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan}{}\right.$$
$$\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

3.901.3.1 Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

3.901. $\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.901.4 Maple [N/A] (verified)

Not integrable

Time = 5.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan (ax)^{\frac{5}{2}}}{x^4\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

3.901.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.901.6 Sympy [N/A]

Not integrable

Time = 177.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^4\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

3.901.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.901.8 Giac [N/A]

Not integrable

Time = 227.33 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.901.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

3.902 $\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.902.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.902.2 Mathematica [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

3.902.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}} dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.902.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.902.4 Maple [N/A] (verified)

Not integrable

Time = 6.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(a x\right)^{\frac{5}{2}}}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.902.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^
2*x^2 + c^2), x)
```

3.902.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.902.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.902.8 Giac [N/A]

Not integrable

Time = 52.53 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.902.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.903 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.903.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.903.2 Mathematica [N/A]

Not integrable

Time = 4.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

3.903.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.903.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.903.4 Maple [N/A] (verified)

Not integrable

Time = 5.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.903.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.903.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.903.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.903.8 Giac [N/A]

Not integrable

Time = 122.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.903.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.904 $\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.904.1 Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{15\sqrt{\arctan(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\arctan(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^2c\sqrt{c+a^2cx^2}}$$

output
```
5/2*x*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^(5/2)/a^2/c/(a
^2*c*x^2+c)^(1/2)-15/8*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2
)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)+15/4*arctan(a*x)^(1
/2)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

3.904.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{4\arctan(ax)\left(15+10ax\arctan(ax)-4\arctan(ax)^2\right)+15i\sqrt{1+a^2x^2}\sqrt{-i\arctan(}}{16a^2c\sqrt{c+a^2cx^2}\sqrt{a}}$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(4*ArcTan[a*x]*(15 + 10*a*x*ArcTan[a*x] - 4*ArcTan[a*x]^2) + (15*I)*Sqrt[1
 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (15*I)*S
qrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(16*a^2*c*
Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.904.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5465, 5433, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{5\left(-\frac{3}{4}\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^(5/2)/(a^2*c*Sqrt[c + a^2*c*x^2])) + (5*((3*Sqrt[ArcTan[a*x]
])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x
^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x
]]])/(2*a*c*Sqrt[c + a^2*c*x^2])))/(2*a)
```

3.904.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.904. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.904.4 Maple [F]

$$\int \frac{x \arctan (ax)^{\frac{5}{2}}}{(a^2 c\,x^2 + c)^{\frac{3}{2}}} dx$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.904.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.904.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.904.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.904.8 Giac [F]

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.904.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

3.905 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$

3.905.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{15x\sqrt{\arctan(ax)}}{4c\sqrt{c+a^2cx^2}} + \frac{5\arctan(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^{5/2}}{c\sqrt{c+a^2cx^2}} + \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4ac\sqrt{c+a^2cx^2}}$$

output
```
5/2*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^(5/2)/c/(a^2*c
*x^2+c)^(1/2)+15/8*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi
^(1/2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-15/4*x*arctan(a*x)^(1/2)/
c/(a^2*c*x^2+c)^(1/2)
```

3.905.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\sqrt{\arctan(ax)}(-15ax+10\arctan(ax)+4ax\arctan(ax)^2)+15\sqrt{2\pi}\sqrt{1+a^2x^2}\,\mathrm{Fresn}}{8ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[ArcTan[a*x]]*(-15*a*x + 10*ArcTan[a*x] + 4*a*x*ArcTan[a*x]^2) + 15
*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a
*c*Sqrt[c + a^2*c*x^2])
```

3.905.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5433, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5433}$$

$$-\frac{15}{4}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{15\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(5*ArcTan[a*x]^(3/2))/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/
(c*Sqrt[c + a^2*c*x^2]) - (15*Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/S
qrt[1 + a^2*x^2] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(4*
a*c*Sqrt[c + a^2*c*x^2])
```

3.905.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.905.4 Maple [F]

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}d x$$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

3.905.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.905.6 Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

3.905.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.905.8 Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.905.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

3.906 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{3/2}}\,dx$

3.906.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

3.906.2 Mathematica [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

3.906.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.906.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.906.4 Maple [N/A] (verified)

Not integrable

Time = 4.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

3.906.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.906.6 Sympy [N/A]

Not integrable

Time = 164.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x\,(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

3.906.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.906.8 Giac [N/A]

Not integrable

Time = 116.35 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.906.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

3.907 $\int \frac{x^m \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx$

3.907.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.907.2 Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx = \int \frac{x^m \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

3.907.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.907.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.907.4 Maple [N/A] (verified)

Not integrable

Time = 11.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.907.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

3.907.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.907.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.907.8 Giac [N/A]

Not integrable

Time = 55.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.907.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.908 $\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.908.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.908.2 Mathematica [N/A]

Not integrable

Time = 7.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

3.908.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^4 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}} dx$$

input
```
Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.908.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.908.4 Maple [N/A] (verified)

Not integrable

Time = 15.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{4} \arctan \left(a x \right)^{\frac{5}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.908.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.908.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.908.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.908.8 Giac [N/A]

Not integrable

Time = 134.80 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.908.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.909 $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.909.1 Optimal result

Integrand size = 26, antiderivative size = 350

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{45\sqrt{\arctan(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}}\\ &+\frac{5x\arctan(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2\arctan(ax)^{5/2}}{3a^4c^2\sqrt{c+a^2cx^2}}\\ &-\frac{5\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\cos(3\arctan(ax))}{144a^4c^2\sqrt{c+a^2cx^2}}\\ &-\frac{45\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16a^4c^2\sqrt{c+a^2cx^2}}\\ &+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
5/18*x^3*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)-1/3*x^2*arctan(a*x)^(5/
2)/a^2/c/(a^2*c*x^2+c)^(3/2)+5/3*x*arctan(a*x)^(3/2)/a^3/c^2/(a^2*c*x^2+c)
^(1/2)-2/3*arctan(a*x)^(5/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+5/864*FresnelC(6^
(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c
^2/(a^2*c*x^2+c)^(1/2)-45/32*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*
2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+45/16*arcta
n(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-5/144*cos(3*arctan(a*x))*(a^2*x^2
+1)^(1/2)*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

3.909.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.06

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{4800 \arctan(ax) + 5040a^2x^2 \arctan(ax) + 2880ax \arctan(ax)^2 + 3360a^3x^3 \arctan(}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(4800*ArcTan[a*x] + 5040*a^2*x^2*ArcTan[a*x] + 2880*a*x*ArcTan[a*x]^2 + 33
60*a^3*x^3*ArcTan[a*x]^2 - 1152*ArcTan[a*x]^3 - 1728*a^2*x^2*ArcTan[a*x]^3
 + (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*Arc
Tan[a*x]] - (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*
ArcTan[a*x]] - (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
 (-3*I)*ArcTan[a*x]] - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[
a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] + (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(1728*a^4*c^2*(1 + a
^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.909.3 Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5475, 5465, 5433, 5440, 5439, 3042, 3785, 3833, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{2\int \frac{x \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2 \arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{5x^3 \arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{5\int \frac{\arctan(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(\frac{5\left(-\frac{3}{4}\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.909. $\int \frac{x^3\arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int\frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3785$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int\frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int\frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3833$$

$$-\frac{5}{12}\int\frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5506$$

$$-\frac{5\sqrt{a^2x^2+1}\int\frac{x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5505$$

3.909. $\int \frac{x^3\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$-\frac{5\sqrt{a^2x^2+1}\int \frac{a^3x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^3d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{5\sqrt{a^2x^2+1}\int \left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(3\arctan(ax))\right)d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

3.909. $\int \frac{x^3\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-$$

$$\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}-$$

$$\frac{5\sqrt{a^2x^2+1}\left(-\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}+\frac{3}{4}\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\,\mathrm{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\mathrm{ar}}\right.}{12a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(5*x^3*ArcTan[a*x]^(3/2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x
]^(5/2))/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^(5/2)/(a^2*c*
Sqrt[c + a^2*c*x^2])) + (5*((3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^
2]) + (x*ArcTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1
 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*
x^2])))/(2*a)))/(3*a^2*c) - (5*Sqrt[1 + a^2*x^2]*((-3*Sqrt[ArcTan[a*x]])/(
4*Sqrt[1 + a^2*x^2]) + (Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/12 + (3*Sqrt
[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelC[Sq
rt[6/Pi]*Sqrt[ArcTan[a*x]]])/12))/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

3.909.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

3.909. $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.909.4 Maple [F]

$$\int \frac{x^3 \arctan (ax)^{\frac{5}{2}}}{(a^2 c\,x^2 + c)^{\frac{5}{2}}}dx$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.909.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.909.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.909.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.909.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.909.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.910 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}} \, dx$

3.910.1 Optimal result

Integrand size = 26, antiderivative size = 295

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}} \, dx = &-\frac{5x^3\sqrt{\arctan(ax)}}{36c\,(c+a^2cx^2)^{3/2}} - \frac{5x\sqrt{\arctan(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{5x^2\arctan(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\arctan(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} \\ &+ \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16a^3c^2\sqrt{c+a^2cx^2}} \\ &- \frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
5/18*x^2*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x^3*arctan(a*x)^(5/
2)/c/(a^2*c*x^2+c)^(3/2)+5/9*arctan(a*x)^(3/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)
-5/864*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*
x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+15/32*FresnelS(2^(1/2)/Pi^(1/2)*a
rctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)
^(1/2)-5/36*x^3*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)-5/6*x*arctan(a*x)^
(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

3.910.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.97

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{-24\arctan(ax)\left(5ax(6+7a^2x^2)-10(2+3a^2x^2)\arctan(ax)-12a^3x^3\arctan(ax)^2\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-24*ArcTan[a*x]*(5*a*x*(6 + 7*a^2*x^2) - 10*(2 + 3*a^2*x^2)*ArcTan[a*x] -
 12*a^3*x^3*ArcTan[a*x]^2) + 35*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan
[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6
/Pi]*Sqrt[ArcTan[a*x]]]) - 15*(1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]
]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan
[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] +
Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(864*a^3*c*(c + a^2*c
*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

3.910.3 Rubi [A] (verified)

Time = 2.71 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5479, 5475, 5465, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{5}{6}a\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\int\frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \ 5465$$

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\int\frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{a}}{6ac\,(a^2}\right.$$

$$\Big\downarrow \ 5440$$

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int\frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^{3/2}}dx}{2ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\right.$$

$$\Big\downarrow \ 5439$$

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int\frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right.}\right.$$

$$\Big\downarrow \ 3042$$

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int\sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\right.$$

3.910. $\int\frac{x^2\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\Big\downarrow \text{3777}$$

$$\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(\right.$$
$$\frac{x^3\arctan(ax)^{5/2}}{3c(a^2cx^2+c)^{3/2}}-$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(a\right.$$
$$\frac{x^3\arctan(ax)^{5/2}}{3c(a^2cx^2+c)^{3/2}}-$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx\right.$$
$$\frac{x^3\arctan(ax)^{5/2}}{3c(a^2cx^2+c)^{3/2}}-$$

$$\Big\downarrow \text{3786}$$

$$\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\right.$$
$$\frac{x^3\arctan(ax)^{5/2}}{3c(a^2cx^2+c)^{3/2}}-$$

$$\Big\downarrow \text{3832}$$

3.910. $\int \frac{x^2\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{5}{6}a\left(-\frac{1}{12}\int \frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/}}{a^2c\sqrt{a^2cx^2+}}\right.}{3a^2c}}{}\right.$$

$$\Big\downarrow \; 5506$$

$$\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int \frac{x^3}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/}}{a^2c\sqrt{a^2cx^2+}}\right.}{3a^2c}}{}\right.$$

$$\Big\downarrow \; 5505$$

$$\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\right.}{3a^2c}}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(a}{a^2c\sqrt{a^2c}}\right.}{3a^2c}}{}\right.$$

$$\Big\downarrow \; 3793$$

3.910. $\int \frac{x^2\arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-$$
$$\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int\left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{}\right.\right.}{2a^2c\sqrt{a^2cx^2+c}}}{3a^2c}\right.$$

$\downarrow$ 2009

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-$$
$$\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan}}{6ac\left(a^2cx^2+\right.}\right.$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) - (5*a*((x^3*Sqrt[ArcT
an[a*x]])/(6*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^(3/2))/(3*a^2*c
*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^(3/2)/(a^2*c*Sqrt[c + a^2*c*x^
2])) + (3*Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^2*c*Sqrt[c + a^2*c
*x^2])))/(3*a^2*c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]
*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/2))/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])))/6
```

3.910.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.910. $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.910. $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}} \, dx$

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.910.4 Maple [F]

$$\int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.910.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.910.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.910.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.910.8 Giac [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.910.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.911 $\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.911.1 Optimal result

Integrand size = 24, antiderivative size = 293

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{5\sqrt{\arctan(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\arctan(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} + \frac{5x\arctan(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\arctan(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16a^2c^2\sqrt{c+a^2cx^2}} - \frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
5/18*x*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)-1/3*arctan(a*x)^(5/2)/a^2
/c/(a^2*c*x^2+c)^(3/2)+5/9*x*arctan(a*x)^(3/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-5
/864*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-15/32*FresnelC(2^(1/2)/Pi^(1/2)*arc
tan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(
1/2)+5/36*arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(3/2)+5/6*arctan(a*x)^(1/2
)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

3.911.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.22

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1680\arctan(ax) + 1440a^2x^2\arctan(ax) + 1440ax\arctan(ax)^2 + 960a^3x^3\arctan(ax}{}$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(1680*ArcTan[a*x] + 1440*a^2*x^2*ArcTan[a*x] + 1440*a*x*ArcTan[a*x]^2 + 96
0*a^3*x^3*ArcTan[a*x]^2 - 576*ArcTan[a*x]^3 + (405*I)*(1 + a^2*x^2)^(3/2)*
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (405*I)*(1 + a^2*x^2
)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a
^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*a^2*
x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*
x]] - (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTa
n[a*x]] - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
 (3*I)*ArcTan[a*x]])/(1728*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[
ArcTan[a*x]])
```

3.911.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5465, 5435, 5433, 5440, 5439, 3042, 3785, 3793, 2009, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \; 5465$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx}{6a} - \frac{\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5435$$

$$\frac{5\left(-\frac{1}{12}\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\arctan(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^{3/2}}{3c(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac(a^2cx^2+c)^{3/2}}\right)}{6a}-\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{5\left(-\frac{1}{12}\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\left(-\frac{3}{4}\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c(a^2cx^2+c)^{3/2}}+\frac{}{6a}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax}{3c(a^2cx^2+c}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.911. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arc}{3c(a}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax}{3c(a^2cx^2+c}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan}}{2ac\sqrt{a^2c}}\right.}{3c}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\right.}{12ac^2\sqrt{a^2cx^2+c}}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3833}$$

3.911. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+x\right.}{3c}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^(5/2)/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (5*(Sqrt[ArcTan[a*x
]]/(6*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^(3/2))/(3*c*(c + a^2*c*x
^2)^(3/2)) + (2*((3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*Ar
cTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]
*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])))/(3*
c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x
]]])/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(12*a*c^2
*Sqrt[c + a^2*c*x^2])))/(6*a)
```

3.911.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.911. $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

3.911. $\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.911.4 Maple [F]

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2 c\, x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.911.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c + a^2 cx^2)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.911.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{(c + a^2 cx^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.911.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.911.8 Giac [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.911.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

3.912 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

3.912.1 Optimal result

Integrand size = 23, antiderivative size = 337

$$\begin{aligned}\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx =& -\frac{45x\sqrt{\arctan(ax)}}{16c^2\sqrt{c+a^2cx^2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}}\\&+\frac{5\arctan(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\arctan(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}}\\&+\frac{45\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16ac^2\sqrt{c+a^2cx^2}}\\&+\frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144ac^2\sqrt{c+a^2cx^2}}\\&-\frac{5\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\sin(3\arctan(ax))}{144ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
5/18*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)+1/3*x*arctan(a*x)^(5/2)/c/(
a^2*c*x^2+c)^(3/2)+5/3*arctan(a*x)^(3/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+2/3*x*a
rctan(a*x)^(5/2)/c^2/(a^2*c*x^2+c)^(1/2)+5/864*FresnelS(6^(1/2)/Pi^(1/2)*a
rctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(
1/2)+45/32*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(
a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-45/16*x*arctan(a*x)^(1/2)/c^2/(
a^2*c*x^2+c)^(1/2)-5/144*sin(3*arctan(a*x))*(a^2*x^2+1)^(1/2)*arctan(a*x)^
(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.912.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.52

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{24\sqrt{\arctan(ax)}(-5ax(21+20a^2x^2)+10(7+6a^2x^2)\arctan(ax)+12ax(3+2a^2x^2)}{}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(24*Sqrt[ArcTan[a*x]]*(-5*a*x*(21 + 20*a^2*x^2) + 10*(7 + 6*a^2*x^2)*ArcTa
n[a*x] + 12*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2) + 1215*Sqrt[2*Pi]*(1 + a^2*
x^2)^(3/2)*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 5*Sqrt[6*Pi]*(1 + a^2*
x^2)^(3/2)*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(864*c^2*(a + a^3*x^2)*
Sqrt[c + a^2*c*x^2])
```

3.912.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5435, 5433, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3793, 2009, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{5}{12}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx+\frac{2\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$-\frac{5}{12}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx+\frac{2\left(-\frac{15}{4}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+$$
$$\frac{x\arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^{3/2}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \textbf{5439}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \textbf{3777}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \textbf{25}$$

3.912. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}dx$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \left(\frac{1}{4}\sqrt{\arctan(ax)}\cos(3\arctan(ax))+\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}\right)d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

3.912. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

$$\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}-$$
$$\frac{5\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{arc}}\right.}{12ac^2\sqrt{a^2cx^2+c}}$$
$$\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 3832}$$

$$-\frac{5\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{a}}\right.}{12ac^2\sqrt{a^2cx^2+c}}$$
$$\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+$$
$$\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(5*ArcTan[a*x]^(3/2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^(5/2
))/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*((5*ArcTan[a*x]^(3/2))/(2*a*c*Sqrt[c +
 a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]) - (15*Sqrt[1
+ a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - Sqrt[Pi/2]*Fresnel
S[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(4*a*c*Sqrt[c + a^2*c*x^2])))/(3*c) - (5
*Sqrt[1 + a^2*x^2]*((3*a*x*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) - (3*S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelS
[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/12 + (Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]]
)/12))/(12*a*c^2*Sqrt[c + a^2*c*x^2])
```

3.912.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.912. $\int\frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

3.912. $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.912.4 Maple [F]

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2 c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

3.912.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.912.6 Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{\left(c\,(a^2x^2+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

3.912.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.912.8 Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.912.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2), x)
```

3.913 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{5/2}}\,dx$

3.913.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Unintegrable(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

3.913.2 Mathematica [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

3.913.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.913.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.913.4 Maple [N/A] (verified)

Not integrable

Time = 3.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x \right)^{\frac{5}{2}}}{x \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

3.913.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.913.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.913.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.913.8 Giac [N/A]

Not integrable

Time = 129.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.913.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

3.914 $\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx$

3.914.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.914.2 Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

3.914.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.914.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.914.4 Maple [N/A] (verified)

Not integrable

Time = 4.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m (a^2c\,x^2+c)}{\sqrt{\arctan(ax)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.914.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

3.914.6 Sympy [N/A]

Not integrable

Time = 13.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^2x^2x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(a**2*x**2*x**m/sqrt(atan(a
*x)), x))
```

3.914.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.914.8 Giac [N/A]

Not integrable

Time = 64.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.914.9 Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(1/2), x)
```

3.915 $\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx$

3.915.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.915.2 Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

3.915.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\sqrt{\arctan(ax)}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x(a^2cx^2+c)}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.915.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.915.4 Maple [N/A] (verified)

Not integrable

Time = 1.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(a^2c\,x^2+c)}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.915.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.915.6 Sympy [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x/sqrt(atan(a*x)), x) + Integral(a**2*x**3/sqrt(atan(a*x)), x)
)
```

3.915.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.915.8 Giac [N/A]

Not integrable

Time = 83.68 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)x}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.915.9 Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(1/2), x)
```

3.916 $\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx$

3.916.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.916.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x]
```

3.916.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{a^2cx^2+c}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.916.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.916.4 Maple [N/A] (verified)

Not integrable

Time = 1.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a^2c\,x^2+c}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.916.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.916.6 Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{c + a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(a**2*x**2/sqrt(atan(a*x)), x) + Integral(1/sqrt(atan(a*x)), x)
)
```

3.916.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.916.8 Giac [N/A]

Not integrable

Time = 75.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{a^2cx^2+c}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.916.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{c\,a^2\,x^2+c}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^(1/2), x)
```

3.917 $\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx$

3.917.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

3.917.2 Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]), x]
```

3.917.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a^2cx^2+c}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.917.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.917.4 Maple [N/A] (verified)

Not integrable

Time = 1.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2c\,x^2+c}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

3.917.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.917.6 Sympy [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(a**2*x/sqrt(atan(a*x)), x
))
```

3.917.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.917.8 Giac [N/A]

Not integrable

Time = 78.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{a^2cx^2+c}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.917.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(1/2)), x)
```

3.918 $\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$

3.918.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.918.2 Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

3.918.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m (a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.918.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.918.4 Maple [N/A] (verified)

Not integrable

Time = 5.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2c\,x^2+c)^2}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.918.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/sqrt(arctan(a*x)), x)
```

3.918.6 Sympy [N/A]

Not integrable

Time = 51.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{2a^2x^2x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^4x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(2*a**2*x**2*x**m/sqrt(a
tan(a*x)), x) + Integral(a**4*x**4*x**m/sqrt(atan(a*x)), x))
```

3.918.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.918.8 Giac [N/A]

Not integrable

Time = 63.97 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.918.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^2}{\sqrt{\mathrm{atan}(a\,x)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2), x)
```

3.919 $\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$

3.919.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.919.2 Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx=\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

3.919.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.919.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.919.4 Maple [N/A] (verified)

Not integrable

Time = 1.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^2}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.919.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.919.6 Sympy [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{2a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x/sqrt(atan(a*x)), x) + Integral(2*a**2*x**3/sqrt(atan(a*x)
), x) + Integral(a**4*x**5/sqrt(atan(a*x)), x))
```

3.919.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.919.8 Giac [N/A]

Not integrable

Time = 91.91 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^2x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.919.9 Mupad [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^2}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2), x)
```

3.920 $\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$

3.920.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.920.2 Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]], x]
```

3.920.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.920.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.920.4 Maple [N/A] (verified)

Not integrable

Time = 1.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.920.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.920.6 Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{2a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^4}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2/sqrt(atan(a*x)), x) + Integral(a**4*x**4/sqrt(a
tan(a*x)), x) + Integral(1/sqrt(atan(a*x)), x))
```

3.920.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.920.8 Giac [N/A]

Not integrable

Time = 85.71 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.920.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^(1/2), x)
```

3.921 $\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx$

3.921.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

3.921.2 Mathematica [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]), x]
```

3.921.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^2}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.921.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.921.4 Maple [N/A] (verified)

Not integrable

Time = 1.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^2}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

3.921.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.921.6 Sympy [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{2a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(2*a**2*x/sqrt(atan(a*x
)), x) + Integral(a**4*x**3/sqrt(atan(a*x)), x))
```

3.921.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.921.8 Giac [N/A]

Not integrable

Time = 88.94 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx=\int \frac{(a^2cx^2+c)^2}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.921.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx=\int \frac{(c\,a^2\,x^2+c)^2}{x\,\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(1/2)), x)
```

3.922 $\int \frac{x^m (c+a^2 c x^2)^3}{\sqrt{\arctan(ax)}} \, dx$

3.922.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2 c x^2)^3}{\sqrt{\arctan(ax)}} \, dx = \text{Int}\left(\frac{x^m (c+a^2 c x^2)^3}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.922.2 Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2 c x^2)^3}{\sqrt{\arctan(ax)}} \, dx = \int \frac{x^m (c+a^2 c x^2)^3}{\sqrt{\arctan(ax)}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

3.922.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\sqrt{\arctan(ax)}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)^3}{\sqrt{\arctan(ax)}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.922.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.922.4 Maple [N/A] (verified)

Not integrable

Time = 5.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2c\,x^2+c)^3}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.922.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/sqrt(arct
an(a*x)), x)
```

3.922.6 Sympy [N/A]

Not integrable

Time = 160.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^2x^2x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^4x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^6x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(3*a**2*x**2*x**m/sqrt(a
tan(a*x)), x) + Integral(3*a**4*x**4*x**m/sqrt(atan(a*x)), x) + Integral(a
**6*x**6*x**m/sqrt(atan(a*x)), x))
```

3.922.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.922.8 Giac [N/A]

Not integrable

Time = 65.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.922.9 Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^3}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2), x)
```

3.923 $\int \frac{x\left(c+a^2cx^2\right)^3}{\sqrt{\arctan(ax)}}\,dx$

3.923.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.923.2 Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

3.923.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.923.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.923.4 Maple [N/A] (verified)

Not integrable

Time = 1.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^3}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.923.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.923.6 Sympy [N/A]

Not integrable

Time = 4.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^7}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(x/sqrt(atan(a*x)), x) + Integral(3*a**2*x**3/sqrt(atan(a*x)
), x) + Integral(3*a**4*x**5/sqrt(atan(a*x)), x) + Integral(a**6*x**7/sqrt
(atan(a*x)), x))
```

3.923.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.923.8 Giac [N/A]

Not integrable

Time = 102.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.923.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2), x)
```

3.924 $\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx$

3.924.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx=\operatorname{Int}\left(\frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.924.2 Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx=\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]], x]
```

3.924.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.924.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.924.4 Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{3}}{\sqrt{\arctan \left(a x \right)}}d x$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.924.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.924.6 Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{3a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^4}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^6}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2/sqrt(atan(a*x)), x) + Integral(3*a**4*x**4/sqrt
(atan(a*x)), x) + Integral(a**6*x**6/sqrt(atan(a*x)), x) + Integral(1/sqrt
(atan(a*x)), x))
```

3.924.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.924.8 Giac [N/A]

Not integrable

Time = 97.83 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.924.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\sqrt{\mathrm{atan}(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^(1/2), x)
```

3.925 $\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx$

3.925.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

3.925.2 Mathematica [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]), x]
```

3.925.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^3}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.925.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.925.4 Maple [N/A] (verified)

Not integrable

Time = 1.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^3}{x\sqrt{\arctan(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

3.925.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.925.6 Sympy [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(3*a**2*x/sqrt(atan(a*x
)), x) + Integral(3*a**4*x**3/sqrt(atan(a*x)), x) + Integral(a**6*x**5/sqr
t(atan(a*x)), x))
```

3.925.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.925.8 Giac [N/A]

Not integrable

Time = 97.80 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.925.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{x\,\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(1/2)), x)
```

3.926 $\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$

3.926.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.926.2 Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

3.926.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.926.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.926.4 Maple [N/A] (verified)

Not integrable

Time = 4.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)\sqrt{\arctan\,(ax)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.926.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

3.926.6 Sympy [N/A]

Not integrable

Time = 12.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^m}{a^2x^2\sqrt{\operatorname{atan}{(ax)}}+\sqrt{\operatorname{atan}{(ax)}}}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

3.926.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.926.8 Giac [N/A]

Not integrable

Time = 65.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.926.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

3.927 $\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$

3.927.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2x\sqrt{\arctan(ax)}}{ac} - \frac{2\mathrm{Int}\left(\sqrt{\arctan(ax)},x\right)}{ac}$$

output
```
2*x*arctan(a*x)^(1/2)/a/c-2*Unintegrable(arctan(a*x)^(1/2),x)/a/c
```

3.927.2 Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

3.927.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{2x\sqrt{\arctan(ax)}}{ac}-\frac{2\int \sqrt{\arctan(ax)}dx}{ac}$$
$$\Big\downarrow \text{5353}$$
$$\frac{2x\sqrt{\arctan(ax)}}{ac}-\frac{2\int \sqrt{\arctan(ax)}dx}{ac}$$

input
```
Int[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.927.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))    Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.927.4 Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(a^{2} c\,x^{2}+c \right) \sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.927.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.927.6 Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x}{a^{2} x^{2} \sqrt{\operatorname{atan}{\left(a x \right)}} + \sqrt{\operatorname{atan}{\left(a x \right)}}}\, dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

3.927.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.927.8 Giac [N/A]

Not integrable

Time = 47.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(a^2cx^2+c)\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.927.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

3.928 $\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$

3.928.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

output
```
2*arctan(a*x)^(1/2)/a/c
```

3.928.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
Integrate[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(2*Sqrt[ArcTan[a*x]])/(a*c)
```

3.928.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \ 5419$$
$$\frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
Int[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(2*Sqrt[ArcTan[a*x]])/(a*c)
```

3.928.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.928.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2\sqrt{\arctan(ax)}}{ac}$	15

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(a*x)^(1/2)/a/c
```

3.928.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(arctan(a*x))/(a*c)
```

3.928.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\operatorname{atan}(ax)}}{ac}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
2*sqrt(atan(a*x))/(a*c)
```

3.928.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.928.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(arctan(a*x))/(a*c)
```

3.928.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\operatorname{atan}(a\,x)}}{a\,c}$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
(2*atan(a*x)^(1/2))/(a*c)
```

3.929 $\int \frac{1}{x(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$

3.929.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx=\operatorname{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.929.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

3.929.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \, 5560$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.929.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.929.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)\sqrt{\arctan\left(a x\right)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

3.929.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.929.6 Sympy [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c
```

3.929.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.929.8 Giac [N/A]

Not integrable

Time = 47.64 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{(a^2cx^2+c)x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.929.9 Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

3.930 $\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$

3.930.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.930.2 Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

3.930.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.930.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.930.4 Maple [N/A] (verified)

Not integrable

Time = 6.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^2\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.930.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(arctan(a*x))), x)
```

3.930.6 Sympy [N/A]

Not integrable

Time = 69.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^m}{a^4x^4\sqrt{\operatorname{atan}{(ax)}}+2a^2x^2\sqrt{\operatorname{atan}{(ax)}}+\sqrt{\operatorname{atan}{(ax)}}}\,dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + s
qrt(atan(a*x))), x)/c**2
```

3.930.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.930.8 Giac [N/A]

Not integrable

Time = 106.93 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.930.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.931 $\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$

3.931.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.931.2 Mathematica [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

3.931.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^3}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.931.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.931.4 Maple [N/A] (verified)

Not integrable

Time = 5.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^2\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.931.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.931.6 Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^3}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x**3/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + s
qrt(atan(a*x))), x)/c**2
```

3.931.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.931.8 Giac [N/A]

Not integrable

Time = 134.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.931.9 Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.932 $\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$

3.932.1 Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{\arctan(ax)}}{a^3c^2}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}$$

output
```
-1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2+arctan(a*x)^(
1/2)/a^3/c^2
```

3.932.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.60

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$$
$$=\frac{16\arctan(ax)-4\sqrt{\pi}\sqrt{\arctan(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+i\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)-}{16a^3c^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(16*ArcTan[a*x] - 4*Sqrt[Pi]*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]] + I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[
a*x]] - I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(16*a
^3*c^2*Sqrt[ArcTan[a*x]])
```

3.932.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \; 5505$$

$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin(\arctan(ax))^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{\int \left(\frac{1}{2\sqrt{\arctan(ax)}}-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{\arctan(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[ArcTan[a*x]] - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2
)/(a^3*c^2)
```

3.932.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.932.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\pi \,\mathrm{FresnelC}\left(\frac{2 \sqrt{\arctan \left(a x \right)}}{\sqrt{\pi}}\right)-2 \sqrt{\arctan \left(a x \right)}\, \sqrt{\pi}}{2 c^{2} a^{3} \sqrt{\pi}}$	38

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2/a^3/Pi^(1/2)*(Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-2*arctan(
a*x)^(1/2)*Pi^(1/2))
```

3.932.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.932.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^2}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + s
qrt(atan(a*x))), x)/c**2
```

3.932.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.932.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^2 \sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.932.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.933 $\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$

3.933.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

output
```
1/2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2
```

3.933.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

3.933.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{3786}$$

$$\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

input
```
Int[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

3.933.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.933.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}}{2a^{2}c^{2}}$	24

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2
```

3.933.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.933.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt
(atan(a*x))), x)/c**2
```

3.933.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.933.8 Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(a^2cx^2+c)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.933.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{atan}(a\,x)}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.934 $\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$

3.934.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\arctan(ax)}}{ac^2} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2ac^2}$$

output
```
1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2+arctan(a*x)^(1/2
)/a/c^2
```

3.934.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.60

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{16\arctan(ax) + 4\sqrt{\pi}\sqrt{\arctan(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) - i\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2}, -2i\arctan(ax)\right) +}{16ac^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(16*ArcTan[a*x] + 4*Sqrt[Pi]*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]] - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[
a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(16*a
*c^2*Sqrt[ArcTan[a*x]])
```

3.934.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \; 5439$$

$$\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2
)/(a*c^2)
```

3.934.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.934.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\pi\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+2\sqrt{\arctan(ax)}\,\sqrt{\pi}}{2c^2a\sqrt{\pi}}$	38

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^2/a/Pi^(1/2)*(Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+2*arctan(a*x
)^(1/2)*Pi^(1/2))
```

3.934.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.934.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt
(atan(a*x))), x)/c**2
```

3.934.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.934.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.934.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.935 $\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$

3.935.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.935.2 Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

3.935.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \ \color{blue}{5560}$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.935.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.935.4 Maple [N/A] (verified)

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(a^2c\,x^2+c)^2\,\sqrt{\arctan\,(ax)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.935.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.935.6 Sympy [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^4x^5\sqrt{\operatorname{atan}(ax)}+2a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**4*x**5*sqrt(atan(a*x)) + 2*a**2*x**3*sqrt(atan(a*x)) + x*sq
rt(atan(a*x))), x)/c**2
```

3.935.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.935.8 Giac [N/A]

Not integrable

Time = 110.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\sqrt{\arctan\left(ax\right)}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.935.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

3.936 $\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.936.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.936.2 Mathematica [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

3.936.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.936.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.936.4 Maple [N/A] (verified)

Not integrable

Time = 6.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^3\,\sqrt{\arctan\,(ax)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.936.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*sqrt(arc
tan(a*x))), x)
```

3.936.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.936.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.936.8 Giac [N/A]

Not integrable

Time = 155.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.936.9 Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.937 $\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.937.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \operatorname{Int}\left(\frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.937.2 Mathematica [N/A]

Not integrable

Time = 3.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

3.937.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^5}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}dx$$

input
```
Int[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.937.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.937.4 Maple [N/A] (verified)

Not integrable

Time = 4.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{\left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.937.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.937.6 Sympy [N/A]

Not integrable

Time = 4.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^5}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**5/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

3.937.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.937.8 Giac [N/A]

Not integrable

Time = 190.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^5}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.937.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^5}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^5/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^5/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.938 $\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx$

3.938.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\arctan(ax)}}{4a^5c^3}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^5c^3}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^5c^3}$$

output
```
1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5/c
^3-1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^5/c^3+3/4*arctan(
a*x)^(1/2)/a^5/c^3
```

3.938.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx$$
$$= \frac{10\sqrt{2\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-80\sqrt{\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+3\sqrt{}}{}$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(10*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
] - 80*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi
]] + 3*Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan
[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*G
amma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTa
n[a*x]] - Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^5*
c^3*Sqrt[ArcTan[a*x]^2])
```

3.938.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^4x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{a^5c^3}$$

input
```
Int[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a
*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(a^5*c^3
)
```

3.938.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.938.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\pi \sqrt{2}\,\operatorname{FresnelC}\left(\frac{2 \sqrt{2}\,\sqrt{\arctan \left(a x \right)}}{\sqrt{\pi}}\right)-8 \pi \,\operatorname{FresnelC}\left(\frac{2 \sqrt{\arctan \left(a x \right)}}{\sqrt{\pi}}\right)+12 \sqrt{\arctan \left(a x \right)}\, \sqrt{\pi}}{16 c^{3} a^{5} \sqrt{\pi}}$	59

input
```
int(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/c^3/a^5*(Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-8*
Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+12*arctan(a*x)^(1/2)*Pi^(1/2))/P
i^(1/2)
```

3.938. $\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.938.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.938.6 Sympy [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^4}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**4/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

3.938.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.938.8 Giac [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.938.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.939 $\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.939.1 Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^4c^3} + \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}$$

output
```
-1/16*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/
c^3+1/4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3
```

3.939.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{-2\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) + \sqrt{-i\arctan(ax)}\cdots}{32a^4c^3\sqrt{\arctan(ax)}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt
[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a
*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)
*ArcTan[a*x]])/(32*a^4*c^3*Sqrt[ArcTan[a*x]])
```

3.939.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^3x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4)/(a^4*c^3)
```

3.939.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.939.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\sqrt{\pi}\left(-\sqrt{2}\,\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{16c^3a^4}$	47

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/c^3/a^4*Pi^(1/2)*(-2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))+4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

3.939.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.939.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^3}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**3/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

3.939.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.939.8 Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.939.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{x^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

input
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.940 $\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.940.1 Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\arctan(ax)}}{4a^3c^3} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^3c^3}$$

output
```
-1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/
c^3+1/4*arctan(a*x)^(1/2)/a^3/c^3
```

3.940.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.95

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$$
$$= \frac{-2\sqrt{2\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + 16\sqrt{\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) + \sqrt{}}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-2*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
] + 16*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi
]] + Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan[a
*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gam
ma[1/2, (2*I)*ArcTan[a*x]] + 7*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTa
n[a*x]] + 7*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^
3*c^3*Sqrt[ArcTan[a*x]^2])
```

3.940.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \text{5505}$$
$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^3}$$
$$\Big\downarrow \text{4906}$$
$$\frac{\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[ArcTan[a*x]]/4 - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/8)/(a^3*c^3)
```

3.940.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.940.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{2}\left(-2\sqrt{2}\,\sqrt{\arctan\left(a x\right)}\,\sqrt{\pi}+\pi\ \operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)\right)}{16 c^{3} a^{3} \sqrt{\pi}}$	47

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^3/a^3*2^(1/2)/Pi^(1/2)*(-2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+Pi*F
resnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2)))
```

3.940.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

3.940.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^2}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

3.940.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.940.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.940.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{atan}\,(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.941 $\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.941.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^2c^3} + \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

output
```
1/16*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c
^3+1/4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3
```

3.941.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{-2\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) - \sqrt{-i\arctan(ax)}}{32a^2c^3\sqrt{\arctan(ax)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt
[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan[a
*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)
*ArcTan[a*x]])/(32*a^2*c^3*Sqrt[ArcTan[a*x]])
```

3.941.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \; 5505$$

$$\frac{\int \frac{ax}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \; 4906$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}$$

input
```
Int[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*Fresn
elS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4)/(a^2*c^3)
```

3.941.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.941.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\sqrt{\pi}\left(\sqrt{2}\,\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{16c^3a^2}$	46

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/c^3/a^2*Pi^(1/2)*(2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/
2))+4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

3.941.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.941.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a*
*2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

3.941.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.941.8 Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.941.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{x}{\sqrt{\operatorname{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.942 $\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.942.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\arctan(ax)}}{4ac^3} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8ac^3} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2ac^3}$$

output
```
1/16*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a/c^3
+1/2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^3+3/4*arctan(a*x)
^(1/2)/a/c^3
```

3.942.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.57

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{-6\sqrt{2\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + 48\sqrt{\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) + \sqrt{}}{}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-6*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
] + 48*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi
]] + Sqrt[ArcTan[a*x]]*(192*Sqrt[ArcTan[a*x]^2] - 20*Sqrt[2]*Sqrt[I*ArcTan
[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 20*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*
Gamma[1/2, (2*I)*ArcTan[a*x]] - 11*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*A
rcTan[a*x]] - 11*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(2
56*a*c^3*Sqrt[ArcTan[a*x]^2])
```

3.942.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a
*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(a*c^3)
```

3.942.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.942.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\pi\sqrt{2}\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\ \sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)+8\pi\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)+12\sqrt{\arctan\left(a x\right)}\,\sqrt{\pi}}{16 c^{3} a \sqrt{\pi}}$	59

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/c^3/a/Pi^(1/2)*(Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))+8*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+12*arctan(a*x)^(1/2)*Pi^(
1/2))
```

3.942.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.942.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a*
*2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

3.942.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.942.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.942.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.943 $\int \frac{1}{x(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$

3.943.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.943.2 Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

3.943.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.943.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.943.4 Maple [N/A] (verified)

Not integrable

Time = 1.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3} \sqrt{\arctan\left(a x\right)}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.943.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.943.6 Sympy [N/A]

Not integrable

Time = 5.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^6x^7\sqrt{\operatorname{atan}(ax)}+3a^4x^5\sqrt{\operatorname{atan}(ax)}+3a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**6*x**7*sqrt(atan(a*x)) + 3*a**4*x**5*sqrt(atan(a*x)) + 3*a*
*2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c**3
```

3.943.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.943.8 Giac [N/A]

Not integrable

Time = 158.51 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\sqrt{\arctan\left(ax\right)}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.943.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

3.944 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$

3.944.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.944.2 Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

3.944.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.944.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.944.4 Maple [N/A] (verified)

Not integrable

Time = 7.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.944.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

3.944.6 Sympy [N/A]

Not integrable

Time = 24.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\sqrt{c\,(a^2x^2+1)}}{\sqrt{\operatorname{atan}{(ax)}}}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

3.944.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.944.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.944.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2), x)
```

3.945 $\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$

3.945.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.945.2 Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

3.945.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.945.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.945.4 Maple [N/A] (verified)

Not integrable

Time = 2.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.945.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.945.6 Sympy [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\sqrt{c(a^2x^2+1)}}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

3.945.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.945.8 Giac [N/A]

Not integrable

Time = 135.76 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.945.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2), x)
```

3.946 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$

3.946.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \operatorname{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.946.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]
```

3.946.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.946.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.946.4 Maple [N/A] (verified)

Not integrable

Time = 2.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.946.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.946.6 Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

3.946.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.946.8 Giac [N/A]

Not integrable

Time = 126.85 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.946.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(1/2), x)
```

3.947 $\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx$

3.947.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

3.947.2 Mathematica [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]), x]
```

3.947.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.947.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.947.4 Maple [N/A] (verified)

Not integrable

Time = 2.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

3.947.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.947.6 Sympy [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*sqrt(atan(a*x))), x)
```

3.947.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.947.8 Giac [N/A]

Not integrable

Time = 129.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.947.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(1/2)), x)
```

3.948 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$

3.948.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \mathrm{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.948.2 Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

3.948.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.948.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.948.4 Maple [N/A] (verified)

Not integrable

Time = 6.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.948.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/sqrt(arctan(a*x)), x)
```

3.948.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.948.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.948.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.948.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2), x)
```

3.949 $\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$

3.949.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.949.2 Mathematica [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

3.949.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.949.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.949.4 Maple [N/A] (verified)

Not integrable

Time = 2.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.949.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.949.6 Sympy [N/A]

Not integrable

Time = 47.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/sqrt(atan(a*x)), x)
```

3.949.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.949.8 Giac [N/A]

Not integrable

Time = 168.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.949.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{3/2}}{\sqrt{\mathrm{atan}(a\,x)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2), x)
```

3.950 $\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}} dx$

3.950.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}} dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.950.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}} dx = \int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}} dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]
```

3.950.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.950.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.950.4 Maple [N/A] (verified)

Not integrable

Time = 2.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.950.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.950.6 Sympy [N/A]

Not integrable

Time = 23.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(atan(a*x)), x)
```

3.950.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.950.8 Giac [N/A]

Not integrable

Time = 161.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.950.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\sqrt{\mathrm{atan}(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(1/2), x)
```

3.951 $\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx$

3.951.1 Optimal result

Integrand size $=26$, antiderivative size $=26$

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

3.951.2 Mathematica [N/A]

Not integrable

Time $=5.07$ (sec) , antiderivative size $=28$, normalized size of antiderivative $=1.08$

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx=\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

3.951.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.951.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.951.4 Maple [N/A] (verified)

Not integrable

Time = 2.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{x \sqrt{\arctan \left(a x \right)}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

3.951.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.951.6 Sympy [N/A]

Not integrable

Time = 21.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*sqrt(atan(a*x))), x)
```

3.951.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.951.8 Giac [N/A]

Not integrable

Time = 163.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.951.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(1/2)), x)
```

3.952 $\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$

3.952.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.952.2 Mathematica [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

3.952.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.952.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.952.4 Maple [N/A] (verified)

Not integrable

Time = 7.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.952.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/sqrt(
arctan(a*x)), x)
```

3.952.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.952.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.952.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.952.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2), x)
```

3.953 $\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx$

3.953.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.953.2 Mathematica [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

3.953.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.953.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.953.4 Maple [N/A] (verified)

Not integrable

Time = 2.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.953.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.953.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.953.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.953.8 Giac [N/A]

Not integrable

Time = 199.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.953.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2), x)
```

3.954 $\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$

3.954.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.954.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]
```

3.954.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.954.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.954.4 Maple [N/A] (verified)

Not integrable

Time = 2.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\arctan(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.954.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.954.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.954.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.954.8 Giac [N/A]

Not integrable

Time = 191.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.954.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(1/2), x)
```

3.955 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx$

3.955.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx=\mathrm{Int}\left(\frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

3.955.2 Mathematica [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

3.955.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.955.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.955.4 Maple [N/A] (verified)

Not integrable

Time = 2.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

3.955.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.955.6 Sympy [N/A]

Not integrable

Time = 153.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c + a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c(a^2x^2 + 1))^{\frac{5}{2}}}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/(x*sqrt(atan(a*x))), x)
```

3.955.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.955.8 Giac [N/A]

Not integrable

Time = 195.85 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}}{x\sqrt{\arctan\left(ax\right)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.955.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(1/2)), x)
```

3.956 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$

3.956.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \mathrm{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.956.2 Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

3.956.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.956.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.956.4 Maple [N/A] (verified)

Not integrable

Time = 7.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.956.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

3.956.6 Sympy [N/A]

Not integrable

Time = 31.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{c(a^2x^2+1)}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

3.956.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.956.8 Giac [N/A]

Not integrable

Time = 64.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.956.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.957 $\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$

3.957.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \mathrm{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.957.2 Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

3.957.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.957.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.957.4 Maple [N/A] (verified)

Not integrable

Time = 2.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.957.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.957.6 Sympy [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

3.957.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.957.8 Giac [N/A]

Not integrable

Time = 133.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.957.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.958 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$

3.958.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.958.2 Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

3.958.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.958.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.958.4 Maple [N/A] (verified)

Not integrable

Time = 3.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.958.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.958.6 Sympy [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

3.958.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.958.8 Giac [N/A]

Not integrable

Time = 133.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.958.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.959 $\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$

3.959.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.959.2 Mathematica [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

3.959.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.959.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.959.4 Maple [N/A] (verified)

Not integrable

Time = 3.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{a^{2} c \,x^{2}+c}\, \sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

3.959.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.959.6 Sympy [N/A]

Not integrable

Time = 4.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

3.959.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.959.8 Giac [N/A]

Not integrable

Time = 131.63 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+cx}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.959.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\,(a\,x)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.960 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$

3.960.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.960.2 Mathematica [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

3.960.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.960.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.960.4 Maple [N/A] (verified)

Not integrable

Time = 6.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.960.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt
(arctan(a*x))), x)
```

3.960.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.960.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.960.8 Giac [N/A]

Not integrable

Time = 36.98 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.960.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.961 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$

3.961.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.961.2 Mathematica [N/A]

Not integrable

Time = 12.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

3.961.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^2}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.961.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.961.4 Maple [N/A] (verified)

Not integrable

Time = 3.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.961.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.961.6 Sympy [N/A]

Not integrable

Time = 8.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

3.961.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.961.8 Giac [N/A]

Not integrable

Time = 102.43 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.961.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.962 $\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$

3.962.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^
(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

3.962.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = -\frac{\sqrt{1+a^2x^2}\left(\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},i\arctan(ax)\right)\right)}{2a^2c\sqrt{c(1+a^2x^2)}\sqrt{\arctan(ax)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
-1/2*(Sqrt[1 + a^2*x^2]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x
]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]]))/(a^2*c*Sqrt[c*(1 + a^
2*x^2)]*Sqrt[ArcTan[a*x]])
```

3.962.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5506, 5505, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3832}$$

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2
*c*Sqrt[c + a^2*c*x^2])
```

3.962.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.962.4 Maple [F]

$$\int \frac{x}{(a^2 c\,x^2 + c)^{\frac{3}{2}} \sqrt{\arctan(ax)}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.962.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.962.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(c(a^2x^2+1))^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

3.962.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.962.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.962.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{atan}(a\,x)}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.963 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$

3.963.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

output
```
FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^
(1/2)/a/c/(a^2*c*x^2+c)^(1/2)
```

3.963.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = -\frac{i\sqrt{1+a^2x^2}\left(\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)-\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},i\arctan(ax)\right)\right)}{2ac\sqrt{c(1+a^2x^2)}\sqrt{\arctan(ax)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
((-1/2*I)*Sqrt[1 + a^2*x^2]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]]))/(a*c*Sqrt[c*(1 +
a^2*x^2)]*Sqrt[ArcTan[a*x]])
```

3.963.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c
*Sqrt[c + a^2*c*x^2])
```

3.963.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.963.4 Maple [F]

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.963.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.963.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

3.963.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.963.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.963.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.964 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$

3.964.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.964.2 Mathematica [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

3.964.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.964.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.964.4 Maple [N/A] (verified)

Not integrable

Time = 2.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{\arctan \left(a x\right)}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.964.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.964.6 Sympy [N/A]

Not integrable

Time = 25.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

3.964.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.964.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.964.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.965 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$

3.965.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.965.2 Mathematica [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

3.965.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.965.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.965.4 Maple [N/A] (verified)

Not integrable

Time = 10.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.965.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*sqrt(arctan(a*x))), x)
```

3.965.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

3.965.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.965.8 Giac [N/A]

Not integrable

Time = 71.76 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.965.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.966 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$

3.966.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.966.2 Mathematica [N/A]

Not integrable

Time = 5.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

3.966.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \, 5560$$
$$\int \frac{x^4}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.966.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.966.4 Maple [N/A] (verified)

Not integrable

Time = 13.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.966.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.966.6 Sympy [N/A]

Not integrable

Time = 61.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{(c(a^2x^2+1))^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

3.966.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.966.8 Giac [N/A]

Not integrable

Time = 195.65 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.966.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.967 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$

3.967.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
-1/12*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x
^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+3/4*FresnelS(2^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1
/2)
```

3.967.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{6}}(1+a^2x^2)^{3/2}\left(3\sqrt{3}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) - \text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\right.\right.}{2a^4c\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi/6]*(1 + a^2*x^2)^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^4*c*(c*(1 + a^2*x^
2))^(3/2))
```

3.967.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/
2 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^4*c^2*Sqrt[
c + a^2*c*x^2])
```

3.967.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.967.4 Maple [F]

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.967.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.967.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{(c\,(a^2x^2+1))^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

3.967.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.967.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.967.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{atan}\,(a\,x)}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.968 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$

3.968.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
-1/12*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x
^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+1/4*FresnelC(2^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1
/2)
```

3.968.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = -\frac{i\sqrt{1+a^2x^2}\left(3\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right) - 3\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},i\arctan(ax)\right) + \sqrt{3}\left(-\sqrt{-i\arc\right.\right.}{24a^3c^2\sqrt{c(1+a^2x^2)}\sqrt{\arctan(ax)}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
((-1/24*I)*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*Arc
Tan[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(-(S
qrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]]) + Sqrt[I*ArcTan[a*x]
]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Sqrt[Arc
Tan[a*x]])
```

3.968.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2
- (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c
+ a^2*c*x^2])
```

3.968.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.968.4 Maple [F]

$$\int \frac{x^2}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \sqrt{\arctan\left(a x\right)}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.968.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.968.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

3.968.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.968.8 Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.968.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.969 $\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$

3.969.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
1/12*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/4*FresnelS(2^(1/2)/Pi^(1/2)*arcta
n(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/
2)
```

3.969.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = -\frac{(1+a^2x^2)^{3/2}\left(3\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+3\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},i\arctan(ax)\right)+\sqrt{3}\left(\sqrt{-i\operatorname{arc}\right.\right.}{24a^2c\,(c\,(1+a^2x^2))^{3/2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
-1/24*((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTa
n[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[
(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gam
ma[1/2, (3*I)*ArcTan[a*x]])))/(a^2*c*(c*(1 + a^2*x^2))^(3/2)*Sqrt[ArcTan[a
*x]])
```

3.969.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \ 5506$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 5505$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 4906$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2
+ (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c
+ a^2*c*x^2])
```

3.969.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.969.4 Maple [F]

$$\int \frac{x}{\left(a^{2} c\,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.969.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.969.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(c\,(a^2x^2+1))^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

3.969.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.969.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.969.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}\,(a\,x)}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.970 $\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$

3.970.1 Optimal result

Integrand size $= 23$, antiderivative size $= 131$

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}}$$

output
```
1/12*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+3/4*FresnelC(2^(1/2)/Pi^(1/2)*arctan(
a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)
```

3.970.2 Mathematica [A] (verified)

Time $= 0.09$ (sec) , antiderivative size $= 93$, normalized size of antiderivative $= 0.71$

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{6}}(1+a^2x^2)^{3/2}\left(3\sqrt{3}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\right.\right.}{2ac\left(c\left(1+a^2x^2\right)\right)^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi/6]*(1 + a^2*x^2)^(3/2)*(3*Sqrt[3]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]] + FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a*c*(c*(1 + a^2*x^2)
)^(3/2))
```

3.970.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/
2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a*c^2*Sqrt[c
+ a^2*c*x^2])
```

3.970.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

3.970.4 Maple [F]

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.970.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.970.6 Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

3.970.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.970.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.970.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.971 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$

3.971.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.971.2 Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

3.971.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.971.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.971.4 Maple [N/A] (verified)

Not integrable

Time = 3.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\,(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\arctan\,(ax)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.971.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.971.6 Sympy [N/A]

Not integrable

Time = 133.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

3.971.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.971.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.971.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.972 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$

3.972.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.972.2 Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

3.972.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a^2cx^2+c)}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m(a^2cx^2+c)}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.972.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.972.4 Maple [N/A] (verified)

Not integrable

Time = 4.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m(a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.972.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

3.972.6 Sympy [N/A]

Not integrable

Time = 33.82 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x**m/atan(a*x)**(3/2), x) + Integral(a**2*x**2*x**m/atan(a*x)*
*(3/2), x))
```

3.972.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.972.8 Giac [N/A]

Not integrable

Time = 62.98 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.972.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(3/2), x)
```

3.973 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$

3.973.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \mathrm{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.973.2 Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

3.973.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.973.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.973.4 Maple [N/A] (verified)

Not integrable

Time = 1.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.973.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.973.6 Sympy [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x/atan(a*x)**(3/2), x) + Integral(a**2*x**3/atan(a*x)**(3/2),
x))
```

3.973.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.973.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.973.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(3/2), x)
```

3.974 $\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx$

3.974.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.974.2 Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2), x]
```

3.974.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{a^2cx^2+c}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.974.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.974.4 Maple [N/A] (verified)

Not integrable

Time = 1.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.974.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.974.6 Sympy [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x
))
```

3.974.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.974.8 Giac [N/A]

Not integrable

Time = 169.69 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{a^2cx^2+c}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.974.9 Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^(3/2), x)
```

3.975 $\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx$

3.975.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx=\operatorname{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

3.975.2 Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx=\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)), x]
```

3.975.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.975.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.975.4 Maple [N/A] (verified)

Not integrable

Time = 1.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^{2} c \,x^{2}+c}{x \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

3.975.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.975.6 Sympy [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{1}{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^2x}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(a**2*x/atan(a*x)**(3/2),
 x))
```

3.975.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.975.8 Giac [N/A]

Not integrable

Time = 234.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx=\int \frac{a^2cx^2+c}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.975.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx=\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(3/2)), x)
```

3.976 $\int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx$

3.976.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx = \text{Int}\left(\frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{3/2}}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.976.2 Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx = \int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

3.976.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan (a x)^{3/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan (a x)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.976.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.976.4 Maple [N/A] (verified)

Not integrable

Time = 5.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{2}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.976.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{\left(a^2cx^2+c\right)^2x^m}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(3/2), x)
```

3.976.6 Sympy [N/A]

Not integrable

Time = 61.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{x^m}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x**m/atan(a*x)**(3/2), x) + Integral(2*a**2*x**2*x**m/atan(
a*x)**(3/2), x) + Integral(a**4*x**4*x**m/atan(a*x)**(3/2), x))
```

3.976.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.976.8 Giac [N/A]

Not integrable

Time = 63.64 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.976.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2), x)
```

3.977 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$

3.977.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.977.2 Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

3.977.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.977.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.977.4 Maple [N/A] (verified)

Not integrable

Time = 1.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.977.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.977.6 Sympy [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x/atan(a*x)**(3/2), x) + Integral(2*a**2*x**3/atan(a*x)**(3
/2), x) + Integral(a**4*x**5/atan(a*x)**(3/2), x))
```

3.977.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.977.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.977.9 Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2), x)
```

3.978 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx$

3.978.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}, x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.978.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx = \int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2), x]
```

3.978.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.978.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.978.4 Maple [N/A] (verified)

Not integrable

Time = 1.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.978.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.978.6 Sympy [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**(3/2), x) + Integral(a**4*x**4/atan(
a*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x))
```

3.978.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.978.8 Giac [N/A]

Not integrable

Time = 209.72 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.978.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^(3/2), x)
```

3.979 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx$

3.979.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

3.979.2 Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)), x]
```

3.979.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.979.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.979.4 Maple [N/A] (verified)

Not integrable

Time = 1.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

3.979.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.979.6 Sympy [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{1}{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{2a^2x}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^4x^3}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(2*a**2*x/atan(a*x)**(
3/2), x) + Integral(a**4*x**3/atan(a*x)**(3/2), x))
```

3.979.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.979.8 Giac [N/A]

Not integrable

Time = 279.87 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.979.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}\,(a\,x)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(3/2)), x)
```

3.980 $\int \frac{x^m \left(c+a^2 c x^2\right)^3}{\arctan(ax)^{3/2}} \, dx$

3.980.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)^{3/2}} \, dx = \text{Int}\left(\frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)^{3/2}}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.980.2 Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)^{3/2}} \, dx = \int \frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

3.980.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^3}{\arctan (a x)^{3/2}} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^3}{\arctan (a x)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.980.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.980.4 Maple [N/A] (verified)

Not integrable

Time = 6.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{3}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.980.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^3x^m}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^(3/2), x)
```

3.980.6 Sympy [N/A]

Not integrable

Time = 176.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx=c^3\left(\int \frac{x^m}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx+\int \frac{3a^2x^2x^m}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx+\int \frac{3a^4x^4x^m}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx+\int \frac{a^6x^6x^m}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(x**m/atan(a*x)**(3/2), x) + Integral(3*a**2*x**2*x**m/atan(
a*x)**(3/2), x) + Integral(3*a**4*x**4*x**m/atan(a*x)**(3/2), x) + Integra
l(a**6*x**6*x**m/atan(a*x)**(3/2), x))
```

3.980.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.980.8 Giac [N/A]

Not integrable

Time = 65.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.980.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2), x)
```

3.981 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}} dx$

3.981.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}} dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}, x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.981.2 Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}} dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

3.981.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.981.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.981.4 Maple [N/A] (verified)

Not integrable

Time = 1.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.981.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.981.6 Sympy [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(x/atan(a*x)**(3/2), x) + Integral(3*a**2*x**3/atan(a*x)**(3
/2), x) + Integral(3*a**4*x**5/atan(a*x)**(3/2), x) + Integral(a**6*x**7/a
tan(a*x)**(3/2), x))
```

3.981.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.981.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.981.9 Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2), x)
```

3.982 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx$

3.982.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.982.2 Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2), x]
```

3.982.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.982.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.982.4 Maple [N/A] (verified)

Not integrable

Time = 1.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.982.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.982.6 Sympy [N/A]

Not integrable

Time = 3.73 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**(3/2), x) + Integral(3*a**4*x**4/ata
n(a*x)**(3/2), x) + Integral(a**6*x**6/atan(a*x)**(3/2), x) + Integral(ata
n(a*x)**(-3/2), x))
```

3.982.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.982.8 Giac [N/A]

Not integrable

Time = 252.60 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.982.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^(3/2), x)
```

3.983 $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx$

3.983.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

3.983.2 Mathematica [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)), x]
```

3.983.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.983.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.983.4 Maple [N/A] (verified)

Not integrable

Time = 1.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

3.983.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.983.6 Sympy [N/A]

Not integrable

Time = 5.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = c^3\left(\int \frac{1}{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^2x}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^4x^3}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^6x^5}{\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(3*a**2*x/atan(a*x)**(
3/2), x) + Integral(3*a**4*x**3/atan(a*x)**(3/2), x) + Integral(a**6*x**5/
atan(a*x)**(3/2), x))
```

3.983.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.983.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.983.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(3/2)), x)
```

3.984 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$

3.984.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2x^m}{ac\sqrt{\arctan(ax)}} + \frac{2m\text{Int}\left(\frac{x^{-1+m}}{\sqrt{\arctan(ax)}},x\right)}{ac}$$

output
```
-2*x^m/a/c/arctan(a*x)^(1/2)+2*m*Unintegrable(x^(-1+m)/arctan(a*x)^(1/2),x
)/a/c
```

3.984.2 Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

3.984.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{5461}$$
$$\frac{2m\int \frac{x^{m-1}}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x^m}{ac\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{5377}$$
$$\frac{2m\int \frac{x^{m-1}}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x^m}{ac\sqrt{\arctan(ax)}}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.984.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.984.4 Maple [N/A] (verified)

Not integrable

Time = 4.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.984.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

3.984.6 Sympy [N/A]

Not integrable

Time = 56.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^m}{a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c
```

3.984.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.984.8 Giac [N/A]

Not integrable

Time = 63.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.984.9 Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

3.985 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$

3.985.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx=-\frac{2x}{ac\sqrt{\arctan(ax)}}+\frac{2\mathrm{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{ac}$$

output
```
-2*x/a/c/arctan(a*x)^(1/2)+2*Unintegrable(1/arctan(a*x)^(1/2),x)/a/c
```

3.985.2 Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx=\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

3.985.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{ 5457}$$
$$\frac{2\int \frac{1}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x}{ac\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{ 5353}$$
$$\frac{2\int \frac{1}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x}{ac\sqrt{\arctan(ax)}}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.985.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.985.4 Maple [N/A] (verified)

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a^2 c\, x^2 + c) \arctan(ax)^{\frac{3}{2}}} dx$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.985.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.985.6 Sympy [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\, dx = \frac{\int \frac{x}{a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax) + \operatorname{atan}^{\frac{3}{2}}(ax)}\, dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c
```

3.985.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.985.8 Giac [N/A]

Not integrable

Time = 123.75 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.985.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

3.986 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$

3.986.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan(ax)}}$$

output
```
-2/a/c/arctan(a*x)^(1/2)
```

3.986.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan(ax)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*Sqrt[ArcTan[a*x]])
```

3.986.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}\,dx$$
$$\Big\downarrow \text{5419}$$
$$-\frac{2}{ac\sqrt{\arctan(ax)}}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*Sqrt[ArcTan[a*x]])
```

3.986.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.986.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2}{ac\sqrt{\arctan(ax)}}$	15

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a/c/arctan(a*x)^(1/2)
```

3.986.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan{(ax)}}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/(a*c*sqrt(arctan(a*x)))
```

3.986.6 Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\operatorname{atan}{(ax)}}}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
-2/(a*c*sqrt(atan(a*x)))
```

3.986.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.986.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan(ax)}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
-2/(a*c*sqrt(arctan(a*x)))
```

3.986.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{a\,c\,\sqrt{\mathrm{atan}\left(a\,x\right)}}$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
-2/(a*c*atan(a*x)^(1/2))
```

3.987 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$

3.987.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx\sqrt{\arctan(ax)}} - \frac{2\text{Int}\left(\frac{1}{x^2\sqrt{\arctan(ax)}},x\right)}{ac}$$

output
```
-2/a/c/x/arctan(a*x)^(1/2)-2*Unintegrable(1/x^2/arctan(a*x)^(1/2),x)/a/c
```

3.987.2 Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

3.987.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2}(a^2cx^2+c)}dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{2\int \frac{1}{x^2\sqrt{\arctan(ax)}}dx}{ac}-\frac{2}{acx\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{2\int \frac{1}{x^2\sqrt{\arctan(ax)}}dx}{ac}-\frac{2}{acx\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.987.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.987.4 Maple [N/A] (verified)

Not integrable

Time = 1.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(a^2c\,x^2+c)\arctan{(ax)}^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

3.987.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.987.6 Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^2x^3\,\mathrm{atan}^{\frac{3}{2}}\,(ax)+x\,\mathrm{atan}^{\frac{3}{2}}\,(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c
```

3.987.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.987.8 Giac [N/A]

Not integrable

Time = 125.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.987.9 Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

3.988 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx$

3.988.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}, x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.988.2 Mathematica [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

3.988.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.988.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.988.4 Maple [N/A] (verified)

Not integrable

Time = 5.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.988.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(3/2)), x)
```

3.988.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.988.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.988.8 Giac [N/A]

Not integrable

Time = 141.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.988.9 Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.989 $\int \frac{x^4}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx$

3.989.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = -\frac{2x^4}{ac^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}} + \frac{8\text{Int}\left(\frac{x^3}{\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)}{a} + 4a\text{Int}\left(\frac{x^5}{\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2*x^4/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+8*Unintegrable(x^3/(a^2*c*x^2+c
)^2/arctan(a*x)^(1/2),x)/a+4*a*Unintegrable(x^5/(a^2*c*x^2+c)^2/arctan(a*x
)^(1/2),x)
```

3.989.2 Mathematica [N/A]

Not integrable

Time = 3.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \int \frac{x^4}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

3.989.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$4a\int \frac{x^5}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{8\int \frac{x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2x^4}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$\frac{4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{8\int \frac{x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2x^4}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

input
```
Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.989.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.989.4 Maple [N/A] (verified)

Not integrable

Time = 9.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a^2 c\,x^2+c\right)^2 \arctan \left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.989.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.989.6 Sympy [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^4}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x**4/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
 atan(a*x)**(3/2)), x)/c**2
```

3.989.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.989.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.989.9 Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^4}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^4/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^4/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.990 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx$

3.990.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2x^3}{ac^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}}+\frac{6\sqrt{\arctan(ax)}}{a^4c^2} - \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^4c^2}+2a\text{Int}\left(\frac{x^4}{\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-3*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^2-2*x^3/a/c^2/(a^
2*x^2+1)/arctan(a*x)^(1/2)+6*arctan(a*x)^(1/2)/a^4/c^2+2*a*Unintegrable(x^
4/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.990.2 Mathematica [N/A]

Not integrable

Time = 6.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

3.990.3 Rubi [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 27, 5505, 3042, 3793, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{6\int \frac{x^2}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}+2a\int \frac{x^4}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\int \frac{x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}+\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{6\int \frac{a^2x^2}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{\sin(\arctan(ax))^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2}+\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{6\int \left(\frac{1}{2\sqrt{\arctan(ax)}}-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2}+\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{6\left(\sqrt{\arctan(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a^4c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

3.990. $\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}dx$

$$\Big\downarrow \text{5560}$$

$$\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{6\left(\sqrt{\arctan(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a^4c^2}-\frac{2x^3}{ac^2\,(a^2x^2+1)\sqrt{\arctan(ax)}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.990.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.990. $\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.990.4 Maple [N/A] (verified)

Not integrable

Time = 5.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.990.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.990.6 Sympy [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^3}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
 atan(a*x)**(3/2)), x)/c**2
```

3.990.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.990.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.990.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.991 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx$

3.991.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx=-\frac{2x^2}{ac^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}}+\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

output
```
2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2-2*x^2/a/c^2/(a^2
*x^2+1)/arctan(a*x)^(1/2)
```

3.991.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx=-\frac{2x^2}{ac^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}}+\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2
*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

3.991.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5477, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{4\int \frac{x}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{4\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2
*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

3.991.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.991. $\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.991.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{2\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\cos(2\arctan(ax))-1}{c^2a^3\sqrt{\arctan(ax)}}$	46

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2/a^3*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1
/2))+cos(2*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

3.991.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.991.6 Sympy [F]

$$\int \frac{x^2}{(c + a^2cx^2)^2 \arctan(ax)^{3/2}} dx = \frac{\int \frac{x^2}{a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax) + 2a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax) + \operatorname{atan}^{\frac{3}{2}}(ax)} dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
 atan(a*x)**(3/2)), x)/c**2
```

3.991.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c + a^2cx^2)^2 \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.991.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(c + a^2cx^2)^2 \arctan(ax)^{3/2}} dx = \text{Timed out}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.991.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.992 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$

3.992.1 Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{4\sqrt{\arctan(ax)}}{a^2c^2} - \frac{8\sqrt{\arctan(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2c^2(1+a^2x^2)} + \frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^2c^2}$$

output
```
2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^2-2*x/a/c^2/(a^2*x
^2+1)/arctan(a*x)^(1/2)+4*arctan(a*x)^(1/2)/a^2/c^2-8*arctan(a*x)^(1/2)/a^
2/c^2/(a^2*x^2+1)+4*(-a^2*x^2+1)*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)
```

3.992.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.14

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{-8ax + 4\sqrt{\pi}(1+a^2x^2)\sqrt{\arctan(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) - i\sqrt{2}(1+}{4a^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8*a*x + 4*Sqrt[Pi]*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]] - I*Sqrt[2]*(1 + a^2*x^2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma
[1/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*(1 + a^2*x^2)*Sqrt[I*ArcTan[a*x]]*Ga
mma[1/2, (2*I)*ArcTan[a*x]])/(4*a^2*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])
```

3.992.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5467, 27, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5467}$$

$$16\int \frac{x\sqrt{\arctan(ax)}}{c^2\left(a^2x^2+1\right)^2}dx-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{16\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{5439}$$

$$\frac{16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{16\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{16\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*(1 - a^2*x^2)*Sqrt[Arc
Tan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (16*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 +
 a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/2)/(4*a^2)))/c^2
```

3.992.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

3.992.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{2\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\arctan(ax)}\,\sqrt{\pi}-\sin(2\arctan(ax))}{c^2a^2\sqrt{\arctan(ax)}}$	47

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2/a^2*(2*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)^(1/2)*Pi^(
1/2)-sin(2*arctan(a*x)))/arctan(a*x)^(1/2)
```

3.992.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.992.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{x}{a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)} \, dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + at
an(a*x)**(3/2)), x)/c**2
```

3.992.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.992.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.992.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{x}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.993 $\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$

3.993.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = -\frac{2}{ac^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

output
```
-2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^2-2/a/c^2/(a^2*x^2+
1)/arctan(a*x)^(1/2)
```

3.993.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \frac{-\frac{2}{(1+a^2x^2)\sqrt{\arctan(ax)}} - 2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2/((1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - 2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(a*c^2)
```

3.993.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5437, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$-4a\int \frac{x}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[
ArcTan[a*x]])/Sqrt[Pi]])/(a*c^2)
```

3.993.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

3.993. $\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.993.4 Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{2\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\cos(2\arctan(ax))+1}{c^2 a\sqrt{\arctan(ax)}}$	47

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2/a*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/
2))+cos(2*arctan(a*x))+1)/arctan(a*x)^(1/2)
```

3.993.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.993.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + at
an(a*x)**(3/2)), x)/c**2
```

3.993.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.993.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.993.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2} \,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.994 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$

3.994.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^2x(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{6\sqrt{\arctan(ax)}}{c^2} - \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^2} - \frac{2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-3*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/c^2-2/a/c^2/x/(a^2*x^2+
1)/arctan(a*x)^(1/2)-6*arctan(a*x)^(1/2)/c^2-2*Unintegrable(1/x^2/(a^2*c*x
^2+c)^2/arctan(a*x)^(1/2),x)/a
```

3.994.2 Mathematica [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

3.994.3 Rubi [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{ 5503}$$

$$-6a \int \frac{1}{c^2 \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}} dx - \frac{2 \int \frac{1}{c^2x^2\left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}} dx}{a} - \frac{2}{ac^2x \left(a^2x^2+1\right) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{6a \int \frac{1}{\left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}} dx}{c^2} - \frac{2 \int \frac{1}{x^2\left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}} dx}{ac^2} - \frac{2}{ac^2x \left(a^2x^2+1\right) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{2 \int \frac{1}{x^2\left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}} dx}{ac^2} - \frac{6 \int \frac{1}{\left(a^2x^2+1\right) \sqrt{\arctan(ax)}} d\arctan(ax)}{c^2} - \frac{2}{ac^2x \left(a^2x^2+1\right) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2 \int \frac{1}{x^2\left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}} dx}{ac^2} - \frac{6 \int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}} d\arctan(ax)}{c^2} - \frac{2}{ac^2x \left(a^2x^2+1\right) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{2 \int \frac{1}{x^2\left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}} dx}{ac^2} - \frac{6 \int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}} + \frac{1}{2\sqrt{\arctan(ax)}}\right) d\arctan(ax)}{c^2} - \frac{2}{ac^2x \left(a^2x^2+1\right) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x\,(a^2x^2+1)\,\sqrt{\arctan(ax)}}-\frac{6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 5560$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x\,(a^2x^2+1)\,\sqrt{\arctan(ax)}}-\frac{6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.994.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.994. $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.994.4 Maple [N/A] (verified)

Not integrable

Time = 1.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(a^2 c\,x^2+c)^2 \arctan\,(ax)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.994.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.994.6 Sympy [N/A]

Not integrable

Time = 4.57 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**(3/2) + 2*a**2*x**3*atan(a*x)**(3/2) + x*
atan(a*x)**(3/2)), x)/c**2
```

3.994.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.994.8 Giac [N/A]

Not integrable

Time = 140.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.994.9 Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.995 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$

3.995.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^2x^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a} - 8a\text{Int}\left(\frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2*c*x^2
+c)^2/arctan(a*x)^(1/2),x)/a-8*a*Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a
*x)^(1/2),x)
```

3.995.2 Mathematica [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

3.995.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \color{blue}{5503}$$

$$-8a\int \frac{1}{c^2x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx - \frac{4\int \frac{1}{c^2x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ac^2x^2(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \color{blue}{27}$$

$$-\frac{8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2} - \frac{4\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2} - \frac{2}{ac^2x^2(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \color{blue}{5560}$$

$$-\frac{8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2} - \frac{4\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2} - \frac{2}{ac^2x^2(a^2x^2+1)\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.995.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.995.4 Maple [N/A] (verified)

Not integrable

Time = 3.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \left(a^2 c\, x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.995.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.995.6 Sympy [N/A]

Not integrable

Time = 7.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{1}{a^4x^6 \operatorname{atan}^{\frac{3}{2}}(ax) + 2a^2x^4 \operatorname{atan}^{\frac{3}{2}}(ax) + x^2 \operatorname{atan}^{\frac{3}{2}}(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**(3/2) + 2*a**2*x**4*atan(a*x)**(3/2) + x*
*2*atan(a*x)**(3/2)), x)/c**2
```

3.995.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.995.8 Giac [N/A]

Not integrable

Time = 148.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.995.9 Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.996 $\int \frac{1}{x^3(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$

3.996.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^2x^3(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a} - 10a\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2*c*x^2
+c)^2/arctan(a*x)^(1/2),x)/a-10*a*Unintegrable(1/x^2/(a^2*c*x^2+c)^2/arcta
n(a*x)^(1/2),x)
```

3.996.2 Mathematica [N/A]

Not integrable

Time = 6.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

3.996.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-10a \int \frac{1}{c^2x^2 (a^2x^2+1)^2 \sqrt{\arctan(ax)}}dx - \frac{6 \int \frac{1}{c^2x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ac^2x^3 (a^2x^2+1) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10a \int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2} - \frac{6 \int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2} - \frac{2}{ac^2x^3 (a^2x^2+1) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{10a \int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2} - \frac{6 \int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2} - \frac{2}{ac^2x^3 (a^2x^2+1) \sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.996.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.996.4 Maple [N/A] (verified)

Not integrable

Time = 2.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 \left(a^2 c\, x^2+c\right)^2 \arctan \left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.996.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.996.6 Sympy [N/A]

Not integrable

Time = 10.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^4x^7\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^5\operatorname{atan}^{\frac{3}{2}}(ax)+x^3\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**(3/2) + 2*a**2*x**5*atan(a*x)**(3/2) + x*
*3*atan(a*x)**(3/2)), x)/c**2
```

3.996.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.996.8 Giac [N/A]

Not integrable

Time = 148.48 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.996.9 Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a x)^{3/2} (c a^2 x^2 + c)^2} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.997 $\int \frac{1}{x^4(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$

3.997.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = -\frac{2}{ac^2x^4(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right)}{a} - 12a\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/a/c^2/x^4/(a^2*x^2+1)/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2*c*x^2
+c)^2/arctan(a*x)^(1/2),x)/a-12*a*Unintegrable(1/x^3/(a^2*c*x^2+c)^2/arcta
n(a*x)^(1/2),x)
```

3.997.2 Mathematica [N/A]

Not integrable

Time = 6.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

3.997.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} (a^2cx^2+c)^2} dx$$

$$\Big\downarrow \; 5503$$

$$-\frac{8\int \frac{1}{c^2x^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a} - 12a\int \frac{1}{c^2x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx - \frac{2}{ac^2x^4(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2} - \frac{12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2} - \frac{2}{ac^2x^4(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \; 5560$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2} - \frac{12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2} - \frac{2}{ac^2x^4(a^2x^2+1)\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.997.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.997.4 Maple [N/A] (verified)

Not integrable

Time = 4.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 \left(a^2 c\, x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

3.997.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(a x)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.997.6 Sympy [N/A]

Not integrable

Time = 13.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^4x^8\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^6\operatorname{atan}^{\frac{3}{2}}(ax)+x^4\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**(3/2) + 2*a**2*x**6*atan(a*x)**(3/2) + x*
*4*atan(a*x)**(3/2)), x)/c**2
```

3.997.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.997.8 Giac [N/A]

Not integrable

Time = 147.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2x^4\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.997.9 Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx=\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

3.998 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$

3.998.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}, x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.998.2 Mathematica [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

3.998.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.998.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.998.4 Maple [N/A] (verified)

Not integrable

Time = 7.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.998.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^(3/2)), x)
```

3.998.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.998.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.998.8 Giac [N/A]

Not integrable

Time = 210.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.998.9 Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.999 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$

3.999.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = -\frac{2x^3}{ac^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^3} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^4c^3}$$

output
```
-1/2*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/c
^3+FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3-2*x^3/a/c^3/(a^
2*x^2+1)^2/arctan(a*x)^(1/2)
```

3.999.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.54

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{-2\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + 16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16a^4}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 16*Sqrt[Pi]*Fres
nelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((-32*a^3*x^3)/(1 + a^2*x^2)^2 + (3
*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - (3*I)*Sqrt[I*A
rcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a*x]])/(16*a^4*c^3)
```

3.999.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.71, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5503, 27, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \ 5503$$

$$\frac{6\int \frac{x^2}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 27$$

$$\frac{6\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 5505$$

$$\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2\int \frac{a^4x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}+\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 3793$$

$$-\frac{2\int \left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^3}+\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{6\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^3}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^3}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^3)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (6*(Sqrt[ArcTan[a*x]]
/4 - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/(a^4*c^3) -
 (2*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/(a^
4*c^3)
```

3.999. $\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx$

3.999.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.999. $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$

3.999.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

method	result
default	$-\frac{2\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}-4\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\arctan(ax)}\,\sqrt{\pi}+2\sin(2\arctan(ax))-\sin(4\arctan(ax))}{4c^3a^4\sqrt{\arctan(ax)}}$

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^3/a^4*(2*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arc
tan(a*x)^(1/2)*Pi^(1/2)-4*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*
x)^(1/2)*Pi^(1/2)+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^(1/
2)
```

3.999.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.999.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^3}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
 3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

3.999.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.999.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.999.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1000 $\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}}\,dx$

3.1000.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}}\,dx=-\frac{2x^2}{ac^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^3}$$

output
```
1/2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^
3-2*x^2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
```

3.1000.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}}\,dx=\frac{-8a^2x^2-\left(1+a^2x^2\right)^2\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-4i\arctan(ax)\right)-\left(1+a^2x^2\right.}{4a^3c^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8*a^2*x^2 - (1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*Arc
Tan[a*x]] - (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*
x]])/(4*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

3.1000.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(67) = 134.

Time = 0.59 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5503, 27, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^3}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{4a\int \frac{x^3}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{4\int \frac{ax}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^3}-\frac{4\int \frac{a^3x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^3}+\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^3}+\frac{4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a^3c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*(-1/8*(Sqrt[Pi/2]*
FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[Arc
Tan[a*x]])/Sqrt[Pi]])/4))/(a^3*c^3) + (4*((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi
]])/4))/(a^3*c^3)
```

3.1000.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1000.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+\cos(4\arctan(ax))-1}{4c^{3}a^{3}\sqrt{\arctan(ax)}}$	53

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/c^3/a^3*(2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arct
an(a*x)^(1/2)*Pi^(1/2)+cos(4*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

3.1000.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1000.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^2}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
 3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

3.1000.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1000.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1000.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1001 $\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}} \, dx$

3.1001.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}} \, dx = -\frac{2x}{ac^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^3} + \frac{\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}$$

output
```
1/2*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c^
3+FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3-2*x/a/c^3/(a^2*x
^2+1)^2/arctan(a*x)^(1/2)
```

3.1001.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}} \, dx = \frac{-\frac{8ax}{\left(1+a^2x^2\right)^2} - i\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2}, -2i\arctan(ax)\right) + i\sqrt{2}\sqrt{i\arctan}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
((-8*a*x)/(1 + a^2*x^2)^2 - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[
a*x]] - I*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I
*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a^2*c^3*Sqrt[ArcTan[a*x]])
```

3.1001.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{1}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-6a\int \frac{x^2}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \; 5505$$

$$-\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \; 4906$$

$$-\frac{6\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (6*(Sqrt[ArcTan[a*x]]/4
 - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/(a^2*c^3) + (
2*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/(a^2*
c^3)
```

3.1001. $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx$

3.1001.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1001.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result
default	$-\frac{-2\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}-4\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\arctan(ax)}\,\sqrt{\pi}+2\sin(2\arctan(ax))+\sin(4\,\mathrm{ar}}{4c^3a^2\sqrt{\arctan(ax)}}$

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^3/a^2/arctan(a*x)^(1/2)*(-2*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)-4*FresnelC(2*arctan(a*x)^(1/2)/
Pi^(1/2))*arctan(a*x)^(1/2)*Pi^(1/2)+2*sin(2*arctan(a*x))+sin(4*arctan(a*x
)))
```

3.1001.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1001.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x}{a^6x^6 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*
a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

3.1001.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1001.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1001.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1002 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$

3.1002.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = -\frac{2}{ac^3 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} - \frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^3} - \frac{2\sqrt{\pi} \operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^3}$$

output
```
-1/2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a/c^3
-2*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a/c^3-2/a/c^3/(a^2*x^2+
1)^2/arctan(a*x)^(1/2)
```

3.1002.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{-\frac{8}{(1+a^2x^2)^2} + 2\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2}, -2i\arctan(ax)\right) + 2\sqrt{2}\sqrt{i\arctan}}{}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8/(1 + a^2*x^2)^2 + 2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*A
rcTan[a*x]] + 2*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]]
+ Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a*c^3*Sqrt[ArcTan[a*x]])
```

3.1002.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5437, 27, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \text{5437}$$
$$-8a\int \frac{x}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{27}$$
$$-\frac{8a\int \frac{x}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{5505}$$
$$-\frac{8\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^3}-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{4906}$$
$$-\frac{8\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^3}-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (8*((Sqrt[Pi/2]*FresnelS[2*
Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]]
)/Sqrt[Pi]])/4))/(a*c^3)
```

3.1002.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1002.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

method	result
default	$-\frac{2\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}+8\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\cos(2\arctan(ax))+\cos(4\,\mathrm{arct}}{4c^3a\sqrt{\arctan(ax)}}$

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^3/a*(2*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*arcta
n(a*x)^(1/2)*Pi^(1/2)+8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^
(1/2)/Pi^(1/2))+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^(1/
2)
```

3.1002.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1002.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*
a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

3.1002.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1002.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}} dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1002.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^3} dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1003 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$

3.1003.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{15\sqrt{\arctan(ax)}}{2c^3} - \frac{5\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4c^3} - \frac{5\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^3} - \frac{2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-5/8*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/c^3-5
*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/c^3-2/a/c^3/x/(a^2*x^2+1)
^2/arctan(a*x)^(1/2)-15/2*arctan(a*x)^(1/2)/c^3-2*Unintegrable(1/x^2/(a^2*
c*x^2+c)^3/arctan(a*x)^(1/2),x)/a
```

3.1003.2 Mathematica [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

3.1003.3 Rubi [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3} \, dx$$

$$\Big\downarrow \text{ 5503}$$

$$-10a \int \frac{1}{c^3 \left(a^2x^2+1\right)^3 \sqrt{\arctan(ax)}}dx - \frac{2\int \frac{1}{c^3x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ac^3x \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{10a\int \frac{1}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{c^3} - \frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{2}{ac^3x \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{10\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{c^3} - \frac{2}{ac^3x \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{10\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c^3} - \frac{2}{ac^3x \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{10\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{c^3}-\frac{2}{ac^3x\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}-\frac{10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}-\frac{10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{c^3}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1003.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.1003. $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx$

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1003.4 Maple [N/A] (verified)

Not integrable

Time = 1.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3} \arctan\left(a x\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.1003.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1003.6 Sympy [N/A]

Not integrable

Time = 13.71 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\frac{\int \frac{1}{a^6x^7\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**(3/2) + 3*a**4*x**5*atan(a*x)**(3/2) + 3*
a**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c**3
```

3.1003.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1003.8 Giac [N/A]

Not integrable

Time = 204.94 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^3 x \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1003.9 Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1004 $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx$

3.1004.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^3x^2(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{a} - 12a\text{Int}\left(\frac{1}{x(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2*c*x
^2+c)^3/arctan(a*x)^(1/2),x)/a-12*a*Unintegrable(1/x/(a^2*c*x^2+c)^3/arcta
n(a*x)^(1/2),x)
```

3.1004.2 Mathematica [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

3.1004.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3} dx$$

$$\Big\downarrow \text{5503}$$

$$-12a\int \frac{1}{c^3x\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{c^3x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ac^3x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{4\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{4\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1004.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1004.4 Maple [N/A] (verified)

Not integrable

Time = 2.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \left(a^2 c\,x^2+c\right)^3 \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.1004.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1004.6 Sympy [N/A]

Not integrable

Time = 20.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^6 x^8 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^6 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**(3/2) + 3*a**4*x**6*atan(a*x)**(3/2) + 3*
a**2*x**4*atan(a*x)**(3/2) + x**2*atan(a*x)**(3/2)), x)/c**3
```

3.1004.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1004.8 Giac [N/A]

Not integrable

Time = 211.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1004.9 Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\, x)^{3/2} \, (c\, a^2\, x^2 + c)^3} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1005 $\int \frac{1}{x^3(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$

3.1005.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = -\frac{2}{ac^3x^3 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^3\sqrt{\arctan(ax)}}, x\right)}{a} - 14a\text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)^3 \sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/a/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2*c*x
^2+c)^3/arctan(a*x)^(1/2),x)/a-14*a*Unintegrable(1/x^2/(a^2*c*x^2+c)^3/arc
tan(a*x)^(1/2),x)
```

3.1005.2 Mathematica [N/A]

Not integrable

Time = 7.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

3.1005.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3} \, dx$$

$$\Big\downarrow \ 5503$$

$$-14a \int \frac{1}{c^3x^2 \left(a^2x^2+1\right)^3 \sqrt{\arctan(ax)}}dx - \frac{6\int \frac{1}{c^3x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ac^3x^3 \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 27$$

$$-\frac{14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3} - \frac{6\int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{2}{ac^3x^3 \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 5560$$

$$-\frac{14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3} - \frac{6\int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{2}{ac^3x^3 \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1005.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1005.4 Maple [N/A] (verified)

Not integrable

Time = 3.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.1005.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1005.6 Sympy [N/A]

Not integrable

Time = 26.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^6 x^9 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^7 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^5 \operatorname{atan}^{\frac{3}{2}}(ax)+x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**(3/2) + 3*a**4*x**7*atan(a*x)**(3/2) + 3*
a**2*x**5*atan(a*x)**(3/2) + x**3*atan(a*x)**(3/2)), x)/c**3
```

3.1005.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1005.8 Giac [N/A]

Not integrable

Time = 211.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{\left(a^2 c x^2+c\right)^3 x^3 \arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1005.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c + a^2cx^2)^3 \arctan(ax)^{3/2}}\, dx = \int \frac{1}{x^3 \operatorname{atan}(a\, x)^{3/2} \left(c\, a^2\, x^2 + c\right)^3}\, dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1006 $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx$

3.1006.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^3x^4(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{a} - 16a\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c^3/x^4/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2*c*x
^2+c)^3/arctan(a*x)^(1/2),x)/a-16*a*Unintegrable(1/x^3/(a^2*c*x^2+c)^3/arc
tan(a*x)^(1/2),x)
```

3.1006.2 Mathematica [N/A]

Not integrable

Time = 7.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

3.1006.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} (a^2cx^2+c)^3} dx$$
$$\Big\downarrow \textcolor{blue}{5503}$$
$$-\frac{8\int \frac{1}{c^3x^5(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a} - 16a\int \frac{1}{c^3x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx - \frac{2}{ac^3x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \textcolor{blue}{27}$$
$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3} - \frac{2}{ac^3x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \textcolor{blue}{5560}$$
$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3} - \frac{2}{ac^3x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1006.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1006.4 Maple [N/A] (verified)

Not integrable

Time = 3.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

3.1006.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1006.6 Sympy [N/A]

Not integrable

Time = 33.80 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^6x^{10}\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^8\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^6\operatorname{atan}^{\frac{3}{2}}(ax)+x^4\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**(3/2) + 3*a**4*x**8*atan(a*x)**(3/2) + 3
*a**2*x**6*atan(a*x)**(3/2) + x**4*atan(a*x)**(3/2)), x)/c**3
```

3.1006.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1006.8 Giac [N/A]

Not integrable

Time = 210.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1006.9 Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2 c x^2)^3 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^3} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

3.1007 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$

3.1007.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

3.1007.2 Mathematica [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

3.1007.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1007.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1007.4 Maple [N/A] (verified)

Not integrable

Time = 10.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

3.1007.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}x^m}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

3.1007.6 Sympy [N/A]

Not integrable

Time = 69.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

3.1007.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1007.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1007.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2), x)
```

3.1008 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$

3.1008.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

3.1008.2 Mathematica [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

3.1008.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1008.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1008.4 Maple [N/A] (verified)

Not integrable

Time = 3.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{a^{2} c \,x^{2}+c}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

3.1008.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1008.6 Sympy [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

3.1008.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1008.8 Giac [N/A]

Not integrable

Time = 267.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1008.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2), x)
```

3.1009 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$

3.1009.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

3.1009.2 Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2), x]
```

3.1009.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1009.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1009.4 Maple [N/A] (verified)

Not integrable

Time = 2.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

3.1009.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1009.6 Sympy [N/A]

Not integrable

Time = 5.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

3.1009.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1009.8 Giac [N/A]

Not integrable

Time = 140.58 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1009.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(3/2), x)
```

3.1010 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx$

3.1010.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \mathrm{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

3.1010.2 Mathematica [N/A]

Not integrable

Time = 5.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)), x]
```

3.1010.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \textcolor{blue}{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1010.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1010.4 Maple [N/A] (verified)

Not integrable

Time = 2.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^{2} c \,x^{2}+c}}{x \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

3.1010.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1010.6 Sympy [N/A]

Not integrable

Time = 9.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(3/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**(3/2)), x)
```

3.1010.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1010.8 Giac [N/A]

Not integrable

Time = 145.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1010.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(3/2)), x)
```

3.1011 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$

3.1011.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1011.2 Mathematica [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

3.1011.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1011.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1011.4 Maple [N/A] (verified)

Not integrable

Time = 8.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1011.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(3/2), x)
```

3.1011.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1011.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1011.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1011.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2), x)
```

3.1012 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$

3.1012.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \mathrm{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1012.2 Mathematica [N/A]

Not integrable

Time = 6.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

3.1012.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1012.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1012.4 Maple [N/A] (verified)

Not integrable

Time = 2.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1012.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1012.6 Sympy [N/A]

Not integrable

Time = 62.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**(3/2), x)
```

3.1012.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1012.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1012.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2), x)
```

3.1013 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$

3.1013.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1013.2 Mathematica [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2), x]
```

3.1013.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1013.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1013.4 Maple [N/A] (verified)

Not integrable

Time = 2.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1013.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1013.6 Sympy [N/A]

Not integrable

Time = 33.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**(3/2), x)
```

3.1013.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1013.8 Giac [N/A]

Not integrable

Time = 187.64 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1013.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\,(a\,x)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(3/2), x)
```

3.1014 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx$

3.1014.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

3.1014.2 Mathematica [N/A]

Not integrable

Time = 11.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)), x]
```

3.1014.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1014.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1014.4 Maple [N/A] (verified)

Not integrable

Time = 2.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

3.1014.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1014.6 Sympy [N/A]

Not integrable

Time = 52.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**(3/2)), x)
```

3.1014.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1014.8 Giac [N/A]

Not integrable

Time = 188.36 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1014.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}(a\,x)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(3/2)), x)
```

3.1015 $\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\arctan (a x)^{3/2}} \, dx$

3.1015.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\arctan (a x)^{3/2}} \, dx=\text{Int}\left(\frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\arctan (a x)^{3/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1015.2 Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\arctan (a x)^{3/2}} \, dx=\int \frac{x^m \left(c+a^2 c x^2\right)^{5/2}}{\arctan (a x)^{3/2}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

3.1015.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan (a x)^{3/2}} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan (a x)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1015.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1015.4 Maple [N/A] (verified)

Not integrable

Time = 7.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1015.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^(3/2), x)
```

3.1015.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1015.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1015.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1015.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2), x)
```

3.1016 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx$

3.1016.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1016.2 Mathematica [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

3.1016.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1016.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1016.4 Maple [N/A] (verified)

Not integrable

Time = 2.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x{\left(a^{2} c \,x^{2}+c \right)}^{\frac{5}{2}}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1016.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1016.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1016.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1016.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1016.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2), x)
```

3.1017 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx$

3.1017.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1017.2 Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2), x]
```

3.1017.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan (ax)^{3/2}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan (ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.1017.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1017.4 Maple [N/A] (verified)

Not integrable

Time = 2.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1017.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1017.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1017.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1017.8 Giac [N/A]

Not integrable

Time = 226.75 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1017.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(3/2), x)
```

3.1018 $\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx$

3.1018.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

3.1018.2 Mathematica [N/A]

Not integrable

Time = 7.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)), x]
```

3.1018.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1018.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1018.4 Maple [N/A] (verified)

Not integrable

Time = 2.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

3.1018.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1018.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1018.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1018.8 Giac [N/A]

Not integrable

Time = 240.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1018.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^{5/2}}{x\,{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(3/2)), x)
```

3.1019 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$

3.1019.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1019.2 Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

3.1019.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1019.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1019.4 Maple [N/A] (verified)

Not integrable

Time = 7.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\arctan\left(ax\right)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1019.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

3.1019.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.1019.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1019.8 Giac [N/A]

Not integrable

Time = 64.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)^{\frac{3}{2}}}}\,dx$$

input
```
integrate(x^m/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1019.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1020 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$

3.1020.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx=\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1020.2 Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx=\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

3.1020.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1020.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1020.4 Maple [N/A] (verified)

Not integrable

Time = 3.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{\arctan(ax)^{\frac{3}{2}}\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1020.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1020.6 Sympy [N/A]

Not integrable

Time = 8.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\sqrt{c(a^2x^2+1)}\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

3.1020.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1020.8 Giac [N/A]

Not integrable

Time = 161.68 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1020.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1021 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$

3.1021.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1021.2 Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

3.1021.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1021.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1021.4 Maple [N/A] (verified)

Not integrable

Time = 3.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\arctan\left(ax\right)^{\frac{3}{2}} \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1021.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1021.6 Sympy [N/A]

Not integrable

Time = 8.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

3.1021.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1021.8 Giac [N/A]

Not integrable

Time = 206.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1021.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{{\mathrm{atan}\left(a\,x\right)}^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1022 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$

3.1022.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = -\frac{2\sqrt{c+a^2cx^2}}{acx\sqrt{\arctan(ax)}} - \frac{2\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-2*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^(1/2)-2*Unintegrable(1/x^2/(a^2*c
*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

3.1022.2 Mathematica [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

3.1022.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5477, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5477}$$

$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{a}-\frac{2\sqrt{a^2cx^2+c}}{acx\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{a}-\frac{2\sqrt{a^2cx^2+c}}{acx\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1022.3.1 Defintions of rubi rules used

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1022.4 Maple [N/A] (verified)

Not integrable

Time = 3.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x \arctan\left(ax\right)^{\frac{3}{2}} \sqrt{a^{2} c \,x^{2}+c}}dx$$

input
```
int(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1022.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1022.6 Sympy [N/A]

Not integrable

Time = 23.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/x/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

3.1022.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1022.8 Giac [N/A]

Not integrable

Time = 209.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1022.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1023 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$

3.1023.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1023.2 Mathematica [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

3.1023.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1023.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1023.4 Maple [N/A] (verified)

Not integrable

Time = 2.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 \arctan\left(ax\right)^{\frac{3}{2}} \sqrt{a^2 c\, x^2+c}}dx$$

input
```
int(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1023.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1023.6 Sympy [N/A]

Not integrable

Time = 41.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/x**2/atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

3.1023.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1023.8 Giac [N/A]

Not integrable

Time = 99.66 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1023.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1024 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1024.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1024.2 Mathematica [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

3.1024.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1024.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1024.4 Maple [N/A] (verified)

Not integrable

Time = 6.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1024.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^(3/2)), x)
```

3.1024.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1024.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1024.8 Giac [N/A]

Not integrable

Time = 89.68 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1024.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1025 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1025.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2x^3}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{6\mathrm{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} + 4a\mathrm{Int}\left(\frac{x^4}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2*x^3/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+6*Unintegrable(x^2/(a^2*c
*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a+4*a*Unintegrable(x^4/(a^2*c*x^2+c)^(3
/2)/arctan(a*x)^(1/2),x)
```

3.1025.2 Mathematica [N/A]

Not integrable

Time = 6.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

3.1025.3 Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \; 5503$$

$$\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5560$$

$$\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1025.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1025.4 Maple [N/A] (verified)

Not integrable

Time = 4.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{3}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1025.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1025.6 Sympy [N/A]

Not integrable

Time = 32.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

3.1025.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1025.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1025.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1026 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1026.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} + 2a\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
4*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1
)^(1/2)/a^3/c/(a^2*c*x^2+c)^(1/2)-2*x^2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x
)^(1/2)+2*a*Unintegrable(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.1026.2 Mathematica [N/A]

Not integrable

Time = 3.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

3.1026.3 Rubi [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5506, 5505, 3042, 3786, 3832, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}+2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1026.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.1026. $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1026.4 Maple [N/A] (verified)

Not integrable

Time = 3.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1026.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1026.6 Sympy [N/A]

Not integrable

Time = 32.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

3.1026.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1026.8 Giac [N/A]

Not integrable

Time = 209.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1026.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1027 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1027.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx=-\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
2*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1
)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)-2*x/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^
(1/2)
```

3.1027.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx=\frac{-2ax-i\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+i\sqrt{1+a^2x^2}}{a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*a*x - I*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTa
n[a*x]] + I*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]
])/(a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.1027.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5477, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[1
+ a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x
^2])
```

3.1027.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.1027.4 Maple [F]

$$\int \frac{x}{(a^2c\,x^2+c)^{\frac{3}{2}} \arctan (ax)^{\frac{3}{2}}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1027.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1027.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \int \frac{x}{(c\,(a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

3.1027.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1027.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1027.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1028 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1028.1 Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

output
```
-2*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+
1)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/
2)
```

3.1028.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \frac{-2+\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+\sqrt{1+a^2x^2}\sqrt{i}}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2 + Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]
] + Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(a*c*
Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.1028.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5437, 5506, 5505, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-2a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (2*Sqrt[2*Pi]*Sqrt[1 + a^
2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```

3.1028.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1028.4 Maple [F]

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1028.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1028.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

3.1028.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1028.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1028.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1029 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1029.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} - \frac{2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-4*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+
1)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-2/a/c/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/
2)-2*Unintegrable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a
```

3.1029.2 Mathematica [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

3.1029.3 Rubi [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 5440, 5439, 3042, 3785, 3833, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-4a\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1029.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1029.4 Maple [N/A] (verified)

Not integrable

Time = 3.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1029.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1029.6 Sympy [N/A]

Not integrable

Time = 76.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,(c\,(a^2x^2+1))^{\frac{3}{2}}\,\mathrm{atan}^{\frac{3}{2}}\,(ax)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

3.1029.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1029.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1029.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1030 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1030.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} - 6a\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2
*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-6*a*Unintegrable(1/x/(a^2*c*x^2+c)^
(3/2)/arctan(a*x)^(1/2),x)
```

3.1030.2 Mathematica [N/A]

Not integrable

Time = 11.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

3.1030.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \text{5503}$$
$$-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{5560}$$
$$-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1030.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1030.4 Maple [N/A] (verified)

Not integrable

Time = 5.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 \left(a^2 c \,x^2+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1030.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1030.6 Sympy [N/A]

Not integrable

Time = 126.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 (c (a^2x^2 + 1))^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}} (ax)} \, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

3.1030.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1030.8 Giac [N/A]

Not integrable

Time = 84.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan (ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1030.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\, dx = \int \frac{1}{x^2\,\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\, dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1031 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1031.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} - 8a\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2
*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-8*a*Unintegrable(1/x^2/(a^2*c*x^2+c
)^(3/2)/arctan(a*x)^(1/2),x)
```

3.1031.2 Mathematica [N/A]

Not integrable

Time = 14.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

3.1031.3 Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}} dx$$

$$\Big\downarrow \text{5503}$$

$$-8a \int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \sqrt{\arctan(ax)}} dx - \frac{6 \int \frac{1}{x^4 (a^2cx^2+c)^{3/2} \sqrt{\arctan(ax)}} dx}{a} - \frac{2}{acx^3 \sqrt{\arctan(ax)} \sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$-8a \int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \sqrt{\arctan(ax)}} dx - \frac{6 \int \frac{1}{x^4 (a^2cx^2+c)^{3/2} \sqrt{\arctan(ax)}} dx}{a} - \frac{2}{acx^3 \sqrt{\arctan(ax)} \sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1031.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1031.4 Maple [N/A] (verified)

Not integrable

Time = 4.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1031.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1031.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1031.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1031.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1031.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1032 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$

3.1032.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^4\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} - 10a\text{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^4/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2
*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-10*a*Unintegrable(1/x^3/(a^2*c*x^2+
c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.1032.2 Mathematica [N/A]

Not integrable

Time = 17.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

3.1032.3 Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}} dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a} - 10a\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx - \frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}}{}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a} - 10a\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx - \frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}}{}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1032.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1032.4 Maple [N/A] (verified)

Not integrable

Time = 6.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

3.1032.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1032.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1032.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1032.8 Giac [N/A]

Not integrable

Time = 85.45 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan(ax)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1032.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1033 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1033.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \mathrm{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1033.2 Mathematica [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

3.1033.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1033.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1033.4 Maple [N/A] (verified)

Not integrable

Time = 8.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1033.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^(3/2)), x)
```

3.1033.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1033.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1033.8 Giac [N/A]

Not integrable

Time = 160.48 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1033.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1034 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1034.1 Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2x^3}{ac(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
3/2*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-1/2*FresnelC(6^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2
)-2*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

3.1034.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{-\frac{8a^3cx^3}{1+a^2x^2} - ic\sqrt{1+a^2x^2}\left(3\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right) - 3\sqrt{i}\right.}{}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
((-8*a^3*c*x^3)/(1 + a^2*x^2) - I*c*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[
a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*Ar
cTan[a*x]] + Sqrt[3]*(-(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*
x]]) + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(4*a^4*c^3*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

3.1034.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5477, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{6\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{6\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{6\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^2\sqrt{a^2cx^2+c}} - \frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^3)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (6*Sqrt[1 + a^2*x
^2]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*F
resnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```

3.1034.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1034.4 Maple [F]

$$\int \frac{x^{3}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1034.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1034.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x^{3}}{\left(c \left(a^{2} x^{2}+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**(3/2)), x)
```

3.1034.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1034.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1034.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1035 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1035.1 Optimal result

Integrand size = 26, antiderivative size = 281

$$\begin{aligned}\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx &= -\frac{2x^2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\\
&\quad-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}\\
&\quad+\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}\\
&\quad+\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}\\
&\quad+\frac{\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/2*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/2*FresnelS(2^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2
)-2*x^2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

3.1035.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{-\frac{12a^2x^2}{\sqrt{\arctan(ax)}}+\sqrt{6\pi}(1+a^2x^2)^{3/2}\left(-3\sqrt{3}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
((-12*a^2*x^2)/Sqrt[ArcTan[a*x]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*(-3*Sqrt
[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[Sqrt[6/Pi]*Sqrt[ArcT
an[a*x]]]) - ((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I
)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]
*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/Sqrt[ArcTan[a*x]])/(6*a^3*c*(c + a^2*
c*x^2)^(3/2))
```

3.1035.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5503, 5506, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{4\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

3.1035. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

$$-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[1 + a^2*x
^2]*((3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]
*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*c*x^2])
 + (4*Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/2 + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sq
rt[c + a^2*c*x^2])
```

3.1035.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1035.4 Maple [F]

$$\int \frac{x^2}{(a^2 c\,x^2+c)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1035.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1035.6 Sympy [F]

$$\int \frac{x^2}{(c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \int \frac{x^2}{(c(a^2x^2 + 1))^{\frac{5}{2}} \operatorname{atan}^{\frac{3}{2}}(ax)} \, dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**(3/2)), x)
```

3.1035.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1035.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1035.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1036 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1036.1 Optimal result

Integrand size = 24, antiderivative size = 280

$$\begin{aligned}\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = & -\frac{2x}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} \\ & + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}} \\ & - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}} \\ & + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}} \\ & + \frac{\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/2*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/2*FresnelC(2^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2
)-2*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

3.1036.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.07

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{i\Big(-8iax+(1+a^2x^2)^{3/2}\sqrt{-i\arctan(ax)}\Gamma\big(\frac{1}{2},-i\arctan(ax)\big)-(1+a^2x^2)^{3/2}\sqrt{i\arctan(ax)}\Gamma\big(\frac{1}{2},i\arctan$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
((-1/4*I)*((-8*I)*a*x + (1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-I)*ArcTan[a*x]] - (1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
 I*ArcTan[a*x]] + Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
3*I)*ArcTan[a*x]] + a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gam
ma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamm
a[1/2, (3*I)*ArcTan[a*x]] - a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]
]*Gamma[1/2, (3*I)*ArcTan[a*x]]))/(a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^
2]*Sqrt[ArcTan[a*x]])
```

3.1036.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{2\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

3.1036. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

$$-\frac{4\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{4\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Sqrt[1 + a^2*x^2
]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*Fre
snelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (
2*Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])
/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt
[c + a^2*c*x^2])
```

3.1036. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1036.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1036. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1036.4 Maple [F]

$$\int \frac{x}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1036.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1036.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{(c\,(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**(3/2)), x)
```

3.1036.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1036.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1036.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1037 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1037.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\,(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}$$

output
```
-3/2*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-1/2*FresnelS(6^(1/2)/Pi^(1/2)*arctan(
a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-2
/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

3.1037.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{-8+(1+a^2x^2)^{3/2}\left(3\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+3\sqrt{i\arctan\right.}{}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8 + (1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(
-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamm
a[1/2, (3*I)*ArcTan[a*x]])))/(4*a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
])
```

3.1037.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5437, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-6a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{6a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{6\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[1 + a^2*x^2]*((
Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]*Fresnel
S[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a*c^2*Sqrt[c + a^2*c*x^2])
```

3.1037.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1037.4 Maple [F]

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1037.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1037.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\left(c\left(a^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**(3/2)), x)
```

3.1037.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1037.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1037.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1038 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1038.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{6\sqrt{2\pi}\sqrt{1+a^2x^2}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\operatorname{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-2/3*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-6*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-2/a/c/x
/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-2*Unintegrable(1/x^2/(a^2*c*x^2+c)^
(5/2)/arctan(a*x)^(1/2),x)/a
```

3.1038.2 Mathematica [N/A]

Not integrable

Time = 6.84 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

3.1038.3 Rubi [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5440, 5439, 3042, 3793, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5503}$$

$$-8a\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5440}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 5560}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1038. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1038.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1038. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1038.4 Maple [N/A] (verified)

Not integrable

Time = 2.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan \left(a x\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1038.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1038.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1038.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1038.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1038.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1039 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1039.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}} - \frac{4\mathrm{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a} - 10a\mathrm{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-4*Unintegrable(1/x^3/(a^2
*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-10*a*Unintegrable(1/x/(a^2*c*x^2+c)
^(5/2)/arctan(a*x)^(1/2),x)
```

3.1039.2 Mathematica [N/A]

Not integrable

Time = 15.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

3.1039.3 Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1039.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1039.4 Maple [N/A] (verified)

Not integrable

Time = 3.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1039.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1039.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1039.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.1039.8 Giac [N/A]

Not integrable

Time = 161.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1039.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2+c\right)^{5/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1040 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1040.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a} - 12a\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-6*Unintegrable(1/x^4/(a^2
*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-12*a*Unintegrable(1/x^2/(a^2*c*x^2+
c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.1040.2 Mathematica [N/A]

Not integrable

Time = 22.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

3.1040.3 Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-12a \int \frac{1}{x^2 (a^2cx^2+c)^{5/2} \sqrt{\arctan(ax)}} dx - \frac{6 \int \frac{1}{x^4 (a^2cx^2+c)^{5/2} \sqrt{\arctan(ax)}} dx}{a} - \frac{2}{acx^3 \sqrt{\arctan(ax)} (a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-12a \int \frac{1}{x^2 (a^2cx^2+c)^{5/2} \sqrt{\arctan(ax)}} dx - \frac{6 \int \frac{1}{x^4 (a^2cx^2+c)^{5/2} \sqrt{\arctan(ax)}} dx}{a} - \frac{2}{acx^3 \sqrt{\arctan(ax)} (a^2cx^2+c)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1040.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1040.4 Maple [N/A] (verified)

Not integrable

Time = 4.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1040.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1040.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1040.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1040.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1040.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}\left(a\,x\right)^{3/2} \left(c\,a^2\,x^2+c\right)^{5/2}} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1041 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$

3.1041.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a} - 14a\text{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-8*Unintegrable(1/x^5/(a^2
*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-14*a*Unintegrable(1/x^3/(a^2*c*x^2+
c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.1041.2 Mathematica [N/A]

Not integrable

Time = 19.89 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

3.1041.3 Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} (a^2cx^2+c)^{5/2}} dx$$
$$\Big\downarrow \text{5503}$$
$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a} - 14a\int \frac{1}{x^3 (a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx - \frac{2}{acx^4\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$
$$\Big\downarrow \text{5560}$$
$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a} - 14a\int \frac{1}{x^3 (a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx - \frac{2}{acx^4\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.1041.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1041.4 Maple [N/A] (verified)

Not integrable

Time = 5.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

3.1041.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}} \, dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1041.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

3.1041.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1041.8 Giac [N/A]

Not integrable

Time = 163.89 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{5}{2}} x^4 \arctan(ax)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1041.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{5/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1042 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$

3.1042.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1042.2 Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

3.1042.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1042.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1042.4 Maple [N/A] (verified)

Not integrable

Time = 4.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m (a^2c\,x^2+c)}{\arctan(ax)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1042.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

3.1042.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1042.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1042.8 Giac [N/A]

Not integrable

Time = 11.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1042.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(5/2), x)
```

3.1043 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$

3.1043.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1043.2 Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

3.1043.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1043.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1043.4 Maple [N/A] (verified)

Not integrable

Time = 1.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1043.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1043.6 Sympy [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = c\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
c*(Integral(x/atan(a*x)**(5/2), x) + Integral(a**2*x**3/atan(a*x)**(5/2),
x))
```

3.1043.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1043.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1043.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(5/2), x)
```

3.1044 $\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx$

3.1044.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\operatorname{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1044.2 Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2), x]
```

3.1044.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a^2cx^2+c}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1044.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1044.4 Maple [N/A] (verified)

Not integrable

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1044.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1044.6 Sympy [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x
))
```

3.1044.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1044.8 Giac [N/A]

Not integrable

Time = 195.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{a^2cx^2+c}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1044.9 Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^(5/2), x)
```

3.1045 $\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx$

3.1045.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

3.1045.2 Mathematica [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx=\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)), x]
```

3.1045.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1045.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1045.4 Maple [N/A] (verified)

Not integrable

Time = 1.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

3.1045.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1045.6 Sympy [N/A]

Not integrable

Time = 7.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = c\left(\int \frac{1}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^2x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**(5/2),x)
```

output
```
c*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(a**2*x/atan(a*x)**(5/2),
 x))
```

3.1045.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1045.8 Giac [N/A]

Not integrable

Time = 294.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \int \frac{a^2cx^2+c}{x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1045.9 Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(5/2)), x)
```

3.1046 $\int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{5/2}} dx$

3.1046.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{5/2}} dx = \operatorname{Int}\left(\frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{5/2}}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1046.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{5/2}} dx = \int \frac{x^m (c+a^2cx^2)^2}{\arctan(ax)^{5/2}} dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

3.1046.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^{5/2}} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^{5/2}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1046.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1046.4 Maple [N/A] (verified)

Not integrable

Time = 5.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2 c\,x^2+c)^2}{\arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1046.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{(a^2cx^2+c)^2x^m}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(5/2), x)
```

3.1046.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1046.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1046.8 Giac [N/A]

Not integrable

Time = 11.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1046.9 Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2), x)
```

3.1047 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$

3.1047.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1047.2 Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

3.1047.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1047.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1047.4 Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1047.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1047.6 Sympy [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(x/atan(a*x)**(5/2), x) + Integral(2*a**2*x**3/atan(a*x)**(5
/2), x) + Integral(a**4*x**5/atan(a*x)**(5/2), x))
```

3.1047.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1047.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1047.9 Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2), x)
```

3.1048 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$

3.1048.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1048.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2), x]
```

3.1048.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1048.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1048.4 Maple [N/A] (verified)

Not integrable

Time = 0.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1048.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1048.6 Sympy [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**(5/2), x) + Integral(a**4*x**4/atan(
a*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x))
```

3.1048.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1048.8 Giac [N/A]

Not integrable

Time = 231.35 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1048.9 Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\mathrm{atan}(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^(5/2), x)
```

3.1049 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx$

3.1049.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

3.1049.2 Mathematica [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)), x]
```

3.1049.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1049.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1049.4 Maple [N/A] (verified)

Not integrable

Time = 1.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

3.1049.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1049.6 Sympy [N/A]

Not integrable

Time = 7.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{1}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{2a^2x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^4x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(2*a**2*x/atan(a*x)**(
5/2), x) + Integral(a**4*x**3/atan(a*x)**(5/2), x))
```

3.1049.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1049.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1049.9 Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(5/2)), x)
```

3.1050 $\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}} \, dx$

3.1050.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}} \, dx = \text{Int}\left(\frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}}, x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1050.2 Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}} \, dx = \int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

3.1050.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^{5/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^{5/2}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1050.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1050.4 Maple [N/A] (verified)

Not integrable

Time = 6.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2 c\, x^2+c)^3}{\arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1050.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^(5/2), x)
```

3.1050.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1050.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1050.8 Giac [N/A]

Not integrable

Time = 11.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1050.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2), x)
```

3.1051 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$

3.1051.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1051.2 Mathematica [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

3.1051.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1051.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1051.4 Maple [N/A] (verified)

Not integrable

Time = 1.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1051.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1051.6 Sympy [N/A]

Not integrable

Time = 8.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(x/atan(a*x)**(5/2), x) + Integral(3*a**2*x**3/atan(a*x)**(5
/2), x) + Integral(3*a**4*x**5/atan(a*x)**(5/2), x) + Integral(a**6*x**7/a
tan(a*x)**(5/2), x))
```

3.1051.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1051.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1051.9 Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2), x)
```

3.1052 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$

3.1052.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1052.2 Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2), x]
```

3.1052.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \, 5560$$
$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1052.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1052.4 Maple [N/A] (verified)

Not integrable

Time = 1.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1052.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1052.6 Sympy [N/A]

Not integrable

Time = 7.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**(5/2), x) + Integral(3*a**4*x**4/ata
n(a*x)**(5/2), x) + Integral(a**6*x**6/atan(a*x)**(5/2), x) + Integral(ata
n(a*x)**(-5/2), x))
```

3.1052.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1052.8 Giac [N/A]

Not integrable

Time = 285.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1052.9 Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\mathrm{atan}(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^(5/2), x)
```

3.1053 $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx$

3.1053.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

3.1053.2 Mathematica [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)), x]
```

3.1053.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1053.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1053.4 Maple [N/A] (verified)

Not integrable

Time = 1.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

3.1053.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1053.6 Sympy [N/A]

Not integrable

Time = 8.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{1}{x\,\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^2x}{\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^4x^3}{\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^6x^5}{\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(3*a**2*x/atan(a*x)**(
5/2), x) + Integral(3*a**4*x**3/atan(a*x)**(5/2), x) + Integral(a**6*x**5/
atan(a*x)**(5/2), x))
```

3.1053.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1053.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1053.9 Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\left(c\,a^2\,x^2+c\right)^3}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(5/2)), x)
```

3.1054 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$

3.1054.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2x^m}{3ac\arctan(ax)^{3/2}} + \frac{2m\text{Int}\left(\frac{x^{-1+m}}{\arctan(ax)^{3/2}},x\right)}{3ac}$$

output
```
-2/3*x^m/a/c/arctan(a*x)^(3/2)+2/3*m*Unintegrable(x^(-1+m)/arctan(a*x)^(3/
2),x)/a/c
```

3.1054.2 Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

3.1054.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\,(a^2cx^2+c)}\,dx$$

$$\Big\downarrow \text{5461}$$

$$\frac{2m\int \frac{x^{m-1}}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x^m}{3ac\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5377}$$

$$\frac{2m\int \frac{x^{m-1}}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x^m}{3ac\arctan(ax)^{3/2}}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1054.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.1054.4 Maple [N/A] (verified)

Not integrable

Time = 3.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m}}{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1054.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^{m}}{{\left(a^{2} c x^{2} + c\right)} \arctan \left(a x\right)^{\frac{5}{2}}} \,d x$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

3.1054.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1054.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1054.8 Giac [N/A]

Not integrable

Time = 11.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1054.9 Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

3.1055 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$

3.1055.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2x}{3ac\arctan(ax)^{3/2}} + \frac{2\text{Int}\left(\frac{1}{\arctan(ax)^{3/2}},x\right)}{3ac}$$

output
```
-2/3*x/a/c/arctan(a*x)^(3/2)+2/3*Unintegrable(1/arctan(a*x)^(3/2),x)/a/c
```

3.1055.2 Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

3.1055.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \ 5457$$
$$\frac{2\int \frac{1}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x}{3ac\arctan(ax)^{3/2}}$$
$$\Big\downarrow \ 5353$$
$$\frac{2\int \frac{1}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x}{3ac\arctan(ax)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1055.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.1055.4 Maple [N/A] (verified)

Not integrable

Time = 1.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1055.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1055.6 Sympy [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x}{a^{2}x^{2}\operatorname{atan}^{\frac{5}{2}}{(ax)}+\operatorname{atan}^{\frac{5}{2}}{(ax)}}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c
```

3.1055.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1055.8 Giac [N/A]

Not integrable

Time = 184.65 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1055.9 Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

3.1056 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$

3.1056.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\arctan(ax)^{3/2}}$$

output
```
-2/3/a/c/arctan(a*x)^(3/2)
```

3.1056.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\arctan(ax)^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c*ArcTan[a*x]^(3/2))
```

3.1056.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \, 5419$$
$$-\frac{2}{3ac\arctan(ax)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c*ArcTan[a*x]^(3/2))
```

3.1056.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.1056.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{2}{3ac\arctan(ax)^{\frac{3}{2}}}$	15

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/c/arctan(a*x)^(3/2)
```

3.1056.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3\,ac\arctan{(ax)}^{\frac{3}{2}}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3/(a*c*arctan(a*x)^(3/2))
```

3.1056.6 Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\,\mathrm{atan}^{\frac{3}{2}}{(ax)}}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
-2/(3*a*c*atan(a*x)**(3/2))
```

3.1056.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1056.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3\,ac\arctan{(ax)}^{\frac{3}{2}}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3/(a*c*arctan(a*x)^(3/2))
```

3.1056.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3\,a\,c\,\mathrm{atan}\left(a\,x\right)^{3/2}}$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
-2/(3*a*c*atan(a*x)^(3/2))
```

3.1057 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$

3.1057.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx=-\frac{2}{3acx\arctan(ax)^{3/2}}-\frac{2\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)^{3/2}},x\right)}{3ac}$$

output
```
-2/3/a/c/x/arctan(a*x)^(3/2)-2/3*Unintegrable(1/x^2/arctan(a*x)^(3/2),x)/a
/c
```

3.1057.2 Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

3.1057.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5461, 5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{5/2} (a^2cx^2+c)} dx$$

$$\Big\downarrow \text{ 5461}$$

$$-\frac{2\int \frac{1}{x^2 \arctan(ax)^{3/2}}dx}{3ac} - \frac{2}{3acx \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5377}$$

$$-\frac{2\int \frac{1}{x^2 \arctan(ax)^{3/2}}dx}{3ac} - \frac{2}{3acx \arctan(ax)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1057.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

rule 5461
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_))/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(f*x)^m*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(
p + 1))), x] - Simp[f*(m/(b*c*d*(p + 1)))   Int[(f*x)^(m - 1)*(a + b*ArcTan
[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d]
&& LtQ[p, -1]
```

3.1057.4 Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right) \arctan\left(a x\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

3.1057.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1057.6 Sympy [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^{2}x^{3}\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)} + x\operatorname{atan}^{\frac{5}{2}}{\left(ax \right)}}\, dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**(5/2) + x*atan(a*x)**(5/2)), x)/c
```

3.1057.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1057.8 Giac [N/A]

Not integrable

Time = 183.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1057.9 Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

3.1058 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$

3.1058.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1058.2 Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

3.1058.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1058.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1058.4 Maple [N/A] (verified)

Not integrable

Time = 6.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1058.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(5/2)), x)
```

3.1058.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1058.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1058.8 Giac [N/A]

Not integrable

Time = 221.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1058.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1059 $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx$

3.1059.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = -\frac{2x^3}{3ac^2(1+a^2x^2)\arctan(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{4x^4}{3c^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^4c^2} + \frac{16}{3}\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right) + \frac{8}{3}a^2\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3*x^3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+4*FresnelS(2*arctan(a*x)^(1/2
)/Pi^(1/2))*Pi^(1/2)/a^4/c^2-4*x^2/a^2/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-4
/3*x^4/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+16/3*Unintegrable(x^3/(a^2*c*x^2+
c)^2/arctan(a*x)^(1/2),x)+8/3*a^2*Unintegrable(x^5/(a^2*c*x^2+c)^2/arctan(
a*x)^(1/2),x)
```

3.1059.2 Mathematica [N/A]

Not integrable

Time = 6.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx=\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

3.1059.3 Rubi [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5477, 5503, 5505, 4906, 27, 3042, 3786, 3832, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{x^2}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{a}+\frac{2}{3}a\int \frac{x^4}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{ac^2}+\frac{2a\int \frac{x^4}{\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}+\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5503$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}+\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5505$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{4\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

3.1059. $\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}+\frac{2\left(\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}+\frac{2\left(\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1059. $\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

3.1059.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1059. $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1059.4 Maple [N/A] (verified)

Not integrable

Time = 4.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{3}}{\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1059.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1059.6 Sympy [N/A]

Not integrable

Time = 11.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\frac{\int \frac{x^3}{a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) +
 atan(a*x)**(5/2)), x)/c**2
```

3.1059.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1059.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1059.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1060 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx$

3.1060.1 Optimal result

Integrand size = 24, antiderivative size = 180

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=-\frac{2x^2}{3ac^2\left(1+a^2x^2\right)\arctan(ax)^{3/2}}-\frac{8x}{3a^2c^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}}+\frac{16\sqrt{\arctan(ax)}}{3a^3c^2}-\frac{32\sqrt{\arctan(ax)}}{3a^3c^2\left(1+a^2x^2\right)}+\frac{16\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{3a^3c^2\left(1+a^2x^2\right)}+\frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^3c^2}$$

output
```
-2/3*x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+8/3*FresnelC(2*arctan(a*x)^(1
/2)/Pi^(1/2))*Pi^(1/2)/a^3/c^2-8/3*x/a^2/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)
+16/3*arctan(a*x)^(1/2)/a^3/c^2-32/3*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)
+16/3*(-a^2*x^2+1)*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)
```

3.1060.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\frac{-2ax(ax+4\arctan(ax))+4\sqrt{\pi}\left(1+a^2x^2\right)\arctan(ax)^{3/2}\,\text{FresnelC}\left(\frac{2\sqrt{a}}{}\right.}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*a*x*(a*x + 4*ArcTan[a*x]) + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)
*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*((-I)*Ar
cTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]] + Sqrt[2]*(1 + a^2*x^2)*(I
*ArcTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/(3*a^3*c^2*(1 + a^2*x^2
)*ArcTan[a*x]^(3/2))
```

3.1060.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5477, 27, 5467, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5477}$$

$$\frac{4\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3a}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3ac^2}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5467}$$

$$\frac{4\left(16\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx-\frac{2x}{a\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2\left(a^2x^2+1\right)}\right)}{3ac^2}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{4\left(16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)-\frac{2x}{a\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2\left(a^2x^2+1\right)}\right)}{3ac^2}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5439$$

$$\frac{4\left(16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(16\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{4\left(16\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{4\left(16\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x^2)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + (4*((-2*x)/(a*(1 + a^
2*x^2)*Sqrt[ArcTan[a*x]]) + (4*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(a^2*(1 +
a^2*x^2)) + 16*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[
a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2))))/
(3*a*c^2)
```

3.1060. $\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx$

3.1060.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.1060.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.34

method	result	size
default	$-\frac{-8\,\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}\,\arctan(ax)^{\frac{3}{2}}+4\sin(2\arctan(ax))\arctan(ax)-\cos(2\arctan(ax))+1}{3c^2a^3\arctan(ax)^{\frac{3}{2}}}$	62

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2/a^3*(-8*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)*arctan(a*
x)^(3/2)+4*sin(2*arctan(a*x))*arctan(a*x)-cos(2*arctan(a*x))+1)/arctan(a*x
)^(3/2)
```

3.1060.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1060.6 Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\, dx = \frac{\int \frac{x^2}{a^4x^4 \operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\, dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) +
 atan(a*x)**(5/2)), x)/c**2
```

3.1060.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1060.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1060.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{x^2}{\mathrm{atan}\,(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1061 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx$

3.1061.1 Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx = -\frac{2x}{3ac^2(1+a^2x^2)\arctan(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}$$

output
```
-2/3*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)-8/3*FresnelS(2*arctan(a*x)^(1/2
)/Pi^(1/2))*Pi^(1/2)/a^2/c^2-4/3*(-a^2*x^2+1)/a^2/c^2/(a^2*x^2+1)/arctan(a
*x)^(1/2)
```

3.1061.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx = -\frac{2\left(ax+(2-2a^2x^2)\arctan(ax)+4\sqrt{\pi}(1+a^2x^2)\arctan(ax)^{3/2}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{3a^2c^2(1+a^2x^2)\arctan(ax)^{3/2}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*(a*x + (2 - 2*a^2*x^2)*ArcTan[a*x] + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a
*x]^(3/2)*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]))/(3*a^2*c^2*(1 + a^2*x
^2)*ArcTan[a*x]^(3/2))
```

3.1061.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5467, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5467}$$

$$-\frac{16}{3}\int \frac{x}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{16\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{3c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 5505}$$

$$-\frac{16\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{16\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{8\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}-\frac{4(1-a^2x^2)}{3a^2c^2(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{16\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{3a^2c^2}-\frac{2x}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}-\frac{4(1-a^2x^2)}{3a^2c^2(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}-\frac{2x}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}-\frac{4(1-a^2x^2)}{3a^2c^2(a^2x^2+1)\sqrt{\arctan(ax)}}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*(1 - a^2*x^2))/(3*a^
2*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]])/(3*a^2*c^2)
```

3.1061.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.1061. $\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1061.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
default	$-\frac{8\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+4\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{3c^{2}a^{2}\arctan(ax)^{\frac{3}{2}}}$	59

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2/a^2*(8*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x
)^(3/2)+4*cos(2*arctan(a*x))*arctan(a*x)+sin(2*arctan(a*x)))/arctan(a*x)^(
3/2)
```

3.1061.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1061.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x}{a^4x^4 \operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + at
an(a*x)**(5/2)), x)/c**2
```

3.1061.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1061.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1061.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1062 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx$

3.1062.1 Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^2\left(1+a^2x^2\right)\arctan(ax)^{3/2}} + \frac{8x}{3c^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}} - \frac{16\sqrt{\arctan(ax)}}{3ac^2} + \frac{32\sqrt{\arctan(ax)}}{3ac^2\left(1+a^2x^2\right)} - \frac{16(1-a^2x^2)\sqrt{\arctan(ax)}}{3ac^2\left(1+a^2x^2\right)} - \frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3ac^2}$$

output
```
-2/3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)-8/3*FresnelC(2*arctan(a*x)^(1/2)/
Pi^(1/2))*Pi^(1/2)/a/c^2+8/3*x/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-16/3*arct
an(a*x)^(1/2)/a/c^2+32/3*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)-16/3*(-a^2*x^
2+1)*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)
```

3.1062.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx = \frac{-2+8ax\arctan(ax)-4\sqrt{\pi}(1+a^2x^2)\arctan(ax)^{3/2}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan}}{\sqrt{\pi}}\right.}{}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2 + 8*a*x*ArcTan[a*x] - 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)*Fresn
elC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*Sqrt[I*ArcTan[
a*x]]*Sqrt[ArcTan[a*x]^2]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (Sqrt[2]*(1 + a
^2*x^2)*ArcTan[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/
(3*c^2*(a + a^3*x^2)*ArcTan[a*x]^(3/2))
```

3.1062.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5437, 27, 5467, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$-\frac{4}{3}a\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5467}$$

$$-\frac{4a\left(16\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx-\frac{2x}{a\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2\left(a^2x^2+1\right)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{4a\left(16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)-\frac{2x}{a\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2\left(a^2x^2+1\right)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5439$$

$$-\frac{4a\left(16\left(\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{4a\left(16\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 3793$$

$$-\frac{4a\left(16\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{4a\left(16\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*a*((-2*x)/(a*(1 + a^2*x^
2)*Sqrt[ArcTan[a*x]]) + (4*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(a^2*(1 + a^2*
x^2)) + 16*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]
] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2))))/(3*c
^2)
```

3.1062. $\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

3.1062.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

3.1062.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.36

method	result	size
default	$\frac{-8 \operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}\, \arctan(ax)^{\frac{3}{2}}+4\sin(2\arctan(ax))\arctan(ax)-\cos(2\arctan(ax))-1}{3c^2 a\arctan(ax)^{\frac{3}{2}}}$	62

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c^2/a*(-8*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)*arctan(a*x)^
(3/2)+4*sin(2*arctan(a*x))*arctan(a*x)-cos(2*arctan(a*x))-1)/arctan(a*x)^(
3/2)
```

3.1062.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1062.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + at
an(a*x)**(5/2)), x)/c**2
```

3.1062.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1062.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1062.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1063 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$

3.1063.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x(1+a^2x^2)\arctan(ax)^{3/2}} + \frac{4}{c^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{4}{3a^2c^2x^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^2} + \frac{8\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{3a^2} + \frac{16}{3}\text{Int}\left(\frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c^2/x/(a^2*x^2+1)/arctan(a*x)^(3/2)+4*FresnelS(2*arctan(a*x)^(1/2)/
Pi^(1/2))*Pi^(1/2)/c^2+4/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+4/3/a^2/c^2/x^2
/(a^2*x^2+1)/arctan(a*x)^(1/2)+8/3*Unintegrable(1/x^3/(a^2*c*x^2+c)^2/arct
an(a*x)^(1/2),x)/a^2+16/3*Unintegrable(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/
2),x)
```

3.1063.2 Mathematica [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

3.1063.3 Rubi [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5437, 5503, 5505, 4906, 27, 3042, 3786, 3832, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5503}$$

$$-2a\int \frac{1}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{c^2x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2a\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{c^2}-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3ac^2}-\frac{2a\left(-4a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2}{a\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5503$$

$$-\frac{2a\left(-4a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx - \frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2} - \frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx - \frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2} - \frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5505$$

$$-\frac{2a\left(-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{a} - \frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2} - \frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx - \frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2} - \frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$-\frac{2a\left(-\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a} - \frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2} - \frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx - \frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2} - \frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2a\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a} - \frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2} - \frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx - \frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a} - \frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2} - \frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3042$$

3.1063. $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

$$-\frac{2a\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3786$$

$$-\frac{2a\left(-\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a}-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3832$$

$$-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2a\left(-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a}\right)}{c^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5560$$

$$-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2a\left(-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a}\right)}{c^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1063. $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

3.1063.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1063.4 Maple [N/A] (verified)

Not integrable

Time = 1.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1063.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1063.6 Sympy [N/A]

Not integrable

Time = 11.87 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^4x^5\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**(5/2) + 2*a**2*x**3*atan(a*x)**(5/2) + x*
atan(a*x)**(5/2)), x)/c**2
```

3.1063.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1063.8 Giac [N/A]

Not integrable

Time = 221.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1063.9 Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1064 $\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

3.1064.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x^2(1+a^2x^2)\arctan(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{16}{3c^2x(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{16a\sqrt{\arctan(ax)}}{c^2} + \frac{8a\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^2} + \frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a^2} + \frac{56}{3}\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+8*a*FresnelC(2*arctan(a*x)^(1
/2)/Pi^(1/2))*Pi^(1/2)/c^2+8/3/a^2/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)+1
6/3/c^2/x/(a^2*x^2+1)/arctan(a*x)^(1/2)+16*a*arctan(a*x)^(1/2)/c^2+8*Unint
egrable(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+56/3*Unintegrable(1
/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

3.1064.2 Mathematica [N/A]

Not integrable

Time = 5.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx=\int \frac{1}{x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

3.1064.3 Rubi [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5503, 5439, 3042, 3793, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\int \frac{1}{c^2x \left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx-\frac{4\int \frac{1}{c^2x^3\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3ac^2x^2 \left(a^2x^2+1\right) \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8a\int \frac{1}{x\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx}{3c^2}-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx}{3ac^2}-\frac{2}{3ac^2x^2 \left(a^2x^2+1\right) \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8a\left(-6a\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-6\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-6\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-6\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

3.1064. $\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

$$\Big\downarrow \text{2009}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}-6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}-6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1064.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.1064. $\int \frac{1}{x^2\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1064.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1064.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1064.6 Sympy [N/A]

Not integrable

Time = 16.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{1}{a^4x^6 \operatorname{atan}^{\frac{5}{2}}(ax) + 2a^2x^4 \operatorname{atan}^{\frac{5}{2}}(ax) + x^2 \operatorname{atan}^{\frac{5}{2}}(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**(5/2) + 2*a**2*x**4*atan(a*x)**(5/2) + x*
*2*atan(a*x)**(5/2)), x)/c**2
```

3.1064.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1064.8 Giac [N/A]

Not integrable

Time = 227.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan{(ax)}^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1064.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^{5/2} \left(c\,a^2\,x^2 + c\right)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1065 $\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

3.1065.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=-\frac{2}{3ac^2x^3\left(1+a^2x^2\right)\arctan(ax)^{3/2}}+\frac{4}{a^2c^2x^4\left(1+a^2x^2\right)\sqrt{\arctan(ax)}}+\frac{20}{3c^2x^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}}+\frac{16\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a^2}+\frac{112}{3}\text{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)+\frac{80}{3}a^2\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(3/2)+4/a^2/c^2/x^4/(a^2*x^2+1)/arc
tan(a*x)^(1/2)+20/3/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+16*Unintegrable(
1/x^5/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+112/3*Unintegrable(1/x^3/(a
^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)+80/3*a^2*Unintegrable(1/x/(a^2*c*x^2+c)
^2/arctan(a*x)^(1/2),x)
```

3.1065.2 Mathematica [N/A]

Not integrable

Time = 5.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c + a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^3 (c + a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

3.1065.3 Rubi [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{5/2} (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3}a\int \frac{1}{c^2x^2 (a^2x^2+1)^2 \arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{c^2x^4(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{a}-\frac{2}{3ac^2x^3 (a^2x^2+1) \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10a\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3c^2}-\frac{2\int \frac{1}{x^4(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{ac^2}-\frac{2}{3ac^2x^3 (a^2x^2+1) \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10a\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}}{2}-\frac{2}{3ac^2x^3\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{10a\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}}{2}-\frac{2}{3ac^2x^3\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1065.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1065. $\int \frac{1}{x^3(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1065.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c\,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1065.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1065.6 Sympy [N/A]

Not integrable

Time = 22.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^4x^7 \operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^5 \operatorname{atan}^{\frac{5}{2}}(ax)+x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**(5/2) + 2*a**2*x**5*atan(a*x)**(5/2) + x*
*3*atan(a*x)**(5/2)), x)/c**2
```

3.1065.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1065.8 Giac [N/A]

Not integrable

Time = 226.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1065.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1066 $\int \frac{1}{x^4(c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx$

3.1066.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx = -\frac{2}{3ac^2x^4 (1+a^2x^2) \arctan(ax)^{3/2}} + \frac{16}{3a^2c^2x^5 (1+a^2x^2) \sqrt{\arctan(ax)}} + \frac{8}{c^2x^3 (1+a^2x^2) \sqrt{\arctan(ax)}} + \frac{80\text{Int}\left(\frac{1}{x^6(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{3a^2} + \frac{184}{3}\text{Int}\left(\frac{1}{x^4 (c+a^2cx^2)^2 \sqrt{\arctan(ax)}},x\right) + 40a^2\text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)^2 \sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c^2/x^4/(a^2*x^2+1)/arctan(a*x)^(3/2)+16/3/a^2/c^2/x^5/(a^2*x^2+1)/
arctan(a*x)^(1/2)+8/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)+80/3*Unintegrabl
e(1/x^6/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+184/3*Unintegrable(1/x^4/
(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)+40*a^2*Unintegrable(1/x^2/(a^2*c*x^2+
c)^2/arctan(a*x)^(1/2),x)
```

3.1066.2 Mathematica [N/A]

Not integrable

Time = 11.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx=\int \frac{1}{x^4 \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

3.1066.3 Rubi [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5503}$$

$$-\frac{8\int \frac{1}{c^2x^5\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3a}-4a\int \frac{1}{c^2x^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx-\frac{2}{3ac^2x^4\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3ac^2}-\frac{4a\int \frac{1}{x^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{c^2}-\frac{2}{3ac^2x^4\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5503}$$

$$-\frac{4a\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-}{2}$$
$$\overline{3ac^2x^4\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \textbf{5560}$$

$$-\frac{4a\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-}{2}$$
$$\overline{3ac^2x^4\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1066.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1066. $\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1066.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

3.1066.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1066.6 Sympy [N/A]

Not integrable

Time = 29.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^4x^8\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^6\operatorname{atan}^{\frac{5}{2}}(ax)+x^4\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**(5/2) + 2*a**2*x**6*atan(a*x)**(5/2) + x*
*4*atan(a*x)**(5/2)), x)/c**2
```

3.1066.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1066.8 Giac [N/A]

Not integrable

Time = 225.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2x^4\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1066.9 Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a x)^{5/2} (c a^2 x^2 + c)^2} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

3.1067 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx$

3.1067.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}, x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1067.2 Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

3.1067.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1067.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1067.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1067.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^(5/2)), x)
```

3.1067.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1067.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1067.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1067.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1068 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx$

3.1068.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\begin{aligned}\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx &= -\frac{2x^3}{3ac^3(1+a^2x^2)^2 \arctan(ax)^{3/2}} \\ &- \frac{4x^2}{a^2c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{4x^4}{3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} \\ &+ \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^4c^3} - \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^4c^3}\end{aligned}$$

output
```
-2/3*x^3/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-4/3*FresnelS(2*arctan(a*x)^
(1/2)/Pi^(1/2))*Pi^(1/2)/a^4/c^3+4/3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4/c^3-4*x^2/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)
^(1/2)+4/3*x^4/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
```

3.1068.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.42

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx = \frac{i\sqrt{2}(1+a^2x^2)^2(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)+\sqrt{2}(1+a^2x^2)}{}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcT
an[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1
/2, (2*I)*ArcTan[a*x]] - 2*(a^2*x^2*(a*x + (6 - 2*a^2*x^2)*ArcTan[a*x]) +
I*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]]
+ (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcTan[
a*x]]))/(3*a^4*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))
```

3.1068.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5503, 27, 5477, 5503, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{x^2}{c^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{a}-\frac{2}{3}a\int \frac{x^4}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{ac^3}-\frac{2a\int \frac{x^4}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{ac^3}-\frac{2a\left(\frac{8\int \frac{x^3}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^3}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2a\left(\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{2\left(\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{4\int \frac{a^3x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2a\left(\frac{8\int \frac{a^3x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^5}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2a\left(\frac{8\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^5}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}+\frac{2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3}+\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right.}{ac^3}$$

$$\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}-\frac{2a\left(\frac{8\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^5}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}+\frac{2\left(-\frac{4\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3}+\frac{4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a}\right.}{}\right.}{a^3}\right.}{ac^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

3.1068. $\int \frac{x^3}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx$

output
```
(-2*x^3)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (2*a*((-2*x^4)/(a*(
1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (8*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[P
i]])/4))/a^5))/(3*c^3) + (2*((-2*x^2)/(a*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]
) - (4*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt
[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4))/a^3 + (4*((Sqrt[Pi/2]*F
resnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[Ar
cTan[a*x]])/Sqrt[Pi]])/4))/a^3))/(a*c^3)
```

3.1068.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1068. $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1068.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

method	result
default	$-\frac{-16\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\cos(2\arctan(ax))\arctan(}{12a^4c^3\arctan(ax)^{\frac{3}{2}}}$

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/a^4/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/
2))*arctan(a*x)^(3/2)+8*cos(2*arctan(a*x))*arctan(a*x)-8*cos(4*arctan(a*x)
)*arctan(a*x)+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

3.1068.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1068.6 Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{x^3}{a^6x^6 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2 \operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)} \, dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) +
 3*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

3.1068.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1068.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1068.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\,(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1069 $\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\, dx$

3.1069.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\begin{aligned}\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\, dx = & -\frac{2x^2}{3ac^3\left(1+a^2x^2\right)^2 \arctan(ax)^{3/2}} \\ & -\frac{8x}{3a^2c^3\left(1+a^2x^2\right)^2 \sqrt{\arctan(ax)}}+\frac{8x^3}{3c^3\left(1+a^2x^2\right)^2 \sqrt{\arctan(ax)}} \\ & +\frac{4\sqrt{2\pi}\, \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^3c^3}\end{aligned}$$

output

```
-2/3*x^2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+4/3*FresnelC(2*2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3/c^3-8/3*x/a^2/c^3/(a^2*x^2+1)
^2/arctan(a*x)^(1/2)+8/3*x^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
```

3.1069.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.01

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\, dx = \frac{\frac{2\sqrt{2\pi}\, \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3}-\frac{16\sqrt{\pi}\, \text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3}+\frac{-\frac{8x^2}{a\left(1+a^2x^2\right)^2}+\frac{32x^3}{\left(1+\cdots\right.}\cdots}{\cdots}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
((2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/a^3 - (16*Sqrt[Pi
]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/a^3 + ((-8*x^2)/(a*(1 + a^2*x^
2)^2) + (32*x^3*ArcTan[a*x])/(1 + a^2*x^2)^2 - (32*x*ArcTan[a*x])/(a + a^3
*x^2)^2 + (4*Sqrt[2]*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x
]])/a^3 + (4*Sqrt[2]*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/
a^3 + (7*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a^3 + (7
*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcTan[a*x]])/a^3)/ArcTan[a*x]^(3/
2))/(12*c^3)
```

3.1069.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. $2(129) = 258$.

Time = 2.11 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.78, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5503, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{c^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3a}-\frac{4}{3}a\int \frac{x^3}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{4a\int \frac{x^3}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

3.1069. $\int \frac{x^2}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx$

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\frac{}{a(a^2x}\right.}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\frac{}{a}\right.}{3ac^3}-\frac{4a\left(\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2\int \frac{a^4x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

3.1069. $\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\cdots\right.}{3ac^3}$$

$$\frac{4a\left(-\frac{2\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}+\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2x^3}{a\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\cdots\right.}{3ac^3}$$

$$\frac{4a\left(-\frac{2\int\left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4}+\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2x^3}{a\left(a^2x^2+1\right)^2\sqrt{\arctan\cdots}}\right.}{3c^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\cdots\right.}{3ac^3}$$

$$\frac{4a\left(\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4}-\cdots\right.}{3c^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 4906$$

3.1069. $\int \frac{x^2}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx$

$$\frac{4\left(-\frac{6\int\left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan\cdots}\right)}{a^2}\cdots\right)}{3ac^3}$$

$$\frac{4a\left(\frac{6\int\left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(\cdots}\right)}{a^4}\cdots\right)}{3c^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{4\left(-\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\cdots}\right)}{a^2}\cdots\right)}{3ac^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}-$$

$$\frac{4a\left(\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\cdots}\right)}{a^4}\cdots\right)}{3c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x^2)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (4*a*((-2*x^3)/(a*(
1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (6*(Sqrt[ArcTan[a*x]]/4 - (Sqrt[Pi/2]*
FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/a^4 - (2*((3*Sqrt[ArcTan[a*x
]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 - (Sqrt[Pi
]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/a^4))/(3*c^3) + (4*((-2*x)
/(a*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (6*(Sqrt[ArcTan[a*x]]/4 - (Sqrt[P
i/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/a^2 + (2*((3*Sqrt[ArcTa
n[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sq
rt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/a^2))/(3*a*c^3)
```

3.1069. $\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

3.1069.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1069. $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1069.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{-16\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\sin(4\arctan(ax))\arctan(ax)-\cos(4\arctan(ax))+1}{12a^3c^3\arctan(ax)^{\frac{3}{2}}}$	68

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/a^3/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))*arctan(a*x)^(3/2)+8*sin(4*arctan(a*x))*arctan(a*x)-cos(4*arctan(a
*x))+1)/arctan(a*x)^(3/2)
```

3.1069.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1069.6 Sympy [F]

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x^2}{a^6x^6\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) +
 3*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

3.1069.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1069.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1069.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1070 $\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx$

3.1070.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2 \arctan(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^2c^3} - \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^2c^3}$$

output
```
-2/3*x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-4/3*FresnelS(2*arctan(a*x)^(1
/2)/Pi^(1/2))*Pi^(1/2)/a^2/c^3-4/3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*2^(1/2)*Pi^(1/2)/a^2/c^3-4/3/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/
2)+4*x^2/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
```

3.1070.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.42

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = \frac{i\sqrt{2}\left(1+a^2x^2\right)^2\left(-i\arctan(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)+\sqrt{2}\left(1+a^2x^2\right)}{}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcT
an[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1
/2, (2*I)*ArcTan[a*x]] + 2*(-(a*x) - 2*ArcTan[a*x] + 6*a^2*x^2*ArcTan[a*x]
 + I*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x
]] + (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcT
an[a*x]]))/(3*c^3*(a + a^3*x^2)^2*ArcTan[a*x]^(3/2))
```

3.1070.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5503, 27, 5437, 5503, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{1}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3a}-2a\int \frac{x^2}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{2a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{c^3}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$-\frac{2a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{c^3}+\frac{2\left(-8a\int \frac{x}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2}{a\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(-8a\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2a\left(\frac{4\int \frac{x}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^3}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5505$$

$$\frac{2\left(-\frac{8\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2a\left(\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{4\int \frac{a^3x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 4906$$

$$\frac{2\left(-\frac{8\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2a\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3}+\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax}}\right.}{c^3}$$

$$\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a}\right)}{3ac^3}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}-$$

$$\frac{2a\left(-\frac{4\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3}+\frac{4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(2\sqrt{\right.\right.}{a^3}\right.}{c^3}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

3.1070. $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

output
```
(-2*x)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (2*a*((-2*x^2)/(a*(1
+ a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/P
i]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]
])/4))/a^3 + (4*((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 +
 (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4))/a^3))/c^3 + (2*(-
2/(a*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (8*((Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqr
t[Pi]])/4))/a))/(3*a*c^3)
```

3.1070.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1070.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

method	result
default	$-\frac{16\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\cos(2\arctan(ax))\arctan(ax}{12a^2c^3\arctan(ax)^{\frac{3}{2}}}$

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/a^2/c^3*(16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2
))*arctan(a*x)^(3/2)+8*cos(2*arctan(a*x))*arctan(a*x)+8*cos(4*arctan(a*x))
*arctan(a*x)+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

3.1070.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1070.6 Sympy [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x}{a^6x^6\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) + 3*
a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

3.1070.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1070.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1070.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1071 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx$

3.1071.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = -\frac{2}{3ac^3(1+a^2x^2)^2 \arctan(ax)^{3/2}} + \frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\,\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3ac^3} - \frac{8\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3ac^3}$$

output
```
-2/3/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-8/3*FresnelC(2*arctan(a*x)^(1/2
)/Pi^(1/2))*Pi^(1/2)/a/c^3-4/3*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/
2))*2^(1/2)*Pi^(1/2)/a/c^3+16/3*x/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)
```

3.1071.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.49

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \frac{2\left(-\frac{1}{a(1+a^2x^2)^2} + \frac{8x\arctan(ax)}{(1+a^2x^2)^2} - \frac{\sqrt{2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)}{a} + \frac{\sqrt{2}\,\mathrm{arctan}\cdots}{\cdots}\cdots\right)}{3c^3\,\mathrm{ar}\cdots}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(2*(-(1/(a*(1 + a^2*x^2)^2)) + (8*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (Sqrt[2
]*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]])/a + (Sqrt[2]*Ar
cTan[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]]) - (((-I
)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a + (ArcTan[a*x]^2*Ga
mma[1/2, (4*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]])))/(3*c^3*ArcTan[a*x]^
(3/2))
```

3.1071.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5437, 27, 5503, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{8}{3}a\int \frac{x}{c^3 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}dx-\frac{2}{3ac^3 \left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{8a\int \frac{x}{\left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}dx}{3c^3}-\frac{2}{3ac^3 \left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5503}$$

$$-\frac{8a\left(\frac{2\int \frac{1}{\left(a^2x^2+1\right)^3 \sqrt{\arctan(ax)}}dx}{a}-6a\int \frac{x^2}{\left(a^2x^2+1\right)^3 \sqrt{\arctan(ax)}}dx-\frac{2x}{a\left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3 \left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5505$$

$$-\frac{8a\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\ldots\right)}{3c^3}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 4906$$

3.1071. $\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

$$-\frac{8a\left(-\frac{6\int\left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\text{arc}}\right.}{a^2}}{3c^3}$$

$$\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8a\left(-\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right.}{a^2}}{3c^3}$$

$$\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (8*a*((-2*x)/(a*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]) - (6*(Sqrt[ArcTan[a*x]]/4 - (Sqrt[Pi/2]*FresnelC
[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/a^2 + (2*((3*Sqrt[ArcTan[a*x]])/4 +
(Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*Fresne
lC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/a^2))/(3*c^3)
```

3.1071.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1071. $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx$

3.1071.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

method	result
default	$\frac{-16\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}-32\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\sin(2\arctan(ax))\arctan(a}{12a\,c^3\arctan(ax)^{\frac{3}{2}}}$

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/a/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))*arctan(a*x)^(3/2)-32*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))
*arctan(a*x)^(3/2)+16*sin(2*arctan(a*x))*arctan(a*x)+8*sin(4*arctan(a*x))*
arctan(a*x)-4*cos(2*arctan(a*x))-cos(4*arctan(a*x))-3)/arctan(a*x)^(3/2)
```

3.1071.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1071.6 Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^6x^6\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) + 3*
a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

3.1071.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1071.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1071.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1072 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$

3.1072.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx &= -\frac{2}{3ac^3x(1+a^2x^2)^2 \arctan(ax)^{3/2}} \\ &+ \frac{20}{3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{4}{3a^2c^3x^2(1+a^2x^2)^2\sqrt{\arctan(ax)}} \\ &+ \frac{5\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3c^3} + \frac{20\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3c^3} \\ &+ \frac{8\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{3a^2} + 8\text{Int}\left(\frac{1}{x(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3/a/c^3/x/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+20/3*FresnelS(2*arctan(a*x)^(
1/2)/Pi^(1/2))*Pi^(1/2)/c^3+5/3*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))*2^(1/2)*Pi^(1/2)/c^3+20/3/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+4/3/a^2
/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+8/3*Unintegrable(1/x^3/(a^2*c*x^2
+c)^3/arctan(a*x)^(1/2),x)/a^2+8*Unintegrable(1/x/(a^2*c*x^2+c)^3/arctan(a
*x)^(1/2),x)
```

3.1072.2 Mathematica [N/A]

Not integrable

Time = 4.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

3.1072.3 Rubi [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5437, 5503, 5505, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \ 5503$$

$$-\frac{10}{3}a\int \frac{1}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{c^3x^2\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{10a\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3c^3}-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5437$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{10a\left(-8a\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5503}$$

$$-\frac{10a\left(-8a\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5505}$$

$$-\frac{10a\left(-\frac{8\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{10a\left(-\frac{8\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

3.1072. $\int \frac{1}{x(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

$$-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{\frac{10a\left(-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a}\right)}{3c^3}}{2}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5560$$

$$-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{\frac{10a\left(-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a}\right)}{3c^3}}{2}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1072.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1072.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\, x^{2}+c\right)^{3} \arctan\left(a x\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1072.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1072.6 Sympy [N/A]

Not integrable

Time = 28.97 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^6x^7\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^5\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**(5/2) + 3*a**4*x**5*atan(a*x)**(5/2) + 3*
a**2*x**3*atan(a*x)**(5/2) + x*atan(a*x)**(5/2)), x)/c**3
```

3.1072.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1072.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1072.9 Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1073 $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

3.1073.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx &= -\frac{2}{3ac^3x^2(1+a^2x^2)^2\arctan(ax)^{3/2}} \\ &+ \frac{8}{3a^2c^3x^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{8}{c^3x(1+a^2x^2)^2\sqrt{\arctan(ax)}} \\ &+ \frac{30a\sqrt{\arctan(ax)}}{c^3} + \frac{5a\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^3} \\ &+ \frac{20a\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^3} + \frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{a^2} \\ &+ \frac{80}{3}\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3/a/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+5/2*a*FresnelC(2*2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/c^3+20*a*FresnelC(2*arctan(a*x)
^(1/2)/Pi^(1/2))*Pi^(1/2)/c^3+8/3/a^2/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1
/2)+8/c^3/x/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+30*a*arctan(a*x)^(1/2)/c^3+8*U
nintegrable(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+80/3*Unintegrab
le(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.1073.2 Mathematica [N/A]

Not integrable

Time = 6.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2}}\,dx=\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

3.1073.3 Rubi [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5503, 5439, 3042, 3793, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} \left(a^2 c x^2+c\right)^3}\,dx$$

$$\Big\downarrow \ 5503$$

$$-4a\int \frac{1}{c^3 x \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}dx-\frac{4\int \frac{1}{c^3x^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{4a\int \frac{1}{x\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{c^3}-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5503$$

$$-\frac{4a\left(-10a\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-10\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-10\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-10\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

3.1073. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

$$\Big\downarrow \text{2009}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}-10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{Fresn}\right.}{c^3}$$

$$\frac{\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}}{}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}-10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{Fresn}\right.}{c^3}$$

$$\frac{\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}}{}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1073.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.1073. $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1073.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \left(a^2 c x^2 + c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}} dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1073.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1073.6 Sympy [N/A]

Not integrable

Time = 40.83 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{1}{a^6 x^8 \operatorname{atan}^{\frac{5}{2}}(ax) + 3a^4 x^6 \operatorname{atan}^{\frac{5}{2}}(ax) + 3a^2 x^4 \operatorname{atan}^{\frac{5}{2}}(ax) + x^2 \operatorname{atan}^{\frac{5}{2}}(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**(5/2) + 3*a**4*x**6*atan(a*x)**(5/2) + 3*
a**2*x**4*atan(a*x)**(5/2) + x**2*atan(a*x)**(5/2)), x)/c**3
```

3.1073.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1073.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2}} \, dx=\text{Timed out}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1073.9 Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2}} \, dx=\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3} \,d x$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1074 $\int \frac{1}{x^3(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$

3.1074.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x^3\left(1+a^2x^2\right)^2\arctan(ax)^{3/2}} + \frac{4}{a^2c^3x^4\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} + \frac{28}{3c^3x^2\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} + \frac{16\mathrm{Int}\left(\frac{1}{x^5(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{a^2} + \frac{152}{3}\mathrm{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}},x\right) + 56a^2\mathrm{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+4/a^2/c^3/x^4/(a^2*x^2+1)^2
/arctan(a*x)^(1/2)+28/3/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+16*Unintegrable(1/x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+152/3*Unintegrable(1/
x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)+56*a^2*Unintegrable(1/x/(a^2*c*x^
2+c)^3/arctan(a*x)^(1/2),x)
```

3.1074.2 Mathematica [N/A]

Not integrable

Time = 6.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^3 \arctan (a x)^{5/2}} \, dx=\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^3 \arctan (a x)^{5/2}} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

3.1074.3 Rubi [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan (a x)^{5/2} \left(a^2 c x^2+c\right)^3} \, dx$$

$$\Big\downarrow \ 5503$$

$$-\frac{14}{3} a \int \frac{1}{c^3 x^2 \left(a^2 x^2+1\right)^3 \arctan (a x)^{3/2}}dx-\frac{2 \int \frac{1}{c^3 x^4 \left(a^2 x^2+1\right)^3 \arctan (a x)^{3/2}}dx}{a}-\frac{2}{3 a c^3 x^3 \left(a^2 x^2+1\right)^2 \arctan (a x)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{14 a \int \frac{1}{x^2 \left(a^2 x^2+1\right)^3 \arctan (a x)^{3/2}}dx}{3 c^3}-\frac{2 \int \frac{1}{x^4 \left(a^2 x^2+1\right)^3 \arctan (a x)^{3/2}}dx}{a c^3}-\frac{2}{3 a c^3 x^3 \left(a^2 x^2+1\right)^2 \arctan (a x)^{3/2}}$$

$$\Big\downarrow \ 5503$$

$$-\frac{14a\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2}{3ac^3x^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5560$$

$$-\frac{14a\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2}{3ac^3x^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1074.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1074. $\int \frac{1}{x^3(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1074.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1074.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1074.6 Sympy [N/A]

Not integrable

Time = 52.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^6x^9\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^7\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^5\operatorname{atan}^{\frac{5}{2}}(ax)+x^3\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**(5/2) + 3*a**4*x**7*atan(a*x)**(5/2) + 3*
a**2*x**5*atan(a*x)**(5/2) + x**3*atan(a*x)**(5/2)), x)/c**3
```

3.1074.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1074.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1074.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1075 $\int \frac{1}{x^4(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx$

3.1075.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx &= -\frac{2}{3ac^3x^4 (1+a^2x^2)^2 \arctan(ax)^{3/2}} \\ &+ \frac{16}{3a^2c^3x^5 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} + \frac{32}{3c^3x^3 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} \\ &+ \frac{80\text{Int}\left(\frac{1}{x^6(c+a^2cx^2)^3\sqrt{\arctan(ax)}}, x\right)}{3a^2} + 80\text{Int}\left(\frac{1}{x^4 (c+a^2cx^2)^3 \sqrt{\arctan(ax)}}, x\right) \\ &+ \frac{224}{3}a^2\text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)^3 \sqrt{\arctan(ax)}}, x\right)\end{aligned}$$

output
```
-2/3/a/c^3/x^4/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+16/3/a^2/c^3/x^5/(a^2*x^2+1
)^2/arctan(a*x)^(1/2)+32/3/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+80/3*Un
integrable(1/x^6/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+80*Unintegrable(
1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)+224/3*a^2*Unintegrable(1/x^2/(a
^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

3.1075.2 Mathematica [N/A]

Not integrable

Time = 13.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

3.1075.3 Rubi [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 27, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} (a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \ 5503$$

$$-\frac{8\int \frac{1}{c^3x^5(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3a}-\frac{16}{3}a\int \frac{1}{c^3x^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx-\frac{2}{3ac^3x^4(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{16a\int \frac{1}{x^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3c^3}-\frac{2}{3ac^3x^4(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5503$$

$$-\frac{16a\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-18a\int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^4\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}}{}$$

$$\Big\downarrow \; 5560$$

$$-\frac{16a\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-18a\int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^4\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}}{}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1075.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1075.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

3.1075.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1075.6 Sympy [N/A]

Not integrable

Time = 64.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^6x^{10}\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^8\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^6\operatorname{atan}^{\frac{5}{2}}(ax)+x^4\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**(5/2) + 3*a**4*x**8*atan(a*x)**(5/2) + 3
*a**2*x**6*atan(a*x)**(5/2) + x**4*atan(a*x)**(5/2)), x)/c**3
```

3.1075.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1075.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1075.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

3.1076 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$

3.1076.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

3.1076.2 Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

3.1076.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1076.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1076.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

3.1076.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

3.1076.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1076.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1076.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1076.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}} \,d x$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2), x)
```

3.1077 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$

3.1077.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

3.1077.2 Mathematica [N/A]

Not integrable

Time = 3.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

3.1077.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1077.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1077.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

3.1077.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1077.6 Sympy [N/A]

Not integrable

Time = 48.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(5/2), x)
```

3.1077.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1077.8 Giac [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1077.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2), x)
```

3.1078 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$

3.1078.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

3.1078.2 Mathematica [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2), x]
```

3.1078.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \textcolor{blue}{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1078.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1078.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

3.1078.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1078.6 Sympy [N/A]

Not integrable

Time = 46.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^{\frac{5}{2}}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(5/2), x)
```

3.1078.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1078.8 Giac [N/A]

Not integrable

Time = 164.75 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1078.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(5/2), x)
```

3.1079 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx$

3.1079.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

3.1079.2 Mathematica [N/A]

Not integrable

Time = 10.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)), x]
```

3.1079.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1079.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1079.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

3.1079.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1079.6 Sympy [N/A]

Not integrable

Time = 61.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(5/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**(5/2)), x)
```

3.1079.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1079.8 Giac [N/A]

Not integrable

Time = 165.70 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1079.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(5/2)), x)
```

3.1080 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$

3.1080.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1080.2 Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

3.1080.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^{5/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^{5/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1080.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1080.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1080.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(5/2), x)
```

3.1080.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1080.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1080.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1080.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2), x)
```

3.1081 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$

3.1081.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1081.2 Mathematica [N/A]

Not integrable

Time = 3.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

3.1081.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1081.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1081.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1081.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1081.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1081.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1081.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1081.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2), x)
```

3.1082 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$

3.1082.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1082.2 Mathematica [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2), x]
```

3.1082.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1082.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1082.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1082.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1082.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1082.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1082.8 Giac [N/A]

Not integrable

Time = 208.94 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1082.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(5/2), x)
```

3.1083 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx$

3.1083.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \mathrm{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

3.1083.2 Mathematica [N/A]

Not integrable

Time = 11.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)), x]
```

3.1083.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1083.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1083.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

3.1083.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1083.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1083.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1083.8 Giac [N/A]

Not integrable

Time = 210.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1083.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x\,{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(5/2)), x)
```

3.1084 $\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\arctan(ax)^{5/2}}\,dx$

3.1084.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1084.2 Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

3.1084.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)^{5/2}}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1084.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1084.4 Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1084.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^(5/2), x)
```

3.1084.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1084.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c + a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1084.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c + a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1084.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c + a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2), x)
```

3.1085 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$

3.1085.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1085.2 Mathematica [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

3.1085.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1085.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1085.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1085.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1085.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1085.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1085.8 Giac [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1085.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2), x)
```

3.1086 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$

3.1086.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1086.2 Mathematica [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2), x]
```

3.1086.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^{5/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

3.1086.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1086.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1086.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1086.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1086.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1086.8 Giac [N/A]

Not integrable

Time = 255.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1086.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(5/2), x)
```

3.1087 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^{5/2}}\,dx$

3.1087.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

3.1087.2 Mathematica [N/A]

Not integrable

Time = 8.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^{5/2}}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)), x]
```

3.1087.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^{5/2}}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1087.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1087.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}{x \arctan\left(a x\right)^{\frac{5}{2}}}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

3.1087.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1087.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1087.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1087.8 Giac [N/A]

Not integrable

Time = 254.81 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1087.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(5/2)), x)
```

3.1088 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$

3.1088.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1088.2 Mathematica [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

3.1088.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1088.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1088.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\arctan(ax)^{\frac{5}{2}}\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1088.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

3.1088.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.1088.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1088.8 Giac [N/A]

Not integrable

Time = 15.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1088.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1089 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$

3.1089.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1089.2 Mathematica [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

3.1089.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \, 5560$$

$$\int \frac{x}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1089.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1089.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{\arctan\left(a x\right)^{\frac{5}{2}} \sqrt{a^{2} c\,x^{2}+c}}dx$$

input
```
int(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1089.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1089.6 Sympy [N/A]

Not integrable

Time = 60.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

3.1089.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1089.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1089.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1090 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$

3.1090.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1090.2 Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

3.1090.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1090.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1090.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\arctan\left(a x\right)^{\frac{5}{2}} \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1090.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1090.6 Sympy [N/A]

Not integrable

Time = 61.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(1/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

3.1090.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1090.8 Giac [N/A]

Not integrable

Time = 263.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1090.9 Mupad [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1091 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$

3.1091.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=-\frac{2\sqrt{c+a^2cx^2}}{3acx\arctan(ax)^{3/2}}-\frac{2\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)}{3a}$$

output
```
-2/3*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^(3/2)-2/3*Unintegrable(1/x^2/ar
ctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

3.1091.2 Mathematica [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

3.1091.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5477, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5477}$$
$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^{3/2}}dx}{3a}-\frac{2\sqrt{a^2cx^2+c}}{3acx\arctan(ax)^{3/2}}$$
$$\Big\downarrow \text{5560}$$
$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^{3/2}}dx}{3a}-\frac{2\sqrt{a^2cx^2+c}}{3acx\arctan(ax)^{3/2}}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1091.3.1 Defintions of rubi rules used

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1091.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x \arctan \left(a x\right)^{\frac{5}{2}} \sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1091.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1091.6 Sympy [N/A]

Not integrable

Time = 126.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^{\frac{5}{2}}\,(ax)}\,dx$$

input
```
integrate(1/x/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

3.1091.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1091.8 Giac [N/A]

Not integrable

Time = 299.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan\,(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1091.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1092 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$

3.1092.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1092.2 Mathematica [N/A]

Not integrable

Time = 11.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

3.1092.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x^2 \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1092.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1092.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 \arctan\left(ax\right)^{\frac{5}{2}} \sqrt{a^2c\,x^2+c}}dx$$

input
```
int(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

3.1092.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1092.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.1092.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1092.8 Giac [N/A]

Not integrable

Time = 261.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^2/arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1092.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

3.1093 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1093.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1093.2 Mathematica [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

3.1093.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1093.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1093.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1093.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^(5/2)), x)
```

3.1093.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1093.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1093.8 Giac [N/A]

Not integrable

Time = 133.75 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1093.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1094 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1094.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} \\ &- \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{8x^4}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} \\ &+ \frac{8\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} \\ &+ \frac{44}{3}\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right) + 8a^2\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3*x^3/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8*FresnelS(2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c/(a^2*c*x^
2+c)^(1/2)-4*x^2/a^2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-8/3*x^4/c/(a^
2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+44/3*Unintegrable(x^3/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)^(1/2),x)+8*a^2*Unintegrable(x^5/(a^2*c*x^2+c)^(3/2)/arctan(
a*x)^(1/2),x)
```

3.1094.2 Mathematica [N/A]

Not integrable

Time = 7.78 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

3.1094.3 Rubi [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5503, 5506, 5505, 3042, 3786, 3832, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{x^2}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{a}+\frac{4}{3}a\int \frac{x^4}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}+\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}+2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}+\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

3.1094. $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1094. $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1094.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1094. $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1094.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{3}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1094.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1094.6 Sympy [N/A]

Not integrable

Time = 162.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

3.1094.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1094.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1094.9 Mupad [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1095 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1095.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = &-\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\\ &-\frac{8x}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}-\frac{4x^3}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\\ &+\frac{8\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^3c\sqrt{c+a^2cx^2}}\\ &+4\text{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)+\frac{8}{3}a^2\text{Int}\left(\frac{x^4}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3*x^2/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/3*FresnelC(2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c/(a^2*c*
x^2+c)^(1/2)-8/3*x/a^2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-4/3*x^3/c/(
a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+4*Unintegrable(x^2/(a^2*c*x^2+c)^(3/2
)/arctan(a*x)^(1/2),x)+8/3*a^2*Unintegrable(x^4/(a^2*c*x^2+c)^(3/2)/arctan
(a*x)^(1/2),x)
```

3.1095.2 Mathematica [N/A]

Not integrable

Time = 6.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

3.1095.3 Rubi [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5477, 5440, 5439, 3042, 3785, 3833, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{3a}+\frac{2}{3}a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5477}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{4\left(\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx+\frac{4\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

3.1095. $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$

$$\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{4\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5560$$

$$\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{4\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1095.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.1095. $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1095.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1095.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1095.6 Sympy [N/A]

Not integrable

Time = 159.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

3.1095.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1095.8 Giac [N/A]

Not integrable

Time = 296.32 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1095.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1096 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1096.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=-\frac{2x}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}-\frac{4}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}-\frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^2c\sqrt{c+a^2cx^2}}$$

output
```
-2/3*x/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)-4/3*FresnelS(2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c/(a^2*c*x^
2+c)^(1/2)-4/3/a^2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)
```

3.1096.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=-\frac{2\left(ax+2\arctan(ax)-i\sqrt{1+a^2x^2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+i\sqrt{1+a^2x^2}(i\arctan(ax))^{3/2}\Gamma\right.}{3a^2c\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*(a*x + 2*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Ga
mma[1/2, (-I)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gam
ma[1/2, I*ArcTan[a*x]]))/(3*a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

3.1096.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5477, 5437, 5506, 5505, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5437}$$

$$\frac{2\left(-2a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{2\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{2\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\left(-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + (2*(-2/(a*c*Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (2*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS
[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])))/(3*a)
```

3.1096.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1096.4 Maple [F]

$$\int \frac{x}{(a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^{\frac{5}{2}}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1096.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1096.6 Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

3.1096.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1096.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1096.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1097 $\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1097.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3ac\sqrt{c+a^2cx^2}}$$

output
```
-2/3/a/c/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)-4/3*FresnelC(2^(1/2)/Pi^(1/
2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c/(a^2*c*x^2+c)
^(1/2)+4/3*x/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)
```

3.1097.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \frac{-2+4ax\arctan(ax)-2\sqrt{1+a^2x^2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan(\right.}{3ac\sqrt{c+a^2cx^2}\arctan(}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2 + 4*a*x*ArcTan[a*x] - 2*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Gam
ma[1/2, (-I)*ArcTan[a*x]] - 2*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamm
a[1/2, I*ArcTan[a*x]])/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

3.1097.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5437, 5477, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-\frac{2}{3}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5477}$$

$$-\frac{2}{3}a\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{2}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int\frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (2*a*((-2*x)/(a*c*Sqrt[
c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*Fresne
lC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])))/3
```

3.1097.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

3.1097.4 Maple [F]

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1097.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1097.6 Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(5/2)), x)
```

3.1097.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1097.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1097.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1098 $\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1098.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2}{3acx\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\\ &+\frac{8}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\\ &+\frac{8\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3c\sqrt{c+a^2cx^2}}\\ &+\frac{8\text{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)}{3a^2}+4\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3/a/c/x/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8/3*FresnelS(2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+c)
^(1/2)+8/3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+4/3/a^2/c/x^2/(a^2*c*x^
2+c)^(1/2)/arctan(a*x)^(1/2)+8/3*Unintegrable(1/x^3/(a^2*c*x^2+c)^(3/2)/ar
ctan(a*x)^(1/2),x)/a^2+4*Unintegrable(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^
(1/2),x)
```

3.1098.2 Mathematica [N/A]

Not integrable

Time = 6.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

3.1098.3 Rubi [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5437, 5503, 5506, 5505, 3042, 3786, 3832, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{4}{3}a\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5437}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{4}{3}a\left(-2a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{4}{3}a\left(-2a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5506

$$-\frac{4}{3}a\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5505

$$-\frac{4}{3}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3042

$$-\frac{4}{3}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3786

3.1098. $\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$

$$-\frac{4}{3}a\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-$$

$$\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-$$

$$\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3832$$

$$-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-$$

$$\frac{4}{3}a\left(-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-$$

$$\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 5560$$

$$-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-$$

$$\frac{4}{3}a\left(-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-$$

$$\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1098. $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1098.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1098. $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1098.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1098.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1098.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1098.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1098.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1098.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1099 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1099.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} \\ &+ \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{4}{cx\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} \\ &+ \frac{8a\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+ \frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a^2} + \frac{44}{3}\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3/a/c/x^2/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+8*a*FresnelC(2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c/(a^2*c*x^2+
c)^(1/2)+8/3/a^2/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+4/c/x/(a^2*c*
x^2+c)^(1/2)/arctan(a*x)^(1/2)+8*Unintegrable(1/x^4/(a^2*c*x^2+c)^(3/2)/ar
ctan(a*x)^(1/2),x)/a^2+44/3*Unintegrable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(
a*x)^(1/2),x)
```

3.1099.2 Mathematica [N/A]

Not integrable

Time = 9.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

3.1099.3 Rubi [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 5503, 5440, 5439, 3042, 3785, 3833, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} (a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5503}$$

$$-2a\int \frac{1}{x (a^2cx^2+c)^{3/2} \arctan(ax)^{3/2}}dx - \frac{4\int \frac{1}{x^3 (a^2cx^2+c)^{3/2} \arctan(ax)^{3/2}}dx}{3a} - \frac{2}{3acx^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5503}$$

$$-\frac{-2a\left(-4a\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}}{} - \frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-2a\left(-\frac{4a\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a} - \frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a} - \frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-2a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a} - \frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

3.1099. $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

$$\Big\downarrow \text{3785}$$

$$-2a\left(-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-$$
$$\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-$$
$$\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$-2a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-$$
$$\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-$$
$$\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$-2a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-$$
$$\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-$$
$$\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1099. $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1099.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1099. $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1099.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a^2 c x^2 + c)^{\frac{3}{2}} \arctan(ax)^{\frac{5}{2}}} dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1099.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1099.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2}} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1099.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1099.8 Giac [N/A]

Not integrable

Time = 214.48 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1099.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{5/2} (c\,a^2\,x^2 + c)^{3/2}}\,dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1100 $\int \frac{1}{x^3 (c+a^2 c x^2)^{3/2} \arctan (a x)^{5/2}} \, dx$

3.1100.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan (a x)^{5/2}} \, dx = -\frac{2}{3 a c x^3 \sqrt{c+a^2 c x^2} \arctan (a x)^{3/2}}+\frac{4}{a^2 c x^4 \sqrt{c+a^2 c x^2} \sqrt{\arctan (a x)}}+\frac{16}{3 c x^2 \sqrt{c+a^2 c x^2} \sqrt{\arctan (a x)}}+\frac{16 \text{Int}\left(\frac{1}{x^5 \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan (a x)}},x\right)}{a^2}+\frac{92}{3} \text{Int}\left(\frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan (a x)}},x\right)+16 a^2 \text{Int}\left(\frac{1}{x \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan (a x)}},x\right)$$

output
```
-2/3/a/c/x^3/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+4/a^2/c/x^4/(a^2*c*x^2+
c)^(1/2)/arctan(a*x)^(1/2)+16/3/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2
)+16*Unintegrable(1/x^5/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a^2+92/3*
Unintegrable(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)+16*a^2*Uninteg
rable(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.1100.2 Mathematica [N/A]

Not integrable

Time = 14.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

3.1100.3 Rubi [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{5/2} (a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \arctan(ax)^{3/2}}dx - \frac{2\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{a} - \frac{2}{3acx^3 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\left(-6a\int\frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int\frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2\left(-\frac{8\int\frac{1}{x^5\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-10a\int\frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{\frac{a}{2}} -$$
$$\frac{2}{3acx^3\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{8}{3}a\left(-6a\int\frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int\frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2\left(-\frac{8\int\frac{1}{x^5\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-10a\int\frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{\frac{a}{2}} -$$
$$\frac{2}{3acx^3\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1100.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1100.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1100.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{5/2}} \, dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1100.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1100.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1100.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1100.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1101 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$

3.1101.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^4\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}+\frac{16}{3a^2cx^5\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{20}{3cx^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{80\text{Int}\left(\frac{1}{x^6(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{3a^2}+52\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)+\frac{80}{3}a^2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x^4/arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2)+16/3/a^2/c/x^5/(a^2*c*x
^2+c)^(1/2)/arctan(a*x)^(1/2)+20/3/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(
1/2)+80/3*Unintegrable(1/x^6/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a^2+
52*Unintegrable(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)+80/3*a^2*Un
integrable(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

3.1101.2 Mathematica [N/A]

Not integrable

Time = 33.78 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

3.1101.3 Rubi [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} (a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{10}{3}a\int \frac{1}{x^3 (a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2}{3acx^4\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3}a\left(-8a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+}}\right.$$

$$\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^4\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{5560}$$

$$-\frac{10}{3}a\left(-8a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+}}\right.$$

$$\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^4\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1101.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1101.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

3.1101.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1101.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1101.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1101.8 Giac [N/A]

Not integrable

Time = 218.81 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1101.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

3.1102 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1102.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}},x\right)$$

output
```
Unintegrable(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1102.2 Mathematica [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

3.1102.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1102.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1102.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1102.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^(5/2)), x)
```

3.1102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1102.8 Giac [N/A]

Not integrable

Time = 241.73 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1102.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1103 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}} \, dx$

3.1103.1 Optimal result

Integrand size = 26, antiderivative size = 190

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = -\frac{2x^3}{3ac\,(c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} - \frac{4x^2}{a^2c\,(c+a^2cx^2)^{3/2} \sqrt{\arctan(ax)}} - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/3*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-FresnelS(2^(1/2)/Pi^(1/
2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^
2+c)^(1/2)+FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(
a^2*x^2+1)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-4*x^2/a^2/c/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)^(1/2)
```

3.1103.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.34

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-2a^2x^2(ax+6\arctan(ax))+\sqrt{6\pi}(1+a^2x^2)^{3/2}\arctan(ax)^{3/2}\left(-3\sqrt{3}\right.}{}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*a^2*x^2*(a*x + 6*ArcTan[a*x]) + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*ArcTan[
a*x]^(3/2)*(-3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]]) - (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*(3*Sqrt[(-
I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma
[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*A
rcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(3*a^4*
c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

3.1103.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5477, 5503, 5506, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5506$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^3}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5505$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\cdots}{ac\sqrt{\arctan(a\cdots}}\right.}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

3.1103. $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\right.}{a^3c^2\sqrt{a^2cx^2+c}}\right.}{a}$$

$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 4906$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\right.\right.}{}\right.}{a}$$

$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\right.}{}\right.}{a}$$

$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x^3)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + (2*((-2*x^2)/(a
*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[1 + a^2*x^2]*((3*Sqr
t[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[
1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqr
t[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*
c*x^2])))/a
```

3.1103. $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

3.1103. $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1103.4 Maple [F]

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1103.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1103.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1103.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1104 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1104.1 Optimal result

Integrand size = 26, antiderivative size = 224

$$\begin{aligned}\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2x^2}{3ac\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}} \\ &\quad-\frac{8x}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}+\frac{4x^3}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}} \\ &\quad-\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^3c^2\sqrt{c+a^2cx^2}} \\ &\quad+\frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/3*x^2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-1/3*FresnelC(2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*
c*x^2+c)^(1/2)+FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/
2)*(a^2*x^2+1)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)-8/3*x/a^2/c/(a^2*c*x^2+c)
^(3/2)/arctan(a*x)^(1/2)+4/3*x^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

3.1104.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-(1+a^2x^2)^{3/2}\,(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+\frac{-4a^2x^2\sqrt{i\arctan}}{}}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-((1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-I)*ArcTan[a*x
]]) + (-4*a^2*x^2*Sqrt[I*ArcTan[a*x]] + (16*I)*a*x*(I*ArcTan[a*x])^(3/2) -
 (8*I)*a^3*x^3*(I*ArcTan[a*x])^(3/2) + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*G
amma[1/2, I*ArcTan[a*x]] - (3*I)*Sqrt[3]*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*S
qrt[ArcTan[a*x]^2]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*
ArcTan[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2
]*ArcTan[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/(6*a^3
*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

3.1104.3 Rubi [A] (verified)

Time = 2.98 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5503, 5477, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5477}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{5503}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{5440}$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{5439}$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

3.1104. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{4\left(-4a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5506$$

$$\frac{4\left(-\frac{4a\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

3.1104. $\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$\Big\downarrow \text{5505}$$

$$\frac{4\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right.\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{3a}$$

$$\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{4\left(-\frac{4\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\right.\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{3a}$$

$$\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(-\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right.}{a^2c^2\sqrt{a^2cx^2}}\right.}{3a}$$

$$\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}-$$

$$\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(\right.$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

3.1104. $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

output
```
(-2*x^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (2*a*((-2*x^3)/
(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (6*Sqrt[1 + a^2*x^2]*((Sqr
t[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelC[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^4*c^2*Sqrt[c + a^2*c*x^2])))/3 + (4*(
(-2*x)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Sqrt[1 + a^2*x^2
]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*Fre
snelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (
2*Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])
/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt
[c + a^2*c*x^2])))/(3*a)
```

3.1104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1104.4 Maple [F]

$$\int \frac{x^2}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan \left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1104.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1104.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1104.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.1104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1105 $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1105.1 Optimal result

Integrand size = 24, antiderivative size = 222

$$\begin{aligned}\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2x}{3ac\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}} \\ &\quad -\frac{4}{3a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}+\frac{8x^2}{3c\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}} \\ &\quad -\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^2c^2\sqrt{c+a^2cx^2}} \\ &\quad -\frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/3*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-1/3*FresnelS(2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*
x^2+c)^(1/2)-FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)
*(a^2*x^2+1)^(1/2)/a^2/c^2/(a^2*c*x^2+c)^(1/2)-4/3/a^2/c/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)^(1/2)+8/3*x^2/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
```

3.1105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.18

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-12(ax+(2-4a^2x^2)\arctan(ax))+4\sqrt{6\pi}(1+a^2x^2)^{3/2}\arctan(ax)^{3/2}}{}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-12*(a*x + (2 - 4*a^2*x^2)*ArcTan[a*x]) + 4*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2
)*ArcTan[a*x]^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Fr
esnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 7*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*
(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a
*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])
))/(18*a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

3.1105.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5503, 5437, 5503, 5506, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{4}{3}a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2x}{3ac\arctan(ax)^{3/2}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$-\frac{4}{3}a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx+\frac{2\left(-6a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5503$$

$$\frac{2\left(-6a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5506$$

$$\frac{2\left(-\frac{6a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^3}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5505$$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

3.1105. $\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(a}}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\,\mathrm{ar}}{a^3c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\mathrm{Fresn}\right.}{}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2\sqrt{a^2}}{}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

3.1105. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}-$$

$$\frac{4}{3}a\left(-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\right.\right.}{a^3c^2\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (4*a*((-2*x^2)/(a
*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[1 + a^2*x^2]*((3*Sqr
t[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[
1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqr
t[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*
c*x^2])))/3 + (2*(-2/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Sq
rt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (
Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a*c^2*Sqrt[c + a^2
*c*x^2])))/(3*a)
```

3.1105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1105. $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1105.4 Maple [F]

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1105.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1105.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1105.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1106 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1106.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\begin{aligned}\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2}{3ac\,(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\\ &+\frac{4x}{c\,(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}-\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}\\ &-\frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-FresnelC(2^(1/2)/Pi^(1/2)*a
rctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(
1/2)-FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^
2+1)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+4*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)
^(1/2)
```

3.1106.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.64

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-4+24ax\arctan(ax)-3\left(1+a^2x^2\right)^{3/2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan\right.}{}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-4 + 24*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*
Gamma[1/2, (-I)*ArcTan[a*x]] - 3*(1 + a^2*x^2)^(3/2)*(I*ArcTan[a*x])^(3/2)
*Gamma[1/2, I*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*((-I)*ArcTan[a*x])^(3/2
)*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*((-I)*Arc
Tan[a*x])^(3/2)*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*(I*
ArcTan[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^
2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]])/(6*c^2*(a + a^
3*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

3.1106.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5437, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-2a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-2a\left(\frac{2\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

3.1106. $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$-2a\left(-4a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5506}$$

$$-2a\left(-\frac{4a\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{Fresne}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5505}$$

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 4906}$$

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\right.\right.}{}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\right.}{}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

3.1106. $\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

output
```
-2/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - 2*a*((-2*x)/(a*c*(c +
 a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*F
resnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]
*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x
^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]
*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2])
)
```

3.1106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

3.1106. $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

3.1106.4 Maple [F]

$$\int \frac{1}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{5}{2}}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1106.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1106.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1106.8 Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1107 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1107.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2}{3acx(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\\
&+\frac{16}{3c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}+\frac{4}{3a^2cx^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\\
&+\frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&+\frac{4\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\mathrm{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&+\frac{8\mathrm{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{3a^2}+\frac{20}{3}\mathrm{Int}\left(\frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3/a/c/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+4/3*FresnelS(6^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+
c)^(1/2)+4*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(
a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+16/3/c/(a^2*c*x^2+c)^(3/2)/arctan
(a*x)^(1/2)+4/3/a^2/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+8/3*Uninte
grable(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+20/3*Unintegrabl
e(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.1107.2 Mathematica [N/A]

Not integrable

Time = 9.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

3.1107.3 Rubi [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5437, 5503, 5506, 5505, 4906, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{8}{3}a\left(-6a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\left(-6a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 5506$$

$$-\frac{8}{3}a\left(-\frac{6a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 5505$$

$$-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 4906$$

$$-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 2009$$

3.1107. $\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$-\frac{2\left(-10a\int \frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}}\right.$$

$$\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\left(-10a\int \frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}}\right.$$

$$\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1107. $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1107.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1107.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1107.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1107.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1107.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\,(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1108 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1108.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2}{3acx^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}} \\ &+ \frac{8}{3a^2cx^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} + \frac{20}{3cx(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} \\ &+ \frac{20a\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{20a\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{3c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a^2} + \frac{68}{3}\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3/a/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+20/9*a*FresnelC(6^(1/2)
/Pi^(1/2)*arctan(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c
*x^2+c)^(1/2)+20*a*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))*2^(1/2)*Pi
^(1/2)*(a^2*x^2+1)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+8/3/a^2/c/x^3/(a^2*c*x^2+
c)^(3/2)/arctan(a*x)^(1/2)+20/3/c/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+
8*Unintegrable(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+68/3*Uni
ntegrable(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.1108.2 Mathematica [N/A]

Not integrable

Time = 9.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

3.1108.3 Rubi [N/A]

Not integrable

Time = 2.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5503, 5503, 5440, 5439, 3042, 3793, 2009, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} \left(a^2 c x^2+c\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3} a \int \frac{1}{x \left(a^2 c x^2+c\right)^{5/2} \arctan(ax)^{3/2}}dx-\frac{4 \int \frac{1}{x^3 \left(a^2 c x^2+c\right)^{5/2} \arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3 a c x^2 \arctan(ax)^{3/2} \left(a^2 c x^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3}a\left(-8a\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\frac{4\left(-12a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{10}{3}a\left(-\frac{8a\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\frac{4\left(-12a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{10}{3}a\left(-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\frac{4\left(-12a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{10}{3}a\left(-\frac{8\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\frac{4\left(-12a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

3.1108. $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$

$$\Big\downarrow \text{3793}$$

$$-\frac{10}{3}a\left(-\frac{8\sqrt{a^2x^2+1}\int\left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int\frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{\ }{a}\right.$$

$$\frac{4\left(-12a\int\frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int\frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{10}{3}a\left(-\frac{2\int\frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\ }\right.\right.}{c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{4\left(-12a\int\frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int\frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{10}{3}a\left(-\frac{2\int\frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\ }\right.\right.}{c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{4\left(-12a\int\frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int\frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1108. $\int\frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1108.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1108.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1108.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1108.8 Giac [N/A]

Not integrable

Time = 254.93 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1108.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{5/2} (c\,a^2\,x^2 + c)^{5/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1109 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1109.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}+\frac{4}{a^2cx^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}+\frac{8}{cx^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}+\frac{16\mathrm{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a^2}+44\mathrm{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)+40a^2\mathrm{Int}\left(\frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+4/a^2/c/x^4/(a^2*c*x^2+
c)^(3/2)/arctan(a*x)^(1/2)+8/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+1
6*Unintegrable(1/x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+44*Unint
egrable(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)+40*a^2*Unintegrable
(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.1109.2 Mathematica [N/A]

Not integrable

Time = 16.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

3.1109.3 Rubi [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{5/2} (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-4a \int \frac{1}{x^2 (a^2cx^2+c)^{5/2} \arctan(ax)^{3/2}} dx - \frac{2 \int \frac{1}{x^4 (a^2cx^2+c)^{5/2} \arctan(ax)^{3/2}} dx}{a} - \frac{2}{3acx^3 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-4a\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right.$$
$$\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-$$
$$\frac{2}{3acx^3\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-4a\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right.$$
$$\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-$$
$$\frac{2}{3acx^3\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1109.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1109.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1109.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1109.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.1109.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right)^{5/2}} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1110 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

3.1110.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}+\frac{16}{3a^2cx^5\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}+\frac{28}{3cx^3\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}+\frac{80\mathrm{Int}\left(\frac{1}{x^6(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{3a^2}+\frac{212}{3}\mathrm{Int}\left(\frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}},x\right)+56a^2\mathrm{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+16/3/a^2/c/x^5/(a^2*c*x
^2+c)^(3/2)/arctan(a*x)^(1/2)+28/3/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(
1/2)+80/3*Unintegrable(1/x^6/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+
212/3*Unintegrable(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)+56*a^2*U
nintegrable(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

3.1110.2 Mathematica [N/A]

Not integrable

Time = 40.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

3.1110.3 Rubi [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5503, 5503, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} (a^2cx^2+c)^{5/2}} dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{14}{3}a\int \frac{1}{x^3 (a^2cx^2+c)^{5/2} \arctan(ax)^{3/2}}dx-\frac{2}{3acx^4 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{14}{3}a\left(-12a\int\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+\right.}\right.$$
$$\frac{8\left(-\frac{10\int\frac{1}{x^6\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-16a\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx^4\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 5560

$$-\frac{14}{3}a\left(-12a\int\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+\right.}\right.$$
$$\frac{8\left(-\frac{10\int\frac{1}{x^6\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-16a\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx^4\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

3.1110.3.1 Defintions of rubi rules used

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

3.1110. $\int\frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1110.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

3.1110.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

3.1110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.1110.8 Giac [N/A]

Not integrable

Time = 257.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^4 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1110.9 Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^{5/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

3.1111 $\int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx$

3.1111.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx = \frac{x \arctan(ax)^{1+n}}{ac(1+n)} - \frac{\mathrm{Int}(\arctan(ax)^{1+n}, x)}{ac(1+n)}$$

output
```
x*arctan(a*x)^(1+n)/a/c/(1+n)-Unintegrable(arctan(a*x)^(1+n),x)/a/c/(1+n)
```

3.1111.2 Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx = \int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx$$

input
```
Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2), x]
```

3.1111.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5457, 5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^n}{a^2cx^2+c}dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{x \arctan(ax)^{n+1}}{ac(n+1)}-\frac{\int \arctan(ax)^{n+1}dx}{ac(n+1)}$$
$$\Big\downarrow \text{5353}$$
$$\frac{x \arctan(ax)^{n+1}}{ac(n+1)}-\frac{\int \arctan(ax)^{n+1}dx}{ac(n+1)}$$

input
```
Int[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

3.1111.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

rule 5457
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2), x
_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp
[1/(b*c*d*(p + 1))   Int[(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b
, c, d, e}, x] && EqQ[e, c^2*d] &&  !IGtQ[p, 0] && NeQ[p, -1]
```

3.1111.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^n}{a^2 c x^2 + c} dx$$

input
```
int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

3.1111.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x \arctan(ax)^n}{c + a^2 c x^2} dx = \int \frac{x \arctan(ax)^n}{a^2 c x^2 + c} dx$$

input
```
integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^n/(a^2*c*x^2 + c), x)
```

3.1111.6 Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x \arctan(ax)^n}{c + a^2 c x^2} dx = \frac{\int \frac{x \operatorname{atan}^n(ax)}{a^2 x^2 + 1} dx}{c}$$

input
```
integrate(x*atan(a*x)**n/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**n/(a**2*x**2 + 1), x)/c
```

3.1111.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^n}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1111.8 Giac [N/A]

Not integrable

Time = 54.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{x\arctan(ax)^n}{c+a^2cx^2}\,dx = \int \frac{x\arctan(ax)^n}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
sage0*x
```

3.1111.9 Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x\arctan(ax)^n}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^n}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^n)/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^n)/(c + a^2*c*x^2), x)
```

3.1112 $\int \frac{\arctan(ax)^n}{c+a^2cx^2} dx$

3.1112.1 Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2} dx = \frac{\arctan(ax)^{1+n}}{ac(1+n)}$$

output
```
arctan(a*x)^(1+n)/a/c/(1+n)
```

3.1112.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2} dx = \frac{\arctan(ax)^{1+n}}{ac(1+n)}$$

input
```
Integrate[ArcTan[a*x]^n/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^(1 + n)/(a*c*(1 + n))
```

3.1112.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^n}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 5419}$$
$$\frac{\arctan(ax)^{n+1}}{ac(n+1)}$$

input
```
Int[ArcTan[a*x]^n/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^(1 + n)/(a*c*(1 + n))
```

3.1112.3.1 Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.1112.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{\arctan(ax)^{1+n}}{ac(1+n)}$	21
parallelrisch	$\frac{\arctan(ax)^n\arctan(ax)}{ca(1+n)}$	23
risch	$\frac{i(\ln(-iax+1)-\ln(iax+1))\left(\frac{i(\ln(-iax+1)-\ln(iax+1))}{2}\right)^n}{2ca(1+n)}$	58

input
```
int(arctan(a*x)^n/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
arctan(a*x)^(1+n)/a/c/(1+n)
```

3.1112.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^n \arctan{(ax)}}{acn+ac}$$

input
```
integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
arctan(a*x)^n*arctan(a*x)/(a*c*n + a*c)
```

3.1112.6 Sympy [F]

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \frac{\int \frac{\operatorname{atan}^n{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**n/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**n/(a**2*x**2 + 1), x)/c
```

3.1112.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1112.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^{n+1}}{ac(n+1)}$$

input
```
integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
arctan(a*x)^(n + 1)/(a*c*(n + 1))
```

3.1112.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \frac{\mathrm{atan}(a\,x)^{n+1}}{a\,c\,\left(n+1\right)}$$

input
```
int(atan(a*x)^n/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^(n + 1)/(a*c*(n + 1))
```

3.1113 $\int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p dx$

3.1113.1 Optimal result

Integrand size = 28, antiderivative size = 28

$$\int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p dx = \operatorname{Int}\left((fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p, x\right)$$

output
```
Unintegrable((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

3.1113.2 Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p dx = \int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p dx$$

input
```
Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

output
```
Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p, x]
```

3.1113.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \left(c^2dx^2+d\right)^q (a+b\arctan(cx))^p\,dx$$

$$\Big\downarrow \; 5560$$

$$\int (fx)^m \left(c^2dx^2+d\right)^q (a+b\arctan(cx))^p dx$$

input
```
Int[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

output
```
$Aborted
```

3.1113.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1113.4 Maple [N/A] (verified)

Not integrable

Time = 1.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(c^2d\,x^2+d\right)^q (a+b\arctan(cx))^p\,dx$$

input
```
int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

output
```
int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

3.1113.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m\left(d+c^2dx^2\right)^q(a+b\arctan(cx))^p\,dx=\int (c^2dx^2+d)^q(fx)^m\,(b\arctan{(cx)}+a)^p\,dx$$

input
```
integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="fricas
")
```

output
```
integral((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

3.1113.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m\left(d+c^2dx^2\right)^q(a+b\arctan(cx))^p\,dx=\text{Timed out}$$

input
```
integrate((f*x)**m*(c**2*d*x**2+d)**q*(a+b*atan(c*x))**p,x)
```

output
```
Timed out
```

3.1113.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m\left(d+c^2dx^2\right)^q(a+b\arctan(cx))^p\,dx=\int (c^2dx^2+d)^q(fx)^m\,(b\arctan{(cx)}+a)^p\,dx$$

input
```
integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="maxima
")
```

output
```
integrate((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

3.1113.8 Giac [N/A]

Not integrable

Time = 80.87 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.11

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx=\int \left(c^2dx^2+d\right)^q (fx)^m \left(b\arctan\left(cx\right)+a\right)^p\,dx$$

input
```
integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="giac")
```

output
```
sage0*x
```

3.1113.9 Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx=\int {\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^p\,{\left(d\,c^2\,x^2+d\right)}^q\,{\left(f\,x\right)}^m \,dx$$

input
```
int((a + b*atan(c*x))^p*(d + c^2*d*x^2)^q*(f*x)^m,x)
```

output
```
int((a + b*atan(c*x))^p*(d + c^2*d*x^2)^q*(f*x)^m, x)
```

3.1114 $\int x^3(d+ex^2)(a+b\arctan(cx))\,dx$

3.1114.1 Optimal result

Integrand size = 19, antiderivative size = 107

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx=\frac{b(3c^2d-2e)\,x}{12c^5}-\frac{b(3c^2d-2e)\,x^3}{36c^3}-\frac{bex^5}{30c}-\frac{b(3c^2d-2e)\arctan(cx)}{12c^6}+\frac{1}{4}dx^4(a+b\arctan(cx))+\frac{1}{6}ex^6(a+b\arctan(cx))$$

output
```
1/12*b*(3*c^2*d-2*e)*x/c^5-1/36*b*(3*c^2*d-2*e)*x^3/c^3-1/30*b*e*x^5/c-1/1
2*b*(3*c^2*d-2*e)*arctan(c*x)/c^6+1/4*d*x^4*(a+b*arctan(c*x))+1/6*e*x^6*(a
+b*arctan(c*x))
```

3.1114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.19

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx=\frac{bdx}{4c^3}-\frac{bex}{6c^5}-\frac{bdx^3}{12c}+\frac{bex^3}{18c^3}+\frac{1}{4}adx^4-\frac{bex^5}{30c}+\frac{1}{6}aex^6-\frac{bd\arctan(cx)}{4c^4}+\frac{be\arctan(cx)}{6c^6}+\frac{1}{4}bdx^4\arctan(cx)+\frac{1}{6}bex^6\arctan(cx)$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(b*d*x)/(4*c^3) - (b*e*x)/(6*c^5) - (b*d*x^3)/(12*c) + (b*e*x^3)/(18*c^3)
+ (a*d*x^4)/4 - (b*e*x^5)/(30*c) + (a*e*x^6)/6 - (b*d*ArcTan[c*x])/(4*c^4)
 + (b*e*ArcTan[c*x])/(6*c^6) + (b*d*x^4*ArcTan[c*x])/4 + (b*e*x^6*ArcTan[c
*x])/6
```

3.1114.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5511, 27, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+e x^2\right) (a+b \arctan (c x)) \, dx$$

$$\Big\downarrow \text{ 5511}$$

$$-b c \int \frac{x^4 \left(2 e x^2+3 d\right)}{12 \left(c^2 x^2+1\right)}dx+\frac{1}{4} d x^4 (a+b \arctan (c x))+\frac{1}{6} e x^6 (a+b \arctan (c x))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{12} b c \int \frac{x^4 \left(2 e x^2+3 d\right)}{c^2 x^2+1}dx+\frac{1}{4} d x^4 (a+b \arctan (c x))+\frac{1}{6} e x^6 (a+b \arctan (c x))$$

$$\Big\downarrow \text{ 363}$$

$$-\frac{1}{12} b c \left(\left(3 d-\frac{2 e}{c^2}\right) \int \frac{x^4}{c^2 x^2+1}dx+\frac{2 e x^5}{5 c^2}\right)+\frac{1}{4} d x^4 (a+b \arctan (c x))+\frac{1}{6} e x^6 (a+b \arctan (c x))$$

$$\Big\downarrow \text{ 254}$$

$$-\frac{1}{12} b c \left(\left(3 d-\frac{2 e}{c^2}\right) \int \left(\frac{x^2}{c^2}+\frac{1}{c^4 \left(c^2 x^2+1\right)}-\frac{1}{c^4}\right)dx+\frac{2 e x^5}{5 c^2}\right)+\frac{1}{4} d x^4 (a+b \arctan (c x))+\frac{1}{6} e x^6 (a+b \arctan (c x))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} d x^4 (a+b \arctan (c x))+\frac{1}{6} e x^6 (a+b \arctan (c x))-\frac{1}{12} b c \left(\left(\frac{\arctan (c x)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3 c^2}\right) \left(3 d-\frac{2 e}{c^2}\right)+\frac{2 e x^5}{5 c^2}\right)$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcTan[c*x]))/4 + (e*x^6*(a + b*ArcTan[c*x]))/6 - (b*c*((2*e
*x^5)/(5*c^2) + (3*d - (2*e)/c^2)*(-(x/c^4) + x^3/(3*c^2) + ArcTan[c*x]/c^
5)))/12
```

3.1114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1114.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\arctan(cx)e\,x^6}{6}+\frac{\arctan(cx)d\,c^4x^4}{4}-\frac{\frac{2e\,c^5x^5}{5}+d\,c^5x^3-\frac{2e\,c^3x^3}{3}-3c^3xd+2ecx+\left(3c^2d-2e\right)\text{arcta}}{12c^2}\right.}{c^4}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b\left(\frac{\arctan(cx)d\,c^6x^4}{4}+\frac{\arctan(cx)e\,c^6x^6}{6}-\frac{d\,c^5x^3}{12}-\frac{e\,c^5x^5}{30}+\frac{c^3xd}{4}+\frac{e\,c^3x^3}{18}-\frac{ecx}{6}-\frac{\left(3c^2d-2e\right)\arctan(cx)}{12}\right)}{c^2}}{c^4}$
default	$\frac{\frac{a\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b\left(\frac{\arctan(cx)d\,c^6x^4}{4}+\frac{\arctan(cx)e\,c^6x^6}{6}-\frac{d\,c^5x^3}{12}-\frac{e\,c^5x^5}{30}+\frac{c^3xd}{4}+\frac{e\,c^3x^3}{18}-\frac{ecx}{6}-\frac{\left(3c^2d-2e\right)\arctan(cx)}{12}\right)}{c^2}}{c^4}$
parallelrisch	$\frac{30x^6\arctan(cx)b\,c^6e+30a\,c^6e\,x^6+45db\arctan(cx)x^4c^6-6b\,c^5e\,x^5+45a\,c^6d\,x^4-15b\,c^5d\,x^3+10b\,c^3e\,x^3+45b\,c^3dx-45b\,c^2}{180c^6}$
risch	$-\frac{ib\left(2e\,x^6+3d\,x^4\right)\ln(icx+1)}{24}+\frac{x^6ea}{6}+\frac{ibe\,x^6\ln(-icx+1)}{12}+\frac{x^4da}{4}-\frac{be\,x^5}{30c}+\frac{ibd\,x^4\ln(-icx+1)}{8}-\frac{bd\,x^3}{12c}+\frac{b}{1}$

input
```
int(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arctan(c*x)*e*x^6+1/4*arctan(c*x)*d
*c^4*x^4-1/12/c^2*(2/5*e*c^5*x^5+d*c^5*x^3-2/3*e*c^3*x^3-3*c^3*x*d+2*e*c*x
+(3*c^2*d-2*e)*arctan(c*x)))
```

3.1114.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int x^3\left(d+ex^2\right)\left(a+b\arctan(cx)\right)dx$$
$$=\frac{30\,ac^6ex^6+45\,ac^6dx^4-6\,bc^5ex^5-5\,(3\,bc^5d-2\,bc^3e)x^3+15\,(3\,bc^3d-2\,bce)x+15\,(2\,bc^6ex^6+3\,bc^6dx^4-}{180\,c^6}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 - 6*b*c^5*e*x^5 - 5*(3*b*c^5*d - 2*
b*c^3*e)*x^3 + 15*(3*b*c^3*d - 2*b*c*e)*x + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*
x^4 - 3*b*c^2*d + 2*b*e)*arctan(c*x))/c^6
```

3.1114.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4\operatorname{atan}(cx)}{4} + \frac{bex^6\operatorname{atan}(cx)}{6} - \frac{bdx^3}{12c} - \frac{bex^5}{30c} + \frac{bdx}{4c^3} + \frac{bex^3}{18c^3} - \frac{bd\operatorname{atan}(cx)}{4c^4} - \frac{bex}{6c^5} + \frac{be\operatorname{atan}(cx)}{6c^6} & \text{for } c \neq \\ a\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) & \text{otherw} \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*atan(c*x)/4 + b*e*x**6*atan(
c*x)/6 - b*d*x**3/(12*c) - b*e*x**5/(30*c) + b*d*x/(4*c**3) + b*e*x**3/(18
*c**3) - b*d*atan(c*x)/(4*c**4) - b*e*x/(6*c**5) + b*e*atan(c*x)/(6*c**6),
 Ne(c, 0)), (a*(d*x**4/4 + e*x**6/6), True))
```

3.1114.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx$$
$$= \frac{1}{6}aex^6 + \frac{1}{4}adx^4 + \frac{1}{12}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bd + \frac{1}{90}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)be$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c
^4 + 3*arctan(c*x)/c^5))*b*d + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 -
5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e
```

3.1114.8 Giac [F]

$$\int x^3 \left(d+ex^2\right) (a+b\arctan (cx)) \, dx = \int (ex^2+d)(b\arctan (cx)+a)x^3 \, dx$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1114.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int x^3 \left(d+ex^2\right) (a+b\arctan (cx)) \, dx = \frac{a\,d\,x^4}{4}+\frac{a\,e\,x^6}{6}+\frac{b\,d\,x}{4\,c^3}-\frac{b\,e\,x}{6\,c^5}-\frac{b\,d\,\mathrm{atan}(c\,x)}{4\,c^4} +\frac{b\,e\,\mathrm{atan}(c\,x)}{6\,c^6}+\frac{b\,d\,x^4\,\mathrm{atan}(c\,x)}{4} +\frac{b\,e\,x^6\,\mathrm{atan}(c\,x)}{6}-\frac{b\,d\,x^3}{12\,c}-\frac{b\,e\,x^5}{30\,c}+\frac{b\,e\,x^3}{18\,c^3}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
(a*d*x^4)/4 + (a*e*x^6)/6 + (b*d*x)/(4*c^3) - (b*e*x)/(6*c^5) - (b*d*atan(
c*x))/(4*c^4) + (b*e*atan(c*x))/(6*c^6) + (b*d*x^4*atan(c*x))/4 + (b*e*x^6
*atan(c*x))/6 - (b*d*x^3)/(12*c) - (b*e*x^5)/(30*c) + (b*e*x^3)/(18*c^3)
```

3.1115 $\int x^2(d+ex^2)(a+b\arctan(cx))\,dx$

3.1115.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx=-\frac{b(5c^2d-3e)\,x^2}{30c^3}-\frac{bex^4}{20c}+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))+\frac{b(5c^2d-3e)\log(1+c^2x^2)}{30c^5}$$

output
```
-1/30*b*(5*c^2*d-3*e)*x^2/c^3-1/20*b*e*x^4/c+1/3*d*x^3*(a+b*arctan(c*x))+1
/5*e*x^5*(a+b*arctan(c*x))+1/30*b*(5*c^2*d-3*e)*ln(c^2*x^2+1)/c^5
```

3.1115.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx=-\frac{bdx^2}{6c}+\frac{bex^2}{10c^3}+\frac{1}{3}adx^3-\frac{bex^4}{20c}+\frac{1}{5}aex^5+\frac{1}{3}bdx^3\arctan(cx)+\frac{1}{5}bex^5\arctan(cx)+\frac{bd\log(1+c^2x^2)}{6c^3}-\frac{be\log(1+c^2x^2)}{10c^5}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/6*(b*d*x^2)/c + (b*e*x^2)/(10*c^3) + (a*d*x^3)/3 - (b*e*x^4)/(20*c) + (
a*e*x^5)/5 + (b*d*x^3*ArcTan[c*x])/3 + (b*e*x^5*ArcTan[c*x])/5 + (b*d*Log[
1 + c^2*x^2])/(6*c^3) - (b*e*Log[1 + c^2*x^2])/(10*c^5)
```

3.1115.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5511, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d+ex^2) (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^3(3ex^2+5d)}{15(c^2x^2+1)}dx+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bc\int \frac{x^3(3ex^2+5d)}{c^2x^2+1}dx+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{30}bc\int \frac{x^2(3ex^2+5d)}{c^2x^2+1}dx^2+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{30}bc\int \left(\frac{3ex^2}{c^2}+\frac{5c^2d-3e}{c^4}+\frac{3e-5c^2d}{c^4(c^2x^2+1)}\right)dx^2+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))-\frac{1}{30}bc\left(\frac{3ex^4}{2c^2}-\frac{(5c^2d-3e)\log(c^2x^2+1)}{c^6}+\frac{x^2(5c^2d-3e)}{c^4}\right)$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcTan[c*x]))/3 + (e*x^5*(a + b*ArcTan[c*x]))/5 - (b*c*(((5*
c^2*d - 3*e)*x^2)/c^4 + (3*e*x^4)/(2*c^2) - ((5*c^2*d - 3*e)*Log[1 + c^2*x
^2])/c^6))/30
```

3.1115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1115.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

method	result
parts	$a\left(\frac{1}{5}e\,x^{5}+\frac{1}{3}d\,x^{3}\right)+\frac{b\left(\frac{c^{3}\arctan(cx)e\,x^{5}}{5}+\frac{\arctan(cx)d\,c^{3}x^{3}}{3}-\frac{\frac{5d\,c^{4}x^{2}}{2}+\frac{3e\,c^{4}x^{4}}{4}-\frac{3e\,c^{2}x^{2}}{2}+\frac{\left(-5c^{2}d+3e\right)\ln\left(c^{2}x^{2}+1\right)}{2}}{15c^{2}}\right)}{c^{3}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^{5}x^{3}+\frac{1}{5}e\,c^{5}x^{5}\right)}{c^{2}}+\frac{b\left(\frac{\arctan(cx)d\,c^{5}x^{3}}{3}+\frac{\arctan(cx)e\,c^{5}x^{5}}{5}-\frac{d\,c^{4}x^{2}}{6}-\frac{e\,c^{4}x^{4}}{20}+\frac{e\,c^{2}x^{2}}{10}-\frac{\left(-5c^{2}d+3e\right)\ln\left(c^{2}x^{2}+1\right)}{30}\right)}{c^{2}}}{c^{3}}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^{5}x^{3}+\frac{1}{5}e\,c^{5}x^{5}\right)}{c^{2}}+\frac{b\left(\frac{\arctan(cx)d\,c^{5}x^{3}}{3}+\frac{\arctan(cx)e\,c^{5}x^{5}}{5}-\frac{d\,c^{4}x^{2}}{6}-\frac{e\,c^{4}x^{4}}{20}+\frac{e\,c^{2}x^{2}}{10}-\frac{\left(-5c^{2}d+3e\right)\ln\left(c^{2}x^{2}+1\right)}{30}\right)}{c^{2}}}{c^{3}}$
parallelrisch	$\frac{12x^{5}\arctan(cx)b\,c^{5}e+12a\,c^{5}e\,x^{5}+20db\arctan(cx)x^{3}c^{5}-3b\,c^{4}e\,x^{4}+20a\,c^{5}d\,x^{3}-10b\,c^{4}d\,x^{2}+6b\,c^{2}e\,x^{2}+10\ln(c^{2}x^{2}+1)b\,c^{2}}{60c^{5}}$
risch	$-\frac{ib\left(3e\,x^{5}+5d\,x^{3}\right)\ln(icx+1)}{30}+\frac{ibe\,x^{5}\ln(-icx+1)}{10}+\frac{ibd\,x^{3}\ln(-icx+1)}{6}+\frac{ae\,x^{5}}{5}+\frac{x^{3}da}{3}-\frac{be\,x^{4}}{20c}-\frac{bd\,x^{2}}{6c}+$

input
```
int(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+b/c^3*(1/5*c^3*arctan(c*x)*e*x^5+1/3*arctan(c*x)*d
*c^3*x^3-1/15/c^2*(5/2*d*c^4*x^2+3/4*e*c^4*x^4-3/2*e*c^2*x^2+1/2*(-5*c^2*d
+3*e)*ln(c^2*x^2+1)))
```

3.1115.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int x^2\left(d+ex^2\right)\left(a+b\arctan(cx)\right)dx$$
$$=\frac{12\,ac^5ex^5+20\,ac^5dx^3-3\,bc^4ex^4-2\,(5\,bc^4d-3\,bc^2e)x^2+4\,(3\,bc^5ex^5+5\,bc^5dx^3)\arctan\left(cx\right)+2\,(5\,bc^2d-}{60\,c^5}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/60*(12*a*c^5*e*x^5 + 20*a*c^5*d*x^3 - 3*b*c^4*e*x^4 - 2*(5*b*c^4*d - 3*b
*c^2*e)*x^2 + 4*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3)*arctan(c*x) + 2*(5*b*c^2*d
 - 3*b*e)*log(c^2*x^2 + 1))/c^5
```

3.1115.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{adx^3}{3} + \frac{aex^5}{5} + \frac{bdx^3\operatorname{atan}(cx)}{3} + \frac{bex^5\operatorname{atan}(cx)}{5} - \frac{bdx^2}{6c} - \frac{bex^4}{20c} + \frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} + \frac{bex^2}{10c^3} - \frac{be\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c \neq 0 \\ a\left(\frac{dx^3}{3} + \frac{ex^5}{5}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*atan(c*x)/3 + b*e*x**5*atan(
c*x)/5 - b*d*x**2/(6*c) - b*e*x**4/(20*c) + b*d*log(x**2 + c**(-2))/(6*c**
3) + b*e*x**2/(10*c**3) - b*e*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a
*(d*x**3/3 + e*x**5/5), True))
```

3.1115.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx$$
$$= \frac{1}{5}aex^5 + \frac{1}{3}adx^3 + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd$$
$$+ \frac{1}{20}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4-2x^2}{c^4} + \frac{2\log(c^2x^2+1)}{c^6}\right)\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*
x^2 + 1)/c^4))*b*d + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 +
2*log(c^2*x^2 + 1)/c^6))*b*e
```

3.1115.8 Giac [F]

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = \int (ex^2+d)(b\arctan(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1115.9 Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = \frac{a\,d\,x^3}{3}+\frac{a\,e\,x^5}{5}+\frac{b\,d\,x^3\,\mathrm{atan}(c\,x)}{3} + \frac{b\,e\,x^5\,\mathrm{atan}(c\,x)}{5}+\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{6\,c^3} - \frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}-\frac{b\,d\,x^2}{6\,c}-\frac{b\,e\,x^4}{20\,c}+\frac{b\,e\,x^2}{10\,c^3}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
(a*d*x^3)/3 + (a*e*x^5)/5 + (b*d*x^3*atan(c*x))/3 + (b*e*x^5*atan(c*x))/5
+ (b*d*log(c^2*x^2 + 1))/(6*c^3) - (b*e*log(c^2*x^2 + 1))/(10*c^5) - (b*d*
x^2)/(6*c) - (b*e*x^4)/(20*c) + (b*e*x^2)/(10*c^3)
```

3.1116 $\int x(d+ex^2)(a+b\arctan(cx))\,dx$

3.1116.1 Optimal result

Integrand size = 17, antiderivative size = 82

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{b(2c^2d-e)x}{4c^3} - \frac{bex^3}{12c} - \frac{b(c^2d-e)^2\arctan(cx)}{4c^4e} + \frac{(d+ex^2)^2(a+b\arctan(cx))}{4e}$$

output
```
-1/4*b*(2*c^2*d-e)*x/c^3-1/12*b*e*x^3/c-1/4*b*(c^2*d-e)^2*arctan(c*x)/c^4/
e+1/4*(e*x^2+d)^2*(a+b*arctan(c*x))/e
```

3.1116.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{bdx}{2c} + \frac{bex}{4c^3} + \frac{1}{2}adx^2 - \frac{bex^3}{12c} + \frac{1}{4}aex^4 + \frac{bd\arctan(cx)}{2c^2} - \frac{be\arctan(cx)}{4c^4} + \frac{1}{2}bdx^2\arctan(cx) + \frac{1}{4}bex^4\arctan(cx)$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*(b*d*x)/c + (b*e*x)/(4*c^3) + (a*d*x^2)/2 - (b*e*x^3)/(12*c) + (a*e*x
^4)/4 + (b*d*ArcTan[c*x])/(2*c^2) - (b*e*ArcTan[c*x])/(4*c^4) + (b*d*x^2*A
rcTan[c*x])/2 + (b*e*x^4*ArcTan[c*x])/4
```

3.1116.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5509, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)(a+b\arctan(cx))\,dx$$
$$\Big\downarrow \text{5509}$$
$$\frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{4e}-\frac{bc\int \frac{\left(ex^2+d\right)^2}{c^2x^2+1}dx}{4e}$$
$$\Big\downarrow \text{300}$$
$$\frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{4e}-\frac{bc\int \left(\frac{e^2x^2}{c^2}+\frac{(2c^2d-e)e}{c^4}+\frac{d^2c^4-2dec^2+e^2}{c^4(c^2x^2+1)}\right)dx}{4e}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{4e}-\frac{bc\left(\frac{\arctan(cx)\left(c^2d-e\right)^2}{c^5}+\frac{e^2x^3}{3c^2}+\frac{ex(2c^2d-e)}{c^4}\right)}{4e}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcTan[c*x]))/(4*e) - (b*c*(((2*c^2*d - e)*e*x)/c^4
+ (e^2*x^3)/(3*c^2) + ((c^2*d - e)^2*ArcTan[c*x])/c^5))/(4*e)
```

3.1116.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1116.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

method	result
parts	$\frac{a(e\,x^2+d)^2}{4e}+\frac{be\arctan(cx)x^4}{4}+\frac{b\arctan(cx)x^2d}{2}-\frac{be\,x^3}{12c}-\frac{bdx}{2c}+\frac{bex}{4c^3}+\frac{bd\arctan(cx)}{2c^2}-\frac{be\arctan(cx)}{4c^4}$
parallelrisch	$\frac{3x^4\arctan(cx)b\,c^4e+3x^4a\,c^4e+6x^2\arctan(cx)b\,c^4d-b\,c^3e\,x^3+6x^2a\,c^4d-6b\,c^3dx+6b\,c^2d\arctan(cx)+3bcex-3eb\arctan(}{12c^4}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{\arctan(cx)b\,c^2d\,x^2}{2}+\frac{\arctan(cx)b\,c^2e\,x^4}{4}-\frac{bcdx}{2}-\frac{bce\,x^3}{12}+\frac{bex}{4c}+\frac{\arctan(cx)bd}{2}-\frac{be\arctan(cx)}{4c^2}}{c^2}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^2}{4c^2e}+\frac{\arctan(cx)b\,c^2d\,x^2}{2}+\frac{\arctan(cx)b\,c^2e\,x^4}{4}-\frac{bcdx}{2}-\frac{bce\,x^3}{12}+\frac{bex}{4c}+\frac{\arctan(cx)bd}{2}-\frac{be\arctan(cx)}{4c^2}}{c^2}$
risch	$-\frac{i(e\,x^2+d)^2b\ln(icx+1)}{8e}+\frac{ibd\,x^2\ln(-icx+1)}{4}+\frac{ib\,d^2\ln(c^2x^2+1)}{16e}+\frac{x^4ea}{4}+\frac{ieb\,x^4\ln(-icx+1)}{8}-\frac{b\,d^2\arctan(c}{8e}$

input
```
int(x*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+1/4*b*e*arctan(c*x)*x^4+1/2*b*arctan(c*x)*x^2*d-1/12*b
*e*x^3/c-1/2*b*d*x/c+1/4*b*e*x/c^3+1/2*b*d*arctan(c*x)/c^2-1/4*b*e*arctan(
c*x)/c^4
```

3.1116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int x(d+ex^2)\,(a+b\arctan(cx))\,dx$$
$$=\frac{3\,ac^4ex^4+6\,ac^4dx^2-bc^3ex^3-3\,(2\,bc^3d-bce)x+3\,(bc^4ex^4+2\,bc^4dx^2+2\,bc^2d-be)\arctan{(cx)}}{12\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 - b*c^3*e*x^3 - 3*(2*b*c^3*d - b*c*e)*
x + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2 + 2*b*c^2*d - b*e)*arctan(c*x))/c^4
```

3.1116.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)\,(a+b\arctan(cx))\,dx$$
$$=\begin{cases} \frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{atan}(cx)}{2}+\frac{bex^4\operatorname{atan}(cx)}{4}-\frac{bdx}{2c}-\frac{bex^3}{12c}+\frac{bd\operatorname{atan}(cx)}{2c^2}+\frac{bex}{4c^3}-\frac{be\operatorname{atan}(cx)}{4c^4} & \text{for } c\neq 0 \\ a\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*atan(c*x)/2 + b*e*x**4*atan(
c*x)/4 - b*d*x/(2*c) - b*e*x**3/(12*c) + b*d*atan(c*x)/(2*c**2) + b*e*x/(4
*c**3) - b*e*atan(c*x)/(4*c**4), Ne(c, 0)), (a*(d*x**2/2 + e*x**4/4), True
))
```

3.1116.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int x(d+ex^2)\,(a+b\arctan(cx))\,dx$$
$$=\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bd$$
$$+\frac{1}{12}\left(3\,x^4\arctan(cx)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan(cx)}{c^5}\right)\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/
c^3))*b*d + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*
x)/c^5))*b*e
```

3.1116.8 Giac [F]

$$\int x\left(d+ex^2\right)(a+b\arctan(cx))\,dx=\int (ex^2+d)(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1116.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int x\left(d+ex^2\right)(a+b\arctan(cx))\,dx=\frac{a\,d\,x^2}{2}+\frac{a\,e\,x^4}{4}-\frac{b\,d\,x}{2\,c}+\frac{b\,e\,x}{4\,c^3}+\frac{b\,d\,\mathrm{atan}(c\,x)}{2\,c^2}-\frac{b\,e\,\mathrm{atan}(c\,x)}{4\,c^4}+\frac{b\,d\,x^2\,\mathrm{atan}(c\,x)}{2}+\frac{b\,e\,x^4\,\mathrm{atan}(c\,x)}{4}-\frac{b\,e\,x^3}{12\,c}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
(a*d*x^2)/2 + (a*e*x^4)/4 - (b*d*x)/(2*c) + (b*e*x)/(4*c^3) + (b*d*atan(c*
x))/(2*c^2) - (b*e*atan(c*x))/(4*c^4) + (b*d*x^2*atan(c*x))/2 + (b*e*x^4*a
tan(c*x))/4 - (b*e*x^3)/(12*c)
```

3.1117 $\int (d+ex^2)(a+b\arctan(cx))\,dx$

3.1117.1 Optimal result

Integrand size = 16, antiderivative size = 68

$$\int (d+ex^2)(a+b\arctan(cx))\,dx=-\frac{bex^2}{6c}+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))-\frac{b(3c^2d-e)\log(1+c^2x^2)}{6c^3}$$

output
```
-1/6*b*e*x^2/c+d*x*(a+b*arctan(c*x))+1/3*e*x^3*(a+b*arctan(c*x))-1/6*b*(3*
c^2*d-e)*ln(c^2*x^2+1)/c^3
```

3.1117.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int (d+ex^2)(a+b\arctan(cx))\,dx=adx-\frac{bex^2}{6c}+\frac{1}{3}aex^3+bdx\arctan(cx)+\frac{1}{3}bex^3\arctan(cx)-\frac{bd\log(1+c^2x^2)}{2c}+\frac{be\log(1+c^2x^2)}{6c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
a*d*x - (b*e*x^2)/(6*c) + (a*e*x^3)/3 + b*d*x*ArcTan[c*x] + (b*e*x^3*ArcTa
n[c*x])/3 - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[1 + c^2*x^2])/(6*c^3)
```

3.1117.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5447, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2) (a+b\arctan(cx)) \, dx$$
$$\Big\downarrow \text{5447}$$
$$-bc\int \frac{x(ex^2+3d)}{3(c^2x^2+1)}dx+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{3}bc\int \frac{x(ex^2+3d)}{c^2x^2+1}dx+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$
$$\Big\downarrow \text{353}$$
$$-\frac{1}{6}bc\int \frac{ex^2+3d}{c^2x^2+1}dx^2+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$
$$\Big\downarrow \text{49}$$
$$-\frac{1}{6}bc\int \left(\frac{3c^2d-e}{c^2(c^2x^2+1)}+\frac{e}{c^2}\right)dx^2+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$
$$\Big\downarrow \text{2009}$$
$$dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{ex^2}{c^2}+\frac{(3c^2d-e)\log(c^2x^2+1)}{c^4}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
d*x*(a + b*ArcTan[c*x]) + (e*x^3*(a + b*ArcTan[c*x]))/3 - (b*c*((e*x^2)/c^
2 + ((3*c^2*d - e)*Log[1 + c^2*x^2])/c^4))/6
```

3.1117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.1117.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

method	result
parts	$a\left(\frac{1}{3}e\,x^{3}+xd\right)+\frac{b\left(\frac{c\arctan\left(cx\right)x^{3}e}{3}+\arctan\left(cx\right)cxd-\frac{\frac{e\,c^{2}x^{2}}{2}+\frac{\left(3c^{2}d-e\right)\ln\left(c^{2}x^{2}+1\right)}{2}}{3c^{2}}\right)}{c}$
derivativedivides	$\frac{\frac{a\left(c^{3}xd+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\arctan\left(cx\right)c^{3}xd+\frac{\arctan\left(cx\right)e\,c^{3}x^{3}}{3}-\frac{e\,c^{2}x^{2}}{6}-\frac{\left(3c^{2}d-e\right)\ln\left(c^{2}x^{2}+1\right)}{6}\right)}{c^{2}}}{c}$
default	$\frac{\frac{a\left(c^{3}xd+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\arctan\left(cx\right)c^{3}xd+\frac{\arctan\left(cx\right)e\,c^{3}x^{3}}{3}-\frac{e\,c^{2}x^{2}}{6}-\frac{\left(3c^{2}d-e\right)\ln\left(c^{2}x^{2}+1\right)}{6}\right)}{c^{2}}}{c}$
parallelrisch	$-\frac{-2x^{3}\arctan\left(cx\right)b\,c^{3}e-2a\,c^{3}e\,x^{3}-6x\arctan\left(cx\right)b\,c^{3}d+b\,c^{2}e\,x^{2}-6a\,c^{3}dx+3\ln\left(c^{2}x^{2}+1\right)b\,c^{2}d-\ln\left(c^{2}x^{2}+1\right)be}{6c^{3}}$
risch	$-\frac{ib\left(e\,x^{3}+3xd\right)\ln\left(icx+1\right)}{6}+\frac{ibe\,x^{3}\ln\left(-icx+1\right)}{6}+\frac{ibdx\ln\left(-icx+1\right)}{2}+\frac{ae\,x^{3}}{3}+adx-\frac{be\,x^{2}}{6c}-\frac{\ln\left(-c^{2}x^{2}-1\right)bc}{2c}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e*x^3+x*d)+b/c*(1/3*c*arctan(c*x)*x^3*e+arctan(c*x)*c*x*d-1/3/c^2*(
1/2*e*c^2*x^2+1/2*(3*c^2*d-e)*ln(c^2*x^2+1)))
```

3.1117.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int (d+ex^2)(a+b\arctan(cx))\,dx = \frac{2\,ac^3ex^3+6\,ac^3dx-bc^2ex^2+2\,(bc^3ex^3+3\,bc^3dx)\arctan(cx)-(3\,bc^2d-be)\log(c^2x^2+1)}{6\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x - b*c^2*e*x^2 + 2*(b*c^3*e*x^3 + 3*b*c^3*
d*x)*arctan(c*x) - (3*b*c^2*d - b*e)*log(c^2*x^2 + 1))/c^3
```

3.1117.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int (d+ex^2)(a+b\arctan(cx))\,dx = \begin{cases} adx+\frac{aex^3}{3}+bdx\operatorname{atan}(cx)+\frac{bex^3\operatorname{atan}(cx)}{3}-\frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bex^2}{6c}+\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for } c\neq 0 \\ a\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**3/3 + b*d*x*atan(c*x) + b*e*x**3*atan(c*x)/3 - b
*d*log(x**2 + c**(-2))/(2*c) - b*e*x**2/(6*c) + b*e*log(x**2 + c**(-2))/(6
*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

3.1117.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int (d+ex^2)(a+b\arctan(cx))\,dx = \frac{1}{3}aex^3 + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)be + adx + \frac{(2cx\arctan(cx) - \log(c^2x^2+1))bd}{2c}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))
*b*e + a*d*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

3.1117.8 Giac [F]

$$\int (d+ex^2)(a+b\arctan(cx))\,dx = \int (ex^2+d)(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1117.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int (d+ex^2)(a+b\arctan(cx))\,dx = a\,d\,x + \frac{a\,e\,x^3}{3} + b\,d\,x\,\mathrm{atan}(c\,x) + \frac{b\,e\,x^3\,\mathrm{atan}(c\,x)}{3} - \frac{b\,d\,\ln(c^2\,x^2+1)}{2\,c} + \frac{b\,e\,\ln(c^2\,x^2+1)}{6\,c^3} - \frac{b\,e\,x^2}{6\,c}$$

input
```
int((a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
a*d*x + (a*e*x^3)/3 + b*d*x*atan(c*x) + (b*e*x^3*atan(c*x))/3 - (b*d*log(c
^2*x^2 + 1))/(2*c) + (b*e*log(c^2*x^2 + 1))/(6*c^3) - (b*e*x^2)/(6*c)
```

3.1118 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx$

3.1118.1 Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = -\frac{bex}{2c}+\frac{be\arctan(cx)}{2c^2}+\frac{1}{2}ex^2(a+b\arctan(cx))+ad\log(x)+\frac{1}{2}ibd\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\mathrm{PolyLog}(2,icx)$$

output
```
-1/2*b*e*x/c+1/2*b*e*arctan(c*x)/c^2+1/2*e*x^2*(a+b*arctan(c*x))+a*d*ln(x)
+1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)
```

3.1118.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = -\frac{bex}{2c}+\frac{1}{2}aex^2+\frac{be\arctan(cx)}{2c^2}+\frac{1}{2}bex^2\arctan(cx)+ad\log(x)+\frac{1}{2}ibd\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\mathrm{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/2*(b*e*x)/c + (a*e*x^2)/2 + (b*e*ArcTan[c*x])/(2*c^2) + (b*e*x^2*ArcTan
[c*x])/2 + a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog
[2, I*c*x]
```

3.1118.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ex^2(a+b\arctan(cx))+ad\log(x)+\frac{be\arctan(cx)}{2c^2}+\frac{1}{2}ibd\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\text{PolyLog}(2,icx)-\frac{bex}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/2*(b*e*x)/c + (b*e*ArcTan[c*x])/(2*c^2) + (e*x^2*(a + b*ArcTan[c*x]))/2
 + a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*
x]
```

3.1118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1118.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln\left(cx\right)+\frac{b\left(\frac{\arctan(cx)e\,c^2x^2}{2}+\arctan(cx)d\,c^2\ln(cx)-\frac{e(cx-\arctan(cx))}{2}-d\,c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(c\ldots}{}\right)\right)}{c^2}$
default	$\frac{ae\,x^2}{2}+ad\ln\left(cx\right)+\frac{b\left(\frac{\arctan(cx)e\,c^2x^2}{2}+\arctan(cx)d\,c^2\ln(cx)-\frac{e(cx-\arctan(cx))}{2}-d\,c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(c\ldots}{}\right)\right)}{c^2}$
parts	$\frac{ae\,x^2}{2}+ad\ln\left(x\right)+b\left(\frac{\arctan(cx)x^2e}{2}+\arctan\left(cx\right)d\ln\left(cx\right)-\frac{e(cx-\arctan(cx))-ic^2d\ln(cx)\ln(icx+1\ldots}{}\right)$
risch	$\frac{ibd\operatorname{dilog}(icx+1)}{2}+\frac{ibe\ln(c^2x^2+1)}{8c^2}+\frac{be\arctan(cx)}{4c^2}-\frac{ibe\ln(icx+1)}{4c^2}-\frac{bex}{2c}-\frac{ibe\ln(icx+1)x^2}{4}-\frac{ibd\operatorname{dilog}(-ic\ldots)}{2}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(c*x)+b/c^2*(1/2*arctan(c*x)*e*c^2*x^2+arctan(c*x)*d*c^2
*ln(c*x)-1/2*e*(c*x-arctan(c*x))-d*c^2*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*l
n(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)))
```

3.1118.5 Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx=\int \frac{(ex^2+d)(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))/x, x)
```

3.1118.6 Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx=\int \frac{(a+b\operatorname{atan}(cx))(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)/x, x)
```

3.1118.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx=\frac{1}{2}aex^2+ad\log(x)-\frac{\pi bc^2d\log(c^2x^2+1)-4\,bc^2d\arctan(cx)\log(cx)+2i\,bc^2d\mathrm{Li}_2(i\,cx+1)-2i\,bc^2d\mathrm{Li}_2(-i\,cx+1)+2\,bcex}{4\,c^2}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) - 1/4*(pi*b*c^2*d*log(c^2*x^2 + 1) - 4*b*c^2*d*ar
ctan(c*x)*log(c*x) + 2*I*b*c^2*d*dilog(I*c*x + 1) - 2*I*b*c^2*d*dilog(-I*c
*x + 1) + 2*b*c*e*x - 2*(b*c^2*e*x^2 + b*e)*arctan(c*x))/c^2
```

3.1118.8 Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.1118.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \begin{cases} \frac{a\,(e\,x^2+2\,d\,\ln(x))}{2} & \text{if } c=0 \\ \frac{a\,(e\,x^2+2\,d\,\ln(x))}{2} - b\,e\left(\frac{x}{2\,c}-\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) - \frac{b\,d\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))\,1\mathrm{i}}{2} & \text{if } c\neq 0 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x,x)
```

output
```
piecewise(c == 0, (a*(e*x^2 + 2*d*log(x)))/2, c ~= 0, (a*(e*x^2 + 2*d*log(
x)))/2 - b*e*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) - (b*d*(dilog(- c*x
*1i + 1) - dilog(c*x*1i + 1))*1i)/2)
```

3.1119 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx$

3.1119.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = -\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx)) + bcd\log(x)-\frac{b(c^2d+e)\log(1+c^2x^2)}{2c}$$

output
```
-d*(a+b*arctan(c*x))/x+e*x*(a+b*arctan(c*x))+b*c*d*ln(x)-1/2*b*(c^2*d+e)*l
n(c^2*x^2+1)/c
```

3.1119.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = -\frac{ad}{x}+aex-\frac{bd\arctan(cx)}{x}+bex\arctan(cx) + bcd\log(x)-\frac{1}{2}bcd\log(1+c^2x^2)-\frac{be\log(1+c^2x^2)}{2c}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x - (b*d*ArcTan[c*x])/x + b*e*x*ArcTan[c*x] + b*c*d*Log[x
] - (b*c*d*Log[1 + c^2*x^2])/2 - (b*e*Log[1 + c^2*x^2])/(2*c)
```

3.1119.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5511, 25, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{d-ex^2}{x(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{25}$$

$$bc\int \frac{d-ex^2}{x(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}bc\int \frac{d-ex^2}{x^2(c^2x^2+1)}dx^2-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}bc\int \left(\frac{d}{x^2}+\frac{-dc^2-e}{c^2x^2+1}\right)dx^2-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))+\frac{1}{2}bc\left(d\log(x^2)-\frac{(c^2d+e)\log(c^2x^2+1)}{c^2}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcTan[c*x]))/x) + e*x*(a + b*ArcTan[c*x]) + (b*c*(d*Log[x^2]
- ((c^2*d + e)*Log[1 + c^2*x^2])/c^2))/2
```

3.1119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1119.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

method	result
derivativedivides	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\left(\arctan(cx)ecx-\frac{\arctan(cx)dc}{x}-\frac{\left(c^2d+e\right)\ln\left(c^2x^2+1\right)}{2}+d\,c^2\ln(cx)\right)}{c^2}\right)$
default	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\left(\arctan(cx)ecx-\frac{\arctan(cx)dc}{x}-\frac{\left(c^2d+e\right)\ln\left(c^2x^2+1\right)}{2}+d\,c^2\ln(cx)\right)}{c^2}\right)$
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\arctan(cx)xe}{c}-\frac{\arctan(cx)d}{cx}-\frac{-d\,c^2\ln(cx)+\frac{\left(c^2d+e\right)\ln\left(c^2x^2+1\right)}{2}}{c^2}\right)$
parallelrisch	$\frac{2b\,c^2d\ln(x)x-\ln(c^2x^2+1)b\,c^2dx+2x^2\arctan(cx)bce+2ae\,x^2c-\ln(c^2x^2+1)bex-2\arctan(cx)bcd-2adc}{2cx}$
risch	$\frac{ib\left(-e\,x^2+d\right)\ln(icx+1)}{2x}+\frac{ibce\,x^2\ln(-icx+1)+2b\,c^2d\ln(x)x-\ln\left(c^2x^2+1\right)b\,c^2dx-ibcd\ln(-icx+1)+2ae\,x^2c-\ln\left(c^2x^2+\right.}{2cx}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(a/c^2*(e*c*x-d*c/x)+b/c^2*(arctan(c*x)*e*c*x-arctan(c*x)*d*c/x-1/2*(c^2
*d+e)*ln(c^2*x^2+1)+d*c^2*ln(c*x)))
```

3.1119.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int\frac{(d+ex^2)\,(a+b\arctan(cx))}{x^2}\,dx$$
$$=\frac{2\,bc^2dx\log{(x)}+2\,acex^2-2\,acd-(bc^2d+be)x\log{(c^2x^2+1)}+2\,(bcex^2-bcd)\arctan{(cx)}}{2\,cx}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*c^2*d*x*log(x) + 2*a*c*e*x^2 - 2*a*c*d - (b*c^2*d + b*e)*x*log(c^
2*x^2 + 1) + 2*(b*c*e*x^2 - b*c*d)*arctan(c*x))/(c*x)
```

3.1119.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx$$
$$= \begin{cases} -\frac{ad}{x} + aex + bcd\log(x) - \frac{bcd\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd\,\mathrm{atan}(cx)}{x} + bex\,\mathrm{atan}(cx) - \frac{be\log\left(x^2+\frac{1}{c^2}\right)}{2c} & \text{for } c \neq 0 \\ a\left(-\frac{d}{x} + ex\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a*d/x + a*e*x + b*c*d*log(x) - b*c*d*log(x**2 + c**(-2))/2 - b
*d*atan(c*x)/x + b*e*x*atan(c*x) - b*e*log(x**2 + c**(-2))/(2*c), Ne(c, 0)
), (a*(-d/x + e*x), True))
```

3.1119.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = -\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd + aex + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))be}{2\,c} - \frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + a*e*x + 1/2
*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e/c - a*d/x
```

3.1119.8 Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1119.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = a\,e\,x - \frac{a\,d}{x} + b\,e\,x\,\mathrm{atan}(c\,x) - \frac{b\,c\,d\,\ln\left(c^2\,x^2+1\right)}{2} + b\,c\,d\,\ln(x) - \frac{b\,d\,\mathrm{atan}(c\,x)}{x} - \frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{2\,c}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^2,x)
```

output
```
a*e*x - (a*d)/x + b*e*x*atan(c*x) - (b*c*d*log(c^2*x^2 + 1))/2 + b*c*d*log
(x) - (b*d*atan(c*x))/x - (b*e*log(c^2*x^2 + 1))/(2*c)
```

3.1120 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3} dx$

3.1120.1 Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3} dx = -\frac{bcd}{2x} - \frac{1}{2}bc^2d\arctan(cx) - \frac{d(a+b\arctan(cx))}{2x^2} + ae\log(x) + \frac{1}{2}ibe\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibe\,\text{PolyLog}(2,icx)$$

output
```
-1/2*b*c*d/x-1/2*b*c^2*d*arctan(c*x)-1/2*d*(a+b*arctan(c*x))/x^2+a*e*ln(x)
+1/2*I*b*e*polylog(2,-I*c*x)-1/2*I*b*e*polylog(2,I*c*x)
```

3.1120.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3} dx = -\frac{ad}{2x^2} - \frac{bd\arctan(cx)}{2x^2} - \frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x} + ae\log(x) + \frac{1}{2}ibe\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibe\,\text{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 - (b*d*ArcTan[c*x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-1/2
, 1, 1/2, -(c^2*x^2)])/(2*x) + a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x]
 - (I/2)*b*e*PolyLog[2, I*c*x]
```

3.1120.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d(a+b\arctan(cx))}{x^3}+\frac{e(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{2x^2}+ae\log(x)-\frac{1}{2}bc^2d\arctan(cx)-\frac{bcd}{2x}+\frac{1}{2}ibe\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\,\text{PolyLog}(2,icx)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d)/x - (b*c^2*d*ArcTan[c*x])/2 - (d*(a + b*ArcTan[c*x]))/(2*x^2)
 + a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x] - (I/2)*b*e*PolyLog[2, I*c*
x]
```

3.1120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1120.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

method	result
derivativedivides	$c^2\left(\frac{ae\ln(cx)}{c^2} - \frac{ad}{2c^2x^2} + \frac{b\left(\arctan(cx)e\ln(cx) - \frac{\arctan(cx)d}{2x^2} + \frac{ie\ln(cx)\ln(icx+1)}{2} - \frac{ie\ln(cx)\ln(-icx+1)}{2} + \frac{ie\,\mathrm{dilog}(icx+1)}{2}\right.}{c^2}\right.$
default	$c^2\left(\frac{ae\ln(cx)}{c^2} - \frac{ad}{2c^2x^2} + \frac{b\left(\arctan(cx)e\ln(cx) - \frac{\arctan(cx)d}{2x^2} + \frac{ie\ln(cx)\ln(icx+1)}{2} - \frac{ie\ln(cx)\ln(-icx+1)}{2} + \frac{ie\,\mathrm{dilog}(icx+1)}{2}\right.}{c^2}\right.$
parts	$ae\ln(x) - \frac{ad}{2x^2} + bc^2\left(\frac{\arctan(cx)\ln(cx)e}{c^2} - \frac{\arctan(cx)d}{2c^2x^2} - \frac{-ie\ln(cx)\ln(icx+1)+ie\ln(cx)\ln(-icx+1)-ie\,\mathrm{di}}{}\right.$
risch	$-\frac{ibe\,\mathrm{dilog}(-icx+1)}{2} + \frac{ic^2bd\ln(-icx)}{4} - \frac{bcd}{2x} - \frac{ic^2bd\ln(c^2x^2+1)}{8} - \frac{b\,c^2d\arctan(cx)}{4} - \frac{ibd\ln(-icx+1)}{4x^2} + ae\,\mathrm{l}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/c^2*e*ln(c*x)-1/2*a*d/c^2/x^2+b/c^2*(arctan(c*x)*e*ln(c*x)-1/2*arct
an(c*x)*d/x^2+1/2*I*e*ln(c*x)*ln(1+I*c*x)-1/2*I*e*ln(c*x)*ln(1-I*c*x)+1/2*
I*e*dilog(1+I*c*x)-1/2*I*e*dilog(1-I*c*x)+1/2*d*c^2*(-1/c/x-arctan(c*x))))
```

3.1120.5 Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))/x^3, x)
```

3.1120.6 Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)/x**3, x)
```

3.1120.7 Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d + b*e*integrate(arcta
n(c*x)/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

3.1120.8 Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1120.9 Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx$$
$$= \begin{cases} a\,e\,\ln(x) - \frac{a\,d}{2\,x^2} & \text{if } c=0 \\ a\,e\,\ln(x) - \frac{a\,d}{2\,x^2} - \frac{b\,d\,\mathrm{atan}(c\,x)}{2\,x^2} - \frac{b\,d\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - \frac{b\,e\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))\,1\mathrm{i}}{2} & \text{if } c\neq 0 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^3,x)
```

output
```
piecewise(c == 0, a*e*log(x) - (a*d)/(2*x^2), c ~= 0, a*e*log(x) - (b*e*(d
ilog(- c*x*1i + 1) - dilog(c*x*1i + 1))*1i)/2 - (a*d)/(2*x^2) - (b*d*atan(
c*x))/(2*x^2) - (b*d*(c^3*atan(c*x) + c^2/x))/(2*c))
```

3.1121 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx$

3.1121.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd}{6x^2}-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x} -\frac{1}{3}bc(c^2d-3e)\log(x)+\frac{1}{6}bc(c^2d-3e)\log(1+c^2x^2)$$

output
```
-1/6*b*c*d/x^2-1/3*d*(a+b*arctan(c*x))/x^3-e*(a+b*arctan(c*x))/x-1/3*b*c*(
c^2*d-3*e)*ln(x)+1/6*b*c*(c^2*d-3*e)*ln(c^2*x^2+1)
```

3.1121.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{x}-\frac{bd\arctan(cx)}{3x^3}-\frac{be\arctan(cx)}{x} +bce\log(x)-\frac{1}{2}bce\log(1+c^2x^2) +\frac{1}{6}bcd\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log(1+c^2x^2)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(a*d)/x^3 - (a*e)/x - (b*d*ArcTan[c*x])/(3*x^3) - (b*e*ArcTan[c*x])/x
 + b*c*e*Log[x] - (b*c*e*Log[1 + c^2*x^2])/2 + (b*c*d*(-x^(-2) - 2*c^2*Log
[x] + c^2*Log[1 + c^2*x^2]))/6
```

3.1121.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5511, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{3ex^2+d}{3x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{3ex^2+d}{x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{6}bc\int \frac{3ex^2+d}{x^4(c^2x^2+1)}dx^2-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{6}bc\int \left(\frac{d}{x^4}+\frac{c^4d-3c^2e}{c^2x^2+1}+\frac{3e-c^2d}{x^2}\right)dx^2-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}+\frac{1}{6}bc\left(-\log(x^2)(c^2d-3e)+(c^2d-3e)\log(c^2x^2+1)-\frac{d}{x^2}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcTan[c*x]))/x^3 - (e*(a + b*ArcTan[c*x]))/x + (b*c*(-(d/x
^2) - (c^2*d - 3*e)*Log[x^2] + (c^2*d - 3*e)*Log[1 + c^2*x^2]))/6
```

3.1121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1121. $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx$

3.1121.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

method	result
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^{3}}\right)+b\,c^{3}\left(-\frac{\arctan(cx)e}{c^{3}x}-\frac{\arctan(cx)d}{3c^{3}x^{3}}-\frac{(c^{2}d-3e)\ln(cx)+\frac{d}{2x^{2}}+\frac{\left(-c^{2}d+3e\right)\ln\left(c^{2}x^{2}+1\right)}{2}}{3c^{2}}\right)$
derivativedivides	$c^{3}\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^{3}}\right)}{c^{2}}+\frac{b\left(-\frac{\arctan(cx)e}{cx}-\frac{\arctan(cx)d}{3c\,x^{3}}-\frac{\left(-c^{2}d+3e\right)\ln\left(c^{2}x^{2}+1\right)}{6}-\frac{\left(c^{2}d-3e\right)\ln(cx)}{3}-\frac{d}{6x^{2}}\right)}{c^{2}}\right)$
default	$c^{3}\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^{3}}\right)}{c^{2}}+\frac{b\left(-\frac{\arctan(cx)e}{cx}-\frac{\arctan(cx)d}{3c\,x^{3}}-\frac{\left(-c^{2}d+3e\right)\ln\left(c^{2}x^{2}+1\right)}{6}-\frac{\left(c^{2}d-3e\right)\ln(cx)}{3}-\frac{d}{6x^{2}}\right)}{c^{2}}\right)$
parallelrisch	$-\frac{2\ln(x)b\,c^{3}d\,x^{3}-\ln(c^{2}x^{2}+1)b\,c^{3}d\,x^{3}-b\,c^{3}d\,x^{3}-6\ln(x)bce\,x^{3}+3\ln(c^{2}x^{2}+1)bce\,x^{3}+6\arctan(cx)be\,x^{2}+6ae\,x^{2}+bcdx+}{6x^{3}}$
risch	$\frac{ib(3e\,x^{2}+d)\ln(icx+1)}{6x^{3}}-\frac{2\ln(x)b\,c^{3}d\,x^{3}-\ln(c^{2}x^{2}+1)b\,c^{3}d\,x^{3}-6\ln(x)bce\,x^{3}+3\ln(c^{2}x^{2}+1)bce\,x^{3}+3ibe\ln(-icx+1)x^{2}}{6x^{3}}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-arctan(c*x)/c^3*e/x-1/3*arctan(c*x)*d/c^3/x^3-1
/3/c^2*((c^2*d-3*e)*ln(c*x)+1/2*d/x^2+1/2*(-c^2*d+3*e)*ln(c^2*x^2+1)))
```

3.1121.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int\frac{(d+ex^{2})\,(a+b\arctan(cx))}{x^{4}}\,dx$$
$$=\frac{(bc^{3}d-3\,bce)x^{3}\log\left(c^{2}x^{2}+1\right)-2\,(bc^{3}d-3\,bce)x^{3}\log\left(x\right)-bcdx-6\,aex^{2}-2\,ad-2\,(3\,bex^{2}+bd)\arctan}{6\,x^{3}}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*((b*c^3*d - 3*b*c*e)*x^3*log(c^2*x^2 + 1) - 2*(b*c^3*d - 3*b*c*e)*x^3*
log(x) - b*c*d*x - 6*a*e*x^2 - 2*a*d - 2*(3*b*e*x^2 + b*d)*arctan(c*x))/x^
3
```

3.1121.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx$$
$$= \begin{cases} -\frac{ad}{3x^3} - \frac{ae}{x} - \frac{bc^3 d\log(x)}{3} + \frac{bc^3 d\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd}{6x^2} + bce\log(x) - \frac{bce\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd\operatorname{atan}(cx)}{3x^3} - \frac{be\operatorname{atan}(cx)}{x} & \text{for } c \\ a\left(-\frac{d}{3x^3} - \frac{e}{x}\right) & \text{othe} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a*d/(3*x**3) - a*e/x - b*c**3*d*log(x)/3 + b*c**3*d*log(x**2 +
 c**(-2))/6 - b*c*d/(6*x**2) + b*c*e*log(x) - b*c*e*log(x**2 + c**(-2))/2
- b*d*atan(c*x)/(3*x**3) - b*e*atan(c*x)/x, Ne(c, 0)), (a*(-d/(3*x**3) - e
/x), True))
```

3.1121.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bd$$
$$- \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)be - \frac{ae}{x} - \frac{ad}{3x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*d - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e - a*e/x
- 1/3*a*d/x^3
```

3.1121.8 Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.1121.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx = b\,c\,e\,\ln(x) - \frac{a\,e}{x} - \frac{b\,c\,e\,\ln(c^2\,x^2+1)}{2} - \frac{b\,c\,d}{6\,x^2} - \frac{a\,d}{3\,x^3} - \frac{b\,d\,\mathrm{atan}(c\,x)}{3\,x^3} - \frac{b\,e\,\mathrm{atan}(c\,x)}{x} + \frac{b\,c^3\,d\,\ln(c^2\,x^2+1)}{6} - \frac{b\,c^3\,d\,\ln(x)}{3}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^4,x)
```

output
```
b*c*e*log(x) - (a*e)/x - (b*c*e*log(c^2*x^2 + 1))/2 - (b*c*d)/(6*x^2) - (a
*d)/(3*x^3) - (b*d*atan(c*x))/(3*x^3) - (b*e*atan(c*x))/x + (b*c^3*d*log(c
^2*x^2 + 1))/6 - (b*c^3*d*log(x))/3
```

3.1122 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx$

3.1122.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd}{12x^3}+\frac{bc(c^2d-2e)}{4x}+\frac{1}{4}bc^2(c^2d-2e)\arctan(cx) - \frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

output
```
-1/12*b*c*d/x^3+1/4*b*c*(c^2*d-2*e)/x+1/4*b*c^2*(c^2*d-2*e)*arctan(c*x)-1/
4*d*(a+b*arctan(c*x))/x^4-1/2*e*(a+b*arctan(c*x))/x^2
```

3.1122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = -\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bd\arctan(cx)}{4x^4}-\frac{be\arctan(cx)}{2x^2} - \frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} - \frac{bce\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(a*d)/x^4 - (a*e)/(2*x^2) - (b*d*ArcTan[c*x])/(4*x^4) - (b*e*ArcTan[c
*x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^
3) - (b*c*e*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x)
```

3.1122.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5511, 27, 359, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{2ex^2+d}{4x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}bc\int \frac{2ex^2+d}{x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{359}$$

$$\frac{1}{4}bc\left(-(c^2d-2e)\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{d}{3x^3}\right)-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4}bc\left(-(c^2d-2e)\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{d}{3x^3}\right)-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}+\frac{1}{4}bc\left(-\left(-c\arctan(cx)-\frac{1}{x}\right)(c^2d-2e)-\frac{d}{3x^3}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d*(a + b*ArcTan[c*x]))/x^4 - (e*(a + b*ArcTan[c*x]))/(2*x^2) + (b*c*
(-1/3*d/x^3 - (c^2*d - 2*e)*(-x^(-1) - c*ArcTan[c*x])))/4
```

3.1122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1122. $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx$

3.1122.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

method	result
parts	$a\left(-\frac{d}{4x^4}-\frac{e}{2x^2}\right)+b\,c^4\left(-\frac{\arctan(cx)d}{4c^4x^4}-\frac{\arctan(cx)e}{2c^4x^2}-\frac{-\frac{c^2d-2e}{cx}+\frac{d}{3c\,x^3}+\left(-c^2d+2e\right)\arctan(cx)}{4c^2}\right)$
parallelrisch	$\frac{3x^4\arctan(cx)b\,c^4d-6\arctan(cx)b\,c^2e\,x^4+6a\,c^2e\,x^4+3b\,c^3d\,x^3-6bce\,x^3-6\arctan(cx)be\,x^2-6ae\,x^2-bcdx-3\arctan(cx)}{12x^4}$
derivativedivides	$c^4\left(\frac{a\left(-\frac{d}{4c^2x^4}-\frac{e}{2c^2x^2}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)d}{4c^2x^4}-\frac{\arctan(cx)e}{2c^2x^2}-\frac{\left(-c^2d+2e\right)\arctan(cx)}{4}+\frac{c^2d-2e}{4cx}-\frac{d}{12c\,x^3}\right)}{c^2}\right)$
default	$c^4\left(\frac{a\left(-\frac{d}{4c^2x^4}-\frac{e}{2c^2x^2}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)d}{4c^2x^4}-\frac{\arctan(cx)e}{2c^2x^2}-\frac{\left(-c^2d+2e\right)\arctan(cx)}{4}+\frac{c^2d-2e}{4cx}-\frac{d}{12c\,x^3}\right)}{c^2}\right)$
risch	$\frac{ib\left(2e\,x^2+d\right)\ln(icx+1)}{8x^4}-\frac{3i\ln(-cx+i)b\,c^4d\,x^4-3i\ln(-cx-i)b\,c^4d\,x^4-6i\ln(-cx+i)b\,c^2e\,x^4+6i\ln(-cx-i)b\,c^2e\,x^4-6b}{24x^4}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/4*d/x^4-1/2*e/x^2)+b*c^4*(-1/4*arctan(c*x)*d/c^4/x^4-1/2*arctan(c*x)
/c^4*e/x^2-1/4/c^2*(-(c^2*d-2*e)/c/x+1/3*d/c/x^3+(-c^2*d+2*e)*arctan(c*x))
)
```

3.1122.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcdx+6\,aex^2-3\,(bc^3d-2\,bce)x^3+3\,ad-3\,((bc^4d-2\,bc^2e)x^4-2\,bex^2-bd)\arctan{(cx)}}{12\,x^4}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
-1/12*(b*c*d*x + 6*a*e*x^2 - 3*(b*c^3*d - 2*b*c*e)*x^3 + 3*a*d - 3*((b*c^4
*d - 2*b*c^2*e)*x^4 - 2*b*e*x^2 - b*d)*arctan(c*x))/x^4
```

3.1122.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = -\frac{ad}{4x^4} - \frac{ae}{2x^2} + \frac{bc^4d\operatorname{atan}(cx)}{4} + \frac{bc^3d}{4x} - \frac{bc^2e\operatorname{atan}(cx)}{2} - \frac{bcd}{12x^3} - \frac{bce}{2x} - \frac{bd\operatorname{atan}(cx)}{4x^4} - \frac{be\operatorname{atan}(cx)}{2x^2}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**5,x)
```

output
```
-a*d/(4*x**4) - a*e/(2*x**2) + b*c**4*d*atan(c*x)/4 + b*c**3*d/(4*x) - b*c
**2*e*atan(c*x)/2 - b*c*d/(12*x**3) - b*c*e/(2*x) - b*d*atan(c*x)/(4*x**4)
 - b*e*atan(c*x)/(2*x**2)
```

3.1122.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = \frac{1}{12}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2-1}{x^3}\right)c - \frac{3\arctan(cx)}{x^4}\right)bd - \frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)be - \frac{ae}{2\,x^2} - \frac{ad}{4\,x^4}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e - 1/2*a*e/x^2 - 1/4
*a*d/x^4
```

3.1122.8 Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.1122.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx= -\frac{\frac{a\,d}{4}+\frac{a\,x^2\,\left(d\,c^2+2\,e\right)}{4}+\frac{b\,d\,\mathrm{atan}(c\,x)}{4}+\frac{b\,c\,d\,x}{12}+\frac{b\,c^3\,x^5\,\left(2\,e-c^2\,d\right)}{4}+\frac{b\,c\,x^3\,\left(3\,e-c^2\,d\right)}{6}-\frac{a\,c^4\,e\,x^6}{2}+\frac{b\,x^2\,\mathrm{atan}(c\,x)\,\left(d\,c^2+2\,e\right)}{4}+\frac{b\,c}{}}{c^2\,x^6+x^4} -\frac{\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,\left(2\,b\,e-b\,c^2\,d\right)\,{\left(c^2\right)}^{3/2}}{4\,c}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^5,x)
```

output
```
- ((a*d)/4 + (a*x^2*(2*e + c^2*d))/4 + (b*d*atan(c*x))/4 + (b*c*d*x)/12 +
(b*c^3*x^5*(2*e - c^2*d))/4 + (b*c*x^3*(3*e - c^2*d))/6 - (a*c^4*e*x^6)/2
+ (b*x^2*atan(c*x)*(2*e + c^2*d))/4 + (b*c^2*e*x^4*atan(c*x))/2)/(x^4 + c^
2*x^6) - (atan((c^2*x)/(c^2)^(1/2))*(2*b*e - b*c^2*d)*(c^2)^(3/2))/(4*c)
```

3.1123 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx$

3.1123.1 Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd}{20x^4}+\frac{bc(3c^2d-5e)}{30x^2}-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}+\frac{1}{15}bc^3(3c^2d-5e)\log(x)-\frac{1}{30}bc^3(3c^2d-5e)\log(1+c^2x^2)$$

output
```
-1/20*b*c*d/x^4+1/30*b*c*(3*c^2*d-5*e)/x^2-1/5*d*(a+b*arctan(c*x))/x^5-1/3
*e*(a+b*arctan(c*x))/x^3+1/15*b*c^3*(3*c^2*d-5*e)*ln(x)-1/30*b*c^3*(3*c^2*
d-5*e)*ln(c^2*x^2+1)
```

3.1123.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = -\frac{ad}{5x^5}-\frac{bcd}{20x^4}-\frac{ae}{3x^3}+\frac{bc^3d}{10x^2}-\frac{bd\arctan(cx)}{5x^5}-\frac{be\arctan(cx)}{3x^3}+\frac{1}{5}bc^5d\log(x)-\frac{1}{10}bc^5d\log(1+c^2x^2)+\frac{1}{6}bce\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log(1+c^2x^2)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(a*d)/x^5 - (b*c*d)/(20*x^4) - (a*e)/(3*x^3) + (b*c^3*d)/(10*x^2) - (
b*d*ArcTan[c*x])/(5*x^5) - (b*e*ArcTan[c*x])/(3*x^3) + (b*c^5*d*Log[x])/5
- (b*c^5*d*Log[1 + c^2*x^2])/10 + (b*c*e*(-x^(-2) - 2*c^2*Log[x] + c^2*Log
[1 + c^2*x^2]))/6
```

3.1123.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5511, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{5ex^2+3d}{15x^5(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}bc\int \frac{5ex^2+3d}{x^5(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{30}bc\int \frac{5ex^2+3d}{x^6(c^2x^2+1)}dx^2-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{30}bc\int \left(\frac{3d}{x^6}+\frac{5c^4e-3c^6d}{c^2x^2+1}+\frac{3c^4d-5c^2e}{x^2}+\frac{5e-3c^2d}{x^4}\right)dx^2-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}+\frac{1}{30}bc\left(\frac{3c^2d-5e}{x^2}+c^2\log\left(x^2\right)\left(3c^2d-5e\right)-c^2\left(3c^2d-5e\right)\log\left(c^2x^2+1\right)-\frac{3d}{2x^4}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d*(a + b*ArcTan[c*x]))/x^5 - (e*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*
((-3*d)/(2*x^4) + (3*c^2*d - 5*e)/x^2 + c^2*(3*c^2*d - 5*e)*Log[x^2] - c^2
*(3*c^2*d - 5*e)*Log[1 + c^2*x^2]))/30
```

3.1123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1123.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

method	result
parts	$a\left(-\frac{e}{3x^3}-\frac{d}{5x^5}\right)+b\,c^5\left(-\frac{\arctan(cx)e}{3c^5x^3}-\frac{\arctan(cx)d}{5c^5x^5}-\frac{(-3c^2d+5e)\ln(cx)-\frac{3c^2d-5e}{2c^2x^2}+\frac{3d}{4c^2x^4}+\frac{\left(3c^2d-5e\right)\ln}{2}}{15c^2}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)d}{5c^3x^5}-\frac{\arctan(cx)e}{3c^3x^3}-\frac{\left(3c^2d-5e\right)\ln\left(c^2x^2+1\right)}{30}-\frac{\left(-3c^2d+5e\right)\ln(cx)}{15}+\frac{3c^2d-5e}{30c^2x^2}-\frac{d}{20c^2}\right.}{c^2}\right.$
default	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)d}{5c^3x^5}-\frac{\arctan(cx)e}{3c^3x^3}-\frac{\left(3c^2d-5e\right)\ln\left(c^2x^2+1\right)}{30}-\frac{\left(-3c^2d+5e\right)\ln(cx)}{15}+\frac{3c^2d-5e}{30c^2x^2}-\frac{d}{20c^2}\right.}{c^2}\right.$
parallelrisch	$\frac{12\ln(x)b\,c^5d\,x^5-6\ln\left(c^2x^2+1\right)x^5b\,c^5d-6b\,c^5d\,x^5-20\ln(x)b\,c^3e\,x^5+10\ln\left(c^2x^2+1\right)x^5b\,c^3e+10b\,c^3e\,x^5+6b\,c^3d\,x^3-10bce}{60x^5}$
risch	$\frac{ib\left(5e\,x^2+3d\right)\ln(icx+1)}{30x^5}-\frac{-12\ln(x)b\,c^5d\,x^5+6\ln\left(-c^2x^2-1\right)b\,c^5d\,x^5+20\ln(x)b\,c^3e\,x^5-10\ln\left(-c^2x^2-1\right)b\,c^3e\,x^5-6b\,c}{60x^5}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*e/x^3-1/5*d/x^5)+b*c^5*(-1/3*arctan(c*x)/c^5*e/x^3-1/5*arctan(c*x)
*d/c^5/x^5-1/15/c^2*((-3*c^2*d+5*e)*ln(c*x)-1/2*(3*c^2*d-5*e)/c^2/x^2+3/4*
d/c^2/x^4+1/2*(3*c^2*d-5*e)*ln(c^2*x^2+1)))
```

3.1123.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int\frac{(d+ex^2)\,(a+b\arctan(cx))}{x^6}\,dx =$$
$$-\frac{2\,(3\,bc^5d-5\,bc^3e)x^5\log\left(c^2x^2+1\right)-4\,(3\,bc^5d-5\,bc^3e)x^5\log\left(x\right)+3\,bcdx+20\,aex^2-2\,(3\,bc^3d-5\,bce}{60\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/60*(2*(3*b*c^5*d - 5*b*c^3*e)*x^5*log(c^2*x^2 + 1) - 4*(3*b*c^5*d - 5*b
*c^3*e)*x^5*log(x) + 3*b*c*d*x + 20*a*e*x^2 - 2*(3*b*c^3*d - 5*b*c*e)*x^3
+ 12*a*d + 4*(5*b*e*x^2 + 3*b*d)*arctan(c*x))/x^5
```

3.1123.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx$$
$$= \begin{cases} -\frac{ad}{5x^5} - \frac{ae}{3x^3} + \frac{bc^5d\log(x)}{5} - \frac{bc^5d\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d}{10x^2} - \frac{bc^3e\log(x)}{3} + \frac{bc^3e\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd}{20x^4} - \frac{bce}{6x^2} - \frac{bd\,\mathrm{atan}(cx)}{5x^5} - b \\ a\left(-\frac{d}{5x^5} - \frac{e}{3x^3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a*d/(5*x**5) - a*e/(3*x**3) + b*c**5*d*log(x)/5 - b*c**5*d*log
(x**2 + c**(-2))/10 + b*c**3*d/(10*x**2) - b*c**3*e*log(x)/3 + b*c**3*e*lo
g(x**2 + c**(-2))/6 - b*c*d/(20*x**4) - b*c*e/(6*x**2) - b*d*atan(c*x)/(5*
x**5) - b*e*atan(c*x)/(3*x**3), Ne(c, 0)), (a*(-d/(5*x**5) - e/(3*x**3)),
True))
```

3.1123.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx$$
$$= -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right) - 2\,c^4\log\left(x^2\right) - \frac{2\,c^2x^2-1}{x^4}\right)c + \frac{4\,\arctan(cx)}{x^5}\right)bd + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\,\arctan(cx)}{x^3}\right)be - \frac{ae}{3\,x^3} - \frac{ad}{5\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^
2)*c - 2*arctan(c*x)/x^3)*b*e - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

3.1123.8 Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.1123.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\begin{aligned}
&\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx \\
&= \frac{b\,c^3\,e\,\ln(c^2\,x^2+1)}{6} - \frac{b\,c^5\,d\,\ln(c^2\,x^2+1)}{10} \\
&\quad - \frac{x^3\left(\frac{b\,c\,e}{6}-\frac{b\,c^3\,d}{10}\right)+\frac{a\,d}{5}+x^2\left(\frac{a\,e}{3}+\frac{b\,e\,\mathrm{atan}(c\,x)}{3}\right)+\frac{b\,d\,\mathrm{atan}(c\,x)}{5}+\frac{b\,c\,d\,x}{20}}{x^5} \\
&\quad + \frac{b\,c^5\,d\,\ln(x)}{5} - \frac{b\,c^3\,e\,\ln(x)}{3}
\end{aligned}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^6,x)
```

output
```
(b*c^3*e*log(c^2*x^2 + 1))/6 - (b*c^5*d*log(c^2*x^2 + 1))/10 - (x^3*((b*c*
e)/6 - (b*c^3*d)/10) + (a*d)/5 + x^2*((a*e)/3 + (b*e*atan(c*x))/3) + (b*d*
atan(c*x))/5 + (b*c*d*x)/20)/x^5 + (b*c^5*d*log(x))/5 - (b*c^3*e*log(x))/3
```

3.1124 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx$

3.1124.1 Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd}{30x^5}+\frac{bc(2c^2d-3e)}{36x^3}-\frac{bc^3(2c^2d-3e)}{12x} -\frac{1}{12}bc^4(2c^2d-3e)\arctan(cx) -\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

output
```
-1/30*b*c*d/x^5+1/36*b*c*(2*c^2*d-3*e)/x^3-1/12*b*c^3*(2*c^2*d-3*e)/x-1/12
*b*c^4*(2*c^2*d-3*e)*arctan(c*x)-1/6*d*(a+b*arctan(c*x))/x^6-1/4*e*(a+b*ar
ctan(c*x))/x^4
```

3.1124.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx = -\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bd\arctan(cx)}{6x^6}-\frac{be\arctan(cx)}{4x^4} -\frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{30x^5} -\frac{bce\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(a*d)/x^6 - (a*e)/(4*x^4) - (b*d*ArcTan[c*x])/(6*x^6) - (b*e*ArcTan[c
*x])/(4*x^4) - (b*c*d*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)])/(30*x^
5) - (b*c*e*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3)
```

3.1124.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5511, 27, 359, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{3ex^2+2d}{12x^6(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bc\int \frac{3ex^2+2d}{x^6(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{359}$$

$$\frac{1}{12}bc\left(-(2c^2d-3e)\int \frac{1}{x^4(c^2x^2+1)}dx-\frac{2d}{5x^5}\right)-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{12}bc\left(-(2c^2d-3e)\left(c^2\left(-\int \frac{1}{x^2(c^2x^2+1)}dx\right)-\frac{1}{3x^3}\right)-\frac{2d}{5x^5}\right)-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{12}bc\left(-(2c^2d-3e)\left(-\left(c^2\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{2d}{5x^5}\right)-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}+\frac{1}{12}bc\left(-\left(-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)\left(2c^2d-3e\right)-\frac{2d}{5x^5}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(d*(a + b*ArcTan[c*x]))/x^6 - (e*(a + b*ArcTan[c*x]))/(4*x^4) + (b*c*
((-2*d)/(5*x^5) - (2*c^2*d - 3*e)*(-1/3*1/x^3 - c^2*(-x^(-1) - c*ArcTan[c*
x]))))/12
```

3.1124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1124.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

method	result
parallelrisch	$-\frac{30x^6\arctan(cx)bc^6d-45x^6\arctan(cx)bc^4e+30bc^5dx^5-45bc^3ex^5-10bc^3dx^3+15bcex^3+45\arctan(cx)bex^2+45ae\,x}{180x^6}$
parts	$a\left(-\frac{d}{6x^6}-\frac{e}{4x^4}\right)+bc^6\left(-\frac{\arctan(cx)d}{6c^6x^6}-\frac{\arctan(cx)e}{4c^6x^4}-\frac{-\frac{-2c^2d+3e}{cx}-\frac{2c^2d-3e}{3c^3x^3}+\frac{2d}{5c^3x^5}+(2c^2d-3e)\arctan(cx}{12c^2}\right.$
derivativedivides	$c^6\left(\frac{a\left(-\frac{e}{4c^4x^4}-\frac{d}{6c^4x^6}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{4c^4x^4}-\frac{\arctan(cx)d}{6c^4x^6}-\frac{(2c^2d-3e)\arctan(cx)}{12}+\frac{-2c^2d+3e}{12cx}+\frac{2c^2d-3e}{36c^3x^3}-\frac{d}{30c^3x^5}\right)}{c^2}\right)$
default	$c^6\left(\frac{a\left(-\frac{e}{4c^4x^4}-\frac{d}{6c^4x^6}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{4c^4x^4}-\frac{\arctan(cx)d}{6c^4x^6}-\frac{(2c^2d-3e)\arctan(cx)}{12}+\frac{-2c^2d+3e}{12cx}+\frac{2c^2d-3e}{36c^3x^3}-\frac{d}{30c^3x^5}\right)}{c^2}\right)$
risch	$\frac{ib(3ex^2+2d)\ln(icx+1)}{24x^6}-\frac{30i\ln(-cx-i)bc^6dx^6-30i\ln(-cx+i)bc^6dx^6-45i\ln(-cx-i)bc^4ex^6+45i\ln(-cx+i)bc^4e\,x}{}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/180*(30*x^6*arctan(c*x)*b*c^6*d-45*x^6*arctan(c*x)*b*c^4*e+30*b*c^5*d*x
^5-45*b*c^3*e*x^5-10*b*c^3*d*x^3+15*b*c*e*x^3+45*arctan(c*x)*b*e*x^2+45*a*
e*x^2+6*b*c*d*x+30*arctan(c*x)*b*d+30*a*d)/x^6
```

3.1124.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^7}\,dx =$$
$$-\frac{15\,(2\,bc^5d-3\,bc^3e)x^5+6\,bcdx+45\,aex^2-5\,(2\,bc^3d-3\,bce)x^3+30\,ad+15\,((2\,bc^6d-3\,bc^4e)x^6+3\,be}{180\,x^6}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
-1/180*(15*(2*b*c^5*d - 3*b*c^3*e)*x^5 + 6*b*c*d*x + 45*a*e*x^2 - 5*(2*b*c
^3*d - 3*b*c*e)*x^3 + 30*a*d + 15*((2*b*c^6*d - 3*b*c^4*e)*x^6 + 3*b*e*x^2
 + 2*b*d)*arctan(c*x))/x^6
```

3.1124.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^7}\,dx = & -\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bc^6d\,\mathrm{atan}\,(cx)}{6}-\frac{bc^5d}{6x}\\ & +\frac{bc^4e\,\mathrm{atan}\,(cx)}{4}+\frac{bc^3d}{18x^3}+\frac{bc^3e}{4x}-\frac{bcd}{30x^5}\\ & -\frac{bce}{12x^3}-\frac{bd\,\mathrm{atan}\,(cx)}{6x^6}-\frac{be\,\mathrm{atan}\,(cx)}{4x^4}\end{aligned}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**7,x)
```

output
```
-a*d/(6*x**6) - a*e/(4*x**4) - b*c**6*d*atan(c*x)/6 - b*c**5*d/(6*x) + b*c
**4*e*atan(c*x)/4 + b*c**3*d/(18*x**3) + b*c**3*e/(4*x) - b*c*d/(30*x**5)
- b*c*e/(12*x**3) - b*d*atan(c*x)/(6*x**6) - b*e*atan(c*x)/(4*x**4)
```

3.1124.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx \\
&= -\frac{1}{90}\left(\left(15\,c^5\arctan\left(cx\right)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\,\arctan\left(cx\right)}{x^6}\right)bd \\
&\quad+\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)be-\frac{ae}{4\,x^4}-\frac{ad}{6\,x^6}
\end{aligned}
$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arct
an(c*x)/x^6)*b*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*a
rctan(c*x)/x^4)*b*e - 1/4*a*e/x^4 - 1/6*a*d/x^6
```

3.1124.8 Giac [F]

$$
\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx=\int \frac{(ex^2+d)(b\arctan\left(cx\right)+a)}{x^7}\,dx
$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
sage0*x
```

3.1124.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$
\begin{aligned}
&\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx \\
&= \frac{b\,c^4\,\mathrm{atan}\left(\frac{b\,c^2\,x\,\left(3\,e-2\,c^2\,d\right)}{3\,b\,c\,e-2\,b\,c^3\,d}\right)\,\left(3\,e-2\,c^2\,d\right)}{12}-\frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,e\,x^2}{4}+\frac{b\,d}{6}\right)}{x^6} \\
&\quad-\frac{x^3\,\left(b\,c\,e-\frac{2\,b\,c^3\,d}{3}\right)+2\,a\,d-c^2\,x^5\,\left(3\,b\,c\,e-2\,b\,c^3\,d\right)+3\,a\,e\,x^2+\frac{2\,b\,c\,d\,x}{5}}{12\,x^6}
\end{aligned}
$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^7,x)
```

output
```
(b*c^4*atan((b*c^2*x*(3*e - 2*c^2*d))/(3*b*c*e - 2*b*c^3*d))*(3*e - 2*c^2*
d))/12 - (atan(c*x)*((b*d)/6 + (b*e*x^2)/4))/x^6 - (x^3*(b*c*e - (2*b*c^3*
d)/3) + 2*a*d - c^2*x^5*(3*b*c*e - 2*b*c^3*d) + 3*a*e*x^2 + (2*b*c*d*x)/5)
/(12*x^6)
```

3.1124. $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx$

3.1125 $\int x^3(d+ex^2)^2(a+b\arctan(cx))\,dx$

3.1125.1 Optimal result

Integrand size = 21, antiderivative size = 185

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))\,dx = \frac{b(6c^4d^2-8c^2de+3e^2)\,x}{24c^7} - \frac{b(6c^4d^2-8c^2de+3e^2)\,x^3}{72c^5} - \frac{b(8c^2d-3e)\,ex^5}{120c^3} - \frac{be^2x^7}{56c} - \frac{b(6c^4d^2-8c^2de+3e^2)\arctan(cx)}{24c^8} + \frac{1}{4}d^2x^4(a+b\arctan(cx)) + \frac{1}{3}dex^6(a+b\arctan(cx)) + \frac{1}{8}e^2x^8(a+b\arctan(cx))$$

output
```
1/24*b*(6*c^4*d^2-8*c^2*d*e+3*e^2)*x/c^7-1/72*b*(6*c^4*d^2-8*c^2*d*e+3*e^2
)*x^3/c^5-1/120*b*(8*c^2*d-3*e)*e*x^5/c^3-1/56*b*e^2*x^7/c-1/24*b*(6*c^4*d
^2-8*c^2*d*e+3*e^2)*arctan(c*x)/c^8+1/4*d^2*x^4*(a+b*arctan(c*x))+1/3*d*e*
x^6*(a+b*arctan(c*x))+1/8*e^2*x^8*(a+b*arctan(c*x))
```

3.1125.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int x^3\left(d+ex^2\right)^2(a+b\arctan(cx))\,dx=\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))+\frac{1}{840}be^2\left(\frac{105x}{c^7}-\frac{35x^3}{c^5}+\frac{21x^5}{c^3}-\frac{15x^7}{c}-\frac{105\arctan(cx)}{c^8}\right)-\frac{1}{45}bde\left(\frac{15x}{c^5}-\frac{5x^3}{c^3}+\frac{3x^5}{c}-\frac{15\arctan(cx)}{c^6}\right)+\frac{1}{12}bd^2\left(\frac{3x}{c^3}-\frac{x^3}{c}-\frac{3\arctan(cx)}{c^4}\right)$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcTan[c*x]))/4 + (d*e*x^6*(a + b*ArcTan[c*x]))/3 + (e^2*x
^8*(a + b*ArcTan[c*x]))/8 + (b*e^2*((105*x)/c^7 - (35*x^3)/c^5 + (21*x^5)/
c^3 - (15*x^7)/c - (105*ArcTan[c*x])/c^8))/840 - (b*d*e*((15*x)/c^5 - (5*x
^3)/c^3 + (3*x^5)/c - (15*ArcTan[c*x])/c^6))/45 + (b*d^2*((3*x)/c^3 - x^3/
c - (3*ArcTan[c*x])/c^4))/12
```

3.1125.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5511, 27, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^4\left(3e^2x^4+8dex^2+6d^2\right)}{24\left(c^2x^2+1\right)}dx+\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{24}bc\int \frac{x^4(3e^2x^4+8dex^2+6d^2)}{c^2x^2+1}dx+\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))$$

$$\Big\downarrow \; 1584$$

$$-\frac{1}{24}bc\int \left(\frac{3e^2x^6}{c^2}+\frac{(8c^2d-3e)\,ex^4}{c^4}+\frac{(6d^2c^4-8dec^2+3e^2)\,x^2}{c^6}-\frac{6d^2c^4-8dec^2+3e^2}{c^8}+\frac{6d^2c^4-8dec^2+3e^2}{c^8\,(c^2x^2+1)}\right)$$
$$\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))-$$
$$\frac{1}{24}bc\left(\frac{\arctan(cx)\,(6c^4d^2-8c^2de+3e^2)}{c^9}+\frac{3e^2x^7}{7c^2}+\frac{ex^5(8c^2d-3e)}{5c^4}-\frac{x(6c^4d^2-8c^2de+3e^2)}{c^8}+\frac{x^3(6c^4d^2-8c^2}{3c^6}\right.$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcTan[c*x]))/4 + (d*e*x^6*(a + b*ArcTan[c*x]))/3 + (e^2*x
^8*(a + b*ArcTan[c*x]))/8 - (b*c*(-(((6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x)/c^
8) + ((6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x^3)/(3*c^6) + ((8*c^2*d - 3*e)*e*x^
5)/(5*c^4) + (3*e^2*x^7)/(7*c^2) + ((6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*ArcTan
[c*x])/c^9))/24
```

3.1125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1125.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}ed\,x^6+\frac{1}{4}d^2x^4\right)+\frac{b\left(\frac{\arctan(cx)c^4e^2x^8}{8}+\frac{\arctan(cx)c^4de\,x^6}{3}+\frac{\arctan(cx)d^2c^4x^4}{4}-\frac{\frac{3e^2c^7x^7}{7}+\frac{8d\,c^7e\,x^5}{5}}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b\left(\frac{\arctan(cx)d^2c^8x^4}{4}+\frac{\arctan(cx)d\,c^8e\,x^6}{3}+\frac{\arctan(cx)e^2c^8x^8}{8}-\frac{d^2c^7x^3}{12}-\frac{d\,c^7e\,x^5}{15}-\frac{e^2c^7x^7}{56}+\right.}{c^4}}{c^4}$
default	$\frac{\frac{a\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b\left(\frac{\arctan(cx)d^2c^8x^4}{4}+\frac{\arctan(cx)d\,c^8e\,x^6}{3}+\frac{\arctan(cx)e^2c^8x^8}{8}-\frac{d^2c^7x^3}{12}-\frac{d\,c^7e\,x^5}{15}-\frac{e^2c^7x^7}{56}+\right.}{c^4}}{c^4}$
parallelrisch	$315x^8\arctan(cx)b\,c^8e^2+315a\,c^8e^2x^8+840x^6\arctan(cx)b\,c^8de-45b\,c^7e^2x^7+840a\,c^8de\,x^6+630d^2b\arctan(cx)x^4c^8-168b$
risch	$-\frac{ib\left(3e^2x^8+8ed\,x^6+6d^2x^4\right)\ln(icx+1)}{48}+\frac{ibde\,x^6\ln(-icx+1)}{6}+\frac{ib\,d^2x^4\ln(-icx+1)}{8}+\frac{x^8e^2a}{8}+\frac{ib\,e^2x^8\ln(-icx+}{16}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*e*d*x^6+1/4*d^2*x^4)+b/c^4*(1/8*arctan(c*x)*c^4*e^2*x^8
+1/3*arctan(c*x)*c^4*d*e*x^6+1/4*arctan(c*x)*d^2*c^4*x^4-1/24/c^4*(3/7*e^2
*c^7*x^7+8/5*d*c^7*e*x^5+2*d^2*c^7*x^3-3/5*e^2*c^5*x^5-8/3*d*c^5*e*x^3-6*c
^5*x*d^2+e^2*c^3*x^3+8*c^3*d*e*x-3*c*x*e^2+(6*c^4*d^2-8*c^2*d*e+3*e^2)*arc
tan(c*x)))
```

3.1125.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.09

$$\int x^3 (d + ex^2)^2 (a + b \arctan(cx)) \, dx$$
$$= \frac{315\, ac^8e^2x^8 + 840\, ac^8dex^6 - 45\, bc^7e^2x^7 + 630\, ac^8d^2x^4 - 21\, (8\, bc^7de - 3\, bc^5e^2)x^5 - 35\, (6\, bc^7d^2 - 8\, bc^5de +}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 - 45*b*c^7*e^2*x^7 + 630*a*c
^8*d^2*x^4 - 21*(8*b*c^7*d*e - 3*b*c^5*e^2)*x^5 - 35*(6*b*c^7*d^2 - 8*b*c^
5*d*e + 3*b*c^3*e^2)*x^3 + 105*(6*b*c^5*d^2 - 8*b*c^3*d*e + 3*b*c*e^2)*x +
 105*(3*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4 - 6*b*c^4*d^2 +
8*b*c^2*d*e - 3*b*e^2)*arctan(c*x))/c^8
```

3.1125.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.41

$$\int x^3 (d + ex^2)^2 (a + b \arctan(cx)) \, dx$$
$$= \begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4 \operatorname{atan}(cx)}{4} + \frac{bdex^6 \operatorname{atan}(cx)}{3} + \frac{be^2x^8 \operatorname{atan}(cx)}{8} - \frac{bd^2x^3}{12c} - \frac{bdex^5}{15c} - \frac{be^2x^7}{56c} + \frac{bd^2x}{4c^3} + \frac{bdex^3}{9c^3} + \\ a\left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*atan
(c*x)/4 + b*d*e*x**6*atan(c*x)/3 + b*e**2*x**8*atan(c*x)/8 - b*d**2*x**3/(
12*c) - b*d*e*x**5/(15*c) - b*e**2*x**7/(56*c) + b*d**2*x/(4*c**3) + b*d*e
*x**3/(9*c**3) + b*e**2*x**5/(40*c**3) - b*d**2*atan(c*x)/(4*c**4) - b*d*e
*x/(3*c**5) - b*e**2*x**3/(24*c**5) + b*d*e*atan(c*x)/(3*c**6) + b*e**2*x/
(8*c**7) - b*e**2*atan(c*x)/(8*c**8), Ne(c, 0)), (a*(d**2*x**4/4 + d*e*x**
6/3 + e**2*x**8/8), True))
```

3.1125.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99

$$\int x^3 (d+ex^2)^2 (a+b\arctan(cx))\,dx$$
$$= \frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{12}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bd^2 + \frac{1}{45}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)bde + \frac{1}{840}\left(105x^8\arctan(cx) - c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8} + \frac{105\arctan(cx)}{c^9}\right)\right)be^2$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arctan(c*x) -
c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2 + 1/45*(15*x^6*arctan(c
*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*d*e +
 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 1
05*x)/c^8 + 105*arctan(c*x)/c^9))*b*e^2
```

3.1125.8 Giac [F]

$$\int x^3 (d+ex^2)^2 (a+b\arctan(cx))\,dx = \int (ex^2+d)^2(b\arctan(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1125.9 Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.02

$$\int x^3 (d+ex^2)^2 (a+b\arctan(cx))\,dx$$
$$= x^4\left(\frac{\frac{a\,e^2}{c^2}-\frac{a\,e\,(2\,d\,c^2+e)}{c^2}}{4\,c^2}+\frac{a\,d\,(d\,c^2+2\,e)}{4\,c^2}\right)-x^6\left(\frac{a\,e^2}{6\,c^2}-\frac{a\,e\,(2\,d\,c^2+e)}{6\,c^2}\right)$$
$$+x^5\left(\frac{b\,e^2}{40\,c^3}-\frac{b\,d\,e}{15\,c}\right)+\mathrm{atan}(c\,x)\,\left(\frac{b\,d^2\,x^4}{4}+\frac{b\,d\,e\,x^6}{3}+\frac{b\,e^2\,x^8}{8}\right)$$
$$-x^2\left(\frac{\frac{\frac{a\,e^2}{c^2}-\frac{a\,e\,\left(2\,d\,c^2+e\right)}{c^2}}{c^2}+\frac{a\,d\,(d\,c^2+2\,e)}{c^2}}{2\,c^2}-\frac{a\,d^2}{2\,c^2}\right)$$
$$-x^3\left(\frac{\frac{b\,e^2}{8\,c^3}-\frac{b\,d\,e}{3\,c}}{3\,c^2}+\frac{b\,d^2}{12\,c}\right)+\frac{x\left(\frac{\frac{b\,e^2}{8\,c^3}-\frac{b\,d\,e}{3\,c}}{c^2}+\frac{b\,d^2}{4\,c}\right)}{c^2}+\frac{a\,e^2\,x^8}{8}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{b\,c\,x\,(6\,c^4\,d^2-8\,c^2\,d\,e+3\,e^2)}{6\,b\,c^4\,d^2-8\,b\,c^2\,d\,e+3\,b\,e^2}\right)\,(6\,c^4\,d^2-8\,c^2\,d\,e+3\,e^2)}{24\,c^8}-\frac{b\,e^2\,x^7}{56\,c}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
x^4*(((a*e^2)/c^2 - (a*e*(e + 2*c^2*d))/c^2)/(4*c^2) + (a*d*(2*e + c^2*d))
/(4*c^2)) - x^6*((a*e^2)/(6*c^2) - (a*e*(e + 2*c^2*d))/(6*c^2)) + x^5*((b*
e^2)/(40*c^3) - (b*d*e)/(15*c)) + atan(c*x)*((b*d^2*x^4)/4 + (b*e^2*x^8)/8
 + (b*d*e*x^6)/3) - x^2*((((a*e^2)/c^2 - (a*e*(e + 2*c^2*d))/c^2)/c^2 + (a
*d*(2*e + c^2*d))/c^2)/(2*c^2) - (a*d^2)/(2*c^2)) - x^3*(((b*e^2)/(8*c^3)
- (b*d*e)/(3*c))/(3*c^2) + (b*d^2)/(12*c)) + (x*(((b*e^2)/(8*c^3) - (b*d*e
)/(3*c))/c^2 + (b*d^2)/(4*c)))/c^2 + (a*e^2*x^8)/8 - (b*atan((b*c*x*(3*e^2
 + 6*c^4*d^2 - 8*c^2*d*e))/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e))*(3*e^2 +
 6*c^4*d^2 - 8*c^2*d*e))/(24*c^8) - (b*e^2*x^7)/(56*c)
```

3.1126 $\int x^2(d+ex^2)^2 (a+b\arctan(cx))\,dx$

3.1126.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\int x^2(d+ex^2)^2 (a+b\arctan(cx))\,dx = -\frac{b(35c^4d^2-42c^2de+15e^2)\,x^2}{210c^5}-\frac{b(14c^2d-5e)\,ex^4}{140c^3} -\frac{be^2x^6}{42c}+\frac{1}{3}d^2x^3(a+b\arctan(cx)) +\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx)) +\frac{b(35c^4d^2-42c^2de+15e^2)\log(1+c^2x^2)}{210c^7}$$

output
```
-1/210*b*(35*c^4*d^2-42*c^2*d*e+15*e^2)*x^2/c^5-1/140*b*(14*c^2*d-5*e)*e*x
^4/c^3-1/42*b*e^2*x^6/c+1/3*d^2*x^3*(a+b*arctan(c*x))+2/5*d*e*x^5*(a+b*arc
tan(c*x))+1/7*e^2*x^7*(a+b*arctan(c*x))+1/210*b*(35*c^4*d^2-42*c^2*d*e+15*
e^2)*ln(c^2*x^2+1)/c^7
```

3.1126.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^2(d+ex^2)^2 (a+b\arctan(cx))\,dx = \frac{c^2x^2(-30be^2+3bc^2e(28d+5ex^2)-2bc^4(35d^2+21dex^2+5e^2x^4)+4ac^5x(35d^2+42dex^2+15e^2x^4))+4}{420c^7}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*x^2*(-30*b*e^2 + 3*b*c^2*e*(28*d + 5*e*x^2) - 2*b*c^4*(35*d^2 + 21*d*
e*x^2 + 5*e^2*x^4) + 4*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)) + 4*b*c
^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcTan[c*x] + 2*b*(35*c^4*d^2 -
42*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/(420*c^7)
```

3.1126.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^3\left(15e^2x^4+42dex^2+35d^2\right)}{105\left(c^2x^2+1\right)}dx+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{105}bc\int \frac{x^3\left(15e^2x^4+42dex^2+35d^2\right)}{c^2x^2+1}dx+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{1578}$$

$$-\frac{1}{210}bc\int \frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{c^2x^2+1}dx^2+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{1195}$$

$$-\frac{1}{210}bc\int \left(\frac{15e^2x^4}{c^2}+\frac{3\left(14c^2d-5e\right)ex^2}{c^4}+\frac{35d^2c^4-42dec^2+15e^2}{c^6}+\frac{-35d^2c^4+42dec^2-15e^2}{c^6\left(c^2x^2+1\right)}\right)dx^2+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))-\frac{1}{210}bc\left(\frac{5e^2x^6}{c^2}+\frac{3ex^4(14c^2d-5e)}{2c^4}-\frac{(35c^4d^2-42c^2de+15e^2)\log(c^2x^2+1)}{c^8}+\frac{x^2(35c^4d^2-42c^2de+15e^2)}{c^6}\right)$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcTan[c*x]))/3 + (2*d*e*x^5*(a + b*ArcTan[c*x]))/5 + (e^2
*x^7*(a + b*ArcTan[c*x]))/7 - (b*c*(((35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*x^
2)/c^6 + (3*(14*c^2*d - 5*e)*e*x^4)/(2*c^4) + (5*e^2*x^6)/c^2 - ((35*c^4*d
^2 - 42*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/c^8))/210
```

3.1126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1126.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}ed\,x^5+\frac{1}{3}d^2x^3\right)+\frac{b\left(\frac{\arctan(cx)c^3e^2x^7}{7}+\frac{2\arctan(cx)c^3de\,x^5}{5}+\frac{\arctan(cx)d^2c^3x^3}{3}-\frac{\frac{35d^2c^6x^2}{2}+\frac{21d\,c^6e}{2}\cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\arctan(cx)d^2c^7x^3}{3}+\frac{2\arctan(cx)d\,c^7e\,x^5}{5}+\frac{\arctan(cx)e^2c^7x^7}{7}-\frac{d^2c^6x^2}{6}-\frac{d\,c^6e\,x^4}{10}+\frac{d\,c^4e\,x^2}{5}\cdots\right.}{c^4}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\arctan(cx)d^2c^7x^3}{3}+\frac{2\arctan(cx)d\,c^7e\,x^5}{5}+\frac{\arctan(cx)e^2c^7x^7}{7}-\frac{d^2c^6x^2}{6}-\frac{d\,c^6e\,x^4}{10}+\frac{d\,c^4e\,x^2}{5}\cdots\right.}{c^4}}{c^3}$
parallelrisch	$\frac{60x^7\arctan(cx)b\,c^7e^2+60a\,c^7e^2x^7+168x^5\arctan(cx)b\,c^7de-10b\,c^6e^2x^6+168a\,c^7de\,x^5+140x^3\arctan(cx)b\,c^7d^2-42b\,c^6\cdots}{\cdots}$
risch	$\frac{ibde\,x^5\ln(-icx+1)}{5}-\frac{ib(15e^2x^7+42ed\,x^5+35d^2x^3)\ln(icx+1)}{210}+\frac{ib\,d^2x^3\ln(-icx+1)}{6}+\frac{x^7e^2a}{7}+\frac{ib\,e^2x^7\ln(-icx\cdots}{14}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*e*d*x^5+1/3*d^2*x^3)+b/c^3*(1/7*arctan(c*x)*c^3*e^2*x^7
+2/5*arctan(c*x)*c^3*d*e*x^5+1/3*arctan(c*x)*d^2*c^3*x^3-1/105/c^4*(35/2*d
^2*c^6*x^2+21/2*d*c^6*e*x^4+5/2*e^2*c^6*x^6-21*d*c^4*e*x^2-15/4*e^2*c^4*x^
4+15/2*e^2*c^2*x^2+1/2*(-35*c^4*d^2+42*c^2*d*e-15*e^2)*ln(c^2*x^2+1)))
```

3.1126.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$=\frac{60\,ac^7e^2x^7+168\,ac^7dex^5-10\,bc^6e^2x^6+140\,ac^7d^2x^3-3\,(14\,bc^6de-5\,bc^4e^2)x^4-2\,(35\,bc^6d^2-42\,bc^4de+}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/420*(60*a*c^7*e^2*x^7 + 168*a*c^7*d*e*x^5 - 10*b*c^6*e^2*x^6 + 140*a*c^7
*d^2*x^3 - 3*(14*b*c^6*d*e - 5*b*c^4*e^2)*x^4 - 2*(35*b*c^6*d^2 - 42*b*c^4
*d*e + 15*b*c^2*e^2)*x^2 + 4*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c
^7*d^2*x^3)*arctan(c*x) + 2*(35*b*c^4*d^2 - 42*b*c^2*d*e + 15*b*e^2)*log(c
^2*x^2 + 1))/c^7
```

3.1126.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.52

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$=\begin{cases}\frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\,\mathrm{atan}\,(cx)}{3}+\frac{2bdex^5\,\mathrm{atan}\,(cx)}{5}+\frac{be^2x^7\,\mathrm{atan}\,(cx)}{7}-\frac{bd^2x^2}{6c}-\frac{bdex^4}{10c}-\frac{be^2x^6}{42c}+\frac{bd^2\log\left(x^2+\frac{1}{c^2}\right.}{6c^3} \\ a\left(\frac{d^2x^3}{3}+\frac{2dex^5}{5}+\frac{e^2x^7}{7}\right)\end{cases}$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*at
an(c*x)/3 + 2*b*d*e*x**5*atan(c*x)/5 + b*e**2*x**7*atan(c*x)/7 - b*d**2*x*
*2/(6*c) - b*d*e*x**4/(10*c) - b*e**2*x**6/(42*c) + b*d**2*log(x**2 + c**(
-2))/(6*c**3) + b*d*e*x**2/(5*c**3) + b*e**2*x**4/(28*c**3) - b*d*e*log(x*
*2 + c**(-2))/(5*c**5) - b*e**2*x**2/(14*c**5) + b*e**2*log(x**2 + c**(-2)
)/(14*c**7), Ne(c, 0)), (a*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), Tru
e))
```

3.1126.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12

$$\int x^2\left(d+ex^2\right)^2(a+b\arctan(cx))\,dx$$
$$=\frac{1}{7}\,ae^2x^7+\frac{2}{5}\,adex^5+\frac{1}{3}\,ad^2x^3+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^2$$
$$+\frac{1}{10}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bde$$
$$+\frac{1}{84}\left(12\,x^7\arctan\left(cx\right)-c\left(\frac{2\,c^4x^6-3\,c^2x^4+6\,x^2}{c^6}-\frac{6\,\log\left(c^2x^2+1\right)}{c^8}\right)\right)be^2$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arctan(c*x) - c
*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2 + 1/10*(4*x^5*arctan(c*x) - c*((c
^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e + 1/84*(12*x^7*arctan
(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*
b*e^2
```

3.1126.8 Giac [F]

$$\int x^2\left(d+ex^2\right)^2(a+b\arctan(cx))\,dx=\int\left(ex^2+d\right)^2(b\arctan\left(cx\right)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1126.9 Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx = \frac{a\,d^2\,x^3}{3}+\frac{a\,e^2\,x^7}{7}+\frac{b\,d^2\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}+\frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{14\,c^7}-\frac{b\,d^2\,x^2}{6\,c}-\frac{b\,e^2\,x^6}{42\,c}+\frac{b\,e^2\,x^4}{28\,c^3}-\frac{b\,e^2\,x^2}{14\,c^5}+\frac{2\,a\,d\,e\,x^5}{5}+\frac{b\,d^2\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,e^2\,x^7\,\mathrm{atan}(c\,x)}{7}-\frac{b\,d\,e\,\ln\left(c^2\,x^2+1\right)}{5\,c^5}-\frac{b\,d\,e\,x^4}{10\,c}+\frac{b\,d\,e\,x^2}{5\,c^3}+\frac{2\,b\,d\,e\,x^5\,\mathrm{atan}(c\,x)}{5}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
(a*d^2*x^3)/3 + (a*e^2*x^7)/7 + (b*d^2*log(c^2*x^2 + 1))/(6*c^3) + (b*e^2*
log(c^2*x^2 + 1))/(14*c^7) - (b*d^2*x^2)/(6*c) - (b*e^2*x^6)/(42*c) + (b*e
^2*x^4)/(28*c^3) - (b*e^2*x^2)/(14*c^5) + (2*a*d*e*x^5)/5 + (b*d^2*x^3*ata
n(c*x))/3 + (b*e^2*x^7*atan(c*x))/7 - (b*d*e*log(c^2*x^2 + 1))/(5*c^5) - (
b*d*e*x^4)/(10*c) + (b*d*e*x^2)/(5*c^3) + (2*b*d*e*x^5*atan(c*x))/5
```

3.1127 $\int x(d+ex^2)^2 (a+b\arctan(cx))\,dx$

3.1127.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int x(d+ex^2)^2 (a+b\arctan(cx))\,dx = -\frac{b(3c^4d^2-3c^2de+e^2)\,x}{6c^5}-\frac{b(3c^2d-e)\,ex^3}{18c^3}-\frac{be^2x^5}{30c}-\frac{b(c^2d-e)^3\arctan(cx)}{6c^6e}+\frac{(d+ex^2)^3(a+b\arctan(cx))}{6e}$$

output
```
-1/6*b*(3*c^4*d^2-3*c^2*d*e+e^2)*x/c^5-1/18*b*(3*c^2*d-e)*e*x^3/c^3-1/30*b
*e^2*x^5/c-1/6*b*(c^2*d-e)^3*arctan(c*x)/c^6/e+1/6*(e*x^2+d)^3*(a+b*arctan
(c*x))/e
```

3.1127.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int x(d+ex^2)^2 (a+b\arctan(cx))\,dx = \frac{cx(-15be^2+5bc^2e(9d+ex^2)+15ac^5x(3d^2+3dex^2+e^2x^4)-3bc^4(15d^2+5dex^2+e^2x^4))+15b(3c^4d^2 -}{90c^6}$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(c*x*(-15*b*e^2 + 5*b*c^2*e*(9*d + e*x^2) + 15*a*c^5*x*(3*d^2 + 3*d*e*x^2
+ e^2*x^4) - 3*b*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)) + 15*b*(3*c^4*d^2 - 3
*c^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x])/(90*c
^6)
```

3.1127.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5509, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{\left(d+ex^2\right)^3 (a+b\arctan(cx))}{6e}-\frac{bc\int \frac{\left(ex^2+d\right)^3}{c^2x^2+1}dx}{6e}$$

$$\Big\downarrow \text{300}$$

$$\frac{\left(d+ex^2\right)^3 (a+b\arctan(cx))}{6e}-\frac{bc\int \left(\frac{e^3x^4}{c^2}+\frac{\left(3c^2d-e\right)e^2x^2}{c^4}+\frac{e\left(3d^2c^4-3dec^2+e^2\right)}{c^6}+\frac{d^3c^6-3d^2ec^4+3de^2c^2-e^3}{c^6\left(c^2x^2+1\right)}\right)dx}{6e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d+ex^2\right)^3 (a+b\arctan(cx))}{6e}-\frac{bc\left(\frac{\arctan(cx)\left(c^2d-e\right)^3}{c^7}+\frac{e^3x^5}{5c^2}+\frac{e^2x^3\left(3c^2d-e\right)}{3c^4}+\frac{ex\left(3c^4d^2-3c^2de+e^2\right)}{c^6}\right)}{6e}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcTan[c*x]))/(6*e) - (b*c*((e*(3*c^4*d^2 - 3*c^2*d*
e + e^2)*x)/c^6 + ((3*c^2*d - e)*e^2*x^3)/(3*c^4) + (e^3*x^5)/(5*c^2) + ((
c^2*d - e)^3*ArcTan[c*x])/c^7))/(6*e)
```

3.1127.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1127.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.62

method	result
parallelrisch	$\frac{15x^6 \arctan(cx)b\,c^6e^2+15x^6a\,c^6e^2+45x^4\arctan(cx)b\,c^6de-3b\,c^5e^2x^5+45x^4a\,c^6de+45x^2\arctan(cx)b\,c^6d^2-15b\,c^5de\,x^3}{90}$
parts	$\frac{a(e\,x^2+d)^3}{6e}+\frac{b\left(\frac{\arctan(cx)c^2e^2x^6}{6}+\frac{\arctan(cx)c^2e\,x^4d}{2}+\frac{\arctan(cx)c^2x^2d^2}{2}+\frac{\arctan(cx)c^2d^3}{6e}-\frac{3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-3}{}\right)}{c^2}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\arctan(cx)c^6d^3}{6e}+\frac{\arctan(cx)c^6d^2x^2}{2}+\frac{\arctan(cx)e\,c^6d\,x^4}{2}+\frac{\arctan(cx)e^2c^6x^6}{6}-\frac{3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-}{}\right)}{c^4}}{c^2}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^3}{6c^4e}+\frac{b\left(\frac{\arctan(cx)c^6d^3}{6e}+\frac{\arctan(cx)c^6d^2x^2}{2}+\frac{\arctan(cx)e\,c^6d\,x^4}{2}+\frac{\arctan(cx)e^2c^6x^6}{6}-\frac{3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-}{}\right)}{c^4}}{c^2}$
risch	$\frac{ie^2b\,x^6\ln(-icx+1)}{12}-\frac{e^2b\arctan\left(\frac{\left(-c^7d^3+6c^5d^2e-6c^3d\,e^2+2c\,e^3\right)x}{c^6d^3-6e\,d^2c^4+6e^2d\,c^2-2e^3}\right)}{12c^6}+\frac{e^2b\arctan\left(\frac{\left(c^7d^3-6c^5d^2e+6c^3d\,e^2-2c\,e^3\right)x}{c^6d^3-6e\,d^2c^4+6e^2d\,c^2-2e^3}\right)}{12c^6}$

input
```
int(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/90*(15*x^6*arctan(c*x)*b*c^6*e^2+15*x^6*a*c^6*e^2+45*x^4*arctan(c*x)*b*c
^6*d*e-3*b*c^5*e^2*x^5+45*x^4*a*c^6*d*e+45*x^2*arctan(c*x)*b*c^6*d^2-15*b*
c^5*d*e*x^3+45*x^2*a*c^6*d^2+5*b*c^3*e^2*x^3-45*b*c^5*d^2*x+45*b*c^4*d^2*a
rctan(c*x)+45*b*c^3*d*e*x-45*b*c^2*d*e*arctan(c*x)-15*b*c*e^2*x+15*b*e^2*a
rctan(c*x))/c^6
```

3.1127.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \frac{15\,ac^6e^2x^6+45\,ac^6dex^4-3\,bc^5e^2x^5+45\,ac^6d^2x^2-5\,(3\,bc^5de-bc^3e^2)x^3-15\,(3\,bc^5d^2-3\,bc^3de+bce^2)x}{90\,c^6}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 - 3*b*c^5*e^2*x^5 + 45*a*c^6*d^2
*x^2 - 5*(3*b*c^5*d*e - b*c^3*e^2)*x^3 - 15*(3*b*c^5*d^2 - 3*b*c^3*d*e + b
*c*e^2)*x + 15*(b*c^6*e^2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2 + 3*b*c^
4*d^2 - 3*b*c^2*d*e + b*e^2)*arctan(c*x))/c^6
```

3.1127.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(102) = 204$.

Time = 0.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\,\mathrm{atan}\,(cx)}{2}+\frac{bdex^4\,\mathrm{atan}\,(cx)}{2}+\frac{be^2x^6\,\mathrm{atan}\,(cx)}{6}-\frac{bd^2x}{2c}-\frac{bdex^3}{6c}-\frac{be^2x^5}{30c}+\frac{bd^2\,\mathrm{atan}\,(cx)}{2c^2}+\frac{bc}{2} \\ a\left(\frac{d^2x^2}{2}+\frac{dex^4}{2}+\frac{e^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*atan
(c*x)/2 + b*d*e*x**4*atan(c*x)/2 + b*e**2*x**6*atan(c*x)/6 - b*d**2*x/(2*c
) - b*d*e*x**3/(6*c) - b*e**2*x**5/(30*c) + b*d**2*atan(c*x)/(2*c**2) + b*
d*e*x/(2*c**3) + b*e**2*x**3/(18*c**3) - b*d*e*atan(c*x)/(2*c**4) - b*e**2
*x/(6*c**5) + b*e**2*atan(c*x)/(6*c**6), Ne(c, 0)), (a*(d**2*x**2/2 + d*e*
x**4/2 + e**2*x**6/6), True))
```

3.1127.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36

$$\begin{aligned}\int x(d+ex^2)^2(a+b\arctan(cx))\,dx &= \frac{1}{6}ae^2x^6+\frac{1}{2}adex^4+\frac{1}{2}ad^2x^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bd^2\\&\quad+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bde\\&\quad+\frac{1}{90}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)be^2\end{aligned}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arctan(c*x) - c*(
x/c^2 - arctan(c*x)/c^3))*b*d^2 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3
*x)/c^4 + 3*arctan(c*x)/c^5))*b*d*e + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4
*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e^2
```

3.1127.8 Giac [F]

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx=\int(ex^2+d)^2(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1127.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.45

$$\int x(d+ex^2)^2 (a+b\arctan(cx))\,dx = \frac{a\,d^2\,x^2}{2}+\frac{a\,e^2\,x^6}{6}-\frac{b\,e^2\,x^5}{30\,c}+\frac{b\,e^2\,x^3}{18\,c^3}+\frac{a\,d\,e\,x^4}{2} -\frac{b\,d^2\,x}{2\,c}-\frac{b\,e^2\,x}{6\,c^5}+\frac{b\,d^2\,\mathrm{atan}(c\,x)}{2\,c^2}+\frac{b\,e^2\,\mathrm{atan}(c\,x)}{6\,c^6} +\frac{b\,d^2\,x^2\,\mathrm{atan}(c\,x)}{2}+\frac{b\,e^2\,x^6\,\mathrm{atan}(c\,x)}{6}-\frac{b\,d\,e\,x^3}{6\,c} +\frac{b\,d\,e\,x}{2\,c^3}-\frac{b\,d\,e\,\mathrm{atan}(c\,x)}{2\,c^4}+\frac{b\,d\,e\,x^4\,\mathrm{atan}(c\,x)}{2}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
(a*d^2*x^2)/2 + (a*e^2*x^6)/6 - (b*e^2*x^5)/(30*c) + (b*e^2*x^3)/(18*c^3)
+ (a*d*e*x^4)/2 - (b*d^2*x)/(2*c) - (b*e^2*x)/(6*c^5) + (b*d^2*atan(c*x))/
(2*c^2) + (b*e^2*atan(c*x))/(6*c^6) + (b*d^2*x^2*atan(c*x))/2 + (b*e^2*x^6
*atan(c*x))/6 - (b*d*e*x^3)/(6*c) + (b*d*e*x)/(2*c^3) - (b*d*e*atan(c*x))/
(2*c^4) + (b*d*e*x^4*atan(c*x))/2
```

3.1128 $\int (d+ex^2)^2 (a+b\arctan(cx))\,dx$

3.1128.1 Optimal result

Integrand size = 18, antiderivative size = 124

$$\begin{aligned}\int (d+ex^2)^2 (a+b\arctan(cx))\,dx &= -\frac{b(10c^2d-3e)\,ex^2}{30c^3} - \frac{be^2x^4}{20c} + d^2x(a+b\arctan(cx)) \\ &\quad + \frac{2}{3}dex^3(a+b\arctan(cx)) + \frac{1}{5}e^2x^5(a+b\arctan(cx)) \\ &\quad - \frac{b(15c^4d^2-10c^2de+3e^2)\log(1+c^2x^2)}{30c^5}\end{aligned}$$

output
```
-1/30*b*(10*c^2*d-3*e)*e*x^2/c^3-1/20*b*e^2*x^4/c+d^2*x*(a+b*arctan(c*x))+
2/3*d*e*x^3*(a+b*arctan(c*x))+1/5*e^2*x^5*(a+b*arctan(c*x))-1/30*b*(15*c^4
*d^2-10*c^2*d*e+3*e^2)*ln(c^2*x^2+1)/c^5
```

3.1128.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05

$$\int (d+ex^2)^2 (a+b\arctan(cx))\,dx$$
$$= \frac{c^2x(4ac^3(15d^2+10dex^2+3e^2x^4)+bex(6e-c^2(20d+3ex^2)))+4bc^5x(15d^2+10dex^2+3e^2x^4)\arctan(c}{60c^5}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*x*(4*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*x*(6*e - c^2*(20*d
 + 3*e*x^2))) + 4*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcTan[c*x] -
2*b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/(60*c^5)
```

3.1128.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5447, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{x\left(3e^2x^4+10dex^2+15d^2\right)}{15\left(c^2x^2+1\right)}dx+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bc\int \frac{x\left(3e^2x^4+10dex^2+15d^2\right)}{c^2x^2+1}dx+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}bc\int \frac{3e^2x^4+10dex^2+15d^2}{c^2x^2+1}dx^2+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}bc\int \left(\frac{3e^2x^2}{c^2}+\frac{\left(10c^2d-3e\right)e}{c^4}+\frac{15d^2c^4-10dec^2+3e^2}{c^4\left(c^2x^2+1\right)}\right)dx^2+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))-\frac{1}{30}bc\left(\frac{3e^2x^4}{2c^2}+\frac{ex^2(10c^2d-3e)}{c^4}+\frac{(15c^4d^2-10c^2de+3e^2)\log(c^2x^2+1)}{c^6}\right)$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
d^2*x*(a + b*ArcTan[c*x]) + (2*d*e*x^3*(a + b*ArcTan[c*x]))/3 + (e^2*x^5*(
a + b*ArcTan[c*x]))/5 - (b*c*(((10*c^2*d - 3*e)*e*x^2)/c^4 + (3*e^2*x^4)/(
2*c^2) + ((15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/c^6))/30
```

3.1128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.1128.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

method	result
parts	$a\left(\frac{1}{5}x^5e^2+\frac{2}{3}x^3ed+x\,d^2\right)+\frac{b\left(\frac{\arctan(cx)c\,e^2x^5}{5}+\frac{2\arctan(cx)cde\,x^3}{3}+\arctan(cx)cx\,d^2-\frac{5d\,c^4e\,x^2+\frac{3e^2c^4x^4}{4}-\frac{3e^2}{}}{}\right.}{c}$
derivativedivides	$\frac{\frac{a\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arctan(cx)c^5x\,d^2+\frac{2\arctan(cx)d\,c^5e\,x^3}{3}+\frac{\arctan(cx)e^2c^5x^5}{5}-\frac{d\,c^4e\,x^2}{3}-\frac{e^2c^4x^4}{20}+\frac{e^2c^2x^2}{10}-\frac{(1}{}\right.}{c^4}}{c}$
default	$\frac{\frac{a\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arctan(cx)c^5x\,d^2+\frac{2\arctan(cx)d\,c^5e\,x^3}{3}+\frac{\arctan(cx)e^2c^5x^5}{5}-\frac{d\,c^4e\,x^2}{3}-\frac{e^2c^4x^4}{20}+\frac{e^2c^2x^2}{10}-\frac{(1}{}\right.}{c^4}}{c}$
parallelrisch	$-\frac{-12x^5\arctan(cx)b\,c^5e^2-12a\,c^5e^2x^5-40x^3\arctan(cx)b\,c^5de+3b\,c^4e^2x^4-40a\,c^5de\,x^3-60x\arctan(cx)b\,c^5d^2+20b\,c^4d}{60c^5}$
risch	$-\frac{ib(3x^5e^2+10x^3ed+15x\,d^2)\ln(icx+1)}{30}+\frac{ib\,e^2x^5\ln(-icx+1)}{10}+\frac{ibde\,x^3\ln(-icx+1)}{3}+\frac{a\,e^2x^5}{5}+\frac{ib\,d^2x\ln(-icx+}{2}$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*x^5*e^2+2/3*x^3*e*d+x*d^2)+b/c*(1/5*arctan(c*x)*c*e^2*x^5+2/3*arcta
n(c*x)*c*d*e*x^3+arctan(c*x)*c*x*d^2-1/15/c^4*(5*d*c^4*e*x^2+3/4*e^2*c^4*x
^4-3/2*e^2*c^2*x^2+1/2*(15*c^4*d^2-10*c^2*d*e+3*e^2)*ln(c^2*x^2+1)))
```

3.1128.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int (d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$=\frac{12\,ac^5e^2x^5+40\,ac^5dex^3-3\,bc^4e^2x^4+60\,ac^5d^2x-2\,(10\,bc^4de-3\,bc^2e^2)x^2+4\,(3\,bc^5e^2x^5+10\,bc^5dex^3+}{60\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/60*(12*a*c^5*e^2*x^5 + 40*a*c^5*d*e*x^3 - 3*b*c^4*e^2*x^4 + 60*a*c^5*d^2
*x - 2*(10*b*c^4*d*e - 3*b*c^2*e^2)*x^2 + 4*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*
e*x^3 + 15*b*c^5*d^2*x)*arctan(c*x) - 2*(15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b
*e^2)*log(c^2*x^2 + 1))/c^5
```

3.1128.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.56

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$
$$= \begin{cases} ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\,\mathrm{atan}\,(cx)+\frac{2bdex^3\,\mathrm{atan}\,(cx)}{3}+\frac{be^2x^5\,\mathrm{atan}\,(cx)}{5}-\frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bdex^2}{3c}-\frac{be^2x^4}{20c}+ \\ a\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*atan(c*x)
+ 2*b*d*e*x**3*atan(c*x)/3 + b*e**2*x**5*atan(c*x)/5 - b*d**2*log(x**2 + c
**(-2))/(2*c) - b*d*e*x**2/(3*c) - b*e**2*x**4/(20*c) + b*d*e*log(x**2 + c
**(-2))/(3*c**3) + b*e**2*x**2/(10*c**3) - b*e**2*log(x**2 + c**(-2))/(10*
c**5), Ne(c, 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

3.1128.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$
$$= \frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{1}{3}\left(2\,x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)bde$$
$$+\frac{1}{20}\left(4\,x^5\arctan(cx)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)be^2$$
$$+ad^2x+\frac{(2\,cx\arctan(cx)-\log(c^2x^2+1))bd^2}{2\,c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(
c^2*x^2 + 1)/c^4))*b*d*e + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/
c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*e^2 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) -
log(c^2*x^2 + 1))*b*d^2/c
```

3.1128.8 Giac [F]

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^2 (b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1128.9 Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx &= \frac{a\,e^2\,x^5}{5} + a\,d^2\,x - \frac{b\,d^2\,\ln\left(c^2\,x^2+1\right)}{2\,c} \\ &\quad - \frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^5} - \frac{b\,e^2\,x^4}{20\,c} + \frac{b\,e^2\,x^2}{10\,c^3} \\ &\quad + \frac{2\,a\,d\,e\,x^3}{3} + b\,d^2\,x\,\mathrm{atan}(c\,x) + \frac{b\,e^2\,x^5\,\mathrm{atan}(c\,x)}{5} \\ &\quad + \frac{b\,d\,e\,\ln\left(c^2\,x^2+1\right)}{3\,c^3} - \frac{b\,d\,e\,x^2}{3\,c} + \frac{2\,b\,d\,e\,x^3\,\mathrm{atan}(c\,x)}{3}\end{aligned}$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
(a*e^2*x^5)/5 + a*d^2*x - (b*d^2*log(c^2*x^2 + 1))/(2*c) - (b*e^2*log(c^2*
x^2 + 1))/(10*c^5) - (b*e^2*x^4)/(20*c) + (b*e^2*x^2)/(10*c^3) + (2*a*d*e*
x^3)/3 + b*d^2*x*atan(c*x) + (b*e^2*x^5*atan(c*x))/5 + (b*d*e*log(c^2*x^2
+ 1))/(3*c^3) - (b*d*e*x^2)/(3*c) + (2*b*d*e*x^3*atan(c*x))/3
```

3.1129 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x}\,dx$

3.1129.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x}\,dx = &-\frac{bdex}{c}+\frac{be^2x}{4c^3}-\frac{be^2x^3}{12c}+\frac{bde\arctan(cx)}{c^2}\\&-\frac{be^2\arctan(cx)}{4c^4}+dex^2(a+b\arctan(cx))\\&+\frac{1}{4}e^2x^4(a+b\arctan(cx))+ad^2\log(x)\\&+\frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-b*d*e*x/c+1/4*b*e^2*x/c^3-1/12*b*e^2*x^3/c+b*d*e*arctan(c*x)/c^2-1/4*b*e^
2*arctan(c*x)/c^4+d*e*x^2*(a+b*arctan(c*x))+1/4*e^2*x^4*(a+b*arctan(c*x))+
a*d^2*ln(x)+1/2*I*b*d^2*polylog(2,-I*c*x)-1/2*I*b*d^2*polylog(2,I*c*x)
```

3.1129.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx = -\frac{bde(cx-\arctan(cx))}{c^2} - \frac{be^2(-3cx+c^3x^3+3\arctan(cx))}{12c^4} + dex^2(a+b\arctan(cx)) + \frac{1}{4}e^2x^4(a+b\arctan(cx)) + ad^2\log(x) + \frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx) - \frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-((b*d*e*(c*x - ArcTan[c*x]))/c^2) - (b*e^2*(-3*c*x + c^3*x^3 + 3*ArcTan[c
*x]))/(12*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2*x^4*(a + b*ArcTan[c*x]
))/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyL
og[2, I*c*x]
```

3.1129.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))}{x} + 2dex(a+b\arctan(cx)) + e^2x^3(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$dex^2(a+b\arctan(cx)) + \frac{1}{4}e^2x^4(a+b\arctan(cx)) + ad^2\log(x) - \frac{be^2\arctan(cx)}{4c^4} + \frac{bde\arctan(cx)}{c^2} + \frac{be^2x}{4c^3} + \frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx) - \frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx) - \frac{bdex}{c} - \frac{be^2x^3}{12c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-((b*d*e*x)/c) + (b*e^2*x)/(4*c^3) - (b*e^2*x^3)/(12*c) + (b*d*e*ArcTan[c*
x])/c^2 - (b*e^2*ArcTan[c*x])/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2
*x^4*(a + b*ArcTan[c*x]))/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c
*x] - (I/2)*b*d^2*PolyLog[2, I*c*x]
```

3.1129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1129.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

method	result
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\arctan(cx)d\,c^4e\,x^2+\frac{\arctan(cx)e^2c^4x^4}{4}+\arctan(cx)c^4d^2\ln(cx)-\frac{e\left(4c^3xd+\frac{e}{}\right.}{}\right.}{}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{b\left(\arctan(cx)d\,c^4e\,x^2+\frac{\arctan(cx)e^2c^4x^4}{4}+\arctan(cx)c^4d^2\ln(cx)-\frac{e\left(4c^3xd+\frac{e}{}\right.}{}\right.}{}$
parts	$a\left(\frac{x^4e^2}{4}+x^2ed+d^2\ln(x)\right)+b\left(\frac{\arctan(cx)e^2x^4}{4}+\arctan(cx)\,de\,x^2+\arctan(cx)\,d^2\ln(cx\right.$
risch	$\frac{b\,e^2x}{4c^3}-\frac{b\,e^2x^3}{12c}-\frac{bdex}{c}+a\,d^2\ln(-icx)+\frac{a\,e^2x^4}{4}-\frac{ib\,d^2\,\mathrm{dilog}(-icx+1)}{2}+\frac{ib\,d^2\,\mathrm{dilog}(icx+1)}{2}+\frac{ade}{c^2}-$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*d*e*x^2+1/4*a*e^2*x^4+a*d^2*ln(c*x)+b/c^4*(arctan(c*x)*d*c^4*e*x^2+1/4*a
rctan(c*x)*e^2*c^4*x^4+arctan(c*x)*c^4*d^2*ln(c*x)-1/4*e*(4*c^3*x*d+1/3*e*
c^3*x^3-e*c*x+(-4*c^2*d+e)*arctan(c*x))-c^4*d^2*(-1/2*I*ln(c*x)*ln(1+I*c*x
)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)))
```

3.1129.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x, x)
```

3.1129.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))\,(d+ex^2)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**2/x, x)
```

3.1129.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx = \frac{1}{4}ae^2x^4 + adex^2 + ad^2\log(x) - \frac{bc^3e^2x^3 + 3\pi bc^4d^2\log(c^2x^2+1) - 12\,bc^4d^2\arctan(cx)\log(cx) + 6i\,bc^4d^2\mathrm{Li}_2(i\,cx+1) - 6i\,bc^4d^2\mathrm{Li}_2(-i}{12\,c^4}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) - 1/12*(b*c^3*e^2*x^3 + 3*pi*b*c^
4*d^2*log(c^2*x^2 + 1) - 12*b*c^4*d^2*arctan(c*x)*log(c*x) + 6*I*b*c^4*d^2
*dilog(I*c*x + 1) - 6*I*b*c^4*d^2*dilog(-I*c*x + 1) + 3*(4*b*c^3*d*e - b*c
*e^2)*x - 3*(b*c^4*e^2*x^4 + 4*b*c^4*d*e*x^2 + 4*b*c^2*d*e - b*e^2)*arctan
(c*x))/c^4
```

3.1129.8 Giac [F]

$$\int \frac{\left(d+ex^2\right)^2 (a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.1129.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

$$\int \frac{\left(d+ex^2\right)^2 (a+b\arctan(cx))}{x}\,dx$$
$$= \begin{cases} \frac{a\,(4\,d^2\,\ln(x)+e^2\,x^4+4\,d\,e\,x^2)}{4} \\ \frac{a\,(4\,d^2\,\ln(x)+e^2\,x^4+4\,d\,e\,x^2)}{4} - 2\,b\,d\,e\left(\frac{x}{2\,c} - \mathrm{atan}(c\,x)\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) - \frac{b\,e^2\,(3\,\mathrm{atan}(c\,x)-3\,c\,x+c^3\,x^3)}{12\,c^4} + \frac{b\,e^2\,x^4\,\mathrm{atan}(c\,x)}{4} - \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x,x)
```

output
```
piecewise(c == 0, (a*(4*d^2*log(x) + e^2*x^4 + 4*d*e*x^2))/4, c ~= 0, (a*(
4*d^2*log(x) + e^2*x^4 + 4*d*e*x^2))/4 - (b*d^2*dilog(- c*x*1i + 1)*1i)/2
+ (b*d^2*dilog(c*x*1i + 1)*1i)/2 - 2*b*d*e*(x/(2*c) - atan(c*x)*(1/(2*c^2)
 + x^2/2)) - (b*e^2*(3*atan(c*x) - 3*c*x + c^3*x^3))/(12*c^4) + (b*e^2*x^4
*atan(c*x))/4)
```

3.1130 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^2}\,dx$

3.1130.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^2}\,dx = -\frac{be^2x^2}{6c}-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))+bcd^2\log(x)-\frac{b(3c^4d^2+6c^2de-e^2)\log(1+c^2x^2)}{6c^3}$$

output
```
-1/6*b*e^2*x^2/c-d^2*(a+b*arctan(c*x))/x+2*d*e*x*(a+b*arctan(c*x))+1/3*e^2
*x^3*(a+b*arctan(c*x))+b*c*d^2*ln(x)-1/6*b*(3*c^4*d^2+6*c^2*d*e-e^2)*ln(c^
2*x^2+1)/c^3
```

3.1130.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^2}\,dx = \frac{1}{6}\left(-\frac{6ad^2}{x}+12adex-\frac{be^2x^2}{c}+2ae^2x^3+\frac{2b(-3d^2+6dex^2+e^2x^4)\arctan(cx)}{x}+6bcd^2\log(x)+\frac{b(-3c^4d^2-6c^2de+e^2)\log(1+c^2x^2)}{c^3}\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
((-6*a*d^2)/x + 12*a*d*e*x - (b*e^2*x^2)/c + 2*a*e^2*x^3 + (2*b*(-3*d^2 +
6*d*e*x^2 + e^2*x^4)*ArcTan[c*x])/x + 6*b*c*d^2*Log[x] + (b*(-3*c^4*d^2 -
6*c^2*d*e + e^2)*Log[1 + c^2*x^2])/c^3)/6
```

3.1130.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2} \, dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{-e^2x^4-6dex^2+3d^2}{3x(c^2x^2+1)}dx-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-e^2x^4-6dex^2+3d^2}{x(c^2x^2+1)}dx-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{6}bc\int \frac{-e^2x^4-6dex^2+3d^2}{x^2(c^2x^2+1)}dx^2-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{6}bc\int \left(\frac{3d^2}{x^2}-\frac{e^2}{c^2}+\frac{-3d^2c^4-6dec^2+e^2}{c^2(c^2x^2+1)}\right)dx^2-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))+\frac{1}{6}bc\left(-\frac{e^2x^2}{c^2}-\frac{(3c^4d^2+6c^2de-e^2)\log(c^2x^2+1)}{c^4}+3d^2\log(x^2)\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcTan[c*x]))/x) + 2*d*e*x*(a + b*ArcTan[c*x]) + (e^2*x^3*(a
 + b*ArcTan[c*x]))/3 + (b*c*(-((e^2*x^2)/c^2) + 3*d^2*Log[x^2] - ((3*c^4*d
^2 + 6*c^2*d*e - e^2)*Log[1 + c^2*x^2])/c^4))/6
```

3.1130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1130.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+bc\left(\frac{\arctan(cx)e^2x^3}{3c}+\frac{2\arctan(cx)xde}{c}-\frac{\arctan(cx)d^2}{cx}-\frac{\frac{e^2c^2x^2}{2}-3c^4d^2\ln(cx)}{}\right.$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\arctan(cx)c^3dex+\frac{\arctan(cx)e^2c^3x^3}{3}-\frac{\arctan(cx)c^3d^2}{x}-\frac{e^2c^2x^2}{6}-\frac{(3c^4d^2+6c^2de-e^2}{6}\right.}{c^4}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\arctan(cx)c^3dex+\frac{\arctan(cx)e^2c^3x^3}{3}-\frac{\arctan(cx)c^3d^2}{x}-\frac{e^2c^2x^2}{6}-\frac{(3c^4d^2+6c^2de-e^2}{6}\right.}{c^4}\right.$
parallelrisch	$\frac{2x^4\arctan(cx)bc^3e^2+2ac^3e^2x^4+6bc^4d^2\ln(x)x-3\ln(c^2x^2+1)bc^4d^2x+12x^2\arctan(cx)bc^3de-bc^2e^2x^3+12ac^3dex^2-}{6xc^3}$
risch	$\frac{ib(-x^4e^2-6x^2ed+3d^2)\ln(icx+1)}{6x}+\frac{ibc^3e^2x^4\ln(-icx+1)+6ibc^3dex^2\ln(-icx+1)+2ac^3e^2x^4+6bc^4d^2\ln(x)x-3\ln(}{}$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3*arctan(c*x)/c*e^2*x^3+2*arctan(c*x)
/c*x*d*e-arctan(c*x)*d^2/c/x-1/3/c^4*(1/2*e^2*c^2*x^2-3*c^4*d^2*ln(c*x)+1/
2*(3*c^4*d^2+6*c^2*d*e-e^2)*ln(c^2*x^2+1)))
```

3.1130.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int\frac{(d+ex^2)^2(a+b\arctan(cx))}{x^2}\,dx$$
$$=\frac{2ac^3e^2x^4+6bc^4d^2x\log(x)+12ac^3dex^2-bc^2e^2x^3-6ac^3d^2-(3bc^4d^2+6bc^2de-be^2)x\log(c^2x^2+1)+}{6c^3x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x*log(x) + 12*a*c^3*d*e*x^2 - b*c^2*e^2
*x^3 - 6*a*c^3*d^2 - (3*b*c^4*d^2 + 6*b*c^2*d*e - b*e^2)*x*log(c^2*x^2 + 1
) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2)*arctan(c*x))/(c^3*x)
```

3.1130.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2}\,dx$$
$$=\begin{cases} -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2\log(x) - \frac{bcd^2\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^2\,\mathrm{atan}\,(cx)}{x} + 2bdex\,\mathrm{atan}\,(cx) + \frac{be^2x^3\,\mathrm{atan}\,(cx)}{3} - \frac{bd\ldots}{} \\ a\left(-\frac{d^2}{x} + 2dex + \frac{e^2x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*log(x) - b*c*d
**2*log(x**2 + c**(-2))/2 - b*d**2*atan(c*x)/x + 2*b*d*e*x*atan(c*x) + b*e
**2*x**3*atan(c*x)/3 - b*d*e*log(x**2 + c**(-2))/c - b*e**2*x**2/(6*c) + b
*e**2*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**2/x + 2*d*e*x + e**
2*x**3/3), True))
```

3.1130.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2}\,dx = \frac{1}{3}ae^2x^3 - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd^2 + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be^2 + 2adex + \frac{(2cx\arctan(cx) - \log(c^2x^2+1))bde}{c} - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*
d^2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*e^2 +
 2*a*d*e*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e/c - a*d^2/x
```

3.1130.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1130.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2}\,dx = \frac{a\,e^2\,x^3}{3} - \frac{a\,d^2}{x} + 2\,a\,d\,e\,x + \frac{b\,e^2\,\ln(c^2\,x^2+1)}{6\,c^3} - \frac{b\,e^2\,x^2}{6\,c} - \frac{b\,c\,d^2\,\ln(c^2\,x^2+1)}{2} + b\,c\,d^2\,\ln(x) - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{x} + \frac{b\,e^2\,x^3\,\mathrm{atan}(c\,x)}{3} - \frac{b\,d\,e\,\ln(c^2\,x^2+1)}{c} + 2\,b\,d\,e\,x\,\mathrm{atan}(c\,x)$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^2,x)
```

output
```
(a*e^2*x^3)/3 - (a*d^2)/x + 2*a*d*e*x + (b*e^2*log(c^2*x^2 + 1))/(6*c^3) -
 (b*e^2*x^2)/(6*c) - (b*c*d^2*log(c^2*x^2 + 1))/2 + b*c*d^2*log(x) - (b*d^
2*atan(c*x))/x + (b*e^2*x^3*atan(c*x))/3 - (b*d*e*log(c^2*x^2 + 1))/c + 2*
b*d*e*x*atan(c*x)
```

3.1131 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx$

3.1131.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx = &-\frac{bcd^2}{2x}-\frac{be^2x}{2c}-\frac{1}{2}bc^2d^2\arctan(cx)\\&+\frac{be^2\arctan(cx)}{2c^2}-\frac{d^2(a+b\arctan(cx))}{2x^2}\\&+\frac{1}{2}e^2x^2(a+b\arctan(cx))+2ade\log(x)\\&+ibde\operatorname{PolyLog}(2,-icx)-ibde\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/2*b*c*d^2/x-1/2*b*e^2*x/c-1/2*b*c^2*d^2*arctan(c*x)+1/2*b*e^2*arctan(c*
x)/c^2-1/2*d^2*(a+b*arctan(c*x))/x^2+1/2*e^2*x^2*(a+b*arctan(c*x))+2*a*d*e
*ln(x)+I*b*d*e*polylog(2,-I*c*x)-I*b*d*e*polylog(2,I*c*x)
```

3.1131.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3}\,dx = \frac{1}{2}\left(-\frac{be^2(cx-\arctan(cx))}{c^2} - \frac{d^2(a+b\arctan(cx))}{x^2} + e^2x^2(a+b\arctan(cx)) - \frac{bcd^2 \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 4ade\log(x) + 2ibde\operatorname{PolyLog}(2,-icx) - 2ibde\operatorname{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
(-((b*e^2*(c*x - ArcTan[c*x]))/c^2) - (d^2*(a + b*ArcTan[c*x]))/x^2 + e^2*
x^2*(a + b*ArcTan[c*x]) - (b*c*d^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x
^2)])/x + 4*a*d*e*Log[x] + (2*I)*b*d*e*PolyLog[2, (-I)*c*x] - (2*I)*b*d*e*
PolyLog[2, I*c*x])/2
```

3.1131.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \ 5515$$

$$\int \left(\frac{d^2(a+b\arctan(cx))}{x^3} + \frac{2de(a+b\arctan(cx))}{x} + e^2x(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{d^2(a+b\arctan(cx))}{2x^2} + \frac{1}{2}e^2x^2(a+b\arctan(cx)) + 2ade\log(x) - \frac{1}{2}bc^2d^2\arctan(cx) + \frac{be^2\arctan(cx)}{2c^2} - \frac{bcd^2}{2x} + ibde\operatorname{PolyLog}(2,-icx) - ibde\operatorname{PolyLog}(2,icx) - \frac{be^2x}{2c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^2)/x - (b*e^2*x)/(2*c) - (b*c^2*d^2*ArcTan[c*x])/2 + (b*e^2*Ar
cTan[c*x])/(2*c^2) - (d^2*(a + b*ArcTan[c*x]))/(2*x^2) + (e^2*x^2*(a + b*A
rcTan[c*x]))/2 + 2*a*d*e*Log[x] + I*b*d*e*PolyLog[2, (-I)*c*x] - I*b*d*e*P
olyLog[2, I*c*x]
```

3.1131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1131.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.39

method	result
parts	$a\left(\frac{e^2x^2}{2}+2ed\ln\left(x\right)-\frac{d^2}{2x^2}\right)+b\,c^2\left(\frac{\arctan(cx)e^2x^2}{2c^2}+\frac{2\arctan(cx)de\ln(cx)}{c^2}-\frac{\arctan(cx)d^2}{2c^2x^2}-\frac{cx\,e^2+}{}\right.$
derivativedivides	$c^2\left(\frac{a\,e^2x^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{b\left(\frac{\arctan(cx)e^2c^2x^2}{2}+2\arctan(cx)c^2de\ln(cx)-\frac{\arctan(cx)c^2d^2}{2x^2}-\frac{cx\,e^2}{2}-\frac{c^3d^2}{2x}+\right.}{}\right.$
default	$c^2\left(\frac{a\,e^2x^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{b\left(\frac{\arctan(cx)e^2c^2x^2}{2}+2\arctan(cx)c^2de\ln(cx)-\frac{\arctan(cx)c^2d^2}{2x^2}-\frac{cx\,e^2}{2}-\frac{c^3d^2}{2x}+\right.}{}\right.$
risch	$-\frac{ib\,d^2\ln(-icx+1)}{4x^2}+\frac{ib\,c^2d^2\ln(icx+1)}{4}+\frac{ib\,d^2\ln(icx+1)}{4x^2}+\frac{ib\,e^2\ln(-icx+1)x^2}{4}+ibed\operatorname{dilog}\left(icx+1\right)+$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2+2*e*d*ln(x)-1/2*d^2/x^2)+b*c^2*(1/2*arctan(c*x)/c^2*e^2*x^2
+2*arctan(c*x)/c^2*d*e*ln(c*x)-1/2*arctan(c*x)*d^2/c^2/x^2-1/2/c^4*(c*x*e^
2+c^3*d^2/x+(c^4*d^2-e^2)*arctan(c*x)+4*c^2*d*e*(-1/2*I*ln(c*x)*ln(1+I*c*x
)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))))
```

3.1131.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x^3, x)
```

3.1131.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3} dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)^2}{x^3} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**3, x)
```

3.1131.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3} dx$$
$$= \frac{1}{2}ae^2x^2 - \frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bd^2 + 2\,ade\log(x) - \frac{ad^2}{2\,x^2}$$
$$- \frac{\pi bc^2de\log(c^2x^2+1) - 4\,bc^2de\arctan(cx)\log(cx) + 2i\,bc^2de\mathrm{Li}_2(i\,cx+1) - 2i\,bc^2de\mathrm{Li}_2(-i\,cx+1) + bc}{2\,c^2}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2 + 2*
a*d*e*log(x) - 1/2*a*d^2/x^2 - 1/2*(pi*b*c^2*d*e*log(c^2*x^2 + 1) - 4*b*c^
2*d*e*arctan(c*x)*log(c*x) + 2*I*b*c^2*d*e*dilog(I*c*x + 1) - 2*I*b*c^2*d*
e*dilog(-I*c*x + 1) + b*c*e^2*x - (b*c^2*e^2*x^2 + b*e^2)*arctan(c*x))/c^2
```

3.1131.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1131.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3}\,dx = \begin{cases} \frac{a\,(e^2\,x^4-d^2+4\,d\,e\,x^2\,\ln(x))}{2\,x^2} \\ \frac{a\,(e^2\,x^4-d^2+4\,d\,e\,x^2\,\ln(x))}{2\,x^2} - b\,e^2\left(\frac{x}{2\,c}-\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) - \frac{b\,d^2\,\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{2\,x^2} - b\,d\,e\,(\mathrm{Li}_2 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^3,x)
```

output
```
piecewise(c == 0, (a*(- d^2 + e^2*x^4 + 4*d*e*x^2*log(x)))/(2*x^2), c ~= 0
, - b*e^2*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) + (a*(- d^2 + e^2*x^4
+ 4*d*e*x^2*log(x)))/(2*x^2) - b*d*e*(dilog(- c*x*1i + 1) - dilog(c*x*1i +
 1))*1i - (b*d^2*(c^3*atan(c*x) + c^2/x))/(2*c) - (b*d^2*atan(c*x))/(2*x^2
))
```

3.1132 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^4}\,dx$

3.1132.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd^2}{6x^2}-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x} + e^2x(a+b\arctan(cx))-\frac{1}{3}bcd(c^2d-6e)\log(x) + \frac{b(c^4d^2-6c^2de-3e^2)\log(1+c^2x^2)}{6c}$$

output
```
-1/6*b*c*d^2/x^2-1/3*d^2*(a+b*arctan(c*x))/x^3-2*d*e*(a+b*arctan(c*x))/x+e
^2*x*(a+b*arctan(c*x))-1/3*b*c*d*(c^2*d-6*e)*ln(x)+1/6*b*(c^4*d^2-6*c^2*d*
e-3*e^2)*ln(c^2*x^2+1)/c
```

3.1132.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^4}\,dx = \frac{1}{6}\left(-\frac{2ad^2}{x^3}-\frac{bcd^2}{x^2}-\frac{12ade}{x}+6ae^2x - \frac{2b(d^2+6dex^2-3e^2x^4)\arctan(cx)}{x^3} - 2bcd(c^2d-6e)\log(x) + \frac{b(c^4d^2-6c^2de-3e^2)\log(1+c^2x^2)}{c}\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
((-2*a*d^2)/x^3 - (b*c*d^2)/x^2 - (12*a*d*e)/x + 6*a*e^2*x - (2*b*(d^2 + 6
*d*e*x^2 - 3*e^2*x^4)*ArcTan[c*x])/x^3 - 2*b*c*d*(c^2*d - 6*e)*Log[x] + (b
*(c^4*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/c)/6
```

3.1132.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^2 (a+b\arctan(cx))}{x^4}dx$$

$$\Big\downarrow \ 5511$$

$$-bc\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^3\left(c^2x^2+1\right)}dx-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}bc\int \frac{-3e^2x^4+6dex^2+d^2}{x^3\left(c^2x^2+1\right)}dx-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$$\Big\downarrow \ 1578$$

$$\frac{1}{6}bc\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\left(c^2x^2+1\right)}dx^2-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$$\Big\downarrow \ 1195$$

$$\frac{1}{6}bc\int \left(\frac{d^2}{x^4}-\frac{\left(c^2d-6e\right)d}{x^2}+\frac{d^2c^4-6dec^2-3e^2}{c^2x^2+1}\right)dx^2-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$$\Big\downarrow \ 2009$$

$$-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))+\frac{1}{6}bc\left(-d\log\left(x^2\right)\left(c^2d-6e\right)+\frac{\left(c^4d^2-6c^2de-3e^2\right)\log\left(c^2x^2+1\right)}{c^2}-\frac{d^2}{x^2}\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcTan[c*x]))/x^3 - (2*d*e*(a + b*ArcTan[c*x]))/x + e^2*x
*(a + b*ArcTan[c*x]) + (b*c*(-(d^2/x^2) - d*(c^2*d - 6*e)*Log[x^2] + ((c^4
*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/c^2))/6
```

3.1132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1132.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

method	result
derivativedivides	$c^{3}\left(\frac{a\left(cx\,e^{2}-\frac{2cde}{x}-\frac{c\,d^{2}}{3x^{3}}\right)}{c^{4}}+\frac{b\left(\arctan(cx)cx\,e^{2}-\frac{2\arctan(cx)cde}{x}-\frac{\arctan(cx)c\,d^{2}}{3x^{3}}-\frac{\left(-c^{4}d^{2}+6c^{2}de+3e^{2}\right)\ln\left(c^{2}x^{2}+1\right)}{6}-\frac{d}{}\right.}{c^{4}}\right.$
default	$c^{3}\left(\frac{a\left(cx\,e^{2}-\frac{2cde}{x}-\frac{c\,d^{2}}{3x^{3}}\right)}{c^{4}}+\frac{b\left(\arctan(cx)cx\,e^{2}-\frac{2\arctan(cx)cde}{x}-\frac{\arctan(cx)c\,d^{2}}{3x^{3}}-\frac{\left(-c^{4}d^{2}+6c^{2}de+3e^{2}\right)\ln\left(c^{2}x^{2}+1\right)}{6}-\frac{d}{}\right.}{c^{4}}\right.$
parts	$a\left(x\,e^{2}-\frac{2ed}{x}-\frac{d^{2}}{3x^{3}}\right)+b\,c^{3}\left(\frac{\arctan(cx)x\,e^{2}}{c^{3}}-\frac{2\arctan(cx)de}{c^{3}x}-\frac{\arctan(cx)d^{2}}{3c^{3}x^{3}}-\frac{d\,c^{2}\left(c^{2}d-6e\right)\ln(cx)+\frac{c^{2}}{2}}{}\right.$
parallelrisch	$-\frac{2\ln(x)b\,c^{4}d^{2}x^{3}-\ln(c^{2}x^{2}+1)x^{3}b\,c^{4}d^{2}-b\,c^{4}d^{2}x^{3}-12\ln(x)b\,c^{2}de\,x^{3}+6\ln(c^{2}x^{2}+1)x^{3}b\,c^{2}de-6x^{4}\arctan(cx)bc\,e^{2}-6x^{4}}{6c\,x^{3}}$
risch	$\frac{ib\left(-3x^{4}e^{2}+6x^{2}ed+d^{2}\right)\ln(icx+1)}{6x^{3}}-\frac{2\ln(x)b\,c^{4}d^{2}x^{3}-\ln(-c^{2}x^{2}-1)b\,c^{4}d^{2}x^{3}-3ibc\,e^{2}x^{4}\ln(-icx+1)-12\ln(x)b\,c^{2}de\,x^{3}}{}$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(a/c^4*(c*x*e^2-2*c*d*e/x-1/3*c*d^2/x^3)+b/c^4*(arctan(c*x)*c*x*e^2-2*
arctan(c*x)*c*d*e/x-1/3*arctan(c*x)*c*d^2/x^3-1/6*(-c^4*d^2+6*c^2*d*e+3*e^
2)*ln(c^2*x^2+1)-1/3*d*c^2*(c^2*d-6*e)*ln(c*x)-1/6*c^2*d^2/x^2))
```

3.1132.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int\frac{\left(d+ex^{2}\right)^{2}\left(a+b\arctan(cx)\right)}{x^{4}}\,dx$$
$$=\frac{6\,ace^{2}x^{4}-bc^{2}d^{2}x-12\,acdex^{2}+(bc^{4}d^{2}-6\,bc^{2}de-3\,be^{2})x^{3}\log\left(c^{2}x^{2}+1\right)-2\,(bc^{4}d^{2}-6\,bc^{2}de)x^{3}\log\left(x\right)}{6\,cx^{3}}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a*c*e^2*x^4 - b*c^2*d^2*x - 12*a*c*d*e*x^2 + (b*c^4*d^2 - 6*b*c^2*d
*e - 3*b*e^2)*x^3*log(c^2*x^2 + 1) - 2*(b*c^4*d^2 - 6*b*c^2*d*e)*x^3*log(x
) - 2*a*c*d^2 + 2*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 - b*c*d^2)*arctan(c*x))/(
c*x^3)
```

3.1132.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \begin{cases} -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x - \frac{bc^3d^2\log(x)}{3} + \frac{bc^3d^2\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd^2}{6x^2} + 2bcde\log(x) - bcde\log\left(x^2+\frac{1}{c^2}\right) - \frac{bd^2\operatorname{atan}(c}{3x^3} \\ a\left(-\frac{d^2}{3x^3} - \frac{2de}{x} + e^2x\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - b*c**3*d**2*log(x)/3
+ b*c**3*d**2*log(x**2 + c**(-2))/6 - b*c*d**2/(6*x**2) + 2*b*c*d*e*log(x)
 - b*c*d*e*log(x**2 + c**(-2)) - b*d**2*atan(c*x)/(3*x**3) - 2*b*d*e*atan(
c*x)/x + b*e**2*x*atan(c*x) - b*e**2*log(x**2 + c**(-2))/(2*c), Ne(c, 0)),
 (a*(-d**2/(3*x**3) - 2*d*e/x + e**2*x), True))
```

3.1132.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bd^2$$
$$- \left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bde + ae^2x$$
$$+ \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))be^2}{2\,c} - \frac{2\,ade}{x} - \frac{ad^2}{3\,x^3}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*d^2 - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d*e + a*e^2*
x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a
*d^2/x^3
```

3.1132.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.1132.9 Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx = a\,e^2\,x - \frac{a\,d^2}{3\,x^3} + \frac{b\,c^3\,d^2\,\ln(c^2\,x^2+1)}{6} - \frac{b\,e^2\,\ln(c^2\,x^2+1)}{2\,c} - \frac{b\,c^3\,d^2\,\ln(x)}{3} - \frac{2\,a\,d\,e}{x} + b\,e^2\,x\,\mathrm{atan}(c\,x) - \frac{b\,c\,d^2}{6\,x^2} - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{3\,x^3} - b\,c\,d\,e\,\ln\left(c^2\,x^2+1\right) + 2\,b\,c\,d\,e\,\ln(x) - \frac{2\,b\,d\,e\,\mathrm{atan}(c\,x)}{x}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^4,x)
```

output
```
a*e^2*x - (a*d^2)/(3*x^3) + (b*c^3*d^2*log(c^2*x^2 + 1))/6 - (b*e^2*log(c^
2*x^2 + 1))/(2*c) - (b*c^3*d^2*log(x))/3 - (2*a*d*e)/x + b*e^2*x*atan(c*x)
 - (b*c*d^2)/(6*x^2) - (b*d^2*atan(c*x))/(3*x^3) - b*c*d*e*log(c^2*x^2 + 1
) + 2*b*c*d*e*log(x) - (2*b*d*e*atan(c*x))/x
```

3.1133 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^5}\,dx$

3.1133.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^5}\,dx = &-\frac{bcd^2}{12x^3}+\frac{bc^3d^2}{4x}-\frac{bcde}{x}+\frac{1}{4}bc^4d^2\arctan(cx)\\ &-bc^2de\arctan(cx)-\frac{d^2(a+b\arctan(cx))}{4x^4}\\ &-\frac{de(a+b\arctan(cx))}{x^2}+ae^2\log(x)\\ &+\frac{1}{2}ibe^2\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibe^2\,\text{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/12*b*c*d^2/x^3+1/4*b*c^3*d^2/x-b*c*d*e/x+1/4*b*c^4*d^2*arctan(c*x)-b*c^
2*d*e*arctan(c*x)-1/4*d^2*(a+b*arctan(c*x))/x^4-d*e*(a+b*arctan(c*x))/x^2+
a*e^2*ln(x)+1/2*I*b*e^2*polylog(2,-I*c*x)-1/2*I*b*e^2*polylog(2,I*c*x)
```

3.1133.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx = -\frac{d^2(a+b\arctan(cx))}{4x^4} - \frac{de(a+b\arctan(cx))}{x^2} - \frac{bcd^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} - \frac{bcde\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + ae^2\log(x) + \frac{1}{2}ibe^2\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibe^2\,\text{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d^2*(a + b*ArcTan[c*x]))/x^4 - (d*e*(a + b*ArcTan[c*x]))/x^2 - (b*c*
d^2*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3) - (b*c*d*e*Hype
rgeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + a*e^2*Log[x] + (I/2)*b*e^2*Po
lyLog[2, (-I)*c*x] - (I/2)*b*e^2*PolyLog[2, I*c*x]
```

3.1133.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))}{x^5}+\frac{2de(a+b\arctan(cx))}{x^3}+\frac{e^2(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{de(a+b\arctan(cx))}{x^2}+ae^2\log(x)+\frac{1}{4}bc^4d^2\arctan(cx)-bc^2de\arctan(cx)+\frac{bc^3d^2}{4x}-\frac{bcd^2}{12x^3}-\frac{bcde}{x}+\frac{1}{2}ibe^2\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibe^2\,\text{PolyLog}(2,icx)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*d^2)/x^3 + (b*c^3*d^2)/(4*x) - (b*c*d*e)/x + (b*c^4*d^2*ArcTan[
c*x])/4 - b*c^2*d*e*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/(4*x^4) - (d*e
*(a + b*ArcTan[c*x]))/x^2 + a*e^2*Log[x] + (I/2)*b*e^2*PolyLog[2, (-I)*c*x
] - (I/2)*b*e^2*PolyLog[2, I*c*x]
```

3.1133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1133.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.40

method	result
derivativedivides	$c^4\left(\frac{a\,e^2\ln(cx)}{c^4}-\frac{a\,d^2}{4c^4x^4}-\frac{ade}{c^4x^2}+\frac{b\left(\arctan(cx)e^2\ln(cx)-\frac{\arctan(cx)d^2}{4x^4}-\frac{\arctan(cx)de}{x^2}+\frac{ie^2\ln(cx)\ln(icx+1)}{2}-\frac{ie^2\ln(\right.}{}\right.$
default	$c^4\left(\frac{a\,e^2\ln(cx)}{c^4}-\frac{a\,d^2}{4c^4x^4}-\frac{ade}{c^4x^2}+\frac{b\left(\arctan(cx)e^2\ln(cx)-\frac{\arctan(cx)d^2}{4x^4}-\frac{\arctan(cx)de}{x^2}+\frac{ie^2\ln(cx)\ln(icx+1)}{2}-\frac{ie^2\ln(\right.}{}\right.$
parts	$a\left(e^2\ln\left(x\right)-\frac{d^2}{4x^4}-\frac{ed}{x^2}\right)+b\,c^4\left(\frac{\arctan(cx)\ln(cx)e^2}{c^4}-\frac{\arctan(cx)d^2}{4c^4x^4}-\frac{\arctan(cx)de}{c^4x^2}-\frac{-2ie^2\ln(cx)\ln(}{}\right.$
risch	$\frac{b\,c^4d^2\arctan(cx)}{8}-\frac{bc\,d^2}{12x^3}+\frac{b\,c^3d^2}{4x}-\frac{bcde}{x}-\frac{b\,c^2de\arctan(cx)}{2}+\frac{ib\,e^2\,\mathrm{dilog}(icx+1)}{2}-\frac{ade}{x^2}-\frac{a\,d^2}{4x^4}+a\,e^2\ln$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^4*(a/c^4*e^2*ln(c*x)-1/4*a*d^2/c^4/x^4-a/c^4*d*e/x^2+b/c^4*(arctan(c*x)*
e^2*ln(c*x)-1/4*arctan(c*x)*d^2/x^4-arctan(c*x)*d*e/x^2+1/2*I*e^2*ln(c*x)*
ln(1+I*c*x)-1/2*I*e^2*ln(c*x)*ln(1-I*c*x)-1/2*I*e^2*dilog(1-I*c*x)+1/2*I*e
^2*dilog(1+I*c*x)+1/4*d*c^2*((c^2*d-4*e)*arctan(c*x)-(-c^2*d+4*e)/c/x-1/3*
d/c/x^3)))
```

3.1133.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x^5, x)
```

3.1133.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^2}{x^5}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**5, x)
```

3.1133.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d*e + b*e^2*integrate(a
rctan(c*x)/x, x) + a*e^2*log(x) - a*d*e/x^2 - 1/4*a*d^2/x^4
```

3.1133.8 Giac [F]

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.1133.9 Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^5}\,dx = \begin{cases} a\,e^2\,\ln(x) - \frac{\frac{a\,d^2}{4}+a\,e\,d\,x^2}{x^4} & \\ a\,e^2\,\ln(x) - \frac{\frac{a\,d^2}{4}+a\,e\,d\,x^2}{x^4} - \frac{b\,d^2\left(\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\mathrm{atan}(c\,x)\right)}{4\,c} - 2\,b\,d\,e\left(\frac{c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}}{2\,c}+\frac{\mathrm{atan}(c\,x)}{2\,x^2}\right) - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{4\,x^4} - b\,e \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^5,x)
```

output
```
piecewise(c == 0, - ((a*d^2)/4 + a*d*e*x^2)/x^4 + a*e^2*log(x), c ~= 0, -
((a*d^2)/4 + a*d*e*x^2)/x^4 + a*e^2*log(x) - (b*e^2*dilog(- c*x*1i + 1)*1i
)/2 + (b*e^2*dilog(c*x*1i + 1)*1i)/2 - (b*d^2*((c^2/3 - c^4*x^2)/x^3 - c^5
*atan(c*x)))/(4*c) - 2*b*d*e*((c^3*atan(c*x) + c^2/x)/(2*c) + atan(c*x)/(2
*x^2)) - (b*d^2*atan(c*x))/(4*x^4))
```

3.1134 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^6}\,dx$

3.1134.1 Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^2}{20x^4}+\frac{bcd(3c^2d-10e)}{30x^2}-\frac{d^2(a+b\arctan(cx))}{5x^5} - \frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x} + \frac{1}{15}bc\left(3c^4d^2-10c^2de+15e^2\right)\log(x) - \frac{1}{30}bc\left(3c^4d^2-10c^2de+15e^2\right)\log\left(1+c^2x^2\right)$$

output
```
-1/20*b*c*d^2/x^4+1/30*b*c*d*(3*c^2*d-10*e)/x^2-1/5*d^2*(a+b*arctan(c*x))/
x^5-2/3*d*e*(a+b*arctan(c*x))/x^3-e^2*(a+b*arctan(c*x))/x+1/15*b*c*(3*c^4*
d^2-10*c^2*d*e+15*e^2)*ln(x)-1/30*b*c*(3*c^4*d^2-10*c^2*d*e+15*e^2)*ln(c^2
*x^2+1)
```

3.1134.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6}\,dx = \frac{1}{60}\left(-\frac{12d^2(a+b\arctan(cx))}{x^5} - \frac{40de(a+b\arctan(cx))}{x^3} - \frac{60e^2(a+b\arctan(cx))}{x} + 30bce^2\left(2\log(x)-\log\left(1+c^2x^2\right)\right) - 20bcde\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log\left(1+c^2x^2\right)\right) - 3bcd^2\left(\frac{1}{x^4}-\frac{2c^2}{x^2}-4c^4\log(x)+2c^4\log\left(1+c^2x^2\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
((-12*d^2*(a + b*ArcTan[c*x]))/x^5 - (40*d*e*(a + b*ArcTan[c*x]))/x^3 - (6
0*e^2*(a + b*ArcTan[c*x]))/x + 30*b*c*e^2*(2*Log[x] - Log[1 + c^2*x^2]) -
20*b*c*d*e*(x^(-2) + 2*c^2*Log[x] - c^2*Log[1 + c^2*x^2]) - 3*b*c*d^2*(x^(
-4) - (2*c^2)/x^2 - 4*c^4*Log[x] + 2*c^4*Log[1 + c^2*x^2]))/60
```

3.1134.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \ 5511$$

$$-bc\int -\frac{15e^2x^4+10dex^2+3d^2}{15x^5(c^2x^2+1)}dx - \frac{d^2(a+b\arctan(cx))}{5x^5} - \frac{2de(a+b\arctan(cx))}{3x^3} - \frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{15}bc\int \frac{15e^2x^4+10dex^2+3d^2}{x^5\left(c^2x^2+1\right)}dx-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{1}{30}bc\int \frac{15e^2x^4+10dex^2+3d^2}{x^6\left(c^2x^2+1\right)}dx^2-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{ 1195}$$

$$\frac{1}{30}bc\int \left(\frac{3d^2}{x^6}-\frac{\left(3c^2d-10e\right)d}{x^4}+\frac{-3d^2c^6+10dec^4-15e^2c^2}{c^2x^2+1}+\frac{3d^2c^4-10dec^2+15e^2}{x^2}\right)dx^2-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}+\frac{1}{30}bc\left(\frac{d\left(3c^2d-10e\right)}{x^2}+\log\left(x^2\right)\left(3c^4d^2-10c^2de+15e^2\right)-\left(3c^4d^2-10c^2de+15e^2\right)\log\left(c^2x^2+1\right)-\frac{3d^2}{2x^4}\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^2*(a + b*ArcTan[c*x]))/x^5 - (2*d*e*(a + b*ArcTan[c*x]))/(3*x^3) -
 (e^2*(a + b*ArcTan[c*x]))/x + (b*c*((-3*d^2)/(2*x^4) + (d*(3*c^2*d - 10*e
))/x^2 + (3*c^4*d^2 - 10*c^2*d*e + 15*e^2)*Log[x^2] - (3*c^4*d^2 - 10*c^2*
d*e + 15*e^2)*Log[1 + c^2*x^2]))/30
```

3.1134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1134.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{2ed}{3x^3}-\frac{d^2}{5x^5}\right)+b\,c^5\left(-\frac{\arctan(cx)e^2}{c^5x}-\frac{2\arctan(cx)de}{3c^5x^3}-\frac{\arctan(cx)d^2}{5c^5x^5}-\frac{(-3c^4d^2+10c^2de-15e}{}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)e^2}{cx}-\frac{\arctan(cx)d^2}{5c\,x^5}-\frac{2\arctan(cx)de}{3c\,x^3}-\frac{\left(3c^4d^2-10c^2de+15e^2\right)\ln\left(c^2x^2+1\right)}{30}-\left(-\right.\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)e^2}{cx}-\frac{\arctan(cx)d^2}{5c\,x^5}-\frac{2\arctan(cx)de}{3c\,x^3}-\frac{\left(3c^4d^2-10c^2de+15e^2\right)\ln\left(c^2x^2+1\right)}{30}-\left(-\right.\right.}{c^4}\right.$
parallelrisch	$\frac{12\ln(x)b\,c^5d^2x^5-6\ln\left(c^2x^2+1\right)x^5b\,c^5d^2-6b\,c^5d^2x^5-40\ln(x)b\,c^3de\,x^5+20\ln\left(c^2x^2+1\right)x^5b\,c^3de+20b\,c^3de\,x^5+60\ln(x)bc}{}$
risch	$\frac{ib\left(15x^4e^2+10x^2ed+3d^2\right)\ln(icx+1)}{30x^5}-\frac{-12\ln(x)b\,c^5d^2x^5+6\ln\left(-c^2x^2-1\right)b\,c^5d^2x^5+40\ln(x)b\,c^3de\,x^5-20\ln\left(-c^2x^2-1\right.}{}$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-e^2/x-2/3*e*d/x^3-1/5*d^2/x^5)+b*c^5*(-arctan(c*x)/c^5*e^2/x-2/3*arcta
n(c*x)/c^5*d*e/x^3-1/5*arctan(c*x)*d^2/c^5/x^5-1/15/c^4*((-3*c^4*d^2+10*c^
2*d*e-15*e^2)*ln(c*x)-1/2*d*(3*c^2*d-10*e)/x^2+3/4*d^2/x^4+1/2*(3*c^4*d^2-
10*c^2*d*e+15*e^2)*ln(c^2*x^2+1)))
```

3.1134.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^6}\,dx = -\frac{60\,ae^2x^4+2\,(3\,bc^5d^2-10\,bc^3de+15\,bce^2)x^5\log\left(c^2x^2+1\right)-4\,(3\,bc^5d^2-10\,bc^3de+15\,bce^2)x^5\log\left(x\right)+}{60\,x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/60*(60*a*e^2*x^4 + 2*(3*b*c^5*d^2 - 10*b*c^3*d*e + 15*b*c*e^2)*x^5*log(
c^2*x^2 + 1) - 4*(3*b*c^5*d^2 - 10*b*c^3*d*e + 15*b*c*e^2)*x^5*log(x) + 3*
b*c*d^2*x + 40*a*d*e*x^2 - 2*(3*b*c^3*d^2 - 10*b*c*d*e)*x^3 + 12*a*d^2 + 4
*(15*b*e^2*x^4 + 10*b*d*e*x^2 + 3*b*d^2)*arctan(c*x))/x^5
```

3.1134.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^6}\,dx = \begin{cases} -\frac{ad^2}{5x^5}-\frac{2ade}{3x^3}-\frac{ae^2}{x}+\frac{bc^5d^2\log\left(x\right)}{5}-\frac{bc^5d^2\log\left(x^2+\frac{1}{c^2}\right)}{10}+\frac{bc^3d^2}{10x^2}-\frac{2bc^3de\log\left(x\right)}{3}+\frac{bc^3de\log\left(x^2+\frac{1}{c^2}\right)}{3}-\frac{bcd^2}{20x^4}-\frac{bcde}{3x^2}+b \\ a\left(-\frac{d^2}{5x^5}-\frac{2de}{3x^3}-\frac{e^2}{x}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c**5*d**2*lo
g(x)/5 - b*c**5*d**2*log(x**2 + c**(-2))/10 + b*c**3*d**2/(10*x**2) - 2*b*
c**3*d*e*log(x)/3 + b*c**3*d*e*log(x**2 + c**(-2))/3 - b*c*d**2/(20*x**4)
- b*c*d*e/(3*x**2) + b*c*e**2*log(x) - b*c*e**2*log(x**2 + c**(-2))/2 - b*
d**2*atan(c*x)/(5*x**5) - 2*b*d*e*atan(c*x)/(3*x**3) - b*e**2*atan(c*x)/x,
 Ne(c, 0)), (a*(-d**2/(5*x**5) - 2*d*e/(3*x**3) - e**2/x), True))
```

3.1134.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6}\,dx \\
&= -\frac{1}{20}\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bd^2 \\
&\quad+\frac{1}{3}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bde \\
&\quad-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)be^2-\frac{ae^2}{x}-\frac{2\,ade}{3\,x^3}-\frac{ad^2}{5\,x^5}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d^2 + 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/
x^2)*c - 2*arctan(c*x)/x^3)*b*d*e - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) +
 2*arctan(c*x)/x)*b*e^2 - a*e^2/x - 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5
```

3.1134.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.1134.9 Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6}\,dx = \frac{b\,c^3\,d^2}{10\,x^2} - \frac{a\,e^2}{x} - \frac{b\,c^5\,d^2\,\ln\left(c^2\,x^2+1\right)}{10} - \frac{a\,d^2}{5\,x^5} + \frac{b\,c^5\,d^2\,\ln\left(x\right)}{5} - \frac{2\,a\,d\,e}{3\,x^3} - \frac{b\,c\,e^2\,\ln\left(c^2\,x^2+1\right)}{2} - \frac{b\,c\,d^2}{20\,x^4} + b\,c\,e^2\,\ln\left(x\right) - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{5\,x^5} - \frac{b\,e^2\,\mathrm{atan}(c\,x)}{x} + \frac{b\,c^3\,d\,e\,\ln\left(c^2\,x^2+1\right)}{3} - \frac{2\,b\,c^3\,d\,e\,\ln\left(x\right)}{3} - \frac{b\,c\,d\,e}{3\,x^2} - \frac{2\,b\,d\,e\,\mathrm{atan}(c\,x)}{3\,x^3}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^6,x)
```

output
```
(b*c^3*d^2)/(10*x^2) - (a*e^2)/x - (b*c^5*d^2*log(c^2*x^2 + 1))/10 - (a*d^
2)/(5*x^5) + (b*c^5*d^2*log(x))/5 - (2*a*d*e)/(3*x^3) - (b*c*e^2*log(c^2*x
^2 + 1))/2 - (b*c*d^2)/(20*x^4) + b*c*e^2*log(x) - (b*d^2*atan(c*x))/(5*x^
5) - (b*e^2*atan(c*x))/x + (b*c^3*d*e*log(c^2*x^2 + 1))/3 - (2*b*c^3*d*e*l
og(x))/3 - (b*c*d*e)/(3*x^2) - (2*b*d*e*atan(c*x))/(3*x^3)
```

3.1135 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^7}\,dx$

3.1135.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd^2}{30x^5}+\frac{bcd(c^2d-3e)}{18x^3}-\frac{bc(c^4d^2-3c^2de+3e^2)}{6x} -\frac{b(c^2d-e)^3\arctan(cx)}{6d} -\frac{(d+ex^2)^3(a+b\arctan(cx))}{6dx^6}$$

output
```
-1/30*b*c*d^2/x^5+1/18*b*c*d*(c^2*d-3*e)/x^3-1/6*b*c*(c^4*d^2-3*c^2*d*e+3*
e^2)/x-1/6*b*(c^2*d-e)^3*arctan(c*x)/d-1/6*(e*x^2+d)^3*(a+b*arctan(c*x))/d
/x^6
```

3.1135.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd^2x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)+5\left((d^2+3dex^2+3e^2x^4)(a+b\arctan(cx))+bcdex^3\,\text{H}\right.}{30x^6}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/30*(b*c*d^2*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + 5*((d^2 +
3*d*e*x^2 + 3*e^2*x^4)*(a + b*ArcTan[c*x]) + b*c*d*e*x^3*Hypergeometric2F1
[-3/2, 1, -1/2, -(c^2*x^2)] + 3*b*c*e^2*x^5*Hypergeometric2F1[-1/2, 1, 1/2
, -(c^2*x^2)]))/x^6
```

3.1135.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5511, 27, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^7}dx$$
$$\Big\downarrow \text{5511}$$
$$-bc\int -\frac{(ex^2+d)^3}{6dx^6(c^2x^2+1)}dx-\frac{(d+ex^2)^3(a+b\arctan(cx))}{6dx^6}$$
$$\Big\downarrow \text{27}$$
$$\frac{bc\int \frac{(ex^2+d)^3}{x^6(c^2x^2+1)}dx}{6d}-\frac{(d+ex^2)^3(a+b\arctan(cx))}{6dx^6}$$
$$\Big\downarrow \text{364}$$
$$\frac{bc\int \left(\frac{d^3}{x^6}-\frac{(c^2d-3e)d^2}{x^4}+\frac{(d^2c^4-3dec^2+3e^2)d}{x^2}-\frac{(c^2d-e)^3}{c^2x^2+1}\right)dx}{6d}-\frac{(d+ex^2)^3(a+b\arctan(cx))}{6dx^6}$$
$$\Big\downarrow \text{2009}$$
$$\frac{bc\left(-\frac{\arctan(cx)(c^2d-e)^3}{c}+\frac{d^2(c^2d-3e)}{3x^3}-\frac{d(c^4d^2-3c^2de+3e^2)}{x}-\frac{d^3}{5x^5}\right)}{6d}-\frac{(d+ex^2)^3(a+b\arctan(cx))}{6dx^6}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*((d + e*x^2)^3*(a + b*ArcTan[c*x]))/(d*x^6) + (b*c*(-1/5*d^3/x^5 + (d
^2*(c^2*d - 3*e))/(3*x^3) - (d*(c^4*d^2 - 3*c^2*d*e + 3*e^2))/x - ((c^2*d
- e)^3*ArcTan[c*x])/c))/(6*d)
```

3.1135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1135. $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^7}\,dx$

3.1135.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.50

method	result
parts	$a\left(-\frac{d^2}{6x^6}-\frac{ed}{2x^4}-\frac{e^2}{2x^2}\right)+b\,c^6\left(-\frac{\arctan(cx)d^2}{6c^6x^6}-\frac{\arctan(cx)de}{2c^6x^4}-\frac{\arctan(cx)e^2}{2c^6x^2}-\frac{-\frac{-c^4d^2+3c^2de-3e^2}{cx}-\right.$
derivativedivides	$c^6\left(\frac{a\left(-\frac{d^2}{6c^2x^6}-\frac{de}{2c^2x^4}-\frac{e^2}{2c^2x^2}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)d^2}{6c^2x^6}-\frac{\arctan(cx)de}{2c^2x^4}-\frac{\arctan(cx)e^2}{2c^2x^2}-\frac{\left(c^4d^2-3c^2de+3e^2\right)\arctan(cx)}{6}+\frac{-c^4}{}\right.}{c^4}\right.$
default	$c^6\left(\frac{a\left(-\frac{d^2}{6c^2x^6}-\frac{de}{2c^2x^4}-\frac{e^2}{2c^2x^2}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)d^2}{6c^2x^6}-\frac{\arctan(cx)de}{2c^2x^4}-\frac{\arctan(cx)e^2}{2c^2x^2}-\frac{\left(c^4d^2-3c^2de+3e^2\right)\arctan(cx)}{6}+\frac{-c^4}{}\right.}{c^4}\right.$
parallelrisch	$-\frac{15x^6\arctan(cx)b\,c^6d^2-45x^6\arctan(cx)b\,c^4de+15b\,c^5d^2x^5+45x^6\arctan(cx)b\,c^2e^2-45a\,c^2e^2x^6-45b\,c^3de\,x^5+45bc\,e^2}{}$
risch	$\frac{ib(3x^4e^2+3x^2ed+d^2)\ln(icx+1)}{12x^6}-\frac{45i\ln(-cx+i)b\,c^4de\,x^6-45i\ln(-cx+i)b\,c^2e^2x^6+15i\ln(-cx-i)b\,c^6d^2x^6-15i\ln(-}{}$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/6*d^2/x^6-1/2*e*d/x^4-1/2*e^2/x^2)+b*c^6*(-1/6*arctan(c*x)*d^2/c^6/x
^6-1/2*arctan(c*x)/c^6*d*e/x^4-1/2*arctan(c*x)/c^6*e^2/x^2-1/6/c^4*(-(-c^4
*d^2+3*c^2*d*e-3*e^2)/c/x-1/3*d/c*(c^2*d-3*e)/x^3+1/5/c*d^2/x^5+(c^4*d^2-3
*c^2*d*e+3*e^2)*arctan(c*x)))
```

3.1135.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int\frac{\left(d+ex^2\right)^2\left(a+b\arctan(cx)\right)}{x^7}\,dx=$$
$$-\frac{45\,ae^2x^4+15\,(bc^5d^2-3\,bc^3de+3\,bce^2)x^5+3\,bcd^2x+45\,adex^2-5\,(bc^3d^2-3\,bcde)x^3+15\,ad^2+15\,(3}{90\,x^6}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
-1/90*(45*a*e^2*x^4 + 15*(b*c^5*d^2 - 3*b*c^3*d*e + 3*b*c*e^2)*x^5 + 3*b*c
*d^2*x + 45*a*d*e*x^2 - 5*(b*c^3*d^2 - 3*b*c*d*e)*x^3 + 15*a*d^2 + 15*(3*b
*e^2*x^4 + (b*c^6*d^2 - 3*b*c^4*d*e + 3*b*c^2*e^2)*x^6 + 3*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x^6
```

3.1135.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx = -\frac{ad^2}{6x^6} - \frac{ade}{2x^4} - \frac{ae^2}{2x^2} - \frac{bc^6d^2\operatorname{atan}(cx)}{6} - \frac{bc^5d^2}{6x} + \frac{bc^4de\operatorname{atan}(cx)}{2} + \frac{bc^3d^2}{18x^3} + \frac{bc^3de}{2x} - \frac{bc^2e^2\operatorname{atan}(cx)}{2} - \frac{bcd^2}{30x^5} - \frac{bcde}{6x^3} - \frac{bce^2}{2x} - \frac{bd^2\operatorname{atan}(cx)}{6x^6} - \frac{bde\operatorname{atan}(cx)}{2x^4} - \frac{be^2\operatorname{atan}(cx)}{2x^2}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**7,x)
```

output
```
-a*d**2/(6*x**6) - a*d*e/(2*x**4) - a*e**2/(2*x**2) - b*c**6*d**2*atan(c*x
)/6 - b*c**5*d**2/(6*x) + b*c**4*d*e*atan(c*x)/2 + b*c**3*d**2/(18*x**3) +
 b*c**3*d*e/(2*x) - b*c**2*e**2*atan(c*x)/2 - b*c*d**2/(30*x**5) - b*c*d*e
/(6*x**3) - b*c*e**2/(2*x) - b*d**2*atan(c*x)/(6*x**6) - b*d*e*atan(c*x)/(
2*x**4) - b*e**2*atan(c*x)/(2*x**2)
```

3.1135.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx = -\frac{1}{90}\left(\left(15\,c^5\arctan(cx) + \frac{15\,c^4x^4 - 5\,c^2x^2 + 3}{x^5}\right)c + \frac{15\arctan(cx)}{x^6}\right)bd^2 + \frac{1}{6}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2 - 1}{x^3}\right)c - \frac{3\arctan(cx)}{x^4}\right)bde - \frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)be^2 - \frac{ae^2}{2\,x^2} - \frac{ade}{2\,x^4} - \frac{ad^2}{6\,x^6}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arct
an(c*x)/x^6)*b*d^2 + 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*
arctan(c*x)/x^4)*b*d*e - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b
*e^2 - 1/2*a*e^2/x^2 - 1/2*a*d*e/x^4 - 1/6*a*d^2/x^6
```

3.1135.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx=\int \frac{(ex^2+d)^2 (b\arctan(cx)+a)}{x^7}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
sage0*x
```

3.1135.9 Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.31

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx=$$

$$-\frac{\frac{a\,d^2}{6}+\frac{b\,d^2\,\mathrm{atan}(c\,x)}{6}-\frac{a\,c^4\,e^2\,x^8}{2}+\frac{a\,e\,x^4\,(d\,c^2+e)}{2}+\frac{b\,c\,x^5\,(2\,c^4\,d^2-6\,c^2\,d\,e+9\,e^2)}{18}+\frac{b\,c\,d^2\,x}{30}+\frac{a\,d\,x^2\,(d\,c^2+3\,e)}{6}+\frac{b\,c^3\,x^7\,(c^4\,d^2-\cdots}{6}\cdots}{c^2\,x^8+x^6}$$

$$-\frac{\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,(c^2)^{5/2}\,(b\,c^4\,d^2-3\,b\,c^2\,d\,e+3\,b\,e^2)}{6\,c^3}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^7,x)
```

output
```
- ((a*d^2)/6 + (b*d^2*atan(c*x))/6 - (a*c^4*e^2*x^8)/2 + (a*e*x^4*(e + c^2
*d))/2 + (b*c*x^5*(9*e^2 + 2*c^4*d^2 - 6*c^2*d*e))/18 + (b*c*d^2*x)/30 + (
a*d*x^2*(3*e + c^2*d))/6 + (b*c^3*x^7*(3*e^2 + c^4*d^2 - 3*c^2*d*e))/6 + (
b*c*d*x^3*(15*e - 2*c^2*d))/90 + (b*d*x^2*atan(c*x)*(3*e + c^2*d))/6 + (b*
c^2*e^2*x^6*atan(c*x))/2 + (b*e*x^4*atan(c*x)*(e + c^2*d))/2)/(x^6 + c^2*x
^8) - (atan((c^2*x)/(c^2)^(1/2))*(c^2)^(5/2)*(3*b*e^2 + b*c^4*d^2 - 3*b*c^
2*d*e))/(6*c^3)
```

3.1136 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^8}\,dx$

3.1136.1 Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^8}\,dx = &-\frac{bcd^2}{42x^6}+\frac{bcd(5c^2d-14e)}{140x^4}\\&-\frac{bc(15c^4d^2-42c^2de+35e^2)}{210x^2}-\frac{d^2(a+b\arctan(cx))}{7x^7}\\&-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}\\&-\frac{1}{105}bc^3(15c^4d^2-42c^2de+35e^2)\log(x)\\&+\frac{1}{210}bc^3(15c^4d^2-42c^2de+35e^2)\log(1+c^2x^2)\end{aligned}$$

output
```
-1/42*b*c*d^2/x^6+1/140*b*c*d*(5*c^2*d-14*e)/x^4-1/210*b*c*(15*c^4*d^2-42*
c^2*d*e+35*e^2)/x^2-1/7*d^2*(a+b*arctan(c*x))/x^7-2/5*d*e*(a+b*arctan(c*x)
)/x^5-1/3*e^2*(a+b*arctan(c*x))/x^3-1/105*b*c^3*(15*c^4*d^2-42*c^2*d*e+35*
e^2)*ln(x)+1/210*b*c^3*(15*c^4*d^2-42*c^2*d*e+35*e^2)*ln(c^2*x^2+1)
```

3.1136.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8} dx = \frac{1}{420}\Bigg(-\frac{60d^2(a+b\arctan(cx))}{x^7} - \frac{168de(a+b\arctan(cx))}{x^5} - \frac{140e^2(a+b\arctan(cx))}{x^3} - 70bce^2\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log\left(1+c^2x^2\right)\right) - 42bcde\left(\frac{1}{x^4}-\frac{2c^2}{x^2}-4c^4\log(x)+2c^4\log\left(1+c^2x^2\right)\right) - 5bcd^2\left(\frac{2-3c^2x^2+6c^4x^4}{x^6}+12c^6\log(x) - 6c^6\log\left(1+c^2x^2\right)\right)\Bigg)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
((-60*d^2*(a + b*ArcTan[c*x]))/x^7 - (168*d*e*(a + b*ArcTan[c*x]))/x^5 - (
140*e^2*(a + b*ArcTan[c*x]))/x^3 - 70*b*c*e^2*(x^(-2) + 2*c^2*Log[x] - c^2
*Log[1 + c^2*x^2]) - 42*b*c*d*e*(x^(-4) - (2*c^2)/x^2 - 4*c^4*Log[x] + 2*c
^4*Log[1 + c^2*x^2]) - 5*b*c*d^2*((2 - 3*c^2*x^2 + 6*c^4*x^4)/x^6 + 12*c^6
*Log[x] - 6*c^6*Log[1 + c^2*x^2]))/420
```

3.1136.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8} dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{35e^2x^4+42dex^2+15d^2}{105x^7\left(c^2x^2+1\right)}dx-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{1}{105}bc\int \frac{35e^2x^4+42dex^2+15d^2}{x^7\left(c^2x^2+1\right)}dx-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{1578}$$

$$\frac{1}{210}bc\int \frac{35e^2x^4+42dex^2+15d^2}{x^8\left(c^2x^2+1\right)}dx^2-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{1195}$$

$$\frac{1}{210}bc\int \left(\frac{15d^2}{x^8}-\frac{3\left(5c^2d-14e\right)d}{x^6}+\frac{15d^2c^8-42dec^6+35e^2c^4}{c^2x^2+1}+\frac{-15d^2c^6+42dec^4-35e^2c^2}{x^2}+\frac{15d^2c^4-42dec}{x^4}\right.$$
$$\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}+$$
$$\frac{1}{210}bc\left(\frac{3d\left(5c^2d-14e\right)}{2x^4}-\frac{15c^4d^2-42c^2de+35e^2}{x^2}-\left(c^2\log\left(x^2\right)\left(15c^4d^2-42c^2de+35e^2\right)\right)+c^2\left(15c^4d^2-42c^2d\right.\right.$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^2*(a + b*ArcTan[c*x]))/x^7 - (2*d*e*(a + b*ArcTan[c*x]))/(5*x^5) -
 (e^2*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*((-5*d^2)/x^6 + (3*d*(5*c^2*d -
14*e))/(2*x^4) - (15*c^4*d^2 - 42*c^2*d*e + 35*e^2)/x^2 - c^2*(15*c^4*d^2
- 42*c^2*d*e + 35*e^2)*Log[x^2] + c^2*(15*c^4*d^2 - 42*c^2*d*e + 35*e^2)*L
og[1 + c^2*x^2]))/210
```

3.1136. $\int \frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{x^8}\,dx$

3.1136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1136.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09

method	result
parts	$a\left(-\frac{d^2}{7x^7}-\frac{e^2}{3x^3}-\frac{2ed}{5x^5}\right)+b\,c^7\left(-\frac{\arctan(cx)d^2}{7c^7x^7}-\frac{\arctan(cx)e^2}{3c^7x^3}-\frac{2\arctan(cx)de}{5c^7x^5}-\frac{-\frac{-15c^4d^2+42c^2de-35}{2c^2x^2}}{}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\arctan(cx)de}{5c^3x^5}-\frac{\arctan(cx)d^2}{7c^3x^7}-\frac{\arctan(cx)e^2}{3c^3x^3}-\frac{(-15c^4d^2+42c^2de-35e^2)\ln(c^2x^2+}{210}\right.}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\arctan(cx)de}{5c^3x^5}-\frac{\arctan(cx)d^2}{7c^3x^7}-\frac{\arctan(cx)e^2}{3c^3x^3}-\frac{(-15c^4d^2+42c^2de-35e^2)\ln(c^2x^2+}{210}\right.}{}\right.$
parallelrisch	$-\frac{60\ln(x)b\,c^7d^2x^7-30\ln(c^2x^2+1)b\,c^7d^2x^7-30b\,c^7d^2x^7-168\ln(x)b\,c^5de\,x^7+84\ln(c^2x^2+1)b\,c^5de\,x^7+84b\,c^5de\,x^7+140}{}$
risch	$\frac{ib(35x^4e^2+42x^2ed+15d^2)\ln(icx+1)}{210x^7}-\frac{60\ln(x)b\,c^7d^2x^7-30\ln(c^2x^2+1)b\,c^7d^2x^7-168\ln(x)b\,c^5de\,x^7+84\ln(c^2x^2+1)}{}$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d^2/x^7-1/3*e^2/x^3-2/5*e*d/x^5)+b*c^7*(-1/7*arctan(c*x)*d^2/c^7/x
^7-1/3*arctan(c*x)/c^7*e^2/x^3-2/5*arctan(c*x)/c^7*d*e/x^5-1/105/c^4*(-1/2
*(-15*c^4*d^2+42*c^2*d*e-35*e^2)/c^2/x^2+(15*c^4*d^2-42*c^2*d*e+35*e^2)*ln
(c*x)+5/2/c^2*d^2/x^6-3/4*d/c^2*(5*c^2*d-14*e)/x^4+1/2*(-15*c^4*d^2+42*c^2
*d*e-35*e^2)*ln(c^2*x^2+1)))
```

3.1136.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^8}\,dx$$
$$=\frac{2\,(15\,bc^7d^2-42\,bc^5de+35\,bc^3e^2)x^7\log(c^2x^2+1)-4\,(15\,bc^7d^2-42\,bc^5de+35\,bc^3e^2)x^7\log(x)-140\,ae^2}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

output
```
1/420*(2*(15*b*c^7*d^2 - 42*b*c^5*d*e + 35*b*c^3*e^2)*x^7*log(c^2*x^2 + 1)
 - 4*(15*b*c^7*d^2 - 42*b*c^5*d*e + 35*b*c^3*e^2)*x^7*log(x) - 140*a*e^2*x
^4 - 2*(15*b*c^5*d^2 - 42*b*c^3*d*e + 35*b*c*e^2)*x^5 - 10*b*c*d^2*x - 168
*a*d*e*x^2 + 3*(5*b*c^3*d^2 - 14*b*c*d*e)*x^3 - 60*a*d^2 - 4*(35*b*e^2*x^4
 + 42*b*d*e*x^2 + 15*b*d^2)*arctan(c*x))/x^7
```

3.1136.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8}\,dx$$
$$=\begin{cases} -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bc^7d^2\log(x)}{7} + \frac{bc^7d^2\log\left(x^2+\frac{1}{c^2}\right)}{14} - \frac{bc^5d^2}{14x^2} + \frac{2bc^5de\log(x)}{5} - \frac{bc^5de\log\left(x^2+\frac{1}{c^2}\right)}{5} + \frac{bc^3d^2}{28x^4} + \frac{bc^3de}{5x^2} - \\ a\left(-\frac{d^2}{7x^7} - \frac{2de}{5x^5} - \frac{e^2}{3x^3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**8,x)
```

output
```
Piecewise((-a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*c**7*
d**2*log(x)/7 + b*c**7*d**2*log(x**2 + c**(-2))/14 - b*c**5*d**2/(14*x**2)
 + 2*b*c**5*d*e*log(x)/5 - b*c**5*d*e*log(x**2 + c**(-2))/5 + b*c**3*d**2/
(28*x**4) + b*c**3*d*e/(5*x**2) - b*c**3*e**2*log(x)/3 + b*c**3*e**2*log(x
**2 + c**(-2))/6 - b*c*d**2/(42*x**6) - b*c*d*e/(10*x**4) - b*c*e**2/(6*x*
*2) - b*d**2*atan(c*x)/(7*x**7) - 2*b*d*e*atan(c*x)/(5*x**5) - b*e**2*atan
(c*x)/(3*x**3), Ne(c, 0)), (a*(-d**2/(7*x**7) - 2*d*e/(5*x**5) - e**2/(3*x
**3)), True))
```

3.1136.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right) - 6\,c^6\log\left(x^2\right) - \frac{6\,c^4x^4 - 3\,c^2x^2 + 2}{x^6}\right)c - \frac{12\arctan(cx)}{x^7}\right)bd^2 - \frac{1}{10}\left(\left(2\,c^4\log\left(c^2x^2+1\right) - 2\,c^4\log\left(x^2\right) - \frac{2\,c^2x^2-1}{x^4}\right)c + \frac{4\arctan(cx)}{x^5}\right)bde + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)be^2 - \frac{ae^2}{3\,x^3} - \frac{2\,ade}{5\,x^5} - \frac{ad^2}{7\,x^7}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 +
2)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^2 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*
c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d*e + 1/6*((c
^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*e^2 -
 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

3.1136.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

output
```
sage0*x
```

3.1136.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8}\,dx = -\frac{60\,a\,d^2+60\,b\,d^2\,\mathrm{atan}(c\,x)+140\,a\,e^2\,x^4-15\,b\,c^3\,d^2\,x^3+30\,b\,c^5\,d^2\,x^5+10\,b\,c\,d^2\,x+168\,a\,d\,e\,x^2+70\,b\,c\,e}{}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^8,x)
```

output
```
-(60*a*d^2 + 60*b*d^2*atan(c*x) + 140*a*e^2*x^4 - 15*b*c^3*d^2*x^3 + 30*b*
c^5*d^2*x^5 + 10*b*c*d^2*x + 168*a*d*e*x^2 + 70*b*c*e^2*x^5 + 140*b*e^2*x^
4*atan(c*x) + 60*b*c^7*d^2*x^7*log(x) + 140*b*c^3*e^2*x^7*log(x) - 84*b*c^
3*d*e*x^5 + 42*b*c*d*e*x^3 - 30*b*c^7*d^2*x^7*log(c^2*x^2 + 1) - 70*b*c^3*
e^2*x^7*log(c^2*x^2 + 1) + 168*b*d*e*x^2*atan(c*x) - 168*b*c^5*d*e*x^7*log
(x) + 84*b*c^5*d*e*x^7*log(c^2*x^2 + 1))/(420*x^7)
```

3.1137 $\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$

3.1137.1 Optimal result

Integrand size = 21, antiderivative size = 240

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx = \frac{b(10c^6d^3-20c^4d^2e+15c^2de^2-4e^3)\,x}{40c^9} - \frac{b(10c^6d^3-20c^4d^2e+15c^2de^2-4e^3)\,x^3}{120c^7} - \frac{be(20c^4d^2-15c^2de+4e^2)\,x^5}{200c^5} - \frac{b(15c^2d-4e)\,e^2x^7}{280c^3} - \frac{be^3x^9}{90c} + \frac{b(c^2d-e)^4(c^2d+4e)\arctan(cx)}{40c^{10}e^2} - \frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2} + \frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}$$

output
```
1/40*b*(10*c^6*d^3-20*c^4*d^2*e+15*c^2*d*e^2-4*e^3)*x/c^9-1/120*b*(10*c^6*
d^3-20*c^4*d^2*e+15*c^2*d*e^2-4*e^3)*x^3/c^7-1/200*b*e*(20*c^4*d^2-15*c^2*
d*e+4*e^2)*x^5/c^5-1/280*b*(15*c^2*d-4*e)*e^2*x^7/c^3-1/90*b*e^3*x^9/c+1/4
0*b*(c^2*d-e)^4*(c^2*d+4*e)*arctan(c*x)/c^10/e^2-1/8*d*(e*x^2+d)^4*(a+b*ar
ctan(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*arctan(c*x))/e^2
```

3.1137.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{be^3(315cx-105c^3x^3+63c^5x^5-45c^7x^7+35c^9x^9-315\arctan(cx))}{3150c^{10}} - \frac{bd^2e(15cx-5c^3x^3+3c^5x^5-15\arctan(cx))}{30c^6} - \frac{bd^3(-3cx+c^3x^3+3\arctan(cx))}{12c^4} - \frac{bde^2(-105cx+35c^3x^3-21c^5x^5+15c^7x^7+105\arctan(cx))}{280c^8} + \frac{1}{4}d^3x^4(a+b\arctan(cx)) + \frac{1}{2}d^2ex^6(a+b\arctan(cx)) + \frac{3}{8}de^2x^8(a+b\arctan(cx)) + \frac{1}{10}e^3x^{10}(a+b\arctan(cx))$$

input
```
Integrate[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
-1/3150*(b*e^3*(315*c*x - 105*c^3*x^3 + 63*c^5*x^5 - 45*c^7*x^7 + 35*c^9*x
^9 - 315*ArcTan[c*x]))/c^10 - (b*d^2*e*(15*c*x - 5*c^3*x^3 + 3*c^5*x^5 - 1
5*ArcTan[c*x]))/(30*c^6) - (b*d^3*(-3*c*x + c^3*x^3 + 3*ArcTan[c*x]))/(12*
c^4) - (b*d*e^2*(-105*c*x + 35*c^3*x^3 - 21*c^5*x^5 + 15*c^7*x^7 + 105*Arc
Tan[c*x]))/(280*c^8) + (d^3*x^4*(a + b*ArcTan[c*x]))/4 + (d^2*e*x^6*(a + b
*ArcTan[c*x]))/2 + (3*d*e^2*x^8*(a + b*ArcTan[c*x]))/8 + (e^3*x^10*(a + b*
ArcTan[c*x]))/10
```

3.1137.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5511, 27, 403, 403, 27, 403, 403, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$\Big\downarrow \; 5511$$
$$-bc\int -\frac{(d-4ex^2)(ex^2+d)^4}{40e^2(c^2x^2+1)}dx + \frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2} - \frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$
$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{(d-4ex^2)(ex^2+d)^4}{c^2x^2+1}dx}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\int \frac{(ex^2+d)^3\left(d(9dc^2+4e)-(23c^2d-36e)ex^2\right)}{c^2x^2+1}dx}{9c^2}-\frac{4ex(d+ex^2)^4}{9c^2}\right)}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{\int \frac{3(ex^2+d)^2\left(d(21d^2c^4+17dec^2-12e^2)-e(25d^2c^4-135dec^2+84e^2)x^2\right)}{c^2x^2+1}dx}{7c^2}-\frac{ex(23c^2d-36e)(d+ex^2)^3}{7c^2}}{9c^2}-\frac{4ex(d+ex^2)^4}{9c^2}\right)}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(\frac{\frac{3\int \frac{(ex^2+d)^2\left(d(21d^2c^4+17dec^2-12e^2)-e(25d^2c^4-135dec^2+84e^2)x^2\right)}{c^2x^2+1}dx}{7c^2}-\frac{ex(23c^2d-36e)(d+ex^2)^3}{7c^2}}{9c^2}-\frac{4ex(d+ex^2)^4}{9c^2}\right)}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{3\left(\frac{\int \frac{(ex^2+d)\left(e(5d^3c^6+750d^2ec^4-1071de^2c^2+420e^3)x^2+d(105d^3c^6+110d^2ec^4-195de^2c^2+84e^3)\right)}{c^2x^2+1}dx}{5c^2}-\frac{ex(25c^4d^2-135c^2de+84e^2)(d+ex^2)^2}{5c^2}\right)}{7c^2}-\frac{ex(2\cdots}{}}{9c^2}\cdots\right)}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

3.1137. $\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$

$$bc\left(\frac{\frac{3\left(\frac{\frac{\int \frac{e\left(325d^4c^8+1815d^3ec^6-4977d^2e^2c^4+4305de^3c^2-1260e^4\right)x^2+d\left(315d^4c^8+325d^3ec^6-1335d^2e^2c^4+1323de^3c^2-420e^4\right)}{c^2x^2+1}dx}{3c^2}+\frac{ex\left(5c^6d^3+750c^4d^2e-107\right.}{3c^2}}{5c^2}\right.}{7c^2}}{9c^2}\right.}{40e^2}$$

$$\frac{\left(d+ex^2\right)^5(a+b\arctan(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e^2}$$

$\Big\downarrow$ 299

$$bc\left(\frac{\frac{3\left(\frac{\frac{\frac{315\left(c^2d+4e\right)\left(c^2d-e\right)^4\int\frac{1}{c^2x^2+1}dx}{c^2}+\frac{ex\left(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4\right)}{c^2}}{3c^2}+\frac{ex\left(5c^6d^3+750c^4d^2e-1071c^2de^2+420e^3\right)\left(d+ex^2\right)}{3c^2}}{5c^2}\right.}{7c^2}}{9c^2}\right.}{40e^2}$$

$$\frac{\left(d+ex^2\right)^5(a+b\arctan(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e^2}$$

$\Big\downarrow$ 216

$$\frac{\left(d+ex^2\right)^5(a+b\arctan(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e^2}+$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{\frac{315\arctan(cx)\left(c^2d+4e\right)\left(c^2d-e\right)^4}{c^3}+\frac{ex\left(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4\right)}{c^2}}{3c^2}+\frac{ex\left(5c^6d^3+750c^4d^2e-1071c^2de^2+420e^3\right)\left(d+ex^2\right)}{3c^2}-}{5c^2}\right.}{7c^2}}{9c^2}\right.}{40e^2}$$

input
```
Int[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

3.1137. $\int x^3\left(d+ex^2\right)^3(a+b\arctan(cx))\,dx$

output
```
-1/8*(d*(d + e*x^2)^4*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^5*(a + b*Arc
Tan[c*x]))/(10*e^2) + (b*c*((-4*e*x*(d + e*x^2)^4)/(9*c^2) + (-1/7*((23*c^
2*d - 36*e)*e*x*(d + e*x^2)^3)/c^2 + (3*(-1/5*(e*(25*c^4*d^2 - 135*c^2*d*e
 + 84*e^2)*x*(d + e*x^2)^2)/c^2 + ((e*(5*c^6*d^3 + 750*c^4*d^2*e - 1071*c^
2*d*e^2 + 420*e^3)*x*(d + e*x^2))/(3*c^2) + ((e*(325*c^8*d^4 + 1815*c^6*d^
3*e - 4977*c^4*d^2*e^2 + 4305*c^2*d*e^3 - 1260*e^4)*x)/c^2 + (315*(c^2*d -
 e)^4*(c^2*d + 4*e)*ArcTan[c*x])/c^3)/(3*c^2))/(5*c^2)))/(7*c^2))/(9*c^2))
)/(40*e^2)
```

3.1137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1137.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.25

method	result
parts	$a\left(\frac{1}{10}e^3x^{10}+\frac{3}{8}e^2d\,x^8+\frac{1}{2}e\,d^2x^6+\frac{1}{4}d^3x^4\right)+\frac{b\left(\frac{\arctan(cx)c^4e^3x^{10}}{10}+\frac{3\arctan(cx)c^4e^2d\,x^8}{8}+\frac{\arctan(cx)c^4d^2e\,x}{2}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{4}d^3c^{10}x^4+\frac{1}{2}d^2c^{10}e\,x^6+\frac{3}{8}d\,c^{10}e^2x^8+\frac{1}{10}e^3c^{10}x^{10}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^{10}x^4}{4}+\frac{\arctan(cx)d^2c^{10}e\,x^6}{2}+\frac{3\arctan(cx)d\,c^{10}e^2x^8}{8}+\right.}{}$
default	$\frac{a\left(\frac{1}{4}d^3c^{10}x^4+\frac{1}{2}d^2c^{10}e\,x^6+\frac{3}{8}d\,c^{10}e^2x^8+\frac{1}{10}e^3c^{10}x^{10}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^{10}x^4}{4}+\frac{\arctan(cx)d^2c^{10}e\,x^6}{2}+\frac{3\arctan(cx)d\,c^{10}e^2x^8}{8}+\right.}{}$
parallelrisch	$-3150b\,c^6d^3\arctan(cx)-4725b\,c^2d\,e^2\arctan(cx)+6300b\,c^4d^2e\arctan(cx)+1260a\,c^{10}e^3x^{10}-140b\,c^9e^3x^9+3150a\,c^{10}d^3x$
risch	$\frac{3ibd\,e^2x^8\ln(-icx+1)}{16}+\frac{ib\,d^2e\,x^6\ln(-icx+1)}{4}-\frac{b\,e^3x^9}{90c}-\frac{3bd\,e^2x^7}{56c}-\frac{b\,d^2e\,x^5}{10c}+\frac{3bd\,e^2x^5}{40c^3}+\frac{b\,d^2e\,x^3}{6c^3}-\frac{bd\,e^2}{8c^5}$

input
```
int(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/10*e^3*x^10+3/8*e^2*d*x^8+1/2*e*d^2*x^6+1/4*d^3*x^4)+b/c^4*(1/10*arct
an(c*x)*c^4*e^3*x^10+3/8*arctan(c*x)*c^4*e^2*d*x^8+1/2*arctan(c*x)*c^4*d^2
*e*x^6+1/4*arctan(c*x)*d^3*c^4*x^4-1/40/c^6*(4/9*e^3*c^9*x^9+15/7*d*c^9*e^
2*x^7+4*d^2*c^9*e*x^5+10/3*d^3*c^9*x^3-4/7*e^3*c^7*x^7-3*d*c^7*e^2*x^5-20/
3*d^2*c^7*e*x^3-10*c^7*x*d^3+4/5*e^3*c^5*x^5+5*c^5*d*e^2*x^3+20*c^5*d^2*e*
x-4/3*e^3*c^3*x^3-15*c^3*x*d*e^2+4*c*x*e^3+(10*c^6*d^3-20*c^4*d^2*e+15*c^2
*d*e^2-4*e^3)*arctan(c*x)))
```

3.1137.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.27

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$=\frac{1260\,ac^{10}e^3x^{10}+4725\,ac^{10}de^2x^8-140\,bc^9e^3x^9+6300\,ac^{10}d^2ex^6+3150\,ac^{10}d^3x^4-45\,(15\,bc^9de^2-4\,bc^7e^3}{}$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12600*(1260*a*c^10*e^3*x^10 + 4725*a*c^10*d*e^2*x^8 - 140*b*c^9*e^3*x^9
+ 6300*a*c^10*d^2*e*x^6 + 3150*a*c^10*d^3*x^4 - 45*(15*b*c^9*d*e^2 - 4*b*c
^7*e^3)*x^7 - 63*(20*b*c^9*d^2*e - 15*b*c^7*d*e^2 + 4*b*c^5*e^3)*x^5 - 105
*(10*b*c^9*d^3 - 20*b*c^7*d^2*e + 15*b*c^5*d*e^2 - 4*b*c^3*e^3)*x^3 + 315*
(10*b*c^7*d^3 - 20*b*c^5*d^2*e + 15*b*c^3*d*e^2 - 4*b*c*e^3)*x + 315*(4*b*
c^10*e^3*x^10 + 15*b*c^10*d*e^2*x^8 + 20*b*c^10*d^2*e*x^6 + 10*b*c^10*d^3*
x^4 - 10*b*c^6*d^3 + 20*b*c^4*d^2*e - 15*b*c^2*d*e^2 + 4*b*e^3)*arctan(c*x
))/c^10
```

3.1137.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.71

$$\int x^3 \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{ad^3x^4}{4} + \frac{ad^2ex^6}{2} + \frac{3ade^2x^8}{8} + \frac{ae^3x^{10}}{10} + \frac{bd^3x^4\operatorname{atan}(cx)}{4} + \frac{bd^2ex^6\operatorname{atan}(cx)}{2} + \frac{3bde^2x^8\operatorname{atan}(cx)}{8} + \frac{be^3x^{10}\operatorname{atan}(cx)}{10} - \frac{bd^3x^3}{12c} - \\ a\left(\frac{d^3x^4}{4} + \frac{d^2ex^6}{2} + \frac{3de^2x^8}{8} + \frac{e^3x^{10}}{10}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x*
*10/10 + b*d**3*x**4*atan(c*x)/4 + b*d**2*e*x**6*atan(c*x)/2 + 3*b*d*e**2*
x**8*atan(c*x)/8 + b*e**3*x**10*atan(c*x)/10 - b*d**3*x**3/(12*c) - b*d**2
*e*x**5/(10*c) - 3*b*d*e**2*x**7/(56*c) - b*e**3*x**9/(90*c) + b*d**3*x/(4
*c**3) + b*d**2*e*x**3/(6*c**3) + 3*b*d*e**2*x**5/(40*c**3) + b*e**3*x**7/
(70*c**3) - b*d**3*atan(c*x)/(4*c**4) - b*d**2*e*x/(2*c**5) - b*d*e**2*x**
3/(8*c**5) - b*e**3*x**5/(50*c**5) + b*d**2*e*atan(c*x)/(2*c**6) + 3*b*d*e
**2*x/(8*c**7) + b*e**3*x**3/(30*c**7) - 3*b*d*e**2*atan(c*x)/(8*c**8) - b
*e**3*x/(10*c**9) + b*e**3*atan(c*x)/(10*c**10), Ne(c, 0)), (a*(d**3*x**4/
4 + d**2*e*x**6/2 + 3*d*e**2*x**8/8 + e**3*x**10/10), True))
```

3.1137.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.12

$$\int x^3 \left(d + ex^2\right)^3 (a + b\arctan(cx))\,dx = \frac{1}{10}\,ae^3x^{10} + \frac{3}{8}\,ade^2x^8 + \frac{1}{2}\,ad^2ex^6 + \frac{1}{4}\,ad^3x^4 + \frac{1}{12}\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd^3 + \frac{1}{30}\left(15\,x^6\arctan\left(cx\right) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bd^2e + \frac{1}{280}\left(105\,x^8\arctan\left(cx\right) - c\left(\frac{15\,c^6x^7 - 21\,c^4x^5 + 35\,c^2x^3 - 105\,x}{c^8} + \frac{105\,\arctan\left(cx\right)}{c^9}\right)\right)bde^2 + \frac{1}{3150}\left(315\,x^{10}\arctan\left(cx\right) - c\left(\frac{35\,c^8x^9 - 45\,c^6x^7 + 63\,c^4x^5 - 105\,c^2x^3 + 315\,x}{c^{10}} - \frac{315\,\arctan\left(cx\right)}{c^{11}}\right)\right)be$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/12
*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^3 +
 1/30*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arc
tan(c*x)/c^7))*b*d^2*e + 1/280*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*
c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*d*e^2 + 1/3150
*(315*x^10*arctan(c*x) - c*((35*c^8*x^9 - 45*c^6*x^7 + 63*c^4*x^5 - 105*c^
2*x^3 + 315*x)/c^10 - 315*arctan(c*x)/c^11))*b*e^3
```

3.1137.8 Giac [F]

$$\int x^3 \left(d + ex^2\right)^3 (a + b\arctan(cx))\,dx = \int \left(ex^2 + d\right)^3 (b\arctan\left(cx\right) + a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1137.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.50

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= x^3\left(\frac{\frac{\frac{b\,e^3}{10\,c^3}-\frac{3\,b\,d\,e^2}{8\,c}}{c^2}+\frac{b\,d^2\,e}{2\,c}}{3\,c^2}-\frac{b\,d^3}{12\,c}\right)-x^8\left(\frac{a\,e^3}{8\,c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{8\,c^2}\right)$$
$$+x^6\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{6\,c^2}+\frac{a\,d\,e\,(d\,c^2+e)}{2\,c^2}\right)+x^7\left(\frac{b\,e^3}{70\,c^3}-\frac{3\,b\,d\,e^2}{56\,c}\right)$$
$$+\operatorname{atan}(c\,x)\left(\frac{b\,d^3\,x^4}{4}+\frac{b\,d^2\,e\,x^6}{2}+\frac{3\,b\,d\,e^2\,x^8}{8}+\frac{b\,e^3\,x^{10}}{10}\right)-x^5\left(\frac{\frac{b\,e^3}{10\,c^3}-\frac{3\,b\,d\,e^2}{8\,c}}{5\,c^2}+\frac{b\,d^2\,e}{10\,c}\right)$$
$$+x^2\left(\frac{\frac{\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}}{c^2}-\frac{a\,d^2\,(d\,c^2+3\,e)}{c^2}}{2\,c^2}+\frac{a\,d^3}{2\,c^2}\right)$$
$$-x^4\left(\frac{\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}}{4\,c^2}-\frac{a\,d^2\,(d\,c^2+3\,e)}{4\,c^2}\right)$$
$$+\frac{a\,e^3\,x^{10}}{10}-\frac{x\left(\frac{\frac{\frac{b\,e^3}{10\,c^3}-\frac{3\,b\,d\,e^2}{8\,c}}{c^2}+\frac{b\,d^2\,e}{2\,c}}{c^2}-\frac{b\,d^3}{4\,c}\right)}{c^2}-\frac{b\,e^3\,x^9}{90\,c}$$
$$+\frac{b\,\operatorname{atan}\left(\frac{b\,c\,x\,(-10\,c^6\,d^3+20\,c^4\,d^2\,e-15\,c^2\,d\,e^2+4\,e^3)}{-10\,b\,c^6\,d^3+20\,b\,c^4\,d^2\,e-15\,b\,c^2\,d\,e^2+4\,b\,e^3}\right)\,(-10\,c^6\,d^3+20\,c^4\,d^2\,e-15\,c^2\,d\,e^2+4\,e^3)}{40\,c^{10}}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
x^3*((((b*e^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/c^2 + (b*d^2*e)/(2*c))/(3*c^2
) - (b*d^3)/(12*c)) - x^8*((a*e^3)/(8*c^2) - (a*e^2*(e + 3*c^2*d))/(8*c^2)
) + x^6*(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/(6*c^2) + (a*d*e*(e + c
^2*d))/(2*c^2)) + x^7*((b*e^3)/(70*c^3) - (3*b*d*e^2)/(56*c)) + atan(c*x)*
((b*d^3*x^4)/4 + (b*e^3*x^10)/10 + (b*d^2*e*x^6)/2 + (3*b*d*e^2*x^8)/8) -
x^5*(((b*e^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/(5*c^2) + (b*d^2*e)/(10*c)) +
x^2*(((((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d
))/c^2)/c^2 - (a*d^2*(3*e + c^2*d))/c^2)/(2*c^2) + (a*d^3)/(2*c^2)) - x^4*
((((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^
2)/(4*c^2) - (a*d^2*(3*e + c^2*d))/(4*c^2)) + (a*e^3*x^10)/10 - (x*((((b*e
^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/c^2 + (b*d^2*e)/(2*c))/c^2 - (b*d^3)/(4*
c)))/c^2 - (b*e^3*x^9)/(90*c) + (b*atan((b*c*x*(4*e^3 - 10*c^6*d^3 - 15*c^
2*d*e^2 + 20*c^4*d^2*e))/(4*b*e^3 - 10*b*c^6*d^3 - 15*b*c^2*d*e^2 + 20*b*c
^4*d^2*e))*(4*e^3 - 10*c^6*d^3 - 15*c^2*d*e^2 + 20*c^4*d^2*e))/(40*c^10)
```

3.1137. $\int x^3(d+ex^2)^3\,(a+b\arctan(cx))\,dx$

3.1138 $\int x^2(d+ex^2)^3 (a+b\arctan(cx))\,dx$

3.1138.1 Optimal result

Integrand size = 21, antiderivative size = 239

$$\begin{aligned}\int x^2(d+ex^2)^3 (a+b\arctan(cx))\,dx &= -\frac{b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\,x^2}{630c^7}-\frac{be(189c^4d^2-135c^2de+35e^2)\,x^4}{1260c^5}\\&\quad-\frac{b(27c^2d-7e)\,e^2x^6}{378c^3}-\frac{be^3x^8}{72c}+\frac{1}{3}d^3x^3(a+b\arctan(cx))\\&\quad+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))\\&\quad+\frac{b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\log(1+c^2x^2)}{630c^9}\end{aligned}$$

output
```
-1/630*b*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*x^2/c^7-1/1260*b
*e*(189*c^4*d^2-135*c^2*d*e+35*e^2)*x^4/c^5-1/378*b*(27*c^2*d-7*e)*e^2*x^6
/c^3-1/72*b*e^3*x^8/c+1/3*d^3*x^3*(a+b*arctan(c*x))+3/5*d^2*e*x^5*(a+b*arc
tan(c*x))+3/7*d*e^2*x^7*(a+b*arctan(c*x))+1/9*e^3*x^9*(a+b*arctan(c*x))+1/
630*b*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*ln(c^2*x^2+1)/c^9
```

3.1138.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.05

$$\int x^2 (d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx)) +\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx)) +\frac{1}{216}be^3\left(\frac{12x^2}{c^7}-\frac{6x^4}{c^5}+\frac{4x^6}{c^3}-\frac{3x^8}{c}-\frac{12\log(1+c^2x^2)}{c^9}\right) -\frac{1}{28}bde^2\left(\frac{6x^2}{c^5}-\frac{3x^4}{c^3}+\frac{2x^6}{c}-\frac{6\log(1+c^2x^2)}{c^7}\right) +\frac{3}{20}bd^2e\left(\frac{2x^2}{c^3}-\frac{x^4}{c}-\frac{2\log(1+c^2x^2)}{c^5}\right) -\frac{1}{6}bd^3\left(\frac{x^2}{c}-\frac{\log(1+c^2x^2)}{c^3}\right)$$

input
```
Integrate[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^3*(a + b*ArcTan[c*x]))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3
*d*e^2*x^7*(a + b*ArcTan[c*x]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 + (b*e
^3*((12*x^2)/c^7 - (6*x^4)/c^5 + (4*x^6)/c^3 - (3*x^8)/c - (12*Log[1 + c^2
*x^2])/c^9))/216 - (b*d*e^2*((6*x^2)/c^5 - (3*x^4)/c^3 + (2*x^6)/c - (6*Lo
g[1 + c^2*x^2])/c^7))/28 + (3*b*d^2*e*((2*x^2)/c^3 - x^4/c - (2*Log[1 + c^
2*x^2])/c^5))/20 - (b*d^3*(x^2/c - Log[1 + c^2*x^2]/c^3))/6
```

3.1138.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d+ex^2)^3 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^3\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{315\left(c^2x^2+1\right)}dx+\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{315}bc\int \frac{x^3\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{c^2x^2+1}dx+\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{630}bc\int \frac{x^2\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{c^2x^2+1}dx^2+\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{2123}$$

$$-\frac{1}{630}bc\int \left(\frac{35e^3x^6}{c^2}+\frac{5\left(27c^2d-7e\right)e^2x^4}{c^4}+\frac{e\left(189d^2c^4-135dec^2+35e^2\right)x^2}{c^6}+\frac{105d^3c^6-189d^2ec^4+135de^2c^2-}{c^8}\right.$$
$$\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))-$$
$$\frac{1}{630}bc\left(\frac{35e^3x^8}{4c^2}+\frac{5e^2x^6\left(27c^2d-7e\right)}{3c^4}+\frac{ex^4\left(189c^4d^2-135c^2de+35e^2\right)}{2c^6}-\frac{\left(105c^6d^3-189c^4d^2e+135c^2de^2-35e\right.}{c^{10}}\right.$$

input
```
Int[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^3*(a + b*ArcTan[c*x]))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3
*d*e^2*x^7*(a + b*ArcTan[c*x]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 - (b*c
*(((105*c^6*d^3 - 189*c^4*d^2*e + 135*c^2*d*e^2 - 35*e^3)*x^2)/c^8 + (e*(1
89*c^4*d^2 - 135*c^2*d*e + 35*e^2)*x^4)/(2*c^6) + (5*(27*c^2*d - 7*e)*e^2*
x^6)/(3*c^4) + (35*e^3*x^8)/(4*c^2) - ((105*c^6*d^3 - 189*c^4*d^2*e + 135*
c^2*d*e^2 - 35*e^3)*Log[1 + c^2*x^2])/c^10))/630
```

3.1138. $\int x^2(d+ex^2)^3\,(a+b\arctan(cx))\,dx$

3.1138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1138.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.14

method	result
parts	$a\left(\frac{1}{9}e^3x^9+\frac{3}{7}e^2d\,x^7+\frac{3}{5}e\,d^2x^5+\frac{1}{3}d^3x^3\right)+\frac{b\left(\frac{\arctan(cx)c^3e^3x^9}{9}+\frac{3\arctan(cx)c^3e^2d\,x^7}{7}+\frac{3\arctan(cx)c^3d^2e\,x^5}{5}\cdots\right.}{\cdots}$
derivativedivides	$\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^9x^3}{3}+\frac{3\arctan(cx)d^2c^9e\,x^5}{5}+\frac{3\arctan(cx)d\,c^9e^2x^7}{7}+\frac{\arctan(cx)\cdots}{9}\cdots\right.}{\cdots}$
default	$\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^9x^3}{3}+\frac{3\arctan(cx)d^2c^9e\,x^5}{5}+\frac{3\arctan(cx)d\,c^9e^2x^7}{7}+\frac{\arctan(cx)\cdots}{9}\cdots\right.}{\cdots}$
parallelrisch	$840x^9\arctan(cx)b\,c^9e^3+840a\,c^9e^3x^9+3240x^7\arctan(cx)b\,c^9d\,e^2-105b\,c^8e^3x^8+3240a\,c^9d\,e^2x^7+4536x^5\arctan(cx)b\,c^9\cdots$
risch	$\frac{ib\,d^3x^3\ln(-icx+1)}{6}+\frac{ib\,e^3x^9\ln(-icx+1)}{18}+\frac{3ibd\,e^2x^7\ln(-icx+1)}{14}+\frac{x^9e^3a}{9}-\frac{ib(35e^3x^9+135e^2d\,x^7+189e\,d^2x^5+\cdots}{630}$

input
```
int(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^3*x^9+3/7*e^2*d*x^7+3/5*e*d^2*x^5+1/3*d^3*x^3)+b/c^3*(1/9*arctan(
c*x)*c^3*e^3*x^9+3/7*arctan(c*x)*c^3*e^2*d*x^7+3/5*arctan(c*x)*c^3*d^2*e*x
^5+1/3*arctan(c*x)*d^3*c^3*x^3-1/315/c^6*(105/2*d^3*c^8*x^2+189/4*d^2*c^8*
e*x^4+45/2*d*c^8*e^2*x^6-189/2*d^2*c^6*e*x^2+35/8*e^3*c^8*x^8-135/4*d*c^6*
e^2*x^4-35/6*e^3*c^6*x^6+135/2*d*c^4*e^2*x^2+35/4*e^3*c^4*x^4-35/2*e^3*c^2
*x^2+1/2*(-105*c^6*d^3+189*c^4*d^2*e-135*c^2*d*e^2+35*e^3)*ln(c^2*x^2+1)))
```

3.1138.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.16

$$\int x^2\left(d+ex^2\right)^3\left(a+b\arctan(cx)\right)dx$$
$$=\frac{840\,ac^9e^3x^9+3240\,ac^9de^2x^7-105\,bc^8e^3x^8+4536\,ac^9d^2ex^5+2520\,ac^9d^3x^3-20\left(27\,bc^8de^2-7\,bc^6e^3\right)x^6\cdots}{\cdots}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/7560*(840*a*c^9*e^3*x^9 + 3240*a*c^9*d*e^2*x^7 - 105*b*c^8*e^3*x^8 + 453
6*a*c^9*d^2*e*x^5 + 2520*a*c^9*d^3*x^3 - 20*(27*b*c^8*d*e^2 - 7*b*c^6*e^3)
*x^6 - 6*(189*b*c^8*d^2*e - 135*b*c^6*d*e^2 + 35*b*c^4*e^3)*x^4 - 12*(105*
b*c^8*d^3 - 189*b*c^6*d^2*e + 135*b*c^4*d*e^2 - 35*b*c^2*e^3)*x^2 + 24*(35
*b*c^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 189*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3
*x^3)*arctan(c*x) + 12*(105*b*c^6*d^3 - 189*b*c^4*d^2*e + 135*b*c^2*d*e^2
- 35*b*e^3)*log(c^2*x^2 + 1))/c^9
```

3.1138.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.63

$$\int x^2(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$=\begin{cases}\frac{ad^3x^3}{3}+\frac{3ad^2ex^5}{5}+\frac{3ade^2x^7}{7}+\frac{ae^3x^9}{9}+\frac{bd^3x^3\operatorname{atan}(cx)}{3}+\frac{3bd^2ex^5\operatorname{atan}(cx)}{5}+\frac{3bde^2x^7\operatorname{atan}(cx)}{7}+\frac{be^3x^9\operatorname{atan}(cx)}{9}-\frac{bd^3x^2}{6c}-\\ a\left(\frac{d^3x^3}{3}+\frac{3d^2ex^5}{5}+\frac{3de^2x^7}{7}+\frac{e^3x^9}{9}\right)\end{cases}$$

input
```
integrate(x**2*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*
x**9/9 + b*d**3*x**3*atan(c*x)/3 + 3*b*d**2*e*x**5*atan(c*x)/5 + 3*b*d*e**
2*x**7*atan(c*x)/7 + b*e**3*x**9*atan(c*x)/9 - b*d**3*x**2/(6*c) - 3*b*d**
2*e*x**4/(20*c) - b*d*e**2*x**6/(14*c) - b*e**3*x**8/(72*c) + b*d**3*log(x
**2 + c**(-2))/(6*c**3) + 3*b*d**2*e*x**2/(10*c**3) + 3*b*d*e**2*x**4/(28*
c**3) + b*e**3*x**6/(54*c**3) - 3*b*d**2*e*log(x**2 + c**(-2))/(10*c**5) -
 3*b*d*e**2*x**2/(14*c**5) - b*e**3*x**4/(36*c**5) + 3*b*d*e**2*log(x**2 +
 c**(-2))/(14*c**7) + b*e**3*x**2/(18*c**7) - b*e**3*log(x**2 + c**(-2))/(
18*c**9), Ne(c, 0)), (a*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e**2*x**7/7 +
 e**3*x**9/9), True))
```

3.1138.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int x^2(d+ex^2)^3\,(a+b\arctan(cx))\,dx \\
&= \frac{1}{9}\,ae^3x^9 + \frac{3}{7}\,ade^2x^7 + \frac{3}{5}\,ad^2ex^5 + \frac{1}{3}\,ad^3x^3 \\
&\quad + \frac{1}{6}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^3 \\
&\quad + \frac{3}{20}\left(4\,x^5\arctan\left(cx\right) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bd^2e \\
&\quad + \frac{1}{28}\left(12\,x^7\arctan\left(cx\right) - c\left(\frac{2\,c^4x^6-3\,c^2x^4+6\,x^2}{c^6} - \frac{6\,\log\left(c^2x^2+1\right)}{c^8}\right)\right)bde^2 \\
&\quad + \frac{1}{216}\left(24\,x^9\arctan\left(cx\right) - c\left(\frac{3\,c^6x^8-4\,c^4x^6+6\,c^2x^4-12\,x^2}{c^8} + \frac{12\,\log\left(c^2x^2+1\right)}{c^{10}}\right)\right)be^3
\end{aligned}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/6*(2
*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3 + 3/20*(4*x^5
*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d^2*e
 + 1/28*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*l
og(c^2*x^2 + 1)/c^8))*b*d*e^2 + 1/216*(24*x^9*arctan(c*x) - c*((3*c^6*x^8
- 4*c^4*x^6 + 6*c^2*x^4 - 12*x^2)/c^8 + 12*log(c^2*x^2 + 1)/c^10))*b*e^3
```

3.1138.8 Giac [F]

$$\int x^2(d+ex^2)^3\,(a+b\arctan(cx))\,dx = \int (ex^2+d)^3(b\arctan\left(cx\right)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1138.9 Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.24

$$\int x^2 (d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{a\,d^3\,x^3}{3} + \frac{a\,e^3\,x^9}{9} + \frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)}{6\,c^3} - \frac{b\,e^3\,\ln\left(c^2\,x^2+1\right)}{18\,c^9} - \frac{b\,d^3\,x^2}{6\,c} - \frac{b\,e^3\,x^8}{72\,c} + \frac{b\,e^3\,x^6}{54\,c^3} - \frac{b\,e^3\,x^4}{36\,c^5} + \frac{b\,e^3\,x^2}{18\,c^7} + \frac{3\,a\,d^2\,e\,x^5}{5} + \frac{3\,a\,d\,e^2\,x^7}{7} + \frac{b\,d^3\,x^3\,\mathrm{atan}(c\,x)}{3} + \frac{b\,e^3\,x^9\,\mathrm{atan}(c\,x)}{9} + \frac{3\,b\,d^2\,e\,x^5\,\mathrm{atan}(c\,x)}{5} + \frac{3\,b\,d\,e^2\,x^7\,\mathrm{atan}(c\,x)}{7} - \frac{3\,b\,d^2\,e\,\ln\left(c^2\,x^2+1\right)}{10\,c^5} + \frac{3\,b\,d\,e^2\,\ln\left(c^2\,x^2+1\right)}{14\,c^7} - \frac{3\,b\,d^2\,e\,x^4}{20\,c} + \frac{3\,b\,d^2\,e\,x^2}{10\,c^3} - \frac{b\,d\,e^2\,x^6}{14\,c} + \frac{3\,b\,d\,e^2\,x^4}{28\,c^3} - \frac{3\,b\,d\,e^2\,x^2}{14\,c^5}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
(a*d^3*x^3)/3 + (a*e^3*x^9)/9 + (b*d^3*log(c^2*x^2 + 1))/(6*c^3) - (b*e^3*
log(c^2*x^2 + 1))/(18*c^9) - (b*d^3*x^2)/(6*c) - (b*e^3*x^8)/(72*c) + (b*e
^3*x^6)/(54*c^3) - (b*e^3*x^4)/(36*c^5) + (b*e^3*x^2)/(18*c^7) + (3*a*d^2*
e*x^5)/5 + (3*a*d*e^2*x^7)/7 + (b*d^3*x^3*atan(c*x))/3 + (b*e^3*x^9*atan(c
*x))/9 + (3*b*d^2*e*x^5*atan(c*x))/5 + (3*b*d*e^2*x^7*atan(c*x))/7 - (3*b*
d^2*e*log(c^2*x^2 + 1))/(10*c^5) + (3*b*d*e^2*log(c^2*x^2 + 1))/(14*c^7) -
 (3*b*d^2*e*x^4)/(20*c) + (3*b*d^2*e*x^2)/(10*c^3) - (b*d*e^2*x^6)/(14*c)
+ (3*b*d*e^2*x^4)/(28*c^3) - (3*b*d*e^2*x^2)/(14*c^5)
```

3.1139 $\int x(d+ex^2)^3 (a+b\arctan(cx))\,dx$

3.1139.1 Optimal result

Integrand size = 19, antiderivative size = 158

$$\int x(d+ex^2)^3 (a+b\arctan(cx))\,dx = -\frac{b(2c^2d-e)\left(2c^4d^2-2c^2de+e^2\right)x}{8c^7} - \frac{be(6c^4d^2-4c^2de+e^2)\,x^3}{24c^5} - \frac{b(4c^2d-e)\,e^2x^5}{40c^3} - \frac{be^3x^7}{56c} - \frac{b(c^2d-e)^4\arctan(cx)}{8c^8e} + \frac{(d+ex^2)^4\,(a+b\arctan(cx))}{8e}$$

output
```
-1/8*b*(2*c^2*d-e)*(2*c^4*d^2-2*c^2*d*e+e^2)*x/c^7-1/24*b*e*(6*c^4*d^2-4*c
^2*d*e+e^2)*x^3/c^5-1/40*b*(4*c^2*d-e)*e^2*x^5/c^3-1/56*b*e^3*x^7/c-1/8*b*
(c^2*d-e)^4*arctan(c*x)/c^8/e+1/8*(e*x^2+d)^4*(a+b*arctan(c*x))/e
```

3.1139.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.37

$$\int x(d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= \frac{cx(105be^3-35bc^2e^2(12d+ex^2)+7bc^4e(90d^2+20dex^2+3e^2x^4)+105ac^7x(4d^3+6d^2ex^2+4de^2x^4+e^3x}{}$$

input
```
Integrate[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(c*x*(105*b*e^3 - 35*b*c^2*e^2*(12*d + e*x^2) + 7*b*c^4*e*(90*d^2 + 20*d*e
*x^2 + 3*e^2*x^4) + 105*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x
^6) - 3*b*c^6*(140*d^3 + 70*d^2*e*x^2 + 28*d*e^2*x^4 + 5*e^3*x^6)) + 105*b
*(4*c^6*d^3 - 6*c^4*d^2*e + 4*c^2*d*e^2 - e^3 + c^8*(4*d^3*x^2 + 6*d^2*e*x
^4 + 4*d*e^2*x^6 + e^3*x^8))*ArcTan[c*x])/(840*c^8)
```

3.1139.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5509, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e}-\frac{bc\int \frac{\left(ex^2+d\right)^4}{c^2x^2+1}dx}{8e}$$

$$\Big\downarrow \text{300}$$

$$\frac{\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e}-\frac{bc\int \left(\frac{e^4x^6}{c^2}+\frac{(4c^2d-e)e^3x^4}{c^4}+\frac{e^2(6d^2c^4-4dec^2+e^2)x^2}{c^6}+\frac{(2c^2d-e)e(2d^2c^4-2dec^2+e^2)}{c^8}+\frac{d^4c^8-4d^3ec^6+6d^2e^2c^4-4de^3c^2+e^4}{c^8(c^2x^2+1)}\right)dx}{8e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e}-\frac{bc\left(\frac{\arctan(cx)(c^2d-e)^4}{c^9}+\frac{e^4x^7}{7c^2}+\frac{e^3x^5(4c^2d-e)}{5c^4}+\frac{ex(2c^2d-e)(2c^4d^2-2c^2de+e^2)}{c^8}+\frac{e^2x^3(6c^4d^2-4c^2de+e^2)}{3c^6}\right)}{8e}$$

input
```
Int[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^4*(a + b*ArcTan[c*x]))/(8*e) - (b*c*(((2*c^2*d - e)*e*(2*c^4*
d^2 - 2*c^2*d*e + e^2)*x)/c^8 + (e^2*(6*c^4*d^2 - 4*c^2*d*e + e^2)*x^3)/(3
*c^6) + ((4*c^2*d - e)*e^3*x^5)/(5*c^4) + (e^4*x^7)/(7*c^2) + ((c^2*d - e)
^4*ArcTan[c*x])/c^9))/(8*e)
```

3.1139.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1139.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.67

method	result
parts	$\frac{a(e\,x^2+d)^4}{8e}+\frac{b\left(\frac{\arctan(cx)c^2e^3x^8}{8}+\frac{\arctan(cx)c^2e^2x^6d}{2}+\frac{3\arctan(cx)c^2e\,x^4d^2}{4}+\frac{\arctan(cx)c^2x^2d^3}{2}+\frac{\arctan(cx)c^2d^4}{8e}-\frac{4c^7d^3e}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^4}{8c^6e}+\frac{b\left(\frac{\arctan(cx)c^8d^4}{8e}+\frac{\arctan(cx)c^8d^3x^2}{2}+\frac{3\arctan(cx)e\,c^8d^2x^4}{4}+\frac{\arctan(cx)e^2c^8d\,x^6}{2}+\frac{\arctan(cx)e^3c^8x^8}{8}-\frac{4c^7d^3}{}\right.}{}}{}$
default	$\frac{\frac{a\left(e\,c^2x^2+c^2d\right)^4}{8c^6e}+\frac{b\left(\frac{\arctan(cx)c^8d^4}{8e}+\frac{\arctan(cx)c^8d^3x^2}{2}+\frac{3\arctan(cx)e\,c^8d^2x^4}{4}+\frac{\arctan(cx)e^2c^8d\,x^6}{2}+\frac{\arctan(cx)e^3c^8x^8}{8}-\frac{4c^7d^3}{}\right.}{}}{}$
parallelrisch	$\frac{105x^8\arctan(cx)b\,c^8e^3+105x^8a\,c^8e^3+420x^6\arctan(cx)b\,c^8d\,e^2-15b\,c^7e^3x^7+420x^6a\,c^8d\,e^2+630x^4\arctan(cx)b\,c^8d^2e-}{}$
risch	Expression too large to display

input
```
int(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*(e*x^2+d)^4/e+b/c^2*(1/8*arctan(c*x)*c^2*e^3*x^8+1/2*arctan(c*x)*c^2
*e^2*x^6*d+3/4*arctan(c*x)*c^2*e*x^4*d^2+1/2*arctan(c*x)*c^2*x^2*d^3+1/8*a
rctan(c*x)*c^2/e*d^4-1/8/c^6/e*(4*c^7*d^3*e*x+2*c^7*d^2*e^2*x^3+4/5*c^7*d*
e^3*x^5+1/7*e^4*c^7*x^7-6*c^5*x*d^2*e^2-4/3*c^5*d*e^3*x^3-1/5*e^4*c^5*x^5+
4*c^3*x*d*e^3+1/3*e^4*c^3*x^3-c*x*e^4+(c^8*d^4-4*c^6*d^3*e+6*c^4*d^2*e^2-4
*c^2*d*e^3+e^4)*arctan(c*x)))
```

3.1139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.63

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= \frac{105\,ac^8e^3x^8 + 420\,ac^8de^2x^6 - 15\,bc^7e^3x^7 + 630\,ac^8d^2ex^4 + 420\,ac^8d^3x^2 - 21\,(4\,bc^7de^2 - bc^5e^3)x^5 - 35\,(6\,b}{}$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/840*(105*a*c^8*e^3*x^8 + 420*a*c^8*d*e^2*x^6 - 15*b*c^7*e^3*x^7 + 630*a*
c^8*d^2*e*x^4 + 420*a*c^8*d^3*x^2 - 21*(4*b*c^7*d*e^2 - b*c^5*e^3)*x^5 - 3
5*(6*b*c^7*d^2*e - 4*b*c^5*d*e^2 + b*c^3*e^3)*x^3 - 105*(4*b*c^7*d^3 - 6*b
*c^5*d^2*e + 4*b*c^3*d*e^2 - b*c*e^3)*x + 105*(b*c^8*e^3*x^8 + 4*b*c^8*d*e
^2*x^6 + 6*b*c^8*d^2*e*x^4 + 4*b*c^8*d^3*x^2 + 4*b*c^6*d^3 - 6*b*c^4*d^2*e
 + 4*b*c^2*d*e^2 - b*e^3)*arctan(c*x))/c^8
```

3.1139.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(144) = 288.

Time = 0.61 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.22

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2\operatorname{atan}(cx)}{2} + \frac{3bd^2ex^4\operatorname{atan}(cx)}{4} + \frac{bde^2x^6\operatorname{atan}(cx)}{2} + \frac{be^3x^8\operatorname{atan}(cx)}{8} - \frac{bd^3x}{2c} - \frac{bd}{} \\ a\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x*
*8/8 + b*d**3*x**2*atan(c*x)/2 + 3*b*d**2*e*x**4*atan(c*x)/4 + b*d*e**2*x*
*6*atan(c*x)/2 + b*e**3*x**8*atan(c*x)/8 - b*d**3*x/(2*c) - b*d**2*e*x**3/
(4*c) - b*d*e**2*x**5/(10*c) - b*e**3*x**7/(56*c) + b*d**3*atan(c*x)/(2*c*
*2) + 3*b*d**2*e*x/(4*c**3) + b*d*e**2*x**3/(6*c**3) + b*e**3*x**5/(40*c**
3) - 3*b*d**2*e*atan(c*x)/(4*c**4) - b*d*e**2*x/(2*c**5) - b*e**3*x**3/(24
*c**5) + b*d*e**2*atan(c*x)/(2*c**6) + b*e**3*x/(8*c**7) - b*e**3*atan(c*x
)/(8*c**8), Ne(c, 0)), (a*(d**3*x**2/2 + 3*d**2*e*x**4/4 + d*e**2*x**6/2 +
 e**3*x**8/8), True))
```

3.1139.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.47

$$\int x(d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= \frac{1}{8}ae^3x^8 + \frac{1}{2}ade^2x^6 + \frac{3}{4}ad^2ex^4 + \frac{1}{2}ad^3x^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^3$$
$$+ \frac{1}{4}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bd^2e$$
$$+ \frac{1}{30}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5 - 5c^2x^3 + 15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)bde^2$$
$$+ \frac{1}{840}\left(105x^8\arctan(cx) - c\left(\frac{15c^6x^7 - 21c^4x^5 + 35c^2x^3 - 105x}{c^8} + \frac{105\arctan(cx)}{c^9}\right)\right)be^3$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/2*(x
^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^3 + 1/4*(3*x^4*arctan(c*
x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2*e + 1/30*(15*x^6*a
rctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*
b*d*e^2 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^
2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*e^3
```

3.1139.8 Giac [F]

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx=\int (ex^2+d)^3(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1139.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.80

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$=x\left(\frac{\frac{\frac{b\,e^3}{8\,c^3}-\frac{b\,d\,e^2}{2\,c}}{c^2}+\frac{3\,b\,d^2\,e}{4\,c}}{c^2}-\frac{b\,d^3}{2\,c}\right)-x^6\left(\frac{a\,e^3}{6\,c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{6\,c^2}\right)$$
$$+x^4\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{4\,c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{4\,c^2}\right)+x^5\left(\frac{b\,e^3}{40\,c^3}-\frac{b\,d\,e^2}{10\,c}\right)$$
$$+\mathrm{atan}(c\,x)\left(\frac{b\,d^3\,x^2}{2}+\frac{3\,b\,d^2\,e\,x^4}{4}+\frac{b\,d\,e^2\,x^6}{2}+\frac{b\,e^3\,x^8}{8}\right)-x^3\left(\frac{\frac{b\,e^3}{8\,c^3}-\frac{b\,d\,e^2}{2\,c}}{3\,c^2}+\frac{b\,d^2\,e}{4\,c}\right)$$
$$-x^2\left(\frac{\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}}{2\,c^2}-\frac{a\,d^2\,(d\,c^2+3\,e)}{2\,c^2}\right)+\frac{a\,e^3\,x^8}{8}-\frac{b\,e^3\,x^7}{56\,c}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{b\,c\,x\,(e-2\,c^2\,d)\,(2\,c^4\,d^2-2\,c^2\,d\,e+e^2)}{-4\,b\,c^6\,d^3+6\,b\,c^4\,d^2\,e-4\,b\,c^2\,d\,e^2+b\,e^3}\right)\,(e-2\,c^2\,d)\,(2\,c^4\,d^2-2\,c^2\,d\,e+e^2)}{8\,c^8}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
x*((((b*e^3)/(8*c^3) - (b*d*e^2)/(2*c))/c^2 + (3*b*d^2*e)/(4*c))/c^2 - (b*
d^3)/(2*c)) - x^6*((a*e^3)/(6*c^2) - (a*e^2*(e + 3*c^2*d))/(6*c^2)) + x^4*
(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/(4*c^2) + (3*a*d*e*(e + c^2*d))
/(4*c^2)) + x^5*((b*e^3)/(40*c^3) - (b*d*e^2)/(10*c)) + atan(c*x)*((b*d^3*
x^2)/2 + (b*e^3*x^8)/8 + (3*b*d^2*e*x^4)/4 + (b*d*e^2*x^6)/2) - x^3*(((b*e
^3)/(8*c^3) - (b*d*e^2)/(2*c))/(3*c^2) + (b*d^2*e)/(4*c)) - x^2*((((a*e^3)
/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^2)/(2*c^2)
 - (a*d^2*(3*e + c^2*d))/(2*c^2)) + (a*e^3*x^8)/8 - (b*e^3*x^7)/(56*c) - (
b*atan((b*c*x*(e - 2*c^2*d)*(e^2 + 2*c^4*d^2 - 2*c^2*d*e))/(b*e^3 - 4*b*c^
6*d^3 - 4*b*c^2*d*e^2 + 6*b*c^4*d^2*e))*(e - 2*c^2*d)*(e^2 + 2*c^4*d^2 - 2
*c^2*d*e))/(8*c^8)
```

3.1139. $\int x(d+ex^2)^3\,(a+b\arctan(cx))\,dx$

3.1140 $\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$

3.1140.1 Optimal result

Integrand size = 18, antiderivative size = 188

$$\begin{aligned}\int (d+ex^2)^3 (a+b\arctan(cx))\,dx &= -\frac{be(35c^4d^2-21c^2de+5e^2)\,x^2}{70c^5}-\frac{b(21c^2d-5e)\,e^2x^4}{140c^3}-\frac{be^3x^6}{42c}\\ &\quad+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))\\ &\quad+\frac{1}{7}e^3x^7(a+b\arctan(cx))-\frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\log(1+c^2x^2)}{70c^7}\end{aligned}$$

output
```
-1/70*b*e*(35*c^4*d^2-21*c^2*d*e+5*e^2)*x^2/c^5-1/140*b*(21*c^2*d-5*e)*e^2
*x^4/c^3-1/42*b*e^3*x^6/c+d^3*x*(a+b*arctan(c*x))+d^2*e*x^3*(a+b*arctan(c*
x))+3/5*d*e^2*x^5*(a+b*arctan(c*x))+1/7*e^3*x^7*(a+b*arctan(c*x))-1/70*b*(
35*c^6*d^3-35*c^4*d^2*e+21*c^2*d*e^2-5*e^3)*ln(c^2*x^2+1)/c^7
```

3.1140.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.02

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{c^2x(12ac^5(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-bex(30e^2-3c^2e(42d+5ex^2)+c^4(210d^2+63dex^2+10}{}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*x*(12*a*c^5*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - b*e*
x*(30*e^2 - 3*c^2*e*(42*d + 5*e*x^2) + c^4*(210*d^2 + 63*d*e*x^2 + 10*e^2*
x^4))) + 12*b*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6)*Arc
Tan[c*x] - 6*b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Log[1 +
c^2*x^2])/(420*c^7)
```

3.1140.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5447, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{35(c^2x^2+1)}dx+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35}bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{c^2x^2+1}dx+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{70}bc\int \frac{5e^3x^6+21de^2x^4+35d^2ex^2+35d^3}{c^2x^2+1}dx^2+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{2389}$$

$$-\frac{1}{70}bc\int\left(\frac{5e^3x^4}{c^2}+\frac{(21c^2d-5e)\,e^2x^2}{c^4}+\frac{e(35d^2c^4-21dec^2+5e^2)}{c^6}+\frac{35d^3c^6-35d^2ec^4+21de^2c^2-5e^3}{c^6\,(c^2x^2+1)}\right)dx^2+$$
$$d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))-$$
$$\frac{1}{70}bc\left(\frac{5e^3x^6}{3c^2}+\frac{e^2x^4(21c^2d-5e)}{2c^4}+\frac{ex^2(35c^4d^2-21c^2de+5e^2)}{c^6}+\frac{(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\log(c^2x^2}{c^8}\right.$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
d^3*x*(a + b*ArcTan[c*x]) + d^2*e*x^3*(a + b*ArcTan[c*x]) + (3*d*e^2*x^5*(
a + b*ArcTan[c*x]))/5 + (e^3*x^7*(a + b*ArcTan[c*x]))/7 - (b*c*((e*(35*c^4
*d^2 - 21*c^2*d*e + 5*e^2)*x^2)/c^6 + ((21*c^2*d - 5*e)*e^2*x^4)/(2*c^4) +
 (5*e^3*x^6)/(3*c^2) + ((35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)
*Log[1 + c^2*x^2])/c^8))/70
```

3.1140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.1140.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

method	result
parts	$a\left(\frac{1}{7}e^3x^7+\frac{3}{5}x^5e^2d+e\,d^2x^3+x\,d^3\right)+\frac{b\Bigg(\frac{\arctan(cx)c\,e^3x^7}{7}+\frac{3\arctan(cx)c\,e^2d\,x^5}{5}+\arctan(cx)c\,d^2e\,x^3+\text{arctar}}{}$
derivativedivides	$\frac{\frac{a\left(c^7x\,d^3+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\Bigg(\arctan(cx)c^7x\,d^3+\arctan(cx)d^2c^7e\,x^3+\frac{3\arctan(cx)d\,c^7e^2x^5}{5}+\frac{\arctan(cx)e^3c^7x^7}{7}}{}}{c}$
default	$\frac{\frac{a\left(c^7x\,d^3+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\Bigg(\arctan(cx)c^7x\,d^3+\arctan(cx)d^2c^7e\,x^3+\frac{3\arctan(cx)d\,c^7e^2x^5}{5}+\frac{\arctan(cx)e^3c^7x^7}{7}}{}}{c}$
parallelrisch	$-\frac{-60x^7\arctan(cx)b\,c^7e^3-60a\,c^7e^3x^7-252x^5\arctan(cx)b\,c^7d\,e^2+10b\,c^6e^3x^6-252a\,c^7d\,e^2x^5-420x^3\arctan(cx)b\,c^7d^2e}{}$
risch	$\frac{ib\,d^3x\ln(-icx+1)}{2}+\frac{3ibd\,e^2x^5\ln(-icx+1)}{10}+\frac{ib\,e^3x^7\ln(-icx+1)}{14}+\frac{a\,e^3x^7}{7}+\frac{ib\,d^2e\,x^3\ln(-icx+1)}{2}+\frac{3ad\,e^2x^5}{5}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^3*x^7+3/5*x^5*e^2*d+e*d^2*x^3+x*d^3)+b/c*(1/7*arctan(c*x)*c*e^3*x
^7+3/5*arctan(c*x)*c*e^2*d*x^5+arctan(c*x)*c*d^2*e*x^3+arctan(c*x)*c*x*d^3
-1/35/c^6*(35/2*d^2*c^6*e*x^2+21/4*d*c^6*e^2*x^4+5/6*e^3*c^6*x^6-21/2*d*c^
4*e^2*x^2-5/4*e^3*c^4*x^4+5/2*e^3*c^2*x^2+1/2*(35*c^6*d^3-35*c^4*d^2*e+21*
c^2*d*e^2-5*e^3)*ln(c^2*x^2+1)))
```

3.1140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= \frac{60\,ac^7e^3x^7 + 252\,ac^7de^2x^5 - 10\,bc^6e^3x^6 + 420\,ac^7d^2ex^3 + 420\,ac^7d^3x - 3\,(21\,bc^6de^2 - 5\,bc^4e^3)x^4 - 6\,(35\,bc}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/420*(60*a*c^7*e^3*x^7 + 252*a*c^7*d*e^2*x^5 - 10*b*c^6*e^3*x^6 + 420*a*c
^7*d^2*e*x^3 + 420*a*c^7*d^3*x - 3*(21*b*c^6*d*e^2 - 5*b*c^4*e^3)*x^4 - 6*
(35*b*c^6*d^2*e - 21*b*c^4*d*e^2 + 5*b*c^2*e^3)*x^2 + 12*(5*b*c^7*e^3*x^7
+ 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*e*x^3 + 35*b*c^7*d^3*x)*arctan(c*x) -
6*(35*b*c^6*d^3 - 35*b*c^4*d^2*e + 21*b*c^2*d*e^2 - 5*b*e^3)*log(c^2*x^2 +
 1))/c^7
```

3.1140.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.63

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= \begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\,\mathrm{atan}\,(cx) + bd^2ex^3\,\mathrm{atan}\,(cx) + \frac{3bde^2x^5\,\mathrm{atan}\,(cx)}{5} + \frac{be^3x^7\,\mathrm{atan}\,(cx)}{7} - \frac{bd}{} \\ a\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 +
b*d**3*x*atan(c*x) + b*d**2*e*x**3*atan(c*x) + 3*b*d*e**2*x**5*atan(c*x)/5
 + b*e**3*x**7*atan(c*x)/7 - b*d**3*log(x**2 + c**(-2))/(2*c) - b*d**2*e*x
**2/(2*c) - 3*b*d*e**2*x**4/(20*c) - b*e**3*x**6/(42*c) + b*d**2*e*log(x**
2 + c**(-2))/(2*c**3) + 3*b*d*e**2*x**2/(10*c**3) + b*e**3*x**4/(28*c**3)
- 3*b*d*e**2*log(x**2 + c**(-2))/(10*c**5) - b*e**3*x**2/(14*c**5) + b*e**
3*log(x**2 + c**(-2))/(14*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d
*e**2*x**5/5 + e**3*x**7/7), True))
```

3.1140.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.18

$$
\begin{aligned}
&\int (d+ex^2)^3 (a+b\arctan(cx))\,dx \\
&= \frac{1}{7}ae^3x^7 + \frac{3}{5}ade^2x^5 + ad^2ex^3 + \frac{1}{2}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^2e \\
&\quad + \frac{3}{20}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4-2x^2}{c^4} + \frac{2\log(c^2x^2+1)}{c^6}\right)\right)bde^2 \\
&\quad + \frac{1}{84}\left(12x^7\arctan(cx) - c\left(\frac{2c^4x^6-3c^2x^4+6x^2}{c^6} - \frac{6\log(c^2x^2+1)}{c^8}\right)\right)be^3 \\
&\quad + ad^3x + \frac{(2cx\arctan(cx) - \log(c^2x^2+1))bd^3}{2c}
\end{aligned}
$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/2*(2*x^3*arctan(c*x) - c
*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2*e + 3/20*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e^2 + 1/84*(12*x^7*ar
ctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^
8))*b*e^3 + a*d^3*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3/c
```

3.1140.8 Giac [F]

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx = \int (ex^2+d)^3(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1140.9 Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.27

$$\int \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx = \frac{a\,e^3\,x^7}{7}+a\,d^3\,x-\frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)}{2\,c}+\frac{b\,e^3\,\ln\left(c^2\,x^2+1\right)}{14\,c^7} -\frac{b\,e^3\,x^6}{42\,c}+\frac{b\,e^3\,x^4}{28\,c^3}-\frac{b\,e^3\,x^2}{14\,c^5}+b\,d^3\,x\,\mathrm{atan}(c\,x) +a\,d^2\,e\,x^3+\frac{3\,a\,d\,e^2\,x^5}{5}+\frac{b\,e^3\,x^7\,\mathrm{atan}(c\,x)}{7} +b\,d^2\,e\,x^3\,\mathrm{atan}(c\,x)+\frac{3\,b\,d\,e^2\,x^5\,\mathrm{atan}(c\,x)}{5} +\frac{b\,d^2\,e\,\ln\left(c^2\,x^2+1\right)}{2\,c^3}-\frac{3\,b\,d\,e^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^5} -\frac{b\,d^2\,e\,x^2}{2\,c}-\frac{3\,b\,d\,e^2\,x^4}{20\,c}+\frac{3\,b\,d\,e^2\,x^2}{10\,c^3}$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
(a*e^3*x^7)/7 + a*d^3*x - (b*d^3*log(c^2*x^2 + 1))/(2*c) + (b*e^3*log(c^2*
x^2 + 1))/(14*c^7) - (b*e^3*x^6)/(42*c) + (b*e^3*x^4)/(28*c^3) - (b*e^3*x^
2)/(14*c^5) + b*d^3*x*atan(c*x) + a*d^2*e*x^3 + (3*a*d*e^2*x^5)/5 + (b*e^3
*x^7*atan(c*x))/7 + b*d^2*e*x^3*atan(c*x) + (3*b*d*e^2*x^5*atan(c*x))/5 +
(b*d^2*e*log(c^2*x^2 + 1))/(2*c^3) - (3*b*d*e^2*log(c^2*x^2 + 1))/(10*c^5)
 - (b*d^2*e*x^2)/(2*c) - (3*b*d*e^2*x^4)/(20*c) + (3*b*d*e^2*x^2)/(10*c^3)
```

3.1141 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x}\,dx$

3.1141.1 Optimal result

Integrand size = 21, antiderivative size = 228

$$
\begin{aligned}
\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x}\,dx = & -\frac{3bd^2ex}{2c}+\frac{3bde^2x}{4c^3}-\frac{be^3x}{6c^5}-\frac{bde^2x^3}{4c} \\
& +\frac{be^3x^3}{18c^3}-\frac{be^3x^5}{30c}+\frac{3bd^2e\arctan(cx)}{2c^2} \\
& -\frac{3bde^2\arctan(cx)}{4c^4}+\frac{be^3\arctan(cx)}{6c^6} \\
& +\frac{3}{2}d^2ex^2(a+b\arctan(cx))+\frac{3}{4}de^2x^4(a+b\arctan(cx)) \\
& +\frac{1}{6}e^3x^6(a+b\arctan(cx))+ad^3\log(x) \\
& +\frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)
\end{aligned}
$$

output

```
-3/2*b*d^2*e*x/c+3/4*b*d*e^2*x/c^3-1/6*b*e^3*x/c^5-1/4*b*d*e^2*x^3/c+1/18*
b*e^3*x^3/c^3-1/30*b*e^3*x^5/c+3/2*b*d^2*e*arctan(c*x)/c^2-3/4*b*d*e^2*arc
tan(c*x)/c^4+1/6*b*e^3*arctan(c*x)/c^6+3/2*d^2*e*x^2*(a+b*arctan(c*x))+3/4
*d*e^2*x^4*(a+b*arctan(c*x))+1/6*e^3*x^6*(a+b*arctan(c*x))+a*d^3*ln(x)+1/2
*I*b*d^3*polylog(2,-I*c*x)-1/2*I*b*d^3*polylog(2,I*c*x)
```

3.1141.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x} dx = -\frac{be^3(15cx-5c^3x^3+3c^5x^5-15\arctan(cx))}{90c^6} - \frac{3bd^2e(cx-\arctan(cx))}{2c^2} - \frac{bde^2(-3cx+c^3x^3+3\arctan(cx))}{4c^4} + \frac{3}{2}d^2ex^2(a+b\arctan(cx)) + \frac{3}{4}de^2x^4(a+b\arctan(cx)) + \frac{1}{6}e^3x^6(a+b\arctan(cx)) + ad^3\log(x) + \frac{1}{2}ibd^3 \operatorname{PolyLog}(2,-icx) - \frac{1}{2}ibd^3 \operatorname{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/90*(b*e^3*(15*c*x - 5*c^3*x^3 + 3*c^5*x^5 - 15*ArcTan[c*x]))/c^6 - (3*b
*d^2*e*(c*x - ArcTan[c*x]))/(2*c^2) - (b*d*e^2*(-3*c*x + c^3*x^3 + 3*ArcTa
n[c*x]))/(4*c^4) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a +
 b*ArcTan[c*x]))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2
)*b*d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

3.1141.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x} dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x} + 3d^2ex(a+b\arctan(cx)) + 3de^2x^3(a+b\arctan(cx)) + e^3x^5(a+b\arctan(cx))\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}d^2ex^2(a+b\arctan(cx))+\frac{3}{4}de^2x^4(a+b\arctan(cx))+\frac{1}{6}e^3x^6(a+b\arctan(cx))+ad^3\log(x)+\frac{be^3\arctan(cx)}{6c^6}-\frac{3bde^2\arctan(cx)}{4c^4}+\frac{3bd^2e\arctan(cx)}{2c^2}-\frac{be^3x}{6c^5}+\frac{3bde^2x}{4c^3}+\frac{be^3x^3}{18c^3}+\frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)-\frac{3bd^2ex}{2c}-\frac{bde^2x^3}{4c}-\frac{be^3x^5}{30c}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(-3*b*d^2*e*x)/(2*c) + (3*b*d*e^2*x)/(4*c^3) - (b*e^3*x)/(6*c^5) - (b*d*e^
2*x^3)/(4*c) + (b*e^3*x^3)/(18*c^3) - (b*e^3*x^5)/(30*c) + (3*b*d^2*e*ArcT
an[c*x])/(2*c^2) - (3*b*d*e^2*ArcTan[c*x])/(4*c^4) + (b*e^3*ArcTan[c*x])/(
6*c^6) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcTan[
c*x]))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2)*b*d^3*Po
lyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

3.1141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1141.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.13

method	result
parts	$a\left(\frac{e^3x^6}{6}+\frac{3x^4e^2d}{4}+\frac{3e\,d^2x^2}{2}+d^3\ln\left(x\right)\right)+b\left(\frac{\arctan(cx)e^3x^6}{6}+\frac{3\arctan(cx)e^2d\,x^4}{4}+\frac{3\arctan(cx)d^2e\,x}{2}\right.$
derivativedivides	$\frac{a\left(\frac{3d^2c^6e\,x^2}{2}+\frac{3d\,c^6e^2x^4}{4}+\frac{e^3c^6x^6}{6}+c^6d^3\ln(cx)\right)}{c^6}+\frac{b\left(\frac{3\arctan(cx)d^2c^6e\,x^2}{2}+\frac{3\arctan(cx)d\,c^6e^2x^4}{4}+\frac{\arctan(cx)e^3c^6x^6}{6}+\mathrm{arc}\right.}{}$
default	$\frac{a\left(\frac{3d^2c^6e\,x^2}{2}+\frac{3d\,c^6e^2x^4}{4}+\frac{e^3c^6x^6}{6}+c^6d^3\ln(cx)\right)}{c^6}+\frac{b\left(\frac{3\arctan(cx)d^2c^6e\,x^2}{2}+\frac{3\arctan(cx)d\,c^6e^2x^4}{4}+\frac{\arctan(cx)e^3c^6x^6}{6}+\mathrm{arc}\right.}{}$
risch	$\frac{3ad\,e^2x^4}{4}+\frac{3a\,d^2e\,x^2}{2}-\frac{3ibe\,d^2\ln(icx+1)x^2}{4}-\frac{3ib\,e^2d\ln(icx+1)x^4}{8}+\frac{3ib\,d^2e\ln(-icx+1)x^2}{4}+\frac{3ib\,e^2d\ln(-icx+}{8}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e^3*x^6+3/4*x^4*e^2*d+3/2*e*d^2*x^2+d^3*ln(x))+b*(1/6*arctan(c*x)*e
^3*x^6+3/4*arctan(c*x)*e^2*d*x^4+3/2*arctan(c*x)*d^2*e*x^2+arctan(c*x)*d^3
*ln(c*x)-1/12/c^6*(e*(18*c^5*x*d^2+3*d*c^5*e*x^3+2/5*e^2*c^5*x^5-9*c^3*d*e
*x-2/3*e^2*c^3*x^3+2*c*x*e^2+(-18*c^4*d^2+9*c^2*d*e-2*e^2)*arctan(c*x))-6*
I*c^6*d^3*ln(c*x)*ln(1+I*c*x)+6*I*c^6*d^3*ln(c*x)*ln(1-I*c*x)-6*I*c^6*d^3*
dilog(1+I*c*x)+6*I*c^6*d^3*dilog(1-I*c*x)))
```

3.1141.5 Fricas [F]

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x}\,dx=\int \frac{(ex^2+d)^3(b\arctan\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x, x)
```

3.1141.6 Sympy [F]

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^3}{x}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x, x)
```

3.1141.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x}\,dx=\frac{1}{6}\,ae^3x^6+\frac{3}{4}\,ade^2x^4+\frac{3}{2}\,ad^2ex^2+ad^3\log(x) - \frac{6\,bc^5e^3x^5+45\,\pi bc^6d^3\log(c^2x^2+1)-180\,bc^6d^3\arctan(cx)\log(cx)+90i\,bc^6d^3\mathrm{Li}_2(i\,cx+1)-90i\,bc^6d^3}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) - 1/180*(
6*b*c^5*e^3*x^5 + 45*pi*b*c^6*d^3*log(c^2*x^2 + 1) - 180*b*c^6*d^3*arctan(
c*x)*log(c*x) + 90*I*b*c^6*d^3*dilog(I*c*x + 1) - 90*I*b*c^6*d^3*dilog(-I*
c*x + 1) + 5*(9*b*c^5*d*e^2 - 2*b*c^3*e^3)*x^3 + 15*(18*b*c^5*d^2*e - 9*b*
c^3*d*e^2 + 2*b*c*e^3)*x - 15*(2*b*c^6*e^3*x^6 + 9*b*c^6*d*e^2*x^4 + 18*b*
c^6*d^2*e*x^2 + 18*b*c^4*d^2*e - 9*b*c^2*d*e^2 + 2*b*e^3)*arctan(c*x))/c^6
```

3.1141.8 Giac [F]

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x}\,dx=\int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.1141.9 Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx$$
$$= \begin{cases} \frac{a e^3 x^6}{6} + a d^3 \ln(x) + \frac{3 a \ldots \\ \frac{a e^3 x^6}{6} + a d^3 \ln(x) - \frac{b e^3 \left(\frac{x}{c^4} - \frac{\operatorname{atan}(c x)}{c^5} + \frac{x^5}{5} - \frac{x^3}{3 c^2}\right)}{6 c} - 3 b d^2 e \left(\frac{x}{2 c} - \operatorname{atan}(c x) \left(\frac{1}{2 c^2} + \frac{x^2}{2}\right)\right) + \frac{3 a d^2 e x^2}{2} + \frac{3 a d e^2 \ldots}{4} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x,x)
```

output
```
piecewise(c == 0, (a*e^3*x^6)/6 + a*d^3*log(x) + (3*a*d^2*e*x^2)/2 + (3*a*
d*e^2*x^4)/4, c ~= 0, (a*e^3*x^6)/6 + a*d^3*log(x) - (b*d^3*dilog(- c*x*1i
 + 1)*1i)/2 + (b*d^3*dilog(c*x*1i + 1)*1i)/2 - (b*e^3*(x/c^4 - atan(c*x)/c
^5 + x^5/5 - x^3/(3*c^2)))/(6*c) - 3*b*d^2*e*(x/(2*c) - atan(c*x)*(1/(2*c^
2) + x^2/2)) + (3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 - 3*b*d*e^2*((3*atan(
c*x) - 3*c*x + c^3*x^3)/(12*c^4) - (x^4*atan(c*x))/4) + (b*e^3*x^6*atan(c*
x))/6)
```

3.1142 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^2}\,dx$

3.1142.1 Optimal result

Integrand size = 21, antiderivative size = 160

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^2}\,dx = & -\frac{b(5c^2d-e)\,e^2x^2}{10c^3}-\frac{be^3x^4}{20c}-\frac{d^3(a+b\arctan(cx))}{x} \\ & +3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx)) \\ & +\frac{1}{5}e^3x^5(a+b\arctan(cx))+bcd^3\log(x) \\ & -\frac{b(5c^6d^3+15c^4d^2e-5c^2de^2+e^3)\log(1+c^2x^2)}{10c^5}\end{aligned}$$

output
```
-1/10*b*(5*c^2*d-e)*e^2*x^2/c^3-1/20*b*e^3*x^4/c-d^3*(a+b*arctan(c*x))/x+3
*d^2*e*x*(a+b*arctan(c*x))+d*e^2*x^3*(a+b*arctan(c*x))+1/5*e^3*x^5*(a+b*ar
ctan(c*x))+b*c*d^3*ln(x)-1/10*b*(5*c^6*d^3+15*c^4*d^2*e-5*c^2*d*e^2+e^3)*l
n(c^2*x^2+1)/c^5
```

3.1142.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx = \frac{1}{20}\left(-\frac{20ad^3}{x}+60ad^2ex+\frac{2be^2(-5c^2d+e)\,x^2}{c^3}+20ade^2x^3-\frac{be^3x^4}{c}+4ae^3x^5+\frac{4b(-5d^3+15d^2ex^2+5de^2x^4+e^3x^6)\arctan(cx)}{x}+20bcd^3\log(x)-\frac{2b(5c^6d^3+15c^4d^2e-5c^2de^2+e^3)\log\left(1+c^2x^2\right)}{c^5}\right)$$

input

```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output

```
((-20*a*d^3)/x + 60*a*d^2*e*x + (2*b*e^2*(-5*c^2*d + e)*x^2)/c^3 + 20*a*d*
e^2*x^3 - (b*e^3*x^4)/c + 4*a*e^3*x^5 + (4*b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*
e^2*x^4 + e^3*x^6)*ArcTan[c*x])/x + 20*b*c*d^3*Log[x] - (2*b*(5*c^6*d^3 +
15*c^4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^5)/20
```

3.1142.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{5x\,(c^2x^2+1)}dx-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}bc\int\frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x\left(c^2x^2+1\right)}dx-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 2331$$

$$\frac{1}{10}bc\int\frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x^2\left(c^2x^2+1\right)}dx^2-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 2123$$

$$\frac{1}{10}bc\int\left(\frac{5d^3}{x^2}-\frac{\left(5c^2d-e\right)e^2}{c^4}-\frac{e^3x^2}{c^2}+\frac{-5d^3c^6-15d^2ec^4+5de^2c^2-e^3}{c^4\left(c^2x^2+1\right)}\right)dx^2-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 2009$$

$$-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))+\frac{1}{10}bc\left(-\frac{e^3x^4}{2c^2}-\frac{e^2x^2\left(5c^2d-e\right)}{c^4}-\frac{\left(5c^6d^3+15c^4d^2e-5c^2de^2+e^3\right)\log\left(c^2x^2+1\right)}{c^6}+5d^3\log\left(x^2\right)\right)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d^3*(a + b*ArcTan[c*x]))/x) + 3*d^2*e*x*(a + b*ArcTan[c*x]) + d*e^2*x^3
*(a + b*ArcTan[c*x]) + (e^3*x^5*(a + b*ArcTan[c*x]))/5 + (b*c*(-(((5*c^2*d
 - e)*e^2*x^2)/c^4) - (e^3*x^4)/(2*c^2) + 5*d^3*Log[x^2] - ((5*c^6*d^3 + 1
5*c^4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^6))/10
```

3.1142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1142.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.21

method	result
parts	$a\left(\frac{e^3x^5}{5}+d\,e^2x^3+3d^2ex-\frac{d^3}{x}\right)+bc\left(\frac{\arctan(cx)e^3x^5}{5c}+\frac{\arctan(cx)x^3d\,e^2}{c}+\frac{3\arctan(cx)x\,d^2e}{c}-\text{arc}\right.$
derivativedivides	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^5d^2ex+\arctan(cx)c^5d\,e^2x^3+\frac{\arctan(cx)e^3c^5x^5}{5}-\frac{\arctan(c}{x}\right.}{}\right.$
default	$c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^5d^2ex+\arctan(cx)c^5d\,e^2x^3+\frac{\arctan(cx)e^3c^5x^5}{5}-\frac{\arctan(c}{x}\right.}{}\right.$
parallelrisch	$\frac{4x^6\arctan(cx)b\,c^5e^3+4a\,c^5e^3x^6+20x^4\arctan(cx)b\,c^5d\,e^2-b\,c^4e^3x^5+20a\,c^5d\,e^2x^4+20b\,c^6d^3\ln(x)x-10\ln(c^2x^2+1)b\,c^6}{}$
risch	$\frac{ib(-e^3x^6-5x^4e^2d-15e\,d^2x^2+5d^3)\ln(icx+1)}{10x}+\frac{2ib\,c^5e^3x^6\ln(-icx+1)+30ib\,c^5d^2e\,x^2\ln(-icx+1)+4a\,c^5e^3x^6-10ib\,c}{}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^3*x^5+d*e^2*x^3+3*d^2*e*x-d^3/x)+b*c*(1/5*arctan(c*x)/c*e^3*x^5+a
rctan(c*x)/c*x^3*d*e^2+3*arctan(c*x)/c*x*d^2*e-arctan(c*x)*d^3/c/x-1/5/c^6
*(1/4*e^3*c^4*x^4+5/2*d*c^4*e^2*x^2-1/2*e^3*c^2*x^2-5*c^6*d^3*ln(c*x)+1/2*
(5*c^6*d^3+15*c^4*d^2*e-5*c^2*d*e^2+e^3)*ln(c^2*x^2+1)))
```

3.1142.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{4\,ac^5e^3x^6 + 20\,ac^5de^2x^4 - bc^4e^3x^5 + 20\,bc^6d^3x\log(x) + 60\,ac^5d^2ex^2 - 20\,ac^5d^3 - 2\,(5\,bc^4de^2 - bc^2e^3)x^3 -$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/20*(4*a*c^5*e^3*x^6 + 20*a*c^5*d*e^2*x^4 - b*c^4*e^3*x^5 + 20*b*c^6*d^3*
x*log(x) + 60*a*c^5*d^2*e*x^2 - 20*a*c^5*d^3 - 2*(5*b*c^4*d*e^2 - b*c^2*e^
3)*x^3 - 2*(5*b*c^6*d^3 + 15*b*c^4*d^2*e - 5*b*c^2*d*e^2 + b*e^3)*x*log(c^
2*x^2 + 1) + 4*(b*c^5*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c^5*d^2*e*x^2 - 5
*b*c^5*d^3)*arctan(c*x))/(c^5*x)
```

3.1142.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \begin{cases} -\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{ae^3x^5}{5} + bcd^3\log(x) - \frac{bcd^3\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^3\operatorname{atan}(cx)}{x} + 3bd^2ex\operatorname{atan}(cx) + bde^2x \\ a\left(-\frac{d^3}{x} + 3d^2ex + de^2x^3 + \frac{e^3x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a*d**3/x + 3*a*d**2*e*x + a*d*e**2*x**3 + a*e**3*x**5/5 + b*c*
d**3*log(x) - b*c*d**3*log(x**2 + c**(-2))/2 - b*d**3*atan(c*x)/x + 3*b*d*
*2*e*x*atan(c*x) + b*d*e**2*x**3*atan(c*x) + b*e**3*x**5*atan(c*x)/5 - 3*b
*d**2*e*log(x**2 + c**(-2))/(2*c) - b*d*e**2*x**2/(2*c) - b*e**3*x**4/(20*
c) + b*d*e**2*log(x**2 + c**(-2))/(2*c**3) + b*e**3*x**2/(10*c**3) - b*e**
3*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*(-d**3/x + 3*d**2*e*x + d*e
**2*x**3 + e**3*x**5/5), True))
```

3.1142.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.23

$$
\begin{aligned}
\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx
&= \frac{1}{5}\,ae^3x^5 + ade^2x^3 - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\,\arctan\left(cx\right)}{x}\right)bd^3 \\
&\quad + \frac{1}{2}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bde^2 \\
&\quad + \frac{1}{20}\left(4\,x^5\arctan\left(cx\right) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)be^3 \\
&\quad + 3\,ad^2ex + \frac{3\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bd^2e}{2\,c} - \frac{ad^3}{x}
\end{aligned}
$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/5*a*e^3*x^5 + a*d*e^2*x^3 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arc
tan(c*x)/x)*b*d^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*b*d*e^2 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*lo
g(c^2*x^2 + 1)/c^6))*b*e^3 + 3*a*d^2*e*x + 3/2*(2*c*x*arctan(c*x) - log(c^
2*x^2 + 1))*b*d^2*e/c - a*d^3/x
```

3.1142.8 Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1142.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.48

$$\begin{aligned}
&\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx \\
&= x\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}\right) \\
&\quad -x^3\left(\frac{a\,e^3}{3\,c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{3\,c^2}\right)+x^2\left(\frac{b\,e^3}{10\,c^3}-\frac{b\,d\,e^2}{2\,c}\right)-\frac{a\,d^3}{x} \\
&\quad +\frac{a\,e^3\,x^5}{5}-\frac{\ln\left(c^2\,x^2+1\right)\,\left(5\,b\,c^6\,d^3+15\,b\,c^4\,d^2\,e-5\,b\,c^2\,d\,e^2+b\,e^3\right)}{10\,c^5} \\
&\quad +\frac{\mathrm{atan}(c\,x)\,\left(-b\,d^3+3\,b\,d^2\,e\,x^2+b\,d\,e^2\,x^4+\frac{b\,e^3\,x^6}{5}\right)}{x}-\frac{b\,e^3\,x^4}{20\,c}+b\,c\,d^3\,\ln\left(x\right)
\end{aligned}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^2,x)
```

output
```
x*(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c
^2) - x^3*((a*e^3)/(3*c^2) - (a*e^2*(e + 3*c^2*d))/(3*c^2)) + x^2*((b*e^3)
/(10*c^3) - (b*d*e^2)/(2*c)) - (a*d^3)/x + (a*e^3*x^5)/5 - (log(c^2*x^2 +
1)*(b*e^3 + 5*b*c^6*d^3 - 5*b*c^2*d*e^2 + 15*b*c^4*d^2*e))/(10*c^5) + (ata
n(c*x)*((b*e^3*x^6)/5 - b*d^3 + 3*b*d^2*e*x^2 + b*d*e^2*x^4))/x - (b*e^3*x
^4)/(20*c) + b*c*d^3*log(x)
```

3.1143 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^3}\,dx$

3.1143.1 Optimal result

Integrand size = 21, antiderivative size = 200

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^3}\,dx = &-\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}+\frac{be^3x}{4c^3}-\frac{be^3x^3}{12c}-\frac{1}{2}bc^2d^3\arctan(cx)\\&+\frac{3bde^2\arctan(cx)}{2c^2}-\frac{be^3\arctan(cx)}{4c^4}\\&-\frac{d^3(a+b\arctan(cx))}{2x^2}+\frac{3}{2}de^2x^2(a+b\arctan(cx))\\&+\frac{1}{4}e^3x^4(a+b\arctan(cx))+3ad^2e\log(x)\\&+\frac{3}{2}ibd^2e\,\text{PolyLog}(2,-icx)-\frac{3}{2}ibd^2e\,\text{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/2*b*c*d^3/x-3/2*b*d*e^2*x/c+1/4*b*e^3*x/c^3-1/12*b*e^3*x^3/c-1/2*b*c^2*
d^3*arctan(c*x)+3/2*b*d*e^2*arctan(c*x)/c^2-1/4*b*e^3*arctan(c*x)/c^4-1/2*
d^3*(a+b*arctan(c*x))/x^2+3/2*d*e^2*x^2*(a+b*arctan(c*x))+1/4*e^3*x^4*(a+b
*arctan(c*x))+3*a*d^2*e*ln(x)+3/2*I*b*d^2*e*polylog(2,-I*c*x)-3/2*I*b*d^2*
e*polylog(2,I*c*x)
```

3.1143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx = \frac{1}{12}\left(-\frac{18bde^2(cx-\arctan(cx))}{c^2} - \frac{be^3(-3cx+c^3x^3+3\arctan(cx))}{c^4} - \frac{6d^3(a+b\arctan(cx))}{x^2} + 18de^2x^2(a+b\arctan(cx)) + 3e^3x^4(a+b\arctan(cx)) - \frac{6bcd^3 \,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 36ad^2e\log(x) + 18ibd^2e\,\text{PolyLog}(2,-icx) - 18ibd^2e\,\text{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
((-18*b*d*e^2*(c*x - ArcTan[c*x]))/c^2 - (b*e^3*(-3*c*x + c^3*x^3 + 3*ArcT
an[c*x]))/c^4 - (6*d^3*(a + b*ArcTan[c*x]))/x^2 + 18*d*e^2*x^2*(a + b*ArcT
an[c*x]) + 3*e^3*x^4*(a + b*ArcTan[c*x]) - (6*b*c*d^3*Hypergeometric2F1[-1
/2, 1, 1/2, -(c^2*x^2)])/x + 36*a*d^2*e*Log[x] + (18*I)*b*d^2*e*PolyLog[2,
 (-I)*c*x] - (18*I)*b*d^2*e*PolyLog[2, I*c*x])/12
```

3.1143.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx$$
$$\Big\downarrow \; 5515$$

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x^3}+\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+e^3x^3(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{2x^2}+\frac{3}{2}de^2x^2(a+b\arctan(cx))+\frac{1}{4}e^3x^4(a+b\arctan(cx))+3ad^2e\log(x)-\frac{be^3\arctan(cx)}{4c^4}-\frac{1}{2}bc^2d^3\arctan(cx)+\frac{3bde^2\arctan(cx)}{2c^2}+\frac{be^3x}{4c^3}-\frac{bcd^3}{2x}+\frac{3}{2}ibd^2e\,\text{PolyLog}(2,-icx)-\frac{3}{2}ibd^2e\,\text{PolyLog}(2,icx)-\frac{3bde^2x}{2c}-\frac{be^3x^3}{12c}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^3)/x - (3*b*d*e^2*x)/(2*c) + (b*e^3*x)/(4*c^3) - (b*e^3*x^3)/(
12*c) - (b*c^2*d^3*ArcTan[c*x])/2 + (3*b*d*e^2*ArcTan[c*x])/(2*c^2) - (b*e
^3*ArcTan[c*x])/(4*c^4) - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) + (3*d*e^2*x^2
*(a + b*ArcTan[c*x]))/2 + (e^3*x^4*(a + b*ArcTan[c*x]))/4 + 3*a*d^2*e*Log[
x] + ((3*I)/2)*b*d^2*e*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d^2*e*PolyLog[2,
 I*c*x]
```

3.1143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1143.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.20

method	result
parts	$a\left(\frac{e^3x^4}{4}+\frac{3d\,e^2x^2}{2}+3e\,d^2\ln\left(x\right)-\frac{d^3}{2x^2}\right)+b\,c^2\left(\frac{\arctan(cx)e^3x^4}{4c^2}+\frac{3\arctan(cx)e^2d\,x^2}{2c^2}+\frac{3\arctan(cx)d}{c^2}\right.$
derivativedivides	$c^2\left(\frac{a\left(\frac{3d\,c^4e^2x^2}{2}+\frac{e^3c^4x^4}{4}+3c^4d^2e\ln(cx)-\frac{c^4d^3}{2x^2}\right)}{c^6}+\frac{b\left(\frac{3\arctan(cx)d\,c^4e^2x^2}{2}+\frac{\arctan(cx)e^3c^4x^4}{4}+3\arctan(cx)c^4d^2e\ln(\right.}{}\right.$
default	$c^2\left(\frac{a\left(\frac{3d\,c^4e^2x^2}{2}+\frac{e^3c^4x^4}{4}+3c^4d^2e\ln(cx)-\frac{c^4d^3}{2x^2}\right)}{c^6}+\frac{b\left(\frac{3\arctan(cx)d\,c^4e^2x^2}{2}+\frac{\arctan(cx)e^3c^4x^4}{4}+3\arctan(cx)c^4d^2e\ln(\right.}{}\right.$
risch	$\frac{b\,e^3x}{4c^3}-\frac{b\,e^3x^3}{12c}-\frac{b\,e^3\arctan(cx)}{4c^4}-\frac{3bd\,e^2x}{2c}+\frac{3bd\,e^2\arctan(cx)}{2c^2}-\frac{bc\,d^3}{2x}-\frac{b\,c^2d^3\arctan(cx)}{2}+\frac{3a\,e^2d}{2c^2}+3a$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^3*x^4+3/2*d*e^2*x^2+3*e*d^2*ln(x)-1/2*d^3/x^2)+b*c^2*(1/4*arctan(
c*x)/c^2*e^3*x^4+3/2*arctan(c*x)/c^2*e^2*d*x^2+3*arctan(c*x)/c^2*d^2*e*ln(
c*x)-1/2*arctan(c*x)*d^3/c^2/x^2-1/4/c^6*(1/3*e^3*c^3*x^3+6*c^3*x*d*e^2-c*
x*e^3+2*c^5*d^3/x+(2*c^6*d^3-6*c^2*d*e^2+e^3)*arctan(c*x)+12*c^4*d^2*e*(-1
/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/
2*I*dilog(1-I*c*x))))
```

3.1143.5 Fricas [F]

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^3(b\arctan\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^3, x)
```

3.1143.6 Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^3}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**3, x)
```

3.1143.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{1}{4}\,ae^3x^4 + \frac{3}{2}\,ade^2x^2 - \frac{1}{2}\left(\left(c\arctan\left(cx\right) + \frac{1}{x}\right)c + \frac{\arctan\left(cx\right)}{x^2}\right)bd^3 + 3\,ad^2e\log\left(x\right) - \frac{ad^3}{2\,x^2}$$
$$- \frac{bc^3e^3x^3 + 9\,\pi bc^4d^2e\log\left(c^2x^2+1\right) - 36\,bc^4d^2e\arctan\left(cx\right)\log\left(cx\right) + 18i\,bc^4d^2e\mathrm{Li}_2(i\,cx+1) - 18i\,bc^4d^2e}{12\,c^4}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*
x)/x^2)*b*d^3 + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 - 1/12*(b*c^3*e^3*x^3 + 9
*pi*b*c^4*d^2*e*log(c^2*x^2 + 1) - 36*b*c^4*d^2*e*arctan(c*x)*log(c*x) + 1
8*I*b*c^4*d^2*e*dilog(I*c*x + 1) - 18*I*b*c^4*d^2*e*dilog(-I*c*x + 1) + 3*
(6*b*c^3*d*e^2 - b*c*e^3)*x - 3*(b*c^4*e^3*x^4 + 6*b*c^4*d*e^2*x^2 + 6*b*c
^2*d*e^2 - b*e^3)*arctan(c*x))/c^4
```

3.1143.8 Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^3(b\arctan\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1143.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3} \, dx$$

$$= \begin{cases} \frac{a\,e^3\,x^4}{4} - \frac{a\,d^3}{2\,x^2} + \frac{3\,a\,d\,e^2\,x^2}{2} + 3 \\ \frac{a\,e^3\,x^4}{4} - \frac{a\,d^3}{2\,x^2} - \frac{b\,d^3\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - 3\,b\,d\,e^2\left(\frac{x}{2\,c} - \mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) + \frac{3\,a\,d\,e^2\,x^2}{2} + 3\,a\,d^2\,e\,\ln(x) - \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^3,x)
```

output
```
piecewise(c == 0, - (a*d^3)/(2*x^2) + (a*e^3*x^4)/4 + (3*a*d*e^2*x^2)/2 +
3*a*d^2*e*log(x), c ~= 0, - (a*d^3)/(2*x^2) + (a*e^3*x^4)/4 - (b*d^3*(c^3*
atan(c*x) + c^2/x))/(2*c) - 3*b*d*e^2*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^
2/2)) + (3*a*d*e^2*x^2)/2 + 3*a*d^2*e*log(x) - (b*e^3*(3*atan(c*x) - 3*c*x
 + c^3*x^3))/(12*c^4) - (b*d^2*e*dilog(- c*x*1i + 1)*3i)/2 + (b*d^2*e*dilo
g(c*x*1i + 1)*3i)/2 - (b*d^3*atan(c*x))/(2*x^2) + (b*e^3*x^4*atan(c*x))/4)
```

3.1144 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^4}\,dx$

3.1144.1 Optimal result

Integrand size = 21, antiderivative size = 158

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^4}\,dx = &-\frac{bcd^3}{6x^2}-\frac{be^3x^2}{6c}-\frac{d^3(a+b\arctan(cx))}{3x^3}\\&-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))\\&+\frac{1}{3}e^3x^3(a+b\arctan(cx))-\frac{1}{3}bcd^2(c^2d-9e)\log(x)\\&+\frac{b(c^2d+e)(c^4d^2-10c^2de+e^2)\log(1+c^2x^2)}{6c^3}\end{aligned}$$

output
```
-1/6*b*c*d^3/x^2-1/6*b*e^3*x^2/c-1/3*d^3*(a+b*arctan(c*x))/x^3-3*d^2*e*(a+
b*arctan(c*x))/x+3*d*e^2*x*(a+b*arctan(c*x))+1/3*e^3*x^3*(a+b*arctan(c*x))
-1/3*b*c*d^2*(c^2*d-9*e)*ln(x)+1/6*b*(c^2*d+e)*(c^4*d^2-10*c^2*d*e+e^2)*ln
(c^2*x^2+1)/c^3
```

3.1144.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx = \frac{1}{6}\left(-\frac{2ad^3}{x^3}-\frac{bcd^3}{x^2}-\frac{18ad^2e}{x}+18ade^2x-\frac{be^3x^2}{c}+2ae^3x^3 + \frac{2b(-d^3-9d^2ex^2+9de^2x^4+e^3x^6)\arctan(cx)}{x^3} - 2bcd^2(c^2d-9e)\log(x) + \frac{b(c^6d^3-9c^4d^2e-9c^2de^2+e^3)\log(1+c^2x^2)}{c^3}\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
((-2*a*d^3)/x^3 - (b*c*d^3)/x^2 - (18*a*d^2*e)/x + 18*a*d*e^2*x - (b*e^3*x
^2)/c + 2*a*e^3*x^3 + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*Ar
cTan[c*x])/x^3 - 2*b*c*d^2*(c^2*d - 9*e)*Log[x] + (b*(c^6*d^3 - 9*c^4*d^2*
e - 9*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^3)/6
```

3.1144.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{3x^3(c^2x^2+1)}dx - \frac{d^3(a+b\arctan(cx))}{3x^3} - \frac{3d^2e(a+b\arctan(cx))}{x} + 3de^2x(a+b\arctan(cx)) + \frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^3(c^2x^2+1)}dx - \frac{d^3(a+b\arctan(cx))}{3x^3} - \frac{3d^2e(a+b\arctan(cx))}{x} + 3de^2x(a+b\arctan(cx)) + \frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{6}bc\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^4(c^2x^2+1)}dx^2-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{6}bc\int \left(\frac{d^3}{x^4}-\frac{(c^2d-9e)\,d^2}{x^2}-\frac{e^3}{c^2}+\frac{(dc^2+e)\,(d^2c^4-10dec^2+e^2)}{c^2\,(c^2x^2+1)}\right)dx^2-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))+\frac{1}{6}bc\left(-d^2\log\left(x^2\right)\left(c^2d-9e\right)-\frac{e^3x^2}{c^2}+\frac{(c^2d+e)\,(c^4d^2-10c^2de+e^2)\log\left(c^2x^2+1\right)}{c^4}-\frac{d^3}{x^2}\right)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d^3*(a + b*ArcTan[c*x]))/x^3 - (3*d^2*e*(a + b*ArcTan[c*x]))/x + 3*d
*e^2*x*(a + b*ArcTan[c*x]) + (e^3*x^3*(a + b*ArcTan[c*x]))/3 + (b*c*(-(d^3
/x^2) - (e^3*x^2)/c^2 - d^2*(c^2*d - 9*e)*Log[x^2] + ((c^2*d + e)*(c^4*d^2
 - 10*c^2*d*e + e^2)*Log[1 + c^2*x^2])/c^4))/6
```

3.1144.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1144.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23

method	result
parts	$a\left(\frac{e^3x^3}{3}+3d\,e^2x-\frac{3e\,d^2}{x}-\frac{d^3}{3x^3}\right)+b\,c^3\left(\frac{\arctan(cx)x^3e^3}{3c^3}+\frac{3\arctan(cx)x\,e^2d}{c^3}-\frac{3\arctan(cx)d^2e}{c^3x}-\text{arc}\right.$
derivativedivides	$c^3\left(\frac{a\left(3c^3xd\,e^2+\frac{e^3c^3x^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^3xd\,e^2+\frac{\arctan(cx)e^3c^3x^3}{3}-\frac{3\arctan(cx)c^3d^2e}{x}-\frac{\arctan(cx)c}{3x^3}\right.}{}\right.$
default	$c^3\left(\frac{a\left(3c^3xd\,e^2+\frac{e^3c^3x^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^3xd\,e^2+\frac{\arctan(cx)e^3c^3x^3}{3}-\frac{3\arctan(cx)c^3d^2e}{x}-\frac{\arctan(cx)c}{3x^3}\right.}{}\right.$
parallelrisch	$-\frac{2\ln(x)bc^6d^3x^3-\ln(c^2x^2+1)x^3bc^6d^3-2x^6\arctan(cx)bc^3e^3-2ac^3e^3x^6-bc^6d^3x^3-18\ln(x)bc^4d^2e\,x^3+9\ln(c^2x^2+1)}{}$
risch	$\frac{ib(-e^3x^6-9x^4e^2d+9e\,d^2x^2+d^3)\ln(icx+1)}{6x^3}-\frac{ibc^3d^3\ln(-icx+1)+2\ln(x)bc^6d^3x^3-\ln(-c^2x^2-1)bc^6d^3x^3-9ibc^3d\,e^2}{}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^3*x^3+3*d*e^2*x-3*e*d^2/x-1/3*d^3/x^3)+b*c^3*(1/3*arctan(c*x)/c^3
*x^3*e^3+3*arctan(c*x)/c^3*x*e^2*d-3*arctan(c*x)/c^3*d^2*e/x-1/3*arctan(c*
x)*d^3/c^3/x^3-1/3/c^6*(1/2*e^3*c^2*x^2+c^4*d^2*(c^2*d-9*e)*ln(c*x)+1/2*c^
4*d^3/x^2+1/2*(-c^6*d^3+9*c^4*d^2*e+9*c^2*d*e^2-e^3)*ln(c^2*x^2+1)))
```

3.1144.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{2\,ac^3e^3x^6+18\,ac^3de^2x^4-bc^2e^3x^5-bc^4d^3x-18\,ac^3d^2ex^2-2\,ac^3d^3+(bc^6d^3-9\,bc^4d^2e-9\,bc^2de^2+be^3)}{6}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^3*x^6 + 18*a*c^3*d*e^2*x^4 - b*c^2*e^3*x^5 - b*c^4*d^3*x -
18*a*c^3*d^2*e*x^2 - 2*a*c^3*d^3 + (b*c^6*d^3 - 9*b*c^4*d^2*e - 9*b*c^2*d*
e^2 + b*e^3)*x^3*log(c^2*x^2 + 1) - 2*(b*c^6*d^3 - 9*b*c^4*d^2*e)*x^3*log(
x) + 2*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^3*d^2*e*x^2 - b*c^3*d^3)
*arctan(c*x))/(c^3*x^3)
```

3.1144.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \begin{cases} -\frac{ad^3}{3x^3}-\frac{3ad^2e}{x}+3ade^2x+\frac{ae^3x^3}{3}-\frac{bc^3d^3\log(x)}{3}+\frac{bc^3d^3\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bcd^3}{6x^2}+3bcd^2e\log(x)-\frac{3bcd^2e\log\left(x^2+\frac{1}{c^2}\right)}{2}- \\ a\left(-\frac{d^3}{3x^3}-\frac{3d^2e}{x}+3de^2x+\frac{e^3x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a*d**3/(3*x**3) - 3*a*d**2*e/x + 3*a*d*e**2*x + a*e**3*x**3/3
- b*c**3*d**3*log(x)/3 + b*c**3*d**3*log(x**2 + c**(-2))/6 - b*c*d**3/(6*x
**2) + 3*b*c*d**2*e*log(x) - 3*b*c*d**2*e*log(x**2 + c**(-2))/2 - b*d**3*a
tan(c*x)/(3*x**3) - 3*b*d**2*e*atan(c*x)/x + 3*b*d*e**2*x*atan(c*x) + b*e*
*3*x**3*atan(c*x)/3 - 3*b*d*e**2*log(x**2 + c**(-2))/(2*c) - b*e**3*x**2/(
6*c) + b*e**3*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**3/(3*x**3)
- 3*d**2*e/x + 3*d*e**2*x + e**3*x**3/3), True))
```

3.1144.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.22

$$\begin{aligned}
&\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx \\
&= \frac{1}{3}ae^3x^3 + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bd^3 \\
&\quad - \frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd^2e \\
&\quad + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be^3 + 3ade^2x \\
&\quad + \frac{3\left(2cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bde^2}{2c} - \frac{3ad^2e}{x} - \frac{ad^3}{3x^3}
\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/3*a*e^3*x^3 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*a
rctan(c*x)/x^3)*b*d^3 - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*
x)/x)*b*d^2*e + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4
))*b*e^3 + 3*a*d*e^2*x + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e^
2/c - 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

3.1144.8 Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.1144.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx = \frac{a\,e^3\,x^3}{3} - \ln(x)\left(\frac{b\,c^3\,d^3}{3} - 3\,b\,c\,d^2\,e\right) - \frac{\frac{b\,c^2\,d^3\,x}{2} + a\,c\,d^3 + 9\,a\,e\,c\,d^2\,x^2}{3\,c\,x^3} - x\left(\frac{a\,e^3}{c^2} - \frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}\right) + \frac{\ln(c^2\,x^2+1)\,\left(b\,c^6\,d^3 - 9\,b\,c^4\,d^2\,e - 9\,b\,c^2\,d\,e^2 + b\,e^3\right)}{6\,c^3} - \frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{3} + 3\,b\,d^2\,e\,x^2 - 3\,b\,d\,e^2\,x^4 - \frac{b\,e^3\,x^6}{3}\right)}{x^3} - \frac{b\,e^3\,x^2}{6\,c}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^4,x)
```

output
```
(a*e^3*x^3)/3 - log(x)*((b*c^3*d^3)/3 - 3*b*c*d^2*e) - (a*c*d^3 + (b*c^2*d
^3*x)/2 + 9*a*c*d^2*e*x^2)/(3*c*x^3) - x*((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*
d))/c^2) + (log(c^2*x^2 + 1)*(b*e^3 + b*c^6*d^3 - 9*b*c^2*d*e^2 - 9*b*c^4*
d^2*e))/(6*c^3) - (atan(c*x)*((b*d^3)/3 - (b*e^3*x^6)/3 + 3*b*d^2*e*x^2 -
3*b*d*e^2*x^4))/x^3 - (b*e^3*x^2)/(6*c)
```

3.1145 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^5}\,dx$

3.1145.1 Optimal result

Integrand size = 21, antiderivative size = 200

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^5}\,dx = &-\frac{bcd^3}{12x^3}+\frac{bc^3d^3}{4x}-\frac{3bcd^2e}{2x}-\frac{be^3x}{2c}+\frac{1}{4}bc^4d^3\arctan(cx)\\ &-\frac{3}{2}bc^2d^2e\arctan(cx)+\frac{be^3\arctan(cx)}{2c^2}\\ &-\frac{d^3(a+b\arctan(cx))}{4x^4}-\frac{3d^2e(a+b\arctan(cx))}{2x^2}\\ &+\frac{1}{2}e^3x^2(a+b\arctan(cx))+3ade^2\log(x)\\ &+\frac{3}{2}ibde^2\operatorname{PolyLog}(2,-icx)-\frac{3}{2}ibde^2\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/12*b*c*d^3/x^3+1/4*b*c^3*d^3/x-3/2*b*c*d^2*e/x-1/2*b*e^3*x/c+1/4*b*c^4*
d^3*arctan(c*x)-3/2*b*c^2*d^2*e*arctan(c*x)+1/2*b*e^3*arctan(c*x)/c^2-1/4*
d^3*(a+b*arctan(c*x))/x^4-3/2*d^2*e*(a+b*arctan(c*x))/x^2+1/2*e^3*x^2*(a+b
*arctan(c*x))+3*a*d*e^2*ln(x)+3/2*I*b*d*e^2*polylog(2,-I*c*x)-3/2*I*b*d*e^
2*polylog(2,I*c*x)
```

3.1145.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5} \, dx = \frac{1}{12}\left(-\frac{6be^3(cx-\arctan(cx))}{c^2} - \frac{3d^3(a+b\arctan(cx))}{x^4} - \frac{18d^2e(a+b\arctan(cx))}{x^2} + 6e^3x^2(a+b\arctan(cx)) - \frac{bcd^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{x^3} - \frac{18bcd^2e\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 36ade^2\log(x) + 18ibde^2\,\text{PolyLog}(2,-icx) - 18ibde^2\,\text{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
((-6*b*e^3*(c*x - ArcTan[c*x]))/c^2 - (3*d^3*(a + b*ArcTan[c*x]))/x^4 - (1
8*d^2*e*(a + b*ArcTan[c*x]))/x^2 + 6*e^3*x^2*(a + b*ArcTan[c*x]) - (b*c*d^
3*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/x^3 - (18*b*c*d^2*e*Hyperg
eometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 36*a*d*e^2*Log[x] + (18*I)*b*d*
e^2*PolyLog[2, (-I)*c*x] - (18*I)*b*d*e^2*PolyLog[2, I*c*x])/12
```

3.1145.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5} \, dx$$

$$\Big\downarrow \; 5515$$

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x^5}+\frac{3d^2e(a+b\arctan(cx))}{x^3}+\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{4x^4}-\frac{3d^2e(a+b\arctan(cx))}{2x^2}+\frac{1}{2}e^3x^2(a+b\arctan(cx))+3ade^2\log(x)+\frac{1}{4}bc^4d^3\arctan(cx)-\frac{3}{2}bc^2d^2e\arctan(cx)+\frac{be^3\arctan(cx)}{2c^2}+\frac{bc^3d^3}{4x}-\frac{bcd^3}{12x^3}-\frac{3bcd^2e}{2x}+\frac{3}{2}ibde^2\operatorname{PolyLog}(2,-icx)-\frac{3}{2}ibde^2\operatorname{PolyLog}(2,icx)-\frac{be^3x}{2c}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*d^3)/x^3 + (b*c^3*d^3)/(4*x) - (3*b*c*d^2*e)/(2*x) - (b*e^3*x)/
(2*c) + (b*c^4*d^3*ArcTan[c*x])/4 - (3*b*c^2*d^2*e*ArcTan[c*x])/2 + (b*e^3
*ArcTan[c*x])/(2*c^2) - (d^3*(a + b*ArcTan[c*x]))/(4*x^4) - (3*d^2*e*(a +
b*ArcTan[c*x]))/(2*x^2) + (e^3*x^2*(a + b*ArcTan[c*x]))/2 + 3*a*d*e^2*Log[
x] + ((3*I)/2)*b*d*e^2*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d*e^2*PolyLog[2,
 I*c*x]
```

3.1145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1145.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.20

method	result
parts	$a\left(\frac{e^3x^2}{2}+3e^2d\ln\left(x\right)-\frac{d^3}{4x^4}-\frac{3e\,d^2}{2x^2}\right)+b\,c^4\left(\frac{\arctan(cx)x^2e^3}{2c^4}+\frac{3\arctan(cx)e^2d\ln(cx)}{c^4}-\frac{\arctan(cx)d^3}{4c^4x^4}\right.$
derivativedivides	$c^4\left(\frac{a\left(\frac{e^3c^2x^2}{2}+3c^2d\,e^2\ln(cx)-\frac{c^2d^3}{4x^4}-\frac{3c^2d^2e}{2x^2}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)e^3c^2x^2}{2}+3\arctan(cx)c^2d\,e^2\ln(cx)-\frac{\arctan(cx)c^2d^3}{4x^4}-\right.}{}\right.$
default	$c^4\left(\frac{a\left(\frac{e^3c^2x^2}{2}+3c^2d\,e^2\ln(cx)-\frac{c^2d^3}{4x^4}-\frac{3c^2d^2e}{2x^2}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)e^3c^2x^2}{2}+3\arctan(cx)c^2d\,e^2\ln(cx)-\frac{\arctan(cx)c^2d^3}{4x^4}-\right.}{}\right.$
risch	$-\frac{b\,e^3x}{2c}+\frac{b\,c^4d^3\arctan(cx)}{8}+\frac{b\,e^3\arctan(cx)}{4c^2}-\frac{3bc\,d^2e}{2x}-\frac{3b\,c^2d^2e\arctan(cx)}{4}-\frac{bc\,d^3}{12x^3}+\frac{b\,c^3d^3}{4x}-\frac{3a\,d^2e}{2x^2}+$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^3*x^2+3*e^2*d*ln(x)-1/4*d^3/x^4-3/2*e*d^2/x^2)+b*c^4*(1/2*arctan(
c*x)/c^4*x^2*e^3+3*arctan(c*x)/c^4*e^2*d*ln(c*x)-1/4*arctan(c*x)*d^3/c^4/x
^4-3/2*arctan(c*x)/c^4*d^2*e/x^2-1/4/c^6*(2*c*x*e^3-c^3*d^2*(c^2*d-6*e)/x+
1/3*c^3*d^3/x^3+(-c^6*d^3+6*c^4*d^2*e-2*e^3)*arctan(c*x)+12*c^2*d*e^2*(-1/
2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2
*I*dilog(1-I*c*x))))
```

3.1145.5 Fricas [F]

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(ex^2+d)^3(b\arctan\left(cx\right)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^5, x)
```

3.1145.6 Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\operatorname{atan}(cx))(d+ex^2)^3}{x^5}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**5, x)
```

3.1145.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.09

$$\begin{aligned}&\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx\\&= \frac{1}{2}ae^3x^2 + \frac{1}{12}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2-1}{x^3}\right)c - \frac{3\,\arctan(cx)}{x^4}\right)bd^3\\&\quad - \frac{3}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)bd^2e + 3\,ade^2\log(x) - \frac{3\,ad^2e}{2\,x^2} - \frac{ad^3}{4\,x^4}\\&\quad - \frac{3\,\pi bc^2de^2\log(c^2x^2+1) - 12\,bc^2de^2\arctan(cx)\log(cx) + 6i\,bc^2de^2\mathrm{Li}_2(i\,cx+1) - 6i\,bc^2de^2\mathrm{Li}_2(-i\,cx+}{4\,c^2}\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/2*a*e^3*x^2 + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arct
an(c*x)/x^4)*b*d^3 - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2
*e + 3*a*d*e^2*log(x) - 3/2*a*d^2*e/x^2 - 1/4*a*d^3/x^4 - 1/4*(3*pi*b*c^2*
d*e^2*log(c^2*x^2 + 1) - 12*b*c^2*d*e^2*arctan(c*x)*log(c*x) + 6*I*b*c^2*d
*e^2*dilog(I*c*x + 1) - 6*I*b*c^2*d*e^2*dilog(-I*c*x + 1) + 2*b*c*e^3*x -
2*(b*c^2*e^3*x^2 + b*e^3)*arctan(c*x))/c^2
```

3.1145.8 Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.1145.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx$$
$$= \begin{cases} -\frac{a\,(d^3-2\,e^3\,x^6+6\,d^2\,e\,x^2-12\,d\,e^2\,x^4 \\ -b\,e^3\left(\frac{x}{2\,c}-\mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right)-\frac{a\,(d^3-2\,e^3\,x^6+6\,d^2\,e\,x^2-12\,d\,e^2\,x^4\,\ln(x))}{4\,x^4}-\frac{b\,d^3\left(\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\mathrm{atan}(c\,x)\right)}{4\,c}-3\,b\,d \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^5,x)
```

output
```
piecewise(c == 0, -(a*(d^3 - 2*e^3*x^6 + 6*d^2*e*x^2 - 12*d*e^2*x^4*log(x)
))/(4*x^4), c ~= 0, - b*e^3*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) - (a
*(d^3 - 2*e^3*x^6 + 6*d^2*e*x^2 - 12*d*e^2*x^4*log(x)))/(4*x^4) - (b*d^3*(
(c^2/3 - c^4*x^2)/x^3 - c^5*atan(c*x)))/(4*c) - 3*b*d^2*e*((c^3*atan(c*x)
+ c^2/x)/(2*c) + atan(c*x)/(2*x^2)) - (b*d*e^2*dilog(- c*x*1i + 1)*3i)/2 +
 (b*d*e^2*dilog(c*x*1i + 1)*3i)/2 - (b*d^3*atan(c*x))/(4*x^4))
```

3.1146 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx$

3.1146.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^3}{20x^4}+\frac{bcd^2(c^2d-5e)}{10x^2} - \frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3} - \frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx)) + \frac{1}{5}bcd(c^4d^2-5c^2de+15e^2)\log(x) - \frac{b(c^6d^3-5c^4d^2e+15c^2de^2+5e^3)\log(1+c^2x^2)}{10c}$$

output
```
-1/20*b*c*d^3/x^4+1/10*b*c*d^2*(c^2*d-5*e)/x^2-1/5*d^3*(a+b*arctan(c*x))/x
^5-d^2*e*(a+b*arctan(c*x))/x^3-3*d*e^2*(a+b*arctan(c*x))/x+e^3*x*(a+b*arct
an(c*x))+1/5*b*c*d*(c^4*d^2-5*c^2*d*e+15*e^2)*ln(x)-1/10*b*(c^6*d^3-5*c^4*
d^2*e+15*c^2*d*e^2+5*e^3)*ln(c^2*x^2+1)/c
```

3.1146.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6} dx = \frac{1}{20}\left(-\frac{4ad^3}{x^5} - \frac{bcd^3}{x^4} - \frac{20ad^2e}{x^3} + \frac{2bcd^2(c^2d-5e)}{x^2} - \frac{60ade^2}{x} + 20ae^3x - \frac{4b(d^3+5d^2ex^2+15de^2x^4-5e^3x^6)\arctan(cx)}{x^5} + 4bcd(c^4d^2-5c^2de+15e^2)\log(x) - \frac{2b(c^6d^3-5c^4d^2e+15c^2de^2+5e^3)\log(1+c^2x^2)}{c}\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
((-4*a*d^3)/x^5 - (b*c*d^3)/x^4 - (20*a*d^2*e)/x^3 + (2*b*c*d^2*(c^2*d - 5
*e))/x^2 - (60*a*d*e^2)/x + 20*a*e^3*x - (4*b*(d^3 + 5*d^2*e*x^2 + 15*d*e^
2*x^4 - 5*e^3*x^6)*ArcTan[c*x])/x^5 + 4*b*c*d*(c^4*d^2 - 5*c^2*d*e + 15*e^
2)*Log[x] - (2*b*(c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^
2*x^2])/c)/20
```

3.1146.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6} dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{-5e^3x^6+15de^2x^4+5d^2ex^2+d^3}{5x^5(c^2x^2+1)}dx - \frac{d^3(a+b\arctan(cx))}{5x^5} - \frac{d^2e(a+b\arctan(cx))}{x^3} - \frac{3de^2(a+b\arctan(cx))}{x} + e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}bc\int \frac{-5e^3x^6+15de^2x^4+5d^2ex^2+d^3}{x^5(c^2x^2+1)}dx-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{10}bc\int \frac{-5e^3x^6+15de^2x^4+5d^2ex^2+d^3}{x^6(c^2x^2+1)}dx^2-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{10}bc\int \left(\frac{d^3}{x^6}-\frac{(c^2d-5e)d^2}{x^4}+\frac{(d^2c^4-5dec^2+15e^2)d}{x^2}+\frac{-d^3c^6+5d^2ec^4-15de^2c^2-5e^3}{c^2x^2+1}\right)dx^2-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))+\frac{1}{10}bc\left(\frac{d^2(c^2d-5e)}{x^2}+d\log(x^2)(c^4d^2-5c^2de+15e^2)-\frac{(c^6d^3-5c^4d^2e+15c^2de^2+5e^3)\log(c^2x^2+1)}{c^2}-\frac{d^3}{2x^4}\right)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^3*(a + b*ArcTan[c*x]))/x^5 - (d^2*e*(a + b*ArcTan[c*x]))/x^3 - (3*
d*e^2*(a + b*ArcTan[c*x]))/x + e^3*x*(a + b*ArcTan[c*x]) + (b*c*(-1/2*d^3/
x^4 + (d^2*(c^2*d - 5*e))/x^2 + d*(c^4*d^2 - 5*c^2*d*e + 15*e^2)*Log[x^2]
- ((c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^2*x^2])/c^2))/
10
```

3.1146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1146.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

method	result
derivativedivides	$c^5 \left(\frac{a\left(cx\,e^3-\frac{c\,d^3}{5x^5}-\frac{3cd\,e^2}{x}-\frac{c\,d^2 e}{x^3}\right)}{c^6} + \frac{b\left(\arctan(cx)cx\,e^3-\frac{\arctan(cx)c\,d^3}{5x^5}-\frac{3\arctan(cx)cd\,e^2}{x}-\frac{\arctan(cx)c\,d^2 e}{x^3}-\frac{(c^6 d^3-5c}{}\right.}{}\right.$
default	$c^5 \left(\frac{a\left(cx\,e^3-\frac{c\,d^3}{5x^5}-\frac{3cd\,e^2}{x}-\frac{c\,d^2 e}{x^3}\right)}{c^6} + \frac{b\left(\arctan(cx)cx\,e^3-\frac{\arctan(cx)c\,d^3}{5x^5}-\frac{3\arctan(cx)cd\,e^2}{x}-\frac{\arctan(cx)c\,d^2 e}{x^3}-\frac{(c^6 d^3-5c}{}\right.}{}\right.$
parts	$a\left(x\,e^3-\frac{3e^2 d}{x}-\frac{e\,d^2}{x^3}-\frac{d^3}{5x^5}\right)+b\,c^5\left(\frac{\arctan(cx)x\,e^3}{c^5}-\frac{3\arctan(cx)e^2 d}{c^5 x}-\frac{\arctan(cx)d^2 e}{c^5 x^3}-\frac{\arctan(cx)d}{5c^5x^5}\right.$
parallelrisch	$\frac{4\ln(x)b\,c^6 d^3 x^5-2\ln(c^2x^2+1)b\,c^6 d^3 x^5-2b\,c^6 d^3 x^5-20\ln(x)b\,c^4 d^2 e\,x^5+10\ln(c^2x^2+1)b\,c^4 d^2 e\,x^5+10b\,c^4 d^2 e\,x^5+60\ln(x}{}$
risch	$\frac{ib\left(-5e^3x^6+15x^4e^2d+5e\,d^2x^2+d^3\right)\ln(icx+1)}{10x^5}+\frac{4\ln(x)b\,c^6 d^3 x^5-2\ln(c^2x^2+1)b\,c^6 d^3 x^5-20\ln(x)b\,c^4 d^2 e\,x^5+10\ln(c^2}{}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
c^5*(a/c^6*(c*x*e^3-1/5*c*d^3/x^5-3*c*d*e^2/x-c*d^2*e/x^3)+b/c^6*(arctan(c
*x)*c*x*e^3-1/5*arctan(c*x)*c*d^3/x^5-3*arctan(c*x)*c*d*e^2/x-arctan(c*x)*
c*d^2*e/x^3-1/10*(c^6*d^3-5*c^4*d^2*e+15*c^2*d*e^2+5*e^3)*ln(c^2*x^2+1)+1/
10*c^2*d^2*(c^2*d-5*e)/x^2-1/20*c^2*d^3/x^4+1/5*d*c^2*(c^4*d^2-5*c^2*d*e+1
5*e^2)*ln(c*x)))
```

3.1146.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{20\,ace^3x^6 - 60\,acde^2x^4 - bc^2d^3x - 20\,acd^2ex^2 - 2\,(bc^6d^3 - 5\,bc^4d^2e + 15\,bc^2de^2 + 5\,be^3)x^5\log(c^2x^2+1)}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/20*(20*a*c*e^3*x^6 - 60*a*c*d*e^2*x^4 - b*c^2*d^3*x - 20*a*c*d^2*e*x^2 -
 2*(b*c^6*d^3 - 5*b*c^4*d^2*e + 15*b*c^2*d*e^2 + 5*b*e^3)*x^5*log(c^2*x^2
+ 1) + 4*(b*c^6*d^3 - 5*b*c^4*d^2*e + 15*b*c^2*d*e^2)*x^5*log(x) - 4*a*c*d
^3 + 2*(b*c^4*d^3 - 5*b*c^2*d^2*e)*x^3 + 4*(5*b*c*e^3*x^6 - 15*b*c*d*e^2*x
^4 - 5*b*c*d^2*e*x^2 - b*c*d^3)*arctan(c*x))/(c*x^5)
```

3.1146.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \begin{cases} -\frac{ad^3}{5x^5} - \frac{ad^2e}{x^3} - \frac{3ade^2}{x} + ae^3x + \frac{bc^5d^3\log(x)}{5} - \frac{bc^5d^3\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^3}{10x^2} - bc^3d^2e\log(x) + \frac{bc^3d^2e\log\left(x^2+\frac{1}{c^2}\right)}{2} - \\ a\left(-\frac{d^3}{5x^5} - \frac{d^2e}{x^3} - \frac{3de^2}{x} + e^3x\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a*d**3/(5*x**5) - a*d**2*e/x**3 - 3*a*d*e**2/x + a*e**3*x + b*
c**5*d**3*log(x)/5 - b*c**5*d**3*log(x**2 + c**(-2))/10 + b*c**3*d**3/(10*
x**2) - b*c**3*d**2*e*log(x) + b*c**3*d**2*e*log(x**2 + c**(-2))/2 - b*c*d
**3/(20*x**4) - b*c*d**2*e/(2*x**2) + 3*b*c*d*e**2*log(x) - 3*b*c*d*e**2*l
og(x**2 + c**(-2))/2 - b*d**3*atan(c*x)/(5*x**5) - b*d**2*e*atan(c*x)/x**3
 - 3*b*d*e**2*atan(c*x)/x + b*e**3*x*atan(c*x) - b*e**3*log(x**2 + c**(-2)
)/(2*c), Ne(c, 0)), (a*(-d**3/(5*x**5) - d**2*e/x**3 - 3*d*e**2/x + e**3*x
), True))
```

3.1146.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\begin{aligned}
&\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6}\,dx \\
&= -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)bd^3 \\
&\quad+\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)bd^2e \\
&\quad-\frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan\left(cx\right)}{x}\right)bde^2+ae^3x \\
&\quad+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)be^3}{2\,c}-\frac{3\,ade^2}{x}-\frac{ad^2e}{x^3}-\frac{ad^3}{5\,x^5}
\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d^3 + 1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/
x^2)*c - 2*arctan(c*x)/x^3)*b*d^2*e - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2))
 + 2*arctan(c*x)/x)*b*d*e^2 + a*e^3*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x
^2 + 1))*b*e^3/c - 3*a*d*e^2/x - a*d^2*e/x^3 - 1/5*a*d^3/x^5
```

3.1146.8 Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.1146.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6}\,dx$$
$$= \ln(x)\left(\frac{b\,c^5\,d^3}{5}-b\,c^3\,d^2\,e+3\,b\,c\,d\,e^2\right) - \frac{a\,d^3-x^3\left(\frac{b\,c^3\,d^3}{2}-\frac{5\,b\,c\,d^2\,e}{2}\right)+\frac{b\,c\,d^3\,x}{4}+5\,a\,d^2\,e\,x^2+15\,a\,d\,e^2\,x^4}{5\,x^5} - \frac{\ln(c^2\,x^2+1)\,(b\,c^6\,d^3-5\,b\,c^4\,d^2\,e+15\,b\,c^2\,d\,e^2+5\,b\,e^3)}{10\,c} - \frac{\mathrm{atan}(c\,x)\left(\frac{b\,d^3}{5}+b\,d^2\,e\,x^2+3\,b\,d\,e^2\,x^4-b\,e^3\,x^6\right)}{x^5} + a\,e^3\,x$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^6,x)
```

output
```
log(x)*((b*c^5*d^3)/5 + 3*b*c*d*e^2 - b*c^3*d^2*e) - (a*d^3 - x^3*((b*c^3*
d^3)/2 - (5*b*c*d^2*e)/2) + (b*c*d^3*x)/4 + 5*a*d^2*e*x^2 + 15*a*d*e^2*x^4
)/(5*x^5) - (log(c^2*x^2 + 1)*(5*b*e^3 + b*c^6*d^3 + 15*b*c^2*d*e^2 - 5*b*
c^4*d^2*e))/(10*c) - (atan(c*x)*((b*d^3)/5 - b*e^3*x^6 + b*d^2*e*x^2 + 3*b
*d*e^2*x^4))/x^5 + a*e^3*x
```

3.1147 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^7}\,dx$

3.1147.1 Optimal result

Integrand size = 21, antiderivative size = 228

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^7}\,dx &= -\frac{bcd^3}{30x^5}+\frac{bc^3d^3}{18x^3}-\frac{bcd^2e}{4x^3}-\frac{bc^5d^3}{6x}\\ &\quad+\frac{3bc^3d^2e}{4x}-\frac{3bcde^2}{2x}-\frac{1}{6}bc^6d^3\arctan(cx)\\ &\quad+\frac{3}{4}bc^4d^2e\arctan(cx)-\frac{3}{2}bc^2de^2\arctan(cx)\\ &\quad-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3d^2e(a+b\arctan(cx))}{4x^4}\\ &\quad-\frac{3de^2(a+b\arctan(cx))}{2x^2}+ae^3\log(x)\\ &\quad+\frac{1}{2}ibe^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibe^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/30*b*c*d^3/x^5+1/18*b*c^3*d^3/x^3-1/4*b*c*d^2*e/x^3-1/6*b*c^5*d^3/x+3/4
*b*c^3*d^2*e/x-3/2*b*c*d*e^2/x-1/6*b*c^6*d^3*arctan(c*x)+3/4*b*c^4*d^2*e*a
rctan(c*x)-3/2*b*c^2*d*e^2*arctan(c*x)-1/6*d^3*(a+b*arctan(c*x))/x^6-3/4*d
^2*e*(a+b*arctan(c*x))/x^4-3/2*d*e^2*(a+b*arctan(c*x))/x^2+a*e^3*ln(x)+1/2
*I*b*e^3*polylog(2,-I*c*x)-1/2*I*b*e^3*polylog(2,I*c*x)
```

3.1147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7}\,dx = \frac{1}{60}\left(-\frac{10d^3(a+b\arctan(cx))}{x^6} - \frac{45d^2e(a+b\arctan(cx))}{x^4} - \frac{90de^2(a+b\arctan(cx))}{x^2} - \frac{2bcd^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{x^5} - \frac{15bcd^2e\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{x^3} - \frac{90bcde^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 60ae^3\log(x) + 30ibe^3\,\text{PolyLog}(2,-icx) - 30ibe^3\,\text{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
((-10*d^3*(a + b*ArcTan[c*x]))/x^6 - (45*d^2*e*(a + b*ArcTan[c*x]))/x^4 -
(90*d*e^2*(a + b*ArcTan[c*x]))/x^2 - (2*b*c*d^3*Hypergeometric2F1[-5/2, 1,
 -3/2, -(c^2*x^2)])/x^5 - (15*b*c*d^2*e*Hypergeometric2F1[-3/2, 1, -1/2, -
(c^2*x^2)])/x^3 - (90*b*c*d*e^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)
])/x + 60*a*e^3*Log[x] + (30*I)*b*e^3*PolyLog[2, (-I)*c*x] - (30*I)*b*e^3*
PolyLog[2, I*c*x])/60
```

3.1147.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7}\,dx$$

$$\Big\downarrow \textcolor{blue}{5515}$$

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x^7}+\frac{3d^2e(a+b\arctan(cx))}{x^5}+\frac{3de^2(a+b\arctan(cx))}{x^3}+\frac{e^3(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3d^2e(a+b\arctan(cx))}{4x^4}-\frac{3de^2(a+b\arctan(cx))}{2x^2}+ae^3\log(x)-\frac{1}{6}bc^6d^3\arctan(cx)+\frac{3}{4}bc^4d^2e\arctan(cx)-\frac{3}{2}bc^2de^2\arctan(cx)-\frac{bc^5d^3}{6x}+\frac{bc^3d^3}{18x^3}+\frac{3bc^3d^2e}{4x}-\frac{bcd^3}{30x^5}-\frac{bcd^2e}{4x^3}-\frac{3bcde^2}{2x}+\frac{1}{2}ibe^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibe^3\operatorname{PolyLog}(2,icx)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/30*(b*c*d^3)/x^5 + (b*c^3*d^3)/(18*x^3) - (b*c*d^2*e)/(4*x^3) - (b*c^5*
d^3)/(6*x) + (3*b*c^3*d^2*e)/(4*x) - (3*b*c*d*e^2)/(2*x) - (b*c^6*d^3*ArcT
an[c*x])/6 + (3*b*c^4*d^2*e*ArcTan[c*x])/4 - (3*b*c^2*d*e^2*ArcTan[c*x])/2
 - (d^3*(a + b*ArcTan[c*x]))/(6*x^6) - (3*d^2*e*(a + b*ArcTan[c*x]))/(4*x^
4) - (3*d*e^2*(a + b*ArcTan[c*x]))/(2*x^2) + a*e^3*Log[x] + (I/2)*b*e^3*Po
lyLog[2, (-I)*c*x] - (I/2)*b*e^3*PolyLog[2, I*c*x]
```

3.1147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1147.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.13

method	result
derivativedivides	$c^6\left(\frac{a\left(e^3\ln(cx)-\frac{3d^2e}{4x^4}-\frac{d^3}{6x^6}-\frac{3d\,e^2}{2x^2}\right)}{c^6}+\frac{b\left(\arctan(cx)e^3\ln(cx)-\frac{3\arctan(cx)d^2e}{4x^4}-\frac{\arctan(cx)d^3}{6x^6}-\frac{3\arctan(cx)d\,e^2}{2x^2}+\frac{ie^3\ln\right.}{}\right.$
default	$c^6\left(\frac{a\left(e^3\ln(cx)-\frac{3d^2e}{4x^4}-\frac{d^3}{6x^6}-\frac{3d\,e^2}{2x^2}\right)}{c^6}+\frac{b\left(\arctan(cx)e^3\ln(cx)-\frac{3\arctan(cx)d^2e}{4x^4}-\frac{\arctan(cx)d^3}{6x^6}-\frac{3\arctan(cx)d\,e^2}{2x^2}+\frac{ie^3\ln\right.}{}\right.$
parts	$a\left(-\frac{d^3}{6x^6}+e^3\ln(x)-\frac{3d^2e}{4x^4}-\frac{3d\,e^2}{2x^2}\right)+b\,c^6\left(-\frac{\arctan(cx)d^3}{6c^6x^6}+\frac{\arctan(cx)\ln(cx)e^3}{c^6}-\frac{3\arctan(cx)d^2e}{4c^6x^4}\right.$
risch	$\frac{3ib\,c^4e\,d^2\ln(icx)}{8}+\frac{3ibe\,d^2\ln(icx+1)}{8x^4}-\frac{3ib\,c^2e^2d\ln(icx)}{4}+\frac{3ib\,e^2d\ln(icx+1)}{4x^2}-\frac{3ib\,d^2e\ln(-icx+1)}{8x^4}+\frac{3ic^2b\,e^2d}{4}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
c^6*(a/c^6*(e^3*ln(c*x)-3/4*d^2*e/x^4-1/6*d^3/x^6-3/2*d*e^2/x^2)+b/c^6*(ar
ctan(c*x)*e^3*ln(c*x)-3/4*arctan(c*x)*d^2*e/x^4-1/6*arctan(c*x)*d^3/x^6-3/
2*arctan(c*x)*d*e^2/x^2+1/2*I*e^3*ln(c*x)*ln(1+I*c*x)-1/2*I*e^3*ln(c*x)*ln
(1-I*c*x)-1/2*I*e^3*dilog(1-I*c*x)+1/2*I*e^3*dilog(1+I*c*x)+1/12*d*c^2*((-
2*c^4*d^2+9*c^2*d*e-18*e^2)*arctan(c*x)-(2*c^4*d^2-9*c^2*d*e+18*e^2)/c/x-2
/5/c*d^2/x^5+1/3*d/c*(2*c^2*d-9*e)/x^3)))
```

3.1147.5 Fricas [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7}\,dx = \int \frac{(ex^2+d)^3 (b\arctan(cx)+a)}{x^7}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^7, x)
```

3.1147.6 Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^3}{x^7}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**7,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**7, x)
```

3.1147.7 Maxima [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7}\,dx = \int \frac{(ex^2+d)^3 (b\arctan(cx)+a)}{x^7}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arct
an(c*x)/x^6)*b*d^3 + 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*
arctan(c*x)/x^4)*b*d^2*e - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)
*b*d*e^2 + b*e^3*integrate(arctan(c*x)/x, x) + a*e^3*log(x) - 3/2*a*d*e^2/
x^2 - 3/4*a*d^2*e/x^4 - 1/6*a*d^3/x^6
```

3.1147.8 Giac [F]

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^7}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^7}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
sage0*x
```

3.1147.9 Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^7}\,dx$$
$$= \begin{cases} a\,e^3\,\ln(x) - \frac{\frac{a\,d^3}{6}+\frac{3\,a\,d^2\,e\,x^2}{4}\cdots}{x^6} \\ a\,e^3\,\ln(x) - \frac{\frac{a\,d^3}{6}+\frac{3\,a\,d^2\,e\,x^2}{4}+\frac{3\,a\,d\,e^2\,x^4}{2}}{x^6} - 3\,b\,d^2\,e\left(\frac{\operatorname{atan}(c\,x)}{4\,x^4}+\frac{\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\operatorname{atan}(c\,x)}{4\,c}\right) - \frac{b\,d^3\left(\frac{c^6\,x^4-\frac{c^4\,x^2}{3}+\frac{c^2}{5}}{x^5}+c^7\,\operatorname{atan}(c\cdots\right.}{6\,c} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^7,x)
```

output
```
piecewise(c == 0, - ((a*d^3)/6 + (3*a*d^2*e*x^2)/4 + (3*a*d*e^2*x^4)/2)/x^
6 + a*e^3*log(x), c ~= 0, - ((a*d^3)/6 + (3*a*d^2*e*x^2)/4 + (3*a*d*e^2*x^
4)/2)/x^6 + a*e^3*log(x) - (b*e^3*dilog(- c*x*1i + 1)*1i)/2 + (b*e^3*dilog
(c*x*1i + 1)*1i)/2 - 3*b*d^2*e*(atan(c*x)/(4*x^4) + ((c^2/3 - c^4*x^2)/x^3
 - c^5*atan(c*x))/(4*c)) - (b*d^3*((c^2/5 - (c^4*x^2)/3 + c^6*x^4)/x^5 + c
^7*atan(c*x)))/(6*c) - 3*b*d*e^2*((c^3*atan(c*x) + c^2/x)/(2*c) + atan(c*x
)/(2*x^2)) - (b*d^3*atan(c*x))/(6*x^6))
```

3.1148 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^8}\,dx$

3.1148.1 Optimal result

Integrand size = 21, antiderivative size = 224

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^8}\,dx = &-\frac{bcd^3}{42x^6}+\frac{bcd^2(5c^2d-21e)}{140x^4}\\ &-\frac{bcd(5c^4d^2-21c^2de+35e^2)}{70x^2}\\ &-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}\\ &-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}\\ &-\frac{1}{35}bc(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)\log(x)\\ &+\frac{1}{70}bc(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)\log(1+c^2x^2)\end{aligned}$$

output
```
-1/42*b*c*d^3/x^6+1/140*b*c*d^2*(5*c^2*d-21*e)/x^4-1/70*b*c*d*(5*c^4*d^2-2
1*c^2*d*e+35*e^2)/x^2-1/7*d^3*(a+b*arctan(c*x))/x^7-3/5*d^2*e*(a+b*arctan(
c*x))/x^5-d*e^2*(a+b*arctan(c*x))/x^3-e^3*(a+b*arctan(c*x))/x-1/35*b*c*(5*
c^6*d^3-21*c^4*d^2*e+35*c^2*d*e^2-35*e^3)*ln(x)+1/70*b*c*(5*c^6*d^3-21*c^4
*d^2*e+35*c^2*d*e^2-35*e^3)*ln(c^2*x^2+1)
```

3.1148.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8} dx = -\frac{d^3(a+b\arctan(cx))}{7x^7} - \frac{3d^2e(a+b\arctan(cx))}{5x^5} - \frac{de^2(a+b\arctan(cx))}{x^3} - \frac{e^3(a+b\arctan(cx))}{x} + \frac{1}{2}bce^3\left(2\log(x)-\log\left(1+c^2x^2\right)\right) - \frac{1}{2}bcde^2\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log\left(1+c^2x^2\right)\right) - \frac{3}{20}bcd^2e\left(\frac{1}{x^4}-\frac{2c^2}{x^2}-4c^4\log(x)+2c^4\log\left(1+c^2x^2\right)\right) - \frac{1}{84}bcd^3\left(\frac{2}{x^6}-\frac{3c^2}{x^4}+\frac{6c^4}{x^2}+12c^6\log(x) - 6c^6\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^3*(a + b*ArcTan[c*x]))/x^7 - (3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5)
 - (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (e^3*(a + b*ArcTan[c*x]))/x + (b*c*e^
3*(2*Log[x] - Log[1 + c^2*x^2]))/2 - (b*c*d*e^2*(x^(-2) + 2*c^2*Log[x] - c
^2*Log[1 + c^2*x^2]))/2 - (3*b*c*d^2*e*(x^(-4) - (2*c^2)/x^2 - 4*c^4*Log[x
] + 2*c^4*Log[1 + c^2*x^2]))/20 - (b*c*d^3*(2/x^6 - (3*c^2)/x^4 + (6*c^4)/
x^2 + 12*c^6*Log[x] - 6*c^6*Log[1 + c^2*x^2]))/84
```

3.1148.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8} dx$$

$$\Big\downarrow \text{ 5511}$$

$$-bc\int -\frac{35e^3x^6+35de^2x^4+21d^2ex^2+5d^3}{35x^7\left(c^2x^2+1\right)}dx-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}bc\int \frac{35e^3x^6+35de^2x^4+21d^2ex^2+5d^3}{x^7\left(c^2x^2+1\right)}dx-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{70}bc\int \frac{35e^3x^6+35de^2x^4+21d^2ex^2+5d^3}{x^8\left(c^2x^2+1\right)}dx^2-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{70}bc\int \left(\frac{5d^3}{x^8}-\frac{\left(5c^2d-21e\right)d^2}{x^6}+\frac{\left(5d^2c^4-21dec^2+35e^2\right)d}{x^4}+\frac{5d^3c^8-21d^2ec^6+35de^2c^4-35e^3c^2}{c^2x^2+1}+\frac{-5d^3c^6+\ldots}{}\right.$$
$$\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}+$$
$$\frac{1}{70}bc\left(\frac{d^2\left(5c^2d-21e\right)}{2x^4}-\frac{d\left(5c^4d^2-21c^2de+35e^2\right)}{x^2}-\log\left(x^2\right)\left(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3\right)+\left(5c^6d^3-21\ldots\right.\right.$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^3*(a + b*ArcTan[c*x]))/x^7 - (3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5)
 - (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (e^3*(a + b*ArcTan[c*x]))/x + (b*c*((
-5*d^3)/(3*x^6) + (d^2*(5*c^2*d - 21*e))/(2*x^4) - (d*(5*c^4*d^2 - 21*c^2*
d*e + 35*e^2))/x^2 - (5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e^3)*Lo
g[x^2] + (5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e^3)*Log[1 + c^2*x^
2]))/70
```

3.1148. $\int \frac{\left(d+ex^2\right)^3(a+b\arctan(cx))}{x^8}\,dx$

3.1148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1148.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.10

method	result
parts	$a\left(-\frac{e^3}{x}-\frac{d^3}{7x^7}-\frac{d\,e^2}{x^3}-\frac{3d^2e}{5x^5}\right)+b\,c^7\left(-\frac{\arctan(cx)e^3}{c^7x}-\frac{\arctan(cx)d^3}{7c^7x^7}-\frac{\arctan(cx)e^2d}{c^7x^3}-\frac{3\arctan(cx)d}{5c^7x^5}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{3d^2e}{5c\,x^5}-\frac{e^3}{cx}-\frac{d^3}{7c\,x^7}-\frac{d\,e^2}{c\,x^3}\right)}{c^6}+\frac{b\left(-\frac{3\arctan(cx)d^2e}{5c\,x^5}-\frac{\arctan(cx)e^3}{cx}-\frac{\arctan(cx)d^3}{7c\,x^7}-\frac{\arctan(cx)d\,e^2}{c\,x^3}-\frac{(-5c^6d^3+21c^4d}{}\right.}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{3d^2e}{5c\,x^5}-\frac{e^3}{cx}-\frac{d^3}{7c\,x^7}-\frac{d\,e^2}{c\,x^3}\right)}{c^6}+\frac{b\left(-\frac{3\arctan(cx)d^2e}{5c\,x^5}-\frac{\arctan(cx)e^3}{cx}-\frac{\arctan(cx)d^3}{7c\,x^7}-\frac{\arctan(cx)d\,e^2}{c\,x^3}-\frac{(-5c^6d^3+21c^4d}{}\right.}{}\right.$
parallelrisch	$-\frac{420ad\,e^2x^4+252a\,d^2e\,x^2+420x^4\arctan(cx)bd\,e^2+252x^2\arctan(cx)b\,d^2e-15x^3d^3c^3b+30x^5c^5d^3b+10bc\,d^3x+60d^3a+}{}$
risch	$\frac{ib(35e^3x^6+35x^4e^2d+21e\,d^2x^2+5d^3)\ln(icx+1)}{70x^7}-\frac{60\ln(x)b\,c^7d^3x^7-30\ln(c^2x^2+1)b\,c^7d^3x^7-252\ln(x)b\,c^5d^2e\,x^7+126}{}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-e^3/x-1/7*d^3/x^7-d*e^2/x^3-3/5*d^2*e/x^5)+b*c^7*(-arctan(c*x)/c^7*e^3
/x-1/7*arctan(c*x)*d^3/c^7/x^7-arctan(c*x)/c^7*e^2*d/x^3-3/5*arctan(c*x)/c
^7*d^2*e/x^5-1/35/c^6*((5*c^6*d^3-21*c^4*d^2*e+35*c^2*d*e^2-35*e^3)*ln(c*x
)-1/4*d^2*(5*c^2*d-21*e)/x^4+5/6*d^3/x^6+1/2*d*(5*c^4*d^2-21*c^2*d*e+35*e^
2)/x^2+1/2*(-5*c^6*d^3+21*c^4*d^2*e-35*c^2*d*e^2+35*e^3)*ln(c^2*x^2+1)))
```

3.1148.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08

$$\int\frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^8}\,dx =$$
$$-\frac{420\,ae^3x^6-6\,(5\,bc^7d^3-21\,bc^5d^2e+35\,bc^3de^2-35\,bce^3)x^7\log\left(c^2x^2+1\right)+12\,(5\,bc^7d^3-21\,bc^5d^2e+35}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/420*(420*a*e^3*x^6 - 6*(5*b*c^7*d^3 - 21*b*c^5*d^2*e + 35*b*c^3*d*e^2 -
 35*b*c*e^3)*x^7*log(c^2*x^2 + 1) + 12*(5*b*c^7*d^3 - 21*b*c^5*d^2*e + 35*
b*c^3*d*e^2 - 35*b*c*e^3)*x^7*log(x) + 420*a*d*e^2*x^4 + 10*b*c*d^3*x + 25
2*a*d^2*e*x^2 + 6*(5*b*c^5*d^3 - 21*b*c^3*d^2*e + 35*b*c*d*e^2)*x^5 + 60*a
*d^3 - 3*(5*b*c^3*d^3 - 21*b*c*d^2*e)*x^3 + 12*(35*b*e^3*x^6 + 35*b*d*e^2*
x^4 + 21*b*d^2*e*x^2 + 5*b*d^3)*arctan(c*x))/x^7
```

3.1148.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^8}\,dx$$
$$= \begin{cases} -\frac{ad^3}{7x^7} - \frac{3ad^2e}{5x^5} - \frac{ade^2}{x^3} - \frac{ae^3}{x} - \frac{bc^7d^3\log(x)}{7} + \frac{bc^7d^3\log\left(x^2+\frac{1}{c^2}\right)}{14} - \frac{bc^5d^3}{14x^2} + \frac{3bc^5d^2e\log(x)}{5} - \frac{3bc^5d^2e\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^3}{28x^4} \\ a\left(-\frac{d^3}{7x^7} - \frac{3d^2e}{5x^5} - \frac{de^2}{x^3} - \frac{e^3}{x}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**8,x)
```

output
```
Piecewise((-a*d**3/(7*x**7) - 3*a*d**2*e/(5*x**5) - a*d*e**2/x**3 - a*e**3
/x - b*c**7*d**3*log(x)/7 + b*c**7*d**3*log(x**2 + c**(-2))/14 - b*c**5*d*
*3/(14*x**2) + 3*b*c**5*d**2*e*log(x)/5 - 3*b*c**5*d**2*e*log(x**2 + c**(-
2))/10 + b*c**3*d**3/(28*x**4) + 3*b*c**3*d**2*e/(10*x**2) - b*c**3*d*e**2
*log(x) + b*c**3*d*e**2*log(x**2 + c**(-2))/2 - b*c*d**3/(42*x**6) - 3*b*c
*d**2*e/(20*x**4) - b*c*d*e**2/(2*x**2) + b*c*e**3*log(x) - b*c*e**3*log(x
**2 + c**(-2))/2 - b*d**3*atan(c*x)/(7*x**7) - 3*b*d**2*e*atan(c*x)/(5*x**
5) - b*d*e**2*atan(c*x)/x**3 - b*e**3*atan(c*x)/x, Ne(c, 0)), (a*(-d**3/(7
*x**7) - 3*d**2*e/(5*x**5) - d*e**2/x**3 - e**3/x), True))
```

3.1148.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right)-6\,c^6\log\left(x^2\right)-\frac{6\,c^4x^4-3\,c^2x^2+2}{x^6}\right)c-\frac{12\arctan\left(cx\right)}{x^7}\right)bd^3$$
$$-\frac{3}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bd^2e$$
$$+\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bde^2$$
$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)be^3-\frac{ae^3}{x}-\frac{ade^2}{x^3}-\frac{3\,ad^2e}{5\,x^5}-\frac{ad^3}{7\,x^7}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 +
2)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^3 - 3/20*((2*c^4*log(c^2*x^2 + 1) - 2*
c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2*e + 1/2*(
(c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d*e
^2 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e^3 - a*e^3
/x - a*d*e^2/x^3 - 3/5*a*d^2*e/x^5 - 1/7*a*d^3/x^7
```

3.1148.8 Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8}\,dx = \int \frac{(ex^2+d)^3(b\arctan\left(cx\right)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

output
```
sage0*x
```

3.1148.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8}\,dx = \ln\left(c^2\,x^2+1\right)\left(\frac{b\,c^7\,d^3}{14}-\frac{3\,b\,c^5\,d^2\,e}{10}+\frac{b\,c^3\,d\,e^2}{2}-\frac{b\,c\,e^3}{2}\right)$$
$$-\ln\left(x\right)\left(\frac{b\,c^7\,d^3}{7}-\frac{3\,b\,c^5\,d^2\,e}{5}+b\,c^3\,d\,e^2-b\,c\,e^3\right)$$
$$-\frac{5\,a\,d^3-x^3\left(\frac{5\,b\,c^3\,d^3}{4}-\frac{21\,b\,c\,d^2\,e}{4}\right)+x^5\left(\frac{5\,b\,c^5\,d^3}{2}-\frac{21\,b\,c^3\,d^2\,e}{2}+\frac{35\,b\,c\,d\,e^2}{2}\right)+35\,a\,e^3\,x^6+\frac{5\,b\,c\,d^3\,x}{6}+21\,a\,d^2\,e\,x^2+}{35\,x^7}$$
$$-\frac{\mathrm{atan}(c\,x)\left(\frac{b\,d^3}{7}+\frac{3\,b\,d^2\,e\,x^2}{5}+b\,d\,e^2\,x^4+b\,e^3\,x^6\right)}{x^7}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^8,x)
```

output
```
log(c^2*x^2 + 1)*((b*c^7*d^3)/14 - (b*c*e^3)/2 + (b*c^3*d*e^2)/2 - (3*b*c^
5*d^2*e)/10) - log(x)*((b*c^7*d^3)/7 - b*c*e^3 + b*c^3*d*e^2 - (3*b*c^5*d^
2*e)/5) - (5*a*d^3 - x^3*((5*b*c^3*d^3)/4 - (21*b*c*d^2*e)/4) + x^5*((5*b*
c^5*d^3)/2 + (35*b*c*d*e^2)/2 - (21*b*c^3*d^2*e)/2) + 35*a*e^3*x^6 + (5*b*
c*d^3*x)/6 + 21*a*d^2*e*x^2 + 35*a*d*e^2*x^4)/(35*x^7) - (atan(c*x)*((b*d^
3)/7 + b*e^3*x^6 + (3*b*d^2*e*x^2)/5 + b*d*e^2*x^4))/x^7
```

3.1149 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^9}\,dx$

3.1149.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^9}\,dx = -\frac{bcd^3}{56x^7}+\frac{bcd^2(c^2d-4e)}{40x^5}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3} + \frac{bc(c^2d-2e)(c^4d^2-2c^2de+2e^2)}{8x} + \frac{b(c^2d-e)^4\arctan(cx)}{8d} - \frac{(d+ex^2)^4(a+b\arctan(cx))}{8dx^8}$$

output
```
-1/56*b*c*d^3/x^7+1/40*b*c*d^2*(c^2*d-4*e)/x^5-1/24*b*c*d*(c^4*d^2-4*c^2*d
*e+6*e^2)/x^3+1/8*b*c*(c^2*d-2*e)*(c^4*d^2-2*c^2*d*e+2*e^2)/x+1/8*b*(c^2*d
-e)^4*arctan(c*x)/d-1/8*(e*x^2+d)^4*(a+b*arctan(c*x))/d/x^8
```

3.1149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^9}\,dx =$$
$$-\frac{5bcd^3x\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-c^2x^2\right)+28bcd^2ex^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)+\cdots}{}$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

output
```
-1/280*(5*b*c*d^3*x*Hypergeometric2F1[-7/2, 1, -5/2, -(c^2*x^2)] + 28*b*c*
d^2*e*x^3*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + 35*((d^3 + 4*d^2*
e*x^2 + 6*d*e^2*x^4 + 4*e^3*x^6)*(a + b*ArcTan[c*x]) + 2*b*c*d*e^2*x^5*Hyp
ergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 4*b*c*e^3*x^7*Hypergeometric2F
1[-1/2, 1, 1/2, -(c^2*x^2)]))/x^8
```

3.1149.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5511, 27, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9} \, dx$$

$$\Big\downarrow \text{ 5511}$$

$$-bc\int -\frac{(ex^2+d)^4}{8dx^8 (c^2x^2+1)}dx-\frac{(d+ex^2)^4 (a+b\arctan(cx))}{8dx^8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bc\int \frac{(ex^2+d)^4}{x^8(c^2x^2+1)}dx}{8d}-\frac{(d+ex^2)^4 (a+b\arctan(cx))}{8dx^8}$$

$$\Big\downarrow \text{ 364}$$

$$\frac{bc\int \left(\frac{d^4}{x^8}-\frac{(c^2d-4e)d^3}{x^6}+\frac{(d^2c^4-4dec^2+6e^2)d^2}{x^4}+\frac{(c^2d-2e)(-d^2c^4+2dec^2-2e^2)d}{x^2}+\frac{(c^2d-e)^4}{c^2x^2+1}\right)dx}{8d}-\frac{(d+ex^2)^4 (a+b\arctan(cx))}{8dx^8}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bc\left(\frac{\arctan(cx)(c^2d-e)^4}{c}+\frac{d^3(c^2d-4e)}{5x^5}-\frac{d^2(c^4d^2-4c^2de+6e^2)}{3x^3}+\frac{d(c^2d-2e)(c^4d^2-2c^2de+2e^2)}{x}-\frac{d^4}{7x^7}\right)}{8d}-\frac{(d+ex^2)^4 (a+b\arctan(cx))}{8dx^8}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

output
```
-1/8*((d + e*x^2)^4*(a + b*ArcTan[c*x]))/(d*x^8) + (b*c*(-1/7*d^4/x^7 + (d
^3*(c^2*d - 4*e))/(5*x^5) - (d^2*(c^4*d^2 - 4*c^2*d*e + 6*e^2))/(3*x^3) +
(d*(c^2*d - 2*e)*(c^4*d^2 - 2*c^2*d*e + 2*e^2))/x + ((c^2*d - e)^4*ArcTan[
c*x])/c))/(8*d)
```

3.1149.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1149.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.64

method	result
parts	$a\left(-\frac{d^{2}e}{2x^{6}}-\frac{3d\,e^{2}}{4x^{4}}-\frac{d^{3}}{8x^{8}}-\frac{e^{3}}{2x^{2}}\right)+b\,c^{8}\left(-\frac{\arctan(cx)d^{2}e}{2c^{8}x^{6}}-\frac{3\arctan(cx)e^{2}d}{4c^{8}x^{4}}-\frac{\arctan(cx)d^{3}}{8c^{8}x^{8}}-\frac{\arctan(c}{2c^{8}x^{2}}\right.$
derivativedivides	$c^{8}\left(\frac{a\left(-\frac{3d\,e^{2}}{4c^{2}x^{4}}-\frac{d^{2}e}{2c^{2}x^{6}}-\frac{e^{3}}{2c^{2}x^{2}}-\frac{d^{3}}{8c^{2}x^{8}}\right)}{c^{6}}+\frac{b\left(-\frac{3\arctan(cx)d\,e^{2}}{4c^{2}x^{4}}-\frac{\arctan(cx)d^{2}e}{2c^{2}x^{6}}-\frac{\arctan(cx)e^{3}}{2c^{2}x^{2}}-\frac{\arctan(cx)d^{3}}{8c^{2}x^{8}}-\frac{\left(-c^{6}d^{3}+\right.}{}\right.}{}\right.$
default	$c^{8}\left(\frac{a\left(-\frac{3d\,e^{2}}{4c^{2}x^{4}}-\frac{d^{2}e}{2c^{2}x^{6}}-\frac{e^{3}}{2c^{2}x^{2}}-\frac{d^{3}}{8c^{2}x^{8}}\right)}{c^{6}}+\frac{b\left(-\frac{3\arctan(cx)d\,e^{2}}{4c^{2}x^{4}}-\frac{\arctan(cx)d^{2}e}{2c^{2}x^{6}}-\frac{\arctan(cx)e^{3}}{2c^{2}x^{2}}-\frac{\arctan(cx)d^{3}}{8c^{2}x^{8}}-\frac{\left(-c^{6}d^{3}+\right.}{}\right.}{}\right.$
parallelrisch	$\underline{105x^{8}\arctan(cx)b\,c^{8}d^{3}-420x^{8}\arctan(cx)b\,c^{6}d^{2}e+105b\,c^{7}d^{3}x^{7}+630x^{8}\arctan(cx)b\,c^{4}d\,e^{2}-420b\,c^{5}d^{2}e\,x^{7}-420x^{8}\arctan}$
risch	$\frac{ib\left(4e^{3}x^{6}+6x^{4}e^{2}d+4e\,d^{2}x^{2}+d^{3}\right)\ln(icx+1)}{16x^{8}}-\frac{1260ad\,e^{2}x^{4}+840a\,d^{2}e\,x^{2}+840bc\,e^{3}x^{7}-42x^{3}d^{3}c^{3}b+70x^{5}c^{5}d^{3}b+420ib\,d}{}$

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*d^2*e/x^6-3/4*d*e^2/x^4-1/8*d^3/x^8-1/2*e^3/x^2)+b*c^8*(-1/2*arcta
n(c*x)/c^8*d^2*e/x^6-3/4*arctan(c*x)/c^8*e^2*d/x^4-1/8*arctan(c*x)*d^3/c^8
/x^8-1/2*arctan(c*x)/c^8*e^3/x^2-1/8/c^6*(-(c^6*d^3-4*c^4*d^2*e+6*c^2*d*e^
2-4*e^3)/c/x-1/5/c*d^2*(c^2*d-4*e)/x^5+1/7/c*d^3/x^7+1/3*d/c*(c^4*d^2-4*c^
2*d*e+6*e^2)/x^3+(-c^6*d^3+4*c^4*d^2*e-6*c^2*d*e^2+4*e^3)*arctan(c*x)))
```

3.1149.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^9}\,dx =$$
$$-\frac{420\,ae^3x^6+630\,ade^2x^4-105\,(bc^7d^3-4\,bc^5d^2e+6\,bc^3de^2-4\,bce^3)x^7+15\,bcd^3x+420\,ad^2ex^2+35\,(bc}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="fricas")
```

output
```
-1/840*(420*a*e^3*x^6 + 630*a*d*e^2*x^4 - 105*(b*c^7*d^3 - 4*b*c^5*d^2*e +
 6*b*c^3*d*e^2 - 4*b*c*e^3)*x^7 + 15*b*c*d^3*x + 420*a*d^2*e*x^2 + 35*(b*c
^5*d^3 - 4*b*c^3*d^2*e + 6*b*c*d*e^2)*x^5 + 105*a*d^3 - 21*(b*c^3*d^3 - 4*
b*c*d^2*e)*x^3 + 105*(4*b*e^3*x^6 - (b*c^8*d^3 - 4*b*c^6*d^2*e + 6*b*c^4*d
*e^2 - 4*b*c^2*e^3)*x^8 + 6*b*d*e^2*x^4 + 4*b*d^2*e*x^2 + b*d^3)*arctan(c*
x))/x^8
```

3.1149.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(139) = 278$.

Time = 0.56 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^9}\,dx = & -\frac{ad^3}{8x^8}-\frac{ad^2e}{2x^6}-\frac{3ade^2}{4x^4}-\frac{ae^3}{2x^2}+\frac{bc^8d^3\operatorname{atan}(cx)}{8}\\ & +\frac{bc^7d^3}{8x}-\frac{bc^6d^2e\operatorname{atan}(cx)}{2}-\frac{bc^5d^3}{24x^3}-\frac{bc^5d^2e}{2x}\\ & +\frac{3bc^4de^2\operatorname{atan}(cx)}{4}+\frac{bc^3d^3}{40x^5}+\frac{bc^3d^2e}{6x^3}\\ & +\frac{3bc^3de^2}{4x}-\frac{bc^2e^3\operatorname{atan}(cx)}{2}-\frac{bcd^3}{56x^7}\\ & -\frac{bcd^2e}{10x^5}-\frac{bcde^2}{4x^3}-\frac{bce^3}{2x}-\frac{bd^3\operatorname{atan}(cx)}{8x^8}\\ & -\frac{bd^2e\operatorname{atan}(cx)}{2x^6}-\frac{3bde^2\operatorname{atan}(cx)}{4x^4}-\frac{be^3\operatorname{atan}(cx)}{2x^2}\end{aligned}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**9,x)
```

output
```
-a*d**3/(8*x**8) - a*d**2*e/(2*x**6) - 3*a*d*e**2/(4*x**4) - a*e**3/(2*x**
2) + b*c**8*d**3*atan(c*x)/8 + b*c**7*d**3/(8*x) - b*c**6*d**2*e*atan(c*x)
/2 - b*c**5*d**3/(24*x**3) - b*c**5*d**2*e/(2*x) + 3*b*c**4*d*e**2*atan(c*
x)/4 + b*c**3*d**3/(40*x**5) + b*c**3*d**2*e/(6*x**3) + 3*b*c**3*d*e**2/(4
*x) - b*c**2*e**3*atan(c*x)/2 - b*c*d**3/(56*x**7) - b*c*d**2*e/(10*x**5)
- b*c*d*e**2/(4*x**3) - b*c*e**3/(2*x) - b*d**3*atan(c*x)/(8*x**8) - b*d**
2*e*atan(c*x)/(2*x**6) - 3*b*d*e**2*atan(c*x)/(4*x**4) - b*e**3*atan(c*x)/
(2*x**2)
```

3.1149.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx = \frac{1}{840}\left(\left(105\,c^7\arctan(cx)+\frac{105\,c^6x^6-35\,c^4x^4+21\,c^2x^2-15}{x^7}\right)c-\frac{105\arctan(cx)}{x^8}\right)bd^3 - \frac{1}{30}\left(\left(15\,c^5\arctan(cx)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan(cx)}{x^6}\right)bd^2e + \frac{1}{4}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bde^2 - \frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)be^3 - \frac{ae^3}{2\,x^2} - \frac{3\,ade^2}{4\,x^4} - \frac{ad^2e}{2\,x^6} - \frac{ad^3}{8\,x^8}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="maxima")
```

output
```
1/840*((105*c^7*arctan(c*x) + (105*c^6*x^6 - 35*c^4*x^4 + 21*c^2*x^2 - 15)
/x^7)*c - 105*arctan(c*x)/x^8)*b*d^3 - 1/30*((15*c^5*arctan(c*x) + (15*c^4
*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^2*e + 1/4*((3*c^3*a
rctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d*e^2 - 1/2*((c
*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e^3 - 1/2*a*e^3/x^2 - 3/4*a*d*e
^2/x^4 - 1/2*a*d^2*e/x^6 - 1/8*a*d^3/x^8
```

3.1149.8 Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^9}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="giac")
```

output
```
sage0*x
```

3.1149.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx$$
$$= \frac{b\,c^2\,\mathrm{atan}\left(\frac{b\,c^2\,x\,(2\,e-c^2\,d)\,(c^4\,d^2-2\,c^2\,d\,e+2\,e^2)}{b\,c^7\,d^3-4\,b\,c^5\,d^2\,e+6\,b\,c^3\,d\,e^2-4\,b\,c\,e^3}\right)\,(2\,e-c^2\,d)\,(c^4\,d^2-2\,c^2\,d\,e+2\,e^2)}{8}$$
$$-\frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{8}+\frac{b\,d^2\,e\,x^2}{2}+\frac{3\,b\,d\,e^2\,x^4}{4}+\frac{b\,e^3\,x^6}{2}\right)}{x^8}$$
$$-\frac{a\,d^3-x^3\,\left(\frac{b\,c^3\,d^3}{5}-\frac{4\,b\,c\,d^2\,e}{5}\right)-x^7\,(b\,c^7\,d^3-4\,b\,c^5\,d^2\,e+6\,b\,c^3\,d\,e^2-4\,b\,c\,e^3)+x^5\,\left(\frac{b\,c^5\,d^3}{3}-\frac{4\,b\,c^3\,d^2\,e}{3}+2\,b\,c\right.}{8\,x^8}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^9,x)
```

output
```
(b*c^2*atan((b*c^2*x*(2*e - c^2*d)*(2*e^2 + c^4*d^2 - 2*c^2*d*e))/(b*c^7*d
^3 - 4*b*c*e^3 + 6*b*c^3*d*e^2 - 4*b*c^5*d^2*e))*(2*e - c^2*d)*(2*e^2 + c^
4*d^2 - 2*c^2*d*e))/8 - (atan(c*x)*((b*d^3)/8 + (b*e^3*x^6)/2 + (b*d^2*e*x
^2)/2 + (3*b*d*e^2*x^4)/4))/x^8 - (a*d^3 - x^3*((b*c^3*d^3)/5 - (4*b*c*d^2
*e)/5) - x^7*(b*c^7*d^3 - 4*b*c*e^3 + 6*b*c^3*d*e^2 - 4*b*c^5*d^2*e) + x^5
*((b*c^5*d^3)/3 + 2*b*c*d*e^2 - (4*b*c^3*d^2*e)/3) + 4*a*e^3*x^6 + (b*c*d^
3*x)/7 + 4*a*d^2*e*x^2 + 6*a*d*e^2*x^4)/(8*x^8)
```

3.1150 $\int (c+dx^2)^4 \arctan(ax)\, dx$

3.1150.1 Optimal result

Integrand size = 14, antiderivative size = 244

$$\int (c+dx^2)^4 \arctan(ax)\, dx = -\frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\, x^2}{630a^7} - \frac{d^2(378a^4c^2-180a^2cd+35d^2)\, x^4}{1260a^5} - \frac{(36a^2c-7d)\, d^3x^6}{378a^3} - \frac{d^4x^8}{72a} + c^4x\arctan(ax) + \frac{4}{3}c^3dx^3\arctan(ax) + \frac{6}{5}c^2d^2x^5\arctan(ax) + \frac{4}{7}cd^3x^7\arctan(ax) + \frac{1}{9}d^4x^9\arctan(ax) - \frac{(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log(1+a^2x^2)}{630a^9}$$

output
```
-1/630*d*(420*a^6*c^3-378*a^4*c^2*d+180*a^2*c*d^2-35*d^3)*x^2/a^7-1/1260*d
^2*(378*a^4*c^2-180*a^2*c*d+35*d^2)*x^4/a^5-1/378*(36*a^2*c-7*d)*d^3*x^6/a
^3-1/72*d^4*x^8/a+c^4*x*arctan(a*x)+4/3*c^3*d*x^3*arctan(a*x)+6/5*c^2*d^2*
x^5*arctan(a*x)+4/7*c*d^3*x^7*arctan(a*x)+1/9*d^4*x^9*arctan(a*x)-1/630*(3
15*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+35*d^4)*ln(a^2*x^2+
1)/a^9
```

3.1150.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int \left(c+dx^2\right)^4 \arctan(ax)\,dx =$$
$$-\frac{a^2dx^2(-420d^3+30a^2d^2(72c+7dx^2)-4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+2}{}$$

input
```
Integrate[(c + d*x^2)^4*ArcTan[a*x],x]
```

output
```
-1/7560*(a^2*d*x^2*(-420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) - 4*a^4*d*(1134
*c^2 + 270*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c
*d^2*x^4 + 35*d^3*x^6)) - 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*
x^4 + 180*c*d^3*x^6 + 35*d^4*x^8)*ArcTan[a*x] + 12*(315*a^8*c^4 - 420*a^6*
c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/a^9
```

3.1150.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5447, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(c+dx^2\right)^4dx$$
$$\Big\downarrow \ 5447$$
$$-a\int \frac{x\left(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4\right)}{315\left(a^2x^2+1\right)}dx+c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$
$$\Big\downarrow \ 27$$
$$-\frac{1}{315}a\int \frac{x\left(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4\right)}{a^2x^2+1}dx+c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$
$$\Big\downarrow \ 2331$$

$$-\frac{1}{630}a\int \frac{35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4}{a^2x^2+1}dx^2+c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

$$\Big\downarrow \text{2389}$$

$$-\frac{1}{630}a\int \left(\frac{35d^4x^6}{a^2}+\frac{5(36a^2c-7d)\,d^3x^4}{a^4}+\frac{d^2(378c^2a^4-180cda^2+35d^2)\,x^2}{a^6}+\frac{d(420c^3a^6-378c^2da^4+180cd^2\right.$$
$$c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{630}a\left(\frac{35d^4x^8}{4a^2}+\frac{5d^3x^6(36a^2c-7d)}{3a^4}+\frac{d^2x^4(378a^4c^2-180a^2cd+35d^2)}{2a^6}+\frac{dx^2(420a^6c^3-378a^4c^2d+180a^2cd\right.$$
$$c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

input
```
Int[(c + d*x^2)^4*ArcTan[a*x],x]
```

output
```
c^4*x*ArcTan[a*x] + (4*c^3*d*x^3*ArcTan[a*x])/3 + (6*c^2*d^2*x^5*ArcTan[a*
x])/5 + (4*c*d^3*x^7*ArcTan[a*x])/7 + (d^4*x^9*ArcTan[a*x])/9 - (a*((d*(42
0*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/a^8 + (d^2*(378*a
^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(2*a^6) + (5*(36*a^2*c - 7*d)*d^3*x^6)
/(3*a^4) + (35*d^4*x^8)/(4*a^2) + ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*
c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/a^10))/630
```

3.1150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.1150.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

method	result
parts	$\frac{d^4x^9\arctan(ax)}{9}+\frac{4c\,d^3x^7\arctan(ax)}{7}+\frac{6c^2d^2x^5\arctan(ax)}{5}+\frac{4c^3d\,x^3\arctan(ax)}{3}+c^4x\arctan(ax)-$
derivativedivides	$\arctan(ax)ax\,c^4+\frac{4\arctan(ax)a\,c^3d\,x^3}{3}+\frac{6\arctan(ax)a\,c^2d^2x^5}{5}+\frac{4\arctan(ax)ac\,d^3x^7}{7}+\frac{\arctan(ax)a\,d^4x^9}{9}-\frac{210c^3a^8d\,x^2+\frac{189c^2a^8}{2}}{}$
default	$\arctan(ax)ax\,c^4+\frac{4\arctan(ax)a\,c^3d\,x^3}{3}+\frac{6\arctan(ax)a\,c^2d^2x^5}{5}+\frac{4\arctan(ax)ac\,d^3x^7}{7}+\frac{\arctan(ax)a\,d^4x^9}{9}-\frac{210c^3a^8d\,x^2+\frac{189c^2a^8}{2}}{}$
parallelrisch	$-\frac{-840x^9\arctan(ax)a^9d^4-4320x^7\arctan(ax)a^9c\,d^3+105d^4a^8x^8-9072x^5\arctan(ax)a^9c^2d^2+720c\,a^8d^3x^6-10080x^3\,a}{}$
meijerg	$\frac{d^4\left(\frac{x^2a^2\left(-15x^6a^6+20a^4x^4-30a^2x^2+60\right)}{270}+\frac{4x^{10}a^{10}\arctan\left(\sqrt{a^2x^2}\right)}{9\sqrt{a^2x^2}}-\frac{2\ln\left(a^2x^2+1\right)}{9}\right)}{4a^9}+\frac{d^3c\left(-\frac{x^2a^2\left(4a^4x^4-6a^2x^2+12\right)}{42}+\right.}{}$
risch	$\frac{i\ln(-iax+1)d^4x^9}{18}+\frac{i\ln(-iax+1)x\,c^4}{2}+\frac{2i\ln(-iax+1)d^3c\,x^7}{7}-\frac{d^4x^8}{72a}+\frac{3i\ln(-iax+1)c^2d^2x^5}{5}-\frac{2c\,d^3x^6}{21a}+$

input
```
int((d*x^2+c)^4*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9*arctan(a*x)+4/7*c*d^3*x^7*arctan(a*x)+6/5*c^2*d^2*x^5*arctan(a
*x)+4/3*c^3*d*x^3*arctan(a*x)+c^4*x*arctan(a*x)-1/315*a*(1/2*d/a^8*(35/4*a
^6*d^3*x^8+60*a^6*c*d^2*x^6+189*a^6*c^2*d*x^4+420*a^6*c^3*x^2-35/3*a^4*d^3
*x^6-90*a^4*c*d^2*x^4-378*a^4*c^2*d*x^2+35/2*a^2*d^3*x^4+180*a^2*c*d^2*x^2
-35*d^3*x^2)+1/2*(315*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+
35*d^4)/a^10*ln(a^2*x^2+1))
```

3.1150.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

$$\int \left(c+dx^2\right)^4 \arctan(ax)\,dx =$$

$$-\frac{105\,a^8d^4x^8+20\,(36\,a^8cd^3-7\,a^6d^4)x^6+6\,(378\,a^8c^2d^2-180\,a^6cd^3+35\,a^4d^4)x^4+12\,(420\,a^8c^3d-378\,a$$

input
```
integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 - 7*a^6*d^4)*x^6 + 6*(378*a^8*
c^2*d^2 - 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d - 378*a^6*c^
2*d^2 + 180*a^4*c*d^3 - 35*a^2*d^4)*x^2 - 24*(35*a^9*d^4*x^9 + 180*a^9*c*d
^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*arctan(a
*x) + 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 +
35*d^4)*log(a^2*x^2 + 1))/a^9
```

3.1150.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.29

$$\int \left(c+dx^2\right)^4 \arctan(ax)\,dx$$

$$= \begin{cases} c^4x\operatorname{atan}(ax)+\frac{4c^3dx^3\operatorname{atan}(ax)}{3}+\frac{6c^2d^2x^5\operatorname{atan}(ax)}{5}+\frac{4cd^3x^7\operatorname{atan}(ax)}{7}+\frac{d^4x^9\operatorname{atan}(ax)}{9}-\frac{c^4\log\left(x^2+\frac{1}{a^2}\right)}{2a}-\frac{2c^3dx^2}{3a}-3c \\ 0 \end{cases}$$

input
```
integrate((d*x**2+c)**4*atan(a*x),x)
```

output
```
Piecewise((c**4*x*atan(a*x) + 4*c**3*d*x**3*atan(a*x)/3 + 6*c**2*d**2*x**5
*atan(a*x)/5 + 4*c*d**3*x**7*atan(a*x)/7 + d**4*x**9*atan(a*x)/9 - c**4*lo
g(x**2 + a**(-2))/(2*a) - 2*c**3*d*x**2/(3*a) - 3*c**2*d**2*x**4/(10*a) -
2*c*d**3*x**6/(21*a) - d**4*x**8/(72*a) + 2*c**3*d*log(x**2 + a**(-2))/(3*
a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**3*x**4/(7*a**3) + d**4*x**6/(54*a
**3) - 3*c**2*d**2*log(x**2 + a**(-2))/(5*a**5) - 2*c*d**3*x**2/(7*a**5) -
 d**4*x**4/(36*a**5) + 2*c*d**3*log(x**2 + a**(-2))/(7*a**7) + d**4*x**2/(
18*a**7) - d**4*log(x**2 + a**(-2))/(18*a**9), Ne(a, 0)), (0, True))
```

3.1150.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int \left(c+dx^2\right)^4 \arctan(ax)\,dx =$$
$$-\frac{1}{7560}a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3-7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2-180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d\right.}{a^8}$$
$$+\frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\arctan\left(ax\right)$$

input
```
integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 - 7*a^4*d^4)*x^6 + 6*(378*a
^6*c^2*d^2 - 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d - 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 - 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 - 420*a^6*c^
3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^10) + 1
/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c
^4*x)*arctan(a*x)
```

3.1150.8 Giac [F]

$$\int \left(c+dx^2\right)^4 \arctan(ax)\,dx = \int \left(dx^2+c\right)^4 \arctan\left(ax\right)\,dx$$

input
```
integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.1150.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$\int (c+dx^2)^4 \arctan(ax)\,dx$$
$$= \operatorname{atan}(a\,x)\,\left(c^4\,x+\frac{4\,c^3\,d\,x^3}{3}+\frac{6\,c^2\,d^2\,x^5}{5}+\frac{4\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{9}\right)$$
$$+x^2\left(\frac{\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}-\frac{2\,c^3\,d}{3\,a}\right)+x^6\left(\frac{d^4}{54\,a^3}-\frac{2\,c\,d^3}{21\,a}\right)-x^4\left(\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)$$
$$-\frac{\ln\left(a^2\,x^2+1\right)\,\left(315\,a^8\,c^4-420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2-180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}-\frac{d^4\,x^8}{72\,a}$$

input
```
int(atan(a*x)*(c + d*x^2)^4,x)
```

output
```
atan(a*x)*(c^4*x + (d^4*x^9)/9 + (4*c^3*d*x^3)/3 + (4*c*d^3*x^7)/7 + (6*c^
2*d^2*x^5)/5) + x^2*(((d^4/(9*a^3) - (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5
*a))/(2*a^2) - (2*c^3*d)/(3*a)) + x^6*(d^4/(54*a^3) - (2*c*d^3)/(21*a)) -
x^4*((d^4/(9*a^3) - (4*c*d^3)/(7*a))/(4*a^2) + (3*c^2*d^2)/(10*a)) - (log(
a^2*x^2 + 1)*(35*d^4 + 315*a^8*c^4 - 180*a^2*c*d^3 - 420*a^6*c^3*d + 378*a
^4*c^2*d^2))/(630*a^9) - (d^4*x^8)/(72*a)
```

3.1151 $\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$

3.1151.1 Optimal result

Integrand size = 21, antiderivative size = 361

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx &= -\frac{bx}{2ce}+\frac{b\arctan(cx)}{2c^2e}+\frac{x^2(a+b\arctan(cx))}{2e}\\&\quad+\frac{d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}\\&\quad-\frac{d(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2}\\&\quad-\frac{d(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2}\\&\quad-\frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}\\&\quad+\frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2}\\&\quad+\frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^2}\end{aligned}$$

output
```
-1/2*b*x/c/e+1/2*b*arctan(c*x)/c^2/e+1/2*x^2*(a+b*arctan(c*x))/e+d*(a+b*ar
ctan(c*x))*ln(2/(1-I*c*x))/e^2-1/2*d*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-
x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2-1/2*d*(a+b*arctan(c*x))
*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2-1/2
*I*b*d*polylog(2,1-2/(1-I*c*x))/e^2+1/4*I*b*d*polylog(2,1-2*c*((-d)^(1/2)-
x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/4*I*b*d*polylog(2,1-2
*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2
```

3.1151.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.32

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$$
$$= \frac{-2bcex+2ac^2ex^2+2be\arctan(cx)+2bc^2ex^2\arctan(cx)+ibc^2d\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)-ibc^2d\,\mathrm{lo}}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
(-2*b*c*e*x + 2*a*c^2*e*x^2 + 2*b*e*ArcTan[c*x] + 2*b*c^2*e*x^2*ArcTan[c*x
] + I*b*c^2*d*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] -
I*Sqrt[e])] - I*b*c^2*d*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] + I*Sqrt[e])] - I*b*c^2*d*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e
]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*c^2*d*Log[1 + I*c*x]*Log[(c*(Sqrt[-d
] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - 2*a*c^2*d*Log[d + e*x^2] + I*b
*c^2*d*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*c^2*
d*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])] + I*b*c^2*d*P
olyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])] - I*b*c^2*d*Poly
Log[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(4*c^2*e^2)
```

3.1151.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5451, 5361, 262, 216, 5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$$
$$\Big\downarrow \text{ 5451}$$
$$\frac{\int x(a+b\arctan(cx))dx}{e}-\frac{d\int \frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$
$$\Big\downarrow \text{ 5361}$$
$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int \frac{x^2}{c^2x^2+1}dx}{e}-\frac{d\int \frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$

$$\Big\downarrow \; 5515$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{e}-\frac{d\int\left(\frac{a+b\arctan(cx)}{2\sqrt{e}(\sqrt{e}x+\sqrt{-d})}-\frac{a+b\arctan(cx)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}\right)dx}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{e}-$$

$$\frac{d\left(\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{ib\operatorname{PolyLog}(2,}{}\right.}{e}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/e - (d*(
-(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(
2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e)
 + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*e) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/e
- ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e))/e
```

3.1151. $\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$

3.1151.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1151. $\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$

3.1151.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{ibd\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{c\sqrt{ed}-e}\right)}{4e^2}-\frac{ib\ln(icx+1)x^2}{4e}+\frac{ibd\ln(icx+1)\ln\left(\frac{c\sqrt{ed}+(icx+1)e-e}{c\sqrt{ed}-e}\right)}{4e^2}-\frac{bx}{2ce}+\frac{ibd}{}$
parts	$\frac{a\,x^2}{2e}-\frac{ad\ln\left(e\,x^2+d\right)}{2e^2}+\frac{b\left(\frac{\arctan(cx)c^4x^2}{2e}-\frac{\arctan(cx)c^4d\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}-\frac{c^2\left(\frac{cx-\arctan(cx)}{e}-\frac{d\,c^2\left(-\frac{i\left(\ln(cx-i)\ln\left(e\,c\right.\right.}{}\right.}{}\right)}{}\right)}{}$

3.1151. $\int\frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$

$i\left(\ln(cx-i)\ln\left(\right.\right.$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*b*d/e^2*ln(1-I*c*x)*ln((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)
-e))-1/4*I*b/e*ln(1+I*c*x)*x^2+1/4*I*b*d/e^2*ln(1+I*c*x)*ln((c*(e*d)^(1/2)
+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-e))-1/2*b*x/c/e+1/4*I*b*d/e^2*dilog((c*(e*d
)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*I*b*d/e^2*ln(1+I*c*x)*ln((c*
(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))-1/4*I*b*d/e^2*dilog((c*(e*d)
^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))-1/4*I*b*d/e^2*dilog((c*(e*d)^(1/2
)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*I/c^2*b/e*ln(1-I*c*x)+1/2*a*x^2/e+
1/2/c^2*a/e-1/2*a*d/e^2*ln((1-I*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+e)+1/4*I*b/e*
ln(1-I*c*x)*x^2-1/4*I*b/c^2/e*ln(1+I*c*x)-1/4*I*b*d/e^2*ln(1-I*c*x)*ln((c*
(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*I*b*d/e^2*dilog((c*(e*d)
^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-e))
```

3.1151.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{(b\arctan{(cx)}+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arctan(c*x) + a*x^3)/(e*x^2 + d), x)
```

3.1151.6 Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{x^3(a+b\operatorname{atan}{(cx)})}{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x**2), x)
```

3.1151.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + 2*b*integrate(1/2*x^3*arctan(c*x)/(
e*x^2 + d), x)
```

3.1151.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1152 $\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx$

3.1152.1 Optimal result

Integrand size = 19, antiderivative size = 311

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx &= -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e} \\ &+ \frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e} \\ &+ \frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e} \\ &+ \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e} \\ &- \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e} \\ &- \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e}\end{aligned}$$

output
```
-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1
/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e+1/2*(a+b*arctan(c*x))
*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e+1/2*I
*b*polylog(2,1-2/(1-I*c*x))/e-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2
))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e-1/4*I*b*polylog(2,1-2*c*((-d)^(1/
2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e
```

3.1152.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.42

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = -\frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e} + \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e} + \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e} - \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e} + \frac{a\log(d+ex^2)}{2e} + \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e} + \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e} - \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
((-1/4*I)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*
Sqrt[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] + I*Sqrt[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt
[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/e - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqr
t[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e + (a*Log[d + e*x^2])/(2*e
) + ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))]
)/e + ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])
/e - ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))
])/e - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]
)/e
```

3.1152.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \ 5515$$

$$\int \left(\frac{a+b\arctan(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\arctan(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
-(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(
2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e)
 + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*e) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/e
- ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e
```

3.1152.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1152.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.27

method	result
risch	$\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4e}+\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{c\sqrt{ed}-e}\right)}{4e}+\frac{ib\,\mathrm{dilog}\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4e}+$
parts	$\frac{a\ln(ex^2+d)}{2e}+\frac{b\left(\frac{\arctan(cx)c^2\ln\left(ec^2x^2+c^2d\right)}{2e}-\frac{c^2\left(-\frac{i\left(\ln(cx-i)\ln\left(ec^2x^2+c^2d\right)-2e\left(\frac{\ln(cx-i)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2+2ie\_\right.}{\mathrm{RootOf}\left(e\_Z^2+2\right.}\right.\right.}{}\right.\right)}{}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{ac^2\ln\left(ec^2x^2+c^2d\right)}{2e}+bc^2\left(\frac{\arctan(cx)\ln\left(ec^2x^2+c^2d\right)}{2e}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(ec^2x^2+c^2d\right)-2e\left(\frac{\ln(cx-i)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2+2ie\_\right.}{\mathrm{RootOf}\left(e\_Z^2+\right.}\right.\right.}{}\right.\right)}{}}{}\right.}{}$
default	$\frac{\frac{ac^2\ln\left(ec^2x^2+c^2d\right)}{2e}+bc^2\left(\frac{\arctan(cx)\ln\left(ec^2x^2+c^2d\right)}{2e}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(ec^2x^2+c^2d\right)-2e\left(\frac{\ln(cx-i)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2+2ie\_\right.}{\mathrm{RootOf}\left(e\_Z^2+\right.}\right.\right.}{}\right.\right)}{}}{}\right.}{}$

3.1152. $\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*b*ln(1-I*c*x)/e*ln((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+
1/4*I*b*ln(1-I*c*x)/e*ln((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))+
1/4*I*b/e*dilog((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*I*b/e
*dilog((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))+1/2*a/e*ln((1-I*c*
x)^2*e-c^2*d-2*(1-I*c*x)*e+e)-1/4*I*b*ln(1+I*c*x)/e*ln((c*(e*d)^(1/2)-(1+I
*c*x)*e+e)/(c*(e*d)^(1/2)+e))-1/4*I*b*ln(1+I*c*x)/e*ln((c*(e*d)^(1/2)+(1+I
*c*x)*e-e)/(c*(e*d)^(1/2)-e))-1/4*I*b/e*dilog((c*(e*d)^(1/2)-(1+I*c*x)*e+e
)/(c*(e*d)^(1/2)+e))-1/4*I*b/e*dilog((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d
)^(1/2)-e))
```

3.1152.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arctan(c*x) + a*x)/(e*x^2 + d), x)
```

3.1152.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x(a+b\operatorname{atan}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x**2), x)
```

3.1152.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*x*arctan(c*x)/(e*x^2 + d), x) + 1/2*a*log(e*x^2 + d)/e
```

3.1152.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1153 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)}\,dx$

3.1153.1 Optimal result

Integrand size = 21, antiderivative size = 353

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x(d+ex^2)}\,dx =\ & \frac{a\log(x)}{d}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d} \\ & -\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d} \\ & -\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2d} \\ & +\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d} \\ & -\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d} \\ & +\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4d}\end{aligned}$$

output
```
a*ln(x)/d+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d-1/2*(a+b*arctan(c*x))*ln(2*c
*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d-1/2*(a+b*arc
tan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2))
)/d+1/2*I*b*polylog(2,-I*c*x)/d-1/2*I*b*polylog(2,I*c*x)/d-1/2*I*b*polylog
(2,1-2/(1-I*c*x))/d+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*
x)/(c*(-d)^(1/2)-I*e^(1/2)))/d+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/
2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d
```

3.1153.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.37

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)}\,dx = \frac{a\log(x)}{d} + \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4d} - \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4d} - \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4d} + \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4d} - \frac{a\log\left(d+ex^2\right)}{2d} + \frac{ib\,\text{PolyLog}(2,-icx)}{2d} - \frac{ib\,\text{PolyLog}(2,icx)}{2d} - \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4d} - \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4d} + \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4d} + \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```

output
```
(a*Log[x])/d + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] - I*Sqrt[e])])/d - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt
[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/d - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqr
t[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/d + ((I/4)*b*Log[1 + I*c*x]
*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/d - (a*Log[d +
e*x^2])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c
*x])/d - ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] - Sqrt[
e]))])/d - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[
e])])/d + ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] - Sqrt
[e]))])/d + ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt
[e])])/d
```

3.1153.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)}dx$$

$$\Big\downarrow \text{5463}$$

$$\int \left(\frac{a+b\arctan(cx)}{dx}-\frac{ex(a+b\arctan(cx))}{d(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d}-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}+\frac{a\log(x)}{d}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d}-\frac{ib\,\text{PolyLog}(2,icx)}{2d}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```

output
```
(a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan
[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(2*d) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x])
/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)
])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d
```

3.1153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

3.1153.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-icx+1)}{2d}-\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4d}-\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{c\sqrt{ed}-e}\right)}{4d}-\frac{ib\,\mathrm{dilog}\left(\right.}{}$
derivativedivides	$-\frac{a\ln(e\,c^2x^2+c^2d)}{2d}+\frac{a\ln(cx)}{d}+b\,c^2\left(-\frac{\arctan(cx)\ln(e\,c^2x^2+c^2d)}{2d\,c^2}+\frac{\arctan(cx)\ln(cx)}{d\,c^2}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}}{}\right.$
default	$-\frac{a\ln(e\,c^2x^2+c^2d)}{2d}+\frac{a\ln(cx)}{d}+b\,c^2\left(-\frac{\arctan(cx)\ln(e\,c^2x^2+c^2d)}{2d\,c^2}+\frac{\arctan(cx)\ln(cx)}{d\,c^2}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}}{}\right.$
parts	$\frac{a\ln(x)}{d}-\frac{a\ln(e\,x^2+d)}{2d}+b\left(\frac{\arctan(cx)\ln(cx)}{d}-\frac{\arctan(cx)\ln(e\,c^2x^2+c^2d)}{2d}-\frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2d}+\frac{i\ln(cx)\ln}{c}\right.}{}\right.$

3.1153. $\int\frac{a+b\arctan(cx)}{x(d+ex^2)}\,dx$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b/d*dilog(1-I*c*x)-1/4*I*b/d*ln(1-I*c*x)*ln((c*(e*d)^(1/2)-(1-I*c*x
)*e+e)/(c*(e*d)^(1/2)+e))-1/4*I*b/d*ln(1-I*c*x)*ln((c*(e*d)^(1/2)+(1-I*c*x
)*e-e)/(c*(e*d)^(1/2)-e))-1/4*I*b/d*dilog((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c
*(e*d)^(1/2)+e))-1/4*I*b/d*dilog((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1
/2)-e))+a/d*ln(-I*c*x)-1/2*a/d*ln((1-I*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+e)+1/2
*I*b/d*dilog(1+I*c*x)+1/4*I*b/d*ln(1+I*c*x)*ln((c*(e*d)^(1/2)-(1+I*c*x)*e+
e)/(c*(e*d)^(1/2)+e))+1/4*I*b/d*ln(1+I*c*x)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-
e)/(c*(e*d)^(1/2)-e))+1/4*I*b/d*dilog((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*
d)^(1/2)+e))+1/4*I*b/d*dilog((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-
e))
```

3.1153.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)}\,dx = \int \frac{b\arctan{(cx)}+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^3 + d*x), x)
```

3.1153.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)}\,dx = \int \frac{a+b\operatorname{atan}{(cx)}}{x\,(d+ex^2)}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x**2)), x)
```

3.1153.7 Maxima [F]

$$\int \frac{a + b \arctan(cx)}{x (d + ex^2)} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)x} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(e*
x^3 + d*x), x)
```

3.1153.8 Giac [F]

$$\int \frac{a + b \arctan(cx)}{x (d + ex^2)} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)x} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x (d + ex^2)} \, dx = \int \frac{a + b \operatorname{atan}(c x)}{x (e x^2 + d)} \, dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)), x)
```

3.1154 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx$

3.1154.1 Optimal result

Integrand size = 21, antiderivative size = 409

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3\,(d+ex^2)}\,dx =& -\frac{bc}{2dx}-\frac{bc^2\arctan(cx)}{2d}-\frac{a+b\arctan(cx)}{2dx^2}\\ &-\frac{ae\log(x)}{d^2}-\frac{e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}\\ &+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^2}\\ &+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^2}\\ &-\frac{ibe\,\mathrm{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\,\mathrm{PolyLog}(2,icx)}{2d^2}\\ &+\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}\\ &-\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^2}\\ &-\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^2}\end{aligned}$$

output
```
-1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d+1/2*(-a-b*arctan(c*x))/d/x^2-a*e*ln(x
)/d^2-e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^2+1/2*e*(a+b*arctan(c*x))*ln(2
*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2+1/2*e*(a
+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^
(1/2)))/d^2-1/2*I*b*e*polylog(2,-I*c*x)/d^2+1/2*I*b*e*polylog(2,I*c*x)/d^2
+1/2*I*b*e*polylog(2,1-2/(1-I*c*x))/d^2-1/4*I*b*e*polylog(2,1-2*c*((-d)^(1
/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-1/4*I*b*e*polylog(2
,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2
```

3.1154.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx =$$

$$-\frac{2ad+2bd\arctan(cx)+2bcdx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+4aex^2\log(x)+ibex^2\log(1+icx}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
-1/4*(2*a*d + 2*b*d*ArcTan[c*x] + 2*b*c*d*x*Hypergeometric2F1[-1/2, 1, 1/2
, -(c^2*x^2)] + 4*a*e*x^2*Log[x] + I*b*e*x^2*Log[1 + I*c*x]*Log[(c*(Sqrt[-
d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] - I*b*e*x^2*Log[1 - I*c*x]*Log[
(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*e*x^2*Log[1 - I
*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*e*x^2
*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] -
 2*a*e*x^2*Log[d + e*x^2] + (2*I)*b*e*x^2*PolyLog[2, (-I)*c*x] - (2*I)*b*e
*x^2*PolyLog[2, I*c*x] + I*b*e*x^2*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[
-d] + I*Sqrt[e])] - I*b*e*x^2*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-
d] + Sqrt[e])] + I*b*e*x^2*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d]
+ Sqrt[e])] - I*b*e*x^2*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqr
t[e])])/(d^2*x^2)
```

3.1154.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5453, 5361, 264, 216, 5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^3}dx}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{5463}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \left(\frac{a+b\arctan(cx)}{dx}-\frac{ex(a+b\arctan(cx))}{d(ex^2+d)}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-$$

$$\frac{e\left(-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d}-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}+\frac{a\log(x)}{d}+\right.}{}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2)/d - (e*
((a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTa
n[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*
c*x))])/(2*d) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*
Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x]
)/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x
)])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d]
- I*Sqrt[e])*(1 - I*c*x))])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d))/d
```

3.1154.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

3.1154.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.40

method	result
risch	$-\frac{ib\ln(-icx+1)}{4d\,x^2}-\frac{bc}{2dx}+\frac{ibe\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{c\sqrt{ed}-e}\right)}{4d^2}+\frac{ic^2b\ln(-icx)}{4d}-\frac{ic^2b\ln(icx)}{4d}-\frac{ic^2b\ln(-ic}{4d}$
parts	$a\left(-\frac{1}{2d\,x^2}-\frac{e\ln(x)}{d^2}+\frac{e\ln(e\,x^2+d)}{2d^2}\right)+b\,c^2\left(-\frac{\arctan(cx)}{2d\,c^2x^2}-\frac{\arctan(cx)e\ln(cx)}{c^2d^2}+\frac{\arctan(cx)e\ln(e\,c^2x^2+}{2c^2d^2}\right.$

3.1154. $\int\frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*b/d*ln(1-I*c*x)/x^2-1/2*b*c/d/x+1/4*I*b*e/d^2*ln(1-I*c*x)*ln((c*(e*
d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))+1/4*I*c^2*b/d*ln(-I*c*x)-1/4*I*
c^2*b/d*ln(I*c*x)-1/4*I*c^2*b/d*ln(1-I*c*x)+1/4*I*b*e/d^2*ln(1-I*c*x)*ln((
c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*I*b*e/d^2*dilog((c*(e*
d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))-1/4*I*b*e/d^2*dilog((c*(e*d)^(1
/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))-1/2*a/d/x^2-a/d^2*e*ln(-I*c*x)+1/2*a
*e/d^2*ln((1-I*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+e)+1/2*I*b/d^2*e*dilog(1-I*c*x
)-1/2*I*b*e/d^2*dilog(1+I*c*x)-1/4*I*b*e/d^2*ln(1+I*c*x)*ln((c*(e*d)^(1/2)
-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*I*c^2*b/d*ln(1+I*c*x)+1/4*I*b*e/d^2
*dilog((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))-1/4*I*b*e/d^2*ln(1
+I*c*x)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-e))+1/4*I*b/d*ln(1
+I*c*x)/x^2-1/4*I*b*e/d^2*dilog((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/
2)-e))
```

3.1154.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{b\arctan\left(cx\right)+a}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^5 + d*x^3), x)
```

3.1154.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\mathrm{atan}\left(cx\right)}{x^3\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*(d + e*x**2)), x)
```

3.1154.7 Maxima [F]

$$\int \frac{a + b\arctan(cx)}{x^3\,(d + ex^2)}\,dx = \int \frac{b\arctan\,(cx) + a}{(ex^2 + d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + 2*b*integrate(
1/2*arctan(c*x)/(e*x^5 + d*x^3), x)
```

3.1154.8 Giac [F]

$$\int \frac{a + b\arctan(cx)}{x^3\,(d + ex^2)}\,dx = \int \frac{b\arctan\,(cx) + a}{(ex^2 + d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^3\,(d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{atan}(c\,x)}{x^3\,\left(e\,x^2 + d\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)), x)
```

3.1155 $\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx$

3.1155.1 Optimal result

Integrand size = 21, antiderivative size = 555

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx &= \frac{ax}{e}+\frac{bx\arctan(cx)}{e}-\frac{a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} \\
&\quad-\frac{ib\sqrt{-d}\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad+\frac{ib\sqrt{-d}\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad-\frac{ib\sqrt{-d}\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad+\frac{ib\sqrt{-d}\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad-\frac{b\log\left(1+c^2x^2\right)}{2ce}+\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad-\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad-\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad+\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}}\end{aligned}$$

output
```
a*x/e+b*x*arctan(c*x)/e-1/2*b*ln(c^2*x^2+1)/c/e-1/4*I*b*ln(1+I*c*x)*ln(c*(
(-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b
*ln(1-I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1
/2)/e^(3/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)-
I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x*e^(1
/2))/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,(I-c*x
)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,(
c*x+I)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)-1/4*I*b*polylo
g(2,(1-I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*(-d)^(1/2)/e^(3/2)-1/4*I*b
*polylog(2,(1+I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*(-d)^(1/2)/e^(3/2)-
a*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(3/2)
```

3.1155.2 Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.38

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \frac{ax}{e} - \frac{a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}$$

$$+ \frac{b\left(4cx\arctan(cx) - 2\log\left(1+c^2x^2\right) + \frac{c^2d\left(-4\arctan(cx)\operatorname{arctanh}\left(\frac{cd}{\sqrt{-c^2dex}}\right) - 2\arccos\left(\frac{c^2d+e}{-c^2d+e}\right)\operatorname{arctanh}\left(\frac{cex}{\sqrt{-c^2de}}\right) - \cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
(a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (b*(4*c*x*ArcT
an[c*x] - 2*Log[1 + c^2*x^2] + (c^2*d*(-4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[
-(c^2*d*e)]*x)] - 2*ArcCos[(c^2*d + e)/(-(c^2*d) + e)]*ArcTanh[(c*e*x)/Sqr
t[-(c^2*d*e)]] - (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[(c*e*
x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^
2*d - e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] - (ArcCos[(c^2*d + e)/(-(c^2*d) +
 e)] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*((-I)*e + Sqrt[
-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] + (Ar
cCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x
)] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)
])/(Sqrt[-(c^2*d) + e]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e
)*Cos[2*ArcTan[c*x]]])] + (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcT
anh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])
*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt
[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])] + I*(-PolyLog[2, ((c^
2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c*d + Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)
*(c*d - Sqrt[-(c^2*d*e)]*x))] + PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*
d*e)])*(c*d + Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d - Sqrt[-(c^2*d*e)]*x)
)])))/Sqrt[-(c^2*d*e)]))/(4*c*e)
```

3.1155.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5451, 2009, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int (a+b\arctan(cx))dx}{e}-\frac{d\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5445}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(a\int\frac{1}{ex^2+d}dx+b\int\frac{\arctan(cx)}{ex^2+d}dx\right)}{e}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(b\int\frac{\arctan(cx)}{ex^2+d}dx+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}\right)}{e}$$

$$\Big\downarrow \text{ 5443}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\frac{\log(1-icx)}{ex^2+d}dx-\frac{1}{2}i\int\frac{\log(icx+1)}{ex^2+d}dx\right)\right)}{e}$$

$$\Big\downarrow \text{ 2856}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(1-icx)}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}\log(1-icx)}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx-\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(icx+1)}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}\log(icx+1)}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx\right)\right)}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log(1-icx)\log\left(\frac{c(\sqrt{\ldots}}{c\sqrt{\ldots}}\right)}{2\sqrt{-d}\sqrt{e}}\right.\right.\right.}{}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

3.1155. $\int\frac{x^2(a+b\arctan(cx))}{d+ex^2}dx$

output
```
(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/e - (d*((a*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + b*((-1/2*I)*((Log[1 + I*c*x]*Log[(c
*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) -
 (Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])
/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqr
t[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-
d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])) + (I/2)*((Log[1 - I*c*x]*Log[(c*(Sqrt
[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (Log[
1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sq
rt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e]
)]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*S
qrt[e])]/(2*Sqrt[-d]*Sqrt[e])))))/e
```

3.1155.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.1155.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{iad \operatorname{arctanh}\left(\frac{2(-icx+1)e-2e}{2c\sqrt{ed}}\right)}{e\sqrt{ed}} - \frac{ib\ln(icx+1)x}{2e} - \frac{b\ln(-icx+1)}{2ce} + \frac{b}{ce} - \frac{bd\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4e\sqrt{ed}}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-I*a*d/e/(e*d)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(e*d)^(1/2))-1/2*I*
b/e*ln(1+I*c*x)*x-1/2/c*b/e*ln(1-I*c*x)+b/c/e-1/4*b*d/e*ln(1-I*c*x)/(e*d)^
(1/2)*ln((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*b*d/e*ln(1-I
*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))-1/4*
b*d/e/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1
/4*b*d/e/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e)
)+a*x/e+I/c*a/e+1/2*I*b/e*ln(1-I*c*x)*x-1/2*b/c/e*ln(1+I*c*x)-1/4*b*d/e*ln
(1+I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+
1/4*b*d/e*ln(1+I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d
)^(1/2)-e))-1/4*b*d/e/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(
e*d)^(1/2)+e))+1/4*b*d/e/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(
c*(e*d)^(1/2)-e))
```

3.1155.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e*x^2 + d), x)
```

3.1155.6 Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^2(a+b\operatorname{atan}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/(d + e*x**2), x)
```

3.1155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1155.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1156 $\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx$

3.1156.1 Optimal result

Integrand size = 18, antiderivative size = 517

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = {} & \frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}-\frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}\\
&+\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}\\
&-\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}\\
&+\frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}\\
&-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
-1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))
/(-d)^(1/2)/e^(1/2)+1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d
)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2
)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*ln(1+I*c
*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/
2)+1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/
e^(1/2)+1/4*I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(
1/2)/e^(1/2)-1/4*I*b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))
/(-d)^(1/2)/e^(1/2)-1/4*I*b*polylog(2,(1+I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^
(1/2)))/(-d)^(1/2)/e^(1/2)+a*arctan(x*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)
```

3.1156.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.89

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx$$
$$= \frac{4a\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) - ib\sqrt{d}\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right) + ib\sqrt{d}\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right) - ib\sqrt{d}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2),x]
```

output
```
(4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - I*b*Sqrt[d]*Log[1 + I*c*x]*Log
[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*Sqrt[d]*Log[1
- I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*Sq
rt[d]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e
])] + I*b*Sqrt[d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d
] + I*Sqrt[e])] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] +
 I*Sqrt[e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d]
+ Sqrt[e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] +
 Sqrt[e])] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sq
rt[e])])/(4*Sqrt[-d^2]*Sqrt[e])
```

3.1156.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 510, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{d+ex^2}dx$$

$$\Big\downarrow \text{5445}$$

$$a\int \frac{1}{ex^2+d}dx+b\int \frac{\arctan(cx)}{ex^2+d}dx$$

$$\Big\downarrow \text{218}$$

$$b\int \frac{\arctan(cx)}{ex^2+d}dx+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{5443}$$

$$\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int \frac{\log(1-icx)}{ex^2+d}dx-\frac{1}{2}i\int \frac{\log(icx+1)}{ex^2+d}dx\right)$$

$$\Big\downarrow \text{2856}$$

$$\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+$$
$$b\left(\frac{1}{2}i\int \left(\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{e}x+\sqrt{-d}\right)}\right)dx-\frac{1}{2}i\int \left(\frac{\sqrt{-d}\log(icx+1)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log(icx+1)}{2d\left(\sqrt{e}x+\sqrt{-d}\right)}\right)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+$$
$$b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log(1-icx)\log}{2\sqrt{-d}}\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2),x]
```

output
```
(a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + b*((-1/2*I)*((Log[1 +
I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sqrt[-
d]*Sqrt[e]) - (Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] +
 I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqr
t[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(1 + I*c*x)
)/(I*c*Sqrt[-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])) + (I/2)*((Log[1 - I*c*x]
*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(2*Sqrt[-d]*Sqr
t[e]) - (Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqr
t[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[
-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(I + c*x))/(c*S
qrt[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])))
```

3.1156.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.1156.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.77

method	result
risch	$\frac{b\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4\sqrt{ed}}-\frac{b\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{c\sqrt{ed}-e}\right)}{4\sqrt{ed}}+\frac{b\,\mathrm{dilog}\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4\sqrt{ed}}-\frac{b\,c}{}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}+\frac{ibc^4\ln\left(1-\frac{(c^2d-e)(icx+1)^2}{(c^2x^2+1)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)\arctan(cx)\sqrt{c^2de}\,d}{2e(c^4d^2-2c^2de+e^2)}-\frac{ibc^2\ln\left(1-\frac{(c^2d-e)(icx+1)^2}{(c^2x^2+1)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)}{c^4d^2-2c^2de+e^2}}{}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}+\frac{ibc^4\ln\left(1-\frac{(c^2d-e)(icx+1)^2}{(c^2x^2+1)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)\arctan(cx)\sqrt{c^2de}\,d}{2e(c^4d^2-2c^2de+e^2)}-\frac{ibc^2\ln\left(1-\frac{(c^2d-e)(icx+1)^2}{(c^2x^2+1)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)}{c^4d^2-2c^2de+e^2}}{}$
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{\sqrt{ed}}-\frac{ib\sqrt{c^2de}\arctan(cx)\ln\left(1-\frac{(c^2d-e)(icx+1)^2}{(c^2x^2+1)\left(-c^2d+2\sqrt{c^2de}-e\right)}\right)}{2cde}+\frac{bc^3\arctan(cx)^2\sqrt{c^2de}\,d}{2e(c^4d^2-2c^2de+e^2)}-\frac{bc\arctan}{c^4d^2-}$

input
```
int((a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*ln(1-I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1
/2)+e))-1/4*b*ln(1-I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*
(e*d)^(1/2)-e))+1/4*b/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(
e*d)^(1/2)+e))-1/4*b/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e
*d)^(1/2)-e))+I*a/(e*d)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(e*d)^(1/2
))+1/4*b*ln(1+I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)
^(1/2)+e))-1/4*b*ln(1+I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/
(c*(e*d)^(1/2)-e))+1/4*b/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(
c*(e*d)^(1/2)+e))-1/4*b/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c
*(e*d)^(1/2)-e))
```

3.1156.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx=\int \frac{b\arctan(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^2 + d), x)
```

3.1156.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = \int \frac{a+b\,\mathrm{atan}\,(cx)}{d+ex^2}\,dx$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))/(d + e*x**2), x)
```

3.1156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1156.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = \int \frac{b\arctan(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{d + ex^2} \, dx = \int \frac{a + b \,\mathrm{atan}(c\,x)}{e\,x^2 + d} \, dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2), x)
```

3.1157 $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}\,dx$

3.1157.1 Optimal result

Integrand size = 21, antiderivative size = 561

$$\begin{aligned}
\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}\,dx &= -\frac{a+b\arctan(cx)}{dx}-\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} \\
&\quad+\frac{bc\log(x)}{d}-\frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}} \\
&\quad+\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}} \\
&\quad-\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}} \\
&\quad+\frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{bc\log(1+c^2x^2)}{2d} \\
&\quad+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4(-d)^{3/2}} \\
&\quad-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}
\end{aligned}$$

output
```
(-a-b*arctan(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(c^2*x^2+1)/d-a*arctan(x*e^(1
/2)/d^(1/2))*e^(1/2)/d^(3/2)-1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2
))/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(3/2)+1/4*I*b*ln(1-I*c*x)*ln(c*(
(-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/4*I*b
*ln(1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)
/(-d)^(3/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+
I*e^(1/2)))*e^(1/2)/(-d)^(3/2)+1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(
1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)+1/4*I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(
-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/4*I*b*polylog(2,(1-I*c*x)*e^(1/
2)/(I*c*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/4*I*b*polylog(2,(1+I*c*x
)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(3/2)
```

3.1157.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}\,dx$$
$$= \frac{-\frac{a+b\arctan(cx)}{x} + bc\left(\log(x)-\frac{1}{2}\log\left(1+c^2x^2\right)\right) - \frac{\sqrt{e}\left(4a\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+ib\sqrt{d}\left(\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)+\text{PolyLog}\left(2,\right.\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(-((a + b*ArcTan[c*x])/x) + b*c*(Log[x] - Log[1 + c^2*x^2]/2) - (Sqrt[e]*(
4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*Sqrt[d]*(Log[1 + I*c*x]*Log
[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[e
]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 - I*c*x]*Log[
(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e]
*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 + I*c*x]*Log
[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e
]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) + I*b*Sqrt[d]*(Log[1 - I*c*x]*Lo
g[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[
e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])))/(4*Sqrt[-d^2]))/d
```

3.1157.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5453, 5361, 243, 47, 14, 16, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^2}dx}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{5445}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(a\int \frac{1}{ex^2+d}dx+b\int \frac{\arctan(cx)}{ex^2+d}dx\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(b\int\frac{\arctan(cx)}{ex^2+d}dx+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}\right)}{d}$$

$$\Big\downarrow \text{5443}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\frac{\log(1-icx)}{ex^2+d}dx-\frac{1}{2}i\int\frac{\log(icx+1)}{ex^2+d}dx\right)\right)}{d}$$

$$\Big\downarrow \text{2856}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(1-icx)}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}\log(1-icx)}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx-\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(icx+1)}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}\log(icx+1)}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx\right)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log(1-icx)\log\left(\frac{c(\sqrt{}}{c\sqrt{}}\right.}{2\sqrt{-d}\sqrt{e}}\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2)/d - (e*
((a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + b*((-1/2*I)*((Log[1 +
 I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sqrt[
-d]*Sqrt[e]) - (Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d]
+ I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sq
rt[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(1 + I*c*x
))/(I*c*Sqrt[-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])) + (I/2)*((Log[1 - I*c*x
]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(2*Sqrt[-d]*Sq
rt[e]) - (Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sq
rt[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt
[-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(I + c*x))/(c*
Sqrt[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])))))/d
```

3.1157. $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}\,dx$

3.1157.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

3.1157. $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}\,dx$

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.1157.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 537, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{be\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4d\sqrt{ed}}+\frac{be\ln(-icx+1)\ln\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{c\sqrt{ed}-e}\right)}{4d\sqrt{ed}}-\frac{be\,\mathrm{dilog}\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{c\sqrt{ed}+e}\right)}{4d\sqrt{ed}}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*e/d*ln(1-I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*
d)^(1/2)+e))+1/4*b*e/d*ln(1-I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1-I*c*x)
*e-e)/(c*(e*d)^(1/2)-e))-1/4*b*e/d/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)-(1-I*c
*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*b*e/d/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)+(1-
I*c*x)*e-e)/(c*(e*d)^(1/2)-e))+1/2*c*b/d*ln(-I*c*x)-1/2*c*b/d*ln(1-I*c*x)+
1/2*I*b/d*ln(1+I*c*x)/x-1/2*I*b/d*ln(1-I*c*x)/x-a/d/x-1/4*b*e/d*ln(1+I*c*x
)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*b*e/
d*ln(1+I*c*x)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-
e))-1/4*b*e/d/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/
2)+e))+1/4*b*e/d/(e*d)^(1/2)*dilog((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^
(1/2)-e))+1/2*b*c/d*ln(I*c*x)-1/2*b*c/d*ln(1+I*c*x)-I*a*e/d/(e*d)^(1/2)*ar
ctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(e*d)^(1/2))
```

3.1157.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{b\arctan\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^4 + d*x^2), x)
```

3.1157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d),x)
```

output
```
Timed out
```

3.1157.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1157.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{b\arctan\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)), x)
```

3.1158 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$

3.1158.1 Optimal result

Integrand size = 21, antiderivative size = 403

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx =& -\frac{bc^2d\arctan(cx)}{2(c^2d-e)e^2}+\frac{d(a+b\arctan(cx))}{2e^2(d+ex^2)}\\ &+\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2(c^2d-e)e^{3/2}}-\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}\\ &+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2}\\ &+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2}\\ &+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}\\ &-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2}\\ &-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^2}\end{aligned}$$

output
```
-1/2*b*c^2*d*arctan(c*x)/(c^2*d-e)/e^2+1/2*d*(a+b*arctan(c*x))/e^2/(e*x^2+
d)-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^2+1/2*(a+b*arctan(c*x))*ln(2*c*((-d
)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/2*(a+b*arctan
(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e
^2+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e^2-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2
)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2-1/4*I*b*polylog(2,1-2
*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2+1/2*b*c*
arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(c^2*d-e)/e^(3/2)
```

3.1158.2 Mathematica [A] (verified)

Time = 7.19 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.30

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{2a\left(\frac{d}{d+ex^2}+\log\left(d+ex^2\right)\right)+b\left(-\frac{2c^2d\arctan(cx)}{c^2d-e}+\frac{2d\arctan(cx)}{d+ex^2}+\frac{2c\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{c^2d-e}+2\arctan(cx)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
(2*a*(d/(d + e*x^2) + Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x])/(c^2*d -
 e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(c^2*d - e) + 2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x]
+ 2*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e
] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqr
t[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Lo
g[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e]
)] + I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] +
 Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e]
)] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] + I*P
olyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*PolyLog[2
, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*e^2)
```

3.1158.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(\frac{x(a+b\arctan(cx))}{e(d+ex^2)}-\frac{dx(a+b\arctan(cx))}{e(d+ex^2)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{d(a+b\arctan(cx))}{2e^2(d+ex^2)}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e^2}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e^2}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^2}+\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}(c^2d-e)}-\frac{bc^2d\arctan(cx)}{2e^2(c^2d-e)}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e^2}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^2) + (d*(a + b*ArcTan[c*x]))/(2*
e^2*(d + e*x^2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(c^2*d - e
)*e^(3/2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 + ((a + b*ArcTan
[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(2*e^2) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) + ((I/2)*b*PolyLog[2, 1 - 2/
(1 - I*c*x)])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c
*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2
```

3.1158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1158.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.78

method	result
parts	$\frac{ad}{2e^2(e\,x^2+d)}+\frac{a\ln\left(e\,x^2+d\right)}{2e^2}+\frac{b\left(\frac{\arctan(cx)c^6d}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{\arctan(cx)c^4\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}-\frac{c^4\left(-\frac{i\left(\ln(cx-i)\ln\left(e\,c^2x^2+c^2d\right)-2e\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$
derivativedivides	$\frac{\frac{a\,c^6d}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^4\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}+b\,c^4\left(\frac{\arctan(cx)d\,c^2}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{\arctan(cx)\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(e\,c^2x^2+c^2d\right)-\cdots\right)}{\cdots}}{\cdots}\right)}{\cdots}$
default	$\frac{\frac{a\,c^6d}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{a\,c^4\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}+b\,c^4\left(\frac{\arctan(cx)d\,c^2}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{\arctan(cx)\ln\left(e\,c^2x^2+c^2d\right)}{2e^2}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(e\,c^2x^2+c^2d\right)-\cdots\right)}{\cdots}}{\cdots}\right)}{\cdots}$

3.1158. $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$

$\frac{ib\,\mathrm{dilog}\left(\frac{c\sqrt{ed}-(-icx+1)e+e}{\cdots}\right)}{\cdots}\cdots\frac{ib\,\mathrm{dilog}\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{\cdots}\right)}{\cdots}\cdots\frac{ib\ln(icx+1)\ln\left(\frac{c\sqrt{ed}-(icx+1)e+e}{\cdots}\right)}{\cdots}\cdots$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e^2*d/(e*x^2+d)+1/2*a/e^2*ln(e*x^2+d)+b/c^4*(1/2*arctan(c*x)*c^6*d/e
^2/(c^2*e*x^2+c^2*d)+1/2*arctan(c*x)*c^4/e^2*ln(c^2*e*x^2+c^2*d)-1/2*c^4*(
1/e^2*(-1/2*I*(ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootO
f(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=1))+ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+
2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z
^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2
)))/e))+1/2*I*(ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootO
f(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=1))+ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-
2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z
^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2
)))/e)))+d*c^2/e^2/(c^2*d-e)*arctan(c*x)-d*c/e/(c^2*d-e)/(e*d)^(1/2)*arcta
n(e*x/(e*d)^(1/2))))
```

3.1158.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan{(cx)}+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arctan(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.1158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1158.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + 2*b*integrate(1/2*x^3*a
rctan(c*x)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.1158.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

3.1159 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$

3.1159.1 Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{bc^2\arctan(cx)}{2(c^2d-e)e} - \frac{a+b\arctan(cx)}{2e(d+ex^2)} - \frac{bc\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}(c^2d-e)\sqrt{e}}$$

output
```
1/2*b*c^2*arctan(c*x)/(c^2*d-e)/e+1/2*(-a-b*arctan(c*x))/e/(e*x^2+d)-1/2*b
*c*arctan(x*e^(1/2)/d^(1/2))/(c^2*d-e)/d^(1/2)/e^(1/2)
```

3.1159.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{a\sqrt{d}(c^2d-e) - b\sqrt{d}e(1+c^2x^2)\arctan(cx) + bc\sqrt{e}(d+ex^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e(-c^2d+e)(d+ex^2)}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a*Sqrt[d]*(c^2*d - e) - b*Sqrt[d]*e*(1 + c^2*x^2)*ArcTan[c*x] + b*c*Sqrt[
e]*(d + e*x^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e*(-(c^2*d) + e)*(d
 + e*x^2))
```

3.1159.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5509, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)(ex^2+d)}dx}{2e}-\frac{a+b\arctan(cx)}{2e\,(d+ex^2)}$$

$$\Big\downarrow \text{303}$$

$$\frac{bc\left(\frac{c^2\int \frac{1}{c^2x^2+1}dx}{c^2d-e}-\frac{e\int \frac{1}{ex^2+d}dx}{c^2d-e}\right)}{2e}-\frac{a+b\arctan(cx)}{2e\,(d+ex^2)}$$

$$\Big\downarrow \text{216}$$

$$\frac{bc\left(\frac{c\arctan(cx)}{c^2d-e}-\frac{e\int \frac{1}{ex^2+d}dx}{c^2d-e}\right)}{2e}-\frac{a+b\arctan(cx)}{2e\,(d+ex^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{bc\left(\frac{c\arctan(cx)}{c^2d-e}-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(c^2d-e)}\right)}{2e}-\frac{a+b\arctan(cx)}{2e\,(d+ex^2)}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])/(e*(d + e*x^2)) + (b*c*((c*ArcTan[c*x])/(c^2*d -
e) - (Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d - e))))/(2*e)
```

3.1159.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1159.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09

method	result
parts	$-\frac{a}{2e(e x^2+d)}-\frac{b c^2 \arctan(cx)}{2e(e c^2 x^2+c^2 d)}+\frac{b c^2 \arctan(cx)}{2(c^2 d-e)e}-\frac{bc \arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2(c^2 d-e)\sqrt{ed}}$
derivativedivides	$\frac{-\frac{a c^4}{2e\left(e c^2 x^2+c^2 d\right)}+b c^4\left(-\frac{\arctan(cx)}{2e\left(e c^2 x^2+c^2 d\right)}+\frac{-\frac{e \arctan\left(\frac{ex}{\sqrt{ed}}\right)}{\left(c^2 d-e\right)c\sqrt{ed}}+\frac{\arctan(cx)}{c^2 d-e}}{2e}\right)}{c^2}$
default	$\frac{-\frac{a c^4}{2e\left(e c^2 x^2+c^2 d\right)}+b c^4\left(-\frac{\arctan(cx)}{2e\left(e c^2 x^2+c^2 d\right)}+\frac{-\frac{e \arctan\left(\frac{ex}{\sqrt{ed}}\right)}{\left(c^2 d-e\right)c\sqrt{ed}}+\frac{\arctan(cx)}{c^2 d-e}}{2e}\right)}{c^2}$
risch	$\frac{ib\ln(icx+1)}{4e(e x^2+d)}-\frac{ic^2 b\ln\left((-icx+1)^2 e-c^2 d-2(-icx+1)e+e\right)}{8(c^2 d-e)e}-\frac{icb\,\operatorname{arctanh}\left(\frac{2(-icx+1)e-2e}{2c\sqrt{ed}}\right)}{4(c^2 d-e)\sqrt{ed}}-\frac{ic^4 b\ln(-icx+1)x}{4(c^2 d-e)(-e c^2 x^2-}$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)-1/2*b*c^2*arctan(c*x)/e/(c^2*e*x^2+c^2*d)+1/2*b*c^2*arc
tan(c*x)/(c^2*d-e)/e-1/2*b*c/(c^2*d-e)/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))
```

3.1159.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.57

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$
$$= \left[-\frac{2\,ac^2d^2 - 2\,ade - (bcex^2 + bcd)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right) - 2\left(bc^2dex^2 + bde\right)\arctan\left(cx\right)}{4\left(c^2d^3e - d^2e^2 + (c^2d^2e^2 - de^3)x^2\right)},\right.$$
$$\left. -\frac{ac^2d^2 - ade + (bcex^2 + bcd)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right) - \left(bc^2dex^2 + bde\right)\arctan\left(cx\right)}{2\left(c^2d^3e - d^2e^2 + (c^2d^2e^2 - de^3)x^2\right)}\right]$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 - 2*a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(-d*e)*log((e*x^2 -
 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(b*c^2*d*e*x^2 + b*d*e)*arctan(c*x))
/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2), -1/2*(a*c^2*d^2 - a*d*
e + (b*c*e*x^2 + b*c*d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (b*c^2*d*e*x^2 +
 b*d*e)*arctan(c*x))/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2)]
```

3.1159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1159.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1159.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1159.9 Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 696, normalized size of antiderivative = 7.65

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{b\,c\,\ln\left(e\,x+\sqrt{-d\,e}\right)\,\sqrt{-d\,e}}{4\,d\,e^2-4\,c^2\,d^2\,e} - \frac{2\,b\,c^2\,\mathrm{atan}\left(-\frac{\frac{c^2\left(c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e+\frac{c^2\,x\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)1\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)1\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)}{4\,e^2-4\,c^2\,d\,e} - \frac{c^2\left(-c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-4\,c \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{c^2\left(c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e+\frac{c^2\,x\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)1\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)1\mathrm{i}}{4\,e^2-4\,c^2\,d\,e}\right)1\mathrm{i}}{4\,e^2-4\,c^2\,d\,e} + \frac{c^2\left(-c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-4\,c \cdots\right.}{\cdots}\right.}{\cdots}}\right.}{4\,e^2-4\,c^2\,d\,e} - \frac{b\,\mathrm{atan}(c\,x)}{2\,e\,\left(e\,x^2+d\right)} - \frac{b\,c\,\ln\left(e\,x-\sqrt{-d\,e}\right)\,\sqrt{-d\,e}}{4\,\left(d\,e^2-c^2\,d^2\,e\right)} - \frac{a}{2\,e^2\,x^2+2\,d\,e}$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
(b*c*log(e*x + (-d*e)^(1/2))*(-d*e)^(1/2))/(4*d*e^2 - 4*c^2*d^2*e) - (2*b*
c^2*atan(-((c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e + (c^2*x*(8*c
^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*
d*e))*1i)/(4*e^2 - 4*c^2*d*e) + c^8*e*x))/(4*e^2 - 4*c^2*d*e) - (c^2*((c^2
*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e - (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4
- 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*
c^2*d*e) - c^8*e*x))/(4*e^2 - 4*c^2*d*e))/((c^2*((c^2*(2*c^5*e^3 - 4*c^7*d
*e^2 + 2*c^9*d^2*e + (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c
^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) + c^8*e*x)*1
i)/(4*e^2 - 4*c^2*d*e) + (c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e
 - (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(
4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) - c^8*e*x)*1i)/(4*e^2 - 4*c^2*
d*e))))/(4*e^2 - 4*c^2*d*e) - (b*atan(c*x))/(2*e*(d + e*x^2)) - (b*c*log(e
*x - (-d*e)^(1/2))*(-d*e)^(1/2))/(4*(d*e^2 - c^2*d^2*e)) - a/(2*d*e + 2*e^
2*x^2)
```

3.1159. $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$

3.1160 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^2}\,dx$

3.1160.1 Optimal result

Integrand size = 21, antiderivative size = 443

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x(d+ex^2)^2}\,dx = &-\frac{bc^2\arctan(cx)}{2d(c^2d-e)}+\frac{a+b\arctan(cx)}{2d(d+ex^2)}+\frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}(c^2d-e)}\\
&+\frac{a\log(x)}{d^2}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}\\
&-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^2}\\
&-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^2}\\
&+\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d^2}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d^2}\\
&-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^2}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^2}\end{aligned}$$

output
```
-1/2*b*c^2*arctan(c*x)/d/(c^2*d-e)+1/2*(a+b*arctan(c*x))/d/(e*x^2+d)+a*ln(
x)/d^2+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^2-1/2*(a+b*arctan(c*x))*ln(2*c*
((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-1/2*(a+b*ar
ctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)
))/d^2+1/2*I*b*polylog(2,-I*c*x)/d^2-1/2*I*b*polylog(2,I*c*x)/d^2-1/2*I*b*
polylog(2,1-2/(1-I*c*x))/d^2+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2)
)/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2+1/4*I*b*polylog(2,1-2*c*((-d)^(1
/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2+1/2*b*c*arctan(x*e^
(1/2)/d^(1/2))*e^(1/2)/d^(3/2)/(c^2*d-e)
```

3.1160.2 Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.33

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx$$
$$= \frac{2a\left(\frac{d}{d+ex^2}+2\log(x)-\log\left(d+ex^2\right)\right)+b\left(-\frac{2c^2d\arctan(cx)}{c^2d-e}+\frac{2d\arctan(cx)}{d+ex^2}+\frac{2c\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{c^2d-e}+4\arctan(cx)\log(x)\ldots\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
(2*a*(d/(d + e*x^2) + 2*Log[x] - Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x
])/(c^2*d - e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(c^2*d - e) + 4*ArcTan[c*x]*Log[x] - 2*ArcTan[c*x
]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - 2*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e]
+ x] - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqr
t[d] - Sqrt[e])] - (2*I)*Log[x]*Log[1 - I*c*x] + I*Log[((-I)*Sqrt[d])/Sqrt
[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] + I*Log[(I*Sqrt[
d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)
*Log[x]*Log[1 + I*c*x] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 +
I*c*x))/(c*Sqrt[d] + Sqrt[e])] + (2*I)*PolyLog[2, (-I)*c*x] - (2*I)*PolyLo
g[2, I*c*x] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e
])] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] - I*
PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[
2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*d^2)
```

3.1160.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{ex(a+b\arctan(cx))}{d^2\left(d+ex^2\right)}+\frac{a+b\arctan(cx)}{d^2x}-\frac{ex(a+b\arctan(cx))}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^2}-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d^2}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}+\frac{a+b\arctan(cx)}{2d\left(d+ex^2\right)}+\frac{a\log(x)}{d^2}+\frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)}-\frac{bc^2\arctan(cx)}{2d\left(c^2d-e\right)}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^2}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{4d^2}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d^2}-\frac{ib\,\text{PolyLog}(2,icx)}{2d^2}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(b*c^2*ArcTan[c*x])/(d*(c^2*d - e)) + (a + b*ArcTan[c*x])/(2*d*(d + e
*x^2)) + (b*c*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c^2*d - e))
 + (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 - ((a + b
*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(
1 - I*c*x))])/(2*d^2) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*
x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + ((I/2)*b*PolyLog[2,
 (-I)*c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 - ((I/2)*b*PolyLog[2, 1
- 2/(1 - I*c*x)])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x
))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 + ((I/4)*b*PolyLog[2, 1 -
(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^2
```

3.1160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1160.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 808, normalized size of antiderivative = 1.82

method	result
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
risch	$-\frac{ib\,\mathrm{dilog}\left(\frac{c\sqrt{ed}+(-icx+1)e-e}{c\sqrt{ed}-e}\right)}{4d^2}-\frac{ib\,c^4e\ln(icx+1)x^2}{4d(c^2d-e)(-e\,c^2x^2-c^2d)}+\frac{ib\,\mathrm{dilog}\left(\frac{c\sqrt{ed}-(icx+1)e+e}{c\sqrt{ed}+e}\right)}{4d^2}+\frac{ib\,\mathrm{dilog}(icx+1)}{2d^2}-$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)/d^2+1/2*a/d/(e*x^2+d)-1/2*a/d^2*ln(e*x^2+d)+b*(arctan(c*x)/d^2*ln(
c*x)+1/2*c^2*arctan(c*x)/d/(c^2*e*x^2+c^2*d)-1/2*arctan(c*x)/d^2*ln(c^2*e*
x^2+c^2*d)-1/2*c^4*(1/d/c^2/(c^2*d-e)*arctan(c*x)-1/d/c^3*e/(c^2*d-e)/(e*d
)^(1/2)*arctan(e*x/(e*d)^(1/2))-I/c^4/d^2*ln(c*x)*ln(1+I*c*x)+I/c^4/d^2*ln
(c*x)*ln(1-I*c*x)-I/c^4/d^2*dilog(1+I*c*x)+I/c^4/d^2*dilog(1-I*c*x)-1/d^2/
c^4*(-1/2*I*(ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf(
e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,ind
ex=1))+ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*
I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,ind
ex=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2
+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))
)/e))+1/2*I*(ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf(
e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,ind
ex=1))+ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*
I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,ind
ex=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2
-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))
)/e)))))
```

3.1160.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.1160.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1160.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + 2*b*integr
ate(1/2*arctan(c*x)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.1160.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^2), x)
```

3.1161 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^2}\,dx$

3.1161.1 Optimal result

Integrand size = 21, antiderivative size = 489

$$
\begin{aligned}
\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^2}\,dx = &-\frac{bc}{2d^2x}-\frac{bc^2\arctan(cx)}{2d^2}+\frac{bc^2e\arctan(cx)}{2d^2(c^2d-e)}-\frac{a+b\arctan(cx)}{2d^2x^2} \\
&-\frac{e(a+b\arctan(cx))}{2d^2(d+ex^2)}-\frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}(c^2d-e)} \\
&-\frac{2ae\log(x)}{d^3}-\frac{2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^3} \\
&+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{d^3} \\
&+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{d^3} \\
&-\frac{ibe\operatorname{PolyLog}(2,-icx)}{d^3}+\frac{ibe\operatorname{PolyLog}(2,icx)}{d^3} \\
&+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3} \\
&-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^3} \\
&-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^3}
\end{aligned}
$$

output
```
-1/2*b*c/d^2/x-1/2*b*c^2*arctan(c*x)/d^2+1/2*b*c^2*e*arctan(c*x)/d^2/(c^2*
d-e)+1/2*(-a-b*arctan(c*x))/d^2/x^2-1/2*e*(a+b*arctan(c*x))/d^2/(e*x^2+d)-
1/2*b*c*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(c^2*d-e)-2*a*e*ln(x)/d^
3-2*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^3+e*(a+b*arctan(c*x))*ln(2*c*((-
d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^3+e*(a+b*arctan(
c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^
3-I*b*e*polylog(2,-I*c*x)/d^3+I*b*e*polylog(2,I*c*x)/d^3+I*b*e*polylog(2,1
-2/(1-I*c*x))/d^3-1/2*I*b*e*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*
x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^3-1/2*I*b*e*polylog(2,1-2*c*((-d)^(1/2)+x*e
^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3
```

3.1161.2 Mathematica [A] (verified)

Time = 9.89 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.31

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx =$$

$$-\frac{a\left(d\left(\frac{1}{x^2}+\frac{e}{d+ex^2}\right)+4e\log(x)-2e\log\left(d+ex^2\right)\right)+b\left(\frac{cd}{x}+\frac{c^2d(c^2d-2e)\arctan(cx)}{c^2d-e}+d\left(\frac{1}{x^2}+\frac{e}{d+ex^2}\right)\arctan(c\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```

output
```
-1/2*(a*(d*(x^(-2) + e/(d + e*x^2)) + 4*e*Log[x] - 2*e*Log[d + e*x^2]) + b
*((c*d)/x + (c^2*d*(c^2*d - 2*e)*ArcTan[c*x])/(c^2*d - e) + d*(x^(-2) + e/
(d + e*x^2))*ArcTan[c*x] + (c*Sqrt[d]*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(c^2*d - e) + 4*e*ArcTan[c*x]*Log[x] - 2*e*ArcTan[c*x]*Log[d + e*x^2] - (
2*I)*e*(Log[x]*(Log[1 - I*c*x] - Log[1 + I*c*x]) - PolyLog[2, (-I)*c*x] +
PolyLog[2, I*c*x]) - e*(2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*
ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] +
x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d]
)/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I
*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] +
 I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqr
t[e])] - 2*ArcTan[c*x]*Log[d + e*x^2] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[
e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(
c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d
] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt
[e])])))/d^3
```

3.1161.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}dx$$

$$\big\downarrow \text{5515}$$

$$\int \left(\frac{2e^2x(a+b\arctan(cx))}{d^3\left(d+ex^2\right)}-\frac{2e(a+b\arctan(cx))}{d^3x}+\frac{e^2x(a+b\arctan(cx))}{d^2\left(d+ex^2\right)^2}+\frac{a+b\arctan(cx)}{d^2x^3}\right)dx$$

$$\big\downarrow \text{2009}$$

$$-\frac{2e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}+\frac{e(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{d^3}+\frac{e(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{d^3}-\frac{e(a+b\arctan(cx))}{2d^2\left(d+ex^2\right)}-\frac{a+b\arctan(cx)}{2d^2x^2}-\frac{2ae\log(x)}{d^3}-\frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\left(c^2d-e\right)}+\frac{bc^2e\arctan(cx)}{2d^2\left(c^2d-e\right)}-\frac{bc^2\arctan(cx)}{2d^2}-\frac{ibe\operatorname{PolyLog}(2,-icx)}{d^3}+\frac{ibe\operatorname{PolyLog}(2,icx)}{d^3}+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3}-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^3}-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{2d^3}-\frac{bc}{2d^2x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```

output
```
-1/2*(b*c)/(d^2*x) - (b*c^2*ArcTan[c*x])/(2*d^2) + (b*c^2*e*ArcTan[c*x])/(
2*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^2*x^2) - (e*(a + b*ArcTan[c*
x]))/(2*d^2*(d + e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^
(5/2)*(c^2*d - e)) - (2*a*e*Log[x])/d^3 - (2*e*(a + b*ArcTan[c*x])*Log[2/(
1 - I*c*x)])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (e*(a + b*ArcTan[c*x])*Log
[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3
 - (I*b*e*PolyLog[2, (-I)*c*x])/d^3 + (I*b*e*PolyLog[2, I*c*x])/d^3 + (I*b
*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 - ((I/2)*b*e*PolyLog[2, 1 - (2*c*(Sq
rt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/2)
*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/d^3
```

3.1161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1161.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.74

method	result
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
risch	$-\frac{bc}{2d^2x}-\frac{ic^4b\,e^2\ln(-icx+1)x^2}{4d^2(c^2d-e)(-e\,c^2x^2-c^2d)}+\frac{ib\,c^4e^2\ln(icx+1)x^2}{4d^2(c^2d-e)(-e\,c^2x^2-c^2d)}-\frac{a}{2d^2x^2}+\frac{c^2ae}{2d^2(-e\,c^2x^2-c^2d)}-\frac{ic^2b\,e^2}{4d^2(c^2d-}$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^2/x^2-2*a*e*ln(x)/d^3-1/2*a*e/d^2/(e*x^2+d)+a*e/d^3*ln(e*x^2+d)+b
*c^2*(-1/2*arctan(c*x)/d^2/c^2/x^2-2/c^2*arctan(c*x)/d^3*e*ln(c*x)-1/2*arc
tan(c*x)*e/d^2/(c^2*e*x^2+c^2*d)+1/c^2*arctan(c*x)*e/d^3*ln(c^2*e*x^2+c^2*
d)-1/2*c^4*(-4/d^3/c^6*e*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*
c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))+2/d^3/c^6*e*(-1/2*I*(ln(c*
x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^
2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(
e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,ind
ex=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootO
f(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,
index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e))+1/2*I*(ln(I+
c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^
2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(
e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,ind
ex=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootO
f(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,
index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e))-1/d^2/c^4*(
-e^2/(c^2*d-e)/c/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))-1/c/x+(-c^2*d+2*e)/(c
^2*d-e)*arctan(c*x))))
```

3.1161.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^2}\,dx=\int \frac{b\arctan(cx)+a}{(ex^2+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

3.1161.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1161.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^2}\,dx=\int \frac{b\arctan(cx)+a}{(ex^2+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e
*log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

3.1161.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^2}\,dx=\int \frac{b\arctan(cx)+a}{(ex^2+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^2), x)
```

3.1162 $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$

3.1162.1 Optimal result

Integrand size = 21, antiderivative size = 1335

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = -\frac{x(a+b\arctan(cx))}{2e(d+ex^2)}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}} - \frac{(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}} - \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}} + \frac{ibc\log\left(-\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}} + \frac{ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}} - \frac{ibc\log\left(\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}} + \frac{bc\log(1+c^2x^2)}{4(c^2d-e)e} - \frac{bc\log(d+ex^2)}{4(c^2d-e)e} + \frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ibc\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}} + \frac{ibc\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}} - \frac{ibc\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}$$

output
```
-1/2*x*(a+b*arctan(c*x))/e/(e*x^2+d)+1/4*b*c*ln(c^2*x^2+1)/(c^2*d-e)/e-1/4
*b*c*ln(e*x^2+d)/(c^2*d-e)/e-1/8*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-
c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/e^(3/2)/(-c^2)^(1/2
)/d^(1/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(
1/2)*d^(1/2)-I*e^(1/2)))/e^(3/2)/(-c^2)^(1/2)/d^(1/2)+1/8*I*b*c*polylog(2,
(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/e^(3/
2)/(-c^2)^(1/2)/d^(1/2)-1/8*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^
(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))/e^(3/2)/(-c^2)^(1/2)/d^(
1/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/
2)))/e^(3/2)/(-d)^(1/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1
/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/e^(3/2)/(-c^2)^(1/2)/d^(1/2)-1/4*I*
b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+e^(1/2)))/e^(3/2)/(-d)^(1/2)
+1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^
(1/2)+a*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/d^(1/2)-1/2*(a+b*arctan(c*x))*ar
ctan(x*e^(1/2)/d^(1/2))/e^(3/2)/d^(1/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/
2)+x*e^(1/2))/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)/(-d)^(1/2)+1/8*I*b*c*polyl
og(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/
e^(3/2)/(-c^2)^(1/2)/d^(1/2)+1/8*I*b*c*ln(-(1-x*(-c^2)^(1/2))*e^(1/2)/(I*(
-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/e^(3/2)/(-c^2)^(1/
2)/d^(1/2)-1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2...
```

3.1162.2 Mathematica [A] (warning: unable to verify)

Time = 9.21 (sec) , antiderivative size = 877, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = -\frac{ax}{2e\,(d+ex^2)} + \frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{3/2}}$$
$$+\frac{bc\left(-\frac{2\log\left(\frac{c^2d+e+\left(c^2d-e\right)\cos(2\arctan(cx))}{c^2d+e}\right)}{c^2d-e}+\frac{-4\arctan(cx)\operatorname{arctanh}\left(\frac{\sqrt{-c^2de}}{cex}\right)+2\arccos\left(\frac{c^2d+e}{-c^2d+e}\right)\operatorname{arctanh}\left(\frac{cex}{\sqrt{-c^2de}}\right)+\left(\operatorname{arcco}\right.}{}\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a*x)/(e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^
(3/2)) + (b*c*((-2*Log[(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d
 + e)])/(c^2*d - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] +
2*ArcCos[(c^2*d + e)/(-(c^2*d) + e)]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (
ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)
]])*Log[(-2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d
 - c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*A
rcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[((2*I)*c^2*d*(e + I*Sqrt[-(c^2*d*e)]
)*(I + c*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[(c^2*
d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + (2*I)*A
rcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[-(c
^2*d) + e]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*ArcT
an[c*x]]])] - (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*ArcTanh[Sqrt[-(c
^2*d*e)]/(c*e*x)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*
Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt[-(c^2*d) - e
+ (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)
*Sqrt[-(c^2*d*e)])*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d - c
*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*
(c^2*d + c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x
))]))/Sqrt[-(c^2*d*e)] - (4*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e ...
```

3.1162.3 Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 1335, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{a+b\arctan(cx)}{e(d+ex^2)}-\frac{d(a+b\arctan(cx))}{e(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2\sqrt{d}e^{3/2}}-\frac{x(a+b\arctan(cx))}{2e(ex^2+d)}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}-\frac{ibc\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{bc\log(c^2x^2+1)}{4(c^2d-e)e}-\frac{bc\log(ex^2+d)}{4(c^2d-e)e}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(icx+1)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}-\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

3.1162. $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$

output
```
-1/2*(x*(a + b*ArcTan[c*x]))/(e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[d]*e^(3/2)) - ((a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*Sqrt[d]*e^(3/2)) - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*
x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 - I*c*x
]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3
/2)) - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d]
- I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-
d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/8)*b*
c*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1
 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-
((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (
I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sq
rt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sq
rt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*Log[(Sqrt[e]*
(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + (b*c*Log[1 + c^2*x^2])/(4*(c^2*
d - e)*e) - (b*c*Log[d + e*x^2])/(4*(c^2*d - e)*e) + ((I/4)*b*PolyLog[2, (
Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*
b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^
(3/2)) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt...
```

3.1162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1162.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2304 vs. 2(991) = 1982.

Time = 1.86 (sec) , antiderivative size = 2305, normalized size of antiderivative = 1.73

method	result	size
parts	Expression too large to display	2305
derivativedivides	Expression too large to display	2344
default	Expression too large to display	2344
risch	Expression too large to display	2391

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4*I*b*c^3*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/
2)-e))*arctan(c*x)*d/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1
/4*I*b*c^5*d^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^
(1/2)-e))*arctan(c*x)/e^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2
)+1/4*I*b/c*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/
2)-e))*arctan(c*x)/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*e*(c^2*d*e)^(1/2)-1
/4*I*b/c*(c^2*d*e)^(1/2)/d/e/(c^2*d-e)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x
)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/2*a/e/(e*d)^(1/2)*arctan(e
*x/(e*d)^(1/2))-1/4*b*c^5*d^2*arctan(c*x)^2/e^2/(c^2*d-e)/(c^4*d^2-2*c^2*d
*e+e^2)*(c^2*d*e)^(1/2)+1/4*b/c*arctan(c*x)^2/d/(c^2*d-e)/(c^4*d^2-2*c^2*d
*e+e^2)*e*(c^2*d*e)^(1/2)+1/8*b/c*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2
+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*e*(c
^2*d*e)^(1/2)+3/4*b*c^3*arctan(c*x)^2*d/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2
)*(c^2*d*e)^(1/2)+1/4*I*b*c*(c^2*d*e)^(1/2)/e^2/(c^2*d-e)*arctan(c*x)*ln(1
-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))-1/2*I*b*c
^3*arctan(c*x)/(c^2*d-e)/e/(c^2*e*x^2+c^2*d)*d-3/4*I*b*c*ln(1-(c^2*d-e)*(1
+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^2*d-e)/
(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/2*b*c^4*arctan(c*x)/(c^2*d-e)/e/
(c^2*e*x^2+c^2*d)*x*d-1/8*b*c^5*d^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x
^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/e^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^...
```

3.1162.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.1162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1162.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1162.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

3.1163 $\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx$

3.1163.1 Optimal result

Integrand size = 18, antiderivative size = 819

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \frac{x(a+b\arctan(cx))}{2d(d+ex^2)} + \frac{(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}$$
$$+ \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\log\left(-\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$+ \frac{ibc\log\left(\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}} - \frac{bc\log(1+c^2x^2)}{4d(c^2d-e)}$$
$$+ \frac{bc\log(d+ex^2)}{4d(c^2d-e)} + \frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$+ \frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$

output
```
1/2*x*(a+b*arctan(c*x))/d/(e*x^2+d)-1/4*b*c*ln(c^2*x^2+1)/d/(c^2*d-e)+1/4*
b*c*ln(e*x^2+d)/d/(c^2*d-e)+1/2*(a+b*arctan(c*x))*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)/e^(1/2)-1/8*I*b*c*ln(-(1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)
*d^(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)+
1/8*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*
ln(1-I*x*e^(1/2)/d^(1/2))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)-1/8*I*b*c*ln(-(1-x*
(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d
^(1/2))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)+1/8*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/
2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/d^(3/2)/(-c
^2)^(1/2)/e^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/(
(-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)-1/8*I*b*c*po
lylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)
))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+
I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d^(3/2)/(-c^2)^(1/2)/e^(1/2
)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(
1/2)+I*e^(1/2)))/d^(3/2)/(-c^2)^(1/2)/e^(1/2)
```

3.1163.2 Mathematica [A] (verified)

Time = 9.12 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \frac{ax}{2d\,(d+ex^2)} + \frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}$$

$$+ \frac{bc\left(\frac{2\log\left(1+\frac{(c^2d-e)\cos(2\arctan(cx))}{c^2d+e}\right)}{c^2d-e} + \frac{-4\arctan(cx)\,\text{arctanh}\left(\frac{\sqrt{-c^2de}}{cex}\right)+2\arccos\left(-\frac{c^2d+e}{c^2d-e}\right)\text{arctanh}\left(\frac{cex}{\sqrt{-c^2de}}\right)-\left(\arccos\left(-\frac{c^2}{c^2}\right.\right.}{}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

output
```
(a*x)/(2*d*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[
e]) + (b*c*((2*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(c^2
*d - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((
c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^
2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c
^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[
-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*
e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/(
(c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(c^2*
d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sq
rt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*Arc
Tan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-((
c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + Ar
cTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTa
n[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]]
)] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-
(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2,
((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^
2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c^2*d*e)] + (4*ArcTan[c
*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])))/...
```

3.1163.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 806, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5447, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx$$
$$\Big\downarrow \ 5447$$
$$-bc\int \frac{\frac{x}{d(ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}}{2\left(c^2x^2+1\right)}dx+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d\left(d+ex^2\right)}$$
$$\Big\downarrow \ 27$$
$$-\frac{1}{2}bc\int \frac{\frac{x}{d(ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}}{c^2x^2+1}dx+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{1}{2}bc\int \left(\frac{x}{d\left(c^2x^2+1\right)\left(ex^2+d\right)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}\left(c^2x^2+1\right)}\right)dx+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d\left(ex^2+d\right)}-\frac{1}{2}bc\left(-\frac{i\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{3/2}\sqrt{e}}+\frac{i\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{3/2}\sqrt{e}}+\frac{i\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)}{4\sqrt{-c^2}d^{3/2}}\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

output
```
(x*(a + b*ArcTan[c*x]))/(2*d*(d + e*x^2)) + ((a + b*ArcTan[c*x])*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]) - (b*c*(((-1/4*I)*Log[(Sqrt[e]*(1
- Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/S
qrt[d]])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/4)*Log[-((Sqrt[e]*(1 + Sqrt[-c
^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/4)*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/
(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-
c^2]*d^(3/2)*Sqrt[e]) - ((I/4)*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c
^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2
)*Sqrt[e]) + Log[1 + c^2*x^2]/(2*d*(c^2*d - e)) - Log[d + e*x^2]/(2*d*(c^2
*d - e)) - ((I/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c
^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/4)*PolyLog[2
, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/
(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*
Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]
) + ((I/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqr
t[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e])))/2
```

3.1163. $\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx$

3.1163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.1163.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2172 vs. 2(611) = 1222.

Time = 1.16 (sec) , antiderivative size = 2173, normalized size of antiderivative = 2.65

method	result	size
risch	Expression too large to display	2173
parts	Expression too large to display	2305
derivativedivides	Expression too large to display	2320
default	Expression too large to display	2320

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*c*b/d/(c^2*d-e)*ln((1-I*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+e)-1/2*c^2*a/d/(-
c^2*e*x^2-c^2*d)*x+1/4*c^2*b/(c^2*d-e)/(e*d)^(1/2)*arctanh(1/2*(2*(1-I*c*x
)*e-2*e)/c/(e*d)^(1/2))+1/4*c^3*b*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)
+1/2*I*a/d/(e*d)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(e*d)^(1/2))-1/8*
c^4*b*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/
2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))*e*x^2+1/4*I*c^2*b*ln(1-I*c*x)/d/(c^2*
d-e)/(-c^2*e*x^2-c^2*d)*e*x-1/8*b*c^4*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^
2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))*e*x^2
+1/8*b*c^4*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d
)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-e))*e*x^2-1/4*I*b*c^2*ln(1+I*c*x)/d/
(c^2*d-e)/(-c^2*e*x^2-c^2*d)*e*x+1/8*c^4*b*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x
^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))*
e*x^2-1/8*c^2*b*ln(1-I*c*x)/d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln(
(c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(c*(e*d)^(1/2)-e))*e^2*x^2+1/8*b*c^2*ln(1+I*
c*x)/d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1+I*c*x
)*e+e)/(c*(e*d)^(1/2)+e))*e^2*x^2-1/8*b*c^2*ln(1+I*c*x)/d/(c^2*d-e)/(-c^2*
e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-e
))*e^2*x^2-1/4*I*c^4*b*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x+1/4*I*b*
c^4*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x+1/8*c^2*b*ln(1-I*c*x)/d/(c^
2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/...
```

3.1163.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.1163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1163.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1163.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^2,x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^2, x)
```

3.1164 $\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

3.1164.1 Optimal result

Integrand size = 21, antiderivative size = 1382

$$
\begin{aligned}
\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx = &-\frac{a+b\arctan(cx)}{d^2x}-\frac{ex(a+b\arctan(cx))}{2d^2\left(d+ex^2\right)} \\
&-\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}-\frac{\sqrt{e}(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}} \\
&+\frac{bc\log(x)}{d^2}+\frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{5/2}} \\
&-\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{5/2}} \\
&+\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{5/2}} \\
&-\frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{5/2}} \\
&-\frac{ibc\sqrt{e}\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}} \\
&+\frac{ibc\sqrt{e}\log\left(-\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}} \\
&+\frac{ibc\sqrt{e}\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}} \\
&-\frac{ibc\sqrt{e}\log\left(\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}} \\
&-\frac{bc\log\left(1+c^2x^2\right)}{2d^2}+\frac{bce\log\left(1+c^2x^2\right)}{4d^2\left(c^2d-e\right)}-\frac{bce\log\left(d+ex^2\right)}{4d^2\left(c^2d-e\right)} \\
&-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{5/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4(-d)^{5/2}} \\
&+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{5/2}} \\
&-\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}} \\
&+\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}} \\
&\quad ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right.
\end{aligned}
$$

output
```
(-a-b*arctan(c*x))/d^2/x-1/2*e*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)+b*c*ln(x)
/d^2-1/2*b*c*ln(c^2*x^2+1)/d^2+1/4*b*c*e*ln(c^2*x^2+1)/d^2/(c^2*d-e)-1/4*b
*c*e*ln(e*x^2+d)/d^2/(c^2*d-e)-a*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)
-1/2*(a+b*arctan(c*x))*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)+1/8*I*b*c
*ln(-(1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x
*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(-c^2)^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^
(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))*e^(1/2)/d^(5
/2)/(-c^2)^(1/2)-1/4*I*b*polylog(2,(I-c*x)*e^(1/2)/(c*(-d)^(1/2)+I*e^(1/2)
))*e^(1/2)/(-d)^(5/2)+1/4*I*b*polylog(2,(1+I*c*x)*e^(1/2)/(I*c*(-d)^(1/2)+
e^(1/2)))*e^(1/2)/(-d)^(5/2)-1/4*I*b*polylog(2,(c*x+I)*e^(1/2)/(c*(-d)^(1/
2)+I*e^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*I*b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c
*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(5/2)+1/8*I*b*c*ln(-(1+x*(-c^2)^(1/2))*
e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))*e^(1/2
)/d^(5/2)/(-c^2)^(1/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/
2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))*e^(1/2)/d^(5/2)/(-c^2)^(1/2)-1/8*I*b
*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*
x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(-c^2)^(1/2)+1/8*I*b*c*polylog(2,(-c^2)
^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))*e^(1/2)/d^(
5/2)/(-c^2)^(1/2)+1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^
(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(5/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/...
```

3.1164.2 Mathematica [A] (verified)

Time = 12.75 (sec) , antiderivative size = 992, normalized size of antiderivative = 0.72

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx = -\frac{a}{d^2x}-\frac{aex}{2d^2(d+ex^2)}-\frac{3a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}}$$
$$+bc^5\left(-\frac{\arctan(cx)}{c^5d^2x}+\frac{\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)}{c^4d^2}-\frac{e\log\left(1-\frac{(-c^2d+e)\cos(2\arctan(cx))}{c^2d+e}\right)}{4c^4d^2(c^2d-e)}\right.$$
$$-\frac{3e\left(4\arctan(cx)\operatorname{arctanh}\left(\frac{cd}{\sqrt{-c^2dex}}\right)+2\arccos\left(\frac{-c^2d-e}{c^2d-e}\right)\operatorname{arctanh}\left(\frac{cex}{\sqrt{-c^2de}}\right)-\left(\arccos\left(\frac{-c^2d-e}{c^2d-e}\right)-2i\operatorname{arct}\right.\right.}{}$$
$$\left.-\frac{e\arctan(cx)\sin(2\arctan(cx))}{2c^4d^2(c^2d+e+c^2d\cos(2\arctan(cx))-e\cos(2\arctan(cx)))}\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
-(a/(d^2*x)) - (a*e*x)/(2*d^2*(d + e*x^2)) - (3*a*Sqrt[e]*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(2*d^(5/2)) + b*c^5*(-(ArcTan[c*x]/(c^5*d^2*x)) + Log[(c*x)/S
qrt[1 + c^2*x^2]]/(c^4*d^2) - (e*Log[1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]
])/(c^2*d + e)])/(4*c^4*d^2*(c^2*d - e)) - (3*e*(4*ArcTan[c*x]*ArcTanh[(c*
d)/(Sqrt[-(c^2*d*e)]*x)] + 2*ArcCos[(-(c^2*d) - e)/(c^2*d - e)]*ArcTanh[(c
*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcT
anh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)
])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(
c^2*d*e)]*x))] + (-ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e
*x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^
2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)
]*x))] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*(ArcTanh[(c*d)/(Sqrt[
-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(
c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)
*Cos[2*ArcTan[c*x]]])] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] + (2*I)*(ArcT
anh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[
(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d +
 e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)
*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2
*d + 2*c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(...
```

3.1164.3 Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 1382, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{e(a+b\arctan(cx))}{d^2\left(d+ex^2\right)}+\frac{a+b\arctan(cx)}{d^2x^2}-\frac{e(a+b\arctan(cx))}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{5/2}}-\frac{a+b\arctan(cx)}{d^2x}-\frac{ex(a+b\arctan(cx))}{2d^2(ex^2+d)}-\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}+\frac{bc\log(x)}{d^2}+\frac{ib\sqrt{e}\log(icx+1)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}+\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ib\sqrt{e}\log(icx+1)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ibc\sqrt{e}\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}d^{5/2}}-\frac{ibc\sqrt{e}\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{bce\log(c^2x^2+1)}{4d^2(c^2d-e)}-\frac{bc\log(c^2x^2+1)}{2d^2}-\frac{bce\log(ex^2+d)}{4d^2(c^2d-e)}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4(-d)^{5/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(icx+1)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}-\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]
```

3.1164. $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx$

output
```
-((a + b*ArcTan[c*x])/(d^2*x)) - (e*x*(a + b*ArcTan[c*x]))/(2*d^2*(d + e*x
^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(5/2) - (Sqrt[e]*(a + b*A
rcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)) + (b*c*Log[x])/d^2 +
((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d]
 - I*Sqrt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[
-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b*Sqrt[e]
*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/
(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x)
)/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*
(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 +
Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sq
rt[d]])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 - Sqrt
[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[-c^2]*d^(5/2)) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 + Sqrt[-c^2]
*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sq
rt[-c^2]*d^(5/2)) - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*e*Log[1 + c^2*x^
2])/(4*d^2*(c^2*d - e)) - (b*c*e*Log[d + e*x^2])/(4*d^2*(c^2*d - e)) - ((I
/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-
d)^(5/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[...
```

3.1164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1164.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2567 vs. $2(1038) = 2076$.

Time = 1.48 (sec) , antiderivative size = 2568, normalized size of antiderivative = 1.86

method	result	size
risch	Expression too large to display	2568
parts	Expression too large to display	3558
derivativedivides	Expression too large to display	3591
default	Expression too large to display	3591

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*c^2/d*e/(c^2*d-e)/(e*d)^(1/2)*arctanh(1/2*(2*(1+I*c*x)*e-2*e)/c/(e*
d)^(1/2))-1/4*b*c^3/d*e*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)+1/8*b*c^4
/d*e^2*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1
/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))*x^2-1/8*b*c^4/d*e^2*ln(1+I*c*x)/(c^2
*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(
e*d)^(1/2)-e))*x^2-1/8*b*c^2/d^2*e^3*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2
*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1+I*c*x)*e+e)/(c*(e*d)^(1/2)+e))*x^2+1/
8*b*c^2/d^2*e^3*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c
*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-e))*x^2-1/8*b*c^2/d*e^2*ln(1+I*
c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1+I*c*x)*
e+e)/(c*(e*d)^(1/2)+e))+1/8*b*c^2/d*e^2*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-
c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1+I*c*x)*e-e)/(c*(e*d)^(1/2)-e))-1/4
*I*b*c^4/d*e*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x+1/4*I*b*c^2/d^2*e^
2*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x+1/8*c^2*b/d*e^2*ln(1-I*c*x)/(
c^2*d-e)/(-c^2*e*x^2-c^2*d)/(e*d)^(1/2)*ln((c*(e*d)^(1/2)+(1-I*c*x)*e-e)/(
c*(e*d)^(1/2)-e))-1/8*c^2*b/d*e^2*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)
/(e*d)^(1/2)*ln((c*(e*d)^(1/2)-(1-I*c*x)*e+e)/(c*(e*d)^(1/2)+e))+1/4*I*c^4
*b*ln(1-I*c*x)/d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*e*x-1/4*I*c^2*b/d^2*e^2*ln(1
-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x-a/x/d^2-1/4*c^2*b/d*e/(c^2*d-e)/(e*
d)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(e*d)^(1/2))-1/4*c^3*b/d*e*l...
```

3.1164.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

3.1164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1164.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1164.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^2), x)
```

3.1165 $\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$

3.1165.1 Optimal result

Integrand size = 21, antiderivative size = 532

$$\begin{aligned}\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx &= -\frac{bcdx}{8(c^2d-e)e^2(d+ex^2)}+\frac{bc^4d^2\arctan(cx)}{4(c^2d-e)^2e^3}-\frac{bc^2d\arctan(cx)}{(c^2d-e)e^3}\\
&-\frac{d^2(a+b\arctan(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\arctan(cx))}{e^3(d+ex^2)}\\
&+\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(c^2d-e)e^{5/2}}-\frac{bc\sqrt{d}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8(c^2d-e)^2e^{5/2}}\\
&-\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^3}\\
&+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^3}\\
&+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^3}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}\\
&-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^3}\\
&-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^3}\end{aligned}$$

output
```
-1/8*b*c*d*x/(c^2*d-e)/e^2/(e*x^2+d)+1/4*b*c^4*d^2*arctan(c*x)/(c^2*d-e)^2
/e^3-b*c^2*d*arctan(c*x)/(c^2*d-e)/e^3-1/4*d^2*(a+b*arctan(c*x))/e^3/(e*x^
2+d)^2+d*(a+b*arctan(c*x))/e^3/(e*x^2+d)-(a+b*arctan(c*x))*ln(2/(1-I*c*x))
/e^3+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)
^(1/2)-I*e^(1/2)))/e^3+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))
/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^3+1/2*I*b*polylog(2,1-2/(1-I*c*x))/
e^3-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)
-I*e^(1/2)))/e^3-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/
(c*(-d)^(1/2)+I*e^(1/2)))/e^3+b*c*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(c^2*d
-e)/e^(5/2)-1/8*b*c*(3*c^2*d-e)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(c^2*d-e
)^2/e^(5/2)
```

3.1165.2 Mathematica [A] (verified)

Time = 9.84 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.11

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{a\left(\frac{d(3d+4ex^2)}{(d+ex^2)^2}+2\log(d+ex^2)\right)+b\left(-\frac{cdex}{2(c^2d-e)(d+ex^2)}+\frac{c^2d(-3c^2d+4e)\arctan(cx)}{(-c^2d+e)^2}+\frac{d(3d+4ex^2)\arctan(cx)}{(d+ex^2)^2}+\frac{c\sqrt{d}(5c^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(a*((d*(3*d + 4*e*x^2))/(d + e*x^2)^2 + 2*Log[d + e*x^2]) + b*(-1/2*(c*d*e
*x)/((c^2*d - e)*(d + e*x^2)) + (c^2*d*(-3*c^2*d + 4*e)*ArcTan[c*x])/(-(c^
2*d) + e)^2 + (d*(3*d + 4*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 + (c*Sqrt[d]*(
5*c^2*d - 7*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(-(c^2*d) + e)^2) +
 2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*ArcTan[c*x]*Log[(I*Sqrt
[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I
*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqr
t[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*
Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*Log[(I*Sqrt[d])/Sqrt
[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*PolyLog[2, (
c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[
d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] + I*S
qrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x
))/(c*Sqrt[d] + Sqrt[e])]))/(4*e^3)
```

3.1165.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2x(a+b\arctan(cx))}{e^2(d+ex^2)^3}-\frac{2dx(a+b\arctan(cx))}{e^2(d+ex^2)^2}+\frac{x(a+b\arctan(cx))}{e^2(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\arctan(cx))}{e^3(d+ex^2)}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e^3}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e^3}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^3}-\frac{bc\sqrt{d}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{5/2}(c^2d-e)^2}+\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(c^2d-e)}-\frac{bc^2d\arctan(cx)}{e^3(c^2d-e)}+\frac{bc^4d^2\arctan(cx)}{4e^3(c^2d-e)^2}-\frac{bcdx}{8e^2(c^2d-e)(d+ex^2)}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^3}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e^3}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}$$

input
```
Int[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/8*(b*c*d*x)/((c^2*d - e)*e^2*(d + e*x^2)) + (b*c^4*d^2*ArcTan[c*x])/(4*
(c^2*d - e)^2*e^3) - (b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^3) - (d^2*(a + b
*ArcTan[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*ArcTan[c*x]))/(e^3*(d + e
*x^2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/((c^2*d - e)*e^(5/2)) -
 (b*c*Sqrt[d]*(3*c^2*d - e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*(c^2*d - e)^2*
e^(5/2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^3 + ((a + b*ArcTan[c
*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x
))])/(2*e^3) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^3) + ((I/2)*b*PolyLog[2, 1 - 2/(1
 - I*c*x)])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(
Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^3
```

3.1165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1165.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.54

method	result	size
parts	Expression too large to display	817
derivativedivides	Expression too large to display	843
default	Expression too large to display	843
risch	Expression too large to display	1666

input
```
int(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/e^3*d/(e*x^2+d)+1/2/e^3*ln(e*x^2+d)-1/4*d^2/e^3/(e*x^2+d)^2)+b/c^6*(1
/2*arctan(c*x)*c^6/e^3*ln(c^2*e*x^2+c^2*d)-1/4*arctan(c*x)*c^10*d^2/e^3/(c
^2*e*x^2+c^2*d)^2+arctan(c*x)*c^8*d/e^3/(c^2*e*x^2+c^2*d)-1/4*c^6*(d*c^2/e
^3*(-1/(c^2*d-e)^2*e*((-1/2*c^2*d+1/2*e)*c*x/(c^2*e*x^2+c^2*d)+1/2*(5*c^2*
d-7*e)/c/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))+1/(c^2*d-e)^2*(3*c^2*d-4*e)*
arctan(c*x))+2/e^3*(-1/2*I*(ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-
I)*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e
*_Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/R
ootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e
*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog
((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^
2*d-e,index=2)))/e))+1/2*I*(ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*
x)*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e
*_Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/R
ootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e
*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog
((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^
2*d-e,index=2)))/e)))))
```

3.1165.5 Fricas [F]

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arctan(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.1165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1165.7 Maxima [F]

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + 2*b*integrate(1/2*x^5*arctan(c*x)/(e^3*x^6 + 3*d*e^2*x^4 + 3*
d^2*e*x^2 + d^3), x)
```

3.1165.8 Giac [F]

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^5*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*atan(c*x)))/(d + e*x^2)^3, x)
```

3.1166 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$

3.1166.1 Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{bcx}{8(c^2d-e)e(d+ex^2)} - \frac{b\arctan(cx)}{4d(c^2d-e)^2} + \frac{x^4(a+b\arctan(cx))}{4d(d+ex^2)^2} - \frac{bc(c^2d-3e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}(c^2d-e)^2e^{3/2}}$$

output
```
1/8*b*c*x/(c^2*d-e)/e/(e*x^2+d)-1/4*b*arctan(c*x)/d/(c^2*d-e)^2+1/4*x^4*(a
+b*arctan(c*x))/d/(e*x^2+d)^2-1/8*b*c*(c^2*d-3*e)*arctan(x*e^(1/2)/d^(1/2)
)/(c^2*d-e)^2/e^(3/2)/d^(1/2)
```

3.1166.2 Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{\frac{2ad}{(d+ex^2)^2} + \frac{-4ac^2d+4ae+bcex}{(c^2d-e)(d+ex^2)} + \frac{2bc^2(c^2d-2e)\arctan(cx)}{(-c^2d+e)^2} - \frac{2b(d+2ex^2)\arctan(cx)}{(d+ex^2)^2} - \frac{bc(c^2d-3e)\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(-c^2d+e)^2}}{8e^2}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
((2*a*d)/(d + e*x^2)^2 + (-4*a*c^2*d + 4*a*e + b*c*e*x)/((c^2*d - e)*(d +
e*x^2)) + (2*b*c^2*(c^2*d - 2*e)*ArcTan[c*x])/(-(c^2*d) + e)^2 - (2*b*(d +
 2*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 - (b*c*(c^2*d - 3*e)*Sqrt[e]*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(-(c^2*d) + e)^2))/(8*e^2)
```

3.1166.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5511, 27, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5511}$$

$$\frac{x^4(a+b\arctan(cx))}{4d(d+ex^2)^2}-bc\int \frac{x^4}{4d(c^2x^2+1)(ex^2+d)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(a+b\arctan(cx))}{4d(d+ex^2)^2}-\frac{bc\int \frac{x^4}{(c^2x^2+1)(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \text{372}$$

$$\frac{x^4(a+b\arctan(cx))}{4d(d+ex^2)^2}-\frac{bc\left(\frac{\int \frac{(c^2d-2e)x^2+d}{(c^2x^2+1)(ex^2+d)}dx}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

$$\Big\downarrow \text{397}$$

$$\frac{x^4(a+b\arctan(cx))}{4d(d+ex^2)^2}-\frac{bc\left(\frac{\frac{2e\int \frac{1}{c^2x^2+1}dx}{c^2d-e}+\frac{d(c^2d-3e)\int \frac{1}{ex^2+d}dx}{c^2d-e}}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^4(a+b\arctan(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\left(\frac{\frac{d\left(c^2d-3e\right)\int\frac{1}{ex^2+d}dx}{c^2d-e}+\frac{2e\arctan(cx)}{c\left(c^2d-e\right)}}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

$$\Big\downarrow \; 218$$

$$\frac{x^4(a+b\arctan(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\left(\frac{\frac{2e\arctan(cx)}{c\left(c^2d-e\right)}+\frac{\sqrt{d}\left(c^2d-3e\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(c^2d-e\right)}}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcTan[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*(-1/2*(d*x)/((c^2*d -
 e)*e*(d + e*x^2)) + ((2*e*ArcTan[c*x])/(c*(c^2*d - e)) + (Sqrt[d]*(c^2*d
- 3*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/((c^2*d - e)*Sqrt[e]))/(2*(c^2*d - e)*
e)))/(4*d)
```

3.1166.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1166.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.62

method	result
parts	$a\left(\frac{d}{4e^2(e\,x^2+d)^2}-\frac{1}{2e^2(e\,x^2+d)}\right)+\frac{b\left(\frac{\arctan(cx)c^8d}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{\arctan(cx)c^6}{2e^2\left(e\,c^2x^2+c^2d\right)}-\frac{c^6\left(\frac{e^2\left(-\frac{\left(c^2d-e\right)cx}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{\left(c^2d-3e\right)a}{2e\right.}}{\left(c^2d-e\right)^2}\right.}{4}\right.}{c^4}$
derivativedivides	$\frac{a\,c^6\left(-\frac{1}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}\right)+b\,c^6\left(-\frac{\arctan(cx)}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{\arctan(cx)d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{e^2\left(-\frac{\left(c^2d-e\right)cx}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{\left(c^2\right.}{}\right.}{\left(c^2d-e\right.}\right.}{c^4}$
default	$\frac{a\,c^6\left(-\frac{1}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}\right)+b\,c^6\left(-\frac{\arctan(cx)}{2e^2\left(e\,c^2x^2+c^2d\right)}+\frac{\arctan(cx)d\,c^2}{4e^2\left(e\,c^2x^2+c^2d\right)^2}-\frac{e^2\left(-\frac{\left(c^2d-e\right)cx}{2e\left(e\,c^2x^2+c^2d\right)}+\frac{\left(c^2\right.}{}\right.}{\left(c^2d-e\right.}\right.}{c^4}$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(1/4*arctan(c*x)*c^8*d/e
^2/(c^2*e*x^2+c^2*d)^2-1/2*arctan(c*x)*c^6/e^2/(c^2*e*x^2+c^2*d)-1/4*c^6/e
^2*(e^2/(c^2*d-e)^2*(-1/2*(c^2*d-e)/e*c*x/(c^2*e*x^2+c^2*d)+1/2*(c^2*d-3*e
)/e/c/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))+(-c^2*d+2*e)/(c^2*d-e)^2*arctan
(c*x)))
```

3.1166. $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$

3.1166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(114) = 228$.

Time = 0.37 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.36

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$
$$= \left[-\frac{4\,ac^4d^4 - 8\,ac^2d^3e + 4\,ad^2e^2 - 2\,(bc^3d^2e^2 - bcde^3)x^3 + 8\,(ac^4d^3e - 2\,ac^2d^2e^2 + ade^3)x^2 - (bc^3d^3 - 3\,bc}{16\,(c^4d^5e^2 - 2\,c^2}\right.$$
$$-\frac{2\,ac^4d^4 - 4\,ac^2d^3e + 2\,ad^2e^2 - (bc^3d^2e^2 - bcde^3)x^3 + 4\,(ac^4d^3e - 2\,ac^2d^2e^2 + ade^3)x^2 + (bc^3d^3 - 3\,bcd^2}{8\,(c^4d^5e^2 - 2\,c^2d^4e^3 +}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 - 2*(b*c^3*d^2*e^2 - b*c
*d*e^3)*x^3 + 8*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^3*d^3
 - 3*b*c*d^2*e + (b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - 3*b*c*d*
e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(b*
c^3*d^3*e - b*c*d^2*e^2)*x + 4*(2*b*d*e^3*x^2 + b*d^2*e^2 - (b*c^4*d^2*e^2
 - 2*b*c^2*d*e^3)*x^4)*arctan(c*x))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4
 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*
e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 - (b*
c^3*d^2*e^2 - b*c*d*e^3)*x^3 + 4*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)
*x^2 + (b*c^3*d^3 - 3*b*c*d^2*e + (b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(b*c^3
*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (b*c^3*d^3*e
- b*c*d^2*e^2)*x + 2*(2*b*d*e^3*x^2 + b*d^2*e^2 - (b*c^4*d^2*e^2 - 2*b*c^2
*d*e^3)*x^4)*arctan(c*x))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^
3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*
e^5)*x^2)]
```

3.1166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1166.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1166.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1166.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.10

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{b\,c^4\,d\,\mathrm{atan}(c\,x)}{4\,e^2\,(e-c^2\,d)^2} - \frac{a\,d}{4\,e^2\,(e\,x^2+d)^2} - \frac{b\,d\,\mathrm{atan}(c\,x)}{4\,e^2\,(e\,x^2+d)^2} - \frac{b\,c\,x^3}{8\,(e-c^2\,d)\,(e\,x^2+d)^2} - \frac{b\,c^2\,\mathrm{atan}(c\,x)}{2\,e\,(e-c^2\,d)^2} - \frac{b\,x^2\,\mathrm{atan}(c\,x)}{2\,e\,(e\,x^2+d)^2} - \frac{b\,c^3\,\mathrm{atan}\left(\frac{x\,\sqrt{-d\,e^3}\,1\mathrm{i}}{d\,e}\right)\,\sqrt{-d\,e^3}\,1\mathrm{i}}{8\,e^3\,(e-c^2\,d)^2} - \frac{a\,x^2}{2\,e\,(e\,x^2+d)^2} - \frac{b\,c\,d\,x}{8\,e\,(e-c^2\,d)\,(e\,x^2+d)^2} + \frac{b\,c\,\mathrm{atan}\left(\frac{x\,\sqrt{-d\,e^3}\,1\mathrm{i}}{d\,e}\right)\,\sqrt{-d\,e^3}\,3\mathrm{i}}{8\,d\,e^2\,(e-c^2\,d)^2}$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
(b*c^4*d*atan(c*x))/(4*e^2*(e - c^2*d)^2) - (a*d)/(4*e^2*(d + e*x^2)^2) -
(b*d*atan(c*x))/(4*e^2*(d + e*x^2)^2) - (b*c*x^3)/(8*(e - c^2*d)*(d + e*x^
2)^2) - (b*c^2*atan(c*x))/(2*e*(e - c^2*d)^2) - (b*x^2*atan(c*x))/(2*e*(d
+ e*x^2)^2) - (b*c^3*atan((x*(-d*e^3)^(1/2)*1i)/(d*e))*(-d*e^3)^(1/2)*1i)/
(8*e^3*(e - c^2*d)^2) - (a*x^2)/(2*e*(d + e*x^2)^2) - (b*c*d*x)/(8*e*(e -
c^2*d)*(d + e*x^2)^2) + (b*c*atan((x*(-d*e^3)^(1/2)*1i)/(d*e))*(-d*e^3)^(1
/2)*3i)/(8*d*e^2*(e - c^2*d)^2)
```

3.1167 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$

3.1167.1 Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = -\frac{bcx}{8d(c^2d-e)(d+ex^2)}+\frac{bc^4\arctan(cx)}{4(c^2d-e)^2e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}-\frac{bc(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}(c^2d-e)^2\sqrt{e}}$$

output
```
-1/8*b*c*x/d/(c^2*d-e)/(e*x^2+d)+1/4*b*c^4*arctan(c*x)/(c^2*d-e)^2/e+1/4*(
-a-b*arctan(c*x))/e/(e*x^2+d)^2-1/8*b*c*(3*c^2*d-e)*arctan(x*e^(1/2)/d^(1/
2))/d^(3/2)/(c^2*d-e)^2/e^(1/2)
```

3.1167.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx(d+ex^2)}{d(c^2d-e)}}{(d+ex^2)^2}+\frac{2b\left(\frac{c^4}{(-c^2d+e)^2}-\frac{1}{(d+ex^2)^2}\right)\arctan(cx)}{e}-\frac{bc(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}(-c^2d+e)^2}\right)$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(-(((2*a)/e + (b*c*x*(d + e*x^2))/(d*(c^2*d - e)))/(d + e*x^2)^2) + (2*b*(
c^4/(-(c^2*d) + e)^2 - (d + e*x^2)^(-2))*ArcTan[c*x])/e - (b*c*(3*c^2*d -
e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e]*(-(c^2*d) + e)^2))/8
```

3.1167.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5509, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)(ex^2+d)^2}dx}{4e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{316}$$

$$\frac{bc\left(\frac{\int \frac{-ex^2c^2+2dc^2-e}{(c^2x^2+1)(ex^2+d)}dx}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{bc\left(\frac{\frac{2c^4d\int \frac{1}{c^2x^2+1}dx}{c^2d-e}-\frac{e(3c^2d-e)\int \frac{1}{ex^2+d}dx}{c^2d-e}}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{bc\left(\frac{\frac{2c^3d\arctan(cx)}{c^2d-e}-\frac{e(3c^2d-e)\int \frac{1}{ex^2+d}dx}{c^2d-e}}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{bc\left(\frac{\frac{2c^3d\arctan(cx)}{c^2d-e}-\frac{\sqrt{e}\left(3c^2d-e\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d-e\right)}}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e\left(d+ex^2\right)^2}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])/(e*(d + e*x^2)^2) + (b*c*(-1/2*(e*x)/(d*(c^2*d -
e)*(d + e*x^2)) + ((2*c^3*d*ArcTan[c*x])/(c^2*d - e) - ((3*c^2*d - e)*Sqrt
[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d - e)))/(2*d*(c^2*d - e)))
)/(4*e)
```

3.1167.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1167.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{a}{4e(e\,x^2+d)^2}+\frac{b\left(-\frac{\arctan(cx)c^6}{4e\left(e\,c^2x^2+c^2d\right)^2}+\frac{c^6\left(-\frac{e\left(\frac{\left(c^2d-e\right)x}{2dc\left(e\,c^2x^2+c^2d\right)}+\frac{\left(3c^2d-e\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\,c^3\sqrt{ed}}\right)}{\left(c^2d-e\right)^2}+\frac{\arctan(cx)}{\left(c^2d-e\right)^2}\right)}{4e}\right)}{c^2}$
derivativedivides	$\frac{-\frac{a\,c^6}{4e\left(e\,c^2x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\arctan(cx)}{4e\left(e\,c^2x^2+c^2d\right)^2}+\frac{-\frac{e\left(\frac{\left(c^2d-e\right)x}{2dc\left(e\,c^2x^2+c^2d\right)}+\frac{\left(3c^2d-e\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\,c^3\sqrt{ed}}\right)}{\left(c^2d-e\right)^2}+\frac{\arctan(cx)}{\left(c^2d-e\right)^2}}{4e}\right)}{c^2}$
default	$\frac{-\frac{a\,c^6}{4e\left(e\,c^2x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\arctan(cx)}{4e\left(e\,c^2x^2+c^2d\right)^2}+\frac{-\frac{e\left(\frac{\left(c^2d-e\right)x}{2dc\left(e\,c^2x^2+c^2d\right)}+\frac{\left(3c^2d-e\right)\arctan\left(\frac{ex}{\sqrt{ed}}\right)}{2d\,c^3\sqrt{ed}}\right)}{\left(c^2d-e\right)^2}+\frac{\arctan(cx)}{\left(c^2d-e\right)^2}}{4e}\right)}{c^2}$
risch	$\frac{ib\ln(icx+1)}{8e(e\,x^2+d)^2}+\frac{c^5bx}{16(c^2d-e)^2(-e\,c^2x^2-c^2d)}-\frac{c^3bex}{16(c^2d-e)^2(-e\,c^2x^2-c^2d)d}+\frac{ic^6b\ln(-icx+1)d}{4(-e\,c^2x^2-c^2d)^2(c^2d-e)^2}+\frac{icb\,\text{arc}}{}$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*arctan(c*x)*c^6/e/(c^2*e*x^2+c^2*d)^2+1/4
*c^6/e*(-1/(c^2*d-e)^2*e*(1/2*(c^2*d-e)/d/c*x/(c^2*e*x^2+c^2*d)+1/2*(3*c^2
*d-e)/d/c^3/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))+1/(c^2*d-e)^2*arctan(c*x)
))
```

3.1167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(115) = 230$.

Time = 0.35 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.86

$$\int \frac{x(a + b\arctan(cx))}{(d + ex^2)^3}\,dx$$

$$= \Bigg[-\frac{4\,ac^4d^4 - 8\,ac^2d^3e + 4\,ad^2e^2 + 2\,(bc^3d^2e^2 - bcde^3)x^3 - (3\,bc^3d^3 - bcd^2e + (3\,bc^3de^2 - bce^3)x^4 + 2\,(3\,bc}{16\,(c^4d^6e - 2\,c^2d^5e^2 + d^4e^3 + (c^4d}$$

$$-\frac{2\,ac^4d^4 - 4\,ac^2d^3e + 2\,ad^2e^2 + (bc^3d^2e^2 - bcde^3)x^3 + (3\,bc^3d^3 - bcd^2e + (3\,bc^3de^2 - bce^3)x^4 + 2\,(3\,bc^3d}{8\,(c^4d^6e - 2\,c^2d^5e^2 + d^4e^3 + (c^4d^4e^3 - 2}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 2*(b*c^3*d^2*e^2 - b*c
*d*e^3)*x^3 - (3*b*c^3*d^3 - b*c*d^2*e + (3*b*c^3*d*e^2 - b*c*e^3)*x^4 + 2
*(3*b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x -
 d)/(e*x^2 + d)) + 2*(b*c^3*d^3*e - b*c*d^2*e^2)*x - 4*(b*c^4*d^2*e^2*x^4
+ 2*b*c^4*d^3*e*x^2 + 2*b*c^2*d^3*e - b*d^2*e^2)*arctan(c*x))/(c^4*d^6*e -
 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2
*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2
*d^3*e + 2*a*d^2*e^2 + (b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (3*b*c^3*d^3 - b*
c*d^2*e + (3*b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(3*b*c^3*d^2*e - b*c*d*e^2)*x^
2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (b*c^3*d^3*e - b*c*d^2*e^2)*x - 2*(b*
c^4*d^2*e^2*x^4 + 2*b*c^4*d^3*e*x^2 + 2*b*c^2*d^3*e - b*d^2*e^2)*arctan(c*
x))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 +
d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

3.1167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1167.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1167.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arctan(cx)+a)x}{(ex^2+d)^3}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1167.9 Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.53

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{b\,c\,x}{8\,\left(e-c^2\,d\right)\,\left(e\,x^2+d\right)^2}-\frac{b\,\mathrm{atan}(c\,x)}{4\,e\,\left(e\,x^2+d\right)^2}-\frac{a}{4\,e\,\left(e\,x^2+d\right)^2}+\frac{b\,c^4\,\mathrm{atan}(c\,x)}{4\,e\,\left(e-c^2\,d\right)^2}+\frac{b\,c\,e\,x^3}{8\,d\,\left(e-c^2\,d\right)\,\left(e\,x^2+d\right)^2}+\frac{b\,c\,\mathrm{atan}\left(\frac{x\,\sqrt{-d^3\,e}\,1\mathrm{i}}{d^2}\right)\,\sqrt{-d^3\,e}\,1\mathrm{i}}{8\,d^3\,\left(e-c^2\,d\right)^2}-\frac{b\,c^3\,\mathrm{atan}\left(\frac{x\,\sqrt{-d^3\,e}\,1\mathrm{i}}{d^2}\right)\,\sqrt{-d^3\,e}\,3\mathrm{i}}{8\,d^2\,e\,\left(e-c^2\,d\right)^2}$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
(b*c*x)/(8*(e - c^2*d)*(d + e*x^2)^2) - (b*atan(c*x))/(4*e*(d + e*x^2)^2)
- a/(4*e*(d + e*x^2)^2) + (b*c^4*atan(c*x))/(4*e*(e - c^2*d)^2) + (b*c*ata
n((x*(-d^3*e)^(1/2)*1i)/d^2)*(-d^3*e)^(1/2)*1i)/(8*d^3*(e - c^2*d)^2) - (b
*c^3*atan((x*(-d^3*e)^(1/2)*1i)/d^2)*(-d^3*e)^(1/2)*3i)/(8*d^2*e*(e - c^2*
d)^2) + (b*c*e*x^3)/(8*d*(e - c^2*d)*(d + e*x^2)^2)
```

3.1168 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^3}\,dx$

3.1168.1 Optimal result

Integrand size = 21, antiderivative size = 574

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx &= \frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)}-\frac{bc^4\arctan(cx)}{4d\left(c^2d-e\right)^2}\\
&\quad-\frac{bc^2\arctan(cx)}{2d^2\left(c^2d-e\right)}+\frac{a+b\arctan(cx)}{4d\left(d+ex^2\right)^2}+\frac{a+b\arctan(cx)}{2d^2\left(d+ex^2\right)}\\
&\quad+\frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\left(c^2d-e\right)}+\frac{bc(3c^2d-e)\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\left(c^2d-e\right)^2}\\
&\quad+\frac{a\log(x)}{d^3}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^3}\\
&\quad-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^3}\\
&\quad-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2d^3}\\
&\quad+\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d^3}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d^3}\\
&\quad-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^3}\\
&\quad+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4d^3}\end{aligned}$$

output
```
1/8*b*c*e*x/d^2/(c^2*d-e)/(e*x^2+d)-1/4*b*c^4*arctan(c*x)/d/(c^2*d-e)^2-1/
2*b*c^2*arctan(c*x)/d^2/(c^2*d-e)+1/4*(a+b*arctan(c*x))/d/(e*x^2+d)^2+1/2*
(a+b*arctan(c*x))/d^2/(e*x^2+d)+a*ln(x)/d^3+(a+b*arctan(c*x))*ln(2/(1-I*c*
x))/d^3-1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(
-d)^(1/2)-I*e^(1/2)))/d^3-1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/
2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3-1/2*I*b*polylog(2,I*c*x)/d^3+1
/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^
(1/2)))/d^3+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-
d)^(1/2)-I*e^(1/2)))/d^3-1/2*I*b*polylog(2,1-2/(1-I*c*x))/d^3+1/2*I*b*poly
log(2,-I*c*x)/d^3+1/2*b*c*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(c^2*d
-e)+1/8*b*c*(3*c^2*d-e)*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(5/2)/(c^2*d-e
)^2
```

3.1168.2 Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.12

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx$$
$$= \frac{2a\left(\frac{d(3d+2ex^2)}{(d+ex^2)^2}+4\log(x)-2\log\left(d+ex^2\right)\right)+b\left(\frac{cdex}{(c^2d-e)(d+ex^2)}+\frac{2c^2d(-3c^2d+2e)\arctan(cx)}{(-c^2d+e)^2}+\frac{2d(3d+2ex^2)\arctan(cx}{(d+ex^2)^2}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
(2*a*((d*(3*d + 2*e*x^2))/(d + e*x^2)^2 + 4*Log[x] - 2*Log[d + e*x^2]) + b
*((c*d*e*x)/((c^2*d - e)*(d + e*x^2)) + (2*c^2*d*(-3*c^2*d + 2*e)*ArcTan[c
*x])/(-(c^2*d) + e)^2 + (2*d*(3*d + 2*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 +
(c*Sqrt[d]*(7*c^2*d - 5*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(-(c^2*d)
+ e)^2 + 8*ArcTan[c*x]*Log[x] - 4*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] - 4*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] - (2*I)*Log[((-I)*Sqrt[d]
)/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*L
og[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt
[e])] + (2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*S
qrt[d] - Sqrt[e])] - (2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 +
I*c*x))/(c*Sqrt[d] + Sqrt[e])] - (4*I)*(Log[x]*(Log[1 - I*c*x] - Log[1 + I
*c*x]) - PolyLog[2, (-I)*c*x] + PolyLog[2, I*c*x]) + (2*I)*PolyLog[2, (c*(
Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt
[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt[d]
+ I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*PolyLog[2, (c*(Sqrt[d] + I*
Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(8*d^3)
```

3.1168.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{ex(a+b\arctan(cx))}{d^3\,(d+ex^2)}+\frac{a+b\arctan(cx)}{d^3x}-\frac{ex(a+b\arctan(cx))}{d^2\,(d+ex^2)^2}-\frac{ex(a+b\arctan(cx))}{d\,(d+ex^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d^3}-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d^3}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}+\frac{a+b\arctan(cx)}{2d^2(d+ex^2)}+\frac{a+b\arctan(cx)}{4d(d+ex^2)^2}+\frac{a\log(x)}{d^3}+\frac{bc\sqrt{e}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}(c^2d-e)^2}+\frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}(c^2d-e)}-\frac{bc^2\arctan(cx)}{2d^2(c^2d-e)}-\frac{bc^4\arctan(cx)}{4d(c^2d-e)^2}+\frac{bcex}{8d^2(c^2d-e)(d+ex^2)}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d^3}+\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d^3}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d^3}-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
(b*c*e*x)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (b*c^4*ArcTan[c*x])/(4*d*(c^2*
d - e)^2) - (b*c^2*ArcTan[c*x])/(2*d^2*(c^2*d - e)) + (a + b*ArcTan[c*x])/
(4*d*(d + e*x^2)^2) + (a + b*ArcTan[c*x])/(2*d^2*(d + e*x^2)) + (b*c*Sqrt[
e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*(c^2*d - e)) + (b*c*(3*c^2*d -
e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*(c^2*d - e)^2) + (a*Log
[x])/d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - ((a + b*ArcTan[c
*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x
))])/(2*d^3) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^3) + ((I/2)*b*PolyLog[2, (-I)*c*x
])/d^3 - ((I/2)*b*PolyLog[2, I*c*x])/d^3 - ((I/2)*b*PolyLog[2, 1 - 2/(1 -
I*c*x)])/d^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqr
t[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3
```

3.1168. $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^3}\,dx$

3.1168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1168.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.57

method	result	size
parts	Expression too large to display	903
derivativedivides	Expression too large to display	920
default	Expression too large to display	920
risch	Expression too large to display	1677

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)/d^3+1/2*a/d^2/(e*x^2+d)+1/4*a/d/(e*x^2+d)^2-1/2*a/d^3*ln(e*x^2+d)+
b*(arctan(c*x)/d^3*ln(c*x)-1/2*arctan(c*x)/d^3*ln(c^2*e*x^2+c^2*d)+1/4*c^4
*arctan(c*x)/d/(c^2*e*x^2+c^2*d)^2+1/2*c^2*arctan(c*x)/d^2/(c^2*e*x^2+c^2*
d)-1/2*c^6*(-I/c^6/d^3*ln(c*x)*ln(1+I*c*x)+I/c^6/d^3*ln(c*x)*ln(1-I*c*x)-I
/c^6/d^3*dilog(1+I*c*x)+I/c^6/d^3*dilog(1-I*c*x)-1/d^3/c^6*(-1/2*I*(ln(c*x
-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^2
*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e
*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,inde
x=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf
(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e))+1/2*I*(ln(I+c
*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^2
*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e
*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,inde
x=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf
(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e)))+1/2/d^2/c^4*
(-1/(c^2*d-e)^2*e*((1/2*c^2*d-1/2*e)*c*x/(c^2*e*x^2+c^2*d)+1/2*(7*c^2*d-5*
e)/c/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))+(3*c^2*d-2*e)/(c^2*d-e)^2*arctan
(c*x))))
```

3.1168.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

3.1168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1168.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \frac{b\arctan(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(e^3*x^7 + 3*d*e^2*x
^5 + 3*d^2*e*x^3 + d^3*x), x)
```

3.1168.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \frac{b\arctan(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^3), x)
```

3.1169 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx$

3.1169.1 Optimal result

Integrand size = 21, antiderivative size = 629

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx &= -\frac{bc}{2d^3x}-\frac{bce^2x}{8d^3(c^2d-e)(d+ex^2)}-\frac{bc^2\arctan(cx)}{2d^3}\\
&+\frac{bc^4e\arctan(cx)}{4d^2(c^2d-e)^2}+\frac{bc^2e\arctan(cx)}{d^3(c^2d-e)}-\frac{a+b\arctan(cx)}{2d^3x^2}\\
&-\frac{e(a+b\arctan(cx))}{4d^2(d+ex^2)^2}-\frac{e(a+b\arctan(cx))}{d^3(d+ex^2)}\\
&-\frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}(c^2d-e)}-\frac{bc(3c^2d-e)\,e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}(c^2d-e)^2}\\
&-\frac{3ae\log(x)}{d^4}-\frac{3e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^4}\\
&+\frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^4}\\
&+\frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^4}\\
&-\frac{3ibe\,\mathrm{PolyLog}(2,-icx)}{2d^4}+\frac{3ibe\,\mathrm{PolyLog}(2,icx)}{2d^4}\\
&+\frac{3ibe\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^4}\\
&-\frac{3ibe\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^4}\\
&-\frac{3ibe\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^4}\end{aligned}$$

output
```
-1/2*b*c/d^3/x-1/8*b*c*e^2*x/d^3/(c^2*d-e)/(e*x^2+d)-1/2*b*c^2*arctan(c*x)
/d^3+1/4*b*c^4*e*arctan(c*x)/d^2/(c^2*d-e)^2+b*c^2*e*arctan(c*x)/d^3/(c^2*
d-e)+1/2*(-a-b*arctan(c*x))/d^3/x^2-1/4*e*(a+b*arctan(c*x))/d^2/(e*x^2+d)^
2-e*(a+b*arctan(c*x))/d^3/(e*x^2+d)-b*c*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/
d^(7/2)/(c^2*d-e)-1/8*b*c*(3*c^2*d-e)*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d^
(7/2)/(c^2*d-e)^2-3*a*e*ln(x)/d^4-3*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^
4+3/2*e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^
(1/2)-I*e^(1/2)))/d^4+3/2*e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2)
)/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^4-3/2*I*b*e*polylog(2,-I*c*x)/d^4+
3/2*I*b*e*polylog(2,1-2/(1-I*c*x))/d^4+3/2*I*b*e*polylog(2,I*c*x)/d^4-3/4*
I*b*e*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(
1/2)))/d^4-3/4*I*b*e*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(
-d)^(1/2)+I*e^(1/2)))/d^4
```

3.1169.2 Mathematica [A] (verified)

Time = 12.59 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx$$
$$= \frac{-a\left(\frac{d(2d^2+9dex^2+6e^2x^4)}{x^2(d+ex^2)^2}+12e\log(x)-6e\log\left(d+ex^2\right)\right)+b\left(-\frac{2cd}{x}-\frac{cde^2x}{2(c^2d-e)(d+ex^2)}+\frac{c^2d\left(-2c^4d^2+9c^2de-6e^2\right)\text{arc}}{(-c^2d+e)^2}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```

output
```
(-(a*((d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4))/(x^2*(d + e*x^2)^2) + 12*e*Log[x
] - 6*e*Log[d + e*x^2])) + b*((-2*c*d)/x - (c*d*e^2*x)/(2*(c^2*d - e)*(d +
 e*x^2)) + (c^2*d*(-2*c^4*d^2 + 9*c^2*d*e - 6*e^2)*ArcTan[c*x])/(-(c^2*d)
+ e)^2 - (d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4)*ArcTan[c*x])/(x^2*(d + e*x^2)^
2) + (c*Sqrt[d]*e^(3/2)*(-11*c^2*d + 9*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*
(-(c^2*d) + e)^2) - 12*e*ArcTan[c*x]*Log[x] + 6*e*ArcTan[c*x]*(Log[((-I)*S
qrt[d])/Sqrt[e] + x] + Log[(I*Sqrt[d])/Sqrt[e] + x] - Log[d + e*x^2]) + 6*
e*ArcTan[c*x]*Log[d + e*x^2] - (6*I)*e*(Log[x]*Log[1 + I*c*x] + PolyLog[2,
 (-I)*c*x]) + (6*I)*e*(Log[x]*Log[1 - I*c*x] + PolyLog[2, I*c*x]) - (3*I)*
e*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sq
rt[e])] + PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])]) +
 (3*I)*e*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d
] + Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e
])]) + (3*I)*e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))
/(c*Sqrt[d] - Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d
] - Sqrt[e])]) - (3*I)*e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1
- I*c*x))/(c*Sqrt[d] + Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/
(c*Sqrt[d] + Sqrt[e])])))/(4*d^4)
```

3.1169.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{3e^2x(a+b\arctan(cx))}{d^4\left(d+ex^2\right)}-\frac{3e(a+b\arctan(cx))}{d^4x}+\frac{2e^2x(a+b\arctan(cx))}{d^3\left(d+ex^2\right)^2}+\frac{a+b\arctan(cx)}{d^3x^3}+\frac{e^2x(a+b\arc$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^4}+\frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d^4}+\frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d^4}-\frac{e(a+b\arctan(cx))}{d^3(d+ex^2)}-\frac{a+b\arctan(cx)}{2d^3x^2}-\frac{e(a+b\arctan(cx))}{4d^2(d+ex^2)^2}-\frac{3ae\log(x)}{d^4}-\frac{bce^{3/2}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}(c^2d-e)^2}-\frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}(c^2d-e)}+\frac{bc^2e\arctan(cx)}{d^3(c^2d-e)}-\frac{bc^2\arctan(cx)}{2d^3}+\frac{bc^4e\arctan(cx)}{4d^2(c^2d-e)^2}-\frac{bce^2x}{8d^3(c^2d-e)(d+ex^2)}-\frac{3ibe\operatorname{PolyLog}(2,-icx)}{2d^4}+\frac{3ibe\operatorname{PolyLog}(2,icx)}{2d^4}+\frac{3ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^4}-\frac{3ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^4}-\frac{3ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d^4}-\frac{bc}{2d^3x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```

output
```
-1/2*(b*c)/(d^3*x) - (b*c*e^2*x)/(8*d^3*(c^2*d - e)*(d + e*x^2)) - (b*c^2*
ArcTan[c*x])/(2*d^3) + (b*c^4*e*ArcTan[c*x])/(4*d^2*(c^2*d - e)^2) + (b*c^
2*e*ArcTan[c*x])/(d^3*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^3*x^2) - (e*
(a + b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a + b*ArcTan[c*x]))/(d^3*
(d + e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(7/2)*(c^2*d -
 e)) - (b*c*(3*c^2*d - e)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*
(c^2*d - e)^2) - (3*a*e*Log[x])/d^4 - (3*e*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/d^4 + (3*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) + (3*e*(a + b*ArcTan[c*x])
*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(2*d^4) - (((3*I)/2)*b*e*PolyLog[2, (-I)*c*x])/d^4 + (((3*I)/2)*b*e*PolyL
og[2, I*c*x])/d^4 + (((3*I)/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^4 - ((
(3*I)/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/d^4 - (((3*I)/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d]
 + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^4
```

3.1169. $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx$

3.1169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1169.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.51

method	result	size
parts	Expression too large to display	951
derivativedivides	Expression too large to display	987
default	Expression too large to display	987
risch	Expression too large to display	1858

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^3/x^2-3*a*e*ln(x)/d^4-a*e/d^3/(e*x^2+d)+3/2*a*e/d^4*ln(e*x^2+d)-1
/4*a*e/d^2/(e*x^2+d)^2+b*c^2*(-1/2*arctan(c*x)/d^3/c^2/x^2-3/c^2*arctan(c*
x)/d^4*e*ln(c*x)+3/2/c^2*arctan(c*x)*e/d^4*ln(c^2*e*x^2+c^2*d)-1/4*c^2*arc
tan(c*x)*e/d^2/(c^2*e*x^2+c^2*d)^2-arctan(c*x)*e/d^3/(c^2*e*x^2+c^2*d)-1/2
*c^6*(-6/d^4/c^8*e*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1
/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))+3/d^4/c^8*e*(-1/2*I*(ln(c*x-I)*l
n(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,
index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2
+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2))
)/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z
^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=
2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e))+1/2*I*(ln(I+c*x)*l
n(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,
index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2
-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2))
)/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z
^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=
2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e)))-1/2/d^3/c^6*(-e^2
/(c^2*d-e)^2*((1/2*c^2*d-1/2*e)*c*x/(c^2*e*x^2+c^2*d)+1/2*(11*c^2*d-9*e)/c
/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2)))+(-2*c^4*d^2+9*c^2*d*e-6*e^2)/(c^2...
```

3.1169.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^
3), x)
```

3.1169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1169.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x
^2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + 2*b*integrate(1/2*arctan
(c*x)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)
```

3.1169.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^3), x)
```

3.1170 $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$

3.1170.1 Optimal result

Integrand size = 21, antiderivative size = 966

$$
\begin{aligned}
\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx &= \frac{bc}{8(c^2d-e)\,e\,(d+ex^2)}-\frac{x(a+b\arctan(cx))}{4e\,(d+ex^2)^2} \\
&+\frac{x(a+b\arctan(cx))}{8de\,(d+ex^2)}+\frac{(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}} \\
&+\frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}} \\
&-\frac{ibc\log\left(-\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}} \\
&-\frac{ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}} \\
&+\frac{ibc\log\left(\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}} \\
&+\frac{bc(5c^2d-3e)\log(1+c^2x^2)}{16d\,(c^2d-e)^2\,e}-\frac{bc\log(1+c^2x^2)}{4d\,(c^2d-e)\,e} \\
&-\frac{bc(5c^2d-3e)\log(d+ex^2)}{16d\,(c^2d-e)^2\,e}+\frac{bc\log(d+ex^2)}{4d\,(c^2d-e)\,e} \\
&+\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}} \\
&-\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}} \\
&+\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}} \\
&-\frac{ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{3/2}e^{3/2}}
\end{aligned}
$$

output
```
1/8*b*c/(c^2*d-e)/e/(e*x^2+d)-1/4*x*(a+b*arctan(c*x))/e/(e*x^2+d)^2+1/8*x*
(a+b*arctan(c*x))/d/e/(e*x^2+d)+1/8*(a+b*arctan(c*x))*arctan(x*e^(1/2)/d^(
1/2))/d^(3/2)/e^(3/2)+1/16*b*c*(5*c^2*d-3*e)*ln(c^2*x^2+1)/d/(c^2*d-e)^2/e
-1/4*b*c*ln(c^2*x^2+1)/d/(c^2*d-e)/e-1/16*b*c*(5*c^2*d-3*e)*ln(e*x^2+d)/d/
(c^2*d-e)^2/e+1/4*b*c*ln(e*x^2+d)/d/(c^2*d-e)/e+1/32*I*b*c*polylog(2,(-c^2
)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d^(3/2)/e^
(3/2)/(-c^2)^(1/2)-1/32*I*b*c*ln(-(1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/
2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(3/2)/(-c^2)^(1/2
)-1/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^
(1/2)+I*e^(1/2)))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)-1/32*I*b*c*ln(-(1+x*(-c^2)^
(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))
/d^(3/2)/e^(3/2)/(-c^2)^(1/2)+1/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I
*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)
+1/32*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2))
)*ln(1-I*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(3/2)/(-c^2)^(1/2)-1/32*I*b*c*polylo
g(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d
^(3/2)/e^(3/2)/(-c^2)^(1/2)+1/32*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-
c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/d^(3/2)/e^(3/2)/(-c
^2)^(1/2)
```

3.1170.2 Mathematica [A] (warning: unable to verify)

Time = 12.54 (sec) , antiderivative size = 1744, normalized size of antiderivative = 1.81

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + (b*c^3*((-2*Log[1 + ((c^2*d - e)*Co
s[2*ArcTan[c*x]])/(c^2*d + e)])/(c^2*d) - (2*Log[1 + ((c^2*d - e)*Cos[2*Ar
cTan[c*x]])/(c^2*d + e)])/e + ((c^2*d - e)*e*(-4*ArcTan[c*x]*ArcTanh[Sqrt[
-(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x
)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[
(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I +
c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e
)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I
*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]
*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-
(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c
^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*
Cos[2*ArcTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTa
nh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(
Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d +
e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*
Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*
Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(
c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]...
```

3.1170.3 Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 966, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{a+b\arctan(cx)}{e\,(d+ex^2)^2}-\frac{d(a+b\arctan(cx))}{e\,(d+ex^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ib\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}+\frac{ib\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{b\log\left(c^2x^2+1\right)c}{4d\left(c^2d-e\right)e}+\frac{b\left(5c^2d-3e\right)\log\left(c^2x^2+1\right)c}{16d\left(c^2d-e\right)^2e}+\frac{b\log\left(ex^2+d\right)c}{4d\left(c^2d-e\right)e}-\frac{b\left(5c^2d-3e\right)\log\left(ex^2+d\right)c}{16d\left(c^2d-e\right)^2e}+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}+\frac{bc}{8\left(c^2d-e\right)e\left(ex^2+d\right)}+\frac{x(a+b\arctan(cx))}{8de\left(ex^2+d\right)}-\frac{x(a+b\arctan(cx))}{4e\left(ex^2+d\right)^2}+\frac{(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(b*c)/(8*(c^2*d - e)*e*(d + e*x^2)) - (x*(a + b*ArcTan[c*x]))/(4*e*(d + e*
x^2)^2) + (x*(a + b*ArcTan[c*x]))/(8*d*e*(d + e*x^2)) + ((a + b*ArcTan[c*x
])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqr
t[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]
*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]
*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1
 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)
/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e]*(1 + Sq
rt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + (b*c*(5*c^2*d - 3*e)*Log[1 + c^2*x^2])
/(16*d*(c^2*d - e)^2*e) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)*e) - (b*
c*(5*c^2*d - 3*e)*Log[d + e*x^2])/(16*d*(c^2*d - e)^2*e) + (b*c*Log[d + e*
x^2])/(4*d*(c^2*d - e)*e) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] -
I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/
2)) - ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c
^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Poly
Log[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e
])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sq
rt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^...
```

3.1170. $\int \frac{x^2(a+b\arctan(cx))}{\left(d+ex^2\right)^3}\,dx$

3.1170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1170.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3773 vs. 2(750) = 1500.

Time = 2.10 (sec) , antiderivative size = 3774, normalized size of antiderivative = 3.91

method	result	size
parts	Expression too large to display	3774
derivativedivides	Expression too large to display	3815
default	Expression too large to display	3815
risch	Expression too large to display	6982

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(e*d)^(1/2)*arctan(e*x/(e*d)^(1
/2)))-1/16*I*b*c^7*d^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c
^2*d*e)^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^2*d*e+e^2)^2/e^2*(c^2*d*e)^(1/2
)+1/4*I*b*c^5*d*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)
^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^2*d*e+e^2)^2/e*(c^2*d*e)^(1/2)-1/8*I*b
*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*e^2*arctan(c*x)*x^4-1/8
*I*b*c*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)*ln(1-(c^2*d
-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/4*I*b*c*e*ln(1
-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*
x)/(c^4*d^2-2*c^2*d*e+e^2)^2/d*(c^2*d*e)^(1/2)-1/16*I*b/c*e^2*ln(1-(c^2*d-
e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^4*
d^2-2*c^2*d*e+e^2)^2/d^2*(c^2*d*e)^(1/2)-3/16*b*c^3*polylog(2,(c^2*d-e)*(1
+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2
)^2*(c^2*d*e)^(1/2)-1/8*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*
d-1/16*b*(e*d)^(1/2)/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+
2*c^2*d+2*e)/c/(e*d)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)+1/32*b/c*(c^2*d*e)^(1/
2)/d^2/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)
/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+1/16*b/c*(c^2*d*e)^(1/2)/d^2/(c^4*d^2-2*c^2
*d*e+e^2)*arctan(c*x)^2+1/32*b*c^3*(c^2*d*e)^(1/2)/e^2/(c^4*d^2-2*c^2*d*e+
e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1...
```

3.1170.5 Fricas [F]

$$\int \frac{x^2(a + b\arctan(cx))}{(d + ex^2)^3}\,dx = \int \frac{(b\arctan(cx) + a)x^2}{(ex^2 + d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.1170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1170.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1170.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^3, x)
```

3.1171 $\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx$

3.1171.1 Optimal result

Integrand size = 18, antiderivative size = 893

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = -\frac{bc}{8d(c^2d-e)(d+ex^2)} + \frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2} + \frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)} + \frac{3(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}} + \frac{3ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\log\left(-\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} + \frac{3ibc\log\left(\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{bc(5c^2d-3e)\log(1+c^2x^2)}{16d^2(c^2d-e)^2} + \frac{bc(5c^2d-3e)\log(d+ex^2)}{16d^2(c^2d-e)^2} + \frac{3ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} + \frac{3ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}$$

output
```
-1/8*b*c/d/(c^2*d-e)/(e*x^2+d)+1/4*x*(a+b*arctan(c*x))/d/(e*x^2+d)^2+3/8*x
*(a+b*arctan(c*x))/d^2/(e*x^2+d)-1/16*b*c*(5*c^2*d-3*e)*ln(c^2*x^2+1)/d^2/
(c^2*d-e)^2+1/16*b*c*(5*c^2*d-3*e)*ln(e*x^2+d)/d^2/(c^2*d-e)^2+3/8*(a+b*ar
ctan(c*x))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)-3/32*I*b*c*ln(-(1+x*(
-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^
(1/2))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)+3/32*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/
2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))/d^(5/2)/(-c
^2)^(1/2)/e^(1/2)-3/32*I*b*c*ln(-(1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2
)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)
+3/32*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2))
)*ln(1+I*x*e^(1/2)/d^(1/2))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)+3/32*I*b*c*polylo
g(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/d
^(5/2)/(-c^2)^(1/2)/e^(1/2)-3/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x
*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)+3
/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/
2)-I*e^(1/2)))/d^(5/2)/(-c^2)^(1/2)/e^(1/2)-3/32*I*b*c*polylog(2,(-c^2)^(1
/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/d^(5/2)/(-c^2)
^(1/2)/e^(1/2)
```

3.1171.2 Mathematica [A] (warning: unable to verify)

Time = 10.77 (sec) , antiderivative size = 1745, normalized size of antiderivative = 1.95

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

output
```
(a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + (b*c*(10*c^2*d*Log[1 + ((c^2*d - e
)*Cos[2*ArcTan[c*x]])/(c^2*d + e)] - 6*e*Log[1 + ((c^2*d - e)*Cos[2*ArcTan
[c*x]])/(c^2*d + e)] + (3*c^2*d*(c^2*d - e)*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-
(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(
c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c
*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)
/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*
e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*
x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(
c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^
2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*C
os[2*ArcTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTan
h[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(S
qrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e
 + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*S
qrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*S
qrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c
^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*...
```

3.1171.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 875, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5447, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{\frac{3ex^3+5dx}{d^2(ex^2+d)^2}+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}}}{8(c^2x^2+1)}dx+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)}+\frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{8}bc\int \frac{\frac{3ex^3+5dx}{d^2(ex^2+d)^2}+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}}}{c^2x^2+1}dx+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)}+\frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{1}{8}bc\int \left(\frac{x(3ex^2+5d)}{d^2(c^2x^2+1)(ex^2+d)^2}+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(c^2x^2+1)}\right)dx+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)}+\frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(ex^2+d)}+\frac{x(a+b\arctan(cx))}{4d(ex^2+d)^2}-\frac{1}{8}bc\left(-\frac{3i\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{5/2}\sqrt{e}}+\frac{3i\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{5/2}\sqrt{e}}+\frac{3i\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right.}{4\sqrt{-c^2}d^{5}}\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

3.1171. $\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx$

output
```
(x*(a + b*ArcTan[c*x]))/(4*d*(d + e*x^2)^2) + (3*x*(a + b*ArcTan[c*x]))/(8
*d^2*(d + e*x^2)) + (3*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8
*d^(5/2)*Sqrt[e]) - (b*c*(1/(d*(c^2*d - e)*(d + e*x^2)) - (((3*I)/4)*Log[(
Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*S
qrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/4)*Log[-((Sqrt[
e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[
e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/4)*Log[-((Sqrt[e]*(
1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/4)*Log[(Sqrt[e]*(1 + Sq
rt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + ((5*c^2*d - 3*e)*Log[1 + c^2*x^2])/(2*
d^2*(c^2*d - e)^2) - ((5*c^2*d - 3*e)*Log[d + e*x^2])/(2*d^2*(c^2*d - e)^2
) - (((3*I)/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]
*Sqrt[d] - I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/4)*PolyLog[
2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])
/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d]
 + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sq
rt[e]) + (((3*I)/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[
-c^2]*Sqrt[d] + I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e])))/8
```

3.1171.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.1171.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4006 vs. 2(681) = 1362.

Time = 1.99 (sec) , antiderivative size = 4007, normalized size of antiderivative = 4.49

method	result	size
parts	Expression too large to display	4007
derivativedivides	Expression too large to display	4032
default	Expression too large to display	4032
risch	Expression too large to display	5059

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e+1/4*a*x/d/(e*x^2+d
)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))+1/
8*b*c^7/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*x^2*e-3/16*b*c*(c^2*d*
e)^(1/2)/d^2/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*
x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))-3/8*b*c*(c^2*d*e)^(1/2)/d^2/(c^4*d^2-
2*c^2*d*e+e^2)*arctan(c*x)^2+1/8*b*c^2*(e*d)^(1/2)/d^2*arctanh(1/4*(2*(c^2
*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(e*d)^(1/2))/(c^4*d^2-2*c^2*d
*e+e^2)-3/16*b*(e*d)^(1/2)/d^3*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2
*x^2+1)+2*c^2*d+2*e)/c/(e*d)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)+1/8*b*c^7/(c^4
*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*x^4*e^2-9/8*b*c^3*arctan(c*x)^2/
d/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)*e-3/32*b*c^7*polylog(2,(c^2*d-
e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/e/(c^4*d^2-2*c^2*
d*e+e^2)^2*(c^2*d*e)^(1/2)*d+3/8*b*c*e^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(
c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)^2/d^2*(c^
2*d*e)^(1/2)-3/4*b*c/d^2/(c^4*d^2-2*c^2*d*e+e^2)*e^2/(c^2*d-e)*ln((1+I*c*x
)/(c^2*x^2+1)^(1/2))+3/16*b*c/d^2/(c^4*d^2-2*c^2*d*e+e^2)*e^2/(c^2*d-e)*ln
(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*
x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-3/4*b*c^6/(c^4*d^2
-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*arctan(c*x)*x^3*e^2+5/16*b*c^2*(e*d)
^(1/2)*e/d^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2...
```

3.1171.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

3.1171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1171.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1171.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^3,x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^3, x)
```

3.1172 $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^3}\,dx$

3.1172.1 Optimal result

Integrand size = 21, antiderivative size = 1518

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
b*c*ln(x)/d^3+(-a-b*arctan(c*x))/d^3/x+1/16*b*c*(5*c^2*d-3*e)*e*ln(c^2*x^2
+1)/d^3/(c^2*d-e)^2-1/16*b*c*(5*c^2*d-3*e)*e*ln(e*x^2+d)/d^3/(c^2*d-e)^2-7
/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/
2)-I*e^(1/2)))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)-7/32*I*b*c*polylog(2,(-c^2)^(1
/2)*(d^(1/2)+I*x*e^(1/2))/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))*e^(1/2)/d^(7/2
)/(-c^2)^(1/2)-7/32*I*b*c*ln((1-x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^
(1/2)+e^(1/2)))*ln(1-I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)-7/3
2*I*b*c*ln((1+x*(-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln
(1+I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)+7/32*I*b*c*ln(-(1+x*(
-c^2)^(1/2))*e^(1/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*x*e^(1/2)/d^
(1/2))*e^(1/2)/d^(7/2)/(-c^2)^(1/2)+7/32*I*b*c*ln(-(1-x*(-c^2)^(1/2))*e^(1
/2)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*x*e^(1/2)/d^(1/2))*e^(1/2)/d^
(7/2)/(-c^2)^(1/2)-1/4*e*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)^2-7/8*e*x*(a+b*
arctan(c*x))/d^3/(e*x^2+d)-1/4*I*b*polylog(2,(1-I*c*x)*e^(1/2)/(I*c*(-d)^(
1/2)+e^(1/2)))*e^(1/2)/(-d)^(7/2)-1/4*I*b*polylog(2,(1+I*c*x)*e^(1/2)/(I*c
*(-d)^(1/2)+e^(1/2)))*e^(1/2)/(-d)^(7/2)-1/2*b*c*ln(c^2*x^2+1)/d^3+1/4*I*b
*ln(1-I*c*x)*ln(c*((-d)^(1/2)-x*e^(1/2))/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)
/(-d)^(7/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+x*e^(1/2))/(c*(-d)^(1/2)+
I*e^(1/2)))*e^(1/2)/(-d)^(7/2)+1/8*b*c*e/d^2/(c^2*d-e)/(e*x^2+d)+1/4*b*c*e
*ln(c^2*x^2+1)/d^3/(c^2*d-e)-1/4*b*c*e*ln(e*x^2+d)/d^3/(c^2*d-e)-1/4*I*...
```

3.1172.2 Mathematica [A] (verified)

Time = 13.08 (sec) , antiderivative size = 2005, normalized size of antiderivative = 1.32

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

output
```
-(a/(d^3*x)) - (a*e*x)/(4*d^2*(d + e*x^2)^2) - (7*a*e*x)/(8*d^3*(d + e*x^2
)) - (15*a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) + b*c^7*(-(Arc
Tan[c*x]/(c^7*d^3*x)) + Log[(c*x)/Sqrt[1 + c^2*x^2]]/(c^6*d^3) - (9*e*Log[
1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(16*c^4*d^2*(c^2*d -
 e)^2) + (7*e^2*Log[1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/
(16*c^6*d^3*(c^2*d - e)^2) - (15*e*(4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^
2*d*e)]*x)] + 2*ArcCos[(-(c^2*d) - e)/(c^2*d - e)]*ArcTanh[(c*e*x)/Sqrt[-(
c^2*d*e)]] - (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/S
qrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d -
 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))
] + (-ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^
2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqr
t[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (Arc
Cos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x
)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(S
qrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan
[c*x]]])] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] + (2*I)*(ArcTanh[(c*d)/(Sq
rt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt
[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d -
 e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^...
```

3.1172.3 Rubi [A] (verified)

Time = 2.71 (sec) , antiderivative size = 1518, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{e(a+b\arctan(cx))}{d^3(d+ex^2)}+\frac{a+b\arctan(cx)}{d^3x^2}-\frac{e(a+b\arctan(cx))}{d^2(d+ex^2)^2}-\frac{e(a+b\arctan(cx))}{d(d+ex^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7x(a+b\arctan(cx))e}{8d^3(ex^2+d)}-\frac{x(a+b\arctan(cx))e}{4d^2(ex^2+d)^2}+\frac{bc\log(c^2x^2+1)e}{4d^3(c^2d-e)}+\frac{bc(5c^2d-3e)\log(c^2x^2+1)e}{16d^3(c^2d-e)^2}-\frac{bc\log(ex^2+d)e}{4d^3(c^2d-e)}-\frac{bc(5c^2d-3e)\log(ex^2+d)e}{16d^3(c^2d-e)^2}+\frac{bce}{8d^2(c^2d-e)(ex^2+d)}-\frac{7(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{8d^{7/2}}-\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{d^{7/2}}-\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}+\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{c\sqrt{-d}-i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}+\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{7ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}-\frac{7ibc\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(icx+1)}{i\sqrt{-d}c+\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}-\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}-\frac{a+b\arctan(cx)}{d^3x}+\frac{bc\log(x)}{d^3}-\frac{bc\log(c^2x^2+1)}{2d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

3.1172. $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^3}\,dx$

output
```
(b*c*e)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (a + b*ArcTan[c*x])/(d^3*x) - (e
*x*(a + b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (7*e*x*(a + b*ArcTan[c*x])
)/(8*d^3*(d + e*x^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(7/2) -
(7*Sqrt[e]*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) +
(b*c*Log[x])/d^3 - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt
[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 - I
*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2)
 - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[
-d] - I*Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sq
rt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b
*c*Sqrt[e]*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e
])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c
*Sqrt[e]*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e
]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*
c*Sqrt[e]*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[
e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (((7*I)/32)*b
*c*Sqrt[e]*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e
])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (b*c*Log[1 + c^
2*x^2])/(2*d^3) + (b*c*(5*c^2*d - 3*e)*e*Log[1 + c^2*x^2])/(16*d^3*(c^2*d
- e)^2) + (b*c*e*Log[1 + c^2*x^2])/(4*d^3*(c^2*d - e)) - (b*c*(5*c^2*d ...
```

3.1172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1172.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.39 (sec) , antiderivative size = 5730, normalized size of antiderivative = 3.77

method	result	size
parts	Expression too large to display	5730
derivativedivides	Expression too large to display	5776
default	Expression too large to display	5776
risch	Expression too large to display	7503

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.1172.5 Fricas [F]

$$\int \frac{a + b \arctan(cx)}{x^2 (d + ex^2)^3} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)^3 x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^8 + 3*d*e^2*x^6 + 3*d^2*e*x^4 + d^3*x^
2), x)
```

3.1172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x^2 (d + ex^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.1172.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1172.8 Giac [F]

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)^3} \, dx = \int \frac{b\arctan(cx) + a}{(ex^2 + d)^3 x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)^3} \, dx = \int \frac{a + b\,\mathrm{atan}(c\,x)}{x^2 \, (e\,x^2 + d)^3} \, dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^3), x)
```

3.1173 $\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1173.1 Optimal result

Integrand size = 23, antiderivative size = 223

$$\begin{aligned}\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx &= -\frac{b(c^2d-12e)\,x\sqrt{d+ex^2}}{120c^3e}-\frac{bx(d+ex^2)^{3/2}}{20ce}\\ &\quad-\frac{d(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{3e^2}\\ &\quad+\frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{5e^2}\\ &\quad+\frac{b(c^2d-e)^{3/2}\,(2c^2d+3e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{15c^5e^2}\\ &\quad+\frac{b(15c^4d^2+20c^2de-24e^2)\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}}\end{aligned}$$

output
```
-1/20*b*x*(e*x^2+d)^(3/2)/c/e-1/3*d*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e^2+
1/5*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/e^2+1/15*b*(c^2*d-e)^(3/2)*(2*c^2*d+
3*e)*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/c^5/e^2+1/120*b*(15*c^4*d^2
+20*c^2*d*e-24*e^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^5/e^(3/2)-1/120*b
*(c^2*d-12*e)*x*(e*x^2+d)^(1/2)/c^3/e
```

3.1173.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.75

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$= \frac{-c^2\sqrt{d+ex^2}(8ac^3(2d^2-dex^2-3e^2x^4)+bex(-12e+c^2(7d+6ex^2)))-8bc^5\sqrt{d+ex^2}(2d^2-dex^2-3e^2}{}$$

input
```
Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
(-(c^2*Sqrt[d + e*x^2]*(8*a*c^3*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*x*(-12
*e + c^2*(7*d + 6*e*x^2)))) - 8*b*c^5*Sqrt[d + e*x^2]*(2*d^2 - d*e*x^2 - 3
*e^2*x^4)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e)*Log[((-6
0*I)*c^6*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d -
e)^(5/2)*(2*c^2*d + 3*e)*(I + c*x))] + (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d
+ 3*e)*Log[((60*I)*c^6*e^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2])
)/(b*(c^2*d - e)^(5/2)*(2*c^2*d + 3*e)*(-I + c*x))] + b*Sqrt[e]*(15*c^4*d^
2 + 20*c^2*d*e - 24*e^2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(120*c^5*e^2)
```

3.1173.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5511, 27, 403, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(2d-3ex^2)\,(ex^2+d)^{3/2}}{15e^2\,(c^2x^2+1)}dx+\frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{c^2x^2+1}dx}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\int \frac{\sqrt{ex^2+d}\left(d\left(8dc^2+3e\right)-\left(c^2d-12e\right)ex^2\right)}{c^2x^2+1}dx}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{\int \frac{e\left(15d^2c^4+20dec^2-24e^2\right)x^2+d\left(16d^2c^4+7dec^2-12e^2\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}-\frac{ex\left(c^2d-12e\right)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{398}$$

$$\frac{bc\left(\frac{\frac{\frac{8\left(2c^2d+3e\right)\left(c^2d-e\right)^2\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(15c^4d^2+20c^2de-24e^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}-\frac{ex\left(c^2d-12e\right)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{bc\left(\frac{\frac{\frac{8\left(2c^2d+3e\right)\left(c^2d-e\right)^2\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(15c^4d^2+20c^2de-24e^2\right)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}-\frac{ex\left(c^2d-12e\right)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{219}$$

3.1173. $\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

$$\frac{bc\left(\frac{\frac{\frac{8(2c^2d+3e)(c^2d-e)^2\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(15c^4d^2+20c^2de-24e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex(c^2d-12e)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \; 291$$

$$\frac{bc\left(\frac{\frac{\frac{8(2c^2d+3e)(c^2d-e)^2\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(15c^4d^2+20c^2de-24e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex(c^2d-12e)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \; 216$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}+\frac{bc\left(\frac{\frac{\frac{8(2c^2d+3e)(c^2d-e)^{3/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}(15c^4d^2+20c^2de-24e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex(c^2d-12e)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}$$

input
```
Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
-1/3*(d*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^(5/2)*(a
 + b*ArcTan[c*x]))/(5*e^2) + (b*c*((-3*e*x*(d + e*x^2)^(3/2))/(4*c^2) + (-
1/2*((c^2*d - 12*e)*e*x*Sqrt[d + e*x^2])/c^2 + ((8*(c^2*d - e)^(3/2)*(2*c^
2*d + 3*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(15
*c^4*d^2 + 20*c^2*d*e - 24*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)
/(2*c^2))/(4*c^2)))/(15*e^2)
```

3.1173. $\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1173.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1173.4 Maple [F]

$$\int x^{3}\sqrt{e\,x^{2}+d}\,(a+b\arctan(cx))\,dx$$

input
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

3.1173.5 Fricas [A] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 1200, normalized size of antiderivative = 5.38

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/240*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*sqrt(e)*log(-2*e*x^2 + 2
*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 4*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sq
rt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d
*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d
^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 6
*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*e^2)*x + 8*(3*b*c^
5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^
5*e^2), 1/240*(8*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sqrt(c^2*d - e)*arcta
n(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e -
e^2)*x^3 + (c^2*d^2 - d*e)*x)) - (15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*
sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(24*a*c^5*e^2*
x^4 + 8*a*c^5*d*e*x^2 - 6*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12
*b*c^2*e^2)*x + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c
*x))*sqrt(e*x^2 + d))/(c^5*e^2), -1/120*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24
*b*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(2*b*c^4*d^2 + b*c
^2*d*e - 3*b*e^2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4
- 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)
*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - (24*a*c^5*e^2*x^4 + 8
*a*c^5*d*e*x^2 - 6*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*
e^2)*x + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))...
```

3.1173.6 Sympy [F]

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^3(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**3*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

3.1173.7 Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1173.8 Giac [F]

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan{(cx)}+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1173.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x^3\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1174 $\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1174.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2}-\frac{ad^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{3/2}}+b\text{Int}\left(x^2\sqrt{d+ex^2}\arctan(cx),x\right)$$

output
```
-1/8*a*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)+1/8*a*d*x*(e*x^2+d)^
(1/2)/e+1/4*a*x^3*(e*x^2+d)^(1/2)+b*Unintegrable(x^2*arctan(c*x)*(e*x^2+d)
^(1/2),x)
```

3.1174.2 Mathematica [N/A]

Not integrable

Time = 14.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

3.1174.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 248, 262, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^2\sqrt{ex^2+d}dx+b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{1}{4}d\int \frac{x^2}{\sqrt{ex^2+d}}dx+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{262}$$

$$a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{219}$$

$$b\int x^2\sqrt{ex^2+d}\arctan(cx)dx+a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int x^2\sqrt{ex^2+d}\arctan(cx)dx+a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)$$

input
```
Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1174.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1174. $\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1174.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{e\,x^2+d}\,(a+b\arctan(cx))\,dx$$

input
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

3.1174.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

3.1174.6 Sympy [N/A]

Not integrable

Time = 23.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x^2(a+b\,\mathrm{atan}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**2*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

3.1174.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1174.8 Giac [N/A]

Not integrable

Time = 95.51 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan{(cx)}+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1174.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x^2\,(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1175 $\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1175.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = -\frac{bx\sqrt{d+ex^2}}{6c}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e} - \frac{b(c^2d-e)^{3/2}\arctan\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{3c^3e} - \frac{b(3c^2d-2e)\operatorname{arctanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e-1/3*b*(c^2*d-e)^(3/2)*arctan(x*(c^
2*d-e)^(1/2)/(e*x^2+d)^(1/2))/c^3/e-1/6*b*(3*c^2*d-2*e)*arctanh(x*e^(1/2)/
(e*x^2+d)^(1/2))/c^3/e^(1/2)-1/6*b*x*(e*x^2+d)^(1/2)/c
```

3.1175.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.99

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$= \frac{c^2\sqrt{d+ex^2}(-bex+2ac(d+ex^2))+2bc^3(d+ex^2)^{3/2}\arctan(cx)-ib(c^2d-e)^{3/2}\log\left(\frac{12c^4e\left(-icd+ex-i\sqrt{c^2d-}\right.}{b(c^2d-e)^{5/2}(-i+}\right.}{6c}$$

input
```
Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*Sqrt[d + e*x^2]*(-(b*e*x) + 2*a*c*(d + e*x^2)) + 2*b*c^3*(d + e*x^2)^
(3/2)*ArcTan[c*x] - I*b*(c^2*d - e)^(3/2)*Log[(12*c^4*e*((-I)*c*d + e*x -
I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))] + I*
b*(c^2*d - e)^(3/2)*Log[(12*c^4*e*(I*c*d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d
+ e*x^2]))/(b*(c^2*d - e)^(5/2)*(I + c*x))] + b*Sqrt[e]*(-3*c^2*d + 2*e)*L
og[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(6*c^3*e)
```

3.1175.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5509, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5509}$$

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\int \frac{(ex^2+d)^{3/2}}{c^2x^2+1}dx}{3e}$$

$$\Big\downarrow \text{ 318}$$

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\int \frac{(3c^2d-2e)ex^2+d(2c^2d-e)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2(c^2d-e)^2\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(3c^2d-2e)\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2\left(c^2d-e\right)^2\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(3c^2d-2e\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2\left(c^2d-e\right)^2\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}\left(3c^2d-2e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$$\Big\downarrow \; 291$$

$$\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2\left(c^2d-e\right)^2\int\frac{1}{1-\frac{\left(e-c^2d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}\left(3c^2d-2e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$$\Big\downarrow \; 216$$

$$\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2\left(c^2d-e\right)^{3/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(3c^2d-2e\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

input
```
Int[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e) - (b*c*((e*x*Sqrt[d + e*x^2]
)/(2*c^2) + ((2*(c^2*d - e)^(3/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^
2]])/c^2 + ((3*c^2*d - 2*e)*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/
c^2)/(2*c^2)))/(3*e)
```

3.1175. $\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1175.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1175.4 Maple [F]

$$\int x\sqrt{e\,x^{2}+d}\,(a+b\arctan(cx))\,dx$$

input
```
int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

3.1175.5 Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 879, normalized size of antiderivative = 6.28

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$
$$=\left[-\frac{(3\,bc^2d-2\,be)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+(bc^2d-be)\sqrt{-c^2d+e}\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-}{\cdots}\right.}{}\right.$$
$$-\frac{2\,(bc^2d-be)\sqrt{c^2d-e}\arctan\left(\frac{\sqrt{c^2d-e}\left((c^2d-2\,e)x^2-d\right)\sqrt{ex^2+d}}{2\,((c^2de-e^2)x^3+(c^2d^2-de)x)}\right)+(3\,bc^2d-2\,be)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+\cdots}\right.}{12\,c^3e}$$
$$-\frac{(bc^2d-be)\sqrt{c^2d-e}\arctan\left(\frac{\sqrt{c^2d-e}\left((c^2d-2\,e)x^2-d\right)\sqrt{ex^2+d}}{2\,((c^2de-e^2)x^3+(c^2d^2-de)x)}\right)-(3\,bc^2d-2\,be)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(2\,a\cdots}{6\,c^3e}$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/12*((3*b*c^2*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(
e)*x - d) + (b*c^2*d - b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8
*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-
c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2*a*c^3*
e*x^2 + 2*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqr
t(e*x^2 + d))/(c^3*e), -1/12*(2*(b*c^2*d - b*e)*sqrt(c^2*d - e)*arctan(1/2
*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*
x^3 + (c^2*d^2 - d*e)*x)) + (3*b*c^2*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(2*a*c^3*e*x^2 + 2*a*c^3*d - b*c^2*e*x +
 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(2*
(3*b*c^2*d - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (b*c^2*d
 - b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2
*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^
2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^3*e*x^2 + 2*a*c^3*d -
b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e
), -1/6*((b*c^2*d - b*e)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*
d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x
)) - (3*b*c^2*d - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*
a*c^3*e*x^2 + 2*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x
))*sqrt(e*x^2 + d))/(c^3*e)]
```

3.1175.6 Sympy [F]

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x(a+b\operatorname{atan}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

3.1175.7 Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

3.1175.8 Giac [F]

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1175.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1176 $\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1176.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\arctan(cx)),x\right)$$

output
```
Unintegrable((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

3.1176.2 Mathematica [N/A]

Not integrable

Time = 5.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

3.1176.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1176.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1176.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \sqrt{e\,x^2+d}\,(a+b\arctan(cx))\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

3.1176.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a), x)
```

3.1176.6 Sympy [N/A]

Not integrable

Time = 7.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int (a+b\operatorname{atan}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

3.1176.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1176.8 Giac [N/A]

Not integrable

Time = 91.78 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1176.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int (a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1177 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x} \, dx$

3.1177.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x} \, dx = a\sqrt{d+ex^2} - a\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\text{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x}, x\right)$$

output
```
-a*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)+a*(e*x^2+d)^(1/2)+b*Unintegrab
le(arctan(c*x)*(e*x^2+d)^(1/2)/x,x)
```

3.1177.2 Mathematica [N/A]

Not integrable

Time = 10.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x} \, dx = \int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x} \, dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x, x]
```

3.1177.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 60, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\sqrt{ex^2+d}}{x}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx+\frac{1}{2}a\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx+\frac{1}{2}a\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

output
```
$Aborted
```

3.1177. $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx$

3.1177.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1177. $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx$

3.1177.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}\,(a+b\arctan(cx))}{x}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

3.1177.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x, x)
```

3.1177.6 Sympy [N/A]

Not integrable

Time = 5.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))\sqrt{d+ex^2}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x, x)
```

3.1177.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1177.8 Giac [N/A]

Not integrable

Time = 283.98 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.1177.9 Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x, x)
```

3.1178 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx$

3.1178.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = -\frac{a\sqrt{d+ex^2}}{x}+a\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x^2},x\right)$$

output
```
a*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)-a*(e*x^2+d)^(1/2)/x+b*Uninteg
rable(arctan(c*x)*(e*x^2+d)^(1/2)/x^2,x)
```

3.1178.2 Mathematica [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2, x]
```

3.1178.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 247, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\sqrt{ex^2+d}}{x^2}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(e\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{\sqrt{d+ex^2}}{x}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{d+ex^2}}{x}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx+a\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx+a\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.1178.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1178.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}\,\left(a+b\arctan\left(cx\right)\right)}{x^{2}}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

3.1178.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^2, x)
```

3.1178.6 Sympy [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**2,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**2, x)
```

3.1178.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1178.8 Giac [N/A]

Not integrable

Time = 281.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1178.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^2, x)
```

3.1179 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx$

3.1179.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = -\frac{a\sqrt{d+ex^2}}{2x^2} - \frac{ae\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2\sqrt{d}} + b\text{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x^3},x\right)$$

output
```
-1/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)-1/2*a*(e*x^2+d)^(1/2)/x^
2+b*Unintegrable(arctan(c*x)*(e*x^2+d)^(1/2)/x^3,x)
```

3.1179.2 Mathematica [N/A]

Not integrable

Time = 11.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3, x]
```

3.1179.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 243, 51, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\sqrt{ex^2+d}}{x^3}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\sqrt{ex^2+d}}{x^4}dx^2+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{1}{2}e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-\frac{\sqrt{d+ex^2}}{x^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}-\frac{\sqrt{d+ex^2}}{x^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx+\frac{1}{2}a\left(-\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx+\frac{1}{2}a\left(-\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.1179.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1179. $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx$

3.1179.4 Maple [N/A] (verified)

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}\,\left(a+b\arctan\left(c x\right)\right)}{x^{3}}d x$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

3.1179.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx=\int \frac{\sqrt{ex^2+d}(b\arctan\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^3, x)
```

3.1179.6 Sympy [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(cx\right)\right)\sqrt{d+ex^2}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**3, x)
```

3.1179.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1179.8 Giac [N/A]

Not integrable

Time = 288.61 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.1179.9 Mupad [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^3, x)
```

3.1180 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx$

3.1180.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = -\frac{bc\sqrt{d+ex^2}}{6x^2} - \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3} + \frac{bc(2c^2d-3e)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6\sqrt{d}} - \frac{b(c^2d-e)^{3/2}\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d}$$

output

```
-1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/d/x^3-1/3*b*(c^2*d-e)^(3/2)*arctanh
(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d+1/6*b*c*(2*c^2*d-3*e)*arctanh((e*x^2
+d)^(1/2)/d^(1/2))/d^(1/2)-1/6*b*c*(e*x^2+d)^(1/2)/x^2
```

3.1180.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = -\frac{\sqrt{d+ex^2}(bcdx+2a(d+ex^2))+2b(d+ex^2)^{3/2}\arctan(cx)+bc\sqrt{d}(2c^2d-3e)\,x^3\log(x)-bc\sqrt{d}(2c^2d-}{}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d + e*x^2)) + 2*b*(d + e*x^2)^(3/2)*
ArcTan[c*x] + b*c*Sqrt[d]*(2*c^2*d - 3*e)*x^3*Log[x] - b*c*Sqrt[d]*(2*c^2*
d - 3*e)*x^3*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*(c^2*d - e)^(3/2)*x^3*Lo
g[(12*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^
(5/2)*(I + c*x))] + b*(c^2*d - e)^(3/2)*x^3*Log[(12*c*d*(c*d + I*e*x + Sqr
t[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))])/(d*x^3)
```

3.1180.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5511, 27, 354, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4} \, dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{\left(ex^2+d\right)^{3/2}}{3dx^3\left(c^2x^2+1\right)}dx-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{\left(ex^2+d\right)^{3/2}}{x^3\left(c^2x^2+1\right)}dx}{3d}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{354}$$

$$\frac{bc\int \frac{\left(ex^2+d\right)^{3/2}}{x^4\left(c^2x^2+1\right)}dx^2}{6d}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{109}$$

$$\frac{bc\left(-\int \frac{\left(c^2d-2e\right)ex^2+d\left(2c^2d-3e\right)}{2x^2\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(-\frac{1}{2}\int \frac{\left(c^2d-2e\right)ex^2+d\left(2c^2d-3e\right)}{x^2\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{174}$$

$$\frac{bc\left(\frac{1}{2}\left(2(c^2d-e)^2\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2-d(2c^2d-3e)\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{bc\left(\frac{1}{2}\left(\frac{4(c^2d-e)^2\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}-\frac{2d(2c^2d-3e)\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{bc\left(\frac{1}{2}\left(2\sqrt{d}(2c^2d-3e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{4(c^2d-e)^{3/2}\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(d*x^3) + (b*c*(-((d*Sqrt[d +
 e*x^2])/x^2) + (2*Sqrt[d]*(2*c^2*d - 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
] - (4*(c^2*d - e)^(3/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/
2))/(6*d)
```

3.1180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1180. $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx$

3.1180.4 Maple [F]

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\arctan(cx))}{x^4}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

3.1180.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 858, normalized size of antiderivative = 6.26

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx$$

$$= \Bigg[-\frac{(bc^2d-be)\sqrt{c^2d-e}x^3\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)x^2+4\,(c^3ex^2+2\,c^3d-ce)\sqrt{c^2d-e}\sqrt{ex^2+d}+e^2}{c^4x^4+2\,c^2x^2+1}\right)+(2\,bc^3}{1}$$

$$-\frac{2\,(bc^2d-be)\sqrt{-c^2d+e}x^3\arctan\left(-\frac{(c^2ex^2+2\,c^2d-e)\sqrt{-c^2d+e}\sqrt{ex^2+d}}{2\,(c^3d^2-cde+(c^3de-ce^2)x^2)}\right)+(2\,bc^3d-3\,bce)\sqrt{d}x^3\log\left(-\frac{ex^2-2\,\sqrt{}}{}\right.}{12\,dx^3}$$

$$-\frac{2\,(2\,bc^3d-3\,bce)\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right)+(bc^2d-be)\sqrt{c^2d-e}x^3\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)}{c^4x^4}\right.}{12\,dx}$$

$$-\frac{(bc^2d-be)\sqrt{-c^2d+e}x^3\arctan\left(-\frac{(c^2ex^2+2\,c^2d-e)\sqrt{-c^2d+e}\sqrt{ex^2+d}}{2\,(c^3d^2-cde+(c^3de-ce^2)x^2)}\right)+(2\,bc^3d-3\,bce)\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-}}{\sqrt{ex^2}}\right.}{6\,dx^3}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
[-1/12*((b*c^2*d - b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 -
 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)
*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b*
c^3*d - 3*b*c*e)*sqrt(d)*x^3*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d
)/x^2) + 2*(b*c*d*x + 2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*s
qrt(e*x^2 + d))/(d*x^3), -1/12*(2*(b*c^2*d - b*e)*sqrt(-c^2*d + e)*x^3*arc
tan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d
^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + (2*b*c^3*d - 3*b*c*e)*sqrt(d)*x^3*l
og(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*x + 2*a*e*x^
2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^3), -1/12
*(2*(2*b*c^3*d - 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(-d)/sqrt(e*x^2 + d)) +
(b*c^2*d - b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d
*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^
2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(b*c*d*x +
2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^3
), -1/6*((b*c^2*d - b*e)*sqrt(-c^2*d + e)*x^3*arctan(-1/2*(c^2*e*x^2 + 2*c
^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c
*e^2)*x^2)) + (2*b*c^3*d - 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(-d)/sqrt(e*x^
2 + d)) + (b*c*d*x + 2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sq
rt(e*x^2 + d))/(d*x^3)]
```

3.1180.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**4,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**4, x)
```

3.1180.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1180.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.1180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^4, x)
```

3.1181 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$

3.1181.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = -\frac{a\sqrt{d+ex^2}}{4x^4} - \frac{ae\sqrt{d+ex^2}}{8dx^2} + \frac{ae^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8d^{3/2}} + b\operatorname{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x^5},x\right)$$

output
```
1/8*a*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/4*a*(e*x^2+d)^(1/2)/x
^4-1/8*a*e*(e*x^2+d)^(1/2)/d/x^2+b*Unintegrable(arctan(c*x)*(e*x^2+d)^(1/2
)/x^5,x)
```

3.1181.2 Mathematica [N/A]

Not integrable

Time = 14.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5, x]
```

3.1181.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 51, 52, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\sqrt{ex^2+d}}{x^5}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\sqrt{ex^2+d}}{x^6}dx^2+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{1}{4}e\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2-\frac{\sqrt{d+ex^2}}{2x^4}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(\frac{1}{4}e\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{1}{4}e\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx+\frac{1}{2}a\left(\frac{1}{4}e\left(\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx+\frac{1}{2}a\left(\frac{1}{4}e\left(\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
$Aborted
```

3.1181.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

3.1181. $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1181.4 Maple [N/A] (verified)

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}\,(a+b\arctan(cx))}{x^{5}}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
```

3.1181.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx=\int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^5, x)
```

3.1181.6 Sympy [N/A]

Not integrable

Time = 8.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}}{x^5}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**5, x)
```

3.1181.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1181.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
Timed out
```

3.1181.9 Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^5,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^5, x)
```

3.1182 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$

3.1182.1 Optimal result

Integrand size = 23, antiderivative size = 224

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx &= \frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2}-\frac{bc(d+ex^2)^{3/2}}{20dx^4}\\&-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}\\&+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}\\&-\frac{bc(24c^4d^2-20c^2de-15e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{120d^{3/2}}\\&+\frac{b(c^2d-e)^{3/2}(3c^2d+2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{15d^2}\end{aligned}$$

output
```
-1/5*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*a
rctan(c*x))/d^2/x^3+1/30*b*c*(3*c^2*d-e)*e*arctanh((e*x^2+d)^(1/2)/d^(1/2)
)/d^(3/2)+1/40*b*c*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/15*b*c*(
c^2*d-e)*(3*c^2*d+2*e)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+1/15*b*(c^
2*d-e)^(3/2)*(3*c^2*d+2*e)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2-
1/20*b*c*(e*x^2+d)^(1/2)/x^4+1/30*b*c*(3*c^2*d-e)*(e*x^2+d)^(1/2)/d/x^2-1/
40*b*c*e*(e*x^2+d)^(1/2)/d/x^2
```

3.1182.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{-\sqrt{d+ex^2}(8a(3d^2+dex^2-2e^2x^4)+bcdx(7ex^2+d(6-12c^2x^2)))-8b\sqrt{d+ex^2}(3d^2+dex^2-2e^2x^4)\,\mathrm{a}}{}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(-(Sqrt[d + e*x^2]*(8*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*d*x*(7*e*x^2 +
 d*(6 - 12*c^2*x^2)))) - 8*b*Sqrt[d + e*x^2]*(3*d^2 + d*e*x^2 - 2*e^2*x^4)
*ArcTan[c*x] + b*c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[x] -
 b*c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[d + Sqrt[d]*Sqrt[d
 + e*x^2]] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c*d^2*(c*d
 - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(3*c^2*d
 + 2*e)*(I + c*x))] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c
*d^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)
*(3*c^2*d + 2*e)*(-I + c*x))])/(120*d^2*x^5)
```

3.1182.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5511, 27, 435, 166, 27, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$$
$$\Big\downarrow \text{5511}$$
$$-bc\int -\frac{(3d-2ex^2)\left(ex^2+d\right)^{3/2}}{15d^2x^5\left(c^2x^2+1\right)}dx+\frac{2e\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\int \frac{(3d-2ex^2)(ex^2+d)^{3/2}}{x^5(c^2x^2+1)}dx}{15d^2}+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \ 435$$

$$\frac{bc\int \frac{(3d-2ex^2)(ex^2+d)^{3/2}}{x^6(c^2x^2+1)}dx^2}{30d^2}+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \ 166$$

$$\frac{bc\left(\frac{1}{2}\int -\frac{\sqrt{ex^2+d}(e(3dc^2+8e)x^2+d(12c^2d-e))}{2x^4(c^2x^2+1)}dx^2-\frac{3d(d+ex^2)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\left(-\frac{1}{4}\int \frac{\sqrt{ex^2+d}(e(3dc^2+8e)x^2+d(12c^2d-e))}{x^4(c^2x^2+1)}dx^2-\frac{3d(d+ex^2)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \ 166$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{d(12c^2d-e)\sqrt{d+ex^2}}{x^2}-\int -\frac{e(12d^2c^4-7dec^2-16e^2)x^2+d(24d^2c^4-20dec^2-15e^2)}{2x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)-\frac{3d(d+ex^2)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{e(12d^2c^4-7dec^2-16e^2)x^2+d(24d^2c^4-20dec^2-15e^2)}{x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2+\frac{d(12c^2d-e)\sqrt{d+ex^2}}{x^2}\right)-\frac{3d(d+ex^2)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \ 174$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(d(24c^4d^2-20c^2de-15e^2)\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-8(c^2d-e)^2(3c^2d+2e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)+\frac{d(12c^2d-e)\sqrt{}}{x^2}\right.\right.}{30d^2}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

3.1182. $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$

$$\Big\downarrow \; 73$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{2d(24c^4d^2-20c^2de-15e^2)\int\frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{16(c^2d-e)^2(3c^2d+2e)\int\frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)+\frac{d(12c^2d-e)\sqrt{d+ex^2}}{x^2}\right)-}{30d^2}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 221$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}+$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{16(c^2d-e)^{3/2}(3c^2d+2e)\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}-2\sqrt{d}(24c^4d^2-20c^2de-15e^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{d(12c^2d-e}{x^2}\right.\right.}{30d^2}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(d*x^5) + (2*e*(d + e*x^2)^(3
/2)*(a + b*ArcTan[c*x]))/(15*d^2*x^3) + (b*c*((-3*d*(d + e*x^2)^(3/2))/(2*
x^4) + ((d*(12*c^2*d - e)*Sqrt[d + e*x^2])/x^2 + (-2*Sqrt[d]*(24*c^4*d^2 -
 20*c^2*d*e - 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (16*(c^2*d - e)^(
3/2)*(3*c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/2)/4
))/(30*d^2)
```

3.1182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1182.4 Maple [F]

$$\int \frac{\sqrt{e\,x^{2}+d}\,(a+b\arctan(cx))}{x^{6}}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
```

3.1182.5 Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 1156, normalized size of antiderivative = 5.16

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
[-1/240*(4*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e^2)*sqrt(c^2*d - e)*x^5*log((c^
4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3
*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 +
2*c^2*x^2 + 1)) + (24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(d)*x^5*l
og(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(16*a*e^2*x^4 - 6*b
*c*d^2*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8*(2*
b*e^2*x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5),
1/240*(8*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e^2)*sqrt(-c^2*d + e)*x^5*arctan(-
1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) - (24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^
2)*sqrt(d)*x^5*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(16
*a*e^2*x^4 - 6*b*c*d^2*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 -
24*a*d^2 + 8*(2*b*e^2*x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 +
 d))/(d^2*x^5), 1/120*((24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(-d)
*x^5*arctan(sqrt(-d)/sqrt(e*x^2 + d)) - 2*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e
^2)*sqrt(c^2*d - e)*x^5*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^
4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqr
t(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (16*a*e^2*x^4 - 6*b*c*d^2
*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8*(2*b*e^2*
x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5), 1/1...
```

3.1182.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(a+b\operatorname{atan}(cx))\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**6,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**6, x)
```

3.1182.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1182.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
Timed out
```

3.1182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^6,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^6, x)
```

3.1183 $\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$

3.1183.1 Optimal result

Integrand size = 23, antiderivative size = 279

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\frac{b(3c^4d^2+54c^2de-40e^2)\,x\sqrt{d+ex^2}}{560c^5e}-\frac{b(13c^2d-30e)\,x(d+ex^2)^{3/2}}{840c^3e}-\frac{bx(d+ex^2)^{5/2}}{42ce}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}+\frac{b(c^2d-e)^{5/2}(2c^2d+5e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{35c^7e^2}+\frac{b(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{560c^7e^{3/2}}$$

output
```
-1/840*b*(13*c^2*d-30*e)*x*(e*x^2+d)^(3/2)/c^3/e-1/42*b*x*(e*x^2+d)^(5/2)/
c/e-1/5*d*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*a
rctan(c*x))/e^2+1/35*b*(c^2*d-e)^(5/2)*(2*c^2*d+5*e)*arctan(x*(c^2*d-e)^(1
/2)/(e*x^2+d)^(1/2))/c^7/e^2+1/560*b*(35*c^6*d^3+70*c^4*d^2*e-168*c^2*d*e^
2+80*e^3)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^7/e^(3/2)+1/560*b*(3*c^4*d^
2+54*c^2*d*e-40*e^2)*x*(e*x^2+d)^(1/2)/c^5/e
```

3.1183.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.50

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx =$$

$$-\frac{c^2\sqrt{d+ex^2}\Big(48ac^5(2d-5ex^2)\,(d+ex^2)^2+bex(120e^2-6c^2e(37d+10ex^2)+c^4(57d^2+106dex^2+40e^2x}{}$$

input
```
Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/1680*(c^2*Sqrt[d + e*x^2]*(48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e
*x*(120*e^2 - 6*c^2*e*(37*d + 10*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40*e
^2*x^4))) + 48*b*c^7*(2*d - 5*e*x^2)*(d + e*x^2)^(5/2)*ArcTan[c*x] + (24*I
)*b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*Log[((-140*I)*c^8*e^2*(c*d - I*e*x +
 Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2*c^2*d + 5*e)*(I
 + c*x))] - (24*I)*b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*Log[((140*I)*c^8*e^
2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2
*c^2*d + 5*e)*(-I + c*x))] - 3*b*Sqrt[e]*(35*c^6*d^3 + 70*c^4*d^2*e - 168*
c^2*d*e^2 + 80*e^3)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(c^7*e^2)
```

3.1183.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5511, 27, 403, 403, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(2d-5ex^2)\,(ex^2+d)^{5/2}}{35e^2\,(c^2x^2+1)}dx+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\downarrow 27$$

$$\frac{bc\int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{c^2x^2+1}dx}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\downarrow 403$$

$$\frac{bc\left(\frac{\int \frac{(ex^2+d)^{3/2}\left(d(12dc^2+5e)-(13c^2d-30e)ex^2\right)}{c^2x^2+1}dx}{6c^2}-\frac{5ex(d+ex^2)^{5/2}}{6c^2}\right)}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\downarrow 403$$

$$\frac{bc\left(\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(e(3d^2c^4+54dec^2-40e^2)x^2+d(16d^2c^4+11dec^2-10e^2)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex(13c^2d-30e)(d+ex^2)^{3/2}}{4c^2}}{6c^2}-\frac{5ex(d+ex^2)^{5/2}}{6c^2}\right)}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\downarrow 27$$

$$\frac{bc\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e(3d^2c^4+54dec^2-40e^2)x^2+d(16d^2c^4+11dec^2-10e^2)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex(13c^2d-30e)(d+ex^2)^{3/2}}{4c^2}}{6c^2}-\frac{5ex(d+ex^2)^{5/2}}{6c^2}\right)}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\downarrow 403$$

$$\frac{bc\left(\frac{\frac{3\left(\frac{\int \frac{e(35d^3c^6+70d^2ec^4-168de^2c^2+80e^3)x^2+d(32d^3c^6+19d^2ec^4-74de^2c^2+40e^3)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex(3c^4d^2+54c^2de-40e^2)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}-\frac{ex(13c^2d-30e)(d+ex^2)^{3/2}}{4c^2}}{6c^2}\right)}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\downarrow 398$$

3.1183. $\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^3\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\int\frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}-\frac{ex\left(13c^2\right.}{}}{6c^2}\right.}{35e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{224}$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^3\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}}{6c^2}\right.}{35e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{219}$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^3\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{2c^2}\right.}{4c^2}}{6c^2}\right.}{35e^2}$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{291}$$

3.1183. $\int x^3\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16(2c^2d+5e)(c^2d-e)^3\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(3c^4d^2+54c^2de-40e^2)\sqrt{d+}}{2c^2}\right.}{4c^2}}{6c^2}\right.$$

$$\overline{35e^2}$$

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow 216$$

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}+$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16(2c^2d+5e)(c^2d-e)^{5/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(3c^4d^2+54c^2de-40e^2)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}}{6c^2}\right.$$

$$\overline{35e^2}$$

input
```
Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/5*(d*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^(7/2)*(a
 + b*ArcTan[c*x]))/(7*e^2) + (b*c*((-5*e*x*(d + e*x^2)^(5/2))/(6*c^2) + (-
1/4*((13*c^2*d - 30*e)*e*x*(d + e*x^2)^(3/2))/c^2 + (3*((e*(3*c^4*d^2 + 54
*c^2*d*e - 40*e^2)*x*Sqrt[d + e*x^2])/(2*c^2) + ((16*(c^2*d - e)^(5/2)*(2*
c^2*d + 5*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(
35*c^6*d^3 + 70*c^4*d^2*e - 168*c^2*d*e^2 + 80*e^3)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/c^2)/(2*c^2)))/(4*c^2))/(6*c^2)))/(35*e^2)
```

3.1183. $\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$

3.1183.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1183.4 Maple [F]

$$\int x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b\arctan \left(c x \right)\right)d x$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

3.1183.5 Fricas [A] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 1566, normalized size of antiderivative = 5.61

$$\int x^3\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[1/3360*(3*(35*b*c^6*d^3 + 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sq
rt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 24*(2*b*c^6*d^3 +
b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*
c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 -
d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) +
2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 - 40*b*c^6*e^3*x^5 + 48*a*c^7*d
^2*e*x^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2 - 30*b*c^4*e^3)*x^3 - 3*(19*b*
c^6*d^2*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*x + 48*(5*b*c^7*e^3*x^6 + 8*b*c
^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d)
)/(c^7*e^2), 1/3360*(48*(2*b*c^6*d^3 + b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e
^3)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqr
t(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) + 3*(35*b*c^6*d^3
+ 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sqrt(e)*log(-2*e*x^2 - 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4
- 40*b*c^6*e^3*x^5 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2
 - 30*b*c^4*e^3)*x^3 - 3*(19*b*c^6*d^2*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*
x + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^
3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e^2), -1/1680*(3*(35*b*c^6*d^3 + 70*
b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) - 12*(2*b*c^6*d^3 + b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*...
```

3.1183.6 Sympy [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx=\int x^3(a+b\operatorname{atan}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

3.1183.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1183.8 Giac [F]

$$\int x^3 (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{3}{2}} (b\arctan(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1183.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int x^3\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1184 $\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$

3.1184.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\frac{ad^2x\sqrt{d+ex^2}}{16e}+\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3(d+ex^2)^{3/2}-\frac{ad^3\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{3/2}}+b\mathrm{Int}\left(x^2(d+ex^2)^{3/2}\arctan(cx),x\right)$$

output
```
1/6*a*x^3*(e*x^2+d)^(3/2)-1/16*a*d^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^
(3/2)+1/16*a*d^2*x*(e*x^2+d)^(1/2)/e+1/8*a*d*x^3*(e*x^2+d)^(1/2)+b*Uninteg
rable(x^2*(e*x^2+d)^(3/2)*arctan(c*x),x)
```

3.1184.2 Mathematica [N/A]

Not integrable

Time = 13.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

3.1184.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 248, 248, 262, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^2 \left(ex^2+d\right)^{3/2}dx+b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{1}{2}d\int x^2\sqrt{ex^2+d}dx+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\int\frac{x^2}{\sqrt{ex^2+d}}dx+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+ b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{262}$$

$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+ b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+ b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{219}$$

$$b\int x^2\left(ex^2+d\right)^{3/2}\arctan(cx)dx+$$
$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int x^2\left(ex^2+d\right)^{3/2}\arctan(cx)dx+$$
$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)$$

input
```
Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1184.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1184.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^{2} \left(e \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arctan \left(c x \right)\right)d x$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

3.1184.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2 \left(d+e x^2\right)^{3/2} (a+b \arctan(c x)) \, dx=\int { {\left(e x^{2} + d\right)}^{\frac{3}{2}} {\left(b \arctan \left(c x\right) + a\right)} x^{2} \,d x }$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arctan(c*x))*sqrt(e*x^2
+ d), x)
```

3.1184.6 Sympy [N/A]

Not integrable

Time = 63.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\int x^2(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

3.1184.7 Maxima [F(-2)]

Exception generated.

$$\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1184.8 Giac [N/A]

Not integrable

Time = 95.98 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}(b\arctan(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1184.9 Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int x^2 (a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1185 $\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$

3.1185.1 Optimal result

Integrand size = 21, antiderivative size = 181

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = -\frac{b(7c^2d-4e)\,x\sqrt{d+ex^2}}{40c^3} - \frac{bx(d+ex^2)^{3/2}}{20c} + \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e} - \frac{b(c^2d-e)^{5/2}\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{5c^5e} - \frac{b(15c^4d^2-20c^2de+8e^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}$$

output
```
-1/20*b*x*(e*x^2+d)^(3/2)/c+1/5*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/e-1/5*b*
(c^2*d-e)^(5/2)*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/c^5/e-1/40*b*(15
*c^4*d^2-20*c^2*d*e+8*e^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^5/e^(1/2)-
1/40*b*(7*c^2*d-4*e)*x*(e*x^2+d)^(1/2)/c^3
```

3.1185.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.73

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \frac{c^2\sqrt{d+ex^2}\left(8ac^3(d+ex^2)^2+bex(4e-c^2(9d+2ex^2))\right)+8bc^5(d+ex^2)^{5/2}\arctan(cx)}{}$$

input
```
Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*x*(4*e - c^2*(9*d + 2*e*
x^2))) + 8*b*c^5*(d + e*x^2)^(5/2)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(5/2)
*Log[(20*c^6*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c
^2*d - e)^(7/2)*(-I + c*x))] + (4*I)*b*(c^2*d - e)^(5/2)*Log[(20*c^6*e*(I*
c*d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(I +
c*x))] - b*Sqrt[e]*(15*c^4*d^2 - 20*c^2*d*e + 8*e^2)*Log[e*x + Sqrt[e]*Sqr
t[d + e*x^2]])/(40*c^5*e)
```

3.1185.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5509, 318, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\int\frac{\left(ex^2+d\right)^{5/2}}{c^2x^2+1}dx}{5e}$$

$$\Big\downarrow \text{318}$$

$$\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\int\frac{\sqrt{ex^2+d}\left(\left(7c^2d-4e\right)ex^2+d\left(4c^2d-e\right)\right)}{c^2x^2+1}dx}{4c^2}+\frac{ex\left(d+ex^2\right)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \text{403}$$

$$\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\int\frac{e\left(15d^2c^4-20dec^2+8e^2\right)x^2+d\left(8d^2c^4-9dec^2+4e^2\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\left(7c^2d-4e\right)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex\left(d+ex^2\right)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \text{398}$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(15c^4d^2-20c^2de+8e^2)\int\frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \; 224$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(15c^4d^2-20c^2de+8e^2)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \; 219$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(15c^4d^2-20c^2de+8e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \; 291$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(15c^4d^2-20c^2de+8e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \; 216$$

3.1185. $\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$

$$\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e} - \frac{bc\left(\frac{\frac{\frac{8\left(c^2d-e\right)^{5/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(15c^4d^2-20c^2de+8e^2\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(7c^2d-4e\right)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex\left(d+ex^2\right)^{3/2}}{4c^2}\right)}{5e}$$

input
```
Int[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e) - (b*c*((e*x*(d + e*x^2)^(3/
2))/(4*c^2) + (((7*c^2*d - 4*e)*e*x*Sqrt[d + e*x^2])/(2*c^2) + ((8*(c^2*d
- e)^(5/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(15
*c^4*d^2 - 20*c^2*d*e + 8*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)/
(2*c^2))/(4*c^2)))/(5*e)
```

3.1185.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1185.4 Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(c x\right)\right)dx$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

3.1185.5 Fricas [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 1192, normalized size of antiderivative = 6.59

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[1/80*((15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(e)*log(-2*e*x^2 + 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) + 4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c
^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x
^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(
c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2*b*c^
4*e^2*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2*x^4
 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e), -1/
80*(8*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^
2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c
^2*d^2 - d*e)*x)) - (15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(e)*log(-2
*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*
d*e*x^2 - 2*b*c^4*e^2*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x +
8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 +
d))/(c^5*e), 1/40*((15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(-e)*arctan
(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c
^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x
^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(
c^4*x^4 + 2*c^2*x^2 + 1)) + (8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2*b*c^4*
e^2*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2*x^4 +
 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e), -1...
```

3.1185.6 Sympy [F]

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \int x(a+b\operatorname{atan}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

3.1185.7 Maxima [F(-2)]

Exception generated.

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

3.1185.8 Giac [F]

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1185.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1186 $\int (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx$

3.1186.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \text{Int}\left((d+ex^2)^{3/2} (a+b\arctan(cx)),x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

3.1186.2 Mathematica [N/A]

Not integrable

Time = 6.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

3.1186.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1186.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1186.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

3.1186.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

3.1186.6 Sympy [N/A]

Not integrable

Time = 30.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int (a+b\operatorname{atan}(cx)) \left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

3.1186.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1186.8 Giac [N/A]

Not integrable

Time = 93.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1186.9 Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int (a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1187 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx$

3.1187.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = ad\sqrt{d+ex^2} + \frac{1}{3}a(d+ex^2)^{3/2} - ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x},x\right)$$

output
```
1/3*a*(e*x^2+d)^(3/2)-a*d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))+a*d*(e*x^
2+d)^(1/2)+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x,x)
```

3.1187.2 Mathematica [N/A]

Not integrable

Time = 10.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x, x]
```

3.1187.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 60, 60, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{x}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\left(ex^2+d\right)^{3/2}}{x}dx+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\left(ex^2+d\right)^{3/2}}{x^2}dx^2+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(d\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx+$$
$$\frac{1}{2}a\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x}dx + \frac{1}{2}a\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}(d+ex^2)^{3/2}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
$Aborted
```

3.1187.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1187.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan(cx))}{x}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

3.1187.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x,
x)
```

3.1187.6 Sympy [N/A]

Not integrable

Time = 22.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x, x)
```

3.1187.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1187.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
Timed out
```

3.1187.9 Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x, x)
```

3.1188 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx$

3.1188.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \frac{3}{2}aex\sqrt{d+ex^2}-\frac{a(d+ex^2)^{3/2}}{x}+\frac{3}{2}ad\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+b\,\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^2},x\right)$$

output
```
-a*(e*x^2+d)^(3/2)/x+3/2*a*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)+3/
2*a*e*x*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x^2,x)
```

3.1188.2 Mathematica [N/A]

Not integrable

Time = 11.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

3.1188.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 247, 211, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{3/2}}{x^2}dx+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(3e\int \sqrt{ex^2+d}dx-\frac{(d+ex^2)^{3/2}}{x}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(3e\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(3e\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)+ \\ b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx+a\left(3e\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx+a\left(3e\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.1188.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1188. $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx$

3.1188.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\arctan\left(cx\right)\right)}{x^{2}}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

3.1188.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\left(d+ex^2\right)^{3/2} (a+b\arctan(cx))}{x^2}\,dx = \int \frac{\left(ex^{2}+d\right)^{\frac{3}{2}}\left(b\arctan\left(cx\right)+a\right)}{x^{2}}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

3.1188.6 Sympy [N/A]

Not integrable

Time = 13.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\left(d+ex^2\right)^{3/2} (a+b\arctan(cx))}{x^2}\,dx = \int \frac{\left(a+b\operatorname{atan}\left(cx\right)\right)\left(d+ex^{2}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**2,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

3.1188.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1188.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.1188.9 Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^2, x)
```

3.1189 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx$

3.1189.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \frac{3}{2}ae\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{2x^2} - \frac{3}{2}a\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^3},x\right)$$

output
```
-1/2*a*(e*x^2+d)^(3/2)/x^2-3/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2
)+3/2*a*e*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x^3,x
)
```

3.1189.2 Mathematica [N/A]

Not integrable

Time = 12.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

3.1189.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 51, 60, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\left(ex^2+d\right)^{3/2}}{x^3}dx+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\left(ex^2+d\right)^{3/2}}{x^4}dx^2+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{3}{2}e\int \frac{\sqrt{ex^2+d}}{x^2}dx^2-\frac{\left(d+ex^2\right)^{3/2}}{x^2}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{2}e\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x^2}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{3}{2}e\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x^2}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx+$$
$$\frac{1}{2}a\left(\frac{3}{2}e\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)-\frac{\left(d+ex^2\right)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^3}dx+$$
$$\frac{1}{2}a\left(\frac{3}{2}e\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)-\frac{(d+ex^2)^{3/2}}{x^2}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.1189.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1189.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan{(cx)})}{x^3}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

3.1189.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x^3}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

3.1189.6 Sympy [N/A]

Not integrable

Time = 10.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

3.1189.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1189.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.1189. $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx$

3.1189.9 Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^3, x)
```

3.1190 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx$

3.1190.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = -\frac{ae\sqrt{d+ex^2}}{x}-\frac{a(d+ex^2)^{3/2}}{3x^3} + ae^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^4},x\right)$$

output
```
-1/3*a*(e*x^2+d)^(3/2)/x^3+a*e^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-a*
e*(e*x^2+d)^(1/2)/x+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)/x^4,x)
```

3.1190.2 Mathematica [N/A]

Not integrable

Time = 35.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

3.1190.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 247, 247, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2} (a+b\arctan(cx))}{x^4} \, dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\left(ex^2+d\right)^{3/2}}{x^4}dx+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(e\int \frac{\sqrt{ex^2+d}}{x^2}dx-\frac{\left(d+ex^2\right)^{3/2}}{3x^3}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(e\left(e\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{\left(d+ex^2\right)^{3/2}}{3x^3}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(e\left(e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{\left(d+ex^2\right)^{3/2}}{3x^3}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx+a\left(e\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{\left(d+ex^2\right)^{3/2}}{3x^3}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx+a\left(e\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{\left(d+ex^2\right)^{3/2}}{3x^3}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
$Aborted
```

3.1190.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1190.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan(cx))}{x^4}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

3.1190.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

3.1190.6 Sympy [N/A]

Not integrable

Time = 9.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**4,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

3.1190.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1190.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.1190.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^4, x)
```

3.1191 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx$

3.1191.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = -\frac{3ae\sqrt{d+ex^2}}{8x^2} - \frac{a(d+ex^2)^{3/2}}{4x^4} - \frac{3ae^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8\sqrt{d}} + b\operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^5},x\right)$$

output
```
-1/4*a*(e*x^2+d)^(3/2)/x^4-3/8*a*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1
/2)-3/8*a*e*(e*x^2+d)^(1/2)/x^2+b*Unintegrable((e*x^2+d)^(3/2)*arctan(c*x)
/x^5,x)
```

3.1191.2 Mathematica [N/A]

Not integrable

Time = 13.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5, x]
```

3.1191.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 51, 51, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{3/2}}{x^5}dx+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{(ex^2+d)^{3/2}}{x^6}dx^2+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{3}{4}e\int \frac{\sqrt{ex^2+d}}{x^4}dx^2-\frac{(d+ex^2)^{3/2}}{2x^4}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{3}{4}e\left(\frac{1}{2}e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{(d+ex^2)^{3/2}}{2x^4}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{3}{4}e\left(\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{(d+ex^2)^{3/2}}{2x^4}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx+\frac{1}{2}a\left(\frac{3}{4}e\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{(d+ex^2)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx+\frac{1}{2}a\left(\frac{3}{4}e\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{(d+ex^2)^{3/2}}{2x^4}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
$Aborted
```

3.1191.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1191. $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx$

3.1191.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan(cx))}{x^5}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

3.1191.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^5
, x)
```

3.1191.6 Sympy [N/A]

Not integrable

Time = 15.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**5, x)
```

3.1191.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1191.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
Timed out
```

3.1191.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^5,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^5, x)
```

3.1192 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx$

3.1192.1 Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2} - \frac{bc(d+ex^2)^{3/2}}{20x^4} - \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5} - \frac{bc(8c^4d^2-20c^2de+15e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40\sqrt{d}} + \frac{b(c^2d-e)^{5/2}\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{5d}$$

output
```
-1/20*b*c*(e*x^2+d)^(3/2)/x^4-1/5*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/d/x^5+
1/5*b*(c^2*d-e)^(5/2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d-1/40*b*
c*(8*c^4*d^2-20*c^2*d*e+15*e^2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+1
/40*b*c*(4*c^2*d-7*e)*(e*x^2+d)^(1/2)/x^2
```

3.1192.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \frac{-\sqrt{d+ex^2}\left(8a(d+ex^2)^2+bcdx(9ex^2+d(2-4c^2x^2))\right)-8b(d+e}{}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(-(Sqrt[d + e*x^2]*(8*a*(d + e*x^2)^2 + b*c*d*x*(9*e*x^2 + d*(2 - 4*c^2*x^
2)))) - 8*b*(d + e*x^2)^(5/2)*ArcTan[c*x] + b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^
2*d*e + 15*e^2)*x^5*Log[x] - b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^2*d*e + 15*e^2)
*x^5*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20
*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)
*(I + c*x))] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20*c*d*(c*d + I*e*x + Sqrt[
c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(-I + c*x))])/(40*d*x^5)
```

3.1192.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5511, 27, 354, 109, 27, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{(ex^2+d)^{5/2}}{5dx^5(c^2x^2+1)}dx-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{(ex^2+d)^{5/2}}{x^5(c^2x^2+1)}dx}{5d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 354$$

$$\frac{bc\int \frac{(ex^2+d)^{5/2}}{x^6(c^2x^2+1)}dx^2}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 109$$

$$\frac{bc\left(-\frac{1}{2}\int \frac{\sqrt{ex^2+d}\left((c^2d-4e)ex^2+d(4c^2d-7e)\right)}{2x^4(c^2x^2+1)}dx^2-\frac{d(d+ex^2)^{3/2}}{2x^4}\right)}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\left(-\frac{1}{4}\int\frac{\sqrt{ex^2+d}\left((c^2d-4e)ex^2+d(4c^2d-7e)\right)}{x^4(c^2x^2+1)}dx^2-\frac{d(d+ex^2)^{3/2}}{2x^4}\right)}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \text{166}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{d(4c^2d-7e)\sqrt{d+ex^2}}{x^2}-\int-\frac{e(4d^2c^4-9dec^2+8e^2)x^2+d(8d^2c^4-20dec^2+15e^2)}{2x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)-\frac{d(d+ex^2)^{3/2}}{2x^4}\right)}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{e(4d^2c^4-9dec^2+8e^2)x^2+d(8d^2c^4-20dec^2+15e^2)}{x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2+\frac{d(4c^2d-7e)\sqrt{d+ex^2}}{x^2}\right)-\frac{d(d+ex^2)^{3/2}}{2x^4}\right)}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \text{174}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(d(8c^4d^2-20c^2de+15e^2)\int\frac{1}{x^2\sqrt{ex^2+d}}dx^2-8(c^2d-e)^3\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)+\frac{d(4c^2d-7e)\sqrt{d+ex^2}}{x^2}\right)-\frac{d(d\ldots}{}\right.}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{2d(8c^4d^2-20c^2de+15e^2)\int\frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{16(c^2d-e)^3\int\frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)+\frac{d(4c^2d-7e)\sqrt{d+ex^2}}{x^2}\right)-\frac{d(d+ex^2)^{3/\ldots}}{2x^4}\right.}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{16(c^2d-e)^{5/2}\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}-2\sqrt{d}(8c^4d^2-20c^2de+15e^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{d(4c^2d-7e)\sqrt{d+ex^2}}{x^2}\right)\ldots\right.}{10d}-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```

3.1192. $\int\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}dx$

output
```
-1/5*((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(d*x^5) + (b*c*(-1/2*(d*(d +
e*x^2)^(3/2))/x^4 + ((d*(4*c^2*d - 7*e)*Sqrt[d + e*x^2])/x^2 + (-2*Sqrt[d]
*(8*c^4*d^2 - 20*c^2*d*e + 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (16*
(c^2*d - e)^(5/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/2)/4))/
(10*d)
```

3.1192.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

3.1192. $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx$

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1192.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

3.1192.5 Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 1145, normalized size of antiderivative = 6.43

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
[1/80*(4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*x^5*log((c^4*e^
2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x
^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^
2*x^2 + 1)) + (8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(d)*x^5*log(-(
e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(8*a*e^2*x^4 + 2*b*c*d^2
*x + 16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e^2*x^4
 + 2*b*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/80*(8*(b*
c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c^2*d + e)*x^5*arctan(-1/2*(c^2*e*x^2
 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d
*e - c*e^2)*x^2)) + (8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(d)*x^5*
log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(8*a*e^2*x^4 + 2*b
*c*d^2*x + 16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e
^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/40*
((8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(-d)*x^5*arctan(sqrt(-d)/sq
rt(e*x^2 + d)) + 2*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*x^5*l
og((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 +
4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*
x^4 + 2*c^2*x^2 + 1)) - (8*a*e^2*x^4 + 2*b*c*d^2*x + 16*a*d*e*x^2 - (4*b*c
^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*ar
ctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/40*(4*(b*c^4*d^2 - 2*b*c^2*d*e +...
```

3.1192.6 Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**6,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**6, x)
```

3.1192.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1192.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
Timed out
```

3.1192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^6,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^6, x)
```

3.1193 $\int x^3 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx$

3.1193.1 Optimal result

Integrand size = 23, antiderivative size = 345

$$\begin{aligned}\int x^3 (d+ex^2)^{5/2} (a &+ b\arctan(cx))\,dx = \frac{b(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3)\,x\sqrt{d+ex^2}}{8064c^7e} \\ &- \frac{b(69c^4d^2-520c^2de+336e^2)\,x(d+ex^2)^{3/2}}{12096c^5e} - \frac{b(33c^2d-56e)\,x(d+ex^2)^{5/2}}{3024c^3e} \\ &- \frac{bx(d+ex^2)^{7/2}}{72ce} - \frac{d(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2} \\ &+ \frac{(d+ex^2)^{9/2}(a+b\arctan(cx))}{9e^2} + \frac{b(c^2d-e)^{7/2}(2c^2d+7e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{63c^9e^2} \\ &+ \frac{b(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8064c^9e^{3/2}}\end{aligned}$$

output
```
-1/12096*b*(69*c^4*d^2-520*c^2*d*e+336*e^2)*x*(e*x^2+d)^(3/2)/c^5/e-1/3024
*b*(33*c^2*d-56*e)*x*(e*x^2+d)^(5/2)/c^3/e-1/72*b*x*(e*x^2+d)^(7/2)/c/e-1/
7*d*(e*x^2+d)^(7/2)*(a+b*arctan(c*x))/e^2+1/9*(e*x^2+d)^(9/2)*(a+b*arctan(
c*x))/e^2+1/63*b*(c^2*d-e)^(7/2)*(2*c^2*d+7*e)*arctan(x*(c^2*d-e)^(1/2)/(e
*x^2+d)^(1/2))/c^9/e^2+1/8064*b*(315*c^8*d^4+840*c^6*d^3*e-3024*c^4*d^2*e^
2+2880*c^2*d*e^3-896*e^4)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^9/e^(3/2)+1
/8064*b*(59*c^6*d^3+712*c^4*d^2*e-1104*c^2*d*e^2+448*e^3)*x*(e*x^2+d)^(1/2
)/c^7/e
```

3.1193.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.36

$$\int x^3(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx =$$

$$-\frac{c^2\sqrt{d+ex^2}\Big(384ac^7(2d-7ex^2)(d+ex^2)^3+bex(-1344e^3+48c^2e^2(83d+14ex^2)-8c^4e(453d^2+242dex}{}$$

input
```
Integrate[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/24192*(c^2*Sqrt[d + e*x^2]*(384*a*c^7*(2*d - 7*e*x^2)*(d + e*x^2)^3 + b
*e*x*(-1344*e^3 + 48*c^2*e^2*(83*d + 14*e*x^2) - 8*c^4*e*(453*d^2 + 242*d*
e*x^2 + 56*e^2*x^4) + 3*c^6*(187*d^3 + 558*d^2*e*x^2 + 424*d*e^2*x^4 + 112
*e^3*x^6))) + 384*b*c^9*(2*d - 7*e*x^2)*(d + e*x^2)^(7/2)*ArcTan[c*x] + (1
92*I)*b*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((-252*I)*c^10*e^2*(c*d - I*
e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(2*c^2*d + 7*
e)*(I + c*x))] - (192*I)*b*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((252*I)*
c^10*e^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(
9/2)*(2*c^2*d + 7*e)*(-I + c*x))] + 3*b*Sqrt[e]*(-315*c^8*d^4 - 840*c^6*d^
3*e + 3024*c^4*d^2*e^2 - 2880*c^2*d*e^3 + 896*e^4)*Log[e*x + Sqrt[e]*Sqrt[
d + e*x^2]])/(c^9*e^2)
```

3.1193.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5511, 27, 403, 403, 403, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(2d-7ex^2)\left(ex^2+d\right)^{7/2}}{63e^2\left(c^2x^2+1\right)}dx+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow 27$$

$$\frac{bc\int \frac{(2d-7ex^2)\left(ex^2+d\right)^{7/2}}{c^2x^2+1}dx}{63e^2}+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow 403$$

$$\frac{bc\left(\frac{\int \frac{\left(ex^2+d\right)^{5/2}\left(d\left(16dc^2+7e\right)-\left(33c^2d-56e\right)ex^2\right)}{c^2x^2+1}dx}{8c^2}-\frac{7ex\left(d+ex^2\right)^{7/2}}{8c^2}\right)}{63e^2}+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow 403$$

$$\frac{bc\left(\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(d\left(96d^2c^4+75dec^2-56e^2\right)-e\left(69d^2c^4-520dec^2+336e^2\right)x^2\right)}{c^2x^2+1}dx}{6c^2}-\frac{ex\left(33c^2d-56e\right)\left(d+ex^2\right)^{5/2}}{6c^2}}{8c^2}-\frac{7ex\left(d+ex^2\right)^{7/2}}{8c^2}\right)}{63e^2}+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow 403$$

$$\frac{bc\left(\frac{\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(e\left(59d^3c^6+712d^2ec^4-1104de^2c^2+448e^3\right)x^2+d\left(128d^3c^6+123d^2ec^4-248de^2c^2+112e^3\right)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex\left(69c^4d^2-520c^2de+336e^2\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}-ex(\cdots}{8c^2}\right.}{63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow 27$$

3.1193. $\int x^3\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx$

$$bc\left(\frac{\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e\left(59d^3c^6+712d^2ec^4-1104de^2c^2+448e^3\right)x^2+d\left(128d^3c^6+123d^2ec^4-248de^2c^2+112e^3\right)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex\left(69c^4d^2-520c^2de+336e^2\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}-\frac{ex(}{}}{8c^2}\right.$$

$$\overline{63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \; 403$$

$$bc\left(\frac{\frac{\frac{3\left(\frac{\int \frac{e\left(315d^4c^8+840d^3ec^6-3024d^2e^2c^4+2880de^3c^2-896e^4\right)x^2+d\left(256d^4c^8+187d^3ec^6-1208d^2e^2c^4+1328de^3c^2-448e^4\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\left(59c^6d^3+712c^4d^2e-1104c\right.}{2c^2}\right.}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\overline{63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \; 398$$

$$bc\left(\frac{\frac{\frac{3\left(\frac{\frac{128\left(2c^2d+7e\right)\left(c^2d-e\right)^4\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\left(59c^6d^3+712c^4d^2e-1104\right.}{2c^2}\right.}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\overline{63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \; 224$$

3.1193. $\int x^3\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx$

$$bc\left(\frac{\frac{\frac{3\left(\frac{\frac{128(2c^2d+7e)(c^2d-e)^4\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex(59c^6d^3+712c^4\cdots}{\cdots}\right)}{4c^2}}{6c^2}}{8c^2}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 63e^2}$$

$$\frac{(d+ex^2)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \; 219$$

$$bc\left(\frac{\frac{\frac{3\left(\frac{\frac{128(2c^2d+7e)(c^2d-e)^4\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(59c^6d^3+712\cdots}{\cdots}\right)}{4c^2}}{6c^2}}{8c^2}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 63e^2}$$

$$\frac{(d+ex^2)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \; 291$$

$$bc\left(\frac{\frac{\frac{3\left(\frac{\frac{128(2c^2d+7e)(c^2d-e)^4\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(59c^6d^3+\cdots}{\cdots}\right)}{4c^2}}{6c^2}}{8c^2}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 63e^2}$$

$$\frac{(d+ex^2)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}$$

3.1193. $\int x^3(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$

$$\Big\downarrow 216$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}+$$

$$\frac{bc\left(\frac{\frac{\frac{3\left(\frac{\frac{128\left(2c^2d+7e\right)\left(c^2d-e\right)^{7/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(59c^6d^3+712c^4\right.}{}\right.}{4c^2}}{6c^2}}{8c^2}\right.}{63e^2}$$

input
```
Int[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/7*(d*(d + e*x^2)^(7/2)*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^(9/2)*(a
 + b*ArcTan[c*x]))/(9*e^2) + (b*c*((-7*e*x*(d + e*x^2)^(7/2))/(8*c^2) + (-
1/6*((33*c^2*d - 56*e)*e*x*(d + e*x^2)^(5/2))/c^2 + (-1/4*(e*(69*c^4*d^2 -
 520*c^2*d*e + 336*e^2)*x*(d + e*x^2)^(3/2))/c^2 + (3*((e*(59*c^6*d^3 + 71
2*c^4*d^2*e - 1104*c^2*d*e^2 + 448*e^3)*x*Sqrt[d + e*x^2])/(2*c^2) + ((128
*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x
^2]])/c^2 + (Sqrt[e]*(315*c^8*d^4 + 840*c^6*d^3*e - 3024*c^4*d^2*e^2 + 288
0*c^2*d*e^3 - 896*e^4)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)/(2*c^2))
)/(4*c^2))/(6*c^2))/(8*c^2)))/(63*e^2)
```

3.1193.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1193.4 Maple [F]

$$\int x^{3}\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

3.1193.5 Fricas [A] (verification not implemented)

Time = 29.10 (sec) , antiderivative size = 1978, normalized size of antiderivative = 5.73

$$\int x^3 \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/48384*(3*(315*b*c^8*d^4 + 840*b*c^6*d^3*e - 3024*b*c^4*d^2*e^2 + 2880*
b*c^2*d*e^3 - 896*b*e^4)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*
x - d) + 192*(2*b*c^8*d^4 + b*c^6*d^3*e - 15*b*c^4*d^2*e^2 + 19*b*c^2*d*e^
3 - 7*b*e^4)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(
3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt
(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2688*a*c^9*e^4*x^8 + 72
96*a*c^9*d*e^3*x^6 - 336*b*c^8*e^4*x^7 + 5760*a*c^9*d^2*e^2*x^4 + 384*a*c^
9*d^3*e*x^2 - 768*a*c^9*d^4 - 8*(159*b*c^8*d*e^3 - 56*b*c^6*e^4)*x^5 - 2*(
837*b*c^8*d^2*e^2 - 968*b*c^6*d*e^3 + 336*b*c^4*e^4)*x^3 - 3*(187*b*c^8*d^
3*e - 1208*b*c^6*d^2*e^2 + 1328*b*c^4*d*e^3 - 448*b*c^2*e^4)*x + 384*(7*b*
c^9*e^4*x^8 + 19*b*c^9*d*e^3*x^6 + 15*b*c^9*d^2*e^2*x^4 + b*c^9*d^3*e*x^2
- 2*b*c^9*d^4)*arctan(c*x))*sqrt(e*x^2 + d))/(c^9*e^2), 1/48384*(384*(2*b*
c^8*d^4 + b*c^6*d^3*e - 15*b*c^4*d^2*e^2 + 19*b*c^2*d*e^3 - 7*b*e^4)*sqrt(
c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 +
 d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - 3*(315*b*c^8*d^4 + 840*b*
c^6*d^3*e - 3024*b*c^4*d^2*e^2 + 2880*b*c^2*d*e^3 - 896*b*e^4)*sqrt(e)*log
(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(2688*a*c^9*e^4*x^8 + 729
6*a*c^9*d*e^3*x^6 - 336*b*c^8*e^4*x^7 + 5760*a*c^9*d^2*e^2*x^4 + 384*a*c^9
*d^3*e*x^2 - 768*a*c^9*d^4 - 8*(159*b*c^8*d*e^3 - 56*b*c^6*e^4)*x^5 - 2*(8
37*b*c^8*d^2*e^2 - 968*b*c^6*d*e^3 + 336*b*c^4*e^4)*x^3 - 3*(187*b*c^8*...
```

3.1193.6 Sympy [F]

$$\int x^3 \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \int x^3(a+b\,\mathrm{atan}\,(cx)) \left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**3*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

3.1193.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1193.8 Giac [F]

$$\int x^3 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{5}{2}} (b\arctan(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1193.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int x^3\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1194 $\int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$

3.1194.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3(d+ex^2)^{3/2} + \frac{1}{8}ax^3(d+ex^2)^{5/2} - \frac{5ad^4\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{3/2}} + b\mathrm{Int}\left(x^2(d+ex^2)^{5/2}\arctan(cx),x\right)$$

output
```
5/48*a*d*x^3*(e*x^2+d)^(3/2)+1/8*a*x^3*(e*x^2+d)^(5/2)-5/128*a*d^4*arctanh
(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)+5/128*a*d^3*x*(e*x^2+d)^(1/2)/e+5/64*a
*d^2*x^3*(e*x^2+d)^(1/2)+b*Unintegrable(x^2*(e*x^2+d)^(5/2)*arctan(c*x),x)
```

3.1194.2 Mathematica [N/A]

Not integrable

Time = 14.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

3.1194.3 Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 248, 248, 248, 262, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^{5/2} (a+b \arctan (cx)) \, dx$$

$$\Big\downarrow \text{5517}$$

$$a \int x^2 \left(ex^2+d\right)^{5/2}dx+b \int x^2 \left(ex^2+d\right)^{5/2} \arctan (cx)dx$$

$$\Big\downarrow \text{248}$$

$$a \left(\frac{5}{8} d \int x^2 \left(ex^2+d\right)^{3/2}dx+\frac{1}{8} x^3 \left(d+ex^2\right)^{5/2}\right)+b \int x^2 \left(ex^2+d\right)^{5/2} \arctan (cx)dx$$

$$\Big\downarrow \text{248}$$

$$a \left(\frac{5}{8} d \left(\frac{1}{2} d \int x^2 \sqrt{ex^2+d}dx+\frac{1}{6} x^3 \left(d+ex^2\right)^{3/2}\right)+\frac{1}{8} x^3 \left(d+ex^2\right)^{5/2}\right)+ \\ b \int x^2 \left(ex^2+d\right)^{5/2} \arctan (cx)dx$$

$$\Big\downarrow \text{248}$$

$$a \left(\frac{5}{8} d \left(\frac{1}{2} d \left(\frac{1}{4} d \int \frac{x^2}{\sqrt{ex^2+d}}dx+\frac{1}{4} x^3 \sqrt{d+ex^2}\right)+\frac{1}{6} x^3 \left(d+ex^2\right)^{3/2}\right)+\frac{1}{8} x^3 \left(d+ex^2\right)^{5/2}\right)+ \\ b \int x^2 \left(ex^2+d\right)^{5/2} \arctan (cx)dx$$

$$\Big\downarrow \text{262}$$

$$a \left(\frac{5}{8} d \left(\frac{1}{2} d \left(\frac{1}{4} d \left(\frac{x \sqrt{d+ex^2}}{2 e}-\frac{d \int \frac{1}{\sqrt{ex^2+d}}dx}{2 e}\right)+\frac{1}{4} x^3 \sqrt{d+ex^2}\right)+\frac{1}{6} x^3 \left(d+ex^2\right)^{3/2}\right)+\frac{1}{8} x^3 \left(d+ex^2\right)^{5/2}\right)+ \\ b \int x^2 \left(ex^2+d\right)^{5/2} \arctan (cx)dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5/2}\right)+$$
$$b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{219}$$

$$b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx+$$
$$a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5/2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx+$$
$$a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5/2}\right)$$

input
```
Int[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1194.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1194.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^{2} \left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

3.1194.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int x^2 (d + ex^2)^{5/2} (a + b \arctan(cx))\, dx = \int (ex^2 + d)^{\frac{5}{2}} (b \arctan(cx) + a) x^2\, dx$$

input
```
integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^6 + 2*a*d*e*x^4 + a*d^2*x^2 + (b*e^2*x^6 + 2*b*d*e*x^4 +
 b*d^2*x^2)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

3.1194.6 Sympy [N/A]

Not integrable

Time = 176.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2 (d + ex^2)^{5/2} (a + b \arctan(cx))\, dx = \int x^2 (a + b \operatorname{atan}(cx)) (d + ex^2)^{\frac{5}{2}}\, dx$$

input
```
integrate(x**2*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

3.1194.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + ex^2)^{5/2} (a + b \arctan(cx))\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1194.8 Giac [N/A]

Not integrable

Time = 98.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int x^2\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx=\int \left(ex^2+d\right)^{\frac{5}{2}}(b\arctan\left(cx\right)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1194.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1195 $\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$

3.1195.1 Optimal result

Integrand size = 21, antiderivative size = 233

$$\begin{aligned}\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx &= -\frac{b(19c^4d^2-22c^2de+8e^2)\,x\sqrt{d+ex^2}}{112c^5} \\ &- \frac{b(11c^2d-6e)\,x(d+ex^2)^{3/2}}{168c^3} - \frac{bx(d+ex^2)^{5/2}}{42c} \\ &+ \frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e} - \frac{b(c^2d-e)^{7/2}\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{7c^7e} \\ &- \frac{b(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3)\,\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{112c^7\sqrt{e}}\end{aligned}$$

output
```
-1/168*b*(11*c^2*d-6*e)*x*(e*x^2+d)^(3/2)/c^3-1/42*b*x*(e*x^2+d)^(5/2)/c+1
/7*(e*x^2+d)^(7/2)*(a+b*arctan(c*x))/e-1/7*b*(c^2*d-e)^(7/2)*arctan(x*(c^2
*d-e)^(1/2)/(e*x^2+d)^(1/2))/c^7/e-1/112*b*(35*c^6*d^3-70*c^4*d^2*e+56*c^2
*d*e^2-16*e^3)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^7/e^(1/2)-1/112*b*(19*
c^4*d^2-22*c^2*d*e+8*e^2)*x*(e*x^2+d)^(1/2)/c^5
```

3.1195.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.52

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \frac{c^2\sqrt{d+ex^2}\Big(48ac^5(d+ex^2)^3 - bex(24e^2 - 6c^2e(13d+2ex^2) + c^4(87d^2+38dex^2+8e^2\ldots$$

input
```
Integrate[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*Sqrt[d + e*x^2]*(48*a*c^5*(d + e*x^2)^3 - b*e*x*(24*e^2 - 6*c^2*e*(13
*d + 2*e*x^2) + c^4*(87*d^2 + 38*d*e*x^2 + 8*e^2*x^4))) + 48*b*c^7*(d + e*
x^2)^(7/2)*ArcTan[c*x] - (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*((-I)*c*
d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(-I + c
*x))] + (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*(I*c*d + e*x + I*Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(I + c*x))] + 3*b*Sqrt[e]*(
-35*c^6*d^3 + 70*c^4*d^2*e - 56*c^2*d*e^2 + 16*e^3)*Log[e*x + Sqrt[e]*Sqrt
[d + e*x^2]])/(336*c^7*e)
```

3.1195.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5509, 318, 403, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e} - \frac{bc\int \frac{(ex^2+d)^{7/2}}{c^2x^2+1}dx}{7e}$$

$$\Big\downarrow \text{318}$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\int\frac{\left(ex^2+d\right)^{3/2}\left(\left(11c^2d-6e\right)ex^2+d\left(6c^2d-e\right)\right)}{c^2x^2+1}dx}{6c^2}+\frac{ex\left(d+ex^2\right)^{5/2}}{6c^2}\right)}{7e}$$

$\downarrow$ 403

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{\int\frac{3\sqrt{ex^2+d}\left(e\left(19d^2c^4-22dec^2+8e^2\right)x^2+d\left(8d^2c^4-5dec^2+2e^2\right)\right)}{c^2x^2+1}dx}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}+\frac{ex\left(d+ex^2\right)^{5/2}}{6c^2}\right)}{7e}$$

$\downarrow$ 27

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\int\frac{\sqrt{ex^2+d}\left(e\left(19d^2c^4-22dec^2+8e^2\right)x^2+d\left(8d^2c^4-5dec^2+2e^2\right)\right)}{c^2x^2+1}dx}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}+\frac{ex\left(d+ex^2\right)^{5/2}}{6c^2}\right)}{7e}$$

$\downarrow$ 403

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\left(\frac{\int\frac{e\left(35d^3c^6-70d^2ec^4+56de^2c^2-16e^3\right)x^2+d\left(16d^3c^6-29d^2ec^4+26de^2c^2-8e^3\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}\right.}{7e}$$

$\downarrow$ 398

3.1195. $\int x\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\int\frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d+e\right.}{4c^2}}{6c^2}\right.}{7e}$$

$$\Big\downarrow \; 224$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2d\right.}{}}{6c^2}\right.}{7e}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2\right.}{}}{6c^2}\right.}{7e}$$

$$\Big\downarrow \; 291$$

3.1195. $\int x\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^4\int\frac{1}{1-\frac{\left(e-c^2d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex(}{}}{6c^2}\right)}{7e}$$

$$\Big\downarrow \; 216$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^{7/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2d\right.}{}}{6c^2}\right)}{7e}$$

input
```
Int[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^(7/2)*(a + b*ArcTan[c*x]))/(7*e) - (b*c*((e*x*(d + e*x^2)^(5/
2))/(6*c^2) + (((11*c^2*d - 6*e)*e*x*(d + e*x^2)^(3/2))/(4*c^2) + (3*((e*(
19*c^4*d^2 - 22*c^2*d*e + 8*e^2)*x*Sqrt[d + e*x^2])/(2*c^2) + ((16*(c^2*d
- e)^(7/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(35
*c^6*d^3 - 70*c^4*d^2*e + 56*c^2*d*e^2 - 16*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/c^2)/(2*c^2)))/(4*c^2))/(6*c^2)))/(7*e)
```

3.1195. $\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$

3.1195.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.1195. $\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1195.4 Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

3.1195.5 Fricas [A] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 1562, normalized size of antiderivative = 6.70

$$\int x\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/672*(3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)*sqr
t(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 24*(b*c^6*d^3 - 3*b
*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2
*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x
)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(
48*a*c^7*e^3*x^6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*x^5 + 144*a*c^7*d^2*e
*x^2 + 48*a*c^7*d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3)*x^3 - 3*(29*b*c^6*d
^2*e - 26*b*c^4*d*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^3*x^6 + 3*b*c^7*d*e^2
*x^4 + 3*b*c^7*d^2*e*x^2 + b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e
), -1/672*(48*(b*c^6*d^3 - 3*b*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(c^2
*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)
/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) + 3*(35*b*c^6*d^3 - 70*b*c^4*d
^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)
*sqrt(e)*x - d) - 2*(48*a*c^7*e^3*x^6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*
x^5 + 144*a*c^7*d^2*e*x^2 + 48*a*c^7*d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3
)*x^3 - 3*(29*b*c^6*d^2*e - 26*b*c^4*d*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^
3*x^6 + 3*b*c^7*d*e^2*x^4 + 3*b*c^7*d^2*e*x^2 + b*c^7*d^3)*arctan(c*x))*sq
rt(e*x^2 + d))/(c^7*e), 1/336*(3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2
*d*e^2 - 16*b*e^3)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 12*(b*c^6
*d^3 - 3*b*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(-c^2*d + e)*log(((c^...
```

3.1195.6 Sympy [F]

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx=\int x(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

3.1195.7 Maxima [F(-2)]

Exception generated.

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

3.1195.8 Giac [F]

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1195.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1196 $\int (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx$

3.1196.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \text{Int}\left(\left(d+ex^2\right)^{5/2} (a+b\arctan(cx)),x\right)$$

output
```
Unintegrable((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

3.1196.2 Mathematica [N/A]

Not integrable

Time = 6.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx$$

input
```
Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

3.1196.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))dx$$

input
```
Int[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1196.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1196.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^2+d\right)^{\frac{5}{2}} (a+b\arctan(cx))\,dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

3.1196.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \left(d+e x^2\right)^{5/2} (a+b \arctan (c x)) \, dx=\int (ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)\, dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

3.1196.6 Sympy [N/A]

Not integrable

Time = 77.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(d+e x^2\right)^{5/2} (a+b \arctan (c x)) \, dx=\int (a+b\operatorname{atan}(cx))\left(d+ex^2\right)^{\frac{5}{2}}\, dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

3.1196.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+e x^2\right)^{5/2} (a+b \arctan (c x)) \, dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1196.8 Giac [N/A]

Not integrable

Time = 94.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^{\frac{5}{2}} (b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1196.9 Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \int (a+b\,\mathrm{atan}(c\,x))\ \left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1197 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx$

3.1197.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = ad^2\sqrt{d+ex^2}+\frac{1}{3}ad(d+ex^2)^{3/2} + \frac{1}{5}a(d+ex^2)^{5/2}-ad^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x},x\right)$$

output
```
1/3*a*d*(e*x^2+d)^(3/2)+1/5*a*(e*x^2+d)^(5/2)-a*d^(5/2)*arctanh((e*x^2+d)^
(1/2)/d^(1/2))+a*d^2*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d)^(5/2)*arctan
(c*x)/x,x)
```

3.1197.2 Mathematica [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x, x]
```

3.1197.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 60, 60, 60, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int \frac{(ex^2+d)^{5/2}}{x}dx+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{(ex^2+d)^{5/2}}{x^2}dx^2+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}a\left(d\int \frac{(ex^2+d)^{3/2}}{x^2}dx^2+\frac{2}{5}(d+ex^2)^{5/2}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}a\left(d\left(d\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+\frac{2}{3}(d+ex^2)^{3/2}\right)+\frac{2}{5}(d+ex^2)^{5/2}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}a\left(d\left(d\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+\frac{2}{3}(d+ex^2)^{3/2}\right)+\frac{2}{5}(d+ex^2)^{5/2}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}a\left(d\left(d\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+\frac{2}{3}(d+ex^2)^{3/2}\right)+\frac{2}{5}(d+ex^2)^{5/2}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x}dx+\frac{1}{2}a\left(d\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+\frac{2}{5}\left(d+ex^2\right)^{5/2}\right)$$

$$\Big\downarrow \textcolor{blue}{5560}$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x}dx+\frac{1}{2}a\left(d\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+\frac{2}{5}\left(d+ex^2\right)^{5/2}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
$Aborted
```

3.1197.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1197.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^{2}+d)^{\frac{5}{2}}\,(a+b\arctan\left(cx\right))}{x}dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
```

3.1197.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{x}\,dx=\int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x, x)
```

3.1197.6 Sympy [N/A]

Not integrable

Time = 40.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x, x)
```

3.1197.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1197.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
Timed out
```

3.1197.9 Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{5/2}}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x, x)
```

3.1198 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx$

3.1198.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \frac{15}{8}adex\sqrt{d+ex^2}+\frac{5}{4}aex(d+ex^2)^{3/2} - \frac{a(d+ex^2)^{5/2}}{x}+\frac{15}{8}ad^2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x^2},x\right)$$

output
```
5/4*a*e*x*(e*x^2+d)^(3/2)-a*(e*x^2+d)^(5/2)/x+15/8*a*d^2*arctanh(x*e^(1/2)
/(e*x^2+d)^(1/2))*e^(1/2)+15/8*a*d*e*x*(e*x^2+d)^(1/2)+b*Unintegrable((e*x
^2+d)^(5/2)*arctan(c*x)/x^2,x)
```

3.1198.2 Mathematica [N/A]

Not integrable

Time = 12.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

3.1198.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 247, 211, 211, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{5/2}}{x^2}dx+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(5e\int (ex^2+d)^{3/2}dx-\frac{(d+ex^2)^{5/2}}{x}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(5e\left(\frac{3}{4}d\int \sqrt{ex^2+d}dx+\frac{1}{4}x(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(5e\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(5e\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^2}dx+$$
$$a\left(5e\left(\frac{3}{4}d\left(\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^2}dx+$$
$$a\left(5e\left(\frac{3}{4}d\left(\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.1198.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

3.1198. $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx$

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1198.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\arctan{(cx)})}{x^2}dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

3.1198.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan{(cx)}+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x^2, x)
```

3.1198.6 Sympy [N/A]

Not integrable

Time = 59.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**2,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**2, x)
```

3.1198.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1198.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.1198.9 Mupad [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^2, x)
```

3.1199 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx$

3.1199.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \frac{5}{2}ade\sqrt{d+ex^2}+\frac{5}{6}ae\left(d+ex^2\right)^{3/2} - \frac{a(d+ex^2)^{5/2}}{2x^2}-\frac{5}{2}ad^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x^3},x\right)$$

output
```
5/6*a*e*(e*x^2+d)^(3/2)-1/2*a*(e*x^2+d)^(5/2)/x^2-5/2*a*d^(3/2)*e*arctanh(
(e*x^2+d)^(1/2)/d^(1/2))+5/2*a*d*e*(e*x^2+d)^(1/2)+b*Unintegrable((e*x^2+d
)^(5/2)*arctan(c*x)/x^3,x)
```

3.1199.2 Mathematica [N/A]

Not integrable

Time = 12.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

3.1199.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 243, 51, 60, 60, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{5/2}}{x^3}dx+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{(ex^2+d)^{5/2}}{x^4}dx^2+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{5}{2}e\int \frac{(ex^2+d)^{3/2}}{x^2}dx^2-\frac{(d+ex^2)^{5/2}}{x^2}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{5}{2}e\left(d\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+\frac{2}{3}(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x^2}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{5}{2}e\left(d\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+\frac{2}{3}(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x^2}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{5}{2}e\left(d\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+\frac{2}{3}(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x^2}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \textcolor{blue}{221}$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^3}dx+\frac{1}{2}a\left(\frac{5}{2}e\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x^2}\right)$$

$$\Big\downarrow \textcolor{blue}{5560}$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^3}dx+\frac{1}{2}a\left(\frac{5}{2}e\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x^2}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.1199.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1199.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{5}{2}}\,(a+b\arctan{(cx)})}{x^3}dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
```

3.1199.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x^3, x)
```

3.1199.6 Sympy [N/A]

Not integrable

Time = 40.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**3, x)
```

3.1199.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1199.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.1199.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^3, x)
```

3.1200 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx$

3.1200.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \frac{5}{2}ae^2x\sqrt{d+ex^2} - \frac{5ae(d+ex^2)^{3/2}}{3x} - \frac{a(d+ex^2)^{5/2}}{3x^3} + \frac{5}{2}ade^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x^4},x\right)$$

output
```
-5/3*a*e*(e*x^2+d)^(3/2)/x-1/3*a*(e*x^2+d)^(5/2)/x^3+5/2*a*d*e^(3/2)*arcta
nh(x*e^(1/2)/(e*x^2+d)^(1/2))+5/2*a*e^2*x*(e*x^2+d)^(1/2)+b*Unintegrable((
e*x^2+d)^(5/2)*arctan(c*x)/x^4,x)
```

3.1200.2 Mathematica [N/A]

Not integrable

Time = 11.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

3.1200.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 247, 247, 211, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{5/2}}{x^4}dx+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(\frac{5}{3}e\int \frac{(ex^2+d)^{3/2}}{x^2}dx-\frac{(d+ex^2)^{5/2}}{3x^3}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(\frac{5}{3}e\left(3e\int \sqrt{ex^2+d}dx-\frac{(d+ex^2)^{3/2}}{x}\right)-\frac{(d+ex^2)^{5/2}}{3x^3}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{5}{3}e\left(3e\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)-\frac{(d+ex^2)^{5/2}}{3x^3}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{5}{3}e\left(3e\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)-\frac{(d+ex^2)^{5/2}}{3x^3}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx+$$
$$a\left(\frac{5}{3}e\left(3e\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)-\frac{(d+ex^2)^{5/2}}{3x^3}\right)$$

$$\Big\downarrow \ 5560$$

$$b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx+$$
$$a\left(\frac{5}{3}e\left(3e\left(\frac{d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)-\frac{(d+ex^2)^{5/2}}{3x^3}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
$Aborted
```

3.1200.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1200.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \arctan \left(c x \right)\right)}{x^{4}}d x$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
```

3.1200.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{x^4}\,dx = \int \frac{{\left(e x^{2} + d\right)}^{\frac{5}{2}} {\left(b \arctan \left(c x\right) + a\right)}}{x^{4}}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x^4, x)
```

3.1200.6 Sympy [N/A]

Not integrable

Time = 42.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**4,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**4, x)
```

3.1200.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1200.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.1200.9 Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^4, x)
```

3.1201 $\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$

3.1201.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{bx\sqrt{d+ex^2}}{6ce} - \frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2} + \frac{b\sqrt{c^2d-e}(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3c^3e^2} + \frac{b(3c^2d+2e)\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6c^3e^{3/2}}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e^2+1/6*b*(3*c^2*d+2*e)*arctanh(x*e^
(1/2)/(e*x^2+d)^(1/2))/c^3/e^(3/2)+1/3*b*(2*c^2*d+e)*arctan(x*(c^2*d-e)^(1
/2)/(e*x^2+d)^(1/2))*(c^2*d-e)^(1/2)/c^3/e^2-1/6*b*x*(e*x^2+d)^(1/2)/c/e-d
*(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/e^2
```

3.1201.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.14

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{-\frac{\sqrt{d+ex^2}(bex+ac(4d-2ex^2))}{c}+2b(-2d+ex^2)\sqrt{d+ex^2}\arctan(cx)-\frac{ib(2c^4d^2-c^2de-e^2)\log\left(\frac{12ic^4e^2\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+e\cdots}\right.}{b\sqrt{c^2d-e}\left(-2c^4d^2+c^2de+e^2\right)(i\cdots}\right.}{c^3\sqrt{c^2d-e}}}{6e^2}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(-((Sqrt[d + e*x^2]*(b*e*x + a*c*(4*d - 2*e*x^2)))/c) + 2*b*(-2*d + e*x^2)
*Sqrt[d + e*x^2]*ArcTan[c*x] - (I*b*(2*c^4*d^2 - c^2*d*e - e^2)*Log[((12*I
)*c^4*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d -
 e]*(-2*c^4*d^2 + c^2*d*e + e^2)*(I + c*x))]])/(c^3*Sqrt[c^2*d - e]) + (I*b
*(2*c^4*d^2 - c^2*d*e - e^2)*Log[((-12*I)*c^4*e^2*(c*d + I*e*x + Sqrt[c^2*
d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-2*c^4*d^2 + c^2*d*e + e^2)*(
-I + c*x))]])/(c^3*Sqrt[c^2*d - e]) + (b*Sqrt[e]*(3*c^2*d + 2*e)*Log[e*x +
Sqrt[e]*Sqrt[d + e*x^2]])/c^3)/(6*e^2)
```

3.1201.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5511, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \ 5511$$
$$-bc\int -\frac{(2d-ex^2)\sqrt{ex^2+d}}{3e^2(c^2x^2+1)}dx+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$
$$\Big\downarrow \ 27$$

$$\frac{bc\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{c^2x^2+1}dx}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \; 403$$

$$\frac{bc\left(\frac{\int \frac{e(3dc^2+2e)x^2+d(4dc^2+e)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \; 398$$

$$\frac{bc\left(\frac{\frac{e(3c^2d+2e)\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}+\frac{2(c^2d-e)(2c^2d+e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \; 224$$

$$\frac{bc\left(\frac{\frac{2(c^2d-e)(2c^2d+e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(3c^2d+2e)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \; 219$$

$$\frac{bc\left(\frac{\frac{2(c^2d-e)(2c^2d+e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(3c^2d+2e)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \; 291$$

$$\frac{bc\left(\frac{\frac{2(c^2d-e)(2c^2d+e)\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(3c^2d+2e)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

3.1201. $\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}dx$

$$\downarrow \text{216}$$

$$\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{bc\left(\frac{\frac{2\sqrt{c^2d-e}\left(2c^2d+e\right)\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(3c^2d+2e\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-((d*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e^2) + ((d + e*x^2)^(3/2)*(a + b
*ArcTan[c*x]))/(3*e^2) + (b*c*(-1/2*(e*x*Sqrt[d + e*x^2])/c^2 + ((2*Sqrt[c
^2*d - e]*(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 +
 (Sqrt[e]*(3*c^2*d + 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)/(2*c^
2)))/(3*e^2)
```

3.1201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1201.4 Maple [F]

$$\int \frac{x^3(a + b\arctan(cx))}{\sqrt{e\,x^2 + d}}dx$$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

3.1201.5 Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 882, normalized size of antiderivative = 5.01

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{(3\,bc^2d+2\,be)\sqrt{e}\log\left(-2\,ex^2-2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+(2\,bc^2d+be)\sqrt{-c^2d+e}\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2}{}\right.}{}\right.$$
$$-\frac{2\,(3\,bc^2d+2\,be)\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(2\,bc^2d+be)\sqrt{-c^2d+e}\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2\,(3\,c^2d^2-4\,de)x^2+}{c^4x^4+2\,c}\right.}{1}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*((3*b*c^2*d + 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e
)*x - d) + (2*b*c^2*d + b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e +
8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(
-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^3
*e*x^2 - 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*
sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*(2*b*c^2*d + b*e)*sqrt(c^2*d - e)*arct
an(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e -
 e^2)*x^3 + (c^2*d^2 - d*e)*x)) + (3*b*c^2*d + 2*b*e)*sqrt(e)*log(-2*e*x^2
 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(2*a*c^3*e*x^2 - 4*a*c^3*d - b*c^2
*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e^2)
, -1/12*(2*(3*b*c^2*d + 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))
 - (2*b*c^2*d + b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x
^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d +
 e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2*a*c^3*e*x^2 -
 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*sqrt(e*x
^2 + d))/(c^3*e^2), 1/6*((2*b*c^2*d + b*e)*sqrt(c^2*d - e)*arctan(1/2*sqrt
(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 +
 (c^2*d^2 - d*e)*x)) - (3*b*c^2*d + 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt
(e*x^2 + d)) + (2*a*c^3*e*x^2 - 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2
*b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e^2)]
```

3.1201.6 Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3(a+b\,\mathrm{atan}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

3.1201.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1201.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1202 $\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$

3.1202.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\frac{ax\sqrt{d+ex^2}}{2e}-\frac{ad\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}+b\text{Int}\left(\frac{x^2\arctan(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
-1/2*a*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)+1/2*a*x*(e*x^2+d)^(1/2
)/e+b*Unintegrable(x^2*arctan(c*x)/(e*x^2+d)^(1/2),x)
```

3.1202.2 Mathematica [N/A]

Not integrable

Time = 12.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

3.1202.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 262, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{5517}$$
$$a\int \frac{x^2}{\sqrt{ex^2+d}}dx+b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx$$
$$\Big\downarrow \text{262}$$
$$a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)+b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx$$
$$\Big\downarrow \text{224}$$
$$a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx$$
$$\Big\downarrow \text{219}$$
$$b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx+a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)$$
$$\Big\downarrow \text{5560}$$
$$b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx+a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.1202.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1202.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

3.1202.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

3.1202.6 Sympy [N/A]

Not integrable

Time = 8.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\,\mathrm{atan}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

3.1202.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1202.8 Giac [N/A]

Not integrable

Time = 86.05 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1202.9 Mupad [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1203 $\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}} dx$

3.1203.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}} dx = \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e} - \frac{b\sqrt{c^2d-e}\arctan\left(\frac{\sqrt{c^2d-ex}}{\sqrt{d+ex^2}}\right)}{ce} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

output
```
-b*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))*(c^2*d-e)^(1/2)/c/e-b*arctanh
(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)+(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/e
```

3.1203.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.44

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}} dx = \frac{2ac\sqrt{d+ex^2} + 2bc\sqrt{d+ex^2}\arctan(cx) - ib\sqrt{c^2d-e}\log\left(\frac{4c^2e\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{3/2}(-i+cx)}\right) + ib\sqrt{c^2d-e}\log}{2ce}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(2*a*c*Sqrt[d + e*x^2] + 2*b*c*Sqrt[d + e*x^2]*ArcTan[c*x] - I*b*Sqrt[c^2*
d - e]*Log[(4*c^2*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/
(b*(c^2*d - e)^(3/2)*(-I + c*x))] + I*b*Sqrt[c^2*d - e]*Log[(4*c^2*e*(I*c*
d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(I + c*
x))] - 2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(2*c*e)
```

3.1203.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5509, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \ 5509$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\int \frac{\sqrt{ex^2+d}}{c^2x^2+1}dx}{e}$$

$$\Big\downarrow \ 301$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}+\frac{(c^2d-e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}\right)}{e}$$

$$\Big\downarrow \ 224$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{(c^2d-e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}\right)}{e}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{(c^2d-e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e}$$

$$\Big\downarrow \ 291$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{(c^2d-e)\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{\sqrt{c^2d-e}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e - (b*c*((Sqrt[c^2*d - e]*ArcTan[(S
qrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/c^2))/e
```

3.1203.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1203.4 Maple [F]

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

3.1203.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 647, normalized size of antiderivative = 6.28

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{2\,b\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+\sqrt{-c^2d+e}b\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2\,(3\,c^2d^2-4\,de)x^2-4\,((c^2d-2\,e)}{c^4x^4+2\,c^2x^2+1}\right.}{4\,ce}\right.$$
$$-\frac{\sqrt{c^2d-e}b\arctan\left(\frac{\sqrt{c^2d-e}((c^2d-2\,e)x^2-d)\sqrt{ex^2+d}}{2\,((c^2de-e^2)x^3+(c^2d^2-de)x)}\right)-b\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\,\sqrt{ex^2+}}{2\,ce}$$
$$-\frac{\sqrt{c^2d-e}b\arctan\left(\frac{\sqrt{c^2d-e}((c^2d-2\,e)x^2-d)\sqrt{ex^2+d}}{2\,((c^2de-e^2)x^3+(c^2d^2-de)x)}\right)-2\,b\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-2\,\sqrt{ex^2+d}(bc\arctan(cx)}{2\,ce}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + sqrt(-
c^2*d + e)*b*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e
)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2
)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^2 + d)*(b*c*arctan(c*x) + a*c))/
(c*e), -1/2*(sqrt(c^2*d - e)*b*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x
^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - b*sqr
t(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*sqrt(e*x^2 + d)*(
b*c*arctan(c*x) + a*c))/(c*e), 1/4*(4*b*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*
x^2 + d)) + sqrt(-c^2*d + e)*b*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*
(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqr
t(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^2 + d)*(b*c*ar
ctan(c*x) + a*c))/(c*e), -1/2*(sqrt(c^2*d - e)*b*arctan(1/2*sqrt(c^2*d - e
)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2
- d*e)*x)) - 2*b*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 2*sqrt(e*x^
2 + d)*(b*c*arctan(c*x) + a*c))/(c*e)]
```

3.1203.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x(a+b\operatorname{atan}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

3.1203.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

3.1203.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1204 $\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx$

3.1204.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \mathrm{Int}\left(\frac{a+b\arctan(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

3.1204.2 Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

3.1204.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.1204.3.1 Defintions of rubi rules used

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1204.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arctan\left(cx\right)}{\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

3.1204.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.1204.6 Sympy [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

3.1204.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1204.8 Giac [N/A]

Not integrable

Time = 52.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\arctan(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1204.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^(1/2), x)
```

3.1205 $\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx$

3.1205.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = -\frac{a\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}} + b\text{Int}\left(\frac{\arctan(cx)}{x\sqrt{d+ex^2}},x\right)$$

output
```
-a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+b*Unintegrable(arctan(c*x)/x/(
e*x^2+d)^(1/2),x)
```

3.1205.2 Mathematica [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]), x]
```

3.1205.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x\sqrt{ex^2+d}}dx+b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{a\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx-\frac{a\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx-\frac{a\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.1205.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1205.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan \left(c x\right)}{x \sqrt{e \,x^{2}+d}}d x$$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.1205.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan{(cx)}+a}{\sqrt{ex^2+dx}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e*x^3 + d*x), x)
```

3.1205.6 Sympy [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}\,(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x*sqrt(d + e*x**2)), x)
```

3.1205.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1205.8 Giac [N/A]

Not integrable

Time = 53.42 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan(cx)+a}{\sqrt{ex^2+d}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1205.9 Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(1/2)), x)
```

3.1206 $\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx$

3.1206.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx} - \frac{bc\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{b\sqrt{c^2d-e}\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d}$$

output
```
-b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+b*arctanh(c*(e*x^2+d)^(1/2)/
(c^2*d-e)^(1/2))*(c^2*d-e)^(1/2)/d-(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/d/x
```

3.1206.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.47

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{-2a\sqrt{d+ex^2} - 2b\sqrt{d+ex^2}\arctan(cx) + 2bc\sqrt{d}x\log(x) - 2bc\sqrt{d}x\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right) + b\sqrt{c^2d-e}}{2dx}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
(-2*a*Sqrt[d + e*x^2] - 2*b*Sqrt[d + e*x^2]*ArcTan[c*x] + 2*b*c*Sqrt[d]*x*
Log[x] - 2*b*c*Sqrt[d]*x*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*Sqrt[c^2*d -
 e]*x*Log[(-4*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2
*d - e)^(3/2)*(I + c*x))] + b*Sqrt[c^2*d - e]*x*Log[(-4*c*d*(c*d + I*e*x +
 Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(-I + c*x))])/(2*d
*x)
```

3.1206.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5511, 25, 27, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{\sqrt{ex^2+d}}{dx\left(c^2x^2+1\right)}dx-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{25}$$

$$bc\int \frac{\sqrt{ex^2+d}}{dx\left(c^2x^2+1\right)}dx-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{\sqrt{ex^2+d}}{x(c^2x^2+1)}dx}{d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{354}$$

$$\frac{bc\int \frac{\sqrt{ex^2+d}}{x^2(c^2x^2+1)}dx^2}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{94}$$

$$\frac{bc\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-\left(c^2d-e\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{73}$$

$$\frac{bc\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{2(c^2d-e)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{bc\left(\frac{2\sqrt{c^2d-e}\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(d*x)) + (b*c*(-2*Sqrt[d]*ArcTanh[
Sqrt[d + e*x^2]/Sqrt[d]] + (2*Sqrt[c^2*d - e]*ArcTanh[(c*Sqrt[d + e*x^2])/
Sqrt[c^2*d - e]])/c))/(2*d)
```

3.1206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1206.4 Maple [F]

$$\int \frac{a+b\arctan\left(cx\right)}{x^{2}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

3.1206.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 660, normalized size of antiderivative = 6.60

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{2\,bc\sqrt{d}x\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right)+\sqrt{c^2d-e}bx\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,\left(4\,c^4de-3\,c^2e^2\right)x^2+4\,\left(c^3ex^2+2\,c^3d-ce\right)\sqrt{}}{c^4x^4+2\,c^2x^2+1}\right.}{4\,dx}\right.$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*c*sqrt(d)*x*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2)
+ sqrt(c^2*d - e)*b*x*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*
d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(
e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*sqrt(e*x^2 + d)*(b*arctan
(c*x) + a))/(d*x), 1/2*(b*c*sqrt(d)*x*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt
(d) + 2*d)/x^2) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d -
e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x
^2)) - 2*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x), 1/4*(4*b*c*sqrt(-d)*x
*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + sqrt(c^2*d - e)*b*x*log((c^4*e^2*x^4 +
 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*
c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 +
 1)) - 4*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x), 1/2*(2*b*c*sqrt(-d)*x
*arctan(sqrt(-d)/sqrt(e*x^2 + d)) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(c^2*
e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (
c^3*d*e - c*e^2)*x^2)) - 2*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x)]
```

3.1206.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{atan}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

3.1206.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1206.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{b\arctan(cx)+a}{\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.1207 $\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx$

3.1207.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx=-\frac{a\sqrt{d+ex^2}}{2dx^2}+\frac{ae\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{3/2}}+b\text{Int}\left(\frac{\arctan(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

output
```
1/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/2*a*(e*x^2+d)^(1/2)/d/x
^2+b*Unintegrable(arctan(c*x)/x^3/(e*x^2+d)^(1/2),x)
```

3.1207.2 Mathematica [N/A]

Not integrable

Time = 12.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

3.1207.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 243, 52, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x^3\sqrt{ex^2+d}}dx+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx+\frac{1}{2}a\left(\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx+\frac{1}{2}a\left(\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.1207.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1207.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.1207.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e*x^5 + d*x^3), x)
```

3.1207.6 Sympy [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{atan}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

3.1207.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1207.8 Giac [N/A]

Not integrable

Time = 57.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1207.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.1208 $\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.1208.1 Optimal result

Integrand size = 23, antiderivative size = 179

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}+\frac{bc(2c^2d+3e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{b\sqrt{c^2d-e}(c^2d+2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2}$$

output
```
1/6*b*c*(2*c^2*d+3*e)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/3*b*(c^2*
d+2*e)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))*(c^2*d-e)^(1/2)/d^2-1/6*
b*c*(e*x^2+d)^(1/2)/d/x^2-1/3*(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*
e*(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/d^2/x
```

3.1208.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.08

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx =$$

$$-\frac{\frac{\sqrt{d+ex^2}(bcdx+2a(d-2ex^2))}{x^3}+\frac{2b(d-2ex^2)\sqrt{d+ex^2}\arctan(cx)}{x^3}+bc\sqrt{d}(2c^2d+3e)\log(x)-bc\sqrt{d}(2c^2d+3e)\log\left(d+\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/6*((Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d - 2*e*x^2)))/x^3 + (2*b*(d - 2*e*
x^2)*Sqrt[d + e*x^2]*ArcTan[c*x])/x^3 + b*c*Sqrt[d]*(2*c^2*d + 3*e)*Log[x]
 - b*c*Sqrt[d]*(2*c^2*d + 3*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*
d^2 + c^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d
 + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e - 2*e^2)*(I + c*x))])/Sq
rt[c^2*d - e] + (b*(c^4*d^2 + c^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d + I*e*x
+ Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e
- 2*e^2)*(-I + c*x))])/Sqrt[c^2*d - e])/d^2
```

3.1208.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5511, 27, 435, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \textbf{5511}$$

$$-bc\int -\frac{(d-2ex^2)\sqrt{ex^2+d}}{3d^2x^3(c^2x^2+1)}dx+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{bc\int \frac{(d-2ex^2)\sqrt{ex^2+d}}{x^3(c^2x^2+1)}dx}{3d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \textbf{435}$$

$$\frac{bc\int \frac{(d-2ex^2)\sqrt{ex^2+d}}{x^4(c^2x^2+1)}dx^2}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \textbf{166}$$

$$\frac{bc\left(\int -\frac{e(dc^2+4e)x^2+d(2dc^2+3e)}{2x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\left(-\frac{1}{2}\int \frac{e(dc^2+4e)x^2+d(2dc^2+3e)}{x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \; 174$$

$$\frac{bc\left(\frac{1}{2}\left(2\left(c^2d-e\right)\left(c^2d+2e\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2-d\left(2c^2d+3e\right)\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \; 73$$

$$\frac{bc\left(\frac{1}{2}\left(\frac{4(c^2d-e)(c^2d+2e)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}-\frac{2d(2c^2d+3e)\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \; 221$$

$$\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}+\frac{bc\left(\frac{1}{2}\left(2\sqrt{d}(2c^2d+3e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{4\sqrt{c^2d-e}(c^2d+2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/3*(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(d*x^3) + (2*e*Sqrt[d + e*x^2]*
(a + b*ArcTan[c*x]))/(3*d^2*x) + (b*c*(-((d*Sqrt[d + e*x^2])/x^2) + (2*Sqr
t[d]*(2*c^2*d + 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] - (4*Sqrt[c^2*d - e]
*(c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/2))/(6*d^2
)
```

3.1208. $\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.1208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1208.4 Maple [F]

$$\int \frac{a + b \arctan(cx)}{x^4 \sqrt{e\,x^2 + d}}dx$$

input
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

3.1208.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 868, normalized size of antiderivative = 4.85

$$\int \frac{a + b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{(bc^2d + 2\,be)\sqrt{c^2d - e}x^3 \log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)x^2-4\,(c^3ex^2+2\,c^3d-ce)\sqrt{c^2d-e}\sqrt{ex^2+d}+e^2}{c^4x^4+2\,c^2x^2+1}\right) + (2\,bc^3 \cdots}{12 \cdots}\right.$$
$$- \frac{2\,(bc^2d + 2\,be)\sqrt{-c^2d + e}x^3 \arctan\left(-\frac{(c^2ex^2+2\,c^2d-e)\sqrt{-c^2d+e}\sqrt{ex^2+d}}{2\,(c^3d^2-cde+(c^3de-ce^2)x^2)}\right) - (2\,bc^3d + 3\,bce)\sqrt{d}x^3 \log\left(-\frac{ex^2+2 \cdots}{\cdots}\right.}{12\,d^2x^3}$$
$$- \frac{2\,(2\,bc^3d + 3\,bce)\sqrt{-d}x^3 \arctan\left(\frac{\sqrt{-d}}{\sqrt{ex^2+d}}\right) - (bc^2d + 2\,be)\sqrt{c^2d - e}x^3 \log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2 \cdots}{c^4x \cdots}\right.}{12\,d \cdots}$$
$$- \frac{(bc^2d + 2\,be)\sqrt{-c^2d + e}x^3 \arctan\left(-\frac{(c^2ex^2+2\,c^2d-e)\sqrt{-c^2d+e}\sqrt{ex^2+d}}{2\,(c^3d^2-cde+(c^3de-ce^2)x^2)}\right) + (2\,bc^3d + 3\,bce)\sqrt{-d}x^3 \arctan\left(\frac{\sqrt{\cdots}}{\sqrt{ex \cdots}}\right.}{6\,d^2x^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*((b*c^2*d + 2*b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2
- 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e
)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b
*c^3*d + 3*b*c*e)*sqrt(d)*x^3*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*
d)/x^2) - 2*(b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x)
)*sqrt(e*x^2 + d))/(d^2*x^3), -1/12*(2*(b*c^2*d + 2*b*e)*sqrt(-c^2*d + e)*
x^3*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)
/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - (2*b*c^3*d + 3*b*c*e)*sqrt(d
)*x^3*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*x - 4
*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*
x^3), -1/12*(2*(2*b*c^3*d + 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(-d)/sqrt(e*x
^2 + d)) - (b*c^2*d + 2*b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*
d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d -
 c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) +
2*(b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*sqrt(e*x
^2 + d))/(d^2*x^3), -1/6*((b*c^2*d + 2*b*e)*sqrt(-c^2*d + e)*x^3*arctan(-1
/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c
*d*e + (c^3*d*e - c*e^2)*x^2)) + (2*b*c^3*d + 3*b*c*e)*sqrt(-d)*x^3*arctan
(sqrt(-d)/sqrt(e*x^2 + d)) + (b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 -
 b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^3)]
```

3.1208.6 Sympy [F]

$$\int \frac{a + b\arctan(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{a + b\operatorname{atan}(cx)}{x^4\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

3.1208.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arctan(cx)}{x^4\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1208.8 Giac [F]

$$\int \frac{a + b \arctan(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{b \arctan(cx) + a}{\sqrt{ex^2 + dx^4}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\mathrm{atan}(c\,x)}{x^4\,\sqrt{e\,x^2 + d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.1209 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$

3.1209.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}-\frac{b(2c^2d-e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{c^2d-e}e^2}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

output
```
-b*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-b*(2*c^2*d-e)*arctan(x*(c^
2*d-e)^(1/2)/(e*x^2+d)^(1/2))/c/e^2/(c^2*d-e)^(1/2)+d*(a+b*arctan(c*x))/e^
2/(e*x^2+d)^(1/2)+(a+b*arctan(c*x))*(e*x^2+d)^(1/2)/e^2
```

3.1209.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.34

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{\frac{2a(2d+ex^2)}{\sqrt{d+ex^2}}+\frac{2b(2d+ex^2)\arctan(cx)}{\sqrt{d+ex^2}}-\frac{ib(2c^2d-e)\log\left(\frac{4c^2e^2\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\left(2c^2d-e\right)(-i+cx)}\right)}{c\sqrt{c^2d-e}}+\frac{ib(2c}{}}{2e^2}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
((2*a*(2*d + e*x^2))/Sqrt[d + e*x^2] + (2*b*(2*d + e*x^2)*ArcTan[c*x])/Sqr
t[d + e*x^2] - (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*((-I)*c*d + e*x - I*Sqrt[
c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(-I + c*x))]
)/(c*Sqrt[c^2*d - e]) + (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*(I*c*d + e*x + I
*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(I + c
*x))])/(c*Sqrt[c^2*d - e]) - (2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2
]])/c)/(2*e^2)
```

3.1209.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5511, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{ex^2+2d}{e^2(c^2x^2+1)\sqrt{ex^2+d}}dx+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{ex^2+2d}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{bc\left(\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}+\left(2d-\frac{e}{c^2}\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{bc\left(\left(2d-\frac{e}{c^2}\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$-\frac{bc\left(\left(2d-\frac{e}{c^2}\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{291}$$

$$-\frac{bc\left(\left(2d-\frac{e}{c^2}\right)\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \textcolor{blue}{216}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}-\frac{bc\left(\frac{\left(2d-\frac{e}{c^2}\right)\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-e}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(d*(a + b*ArcTan[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x]))/e^2 - (b*c*(((2*d - e/c^2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d +
 e*x^2]])/Sqrt[c^2*d - e] + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/c^2))/e^2
```

3.1209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.1209. $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
 !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] && !ILt
Q[(m - 1)/2, 0]))
```

3.1209.4 Maple [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

3.1209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(121) = 242$.

Time = 0.55 (sec) , antiderivative size = 1291, normalized size of antiderivative = 9.42

$$\int \frac{x^3(a + b\arctan(cx))}{(d + ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(-2*e*x^2
 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + (2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e
- b*e^2)*x^2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*
(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqr
t(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(2*a*c^3*d^2 - 2*a*c*d*
e + (a*c^3*d*e - a*c*e^2)*x^2 + (2*b*c^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*
c*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*d^2*e^2 - c*d*e^3 + (c^3*d*
e^3 - c*e^4)*x^2), -1/2*((2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e - b*e^2)*x^2)
*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e
*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - (b*c^2*d^2 - b*d*e
+ (b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e
)*x - d) - 2*(2*a*c^3*d^2 - 2*a*c*d*e + (a*c^3*d*e - a*c*e^2)*x^2 + (2*b*c
^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*c*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 +
d))/(c^3*d^2*e^2 - c*d*e^3 + (c^3*d*e^3 - c*e^4)*x^2), 1/4*(4*(b*c^2*d^2 -
 b*d*e + (b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 +
d)) + (2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e - b*e^2)*x^2)*sqrt(-c^2*d + e)*l
og(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^
2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2
*c^2*x^2 + 1)) + 4*(2*a*c^3*d^2 - 2*a*c*d*e + (a*c^3*d*e - a*c*e^2)*x^2 +
(2*b*c^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*c*e^2)*x^2)*arctan(c*x))*sqrt...
```

3.1209.6 Sympy [F]

$$\int \frac{x^3(a + b\arctan(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x^3(a + b\operatorname{atan}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

3.1209.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1209.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1210 $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$

3.1210.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{ax}{e\sqrt{d+ex^2}}+\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}+b\mathrm{Int}\left(\frac{x^2\arctan(cx)}{(d+ex^2)^{3/2}},x\right)$$

output
```
a*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(3/2)-a*x/e/(e*x^2+d)^(1/2)+b*Unint
egrable(x^2*arctan(c*x)/(e*x^2+d)^(3/2),x)
```

3.1210.2 Mathematica [N/A]

Not integrable

Time = 23.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

3.1210.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 252, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^2}{(ex^2+d)^{3/2}}dx+b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{252}$$

$$a\left(\frac{\int \frac{1}{\sqrt{ex^2+d}}dx}{e}-\frac{x}{e\sqrt{d+ex^2}}\right)+b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{x}{e\sqrt{d+ex^2}}\right)+b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx+a\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx+a\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}\right)$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.1210.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1210.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

3.1210.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

3.1210.6 Sympy [N/A]

Not integrable

Time = 35.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^2(a+b\,\mathrm{atan}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

3.1210.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1210.8 Giac [N/A]

Not integrable

Time = 136.84 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1210.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1211 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$

3.1211.1 Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}}+\frac{bc\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-e}e}$$

output
```
b*c*arctan(x*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/e/(c^2*d-e)^(1/2)+(-a-b*arct
an(c*x))/e/(e*x^2+d)^(1/2)
```

3.1211.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.96

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{\frac{2a}{\sqrt{d+ex^2}}+\frac{2b\arctan(cx)}{\sqrt{d+ex^2}}+\frac{ibc\log\left(-\frac{4ie\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}(i+cx)}\right)}{\sqrt{c^2d-e}}-\frac{ibc\log\left(\frac{4ie\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}(-i+cx)}\right)}{\sqrt{c^2d-e}}}{2e}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-1/2*((2*a)/Sqrt[d + e*x^2] + (2*b*ArcTan[c*x])/Sqrt[d + e*x^2] + (I*b*c*L
og[((-4*I)*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*
d - e]*(I + c*x))])/Sqrt[c^2*d - e] - (I*b*c*Log[((4*I)*e*(c*d + I*e*x + S
qrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2
*d - e])/e
```

3.1211.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5509, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{e}-\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{bc\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{bc\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{e\sqrt{c^2d-e}}-\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcTan[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*ArcTan[(Sqrt[c^2*d - e]*
x)/Sqrt[d + e*x^2]])/(Sqrt[c^2*d - e]*e)
```

3.1211.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1211.4 Maple [F]

$$\int \frac{x(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

3.1211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(63) = 126$.

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 5.34

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bcex^2+bcd)\sqrt{-c^2d+e}\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2\,(3\,c^2d^2-4\,de)x^2-4\,((c^2d-2\,e)x^3-d}{c^4x^4+2\,c^2x^2+1}\right.}{4\,(c^2d^2e-de^2+(c}\right.$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*c*e*x^2 + b*c*d)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*
e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c
^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(a*c^2*d -
 a*e + (b*c^2*d - b*e)*arctan(c*x))*sqrt(e*x^2 + d))/(c^2*d^2*e - d*e^2 +
(c^2*d*e^2 - e^3)*x^2), 1/2*((b*c*e*x^2 + b*c*d)*sqrt(c^2*d - e)*arctan(1/
2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)
*x^3 + (c^2*d^2 - d*e)*x)) - 2*(a*c^2*d - a*e + (b*c^2*d - b*e)*arctan(c*x
))*sqrt(e*x^2 + d))/(c^2*d^2*e - d*e^2 + (c^2*d*e^2 - e^3)*x^2)]
```

3.1211.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{atan}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

3.1211.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

3.1211.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1212 $\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx$

3.1212.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}+\frac{b\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

output
```
b*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(1/2)+x*(a+b*arct
an(c*x))/d/(e*x^2+d)^(1/2)
```

3.1212.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.89

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{\frac{2ax}{\sqrt{d+ex^2}}+\frac{2bx\arctan(cx)}{\sqrt{d+ex^2}}+\frac{b\log\left(-\frac{4cd\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}(i+cx)}\right)}{\sqrt{c^2d-e}}+\frac{b\log\left(-\frac{4cd\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right.}{b\sqrt{c^2d-e}(-i+cx)}\right.}{\sqrt{c^2d-e}}}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
((2*a*x)/Sqrt[d + e*x^2] + (2*b*x*ArcTan[c*x])/Sqrt[d + e*x^2] + (b*Log[(-
4*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*
(I + c*x))])/Sqrt[c^2*d - e] + (b*Log[(-4*c*d*(c*d + I*e*x + Sqrt[c^2*d -
e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*d - e])/(2*
d)
```

3.1212.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5447, 27, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{5447}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-bc\int \frac{x}{d\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{x}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2}{2d}$$

$$\Big\downarrow \text{73}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{de}$$

$$\Big\downarrow \text{221}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}+\frac{b\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

3.1212. $\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}dx$

output
```
(x*(a + b*ArcTan[c*x]))/(d*Sqrt[d + e*x^2]) + (b*ArcTanh[(c*Sqrt[d + e*x^2
])/Sqrt[c^2*d - e]])/(d*Sqrt[c^2*d - e])
```

3.1212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.1212.4 Maple [F]

$$\int \frac{a+b\arctan(cx)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

3.1212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(62) = 124$.

Time = 0.32 (sec) , antiderivative size = 388, normalized size of antiderivative = 5.54

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \left[\frac{(bex^2+bd)\sqrt{c^2d-e}\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)\,x^2+4\,(c^3ex^2+2\,c^3d-ce)\sqrt{c^2d-e}}{c^4x^4+2\,c^2x^2+1}\right.}{4\,(c^2d^3-d^2e+(c^2d^2e-}\right.$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*e*x^2 + b*d)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2
*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(
c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^
2 + d)*((b*c^2*d - b*e)*x*arctan(c*x) + (a*c^2*d - a*e)*x))/(c^2*d^3 - d^2
*e + (c^2*d^2*e - d*e^2)*x^2), 1/2*((b*e*x^2 + b*d)*sqrt(-c^2*d + e)*arcta
n(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2
 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*sqrt(e*x^2 + d)*((b*c^2*d - b*e)*x*
arctan(c*x) + (a*c^2*d - a*e)*x))/(c^2*d^3 - d^2*e + (c^2*d^2*e - d*e^2)*x
^2)]
```

3.1212.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{atan}\,(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

3.1212.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

3.1212.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^(3/2), x)
```

3.1213 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{3/2}}\,dx$

3.1213.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{3/2}}\,dx = \frac{a}{d\sqrt{d+ex^2}} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + b\text{Int}\left(\frac{\arctan(cx)}{x(d+ex^2)^{3/2}},x\right)$$

output
```
-a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+a/d/(e*x^2+d)^(1/2)+b*Unintegr
able(arctan(c*x)/x/(e*x^2+d)^(3/2),x)
```

3.1213.2 Mathematica [N/A]

Not integrable

Time = 10.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\arctan(cx)}{x(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

3.1213.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 61, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x(ex^2+d)^{3/2}}dx+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2(ex^2+d)^{3/2}}dx^2+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{2\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx+\frac{1}{2}a\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx+\frac{1}{2}a\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

3.1213. $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{3/2}}dx$

output
```
$Aborted
```

3.1213.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1213.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.1213.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^{2}+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

3.1213.6 Sympy [N/A]

Not integrable

Time = 32.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}\left(cx\right)}{x\left(d+ex^{2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

3.1213.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1213.8 Giac [N/A]

Not integrable

Time = 42.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1213.9 Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(3/2)), x)
```

3.1214 $\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

3.1214.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{b\text{carctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{b(c^2d-2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d^2\sqrt{c^2d-e}}$$

output
```
-b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+b*(c^2*d-2*e)*arctanh(c*(e*x
^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(1/2)+(-a-b*arctan(c*x))/d/x/(e
*x^2+d)^(1/2)-2*e*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)^(1/2)
```

3.1214.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.27

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\frac{-\frac{2a(d+2ex^2)}{x\sqrt{d+ex^2}}-\frac{2b(d+2ex^2)\arctan(cx)}{x\sqrt{d+ex^2}}+2bc\sqrt{d}\log(x)-2bc\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)-}{2d^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
((-2*a*(d + 2*e*x^2))/(x*Sqrt[d + e*x^2]) - (2*b*(d + 2*e*x^2)*ArcTan[c*x]
)/(x*Sqrt[d + e*x^2]) + 2*b*c*Sqrt[d]*Log[x] - 2*b*c*Sqrt[d]*Log[d + Sqrt[
d]*Sqrt[d + e*x^2]] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d - I*e*x + Sqrt[c
^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(I + c*x))])/
Sqrt[c^2*d - e] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d + I*e*x + Sqrt[c^2*d
 - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(-I + c*x))])/Sqr
t[c^2*d - e])/(2*d^2)
```

3.1214.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5511, 25, 27, 435, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2 (d+ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{2ex^2+d}{d^2x(c^2x^2+1)\sqrt{ex^2+d}}dx-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$bc\int \frac{2ex^2+d}{d^2x(c^2x^2+1)\sqrt{ex^2+d}}dx-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{2ex^2+d}{x(c^2x^2+1)\sqrt{ex^2+d}}dx}{d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{bc\int \frac{2ex^2+d}{x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2}{2d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{174}$$

$$\frac{bc\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-(c^2d-2e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)}{2d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{bc\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{2(c^2d-2e)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)}{2d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}+\frac{bc\left(\frac{2(c^2d-2e)\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c\sqrt{c^2d-e}}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)}{2d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcTan[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcTan[c*x]))
/(d^2*Sqrt[d + e*x^2]) + (b*c*(-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]
 + (2*(c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*Sqrt[
c^2*d - e])))/(2*d^2)
```

3.1214.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1214.4 Maple [F]

$$\int \frac{a+b\arctan\left(cx\right)}{x^{2}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

3.1214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(119) = 238$.

Time = 0.42 (sec) , antiderivative size = 1317, normalized size of antiderivative = 9.76

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 - 2*b*d*e)*x)*sqrt(c^2*d -
e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^
2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(
c^4*x^4 + 2*c^2*x^2 + 1)) - 2*((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*
c*d*e)*x)*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 4*
(a*c^2*d^2 - a*d*e + 2*(a*c^2*d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b
*c^2*d*e - b*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^3*e - d^2*e^2
)*x^3 + (c^2*d^4 - d^3*e)*x), 1/2*(((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2
 - 2*b*d*e)*x)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt
(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) +
((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*c*d*e)*x)*sqrt(d)*log(-(e*x^2
- 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(a*c^2*d^2 - a*d*e + 2*(a*c^2*
d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d*e - b*e^2)*x^2)*arctan(
c*x))*sqrt(e*x^2 + d))/((c^2*d^3*e - d^2*e^2)*x^3 + (c^2*d^4 - d^3*e)*x),
1/4*(4*((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*c*d*e)*x)*sqrt(-d)*arct
an(sqrt(-d)/sqrt(e*x^2 + d)) - ((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 - 2
*b*d*e)*x)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4
*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*
sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(a*c^2*d^2 - a*d*e +
 2*(a*c^2*d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d*e - b*e^2)...
```

3.1214.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}\,(cx)}{x^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

3.1214.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1214.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.1215 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

3.1215.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx = -\frac{3ae}{2d^2\sqrt{d+ex^2}} - \frac{a}{2dx^2\sqrt{d+ex^2}} + \frac{3ae\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{5/2}} + b\text{Int}\left(\frac{\arctan(cx)}{x^3(d+ex^2)^{3/2}},x\right)$$

output
```
3/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)-3/2*a*e/d^2/(e*x^2+d)^(1/
2)-1/2*a/d/x^2/(e*x^2+d)^(1/2)+b*Unintegrable(arctan(c*x)/x^3/(e*x^2+d)^(3
/2),x)
```

3.1215.2 Mathematica [N/A]

Not integrable

Time = 14.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

3.1215.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 52, 61, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x^3\left(ex^2+d\right)^{3/2}}dx+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^4\left(ex^2+d\right)^{3/2}}dx^2+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(-\frac{3e\int \frac{1}{x^2(ex^2+d)^{3/2}}dx^2}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(-\frac{3e\left(\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(-\frac{3e\left(\frac{2\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx+\frac{1}{2}a\left(-\frac{3e\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx+\frac{1}{2}a\left(-\frac{3e\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.1215.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1215.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx)}{x^3\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.1215.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a + b\arctan(cx)}{x^3\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{b\arctan\left(cx\right) + a}{\left(ex^2 + d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

3.1215.6 Sympy [N/A]

Not integrable

Time = 53.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a + b\arctan(cx)}{x^3\left(d + ex^2\right)^{3/2}}\,dx = \int \frac{a + b\operatorname{atan}\left(cx\right)}{x^3\left(d + ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*(d + e*x**2)**(3/2)), x)
```

3.1215.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{x^3\left(d + ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1215.8 Giac [N/A]

Not integrable

Time = 43.67 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1215.9 Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.1216 $\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx$

3.1216.1 Optimal result

Integrand size = 23, antiderivative size = 249

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx &= -\frac{bc\sqrt{d+ex^2}}{6d^2x^2}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}\\ &\quad+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{bce\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{5/2}}\\ &\quad+\frac{bc(c^2d+4e)\,\mathrm{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{b(c^4d^2+4c^2de-8e^2)\,\mathrm{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3\sqrt{c^2d-e}}\end{aligned}$$

output
```
1/6*b*c*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+1/3*b*c*(c^2*d+4*e)*arc
tanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)-1/3*b*(c^4*d^2+4*c^2*d*e-8*e^2)*arct
anh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^3/(c^2*d-e)^(1/2)+1/3*(-a-b*arcta
n(c*x))/d/x^3/(e*x^2+d)^(1/2)+4/3*e*(a+b*arctan(c*x))/d^2/x/(e*x^2+d)^(1/2
)+8/3*e^2*x*(a+b*arctan(c*x))/d^3/(e*x^2+d)^(1/2)-1/6*b*c*(e*x^2+d)^(1/2)/
d^2/x^2
```

3.1216.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.63

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx =$$

$$-\frac{\frac{bcdx(d+ex^2)+2a(d^2-4dex^2-8e^2x^4)}{x^3\sqrt{d+ex^2}}+\frac{2b(d^2-4dex^2-8e^2x^4)\arctan(cx)}{x^3\sqrt{d+ex^2}}+bc\sqrt{d}(2c^2d+9e)\log(x)-bc\sqrt{d}(2c^2d+9e)\log}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
-1/6*((b*c*d*x*(d + e*x^2) + 2*a*(d^2 - 4*d*e*x^2 - 8*e^2*x^4))/(x^3*Sqrt[
d + e*x^2]) + (2*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*ArcTan[c*x])/(x^3*Sqrt[d
+ e*x^2]) + b*c*Sqrt[d]*(2*c^2*d + 9*e)*Log[x] - b*c*Sqrt[d]*(2*c^2*d + 9*
e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*Log
[(12*c*d^3*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d
- e]*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*(I + c*x))]])/Sqrt[c^2*d - e] + (b*(c^4*
d^2 + 4*c^2*d*e - 8*e^2)*Log[(12*c*d^3*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt
[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*(-I + c*x))
])/Sqrt[c^2*d - e])/d^3
```

3.1216.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{-8e^2x^4-4dex^2+d^2}{3d^3x^3(c^2x^2+1)\sqrt{ex^2+d}}dx+\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{-8e^2x^4-4dex^2+d^2}{x^3(c^2x^2+1)\sqrt{ex^2+d}}dx}{3d^3}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{7276}$$

$$\frac{bc\int \left(\frac{d^2}{x^3\sqrt{ex^2+d}}-\frac{(dc^2+4e)d}{x\sqrt{ex^2+d}}+\frac{(d^2c^4+4dec^2-8e^2)x}{(c^2x^2+1)\sqrt{ex^2+d}}\right)dx}{3d^3}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{bc\left(\sqrt{d}(c^2d+4e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{(c^4d^2+4c^2de-8e^2)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c\sqrt{c^2d-e}}+\frac{1}{2}\sqrt{d}e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{d\sqrt{d+ex^2}}{2x^2}\right)}{3d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/(d*x^3*Sqrt[d + e*x^2]) + (4*e*(a + b*ArcTan[c*x]
))/(3*d^2*x*Sqrt[d + e*x^2]) + (8*e^2*x*(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d
 + e*x^2]) + (b*c*(-1/2*(d*Sqrt[d + e*x^2])/x^2 + (Sqrt[d]*e*ArcTanh[Sqrt[
d + e*x^2]/Sqrt[d]])/2 + Sqrt[d]*(c^2*d + 4*e)*ArcTanh[Sqrt[d + e*x^2]/Sqr
t[d]] - ((c^4*d^2 + 4*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^
2*d - e]])/(c*Sqrt[c^2*d - e])))/(3*d^3)
```

3.1216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.1216.4 Maple [F]

$$\int \frac{a+b\arctan\left(cx\right)}{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

3.1216.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(209) = 418.

Time = 0.55 (sec) , antiderivative size = 1920, normalized size of antiderivative = 7.71

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(((b*c^4*d^2*e + 4*b*c^2*d*e^2 - 8*b*e^3)*x^5 + (b*c^4*d^3 + 4*b*c^
2*d^2*e - 8*b*d*e^2)*x^3)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8
*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*s
qrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - ((2*b*c
^5*d^2*e + 7*b*c^3*d*e^2 - 9*b*c*e^3)*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d^2*e -
 9*b*c*d*e^2)*x^3)*sqrt(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/
x^2) + 2*(2*a*c^2*d^3 - 16*(a*c^2*d*e^2 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*
d^2*e - b*c*d*e^2)*x^3 - 8*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*
d^2*e)*x + 2*(b*c^2*d^3 - 8*(b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e - 4*(b*c^2
*d^2*e - b*d*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^4*e - d^3*e^2
)*x^5 + (c^2*d^5 - d^4*e)*x^3), -1/12*(2*((b*c^4*d^2*e + 4*b*c^2*d*e^2 - 8
*b*e^3)*x^5 + (b*c^4*d^3 + 4*b*c^2*d^2*e - 8*b*d*e^2)*x^3)*sqrt(-c^2*d + e
)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(
c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - ((2*b*c^5*d^2*e + 7*b*c^3*d*e^
2 - 9*b*c*e^3)*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d^2*e - 9*b*c*d*e^2)*x^3)*sqrt
(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(2*a*c^2*d^3 -
 16*(a*c^2*d*e^2 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*d^2*e - b*c*d*e^2)*x^3
- 8*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*d^2*e)*x + 2*(b*c^2*d^3
 - 8*(b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e - 4*(b*c^2*d^2*e - b*d*e^2)*x^2)*
arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^4*e - d^3*e^2)*x^5 + (c^2*d^5 - d...
```

3.1216.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{atan}\left(cx\right)}{x^4\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**4*(d + e*x**2)**(3/2)), x)
```

3.1216.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1216.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(3/2)), x)
```

3.1217 $\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$

3.1217.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{ax^3}{3e(d+ex^2)^{3/2}} - \frac{ax}{e^2\sqrt{d+ex^2}} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}} + b\text{Int}\left(\frac{x^4\arctan(cx)}{(d+ex^2)^{5/2}},x\right)$$

output
```
-1/3*a*x^3/e/(e*x^2+d)^(3/2)+a*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)-
a*x/e^2/(e*x^2+d)^(1/2)+b*Unintegrable(x^4*arctan(c*x)/(e*x^2+d)^(5/2),x)
```

3.1217.2 Mathematica [N/A]

Not integrable

Time = 17.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

3.1217.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 252, 252, 224, 219, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^4}{(ex^2+d)^{5/2}}dx+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{252}$$

$$a\left(\frac{\int \frac{x^2}{(ex^2+d)^{3/2}}dx}{e}-\frac{x^3}{3e\,(d+ex^2)^{3/2}}\right)+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{252}$$

$$a\left(\frac{\frac{\int \frac{1}{\sqrt{ex^2+d}}dx}{e}-\frac{x}{e\sqrt{d+ex^2}}}{e}-\frac{x^3}{3e\,(d+ex^2)^{3/2}}\right)+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{\frac{\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{x}{e\sqrt{d+ex^2}}}{e}-\frac{x^3}{3e\,(d+ex^2)^{3/2}}\right)+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx+a\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}}{e}-\frac{x^3}{3e\,(d+ex^2)^{3/2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^4 \arctan(cx)}{(ex^2+d)^{5/2}}dx + a\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} - \frac{x}{e\sqrt{d+ex^2}}}{e} - \frac{x^3}{3e(d+ex^2)^{3/2}}\right)$$

input
```
Int[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.1217.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1217.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

3.1217.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arctan(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

3.1217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.1217.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1217.8 Giac [N/A]

Not integrable

Time = 168.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1217.9 Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1218 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$

3.1218.1 Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx}{3(c^2d-e)e\sqrt{d+ex^2}} + \frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}} + \frac{bc(2c^2d-3e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3(c^2d-e)^{3/2}e^2}$$

output
```
1/3*d*(a+b*arctan(c*x))/e^2/(e*x^2+d)^(3/2)+1/3*b*c*(2*c^2*d-3*e)*arctan(x
*(c^2*d-e)^(1/2)/(e*x^2+d)^(1/2))/(c^2*d-e)^(3/2)/e^2+1/3*b*c*x/(c^2*d-e)/
e/(e*x^2+d)^(1/2)+(-a-b*arctan(c*x))/e^2/(e*x^2+d)^(1/2)
```

3.1218.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.28

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{2\sqrt{c^2d-e}(bcex(d+ex^2)-a(c^2d-e)(2d+3ex^2))-2b(c^2d-e)^{3/2}(2d+3ex}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c^2*d - e]*(b*c*e*x*(d + e*x^2) - a*(c^2*d - e)*(2*d + 3*e*x^2)) -
 2*b*(c^2*d - e)^(3/2)*(2*d + 3*e*x^2)*ArcTan[c*x] - I*b*c*(2*c^2*d - 3*e)
*(d + e*x^2)^(3/2)*Log[((-12*I)*Sqrt[c^2*d - e]*e^2*(c*d - I*e*x + Sqrt[c^
2*d - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(I + c*x))] + I*b*c*(2*c^2*d
 - 3*e)*(d + e*x^2)^(3/2)*Log[((12*I)*Sqrt[c^2*d - e]*e^2*(c*d + I*e*x + S
qrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(-I + c*x))])/(6*(c^2*
d - e)^(3/2)*e^2*(d + e*x^2)^(3/2))
```

3.1218.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5511, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{3ex^2+2d}{3e^2(c^2x^2+1)(ex^2+d)^{3/2}}dx-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{3ex^2+2d}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{bc\left(\frac{\int \frac{d(2c^2d-3e)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{d(c^2d-e)}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(\frac{(2c^2d-3e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2d-e}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{bc\left(\frac{(2c^2d-3e)\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2d-e}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{bc\left(\frac{(2c^2d-3e)\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{(c^2d-e)^{3/2}}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(d*(a + b*ArcTan[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcTan[c*x])/(e
^2*Sqrt[d + e*x^2]) + (b*c*((e*x)/((c^2*d - e)*Sqrt[d + e*x^2]) + ((2*c^2*
d - 3*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(c^2*d - e)^(3/2)))/
(3*e^2)
```

3.1218.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1218.4 Maple [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

3.1218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(125) = 250$.

Time = 0.60 (sec) , antiderivative size = 863, normalized size of antiderivative = 6.03

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(2\,bc^3d^3 - 3\,bcd^2e + (2\,bc^3de^2 - 3\,bce^3)x^4 + 2\,(2\,bc^3d^2e - 3\,bcde^2)x^2)\sqrt{-c^2d}}{}\right.$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*((2*b*c^3*d^3 - 3*b*c*d^2*e + (2*b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(
2*b*c^3*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d
*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*
sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(2*
a*c^4*d^3 - 4*a*c^2*d^2*e + 2*a*d*e^2 - (b*c^3*d*e^2 - b*c*e^3)*x^3 + 3*(a
*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^2 - (b*c^3*d^2*e - b*c*d*e^2)*x + (2
*b*c^4*d^3 - 4*b*c^2*d^2*e + 2*b*d*e^2 + 3*(b*c^4*d^2*e - 2*b*c^2*d*e^2 +
b*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d
^2*e^4 + (c^4*d^2*e^4 - 2*c^2*d*e^5 + e^6)*x^4 + 2*(c^4*d^3*e^3 - 2*c^2*d^
2*e^4 + d*e^5)*x^2), 1/6*((2*b*c^3*d^3 - 3*b*c*d^2*e + (2*b*c^3*d*e^2 - 3*
b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(c^2*d - e)*arctan
(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e
^2)*x^3 + (c^2*d^2 - d*e)*x)) - 2*(2*a*c^4*d^3 - 4*a*c^2*d^2*e + 2*a*d*e^2
 - (b*c^3*d*e^2 - b*c*e^3)*x^3 + 3*(a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x
^2 - (b*c^3*d^2*e - b*c*d*e^2)*x + (2*b*c^4*d^3 - 4*b*c^2*d^2*e + 2*b*d*e^
2 + 3*(b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 +
 d))/(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4 + (c^4*d^2*e^4 - 2*c^2*d*e^5 +
 e^6)*x^4 + 2*(c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^2)]
```

3.1218.6 Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^3(a+b\,\mathrm{atan}\,(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

3.1218.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1218.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1219 $\int \frac{x^2(a+b\arctan(cx))}{\left(d+ex^2\right)^{5/2}}\,dx$

3.1219.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{x^2(a+b\arctan(cx))}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{bc}{3\left(c^2d-e\right)e\sqrt{d+ex^2}}+\frac{x^3(a+b\arctan(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{b\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d\left(c^2d-e\right)^{3/2}}$$

output
```
1/3*x^3*(a+b*arctan(c*x))/d/(e*x^2+d)^(3/2)-1/3*b*arctanh(c*(e*x^2+d)^(1/2
)/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(3/2)+1/3*b*c/(c^2*d-e)/e/(e*x^2+d)^(1/2)
```

3.1219.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.31

$$\int \frac{x^2(a+b\arctan(cx))}{\left(d+ex^2\right)^{5/2}}\,dx=$$

$$-\frac{\frac{2adx}{e(d+ex^2)^{3/2}}-\frac{2\left(bcd+a\left(c^2d-e\right)x\right)}{(c^2d-e)e\sqrt{d+ex^2}}-\frac{2bx^3\arctan(cx)}{(d+ex^2)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(i+cx)}\right)}{(c^2d-e)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(cd+iex+\sqrt{c^2 \cdots}\right.}{b(-i+cx)}\right.}{(c^2d-e)^{3/2}}\cdots}{6d}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/6*((2*a*d*x)/(e*(d + e*x^2)^(3/2)) - (2*(b*c*d + a*(c^2*d - e)*x))/((c^
2*d - e)*e*Sqrt[d + e*x^2]) - (2*b*x^3*ArcTan[c*x])/(d + e*x^2)^(3/2) + (b
*Log[(12*c*d*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2
]))/(b*(I + c*x))])/(c^2*d - e)^(3/2) + (b*Log[(12*c*d*Sqrt[c^2*d - e]*(c*
d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(-I + c*x))])/(c^2*d - e)
^(3/2))/d
```

3.1219.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5511, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5511}$$

$$\frac{x^3(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}-bc\int \frac{x^3}{3d(c^2x^2+1)(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}-\frac{bc\int \frac{x^3}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{354}$$

$$\frac{x^3(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}-\frac{bc\int \frac{x^2}{(c^2x^2+1)(ex^2+d)^{3/2}}dx^2}{6d}$$

$$\Big\downarrow \text{87}$$

$$\frac{x^3(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}-\frac{bc\left(-\frac{\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2}{c^2d-e}-\frac{2d}{e(c^2d-e)\sqrt{d+ex^2}}\right)}{6d}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^3(a+b\arctan(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\left(-\frac{2\int\frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e(c^2d-e)}-\frac{2d}{e(c^2d-e)\sqrt{d+ex^2}}\right)}{6d}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{x^3(a+b\arctan(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\left(\frac{2\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}-\frac{2d}{e(c^2d-e)\sqrt{d+ex^2}}\right)}{6d}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcTan[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c*((-2*d)/((c^2*d -
 e)*e*Sqrt[d + e*x^2]) + (2*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/
(c*(c^2*d - e)^(3/2))))/(6*d)
```

3.1219.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.1219. $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1219.4 Maple [F]

$$\int \frac{x^{2} \left(a +b \arctan \left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{\frac{5}{2}}}d x$$

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

3.1219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(93) = 186.

Time = 0.42 (sec) , antiderivative size = 676, normalized size of antiderivative = 6.20

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(be^3x^4+2\,bde^2x^2+bd^2e)\sqrt{c^2d-e}\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)x^2+4}{c^4x^4+2\,c^2x}\right.}{12\,(c^4}\right.$$

$$-\frac{(be^3x^4+2\,bde^2x^2+bd^2e)\sqrt{-c^2d+e}\arctan\left(-\frac{(c^2ex^2+2\,c^2d-e)\sqrt{-c^2d+e}\sqrt{ex^2+d}}{2\,(c^3d^2-cde+(c^3de-ce^2)x^2)}\right)-2\,(bc^3d^3-bcd^2e+(bc^4d^2e}{6\,(c^4d^5e-2\,c^2d^4e^2+d^3e^3+(c^4d^3e^3-2\,c^2d^2e^4+d}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*((b*e^3*x^4 + 2*b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*d - e)*log((c^4*e^2
*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^
2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2
*x^2 + 1)) - 4*(b*c^3*d^3 - b*c*d^2*e + (b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e
^3)*x^3*arctan(c*x) + (a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d
^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e
^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^
3*e^3 + d^2*e^4)*x^2), -1/6*((b*e^3*x^4 + 2*b*d*e^2*x^2 + b*d^2*e)*sqrt(-c
^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^
2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(b*c^3*d^3 - b*c*d^2
*e + (b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^3*arctan(c*x) + (a*c^4*d^2*e
- 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 +
 d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 +
 d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

3.1219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.1219.7 Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + 2*b*integrate
(1/2*x^2*arctan(c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

3.1219.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1220 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$

3.1220.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx}{3d(c^2d-e)\sqrt{d+ex^2}} - \frac{a+b\arctan(cx)}{3e(d+ex^2)^{3/2}} + \frac{bc^3\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3(c^2d-e)^{3/2}e}$$

output
```
1/3*(-a-b*arctan(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*c^3*arctan(x*(c^2*d-e)^(1/2
)/(e*x^2+d)^(1/2))/(c^2*d-e)^(3/2)/e-1/3*b*c*x/d/(c^2*d-e)/(e*x^2+d)^(1/2)
```

3.1220.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.35

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{1}{6}\left(-\frac{2a}{e(d+ex^2)^{3/2}} - \frac{2bcx}{(c^2d^2-de)\sqrt{d+ex^2}} - \frac{2b\arctan(cx)}{e(d+ex^2)^{3/2}} - \frac{ibc^3\log\left(-\frac{12i\sqrt{c^2d-e}e\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{bc^2(i+cx)}\right)}{(c^2d-e)^{3/2}e} + \frac{ibc^3\log\left(\frac{12i\sqrt{c^2d-e}e\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{bc^2(-i+cx)}\right)}{(c^2d-e)^{3/2}e}\right)$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
((-2*a)/(e*(d + e*x^2)^(3/2)) - (2*b*c*x)/((c^2*d^2 - d*e)*Sqrt[d + e*x^2]
) - (2*b*ArcTan[c*x])/(e*(d + e*x^2)^(3/2)) - (I*b*c^3*Log[((-12*I)*Sqrt[c
^2*d - e]*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(I + c
*x))])/((c^2*d - e)^(3/2)*e) + (I*b*c^3*Log[((12*I)*Sqrt[c^2*d - e]*e*(c*d
 + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(-I + c*x))])/((c^2*d
- e)^(3/2)*e))/6
```

3.1220.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5509, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3e}-\frac{a+b\arctan(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{296}$$

$$\frac{bc\left(\frac{c^2\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2d-e}-\frac{ex}{d(c^2d-e)\sqrt{d+ex^2}}\right)}{3e}-\frac{a+b\arctan(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{bc\left(\frac{c^2\int \frac{1}{1-\frac{\left(e-c^2d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2d-e}-\frac{ex}{d(c^2d-e)\sqrt{d+ex^2}}\right)}{3e}-\frac{a+b\arctan(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{bc\left(\frac{c^2\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{(c^2d-e)^{3/2}}-\frac{ex}{d(c^2d-e)\sqrt{d+ex^2}}\right)}{3e}-\frac{a+b\arctan(cx)}{3e\,(d+ex^2)^{3/2}}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/(e*(d + e*x^2)^(3/2)) + (b*c*(-((e*x)/(d*(c^2*d -
 e)*Sqrt[d + e*x^2])) + (c^2*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/
(c^2*d - e)^(3/2)))/(3*e)
```

3.1220.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

3.1220.4 Maple [F]

$$\int \frac{x(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

3.1220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(94) = 188$.

Time = 0.57 (sec) , antiderivative size = 679, normalized size of antiderivative = 6.17

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(bc^3de^2x^4+2\,bc^3d^2ex^2+bc^3d^3)\sqrt{-c^2d+e}\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2\,(3\,c^2d^2-4\,de)}{c^4x^4}\right.}{12\,(c^4}\right.$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((b*c^3*d*e^2*x^4 + 2*b*c^3*d^2*e*x^2 + b*c^3*d^3)*sqrt(-c^2*d + e)*
log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c
^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 +
2*c^2*x^2 + 1)) - 4*(a*c^4*d^3 - 2*a*c^2*d^2*e + a*d*e^2 + (b*c^3*d*e^2 -
b*c*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x + (b*c^4*d^3 - 2*b*c^2*d^2*e +
b*d*e^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^
3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3
*e^3 + d^2*e^4)*x^2), 1/6*((b*c^3*d*e^2*x^4 + 2*b*c^3*d^2*e*x^2 + b*c^3*d^
3)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt
(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - 2*(a*c^4*d^3 - 2*
a*c^2*d^2*e + a*d*e^2 + (b*c^3*d*e^2 - b*c*e^3)*x^3 + (b*c^3*d^2*e - b*c*d
*e^2)*x + (b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e^2)*arctan(c*x))*sqrt(e*x^2 +
d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 +
d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

3.1220.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x(a+b\,\mathrm{atan}\,(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

3.1220.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

3.1220.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1221 $\int \frac{a+b\arctan(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.1221.1 Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bc}{3d\,(c^2d-e)\sqrt{d+ex^2}} + \frac{x(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}} + \frac{b(3c^2d-2e)\,\mathrm{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2\,(c^2d-e)^{3/2}}$$

output
```
1/3*x*(a+b*arctan(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*(3*c^2*d-2*e)*arctanh(c*(e
*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(3/2)-1/3*b*c/d/(c^2*d-e)/(e*
x^2+d)^(1/2)+2/3*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)^(1/2)
```

3.1221.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.20

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{2\sqrt{c^2d-e}(-bcd(d+ex^2)+a(c^2d-e)\,x(3d+2ex^2))+2b(c^2d-e)^{3/2}\,x(3d+2ex}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c^2*d - e]*(-(b*c*d*(d + e*x^2)) + a*(c^2*d - e)*x*(3*d + 2*e*x^2)
) + 2*b*(c^2*d - e)^(3/2)*x*(3*d + 2*e*x^2)*ArcTan[c*x] + b*(3*c^2*d - 2*e
)*(d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(I + c*x))] + b*(3*c^2*d - 2*
e)*(d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d + I*e*x + Sqrt[c^
2*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(-I + c*x))])/(6*d^2*(c^2*d
- e)^(3/2)*(d + e*x^2)^(3/2))
```

3.1221.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5447, 27, 435, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 5447$$

$$-bc\int \frac{x(2ex^2+3d)}{3d^2(c^2x^2+1)(ex^2+d)^{3/2}}dx+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{bc\int \frac{x(2ex^2+3d)}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \; 435$$

$$-\frac{bc\int \frac{2ex^2+3d}{(c^2x^2+1)(ex^2+d)^{3/2}}dx^2}{6d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \; 87$$

$$-\frac{bc\left(\frac{(3c^2d-2e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2}{c^2d-e}+\frac{2d}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{bc\left(\frac{2(3c^2d-2e)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e(c^2d-e)}+\frac{2d}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}-\frac{bc\left(\frac{2d}{(c^2d-e)\sqrt{d+ex^2}}-\frac{2(3c^2d-2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}\right)}{6d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcTan[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcTan[c*x])
)/(3*d^2*Sqrt[d + e*x^2]) - (b*c*((2*d)/((c^2*d - e)*Sqrt[d + e*x^2]) - (2
*(3*c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*(c^2*d -
 e)^(3/2))))/(6*d^2)
```

3.1221.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

3.1221.4 Maple [F]

$$\int \frac{a+b\arctan(cx)}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

3.1221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(124) = 248.

Time = 0.63 (sec) , antiderivative size = 864, normalized size of antiderivative = 6.00

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(3\,bc^2d^3 + (3\,bc^2de^2 - 2\,be^3)x^4 - 2\,bd^2e + 2\,(3\,bc^2d^2e - 2\,bde^2)x^2)\sqrt{c^2d-e}\,\log\left(\cdots\right.}{\cdots}\right.$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((3*b*c^2*d^3 + (3*b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c
^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 -
8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*
sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(b*c
^3*d^3 - b*c*d^2*e - 2*(a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*
d^2*e - b*c*d*e^2)*x^2 - 3*(a*c^4*d^3 - 2*a*c^2*d^2*e + a*d*e^2)*x - (2*(b
*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^4*d^3 - 2*b*c^2*d^2*e + b
*d*e^2)*x)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^6 - 2*c^2*d^5*e + d^4*e^2
+ (c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^4*d^5*e - 2*c^2*d^4*e
^2 + d^3*e^3)*x^2), 1/6*((3*b*c^2*d^3 + (3*b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*
b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-c^2*d + e)*arctan(-1/2*
(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*
e + (c^3*d*e - c*e^2)*x^2)) - 2*(b*c^3*d^3 - b*c*d^2*e - 2*(a*c^4*d^2*e -
2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x^2 - 3*(a*c^4*d^3
- 2*a*c^2*d^2*e + a*d*e^2)*x - (2*(b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^
3 + 3*(b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e^2)*x)*arctan(c*x))*sqrt(e*x^2 + d
))/(c^4*d^6 - 2*c^2*d^5*e + d^4*e^2 + (c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e
^4)*x^4 + 2*(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

3.1221.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.1221.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + 2*b*integrat
e(1/2*arctan(c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

3.1221.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^(5/2), x)
```

3.1222 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{5/2}}\,dx$

3.1222.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{5/2}}\,dx = \frac{a}{3d(d+ex^2)^{3/2}}+\frac{a}{d^2\sqrt{d+ex^2}} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}+b\text{Int}\left(\frac{\arctan(cx)}{x(d+ex^2)^{5/2}},x\right)$$

output
```
1/3*a/d/(e*x^2+d)^(3/2)-a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+a/d^2/(
e*x^2+d)^(1/2)+b*Unintegrable(arctan(c*x)/x/(e*x^2+d)^(5/2),x)
```

3.1222.2 Mathematica [N/A]

Not integrable

Time = 14.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\arctan(cx)}{x(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

3.1222.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 243, 61, 61, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x\left(ex^2+d\right)^{5/2}}dx+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\left(ex^2+d\right)^{5/2}}dx^2+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{x^2(ex^2+d)^{3/2}}dx^2}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{\frac{2\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.1222.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1222.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.1222.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^{2}+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

3.1222.6 Sympy [N/A]

Not integrable

Time = 65.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\operatorname{atan}\left(cx\right)}{x\left(d+ex^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x**2)**(5/2)), x)
```

3.1222.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1222.8 Giac [N/A]

Not integrable

Time = 54.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1222.9 Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(5/2)), x)
```

3.1223 $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx$

3.1223.1 Optimal result

Integrand size = 23, antiderivative size = 274

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx &= \frac{bc}{d^2\sqrt{d+ex^2}}-\frac{8be}{3cd^3\sqrt{d+ex^2}}-\frac{b(3c^4d^2-12c^2de+8e^2)}{3cd^3(c^2d-e)\sqrt{d+ex^2}}\\ &\quad-\frac{a+b\arctan(cx)}{dx(d+ex^2)^{3/2}}-\frac{4ex(a+b\arctan(cx))}{3d^2(d+ex^2)^{3/2}}-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}\\ &\quad-\frac{bc\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}+\frac{b(3c^4d^2-12c^2de+8e^2)\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3(c^2d-e)^{3/2}}\end{aligned}$$

output
```
(-a-b*arctan(c*x))/d/x/(e*x^2+d)^(3/2)-4/3*e*x*(a+b*arctan(c*x))/d^2/(e*x^
2+d)^(3/2)-b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+1/3*b*(3*c^4*d^2-1
2*c^2*d*e+8*e^2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^3/(c^2*d-e)^
(3/2)+b*c/d^2/(e*x^2+d)^(1/2)-8/3*b*e/c/d^3/(e*x^2+d)^(1/2)-1/3*b*(3*c^4*d
^2-12*c^2*d*e+8*e^2)/c/d^3/(c^2*d-e)/(e*x^2+d)^(1/2)-8/3*e*x*(a+b*arctan(c
*x))/d^3/(e*x^2+d)^(1/2)
```

3.1223.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.53

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \frac{-\frac{2adex}{(d+ex^2)^{3/2}}+\frac{2e(bcd+5a(-c^2d+e)x)}{(c^2d-e)\sqrt{d+ex^2}}-\frac{6a\sqrt{d+ex^2}}{x}-\frac{2b(3d^2+12dex^2+8e^2x^4)\arctan(cx)}{x(d+ex^2)^{3/2}}+6bc\sqrt{d}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output
```
((-2*a*d*e*x)/(d + e*x^2)^(3/2) + (2*e*(b*c*d + 5*a*(-(c^2*d) + e)*x))/((c
^2*d - e)*Sqrt[d + e*x^2]) - (6*a*Sqrt[d + e*x^2])/x - (2*b*(3*d^2 + 12*d*
e*x^2 + 8*e^2*x^4)*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)) + 6*b*c*Sqrt[d]*Log[
x] - 6*b*c*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(3*c^4*d^2 - 12*c
^2*d*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d -
 e]*Sqrt[d + e*x^2]))/(b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*(I + c*x))]])/(c^
2*d - e)^(3/2) + (b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c
^2*d - e]*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^4*d^2 -
 12*c^2*d*e + 8*e^2)*(-I + c*x))]])/(c^2*d - e)^(3/2))/(6*d^3)
```

3.1223.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{8e^2x^4+12dex^2+3d^2}{3d^3x\left(c^2x^2+1\right)\left(ex^2+d\right)^{3/2}}dx-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{8e^2x^4+12dex^2+3d^2}{x(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d^3}-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 7276}$$

$$\frac{bc\int \left(\frac{3d^2}{x(ex^2+d)^{3/2}}+\frac{8e^2x}{c^2(ex^2+d)^{3/2}}-\frac{(3d^2c^4-12dec^2+8e^2)x}{c^2(c^2x^2+1)(ex^2+d)^{3/2}}\right)dx}{3d^3}-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}+\frac{bc\left(\frac{(3c^4d^2-12c^2de+8e^2)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}-3\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{8e}{c^2\sqrt{d+ex^2}}-\frac{3c^4d^2-12c^2de+8e^2}{c^2(c^2d-e)\sqrt{d+ex^2}}+\frac{3d}{\sqrt{d+ex^2}}\right)}{3d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcTan[c*x])/(d*x*(d + e*x^2)^(3/2))) - (4*e*x*(a + b*ArcTan[c*x]
))/(3*d^2*(d + e*x^2)^(3/2)) - (8*e*x*(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d +
 e*x^2]) + (b*c*((3*d)/Sqrt[d + e*x^2] - (8*e)/(c^2*Sqrt[d + e*x^2]) - (3*
c^4*d^2 - 12*c^2*d*e + 8*e^2)/(c^2*(c^2*d - e)*Sqrt[d + e*x^2]) - 3*Sqrt[d
]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + ((3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*Arc
Tanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*(c^2*d - e)^(3/2))))/(3*d^3)
```

3.1223.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.1223.4 Maple [F]

$$\int \frac{a + b\arctan\left(cx\right)}{x^{2} \left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

3.1223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(240) = 480.

Time = 1.10 (sec) , antiderivative size = 2714, normalized size of antiderivative = 9.91

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(((3*b*c^4*d^2*e^2 - 12*b*c^2*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^4*d^3
*e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^4*d^4 - 12*b*c^2*d^3*e + 8
*b*d^2*e^2)*x)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e +
2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d -
 e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 6*((b*c^5*d^2*e^2
- 2*b*c^3*d*e^3 + b*c*e^4)*x^5 + 2*(b*c^5*d^3*e - 2*b*c^3*d^2*e^2 + b*c*d*
e^3)*x^3 + (b*c^5*d^4 - 2*b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(d)*log(-(e*x^
2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 4*(3*a*c^4*d^4 - 6*a*c^2*d^3*e
 + 3*a*d^2*e^2 + 8*(a*c^4*d^2*e^2 - 2*a*c^2*d*e^3 + a*e^4)*x^4 - (b*c^3*d^
2*e^2 - b*c*d*e^3)*x^3 + 12*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2
- (b*c^3*d^3*e - b*c*d^2*e^2)*x + (3*b*c^4*d^4 - 6*b*c^2*d^3*e + 3*b*d^2*e
^2 + 8*(b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 12*(b*c^4*d^3*e - 2*b
*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^4*d^5*e^2 -
 2*c^2*d^4*e^3 + d^3*e^4)*x^5 + 2*(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3)*x^
3 + (c^4*d^7 - 2*c^2*d^6*e + d^5*e^2)*x), 1/6*(((3*b*c^4*d^2*e^2 - 12*b*c^
2*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^4*d^3*e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*
x^3 + (3*b*c^4*d^4 - 12*b*c^2*d^3*e + 8*b*d^2*e^2)*x)*sqrt(-c^2*d + e)*arc
tan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d
^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 3*((b*c^5*d^2*e^2 - 2*b*c^3*d*e^3 +
 b*c*e^4)*x^5 + 2*(b*c^5*d^3*e - 2*b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*...
```

3.1223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.1223.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1223.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(5/2)), x)
```

3.1224 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

3.1224.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = &-\frac{5ae}{6d^2\left(d+ex^2\right)^{3/2}}-\frac{a}{2dx^2\left(d+ex^2\right)^{3/2}}\\ &-\frac{5ae}{2d^3\sqrt{d+ex^2}}+\frac{5ae\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}}+b\operatorname{Int}\left(\frac{\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)\end{aligned}$$

output
```
-5/6*a*e/d^2/(e*x^2+d)^(3/2)-1/2*a/d/x^2/(e*x^2+d)^(3/2)+5/2*a*e*arctanh((
e*x^2+d)^(1/2)/d^(1/2))/d^(7/2)-5/2*a*e/d^3/(e*x^2+d)^(1/2)+b*Unintegrable
(arctan(c*x)/x^3/(e*x^2+d)^(5/2),x)
```

3.1224.2 Mathematica [N/A]

Not integrable

Time = 18.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

3.1224.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 243, 52, 61, 61, 73, 221, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x^3\left(ex^2+d\right)^{5/2}}dx+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^4\left(ex^2+d\right)^{5/2}}dx^2+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(-\frac{5e\int \frac{1}{x^2(ex^2+d)^{5/2}}dx^2}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(-\frac{5e\left(\frac{\int \frac{1}{x^2\left(ex^2+d\right)^{3/2}}dx^2}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{2\int\frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int\frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int\frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int\frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.1224. $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

3.1224.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1224.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan\left(cx\right)}{x^{3}\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.1224.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^{2}+d\right)^{\frac{5}{2}}x^{3}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

3.1224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.1224.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1224.8 Giac [N/A]

Not integrable

Time = 55.52 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1224.9 Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.1225 $\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{5/2}}\,dx$

3.1225.1 Optimal result

Integrand size = 23, antiderivative size = 423

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{5/2}}\,dx=-\frac{bce}{2d^3\sqrt{d+ex^2}}+\frac{16be^2}{3cd^4\sqrt{d+ex^2}}-\frac{bc(c^2d+6e)}{3d^3\sqrt{d+ex^2}}+\frac{b(c^2d-2e)(c^4d^2+8c^2de-8e^2)}{3cd^4(c^2d-e)\sqrt{d+ex^2}}-\frac{bc}{6d^2x^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3(d+ex^2)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x(d+ex^2)^{3/2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3(d+ex^2)^{3/2}}+\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{bce\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}}+\frac{bc(c^2d+6e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{3d^{7/2}}-\frac{b(c^2d-2e)(c^4d^2+8c^2de-8e^2)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^4(c^2d-e)^{3/2}}$$

output
```
1/3*(-a-b*arctan(c*x))/d/x^3/(e*x^2+d)^(3/2)+2*e*(a+b*arctan(c*x))/d^2/x/(
e*x^2+d)^(3/2)+8/3*e^2*x*(a+b*arctan(c*x))/d^3/(e*x^2+d)^(3/2)+1/2*b*c*e*a
rctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(7/2)+1/3*b*c*(c^2*d+6*e)*arctanh((e*x^2
+d)^(1/2)/d^(1/2))/d^(7/2)-1/3*b*(c^2*d-2*e)*(c^4*d^2+8*c^2*d*e-8*e^2)*arc
tanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^4/(c^2*d-e)^(3/2)-1/2*b*c*e/d^3/
(e*x^2+d)^(1/2)+16/3*b*e^2/c/d^4/(e*x^2+d)^(1/2)-1/3*b*c*(c^2*d+6*e)/d^3/(
e*x^2+d)^(1/2)+1/3*b*(c^2*d-2*e)*(c^4*d^2+8*c^2*d*e-8*e^2)/c/d^4/(c^2*d-e)
/(e*x^2+d)^(1/2)-1/6*b*c/d^2/x^2/(e*x^2+d)^(1/2)+16/3*e^2*x*(a+b*arctan(c*
x))/d^4/(e*x^2+d)^(1/2)
```

3.1225.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.21

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx =$$

$$-\frac{\frac{2a\left(d^3-6d^2ex^2-24de^2x^4-16e^3x^6\right)}{x^3\left(d+ex^2\right)^{3/2}}+\frac{bcd\left(e\left(-d+ex^2\right)+c^2d\left(d+ex^2\right)\right)}{\left(c^2d-e\right)x^2\sqrt{d+ex^2}}+\frac{2b\left(d^3-6d^2ex^2-24de^2x^4-16e^3x^6\right)\arctan(cx)}{x^3\left(d+ex^2\right)^{3/2}}+bc\sqrt{d}(2c^2d+1}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

output
```
-1/6*((2*a*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6))/(x^3*(d + e*x^
2)^(3/2)) + (b*c*d*(e*(-d + e*x^2) + c^2*d*(d + e*x^2)))/((c^2*d - e)*x^2*
Sqrt[d + e*x^2]) + (2*b*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6)*Ar
cTan[c*x])/(x^3*(d + e*x^2)^(3/2)) + b*c*Sqrt[d]*(2*c^2*d + 15*e)*Log[x] -
 b*c*Sqrt[d]*(2*c^2*d + 15*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^6*d
^3 + 6*c^4*d^2*e - 24*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c
*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e -
 24*c^2*d*e^2 + 16*e^3)*(I + c*x))])/(c^2*d - e)^(3/2) + (b*(c^6*d^3 + 6*c
^4*d^2*e - 24*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c*d + I*e
*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e - 24*c^2*
d*e^2 + 16*e^3)*(-I + c*x))])/(c^2*d - e)^(3/2))/d^4
```

3.1225.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{-16e^3x^6-24de^2x^4-6d^2ex^2+d^3}{3d^4x^3\left(c^2x^2+1\right)\left(ex^2+d\right)^{3/2}}dx+\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{bc\int \frac{-16e^3x^6-24de^2x^4-6d^2ex^2+d^3}{x^3(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d^4}+\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow 7276$$

$$\frac{bc\int \left(\frac{d^3}{x^3(ex^2+d)^{3/2}}-\frac{(dc^2+6e)d^2}{x(ex^2+d)^{3/2}}-\frac{16e^3x}{c^2(ex^2+d)^{3/2}}+\frac{(c^2d-2e)\left(d^2c^4+8dec^2-8e^2\right)x}{c^2(c^2x^2+1)(ex^2+d)^{3/2}}\right)dx}{3d^4}+\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow 2009$$

$$\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{bc\left(\sqrt{d}(c^2d+6e)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{(c^2d-2e)(c^4d^2+8c^2de-8e^2)\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}+\frac{3}{2}\sqrt{d}e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+\frac{16}{c^2\sqrt{d}}\right.}{3d^4}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/(d*x^3*(d + e*x^2)^(3/2)) + (2*e*(a + b*ArcTan[c*
x]))/(d^2*x*(d + e*x^2)^(3/2)) + (8*e^2*x*(a + b*ArcTan[c*x]))/(3*d^3*(d +
 e*x^2)^(3/2)) + (16*e^2*x*(a + b*ArcTan[c*x]))/(3*d^4*Sqrt[d + e*x^2]) +
(b*c*((-3*d*e)/(2*Sqrt[d + e*x^2]) + (16*e^2)/(c^2*Sqrt[d + e*x^2]) - (d*(
c^2*d + 6*e))/Sqrt[d + e*x^2] + ((c^2*d - 2*e)*(c^4*d^2 + 8*c^2*d*e - 8*e^
2))/(c^2*(c^2*d - e)*Sqrt[d + e*x^2]) - d^2/(2*x^2*Sqrt[d + e*x^2]) + (3*S
qrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + Sqrt[d]*(c^2*d + 6*e)*ArcTa
nh[Sqrt[d + e*x^2]/Sqrt[d]] - ((c^2*d - 2*e)*(c^4*d^2 + 8*c^2*d*e - 8*e^2)
*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*(c^2*d - e)^(3/2))))/(3*
d^4)
```

3.1225. $\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{5/2}}\,dx$

3.1225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.1225.4 Maple [F]

$$\int \frac{a+b\arctan\left(cx\right)}{x^{4} \left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

3.1225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(365) = 730$.

Time = 1.77 (sec) , antiderivative size = 3460, normalized size of antiderivative = 8.18

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(((b*c^6*d^3*e^2 + 6*b*c^4*d^2*e^3 - 24*b*c^2*d*e^4 + 16*b*e^5)*x^7
 + 2*(b*c^6*d^4*e + 6*b*c^4*d^3*e^2 - 24*b*c^2*d^2*e^3 + 16*b*d*e^4)*x^5 +
 (b*c^6*d^5 + 6*b*c^4*d^4*e - 24*b*c^2*d^3*e^2 + 16*b*d^2*e^3)*x^3)*sqrt(c
^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*
e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) +
 e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - ((2*b*c^7*d^3*e^2 + 11*b*c^5*d^2*e^3 -
28*b*c^3*d*e^4 + 15*b*c*e^5)*x^7 + 2*(2*b*c^7*d^4*e + 11*b*c^5*d^3*e^2 - 2
8*b*c^3*d^2*e^3 + 15*b*c*d*e^4)*x^5 + (2*b*c^7*d^5 + 11*b*c^5*d^4*e - 28*b
*c^3*d^3*e^2 + 15*b*c*d^2*e^3)*x^3)*sqrt(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d
)*sqrt(d) + 2*d)/x^2) + 2*(2*a*c^4*d^5 - 4*a*c^2*d^4*e - 32*(a*c^4*d^2*e^3
 - 2*a*c^2*d*e^4 + a*e^5)*x^6 + 2*a*d^3*e^2 + (b*c^5*d^3*e^2 - b*c*d*e^4)*
x^5 - 48*(a*c^4*d^3*e^2 - 2*a*c^2*d^2*e^3 + a*d*e^4)*x^4 + 2*(b*c^5*d^4*e
- b*c^3*d^3*e^2)*x^3 - 12*(a*c^4*d^4*e - 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x^2
+ (b*c^5*d^5 - 2*b*c^3*d^4*e + b*c*d^3*e^2)*x + 2*(b*c^4*d^5 - 2*b*c^2*d^4
*e - 16*(b*c^4*d^2*e^3 - 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*d^3*e^2 - 24*(b*c^
4*d^3*e^2 - 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 - 6*(b*c^4*d^4*e - 2*b*c^2*d^3*
e^2 + b*d^2*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^4*d^6*e^2 - 2*c^2*
d^5*e^3 + d^4*e^4)*x^7 + 2*(c^4*d^7*e - 2*c^2*d^6*e^2 + d^5*e^3)*x^5 + (c^
4*d^8 - 2*c^2*d^7*e + d^6*e^2)*x^3), -1/12*(2*((b*c^6*d^3*e^2 + 6*b*c^4*d^
2*e^3 - 24*b*c^2*d*e^4 + 16*b*e^5)*x^7 + 2*(b*c^6*d^4*e + 6*b*c^4*d^3*e...
```

3.1225.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^4 (d + ex^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.1225.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arctan(cx)}{x^4 (d + ex^2)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1225.8 Giac [F]

$$\int \frac{a + b \arctan(cx)}{x^4 (d + ex^2)^{5/2}} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x^4} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x^4 (d + ex^2)^{5/2}} \, dx = \int \frac{a + b \operatorname{atan}(c x)}{x^4 (e x^2 + d)^{5/2}} \, dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(5/2)), x)
```

3.1226 $\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx$

3.1226.1 Optimal result

Integrand size = 16, antiderivative size = 208

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = -\frac{a}{15c(a^2c-d)(c+dx^2)^{3/2}} - \frac{a(7a^2c-4d)}{15c^2(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\arctan(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\arctan(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}} + \frac{(15a^4c^2-20a^2cd+8d^2)\,\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3(a^2c-d)^{5/2}}$$

output
```
-1/15*a/c/(a^2*c-d)/(d*x^2+c)^(3/2)+1/5*x*arctan(a*x)/c/(d*x^2+c)^(5/2)+4/
15*x*arctan(a*x)/c^2/(d*x^2+c)^(3/2)+1/15*(15*a^4*c^2-20*a^2*c*d+8*d^2)*ar
ctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^3/(a^2*c-d)^(5/2)-1/15*a*(7*a^2
*c-4*d)/c^2/(a^2*c-d)^2/(d*x^2+c)^(1/2)+8/15*x*arctan(a*x)/c^3/(d*x^2+c)^(
1/2)
```

3.1226.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.66

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{-\frac{2ac\left(-d(5c+4dx^2)+a^2c(8c+7dx^2)\right)}{(-a^2c+d)^2(c+dx^2)^{3/2}} + \frac{2x(15c^2+20cdx^2+8d^2x^4)\arctan(ax)}{(c+dx^2)^{5/2}} + \frac{(15a^4c^2-20a^2cd+8d^2)\log\left(-\frac{60a}{\cdots}\right.}{(\cdots}}{3\cdots}$$

input
```
Integrate[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
((-2*a*c*(-(d*(5*c + 4*d*x^2)) + a^2*c*(8*c + 7*d*x^2)))/((-(a^2*c) + d)^2
*(c + d*x^2)^(3/2)) + (2*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTan[a*x])/
(c + d*x^2)^(5/2) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*Log[(-60*a*c^3*(a^2
*c - d)^(3/2)*(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c^
2 - 20*a^2*c*d + 8*d^2)*(I + a*x))])/(a^2*c - d)^(5/2) + ((15*a^4*c^2 - 20
*a^2*c*d + 8*d^2)*Log[(-60*a*c^3*(a^2*c - d)^(3/2)*(a*c + I*d*x + Sqrt[a^2
*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*(-I + a*x))])
/(a^2*c - d)^(5/2))/(30*c^3)
```

3.1226.3 Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5447, 27, 7266, 1192, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 5447$$

$$-a\int \frac{x\left(8d^2x^4+20cdx^2+15c^2\right)}{15c^3\left(a^2x^2+1\right)\left(dx^2+c\right)^{5/2}}dx+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\int \frac{x\left(8d^2x^4+20cdx^2+15c^2\right)}{\left(a^2x^2+1\right)\left(dx^2+c\right)^{5/2}}dx}{15c^3}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{7266}$$

$$-\frac{a\int \frac{8d^2x^4+20cdx^2+15c^2}{(a^2x^2+1)(dx^2+c)^{5/2}}dx^2}{30c^3}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{a\int -\frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c-d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\int \frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c-d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1584}$$

$$\frac{a\int \left(-\frac{(15c^2a^4-20cda^2+8d^2)d^2}{(d-a^2c)^2(a^2x^4-a^2c+d)}+\frac{c(7a^2c-4d)d^2}{(a^2c-d)^2x^4}+\frac{3c^2d^2}{(a^2c-d)x^8}\right)d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{c^2d^2}{x^6(a^2c-d)}+\frac{cd^2(7a^2c-4d)}{x^2(a^2c-d)^2}-\frac{d^2(15a^4c^2-20a^2cd+8d^2)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{a(a^2c-d)^{5/2}}\right)}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

input
```
Int[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
(x*ArcTan[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*ArcTan[a*x])/(15*c^2*(c + d
*x^2)^(3/2)) + (8*x*ArcTan[a*x])/(15*c^3*Sqrt[c + d*x^2]) - (a*((c^2*d^2)/
((a^2*c - d)*x^6) + (c*(7*a^2*c - 4*d)*d^2)/((a^2*c - d)^2*x^2) - (d^2*(15
*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c - d]
])/(a*(a^2*c - d)^(5/2))))/(15*c^3*d^2)
```

3.1226. $\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx$

3.1226.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.1226.4 Maple [F]

$$\int \frac{\arctan \left(a x \right)}{\left(d \,x^{2}+c \right)^{\frac{7}{2}}}d x$$

input
```
int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

output
```
int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

3.1226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(180) = 360$.

Time = 0.35 (sec) , antiderivative size = 1280, normalized size of antiderivative = 6.15

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\, dx = \text{Too large to display}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^5
)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*
(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c - d)*log((a^4*
d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d
*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*
a^2*x^2 + 1)) - 4*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2
 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*
c^2*d^3)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4 - d^5)*x^5 +
20*(a^6*c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 15*(a^6*c^5 -
 3*a^4*c^4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arctan(a*x))*sqrt(d*x^2 + c))/(
a^6*c^9 - 3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 - 3*a^4*c^5
*d^4 + 3*a^2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*d^3 + 3*a
^2*c^5*d^4 - c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 -
 c^5*d^4)*x^2), 1/30*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^
2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*
c*d^4)*x^4 + 3*(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2*
c + d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 +
 c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(8*a^5*c^5 - 13*a^3*c^4
*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5
*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4...
```

3.1226.6 Sympy [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(atan(a*x)/(c + d*x**2)**(7/2), x)
```

3.1226.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

3.1226.8 Giac [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\arctan(ax)}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx=\int \frac{\mathrm{atan}(a\,x)}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(atan(a*x)/(c + d*x^2)^(7/2),x)
```

output
```
int(atan(a*x)/(c + d*x^2)^(7/2), x)
```

3.1227 $\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx$

3.1227.1 Optimal result

Integrand size = 16, antiderivative size = 293

$$\begin{aligned}\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx &= -\frac{a}{35c(a^2c-d)(c+dx^2)^{5/2}}-\frac{a(11a^2c-6d)}{105c^2(a^2c-d)^2(c+dx^2)^{3/2}}\\ &\quad-\frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3(a^2c-d)^3\sqrt{c+dx^2}}+\frac{x\arctan(ax)}{7c(c+dx^2)^{7/2}}+\frac{6x\arctan(ax)}{35c^2(c+dx^2)^{5/2}}+\frac{8x\arctan(ax)}{35c^3(c+dx^2)^{3/2}}\\ &\quad+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\,\mathrm{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4(a^2c-d)^{7/2}}\end{aligned}$$

output
```
-1/35*a/c/(a^2*c-d)/(d*x^2+c)^(5/2)-1/105*a*(11*a^2*c-6*d)/c^2/(a^2*c-d)^2
/(d*x^2+c)^(3/2)+1/7*x*arctan(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arctan(a*x)/c^
2/(d*x^2+c)^(5/2)+8/35*x*arctan(a*x)/c^3/(d*x^2+c)^(3/2)+1/35*(35*a^6*c^3-
70*a^4*c^2*d+56*a^2*c*d^2-16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2
))/c^4/(a^2*c-d)^(7/2)-1/35*a*(19*a^4*c^2-22*a^2*c*d+8*d^2)/c^3/(a^2*c-d)^
3/(d*x^2+c)^(1/2)+16/35*x*arctan(a*x)/c^4/(d*x^2+c)^(1/2)
```

3.1227.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{-\frac{2ac\left(3c^2(-a^2c+d)^2+c(11a^2c-6d)(a^2c-d)(c+dx^2)+3(19a^4c^2-22a^2cd+8d^2)(c+dx^2)^2\right)}{(a^2c-d)^3(c+dx^2)^{5/2}} + \frac{6x(35c^3+70c^2dx^2+56}{(c-}}{}$$

input
```
Integrate[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
((-2*a*c*(3*c^2*(-(a^2*c) + d)^2 + c*(11*a^2*c - 6*d)*(a^2*c - d)*(c + d*x
^2) + 3*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2)*(c + d*x^2)^2))/((a^2*c - d)^3*(
c + d*x^2)^(5/2)) + (6*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^
6)*ArcTan[a*x])/(c + d*x^2)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2
*c*d^2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c - I*d*x + Sqrt[a^2
*c - d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*
d^3)*(I + a*x))])/(a^2*c - d)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a
^2*c*d^2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c + I*d*x + Sqrt[a
^2*c - d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 1
6*d^3)*(-I + a*x))])/(a^2*c - d)^(7/2))/(210*c^4)
```

3.1227.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5447, 27, 7266, 2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow \; 5447$$

$$-a\int \frac{x(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3)}{35c^4(a^2x^2+1)(dx^2+c)^{7/2}}dx + \frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}} + \frac{8x\arctan(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{6x\arctan(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{x\arctan(ax)}{7c(c+dx^2)^{7/2}}$$

$$\Big\downarrow 27$$

$$-\frac{a\int \frac{x(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3)}{(a^2x^2+1)(dx^2+c)^{7/2}}dx}{35c^4}+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow 7266$$

$$-\frac{a\int \frac{16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3}{(a^2x^2+1)(dx^2+c)^{7/2}}dx^2}{70c^4}+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow 2122$$

$$-\frac{a\int \left(-\frac{5dc^3}{(a^2c-d)(dx^2+c)^{7/2}}-\frac{(11a^2c-6d)dc^2}{(d-a^2c)^2(dx^2+c)^{5/2}}+\frac{d(19c^2a^4-22cda^2+8d^2)c}{(d-a^2c)^3(dx^2+c)^{3/2}}+\frac{35c^3a^6-70c^2da^4+56cd^2a^2-16d^3}{(a^2c-d)^3(a^2x^2+1)\sqrt{dx^2+c}}\right)dx^2}{70c^4}+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow 2009$$

$$-\frac{a\left(\frac{2c^3}{(a^2c-d)(c+dx^2)^{5/2}}+\frac{2c^2(11a^2c-6d)}{3(a^2c-d)^2(c+dx^2)^{3/2}}+\frac{2c(19a^4c^2-22a^2cd+8d^2)}{(a^2c-d)^3\sqrt{c+dx^2}}-\frac{2(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{a(a^2c-d)^{7/2}}\right)}{70c^4}+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

input
```
Int[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
(x*ArcTan[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcTan[a*x])/(35*c^2*(c + d
*x^2)^(5/2)) + (8*x*ArcTan[a*x])/(35*c^3*(c + d*x^2)^(3/2)) + (16*x*ArcTan
[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (a*((2*c^3)/((a^2*c - d)*(c + d*x^2)^(5/
2)) + (2*c^2*(11*a^2*c - 6*d))/(3*(a^2*c - d)^2*(c + d*x^2)^(3/2)) + (2*c*
(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/((a^2*c - d)^3*Sqrt[c + d*x^2]) - (2*(3
5*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^
2])/Sqrt[a^2*c - d]])/(a*(a^2*c - d)^(7/2))))/(70*c^4)
```

3.1227. $\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx$

3.1227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.1227.4 Maple [F]

$$\int \frac{\arctan(ax)}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

output
```
int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

3.1227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(257) = 514$.

Time = 0.63 (sec) , antiderivative size = 1986, normalized size of antiderivative = 6.78

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(3*(35*a^6*c^7 - 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 -
70*a^4*c^2*d^5 + 56*a^2*c*d^6 - 16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d
^3 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 -
 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 - 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*
a^4*c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^3*d^4)*x^2)*sqrt(a^2*c - d)*log((a^4*d
^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d*
x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a
^2*x^2 + 1)) - 4*(71*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*
d^3 + 3*(19*a^7*c^4*d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6
 + (182*a^7*c^5*d^2 - 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^
4 + (196*a^7*c^6*d - 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2
 - 3*(16*(a^8*c^4*d^3 - 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)
*x^7 + 56*(a^8*c^5*d^2 - 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c
*d^6)*x^5 + 70*(a^8*c^6*d - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4
+ c^2*d^5)*x^3 + 35*(a^8*c^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3
 + c^3*d^4)*x)*arctan(a*x))*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*
a^4*c^10*d^2 - 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*
a^4*c^6*d^6 - 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^
4 + 6*a^4*c^7*d^5 - 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6
*c^9*d^3 + 6*a^4*c^8*d^4 - 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d...
```

3.1227.6 Sympy [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(d*x**2+c)**(9/2),x)
```

output
```
Integral(atan(a*x)/(c + d*x**2)**(9/2), x)
```

3.1227.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

3.1227.8 Giac [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\arctan(ax)}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.1227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(atan(a*x)/(c + d*x^2)^(9/2),x)
```

output
```
int(atan(a*x)/(c + d*x^2)^(9/2), x)
```

3.1228 $\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx$

3.1228.1 Optimal result

Integrand size = 21, antiderivative size = 378

$$\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx = -\frac{be(e^2(15+8m+m^2)-3c^2de(21+10m+m^2)+3c^4d^2(35+12m+m^2))\,x^{2+m}}{c^5(2+m)(3+m)(5+m)(7+m)} + \frac{be^2(e(5+m)-3c^2d(7+m))\,x^{4+m}}{c^3(4+m)(5+m)(7+m)} - \frac{be^3x^{6+m}}{c(6+m)(7+m)} + \frac{d^3x^{1+m}(a+b\arctan(cx))}{1+m} + \frac{3d^2ex^{3+m}(a+b\arctan(cx))}{3+m} + \frac{3de^2x^{5+m}(a+b\arctan(cx))}{5+m} + \frac{e^3x^{7+m}(a+b\arctan(cx))}{7+m} + \frac{b(e^3(15+23m+9m^2+m^3)-3c^2de^2(21+31m+11m^2+m^3)+3c^4d^2e(35+47m+13m^2+m^3)-c}{c^5(1+m)(2+m)(3+m)(5+m}$$

output
```
-b*e*(e^2*(m^2+8*m+15)-3*c^2*d*e*(m^2+10*m+21)+3*c^4*d^2*(m^2+12*m+35))*x^
(2+m)/c^5/(2+m)/(7+m)/(m^2+8*m+15)+b*e^2*(e*(5+m)-3*c^2*d*(7+m))*x^(4+m)/c
^3/(4+m)/(5+m)/(7+m)-b*e^3*x^(6+m)/c/(6+m)/(7+m)+d^3*x^(1+m)*(a+b*arctan(c
*x))/(1+m)+3*d^2*e*x^(3+m)*(a+b*arctan(c*x))/(3+m)+3*d*e^2*x^(5+m)*(a+b*ar
ctan(c*x))/(5+m)+e^3*x^(7+m)*(a+b*arctan(c*x))/(7+m)+b*(e^3*(m^3+9*m^2+23*
m+15)-3*c^2*d*e^2*(m^3+11*m^2+31*m+21)+3*c^4*d^2*e*(m^3+13*m^2+47*m+35)-c^
6*d^3*(m^3+15*m^2+71*m+105))*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2
*x^2)/c^5/(m^2+12*m+35)/(m^3+6*m^2+11*m+6)
```

3.1228.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.70

$$\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= x^{1+m}\left(\frac{d^3(a+b\arctan(cx))}{1+m}+\frac{3d^2ex^2(a+b\arctan(cx))}{3+m}+\frac{3de^2x^4(a+b\arctan(cx))}{5+m}+\frac{e^3x^6(a+b\arctan(cx))}{7+m}-\frac{bce^3x^7\,\text{Hypergeometric2F1}\left(1,4+\frac{m}{2},5+\frac{m}{2},-c^2x^2\right)}{(7+m)(8+m)}-\frac{bcd^3x\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2+3m+m^2}-\frac{3bcd^2ex^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-c^2x^2\right)}{12+7m+m^2}-\frac{3bcde^2x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-c^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
x^(1 + m)*((d^3*(a + b*ArcTan[c*x]))/(1 + m) + (3*d^2*e*x^2*(a + b*ArcTan[
c*x]))/(3 + m) + (3*d*e^2*x^4*(a + b*ArcTan[c*x]))/(5 + m) + (e^3*x^6*(a +
 b*ArcTan[c*x]))/(7 + m) - (b*c*e^3*x^7*Hypergeometric2F1[1, 4 + m/2, 5 +
m/2, -(c^2*x^2)])/((7 + m)*(8 + m)) - (b*c*d^3*x*Hypergeometric2F1[1, (2 +
 m)/2, (4 + m)/2, -(c^2*x^2)])/(2 + 3*m + m^2) - (3*b*c*d^2*e*x^3*Hypergeo
metric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)])/(12 + 7*m + m^2) - (3*b*c*
d*e^2*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(c^2*x^2)])/((5 + m)
*(6 + m)))
```

3.1228.3 Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5511, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^{m+1}\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{c^2x^2+1}dx+\frac{d^3x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{3d^2ex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{3de^2x^{m+5}(a+b\arctan(cx))}{m+5}+\frac{e^3x^{m+7}(a+b\arctan(cx))}{m+7}$$

$$\Big\downarrow \text{2333}$$

$$-bc\int \left(\frac{e\left(3d^2\left(m^2+12m+35\right)c^4-3de\left(m^2+10m+21\right)c^2+e^2\left(m^2+8m+15\right)\right)x^{m+1}}{c^6(m+3)(m+5)(m+7)}+\frac{\left(105d^3c^6+d^3m^3c^6\right.}{}\right.$$
$$\frac{d^3x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{3d^2ex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{3de^2x^{m+5}(a+b\arctan(cx))}{m+5}+\frac{e^3x^{m+7}(a+b\arctan(cx))}{m+7}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{3d^2ex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{3de^2x^{m+5}(a+b\arctan(cx))}{m+5}+\frac{e^3x^{m+7}(a+b\arctan(cx))}{m+7}-$$
$$bc\left(\frac{e^3x^{m+6}}{c^2(m+6)(m+7)}+\frac{e^2x^{m+4}\left(\frac{3c^2d}{m+5}-\frac{e}{m+7}\right)}{c^4(m+4)}+\frac{ex^{m+2}\left(3c^4d^2\left(m^2+12m+35\right)-3c^2de\left(m^2+10m+21\right)+e^2\right.}{c^6(m+2)(m+3)(m+5)(m+7)}\right.$$

input
```
Int[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (3*d^2*e*x^(3 + m)*(a + b*Ar
cTan[c*x]))/(3 + m) + (3*d*e^2*x^(5 + m)*(a + b*ArcTan[c*x]))/(5 + m) + (e
^3*x^(7 + m)*(a + b*ArcTan[c*x]))/(7 + m) - b*c*((e*(e^2*(15 + 8*m + m^2)
- 3*c^2*d*e*(21 + 10*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2))*x^(2 + m))/(c
^6*(2 + m)*(3 + m)*(5 + m)*(7 + m)) + (e^2*((3*c^2*d)/(5 + m) - e/(7 + m))
*x^(4 + m))/(c^4*(4 + m)) + (e^3*x^(6 + m))/(c^2*(6 + m)*(7 + m)) - ((e^3*
(15 + 23*m + 9*m^2 + m^3) - 3*c^2*d*e^2*(21 + 31*m + 11*m^2 + m^3) + 3*c^4
*d^2*e*(35 + 47*m + 13*m^2 + m^3) - c^6*d^3*(105 + 71*m + 15*m^2 + m^3))*x
^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(c^6*(1 +
 m)*(2 + m)*(3 + m)*(5 + m)*(7 + m)))
```

3.1228. $\int x^m\left(d+ex^2\right)^3(a+b\arctan(cx))\,dx$

3.1228.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1228.4 Maple [F]

$$\int x^{m} \left(e\,x^{2}+d\right)^{3} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

3.1228.5 Fricas [F]

$$\int x^{m}\left(d+ex^{2}\right)^{3}\left(a+b\arctan(cx)\right)dx=\int\left(ex^{2}+d\right)^{3}\left(b\arctan\left(cx\right)+a\right)x^{m}\,dx$$

input
```
integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))*x^m, x)
```

3.1228.6 Sympy [F]

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx = \int x^m(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^3\,dx$$

input
```
integrate(x**m*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**3, x)
```

3.1228.7 Maxima [F]

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^3 (b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*x^(m + 7)/(m + 7) + 3*a*d*e^2*x^(m + 5)/(m + 5) + 3*a*d^2*e*x^(m + 3
)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) + (((b*e^3*m^3 + 9*b*e^3*m^2 + 23*b*e^
3*m + 15*b*e^3)*x^7 + 3*(b*d*e^2*m^3 + 11*b*d*e^2*m^2 + 31*b*d*e^2*m + 21*
b*d*e^2)*x^5 + 3*(b*d^2*e*m^3 + 13*b*d^2*e*m^2 + 47*b*d^2*e*m + 35*b*d^2*e
)*x^3 + (b*d^3*m^3 + 15*b*d^3*m^2 + 71*b*d^3*m + 105*b*d^3)*x)*x^m*arctan(
c*x) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*integrate(((b*c*e^3*m^3 + 9*b
*c*e^3*m^2 + 23*b*c*e^3*m + 15*b*c*e^3)*x^7 + 3*(b*c*d*e^2*m^3 + 11*b*c*d*
e^2*m^2 + 31*b*c*d*e^2*m + 21*b*c*d*e^2)*x^5 + 3*(b*c*d^2*e*m^3 + 13*b*c*d
^2*e*m^2 + 47*b*c*d^2*e*m + 35*b*c*d^2*e)*x^3 + (b*c*d^3*m^3 + 15*b*c*d^3*
m^2 + 71*b*c*d^3*m + 105*b*c*d^3)*x)*x^m/(m^4 + 16*m^3 + (c^2*m^4 + 16*c^2
*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/
(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.1228.8 Giac [F]

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx=\int \left(ex^2+d\right)^3(b\arctan\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1228.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx=\int x^m\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^3, x)
```

3.1229 $\int x^m (d+ex^2)^2 (a+b\arctan(cx))\,dx$

3.1229.1 Optimal result

Integrand size = 21, antiderivative size = 230

$$\int x^m (d+ex^2)^2 (a+b\arctan(cx))\,dx = \frac{be(e(3+m)-2c^2d(5+m))\,x^{2+m}}{c^3(2+m)(3+m)(5+m)} - \frac{be^2x^{4+m}}{c(4+m)(5+m)} + \frac{d^2x^{1+m}(a+b\arctan(cx))}{1+m} + \frac{2dex^{3+m}(a+b\arctan(cx))}{3+m} + \frac{e^2x^{5+m}(a+b\arctan(cx))}{5+m} - \frac{b(e^2(3+4m+m^2)-2c^2de(5+6m+m^2)+c^4d^2(15+8m+m^2))\,x^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\right.}{c^3(1+m)(2+m)(3+m)(5+m)}$$

output
```
b*e*(e*(3+m)-2*c^2*d*(5+m))*x^(2+m)/c^3/(5+m)/(m^2+5*m+6)-b*e^2*x^(4+m)/c/
(4+m)/(5+m)+d^2*x^(1+m)*(a+b*arctan(c*x))/(1+m)+2*d*e*x^(3+m)*(a+b*arctan(
c*x))/(3+m)+e^2*x^(5+m)*(a+b*arctan(c*x))/(5+m)-b*(e^2*(m^2+4*m+3)-2*c^2*d
*e*(m^2+6*m+5)+c^4*d^2*(m^2+8*m+15))*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2
*m],-c^2*x^2)/c^3/(m^2+3*m+2)/(m^2+8*m+15)
```

3.1229.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.84

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$
$$= x^{1+m}\left(\frac{d^2(a+b\arctan(cx))}{1+m}+\frac{2dex^2(a+b\arctan(cx))}{3+m}+\frac{e^2x^4(a+b\arctan(cx))}{5+m} - \frac{bcd^2x\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2+3m+m^2} - \frac{2bcdex^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-c^2x^2\right)}{12+7m+m^2} - \frac{bce^2x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-c^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
x^(1 + m)*((d^2*(a + b*ArcTan[c*x]))/(1 + m) + (2*d*e*x^2*(a + b*ArcTan[c*
x]))/(3 + m) + (e^2*x^4*(a + b*ArcTan[c*x]))/(5 + m) - (b*c*d^2*x*Hypergeo
metric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2 + 3*m + m^2) - (2*b*c*d
*e*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)])/(12 + 7*m +
 m^2) - (b*c*e^2*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(c^2*x^2)
])/((5 + m)*(6 + m)))
```

3.1229.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5511, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$
$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^{m+1}\left(\frac{e^2x^4}{m+5}+\frac{2dex^2}{m+3}+\frac{d^2}{m+1}\right)}{c^2x^2+1}dx+\frac{d^2x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{2dex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{e^2x^{m+5}(a+b\arctan(cx))}{m+5}$$

$$\Big\downarrow \text{ 1584}$$

$$-bc\int \left(\frac{e\left(\frac{2c^2d}{m+3}-\frac{e}{m+5}\right)x^{m+1}}{c^4}+\frac{(15d^2c^4+d^2m^2c^4+8d^2mc^4-2dem^2c^2-10dec^2-12demc^2+3e^2+e^2m^2+4}{c^4(m+1)(m+3)(m+5)\left(c^2x^2+1\right)}\right.$$
$$\frac{d^2x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{2dex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{e^2x^{m+5}(a+b\arctan(cx))}{m+5}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^2x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{2dex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{e^2x^{m+5}(a+b\arctan(cx))}{m+5}-$$
$$bc\left(\frac{e^2x^{m+4}}{c^2(m+4)(m+5)}+\frac{x^{m+2}\left(c^4d^2\left(m^2+8m+15\right)-2c^2de\left(m^2+6m+5\right)+e^2\left(m^2+4m+3\right)\right)\text{Hypergeometri}}{c^4(m+1)(m+2)(m+3)(m+5)}\right.$$

input
```
Int[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (2*d*e*x^(3 + m)*(a + b*ArcT
an[c*x]))/(3 + m) + (e^2*x^(5 + m)*(a + b*ArcTan[c*x]))/(5 + m) - b*c*((e*
((2*c^2*d)/(3 + m) - e/(5 + m))*x^(2 + m))/(c^4*(2 + m)) + (e^2*x^(4 + m))
/(c^2*(4 + m)*(5 + m)) + ((e^2*(3 + 4*m + m^2) - 2*c^2*d*e*(5 + 6*m + m^2)
 + c^4*d^2*(15 + 8*m + m^2))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4
+ m)/2, -(c^2*x^2)])/(c^4*(1 + m)*(2 + m)*(3 + m)*(5 + m)))
```

3.1229.3.1 Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1229.4 Maple [F]

$$\int x^m (e\,x^2+d)^2 (a+b\arctan(cx))\,dx$$

input
```
int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

3.1229.5 Fricas [F]

$$\int x^m (d+ex^2)^2 (a+b\arctan(cx))\,dx = \int (ex^2+d)^2 (b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*x^m, x)
```

3.1229.6 Sympy [F]

$$\int x^m (d+ex^2)^2 (a+b\arctan(cx))\,dx = \int x^m (a+b\,\mathrm{atan}(cx)) (d+ex^2)^2\,dx$$

input
```
integrate(x**m*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**2, x)
```

3.1229.7 Maxima [F]

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\int \left(ex^2+d\right)^2 (b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*x^(m + 5)/(m + 5) + 2*a*d*e*x^(m + 3)/(m + 3) + a*d^2*x^(m + 1)/(m +
 1) + (((b*e^2*m^2 + 4*b*e^2*m + 3*b*e^2)*x^5 + 2*(b*d*e*m^2 + 6*b*d*e*m +
 5*b*d*e)*x^3 + (b*d^2*m^2 + 8*b*d^2*m + 15*b*d^2)*x)*x^m*arctan(c*x) - (m
^3 + 9*m^2 + 23*m + 15)*integrate(((b*c*e^2*m^2 + 4*b*c*e^2*m + 3*b*c*e^2)
*x^5 + 2*(b*c*d*e*m^2 + 6*b*c*d*e*m + 5*b*c*d*e)*x^3 + (b*c*d^2*m^2 + 8*b*
c*d^2*m + 15*b*c*d^2)*x)*x^m/(m^3 + (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c
^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

3.1229.8 Giac [F]

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\int \left(ex^2+d\right)^2 (b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1229.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^2, x)
```

3.1230 $\int x^m(d+ex^2)(a+b\arctan(cx))\,dx$

3.1230.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{bex^{2+m}}{c(6+5m+m^2)}+\frac{dx^{1+m}(a+b\arctan(cx))}{1+m}+\frac{ex^{3+m}(a+b\arctan(cx))}{3+m} - \frac{b\left(\frac{c^2d}{1+m}-\frac{e}{3+m}\right)x^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{c(2+m)}$$

output
```
-b*e*x^(2+m)/c/(m^2+5*m+6)+d*x^(1+m)*(a+b*arctan(c*x))/(1+m)+e*x^(3+m)*(a+
b*arctan(c*x))/(3+m)-b*(c^2*d/(1+m)-e/(3+m))*x^(2+m)*hypergeom([1, 1+1/2*m
],[2+1/2*m],-c^2*x^2)/c/(2+m)
```

3.1230.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx = x^{1+m}\left(-\frac{bcdx\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2+3m+m^2} + \frac{\frac{(d(3+m)+e(1+m)x^2)(a+b\arctan(cx))}{1+m}-\frac{bcex^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-c^2x^2\right)}{4+m}}{3+m}\right)$$

input
```
Integrate[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
x^(1 + m)*(-((b*c*d*x*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2
)])/(2 + 3*m + m^2)) + (((d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcTan[c*x]))/
(1 + m) - (b*c*e*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)
])/(4 + m))/(3 + m))
```

3.1230.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5511, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d+ex^2) (a+b\arctan(cx)) \, dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^{m+1}\left(\frac{ex^2}{m+3}+\frac{d}{m+1}\right)}{c^2x^2+1}dx+\frac{dx^{m+1}(a+b\arctan(cx))}{m+1}+\frac{ex^{m+3}(a+b\arctan(cx))}{m+3}$$

$$\Big\downarrow \text{363}$$

$$-bc\left(\left(\frac{d}{m+1}-\frac{e}{c^2(m+3)}\right)\int \frac{x^{m+1}}{c^2x^2+1}dx+\frac{ex^{m+2}}{c^2(m+2)(m+3)}\right)+\frac{dx^{m+1}(a+b\arctan(cx))}{m+1}+\frac{ex^{m+3}(a+b\arctan(cx))}{m+3}$$

$$\Big\downarrow \text{278}$$

$$\frac{dx^{m+1}(a+b\arctan(cx))}{m+1}+\frac{ex^{m+3}(a+b\arctan(cx))}{m+3}-bc\left(\frac{x^{m+2}\left(\frac{d}{m+1}-\frac{e}{c^2(m+3)}\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{m+2}+\frac{ex^{m+2}}{c^2(m+2)(m+3)}\right)$$

input
```
Int[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (e*x^(3 + m)*(a + b*ArcTan[c*x
]))/(3 + m) - b*c*((e*x^(2 + m))/(c^2*(2 + m)*(3 + m)) + ((d/(1 + m) - e/(
c^2*(3 + m)))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x
^2)])/(2 + m))
```

3.1230.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1230.4 Maple [F]

$$\int x^m (e\,x^2+d)\,(a+b\arctan(cx))\,dx$$

input
```
int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

3.1230.5 Fricas [F]

$$\int x^m (d+ex^2) (a+b\arctan(cx))\,dx = \int (ex^2+d)(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*x^m, x)
```

3.1230.6 Sympy [F]

$$\int x^m (d+ex^2) (a+b\arctan(cx))\,dx = \int x^m(a+b\,\mathrm{atan}(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**m*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*(d + e*x**2), x)
```

3.1230.7 Maxima [F]

$$\int x^m (d+ex^2) (a+b\arctan(cx))\,dx = \int (ex^2+d)(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
a*e*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) + (((b*e*m + b*e)*x^3 + (b*d
*m + 3*b*d)*x)*x^m*arctan(c*x) - (m^2 + 4*m + 3)*integrate(((b*c*e*m + b*c
*e)*x^3 + (b*c*d*m + 3*b*c*d)*x)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 + m^
2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

3.1230.8 Giac [F]

$$\int x^m \left(d+ex^2\right) (a+b\arctan(cx))\,dx = \int (ex^2+d)(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1230.9 Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right) (a+b\arctan(cx))\,dx = \int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2), x)
```

3.1231 $\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx$

3.1231.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \frac{ax^{1+m}\,\mathrm{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)} + b\mathrm{Int}\left(\frac{x^m\arctan(cx)}{d+ex^2},x\right)$$

output
```
a*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1+m)+b*Uninteg
rable(x^m*arctan(c*x)/(e*x^2+d),x)
```

3.1231.2 Mathematica [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2), x]
```

3.1231.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx$$
$$\Big\downarrow \text{5517}$$
$$a\int \frac{x^m}{ex^2+d}dx+b\int \frac{x^m\arctan(cx)}{ex^2+d}dx$$
$$\Big\downarrow \text{278}$$
$$b\int \frac{x^m\arctan(cx)}{ex^2+d}dx+\frac{ax^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}$$
$$\Big\downarrow \text{5560}$$
$$b\int \frac{x^m\arctan(cx)}{ex^2+d}dx+\frac{ax^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

3.1231.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1231.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{m}(a+b\arctan(cx))}{e\,x^{2}+d}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

3.1231.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{ex^2+d}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

3.1231.6 Sympy [N/A]

Not integrable

Time = 160.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^m(a+b\,\mathrm{atan}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))/(d + e*x**2), x)
```

3.1231.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{ex^2+d}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

3.1231.8 Giac [N/A]

Not integrable

Time = 154.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{ex^2+d}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1231.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2), x)
```

3.1232 $\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$

3.1232.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{ax^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d^2(1+m)} + b\text{Int}\left(\frac{x^m\arctan(cx)}{(d+ex^2)^2},x\right)$$

output
```
a*x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d^2/(1+m)+b*Unint
egrable(x^m*arctan(c*x)/(e*x^2+d)^2,x)
```

3.1232.2 Mathematica [N/A]

Not integrable

Time = 3.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2, x]
```

3.1232.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^m}{(ex^2+d)^2}dx+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^2}dx$$

$$\Big\downarrow \text{278}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^2}dx+\frac{ax^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)}$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^2}dx+\frac{ax^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

3.1232.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1232.4 Maple [N/A] (verified)

Not integrable

Time = 1.64 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(e\,x^2+d)^2}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

3.1232.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*x^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.1232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1232.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^2, x)
```

3.1232.8 Giac [N/A]

Not integrable

Time = 175.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1232.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

3.1233 $\int x^m (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx$

3.1233.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^m (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \frac{ax^{1+m}(d+ex^2)^{7/2}\,\text{Hypergeometric2F1}\left(1,\frac{8+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)} + b\text{Int}\left(x^m (d+ex^2)^{5/2}\arctan(cx),x\right)$$

output
```
a*x^(1+m)*(e*x^2+d)^(7/2)*hypergeom([1, 4+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(
1+m)+b*Unintegrable(x^m*(e*x^2+d)^(5/2)*arctan(c*x),x)
```

3.1233.2 Mathematica [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^m (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int x^m (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

3.1233.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d+ex^2\right)^{5/2} (a+b\arctan(cx)) \, dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^m \left(ex^2+d\right)^{5/2}dx+b\int x^m \left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$\frac{ad^2\sqrt{d+ex^2}\int x^m \left(\frac{ex^2}{d}+1\right)^{5/2}dx}{\sqrt{\frac{ex^2}{d}+1}}+b\int x^m \left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^m \left(ex^2+d\right)^{5/2}\arctan(cx)dx+\frac{ad^2x^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^m \left(ex^2+d\right)^{5/2}\arctan(cx)dx+\frac{ad^2x^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

input
```
Int[x^m*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1233.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1233.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^m \left(e\,x^2+d\right)^{\frac{5}{2}} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

3.1233.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int x^m (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)*x^m, x)
```

3.1233.6 Sympy [F(-1)]

Timed out.

$$\int x^m (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**m*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1233.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)*x^m, x)
```

3.1233.8 Giac [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^{\frac{5}{2}}(b\arctan\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)*x^m, x)
```

3.1233.9 Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx = \int x^m\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,{\left(e\,x^2+d\right)}^{5/2}\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

3.1234 $\int x^m (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx$

3.1234.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^m (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \frac{a x^{1+m} (d+ex^2)^{5/2} \operatorname{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)} + b\operatorname{Int}\left(x^m (d+ex^2)^{3/2} \arctan(cx),x\right)$$

output
```
a*x^(1+m)*(e*x^2+d)^(5/2)*hypergeom([1, 3+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(
1+m)+b*Unintegrable(x^m*(e*x^2+d)^(3/2)*arctan(c*x),x)
```

3.1234.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^m (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int x^m (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

3.1234.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^m \left(ex^2+d\right)^{3/2}dx+b\int x^m \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$\frac{ad\sqrt{d+ex^2}\int x^m \left(\frac{ex^2}{d}+1\right)^{3/2}dx}{\sqrt{\frac{ex^2}{d}+1}}+b\int x^m \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^m \left(ex^2+d\right)^{3/2}\arctan(cx)dx+\frac{adx^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^m \left(ex^2+d\right)^{3/2}\arctan(cx)dx+\frac{adx^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

input
```
Int[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1234.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1234.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

3.1234.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\arctan\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)*x^m
, x)
```

3.1234.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**m*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1234.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}}(b\arctan\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

3.1234.8 Giac [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\int(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

3.1234.9 Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

3.1235 $\int x^m \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$

3.1235.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^m \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \frac{ax^{1+m}(d+ex^2)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)} + b\text{Int}\left(x^m\sqrt{d+ex^2}\arctan(cx),x\right)$$

output
```
a*x^(1+m)*(e*x^2+d)^(3/2)*hypergeom([1, 2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(
1+m)+b*Unintegrable(x^m*arctan(c*x)*(e*x^2+d)^(1/2),x)
```

3.1235.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^m \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x^m \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

3.1235.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int x^m\sqrt{ex^2+d}dx+b\int x^m\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{a\sqrt{d+ex^2}\int x^m\sqrt{\frac{ex^2}{d}+1}dx}{\sqrt{\frac{ex^2}{d}+1}}+b\int x^m\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{ 278}$$

$$b\int x^m\sqrt{ex^2+d}\arctan(cx)dx+\frac{ax^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

$$\Big\downarrow \text{ 5560}$$

$$b\int x^m\sqrt{ex^2+d}\arctan(cx)dx+\frac{ax^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

input
```
Int[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1235.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1235.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^m \sqrt{e\,x^2+d}\,(a+b\arctan(cx))\,dx$$

input
```
int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

3.1235.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int \sqrt{ex^2+d}(b\arctan{(cx)}+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

3.1235.6 Sympy [N/A]

Not integrable

Time = 42.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^m(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**m*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

3.1235.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int \sqrt{ex^2+d}(b\arctan{(cx)}+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

3.1235.8 Giac [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int\sqrt{ex^2+d}(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

3.1235.9 Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^m\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

3.1236 $\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$

3.1236.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\frac{ax^{1+m}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)}+b\text{Int}\left(\frac{x^m\arctan(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
a*x^(1+m)*hypergeom([1, 1+1/2*m],[3/2+1/2*m],-e*x^2/d)*(e*x^2+d)^(1/2)/d/(
1+m)+b*Unintegrable(x^m*arctan(c*x)/(e*x^2+d)^(1/2),x)
```

3.1236.2 Mathematica [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

3.1236.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^m}{\sqrt{ex^2+d}}dx+b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{a\sqrt{\frac{ex^2}{d}+1}\int \frac{x^m}{\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}+b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{278}$$

$$b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{d+ex^2}}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.1236.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1236.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{m}(a+b\arctan(cx))}{\sqrt{e\,x^{2}+d}}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

3.1236.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

3.1236.6 Sympy [N/A]

Not integrable

Time = 22.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^m(a+b\,\mathrm{atan}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

3.1236.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

3.1236.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan{(cx)}+a)x^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

3.1236.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

3.1237 $\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$

3.1237.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{ax^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)\sqrt{d+ex^2}} + b\text{Int}\left(\frac{x^m\arctan(cx)}{(d+ex^2)^{3/2}},x\right)$$

output
```
a*x^(1+m)*hypergeom([1, 1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1+m)/(e*x^2+d)^(1/
2)+b*Unintegrable(x^m*arctan(c*x)/(e*x^2+d)^(3/2),x)
```

3.1237.2 Mathematica [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

3.1237.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^m}{(ex^2+d)^{3/2}}dx+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{a\sqrt{\frac{ex^2}{d}+1}\int \frac{x^m}{\left(\frac{ex^2}{d}+1\right)^{3/2}}dx}{d\sqrt{d+ex^2}}+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{278}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)\sqrt{d+ex^2}}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.1237.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1237.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{m}(a+b\arctan\left(cx\right))}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

3.1237.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m/(e^2*x^4 + 2*d*e*x^2 + d^
2), x)
```

3.1237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

3.1237.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(3/2), x)
```

3.1237.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(3/2), x)
```

3.1237.9 Mupad [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

3.1238 $\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$

3.1238.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{ax^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-2+m),\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)\,(d+ex^2)^{3/2}} + b\text{Int}\left(\frac{x^m\arctan(cx)}{(d+ex^2)^{5/2}},x\right)$$

output
```
a*x^(1+m)*hypergeom([1, -1+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1+m)/(e*x^2+d)^
(3/2)+b*Unintegrable(x^m*arctan(c*x)/(e*x^2+d)^(5/2),x)
```

3.1238.2 Mathematica [N/A]

Not integrable

Time = 5.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

3.1238.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^m}{(ex^2+d)^{5/2}}dx+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{a\sqrt{\frac{ex^2}{d}+1}\int \frac{x^m}{\left(\frac{ex^2}{d}+1\right)^{5/2}}dx}{d^2\sqrt{d+ex^2}}+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{278}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)\sqrt{d+ex^2}}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.1238.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1238.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

3.1238.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m/(e^3*x^6 + 3*d*e^2*x^4 +
3*d^2*e*x^2 + d^3), x)
```

3.1238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.1238.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(5/2), x)
```

3.1238.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan{(cx)}+a)x^m}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(5/2), x)
```

3.1238.9 Mupad [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

3.1239 $\int x^m(d+ex^2)^p (a+b\arctan(cx))\,dx$

3.1239.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \frac{ax^{1+m}(d+ex^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3+m+2p),\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)} + b\text{Int}\left(x^m(d+ex^2)^p\arctan(cx),x\right)$$

output
```
a*x^(1+m)*(e*x^2+d)^(p+1)*hypergeom([1, 3/2+1/2*m+p],[3/2+1/2*m],-e*x^2/d)
/d/(1+m)+b*Unintegrable(x^m*(e*x^2+d)^p*arctan(c*x),x)
```

3.1239.2 Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

3.1239.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^m (ex^2+d)^p\,dx+b\int x^m (ex^2+d)^p \arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a(d+ex^2)^p \left(\frac{ex^2}{d}+1\right)^{-p}\int x^m \left(\frac{ex^2}{d}+1\right)^p dx+b\int x^m (ex^2+d)^p \arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^m (ex^2+d)^p \arctan(cx)dx+\frac{ax^{m+1}(d+ex^2)^p \left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{ex^2}{d}\right)}{m+1}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^m (ex^2+d)^p \arctan(cx)dx+\frac{ax^{m+1}(d+ex^2)^p \left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{ex^2}{d}\right)}{m+1}$$

input
```
Int[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1239.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1239.4 Maple [N/A] (verified)

Not integrable

Time = 0.74 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{m} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1239.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m \left(d+ex^2\right)^p (a+b\arctan(cx))\, dx = \int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^m\, dx$$

input
```
integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

3.1239.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right)^p (a+b\arctan(cx))\, dx = \text{Timed out}$$

input
```
integrate(x**m*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1239.7 Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m \left(d+ex^2\right)^p (a+b\arctan(cx))\, dx = \int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^m\, dx$$

input
```
integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

3.1239.8 Giac [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(ex^2+d)^p x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

3.1239.9 Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^p,x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^p, x)
```

3.1240 $\int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$

3.1240.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = -\frac{ax^{-1-2p}(d+ex^2)^{1+p}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1,\frac{1}{2}(1-2p),-\frac{ex^2}{d}\right)}{d(1+2p)} + b\text{Int}\left(x^{-2-2p}(d+ex^2)^p\arctan(cx),x\right)$$

output
```
-a*x^(-1-2*p)*(e*x^2+d)^(p+1)*hypergeom([1/2, 1],[1/2-p],-e*x^2/d)/d/(1+2*
p)+b*Unintegrable(x^(-2-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

3.1240.2 Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = \int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

3.1240.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-2}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^{-2(p+1)}\left(ex^2+d\right)^pdx+b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+1)}\left(\frac{ex^2}{d}+1\right)^pdx+b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-1}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-1),-p,\frac{1}{2}(1-2p),-\frac{ex^2}{d}\right)}{2p+1}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-1}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-1),-p,\frac{1}{2}(1-2p),-\frac{ex^2}{d}\right)}{2p+1}$$

input
```
Int[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1240.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1240.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-2-2p} \left(e\,x^2+d\right)^p \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1240.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

3.1240.6 Sympy [F(-1)]

Timed out.

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-2-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1240.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

3.1240.8 Giac [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(ex^2+d)^p x^{-2p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

3.1240.9 Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-2-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+2}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 2),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 2), x)
```

3.1241 $\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx$

3.1241.1 Optimal result

Integrand size = 25, antiderivative size = 129

$$\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = -\frac{bcx^{-1-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}(-1-2p),1,-1-p,\frac{1}{2}(1-2p),-c^2x^2,-\frac{ex^2}{d}\right)}{2\left(1+3p+2p^2\right)} - \frac{x^{-2(1+p)}(d+ex^2)^{1+p}\,(a+b\arctan(cx))}{2d(1+p)}$$

output
```
-1/2*b*c*x^(-1-2*p)*(e*x^2+d)^p*AppellF1(-1/2-p,1,-1-p,1/2-p,-c^2*x^2,-e*x
^2/d)/(2*p^2+3*p+1)/((1+e*x^2/d)^p)-1/2*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/
d/(p+1)/(x^(2*p+2))
```

3.1241.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.29

$$\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = -\frac{x^{-2(1+p)}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\left(b(c^2d-e)\,x\operatorname{AppellF1}\left(-\frac{1}{2}-p,-p,1,\frac{1}{2}-p,-\frac{ex^2}{d},-c^2x^2\right)+c(1+2p\cdots\right.}{2cd(1+p)(1\cdots}$$

input
```
Integrate[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*((d + e*x^2)^p*(b*(c^2*d - e)*x*AppellF1[-1/2 - p, -p, 1, 1/2 - p, -(
(e*x^2)/d), -(c^2*x^2)] + c*(1 + 2*p)*(d + e*x^2)*(1 + (e*x^2)/d)^p*(a + b
*ArcTan[c*x]) + b*e*x*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((e*x^2)/d
)]))/(c*d*(1 + p)*(1 + 2*p)*x^(2*(1 + p))*(1 + (e*x^2)/d)^p)
```

3.1241.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5511, 27, 393, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-3}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5511$$

$$-bc\int -\frac{x^{-2(p+1)}\left(ex^2+d\right)^{p+1}}{2d(p+1)\left(c^2x^2+1\right)}dx-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\int \frac{x^{-2(p+1)}\left(ex^2+d\right)^{p+1}}{c^2x^2+1}dx}{2d(p+1)}-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow \ 393$$

$$\frac{bcx^{-2p-3}\left(x^2\right)^{p+\frac{3}{2}}\int \frac{\left(x^2\right)^{-p-\frac{3}{2}}\left(ex^2+d\right)^{p+1}}{c^2x^2+1}dx^2}{4d(p+1)}-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow \ 152$$

$$\frac{bcx^{-2p-3}\left(x^2\right)^{p+\frac{3}{2}}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int \frac{\left(x^2\right)^{-p-\frac{3}{2}}\left(\frac{ex^2}{d}+1\right)^{p+1}}{c^2x^2+1}dx^2}{4(p+1)}-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow \ 150$$

$$-\frac{x^{-2(p+1)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d(p+1)}-\frac{bcx^{-2p-1}(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{AppellF1}\left(-p-\frac{1}{2},-p-1,1,\frac{1}{2}-p,-\frac{ex^2}{d},-c^2x^2\right)}{2(p+1)(2p+1)}$$

input
```
Int[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*(b*c*x^(-1 - 2*p)*(d + e*x^2)^p*AppellF1[-1/2 - p, -1 - p, 1, 1/2 - p
, -((e*x^2)/d), -(c^2*x^2)])/((1 + p)*(1 + 2*p)*(1 + (e*x^2)/d)^p) - ((d +
 e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(1 + p)*x^(2*(1 + p)))
```

3.1241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1241.4 Maple [F]

$$\int x^{-3-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1241.5 Fricas [F]

$$\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx=\int \left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^p x^{-2\,p-3}\,dx$$

input
```
integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 3), x)
```

3.1241.6 Sympy [F(-1)]

Timed out.

$$\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**(-3-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1241.7 Maxima [F]

$$\int x^{-3-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2p-3}\,dx$$

input
```
integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^3, x) - 1/2*(e
*x^2 + d)*a*e^(p*log(e*x^2 + d) - 2*p*log(x))/(d*(p + 1)*x^2)
```

3.1241.8 Giac [F]

$$\int x^{-3-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2p-3}\,dx$$

input
```
integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 3), x)
```

3.1241.9 Mupad [F(-1)]

Timed out.

$$\int x^{-3-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(e\,x^2+d\right)^p}{x^{2\,p+3}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 3),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 3), x)
```

3.1242 $\int x^{-4-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx$

3.1242.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-4-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = -\frac{ax^{-3-2p}(d+ex^2)^{1+p}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2}(-1-2p),-\frac{ex^2}{d}\right)}{d(3+2p)} + b\text{Int}\left(x^{-4-2p}(d+ex^2)^p\arctan(cx),x\right)$$

output
```
-a*x^(-3-2*p)*(e*x^2+d)^(p+1)*hypergeom([-1/2, 1],[-1/2-p],-e*x^2/d)/d/(3+
2*p)+b*Unintegrable(x^(-4-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

3.1242.2 Mathematica [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = \int x^{-4-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

3.1242.3 Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-4}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^{-2(p+2)}\left(ex^2+d\right)^p dx+b\int x^{-2(p+2)}\left(ex^2+d\right)^p \arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+2)}\left(\frac{ex^2}{d}+1\right)^p dx+b\int x^{-2(p+2)}\left(ex^2+d\right)^p \arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^{-2(p+2)}\left(ex^2+d\right)^p \arctan(cx)dx-\frac{ax^{-2p-3}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{2p+3}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^{-2(p+2)}\left(ex^2+d\right)^p \arctan(cx)dx-\frac{ax^{-2p-3}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{2p+3}$$

input
```
Int[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1242.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1242.4 Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-4-2p}\left(e\,x^2+d\right)^p\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1242.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int(b\arctan(cx)+a)\left(ex^2+d\right)^px^{-2p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

3.1242.6 Sympy [F(-1)]

Timed out.

$$\int x^{-4-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**(-4-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1242.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int(b\arctan(cx)+a)\left(ex^2+d\right)^px^{-2p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

3.1242.8 Giac [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p}\left(d+ex^2\right)^p (a+b\arctan(cx))\,dx=\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2\,p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

3.1242.9 Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-4-2p}\left(d+ex^2\right)^p (a+b\arctan(cx))\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+4}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 4),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 4), x)
```

3.1243 $\int x^{-5-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx$

3.1243.1 Optimal result

Integrand size = 25, antiderivative size = 285

$$\int x^{-5-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx =$$

$$-\frac{b(e+c^2d(1+p))\,x^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}(-3-2p),1,-1-p,\frac{1}{2}(-1-2p),-c^2x^2,-\right.}{2cd(1+p)(2+p)(3+2p)}$$

$$+\frac{ex^{-2(1+p)}(d+ex^2)^{1+p}\,(a+b\arctan(cx))}{2d^2(1+p)(2+p)}-\frac{x^{-2(2+p)}(d+ex^2)^{1+p}\,(a+b\arctan(cx))}{2d(2+p)}$$

$$+\frac{bex^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}(-3-2p),-1-p,\frac{1}{2}(-1-2p),-\frac{ex^2}{d}\right)}{2cd\,(6+13p+9p^2+2p^3)}$$

output
```
-1/2*b*(e+c^2*d*(p+1))*x^(-3-2*p)*(e*x^2+d)^p*AppellF1(-3/2-p,1,-1-p,-1/2-
p,-c^2*x^2,-e*x^2/d)/c/d/(3+2*p)/(p^2+3*p+2)/((1+e*x^2/d)^p)+1/2*e*(e*x^2+
d)^(p+1)*(a+b*arctan(c*x))/d^2/(p+1)/(2+p)/(x^(2*p+2))-1/2*(e*x^2+d)^(p+1)
*(a+b*arctan(c*x))/d/(2+p)/(x^(4+2*p))+1/2*b*e*x^(-3-2*p)*(e*x^2+d)^p*hype
rgeom([-1-p, -3/2-p],[-1/2-p],-e*x^2/d)/c/d/(2*p^3+9*p^2+13*p+6)/((1+e*x^2
/d)^p)
```

3.1243.2 Mathematica [F]

$$\int x^{-5-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx=\int x^{-5-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

3.1243.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5511, 27, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-5}(d+ex^2)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{x^{-2(p+2)}(d(p+1)-ex^2)(ex^2+d)^{p+1}}{2d^2(p+1)(p+2)(c^2x^2+1)}dx+\frac{ex^{-2(p+1)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d(p+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x^{-2(p+2)}(d(p+1)-ex^2)(ex^2+d)^{p+1}}{c^2x^2+1}dx}{2d^2(p+1)(p+2)}+\frac{ex^{-2(p+1)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d(p+2)}$$

$$\Big\downarrow \text{446}$$

$$\frac{bc\int \left(\frac{(d(p+1)c^2+e)x^{-2(p+2)}(ex^2+d)^{p+1}}{c^2(c^2x^2+1)}-\frac{ex^{-2(p+2)}(ex^2+d)^{p+1}}{c^2}\right)dx}{2d^2(p+1)(p+2)}+\frac{ex^{-2(p+1)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d(p+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ex^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+2)}+\frac{bc\left(\frac{dex^{-2p-3}(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p-1,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{c^2(2p+3)}-\frac{dx^{-2p-3}(c^2d(p+1)+e)(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-1}}{c^2(}\right.}{2d^2(p+1)(p+2)}$$

input
```
Int[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
(e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d^2*(1 + p)*(2 + p)*x^(2*(1
 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(2 + p)*x^(2*(2 +
 p))) + (b*c*(-((d*(e + c^2*d*(1 + p))*x^(-3 - 2*p)*(d + e*x^2)^p*AppellF1
[-3/2 - p, -1 - p, 1, -1/2 - p, -((e*x^2)/d), -(c^2*x^2)])/(c^2*(3 + 2*p)*
(1 + (e*x^2)/d)^p)) + (d*e*x^(-3 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-
3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((e*x^2)/d)])/(c^2*(3 + 2*p)*(1 + (e*x^
2)/d)^p)))/(2*d^2*(1 + p)*(2 + p))
```

3.1243.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

3.1243.4 Maple [F]

$$\int x^{-5-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(c x\right)\right)dx$$

input
```
int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1243.5 Fricas [F]

$$\int x^{-5-2p}\left(d+ex^{2}\right)^{p}\left(a+b\arctan(cx)\right)dx=\int \left(b\arctan\left(cx\right)+a\right)\left(ex^{2}+d\right)^{p}x^{-2\,p-5}\,dx$$

input
```
integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 5), x)
```

3.1243.6 Sympy [F(-1)]

Timed out.

$$\int x^{-5-2p}\left(d+ex^{2}\right)^{p}\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-5-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1243.7 Maxima [F]

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2p-5}\,dx$$

input
```
integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^5, x) + 1/2*(e
^2*x^4 - d*e*p*x^2 - d^2*(p + 1))*a*e^(p*log(e*x^2 + d) - 2*p*log(x))/((p^
2 + 3*p + 2)*d^2*x^4)
```

3.1243.8 Giac [F]

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2p-5}\,dx$$

input
```
integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 5), x)
```

3.1243.9 Mupad [F(-1)]

Timed out.

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(e\,x^2+d\right)^p}{x^{2\,p+5}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 5),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 5), x)
```

3.1244 $\int x^{-6-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$

3.1244.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-6-2p}\left(d+ex^2\right)^p (a+b\arctan(cx))\,dx = -\frac{ax^{-5-2p}(d+ex^2)^{1+p}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,\frac{1}{2}(-3-2p),-\frac{ex^2}{d}\right)}{d(5+2p)} + b\text{Int}\left(x^{-6-2p}\left(d+ex^2\right)^p\arctan(cx),x\right)$$

output
```
-a*x^(-5-2*p)*(e*x^2+d)^(p+1)*hypergeom([-3/2, 1],[-3/2-p],-e*x^2/d)/d/(5+
2*p)+b*Unintegrable(x^(-6-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

3.1244.2 Mathematica [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}\left(d+ex^2\right)^p (a+b\arctan(cx))\,dx = \int x^{-6-2p}\left(d+ex^2\right)^p (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

3.1244.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-6}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5517$$

$$a\int x^{-2(p+3)}\left(ex^2+d\right)^p dx+b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \ 279$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+3)}\left(\frac{ex^2}{d}+1\right)^p dx+b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \ 278$$

$$b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-5}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-5),-p,\frac{1}{2}(-2p-3),-\frac{ex^2}{d}\right)}{2p+5}$$

$$\Big\downarrow \ 5560$$

$$b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-5}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-5),-p,\frac{1}{2}(-2p-3),-\frac{ex^2}{d}\right)}{2p+5}$$

input
```
Int[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1244.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1244.4 Maple [N/A] (verified)

Not integrable

Time = 0.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-6-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1244.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx=\int(b\arctan(cx)+a)(ex^2+d)^px^{-2p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

3.1244.6 Sympy [F(-1)]

Timed out.

$$\int x^{-6-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**(-6-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1244.7 Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx=\int(b\arctan(cx)+a)(ex^2+d)^px^{-2p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

3.1244.8 Giac [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx=\int(b\arctan{(cx)}+a)(ex^2+d)^px^{-2\,p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

3.1244.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-6-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx=\int\frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+6}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 6),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 6), x)
```

3.1245 $\int x^{-7-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx$

3.1245.1 Optimal result

Integrand size = 25, antiderivative size = 466

$$\int x^{-7-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx =$$

$$-\frac{b(2e^2+2c^2de(1+p)+c^4d^2(2+3p+p^2))\,x^{-5-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{AppellF1}\left(\frac{1}{2}(-5-2p),1,-1\right.}{2c^3d^2(1+p)(2+p)(3+p)(5+2p)}$$

$$-\frac{e^2x^{-2(1+p)}(d+ex^2)^{1+p}(a+b\arctan(cx))}{d^3(1+p)(2+p)(3+p)}$$

$$+\frac{ex^{-2(2+p)}(d+ex^2)^{1+p}(a+b\arctan(cx))}{d^2(2+p)(3+p)}-\frac{x^{-2(3+p)}(d+ex^2)^{1+p}(a+b\arctan(cx))}{2d(3+p)}$$

$$+\frac{be(e+c^2d(1+p))\,x^{-5-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-2p),-1-p,\frac{1}{2}(-3-2p\right.}{c^3d^2(1+p)(2+p)(3+p)(5+2p)}$$

$$-\frac{be^2x^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-2p),-1-p,\frac{1}{2}(-1-2p),-\frac{ex^2}{d}\right)}{cd^2(1+p)(2+p)(3+p)(3+2p)}$$

output
```
-1/2*b*(2*e^2+2*c^2*d*e*(p+1)+c^4*d^2*(p^2+3*p+2))*x^(-5-2*p)*(e*x^2+d)^p*
AppellF1(-5/2-p,1,-1-p,-3/2-p,-c^2*x^2,-e*x^2/d)/c^3/d^2/(3+p)/(5+2*p)/(p^
2+3*p+2)/((1+e*x^2/d)^p)-e^2*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/d^3/(2+p)/(
p^2+4*p+3)/(x^(2*p+2))+e*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/d^2/(2+p)/(3+p)
/(x^(4+2*p))-1/2*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/d/(3+p)/(x^(6+2*p))+b*e
*(e+c^2*d*(p+1))*x^(-5-2*p)*(e*x^2+d)^p*hypergeom([-1-p, -5/2-p],[-3/2-p],
-e*x^2/d)/c^3/d^2/(3+p)/(5+2*p)/(p^2+3*p+2)/((1+e*x^2/d)^p)-b*e^2*x^(-3-2*
p)*(e*x^2+d)^p*hypergeom([-1-p, -3/2-p],[-1/2-p],-e*x^2/d)/c/d^2/(p^2+3*p+
2)/(2*p^2+9*p+9)/((1+e*x^2/d)^p)
```

3.1245.2 Mathematica [F]

$$\int x^{-7-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int x^{-7-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

3.1245.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-7}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{x^{-2(p+3)}\left(ex^2+d\right)^{p+1}\left(2e^2x^4-2de(p+1)x^2+d^2(p+1)(p+2)\right)}{2d^3(p+1)(p+2)(p+3)\left(c^2x^2+1\right)}dx-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x^{-2(p+3)}\left(ex^2+d\right)^{p+1}\left(2e^2x^4-2de(p+1)x^2+d^2(p+1)(p+2)\right)}{c^2x^2+1}dx}{2d^3(p+1)(p+2)(p+3)}-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}$$

$$\Big\downarrow \text{7276}$$

$$\frac{bc\int \left(-\frac{2e\left(d(p+1)c^2+e\right)x^{-2(p+3)}\left(ex^2+d\right)^{p+1}}{c^4}+\frac{2e^2x^{2-2(p+3)}\left(ex^2+d\right)^{p+1}}{c^2}+\frac{\left(2d^2c^4+d^2p^2c^4+3d^2pc^4+2dec^2+2depc^2+2e^2\right)x^{-2(p+3)}\left(ex^2+\right.}{c^4\left(c^2x^2+1\right)}}{2d^3(p+1)(p+2)(p+3)}-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}+\frac{bc\left(-\frac{dx^{-2p-5}\left(c^4d^2\left(p^2+3p+2\right)+2c^2de(p+1)+2e^2\right)\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\operatorname{AppellF1}\left(-p-\frac{5}{2},-p-1,1,-p-\frac{3}{2},-\frac{ex^2}{d},-c^2x^2\right)}{c^4(2p+5)}-\frac{2de^2x^{-2p-3}\left(d+\right.}{}\right.}{}$$

input
```
Int[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

3.1245. $\int x^{-7-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$

output
```
-((e^2*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^3*(1 + p)*(2 + p)*(3 +
p)*x^(2*(1 + p)))) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^2*(2 +
 p)*(3 + p)*x^(2*(2 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*
d*(3 + p)*x^(2*(3 + p))) + (b*c*(-((d*(2*e^2 + 2*c^2*d*e*(1 + p) + c^4*d^2
*(2 + 3*p + p^2))*x^(-5 - 2*p)*(d + e*x^2)^p*AppellF1[-5/2 - p, -1 - p, 1,
 -3/2 - p, -((e*x^2)/d), -(c^2*x^2)])/(c^4*(5 + 2*p)*(1 + (e*x^2)/d)^p)) +
 (2*d*e*(e + c^2*d*(1 + p))*x^(-5 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(
-5 - 2*p)/2, -1 - p, (-3 - 2*p)/2, -((e*x^2)/d)])/(c^4*(5 + 2*p)*(1 + (e*x
^2)/d)^p) - (2*d*e^2*x^(-3 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-3 - 2*
p)/2, -1 - p, (-1 - 2*p)/2, -((e*x^2)/d)])/(c^2*(3 + 2*p)*(1 + (e*x^2)/d)^
p)))/(2*d^3*(1 + p)*(2 + p)*(3 + p))
```

3.1245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.1245.4 Maple [F]

$$\int x^{-7-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(c x\right)\right)dx$$

input
```
int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1245.5 Fricas [F]

$$\int x^{-7-2p} \left(d+ex^2\right)^p (a+b\arctan(cx))\,dx = \int \left(b\arctan\left(cx\right)+a\right)\left(ex^{2}+d\right)^{p} x^{-2\,p-7}\,dx$$

input
```
integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 7), x)
```

3.1245.6 Sympy [F(-1)]

Timed out.

$$\int x^{-7-2p} \left(d+ex^2\right)^p (a+b\arctan(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**(-7-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1245.7 Maxima [F]

$$\int x^{-7-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(ex^2+d)^p x^{-2p-7}\,dx$$

input
```
integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^7, x) - 1/2*(2
*e^3*x^6 - 2*d*e^2*p*x^4 + (p^2 + p)*d^2*e*x^2 + (p^2 + 3*p + 2)*d^3)*a*e^
(p*log(e*x^2 + d) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*d^3*x^6)
```

3.1245.8 Giac [F]

$$\int x^{-7-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(ex^2+d)^p x^{-2p-7}\,dx$$

input
```
integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 7), x)
```

3.1245.9 Mupad [F(-1)]

Timed out.

$$\int x^{-7-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^p}{x^{2\,p+7}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 7),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 7), x)
```

3.1246 $\int x^{-8-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$

3.1246.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-8-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = -\frac{ax^{-7-2p}(d+ex^2)^{1+p}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,\frac{1}{2}(-5-2p),-\frac{ex^2}{d}\right)}{d(7+2p)} + b\text{Int}\left(x^{-8-2p}(d+ex^2)^p\arctan(cx),x\right)$$

output
```
-a*x^(-7-2*p)*(e*x^2+d)^(p+1)*hypergeom([-5/2, 1],[-5/2-p],-e*x^2/d)/d/(7+
2*p)+b*Unintegrable(x^(-8-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

3.1246.2 Mathematica [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = \int x^{-8-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

3.1246.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5517, 279, 278, 5560}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-8}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^{-2(p+4)}\left(ex^2+d\right)^p dx+b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+4)}\left(\frac{ex^2}{d}+1\right)^p dx+b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-7}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-7),-p,\frac{1}{2}(-2p-5),-\frac{ex^2}{d}\right)}{2p+7}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-7}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-7),-p,\frac{1}{2}(-2p-5),-\frac{ex^2}{d}\right)}{2p+7}$$

input
```
Int[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.1246.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 5517
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a   Int[(f*x)^m*(d + e*x^2)^q, x], x] + Simp[b
   Int[(f*x)^m*(d + e*x^2)^q*ArcTan[c*x], x], x] /; FreeQ[{a, b, c, d, e, f
, m, q}, x]
```

rule 5560
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcTan[c*x])^p, x] /; FreeQ[{a, b, c, p}, x] && (EqQ[u, 1] || M
atchQ[u, ((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.
)*x)^(m_.)*((d_.) + (e_.)*x)^(q_.) /; FreeQ[{d, e, f, m, q}, x]] || MatchQ[
u, ((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, q}, x]] || MatchQ[u, ((f_.)*x)
^(m_.)*((d_.) + (e_.)*x^2)^(q_.) /; FreeQ[{d, e, f, m, q}, x]])
```

3.1246.4 Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-8-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

3.1246.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int \left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-8}\,dx$$

input
```
integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

3.1246.6 Sympy [F(-1)]

Timed out.

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-8-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

3.1246.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int \left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-8}\,dx$$

input
```
integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

3.1246.8 Giac [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^p x^{-2\,p-8}\,dx$$

input
```
integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

3.1246.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(e\,x^2+d\right)^p}{x^{2\,p+8}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 8),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 8), x)
```

3.1247 $\int x^3(d+ex^2)\left(a+b\arctan(cx)\right)^2 dx$

3.1247.1 Optimal result

Integrand size = 21, antiderivative size = 271

$$\begin{aligned}\int x^3(d+ex^2)\left(a+b\arctan(cx)\right)^2 dx &= \frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx^2}{12c^2}-\frac{4b^2ex^2}{45c^4}+\frac{b^2ex^4}{60c^2}\\ &\quad+\frac{b^2dx\arctan(cx)}{2c^3}-\frac{b^2ex\arctan(cx)}{3c^5}\\ &\quad-\frac{bdx^3(a+b\arctan(cx))}{6c}+\frac{bex^3(a+b\arctan(cx))}{9c^3}\\ &\quad-\frac{bex^5(a+b\arctan(cx))}{15c}-\frac{d(a+b\arctan(cx))^2}{4c^4}\\ &\quad+\frac{e(a+b\arctan(cx))^2}{6c^6}+\frac{1}{4}dx^4(a+b\arctan(cx))^2\\ &\quad+\frac{1}{6}ex^6(a+b\arctan(cx))^2\\ &\quad-\frac{b^2d\log(1+c^2x^2)}{3c^4}+\frac{23b^2e\log(1+c^2x^2)}{90c^6}\end{aligned}$$

output
```
1/2*a*b*d*x/c^3-1/3*a*b*e*x/c^5+1/12*b^2*d*x^2/c^2-4/45*b^2*e*x^2/c^4+1/60
*b^2*e*x^4/c^2+1/2*b^2*d*x*arctan(c*x)/c^3-1/3*b^2*e*x*arctan(c*x)/c^5-1/6
*b*d*x^3*(a+b*arctan(c*x))/c+1/9*b*e*x^3*(a+b*arctan(c*x))/c^3-1/15*b*e*x^
5*(a+b*arctan(c*x))/c-1/4*d*(a+b*arctan(c*x))^2/c^4+1/6*e*(a+b*arctan(c*x)
)^2/c^6+1/4*d*x^4*(a+b*arctan(c*x))^2+1/6*e*x^6*(a+b*arctan(c*x))^2-1/3*b^
2*d*ln(c^2*x^2+1)/c^4+23/90*b^2*e*ln(c^2*x^2+1)/c^6
```

3.1247.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.89

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{cx(15a^2c^5x^3(3d+2ex^2)+b^2cx(-16e+3c^2(5d+ex^2))-2ab(30e-5c^2(9d+2ex^2)+3c^4(5dx^2+2ex^4)))}{}$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(15*a^2*c^5*x^3*(3*d + 2*e*x^2) + b^2*c*x*(-16*e + 3*c^2*(5*d + e*x^2
)) - 2*a*b*(30*e - 5*c^2*(9*d + 2*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4))) + 2
*b*(b*c*x*(-30*e + 5*c^2*(9*d + 2*e*x^2) - 3*c^4*(5*d*x^2 + 2*e*x^4)) + 15
*a*(-3*c^2*d + 2*e + c^6*(3*d*x^4 + 2*e*x^6)))*ArcTan[c*x] + 15*b^2*(-3*c^
2*d + 2*e + c^6*(3*d*x^4 + 2*e*x^6))*ArcTan[c*x]^2 + 2*b^2*(-30*c^2*d + 23
*e)*Log[1 + c^2*x^2])/(180*c^6)
```

3.1247.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(dx^3(a+b\arctan(cx))^2+ex^5(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{e(a+b\arctan(cx))^2}{6c^6}-\frac{d(a+b\arctan(cx))^2}{4c^4}+\frac{bex^3(a+b\arctan(cx))}{9c^3}+\frac{1}{4}dx^4(a+b\arctan(cx))^2-\frac{bdx^3(a+b\arctan(cx))}{6c}+\frac{1}{6}ex^6(a+b\arctan(cx))^2-\frac{bex^5(a+b\arctan(cx))}{15c}-\frac{abex}{3c^5}+\frac{abdx}{2c^3}-\frac{b^2ex\arctan(cx)}{3c^5}+\frac{b^2dx\arctan(cx)}{2c^3}-\frac{4b^2ex^2}{45c^4}+\frac{b^2dx^2}{12c^2}+\frac{b^2ex^4}{60c^2}+\frac{23b^2e\log(c^2x^2+1)}{90c^6}-\frac{b^2d\log(c^2x^2+1)}{3c^4}$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*b*d*x)/(2*c^3) - (a*b*e*x)/(3*c^5) + (b^2*d*x^2)/(12*c^2) - (4*b^2*e*x^
2)/(45*c^4) + (b^2*e*x^4)/(60*c^2) + (b^2*d*x*ArcTan[c*x])/(2*c^3) - (b^2*
e*x*ArcTan[c*x])/(3*c^5) - (b*d*x^3*(a + b*ArcTan[c*x]))/(6*c) + (b*e*x^3*
(a + b*ArcTan[c*x]))/(9*c^3) - (b*e*x^5*(a + b*ArcTan[c*x]))/(15*c) - (d*(
a + b*ArcTan[c*x])^2)/(4*c^4) + (e*(a + b*ArcTan[c*x])^2)/(6*c^6) + (d*x^4
*(a + b*ArcTan[c*x])^2)/4 + (e*x^6*(a + b*ArcTan[c*x])^2)/6 - (b^2*d*Log[1
 + c^2*x^2])/(3*c^4) + (23*b^2*e*Log[1 + c^2*x^2])/(90*c^6)
```

3.1247.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1247.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.11

method	result
parts	$a^2\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b^2\left(\frac{\arctan(cx)^2c^4e\,x^6}{6}+\frac{\arctan(cx)^2c^4x^4d}{4}-\frac{2\arctan(cx)e\,c^5x^5}{5}+\arctan(cx)d\,c^5x^3-\frac{2\arctan(cx)e\,c}{3}\right.}{}$
derivativedivides	$\frac{\frac{a^2\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2d\,c^6x^4}{4}+\frac{\arctan(cx)^2e\,c^6x^6}{6}-\frac{\arctan(cx)d\,c^5x^3}{6}-\frac{\arctan(cx)e\,c^5x^5}{15}+\frac{\arctan(cx)c^3xd}{2}+\mathrm{arct}\right.}{}}{}$
default	$\frac{\frac{a^2\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2d\,c^6x^4}{4}+\frac{\arctan(cx)^2e\,c^6x^6}{6}-\frac{\arctan(cx)d\,c^5x^3}{6}-\frac{\arctan(cx)e\,c^5x^5}{15}+\frac{\arctan(cx)c^3xd}{2}+\mathrm{arct}\right.}{}}{}$
parallelrisch	$-\frac{-60x^6\arctan(cx)ab\,c^6e-90adb\arctan(cx)x^4c^6-30c^6a^2e\,x^6-45c^6a^2d\,x^4-30\arctan(cx)^2b^2e-16b^2e+60b^2c^2d\ln(c^2x}{}$
risch	$-\frac{4b^2e\,x^2}{45c^4}+\frac{b^2e\,x^4}{60c^2}+\frac{23b^2e\ln(c^2x^2+1)}{90c^6}-\frac{abex}{3c^5}+\frac{x^4d\,a^2}{4}+\frac{x^6e\,a^2}{6}+\frac{b^2d\,x^2}{12c^2}-\frac{b^2d\ln(c^2x^2+1)}{3c^4}+\frac{abdx}{2c^3}-$

input
```
int(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/6*e*x^6+1/4*d*x^4)+b^2/c^4*(1/6*arctan(c*x)^2*c^4*e*x^6+1/4*arctan(
c*x)^2*c^4*x^4*d-1/6/c^2*(2/5*arctan(c*x)*e*c^5*x^5+arctan(c*x)*d*c^5*x^3-
2/3*arctan(c*x)*e*c^3*x^3-3*arctan(c*x)*c^3*x*d+2*arctan(c*x)*e*c*x+3*arct
an(c*x)^2*c^2*d-2*arctan(c*x)^2*e-1/2*d*c^4*x^2-1/10*e*c^4*x^4+8/15*e*c^2*
x^2-1/30*(-60*c^2*d+46*e)*ln(c^2*x^2+1)-1/30*(45*c^2*d-30*e)*arctan(c*x)^2
))+2*a*b/c^4*(1/6*c^4*arctan(c*x)*e*x^6+1/4*arctan(c*x)*d*c^4*x^4-1/12/c^2
*(2/5*e*c^5*x^5+d*c^5*x^3-2/3*e*c^3*x^3-3*c^3*x*d+2*e*c*x+(3*c^2*d-2*e)*ar
ctan(c*x)))
```

3.1247.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{30\,a^2c^6ex^6 - 12\,abc^5ex^5 + 3\,(15\,a^2c^6d + b^2c^4e)x^4 - 10\,(3\,abc^5d - 2\,abc^3e)x^3 + (15\,b^2c^4d - 16\,b^2c^2e)x^2 + 1}{}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/180*(30*a^2*c^6*e*x^6 - 12*a*b*c^5*e*x^5 + 3*(15*a^2*c^6*d + b^2*c^4*e)*
x^4 - 10*(3*a*b*c^5*d - 2*a*b*c^3*e)*x^3 + (15*b^2*c^4*d - 16*b^2*c^2*e)*x
^2 + 15*(2*b^2*c^6*e*x^6 + 3*b^2*c^6*d*x^4 - 3*b^2*c^2*d + 2*b^2*e)*arctan
(c*x)^2 + 30*(3*a*b*c^3*d - 2*a*b*c*e)*x + 2*(30*a*b*c^6*e*x^6 + 45*a*b*c^
6*d*x^4 - 6*b^2*c^5*e*x^5 - 45*a*b*c^2*d - 5*(3*b^2*c^5*d - 2*b^2*c^3*e)*x
^3 + 30*a*b*e + 15*(3*b^2*c^3*d - 2*b^2*c*e)*x)*arctan(c*x) - 2*(30*b^2*c^
2*d - 23*b^2*e)*log(c^2*x^2 + 1))/c^6
```

3.1247.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.47

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2dx^4}{4} + \frac{a^2ex^6}{6} + \frac{abdx^4\operatorname{atan}(cx)}{2} + \frac{abex^6\operatorname{atan}(cx)}{3} - \frac{abdx^3}{6c} - \frac{abex^5}{15c} + \frac{abdx}{2c^3} + \frac{abex^3}{9c^3} - \frac{abd\operatorname{atan}(cx)}{2c^4} - \frac{abex}{3c^5} + \frac{abe\operatorname{atan}(cx)}{3c^6} \\ a^2\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x**4/4 + a**2*e*x**6/6 + a*b*d*x**4*atan(c*x)/2 + a*b*e*
x**6*atan(c*x)/3 - a*b*d*x**3/(6*c) - a*b*e*x**5/(15*c) + a*b*d*x/(2*c**3)
 + a*b*e*x**3/(9*c**3) - a*b*d*atan(c*x)/(2*c**4) - a*b*e*x/(3*c**5) + a*b
*e*atan(c*x)/(3*c**6) + b**2*d*x**4*atan(c*x)**2/4 + b**2*e*x**6*atan(c*x)
**2/6 - b**2*d*x**3*atan(c*x)/(6*c) - b**2*e*x**5*atan(c*x)/(15*c) + b**2*
d*x**2/(12*c**2) + b**2*e*x**4/(60*c**2) + b**2*d*x*atan(c*x)/(2*c**3) + b
**2*e*x**3*atan(c*x)/(9*c**3) - b**2*d*log(x**2 + c**(-2))/(3*c**4) - b**2
*d*atan(c*x)**2/(4*c**4) - 4*b**2*e*x**2/(45*c**4) - b**2*e*x*atan(c*x)/(3
*c**5) + 23*b**2*e*log(x**2 + c**(-2))/(90*c**6) + b**2*e*atan(c*x)**2/(6*
c**6), Ne(c, 0)), (a**2*(d*x**4/4 + e*x**6/6), True))
```

3.1247.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.13

$$\begin{aligned}\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx &= \frac{1}{6}b^2ex^6\arctan(cx)^2+\frac{1}{6}a^2ex^6+\frac{1}{4}b^2dx^4\arctan(cx)^2\\&+\frac{1}{4}a^2dx^4+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)abd\\&-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2d\\&+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)abe\\&-\frac{1}{180}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\arctan(cx)-\frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+}{c^6}\right.\end{aligned}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*e*x^6*arctan(c*x)^2 + 1/6*a^2*e*x^6 + 1/4*b^2*d*x^4*arctan(c*x)^2
+ 1/4*a^2*d*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arct
an(c*x)/c^5))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*
arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d
+ 1/45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*ar
ctan(c*x)/c^7))*a*b*e - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 1
5*arctan(c*x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^
2 + 46*log(c^2*x^2 + 1))/c^6)*b^2*e
```

3.1247.8 Giac [F]

$$\int x^3 (d+ex^2) (a+b\arctan(cx))^2 \, dx = \int (ex^2+d)(b\arctan(cx)+a)^2 x^3 \, dx$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1247.9 Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.25

$$\int x^3 (d+ex^2) (a+b\arctan(cx))^2 \, dx$$
$$= \frac{46\,b^2\,e\,\ln\left(c^2\,x^2+1\right)+30\,b^2\,e\,\mathrm{atan}(c\,x)^2-60\,b^2\,c^2\,d\,\ln\left(c^2\,x^2+1\right)+45\,a^2\,c^6\,d\,x^4+15\,b^2\,c^4\,d\,x^2+30\,a^2\,c^6\,e}{}$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
(46*b^2*e*log(c^2*x^2 + 1) + 30*b^2*e*atan(c*x)^2 - 60*b^2*c^2*d*log(c^2*x
^2 + 1) + 45*a^2*c^6*d*x^4 + 15*b^2*c^4*d*x^2 + 30*a^2*c^6*e*x^6 - 16*b^2*
c^2*e*x^2 + 3*b^2*c^4*e*x^4 + 60*a*b*e*atan(c*x) - 45*b^2*c^2*d*atan(c*x)^
2 + 45*b^2*c^6*d*x^4*atan(c*x)^2 + 30*b^2*c^6*e*x^6*atan(c*x)^2 - 30*a*b*c
^5*d*x^3 + 20*a*b*c^3*e*x^3 - 12*a*b*c^5*e*x^5 + 90*b^2*c^3*d*x*atan(c*x)
- 60*a*b*c*e*x - 30*b^2*c^5*d*x^3*atan(c*x) + 20*b^2*c^3*e*x^3*atan(c*x) -
 12*b^2*c^5*e*x^5*atan(c*x) + 90*a*b*c^3*d*x - 90*a*b*c^2*d*atan(c*x) - 60
*b^2*c*e*x*atan(c*x) + 90*a*b*c^6*d*x^4*atan(c*x) + 60*a*b*c^6*e*x^6*atan(
c*x))/(180*c^6)
```

3.1248 $\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx$

3.1248.1 Optimal result

Integrand size = 21, antiderivative size = 323

$$\begin{aligned}\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx = {} & \frac{b^2dx}{3c^2}-\frac{3b^2ex}{10c^4}+\frac{b^2ex^3}{30c^2}-\frac{b^2d\arctan(cx)}{3c^3}\\ & +\frac{3b^2e\arctan(cx)}{10c^5}-\frac{bdx^2(a+b\arctan(cx))}{3c}\\ & +\frac{bex^2(a+b\arctan(cx))}{5c^3}-\frac{bex^4(a+b\arctan(cx))}{10c}\\ & -\frac{id(a+b\arctan(cx))^2}{3c^3}+\frac{ie(a+b\arctan(cx))^2}{5c^5}\\ & +\frac{1}{3}dx^3(a+b\arctan(cx))^2+\frac{1}{5}ex^5(a+b\arctan(cx))^2\\ & -\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\ & +\frac{2be(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5}\\ & -\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\\ & +\frac{ib^2e\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}\end{aligned}$$

output
```
1/3*b^2*d*x/c^2-3/10*b^2*e*x/c^4+1/30*b^2*e*x^3/c^2-1/3*b^2*d*arctan(c*x)/
c^3+3/10*b^2*e*arctan(c*x)/c^5-1/3*b*d*x^2*(a+b*arctan(c*x))/c+1/5*b*e*x^2
*(a+b*arctan(c*x))/c^3-1/10*b*e*x^4*(a+b*arctan(c*x))/c-1/3*I*d*(a+b*arcta
n(c*x))^2/c^3-1/3*I*b^2*d*polylog(2,1-2/(1+I*c*x))/c^3+1/3*d*x^3*(a+b*arct
an(c*x))^2+1/5*e*x^5*(a+b*arctan(c*x))^2-2/3*b*d*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c^3+2/5*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5+1/5*I*b^2*e*pol
ylog(2,1-2/(1+I*c*x))/c^5+1/5*I*e*(a+b*arctan(c*x))^2/c^5
```

3.1248.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\int x^2(d+ex^2)\,(a+b\arctan(cx))^2\,dx$$
$$= \frac{9abe+10b^2c^3dx-9b^2cex-10abc^4dx^2+6abc^2ex^2+10a^2c^5dx^3+b^2c^3ex^3-3abc^4ex^4+6a^2c^5ex^5+2b^2(5}{}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(9*a*b*e + 10*b^2*c^3*d*x - 9*b^2*c*e*x - 10*a*b*c^4*d*x^2 + 6*a*b*c^2*e*x
^2 + 10*a^2*c^5*d*x^3 + b^2*c^3*e*x^3 - 3*a*b*c^4*e*x^4 + 6*a^2*c^5*e*x^5
+ 2*b^2*((5*I)*c^2*d - (3*I)*e + c^5*(5*d*x^3 + 3*e*x^5))*ArcTan[c*x]^2 -
b*ArcTan[c*x]*(-4*a*c^5*x^3*(5*d + 3*e*x^2) + b*(1 + c^2*x^2)*(-9*e + c^2*
(10*d + 3*e*x^2)) + 4*b*(5*c^2*d - 3*e)*Log[1 + E^((2*I)*ArcTan[c*x])]) +
10*a*b*c^2*d*Log[1 + c^2*x^2] - 6*a*b*e*Log[1 + c^2*x^2] + (2*I)*b^2*(5*c^
2*d - 3*e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

3.1248.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex^2)\,(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \; 5515$$

$$\int \left(dx^2(a+b\arctan(cx))^2+ex^4(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie(a+b\arctan(cx))^2}{5c^5}+\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^5}-\frac{id(a+b\arctan(cx))^2}{3c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{bex^2(a+b\arctan(cx))}{5c^3}+\frac{1}{3}dx^3(a+b\arctan(cx))^2-\frac{bdx^2(a+b\arctan(cx))}{3c}+\frac{1}{5}ex^5(a+b\arctan(cx))^2-\frac{bex^4(a+b\arctan(cx))}{10c}+\frac{3b^2e\arctan(cx)}{10c^5}-\frac{b^2d\arctan(cx)}{3c^3}+\frac{ib^2e\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2ex}{10c^4}-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2dx}{3c^2}+\frac{b^2ex^3}{30c^2}$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*d*x)/(3*c^2) - (3*b^2*e*x)/(10*c^4) + (b^2*e*x^3)/(30*c^2) - (b^2*d*A
rcTan[c*x])/(3*c^3) + (3*b^2*e*ArcTan[c*x])/(10*c^5) - (b*d*x^2*(a + b*Arc
Tan[c*x]))/(3*c) + (b*e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e*x^4*(a + b
*ArcTan[c*x]))/(10*c) - ((I/3)*d*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e*(a
+ b*ArcTan[c*x])^2)/c^5 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (e*x^5*(a + b*
ArcTan[c*x])^2)/5 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3)
 + (2*b*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) - ((I/3)*b^2*d*P
olyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + ((I/5)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c
*x)])/c^5
```

3.1248.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1248.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.26

method	result
parts	$a^2\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b^2\left(\frac{\arctan(cx)^2 c^3 e\,x^5}{5}+\frac{\arctan(cx)^2 d\,c^3 x^3}{3}-\frac{2\left(\frac{5\arctan(cx)c^4 d\,x^2}{2}+\frac{3\arctan(cx)c^4 e\,x^4}{4}-\frac{3\arctan(\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2 d\,c^5x^3}{3}+\frac{\arctan(cx)^2 e\,c^5x^5}{5}-\frac{\arctan(cx)c^4 d\,x^2}{3}-\frac{\arctan(cx)c^4 e\,x^4}{10}+\frac{\arctan(cx)e\,c^2x^2}{5}+\right.}{}}{}$
default	$\frac{\frac{a^2\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2 d\,c^5x^3}{3}+\frac{\arctan(cx)^2 e\,c^5x^5}{5}-\frac{\arctan(cx)c^4 d\,x^2}{3}-\frac{\arctan(cx)c^4 e\,x^4}{10}+\frac{\arctan(cx)e\,c^2x^2}{5}+\right.}{}}{}$
risch	$\frac{ib^2e\ln(icx+1)x^4}{20c}-\frac{ie\,b^2\ln(-icx+1)x^4}{20c}+\frac{ie\,b^2\ln(-icx+1)x^2}{10c^3}-\frac{id\,b^2\ln(-icx+1)x^2}{6c}+\frac{ib^2e\ln(icx+1)\ln(-icx+1}{10c^5}$

input
```
int(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/5*e*x^5+1/3*d*x^3)+b^2/c^3*(1/5*arctan(c*x)^2*c^3*e*x^5+1/3*arctan(
c*x)^2*d*c^3*x^3-2/15/c^2*(5/2*arctan(c*x)*c^4*d*x^2+3/4*arctan(c*x)*c^4*e
*x^4-3/2*arctan(c*x)*e*c^2*x^2-5/2*arctan(c*x)*ln(c^2*x^2+1)*c^2*d+3/2*arc
tan(c*x)*ln(c^2*x^2+1)*e-1/4*e*c^3*x^3-5/2*c^3*x*d+9/4*e*c*x-1/4*(-10*c^2*
d+9*e)*arctan(c*x)-1/4*(-10*c^2*d+6*e)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/
2*ln(c*x-I)^2-dilog(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x)))+1/2*I*(l
n(I+c*x)*ln(c^2*x^2+1)-1/2*ln(I+c*x)^2-dilog(1/2*I*(c*x-I))-ln(I+c*x)*ln(1
/2*I*(c*x-I))))))+2*a*b/c^3*(1/5*c^3*arctan(c*x)*e*x^5+1/3*arctan(c*x)*d*c
^3*x^3-1/15/c^2*(5/2*d*c^4*x^2+3/4*e*c^4*x^4-3/2*e*c^2*x^2+1/2*(-5*c^2*d+3
*e)*ln(c^2*x^2+1)))
```

3.1248.5 Fricas [F]

$$\int x^2\left(d+ex^2\right)(a+b\arctan(cx))^2\,dx=\int\left(ex^2+d\right)(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e*x^4 + a^2*d*x^2 + (b^2*e*x^4 + b^2*d*x^2)*arctan(c*x)^2 + 2
*(a*b*e*x^4 + a*b*d*x^2)*arctan(c*x), x)
```

3.1248.6 Sympy [F]

$$\int x^2\left(d+ex^2\right)(a+b\arctan(cx))^2\,dx=\int x^2(a+b\operatorname{atan}{(cx)})^2\left(d+ex^2\right)\,dx$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2), x)
```

3.1248.7 Maxima [F]

$$\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*e*x^5 + 1/3*a^2*d*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(
c^2*x^2 + 1)/c^4))*a*b*d + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/
c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*e + 1/60*(3*b^2*e*x^5 + 5*b^2*d*x^3)*ar
ctan(c*x)^2 - 1/240*(3*b^2*e*x^5 + 5*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + integ
rate(1/240*(180*(b^2*c^2*e*x^6 + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*arct
an(c*x)^2 + 15*(b^2*c^2*e*x^6 + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*log(c
^2*x^2 + 1)^2 - 8*(3*b^2*c*e*x^5 + 5*b^2*c*d*x^3)*arctan(c*x) + 4*(3*b^2*c
^2*e*x^6 + 5*b^2*c^2*d*x^4)*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

3.1248.8 Giac [F]

$$\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1248.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx = \int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2), x)
```

3.1249 $\int x(d+ex^2)(a+b\arctan(cx))^2\,dx$

3.1249.1 Optimal result

Integrand size = 19, antiderivative size = 199

$$\begin{aligned}\int x(d+ex^2)(a+b\arctan(cx))^2\,dx = &-\frac{abdx}{c}+\frac{abex}{2c^3}+\frac{b^2ex^2}{12c^2}-\frac{b^2dx\arctan(cx)}{c}\\&+\frac{b^2ex\arctan(cx)}{2c^3}-\frac{bex^3(a+b\arctan(cx))}{6c}\\&+\frac{d(a+b\arctan(cx))^2}{2c^2}-\frac{e(a+b\arctan(cx))^2}{4c^4}\\&+\frac{1}{2}dx^2(a+b\arctan(cx))^2+\frac{1}{4}ex^4(a+b\arctan(cx))^2\\&+\frac{b^2d\log(1+c^2x^2)}{2c^2}-\frac{b^2e\log(1+c^2x^2)}{3c^4}\end{aligned}$$

output
```
-a*b*d*x/c+1/2*a*b*e*x/c^3+1/12*b^2*e*x^2/c^2-b^2*d*x*arctan(c*x)/c+1/2*b^
2*e*x*arctan(c*x)/c^3-1/6*b*e*x^3*(a+b*arctan(c*x))/c+1/2*d*(a+b*arctan(c*
x))^2/c^2-1/4*e*(a+b*arctan(c*x))^2/c^4+1/2*d*x^2*(a+b*arctan(c*x))^2+1/4*
e*x^4*(a+b*arctan(c*x))^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2-1/3*b^2*e*ln(c^2*x^2
+1)/c^4
```

3.1249.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{cx(6abe+b^2cex+3a^2c^3x(2d+ex^2)-2abc^2(6d+ex^2))+2b(6ac^2d-3ae+3bcex-bc^3x(6d+ex^2)+3a}{12}$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(6*a*b*e + b^2*c*e*x + 3*a^2*c^3*x*(2*d + e*x^2) - 2*a*b*c^2*(6*d + e
*x^2)) + 2*b*(6*a*c^2*d - 3*a*e + 3*b*c*e*x - b*c^3*x*(6*d + e*x^2) + 3*a*
c^4*(2*d*x^2 + e*x^4))*ArcTan[c*x] + 3*b^2*(2*c^2*d - e + c^4*(2*d*x^2 + e
*x^4))*ArcTan[c*x]^2 + 2*b^2*(3*c^2*d - 2*e)*Log[1 + c^2*x^2])/(12*c^4)
```

3.1249.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(dx(a+b\arctan(cx))^2+ex^3(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{e(a+b\arctan(cx))^2}{4c^4}+\frac{d(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))^2+\frac{1}{4}ex^4(a+b\arctan(cx))^2-\frac{bex^3(a+b\arctan(cx))}{6c}+\frac{abex}{2c^3}-\frac{abdx}{c}+\frac{b^2ex\arctan(cx)}{2c^3}-\frac{b^2dx\arctan(cx)}{c}+\frac{b^2d\log(c^2x^2+1)}{2c^2}+\frac{b^2ex^2}{12c^2}-\frac{b^2e\log(c^2x^2+1)}{3c^4}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
-((a*b*d*x)/c) + (a*b*e*x)/(2*c^3) + (b^2*e*x^2)/(12*c^2) - (b^2*d*x*ArcTa
n[c*x])/c + (b^2*e*x*ArcTan[c*x])/(2*c^3) - (b*e*x^3*(a + b*ArcTan[c*x]))/
(6*c) + (d*(a + b*ArcTan[c*x])^2)/(2*c^2) - (e*(a + b*ArcTan[c*x])^2)/(4*c
^4) + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (e*x^4*(a + b*ArcTan[c*x])^2)/4 +
(b^2*d*Log[1 + c^2*x^2])/(2*c^2) - (b^2*e*Log[1 + c^2*x^2])/(3*c^4)
```

3.1249.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1249.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.25

method	result
parts	$\frac{a^2(e\,x^2+d)^2}{4e}+\frac{b^2\arctan(cx)^2 e\,x^4}{4}+\frac{b^2\arctan(cx)^2 x^2 d}{2}-\frac{b^2 e\arctan(cx)x^3}{6c}-\frac{b^2 dx\arctan(cx)}{c}+\frac{b^2 ex\arctan(c$
derivativedivides	$\frac{\frac{a^2(e\,c^2x^2+c^2d)^2}{4c^2e}+\frac{b^2\arctan(cx)^2 d\,c^2x^2}{2}+\frac{b^2c^2\arctan(cx)^2 e\,x^4}{4}-b^2\arctan(cx)dcx-\frac{b^2ce\arctan(cx)x^3}{6}+\frac{b^2ex\arctan(cx)}{2c}+\frac{b^2\text{ar}}{}}{}$
default	$\frac{\frac{a^2(e\,c^2x^2+c^2d)^2}{4c^2e}+\frac{b^2\arctan(cx)^2 d\,c^2x^2}{2}+\frac{b^2c^2\arctan(cx)^2 e\,x^4}{4}-b^2\arctan(cx)dcx-\frac{b^2ce\arctan(cx)x^3}{6}+\frac{b^2ex\arctan(cx)}{2c}+\frac{b^2\text{ar}}{}}{}$
parallelrisch	$\frac{3x^4\arctan(cx)^2b^2c^4e+6x^4\arctan(cx)ab\,c^4e+3c^4a^2e\,x^4+6x^2\arctan(cx)^2b^2c^4d-2x^3\arctan(cx)b^2c^3e+12x^2\arctan(cx)c}{}$
risch	$-\frac{b^2(e\,c^4x^4+2d\,c^4x^2+2c^2d-e)\ln(icx+1)^2}{16c^4}-\frac{ib^2dx\ln(-icx+1)}{2c}-\frac{b^2e\,x^4\ln(-icx+1)^2}{16}-\frac{ib(6x^4a\,c^4e+3ib\,c^4e\,x^4\ln}{}$

input
```
int(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*(e*x^2+d)^2/e+1/4*b^2*arctan(c*x)^2*e*x^4+1/2*b^2*arctan(c*x)^2*x^
2*d-1/6*b^2/c*e*arctan(c*x)*x^3-b^2*d*x*arctan(c*x)/c+1/2*b^2*e*x*arctan(c
*x)/c^3+1/2*b^2/c^2*arctan(c*x)^2*d-1/4*b^2/c^4*e*arctan(c*x)^2+1/12*b^2*e
*x^2/c^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2-1/3*b^2*e*ln(c^2*x^2+1)/c^4+1/2*a*b*a
rctan(c*x)*e*x^4+a*b*arctan(c*x)*x^2*d-1/6/c*a*b*e*x^3-a*b*d*x/c+1/2*a*b*e
*x/c^3+1/c^2*a*b*d*arctan(c*x)-1/2/c^4*a*b*e*arctan(c*x)
```

3.1249.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.10

$$\int x\left(d+ex^2\right)(a+b\arctan(cx))^2\,dx$$
$$= \frac{3\,a^2c^4ex^4-2\,abc^3ex^3+(6\,a^2c^4d+b^2c^2e)x^2+3\,(b^2c^4ex^4+2\,b^2c^4dx^2+2\,b^2c^2d-b^2e)\arctan{(cx)}^2-6\,(2\,a}{}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*a^2*c^4*e*x^4 - 2*a*b*c^3*e*x^3 + (6*a^2*c^4*d + b^2*c^2*e)*x^2 +
3*(b^2*c^4*e*x^4 + 2*b^2*c^4*d*x^2 + 2*b^2*c^2*d - b^2*e)*arctan(c*x)^2 -
6*(2*a*b*c^3*d - a*b*c*e)*x + 2*(3*a*b*c^4*e*x^4 + 6*a*b*c^4*d*x^2 - b^2*c
^3*e*x^3 + 6*a*b*c^2*d - 3*a*b*e - 3*(2*b^2*c^3*d - b^2*c*e)*x)*arctan(c*x
) + 2*(3*b^2*c^2*d - 2*b^2*e)*log(c^2*x^2 + 1))/c^4
```

3.1249.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.49

$$\int x\left(d+ex^2\right)(a+b\arctan(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2dx^2}{2}+\frac{a^2ex^4}{4}+abdx^2\operatorname{atan}{(cx)}+\frac{abex^4\operatorname{atan}{(cx)}}{2}-\frac{abdx}{c}-\frac{abex^3}{6c}+\frac{abd\operatorname{atan}{(cx)}}{c^2}+\frac{abex}{2c^3}-\frac{abe\operatorname{atan}{(cx)}}{2c^4}+\frac{b^2dx^2\operatorname{atan}}{2} \\ a^2\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x**2/2 + a**2*e*x**4/4 + a*b*d*x**2*atan(c*x) + a*b*e*x*
*4*atan(c*x)/2 - a*b*d*x/c - a*b*e*x**3/(6*c) + a*b*d*atan(c*x)/c**2 + a*b
*e*x/(2*c**3) - a*b*e*atan(c*x)/(2*c**4) + b**2*d*x**2*atan(c*x)**2/2 + b*
*2*e*x**4*atan(c*x)**2/4 - b**2*d*x*atan(c*x)/c - b**2*e*x**3*atan(c*x)/(6
*c) + b**2*d*log(x**2 + c**(-2))/(2*c**2) + b**2*d*atan(c*x)**2/(2*c**2) +
 b**2*e*x**2/(12*c**2) + b**2*e*x*atan(c*x)/(2*c**3) - b**2*e*log(x**2 + c
**(-2))/(3*c**4) - b**2*e*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*(d*x**2/
2 + e*x**4/4), True))
```

3.1249.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.24

$$
\begin{aligned}
\int x(d+ex^2)\,(a+b\arctan(cx))^2\,dx &= \frac{1}{4}\,b^2ex^4\arctan\left(cx\right)^2+\frac{1}{4}\,a^2ex^4 \\
&\quad+\frac{1}{2}\,b^2dx^2\arctan\left(cx\right)^2+\frac{1}{2}\,a^2dx^2+\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)abd \\
&\quad-\frac{1}{2}\left(2\,c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\arctan\left(cx\right)+\frac{\arctan\left(cx\right)^2-\log\left(c^2x^2+1\right)}{c^2}\right)b^2d \\
&\quad+\frac{1}{6}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)abe \\
&\quad-\frac{1}{12}\left(2\,c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\arctan\left(cx\right)-\frac{c^2x^2+3\,\arctan\left(cx\right)^2-4\,\log\left(c^2x^2+1\right)}{c^4}\right)b^2e
\end{aligned}
$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*e*x^4*arctan(c*x)^2 + 1/4*a^2*e*x^4 + 1/2*b^2*d*x^2*arctan(c*x)^2
+ 1/2*a^2*d*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d -
1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*
x^2 + 1))/c^2)*b^2*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3
*arctan(c*x)/c^5))*a*b*e - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/
c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b
^2*e
```

3.1249.8 Giac [F]

$$\int x(d+ex^2)\,(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1249.9 Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.25

$$\begin{aligned}\int x(d+ex^2)\,(a+b\arctan(cx))^2\,dx &= \frac{a^2\,d\,x^2}{2}+\frac{a^2\,e\,x^4}{4}+\frac{b^2\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c^2}\\&\quad-\frac{b^2\,e\,\ln\left(c^2\,x^2+1\right)}{3\,c^4}+\frac{b^2\,e\,x^2}{12\,c^2}+\frac{b^2\,d\,\mathrm{atan}(c\,x)^2}{2\,c^2}\\&\quad-\frac{b^2\,e\,\mathrm{atan}(c\,x)^2}{4\,c^4}+\frac{b^2\,d\,x^2\,\mathrm{atan}(c\,x)^2}{2}\\&\quad+\frac{b^2\,e\,x^4\,\mathrm{atan}(c\,x)^2}{4}-\frac{a\,b\,e\,x^3}{6\,c}-\frac{b^2\,d\,x\,\mathrm{atan}(c\,x)}{c}\\&\quad+\frac{b^2\,e\,x\,\mathrm{atan}(c\,x)}{2\,c^3}-\frac{b^2\,e\,x^3\,\mathrm{atan}(c\,x)}{6\,c}-\frac{a\,b\,d\,x}{c}\\&\quad+\frac{a\,b\,e\,x}{2\,c^3}+\frac{a\,b\,d\,\mathrm{atan}(c\,x)}{c^2}-\frac{a\,b\,e\,\mathrm{atan}(c\,x)}{2\,c^4}\\&\quad+a\,b\,d\,x^2\,\mathrm{atan}(c\,x)+\frac{a\,b\,e\,x^4\,\mathrm{atan}(c\,x)}{2}\end{aligned}$$

input
```
int(x*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
(a^2*d*x^2)/2 + (a^2*e*x^4)/4 + (b^2*d*log(c^2*x^2 + 1))/(2*c^2) - (b^2*e*
log(c^2*x^2 + 1))/(3*c^4) + (b^2*e*x^2)/(12*c^2) + (b^2*d*atan(c*x)^2)/(2*
c^2) - (b^2*e*atan(c*x)^2)/(4*c^4) + (b^2*d*x^2*atan(c*x)^2)/2 + (b^2*e*x^
4*atan(c*x)^2)/4 - (a*b*e*x^3)/(6*c) - (b^2*d*x*atan(c*x))/c + (b^2*e*x*at
an(c*x))/(2*c^3) - (b^2*e*x^3*atan(c*x))/(6*c) - (a*b*d*x)/c + (a*b*e*x)/(
2*c^3) + (a*b*d*atan(c*x))/c^2 - (a*b*e*atan(c*x))/(2*c^4) + a*b*d*x^2*ata
n(c*x) + (a*b*e*x^4*atan(c*x))/2
```

3.1250 $\int (d+ex^2)(a+b\arctan(cx))^2\,dx$

3.1250.1 Optimal result

Integrand size = 18, antiderivative size = 231

$$\begin{aligned}\int (d+ex^2)(a+b\arctan(cx))^2\,dx &= \frac{b^2ex}{3c^2}-\frac{b^2e\arctan(cx)}{3c^3}-\frac{bex^2(a+b\arctan(cx))}{3c}\\&\quad+\frac{id(a+b\arctan(cx))^2}{c}-\frac{ie(a+b\arctan(cx))^2}{3c^3}\\&\quad+dx(a+b\arctan(cx))^2+\frac{1}{3}ex^3(a+b\arctan(cx))^2\\&\quad+\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}\\&\quad-\frac{2be(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\&\quad+\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}\\&\quad-\frac{ib^2e\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\end{aligned}$$

output
```
1/3*b^2*e*x/c^2-1/3*b^2*e*arctan(c*x)/c^3-1/3*b*e*x^2*(a+b*arctan(c*x))/c+
I*d*(a+b*arctan(c*x))^2/c-1/3*I*e*(a+b*arctan(c*x))^2/c^3+d*x*(a+b*arctan(
c*x))^2+1/3*e*x^3*(a+b*arctan(c*x))^2+2*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))/c-2/3*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+I*b^2*d*polylog(2,1-2/
(1+I*c*x))/c-1/3*I*b^2*e*polylog(2,1-2/(1+I*c*x))/c^3
```

3.1250.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.90

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$=\frac{3a^2c^3dx+b^2cex-abc^2ex^2+a^2c^3ex^3+b^2(-3ic^2d+ie+c^3(3dx+ex^3))\arctan(cx)^2-b\arctan(cx)(-2a}{}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(3*a^2*c^3*d*x + b^2*c*e*x - a*b*c^2*e*x^2 + a^2*c^3*e*x^3 + b^2*((-3*I)*c
^2*d + I*e + c^3*(3*d*x + e*x^3))*ArcTan[c*x]^2 - b*ArcTan[c*x]*(-2*a*c^3*
x*(3*d + e*x^2) + b*(e + c^2*e*x^2) + 2*b*(-3*c^2*d + e)*Log[1 + E^((2*I)*
ArcTan[c*x])]) - 3*a*b*c^2*d*Log[1 + c^2*x^2] + a*b*e*Log[1 + c^2*x^2] - I
*b^2*(3*c^2*d - e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c^3)
```

3.1250.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5449}$$
$$\int \left(d(a+b\arctan(cx))^2+ex^2(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{ie(a+b\arctan(cx))^2}{3c^3}-\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+dx(a+b\arctan(cx))^2+\frac{id(a+b\arctan(cx))^2}{c}+\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{1}{3}ex^3(a+b\arctan(cx))^2-\frac{bex^2(a+b\arctan(cx))}{3c}-\frac{b^2e\arctan(cx)}{3c^3}-\frac{ib^2e\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2ex}{3c^2}+\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*e*x)/(3*c^2) - (b^2*e*ArcTan[c*x])/(3*c^3) - (b*e*x^2*(a + b*ArcTan[c
*x]))/(3*c) + (I*d*(a + b*ArcTan[c*x])^2)/c - ((I/3)*e*(a + b*ArcTan[c*x])
^2)/c^3 + d*x*(a + b*ArcTan[c*x])^2 + (e*x^3*(a + b*ArcTan[c*x])^2)/3 + (2
*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e*(a + b*ArcTan[c*x]
)*Log[2/(1 + I*c*x)])/(3*c^3) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
- ((I/3)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

3.1250.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

3.1250.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.39

method	result
parts	$a^2\left(\frac{1}{3}e\,x^3+xd\right)+\frac{b^2\left(\frac{\arctan(cx)^2 c\,x^3 e}{3}+\arctan(cx)^2 cxd-\frac{2\left(\frac{\arctan(cx)e\,c^2x^2}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)c^2d}{2}-\arctan(c\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2\left(c^3xd+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2c^3xd+\frac{\arctan(cx)^2e\,c^3x^3}{3}-\frac{\arctan(cx)e\,c^2x^2}{3}-\arctan(cx)\ln\left(c^2x^2+1\right)c^2d+\frac{\arctan(cx)\ln\left(c^2x^2\right.}{3}\right.}{}}{}$
default	$\frac{\frac{a^2\left(c^3xd+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2c^3xd+\frac{\arctan(cx)^2e\,c^3x^3}{3}-\frac{\arctan(cx)e\,c^2x^2}{3}-\arctan(cx)\ln\left(c^2x^2+1\right)c^2d+\frac{\arctan(cx)\ln\left(c^2x^2\right.}{3}\right.}{}}{}$
risch	$-\frac{abe\,x^2}{3c}+\frac{2abd}{c}-\frac{11abe}{9c^3}+\frac{ib^2d\ln(c^2x^2+1)}{2c}+\frac{ib^2d\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{c}+\frac{b^2d\ln(icx+1)\ln(-icx+1)x}{2}+\frac{b^2e\ln(icx}{}$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*e*x^3+x*d)+b^2/c*(1/3*arctan(c*x)^2*c*x^3*e+arctan(c*x)^2*c*x*d-2
/3/c^2*(1/2*arctan(c*x)*e*c^2*x^2+3/2*arctan(c*x)*ln(c^2*x^2+1)*c^2*d-1/2*
arctan(c*x)*ln(c^2*x^2+1)*e-1/2*e*(c*x-arctan(c*x))-1/2*(3*c^2*d-e)*(-1/2*
I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(I+c*x))-ln(c*x-I)
*ln(-1/2*I*(I+c*x)))+1/2*I*(ln(I+c*x)*ln(c^2*x^2+1)-1/2*ln(I+c*x)^2-dilog(
1/2*I*(c*x-I))-ln(I+c*x)*ln(1/2*I*(c*x-I))))))+2*a*b/c*(1/3*c*arctan(c*x)*
x^3*e+arctan(c*x)*c*x*d-1/3/c^2*(1/2*e*c^2*x^2+1/2*(3*c^2*d-e)*ln(c^2*x^2+
1)))
```

3.1250.5 Fricas [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*
x^2 + a*b*d)*arctan(c*x), x)
```

3.1250.6 Sympy [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(cx))^2\,(d+ex^2)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2), x)
```

3.1250.7 Maxima [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*e*x^3 + 36*b^2*c^2*e*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^2 + 1
), x) + 3*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x
) + 4*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 36
*b^2*c^2*d*integrate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*c^2*
d*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*i
ntegrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 1/4*b^2*d*arctan(c*
x)^3/c - 8*b^2*c*e*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1), x) - 24*b
^2*c*d*integrate(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + 1/3*(2*x^3*arctan(
c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e + a^2*d*x + 36*b^2*e*inte
grate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*e*integrate(1/48*x^
2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3*b^2*d*integrate(1/48*log(c^2*x^
2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d/
c + 1/12*(b^2*e*x^3 + 3*b^2*d*x)*arctan(c*x)^2 - 1/48*(b^2*e*x^3 + 3*b^2*d
*x)*log(c^2*x^2 + 1)^2
```

3.1250.8 Giac [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1250.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
int((a + b*atan(c*x))^2*(d + e*x^2), x)
```

3.1251 $\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx$

3.1251.1 Optimal result

Integrand size = 21, antiderivative size = 217

$$\begin{aligned}\int \frac{(d+ex^2)\,(a+b\arctan(cx))^2}{x}\,dx = &-\frac{abex}{c}-\frac{b^2ex\arctan(cx)}{c}+\frac{e(a+b\arctan(cx))^2}{2c^2}\\&+\frac{1}{2}ex^2(a+b\arctan(cx))^2\\&+2d(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\&+\frac{b^2e\log(1+c^2x^2)}{2c^2}\\&-ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\&+ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\&-\frac{1}{2}b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\&+\frac{1}{2}b^2d\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-a*b*e*x/c-b^2*e*x*arctan(c*x)/c+1/2*e*(a+b*arctan(c*x))^2/c^2+1/2*e*x^2*(
a+b*arctan(c*x))^2-2*d*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+1/2*b^2
*e*ln(c^2*x^2+1)/c^2-I*b*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*
d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*d*polylog(3,1-2/(1+I
*c*x))+1/2*b^2*d*polylog(3,-1+2/(1+I*c*x))
```

3.1251.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.21

$$
\begin{aligned}
&\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx \\
&= \frac{1}{2}a^2ex^2 + \frac{abe(-cx+(1+c^2x^2)\arctan(cx))}{c^2} + a^2d\log(x) \\
&\quad + \frac{b^2e(-2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2+\log(1+c^2x^2))}{2c^2} \\
&\quad + iabd(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + b^2d\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3\right. \\
&\quad + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) \\
&\quad + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) \\
&\quad \left. + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)
\end{aligned}
$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
(a^2*e*x^2)/2 + (a*b*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]))/c^2 + a^2*d*L
og[x] + (b^2*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 +
 c^2*x^2]))/(2*c^2) + I*a*b*d*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) +
 b^2*d*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E
^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*
ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2,
-E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3
, -E^((2*I)*ArcTan[c*x])]/2)
```

3.1251.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx$$

$$\Big\downarrow \; 5515$$

$$\int \left(\frac{d(a+b\arctan(cx))^2}{x}+ex(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2d\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2+\frac{e(a+b\arctan(cx))^2}{2c^2}-ibd\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{1}{2}ex^2(a+b\arctan(cx))^2-\frac{abex}{c}-\frac{b^2ex\arctan(cx)}{c}+\frac{b^2e\log(c^2x^2+1)}{2c^2}-\frac{1}{2}b^2d\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
-((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (e*(a + b*ArcTan[c*x])^2)/(2*c^
2) + (e*x^2*(a + b*ArcTan[c*x])^2)/2 + 2*d*(a + b*ArcTan[c*x])^2*ArcTanh[1
 - 2/(1 + I*c*x)] + (b^2*e*Log[1 + c^2*x^2])/(2*c^2) - I*b*d*(a + b*ArcTan
[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2
, -1 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*P
olyLog[3, -1 + 2/(1 + I*c*x)])/2
```

3.1251.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1251.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.62 (sec) , antiderivative size = 1262, normalized size of antiderivative = 5.82

method	result	size
parts	Expression too large to display	1262
derivativedivides	Expression too large to display	1263
default	Expression too large to display	1263

input
```
int((e*x^2+d)*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*e*x^2+a^2*d*ln(x)+b^2*(1/2*arctan(c*x)^2*x^2*e+arctan(c*x)^2*d*ln(
c*x)-1/c^2*(-1/2*I*c^2*d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/
(c^2*x^2+1)+1))^3*arctan(c*x)^2+2*I*c^2*d*arctan(c*x)*polylog(2,(1+I*c*x)/
(c^2*x^2+1)^(1/2))+1/2*I*c^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^
2-I*c^2*d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*c^2*d*Pi*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2
-1/2*arctan(c*x)^2*e-1/2*I*c^2*d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))*arctan(c*x)^2+e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/2*I*c^2*d*Pi
*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*c^2*d*Pi*csgn(((1+I*c*x)^
2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+e*arctan(c*x
)*(c*x-I)-1/2*I*c^2*d*Pi*arctan(c*x)^2+ln((1+I*c*x)^2/(c^2*x^2+1)-1)*c^2*d
*arctan(c*x)^2-ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))*c^2*d*arctan(c*x)^2-1/2*I
*c^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^
3*arctan(c*x)^2-2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*c^2*d-ln(1-(1+I*
c*x)/(c^2*x^2+1)^(1/2))*c^2*d*arctan(c*x)^2+2*I*c^2*d*arctan(c*x)*polylog(
2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2)...
```

3.1251.5 Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e
*x^2 + a*b*d)*arctan(c*x))/x, x)
```

3.1251.6 Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2/x,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)/x, x)
```

3.1251.7 Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/8*b^2*e*x^2*arctan(c*x)^2 - 1/32*b^2*e*x^2*log(c^2*x^2 + 1)^2 + 12*b^2*c
^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*c^2*e*integr
ate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*e*integrate
(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 2*b^2*c^2*e*integrate(1/16*x^4*l
og(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d*integrate(1/16*x^2*arctan
(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1/16*x^2*arctan(c*x)/(c
^2*x^3 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + 1/2*a^2*e*x^2 - 4*b^2*c*
e*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*e*integrate(1/
16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*e*integrate(1/16*x^2*arcta
n(c*x)/(c^2*x^3 + x), x) + 12*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^3
+ x), x) + b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*
a*b*d*integrate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*e*log(c^2*x^
2 + 1)^3/c^2 + a^2*d*log(x)
```

3.1251.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
Timed out
```

3.1251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x, x)
```

3.1252 $\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx$

3.1252.1 Optimal result

Integrand size = 21, antiderivative size = 172

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx &= -icd(a+b\arctan(cx))^2+\frac{ie(a+b\arctan(cx))^2}{c}\\&\quad-\frac{d(a+b\arctan(cx))^2}{x}+ex(a+b\arctan(cx))^2\\&\quad+\frac{2be(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}\\&\quad+2bcd(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\&\quad-ib^2cd\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\&\quad+\frac{ib^2e\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}\end{aligned}$$

output
```
-I*c*d*(a+b*arctan(c*x))^2+I*e*(a+b*arctan(c*x))^2/c-d*(a+b*arctan(c*x))^2
/x+e*x*(a+b*arctan(c*x))^2+2*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+2*b*c
*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-I*b^2*c*d*polylog(2,-1+2/(1-I*c*x))
+I*b^2*e*polylog(2,1-2/(1+I*c*x))/c
```

3.1252.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{-a^2cd+a^2cex^2+abcd(-2\arctan(cx)+cx(2\log(cx)-\log(1+c^2x^2)))+abex(2cx\arctan(cx)-\log(1+}{}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-(a^2*c*d) + a^2*c*e*x^2 + a*b*c*d*(-2*ArcTan[c*x] + c*x*(2*Log[c*x] - Lo
g[1 + c^2*x^2])) + a*b*e*x*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) + b^2*e*
x*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])])
 - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) - b^2*c*d*(ArcTan[c*x]^2 - 2*c*x*
ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLo
g[2, E^((2*I)*ArcTan[c*x])])))/(c*x)
```

3.1252.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \ 5515$$
$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^2}+e(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \ 2009$$
$$-icd(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{x}+2bcd\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+\frac{ie(a+b\arctan(cx))^2}{c}+ex(a+b\arctan(cx))^2+\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib^2cd\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{ib^2e\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-I)*c*d*(a + b*ArcTan[c*x])^2 + (I*e*(a + b*ArcTan[c*x])^2)/c - (d*(a + b
*ArcTan[c*x])^2)/x + e*x*(a + b*ArcTan[c*x])^2 + (2*b*e*(a + b*ArcTan[c*x]
)*Log[2/(1 + I*c*x)])/c + 2*b*c*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x
)] - I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + (I*b^2*e*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c
```

3.1252.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1252.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(164) = 328$.

Time = 0.68 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.07

method	result
derivativedivides	$c\left(\frac{a^2\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2cxe-\frac{\arctan(cx)^2dc}{x}-\arctan(cx)\ln(c^2x^2+1)c^2d-\arctan(cx)\ln(c^2x^2+1)e+2\,\mathrm{arcta}\right.}{}\right.$
default	$c\left(\frac{a^2\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2cxe-\frac{\arctan(cx)^2dc}{x}-\arctan(cx)\ln(c^2x^2+1)c^2d-\arctan(cx)\ln(c^2x^2+1)e+2\,\mathrm{arcta}\right.}{}\right.$
parts	$a^2\left(ex-\frac{d}{x}\right)+b^2c\left(\frac{\arctan(cx)^2xe}{c}-\frac{\arctan(cx)^2d}{cx}-\frac{2\left(-\arctan(cx)d\,c^2\ln(cx)+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)c^2d}{2}+\right.}{}\right.$

input
```
int((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(a^2/c^2*(e*c*x-d*c/x)+b^2/c^2*(arctan(c*x)^2*c*x*e-arctan(c*x)^2*d*c/x-
arctan(c*x)*ln(c^2*x^2+1)*c^2*d-arctan(c*x)*ln(c^2*x^2+1)*e+2*arctan(c*x)*
d*c^2*ln(c*x)+(c^2*d+e)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-d
ilog(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x)))+1/2*I*(ln(I+c*x)*ln(c^2
*x^2+1)-1/2*ln(I+c*x)^2-dilog(1/2*I*(c*x-I))-ln(I+c*x)*ln(1/2*I*(c*x-I))))
-2*d*c^2*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog
(1+I*c*x)+1/2*I*dilog(1-I*c*x)))+2*a*b/c^2*(arctan(c*x)*e*c*x-arctan(c*x)*
d*c/x-1/2*(c^2*d+e)*ln(c^2*x^2+1)+d*c^2*ln(c*x)))
```

3.1252. $\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx$

3.1252.5 Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e
*x^2 + a*b*d)*arctan(c*x))/x^2, x)
```

3.1252.6 Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2(d+ex^2)}{x^2}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**2, x)
```

3.1252.7 Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*d + a^2*e*x + (2*
c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*e/c - a^2*d/x + 1/16*(4*(b^2*e*x^2
 - b^2*d)*arctan(c*x)^2 - (b^2*e*x^2 - b^2*d)*log(c^2*x^2 + 1)^2 + 4*(b^2*
c*d*arctan(c*x)^3 + 48*b^2*c^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^4
 + x^2), x) + 4*b^2*c^2*e*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4 +
 x^2), x) + 16*b^2*c^2*e*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^4 + x^
2), x) + 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2)
, x) - 16*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x
) + b^2*e*arctan(c*x)^3/c - 32*b^2*c*e*integrate(1/16*x^3*arctan(c*x)/(c^2
*x^4 + x^2), x) + 32*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2),
 x) + 4*b^2*e*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) +
48*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*b^2*d*integr
ate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x)/x
```

3.1252.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.1252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^2, x)
```

3.1253 $\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx$

3.1253.1 Optimal result

Integrand size = 21, antiderivative size = 220

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = &-\frac{bcd(a+b\arctan(cx))}{x}\\
&-\frac{1}{2}c^2d(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{2x^2}\\
&+2e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\
&+b^2c^2d\log(x)-\frac{1}{2}b^2c^2d\log\left(1+c^2x^2\right)\\
&-ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\
&+ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\
&-\frac{1}{2}b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\
&+\frac{1}{2}b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-b*c*d*(a+b*arctan(c*x))/x-1/2*c^2*d*(a+b*arctan(c*x))^2-1/2*d*(a+b*arctan
(c*x))^2/x^2-2*e*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d*ln(
x)-1/2*b^2*c^2*d*ln(c^2*x^2+1)-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*
c*x))+I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*e*polylog(
3,1-2/(1+I*c*x))+1/2*b^2*e*polylog(3,-1+2/(1+I*c*x))
```

3.1253.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{a^2d}{2x^2} - \frac{abd(\arctan(cx)+cx(1+cx\arctan(cx)))}{x^2} + a^2e\log(x) - \frac{b^2d\left(2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{2x^2} + iabe(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx)) + \frac{1}{24}b^2e\left(-i\pi^3+16i\arctan(cx)^3 + 24\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - 24\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + 24i\arctan(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + 24i\arctan(cx)\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + 12\,\mathrm{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - 12\,\mathrm{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-1/2*(a^2*d)/x^2 - (a*b*d*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 +
 a^2*e*Log[x] - (b^2*d*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 -
2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/(2*x^2) + I*a*b*e*(PolyLog[2, (-I
)*c*x] - PolyLog[2, I*c*x]) + (b^2*e*((-I)*Pi^3 + (16*I)*ArcTan[c*x]^3 + 2
4*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]^2*Log[1 +
 E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c
*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 12*PolyLog
[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/24
```

3.1253.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx$$
$$\Big\downarrow \; 5515$$

$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^3}+\frac{e(a+b\arctan(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2e\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}c^2d(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{2x^2}-\frac{bcd(a+b\arctan(cx))}{x}-ibe\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibe\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-\frac{1}{2}b^2c^2d\log\left(c^2x^2+1\right)+b^2c^2d\log(x)-\frac{1}{2}b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2e\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d*(a + b*ArcTan[c*x]))/x) - (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
 + b*ArcTan[c*x])^2)/(2*x^2) + 2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
+ I*c*x)] + b^2*c^2*d*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 - I*b*e*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*e*(a + b*ArcTan[c*x])*
PolyLog[2, -1 + 2/(1 + I*c*x)] - (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 +
 (b^2*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

3.1253.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1253.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.21 (sec) , antiderivative size = 1289, normalized size of antiderivative = 5.86

method	result	size
derivativedivides	Expression too large to display	1289
default	Expression too large to display	1289
parts	Expression too large to display	1318

input
```
int((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a^2/c^2*e*ln(c*x)-1/2*a^2*d/c^2/x^2+b^2/c^2*(-1/2*I*e*Pi*csgn(I/((1+I
*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^2*arctan(c*x)^2+arctan(c*x)^2*e*ln(c*x)+d*c^2*ln(1+(1+I*c*x)
/(c^2*x^2+1)^(1/2))+d*c^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+1/2*I*e*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1
/2*c*d*arctan(c*x)*(I*c*x-(c^2*x^2+1)^(1/2)+1)/x-2*I*e*arctan(c*x)*polylog
(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*e*Pi*arctan(c*x)^2-2*I*e*arctan(c*x
)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*e*arctan(c*x)*polylog(2,-(1+I*c
*x)^2/(c^2*x^2+1))-1/2*I*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/
2*c*d*arctan(c*x)*(I*c*x+(c^2*x^2+1)^(1/2)+1)/x-1/2*I*e*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^
2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*e*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+e*arctan(c*x)^2*ln
(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-e*ln((1+I*c*x)^2/(c^2*x^2+1)-1)*arctan(c*x
)^2+e*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*e*polylog(3,-(1+
I*c*x)^2/(c^2*x^2+1))+2*e*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*e*poly
log(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^...
```

3.1253.5 Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e
*x^2 + a*b*d)*arctan(c*x))/x^3, x)
```

3.1253.6 Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^2(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**3, x)
```

3.1253.7 Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d + a^2*e*log(x) - 1/2*a^
2*d/x^2 - 1/96*(12*b^2*d*arctan(c*x)^2 - 3*b^2*d*log(c^2*x^2 + 1)^2 - (115
2*b^2*c^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*
b*c^2*e*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*
d*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*d*inte
grate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 192*b^2*c^2*d*inte
grate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + b^2*e*log(c^2*x^2 +
1)^3 + 384*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152
*b^2*e*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*e*int
egrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e*integr
ate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*d*integrate(1/16*a
rctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*d*integrate(1/16*log(c^2*x^2 + 1
)^2/(c^2*x^5 + x^3), x))*x^2)/x^2
```

3.1253.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.1253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^3, x)
```

3.1254 $\int x^3(d+ex^2)^2 (a+b\arctan(cx))^2\,dx$

3.1254.1 Optimal result

Integrand size = 23, antiderivative size = 502

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx = \frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{8b^2dex^2}{45c^4}+\frac{71b^2e^2x^2}{840c^6}+\frac{b^2dex^4}{30c^2}-\frac{3b^2e^2x^4}{140c^4}+\frac{b^2e^2x^6}{168c^2}+\frac{b^2d^2x\arctan(cx)}{2c^3}-\frac{2b^2dex\arctan(cx)}{3c^5}+\frac{b^2e^2x\arctan(cx)}{4c^7}-\frac{bd^2x^3(a+b\arctan(cx))}{6c}+\frac{2bdex^3(a+b\arctan(cx))}{9c^3}-\frac{be^2x^3(a+b\arctan(cx))}{12c^5}-\frac{2bdex^5(a+b\arctan(cx))}{15c}+\frac{be^2x^5(a+b\arctan(cx))}{20c^3}-\frac{be^2x^7(a+b\arctan(cx))}{28c}-\frac{d^2(a+b\arctan(cx))^2}{4c^4}+\frac{de(a+b\arctan(cx))^2}{3c^6}-\frac{e^2(a+b\arctan(cx))^2}{8c^8}+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2+\frac{1}{3}dex^6(a+b\arctan(cx))^2+\frac{1}{8}e^2x^8(a+b\arctan(cx))^2-\frac{b^2d^2\log(1+c^2x^2)}{3c^4}+\frac{23b^2de\log(1+c^2x^2)}{45c^6}-\frac{22b^2e^2\log(1+c^2x^2)}{105c^8}$$

output
```
1/2*a*b*d^2*x/c^3+1/2*b^2*d^2*x*arctan(c*x)/c^3-1/6*b*d^2*x^3*(a+b*arctan(
c*x))/c-1/8*e^2*(a+b*arctan(c*x))^2/c^8+1/8*e^2*x^8*(a+b*arctan(c*x))^2+23
/45*b^2*d*e*ln(c^2*x^2+1)/c^6+1/4*a*b*e^2*x/c^7-8/45*b^2*d*e*x^2/c^4+1/30*
b^2*d*e*x^4/c^2+1/4*b^2*e^2*x*arctan(c*x)/c^7-1/12*b*e^2*x^3*(a+b*arctan(c
*x))/c^5+1/20*b*e^2*x^5*(a+b*arctan(c*x))/c^3-1/28*b*e^2*x^7*(a+b*arctan(c
*x))/c-2/3*a*b*d*e*x/c^5-2/3*b^2*d*e*x*arctan(c*x)/c^5+2/9*b*d*e*x^3*(a+b*
arctan(c*x))/c^3-1/4*d^2*(a+b*arctan(c*x))^2/c^4+1/4*d^2*x^4*(a+b*arctan(c
*x))^2-22/105*b^2*e^2*ln(c^2*x^2+1)/c^8+71/840*b^2*e^2*x^2/c^6-3/140*b^2*e
^2*x^4/c^4+1/168*b^2*e^2*x^6/c^2+1/3*d*e*(a+b*arctan(c*x))^2/c^6+1/3*d*e*x
^6*(a+b*arctan(c*x))^2-2/15*b*d*e*x^5*(a+b*arctan(c*x))/c+1/12*b^2*d^2*x^2
/c^2-1/3*b^2*d^2*ln(c^2*x^2+1)/c^4
```

3.1254.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.82

$$\int x^3\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{cx(105a^2c^7x^3(6d^2+8dex^2+3e^2x^4)+b^2cx(213e^2-2c^2e(224d+27ex^2)+3c^4(70d^2+28dex^2+5e^2x^4))-$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(105*a^2*c^7*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + b^2*c*x*(213*e^2 -
 2*c^2*e*(224*d + 27*e*x^2) + 3*c^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)) - 2
*a*b*(-315*e^2 + 105*c^2*e*(8*d + e*x^2) - 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*
e^2*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6))) + 2*b*(b*c*x*(31
5*e^2 - 105*c^2*e*(8*d + e*x^2) + 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2*x^4)
- 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)) + 105*a*(-6*c^4*d^2 + 8*c^
2*d*e - 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8)))*ArcTan[c*x] + 10
5*b^2*(-6*c^4*d^2 + 8*c^2*d*e - 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2
*x^8))*ArcTan[c*x]^2 - 8*b^2*(105*c^4*d^2 - 161*c^2*d*e + 66*e^2)*Log[1 +
c^2*x^2])/(2520*c^8)
```

3.1254.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \ 5515$$
$$\int \left(d^2x^3(a+b\arctan(cx))^2+2dex^5(a+b\arctan(cx))^2+e^2x^7(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{e^2(a+b\arctan(cx))^2}{8c^8}+\frac{de(a+b\arctan(cx))^2}{3c^6}-\frac{be^2x^3(a+b\arctan(cx))}{12c^5}-\frac{d^2(a+b\arctan(cx))^2}{4c^4}+\frac{2bdex^3(a+b\arctan(cx))}{9c^3}+\frac{be^2x^5(a+b\arctan(cx))}{20c^3}+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2-\frac{bd^2x^3(a+b\arctan(cx))}{6c}+\frac{1}{3}dex^6(a+b\arctan(cx))^2-\frac{2bdex^5(a+b\arctan(cx))}{15c}+\frac{1}{8}e^2x^8(a+b\arctan(cx))^2-\frac{be^2x^7(a+b\arctan(cx))}{28c}+\frac{abe^2x}{4c^7}-\frac{2abdex}{3c^5}+\frac{abd^2x}{2c^3}+\frac{b^2e^2x\arctan(cx)}{4c^7}-\frac{2b^2dex\arctan(cx)}{3c^5}+\frac{b^2d^2x\arctan(cx)}{2c^3}+\frac{71b^2e^2x^2}{840c^6}-\frac{8b^2dex^2}{45c^4}-\frac{3b^2e^2x^4}{140c^4}+\frac{b^2d^2x^2}{12c^2}+\frac{b^2dex^4}{30c^2}+\frac{b^2e^2x^6}{168c^2}-\frac{22b^2e^2\log(c^2x^2+1)}{105c^8}+\frac{23b^2de\log(c^2x^2+1)}{45c^6}-\frac{b^2d^2\log(c^2x^2+1)}{3c^4}$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*b*d^2*x)/(2*c^3) - (2*a*b*d*e*x)/(3*c^5) + (a*b*e^2*x)/(4*c^7) + (b^2*d
^2*x^2)/(12*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (71*b^2*e^2*x^2)/(840*c^6) +
 (b^2*d*e*x^4)/(30*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (b^2*e^2*x^6)/(168*c
^2) + (b^2*d^2*x*ArcTan[c*x])/(2*c^3) - (2*b^2*d*e*x*ArcTan[c*x])/(3*c^5)
+ (b^2*e^2*x*ArcTan[c*x])/(4*c^7) - (b*d^2*x^3*(a + b*ArcTan[c*x]))/(6*c)
+ (2*b*d*e*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*e^2*x^3*(a + b*ArcTan[c*x
]))/(12*c^5) - (2*b*d*e*x^5*(a + b*ArcTan[c*x]))/(15*c) + (b*e^2*x^5*(a +
b*ArcTan[c*x]))/(20*c^3) - (b*e^2*x^7*(a + b*ArcTan[c*x]))/(28*c) - (d^2*(
a + b*ArcTan[c*x])^2)/(4*c^4) + (d*e*(a + b*ArcTan[c*x])^2)/(3*c^6) - (e^2
*(a + b*ArcTan[c*x])^2)/(8*c^8) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + (d*e
*x^6*(a + b*ArcTan[c*x])^2)/3 + (e^2*x^8*(a + b*ArcTan[c*x])^2)/8 - (b^2*d
^2*Log[1 + c^2*x^2])/(3*c^4) + (23*b^2*d*e*Log[1 + c^2*x^2])/(45*c^6) - (2
2*b^2*e^2*Log[1 + c^2*x^2])/(105*c^8)
```

3.1254.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1254.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.10

method	result
parts	$a^2\left(\frac{1}{8}e^2x^8+\frac{1}{3}ed\,x^6+\frac{1}{4}d^2x^4\right)+\frac{b^2\left(\frac{\arctan(cx)^2c^4e^2x^8}{8}+\frac{\arctan(cx)^2c^4de\,x^6}{3}+\frac{\arctan(cx)^2d^2c^4x^4}{4}-\frac{3\arctan(cx)e^2}{7}\right.}{}$
derivativedivides	$\frac{a^2\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^8x^4}{4}+\frac{\arctan(cx)^2d\,c^8e\,x^6}{3}+\frac{\arctan(cx)^2e^2c^8x^8}{8}-\frac{\arctan(cx)d^2c^7x^3}{6}-2a\right.}{}$
default	$\frac{a^2\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^8x^4}{4}+\frac{\arctan(cx)^2d\,c^8e\,x^6}{3}+\frac{\arctan(cx)^2e^2c^8x^8}{8}-\frac{\arctan(cx)d^2c^7x^3}{6}-2a\right.}{}$
parallelrisch	$-\frac{-315c^8a^2e^2x^8-630c^8a^2d^2x^4+528e^2b^2\ln(c^2x^2+1)+336ab\,c^7de\,x^5+315\arctan(cx)^2b^2e^2+210b^2c^4d^2+213e^2b^2+1260}{}$
risch	$-\frac{b^2e^2x^8\ln(-icx+1)^2}{32}-\frac{b^2d^2x^4\ln(-icx+1)^2}{16}+\frac{b^2d^2\ln(-icx+1)^2}{16c^4}+\frac{b^2e^2\ln(-icx+1)^2}{32c^8}-\frac{b^2\left(3e^2c^8x^8+8d\,c^8e\,x^6\right.}{}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/8*e^2*x^8+1/3*e*d*x^6+1/4*d^2*x^4)+b^2/c^4*(1/8*arctan(c*x)^2*c^4*e
^2*x^8+1/3*arctan(c*x)^2*c^4*d*e*x^6+1/4*arctan(c*x)^2*d^2*c^4*x^4-1/12/c^
4*(3/7*arctan(c*x)*e^2*c^7*x^7+8/5*arctan(c*x)*d*c^7*e*x^5+2*arctan(c*x)*d
^2*c^7*x^3-3/5*arctan(c*x)*e^2*c^5*x^5-8/3*arctan(c*x)*d*c^5*e*x^3-6*arcta
n(c*x)*c^5*x*d^2+arctan(c*x)*e^2*c^3*x^3+8*arctan(c*x)*c^3*d*e*x-3*arctan(
c*x)*c*x*e^2+6*arctan(c*x)^2*c^4*d^2-8*arctan(c*x)^2*c^2*d*e+3*arctan(c*x)
^2*e^2-d^2*c^6*x^2-2/5*d*c^6*e*x^4-1/14*e^2*c^6*x^6+32/15*d*c^4*e*x^2+9/35
*e^2*c^4*x^4-71/70*e^2*c^2*x^2-1/210*(-840*c^4*d^2+1288*c^2*d*e-528*e^2)*l
n(c^2*x^2+1)-1/210*(630*c^4*d^2-840*c^2*d*e+315*e^2)*arctan(c*x)^2))+2*a*b
/c^4*(1/8*arctan(c*x)*c^4*e^2*x^8+1/3*arctan(c*x)*c^4*d*e*x^6+1/4*arctan(c
*x)*d^2*c^4*x^4-1/24/c^4*(3/7*e^2*c^7*x^7+8/5*d*c^7*e*x^5+2*d^2*c^7*x^3-3/
5*e^2*c^5*x^5-8/3*d*c^5*e*x^3-6*c^5*x*d^2+e^2*c^3*x^3+8*c^3*d*e*x-3*c*x*e^
2+(6*c^4*d^2-8*c^2*d*e+3*e^2)*arctan(c*x)))
```

3.1254.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.06

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{315\,a^2c^8e^2x^8-90\,abc^7e^2x^7+15\,(56\,a^2c^8de+b^2c^6e^2)x^6-42\,(8\,abc^7de-3\,abc^5e^2)x^5+6\,(105\,a^2c^8d^2+14\,b}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/2520*(315*a^2*c^8*e^2*x^8 - 90*a*b*c^7*e^2*x^7 + 15*(56*a^2*c^8*d*e + b^
2*c^6*e^2)*x^6 - 42*(8*a*b*c^7*d*e - 3*a*b*c^5*e^2)*x^5 + 6*(105*a^2*c^8*d
^2 + 14*b^2*c^6*d*e - 9*b^2*c^4*e^2)*x^4 - 70*(6*a*b*c^7*d^2 - 8*a*b*c^5*d
*e + 3*a*b*c^3*e^2)*x^3 + (210*b^2*c^6*d^2 - 448*b^2*c^4*d*e + 213*b^2*c^2
*e^2)*x^2 + 105*(3*b^2*c^8*e^2*x^8 + 8*b^2*c^8*d*e*x^6 + 6*b^2*c^8*d^2*x^4
 - 6*b^2*c^4*d^2 + 8*b^2*c^2*d*e - 3*b^2*e^2)*arctan(c*x)^2 + 210*(6*a*b*c
^5*d^2 - 8*a*b*c^3*d*e + 3*a*b*c*e^2)*x + 2*(315*a*b*c^8*e^2*x^8 + 840*a*b
*c^8*d*e*x^6 - 45*b^2*c^7*e^2*x^7 + 630*a*b*c^8*d^2*x^4 - 630*a*b*c^4*d^2
+ 840*a*b*c^2*d*e - 21*(8*b^2*c^7*d*e - 3*b^2*c^5*e^2)*x^5 - 315*a*b*e^2 -
 35*(6*b^2*c^7*d^2 - 8*b^2*c^5*d*e + 3*b^2*c^3*e^2)*x^3 + 105*(6*b^2*c^5*d
^2 - 8*b^2*c^3*d*e + 3*b^2*c*e^2)*x)*arctan(c*x) - 8*(105*b^2*c^4*d^2 - 16
1*b^2*c^2*d*e + 66*b^2*e^2)*log(c^2*x^2 + 1))/c^8
```

3.1254.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.51

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2d^2x^4}{4}+\frac{a^2dex^6}{3}+\frac{a^2e^2x^8}{8}+\frac{abd^2x^4\operatorname{atan}(cx)}{2}+\frac{2abdex^6\operatorname{atan}(cx)}{3}+\frac{abe^2x^8\operatorname{atan}(cx)}{4}-\frac{abd^2x^3}{6c}-\frac{2abdex^5}{15c}-\frac{abe^2x^7}{28c}+\frac{abd^2}{2c^3} \\ a^2\left(\frac{d^2x^4}{4}+\frac{dex^6}{3}+\frac{e^2x^8}{8}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x**4/4 + a**2*d*e*x**6/3 + a**2*e**2*x**8/8 + a*b*d**
2*x**4*atan(c*x)/2 + 2*a*b*d*e*x**6*atan(c*x)/3 + a*b*e**2*x**8*atan(c*x)/
4 - a*b*d**2*x**3/(6*c) - 2*a*b*d*e*x**5/(15*c) - a*b*e**2*x**7/(28*c) + a
*b*d**2*x/(2*c**3) + 2*a*b*d*e*x**3/(9*c**3) + a*b*e**2*x**5/(20*c**3) - a
*b*d**2*atan(c*x)/(2*c**4) - 2*a*b*d*e*x/(3*c**5) - a*b*e**2*x**3/(12*c**5
) + 2*a*b*d*e*atan(c*x)/(3*c**6) + a*b*e**2*x/(4*c**7) - a*b*e**2*atan(c*x
)/(4*c**8) + b**2*d**2*x**4*atan(c*x)**2/4 + b**2*d*e*x**6*atan(c*x)**2/3
+ b**2*e**2*x**8*atan(c*x)**2/8 - b**2*d**2*x**3*atan(c*x)/(6*c) - 2*b**2*
d*e*x**5*atan(c*x)/(15*c) - b**2*e**2*x**7*atan(c*x)/(28*c) + b**2*d**2*x*
*2/(12*c**2) + b**2*d*e*x**4/(30*c**2) + b**2*e**2*x**6/(168*c**2) + b**2*
d**2*x*atan(c*x)/(2*c**3) + 2*b**2*d*e*x**3*atan(c*x)/(9*c**3) + b**2*e**2
*x**5*atan(c*x)/(20*c**3) - b**2*d**2*log(x**2 + c**(-2))/(3*c**4) - b**2*
d**2*atan(c*x)**2/(4*c**4) - 8*b**2*d*e*x**2/(45*c**4) - 3*b**2*e**2*x**4/
(140*c**4) - 2*b**2*d*e*x*atan(c*x)/(3*c**5) - b**2*e**2*x**3*atan(c*x)/(1
2*c**5) + 23*b**2*d*e*log(x**2 + c**(-2))/(45*c**6) + b**2*d*e*atan(c*x)**
2/(3*c**6) + 71*b**2*e**2*x**2/(840*c**6) + b**2*e**2*x*atan(c*x)/(4*c**7)
 - 22*b**2*e**2*log(x**2 + c**(-2))/(105*c**8) - b**2*e**2*atan(c*x)**2/(8
*c**8), Ne(c, 0)), (a**2*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), True))
```

3.1254.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.03

$$
\begin{aligned}
&\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx \\
&= \frac{1}{8}b^2e^2x^8\arctan(cx)^2+\frac{1}{8}a^2e^2x^8+\frac{1}{3}b^2dex^6\arctan(cx)^2+\frac{1}{3}a^2dex^6+\frac{1}{4}b^2d^2x^4\arctan(cx)^2 \\
&\quad+\frac{1}{4}a^2d^2x^4+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)abd^2 \\
&\quad-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2d^2 \\
&\quad+\frac{2}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)abde \\
&\quad-\frac{1}{90}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\arctan(cx)-\frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+}{c^6}\right. \\
&\quad+\frac{1}{420}\left(105x^8\arctan(cx)-c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8}+\frac{105\arctan(cx)}{c^9}\right)\right)abe^2 \\
&\quad-\frac{1}{840}\left(2c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8}+\frac{105\arctan(cx)}{c^9}\right)\arctan(cx)-\frac{5c^6x^6-18c^4x^4+71}{}\right.
\end{aligned}
$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*e^2*x^8*arctan(c*x)^2 + 1/8*a^2*e^2*x^8 + 1/3*b^2*d*e*x^6*arctan(c
*x)^2 + 1/3*a^2*d*e*x^6 + 1/4*b^2*d^2*x^4*arctan(c*x)^2 + 1/4*a^2*d^2*x^4
+ 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*
b*d^2 - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) -
(c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 + 2/45*(15*x
^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^
7))*a*b*d*e - 1/90*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*
x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(
c^2*x^2 + 1))/c^6)*b^2*d*e + 1/420*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 -
 21*c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*a*b*e^2 - 1/
840*(2*c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(
c*x)/c^9)*arctan(c*x) - (5*c^6*x^6 - 18*c^4*x^4 + 71*c^2*x^2 + 105*arctan(
c*x)^2 - 176*log(c^2*x^2 + 1))/c^8)*b^2*e^2
```

3.1254.8 Giac [F]

$$\int x^3 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx = \int \left(ex^2+d\right)^2 (b\arctan(cx)+a)^2 x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1254.9 Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.85

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$\begin{aligned}
&= \frac{a^2 d^2 x^4}{4} + \frac{a^2 e^2 x^8}{8} - \frac{b^2 d^2 \ln(c^2 x^2+1)}{3 c^4} - \frac{22 b^2 e^2 \ln(c^2 x^2+1)}{105 c^8} + \frac{b^2 d^2 x^2}{12 c^2} \\
&+ \frac{b^2 e^2 x^6}{168 c^2} - \frac{3 b^2 e^2 x^4}{140 c^4} + \frac{71 b^2 e^2 x^2}{840 c^6} - \frac{b^2 d^2 \operatorname{atan}(c x)^2}{4 c^4} - \frac{b^2 e^2 \operatorname{atan}(c x)^2}{8 c^8} \\
&+ \frac{b^2 d^2 x^4 \operatorname{atan}(c x)^2}{4} + \frac{b^2 e^2 x^8 \operatorname{atan}(c x)^2}{8} + \frac{a^2 d e x^6}{3} - \frac{b^2 d^2 x^3 \operatorname{atan}(c x)}{6 c} \\
&- \frac{b^2 e^2 x^7 \operatorname{atan}(c x)}{28 c} + \frac{b^2 e^2 x^5 \operatorname{atan}(c x)}{20 c^3} - \frac{b^2 e^2 x^3 \operatorname{atan}(c x)}{12 c^5} + \frac{a b d^2 x}{2 c^3} \\
&+ \frac{a b e^2 x}{4 c^7} + \frac{a b d^2 x^4 \operatorname{atan}(c x)}{2} + \frac{a b e^2 x^8 \operatorname{atan}(c x)}{4} + \frac{23 b^2 d e \ln(c^2 x^2+1)}{45 c^6} \\
&- \frac{a b d^2 x^3}{6 c} - \frac{a b e^2 x^7}{28 c} + \frac{a b e^2 x^5}{20 c^3} - \frac{a b e^2 x^3}{12 c^5} + \frac{b^2 d e x^4}{30 c^2} - \frac{8 b^2 d e x^2}{45 c^4} \\
&+ \frac{b^2 d e \operatorname{atan}(c x)^2}{3 c^6} + \frac{b^2 d^2 x \operatorname{atan}(c x)}{2 c^3} + \frac{b^2 e^2 x \operatorname{atan}(c x)}{4 c^7} + \frac{b^2 d e x^6 \operatorname{atan}(c x)^2}{3} \\
&- \frac{a b d^2 \operatorname{atan}\left(\frac{3 b c e^2 x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2} + \frac{6 b c^5 d^2 x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2} - \frac{8 b c^3 d e x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2}\right)}{2 c^4} \\
&- \frac{a b e^2 \operatorname{atan}\left(\frac{3 b c e^2 x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2} + \frac{6 b c^5 d^2 x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2} - \frac{8 b c^3 d e x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2}\right)}{4 c^8} \\
&- \frac{2 b^2 d e x^5 \operatorname{atan}(c x)}{15 c} + \frac{2 b^2 d e x^3 \operatorname{atan}(c x)}{9 c^3} - \frac{2 a b d e x}{3 c^5} \\
&+ \frac{2 a b d e x^6 \operatorname{atan}(c x)}{3} - \frac{2 a b d e x^5}{15 c} + \frac{2 a b d e x^3}{9 c^3} - \frac{2 b^2 d e x \operatorname{atan}(c x)}{3 c^5} \\
&+ \frac{2 a b d e \operatorname{atan}\left(\frac{3 b c e^2 x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2} + \frac{6 b c^5 d^2 x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2} - \frac{8 b c^3 d e x}{6 b c^4 d^2 - 8 b c^2 d e + 3 b e^2}\right)}{3 c^6}
\end{aligned}$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
(a^2*d^2*x^4)/4 + (a^2*e^2*x^8)/8 - (b^2*d^2*log(c^2*x^2 + 1))/(3*c^4) - (
22*b^2*e^2*log(c^2*x^2 + 1))/(105*c^8) + (b^2*d^2*x^2)/(12*c^2) + (b^2*e^2
*x^6)/(168*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (71*b^2*e^2*x^2)/(840*c^6) -
 (b^2*d^2*atan(c*x)^2)/(4*c^4) - (b^2*e^2*atan(c*x)^2)/(8*c^8) + (b^2*d^2*
x^4*atan(c*x)^2)/4 + (b^2*e^2*x^8*atan(c*x)^2)/8 + (a^2*d*e*x^6)/3 - (b^2*
d^2*x^3*atan(c*x))/(6*c) - (b^2*e^2*x^7*atan(c*x))/(28*c) + (b^2*e^2*x^5*a
tan(c*x))/(20*c^3) - (b^2*e^2*x^3*atan(c*x))/(12*c^5) + (a*b*d^2*x)/(2*c^3
) + (a*b*e^2*x)/(4*c^7) + (a*b*d^2*x^4*atan(c*x))/2 + (a*b*e^2*x^8*atan(c*
x))/4 + (23*b^2*d*e*log(c^2*x^2 + 1))/(45*c^6) - (a*b*d^2*x^3)/(6*c) - (a*
b*e^2*x^7)/(28*c) + (a*b*e^2*x^5)/(20*c^3) - (a*b*e^2*x^3)/(12*c^5) + (b^2
*d*e*x^4)/(30*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (b^2*d*e*atan(c*x)^2)/(3*c
^6) + (b^2*d^2*x*atan(c*x))/(2*c^3) + (b^2*e^2*x*atan(c*x))/(4*c^7) + (b^2
*d*e*x^6*atan(c*x)^2)/3 - (a*b*d^2*atan((3*b*c*e^2*x)/(3*b*e^2 + 6*b*c^4*d
^2 - 8*b*c^2*d*e) + (6*b*c^5*d^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e)
- (8*b*c^3*d*e*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e)))/(2*c^4) - (a*b*e
^2*atan((3*b*c*e^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) + (6*b*c^5*d^2
*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) - (8*b*c^3*d*e*x)/(3*b*e^2 + 6*b
*c^4*d^2 - 8*b*c^2*d*e)))/(4*c^8) - (2*b^2*d*e*x^5*atan(c*x))/(15*c) + (2*
b^2*d*e*x^3*atan(c*x))/(9*c^3) - (2*a*b*d*e*x)/(3*c^5) + (2*a*b*d*e*x^6*at
an(c*x))/3 - (2*a*b*d*e*x^5)/(15*c) + (2*a*b*d*e*x^3)/(9*c^3) - (2*b^2*...
```

3.1254. $\int x^3(d+ex^2)^2 \left(a+b\arctan(cx)\right)^2 dx$

3.1255 $\int x^2(d+ex^2)^2 (a+b\arctan(cx))^2 dx$

3.1255.1 Optimal result

Integrand size = 23, antiderivative size = 580

$$\int x^2\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx=\frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\arctan(cx)}{3c^3}+\frac{3b^2de\arctan(cx)}{5c^5}-\frac{11b^2e^2\arctan(cx)}{42c^7}-\frac{bd^2x^2(a+b\arctan(cx))}{3c}+\frac{2bdex^2(a+b\arctan(cx))}{5c^3}-\frac{be^2x^2(a+b\arctan(cx))}{7c^5}-\frac{bdex^4(a+b\arctan(cx))}{5c}+\frac{be^2x^4(a+b\arctan(cx))}{14c^3}-\frac{be^2x^6(a+b\arctan(cx))}{21c}-\frac{id^2(a+b\arctan(cx))^2}{3c^3}+\frac{2ide(a+b\arctan(cx))^2}{5c^5}-\frac{ie^2(a+b\arctan(cx))^2}{7c^7}+\frac{1}{3}d^2x^3(a+b\arctan(cx))^2+\frac{2}{5}dex^5(a+b\arctan(cx))^2+\frac{1}{7}e^2x^7(a+b\arctan(cx))^2-\frac{2bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}+\frac{4bde(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5}-\frac{2be^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{7c^7}-\frac{ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{2ib^2de\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{7c^7}$$

output
```
2/5*I*d*e*(a+b*arctan(c*x))^2/c^5+2/5*I*b^2*d*e*polylog(2,1-2/(1+I*c*x))/c
^5-1/3*b*d^2*x^2*(a+b*arctan(c*x))/c-2/3*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I
*c*x))/c^3+1/7*e^2*x^7*(a+b*arctan(c*x))^2-2/7*b*e^2*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c^7-3/5*b^2*d*e*x/c^4+1/15*b^2*d*e*x^3/c^2+3/5*b^2*d*e*arctan
(c*x)/c^5-1/7*b*e^2*x^2*(a+b*arctan(c*x))/c^5+1/14*b*e^2*x^4*(a+b*arctan(c
*x))/c^3-1/21*b*e^2*x^6*(a+b*arctan(c*x))/c-1/3*I*b^2*d^2*polylog(2,1-2/(1
+I*c*x))/c^3-1/7*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c^7+1/3*d^2*x^3*(a+b*a
rctan(c*x))^2+1/3*b^2*d^2*x/c^2-1/3*b^2*d^2*arctan(c*x)/c^3+11/42*b^2*e^2*
x/c^6-5/126*b^2*e^2*x^3/c^4+1/105*b^2*e^2*x^5/c^2-11/42*b^2*e^2*arctan(c*x
)/c^7+2/5*d*e*x^5*(a+b*arctan(c*x))^2-1/3*I*d^2*(a+b*arctan(c*x))^2/c^3-1/
7*I*e^2*(a+b*arctan(c*x))^2/c^7+2/5*b*d*e*x^2*(a+b*arctan(c*x))/c^3-1/5*b*
d*e*x^4*(a+b*arctan(c*x))/c+4/5*b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^
5
```

3.1255.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.88

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{378abc^2de-165abe^2+210b^2c^5d^2x-378b^2c^3dex+165b^2ce^2x-210abc^6d^2x^2+252abc^4dex^2-90abc^2e^2x}{}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(378*a*b*c^2*d*e - 165*a*b*e^2 + 210*b^2*c^5*d^2*x - 378*b^2*c^3*d*e*x + 1
65*b^2*c*e^2*x - 210*a*b*c^6*d^2*x^2 + 252*a*b*c^4*d*e*x^2 - 90*a*b*c^2*e^
2*x^2 + 210*a^2*c^7*d^2*x^3 + 42*b^2*c^5*d*e*x^3 - 25*b^2*c^3*e^2*x^3 - 12
6*a*b*c^6*d*e*x^4 + 45*a*b*c^4*e^2*x^4 + 252*a^2*c^7*d*e*x^5 + 6*b^2*c^5*e
^2*x^5 - 30*a*b*c^6*e^2*x^6 + 90*a^2*c^7*e^2*x^7 + 6*b^2*((35*I)*c^4*d^2 -
 (42*I)*c^2*d*e + (15*I)*e^2 + c^7*(35*d^2*x^3 + 42*d*e*x^5 + 15*e^2*x^7))
*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e
^2*x^4) + b*(1 + c^2*x^2)*(55*e^2 - c^2*e*(126*d + 25*e*x^2) + 2*c^4*(35*d
^2 + 21*d*e*x^2 + 5*e^2*x^4)) + 4*b*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*Log
[1 + E^((2*I)*ArcTan[c*x])]) + 210*a*b*c^4*d^2*Log[1 + c^2*x^2] - 252*a*b*
c^2*d*e*Log[1 + c^2*x^2] + 90*a*b*e^2*Log[1 + c^2*x^2] + (6*I)*b^2*(35*c^4
*d^2 - 42*c^2*d*e + 15*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(630*c^7)
```

3.1255.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^2(a+b\arctan(cx))^2dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(d^2x^2(a+b\arctan(cx))^2+2dex^4(a+b\arctan(cx))^2+e^2x^6(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^2(a+b\arctan(cx))^2}{7c^7}-\frac{2be^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{7c^7}+\frac{2ide(a+b\arctan(cx))^2}{5c^5}+\frac{4bde\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^5}-\frac{be^2x^2(a+b\arctan(cx))}{7c^5}-\frac{id^2(a+b\arctan(cx))^2}{3c^3}-\frac{2bd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{2bdex^2(a+b\arctan(cx))}{5c^3}+\frac{be^2x^4(a+b\arctan(cx))}{14c^3}+\frac{1}{3}d^2x^3(a+b\arctan(cx))^2-\frac{bd^2x^2(a+b\arctan(cx))}{3c}+\frac{2}{5}dex^5(a+b\arctan(cx))^2-\frac{bdex^4(a+b\arctan(cx))}{5c}+\frac{1}{7}e^2x^7(a+b\arctan(cx))^2-\frac{be^2x^6(a+b\arctan(cx))}{21c}-\frac{11b^2e^2\arctan(cx)}{42c^7}+\frac{3b^2de\arctan(cx)}{5c^5}-\frac{b^2d^2\arctan(cx)}{3c^3}-\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{7c^7}+\frac{11b^2e^2x}{42c^6}+\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2dex}{5c^4}-\frac{5b^2e^2x^3}{126c^4}-\frac{ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2d^2x}{3c^2}+\frac{b^2dex^3}{15c^2}+\frac{b^2e^2x^5}{105c^2}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*d^2*x)/(3*c^2) - (3*b^2*d*e*x)/(5*c^4) + (11*b^2*e^2*x)/(42*c^6) + (b
^2*d*e*x^3)/(15*c^2) - (5*b^2*e^2*x^3)/(126*c^4) + (b^2*e^2*x^5)/(105*c^2)
 - (b^2*d^2*ArcTan[c*x])/(3*c^3) + (3*b^2*d*e*ArcTan[c*x])/(5*c^5) - (11*b
^2*e^2*ArcTan[c*x])/(42*c^7) - (b*d^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + (2*
b*d*e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(
7*c^5) - (b*d*e*x^4*(a + b*ArcTan[c*x]))/(5*c) + (b*e^2*x^4*(a + b*ArcTan[
c*x]))/(14*c^3) - (b*e^2*x^6*(a + b*ArcTan[c*x]))/(21*c) - ((I/3)*d^2*(a +
 b*ArcTan[c*x])^2)/c^3 + (((2*I)/5)*d*e*(a + b*ArcTan[c*x])^2)/c^5 - ((I/7
)*e^2*(a + b*ArcTan[c*x])^2)/c^7 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*
d*e*x^5*(a + b*ArcTan[c*x])^2)/5 + (e^2*x^7*(a + b*ArcTan[c*x])^2)/7 - (2*
b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (4*b*d*e*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) - (2*b*e^2*(a + b*ArcTan[c*x])*Log[
2/(1 + I*c*x)])/(7*c^7) - ((I/3)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^
3 + (((2*I)/5)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5 - ((I/7)*b^2*e^2
*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^7
```

3.1255.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1255.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.09

method	result
parts	$a^2\left(\frac{1}{7}e^2x^7+\frac{2}{5}ed\,x^5+\frac{1}{3}d^2x^3\right)+\frac{b^2\left(\frac{\arctan(cx)^2c^3e^2x^7}{7}+\frac{2\arctan(cx)^2c^3de\,x^5}{5}+\frac{\arctan(cx)^2d^2c^3x^3}{3}-\frac{2\left(\frac{35\arctan}{}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^7x^3}{3}+\frac{2\arctan(cx)^2d\,c^7e\,x^5}{5}+\frac{\arctan(cx)^2e^2c^7x^7}{7}-\frac{\arctan(cx)c^6d^2x^2}{3}-\right.}{}}{}$
default	$\frac{\frac{a^2\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^7x^3}{3}+\frac{2\arctan(cx)^2d\,c^7e\,x^5}{5}+\frac{\arctan(cx)^2e^2c^7x^7}{7}-\frac{\arctan(cx)c^6d^2x^2}{3}-\right.}{}}{}$
risch	Expression too large to display

input
```
int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

3.1255. $\int x^2(d+ex^2)^2\,(a+b\arctan(cx))^2\,dx$

output
```
a^2*(1/7*e^2*x^7+2/5*e*d*x^5+1/3*d^2*x^3)+b^2/c^3*(1/7*arctan(c*x)^2*c^3*e
^2*x^7+2/5*arctan(c*x)^2*c^3*d*e*x^5+1/3*arctan(c*x)^2*d^2*c^3*x^3-2/105/c
^4*(35/2*arctan(c*x)*c^6*d^2*x^2+21/2*arctan(c*x)*e*c^6*d*x^4+5/2*arctan(c
*x)*e^2*c^6*x^6-21*arctan(c*x)*d*c^4*e*x^2-15/4*arctan(c*x)*e^2*c^4*x^4+15
/2*arctan(c*x)*e^2*c^2*x^2-35/2*arctan(c*x)*ln(c^2*x^2+1)*c^4*d^2+21*arcta
n(c*x)*ln(c^2*x^2+1)*c^2*d*e-15/2*arctan(c*x)*ln(c^2*x^2+1)*e^2-1/2*e^2*c^
5*x^5-7/2*d*c^5*e*x^3-35/2*c^5*x*d^2+25/12*e^2*c^3*x^3+63/2*c^3*d*e*x-55/4
*c*x*e^2-1/4*(-70*c^4*d^2+126*c^2*d*e-55*e^2)*arctan(c*x)-1/4*(-70*c^4*d^2
+84*c^2*d*e-30*e^2)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog
(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x)))+1/2*I*(ln(I+c*x)*ln(c^2*x^2
+1)-1/2*ln(I+c*x)^2-dilog(1/2*I*(c*x-I))-ln(I+c*x)*ln(1/2*I*(c*x-I))))))+2
*a*b/c^3*(1/7*arctan(c*x)*c^3*e^2*x^7+2/5*arctan(c*x)*c^3*d*e*x^5+1/3*arct
an(c*x)*d^2*c^3*x^3-1/105/c^4*(35/2*d^2*c^6*x^2+21/2*d*c^6*e*x^4+5/2*e^2*c
^6*x^6-21*d*c^4*e*x^2-15/4*e^2*c^4*x^4+15/2*e^2*c^2*x^2+1/2*(-35*c^4*d^2+4
2*c^2*d*e-15*e^2)*ln(c^2*x^2+1)))
```

3.1255.5 Fricas [F]

$$\int x^2\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx=\int \left(ex^2+d\right)^2(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e^2*x^6 + 2*a^2*d*e*x^4 + a^2*d^2*x^2 + (b^2*e^2*x^6 + 2*b^2*
d*e*x^4 + b^2*d^2*x^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^6 + 2*a*b*d*e*x^4 + a*
b*d^2*x^2)*arctan(c*x), x)
```

3.1255.6 Sympy [F]

$$\int x^2\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx=\int x^2(a+b\,\mathrm{atan}\,(cx))^2\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

3.1255.7 Maxima [F]

$$\int x^2 (d + ex^2)^2 (a + b \arctan(cx))^2 \, dx = \int (ex^2 + d)^2 (b \arctan(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*a^2*e^2*x^7 + 2/5*a^2*d*e*x^5 + 1/3*a^2*d^2*x^3 + 1/3*(2*x^3*arctan(c*
x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^2 + 1/5*(4*x^5*arctan(c*x)
- c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*d*e + 1/42*(12*x
^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 +
1)/c^8))*a*b*e^2 + 1/420*(15*b^2*e^2*x^7 + 42*b^2*d*e*x^5 + 35*b^2*d^2*x^3
)*arctan(c*x)^2 - 1/1680*(15*b^2*e^2*x^7 + 42*b^2*d*e*x^5 + 35*b^2*d^2*x^3
)*log(c^2*x^2 + 1)^2 + integrate(1/1680*(1260*(b^2*c^2*e^2*x^8 + (2*b^2*c^
2*d*e + b^2*e^2)*x^6 + b^2*d^2*x^2 + (b^2*c^2*d^2 + 2*b^2*d*e)*x^4)*arctan
(c*x)^2 + 105*(b^2*c^2*e^2*x^8 + (2*b^2*c^2*d*e + b^2*e^2)*x^6 + b^2*d^2*x
^2 + (b^2*c^2*d^2 + 2*b^2*d*e)*x^4)*log(c^2*x^2 + 1)^2 - 8*(15*b^2*c*e^2*x
^7 + 42*b^2*c*d*e*x^5 + 35*b^2*c*d^2*x^3)*arctan(c*x) + 4*(15*b^2*c^2*e^2*
x^8 + 42*b^2*c^2*d*e*x^6 + 35*b^2*c^2*d^2*x^4)*log(c^2*x^2 + 1))/(c^2*x^2
+ 1), x)
```

3.1255.8 Giac [F]

$$\int x^2 (d + ex^2)^2 (a + b \arctan(cx))^2 \, dx = \int (ex^2 + d)^2 (b \arctan(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1255.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = \int x^2 (a+b\,\mathrm{atan}(c\,x))^2 \left(e\,x^2+d\right)^2 dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2)^2, x)
```

3.1256 $\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx$

3.1256.1 Optimal result

Integrand size = 21, antiderivative size = 380

$$\begin{aligned}\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = & -\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}+\frac{b^2dex^2}{6c^2}-\frac{4b^2e^2x^2}{45c^4} \\ & +\frac{b^2e^2x^4}{60c^2}-\frac{b^2d^2x\arctan(cx)}{c}+\frac{b^2dex\arctan(cx)}{c^3} \\ & -\frac{b^2e^2x\arctan(cx)}{3c^5}-\frac{bdex^3(a+b\arctan(cx))}{3c} \\ & +\frac{be^2x^3(a+b\arctan(cx))}{9c^3}-\frac{be^2x^5(a+b\arctan(cx))}{15c} \\ & +\frac{d^2(a+b\arctan(cx))^2}{2c^2}-\frac{de(a+b\arctan(cx))^2}{2c^4} \\ & +\frac{e^2(a+b\arctan(cx))^2}{6c^6}+\frac{1}{2}d^2x^2(a+b\arctan(cx))^2 \\ & +\frac{1}{2}dex^4(a+b\arctan(cx))^2 \\ & +\frac{1}{6}e^2x^6(a+b\arctan(cx))^2+\frac{b^2d^2\log(1+c^2x^2)}{2c^2} \\ & -\frac{2b^2de\log(1+c^2x^2)}{3c^4}+\frac{23b^2e^2\log(1+c^2x^2)}{90c^6}\end{aligned}$$

output
```
-a*b*d^2*x/c+a*b*d*e*x/c^3-1/3*a*b*e^2*x/c^5+1/6*b^2*d*e*x^2/c^2-4/45*b^2*
e^2*x^2/c^4+1/60*b^2*e^2*x^4/c^2-b^2*d^2*x*arctan(c*x)/c+b^2*d*e*x*arctan(
c*x)/c^3-1/3*b^2*e^2*x*arctan(c*x)/c^5-1/3*b*d*e*x^3*(a+b*arctan(c*x))/c+1
/9*b*e^2*x^3*(a+b*arctan(c*x))/c^3-1/15*b*e^2*x^5*(a+b*arctan(c*x))/c+1/2*
d^2*(a+b*arctan(c*x))^2/c^2-1/2*d*e*(a+b*arctan(c*x))^2/c^4+1/6*e^2*(a+b*a
rctan(c*x))^2/c^6+1/2*d^2*x^2*(a+b*arctan(c*x))^2+1/2*d*e*x^4*(a+b*arctan(
c*x))^2+1/6*e^2*x^6*(a+b*arctan(c*x))^2+1/2*b^2*d^2*ln(c^2*x^2+1)/c^2-2/3*
b^2*d*e*ln(c^2*x^2+1)/c^4+23/90*b^2*e^2*ln(c^2*x^2+1)/c^6
```

3.1256.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.83

$$\int x\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{cx(30a^2c^5x(3d^2+3dex^2+e^2x^4)+b^2cex(-16e+3c^2(10d+ex^2))-4ab(15e^2-5c^2e(9d+ex^2)+3c^4(15c}{}$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(30*a^2*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + b^2*c*e*x*(-16*e + 3*c^
2*(10*d + e*x^2)) - 4*a*b*(15*e^2 - 5*c^2*e*(9*d + e*x^2) + 3*c^4*(15*d^2
+ 5*d*e*x^2 + e^2*x^4))) + 4*b*(-(b*c*x*(15*e^2 - 5*c^2*e*(9*d + e*x^2) +
3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4))) + 15*a*(3*c^4*d^2 - 3*c^2*d*e + e^2
 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6)))*ArcTan[c*x] + 30*b^2*(3*c^4*d^2
 - 3*c^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x]^2
+ 2*b^2*(45*c^4*d^2 - 60*c^2*d*e + 23*e^2)*Log[1 + c^2*x^2])/(180*c^6)
```

3.1256.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(d^2x(a+b\arctan(cx))^2+2dex^3(a+b\arctan(cx))^2+e^2x^5(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2(a+b\arctan(cx))^2}{6c^6}-\frac{de(a+b\arctan(cx))^2}{2c^4}+\frac{be^2x^3(a+b\arctan(cx))}{9c^3}+\frac{d^2(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}d^2x^2(a+b\arctan(cx))^2+\frac{1}{2}dex^4(a+b\arctan(cx))^2-\frac{bdex^3(a+b\arctan(cx))}{3c}+\frac{1}{6}e^2x^6(a+b\arctan(cx))^2-\frac{be^2x^5(a+b\arctan(cx))}{15c}-\frac{abe^2x}{3c^5}+\frac{abdex}{c^3}-\frac{abd^2x}{c}-\frac{b^2e^2x\arctan(cx)}{3c^5}+\frac{b^2dex\arctan(cx)}{c^3}-\frac{b^2d^2x\arctan(cx)}{c}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2d^2\log\left(c^2x^2+1\right)}{2c^2}+\frac{b^2dex^2}{6c^2}+\frac{b^2e^2x^4}{60c^2}+\frac{23b^2e^2\log\left(c^2x^2+1\right)}{90c^6}-\frac{2b^2de\log\left(c^2x^2+1\right)}{3c^4}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-((a*b*d^2*x)/c) + (a*b*d*e*x)/c^3 - (a*b*e^2*x)/(3*c^5) + (b^2*d*e*x^2)/(
6*c^2) - (4*b^2*e^2*x^2)/(45*c^4) + (b^2*e^2*x^4)/(60*c^2) - (b^2*d^2*x*Ar
cTan[c*x])/c + (b^2*d*e*x*ArcTan[c*x])/c^3 - (b^2*e^2*x*ArcTan[c*x])/(3*c^
5) - (b*d*e*x^3*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^3*(a + b*ArcTan[c*x]
))/(9*c^3) - (b*e^2*x^5*(a + b*ArcTan[c*x]))/(15*c) + (d^2*(a + b*ArcTan[c
*x])^2)/(2*c^2) - (d*e*(a + b*ArcTan[c*x])^2)/(2*c^4) + (e^2*(a + b*ArcTan
[c*x])^2)/(6*c^6) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + (d*e*x^4*(a + b*Ar
cTan[c*x])^2)/2 + (e^2*x^6*(a + b*ArcTan[c*x])^2)/6 + (b^2*d^2*Log[1 + c^2
*x^2])/(2*c^2) - (2*b^2*d*e*Log[1 + c^2*x^2])/(3*c^4) + (23*b^2*e^2*Log[1
+ c^2*x^2])/(90*c^6)
```

3.1256.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1256.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.29

method	result
parts	$\frac{a^2(e\,x^2+d)^3}{6e}+\frac{b^2\arctan(cx)^2e^2x^6}{6}+\frac{b^2\arctan(cx)^2e\,x^4d}{2}+\frac{b^2\arctan(cx)^2x^2d^2}{2}-\frac{b^2e^2\arctan(cx)x^5}{15c}-\frac{b^2e\,\mathrm{arc}}{}$
derivativedivides	$\frac{a^2(e\,c^2x^2+c^2d)^3}{6c^4e}+\frac{b^2\arctan(cx)^2d^2c^2x^2}{2}+\frac{b^2c^2\arctan(cx)^2ed\,x^4}{2}+\frac{b^2c^2\arctan(cx)^2e^2x^6}{6}-b^2c\,d^2x\arctan(cx)-\frac{b^2ce\arctan(cx)}{3}$
default	$\frac{a^2(e\,c^2x^2+c^2d)^3}{6c^4e}+\frac{b^2\arctan(cx)^2d^2c^2x^2}{2}+\frac{b^2c^2\arctan(cx)^2ed\,x^4}{2}+\frac{b^2c^2\arctan(cx)^2e^2x^6}{6}-b^2c\,d^2x\arctan(cx)-\frac{b^2ce\arctan(cx)}{3}$
parallelrisch	$46e^2b^2\ln(c^2x^2+1)+30\arctan(cx)^2b^2e^2+180x^4\arctan(cx)ab\,c^6de+16e^2b^2+180ab\,c^4d^2\arctan(cx)-180ab\,c^2de\arctan($
risch	$\frac{x^6e^2a^2}{6}+\frac{x^2d^2a^2}{2}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2e^2x^4}{60c^2}+\frac{23b^2e^2\ln(c^2x^2+1)}{90c^6}-\frac{ab\,e^2x}{3c^5}+\frac{b^2de\,x^2}{6c^2}-\frac{2b^2de\ln(c^2x^2+1)}{3c^4}+b^2$

input
```
int(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*(e*x^2+d)^3/e+1/6*b^2*arctan(c*x)^2*e^2*x^6+1/2*b^2*arctan(c*x)^2*
e*x^4*d+1/2*b^2*arctan(c*x)^2*x^2*d^2-1/15*b^2/c*e^2*arctan(c*x)*x^5-1/3*b
^2/c*e*arctan(c*x)*x^3*d-b^2*d^2*x*arctan(c*x)/c+1/9*b^2/c^3*e^2*arctan(c*
x)*x^3+b^2*d*e*x*arctan(c*x)/c^3-1/3*b^2*e^2*x*arctan(c*x)/c^5+1/2*b^2/c^2
*arctan(c*x)^2*d^2-1/2*b^2/c^4*e*arctan(c*x)^2*d+1/6*b^2/c^6*e^2*arctan(c*
x)^2+1/6*b^2*d*e*x^2/c^2+1/60*b^2*e^2*x^4/c^2-4/45*b^2*e^2*x^2/c^4+1/2*b^2
*d^2*ln(c^2*x^2+1)/c^2-2/3*b^2*d*e*ln(c^2*x^2+1)/c^4+23/90*b^2*e^2*ln(c^2*
x^2+1)/c^6+2*a*b/c^2*(1/6*arctan(c*x)*c^2*e^2*x^6+1/2*arctan(c*x)*c^2*e*x^
4*d+1/2*arctan(c*x)*c^2*x^2*d^2+1/6*arctan(c*x)*c^2/e*d^3-1/6/c^4/e*(3*c^5
*d^2*e*x+c^5*d*e^2*x^3+1/5*e^3*c^5*x^5-3*c^3*x*d*e^2-1/3*e^3*c^3*x^3+c*x*e
^3+(c^6*d^3-3*c^4*d^2*e+3*c^2*d*e^2-e^3)*arctan(c*x)))
```

3.1256.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.09

$$\int x(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$

$$=\frac{30\,a^2c^6e^2x^6-12\,abc^5e^2x^5+3\,(30\,a^2c^6de+b^2c^4e^2)x^4-20\,(3\,abc^5de-abc^3e^2)x^3+2\,(45\,a^2c^6d^2+15\,b^2c^4d}{}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/180*(30*a^2*c^6*e^2*x^6 - 12*a*b*c^5*e^2*x^5 + 3*(30*a^2*c^6*d*e + b^2*c
^4*e^2)*x^4 - 20*(3*a*b*c^5*d*e - a*b*c^3*e^2)*x^3 + 2*(45*a^2*c^6*d^2 + 1
5*b^2*c^4*d*e - 8*b^2*c^2*e^2)*x^2 + 30*(b^2*c^6*e^2*x^6 + 3*b^2*c^6*d*e*x
^4 + 3*b^2*c^6*d^2*x^2 + 3*b^2*c^4*d^2 - 3*b^2*c^2*d*e + b^2*e^2)*arctan(c
*x)^2 - 60*(3*a*b*c^5*d^2 - 3*a*b*c^3*d*e + a*b*c*e^2)*x + 4*(15*a*b*c^6*e
^2*x^6 + 45*a*b*c^6*d*e*x^4 - 3*b^2*c^5*e^2*x^5 + 45*a*b*c^6*d^2*x^2 + 45*
a*b*c^4*d^2 - 45*a*b*c^2*d*e + 15*a*b*e^2 - 5*(3*b^2*c^5*d*e - b^2*c^3*e^2
)*x^3 - 15*(3*b^2*c^5*d^2 - 3*b^2*c^3*d*e + b^2*c*e^2)*x)*arctan(c*x) + 2*
(45*b^2*c^4*d^2 - 60*b^2*c^2*d*e + 23*b^2*e^2)*log(c^2*x^2 + 1))/c^6
```

3.1256.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.51

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2d^2x^2}{2} + \frac{a^2dex^4}{2} + \frac{a^2e^2x^6}{6} + abd^2x^2\operatorname{atan}(cx) + abdex^4\operatorname{atan}(cx) + \frac{abe^2x^6\operatorname{atan}(cx)}{3} - \frac{abd^2x}{c} - \frac{abdex^3}{3c} - \frac{abe^2x^5}{15c} + \\ a^2\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x**2/2 + a**2*d*e*x**4/2 + a**2*e**2*x**6/6 + a*b*d**
2*x**2*atan(c*x) + a*b*d*e*x**4*atan(c*x) + a*b*e**2*x**6*atan(c*x)/3 - a*
b*d**2*x/c - a*b*d*e*x**3/(3*c) - a*b*e**2*x**5/(15*c) + a*b*d**2*atan(c*x
)/c**2 + a*b*d*e*x/c**3 + a*b*e**2*x**3/(9*c**3) - a*b*d*e*atan(c*x)/c**4
- a*b*e**2*x/(3*c**5) + a*b*e**2*atan(c*x)/(3*c**6) + b**2*d**2*x**2*atan(
c*x)**2/2 + b**2*d*e*x**4*atan(c*x)**2/2 + b**2*e**2*x**6*atan(c*x)**2/6 -
 b**2*d**2*x*atan(c*x)/c - b**2*d*e*x**3*atan(c*x)/(3*c) - b**2*e**2*x**5*
atan(c*x)/(15*c) + b**2*d**2*log(x**2 + c**(-2))/(2*c**2) + b**2*d**2*atan
(c*x)**2/(2*c**2) + b**2*d*e*x**2/(6*c**2) + b**2*e**2*x**4/(60*c**2) + b*
*2*d*e*x*atan(c*x)/c**3 + b**2*e**2*x**3*atan(c*x)/(9*c**3) - 2*b**2*d*e*l
og(x**2 + c**(-2))/(3*c**4) - b**2*d*e*atan(c*x)**2/(2*c**4) - 4*b**2*e**2
*x**2/(45*c**4) - b**2*e**2*x*atan(c*x)/(3*c**5) + 23*b**2*e**2*log(x**2 +
 c**(-2))/(90*c**6) + b**2*e**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*(d
**2*x**2/2 + d*e*x**4/2 + e**2*x**6/6), True))
```

3.1256.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.14

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2\,dx$$
$$\begin{aligned}&=\frac{1}{6}b^2e^2x^6\arctan(cx)^2+\frac{1}{6}a^2e^2x^6+\frac{1}{2}b^2dex^4\arctan(cx)^2+\frac{1}{2}a^2dex^4\\&+\frac{1}{2}b^2d^2x^2\arctan(cx)^2+\frac{1}{2}a^2d^2x^2+\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)abd^2\\&-\frac{1}{2}\left(2c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\arctan(cx)+\frac{\arctan(cx)^2-\log(c^2x^2+1)}{c^2}\right)b^2d^2\\&+\frac{1}{3}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)abde\\&-\frac{1}{6}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2de\\&+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)abe^2\\&-\frac{1}{180}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\arctan(cx)-\frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+}{c^6}\right.\end{aligned}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*e^2*x^6*arctan(c*x)^2 + 1/6*a^2*e^2*x^6 + 1/2*b^2*d*e*x^4*arctan(c
*x)^2 + 1/2*a^2*d*e*x^4 + 1/2*b^2*d^2*x^2*arctan(c*x)^2 + 1/2*a^2*d^2*x^2
+ (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^2 - 1/2*(2*c*(x/c^
2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)
*b^2*d^2 + 1/3*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)
/c^5))*a*b*d*e - 1/6*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan
(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d*e + 1/
45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan
(c*x)/c^7))*a*b*e^2 - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*
arctan(c*x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2
+ 46*log(c^2*x^2 + 1))/c^6)*b^2*e^2
```

3.1256.8 Giac [F]

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2\,dx = \int (ex^2+d)^2(b\arctan(cx)+a)^2 x\,dx$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1256.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.05

$$\begin{aligned}\int x(d+ex^2)^2 (a+b\arctan(cx))^2\,dx &= \frac{a^2\,d^2\,x^2}{2}+\frac{a^2\,e^2\,x^6}{6}+\frac{b^2\,d^2\,\ln\left(c^2\,x^2+1\right)}{2\,c^2}+\frac{23\,b^2\,e^2\,\ln\left(c^2\,x^2+1\right)}{90\,c^6}+\frac{b^2\,e^2\,x^4}{60\,c^2}-\frac{4\,b^2\,e^2\,x^2}{45\,c^4}\\&\quad+\frac{b^2\,d^2\,\mathrm{atan}(c\,x)^2}{2\,c^2}+\frac{b^2\,e^2\,\mathrm{atan}(c\,x)^2}{6\,c^6}+\frac{b^2\,d^2\,x^2\,\mathrm{atan}(c\,x)^2}{2}+\frac{b^2\,e^2\,x^6\,\mathrm{atan}(c\,x)^2}{6}+\frac{a^2\,d\,e\,x^4}{2}\\&\quad-\frac{b^2\,e^2\,x^5\,\mathrm{atan}(c\,x)}{15\,c}+\frac{b^2\,e^2\,x^3\,\mathrm{atan}(c\,x)}{9\,c^3}-\frac{a\,b\,d^2\,x}{c}-\frac{a\,b\,e^2\,x}{3\,c^5}+a\,b\,d^2\,x^2\,\mathrm{atan}(c\,x)\\&\quad+\frac{a\,b\,e^2\,x^6\,\mathrm{atan}(c\,x)}{3}-\frac{2\,b^2\,d\,e\,\ln\left(c^2\,x^2+1\right)}{3\,c^4}-\frac{a\,b\,e^2\,x^5}{15\,c}+\frac{a\,b\,e^2\,x^3}{9\,c^3}+\frac{b^2\,d\,e\,x^2}{6\,c^2}\\&\quad-\frac{b^2\,d\,e\,\mathrm{atan}(c\,x)^2}{2\,c^4}-\frac{b^2\,d^2\,x\,\mathrm{atan}(c\,x)}{c}-\frac{b^2\,e^2\,x\,\mathrm{atan}(c\,x)}{3\,c^5}+\frac{b^2\,d\,e\,x^4\,\mathrm{atan}(c\,x)^2}{2}\\&\quad+\frac{a\,b\,d^2\,\mathrm{atan}\left(\frac{b\,c\,e^2\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}+\frac{3\,b\,c^5\,d^2\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}-\frac{3\,b\,c^3\,d\,e\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}\right)}{c^2}\\&\quad+\frac{a\,b\,e^2\,\mathrm{atan}\left(\frac{b\,c\,e^2\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}+\frac{3\,b\,c^5\,d^2\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}-\frac{3\,b\,c^3\,d\,e\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}\right)}{3\,c^6}\\&\quad-\frac{b^2\,d\,e\,x^3\,\mathrm{atan}(c\,x)}{3\,c}+\frac{a\,b\,d\,e\,x}{c^3}+a\,b\,d\,e\,x^4\,\mathrm{atan}(c\,x)-\frac{a\,b\,d\,e\,x^3}{3\,c}+\frac{b^2\,d\,e\,x\,\mathrm{atan}(c\,x)}{c^3}\\&\quad-\frac{a\,b\,d\,e\,\mathrm{atan}\left(\frac{b\,c\,e^2\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}+\frac{3\,b\,c^5\,d^2\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}-\frac{3\,b\,c^3\,d\,e\,x}{3\,b\,c^4\,d^2-3\,b\,c^2\,d\,e+b\,e^2}\right)}{c^4}\end{aligned}$$

input
```
int(x*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
(a^2*d^2*x^2)/2 + (a^2*e^2*x^6)/6 + (b^2*d^2*log(c^2*x^2 + 1))/(2*c^2) + (
23*b^2*e^2*log(c^2*x^2 + 1))/(90*c^6) + (b^2*e^2*x^4)/(60*c^2) - (4*b^2*e^
2*x^2)/(45*c^4) + (b^2*d^2*atan(c*x)^2)/(2*c^2) + (b^2*e^2*atan(c*x)^2)/(6
*c^6) + (b^2*d^2*x^2*atan(c*x)^2)/2 + (b^2*e^2*x^6*atan(c*x)^2)/6 + (a^2*d
*e*x^4)/2 - (b^2*e^2*x^5*atan(c*x))/(15*c) + (b^2*e^2*x^3*atan(c*x))/(9*c^
3) - (a*b*d^2*x)/c - (a*b*e^2*x)/(3*c^5) + a*b*d^2*x^2*atan(c*x) + (a*b*e^
2*x^6*atan(c*x))/3 - (2*b^2*d*e*log(c^2*x^2 + 1))/(3*c^4) - (a*b*e^2*x^5)/
(15*c) + (a*b*e^2*x^3)/(9*c^3) + (b^2*d*e*x^2)/(6*c^2) - (b^2*d*e*atan(c*x
)^2)/(2*c^4) - (b^2*d^2*x*atan(c*x))/c - (b^2*e^2*x*atan(c*x))/(3*c^5) + (
b^2*d*e*x^4*atan(c*x)^2)/2 + (a*b*d^2*atan((b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^
2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) - (
3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e)))/c^2 + (a*b*e^2*atan((
b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)/(b*e^2 +
3*b*c^4*d^2 - 3*b*c^2*d*e) - (3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^
2*d*e)))/(3*c^6) - (b^2*d*e*x^3*atan(c*x))/(3*c) + (a*b*d*e*x)/c^3 + a*b*d
*e*x^4*atan(c*x) - (a*b*d*e*x^3)/(3*c) + (b^2*d*e*x*atan(c*x))/c^3 - (a*b*
d*e*atan((b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)
/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) - (3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^
2 - 3*b*c^2*d*e)))/c^4
```

3.1256. $\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx$

3.1257 $\int (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx$

3.1257.1 Optimal result

Integrand size = 20, antiderivative size = 442

$$
\begin{aligned}
\int (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = {} & \frac{2b^2dex}{3c^2} - \frac{3b^2e^2x}{10c^4} + \frac{b^2e^2x^3}{30c^2} - \frac{2b^2de\arctan(cx)}{3c^3} \\
& + \frac{3b^2e^2\arctan(cx)}{10c^5} - \frac{2bdex^2(a+b\arctan(cx))}{3c} \\
& + \frac{be^2x^2(a+b\arctan(cx))}{5c^3} - \frac{be^2x^4(a+b\arctan(cx))}{10c} \\
& + \frac{id^2(a+b\arctan(cx))^2}{c} - \frac{2ide(a+b\arctan(cx))^2}{3c^3} \\
& + \frac{ie^2(a+b\arctan(cx))^2}{5c^5} + d^2x(a+b\arctan(cx))^2 \\
& + \frac{2}{3}dex^3(a+b\arctan(cx))^2 + \frac{1}{5}e^2x^5(a+b\arctan(cx))^2 \\
& + \frac{2bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} \\
& - \frac{4bde(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} \\
& + \frac{2be^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5} \\
& + \frac{ib^2d^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} \\
& - \frac{2ib^2de\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3} \\
& + \frac{ib^2e^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}
\end{aligned}
$$

output
```
2/3*b^2*d*e*x/c^2-3/10*b^2*e^2*x/c^4+1/30*b^2*e^2*x^3/c^2-2/3*b^2*d*e*arct
an(c*x)/c^3+3/10*b^2*e^2*arctan(c*x)/c^5-2/3*b*d*e*x^2*(a+b*arctan(c*x))/c
+1/5*b*e^2*x^2*(a+b*arctan(c*x))/c^3-1/10*b*e^2*x^4*(a+b*arctan(c*x))/c+I*
b^2*d^2*polylog(2,1-2/(1+I*c*x))/c-2/3*I*b^2*d*e*polylog(2,1-2/(1+I*c*x))/
c^3-2/3*I*d*e*(a+b*arctan(c*x))^2/c^3+d^2*x*(a+b*arctan(c*x))^2+2/3*d*e*x^
3*(a+b*arctan(c*x))^2+1/5*e^2*x^5*(a+b*arctan(c*x))^2+2*b*d^2*(a+b*arctan(
c*x))*ln(2/(1+I*c*x))/c-4/3*b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+2/
5*b*e^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5+I*d^2*(a+b*arctan(c*x))^2/c+
1/5*I*e^2*(a+b*arctan(c*x))^2/c^5+1/5*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c
^5
```

3.1257.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx$$
$$= \frac{9abe^2+30a^2c^5d^2x+20b^2c^3dex-9b^2ce^2x-20abc^4dex^2+6abc^2e^2x^2+20a^2c^5dex^3+b^2c^3e^2x^3-3abc^4e^2x}{}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(9*a*b*e^2 + 30*a^2*c^5*d^2*x + 20*b^2*c^3*d*e*x - 9*b^2*c*e^2*x - 20*a*b*
c^4*d*e*x^2 + 6*a*b*c^2*e^2*x^2 + 20*a^2*c^5*d*e*x^3 + b^2*c^3*e^2*x^3 - 3
*a*b*c^4*e^2*x^4 + 6*a^2*c^5*e^2*x^5 + 2*b^2*((-15*I)*c^4*d^2 + (10*I)*c^2
*d*e - (3*I)*e^2 + c^5*(15*d^2*x + 10*d*e*x^3 + 3*e^2*x^5))*ArcTan[c*x]^2
+ b*ArcTan[c*x]*(4*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*e*(1 + c^
2*x^2)*(-9*e + c^2*(20*d + 3*e*x^2)) + 4*b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^
2)*Log[1 + E^((2*I)*ArcTan[c*x])]) - 30*a*b*c^4*d^2*Log[1 + c^2*x^2] + 20*
a*b*c^2*d*e*Log[1 + c^2*x^2] - 6*a*b*e^2*Log[1 + c^2*x^2] - (2*I)*b^2*(15*
c^4*d^2 - 10*c^2*d*e + 3*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

3.1257.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2 \, dx$$

$$\Big\downarrow \ 5449$$

$$\int \left(d^2(a+b\arctan(cx))^2+2dex^2(a+b\arctan(cx))^2+e^2x^4(a+b\arctan(cx))^2\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ie^2(a+b\arctan(cx))^2}{5c^5}+\frac{2be^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^5}-\frac{2ide(a+b\arctan(cx))^2}{3c^3}-\frac{4bde\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{be^2x^2(a+b\arctan(cx))}{5c^3}+d^2x(a+b\arctan(cx))^2+\frac{id^2(a+b\arctan(cx))^2}{c}+\frac{2bd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{2}{3}dex^3(a+b\arctan(cx))^2-\frac{2bdex^2(a+b\arctan(cx))}{3c}+\frac{1}{5}e^2x^5(a+b\arctan(cx))^2-\frac{be^2x^4(a+b\arctan(cx))}{10c}+\frac{3b^2e^2\arctan(cx)}{10c^5}-\frac{2b^2de\arctan(cx)}{3c^3}+\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2e^2x}{10c^4}-\frac{2ib^2de\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{2b^2dex}{3c^2}+\frac{b^2e^2x^3}{30c^2}+\frac{ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(2*b^2*d*e*x)/(3*c^2) - (3*b^2*e^2*x)/(10*c^4) + (b^2*e^2*x^3)/(30*c^2) -
(2*b^2*d*e*ArcTan[c*x])/(3*c^3) + (3*b^2*e^2*ArcTan[c*x])/(10*c^5) - (2*b*
d*e*x^2*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^2*(a + b*ArcTan[c*x]))/(5*c^
3) - (b*e^2*x^4*(a + b*ArcTan[c*x]))/(10*c) + (I*d^2*(a + b*ArcTan[c*x])^2
)/c - (((2*I)/3)*d*e*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e^2*(a + b*ArcTan
[c*x])^2)/c^5 + d^2*x*(a + b*ArcTan[c*x])^2 + (2*d*e*x^3*(a + b*ArcTan[c*x
])^2)/3 + (e^2*x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*d^2*(a + b*ArcTan[c*x])
*Log[2/(1 + I*c*x)])/c - (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
(3*c^3) + (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + (I*b^
2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (((2*I)/3)*b^2*d*e*PolyLog[2, 1 -
 2/(1 + I*c*x)])/c^3 + ((I/5)*b^2*e^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5
```

3.1257.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

3.1257.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.12

method	result
parts	$a^2\left(\frac{1}{5}x^5e^2+\frac{2}{3}x^3ed+x\,d^2\right)+\frac{b^2\left(\frac{\arctan(cx)^2 c\,e^2x^5}{5}+\frac{2\arctan(cx)^2 cde\,x^3}{3}+\arctan(cx)^2 cx\,d^2-\frac{2\left(5\arctan(cx)d\,c^4\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(\arctan(cx)^2c^5x\,d^2+\frac{2\arctan(cx)^2d\,c^5e\,x^3}{3}+\frac{\arctan(cx)^2e^2c^5x^5}{5}-\frac{2\arctan(cx)d\,c^4e\,x^2}{3}-\right.}{}}{}$
default	$\frac{\frac{a^2\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(\arctan(cx)^2c^5x\,d^2+\frac{2\arctan(cx)^2d\,c^5e\,x^3}{3}+\frac{\arctan(cx)^2e^2c^5x^5}{5}-\frac{2\arctan(cx)d\,c^4e\,x^2}{3}-\right.}{}}{}$
risch	Expression too large to display

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/5*x^5*e^2+2/3*x^3*e*d+x*d^2)+b^2/c*(1/5*arctan(c*x)^2*c*e^2*x^5+2/3
*arctan(c*x)^2*c*d*e*x^3+arctan(c*x)^2*c*x*d^2-2/15/c^4*(5*arctan(c*x)*d*c
^4*e*x^2+3/4*arctan(c*x)*e^2*c^4*x^4-3/2*arctan(c*x)*e^2*c^2*x^2+15/2*arct
an(c*x)*ln(c^2*x^2+1)*c^4*d^2-5*arctan(c*x)*ln(c^2*x^2+1)*c^2*d*e+3/2*arct
an(c*x)*ln(c^2*x^2+1)*e^2-1/4*e*(20*c^3*x*d+e*c^3*x^3-9*e*c*x+(-20*c^2*d+9
*e)*arctan(c*x))-1/4*(30*c^4*d^2-20*c^2*d*e+6*e^2)*(-1/2*I*(ln(c*x-I)*ln(c
^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x
)))+1/2*I*(ln(I+c*x)*ln(c^2*x^2+1)-1/2*ln(I+c*x)^2-dilog(1/2*I*(c*x-I))-ln
(I+c*x)*ln(1/2*I*(c*x-I))))))+2*a*b/c*(1/5*arctan(c*x)*c*e^2*x^5+2/3*arcta
n(c*x)*c*d*e*x^3+arctan(c*x)*c*x*d^2-1/15/c^4*(5*d*c^4*e*x^2+3/4*e^2*c^4*x
^4-3/2*e^2*c^2*x^2+1/2*(15*c^4*d^2-10*c^2*d*e+3*e^2)*ln(c^2*x^2+1)))
```

3.1257. $\int (d+ex^2)^2\,(a+b\arctan(cx))^2\,dx$

3.1257.5 Fricas [F]

$$\int \left(d+e x^2\right)^2 (a+b \arctan (c x))^2 \, dx=\int { {\left(e x^{2} + d\right)}^{2} {\left(b \arctan \left(c x\right) + a\right)}^{2} \,d x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e*
x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*a
rctan(c*x), x)
```

3.1257.6 Sympy [F]

$$\int \left(d+e x^2\right)^2 (a+b \arctan (c x))^2 \, dx=\int \left(a + b \operatorname{atan}{\left(c x \right)}\right)^{2} \left(d + e x^{2}\right)^{2}\, dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

3.1257.7 Maxima [F]

$$\int \left(d+e x^2\right)^2 (a+b \arctan (c x))^2 \, dx=\int { {\left(e x^{2} + d\right)}^{2} {\left(b \arctan \left(c x\right) + a\right)}^{2} \,d x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*e^2*x^5 + 2/3*a^2*d*e*x^3 + 180*b^2*c^2*e^2*integrate(1/240*x^6*ar
ctan(c*x)^2/(c^2*x^2 + 1), x) + 15*b^2*c^2*e^2*integrate(1/240*x^6*log(c^2
*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*e^2*integrate(1/240*x^6*log(c^2
*x^2 + 1)/(c^2*x^2 + 1), x) + 360*b^2*c^2*d*e*integrate(1/240*x^4*arctan(c
*x)^2/(c^2*x^2 + 1), x) + 30*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x^2 +
 1)^2/(c^2*x^2 + 1), x) + 40*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x^2 +
 1)/(c^2*x^2 + 1), x) + 180*b^2*c^2*d^2*integrate(1/240*x^2*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 15*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)^2/
(c^2*x^2 + 1), x) + 60*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)/(c
^2*x^2 + 1), x) + 1/4*b^2*d^2*arctan(c*x)^3/c - 24*b^2*c*e^2*integrate(1/2
40*x^5*arctan(c*x)/(c^2*x^2 + 1), x) - 80*b^2*c*d*e*integrate(1/240*x^3*ar
ctan(c*x)/(c^2*x^2 + 1), x) - 120*b^2*c*d^2*integrate(1/240*x*arctan(c*x)/
(c^2*x^2 + 1), x) + 2/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*a*b*d*e + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*lo
g(c^2*x^2 + 1)/c^6))*a*b*e^2 + a^2*d^2*x + 180*b^2*e^2*integrate(1/240*x^4
*arctan(c*x)^2/(c^2*x^2 + 1), x) + 15*b^2*e^2*integrate(1/240*x^4*log(c^2*
x^2 + 1)^2/(c^2*x^2 + 1), x) + 360*b^2*d*e*integrate(1/240*x^2*arctan(c*x)
^2/(c^2*x^2 + 1), x) + 30*b^2*d*e*integrate(1/240*x^2*log(c^2*x^2 + 1)^2/(
c^2*x^2 + 1), x) + 15*b^2*d^2*integrate(1/240*log(c^2*x^2 + 1)^2/(c^2*x^2
+ 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2/c + 1/60*(3*b...
```

3.1257.8 Giac [F]

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx=\int \left(ex^2+d\right)^2(b\arctan\left(cx\right)+a)^2\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1257.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = \int (a+b\,\mathrm{atan}(c\,x))^2 \left(e\,x^2+d\right)^2 dx$$

input
```
int((a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
int((a + b*atan(c*x))^2*(d + e*x^2)^2, x)
```

3.1258 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x}\,dx$

3.1258.1 Optimal result

Integrand size = 23, antiderivative size = 355

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x}\,dx &= -\frac{2abdex}{c}+\frac{abe^2x}{2c^3}+\frac{b^2e^2x^2}{12c^2}-\frac{2b^2dex\arctan(cx)}{c}\\
&+\frac{b^2e^2x\arctan(cx)}{2c^3}-\frac{be^2x^3(a+b\arctan(cx))}{6c}\\
&+\frac{de(a+b\arctan(cx))^2}{c^2}-\frac{e^2(a+b\arctan(cx))^2}{4c^4}\\
&+dex^2(a+b\arctan(cx))^2+\frac{1}{4}e^2x^4(a+b\arctan(cx))^2\\
&+2d^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\
&+\frac{b^2de\log(1+c^2x^2)}{c^2}-\frac{b^2e^2\log(1+c^2x^2)}{3c^4}\\
&-ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\
&+ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\
&-\frac{1}{2}b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\
&+\frac{1}{2}b^2d^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-2*a*b*d*e*x/c+1/2*a*b*e^2*x/c^3+1/12*b^2*e^2*x^2/c^2-2*b^2*d*e*x*arctan(c
*x)/c+1/2*b^2*e^2*x*arctan(c*x)/c^3-1/6*b*e^2*x^3*(a+b*arctan(c*x))/c+d*e*
(a+b*arctan(c*x))^2/c^2-1/4*e^2*(a+b*arctan(c*x))^2/c^4+d*e*x^2*(a+b*arcta
n(c*x))^2+1/4*e^2*x^4*(a+b*arctan(c*x))^2-2*d^2*(a+b*arctan(c*x))^2*arctan
h(-1+2/(1+I*c*x))+b^2*d*e*ln(c^2*x^2+1)/c^2-1/3*b^2*e^2*ln(c^2*x^2+1)/c^4+
I*b*d^2*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-I*b*d^2*(a+b*arctan(c*
x))*polylog(2,1-2/(1+I*c*x))-1/2*b^2*d^2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*
d^2*polylog(3,-1+2/(1+I*c*x))
```

3.1258.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.10

$$
\begin{aligned}
&\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx \\
&= a^2dex^2 + \frac{1}{4}a^2e^2x^4 + \frac{2abde(-cx+(1+c^2x^2)\arctan(cx))}{c^2} \\
&\quad + \frac{abe^2(3cx-c^3x^3+3(-1+c^4x^4)\arctan(cx))}{6c^4} + a^2d^2\log(x) \\
&\quad + \frac{b^2e^2(1+c^2x^2+(6cx-2c^3x^3)\arctan(cx)+3(-1+c^4x^4)\arctan(cx)^2-4\log(1+c^2x^2))}{12c^4} \\
&\quad + \frac{b^2de(-2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2+\log(1+c^2x^2))}{c^2} \\
&\quad + iabd^2(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx)) + b^2d^2\bigg(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3 \\
&\quad + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) \\
&\quad + i\arctan(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) \\
&\quad + \frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\bigg)
\end{aligned}
$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
a^2*d*e*x^2 + (a^2*e^2*x^4)/4 + (2*a*b*d*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[
c*x]))/c^2 + (a*b*e^2*(3*c*x - c^3*x^3 + 3*(-1 + c^4*x^4)*ArcTan[c*x]))/(6
*c^4) + a^2*d^2*Log[x] + (b^2*e^2*(1 + c^2*x^2 + (6*c*x - 2*c^3*x^3)*ArcTa
n[c*x] + 3*(-1 + c^4*x^4)*ArcTan[c*x]^2 - 4*Log[1 + c^2*x^2]))/(12*c^4) +
(b^2*d*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x
^2]))/c^2 + I*a*b*d^2*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^2*d^2
*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*
I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan
[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2
*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^(
(2*I)*ArcTan[c*x])]/2)
```

3.1258.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x} \, dx$$

$$\Big\downarrow \text{ 5515}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{x}+2dex(a+b\arctan(cx))^2+e^2x^3(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2d^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{e^2(a+b\arctan(cx))^2}{4c^4}+\frac{de(a+b\arctan(cx))^2}{c^2}-ibd^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+dex^2(a+b\arctan(cx))^2+\frac{1}{4}e^2x^4(a+b\arctan(cx))^2-\frac{be^2x^3(a+b\arctan(cx))}{6c}+\frac{abe^2x}{2c^3}-\frac{2abdex}{c}+\frac{b^2e^2x\arctan(cx)}{2c^3}-\frac{2b^2dex\arctan(cx)}{c}+\frac{b^2de\log(c^2x^2+1)}{c^2}+\frac{b^2e^2x^2}{12c^2}-\frac{b^2e^2\log(c^2x^2+1)}{3c^4}-\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

3.1258. $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x}\,dx$

output
```
(-2*a*b*d*e*x)/c + (a*b*e^2*x)/(2*c^3) + (b^2*e^2*x^2)/(12*c^2) - (2*b^2*d
*e*x*ArcTan[c*x])/c + (b^2*e^2*x*ArcTan[c*x])/(2*c^3) - (b*e^2*x^3*(a + b*
ArcTan[c*x]))/(6*c) + (d*e*(a + b*ArcTan[c*x])^2)/c^2 - (e^2*(a + b*ArcTan
[c*x])^2)/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x])^2 + (e^2*x^4*(a + b*ArcTan
[c*x])^2)/4 + 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (b^
2*d*e*Log[1 + c^2*x^2])/c^2 - (b^2*e^2*Log[1 + c^2*x^2])/(3*c^4) - I*b*d^2
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan
[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c
*x)])/2 + (b^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

3.1258.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1258.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.60 (sec) , antiderivative size = 1524, normalized size of antiderivative = 4.29

method	result	size
derivativedivides	Expression too large to display	1524
default	Expression too large to display	1524
parts	Expression too large to display	1526

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*ln(c*x)+b^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2*arctan(c*x)^2+2*
I*b^2/c^2*arctan(c*x)*d*e-1/2*I*b^2*Pi*d^2*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1
)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*Pi*d^2*csgn(((1+I
*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*
b^2*Pi*d^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
^3*arctan(c*x)^2+a^2*d*e*x^2+1/4*b^2*arctan(c*x)^2*e^2*x^4+1/12*b^2*e^2*x^
2/c^2+1/2*b^2*e^2*x*arctan(c*x)/c^3+b^2/c^2*e*arctan(c*x)^2*d-2*b^2*d*e*x*
arctan(c*x)/c+2*a*b/c^4*(arctan(c*x)*d*c^4*e*x^2+1/4*arctan(c*x)*e^2*c^4*x
^4+arctan(c*x)*c^4*d^2*ln(c*x)-1/4*e*(4*c^3*x*d+1/3*e*c^3*x^3-e*c*x+(-4*c^
2*d+e)*arctan(c*x))-c^4*d^2*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1
-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)))-1/4*b^2/c^4*e^2*arctan
(c*x)^2-1/6*b^2/c*e^2*arctan(c*x)*x^3+I*b^2*d^2*arctan(c*x)*polylog(2,-(1+
I*c*x)^2/(c^2*x^2+1))-2*b^2/c^2*d*e*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-2*I*b^2*
polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2*arctan(c*x)+1/2*I*b^2*Pi*d^2*ar
ctan(c*x)^2-2*I*b^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2*arctan(c*x
)+1/12*b^2/c^4*e^2+2*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2-1/2*b
^2*d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2*polylog(3,(1+I*c*x)/(c^2*
x^2+1)^(1/2))*d^2+2/3*b^2/c^4*e^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+b^2*arctan
(c*x)^2*d^2*ln(c*x)-b^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)*d^2*arctan(c*x)^2+b^
2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2*arctan(c*x)^2+b^2*arctan(c*x)^2...
```

3.1258.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)^2 (b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e
*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*
arctan(c*x))/x, x)
```

3.1258.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2 (d+ex^2)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x, x)
```

3.1258.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/4*a^2*e^2*x^4 + 12*b^2*c^2*e^2*integrate(1/16*x^6*arctan(c*x)^2/(c^2*x^3
 + x), x) + b^2*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)^2/(c^2*x^3 + x
), x) + 32*a*b*c^2*e^2*integrate(1/16*x^6*arctan(c*x)/(c^2*x^3 + x), x) +
b^2*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 24*b^2
*c^2*d*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*b^2*c^2*d*
e*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 64*a*b*c^2*d*e
*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 4*b^2*c^2*d*e*integrat
e(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d^2*integrate(1
/16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d^2*integrate(1/16*x^
2*arctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*d^2*log(c^2*x^2 + 1)^3 + a^2*d*
e*x^2 - 2*b^2*c*e^2*integrate(1/16*x^5*arctan(c*x)/(c^2*x^3 + x), x) - 8*b
^2*c*d*e*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*e^2*int
egrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*e^2*integrate(1/16*x
^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*e^2*integrate(1/16*x^4*ar
ctan(c*x)/(c^2*x^3 + x), x) + 24*b^2*d*e*integrate(1/16*x^2*arctan(c*x)^2/
(c^2*x^3 + x), x) + 64*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^3 + x
), x) + 12*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*d^
2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d^2*integra
te(1/16*arctan(c*x)/(c^2*x^3 + x), x) + 1/48*b^2*d*e*log(c^2*x^2 + 1)^3/c^
2 + a^2*d^2*log(x) + 1/16*(b^2*e^2*x^4 + 4*b^2*d*e*x^2)*arctan(c*x)^2 -...
```

3.1258.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
Timed out
```

3.1258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)^2}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x, x)
```

3.1259 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^2}\,dx$

3.1259.1 Optimal result

Integrand size = 23, antiderivative size = 343

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^2}\,dx &= \frac{b^2e^2x}{3c^2}-\frac{b^2e^2\arctan(cx)}{3c^3}-\frac{be^2x^2(a+b\arctan(cx))}{3c}\\ &\quad-icd^2(a+b\arctan(cx))^2+\frac{2ide(a+b\arctan(cx))^2}{c}\\ &\quad-\frac{ie^2(a+b\arctan(cx))^2}{3c^3}-\frac{d^2(a+b\arctan(cx))^2}{x}\\ &\quad+2dex(a+b\arctan(cx))^2+\frac{1}{3}e^2x^3(a+b\arctan(cx))^2\\ &\quad+\frac{4bde(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}\\ &\quad-\frac{2be^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\ &\quad+2bcd^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ &\quad-ib^2cd^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\ &\quad+\frac{2ib^2de\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}\\ &\quad-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\end{aligned}$$

output
```
1/3*b^2*e^2*x/c^2-1/3*b^2*e^2*arctan(c*x)/c^3-1/3*b*e^2*x^2*(a+b*arctan(c*
x))/c-I*c*d^2*(a+b*arctan(c*x))^2+2*I*d*e*(a+b*arctan(c*x))^2/c-1/3*I*e^2*
(a+b*arctan(c*x))^2/c^3-d^2*(a+b*arctan(c*x))^2/x+2*d*e*x*(a+b*arctan(c*x)
)^2+1/3*e^2*x^3*(a+b*arctan(c*x))^2+4*b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))/c-2/3*b*e^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+2*b*c*d^2*(a+b*arcta
n(c*x))*ln(2-2/(1-I*c*x))-I*b^2*c*d^2*polylog(2,-1+2/(1-I*c*x))+2*I*b^2*d*
e*polylog(2,1-2/(1+I*c*x))/c-1/3*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c^3
```

3.1259.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx$$

$$= \frac{1}{3}\left(-\frac{3a^2d^2}{x} + 6a^2dex + a^2e^2x^3 + \frac{6abde(2cx\arctan(cx) - \log(1+c^2x^2))}{c} + \frac{abe^2(-c^2x^2 + 2c^3x^3\arctan(cx) + \log(1+c^2x^2))}{c^3} - \frac{3abd^2(2\arctan(cx) + cx(-2\log(cx) + \log(1+c^2x^2)))}{x} + \frac{6b^2de\left(\arctan(cx)\left((-i+cx)\arctan(cx) + 2\log\left(1+e^{2i\arctan(cx)}\right)\right) - i\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right)}{c} + \frac{b^2e^2\left(cx + (i+c^3x^3)\arctan(cx)^2 - \arctan(cx)\left(1+c^2x^2+2\log\left(1+e^{2i\arctan(cx)}\right)\right) + i\,\text{PolyLog}\left(2,-e^{2i}\right.\right.}{c^3} + 3b^2cd^2\left(\arctan(cx)\left(\left(-i-\frac{1}{cx}\right)\arctan(cx) + 2\log\left(1-e^{2i\arctan(cx)}\right)\right) - i\,\text{PolyLog}\left(2,e^{2i\arctan(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
((-3*a^2*d^2)/x + 6*a^2*d*e*x + a^2*e^2*x^3 + (6*a*b*d*e*(2*c*x*ArcTan[c*x
] - Log[1 + c^2*x^2]))/c + (a*b*e^2*(-(c^2*x^2) + 2*c^3*x^3*ArcTan[c*x] +
Log[1 + c^2*x^2]))/c^3 - (3*a*b*d^2*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Lo
g[1 + c^2*x^2])))/x + (6*b^2*d*e*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*
Log[1 + E^((2*I)*ArcTan[c*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
 + (b^2*e^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2
+ 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
))/c^3 + 3*b^2*c*d^2*(ArcTan[c*x]*((-I - 1/(c*x))*ArcTan[c*x] + 2*Log[1 -
E^((2*I)*ArcTan[c*x])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/3
```

3.1259.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2} \, dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{x^2}+2de(a+b\arctan(cx))^2+e^2x^2(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ie^2(a+b\arctan(cx))^2}{3c^3}-\frac{2be^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}-icd^2(a+b\arctan(cx))^2-\frac{d^2(a+b\arctan(cx))^2}{x}+2bcd^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+2dex(a+b\arctan(cx))^2+\frac{2ide(a+b\arctan(cx))^2}{c}+\frac{4bde\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{1}{3}e^2x^3(a+b\arctan(cx))^2-\frac{be^2x^2(a+b\arctan(cx))}{3c}-\frac{b^2e^2\arctan(cx)}{3c^3}-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2e^2x}{3c^2}-ib^2cd^2\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{2ib^2de\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (b*e^2*x^2*(a + b*Ar
cTan[c*x]))/(3*c) - I*c*d^2*(a + b*ArcTan[c*x])^2 + ((2*I)*d*e*(a + b*ArcT
an[c*x])^2)/c - ((I/3)*e^2*(a + b*ArcTan[c*x])^2)/c^3 - (d^2*(a + b*ArcTan
[c*x])^2)/x + 2*d*e*x*(a + b*ArcTan[c*x])^2 + (e^2*x^3*(a + b*ArcTan[c*x])
^2)/3 + (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + 2*b*c*d^2*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)] + ((
2*I)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I/3)*b^2*e^2*PolyLog[2,
1 - 2/(1 + I*c*x)])/c^3
```

3.1259.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1259.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.53

method	result
derivativedivides	$c\left(\frac{a^2\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b^2\left(2\arctan(cx)^2c^3xde+\frac{\arctan(cx)^2e^2c^3x^3}{3}-\frac{\arctan(cx)^2c^3d^2}{x}-\frac{\arctan(cx)e^2c^2x^2}{3}+2\right.}{}\right.$
default	$c\left(\frac{a^2\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b^2\left(2\arctan(cx)^2c^3xde+\frac{\arctan(cx)^2e^2c^3x^3}{3}-\frac{\arctan(cx)^2c^3d^2}{x}-\frac{\arctan(cx)e^2c^2x^2}{3}+2\right.}{}\right.$
parts	$a^2\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+b^2c\left(\frac{\arctan(cx)^2e^2x^3}{3c}+\frac{2\arctan(cx)^2xde}{c}-\frac{\arctan(cx)^2d^2}{cx}-\frac{2\left(\frac{\arctan(cx)e^2c}{2}\right.}{}\right.$

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

3.1259. $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^2}\,dx$

output
```
c*(a^2/c^4*(2*c^3*d*e*x+1/3*e^2*c^3*x^3-c^3*d^2/x)+b^2/c^4*(2*arctan(c*x)^
2*c^3*x*d*e+1/3*arctan(c*x)^2*e^2*c^3*x^3-arctan(c*x)^2*c^3*d^2/x-1/3*arct
an(c*x)*e^2*c^2*x^2+2*arctan(c*x)*c^4*d^2*ln(c*x)-arctan(c*x)*ln(c^2*x^2+1
)*c^4*d^2-2*arctan(c*x)*ln(c^2*x^2+1)*c^2*d*e+1/3*arctan(c*x)*ln(c^2*x^2+1
)*e^2+1/3*e^2*(c*x-arctan(c*x))+1/3*(3*c^4*d^2+6*c^2*d*e-e^2)*(-1/2*I*(ln(
c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1
/2*I*(I+c*x)))+1/2*I*(ln(I+c*x)*ln(c^2*x^2+1)-1/2*ln(I+c*x)^2-dilog(1/2*I*
(c*x-I))-ln(I+c*x)*ln(1/2*I*(c*x-I))))-2*c^4*d^2*(-1/2*I*ln(c*x)*ln(1+I*c*
x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)))+2
*a*b/c^4*(2*arctan(c*x)*c^3*d*e*x+1/3*arctan(c*x)*e^2*c^3*x^3-arctan(c*x)*
c^3*d^2/x-1/6*e^2*c^2*x^2-1/6*(3*c^4*d^2+6*c^2*d*e-e^2)*ln(c^2*x^2+1)+c^4*
d^2*ln(c*x)))
```

3.1259.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e
*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*
arctan(c*x))/x^2, x)
```

3.1259.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^2 (d+ex^2)^2}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**2, x)
```

3.1259.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
1/3*a^2*e^2*x^3 - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*
d^2 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e^2
 + 2*a^2*d*e*x + 2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d*e/c - a^2*
d^2/x + 1/48*(4*(b^2*e^2*x^4 + 6*b^2*d*e*x^2 - 3*b^2*d^2)*arctan(c*x)^2 -
(b^2*e^2*x^4 + 6*b^2*d*e*x^2 - 3*b^2*d^2)*log(c^2*x^2 + 1)^2 + 12*(b^2*c*d
^2*arctan(c*x)^3 + 144*b^2*c^2*e^2*integrate(1/48*x^6*arctan(c*x)^2/(c^2*x
^4 + x^2), x) + 12*b^2*c^2*e^2*integrate(1/48*x^6*log(c^2*x^2 + 1)^2/(c^2*
x^4 + x^2), x) + 16*b^2*c^2*e^2*integrate(1/48*x^6*log(c^2*x^2 + 1)/(c^2*x
^4 + x^2), x) + 288*b^2*c^2*d*e*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^4
+ x^2), x) + 24*b^2*c^2*d*e*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4
 + x^2), x) + 96*b^2*c^2*d*e*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^4
+ x^2), x) + 12*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4
 + x^2), x) - 48*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^4
+ x^2), x) + 2*b^2*d*e*arctan(c*x)^3/c - 32*b^2*c*e^2*integrate(1/48*x^5*a
rctan(c*x)/(c^2*x^4 + x^2), x) - 192*b^2*c*d*e*integrate(1/48*x^3*arctan(c
*x)/(c^2*x^4 + x^2), x) + 96*b^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x
^4 + x^2), x) + 144*b^2*e^2*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^4 + x^
2), x) + 12*b^2*e^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2),
 x) + 24*b^2*d*e*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x)
 + 144*b^2*d^2*integrate(1/48*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 12*b^...
```

3.1259.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.1259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)^2}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^2, x)
```

3.1260 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^3}\,dx$

3.1260.1 Optimal result

Integrand size = 23, antiderivative size = 320

$$\begin{aligned}
\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^3}\,dx = & -\frac{abe^2x}{c}-\frac{b^2e^2x\arctan(cx)}{c}-\frac{bcd^2(a+b\arctan(cx))}{x} \\
& -\frac{1}{2}c^2d^2(a+b\arctan(cx))^2+\frac{e^2(a+b\arctan(cx))^2}{2c^2} \\
& -\frac{d^2(a+b\arctan(cx))^2}{2x^2}+\frac{1}{2}e^2x^2(a+b\arctan(cx))^2 \\
& +4de(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
& +b^2c^2d^2\log(x)-\frac{1}{2}b^2c^2d^2\log\left(1+c^2x^2\right) \\
& +\frac{b^2e^2\log\left(1+c^2x^2\right)}{2c^2} \\
& -2ibde(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\
& +2ibde(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\
& -b^2de\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) \\
& +b^2de\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)
\end{aligned}$$

output
```
-a*b*e^2*x/c-b^2*e^2*x*arctan(c*x)/c-b*c*d^2*(a+b*arctan(c*x))/x-1/2*c^2*d
^2*(a+b*arctan(c*x))^2+1/2*e^2*(a+b*arctan(c*x))^2/c^2-1/2*d^2*(a+b*arctan
(c*x))^2/x^2+1/2*e^2*x^2*(a+b*arctan(c*x))^2-4*d*e*(a+b*arctan(c*x))^2*arc
tanh(-1+2/(1+I*c*x))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(c^2*x^2+1)+1/2*b
^2*e^2*ln(c^2*x^2+1)/c^2-2*I*b*d*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*
x))+2*I*b*d*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-b^2*d*e*polylog(
3,1-2/(1+I*c*x))+b^2*d*e*polylog(3,-1+2/(1+I*c*x))
```

3.1260.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx \\
&= \frac{1}{2}\Bigg(-\frac{a^2d^2}{x^2}+a^2e^2x^2+\frac{2abe^2(-cx+(1+c^2x^2)\arctan(cx))}{c^2} \\
&\quad -\frac{2abd^2(\arctan(cx)+cx(1+cx\arctan(cx)))}{x^2}+4a^2de\log(x) \\
&\quad -\frac{b^2d^2\left(2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{x^2} \\
&\quad +\frac{b^2e^2(-2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2+\log(1+c^2x^2))}{c^2} \\
&\quad +4iabde(\operatorname{PolyLog}(2,-icx)-\operatorname{PolyLog}(2,icx))+\frac{1}{6}b^2de\big(-i\pi^3+16i\arctan(cx)^3 \\
&\quad +24\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)-24\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) \\
&\quad +24i\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)+24i\arctan(cx)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) \\
&\quad +12\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)-12\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\big)\Bigg)
\end{aligned}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
(-((a^2*d^2)/x^2) + a^2*e^2*x^2 + (2*a*b*e^2*(-(c*x) + (1 + c^2*x^2)*ArcTa
n[c*x]))/c^2 - (2*a*b*d^2*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 +
 4*a^2*d*e*Log[x] - (b^2*d^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x
]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/x^2 + (b^2*e^2*(-2*c*x*ArcT
an[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]))/c^2 + (4*I)*a*b
*d*e*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*d*e*((-I)*Pi^3 + (1
6*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24
*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog
[2, E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcT
an[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I
)*ArcTan[c*x])]))/6)/2
```

3.1260.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3} dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{x^3}+\frac{2de(a+b\arctan(cx))^2}{x}+e^2x(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$4de\,\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}c^2d^2(a+b\arctan(cx))^2+\frac{e^2(a+b\arctan(cx))^2}{2c^2}-\frac{d^2(a+b\arctan(cx))^2}{2x^2}-\frac{bcd^2(a+b\arctan(cx))}{x}-2ibde\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+2ibde\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{1}{2}e^2x^2(a+b\arctan(cx))^2-\frac{abe^2x}{c}-\frac{b^2e^2x\arctan(cx)}{c}-\frac{1}{2}b^2c^2d^2\log\left(c^2x^2+1\right)+b^2c^2d^2\log(x)+\frac{b^2e^2\log\left(c^2x^2+1\right)}{2c^2}-b^2de\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+b^2de\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

3.1260. $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^3}dx$

output
```
-((a*b*e^2*x)/c) - (b^2*e^2*x*ArcTan[c*x])/c - (b*c*d^2*(a + b*ArcTan[c*x]
))/x - (c^2*d^2*(a + b*ArcTan[c*x])^2)/2 + (e^2*(a + b*ArcTan[c*x])^2)/(2*
c^2) - (d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) + (e^2*x^2*(a + b*ArcTan[c*x])^
2)/2 + 4*d*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^
2*Log[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (b^2*e^2*Log[1 + c^2*x^2])/(
2*c^2) - (2*I)*b*d*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + (
2*I)*b*d*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - b^2*d*e*Po
lyLog[3, 1 - 2/(1 + I*c*x)] + b^2*d*e*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

3.1260.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1260.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.75 (sec) , antiderivative size = 1500, normalized size of antiderivative = 4.69

method	result	size
parts	Expression too large to display	1500
derivativedivides	Expression too large to display	1521
default	Expression too large to display	1521

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
I*b^2*d*e*Pi*arctan(c*x)^2+2*I*b^2*d*e*arctan(c*x)*polylog(2,-(1+I*c*x)^2/
(c^2*x^2+1))-4*I*b^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)*d
*e-4*I*b^2*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)*d*e-b^2*e^2*
x*arctan(c*x)/c+2*a*b*c^2*(1/2*arctan(c*x)/c^2*e^2*x^2+2*arctan(c*x)/c^2*d
*e*ln(c*x)-1/2*arctan(c*x)*d^2/c^2/x^2-1/2/c^4*(c*x*e^2+c^3*d^2/x+(c^4*d^2
-e^2)*arctan(c*x)+4*c^2*d*e*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1
-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))))+I*b^2*d*e*Pi*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+4*b^2*
polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))*d*e-b^2*d*e*polylog(3,-(1+I*c*x)^2/
(c^2*x^2+1))+4*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*d*e+1/2*b^2*arc
tan(c*x)^2*e^2*x^2-1/2*b^2*arctan(c*x)^2*d^2/x^2-b^2/c^2*e^2*ln((1+I*c*x)^
2/(c^2*x^2+1)+1)+1/2*b^2/c^2*e^2*arctan(c*x)^2+b^2*c^2*ln(1+(1+I*c*x)/(c^2
*x^2+1)^(1/2))*d^2+b^2*c^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)*d^2-1/2*b^2*c
^2*arctan(c*x)^2*d^2-I*b^2*d*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c
*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*b^2*d*e*Pi*csgn(((1+I*c*x)^2/(c^2*
x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+I*b^2*d*e*Pi*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+
I*b^2*d*e*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan...
```

3.1260.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e
*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*
arctan(c*x))/x^3, x)
```

3.1260.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3} dx = \int \frac{(a+b\operatorname{atan}(cx))^2 (d+ex^2)^2}{x^3} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**3, x)
```

3.1260.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3} dx = \int \frac{(ex^2+d)^2 (b\arctan(cx)+a)^2}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*a^2*e^2*x^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^2 + 2*
a^2*d*e*log(x) - 1/2*a^2*d^2/x^2 + 1/96*((1152*b^2*c^2*e^2*integrate(1/16*
x^6*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*e^2*integrate(1/16*x^6*
log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^2*e^2*integrate(1/16*x
^6*arctan(c*x)/(c^2*x^5 + x^3), x) + 192*b^2*c^2*e^2*integrate(1/16*x^6*lo
g(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 2304*b^2*c^2*d*e*integrate(1/16*x^4*a
rctan(c*x)^2/(c^2*x^5 + x^3), x) + 6144*a*b*c^2*d*e*integrate(1/16*x^4*arc
tan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*d^2*integrate(1/16*x^2*arctan(
c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x^2
 + 1)^2/(c^2*x^5 + x^3), x) - 192*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x
^2 + 1)/(c^2*x^5 + x^3), x) + 2*b^2*d*e*log(c^2*x^2 + 1)^3 - 384*b^2*c*e^2
*integrate(1/16*x^5*arctan(c*x)/(c^2*x^5 + x^3), x) + 384*b^2*c*d^2*integr
ate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*e^2*integrate(1/16*x
^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e^2*integrate(1/16*x^4*arc
tan(c*x)/(c^2*x^5 + x^3), x) + 2304*b^2*d*e*integrate(1/16*x^2*arctan(c*x)
^2/(c^2*x^5 + x^3), x) + 192*b^2*d*e*integrate(1/16*x^2*log(c^2*x^2 + 1)^2
/(c^2*x^5 + x^3), x) + 6144*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^
5 + x^3), x) + 1152*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^5 + x^3),
x) + 96*b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + b^
2*e^2*log(c^2*x^2 + 1)^3/c^2)*x^2 + 12*(b^2*e^2*x^4 - b^2*d^2)*arctan(c...
```

3.1260.8 Giac [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3} dx = \text{Timed out}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.1260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3} dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)^2}{x^3} dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^3, x)
```

3.1261 $\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx$

3.1261.1 Optimal result

Integrand size = 23, antiderivative size = 590

$$
\begin{aligned}
\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = & -\frac{abx}{ce}-\frac{b^2x\arctan(cx)}{ce}+\frac{(a+b\arctan(cx))^2}{2c^2e} \\
& +\frac{x^2(a+b\arctan(cx))^2}{2e}+\frac{d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} \\
& -\frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2} \\
& -\frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2} \\
& +\frac{b^2\log(1+c^2x^2)}{2c^2e} \\
& -\frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^2} \\
& +\frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2} \\
& +\frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2} \\
& +\frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^2} \\
& -\frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2} \\
& -\frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^2}
\end{aligned}
$$

output
```
-a*b*x/c/e-b^2*x*arctan(c*x)/c/e+1/2*(a+b*arctan(c*x))^2/c^2/e+1/2*x^2*(a+
b*arctan(c*x))^2/e+d*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^2+1/2*b^2*ln(c^
2*x^2+1)/c^2/e-1/2*d*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-
I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2-1/2*d*(a+b*arctan(c*x))^2*ln(2*c*((-d
)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2-I*b*d*(a+b*arct
an(c*x))*polylog(2,1-2/(1-I*c*x))/e^2+1/2*I*b*d*(a+b*arctan(c*x))*polylog(
2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/2
*I*b*d*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/
(c*(-d)^(1/2)+I*e^(1/2)))/e^2+1/2*b^2*d*polylog(3,1-2/(1-I*c*x))/e^2-1/4*b
^2*d*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1
/2)))/e^2-1/4*b^2*d*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-
d)^(1/2)+I*e^(1/2)))/e^2
```

3.1261.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1520 vs. 2(590) = 1180.

Time = 13.44 (sec) , antiderivative size = 1520, normalized size of antiderivative = 2.58

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
(-4*a*b*c*e*x + 2*a^2*c^2*e*x^2 + 4*a*b*e*ArcTan[c*x] - 4*b^2*c*e*x*ArcTan
[c*x] + 4*a*b*c^2*e*x^2*ArcTan[c*x] + 2*b^2*e*ArcTan[c*x]^2 + 2*b^2*c^2*e*
x^2*ArcTan[c*x]^2 - (8*I)*a*b*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcT
an[(c*e*x)/Sqrt[c^2*d*e]] + 8*a*b*c^2*d*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTa
n[c*x])] + 4*b^2*c^2*d*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*a*
b*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2
*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 4*a*b*c^2*d*ArcTan[c*x]*Log[1
 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 4*
b^2*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d +
e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 4*b^2*c^2*d*Arc
Tan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(
c^2*d - e)] - 4*a*b*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[(-2*Sqrt[c
^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1
+ E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 4*a*b*c^2*d*ArcTan[c*x]*Log[(-2*S
qrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*
d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 4*b^2*c^2*d*ArcSin[Sqrt[(c^2
*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x])
+ e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2
*d - e)] - 4*b^2*c^2*d*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan
[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*...
```

3.1261.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5451, 5361, 5451, 2009, 5419, 5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x(a+b\arctan(cx))^2dx}{e}-\frac{d\int \frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\int \frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{ 5515}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{e}-\frac{d\int\left(\frac{(a+b\arctan(cx))^2}{2\sqrt{e}(\sqrt{e}x+\sqrt{-d})}-\frac{(a+b\arctan(cx))^2}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}\right)dx}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e}-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2e}+\frac{(a+b\arctan(cx))^2\log\cdots}{\cdots}\right.}{\cdots}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

3.1261. $\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx$

output
```
((x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) +
 (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2))/e - (d*(-(((a
+ b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e) +
 ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*e) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/
(1 - I*c*x)])/e - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e - ((I/2)*b*(a
+ b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e)
+ (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e
])*(1 - I*c*x))])/(4*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))
/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e)))/e
```

3.1261.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.1261. $\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx$

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1261.4 Maple [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{e\,x^2+d}dx$$

input
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

output
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

3.1261.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e*x^2
+ d), x)
```

3.1261.6 Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x^3(a+b\operatorname{atan}(cx))^2}{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

3.1261.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(x^2/e - d*log(e*x^2 + d)/e^2) + integrate((b^2*x^3*arctan(c*x)^2
+ 2*a*b*x^3*arctan(c*x))/(e*x^2 + d), x)
```

3.1261.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

3.1262 $\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx$

3.1262.1 Optimal result

Integrand size = 23, antiderivative size = 554

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx &= \frac{i(a+b\arctan(cx))^2}{ce}+\frac{x(a+b\arctan(cx))^2}{e}\\
&+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce}\\
&+\frac{\sqrt{-d}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}\\
&-\frac{\sqrt{-d}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}\\
&+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce}\\
&-\frac{ib\sqrt{-d}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}\\
&+\frac{ib\sqrt{-d}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}\\
&+\frac{b^2\sqrt{-d}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^{3/2}}\\
&-\frac{b^2\sqrt{-d}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^{3/2}}\end{aligned}$$

output
```
I*(a+b*arctan(c*x))^2/c/e+x*(a+b*arctan(c*x))^2/e+2*b*(a+b*arctan(c*x))*ln
(2/(1+I*c*x))/c/e+I*b^2*polylog(2,1-2/(1+I*c*x))/c/e+1/2*(a+b*arctan(c*x))
^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^
(1/2)/e^(3/2)-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c
*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*(a+b*arctan(c*x))
*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2))
)*(-d)^(1/2)/e^(3/2)+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)
+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)+1/4*b^2
*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2))
)*(-d)^(1/2)/e^(3/2)-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c
*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/e^(3/2)
```

3.1262.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx=\$\text{Aborted}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
$Aborted
```

3.1262.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5451, 5345, 5449, 2009, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int (a+b\arctan(cx))^2dx}{e}-\frac{d\int \frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5345}$$

$$\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\int\frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5449}$$

$$\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\int\left(\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

$$\Big\downarrow \text{5455}$$

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

$$\Big\downarrow \text{5379}$$

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

$$\Big\downarrow \text{2849}$$

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

$$\Big\downarrow \text{2752}$$

3.1262. $\int\frac{x^2(a+b\arctan(cx))^2}{d+ex^2}dx$

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)}{e}-$$

$$\frac{d\left(-\frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
(x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2)
 - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2
/(1 + I*c*x)])/c)/c))/e - (d*(((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e])
- ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I
*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(
1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2,
 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[
3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))
])/(4*Sqrt[-d]*Sqrt[e])))/e
```

3.1262.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.1262.4 Maple [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{e\,x^2+d}dx$$

input
```
int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

output
```
int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

3.1262.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e*x^2
+ d), x)
```

3.1262.6 Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x^2(a+b\operatorname{atan}(cx))^2}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

3.1262.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1262.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

3.1263 $\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx$

3.1263.1 Optimal result

Integrand size = 21, antiderivative size = 492

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = &-\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e} \\
&+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e} \\
&+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e} \\
&+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e} \\
&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e} \\
&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e} \\
&-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e} \\
&+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e} \\
&+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e}\end{aligned}$$

output
```
-(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d
)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e+1/2*(a+b*arctan(c
*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e
+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e-1/2*I*b*(a+b*arctan(c*x)
)*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)
))/e-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I
*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e-1/2*b^2*polylog(3,1-2/(1-I*c*x))/e+1/4*b
^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2
)))/e+1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/
2)+I*e^(1/2)))/e
```

3.1263.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1322 vs. $2(492) = 984$.

Time = 13.38 (sec) , antiderivative size = 1322, normalized size of antiderivative = 2.69

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
((8*I)*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]]
 - 8*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 4*b^2*ArcTan[c*x]^2*
Log[1 + E^((2*I)*ArcTan[c*x])] - 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*L
og[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)]
+ 4*a*b*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan
[c*x]))/(c^2*d - e)] - 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]
*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)
] + 4*b^2*ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*Ar
cTan[c*x]))/(c^2*d - e)] + 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[(-2
*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^
2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 4*a*b*ArcTan[c*x]*Log[(-2*
Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2
*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 4*b^2*ArcSin[Sqrt[(c^2*d)/(
c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(
-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d -
e)] + 4*b^2*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*
(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d -
 e)] - 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*
d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*
x))] - 2*b^2*ArcTan[c*x]^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c...
```

3.1263.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{(a+b\arctan(cx))^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e}-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2e}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
-(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(
2*e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d
] + I*Sqrt[e])*(1 - I*c*x))])/(2*e) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 - I*c*x)])/e - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(
Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e - ((I/2)
*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqr
t[-d] + I*Sqrt[e])*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/
(2*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/(4*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e)
```

3.1263.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1263. $\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx$

3.1263.4 Maple [F]

$$\int \frac{x(a+b\arctan(cx))^2}{e\,x^2+d}dx$$

input
```
int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

output
```
int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

3.1263.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e*x^2 + d),
x)
```

3.1263.6 Sympy [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x(a+b\,\mathrm{atan}(cx))^2}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

3.1263.7 Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*log(e*x^2 + d)/e + integrate((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan
(c*x))/(e*x^2 + d), x)
```

3.1263.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2),x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

3.1264 $\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx$

3.1264.1 Optimal result

Integrand size = 20, antiderivative size = 460

$$
\begin{aligned}
\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx &= \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\
&\quad + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}} \\
&\quad - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

output
```
1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1
/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1
/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*
I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*
(-d)^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*(a+b*arctan(c*x))*polylo
g(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^
(1/2)/e^(1/2)+1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*
(-d)^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*b^2*polylog(3,1-2*c*((-d)^(1
/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

3.1264.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
```

output
```
$Aborted
```

3.1264.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5449}$$

$$\int \left(\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}}-\frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}}
$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
```

output
```
((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*S
qrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcTan[c*x])^2*Log[(
2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*Sq
rt[-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e
]) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*
x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) + (b^2*Pol
yLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*
c*x))])/(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e
]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e])
```

3.1264.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

3.1264.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2343 vs. $2(362) = 724$.

Time = 225.41 (sec) , antiderivative size = 2344, normalized size of antiderivative = 5.10

method	result	size
parts	Expression too large to display	2344
derivativedivides	Expression too large to display	5222
default	Expression too large to display	5222

input
```
int((a+b*arctan(c*x))^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a^2/(e*d)^(1/2)*arctan(e*x/(e*d)^(1/2))+b^2/c*(-1/2*(-(c^2*d*e)^(1/2)*c^2*
d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)*c^2/e/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^
2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)-1
/4*I*(-(c^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)*c^2*polylog(3,(c
^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))/e/(c^4*d^2-2
*c^2*d*e+e^2)+(c^2*d-2*(c^2*d*e)^(1/2)+e)/(c^4*d^2-2*c^2*d*e+e^2)*polylog(
2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c
*x)*c^2-1/4*I*(-(c^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)*polylog
(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))/d/(c^4*
d^2-2*c^2*d*e+e^2)-1/3*(-(c^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e
)*c^2/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^3-1/2*(c^2*d*e)^(1/2)/d/e*arct
an(c*x)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1
/2)-e))-1/3*(-(c^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)/d/(c^4*d^
2-2*c^2*d*e+e^2)*arctan(c*x)^3-1/2*I*(-(c^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^
2*d*e)^(1/2)*e)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)
^(1/2)-e))*arctan(c*x)^2/d/(c^4*d^2-2*c^2*d*e+e^2)-1/4*I*(c^2*d*e)^(1/2)/d
/e*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e
))+I*(c^2*d-2*(c^2*d*e)^(1/2)+e)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-
c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)^2*c^2/(c^4*d^2-2*c^2*d*e+e^2)-1/3*
(c^2*d*e)^(1/2)/d/e*arctan(c*x)^3+2/3*(c^2*d-2*(c^2*d*e)^(1/2)+e)/(c^4*...
```

3.1264.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx=\int \frac{(b\arctan(cx)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^2 + d), x)
```

3.1264.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\,\mathrm{atan}(cx))^2}{d+ex^2}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(d + e*x**2), x)
```

3.1264.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1264.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + e*x^2),x)
```

output
```
int((a + b*atan(c*x))^2/(d + e*x^2), x)
```

3.1265 $\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx$

3.1265.1 Optimal result

Integrand size = 23, antiderivative size = 637

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d} - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d} + \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d}$$

output
```
-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d+(a+b*arctan(c*x))^2*ln(2/
(1-I*c*x))/d-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*
x)/(c*(-d)^(1/2)-I*e^(1/2)))/d-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+
x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d-I*b*(a+b*arctan(c*x))*pol
ylog(2,1-2/(1-I*c*x))/d-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d+I
*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d+1/2*I*b*(a+b*arctan(c*x))
*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2))
)/d+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*
c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d+1/2*b^2*polylog(3,1-2/(1-I*c*x))/d-1/2*b^
2*polylog(3,1-2/(1+I*c*x))/d+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d-1/4*b^2*p
olylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/
d-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I
*e^(1/2)))/d
```

3.1265.2 Mathematica [A] (warning: unable to verify)

Time = 10.89 (sec) , antiderivative size = 1264, normalized size of antiderivative = 1.98

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
```

output
```
(24*a^2*Log[x] - 12*a^2*Log[d + e*x^2] - 24*a*b*((-I)*ArcTan[c*x]^2 + (2*I
)*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] - 2*ArcT
an[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)
]] + ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c
*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log
[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x]))
+ c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + I*(ArcTan[c*x]^2 + Pol
yLog[2, E^((2*I)*ArcTan[c*x])]) - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqr
t[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e
 + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))])) + b^2*((-I)*Pi^
3 + (16*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])
] + 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e +
 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 24*ArcTan[c*x]^2*L
og[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)]
- 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E
^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I
)*ArcTan[c*x])))/(c^2*d - e)] - 24*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^(
(2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*
ArcTan[c*x])))/(c^2*d - e)] + 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[
c*x]*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*...
```

3.1265.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(\frac{(a+b\arctan(cx))^2}{dx}-\frac{ex(a+b\arctan(cx))^2}{d\,(d+ex^2)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2d}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d}+\frac{ib\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d}+\frac{b^2\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
```

output
```
(2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d) - ((a + b*ArcTa
n[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 -
I*c*x))])/(2*d) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/
d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (I*b*(a +
b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b*(a + b*ArcTan[
c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/d + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqr
t[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d + (b^2*Poly
Log[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*
d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - (2*c
*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d) -
(b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/(4*d)
```

3.1265. $\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx$

3.1265.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1265.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(e\,x^2+d)}dx$$

input
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

output
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

3.1265.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^3 + d*x), x)
```

3.1265.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2}{x(d+ex^2)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x*(d + e*x**2)), x)
```

3.1265.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(log(e*x^2 + d)/d - 2*log(x)/d) + integrate((b^2*arctan(c*x)^2 +
2*a*b*arctan(c*x))/(e*x^3 + d*x), x)
```

3.1265.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(e\,x^2+d)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)), x)
```

3.1266 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx$

3.1266.1 Optimal result

Integrand size = 23, antiderivative size = 553

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx =& -\frac{ic(a+b\arctan(cx))^2}{d}-\frac{(a+b\arctan(cx))^2}{dx}\\
&+\frac{\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&-\frac{\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d}\\
&-\frac{ib^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}\\
&-\frac{ib\sqrt{e}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&+\frac{ib\sqrt{e}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&+\frac{b^2\sqrt{e}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}}\\
&-\frac{b^2\sqrt{e}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}}\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*b*c*(a+b*arctan(c*x))
*ln(2-2/(1-I*c*x))/d-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d+1/2*(a+b*arctan(c
*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*e
^(1/2)/(-d)^(3/2)-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1
-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*(a+b*arctan(c
*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1
/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(
1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)+1/4
*b^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1
/2)))*e^(1/2)/(-d)^(3/2)-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1
-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)
```

3.1266.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

output
```
$Aborted
```

3.1266.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5453, 5361, 5449, 2009, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx$$

$$\Big\downarrow \; 5453$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^2}dx}{d} - \frac{e\int \frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{d}$$

$$\Big\downarrow \; 5361$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{5449}$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{e\int \left(\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-$$

$$\frac{e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-$$

$$\frac{e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-$$

$$\frac{e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

$$\Big\downarrow \text{2897}$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}$$

$$\frac{e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log}{2\sqrt{}}\right.}{}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

3.1266. $\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx$

output
```
(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/
(1 - I*c*x)])/2)))/d - (e*(((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqr
t[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - (
(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sq
rt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
 I*c*x))])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1
- (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(S
qrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3,
1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/
(4*Sqrt[-d]*Sqrt[e])))/d
```

3.1266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.1266.4 Maple [F]

$$\int \frac{(a + b \arctan(cx))^2}{x^2 (e x^2 + d)} dx$$

input
```
int((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x)
```

output
```
int((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x)
```

3.1266.5 Fricas [F]

$$\int \frac{(a + b \arctan(cx))^2}{x^2 (d + ex^2)} dx = \int \frac{(b \arctan(cx) + a)^2}{(ex^2 + d)x^2} dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^4 + d*x^2), x)
```

3.1266.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d),x)
```

output
```
Timed out
```

3.1266.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1266.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{x^2\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)), x)
```

3.1267 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx$

3.1267.1 Optimal result

Integrand size = 23, antiderivative size = 745

$$
\begin{aligned}
\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = & -\frac{bc(a+b\arctan(cx))}{dx} \\
& -\frac{c^2(a+b\arctan(cx))^2}{2d}-\frac{(a+b\arctan(cx))^2}{2dx^2} \\
& -\frac{2e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2} \\
& +\frac{b^2c^2\log(x)}{d}-\frac{e(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^2} \\
& +\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^2} \\
& +\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^2} \\
& -\frac{b^2c^2\log(1+c^2x^2)}{2d} \\
& +\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^2} \\
& +\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2} \\
& -\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2} \\
& -\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^2} \\
& -\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^2} \\
& -\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^2}+\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d^2} \\
& -\frac{b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2} \\
& +\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^2} \\
& +\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^2}
\end{aligned}
$$

output
```
-b*c*(a+b*arctan(c*x))/d/x-1/2*c^2*(a+b*arctan(c*x))^2/d-1/2*(a+b*arctan(c
*x))^2/d/x^2+2*e*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^2+b^2*c^2*l
n(x)/d-e*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)
/d+1/2*e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-
d)^(1/2)-I*e^(1/2)))/d^2+1/2*e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^
(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2-1/2*I*b*e*(a+b*arctan(c*x))
*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2))
)/d^2+I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d^2-1/2*I*b*e*(a+b*
arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2
)-I*e^(1/2)))/d^2-I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2+I*
b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d^2-1/2*b^2*e*polylog(3,1-2
/(1-I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2/(1+I*c*x))/d^2-1/2*b^2*e*polylog(3
,-1+2/(1+I*c*x))/d^2+1/4*b^2*e*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I
*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2+1/4*b^2*e*polylog(3,1-2*c*((-d)^(1/2)+
x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2
```

3.1267.2 Mathematica [A] (warning: unable to verify)

Time = 14.31 (sec) , antiderivative size = 1412, normalized size of antiderivative = 1.90

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

output
```
-1/24*((12*a^2*d)/x^2 + (24*a*b*c*d)/x + (24*a*b*d*(1 + c^2*x^2)*ArcTan[c*
x])/x^2 + 24*a^2*e*Log[x] - 12*a^2*e*Log[d + e*x^2] - (24*I)*a*b*e*(ArcTan
[c*x]*(ArcTan[c*x] + (2*I)*Log[1 - E^((2*I)*ArcTan[c*x])]) + PolyLog[2, E^
((2*I)*ArcTan[c*x])]) - (48*a*b*(c^2*d - e)*e*((-I)*ArcTan[c*x]^2 + (2*I)*
ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] + (-ArcSin
[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^
2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d
- e)]] + ArcTan[c*x])*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1
+ E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)]
 - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x
]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*Ar
cTan[c*x]))/(c^2*d - e))])))/(2*c^2*d - 2*e) + b^2*((-I)*e*Pi^3 + (24*c*d*
ArcTan[c*x])/x + (12*d*(1 + c^2*x^2)*ArcTan[c*x]^2)/x^2 + (8*I)*e*ArcTan[c
*x]^3 + 24*e*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*c^2*d*Log[
c*x] + 12*c^2*d*Log[1 + c^2*x^2] + (24*I)*e*ArcTan[c*x]*PolyLog[2, E^((-2*
I)*ArcTan[c*x])] + 12*e*PolyLog[3, E^((-2*I)*ArcTan[c*x])]) + 2*b^2*e*((4*
I)*ArcTan[c*x]^3 + 12*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1
+ ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 12*
ArcTan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x])
)/(c^2*d - e)] - 12*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[(...
```

3.1267.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 724, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3\,(d+ex^2)}\,dx$$

$$\Big\downarrow \; 5453$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^3}dx}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 5361$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 5453$$

$$\frac{bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 5361$$

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 243$$

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 47$$

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 14$$

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 16$$

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 5419$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \; 5515$$

3.1267. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int\left(\frac{(a+b\arctan(cx))^2}{dx}-\frac{ex(a+b\arctan(cx))^2}{d(ex^2+d)}\right)dx}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$
$$e\left(\frac{2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{(\sqrt{}}{}\right.}{2d}\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(-((a + b*ArcTan[c*x])/x) - (c*(a +
b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2))/d - (e*(
(2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d) - ((a + b*ArcTa
n[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 -
I*c*x))])/(2*d) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/
d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (I*b*(a +
b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b*(a + b*ArcTan[
c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/d + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqr
t[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d + (b^2*Poly
Log[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*
d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - (2*c
*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d) -
(b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/(4*d)))/d
```

3.1267. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx$

3.1267.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.1267. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx$

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1267.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(e\,x^2+d)}dx$$

input
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

output
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

3.1267.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^5 + d*x^3), x)
```

3.1267.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^2}{x^3(d+ex^2)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x**3*(d + e*x**2)), x)
```

3.1267.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + integrate((b
^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e*x^5 + d*x^3), x)
```

3.1267.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.1267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^3\,(e\,x^2+d)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)), x)
```

3.1268 $\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

3.1268.1 Optimal result

Integrand size = 23, antiderivative size = 943

$$\begin{aligned}
\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx =& -\frac{c^2d(a+b\arctan(cx))^2}{2(c^2d-e)e^2}+\frac{(a+b\arctan(cx))^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} \\
&+\frac{(a+b\arctan(cx))^2}{4e^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} \\
&-\frac{bc\sqrt{-d}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2(c^2d-e)e^{3/2}} \\
&+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2} \\
&+\frac{bc\sqrt{-d}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2(c^2d-e)e^{3/2}} \\
&+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2} \\
&+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^2} \\
&+\frac{ib^2c\sqrt{-d}\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(c^2d-e)e^{3/2}} \\
&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2} \\
&-\frac{ib^2c\sqrt{-d}\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4(c^2d-e)e^{3/2}} \\
&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2} \\
&-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^2} \\
&+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2} \\
&+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^2}
\end{aligned}$$

output
```
-1/2*c^2*d*(a+b*arctan(c*x))^2/(c^2*d-e)/e^2-(a+b*arctan(c*x))^2*ln(2/(1-I
*c*x))/e^2+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)
/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+
x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2+1/4*I*b^2*c*polylog(2,1
-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1/2)
/(c^2*d-e)/e^(3/2)-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x
*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^2-1/4*I*b^2*c*polylog(2,1-
2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-d)^(1/2)/
(c^2*d-e)/e^(3/2)-1/2*b^2*polylog(3,1-2/(1-I*c*x))/e^2+1/4*b^2*polylog(3,1
-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/4*b^
2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)
))/e^2-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(
c*(-d)^(1/2)-I*e^(1/2)))*(-d)^(1/2)/(c^2*d-e)/e^(3/2)+1/2*b*c*(a+b*arctan(
c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*(-
d)^(1/2)/(c^2*d-e)/e^(3/2)+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/
e^2-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*
c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^2+1/4*(a+b*arctan(c*x))^2/e^2/(1-x*e^(1/2
)/(-d)^(1/2))+1/4*(a+b*arctan(c*x))^2/e^2/(1+x*e^(1/2)/(-d)^(1/2))
```

3.1268.2 Mathematica [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

3.1268.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \; 5515$$

$$\int \left(\frac{x(a+b\arctan(cx))^2}{e(d+ex^2)}-\frac{dx(a+b\arctan(cx))^2}{e(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ic\sqrt{-d}\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4(c^2d-e)e^{3/2}}-\frac{ic\sqrt{-d}\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4(c^2d-e)e^{3/2}}-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{2e^2}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4e^2}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4e^2}-\frac{c\sqrt{-d}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2(c^2d-e)e^{3/2}}+\frac{c\sqrt{-d}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2(c^2d-e)e^{3/2}}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b}{e^2}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2e^2}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2e^2}+\frac{(a+b\arctan(cx))^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}+\frac{(a+b\arctan(cx))^2}{4e^2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}-\frac{c^2d(a+b\arctan(cx))^2}{2(c^2d-e)e^2}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

3.1268. $\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

output
```
-1/2*(c^2*d*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e^2) + (a + b*ArcTan[c*x])
^2/(4*e^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*e^2*(1 +
(Sqrt[e]*x)/Sqrt[-d])) - ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 -
(b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqr
t[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2)) + ((a + b*ArcTan
[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))])/(2*e^2) + (b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] +
Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e^(3/2
)) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 - I*c*x)])/e^2 + ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2, 1 - (2*c*(Sqrt[
-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e^
(3/2)) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt
[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b^2*c*Sqrt[-
d]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(
1 - I*c*x))])/((c^2*d - e)*e^(3/2)) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog
[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x)
)])/e^2 - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) + (b^2*PolyLog[3, 1
- (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4
*e^2) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +...
```

3.1268.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1268. $\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

3.1268.4 Maple [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(e\,x^2+d)^2}dx$$

input
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

output
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

3.1268.5 Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e^2*x^
4 + 2*d*e*x^2 + d^2), x)
```

3.1268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1268.7 Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + integrate((b^2*x^3*ar
ctan(c*x)^2 + 2*a*b*x^3*arctan(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.1268.8 Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

3.1269 $\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

3.1269.1 Optimal result

Integrand size = 23, antiderivative size = 1033

$$
\begin{aligned}
\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx &= -\frac{ic(a+b\arctan(cx))^2}{2(c^2d-e)e}+\frac{(a+b\arctan(cx))^2}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} \\
&\quad-\frac{(a+b\arctan(cx))^2}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{(c^2d-e)e} \\
&\quad-\frac{bc(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{(c^2d-e)e} \\
&\quad-\frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2(c^2d-e)e} \\
&\quad+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad-\frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2(c^2d-e)e} \\
&\quad-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad-\frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2(c^2d-e)e}-\frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2(c^2d-e)e} \\
&\quad+\frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(c^2d-e)e} \\
&\quad-\frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad+\frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4(c^2d-e)e} \\
&\quad+\frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}} \\
&\quad+\frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{8\sqrt{-d}e^{3/2}} \\
&\quad-\frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{8\sqrt{-d}e^{3/2}}
\end{aligned}
$$

output
```
1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)
-I*e^(1/2)))/(c^2*d-e)/e+b*c*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/(c^2*d-e)/e
-b*c*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/(c^2*d-e)/e-1/2*b*c*(a+b*arctan(c*x
))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(c^2*
d-e)/e-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(
c*(-d)^(1/2)+I*e^(1/2)))/(c^2*d-e)/e+1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/
2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(c^2*d-e)/e-1/2*I*b^2*c*
polylog(2,1-2/(1-I*c*x))/(c^2*d-e)/e-1/4*I*b*(a+b*arctan(c*x))*polylog(2,1
-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)/(-
d)^(1/2)+1/4*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/
(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arctan(c*x
))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^(
3/2)/(-d)^(1/2)-1/4*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I
*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^(1/2)-1/2*I*b^2*c*polylog(2,1
-2/(1+I*c*x))/(c^2*d-e)/e-1/2*I*c*(a+b*arctan(c*x))^2/(c^2*d-e)/e+1/8*b^2*
polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))
/e^(3/2)/(-d)^(1/2)-1/8*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*
x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arctan(c*x))^2/e^
(3/2)/((-d)^(1/2)-x*e^(1/2))-1/4*(a+b*arctan(c*x))^2/e^(3/2)/((-d)^(1/2)+x
*e^(1/2))
```

3.1269.2 Mathematica [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

3.1269.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 1033, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{e(d+ex^2)}-\frac{d(a+b\arctan(cx))^2}{e(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2(c^2d-e)e}-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2(c^2d-e)e}+\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4(c^2d-e)e}+\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4(c^2d-e)e}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{8\sqrt{-d}e^{3/2}}-\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{8\sqrt{-d}e^{3/2}}+\frac{c(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)b}{(c^2d-e)e}-\frac{c(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)b}{(c^2d-e)e}-\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2(c^2d-e)e}-\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2(c^2d-e)e}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{4\sqrt{-d}e^{3/2}}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{4\sqrt{-d}e^{3/2}}-\frac{ic(a+b\arctan(cx))^2}{2(c^2d-e)e}+\frac{(a+b\arctan(cx))^2}{4e^{3/2}(\sqrt{-d}-\sqrt{e}x)}-\frac{(a+b\arctan(cx))^2}{4e^{3/2}(\sqrt{e}x+\sqrt{-d})}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
((-1/2*I)*c*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e) + (a + b*ArcTan[c*x])^2
/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcTan[c*x])^2/(4*e^(3/2)*(Sq
rt[-d] + Sqrt[e]*x)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/((c^2*
d - e)*e) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/((c^2*d - e)*e) -
 (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) + ((a + b*ArcTan[c*x])^2*Log[(
2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sq
rt[-d]*e^(3/2)) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x)
)/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) - ((a + b*Arc
Tan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1
- I*c*x))])/(4*Sqrt[-d]*e^(3/2)) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*
x)])/((c^2*d - e)*e) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/((c^2*d
 - e)*e) + ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e) - ((I/4)*b*(a + b*ArcTa
n[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[
e])*(1 - I*c*x))])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(
Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((c^2*d -
e)*e) + ((I/4)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*e^(3/2)) + (b^2*
PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(...
```

3.1269.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1269.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 157.17 (sec) , antiderivative size = 6565, normalized size of antiderivative = 6.36

method	result	size
parts	Expression too large to display	6565
derivativedivides	Expression too large to display	6636
default	Expression too large to display	6636

input
```
int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.1269.5 Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e^2*x^
4 + 2*d*e*x^2 + d^2), x)
```

3.1269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1269.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1269.8 Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

3.1270 $\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

3.1270.1 Optimal result

Integrand size = 21, antiderivative size = 457

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx &= \frac{c^2(a+b\arctan(cx))^2}{2(c^2d-e)e}-\frac{(a+b\arctan(cx))^2}{4de\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}\\ &-\frac{(a+b\arctan(cx))^2}{4de\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}\\ &-\frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\,(c^2d-e)\sqrt{e}}\\ &+\frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\,(c^2d-e)\sqrt{e}}\\ &+\frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\,(c^2d-e)\sqrt{e}}\\ &-\frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\,(c^2d-e)\sqrt{e}}\end{aligned}$$

output
```
1/2*c^2*(a+b*arctan(c*x))^2/(c^2*d-e)/e-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*(
(-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(c^2*d-e)/(-d)^(
1/2)/e^(1/2)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*
c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(c^2*d-e)/(-d)^(1/2)/e^(1/2)+1/4*I*b^2*c*po
lylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(
c^2*d-e)/(-d)^(1/2)/e^(1/2)-1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1
/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(c^2*d-e)/(-d)^(1/2)/e^(1/2)-1/4*
(a+b*arctan(c*x))^2/d/e/(1-x*e^(1/2)/(-d)^(1/2))-1/4*(a+b*arctan(c*x))^2/d
/e/(1+x*e^(1/2)/(-d)^(1/2))
```

3.1270.2 Mathematica [A] (verified)

Time = 6.93 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.83

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \frac{1}{4}\left(-\frac{2a^2}{e(d+ex^2)} + \frac{4ab\left(-\frac{(1+c^2x^2)\arctan(cx)}{d+ex^2} + \frac{c\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}\right)}{-c^2d+e} + \frac{b^2c^2\left(\frac{4\arctan(cx)^2}{c^2d+e+(c^2d-e)\cos(2\arctan(cx))} + \frac{-4\arctan(cx)\operatorname{arctanh}\left(\frac{cd}{\sqrt{-c^2dex}}\right) - 2\arccos\left(-\frac{c^2d+e}{c^2d-e}\right)\operatorname{arctanh}\left(\frac{cex}{\sqrt{-c^2de}}\right) + \left(\arccos\left(-\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
((-2*a^2)/(e*(d + e*x^2)) + (4*a*b*(-(((1 + c^2*x^2)*ArcTan[c*x])/(d + e*x
^2)) + (c*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])))/(-(c^2*d) + e)
+ (b^2*c^2*((4*ArcTan[c*x]^2)/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])
 + (-4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] - 2*ArcCos[-((c^2*d
 + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (ArcCos[-((c^2*d +
 e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*((
-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c*d + Sqrt[-(c^2*d*e)]
*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-
(c^2*d*e)]])*Log[(2*c*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(
c*d + Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*
(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))
*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^
2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] - (ArcCos[-((c^2*d + e)/(c^2*d
 - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqr
t[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^
2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] - I*(PolyLog[2
, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c*d - Sqrt[-(c^2*d*e)]*x))/((c^2*
d - e)*(c*d + Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[
-(c^2*d*e)])*(c*d - Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d + Sqrt[-(c^2*d*
e)]*x))]))/Sqrt[-(c^2*d*e)]))/(c^2*d - e))/4
```

3.1270.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 996 vs. 2(457) = 914.

Time = 1.46 (sec) , antiderivative size = 996, normalized size of antiderivative = 2.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5513, 5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \ 5513$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)^2}dx}{4(-d)^{3/2}\sqrt{e}}-\frac{\int \frac{(a+b\arctan(cx))^2}{\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)^2}dx}{4(-d)^{3/2}\sqrt{e}}$$

$$\Big\downarrow \ 5389$$

$$\frac{\frac{\sqrt{-d}(a+b\arctan(cx))^2}{\sqrt{e}\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}-\frac{2bc\sqrt{-d}\int\left(-\frac{\sqrt{-d}(\sqrt{e}x+\sqrt{-d})(a+b\arctan(cx))c^2}{(c^2d-e)(c^2x^2+1)}-\frac{\sqrt{-d}e(a+b\arctan(cx))}{(c^2d-e)(\sqrt{-d}-\sqrt{e}x)}\right)dx}{\sqrt{e}}}{4(-d)^{3/2}\sqrt{e}}-\frac{\frac{2bc\sqrt{-d}\int\left(\frac{c^2(d+\sqrt{-d}\sqrt{e}x)(a+b\arctan(cx))}{(c^2d-e)(c^2x^2+1)}-\frac{\sqrt{-d}e(a+b\arctan(cx))}{(c^2d-e)(\sqrt{e}x+\sqrt{-d})}\right)dx}{\sqrt{e}}-\frac{\sqrt{-d}(a+b\arctan(cx))^2}{\sqrt{e}\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}}{4(-d)^{3/2}\sqrt{e}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{-d}(a+b\arctan(cx))^2}{\sqrt{e}\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}-\frac{2bc\sqrt{-d}\left(\frac{i\sqrt{-d}\sqrt{e}(a+b\arctan(cx))^2}{2b(c^2d-e)}+\frac{cd(a+b\arctan(cx))^2}{2b(c^2d-e)}-\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c^2d-e}+\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{icx+1}\right)(a+b\operatorname{arc}\right.}{c^2d-e}}{}}{}$$

$$\frac{2bc\sqrt{-d}\left(-\frac{i\sqrt{-d}\sqrt{e}(a+b\arctan(cx))^2}{2b(c^2d-e)}+\frac{cd(a+b\arctan(cx))^2}{2b(c^2d-e)}+\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c^2d-e}-\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^2d-e}-\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{(\sqrt{-}\right.}{}\right.}{\sqrt{e}}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

3.1270. $\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

output
```
((Sqrt[-d]*(a + b*ArcTan[c*x])^2)/(Sqrt[e]*(1 - (Sqrt[e]*x)/Sqrt[-d])) - (
2*b*c*Sqrt[-d]*((c*d*(a + b*ArcTan[c*x])^2)/(2*b*(c^2*d - e)) + ((I/2)*Sqr
t[-d]*Sqrt[e]*(a + b*ArcTan[c*x])^2)/(b*(c^2*d - e)) - (Sqrt[-d]*Sqrt[e]*(
a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/(c^2*d - e) + (Sqrt[-d]*Sqrt[e]*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2*d - e) + (Sqrt[-d]*Sqrt[e]*(a +
b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*
(1 - I*c*x))])/(c^2*d - e) + ((I/2)*b*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1
 - I*c*x)])/(c^2*d - e) + ((I/2)*b*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^2*d - e) - ((I/2)*b*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt
[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(c^2*d - e)))/
Sqrt[e])/(4*(-d)^(3/2)*Sqrt[e]) - (-((Sqrt[-d]*(a + b*ArcTan[c*x])^2)/(Sqr
t[e]*(1 + (Sqrt[e]*x)/Sqrt[-d]))) + (2*b*c*Sqrt[-d]*((c*d*(a + b*ArcTan[c*
x])^2)/(2*b*(c^2*d - e)) - ((I/2)*Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])^2)/
(b*(c^2*d - e)) + (Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)]
)/(c^2*d - e) - (Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
(c^2*d - e) - (Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(c^2*d - e) - ((I/2)*b
*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d - e) - ((I/2)*b*Sq
rt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d - e) + ((I/2)*b*Sqrt[
-d]*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + ...
```

3.1270.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5513
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[1/(4*d^2*Rt[-e/d, 2])   Int[(a + b*ArcTan[c*x])^p/(1 -
Rt[-e/d, 2]*x)^2, x], x] - Simp[1/(4*d^2*Rt[-e/d, 2])   Int[(a + b*ArcTan[c
*x])^p/(1 + Rt[-e/d, 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p
, 0]
```

3.1270. $\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

3.1270.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(377) = 754$.

Time = 2.58 (sec) , antiderivative size = 1175, normalized size of antiderivative = 2.57

method	result	size
parts	Expression too large to display	1175
derivativedivides	Expression too large to display	1209
default	Expression too large to display	1209

input
```
int(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/e/(e*x^2+d)-1/2*b^2*c^2*arctan(c*x)^2/e/(c^2*e*x^2+c^2*d)+I*b^2*c
^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*ar
ctan(c*x)/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/2*I*b^2*e*ln
(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(
c*x)/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1/2*I*b^2/e*(c^2*
d*e)^(1/2)/d/(c^2*d-e)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/
(-c^2*d+2*(c^2*d*e)^(1/2)-e))-1/2*b^2*c^4/e/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e
^2)*arctan(c*x)^2*(c^2*d*e)^(1/2)*d-1/4*b^2*c^4/e/(c^2*d-e)/(c^4*d^2-2*c^2
*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^
(1/2)-e))*(c^2*d*e)^(1/2)*d-1/2*I*b^2*c^4/e*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^
2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^2*d-e)/(c^4*d^2-2*c^
2*d*e+e^2)*(c^2*d*e)^(1/2)*d-1/2*b^2*e/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)
*arctan(c*x)^2*(c^2*d*e)^(1/2)-1/4*b^2*e/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^
2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e
))*(c^2*d*e)^(1/2)+1/2*b^2/e*(c^2*d*e)^(1/2)/d/(c^2*d-e)*arctan(c*x)^2+1/4
*b^2/e*(c^2*d*e)^(1/2)/d/(c^2*d-e)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^
2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))+b^2*c^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^
2)*arctan(c*x)^2*(c^2*d*e)^(1/2)+1/2*b^2*c^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+
e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)
-e))*(c^2*d*e)^(1/2)+1/2*b^2*c^2/e/(c^2*d-e)*arctan(c*x)^2-a*b*c^2*arct...
```

3.1270.5 Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e^2*x^4 + 2*
d*e*x^2 + d^2), x)
```

3.1270.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1270.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1270.8 Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

3.1271 $\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$

3.1271.1 Optimal result

Integrand size = 20, antiderivative size = 1039

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \frac{ic(a+b\arctan(cx))^2}{2d(c^2d-e)} - \frac{(a+b\arctan(cx))^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{(a+b\arctan(cx))^2}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d(c^2d-e)} + \frac{bc(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d(c^2d-e)} + \frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d(c^2d-e)} - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d(c^2d-e)} + \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d(c^2d-e)} + \frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d(c^2d-e)} - \frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d(c^2d-e)} + \frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d(c^2d-e)} - \frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{8(-d)^{3/2}\sqrt{e}} + \frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{8(-d)^{3/2}\sqrt{e}}$$

output
```
1/2*I*b^2*c*polylog(2,1-2/(1+I*c*x))/d/(c^2*d-e)-b*c*(a+b*arctan(c*x))*ln(
2/(1-I*c*x))/d/(c^2*d-e)+b*c*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d/(c^2*d-e)
+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)
^(1/2)-I*e^(1/2)))/d/(c^2*d-e)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2
)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d/(c^2*d-e)-1/4*I*b^2*c*p
olylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/
d/(c^2*d-e)+1/4*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2
))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b^2*c*poly
log(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d/(
c^2*d-e)+1/2*I*c*(a+b*arctan(c*x))^2/d/(c^2*d-e)-1/4*(a+b*arctan(c*x))^2*l
n(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(3/2
)/e^(1/2)+1/4*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/
(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(3/2)/e^(1/2)+1/2*I*b^2*c*polylog(2,1-2/(1-
I*c*x))/d/(c^2*d-e)-1/4*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+
x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(3/2)/e^(1/2)-1/8*b^2*
polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))
/(-d)^(3/2)/e^(1/2)+1/8*b^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*
x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arctan(c*x))^2/d/
e^(1/2)/((-d)^(1/2)-x*e^(1/2))+1/4*(a+b*arctan(c*x))^2/d/e^(1/2)/((-d)^(1/
2)+x*e^(1/2))
```

3.1271.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2, x]
```

3.1271.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 1039, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{5449}$$

$$\int \left(-\frac{e(a+b\arctan(cx))^2}{2d(-de-e^2x^2)}-\frac{e(a+b\arctan(cx))^2}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\arctan(cx))^2}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2d(c^2d-e)}+\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2d(c^2d-e)}-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4d(c^2d-e)}-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4d(c^2d-e)}-\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{3/2}\sqrt{e}}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{3/2}\sqrt{e}}-\frac{c(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)b}{d(c^2d-e)}+\frac{c(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)b}{d(c^2d-e)}+\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2d(c^2d-e)}+\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2d(c^2d-e)}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{4(-d)^{3/2}\sqrt{e}}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{4(-d)^{3/2}\sqrt{e}}+\frac{ic(a+b\arctan(cx))^2}{2d(c^2d-e)}-\frac{(a+b\arctan(cx))^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{(a+b\arctan(cx))^2}{4d\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

output
```
((I/2)*c*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(4
*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcTan[c*x])^2/(4*d*Sqrt[e]*(S
qrt[-d] + Sqrt[e]*x)) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/(d*(c
^2*d - e)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d*(c^2*d - e))
+ (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[
(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*(
-d)^(3/2)*Sqrt[e]) + (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) + ((a + b*
ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*
(1 - I*c*x))])/(4*(-d)^(3/2)*Sqrt[e]) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 -
 I*c*x)])/(d*(c^2*d - e)) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d
*(c^2*d - e)) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) + ((I/4)*b*(a + b*
ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 -
 (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d*
(c^2*d - e)) - ((I/4)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d]
+ Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*Sqrt[e]
) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*S...
```

3.1271.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

3.1271.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.79 (sec) , antiderivative size = 6565, normalized size of antiderivative = 6.32

method	result	size
parts	Expression too large to display	6565
derivativedivides	Expression too large to display	6570
default	Expression too large to display	6570

input
```
int((a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.1271.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^4 + 2*d*e*x^
2 + d^2), x)
```

3.1271.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1271.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1271.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + e*x^2)^2,x)
```

output
```
int((a + b*atan(c*x))^2/(d + e*x^2)^2, x)
```

3.1272 $\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx$

3.1272.1 Optimal result

Integrand size = 23, antiderivative size = 1087

$$\int \frac{(a+b\arctan(cx))^2}{x\left(d+ex^2\right)^2}\,dx = -\frac{c^2(a+b\arctan(cx))^2}{2d\left(c^2d-e\right)}+\frac{(a+b\arctan(cx))^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} + \frac{(a+b\arctan(cx))^2}{4d^2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)}+\frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^2} - \frac{bc\sqrt{e}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2(-d)^{3/2}\left(c^2d-e\right)} - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^2} + \frac{bc\sqrt{e}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2(-d)^{3/2}\left(c^2d-e\right)} - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2d^2} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^2} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2} + \frac{ib^2c\sqrt{e}\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{3/2}\left(c^2d-e\right)} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^2} - \frac{ib^2c\sqrt{e}\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{3/2}\left(c^2d-e\right)} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2d^2} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^2} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d^2} + \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^2} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4d^2}$$

output
```
-1/2*c^2*(a+b*arctan(c*x))^2/d/(c^2*d-e)-2*(a+b*arctan(c*x))^2*arctanh(-1+
2/(1+I*c*x))/d^2+(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2-1/2*(a+b*arctan(c
*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d
^2-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)
^(1/2)+I*e^(1/2)))/d^2-1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/
(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(3/2)/(c^2*d-e)+1/4*I*b^2
*c*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2
)))*e^(1/2)/(-d)^(3/2)/(c^2*d-e)-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*
c*x))/d^2-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d^2+1/2*I*b*(a+b*
arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2
)-I*e^(1/2)))/d^2+1/2*b^2*polylog(3,1-2/(1-I*c*x))/d^2-1/2*b^2*polylog(3,1
-2/(1+I*c*x))/d^2+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d^2-1/4*b^2*polylog(3,
1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/d^2-1/4*b
^2*polylog(3,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2
)))/d^2-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/
(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(3/2)/(c^2*d-e)+1/2*b*c*(a+b*arctan
(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e
^(1/2)/(-d)^(3/2)/(c^2*d-e)+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d
)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2+I*b*(a+b*arctan
(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2+1/4*(a+b*arctan(c*x))^2/d^2/(1-x*e...
```

3.1272.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx=\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2), x]
```

3.1272.3 Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 1087, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x\left(d+ex^2\right)^2}dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(-\frac{ex(a+b\arctan(cx))^2}{d^2\left(d+ex^2\right)}+\frac{(a+b\arctan(cx))^2}{d^2x}-\frac{ex(a+b\arctan(cx))^2}{d\left(d+ex^2\right)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ic\sqrt{e}\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{3/2}\left(c^2d-e\right)}-\frac{ic\sqrt{e}\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{3/2}\left(c^2d-e\right)}+\frac{\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{2d^2}-\frac{\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)b^2}{2d^2}+\frac{\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)b^2}{2d^2}-\frac{\text{PolyLog}\left(3,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4d^2}-\frac{\text{PolyLog}\left(3,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{4d^2}-\frac{c\sqrt{e}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b}{2(-d)^{3/2}\left(c^2d-e\right)}+\frac{c\sqrt{e}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b}{2(-d)^{3/2}\left(c^2d-e\right)}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b}{d^2}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b}{d^2}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)b}{d^2}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b}{2d^2}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b}{2d^2}-\frac{c^2(a+b\arctan(cx))^2}{2d\left(c^2d-e\right)}+\frac{(a+b\arctan(cx))^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}+\frac{(a+b\arctan(cx))^2}{4d^2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}+\frac{2(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{icx+1}\right)}{d^2}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^2}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^2}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{2d^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(c^2*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) + (a + b*ArcTan[c*x])^2/(
4*d^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*d^2*(1 + (Sqr
t[e]*x)/Sqrt[-d])) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/
d^2 + ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^2 - (b*c*Sqrt[e]*(a + b
*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(
1 - I*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c
*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2)
+ (b*c*Sqrt[e]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sq
rt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*Ar
cTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1
 - I*c*x))])/(2*d^2) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*
x)])/d^2 - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2 + (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + ((I/4)*b^2*c
*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt
[e])*(1 - I*c*x))])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1
 - I*c*x))])/d^2 - ((I/4)*b^2*c*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*(c^2*d - e)
) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x
))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^2 + (b^2*PolyLog[3, 1 - 2...
```

3.1272.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1272.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(e\,x^2+d)^2}dx$$

input
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

output
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

3.1272.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^5 + 2*d*e*x^
3 + d^2*x), x)
```

3.1272.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1272.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\left(d+ex^2\right)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(ex^2+d\right)^2 x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + integrat
e((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x
)
```

3.1272.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\left(d+ex^2\right)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(ex^2+d\right)^2 x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x\left(d+ex^2\right)^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)^2), x)
```

3.1273 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx$

3.1273.1 Optimal result

Integrand size = 23, antiderivative size = 1141

$$
\begin{aligned}
\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = & -\frac{ic(a+b\arctan(cx))^2}{d^2}-\frac{ice(a+b\arctan(cx))^2}{2d^2(c^2d-e)} \\
& -\frac{(a+b\arctan(cx))^2}{d^2x}+\frac{\sqrt{e}(a+b\arctan(cx))^2}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} \\
& -\frac{\sqrt{e}(a+b\arctan(cx))^2}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bce(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^2(c^2d-e)} \\
& -\frac{bce(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^2(c^2d-e)} \\
& -\frac{bce(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^2(c^2d-e)} \\
& -\frac{3\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{5/2}} \\
& -\frac{bce(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2d^2(c^2d-e)} \\
& +\frac{3\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{5/2}} \\
& +\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^2} \\
& -\frac{ib^2ce\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2(c^2d-e)} \\
& -\frac{ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}-\frac{ib^2ce\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2(c^2d-e)} \\
& +\frac{ib^2ce\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^2(c^2d-e)} \\
& +\frac{3ib\sqrt{e}(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{5/2}} \\
& +\frac{ib^2ce\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4d^2(c^2d-e)} \\
& -\frac{3ib\sqrt{e}(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4(-d)^{5/2}} \\
& -\frac{3b^2\sqrt{e}\,\mathrm{PolyLog}\left(3,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{8(-d)^{5/2}} \\
& +\frac{3b^2\sqrt{e}\,\mathrm{PolyLog}\left(3,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{8(-d)^{5/2}}
\end{aligned}
$$

output
```
-1/2*I*b^2*c*e*polylog(2,1-2/(1-I*c*x))/d^2/(c^2*d-e)-1/2*I*b^2*c*e*polylo
g(2,1-2/(1+I*c*x))/d^2/(c^2*d-e)-(a+b*arctan(c*x))^2/d^2/x+b*c*e*(a+b*arct
an(c*x))*ln(2/(1-I*c*x))/d^2/(c^2*d-e)-b*c*e*(a+b*arctan(c*x))*ln(2/(1+I*c
*x))/d^2/(c^2*d-e)+2*b*c*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^2-1/2*b*c*e
*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I
*e^(1/2)))/d^2/(c^2*d-e)-1/2*b*c*e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*
e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2/(c^2*d-e)+3/4*I*b*(a+b*ar
ctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-
I*e^(1/2)))*e^(1/2)/(-d)^(5/2)-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d^2-3/4*I
*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(
-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(5/2)-I*c*(a+b*arctan(c*x))^2/d^2+1/4*I
*b^2*c*e*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*
e^(1/2)))/d^2/(c^2*d-e)-3/4*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/
2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arctan
(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))
*e^(1/2)/(-d)^(5/2)+1/4*I*b^2*c*e*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(
1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^2/(c^2*d-e)-1/2*I*c*e*(a+b*arctan(c*x
))^2/d^2/(c^2*d-e)-3/8*b^2*polylog(3,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x
)/(c*(-d)^(1/2)-I*e^(1/2)))*e^(1/2)/(-d)^(5/2)+3/8*b^2*polylog(3,1-2*c*((-
d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))*e^(1/2)/(-d)^(5...
```

3.1273.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

output
```
$Aborted
```

3.1273.3 Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(-\frac{e(a+b\arctan(cx))^2}{d^2(d+ex^2)}+\frac{(a+b\arctan(cx))^2}{d^2x^2}-\frac{e(a+b\arctan(cx))^2}{d(d+ex^2)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ice\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2d^2(c^2d-e)}-\frac{ic\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)b^2}{d^2}-\frac{ice\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2d^2(c^2d-e)}+\frac{ice\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4d^2(c^2d-e)}+\frac{ice\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4d^2(c^2d-e)}-\frac{3\sqrt{e}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{5/2}}+\frac{3\sqrt{e}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{5/2}}+\frac{ce(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)b}{d^2(c^2d-e)}-\frac{ce(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)b}{d^2(c^2d-e)}-\frac{ce(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2d^2(c^2d-e)}-\frac{ce(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2d^2(c^2d-e)}+\frac{2c(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)b}{d^2}+\frac{3i\sqrt{e}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{4(-d)^{5/2}}-\frac{3i\sqrt{e}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{4(-d)^{5/2}}-\frac{ice(a+b\arctan(cx))^2}{2d^2(c^2d-e)}-\frac{(a+b\arctan(cx))^2}{d^2x}+\frac{\sqrt{e}(a+b\arctan(cx))^2}{4d^2(\sqrt{-d}-\sqrt{e}x)}-\frac{\sqrt{e}(a+b\arctan(cx))^2}{4d^2(\sqrt{e}x+\sqrt{-d})}-\frac{ic(a+b\arctan(cx))^2}{d^2}-\frac{3\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4(-d)^{5/2}}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

output
```
((-I)*c*(a + b*ArcTan[c*x])^2)/d^2 - ((I/2)*c*e*(a + b*ArcTan[c*x])^2)/(d^
2*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(d^2*x) + (Sqrt[e]*(a + b*ArcTan[c*
x])^2)/(4*d^2*(Sqrt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2)/(4
*d^2*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x
)])/(d^2*(c^2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d^
2*(c^2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x
))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d - e)) - (3*Sqrt[
e]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I
*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(5/2)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(
2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^
2*(c^2*d - e)) + (3*Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqr
t[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(5/2)) + (2*b*c*
(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b^2*c*e*PolyLog[2
, 1 - 2/(1 - I*c*x)])/(d^2*(c^2*d - e)) - (I*b^2*c*PolyLog[2, -1 + 2/(1 -
I*c*x)])/d^2 - ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d^2*(c^2*d -
 e)) + ((I/4)*b^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt
[-d] - I*Sqrt[e])*(1 - I*c*x))])/(d^2*(c^2*d - e)) + (((3*I)/4)*b*Sqrt[e]*
(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-
d] - I*Sqrt[e])*(1 - I*c*x))])/(-d)^(5/2) + ((I/4)*b^2*c*e*PolyLog[2, 1 -
(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(...
```

3.1273.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1273.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(e\,x^2+d)^2}dx$$

input
```
int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

output
```
int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

3.1273.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^6 + 2*d*e*x^
4 + d^2*x^2), x)
```

3.1273.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1273.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.1273.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)^2), x)
```

3.1274 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx$

3.1274.1 Optimal result

Integrand size = 23, antiderivative size = 1181

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx = -\frac{bc(a+b\arctan(cx))}{d^2x} - \frac{c^2(a+b\arctan(cx))^2}{2d^2} + \frac{c^2e(a+b\arctan(cx))^2}{2d^2(c^2d-e)} - \frac{(a+b\arctan(cx))^2}{2d^2x^2} - \frac{e(a+b\arctan(cx))^2}{4d^3\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{e(a+b\arctan(cx))^2}{4d^3\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} - \frac{4e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^3} + \frac{b^2c^2\log(x)}{d^2} - \frac{2e(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^3} - \frac{bce^{3/2}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2(-d)^{5/2}(c^2d-e)} + \frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{d^3} + \frac{bce^{3/2}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2(-d)^{5/2}(c^2d-e)} + \frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{d^3} - \frac{b^2c^2\log(1+c^2x^2)}{2d^2} + \frac{2ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3} + \frac{2ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^3} - \frac{2ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^3} + \frac{ib^2ce^{3/2}\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{5/2}(c^2d-e)} - \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{d^3} - \frac{ib^2ce^{3/2}\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4(-d)^{5/2}(c^2d-e)} - \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{d^3} - \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{d^3} + \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{d^3} - \frac{b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{d^3}$$

output
```
e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2
)-I*e^(1/2)))/d^3+b^2*e*polylog(3,1-2/(1+I*c*x))/d^3-b^2*e*polylog(3,-1+2/
(1+I*c*x))/d^3+e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*
x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3-1/4*e*(a+b*arctan(c*x))^2/d^3/(1-x*e^(1/2
)/(-d)^(1/2))-1/4*e*(a+b*arctan(c*x))^2/d^3/(1+x*e^(1/2)/(-d)^(1/2))+4*e*(
a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3-1/2*b^2*c^2*ln(c^2*x^2+1)/d
^2-b^2*e*polylog(3,1-2/(1-I*c*x))/d^3-1/2*b*c*e^(3/2)*(a+b*arctan(c*x))*ln
(2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(5/2)
/(c^2*d-e)+1/2*b*c*e^(3/2)*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+x*e^(1/2))
/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(5/2)/(c^2*d-e)-1/4*I*b^2*c*e^(3
/2)*polylog(2,1-2*c*((-d)^(1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/
2)))/(-d)^(5/2)/(c^2*d-e)-1/2*c^2*(a+b*arctan(c*x))^2/d^2-1/2*(a+b*arctan(
c*x))^2/d^2/x^2-2*I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^3-I*
b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*
(-d)^(1/2)-I*e^(1/2)))/d^3-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(
1/2)+x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/d^3+1/4*I*b^2*c*e^(3/2
)*polylog(2,1-2*c*((-d)^(1/2)-x*e^(1/2))/(1-I*c*x)/(c*(-d)^(1/2)-I*e^(1/2)
))/(-d)^(5/2)/(c^2*d-e)+2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))
/d^3+2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d^3-b*c*(a+b*arcta
n(c*x))/d^2/x+b^2*c^2*ln(x)/d^2-2*e*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))...
```

3.1274.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2), x]
```

3.1274.3 Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 1181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{ 5515}$$

$$\int \left(\frac{2e^2x(a+b\arctan(cx))^2}{d^3(d+ex^2)}-\frac{2e(a+b\arctan(cx))^2}{d^3x}+\frac{e^2x(a+b\arctan(cx))^2}{d^2(d+ex^2)^2}+\frac{(a+b\arctan(cx))^2}{d^2x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\log(x)b^2}{d^2}-\frac{c^2\log\left(c^2x^2+1\right)b^2}{2d^2}+\frac{ice^{3/2}\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{5/2}\left(c^2d-e\right)}-\frac{ice^{3/2}\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{5/2}\left(c^2d-e\right)}-\frac{e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{d^3}+\frac{e\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)b^2}{d^3}-\frac{e\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)b^2}{d^3}+\frac{e\operatorname{PolyLog}\left(3,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{2d^3}+\frac{e\operatorname{PolyLog}\left(3,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{2d^3}-\frac{c(a+b\arctan(cx))b}{d^2x}-\frac{ce^{3/2}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b}{2(-d)^{5/2}\left(c^2d-e\right)}+\frac{ce^{3/2}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b}{2(-d)^{5/2}\left(c^2d-e\right)}+\frac{2ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b}{d^3}+\frac{2ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b}{d^3}-\frac{2ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)b}{d^3}-\frac{ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b}{d^3}-\frac{ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b}{d^3}+\frac{c^2e(a+b\arctan(cx))^2}{2d^2\left(c^2d-e\right)}-\frac{e(a+b\arctan(cx))^2}{4d^3\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}-\frac{e(a+b\arctan(cx))^2}{4d^3\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}-\frac{c^2(a+b\arctan(cx))^2}{2d^2}-\frac{(a+b\arctan(cx))^2}{2d^2x^2}-\frac{4e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{icx+1}\right)}{d^3}-\frac{2e(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^3}+\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{d^3}+\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{d^3}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

3.1274. $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx$

output
```
-((b*c*(a + b*ArcTan[c*x]))/(d^2*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d^2)
 + (c^2*e*(a + b*ArcTan[c*x])^2)/(2*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])
^2/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 - (Sqrt[e]*x)/Sqrt[-d
])) - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 + (Sqrt[e]*x)/Sqrt[-d])) - (4*e*
(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 + (b^2*c^2*Log[x])/d
^2 - (2*e*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^3 - (b*c*e^(3/2)*(a
+ b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x])^2*Log
[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3
 + (b*c*e^(3/2)*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b
*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + ((2*I)*b*e*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + ((2*I)*b*e*(a + b*Arc
Tan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3 - ((2*I)*b*e*(a + b*ArcTan[c*
x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + ((I/4)*b^2*c*e^(3/2)*PolyLog[2,
1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
((-d)^(5/2)*(c^2*d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*
(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I
/4)*b^2*c*e^(3/2)*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[...
```

3.1274.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

3.1274.4 Maple [F]

$$\int \frac{\left(a+b\arctan\left(cx\right)\right)^{2}}{x^{3}\left(e\,x^{2}+d\right)^{2}}dx$$

input
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

output
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

3.1274.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^{2}}{\left(ex^{2}+d\right)^{2}x^{3}}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^7 + 2*d*e*x^
5 + d^2*x^3), x)
```

3.1274.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.1274.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4
*e*log(x)/d^3) + integrate((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e^2*x^
7 + 2*d*e*x^5 + d^2*x^3), x)
```

3.1274.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.1274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^3\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)^2), x)
```

3.1275 $\int x^4 \arctan(x) \log(1+x^2)\, dx$

3.1275.1 Optimal result

Integrand size = 12, antiderivative size = 111

$$\int x^4 \arctan(x) \log(1+x^2)\, dx = -\frac{77x^2}{300} + \frac{9x^4}{200} - \frac{2}{5}x\arctan(x) + \frac{2}{15}x^3\arctan(x) - \frac{2}{25}x^5\arctan(x) + \frac{\arctan(x)^2}{5} + \frac{137}{300}\log(1+x^2) + \frac{1}{10}x^2\log(1+x^2) - \frac{1}{20}x^4\log(1+x^2) + \frac{1}{5}x^5\arctan(x)\log(1+x^2) - \frac{1}{20}\log^2(1+x^2)$$

output
```
-77/300*x^2+9/200*x^4-2/5*x*arctan(x)+2/15*x^3*arctan(x)-2/25*x^5*arctan(x
)+1/5*arctan(x)^2+137/300*ln(x^2+1)+1/10*x^2*ln(x^2+1)-1/20*x^4*ln(x^2+1)+
1/5*x^5*arctan(x)*ln(x^2+1)-1/20*ln(x^2+1)^2
```

3.1275.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int x^4 \arctan(x) \log(1+x^2)\, dx = \frac{1}{600}\left(x^2(-154+27x^2) + 120\arctan(x)^2 + (274+60x^2-30x^4)\log(1+x^2) - 30\log^2(1+x^2) + 8x\arctan(x)\left(-30+10x^2-6x^4+15x^4\log(1+x^2)\right)\right)$$

input
```
Integrate[x^4*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(x^2*(-154 + 27*x^2) + 120*ArcTan[x]^2 + (274 + 60*x^2 - 30*x^4)*Log[1 + x
^2] - 30*Log[1 + x^2]^2 + 8*x*ArcTan[x]*(-30 + 10*x^2 - 6*x^4 + 15*x^4*Log
[1 + x^2]))/600
```

3.1275.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arctan(x) \log \left(x^2+1\right) \, dx$$

$$\Big\downarrow \ 5556$$

$$-2 \int \left(\frac{x^3 \left(4 \arctan(x) x^3-x^2+2\right)}{20 \left(x^2+1\right)}-\frac{x \log \left(x^2+1\right)}{10 \left(x^2+1\right)}\right)dx+\frac{1}{5} x^5 \arctan(x) \log \left(x^2+1\right)-\frac{1}{10} \log ^2\left(x^2+1\right)+\frac{1}{10} x^2 \log \left(x^2+1\right)-\frac{1}{20} x^4 \log \left(x^2+1\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5} x^5 \arctan(x) \log \left(x^2+1\right)-2 \left(\frac{1}{25} x^5 \arctan(x)-\frac{1}{15} x^3 \arctan(x)+\frac{1}{5} x \arctan(x)-\frac{\arctan(x)^2}{10}-\frac{9 x^4}{400}+\frac{77 x^2}{600}-\frac{1}{40} \log ^2\left(x^2+1\right)-\frac{137}{600} \log (x\right.$$
$$-\frac{1}{10} \log ^2\left(x^2+1\right)+\frac{1}{10} x^2 \log \left(x^2+1\right)-\frac{1}{20} x^4 \log \left(x^2+1\right)$$

input
```
Int[x^4*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(x^2*Log[1 + x^2])/10 - (x^4*Log[1 + x^2])/20 + (x^5*ArcTan[x]*Log[1 + x^2
])/5 - Log[1 + x^2]^2/10 - 2*((77*x^2)/600 - (9*x^4)/400 + (x*ArcTan[x])/5
 - (x^3*ArcTan[x])/15 + (x^5*ArcTan[x])/25 - ArcTan[x]^2/10 - (137*Log[1 +
 x^2])/600 - Log[1 + x^2]^2/40)
```

3.1275.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1275.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{x^5 \arctan(x)\ln(x^2+1)}{5} - \frac{2x^5\arctan(x)}{25} - \frac{x^4\ln(x^2+1)}{20} + \frac{9x^4}{200} + \frac{2x^3\arctan(x)}{15} + \frac{x^2\ln(x^2+1)}{10} - \frac{77x^2}{300} - \frac{2x\,\mathrm{arctan}}{5}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^4*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arctan(x)*ln(x^2+1)-2/25*x^5*arctan(x)-1/20*x^4*ln(x^2+1)+9/200*x^
4+2/15*x^3*arctan(x)+1/10*x^2*ln(x^2+1)-77/300*x^2-2/5*x*arctan(x)+1/5*arc
tan(x)^2-1/20*ln(x^2+1)^2+137/300*ln(x^2+1)+77/300
```

3.1275.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65

$$\begin{aligned}\int x^4 \arctan(x)\log\left(1+x^2\right)\,dx = & \frac{9}{200}\,x^4 - \frac{77}{300}\,x^2 \\ & - \frac{2}{75}\left(3\,x^5 - 5\,x^3 + 15\,x\right)\arctan\left(x\right) + \frac{1}{5}\,\arctan\left(x\right)^2 \\ & + \frac{1}{300}\left(60\,x^5\arctan\left(x\right) - 15\,x^4 + 30\,x^2 + 137\right)\log\left(x^2+1\right) \\ & - \frac{1}{20}\,\log\left(x^2+1\right)^2\end{aligned}$$

input
```
integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
9/200*x^4 - 77/300*x^2 - 2/75*(3*x^5 - 5*x^3 + 15*x)*arctan(x) + 1/5*arcta
n(x)^2 + 1/300*(60*x^5*arctan(x) - 15*x^4 + 30*x^2 + 137)*log(x^2 + 1) - 1
/20*log(x^2 + 1)^2
```

3.1275.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int x^4 \arctan(x) \log\left(1+x^2\right)\,dx = \frac{x^5 \log\left(x^2+1\right) \operatorname{atan}(x)}{5} - \frac{2x^5 \operatorname{atan}(x)}{25} - \frac{x^4 \log\left(x^2+1\right)}{20} + \frac{9x^4}{200} + \frac{2x^3 \operatorname{atan}(x)}{15} + \frac{x^2 \log\left(x^2+1\right)}{10} - \frac{77x^2}{300} - \frac{2x \operatorname{atan}(x)}{5} - \frac{\log\left(x^2+1\right)^2}{20} + \frac{137 \log\left(x^2+1\right)}{300} + \frac{\operatorname{atan}^2(x)}{5}$$

input
```
integrate(x**4*atan(x)*ln(x**2+1),x)
```

output
```
x**5*log(x**2 + 1)*atan(x)/5 - 2*x**5*atan(x)/25 - x**4*log(x**2 + 1)/20 +
 9*x**4/200 + 2*x**3*atan(x)/15 + x**2*log(x**2 + 1)/10 - 77*x**2/300 - 2*
x*atan(x)/5 - log(x**2 + 1)**2/20 + 137*log(x**2 + 1)/300 + atan(x)**2/5
```

3.1275.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int x^4 \arctan(x) \log\left(1+x^2\right)\,dx = \frac{9}{200}\,x^4 - \frac{77}{300}\,x^2 + \frac{1}{75}\left(15\,x^5 \log\left(x^2+1\right) - 6\,x^5 + 10\,x^3 - 30\,x + 30\,\arctan(x)\right)\arctan(x) - \frac{1}{5}\,\arctan(x)^2 - \frac{1}{300}\left(15\,x^4 - 30\,x^2 - 137\right)\log\left(x^2+1\right) - \frac{1}{20}\,\log\left(x^2+1\right)^2$$

input
```
integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
9/200*x^4 - 77/300*x^2 + 1/75*(15*x^5*log(x^2 + 1) - 6*x^5 + 10*x^3 - 30*x
 + 30*arctan(x))*arctan(x) - 1/5*arctan(x)^2 - 1/300*(15*x^4 - 30*x^2 - 13
7)*log(x^2 + 1) - 1/20*log(x^2 + 1)^2
```

3.1275.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.51

$$
\begin{aligned}
\int x^4 \arctan(x)\log\left(1+x^2\right)\,dx = {} & \frac{1}{10}\,\pi x^5\log\left(x^2+1\right)\operatorname{sgn}(x) - \frac{1}{5}\,x^5\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) \\
& - \frac{1}{25}\,\pi x^5\operatorname{sgn}(x) + \frac{2}{25}\,x^5\arctan\left(\frac{1}{x}\right) - \frac{1}{20}\,x^4\log\left(x^2+1\right) \\
& + \frac{1}{15}\,\pi x^3\operatorname{sgn}(x) + \frac{9}{200}\,x^4 - \frac{2}{15}\,x^3\arctan\left(\frac{1}{x}\right) \\
& + \frac{1}{10}\,x^2\log\left(x^2+1\right) - \frac{3}{10}\,\pi^2\operatorname{sgn}(x) - \frac{1}{5}\,\pi x\operatorname{sgn}(x) \\
& - \frac{1}{5}\,\pi\arctan\left(\frac{1}{x}\right)\operatorname{sgn}(x) + \frac{1}{10}\,\pi^2 - \frac{77}{300}\,x^2 \\
& + \frac{1}{5}\,\pi\arctan\left(x\right) + \frac{1}{5}\,\pi\arctan\left(\frac{1}{x}\right) + \frac{2}{5}\,x\arctan\left(\frac{1}{x}\right) \\
& + \frac{1}{5}\,\arctan\left(\frac{1}{x}\right)^2 - \frac{1}{20}\,\log\left(x^2+1\right)^2 + \frac{137}{300}\,\log\left(x^2+1\right)
\end{aligned}
$$

input
```
integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/10*pi*x^5*log(x^2 + 1)*sgn(x) - 1/5*x^5*arctan(1/x)*log(x^2 + 1) - 1/25*
pi*x^5*sgn(x) + 2/25*x^5*arctan(1/x) - 1/20*x^4*log(x^2 + 1) + 1/15*pi*x^3
*sgn(x) + 9/200*x^4 - 2/15*x^3*arctan(1/x) + 1/10*x^2*log(x^2 + 1) - 3/10*
pi^2*sgn(x) - 1/5*pi*x*sgn(x) - 1/5*pi*arctan(1/x)*sgn(x) + 1/10*pi^2 - 77
/300*x^2 + 1/5*pi*arctan(x) + 1/5*pi*arctan(1/x) + 2/5*x*arctan(1/x) + 1/5
*arctan(1/x)^2 - 1/20*log(x^2 + 1)^2 + 137/300*log(x^2 + 1)
```

3.1275.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int x^4 \arctan(x) \log\left(1+x^2\right) \, dx = \frac{137 \ln\left(x^2+1\right)}{300} - \frac{\ln\left(x^2+1\right)^2}{20} + \frac{\operatorname{atan}(x)^2}{5} - \operatorname{atan}(x) \left(\frac{2\,x}{5} - \frac{2\,x^3}{15} + \frac{2\,x^5}{25} - \frac{x^5 \ln\left(x^2+1\right)}{5}\right) + \ln\left(x^2+1\right) \left(\frac{x^2}{10} - \frac{x^4}{20}\right) - \frac{77\,x^2}{300} + \frac{9\,x^4}{200}$$

input
```
int(x^4*log(x^2 + 1)*atan(x),x)
```

output
```
(137*log(x^2 + 1))/300 - log(x^2 + 1)^2/20 + atan(x)^2/5 - atan(x)*((2*x)/
5 - (2*x^3)/15 + (2*x^5)/25 - (x^5*log(x^2 + 1))/5) + log(x^2 + 1)*(x^2/10
 - x^4/20) - (77*x^2)/300 + (9*x^4)/200
```

3.1276 $\int x^3 \arctan(x) \log(1+x^2) \, dx$

3.1276.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int x^3 \arctan(x) \log(1+x^2) \, dx = -\frac{25x}{24} + \frac{7x^3}{72} + \frac{25\arctan(x)}{24} + \frac{1}{4}x^2\arctan(x) - \frac{1}{8}x^4\arctan(x) + \frac{1}{4}x\log(1+x^2) - \frac{1}{12}x^3\log(1+x^2) - \frac{1}{4}\arctan(x)\log(1+x^2) + \frac{1}{4}x^4\arctan(x)\log(1+x^2)$$

output
```
-25/24*x+7/72*x^3+25/24*arctan(x)+1/4*x^2*arctan(x)-1/8*x^4*arctan(x)+1/4*
x*ln(x^2+1)-1/12*x^3*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)+1/4*x^4*arctan(x)*l
n(x^2+1)
```

3.1276.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int x^3 \arctan(x) \log(1+x^2) \, dx = \frac{1}{72}\left(x\left(-75+7x^2-6\left(-3+x^2\right)\log\left(1+x^2\right)\right) + 3\arctan(x)\left(25+6x^2-3x^4+6\left(-1+x^4\right)\log\left(1+x^2\right)\right)\right)$$

input
```
Integrate[x^3*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(x*(-75 + 7*x^2 - 6*(-3 + x^2)*Log[1 + x^2]) + 3*ArcTan[x]*(25 + 6*x^2 - 3
*x^4 + 6*(-1 + x^4)*Log[1 + x^2]))/72
```

3.1276.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(x) \log\left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{5554}$$

$$-\int \left(\frac{x^2 (2-x^2)}{8 (x^2+1)}-\frac{1}{4} (1-x^2) \log\left(x^2+1\right)\right)dx-\frac{1}{8} x^4 \arctan(x)+\frac{1}{4} x^2 \arctan(x)-\frac{1}{4} \arctan(x) \log\left(x^2+1\right)+\frac{1}{4} x^4 \arctan(x) \log\left(x^2+1\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8} x^4 \arctan(x)+\frac{1}{4} x^2 \arctan(x)-\frac{1}{4} \arctan(x) \log\left(x^2+1\right)+\frac{1}{4} x^4 \arctan(x) \log\left(x^2+1\right)+\frac{25 \arctan(x)}{24}+\frac{7 x^3}{72}+\frac{1}{4} x \log\left(x^2+1\right)-\frac{1}{12} x^3 \log\left(x^2+1\right)-\frac{25 x}{24}$$

input
```
Int[x^3*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(-25*x)/24 + (7*x^3)/72 + (25*ArcTan[x])/24 + (x^2*ArcTan[x])/4 - (x^4*Arc
Tan[x])/8 + (x*Log[1 + x^2])/4 - (x^3*Log[1 + x^2])/12 - (ArcTan[x]*Log[1
+ x^2])/4 + (x^4*ArcTan[x]*Log[1 + x^2])/4
```

3.1276.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

3.1276.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

method	result
parallelrisch	$-\frac{25x}{24}+\frac{7x^3}{72}+\frac{25\arctan(x)}{24}+\frac{x^2\arctan(x)}{4}-\frac{x^4\arctan(x)}{8}+\frac{x\ln(x^2+1)}{4}-\frac{x^3\ln(x^2+1)}{12}-\frac{\arctan(x)\ln(x^2+1)}{4}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^3*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-25/24*x+7/72*x^3+25/24*arctan(x)+1/4*x^2*arctan(x)-1/8*x^4*arctan(x)+1/4*
x*ln(x^2+1)-1/12*x^3*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)+1/4*x^4*arctan(x)*l
n(x^2+1)
```

3.1276.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int x^3\arctan(x)\log\left(1+x^2\right)\,dx=\frac{7}{72}\,x^3-\frac{1}{24}\left(3\,x^4-6\,x^2-25\right)\arctan\left(x\right)-\frac{1}{12}\left(x^3-3\left(x^4-1\right)\arctan\left(x\right)-3\,x\right)\log\left(x^2+1\right)-\frac{25}{24}\,x$$

input
```
integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
7/72*x^3 - 1/24*(3*x^4 - 6*x^2 - 25)*arctan(x) - 1/12*(x^3 - 3*(x^4 - 1)*a
rctan(x) - 3*x)*log(x^2 + 1) - 25/24*x
```

3.1276.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int x^3\arctan(x)\log\left(1+x^2\right)\,dx=\frac{x^4\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{4}-\frac{x^4\operatorname{atan}\left(x\right)}{8}-\frac{x^3\log\left(x^2+1\right)}{12}+\frac{7x^3}{72}+\frac{x^2\operatorname{atan}\left(x\right)}{4}+\frac{x\log\left(x^2+1\right)}{4}-\frac{25x}{24}-\frac{\log\left(x^2+1\right)\operatorname{atan}\left(x\right)}{4}+\frac{25\operatorname{atan}\left(x\right)}{24}$$

input
```
integrate(x**3*atan(x)*ln(x**2+1),x)
```

output
```
x**4*log(x**2 + 1)*atan(x)/4 - x**4*atan(x)/8 - x**3*log(x**2 + 1)/12 + 7*
x**3/72 + x**2*atan(x)/4 + x*log(x**2 + 1)/4 - 25*x/24 - log(x**2 + 1)*ata
n(x)/4 + 25*atan(x)/24
```

3.1276.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int x^3\arctan(x)\log\left(1+x^2\right)\,dx = \frac{7}{72}\,x^3+\frac{1}{8}\left(2\,x^4\log\left(x^2+1\right)-x^4+2\,x^2-2\,\log\left(x^2+1\right)\right)\arctan\left(x\right) - \frac{1}{12}\left(x^3-3\,x\right)\log\left(x^2+1\right)-\frac{25}{24}\,x+\frac{25}{24}\,\arctan\left(x\right)$$

input
```
integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
7/72*x^3 + 1/8*(2*x^4*log(x^2 + 1) - x^4 + 2*x^2 - 2*log(x^2 + 1))*arctan(
x) - 1/12*(x^3 - 3*x)*log(x^2 + 1) - 25/24*x + 25/24*arctan(x)
```

3.1276.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int x^3\arctan(x)\log\left(1+x^2\right)\,dx = \frac{1}{8}\,\pi x^4\log\left(x^2+1\right)\mathrm{sgn}(x)-\frac{1}{4}\,x^4\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{1}{16}\,\pi x^4\mathrm{sgn}(x)+\frac{1}{8}\,x^4\arctan\left(\frac{1}{x}\right) - \frac{1}{12}\,x^3\log\left(x^2+1\right)+\frac{1}{8}\,\pi x^2\mathrm{sgn}(x)+\frac{7}{72}\,x^3 - \frac{1}{4}\,x^2\arctan\left(\frac{1}{x}\right)-\frac{1}{8}\,\pi\log\left(x^2+1\right)\mathrm{sgn}(x) + \frac{1}{4}\,x\log\left(x^2+1\right)+\frac{1}{4}\,\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{25}{24}\,\pi\mathrm{sgn}(x)-\frac{25}{24}\,x+\frac{25}{24}\,\arctan\left(x\right)$$

input
```
integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/8*pi*x^4*log(x^2 + 1)*sgn(x) - 1/4*x^4*arctan(1/x)*log(x^2 + 1) - 1/16*p
i*x^4*sgn(x) + 1/8*x^4*arctan(1/x) - 1/12*x^3*log(x^2 + 1) + 1/8*pi*x^2*sg
n(x) + 7/72*x^3 - 1/4*x^2*arctan(1/x) - 1/8*pi*log(x^2 + 1)*sgn(x) + 1/4*x
*log(x^2 + 1) + 1/4*arctan(1/x)*log(x^2 + 1) - 25/24*pi*sgn(x) - 25/24*x +
 25/24*arctan(x)
```

3.1276.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\begin{aligned}\int x^3 \arctan(x) \log\left(1+x^2\right) \, dx &= \frac{25\,\mathrm{atan}(x)}{24} + \frac{x^2\,\mathrm{atan}(x)}{4} + x\left(\frac{\ln\left(x^2+1\right)}{4} - \frac{25}{24}\right) \\ &\quad - x^3\left(\frac{\ln\left(x^2+1\right)}{12} - \frac{7}{72}\right) \\ &\quad - x^4\left(\frac{\mathrm{atan}(x)}{8} - \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{4}\right) \\ &\quad - \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{4}\end{aligned}$$

input
```
int(x^3*log(x^2 + 1)*atan(x),x)
```

output
```
(25*atan(x))/24 + (x^2*atan(x))/4 + x*(log(x^2 + 1)/4 - 25/24) - x^3*(log(
x^2 + 1)/12 - 7/72) - x^4*(atan(x)/8 - (log(x^2 + 1)*atan(x))/4) - (log(x^
2 + 1)*atan(x))/4
```

3.1277 $\int x^2 \arctan(x) \log(1+x^2)\,dx$

3.1277.1 Optimal result

Integrand size = 12, antiderivative size = 82

$$\int x^2 \arctan(x) \log(1+x^2)\,dx = \frac{5x^2}{18} + \frac{2}{3}x\arctan(x) - \frac{2}{9}x^3\arctan(x) - \frac{\arctan(x)^2}{3} - \frac{11}{18}\log(1+x^2) - \frac{1}{6}x^2\log(1+x^2) + \frac{1}{3}x^3\arctan(x)\log(1+x^2) + \frac{1}{12}\log^2(1+x^2)$$

output
```
5/18*x^2+2/3*x*arctan(x)-2/9*x^3*arctan(x)-1/3*arctan(x)^2-11/18*ln(x^2+1)
-1/6*x^2*ln(x^2+1)+1/3*x^3*arctan(x)*ln(x^2+1)+1/12*ln(x^2+1)^2
```

3.1277.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x) \log(1+x^2)\,dx = \frac{1}{36}\left(10x^2 - 12\arctan(x)^2 - 2(11+3x^2)\log(1+x^2) + 3\log^2(1+x^2) + 4x\arctan(x)\left(6-2x^2+3x^2\log(1+x^2)\right)\right)$$

input
```
Integrate[x^2*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(10*x^2 - 12*ArcTan[x]^2 - 2*(11 + 3*x^2)*Log[1 + x^2] + 3*Log[1 + x^2]^2
+ 4*x*ArcTan[x]*(6 - 2*x^2 + 3*x^2*Log[1 + x^2]))/36
```

3.1277.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(x) \log\left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{5556}$$

$$-2 \int \left(\frac{x \log\left(x^2+1\right)}{6\left(x^2+1\right)}-\frac{x^3(1-2 x \arctan(x))}{6\left(x^2+1\right)}\right) dx+\frac{1}{3} x^3 \arctan(x) \log\left(x^2+1\right)+\frac{1}{6} \log^2\left(x^2+1\right)-\frac{1}{6} x^2 \log\left(x^2+1\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{1}{9} x^3 \arctan(x)-\frac{1}{3} x \arctan(x)+\frac{\arctan(x)^2}{6}-\frac{5 x^2}{36}+\frac{1}{24} \log^2\left(x^2+1\right)+\frac{11}{36} \log\left(x^2+1\right)\right)+\frac{1}{3} x^3 \arctan(x) \log\left(x^2+1\right)+\frac{1}{6} \log^2\left(x^2+1\right)-\frac{1}{6} x^2 \log\left(x^2+1\right)$$

input
```
Int[x^2*ArcTan[x]*Log[1 + x^2],x]
```

output
```
-1/6*(x^2*Log[1 + x^2]) + (x^3*ArcTan[x]*Log[1 + x^2])/3 + Log[1 + x^2]^2/
6 - 2*((-5*x^2)/36 - (x*ArcTan[x])/3 + (x^3*ArcTan[x])/9 + ArcTan[x]^2/6 +
 (11*Log[1 + x^2])/36 + Log[1 + x^2]^2/24)
```

3.1277.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1277.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{x^{3}\arctan(x)\ln(x^{2}+1)}{3}-\frac{2x^{3}\arctan(x)}{9}-\frac{x^{2}\ln(x^{2}+1)}{6}+\frac{5x^{2}}{18}+\frac{2x\arctan(x)}{3}-\frac{\arctan(x)^{2}}{3}+\frac{\ln(x^{2}+1)^{2}}{12}-\frac{11\ln(\ldots}{1\ldots}$
default	Expression too large to display
risch	Expression too large to display

input
```
int(x^2*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arctan(x)*ln(x^2+1)-2/9*x^3*arctan(x)-1/6*x^2*ln(x^2+1)+5/18*x^2+2
/3*x*arctan(x)-1/3*arctan(x)^2+1/12*ln(x^2+1)^2-11/18*ln(x^2+1)-5/18
```

3.1277.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

$$\int x^2\arctan(x)\log\left(1+x^2\right)\,dx=\frac{5}{18}\,x^{2}-\frac{2}{9}\left(x^{3}-3\,x\right)\arctan\left(x\right)-\frac{1}{3}\arctan\left(x\right)^{2}+\frac{1}{18}\left(6\,x^{3}\arctan\left(x\right)-3\,x^{2}-11\right)\log\left(x^{2}+1\right)+\frac{1}{12}\log\left(x^{2}+1\right)^{2}$$

input
```
integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
5/18*x^2 - 2/9*(x^3 - 3*x)*arctan(x) - 1/3*arctan(x)^2 + 1/18*(6*x^3*arcta
n(x) - 3*x^2 - 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

3.1277.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int x^2\arctan(x)\log\left(1+x^2\right)\,dx = \frac{x^3\log\left(x^2+1\right)\operatorname{atan}(x)}{3} - \frac{2x^3\operatorname{atan}(x)}{9} - \frac{x^2\log\left(x^2+1\right)}{6} + \frac{5x^2}{18} + \frac{2x\operatorname{atan}(x)}{3} + \frac{\log\left(x^2+1\right)^2}{12} - \frac{11\log\left(x^2+1\right)}{18} - \frac{\operatorname{atan}^2(x)}{3}$$

input
```
integrate(x**2*atan(x)*ln(x**2+1),x)
```

output
```
x**3*log(x**2 + 1)*atan(x)/3 - 2*x**3*atan(x)/9 - x**2*log(x**2 + 1)/6 + 5
*x**2/18 + 2*x*atan(x)/3 + log(x**2 + 1)**2/12 - 11*log(x**2 + 1)/18 - ata
n(x)**2/3
```

3.1277.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int x^2\arctan(x)\log\left(1+x^2\right)\,dx = \frac{5}{18}\,x^2 + \frac{1}{9}\left(3\,x^3\log\left(x^2+1\right) - 2\,x^3 + 6\,x - 6\arctan(x)\right)\arctan(x) + \frac{1}{3}\arctan(x)^2 - \frac{1}{18}\left(3\,x^2+11\right)\log\left(x^2+1\right) + \frac{1}{12}\log\left(x^2+1\right)^2$$

input
```
integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
5/18*x^2 + 1/9*(3*x^3*log(x^2 + 1) - 2*x^3 + 6*x - 6*arctan(x))*arctan(x)
+ 1/3*arctan(x)^2 - 1/18*(3*x^2 + 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

3.1277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(66) = 132$.

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

$$\int x^2 \arctan(x) \log(1+x^2)\,dx = \frac{1}{6}\pi x^3 \log(x^2+1)\,\mathrm{sgn}(x) - \frac{1}{3}x^3 \arctan\left(\frac{1}{x}\right)\log(x^2+1) - \frac{1}{9}\pi x^3 \mathrm{sgn}(x) + \frac{2}{9}x^3 \arctan\left(\frac{1}{x}\right) - \frac{1}{6}x^2\log(x^2+1) + \frac{1}{6}\pi^2 \mathrm{sgn}(x) + \frac{1}{3}\pi x \mathrm{sgn}(x) + \frac{1}{3}\pi \arctan\left(\frac{1}{x}\right)\mathrm{sgn}(x) - \frac{1}{6}\pi^2 + \frac{5}{18}x^2 - \frac{1}{3}\pi\arctan(x) - \frac{1}{3}\pi\arctan\left(\frac{1}{x}\right) - \frac{2}{3}x\arctan\left(\frac{1}{x}\right) - \frac{1}{3}\arctan\left(\frac{1}{x}\right)^2 + \frac{1}{12}\log(x^2+1)^2 - \frac{11}{18}\log(x^2+1)$$

input
```
integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/6*pi*x^3*log(x^2 + 1)*sgn(x) - 1/3*x^3*arctan(1/x)*log(x^2 + 1) - 1/9*pi
*x^3*sgn(x) + 2/9*x^3*arctan(1/x) - 1/6*x^2*log(x^2 + 1) + 1/6*pi^2*sgn(x)
 + 1/3*pi*x*sgn(x) + 1/3*pi*arctan(1/x)*sgn(x) - 1/6*pi^2 + 5/18*x^2 - 1/3
*pi*arctan(x) - 1/3*pi*arctan(1/x) - 2/3*x*arctan(1/x) - 1/3*arctan(1/x)^2
 + 1/12*log(x^2 + 1)^2 - 11/18*log(x^2 + 1)
```

3.1277.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int x^2 \arctan(x) \log(1+x^2)\,dx = \frac{\ln(x^2+1)^2}{12} - \frac{11\ln(x^2+1)}{18} - \frac{\mathrm{atan}(x)^2}{3} - x^2\left(\frac{\ln(x^2+1)}{6} - \frac{5}{18}\right) - x^3\left(\frac{2\,\mathrm{atan}(x)}{9} - \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{3}\right) + \frac{2\,x\,\mathrm{atan}(x)}{3}$$

input
```
int(x^2*log(x^2 + 1)*atan(x),x)
```

output
```
log(x^2 + 1)^2/12 - (11*log(x^2 + 1))/18 - atan(x)^2/3 - x^2*(log(x^2 + 1)
/6 - 5/18) - x^3*((2*atan(x))/9 - (log(x^2 + 1)*atan(x))/3) + (2*x*atan(x)
)/3
```

3.1277. $\int x^2 \arctan(x) \log(1 + x^2)\, dx$

3.1278 $\int x \arctan(x) \log(1+x^2)\, dx$

3.1278.1 Optimal result

Integrand size = 10, antiderivative size = 49

$$\int x \arctan(x) \log(1+x^2)\, dx = \frac{3x}{2} - \frac{3\arctan(x)}{2} - \frac{1}{2}x^2\arctan(x) - \frac{1}{2}x\log(1+x^2) + \frac{1}{2}(1+x^2)\arctan(x)\log(1+x^2)$$

output
```
3/2*x-3/2*arctan(x)-1/2*x^2*arctan(x)-1/2*x*ln(x^2+1)+1/2*(x^2+1)*arctan(x
)*ln(x^2+1)
```

3.1278.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int x \arctan(x) \log(1+x^2)\, dx = \frac{1}{2}\left(3x - 3\arctan(x) - x^2\arctan(x) + \left(-x + (1+x^2)\arctan(x)\right)\log(1+x^2)\right)$$

input
```
Integrate[x*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(3*x - 3*ArcTan[x] - x^2*ArcTan[x] + (-x + (1 + x^2)*ArcTan[x])*Log[1 + x^
2])/2
```

3.1278.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(x) \log\left(x^2+1\right) \, dx$$

$$\Big\downarrow \ 5554$$

$$-\int \left(\frac{1}{2}\log\left(x^2+1\right)-\frac{x^2}{2\left(x^2+1\right)}\right)dx-\frac{1}{2}x^2\arctan(x)+\frac{1}{2}\left(x^2+1\right)\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{2}x^2\arctan(x)+\frac{1}{2}\left(x^2+1\right)\arctan(x)\log\left(x^2+1\right)-\frac{3\arctan(x)}{2}-\frac{1}{2}x\log\left(x^2+1\right)+\frac{3x}{2}$$

input
```
Int[x*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(3*x)/2 - (3*ArcTan[x])/2 - (x^2*ArcTan[x])/2 - (x*Log[1 + x^2])/2 + ((1 +
 x^2)*ArcTan[x]*Log[1 + x^2])/2
```

3.1278.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

3.1278.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

method	result	size
parallelrisch	$\frac{x^2 \arctan(x) \ln(x^2+1)}{2} - \frac{x^2 \arctan(x)}{2} - \frac{x \ln(x^2+1)}{2} + \frac{\arctan(x) \ln(x^2+1)}{2} + \frac{3x}{2} - \frac{3 \arctan(x)}{2}$	48
default	Expression too large to display	2222
risch	Expression too large to display	15978

input
```
int(x*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(x)*ln(x^2+1)-1/2*x^2*arctan(x)-1/2*x*ln(x^2+1)+1/2*arctan(x
)*ln(x^2+1)+3/2*x-3/2*arctan(x)
```

3.1278.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int x \arctan(x) \log(1+x^2)\, dx = -\frac{1}{2}(x^2+3)\arctan(x) + \frac{1}{2}\left((x^2+1)\arctan(x) - x\right)\log(x^2+1) + \frac{3}{2}x$$

input
```
integrate(x*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*(x^2 + 3)*arctan(x) + 1/2*((x^2 + 1)*arctan(x) - x)*log(x^2 + 1) + 3/
2*x
```

3.1278.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int x \arctan(x) \log(1+x^2)\, dx = \frac{x^2 \log(x^2+1) \operatorname{atan}(x)}{2} - \frac{x^2 \operatorname{atan}(x)}{2} - \frac{x \log(x^2+1)}{2} + \frac{3x}{2} + \frac{\log(x^2+1) \operatorname{atan}(x)}{2} - \frac{3 \operatorname{atan}(x)}{2}$$

input
```
integrate(x*atan(x)*ln(x**2+1),x)
```

output
```
x**2*log(x**2 + 1)*atan(x)/2 - x**2*atan(x)/2 - x*log(x**2 + 1)/2 + 3*x/2
+ log(x**2 + 1)*atan(x)/2 - 3*atan(x)/2
```

3.1278.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx=-\frac{1}{2}\left(x^2-\left(x^2+1\right)\log\left(x^2+1\right)+1\right)\arctan\left(x\right)-\frac{1}{2}\,x\log\left(x^2+1\right)+\frac{3}{2}\,x-\arctan\left(x\right)$$

input
```
integrate(x*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*(x^2 - (x^2 + 1)*log(x^2 + 1) + 1)*arctan(x) - 1/2*x*log(x^2 + 1) + 3
/2*x - arctan(x)
```

3.1278.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx=\frac{1}{4}\,\pi x^2\log\left(x^2+1\right)\operatorname{sgn}(x)-\frac{1}{2}\,x^2\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right)-\frac{1}{4}\,\pi x^2\operatorname{sgn}(x)+\frac{1}{2}\,x^2\arctan\left(\frac{1}{x}\right)+\frac{1}{4}\,\pi\log\left(x^2+1\right)\operatorname{sgn}(x)-\frac{1}{2}\,x\log\left(x^2+1\right)-\frac{1}{2}\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right)+\frac{3}{2}\,x-\frac{3}{2}\arctan\left(x\right)$$

input
```
integrate(x*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/4*pi*x^2*log(x^2 + 1)*sgn(x) - 1/2*x^2*arctan(1/x)*log(x^2 + 1) - 1/4*pi
*x^2*sgn(x) + 1/2*x^2*arctan(1/x) + 1/4*pi*log(x^2 + 1)*sgn(x) - 1/2*x*log
(x^2 + 1) - 1/2*arctan(1/x)*log(x^2 + 1) + 3/2*x - 3/2*arctan(x)
```

3.1278.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx=\frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{2}-x\left(\frac{\ln\left(x^2+1\right)}{2}-\frac{3}{2}\right)-x^2\left(\frac{\mathrm{atan}(x)}{2}-\frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{2}\right)-\frac{3\,\mathrm{atan}(x)}{2}$$

input
```
int(x*log(x^2 + 1)*atan(x),x)
```

output
```
(log(x^2 + 1)*atan(x))/2 - x*(log(x^2 + 1)/2 - 3/2) - x^2*(atan(x)/2 - (lo
g(x^2 + 1)*atan(x))/2) - (3*atan(x))/2
```

3.1279 $\int \arctan(x)\log(1+x^2)\,dx$

3.1279.1 Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \arctan(x)\log(1+x^2)\,dx = -2x\arctan(x)+\arctan(x)^2+\log(1+x^2)+x\arctan(x)\log(1+x^2)-\frac{1}{4}\log^2(1+x^2)$$

output
```
-2*x*arctan(x)+arctan(x)^2+ln(x^2+1)+x*arctan(x)*ln(x^2+1)-1/4*ln(x^2+1)^2
```

3.1279.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \arctan(x)\log(1+x^2)\,dx = -2x\arctan(x)+\arctan(x)^2+\log(1+x^2)+x\arctan(x)\log(1+x^2)-\frac{1}{4}\log^2(1+x^2)$$

input
```
Integrate[ArcTan[x]*Log[1 + x^2],x]
```

output
```
-2*x*ArcTan[x] + ArcTan[x]^2 + Log[1 + x^2] + x*ArcTan[x]*Log[1 + x^2] - L
og[1 + x^2]^2/4
```

3.1279.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {5544, 2925, 2837, 2738, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(x)\log\left(x^2+1\right)\,dx$$

$$\Big\downarrow \text{5544}$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx-\int \frac{x\log\left(x^2+1\right)}{x^2+1}dx+x\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \text{2925}$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx-\frac{1}{2}\int \frac{\log\left(x^2+1\right)}{x^2+1}dx^2+x\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \text{2837}$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx-\frac{1}{2}\int \frac{\log\left(x^2+1\right)}{x^2}d\left(x^2+1\right)+x\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \text{2738}$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \text{5451}$$

$$-2\left(\int \arctan(x)dx-\int \frac{\arctan(x)}{x^2+1}dx\right)+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \text{5345}$$

$$-2\left(-\int \frac{\arctan(x)}{x^2+1}dx-\int \frac{x}{x^2+1}dx+x\arctan(x)\right)+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \text{240}$$

$$-2\left(-\int \frac{\arctan(x)}{x^2+1}dx+x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \text{5419}$$

$$x\arctan(x)\log\left(x^2+1\right)-2\left(-\frac{1}{2}\arctan(x)^2+x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

input
```
Int[ArcTan[x]*Log[1 + x^2],x]
```

output
```
-2*(x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2) + x*ArcTan[x]*Log[1 + x^
2] - Log[1 + x^2]^2/4
```

3.1279.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5544
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2
*e*g   Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

3.1279.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$-2x\arctan\left(x\right)+\arctan\left(x\right)^{2}+\ln\left(x^{2}+1\right)+x\arctan\left(x\right)\ln\left(x^{2}+1\right)-\frac{\ln\left(x^{2}+1\right)^{2}}{4}$	37
default	Expression too large to display	1913
risch	Expression too large to display	4618

input
```
int(arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x*arctan(x)+arctan(x)^2+ln(x^2+1)+x*arctan(x)*ln(x^2+1)-1/4*ln(x^2+1)^2
```

3.1279.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \arctan(x)\log\left(1+x^2\right)\,dx=-2\,x\arctan\left(x\right)+\arctan\left(x\right)^2+\left(x\arctan\left(x\right)+1\right)\log\left(x^2+1\right)-\frac{1}{4}\,\log\left(x^2+1\right)^2$$

input
```
integrate(arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
-2*x*arctan(x) + arctan(x)^2 + (x*arctan(x) + 1)*log(x^2 + 1) - 1/4*log(x^
2 + 1)^2
```

3.1279.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \arctan(x)\log\left(1+x^2\right)\,dx = x\log\left(x^2+1\right)\operatorname{atan}(x) - 2x\operatorname{atan}(x) - \frac{\log\left(x^2+1\right)^2}{4} + \log\left(x^2+1\right) + \operatorname{atan}^2(x)$$

input
```
integrate(atan(x)*ln(x**2+1),x)
```

output
```
x*log(x**2 + 1)*atan(x) - 2*x*atan(x) - log(x**2 + 1)**2/4 + log(x**2 + 1)
 + atan(x)**2
```

3.1279.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \arctan(x)\log\left(1+x^2\right)\,dx = \left(x\log\left(x^2+1\right) - 2\,x + 2\,\arctan(x)\right)\arctan(x) - \arctan(x)^2 - \frac{1}{4}\,\log\left(x^2+1\right)^2 + \log\left(x^2+1\right)$$

input
```
integrate(arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
(x*log(x^2 + 1) - 2*x + 2*arctan(x))*arctan(x) - arctan(x)^2 - 1/4*log(x^2
 + 1)^2 + log(x^2 + 1)
```

3.1279.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(36) = 72.

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.42

$$\int \arctan(x)\log\left(1+x^2\right)\,dx = \frac{1}{2}\,\pi x\log\left(x^2+1\right)\operatorname{sgn}(x) - x\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{3}{2}\,\pi^2\operatorname{sgn}(x) - \pi x\operatorname{sgn}(x) - \pi\arctan\left(\frac{1}{x}\right)\operatorname{sgn}(x) + \frac{1}{2}\,\pi^2 + \pi\arctan(x) + \pi\arctan\left(\frac{1}{x}\right) + 2\,x\arctan\left(\frac{1}{x}\right) + \arctan\left(\frac{1}{x}\right)^2 - \frac{1}{4}\,\log\left(x^2+1\right)^2 + \log\left(x^2+1\right)$$

input
```
integrate(arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/2*pi*x*log(x^2 + 1)*sgn(x) - x*arctan(1/x)*log(x^2 + 1) - 3/2*pi^2*sgn(x
) - pi*x*sgn(x) - pi*arctan(1/x)*sgn(x) + 1/2*pi^2 + pi*arctan(x) + pi*arc
tan(1/x) + 2*x*arctan(1/x) + arctan(1/x)^2 - 1/4*log(x^2 + 1)^2 + log(x^2
+ 1)
```

3.1279.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \arctan(x)\log\left(1+x^2\right)\,dx = \ln\left(x^2+1\right) - \frac{\ln\left(x^2+1\right)^2}{4} + \mathrm{atan}(x)^2 - x\left(2\,\mathrm{atan}(x) - \ln\left(x^2+1\right)\,\mathrm{atan}(x)\right)$$

input
```
int(log(x^2 + 1)*atan(x),x)
```

output
```
log(x^2 + 1) - log(x^2 + 1)^2/4 + atan(x)^2 - x*(2*atan(x) - log(x^2 + 1)*
atan(x))
```

3.1280 $\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx$

3.1280.1 Optimal result

Integrand size = 12, antiderivative size = 189

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = &-\frac{1}{2}i\log^2(1+ix)\log(-ix)+\frac{1}{2}i\log^2(1-ix)\log(ix)\\&+i\log(1-ix)\operatorname{PolyLog}(2,1-ix)\\&-i\log(1+ix)\operatorname{PolyLog}(2,1+ix)\\&-\frac{1}{2}i\left(\log(1-ix)+\log(1+ix)-\log\left(1+x^2\right)\right)\operatorname{PolyLog}(2,-ix)\\&+\frac{1}{2}i\left(\log(1-ix)+\log(1+ix)-\log\left(1+x^2\right)\right)\operatorname{PolyLog}(2,ix)\\&-i\operatorname{PolyLog}(3,1-ix)+i\operatorname{PolyLog}(3,1+ix)\end{aligned}$$

output
```
-1/2*I*ln(1+I*x)^2*ln(-I*x)+1/2*I*ln(1-I*x)^2*ln(I*x)+I*ln(1-I*x)*polylog(
2,1-I*x)-I*ln(1+I*x)*polylog(2,1+I*x)-1/2*I*(ln(1-I*x)+ln(1+I*x)-ln(x^2+1)
)*polylog(2,-I*x)+1/2*I*(ln(1-I*x)+ln(1+I*x)-ln(x^2+1))*polylog(2,I*x)-I*p
olylog(3,1-I*x)+I*polylog(3,1+I*x)
```

3.1280.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.61

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \frac{1}{2}i\left(-\log^2(1-ix)\log(x)+\log^2(1+ix)\log(x) - 2\log(1+ix)\log(x)\log(-i+x)-\log(-ix)\log^2(-i+x) + \log(x)\log^2(-i+x)+2\log(1-ix)\log(x)\log(i+x) + \log(ix)\log^2(i+x)-\log(x)\log^2(i+x) + 2\log(i+x)\operatorname{PolyLog}(2,1-ix) - 2\log(-i+x)\operatorname{PolyLog}(2,1+ix) - \log(1-ix)\operatorname{PolyLog}(2,-ix)+\log(1+ix)\operatorname{PolyLog}(2,-ix) - 2\log(-i+x)\operatorname{PolyLog}(2,-ix) + \log\left(1+x^2\right)\operatorname{PolyLog}(2,-ix)-\log(1-ix)\operatorname{PolyLog}(2,ix) + \log(1+ix)\operatorname{PolyLog}(2,ix)+2\log(i+x)\operatorname{PolyLog}(2,ix) - \log\left(1+x^2\right)\operatorname{PolyLog}(2,ix)-2\operatorname{PolyLog}(3,1-ix) + 2\operatorname{PolyLog}(3,1+ix)\right)$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x,x]
```

output
```
(I/2)*(-(Log[1 - I*x]^2*Log[x]) + Log[1 + I*x]^2*Log[x] - 2*Log[1 + I*x]*L
og[x]*Log[-I + x] - Log[(-I)*x]*Log[-I + x]^2 + Log[x]*Log[-I + x]^2 + 2*L
og[1 - I*x]*Log[x]*Log[I + x] + Log[I*x]*Log[I + x]^2 - Log[x]*Log[I + x]^
2 + 2*Log[I + x]*PolyLog[2, 1 - I*x] - 2*Log[-I + x]*PolyLog[2, 1 + I*x] -
 Log[1 - I*x]*PolyLog[2, (-I)*x] + Log[1 + I*x]*PolyLog[2, (-I)*x] - 2*Log
[-I + x]*PolyLog[2, (-I)*x] + Log[1 + x^2]*PolyLog[2, (-I)*x] - Log[1 - I*
x]*PolyLog[2, I*x] + Log[1 + I*x]*PolyLog[2, I*x] + 2*Log[I + x]*PolyLog[2
, I*x] - Log[1 + x^2]*PolyLog[2, I*x] - 2*PolyLog[3, 1 - I*x] + 2*PolyLog[
3, 1 + I*x])
```

3.1280.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5546, 2843, 2881, 2821, 5355, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x}dx$$

$$\Big\downarrow \; 5546$$

$$-\left(\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\int\frac{\arctan(x)}{x}dx\right)+\frac{1}{2}i\int\frac{\log^2(1-ix)}{x}dx-\frac{1}{2}i\int\frac{\log^2(ix+1)}{x}dx$$

$$\Big\downarrow \; 2843$$

$$-\left(\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\int\frac{\arctan(x)}{x}dx\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2i\int\frac{\log(ix+1)\log(-ix)}{ix+1}dx\right)+\frac{1}{2}i\left(2i\int\frac{\log(1-ix)\log(ix)}{1-ix}dx+\log(ix)\log^2(1-ix)\right)$$

$$\Big\downarrow \; 2881$$

$$-\left(\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\int\frac{\arctan(x)}{x}dx\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\int\frac{\log(ix+1)\log(-ix)}{ix+1}d(ix+1)\right)+\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\int\frac{\log(1-ix)\log(ix)}{1-ix}d(1-ix)\right)$$

$$\Big\downarrow \; 2821$$

$$-\left(\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\int\frac{\arctan(x)}{x}dx\right)+\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\left(\int\frac{\text{PolyLog}(2,1-ix)}{1-ix}d(1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix)\right)\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\left(\int\frac{\text{PolyLog}(2,ix+1)}{ix+1}d(ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix)\right)\right)$$

$$\Big\downarrow \; 5355$$

$$\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\left(\int\frac{\text{PolyLog}(2,1-ix)}{1-ix}d(1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix)\right)\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\left(\int\frac{\text{PolyLog}(2,ix+1)}{ix+1}d(ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix)\right)\right)-\left(\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\left(\frac{1}{2}i\int\frac{\log(1-ix)}{x}dx-\frac{1}{2}i\int\frac{\log(ix+1)}{x}dx\right)\right)$$

$$\Big\downarrow \; 2838$$

3.1280. $\int \frac{\arctan(x)\log(1+x^2)}{x}dx$

$$\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\left(\int\frac{\text{PolyLog}(2,1-ix)}{1-ix}d(1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix)\right)\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\left(\int\frac{\text{PolyLog}(2,ix+1)}{ix+1}d(ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix)\right)\right)-\left(\left(\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)\right)\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$-\left(\left(\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)\right)\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\right)+\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2(\text{PolyLog}(3,1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix))\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2(\text{PolyLog}(3,ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix))\right)$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x,x]
```

output
```
-((Log[1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*((I/2)*PolyLog[2, (-I)*x] -
 (I/2)*PolyLog[2, I*x])) + (I/2)*(Log[1 - I*x]^2*Log[I*x] - 2*(-(Log[1 - I
*x]*PolyLog[2, 1 - I*x]) + PolyLog[3, 1 - I*x])) - (I/2)*(Log[1 + I*x]^2*L
og[(-I)*x] - 2*(-(Log[1 + I*x]*PolyLog[2, 1 + I*x]) + PolyLog[3, 1 + I*x])
)
```

3.1280.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5546
```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Simp[
(Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x])   Int[ArcTan[c*x]/x, x],
 x] + (Simp[I/2   Int[Log[1 - I*c*x]^2/x, x], x] - Simp[I/2   Int[Log[1 + I
*c*x]^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.1280.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.67 (sec) , antiderivative size = 2965, normalized size of antiderivative = 15.69

method	result	size
risch	Expression too large to display	2965

input
```
int(arctan(x)*ln(x^2+1)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(x+I)^2*ln(1+I*(x+I))+I*ln(x+I)*polylog(2,-I*(x+I))-I*polylog(3,-I
*(x+I))-1/2*I*ln(x-I)^2*ln(1-I*(x-I))-I*ln(x-I)*polylog(2,I*(x-I))+I*polyl
og(3,I*(x-I))-1/8*I*Pi^2*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+
1,index=1)))*(-csgn(I*(x-RootOf(_Z^2+1,index=1)))^2*csgn(I*(x+RootOf(_Z^2+
1,index=1)))*csgn(x-RootOf(_Z^2+1,index=1))+csgn(I*(x-RootOf(_Z^2+1,index=
1)))^2*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,index=1)))*csgn(
x-RootOf(_Z^2+1,index=1))-csgn(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x+Roo
tOf(_Z^2+1,index=1)))*csgn(x+RootOf(_Z^2+1,index=1))*csgn(I*(x-RootOf(_Z^2
+1,index=1))*(x+RootOf(_Z^2+1,index=1)))-csgn(I*(x-RootOf(_Z^2+1,index=1))
)*csgn(I*(x+RootOf(_Z^2+1,index=1)))*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+
RootOf(_Z^2+1,index=1)))*csgn(x-RootOf(_Z^2+1,index=1))^2-csgn(I*(x-RootOf
(_Z^2+1,index=1)))*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,inde
x=1)))^2*csgn(x-RootOf(_Z^2+1,index=1))+csgn(I*(x-RootOf(_Z^2+1,index=1)))
*csgn(I*(x+RootOf(_Z^2+1,index=1)))*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+R
ootOf(_Z^2+1,index=1)))*csgn(x-RootOf(_Z^2+1,index=1))-csgn(I*(x-RootOf(_Z
^2+1,index=1)))*csgn(I*(x+RootOf(_Z^2+1,index=1)))^2*csgn(x+RootOf(_Z^2+1,
index=1))^2+csgn(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x+RootOf(_Z^2+1,ind
ex=1)))*csgn(x+RootOf(_Z^2+1,index=1))^3+csgn(I*(x-RootOf(_Z^2+1,index=1))
)*csgn(I*(x+RootOf(_Z^2+1,index=1)))*csgn(x-RootOf(_Z^2+1,index=1))^3-csgn
(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x+RootOf(_Z^2+1,index=1)))*csgn(...
```

3.1280.5 Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x, x)
```

3.1280.6 Sympy [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\log(x^2+1)\,\mathrm{atan}\,(x)}{x}\,dx$$

input
```
integrate(atan(x)*ln(x**2+1)/x,x)
```

output
```
Integral(log(x**2 + 1)*atan(x)/x, x)
```

3.1280.7 Maxima [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x,x, algorithm="maxima")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x, x)
```

3.1280.8 Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x, x)
```

3.1280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log\left(1+x^2\right)}{x}\,dx = \int \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{x}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x, x)
```

3.1281 $\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx$

3.1281.1 Optimal result

Integrand size = 12, antiderivative size = 41

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \arctan(x)^2 - \frac{\arctan(x)\log(1+x^2)}{x} - \frac{1}{4}\log^2(1+x^2) - \frac{\text{PolyLog}(2,-x^2)}{2}$$

output
```
arctan(x)^2-arctan(x)*ln(x^2+1)/x-1/4*ln(x^2+1)^2-1/2*polylog(2,-x^2)
```

3.1281.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \arctan(x)^2 - \frac{\arctan(x)\log(1+x^2)}{x} - \frac{1}{4}\log^2(1+x^2) - \frac{\text{PolyLog}(2,-x^2)}{2}$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

output
```
ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x - Log[1 + x^2]^2/4 - PolyLog[2, -
x^2]/2
```

3.1281.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5552, 2925, 2857, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^2}dx$$

$$\Big\downarrow \text{ 5552}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx+\int \frac{\log\left(x^2+1\right)}{x\left(x^2+1\right)}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{x}$$

$$\Big\downarrow \text{ 2925}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \frac{\log\left(x^2+1\right)}{x^2\left(x^2+1\right)}dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{x}$$

$$\Big\downarrow \text{ 2857}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \left(\frac{\log\left(x^2+1\right)}{-x^2-1}+\frac{\log\left(x^2+1\right)}{x^2}\right)dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{x}+\frac{1}{2}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2}\log^2\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{ 5419}$$

$$-\frac{\arctan(x)\log\left(x^2+1\right)}{x}+\arctan(x)^2+\frac{1}{2}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2}\log^2\left(x^2+1\right)\right)$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

output
```
ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x + (-1/2*Log[1 + x^2]^2 - PolyLog[
2, -x^2])/2
```

3.1281.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

3.1281.4 Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^2}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^2,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^2,x)
```

3.1281.5 Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^2}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^2, x)
```

3.1281.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = -\frac{\log(x^2+1)^2}{4} + \operatorname{atan}^2(x) - \frac{\operatorname{Li}_2(x^2e^{i\pi})}{2} - \frac{\log(x^2+1)\operatorname{atan}(x)}{x}$$

input
```
integrate(atan(x)*ln(x**2+1)/x**2,x)
```

output
```
-log(x**2 + 1)**2/4 + atan(x)**2 - polylog(2, x**2*exp_polar(I*pi))/2 - lo
g(x**2 + 1)*atan(x)/x
```

3.1281.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = -\left(\frac{\log(x^2+1)}{x} - 2\arctan(x)\right)\arctan(x) - \arctan(x)^2 + \frac{1}{2}\log(-x^2)\log(x^2+1) - \frac{1}{4}\log(x^2+1)^2 + \frac{1}{2}\operatorname{Li}_2(x^2+1)$$

input
```
integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="maxima")
```

output
```
-(log(x^2 + 1)/x - 2*arctan(x))*arctan(x) - arctan(x)^2 + 1/2*log(-x^2)*lo
g(x^2 + 1) - 1/4*log(x^2 + 1)^2 + 1/2*dilog(x^2 + 1)
```

3.1281.8 Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^2}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^2, x)
```

3.1281.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \operatorname{atan}(x)^2 - \frac{\ln(x^2+1)^2}{4} - \frac{\operatorname{Li}_2(x^2+1)}{2} - \frac{\ln(x^2+1)\operatorname{atan}(x)}{x}$$

input
```
int((log(x^2 + 1)*atan(x))/x^2,x)
```

output
```
atan(x)^2 - log(x^2 + 1)^2/4 - dilog(x^2 + 1)/2 - (log(x^2 + 1)*atan(x))/x
```

3.1282 $\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx$

3.1282.1 Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \arctan(x) - \frac{\log(1+x^2)}{2x} - \frac{1}{2}\arctan(x)\log(1+x^2) - \frac{\arctan(x)\log(1+x^2)}{2x^2} + \frac{1}{2}i\,\mathrm{PolyLog}(2,-ix) - \frac{1}{2}i\,\mathrm{PolyLog}(2,ix)$$

output
```
arctan(x)-1/2*ln(x^2+1)/x-1/2*arctan(x)*ln(x^2+1)-1/2*arctan(x)*ln(x^2+1)/
x^2+1/2*I*polylog(2,-I*x)-1/2*I*polylog(2,I*x)
```

3.1282.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \arctan(x) - \frac{(x+\arctan(x)+x^2\arctan(x))\log(1+x^2)}{2x^2} + \frac{1}{2}i(\mathrm{PolyLog}(2,-ix) - \mathrm{PolyLog}(2,ix))$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

output
```
ArcTan[x] - ((x + ArcTan[x] + x^2*ArcTan[x])*Log[1 + x^2])/(2*x^2) + (I/2)
*(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

3.1282.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log(x^2+1)}{x^3}\,dx$$

$$\Big\downarrow \text{5556}$$

$$-2\int \left(-\frac{\arctan(x)}{2x}-\frac{1}{2(x^2+1)}\right)dx-\frac{\arctan(x)\log(x^2+1)}{2x^2}-\frac{1}{2}\arctan(x)\log(x^2+1)-\frac{\log(x^2+1)}{2x}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{\arctan(x)}{2}-\frac{1}{4}i\,\text{PolyLog}(2,-ix)+\frac{1}{4}i\,\text{PolyLog}(2,ix)\right)-\frac{\arctan(x)\log(x^2+1)}{2x^2}-\frac{1}{2}\arctan(x)\log(x^2+1)-\frac{\log(x^2+1)}{2x}$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

output
```
-1/2*Log[1 + x^2]/x - (ArcTan[x]*Log[1 + x^2])/2 - (ArcTan[x]*Log[1 + x^2]
)/(2*x^2) - 2*(-1/2*ArcTan[x] - (I/4)*PolyLog[2, (-I)*x] + (I/4)*PolyLog[2
, I*x])
```

3.1282.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1282.4 Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^3}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^3,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^3,x)
```

3.1282.5 Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^3}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^3, x)
```

3.1282.6 Sympy [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \int \frac{\log(x^2+1)\,\mathrm{atan}(x)}{x^3}\,dx$$

input
```
integrate(atan(x)*ln(x**2+1)/x**3,x)
```

output
```
Integral(log(x**2 + 1)*atan(x)/x**3, x)
```

3.1282.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx$$
$$= \frac{4\,x^2\arctan(x)\log(x) + 4\,x^2\arctan(x) - 2i\,x^2\mathrm{Li}_2(i\,x+1) + 2i\,x^2\mathrm{Li}_2(-i\,x+1) - (\pi x^2 + 2\,(x^2+1)\,\mathrm{arcta}}{4\,x^2}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="maxima")
```

output
```
1/4*(4*x^2*arctan(x)*log(x) + 4*x^2*arctan(x) - 2*I*x^2*dilog(I*x + 1) + 2
*I*x^2*dilog(-I*x + 1) - (pi*x^2 + 2*(x^2 + 1)*arctan(x) + 2*x)*log(x^2 +
1))/x^2
```

3.1282.8 Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^3}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^3, x)
```

3.1282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \int \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x^3}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^3,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^3, x)
```

3.1283 $\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx$

3.1283.1 Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = -\frac{2\arctan(x)}{3x}-\frac{\arctan(x)^2}{3}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)-\frac{\log(1+x^2)}{6x^2}-\frac{\arctan(x)\log(1+x^2)}{3x^3}+\frac{1}{12}\log^2\left(1+x^2\right)+\frac{\text{PolyLog}(2,-x^2)}{6}$$

output
```
-2/3*arctan(x)/x-1/3*arctan(x)^2+ln(x)-1/2*ln(x^2+1)-1/6*ln(x^2+1)/x^2-1/3
*arctan(x)*ln(x^2+1)/x^3+1/12*ln(x^2+1)^2+1/6*polylog(2,-x^2)
```

3.1283.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = -\frac{2\arctan(x)}{3x}-\frac{\arctan(x)^2}{3}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)-\frac{\log(1+x^2)}{6x^2}-\frac{\arctan(x)\log(1+x^2)}{3x^3}+\frac{1}{12}\log^2\left(1+x^2\right)+\frac{\text{PolyLog}(2,-x^2)}{6}$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^4,x]
```

output
```
(-2*ArcTan[x])/(3*x) - ArcTan[x]^2/3 + Log[x] - Log[1 + x^2]/2 - Log[1 + x
^2]/(6*x^2) - (ArcTan[x]*Log[1 + x^2])/(3*x^3) + Log[1 + x^2]^2/12 + PolyL
og[2, -x^2]/6
```

3.1283.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5552, 2925, 2857, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^4}dx$$

$$\Big\downarrow \text{5552}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx+\frac{1}{3}\int \frac{\log\left(x^2+1\right)}{x^3\left(x^2+1\right)}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}$$

$$\Big\downarrow \text{2925}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx+\frac{1}{6}\int \frac{\log\left(x^2+1\right)}{x^4\left(x^2+1\right)}dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}$$

$$\Big\downarrow \text{2857}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx+\frac{1}{6}\int \left(\frac{\log\left(x^2+1\right)}{x^2+1}-\frac{\log\left(x^2+1\right)}{x^2}+\frac{\log\left(x^2+1\right)}{x^4}\right)dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{2}{3}\left(\int \frac{\arctan(x)}{x^2}dx-\int \frac{\arctan(x)}{x^2+1}dx\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\int \frac{1}{x\left(x^2+1\right)}dx-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \frac{1}{x^2\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\log\left(x^2\right)-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{2}{3}\left(-\frac{1}{2}\arctan(x)^2-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^4,x]
```

3.1283. $\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx$

output
```
(2*(-(ArcTan[x]/x) - ArcTan[x]^2/2 + (Log[x^2] - Log[1 + x^2])/2))/3 - (Ar
cTan[x]*Log[1 + x^2])/(3*x^3) + (Log[x^2] - Log[1 + x^2] - Log[1 + x^2]/x^
2 + Log[1 + x^2]^2/2 + PolyLog[2, -x^2])/6
```

3.1283.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

3.1283. $\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

3.1283.4 Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^4}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^4,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^4,x)
```

3.1283.5 Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^4}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^4, x)
```

3.1283.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = {} & \frac{2\log(x)}{3} + \frac{\log(2x^2)}{6} + \frac{\log(x^2+1)^2}{12} - \frac{\log(x^2+1)}{3} \\ & - \frac{\log(2x^2+2)}{6} - \frac{\operatorname{atan}^2(x)}{3} + \frac{\operatorname{Li}_2(x^2e^{i\pi})}{6} \\ & - \frac{2\operatorname{atan}(x)}{3x} - \frac{\log(x^2+1)}{6x^2} - \frac{\log(x^2+1)\operatorname{atan}(x)}{3x^3}\end{aligned}$$

input
```
integrate(atan(x)*ln(x**2+1)/x**4,x)
```

output
```
2*log(x)/3 + log(2*x**2)/6 + log(x**2 + 1)**2/12 - log(x**2 + 1)/3 - log(2
*x**2 + 2)/6 - atan(x)**2/3 + polylog(2, x**2*exp_polar(I*pi))/6 - 2*atan(
x)/(3*x) - log(x**2 + 1)/(6*x**2) - log(x**2 + 1)*atan(x)/(3*x**3)
```

3.1283.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = {} & -\frac{1}{3}\left(\frac{2}{x} + \frac{\log(x^2+1)}{x^3} + 2\arctan(x)\right)\arctan(x) \\ & + \frac{4x^2\arctan(x)^2 + x^2\log(x^2+1)^2 - 2x^2\operatorname{Li}_2(x^2+1) + 12x^2\log(x) - 2(x^2\log(-x^2) + 3x^2 + 1)\log(x^2}{12x^2}\end{aligned}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(2/x + log(x^2 + 1)/x^3 + 2*arctan(x))*arctan(x) + 1/12*(4*x^2*arctan
(x)^2 + x^2*log(x^2 + 1)^2 - 2*x^2*dilog(x^2 + 1) + 12*x^2*log(x) - 2*(x^2
*log(-x^2) + 3*x^2 + 1)*log(x^2 + 1))/x^2
```

3.1283.8 Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^4}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^4, x)
```

3.1283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = \int \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x^4}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^4,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^4, x)
```

3.1284 $\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx$

3.1284.1 Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = -\frac{5}{12x}-\frac{11\arctan(x)}{12}-\frac{\arctan(x)}{4x^2}-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x} + \frac{1}{4}\arctan(x)\log(1+x^2)-\frac{\arctan(x)\log(1+x^2)}{4x^4} - \frac{1}{4}i\,\mathrm{PolyLog}(2,-ix)+\frac{1}{4}i\,\mathrm{PolyLog}(2,ix)$$

output
```
-5/12/x-11/12*arctan(x)-1/4*arctan(x)/x^2-1/12*ln(x^2+1)/x^3+1/4*ln(x^2+1)
/x+1/4*arctan(x)*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)/x^4-1/4*I*polylog(2,-I*
x)+1/4*I*polylog(2,I*x)
```

3.1284.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = -\frac{1}{6x}-\frac{2\arctan(x)}{3}+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{x}-\arctan(x)\right)-\frac{\arctan(x)}{2x^2}\right) + \frac{(-x+3x^3-3\arctan(x)+3x^4\arctan(x))\log(1+x^2)}{12x^4} - \frac{1}{4}i(\mathrm{PolyLog}(2,-ix)-\mathrm{PolyLog}(2,ix))$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```

output
```
-1/6*1/x - (2*ArcTan[x])/3 + ((-x^(-1) - ArcTan[x])/2 - ArcTan[x]/(2*x^2))
/2 + ((-x + 3*x^3 - 3*ArcTan[x] + 3*x^4*ArcTan[x])*Log[1 + x^2])/(12*x^4)
- (I/4)*(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

3.1284.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^5}dx$$

$$\Big\downarrow \text{5556}$$

$$-2\int \left(-\frac{1-3x^2}{12x^2\left(x^2+1\right)}-\frac{\left(1-x^2\right)\arctan(x)}{4x^3}\right)dx+\frac{1}{4}\arctan(x)\log\left(x^2+1\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{4x^4}+\frac{\log\left(x^2+1\right)}{4x}-\frac{\log\left(x^2+1\right)}{12x^3}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{\arctan(x)}{8x^2}+\frac{11\arctan(x)}{24}+\frac{1}{8}i\,\text{PolyLog}(2,-ix)-\frac{1}{8}i\,\text{PolyLog}(2,ix)+\frac{5}{24x}\right)+\frac{1}{4}\arctan(x)\log\left(x^2+1\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{4x^4}+\frac{\log\left(x^2+1\right)}{4x}-\frac{\log\left(x^2+1\right)}{12x^3}$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```

output
```
-1/12*Log[1 + x^2]/x^3 + Log[1 + x^2]/(4*x) + (ArcTan[x]*Log[1 + x^2])/4 -
 (ArcTan[x]*Log[1 + x^2])/(4*x^4) - 2*(5/(24*x) + (11*ArcTan[x])/24 + ArcT
an[x]/(8*x^2) + (I/8)*PolyLog[2, (-I)*x] - (I/8)*PolyLog[2, I*x])
```

3.1284.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1284.4 Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^5}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^5,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^5,x)
```

3.1284.5 Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^5}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^5, x)
```

3.1284.6 Sympy [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\log(x^2+1)\,\mathrm{atan}(x)}{x^5}\,dx$$

input
```
integrate(atan(x)*ln(x**2+1)/x**5,x)
```

output
```
Integral(log(x**2 + 1)*atan(x)/x**5, x)
```

3.1284.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = -\frac{12\,x^4\arctan(x)\log(x) - 6i\,x^4\mathrm{Li}_2(i\,x+1) + 6i\,x^4\mathrm{Li}_2(-i\,x+1) + 10\,x^3 + 2\,(11\,x^4+3\,x^2)\arctan(x) -}{24\,x^4}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="maxima")
```

output
```
-1/24*(12*x^4*arctan(x)*log(x) - 6*I*x^4*dilog(I*x + 1) + 6*I*x^4*dilog(-I
*x + 1) + 10*x^3 + 2*(11*x^4 + 3*x^2)*arctan(x) - (3*pi*x^4 + 6*x^3 + 6*(x
^4 - 1)*arctan(x) - 2*x)*log(x^2 + 1))/x^4
```

3.1284.8 Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^5}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^5, x)
```

3.1284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\ln(x^2+1)\,\operatorname{atan}(x)}{x^5}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^5,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^5, x)
```

3.1285 $\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx$

3.1285.1 Optimal result

Integrand size = 12, antiderivative size = 114

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = & -\frac{7}{60x^2}-\frac{2\arctan(x)}{15x^3}+\frac{2\arctan(x)}{5x}+\frac{\arctan(x)^2}{5} \\ & -\frac{5\log(x)}{6}+\frac{5}{12}\log\left(1+x^2\right)-\frac{\log(1+x^2)}{20x^4} \\ & +\frac{\log(1+x^2)}{10x^2}-\frac{\arctan(x)\log(1+x^2)}{5x^5} \\ & -\frac{1}{20}\log^2\left(1+x^2\right)-\frac{\operatorname{PolyLog}(2,-x^2)}{10}\end{aligned}$$

output
```
-7/60/x^2-2/15*arctan(x)/x^3+2/5*arctan(x)/x+1/5*arctan(x)^2-5/6*ln(x)+5/1
2*ln(x^2+1)-1/20*ln(x^2+1)/x^4+1/10*ln(x^2+1)/x^2-1/5*arctan(x)*ln(x^2+1)/
x^5-1/20*ln(x^2+1)^2-1/10*polylog(2,-x^2)
```

3.1285.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = & -\frac{7}{60x^2}-\frac{2\arctan(x)}{15x^3}+\frac{2\arctan(x)}{5x}+\frac{\arctan(x)^2}{5} \\ & -\frac{5\log(x)}{6}+\frac{5}{12}\log\left(1+x^2\right)-\frac{\log(1+x^2)}{20x^4} \\ & +\frac{\log(1+x^2)}{10x^2}-\frac{\arctan(x)\log(1+x^2)}{5x^5} \\ & -\frac{1}{20}\log^2\left(1+x^2\right)-\frac{\operatorname{PolyLog}(2,-x^2)}{10}\end{aligned}$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```

output
```
-7/(60*x^2) - (2*ArcTan[x])/(15*x^3) + (2*ArcTan[x])/(5*x) + ArcTan[x]^2/5
 - (5*Log[x])/6 + (5*Log[1 + x^2])/12 - Log[1 + x^2]/(20*x^4) + Log[1 + x^
2]/(10*x^2) - (ArcTan[x]*Log[1 + x^2])/(5*x^5) - Log[1 + x^2]^2/20 - PolyL
og[2, -x^2]/10
```

3.1285.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {5552, 2925, 2857, 2009, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^6}dx$$

$$\Big\downarrow \text{5552}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4\left(x^2+1\right)}dx+\frac{1}{5}\int \frac{\log\left(x^2+1\right)}{x^5\left(x^2+1\right)}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}$$

$$\Big\downarrow \text{2925}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4\left(x^2+1\right)}dx+\frac{1}{10}\int \frac{\log\left(x^2+1\right)}{x^6\left(x^2+1\right)}dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}$$

$$\Big\downarrow \text{2857}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4\left(x^2+1\right)}dx+\frac{1}{10}\int \left(\frac{\log\left(x^2+1\right)}{-x^2-1}+\frac{\log\left(x^2+1\right)}{x^2}-\frac{\log\left(x^2+1\right)}{x^4}+\frac{\log\left(x^2+1\right)}{x^6}\right)dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4\left(x^2+1\right)}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2x^2}-\frac{1}{2}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{x^2}+\frac{3}{2}\log\left(x^2+1\right)-\frac{3\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^4}dx-\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2x^2}-\frac{1}{2}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{x^2}+\frac{3}{2}\log\left(x^2+1\right)-\frac{3\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx+\frac{1}{3}\int \frac{1}{x^3\left(x^2+1\right)}dx-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2x^2}-\frac{1}{2}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{x^2}+\frac{3}{2}\log\left(x^2+1\right)-\frac{3\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx+\frac{1}{6}\int \frac{1}{x^4\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2x^2}-\frac{1}{2}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{x^2}+\frac{3}{2}\log\left(x^2+1\right)-\frac{3\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \text{54}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx+\frac{1}{6}\int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^2+1}\right)dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2x^2}-\frac{1}{2}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{x^2}+\frac{3}{2}\log\left(x^2+1\right)-\frac{3\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2\left(x^2+1\right)}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2x^2}-\frac{1}{2}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{x^2}+\frac{3}{2}\log\left(x^2+1\right)-\frac{3\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2}dx+\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log\left(x^2\right)+\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2x^2}-\frac{1}{2}\log^2\left(x^2+1\right)+\frac{\log\left(x^2+1\right)}{x^2}+\frac{3}{2}\log\left(x^2+1\right)-\frac{3\log\left(x^2\right)}{2}-\frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \text{5361}$$

3.1285. $\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx$

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx - \int \frac{1}{x\left(x^2+1\right)}dx - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2+1\right)\right)\right) - \frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} + \frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \; 243$$

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx - \frac{1}{2}\int \frac{1}{x^2\left(x^2+1\right)}dx^2 - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2+1\right)\right)\right) - \frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} + \frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \; 47$$

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx + \frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2 - \int \frac{1}{x^2}dx^2\right) - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2+1\right.\right.\right. - \frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} + \frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \; 14$$

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx + \frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2 - \log\left(x^2\right)\right) - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2+1\right)\right.\right. - \frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} + \frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \; 16$$

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{2}\left(\log\left(x^2+1\right) - \log\left(x^2\right)\right) + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2+1\right)\right)\right. - \frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} + \frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$$\Big\downarrow \; 5419$$

3.1285. $\int \frac{\arctan(x)\log\left(1+x^2\right)}{x^6}\,dx$

$$-\frac{\arctan(x)\log(x^2+1)}{5x^5}+$$
$$\frac{2}{5}\left(-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)^2}{2}+\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log(x^2+1)-\log(x^2)\right)+\frac{1}{6}\left(-\frac{1}{x^2}-\log(x^2)+\log(x^2+1)\right)\right)+$$
$$\frac{1}{10}\left(-\operatorname{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```

output
```
-1/5*(ArcTan[x]*Log[1 + x^2])/x^5 + (2*(-1/3*ArcTan[x]/x^3 + ArcTan[x]/x +
 ArcTan[x]^2/2 + (-Log[x^2] + Log[1 + x^2])/2 + (-x^(-2) - Log[x^2] + Log[
1 + x^2])/6))/5 + (-1/2*1/x^2 - (3*Log[x^2])/2 + (3*Log[1 + x^2])/2 - Log[
1 + x^2]/(2*x^4) + Log[1 + x^2]/x^2 - Log[1 + x^2]^2/2 - PolyLog[2, -x^2])
/10
```

3.1285.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1285. $\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx$

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

3.1285. $\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx$

3.1285.4 Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^6}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^6,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^6,x)
```

3.1285.5 Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^6}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^6, x)
```

3.1285.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.97 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = &-\frac{8\log(x)}{15} - \frac{\log(x^2)}{20} - \frac{\log(2x^2)}{10} - \frac{\log(x^2+1)^2}{20} \\ &+ \frac{19\log(x^2+1)}{60} + \frac{\log(2x^2+2)}{10} + \frac{\operatorname{atan}^2(x)}{5} \\ &- \frac{\operatorname{Li}_2(x^2e^{i\pi})}{10} + \frac{2\operatorname{atan}(x)}{5x} + \frac{\log(x^2+1)}{10x^2} - \frac{7}{60x^2} \\ &- \frac{2\operatorname{atan}(x)}{15x^3} - \frac{\log(x^2+1)}{20x^4} - \frac{\log(x^2+1)\operatorname{atan}(x)}{5x^5}\end{aligned}$$

input
```
integrate(atan(x)*ln(x**2+1)/x**6,x)
```

output
```
-8*log(x)/15 - log(x**2)/20 - log(2*x**2)/10 - log(x**2 + 1)**2/20 + 19*lo
g(x**2 + 1)/60 + log(2*x**2 + 2)/10 + atan(x)**2/5 - polylog(2, x**2*exp_p
olar(I*pi))/10 + 2*atan(x)/(5*x) + log(x**2 + 1)/(10*x**2) - 7/(60*x**2) -
 2*atan(x)/(15*x**3) - log(x**2 + 1)/(20*x**4) - log(x**2 + 1)*atan(x)/(5*
x**5)
```

3.1285.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \frac{1}{15}\left(\frac{2\,(3\,x^2-1)}{x^3} - \frac{3\,\log(x^2+1)}{x^5} + 6\,\arctan(x)\right)\arctan(x) - \frac{12\,x^4\arctan(x)^2 + 3\,x^4\log(x^2+1)^2 - 6\,x^4\mathrm{Li}_2(x^2+1) + 50\,x^4\log(x) + 7\,x^2 - (6\,x^4\log(-x^2) + 25\,x^4}{60\,x^4}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="maxima")
```

output
```
1/15*(2*(3*x^2 - 1)/x^3 - 3*log(x^2 + 1)/x^5 + 6*arctan(x))*arctan(x) - 1/
60*(12*x^4*arctan(x)^2 + 3*x^4*log(x^2 + 1)^2 - 6*x^4*dilog(x^2 + 1) + 50*
x^4*log(x) + 7*x^2 - (6*x^4*log(-x^2) + 25*x^4 + 6*x^2 - 3)*log(x^2 + 1))/
x^4
```

3.1285.8 Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^6}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^6, x)
```

3.1285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \int \frac{\ln(x^2+1)\ \mathrm{atan}(x)}{x^6}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^6,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^6, x)
```

3.1286 $\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

3.1286.1 Optimal result

Integrand size = 26, antiderivative size = 278

$$\begin{aligned}&\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx\\&=-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}+\frac{2aex^3}{15c^2}+\frac{9bex^4}{200c}-\frac{2}{25}aex^5+\frac{2ae\arctan(cx)}{5c^5}-\frac{2bex\arctan(cx)}{5c^4}\\&\quad+\frac{2bex^3\arctan(cx)}{15c^2}-\frac{2}{25}bex^5\arctan(cx)+\frac{be\arctan(cx)^2}{5c^5}+\frac{137be\log\left(1+c^2x^2\right)}{300c^5}\\&\quad+\frac{be\log^2\left(1+c^2x^2\right)}{20c^5}+\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^3}-\frac{bx^4\left(d+e\log\left(1+c^2x^2\right)\right)}{20c}\\&\quad+\frac{1}{5}x^5(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{b\log\left(1+c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^5}\end{aligned}$$

output
```
-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3+2/15*a*e*x^3/c^2+9/200*b*e*x^4/c-2/25*a*
e*x^5+2/5*a*e*arctan(c*x)/c^5-2/5*b*e*x*arctan(c*x)/c^4+2/15*b*e*x^3*arcta
n(c*x)/c^2-2/25*b*e*x^5*arctan(c*x)+1/5*b*e*arctan(c*x)^2/c^5+137/300*b*e*
ln(c^2*x^2+1)/c^5+1/20*b*e*ln(c^2*x^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(c^2*x^2+
1))/c^3-1/20*b*x^4*(d+e*ln(c^2*x^2+1))/c+1/5*x^5*(a+b*arctan(c*x))*(d+e*ln
(c^2*x^2+1))-1/10*b*ln(c^2*x^2+1)*(d+e*ln(c^2*x^2+1))/c^5
```

3.1286.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.77

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{cx(bcx(-30d(-2+c^2x^2)+e(-154+27c^2x^2))+8a(15c^4dx^4-2e(15-5c^2x^2+3c^4x^4)))+120be\arctan(}{}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(c*x*(b*c*x*(-30*d*(-2 + c^2*x^2) + e*(-154 + 27*c^2*x^2)) + 8*a*(15*c^4*d
*x^4 - 2*e*(15 - 5*c^2*x^2 + 3*c^4*x^4))) + 120*b*e*ArcTan[c*x]^2 + (-60*b
*d + 120*a*c^5*e*x^5 + 2*b*e*(137 + 30*c^2*x^2 - 15*c^4*x^4))*Log[1 + c^2*
x^2] - 30*b*e*Log[1 + c^2*x^2]^2 + 8*ArcTan[c*x]*(30*a*e + 15*b*c^5*d*x^5
- 2*b*c*e*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 15*b*c^5*e*x^5*Log[1 + c^2*x^2]
))/(600*c^5)
```

3.1286.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$
$$\Big\downarrow \text{5556}$$
$$-2c^2e\int\left(\frac{4ac^3x^6+4bc^3\arctan(cx)x^6-bc^2x^5+2bx^3}{20c^3\left(c^2x^2+1\right)}-\frac{bx\log\left(c^2x^2+1\right)}{10c^5\left(c^2x^2+1\right)}\right)dx+\frac{1}{5}x^5(a+ b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bx^4\left(e\log\left(c^2x^2+1\right)+d\right)}{20c}-\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^3}$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-$$
$$2c^2e\left(-\frac{a\arctan(cx)}{5c^7}+\frac{ax}{5c^6}-\frac{ax^3}{15c^4}+\frac{ax^5}{25c^2}-\frac{b\arctan(cx)^2}{10c^7}+\frac{bx\arctan(cx)}{5c^6}-\frac{bx^3\arctan(cx)}{15c^4}+\frac{bx^5\arctan(cx)}{25c^2}-\right.$$
$$\frac{bx^4\left(e\log\left(c^2x^2+1\right)+d\right)}{20c}-\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^3}$$

input
```
Int[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 + c^2*x^2]))/(10*c^3) - (b*x^4*(d + e*Log[1 + c^2*x^2]
))/(20*c) + (x^5*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/5 - (b*Log[
1 + c^2*x^2]*(d + e*Log[1 + c^2*x^2]))/(10*c^5) - 2*c^2*e*((a*x)/(5*c^6) +
 (77*b*x^2)/(600*c^5) - (a*x^3)/(15*c^4) - (9*b*x^4)/(400*c^3) + (a*x^5)/(
25*c^2) - (a*ArcTan[c*x])/(5*c^7) + (b*x*ArcTan[c*x])/(5*c^6) - (b*x^3*Arc
Tan[c*x])/(15*c^4) + (b*x^5*ArcTan[c*x])/(25*c^2) - (b*ArcTan[c*x]^2)/(10*
c^7) - (137*b*Log[1 + c^2*x^2])/(600*c^7) - (b*Log[1 + c^2*x^2]^2)/(40*c^7
))
```

3.1286.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1286.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{120a\,c^5d\,x^5+80x^3\arctan(cx)b\,c^3e+154eb-154b\,c^2e\,x^2-48a\,c^5e\,x^5+27b\,c^4e\,x^4+80a\,c^3e\,x^3+274\ln\left(c^2x^2+1\right)be-48x^5\arctan(c}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^4*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/600*(120*a*c^5*d*x^5+80*x^3*arctan(c*x)*b*c^3*e+154*e*b-154*b*c^2*e*x^2-
48*a*c^5*e*x^5+27*b*c^4*e*x^4+80*a*c^3*e*x^3+274*ln(c^2*x^2+1)*b*e-48*x^5*
arctan(c*x)*b*c^5*e-240*x*a*c*e-30*e*b*ln(c^2*x^2+1)^2-30*b*c^4*d*x^4+240*
e*a*arctan(c*x)+120*e*b*arctan(c*x)^2-60*ln(c^2*x^2+1)*b*d+120*e*a*ln(c^2*
x^2+1)*x^5*c^5+60*x^2*ln(c^2*x^2+1)*b*c^2*e-240*e*b*arctan(c*x)*x*c-30*e*b
*ln(c^2*x^2+1)*x^4*c^4+120*b*arctan(c*x)*x^5*c^5*d+120*e*b*ln(c^2*x^2+1)*a
rctan(c*x)*x^5*c^5+60*c^2*x^2*b*d-60*b*d)/c^5
```

3.1286.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.79

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{80\,ac^3ex^3+24\,(5\,ac^5d-2\,ac^5e)x^5-3\,(10\,bc^4d-9\,bc^4e)x^4-240\,acex+120\,be\arctan\left(cx\right)^2-30\,be\log\left(c\right.}{}$$

input
```
integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/600*(80*a*c^3*e*x^3 + 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9
*b*c^4*e)*x^4 - 240*a*c*e*x + 120*b*e*arctan(c*x)^2 - 30*b*e*log(c^2*x^2 +
 1)^2 + 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 + 8*(10*b*c^3*e*x^3 + 3*(5*b*c^5*d
 - 2*b*c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*arctan(c*x) + 2*(60*b*c^5*e*x^5*a
rctan(c*x) + 60*a*c^5*e*x^5 - 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 - 30*b*d + 1
37*b*e)*log(c^2*x^2 + 1))/c^5
```

3.1286.6 Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.22

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^5}{5}+\frac{aex^5\log\left(c^2x^2+1\right)}{5}-\frac{2aex^5}{25}+\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\,\mathrm{atan}\,(cx)}{5c^5}+\frac{bdx^5\,\mathrm{atan}\,(cx)}{5}+\frac{bex^5\log\left(c^2x^2+1\right)\mathrm{atan}\,(cx)}{5}-\frac{2bex^5\,\mathrm{ata}}{25} \\ \frac{adx^5}{5}\end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**5/5 + a*e*x**5*log(c**2*x**2 + 1)/5 - 2*a*e*x**5/25 + 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atan(c*x)/(5*c**5) + b*d*x**
5*atan(c*x)/5 + b*e*x**5*log(c**2*x**2 + 1)*atan(c*x)/5 - 2*b*e*x**5*atan(
c*x)/25 - b*d*x**4/(20*c) - b*e*x**4*log(c**2*x**2 + 1)/(20*c) + 9*b*e*x**
4/(200*c) + 2*b*e*x**3*atan(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x**2
*log(c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*atan(c*x)
/(5*c**4) - b*d*log(c**2*x**2 + 1)/(10*c**5) - b*e*log(c**2*x**2 + 1)**2/(
20*c**5) + 137*b*e*log(c**2*x**2 + 1)/(300*c**5) + b*e*atan(c*x)**2/(5*c**
5), Ne(c, 0)), (a*d*x**5/5, True))
```

3.1286.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.92

$$
\begin{aligned}
\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx &= \frac{1}{5}\,adx^5 \\
&+\frac{1}{75}\left(15\,x^5\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)be\arctan\left(cx\right) \\
&+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bd \\
&+\frac{1}{75}\left(15\,x^5\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)ae \\
&+\frac{\left(27\,c^4x^4-154\,c^2x^2-120\,\arctan\left(cx\right)^2-2\left(15\,c^4x^4-30\,c^2x^2-137\right)\log\left(c^2x^2+1\right)-30\,\log\left(c^2x^2+1\right)\right.}{600\,c^5}
\end{aligned}
$$

input
```
integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/5*a*d*x^5 + 1/75*(15*x^5*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^
3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e*arctan(c*x) + 1/20*(4*x^5*arctan(
c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d + 1/75*(15*
x^5*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arct
an(c*x)/c^7))*a*e + 1/600*(27*c^4*x^4 - 154*c^2*x^2 - 120*arctan(c*x)^2 -
2*(15*c^4*x^4 - 30*c^2*x^2 - 137)*log(c^2*x^2 + 1) - 30*log(c^2*x^2 + 1)^2
)*b*e/c^5
```

3.1286.8 Giac [F]

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\int (b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1286.9 Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.99

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{a\,d\,x^5}{5}-\frac{2\,a\,e\,x^5}{25}-\frac{b\,e\ln\left(c^2\,x^2+1\right)^2}{20\,c^5}-\frac{2\,a\,e\,x}{5\,c^4}+\frac{2\,a\,e\,\mathrm{atan}(c\,x)}{5\,c^5}+\frac{b\,d\,x^5\,\mathrm{atan}(c\,x)}{5}-\frac{2\,b\,e\,x^5\,\mathrm{atan}(c\,x)}{25}-\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}+\frac{137\,b\,e\,\ln\left(c^2\,x^2+1\right)}{300\,c^5}+\frac{2\,a\,e\,x^3}{15\,c^2}-\frac{b\,d\,x^4}{20\,c}+\frac{b\,d\,x^2}{10\,c^3}+\frac{9\,b\,e\,x^4}{200\,c}-\frac{77\,b\,e\,x^2}{300\,c^3}+\frac{a\,e\,x^5\,\ln\left(c^2\,x^2+1\right)}{5}+\frac{b\,e\,\mathrm{atan}(c\,x)^2}{5\,c^5}+\frac{2\,b\,e\,x^3\,\mathrm{atan}(c\,x)}{15\,c^2}+\frac{b\,e\,x^5\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{5}-\frac{b\,e\,x^4\,\ln\left(c^2\,x^2+1\right)}{20\,c}+\frac{b\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^3}-\frac{2\,b\,e\,x\,\mathrm{atan}(c\,x)}{5\,c^4}$$

input
```
int(x^4*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^5)/5 - (2*a*e*x^5)/25 - (b*e*log(c^2*x^2 + 1)^2)/(20*c^5) - (2*a*e*
x)/(5*c^4) + (2*a*e*atan(c*x))/(5*c^5) + (b*d*x^5*atan(c*x))/5 - (2*b*e*x^
5*atan(c*x))/25 - (b*d*log(c^2*x^2 + 1))/(10*c^5) + (137*b*e*log(c^2*x^2 +
 1))/(300*c^5) + (2*a*e*x^3)/(15*c^2) - (b*d*x^4)/(20*c) + (b*d*x^2)/(10*c
^3) + (9*b*e*x^4)/(200*c) - (77*b*e*x^2)/(300*c^3) + (a*e*x^5*log(c^2*x^2
+ 1))/5 + (b*e*atan(c*x)^2)/(5*c^5) + (2*b*e*x^3*atan(c*x))/(15*c^2) + (b*
e*x^5*atan(c*x)*log(c^2*x^2 + 1))/5 - (b*e*x^4*log(c^2*x^2 + 1))/(20*c) +
(b*e*x^2*log(c^2*x^2 + 1))/(10*c^3) - (2*b*e*x*atan(c*x))/(5*c^4)
```

3.1287 $\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

3.1287.1 Optimal result

Integrand size = 26, antiderivative size = 221

$$\begin{aligned}
&\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx \\
&=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}-\frac{b(2d-e)x^3}{24c}+\frac{bex^3}{18c}-\frac{b(2d-3e)\arctan(cx)}{8c^4} \\
&\quad+\frac{2be\arctan(cx)}{3c^4}+\frac{ex^2(a+b\arctan(cx))}{4c^2}-\frac{1}{8}ex^4(a+b\arctan(cx)) \\
&\quad+\frac{bex\log\left(1+c^2x^2\right)}{4c^3}-\frac{bex^3\log\left(1+c^2x^2\right)}{12c}-\frac{e(a+b\arctan(cx))\log\left(1+c^2x^2\right)}{4c^4} \\
&\quad+\frac{1}{4}x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)
\end{aligned}$$

output
```
1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3-1/24*b*(2*d-e)*x^3/c+1/18*b*e*x^3/c-1/
8*b*(2*d-3*e)*arctan(c*x)/c^4+2/3*b*e*arctan(c*x)/c^4+1/4*e*x^2*(a+b*arcta
n(c*x))/c^2-1/8*e*x^4*(a+b*arctan(c*x))+1/4*b*e*x*ln(c^2*x^2+1)/c^3-1/12*b
*e*x^3*ln(c^2*x^2+1)/c-1/4*e*(a+b*arctan(c*x))*ln(c^2*x^2+1)/c^4+1/4*x^4*(
a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))
```

3.1287.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.74

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{cx(18ac^3dx^3-6bd(-3+c^2x^2)-9acex(-2+c^2x^2)+be(-75+7c^2x^2))-6e(bcx(-3+c^2x^2)+a(3-3c}{7}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(c*x*(18*a*c^3*d*x^3 - 6*b*d*(-3 + c^2*x^2) - 9*a*c*e*x*(-2 + c^2*x^2) + b
*e*(-75 + 7*c^2*x^2)) - 6*e*(b*c*x*(-3 + c^2*x^2) + a*(3 - 3*c^4*x^4))*Log
[1 + c^2*x^2] + 3*b*ArcTan[c*x]*(e*(25 + 6*c^2*x^2 - 3*c^4*x^4) + 6*d*(-1
+ c^4*x^4) + 6*e*(-1 + c^4*x^4)*Log[1 + c^2*x^2]))/(72*c^4)
```

3.1287.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$
$$\Big\downarrow \text{5554}$$
$$-bc\int\left(\frac{x^2\left(c^2(2d-e)x^2+2e\right)}{8c^2\left(c^2x^2+1\right)}-\frac{e\left(1-c^2x^2\right)\log\left(c^2x^2+1\right)}{4c^4}\right)dx+\frac{1}{4}x^4(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{ex^2(a+b\arctan(cx))}{4c^2}-\frac{e\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\arctan(cx))$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{ex^2(a+b\arctan(cx))}{4c^2}-\frac{e\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\arctan(cx))-bc\left(\frac{(2d-3e)\arctan(cx)}{8c^5}-\frac{2e\arctan(cx)}{3c^5}-\frac{x(2d-3e)}{8c^4}+\frac{2ex}{3c^4}+\frac{x^3(2d-e)}{24c^2}-\frac{ex^3}{18c^2}+\frac{ex^3\log\left(c^2x^2+1\right)}{12c^2}-\frac{ex\log\right.$$

input
```
Int[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(e*x^2*(a + b*ArcTan[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTan[c*x]))/8 - (e*(
a + b*ArcTan[c*x])*Log[1 + c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcTan[c*x])*(d
 + e*Log[1 + c^2*x^2]))/4 - b*c*(-1/8*((2*d - 3*e)*x)/c^4 + (2*e*x)/(3*c^4
) + ((2*d - e)*x^3)/(24*c^2) - (e*x^3)/(18*c^2) + ((2*d - 3*e)*ArcTan[c*x]
)/(8*c^5) - (2*e*ArcTan[c*x])/(3*c^5) - (e*x*Log[1 + c^2*x^2])/(4*c^4) + (
e*x^3*Log[1 + c^2*x^2])/(12*c^2))
```

3.1287.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

3.1287.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{18eb\ln(c^2x^2+1)\arctan(cx)x^4c^4+18x^4\arctan(cx)bc^4d-9x^4\arctan(cx)bc^4e+18ea\ln(c^2x^2+1)x^4c^4+18c^4adx^4-9x^4ac^4e-6e}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/72*(18*e*b*ln(c^2*x^2+1)*arctan(c*x)*x^4*c^4+18*x^4*arctan(c*x)*b*c^4*d-
9*x^4*arctan(c*x)*b*c^4*e+18*e*a*ln(c^2*x^2+1)*x^4*c^4+18*c^4*a*d*x^4-9*x^
4*a*c^4*e-6*e*b*ln(c^2*x^2+1)*x^3*c^3-6*b*c^3*d*x^3+7*b*c^3*e*x^3+18*arcta
n(c*x)*b*c^2*e*x^2+18*a*c^2*e*x^2+18*ln(c^2*x^2+1)*b*c*e*x+18*b*c*d*x-75*b
*c*e*x-18*arctan(c*x)*ln(c^2*x^2+1)*b*e-18*arctan(c*x)*b*d+75*e*b*arctan(c
*x)-18*ln(c^2*x^2+1)*a*e-18*e*a)/c^4
```

3.1287.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.81

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{18\,ac^2ex^2+9\,(2\,ac^4d-ac^4e)x^4-(6\,bc^3d-7\,bc^3e)x^3+3\,(6\,bcd-25\,bce)x+3\,(6\,bc^2ex^2+3\,(2\,bc^4d-bc^4e$$

input
```
integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/72*(18*a*c^2*e*x^2 + 9*(2*a*c^4*d - a*c^4*e)*x^4 - (6*b*c^3*d - 7*b*c^3*
e)*x^3 + 3*(6*b*c*d - 25*b*c*e)*x + 3*(6*b*c^2*e*x^2 + 3*(2*b*c^4*d - b*c^
4*e)*x^4 - 6*b*d + 25*b*e)*arctan(c*x) + 6*(3*a*c^4*e*x^4 - b*c^3*e*x^3 +
3*b*c*e*x - 3*a*e + 3*(b*c^4*e*x^4 - b*e)*arctan(c*x))*log(c^2*x^2 + 1))/c
^4
```

3.1287.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.26

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^4}{4}+\frac{aex^4\log\left(c^2x^2+1\right)}{4}-\frac{aex^4}{8}+\frac{aex^2}{4c^2}-\frac{ae\log\left(c^2x^2+1\right)}{4c^4}+\frac{bdx^4\operatorname{atan}(cx)}{4}+\frac{bex^4\log\left(c^2x^2+1\right)\operatorname{atan}(cx)}{4}-\frac{bex^4\operatorname{atan}(cx)}{8}-\\ \frac{adx^4}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**4*log(c**2*x**2 + 1)/4 - a*e*x**4/8 + a*e*x
**2/(4*c**2) - a*e*log(c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atan(c*x)/4 + b*
e*x**4*log(c**2*x**2 + 1)*atan(c*x)/4 - b*e*x**4*atan(c*x)/8 - b*d*x**3/(1
2*c) - b*e*x**3*log(c**2*x**2 + 1)/(12*c) + 7*b*e*x**3/(72*c) + b*e*x**2*a
tan(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(c**2*x**2 + 1)/(4*c**3) - 2
5*b*e*x/(24*c**3) - b*d*atan(c*x)/(4*c**4) - b*e*log(c**2*x**2 + 1)*atan(c
*x)/(4*c**4) + 25*b*e*atan(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

3.1287.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01

$$\begin{aligned}
&\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx\\
&=\frac{1}{4}\,adx^4+\frac{1}{72}\,bce\left(\frac{7\,c^2x^3-6\,(c^2x^3-3\,x)\log\left(c^2x^2+1\right)-75\,x}{c^4}+\frac{75\,\arctan\left(cx\right)}{c^5}\right)\\
&\quad+\frac{1}{8}\left(2\,x^4\log\left(c^2x^2+1\right)-c^2\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)be\arctan\left(cx\right)\\
&\quad+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd\\
&\quad+\frac{1}{8}\left(2\,x^4\log\left(c^2x^2+1\right)-c^2\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)ae
\end{aligned}$$

input
```
integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/4*a*d*x^4 + 1/72*b*c*e*((7*c^2*x^3 - 6*(c^2*x^3 - 3*x)*log(c^2*x^2 + 1)
- 75*x)/c^4 + 75*arctan(c*x)/c^5) + 1/8*(2*x^4*log(c^2*x^2 + 1) - c^2*((c^
2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*e*arctan(c*x) + 1/12*(3*x^
4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d + 1/8*(2*
x^4*log(c^2*x^2 + 1) - c^2*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6
))*a*e
```

3.1287.8 Giac [F]

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\int(b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1287.9 Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.34

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{a\,d\,x^4}{4}-\frac{a\,e\,x^4}{8}+\frac{b\,d\,x}{4\,c^3}-\frac{25\,b\,e\,x}{24\,c^3}+\frac{b\,d\,x^4\,\mathrm{atan}(c\,x)}{4}-\frac{b\,e\,x^4\,\mathrm{atan}(c\,x)}{8}$$
$$-\frac{a\,e\,\ln\left(c^2\,x^2+1\right)}{4\,c^4}+\frac{a\,e\,x^2}{4\,c^2}-\frac{b\,d\,x^3}{12\,c}-\frac{b\,d\,\mathrm{atan}\left(\frac{6\,b\,c\,d\,x}{6\,b\,d-25\,b\,e}-\frac{25\,b\,c\,e\,x}{6\,b\,d-25\,b\,e}\right)}{4\,c^4}$$
$$+\frac{7\,b\,e\,x^3}{72\,c}+\frac{25\,b\,e\,\mathrm{atan}\left(\frac{6\,b\,c\,d\,x}{6\,b\,d-25\,b\,e}-\frac{25\,b\,c\,e\,x}{6\,b\,d-25\,b\,e}\right)}{24\,c^4}+\frac{a\,e\,x^4\,\ln\left(c^2\,x^2+1\right)}{4}$$
$$+\frac{b\,e\,x\,\ln\left(c^2\,x^2+1\right)}{4\,c^3}-\frac{b\,e\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{4\,c^4}+\frac{b\,e\,x^2\,\mathrm{atan}(c\,x)}{4\,c^2}$$
$$+\frac{b\,e\,x^4\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{4}-\frac{b\,e\,x^3\,\ln\left(c^2\,x^2+1\right)}{12\,c}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^4)/4 - (a*e*x^4)/8 + (b*d*x)/(4*c^3) - (25*b*e*x)/(24*c^3) + (b*d*x
^4*atan(c*x))/4 - (b*e*x^4*atan(c*x))/8 - (a*e*log(c^2*x^2 + 1))/(4*c^4) +
 (a*e*x^2)/(4*c^2) - (b*d*x^3)/(12*c) - (b*d*atan((6*b*c*d*x)/(6*b*d - 25*
b*e) - (25*b*c*e*x)/(6*b*d - 25*b*e)))/(4*c^4) + (7*b*e*x^3)/(72*c) + (25*
b*e*atan((6*b*c*d*x)/(6*b*d - 25*b*e) - (25*b*c*e*x)/(6*b*d - 25*b*e)))/(2
4*c^4) + (a*e*x^4*log(c^2*x^2 + 1))/4 + (b*e*x*log(c^2*x^2 + 1))/(4*c^3) -
 (b*e*atan(c*x)*log(c^2*x^2 + 1))/(4*c^4) + (b*e*x^2*atan(c*x))/(4*c^2) +
(b*e*x^4*atan(c*x)*log(c^2*x^2 + 1))/4 - (b*e*x^3*log(c^2*x^2 + 1))/(12*c)
```

3.1288 $\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

3.1288.1 Optimal result

Integrand size = 26, antiderivative size = 213

$$\begin{aligned}\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx &= \frac{2aex}{3c^2}+\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2ae\arctan(cx)}{3c^3}+\frac{2bex\arctan(cx)}{3c^2}-\frac{2}{9}bex^3\arctan(cx)\\ &\quad -\frac{be\arctan(cx)^2}{3c^3}-\frac{11be\log\left(1+c^2x^2\right)}{18c^3}-\frac{be\log^2\left(1+c^2x^2\right)}{12c^3}-\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{6c}\\ &\quad +\frac{1}{3}x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)+\frac{b\log\left(1+c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{6c^3}\end{aligned}$$

output
```
2/3*a*e*x/c^2+5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*a*e*arctan(c*x)/c^3+2/3*b*e*x
*arctan(c*x)/c^2-2/9*b*e*x^3*arctan(c*x)-1/3*b*e*arctan(c*x)^2/c^3-11/18*b
*e*ln(c^2*x^2+1)/c^3-1/12*b*e*ln(c^2*x^2+1)^2/c^3-1/6*b*x^2*(d+e*ln(c^2*x^
2+1))/c+1/3*x^3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))+1/6*b*ln(c^2*x^2+1)*
(d+e*ln(c^2*x^2+1))/c^3
```

3.1288.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{2cx(bc(-3d+5e)x+6ac^2dx^2-4ae(-3+c^2x^2))-12be\arctan(cx)^2+2(3bd+6ac^3ex^3-be(11+3c^2x^2)}{}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(2*c*x*(b*c*(-3*d + 5*e)*x + 6*a*c^2*d*x^2 - 4*a*e*(-3 + c^2*x^2)) - 12*b*
e*ArcTan[c*x]^2 + 2*(3*b*d + 6*a*c^3*e*x^3 - b*e*(11 + 3*c^2*x^2))*Log[1 +
 c^2*x^2] + 3*b*e*Log[1 + c^2*x^2]^2 - 4*ArcTan[c*x]*(6*a*e + b*c*x*(-6*e
- 3*c^2*d*x^2 + 2*c^2*e*x^2) - 3*b*c^3*e*x^3*Log[1 + c^2*x^2]))/(36*c^3)
```

3.1288.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$
$$\Big\downarrow \text{5556}$$
$$-2c^2e\int\left(\frac{bx\log\left(c^2x^2+1\right)}{6c^3\left(c^2x^2+1\right)}-\frac{x^3(-2cx\arctan(cx)b+b-2acx)}{6c\left(c^2x^2+1\right)}\right)dx+\frac{1}{3}x^3(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{6c}+\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{6c^3}$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-$$
$$2c^2e\left(\frac{a\arctan(cx)}{3c^5}-\frac{ax}{3c^4}+\frac{ax^3}{9c^2}+\frac{b\arctan(cx)^2}{6c^5}-\frac{bx\arctan(cx)}{3c^4}+\frac{bx^3\arctan(cx)}{9c^2}-\frac{5bx^2}{36c^3}+\frac{b\log^2\left(c^2x^2+1\right)}{24c^5}+\right.$$
$$\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{6c}+\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{6c^3}$$

input
```
Int[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
-1/6*(b*x^2*(d + e*Log[1 + c^2*x^2]))/c + (x^3*(a + b*ArcTan[c*x])*(d + e*
Log[1 + c^2*x^2]))/3 + (b*Log[1 + c^2*x^2]*(d + e*Log[1 + c^2*x^2]))/(6*c^
3) - 2*c^2*e*(-1/3*(a*x)/c^4 - (5*b*x^2)/(36*c^3) + (a*x^3)/(9*c^2) + (a*A
rcTan[c*x])/(3*c^5) - (b*x*ArcTan[c*x])/(3*c^4) + (b*x^3*ArcTan[c*x])/(9*c
^2) + (b*ArcTan[c*x]^2)/(6*c^5) + (11*b*Log[1 + c^2*x^2])/(36*c^5) + (b*Lo
g[1 + c^2*x^2]^2)/(24*c^5))
```

3.1288.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1288.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{12eb\ln\left(c^2x^2+1\right)\arctan(cx)x^3c^3+12b\arctan(cx)x^3c^3d-8x^3\arctan(cx)bc^3e+12ea\ln\left(c^2x^2+1\right)x^3c^3+12ac^3dx^3-8ac^3ex^3-6}{}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x^2*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/36*(12*e*b*ln(c^2*x^2+1)*arctan(c*x)*x^3*c^3+12*b*arctan(c*x)*x^3*c^3*d-
8*x^3*arctan(c*x)*b*c^3*e+12*e*a*ln(c^2*x^2+1)*x^3*c^3+12*a*c^3*d*x^3-8*a*
c^3*e*x^3-6*x^2*ln(c^2*x^2+1)*b*c^2*e-6*c^2*x^2*b*d+10*b*c^2*e*x^2+24*e*b*
arctan(c*x)*x*c+24*x*a*c*e-12*e*b*arctan(c*x)^2+3*e*b*ln(c^2*x^2+1)^2-24*e
*a*arctan(c*x)+6*ln(c^2*x^2+1)*b*d-22*ln(c^2*x^2+1)*b*e)/c^3
```

3.1288.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{24\,acex+4\,(3\,ac^3d-2\,ac^3e)x^3-12\,be\arctan{(cx)}^2+3\,be\log{(c^2x^2+1)}^2-2\,(3\,bc^2d-5\,bc^2e)x^2+4\,(6\,bc$$

input
```
integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/36*(24*a*c*e*x + 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 12*b*e*arctan(c*x)^2 +
3*b*e*log(c^2*x^2 + 1)^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 + 4*(6*b*c*e*x +
(3*b*c^3*d - 2*b*c^3*e)*x^3 - 6*a*e)*arctan(c*x) + 2*(6*b*c^3*e*x^3*arctan
(c*x) + 6*a*c^3*e*x^3 - 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(c^2*x^2 + 1))/
c^3
```

3.1288.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^3}{3}+\frac{aex^3\log{(c^2x^2+1)}}{3}-\frac{2aex^3}{9}+\frac{2aex}{3c^2}-\frac{2ae\operatorname{atan}{(cx)}}{3c^3}+\frac{bdx^3\operatorname{atan}{(cx)}}{3}+\frac{bex^3\log{(c^2x^2+1)}\operatorname{atan}{(cx)}}{3}-\frac{2bex^3\operatorname{atan}{(cx)}}{9}-\\ \frac{adx^3}{3}\end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**3*log(c**2*x**2 + 1)/3 - 2*a*e*x**3/9 + 2*a
*e*x/(3*c**2) - 2*a*e*atan(c*x)/(3*c**3) + b*d*x**3*atan(c*x)/3 + b*e*x**3
*log(c**2*x**2 + 1)*atan(c*x)/3 - 2*b*e*x**3*atan(c*x)/9 - b*d*x**2/(6*c)
- b*e*x**2*log(c**2*x**2 + 1)/(6*c) + 5*b*e*x**2/(18*c) + 2*b*e*x*atan(c*x
)/(3*c**2) + b*d*log(c**2*x**2 + 1)/(6*c**3) + b*e*log(c**2*x**2 + 1)**2/(
12*c**3) - 11*b*e*log(c**2*x**2 + 1)/(18*c**3) - b*e*atan(c*x)**2/(3*c**3)
, Ne(c, 0)), (a*d*x**3/3, True))
```

3.1288.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx \\
&= \frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)be\arctan\left(cx\right) \\
&\quad+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd \\
&\quad+\frac{1}{9}\left(3\,x^3\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)ae \\
&\quad+\frac{\left(10\,c^2x^2+12\,\arctan\left(cx\right)^2-2\left(3\,c^2x^2+11\right)\log\left(c^2x^2+1\right)+3\,\log\left(c^2x^2+1\right)^2\right)be}{36\,c^3}
\end{aligned}$$

input
```
integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/3*a*d*x^3 + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^3 - 3*x)/c^4 + 3
*arctan(c*x)/c^5))*b*e*arctan(c*x) + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 -
 log(c^2*x^2 + 1)/c^4))*b*d + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^
3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*e + 1/36*(10*c^2*x^2 + 12*arctan(c*x)
^2 - 2*(3*c^2*x^2 + 11)*log(c^2*x^2 + 1) + 3*log(c^2*x^2 + 1)^2)*b*e/c^3
```

3.1288.8 Giac [F]

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = \int (b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1288.9 Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = \frac{a\,d\,x^3}{3}-\frac{2\,a\,e\,x^3}{9}+\frac{b\,e\ln\left(c^2\,x^2+1\right)^2}{12\,c^3}+\frac{2\,a\,e\,x}{3\,c^2}-\frac{2\,a\,e\,\mathrm{atan}(c\,x)}{3\,c^3}+\frac{b\,d\,x^3\,\mathrm{atan}(c\,x)}{3}-\frac{2\,b\,e\,x^3\,\mathrm{atan}(c\,x)}{9}+\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{11\,b\,e\,\ln\left(c^2\,x^2+1\right)}{18\,c^3}-\frac{b\,d\,x^2}{6\,c}+\frac{5\,b\,e\,x^2}{18\,c}+\frac{a\,e\,x^3\,\ln\left(c^2\,x^2+1\right)}{3}-\frac{b\,e\,\mathrm{atan}(c\,x)^2}{3\,c^3}+\frac{b\,e\,x^3\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{3}-\frac{b\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{6\,c}+\frac{2\,b\,e\,x\,\mathrm{atan}(c\,x)}{3\,c^2}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^3)/3 - (2*a*e*x^3)/9 + (b*e*log(c^2*x^2 + 1)^2)/(12*c^3) + (2*a*e*x
)/(3*c^2) - (2*a*e*atan(c*x))/(3*c^3) + (b*d*x^3*atan(c*x))/3 - (2*b*e*x^3
*atan(c*x))/9 + (b*d*log(c^2*x^2 + 1))/(6*c^3) - (11*b*e*log(c^2*x^2 + 1))
/(18*c^3) - (b*d*x^2)/(6*c) + (5*b*e*x^2)/(18*c) + (a*e*x^3*log(c^2*x^2 +
1))/3 - (b*e*atan(c*x)^2)/(3*c^3) + (b*e*x^3*atan(c*x)*log(c^2*x^2 + 1))/3
 - (b*e*x^2*log(c^2*x^2 + 1))/(6*c) + (2*b*e*x*atan(c*x))/(3*c^2)
```

3.1289 $\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$

3.1289.1 Optimal result

Integrand size = 24, antiderivative size = 137

$$\begin{aligned}
&\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx\\
&=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\arctan(cx)}{2c^2}-\frac{be\arctan(cx)}{c^2}\\
&\quad+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{2}ex^2(a+b\arctan(cx))\\
&\quad-\frac{bex\log\left(1+c^2x^2\right)}{2c}+\frac{e\left(1+c^2x^2\right)(a+b\arctan(cx))\log\left(1+c^2x^2\right)}{2c^2}
\end{aligned}$$

output
```
-1/2*b*(d-e)*x/c+b*e*x/c+1/2*b*(d-e)*arctan(c*x)/c^2-b*e*arctan(c*x)/c^2+1
/2*d*x^2*(a+b*arctan(c*x))-1/2*e*x^2*(a+b*arctan(c*x))-1/2*b*e*x*ln(c^2*x^
2+1)/c+1/2*e*(c^2*x^2+1)*(a+b*arctan(c*x))*ln(c^2*x^2+1)/c^2
```

3.1289.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx\\
&=\frac{cx(-b(d-3e)+ac(d-e)x)+e\left(a-bcx+ac^2x^2\right)\log\left(1+c^2x^2\right)+b\arctan(cx)\left(d+c^2dx^2-e\left(3+c^2x^2\right)\right.}{2c^2}
\end{aligned}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(c*x*(-(b*(d - 3*e)) + a*c*(d - e)*x) + e*(a - b*c*x + a*c^2*x^2)*Log[1 +
c^2*x^2] + b*ArcTan[c*x]*(d + c^2*d*x^2 - e*(3 + c^2*x^2) + (e + c^2*e*x^2
)*Log[1 + c^2*x^2]))/(2*c^2)
```

3.1289.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$
$$\Big\downarrow \text{5554}$$
$$-bc\int\left(\frac{(d-e)x^2}{2\left(c^2x^2+1\right)}+\frac{e\log\left(c^2x^2+1\right)}{2c^2}\right)dx+\frac{e\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{2}ex^2(a+b\arctan(cx))$$
$$\Big\downarrow \text{2009}$$
$$\frac{e\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{2}ex^2(a+b\arctan(cx))-bc\left(-\frac{(d-e)\arctan(cx)}{2c^3}+\frac{e\arctan(cx)}{c^3}+\frac{x(d-e)}{2c^2}+\frac{ex\log\left(c^2x^2+1\right)}{2c^2}-\frac{ex}{c^2}\right)$$

input
```
Int[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]))/2 + (e*(1 + c^
2*x^2)*(a + b*ArcTan[c*x])*Log[1 + c^2*x^2])/(2*c^2) - b*c*(((d - e)*x)/(2
*c^2) - (e*x)/c^2 - ((d - e)*ArcTan[c*x])/(2*c^3) + (e*ArcTan[c*x])/c^3 +
(e*x*Log[1 + c^2*x^2])/(2*c^2))
```

3.1289.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

3.1289.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{\arctan(cx)\ln(c^2x^2+1)b\,c^2e\,x^2+\arctan(cx)b\,c^2d\,x^2-\arctan(cx)b\,c^2e\,x^2+\ln(c^2x^2+1)a\,c^2e\,x^2+a\,c^2d\,x^2-a\,c^2e\,x^2-\ln(c^2x^2+1)b}{2c^2}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int(x*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(c*x)*ln(c^2*x^2+1)*b*c^2*e*x^2+arctan(c*x)*b*c^2*d*x^2-arctan(
c*x)*b*c^2*e*x^2+ln(c^2*x^2+1)*a*c^2*e*x^2+a*c^2*d*x^2-a*c^2*e*x^2-ln(c^2*
x^2+1)*b*c*e*x-b*c*d*x+3*b*c*e*x+arctan(c*x)*ln(c^2*x^2+1)*b*e+arctan(c*x)
*b*d-3*e*b*arctan(c*x)+ln(c^2*x^2+1)*a*e)/c^2
```

3.1289.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{(ac^2d-ac^2e)x^2-(bcd-3\,bce)x+((bc^2d-bc^2e)x^2+bd-3\,be)\arctan(cx)+(ac^2ex^2-bcex+ae+(bc}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

output
```
1/2*((a*c^2*d - a*c^2*e)*x^2 - (b*c*d - 3*b*c*e)*x + ((b*c^2*d - b*c^2*e)*
x^2 + b*d - 3*b*e)*arctan(c*x) + (a*c^2*e*x^2 - b*c*e*x + a*e + (b*c^2*e*x
^2 + b*e)*arctan(c*x))*log(c^2*x^2 + 1))/c^2
```

3.1289.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^2\log\left(c^2x^2+1\right)}{2}-\frac{aex^2}{2}+\frac{ae\log\left(c^2x^2+1\right)}{2c^2}+\frac{bdx^2\operatorname{atan}(cx)}{2}+\frac{bex^2\log\left(c^2x^2+1\right)\operatorname{atan}(cx)}{2}-\frac{bex^2\operatorname{atan}(cx)}{2}-\frac{bdx}{2c}- \\ \frac{adx^2}{2}\end{cases}$$

input
```
integrate(x*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**2*log(c**2*x**2 + 1)/2 - a*e*x**2/2 + a*e*l
og(c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atan(c*x)/2 + b*e*x**2*log(c**2*x**2
 + 1)*atan(c*x)/2 - b*e*x**2*atan(c*x)/2 - b*d*x/(2*c) - b*e*x*log(c**2*x*
*2 + 1)/(2*c) + 3*b*e*x/(2*c) + b*d*atan(c*x)/(2*c**2) + b*e*log(c**2*x**2
 + 1)*atan(c*x)/(2*c**2) - 3*b*e*atan(c*x)/(2*c**2), Ne(c, 0)), (a*d*x**2/
2, True))
```

3.1289.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bd$$
$$-\frac{\left(x\log\left(c^2x^2+1\right)-3\,x+\frac{2\,\arctan(cx)}{c}\right)be}{2\,c}$$
$$-\frac{\left(c^2x^2-\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)+1\right)be\arctan(cx)}{2\,c^2}$$
$$-\frac{\left(c^2x^2-\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)+1\right)ae}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d - 1/
2*(x*log(c^2*x^2 + 1) - 3*x + 2*arctan(c*x)/c)*b*e/c - 1/2*(c^2*x^2 - (c^2
*x^2 + 1)*log(c^2*x^2 + 1) + 1)*b*e*arctan(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*
x^2 + 1)*log(c^2*x^2 + 1) + 1)*a*e/c^2
```

3.1289.8 Giac [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = \int (b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1289.9 Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.66

$$\begin{aligned}\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx &= \frac{a\,d\,x^2}{2}-\frac{a\,e\,x^2}{2}-\frac{b\,d\,x}{2\,c}+\frac{3\,b\,e\,x}{2\,c}+\frac{b\,d\,x^2\,\mathrm{atan}(c\,x)}{2}-\frac{b\,e\,x^2\,\mathrm{atan}(c\,x)}{2}+\frac{a\,e\,\ln\left(c^2\,x^2+1\right)}{2\,c^2}\\&\quad+\frac{b\,d\,\mathrm{atan}\left(\frac{b\,c\,d\,x}{b\,d-3\,b\,e}-\frac{3\,b\,c\,e\,x}{b\,d-3\,b\,e}\right)}{2\,c^2}-\frac{3\,b\,e\,\mathrm{atan}\left(\frac{b\,c\,d\,x}{b\,d-3\,b\,e}-\frac{3\,b\,c\,e\,x}{b\,d-3\,b\,e}\right)}{2\,c^2}+\frac{a\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{2}\\&\quad-\frac{b\,e\,x\,\ln\left(c^2\,x^2+1\right)}{2\,c}+\frac{b\,e\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{2\,c^2}+\frac{b\,e\,x^2\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{2}\end{aligned}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^2)/2 - (a*e*x^2)/2 - (b*d*x)/(2*c) + (3*b*e*x)/(2*c) + (b*d*x^2*ata
n(c*x))/2 - (b*e*x^2*atan(c*x))/2 + (a*e*log(c^2*x^2 + 1))/(2*c^2) + (b*d*
atan((b*c*d*x)/(b*d - 3*b*e) - (3*b*c*e*x)/(b*d - 3*b*e)))/(2*c^2) - (3*b*
e*atan((b*c*d*x)/(b*d - 3*b*e) - (3*b*c*e*x)/(b*d - 3*b*e)))/(2*c^2) + (a*
e*x^2*log(c^2*x^2 + 1))/2 - (b*e*x*log(c^2*x^2 + 1))/(2*c) + (b*e*atan(c*x
)*log(c^2*x^2 + 1))/(2*c^2) + (b*e*x^2*atan(c*x)*log(c^2*x^2 + 1))/2
```

3.1289. $\int x(a + b\arctan(cx))\left(d + e\log\left(1 + c^2x^2\right)\right)\,dx$

3.1290 $\int (a+b\arctan(cx)) \left(d+e\log\left(1+c^2x^2\right)\right) \, dx$

3.1290.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int (a+b\arctan(cx)) \left(d+e\log\left(1+c^2x^2\right)\right) \, dx = -2aex-2bex\arctan(cx)+\frac{e(a+b\arctan(cx))^2}{bc}+\frac{be\log\left(1+c^2x^2\right)}{c} + x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{b\left(d+e\log\left(1+c^2x^2\right)\right)^2}{4ce}$$

output
```
-2*a*e*x-2*b*e*x*arctan(c*x)+e*(a+b*arctan(c*x))^2/b/c+b*e*ln(c^2*x^2+1)/c
+x*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))-1/4*b*(d+e*ln(c^2*x^2+1))^2/c/e
```

3.1290.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int (a+b\arctan(cx)) \left(d+e\log\left(1+c^2x^2\right)\right) \, dx = adx-2aex+\frac{2ae\arctan(cx)}{c}+bdx\arctan(cx)-2bex\arctan(cx) + \frac{be\arctan(cx)^2}{c}-\frac{bd\log\left(1+c^2x^2\right)}{2c}+\frac{be\log\left(1+c^2x^2\right)}{c} + aex\log\left(1+c^2x^2\right)+bex\arctan(cx)\log\left(1+c^2x^2\right)-\frac{be\log^2\left(1+c^2x^2\right)}{4c}$$

input
```
Integrate[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + (2*a*e*ArcTan[c*x])/c + b*d*x*ArcTan[c*x] - 2*b*e*x*ArcT
an[c*x] + (b*e*ArcTan[c*x]^2)/c - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[
1 + c^2*x^2])/c + a*e*x*Log[1 + c^2*x^2] + b*e*x*ArcTan[c*x]*Log[1 + c^2*x
^2] - (b*e*Log[1 + c^2*x^2]^2)/(4*c)
```

3.1290.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5544, 2925, 2837, 2738, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)dx$$

$$\Big\downarrow \text{5544}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx-bc\int \frac{x\left(d+e\log\left(c^2x^2+1\right)\right)}{c^2x^2+1}dx+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)$$

$$\Big\downarrow \text{2925}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{c^2x^2+1}dx^2+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)$$

$$\Big\downarrow \text{2837}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx-\frac{b\int \frac{d+e\log(c^2x^2+1)}{x^2}d(c^2x^2+1)}{2c}+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)$$

$$\Big\downarrow \text{2738}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{5451}$$

$$-2c^2e\left(\frac{\int (a+b\arctan(cx))dx}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{2009}$$

$$-2c^2e\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{5419}$$

$$x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-2c^2e\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

input
```
Int[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]) - (b*(d + e*Log[1 + c^2*x^2
])^2)/(4*c*e) - 2*c^2*e*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*A
rcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)
```

3.1290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5544
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2
*e*g   Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

3.1290.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.37

method	result
parallelrisch	$\frac{4eb\ln(c^2x^2+1)x\arctan(cx)c+4b\arctan(cx)xcd-8eb\arctan(cx)xc+4ea\ln(c^2x^2+1)xc+4acdx-8xace+4eb\arctan(cx)^2-eb\ln}{4c}$
default	Expression too large to display
parts	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*e*b*ln(c^2*x^2+1)*x*arctan(c*x)*c+4*b*arctan(c*x)*x*c*d-8*e*b*arcta
n(c*x)*x*c+4*e*a*ln(c^2*x^2+1)*x*c+4*a*c*d*x-8*x*a*c*e+4*e*b*arctan(c*x)^2
-e*b*ln(c^2*x^2+1)^2+8*e*a*arctan(c*x)-2*ln(c^2*x^2+1)*b*d+4*ln(c^2*x^2+1)
*b*e)/c
```

3.1290.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{4\,be\arctan\left(cx\right)^2-be\log\left(c^2x^2+1\right)^2+4\left(acd-2\,ace\right)x+4\left(2\,ae+\left(bcd-2\,bce\right)x\right)\arctan\left(cx\right)+2\left(2\,bcex\right.}{4\,c}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

output
```
1/4*(4*b*e*arctan(c*x)^2 - b*e*log(c^2*x^2 + 1)^2 + 4*(a*c*d - 2*a*c*e)*x
+ 4*(2*a*e + (b*c*d - 2*b*c*e)*x)*arctan(c*x) + 2*(2*b*c*e*x*arctan(c*x) +
 2*a*c*e*x - b*d + 2*b*e)*log(c^2*x^2 + 1))/c
```

3.1290.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases} adx+aex\log\left(c^2x^2+1\right)-2aex+\frac{2ae\,\mathrm{atan}\,(cx)}{c}+bdx\,\mathrm{atan}\,(cx)+bex\log\left(c^2x^2+1\right)\mathrm{atan}\,(cx)-2bex\,\mathrm{atan} \\ adx \end{cases}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x + a*e*x*log(c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atan(c*x)/c
+ b*d*x*atan(c*x) + b*e*x*log(c**2*x**2 + 1)*atan(c*x) - 2*b*e*x*atan(c*x)
 - b*d*log(c**2*x**2 + 1)/(2*c) - b*e*log(c**2*x**2 + 1)**2/(4*c) + b*e*lo
g(c**2*x**2 + 1)/c + b*e*atan(c*x)**2/c, Ne(c, 0)), (a*d*x, True))
```

3.1290.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.53

$$
\begin{aligned}
&\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx \\
&= -\left(2\,c^2\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)-x\log\left(c^2x^2+1\right)\right)be\arctan(cx) \\
&\quad -\left(2\,c^2\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)-x\log\left(c^2x^2+1\right)\right)ae \\
&\quad + adx+\frac{(2\,cx\arctan(cx)-\log(c^2x^2+1))bd}{2\,c} \\
&\quad - \frac{\left(4\,\arctan(cx)^2+\log(c^2x^2+1)^2-4\,\log(c^2x^2+1)\right)be}{4\,c}
\end{aligned}
$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

output
```
-(2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*b*e*arctan(c*x) -
(2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*a*e + a*d*x + 1/2*(
2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c - 1/4*(4*arctan(c*x)^2 + log(c
^2*x^2 + 1)^2 - 4*log(c^2*x^2 + 1))*b*e/c
```

3.1290.8 Giac [F]

$$
\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx=\int (b\arctan(cx)+a)\left(e\log\left(c^2x^2+1\right)+d\right)dx
$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1290.9 Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.34

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$= a\,d\,x-2\,a\,e\,x-\frac{b\,e\ln\left(c^2\,x^2+1\right)^2}{4\,c}+b\,d\,x\,\mathrm{atan}(c\,x)-2\,b\,e\,x\,\mathrm{atan}(c\,x)$$
$$+a\,e\,x\,\ln\left(c^2\,x^2+1\right)+\frac{2\,a\,e\,\mathrm{atan}(c\,x)}{c}-\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c}$$
$$+\frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{c}+\frac{b\,e\,\mathrm{atan}(c\,x)^2}{c}+b\,e\,x\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)$$

input
```
int((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
a*d*x - 2*a*e*x - (b*e*log(c^2*x^2 + 1)^2)/(4*c) + b*d*x*atan(c*x) - 2*b*e
*x*atan(c*x) + a*e*x*log(c^2*x^2 + 1) + (2*a*e*atan(c*x))/c - (b*d*log(c^2
*x^2 + 1))/(2*c) + (b*e*log(c^2*x^2 + 1))/c + (b*e*atan(c*x)^2)/c + b*e*x*
atan(c*x)*log(c^2*x^2 + 1)
```

3.1291 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx$

3.1291.1 Optimal result

Integrand size = 26, antiderivative size = 282

$$\begin{aligned}
&\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx \\
&= ad\log(x)+\frac{1}{2}ibe\log(icx)\log^2(1-icx) \\
&\quad -\frac{1}{2}ibe\log(-icx)\log^2(1+icx)+\frac{1}{2}ibd\,\text{PolyLog}(2,-icx) \\
&\quad -\frac{1}{2}ibe\left(\log(1-icx)+\log(1+icx)-\log\left(1+c^2x^2\right)\right)\text{PolyLog}(2,-icx) \\
&\quad -\frac{1}{2}ibd\,\text{PolyLog}(2,icx)+\frac{1}{2}ibe\left(\log(1-icx)+\log(1+icx)-\log\left(1+c^2x^2\right)\right)\text{PolyLog}(2,icx) \\
&\quad -\frac{1}{2}ae\,\text{PolyLog}\left(2,-c^2x^2\right)+ibe\log(1-icx)\,\text{PolyLog}(2,1-icx) \\
&\quad -ibe\log(1+icx)\,\text{PolyLog}(2,1+icx)-ibe\,\text{PolyLog}(3,1-icx)+ibe\,\text{PolyLog}(3,1+icx)
\end{aligned}$$

output
```
a*d*ln(x)+1/2*I*b*e*ln(I*c*x)*ln(1-I*c*x)^2-1/2*I*b*e*ln(-I*c*x)*ln(1+I*c*
x)^2+1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*e*(ln(1-I*c*x)+ln(1+I*c*x)-ln(c^2
*x^2+1))*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)+1/2*I*b*e*(ln(1-I*c*
x)+ln(1+I*c*x)-ln(c^2*x^2+1))*polylog(2,I*c*x)-1/2*a*e*polylog(2,-c^2*x^2)
+I*b*e*ln(1-I*c*x)*polylog(2,1-I*c*x)-I*b*e*ln(1+I*c*x)*polylog(2,1+I*c*x)
-I*b*e*polylog(3,1-I*c*x)+I*b*e*polylog(3,1+I*c*x)
```

3.1291.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = \int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x, x]
```

3.1291.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5550, 5355, 2838, 5548, 2838, 5546, 2843, 2881, 2821, 5355, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}\,dx$$

$$\Big\downarrow \text{5550}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(c^2x^2+1\right)}{x}dx+d\int \frac{a+b\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{5355}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(c^2x^2+1\right)}{x}dx+ d\left(\frac{1}{2}ib\int \frac{\log(1-icx)}{x}dx-\frac{1}{2}ib\int \frac{\log(icx+1)}{x}dx+a\log(x)\right)$$

$$\Big\downarrow \text{2838}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(c^2x^2+1\right)}{x}dx+ d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{5548}$$

$$e\left(a\int \frac{\log\left(c^2x^2+1\right)}{x}dx+b\int \frac{\arctan(cx)\log\left(c^2x^2+1\right)}{x}dx\right)+$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \; 2838$$

$$e\left(b\int \frac{\arctan(cx)\log\left(c^2x^2+1\right)}{x}dx-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)\right)+$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \; 5546$$

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int \frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\int \frac{\text{lo}}{}\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \; 2843$$

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int \frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\left(2ic\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \; 2881$$

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int \frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\left(\log\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \; 2821$$

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int \frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\left(\log\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \; 5355$$

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\left(\frac{1}{2}i\int \frac{\log(1-icx)}{x}dx-\frac{1}{2}i\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

3.1291. $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx$

$$\Big\downarrow \text{2838}$$

$$e\left(-\frac{1}{2}a\operatorname{PolyLog}\left(2,-c^2x^2\right)+b\left(\frac{1}{2}i\left(\log(icx)\log^2(1-icx)-2\left(\int\frac{\operatorname{PolyLog}(2,1-icx)}{1-icx}d(1-icx)-\operatorname{PolyLog}(2,1\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ib\operatorname{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{7143}$$

$$e\left(-\frac{1}{2}a\operatorname{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(\frac{1}{2}i\operatorname{PolyLog}(2,-icx)-\frac{1}{2}i\operatorname{PolyLog}(2,icx)\right)\left(-\log\left(c^2x^2+1\right)+\log(1-icx\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ib\operatorname{PolyLog}(2,icx)\right)$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
```

output
```
d*(a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]) +
e*(-1/2*(a*PolyLog[2, -(c^2*x^2)]) + b*(-((Log[1 - I*c*x] + Log[1 + I*c*x]
 - Log[1 + c^2*x^2])*((I/2)*PolyLog[2, (-I)*c*x] - (I/2)*PolyLog[2, I*c*x]
)) + (I/2)*(Log[I*c*x]*Log[1 - I*c*x]^2 - 2*(-(Log[1 - I*c*x]*PolyLog[2, 1
 - I*c*x]) + PolyLog[3, 1 - I*c*x])) - (I/2)*(Log[(-I)*c*x]*Log[1 + I*c*x]
^2 - 2*(-(Log[1 + I*c*x]*PolyLog[2, 1 + I*c*x]) + PolyLog[3, 1 + I*c*x])))
)
```

3.1291.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5546
```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Simp[
(Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x])   Int[ArcTan[c*x]/x, x],
 x] + (Simp[I/2   Int[Log[1 - I*c*x]^2/x, x], x] - Simp[I/2   Int[Log[1 + I
*c*x]^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

rule 5548
```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTan[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_S
ymbol] :> Simp[a   Int[Log[f + g*x^2]/x, x], x] + Simp[b   Int[Log[f + g*x^
2]*(ArcTan[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 5550
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
(d_)))/(x_), x_Symbol] :> Simp[d   Int[(a + b*ArcTan[c*x])/x, x], x] + Simp
[e   Int[Log[f + g*x^2]*((a + b*ArcTan[c*x])/x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.1291. $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x}\,dx$

3.1291.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.84 (sec) , antiderivative size = 5420, normalized size of antiderivative = 19.22

method	result	size
risch	Expression too large to display	5420

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.1291.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x, x)
```

3.1291.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\left(d+e\log\left(c^2x^2+1\right)\right)}{x}\,dx$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x, x)
```

3.1291.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="maxima")
```

output
```
a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*e*arctan(c*x) + a*e)*lo
g(c^2*x^2 + 1))/x, x)
```

3.1291.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="giac")
```

output
```
Timed out
```

3.1291.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x, x)
```

3.1292 $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^2}\,dx$

3.1292.1 Optimal result

Integrand size = 26, antiderivative size = 100

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx &= \frac{ce(a+b\arctan(cx))^2}{b}-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\\ &\quad+\frac{1}{2}bc\left(d+e\log\left(1+c^2x^2\right)\right)\log\left(1-\frac{1}{1+c^2x^2}\right)-\frac{1}{2}bce\operatorname{PolyLog}\left(2,\frac{1}{1+c^2x^2}\right)\end{aligned}$$

output
```
c*e*(a+b*arctan(c*x))^2/b-(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x+1/2*b*c*
(d+e*ln(c^2*x^2+1))*ln(1-1/(c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(c^2*x^2+1))
```

3.1292.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx &= \frac{ce(a+b\arctan(cx))^2}{b}-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\\ &\quad+bc\left(-\frac{\left(d+e\log\left(1+c^2x^2\right)\right)\left(d-2e\log\left(-c^2x^2\right)+e\log\left(1+c^2x^2\right)\right)}{4e}\right.\\ &\quad\left.+\frac{1}{2}e\operatorname{PolyLog}\left(2,1+c^2x^2\right)\right)\end{aligned}$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

output
```
(c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^
2]))/x + b*c*(-1/4*((d + e*Log[1 + c^2*x^2])*(d - 2*e*Log[-(c^2*x^2)] + e*
Log[1 + c^2*x^2]))/e + (e*PolyLog[2, 1 + c^2*x^2])/2)
```

3.1292.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5552, 2925, 2858, 25, 27, 2779, 2838, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^2}dx$$

$$\Big\downarrow \text{5552}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx+bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx+\frac{1}{2}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^2\left(c^2x^2+1\right)}dx^2-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \text{2858}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx+\frac{b\int \frac{d+e\log\left(c^2x^2+1\right)}{x^4}d\left(c^2x^2+1\right)}{2c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \text{25}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{b\int -\frac{d+e\log\left(c^2x^2+1\right)}{x^4}d\left(c^2x^2+1\right)}{2c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \textcolor{blue}{2779}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(c^2x^2+1)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \textcolor{blue}{2838}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}-\frac{1}{2}bc\left(e\,\mathrm{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)$$

$$\Big\downarrow \textcolor{blue}{5419}$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}+\frac{ce(a+b\arctan(cx))^2}{b}-\frac{1}{2}bc\left(e\,\mathrm{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

output
```
(c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^
2]))/x - (b*c*(-(Log[1 - x^(-2)]*(d + e*Log[1 + c^2*x^2])) + e*PolyLog[2,
x^(-2)]))/2
```

3.1292.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

3.1292. $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^2}\,dx$

3.1292.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^2}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

3.1292.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^2, x)
```

3.1292.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.1292.7 Maxima [F]

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^2}\,dx = \int \frac{(b\arctan(cx) + a)(e\log(c^2x^2 + 1) + d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="maxima"
)
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + (2*c*arctan
(c*x) - log(c^2*x^2 + 1)/x)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^2 +
1)/x^2, x) - a*d/x
```

3.1292.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.1292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^2}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x))\,\left(d + e\,\ln\left(c^2\,x^2 + 1\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^2, x)
```

3.1293 $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^3}\,dx$

3.1293.1 Optimal result

Integrand size = 26, antiderivative size = 154

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx &= bc^2e\arctan(cx)+ac^2e\log(x)-\frac{1}{2}ac^2e\log\left(1+c^2x^2\right)-\frac{bc(d+e\log\left(1+c^2x^2\right))}{2x}\\ &\quad-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{2x^2}\\ &\quad+\frac{1}{2}ibc^2e\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ibc^2e\,\mathrm{PolyLog}(2,icx)\end{aligned}$$

output
```
b*c^2*e*arctan(c*x)+a*c^2*e*ln(x)-1/2*a*c^2*e*ln(c^2*x^2+1)-1/2*b*c*(d+e*l
n(c^2*x^2+1))/x-1/2*b*c^2*arctan(c*x)*(d+e*ln(c^2*x^2+1))-1/2*(a+b*arctan(
c*x))*(d+e*ln(c^2*x^2+1))/x^2+1/2*I*b*c^2*e*polylog(2,-I*c*x)-1/2*I*b*c^2*
e*polylog(2,I*c*x)
```

3.1293.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx = -\frac{ad+bcdx+bd\arctan(cx)+bc^2dx^2\arctan(cx)-2bc^2ex^2\arctan(cx)-2ac^2ex^2\log(x)+ae\log\left(1+c^2x^2\right)\cdots}{}$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

output
```
-1/2*(a*d + b*c*d*x + b*d*ArcTan[c*x] + b*c^2*d*x^2*ArcTan[c*x] - 2*b*c^2*
e*x^2*ArcTan[c*x] - 2*a*c^2*e*x^2*Log[x] + a*e*Log[1 + c^2*x^2] + b*c*e*x*
Log[1 + c^2*x^2] + a*c^2*e*x^2*Log[1 + c^2*x^2] + b*e*ArcTan[c*x]*Log[1 +
c^2*x^2] + b*c^2*e*x^2*ArcTan[c*x]*Log[1 + c^2*x^2] - I*b*c^2*e*x^2*PolyLo
g[2, (-I)*c*x] + I*b*c^2*e*x^2*PolyLog[2, I*c*x])/x^2
```

3.1293.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^3}dx$$

$$\Big\downarrow \text{5556}$$

$$-2c^2e\int \left(-\frac{a+bcx}{2x\left(c^2x^2+1\right)}-\frac{b\arctan(cx)}{2x}\right)dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{2x^2}-\frac{1}{2}bc^2\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{2x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{2x^2}-2c^2e\left(\frac{1}{4}a\log\left(c^2x^2+1\right)-\frac{1}{2}a\log(x)-\frac{1}{2}b\arctan(cx)-\frac{1}{4}ib\,\text{PolyLog}(2,-icx)+\frac{1}{4}ib\,\text{PolyLog}(2,icx)\right)-\frac{1}{2}bc^2\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{2x}$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[1 + c^2*x^2]))/x - (b*c^2*ArcTan[c*x]*(d + e*Log[1 +
c^2*x^2]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(2*x^2) - 2*
c^2*e*(-1/2*(b*ArcTan[c*x]) - (a*Log[x])/2 + (a*Log[1 + c^2*x^2])/4 - (I/4
)*b*PolyLog[2, (-I)*c*x] + (I/4)*b*PolyLog[2, I*c*x])
```

3.1293.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1293.4 Maple [F]

$$\int \frac{(a+b\arctan (cx)) \left(d+e\ln \left(c^2x^2+1\right)\right)}{x^3}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

3.1293.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx)) \left(d+e\log \left(1+c^2x^2\right)\right)}{x^3}\,dx = \int \frac{(b\arctan (cx)+a)(e\log \left(c^2x^2+1\right)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^3, x)
```

3.1293.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+e\log(c^2x^2+1))}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**3, x)
```

3.1293.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="maxima"
)
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - 1/2*(c^2*(log(c^2*x
^2 + 1) - log(x^2)) + log(c^2*x^2 + 1)/x^2)*a*e + 1/2*(4*c^4*x^2*integrate
(1/2*x*arctan(c*x)/(c^2*x^2 + 1), x) + 2*c^2*x^2*arctan(c*x) + 4*c^2*x^2*i
ntegrate(1/2*arctan(c*x)/(c^2*x^3 + x), x) - (c*x + (c^2*x^2 + 1)*arctan(c
*x))*log(c^2*x^2 + 1))*b*e/x^2 - 1/2*a*d/x^2
```

3.1293.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.1293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^3, x)
```

3.1294 $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^4}\,dx$

3.1294.1 Optimal result

Integrand size = 26, antiderivative size = 189

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx &= -\frac{2c^2e(a+b\arctan(cx))}{3x}-\frac{c^3e(a+b\arctan(cx))^2}{3b}+bc^3e\log(x)-\frac{1}{3}bc^3e\log\left(1+c^2x^2\right)\\ &\quad-\frac{bc(1+c^2x^2)\left(d+e\log\left(1+c^2x^2\right)\right)}{6x^2}-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{3x^3}\\ &\quad-\frac{1}{6}bc^3\left(d+e\log\left(1+c^2x^2\right)\right)\log\left(1-\frac{1}{1+c^2x^2}\right)+\frac{1}{6}bc^3e\,\mathrm{PolyLog}\left(2,\frac{1}{1+c^2x^2}\right)\end{aligned}$$

output
```
-2/3*c^2*e*(a+b*arctan(c*x))/x-1/3*c^3*e*(a+b*arctan(c*x))^2/b+b*c^3*e*ln(
x)-1/3*b*c^3*e*ln(c^2*x^2+1)-1/6*b*c*(c^2*x^2+1)*(d+e*ln(c^2*x^2+1))/x^2-1
/3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3-1/6*b*c^3*(d+e*ln(c^2*x^2+1))
*ln(1-1/(c^2*x^2+1))+1/6*b*c^3*e*polylog(2,1/(c^2*x^2+1))
```

3.1294.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx$$
$$=\frac{1}{12}\left(-\frac{8c^2e(a+b\arctan(cx))}{x}-\frac{4c^3e(a+b\arctan(cx))^2}{b}+6bc^3e\left(2\log(x)-\log\left(1+c^2x^2\right)\right)-\frac{2bc\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}-\frac{4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}+\frac{bc^3\left(d+e\log\left(1+c^2x^2\right)\right)^2}{e}-2bc^3\left(\log\left(-c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)+e\,\mathrm{PolyLog}\left(2,1+c^2x^2\right)\right)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

output
```
((-8*c^2*e*(a + b*ArcTan[c*x]))/x - (4*c^3*e*(a + b*ArcTan[c*x])^2)/b + 6*
b*c^3*e*(2*Log[x] - Log[1 + c^2*x^2]) - (2*b*c*(d + e*Log[1 + c^2*x^2]))/x
^2 - (4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3 + (b*c^3*(d + e*
Log[1 + c^2*x^2])^2)/e - 2*b*c^3*(Log[-(c^2*x^2)]*(d + e*Log[1 + c^2*x^2])
 + e*PolyLog[2, 1 + c^2*x^2]))/12
```

3.1294.3 Rubi [A] (warning: unable to verify)

Time = 1.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5552, 2925, 2858, 27, 2789, 2751, 16, 2779, 2838, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^4}\,dx$$
$$\Big\downarrow \text{5552}$$
$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+\frac{1}{3}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^3\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{2925}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx+\frac{1}{6}bc\int \frac{d+e\log(c^2x^2+1)}{x^4(c^2x^2+1)}dx^2-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{3x^3}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx+\frac{b\int \frac{d+e\log(c^2x^2+1)}{x^6}d(c^2x^2+1)}{6c}-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx+\frac{1}{6}bc^3\int \frac{d+e\log(c^2x^2+1)}{c^4x^6}d(c^2x^2+1)-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{3x^3}$$

$$\Big\downarrow \text{2789}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx+\frac{1}{6}bc^3\left(\int -\frac{d+e\log(c^2x^2+1)}{c^2x^4}d(c^2x^2+1)+\int \frac{d+e\log(c^2x^2+1)}{c^4x^4}d(c^2x^2+1)\right)-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{3x^3}$$

$$\Big\downarrow \text{2751}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx+\frac{1}{6}bc^3\left(\int -\frac{d+e\log(c^2x^2+1)}{c^2x^4}d(c^2x^2+1)-e\int -\frac{1}{c^2x^2}d(c^2x^2+1)-\frac{(c^2x^2+1)(e\log(c^2x^2+1)+d)}{c^2x^2}\right)-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{3x^3}$$

$$\Big\downarrow \text{16}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx+\frac{1}{6}bc^3\left(\int -\frac{d+e\log(c^2x^2+1)}{c^2x^4}d(c^2x^2+1)-\frac{(c^2x^2+1)(e\log(c^2x^2+1)+d)}{c^2x^2}+e\log(-c^2x^2)\right)-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{3x^3}$$

$$\Big\downarrow \text{2779}$$

3.1294. $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^4}\,dx$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+$$
$$\frac{1}{6}bc^3\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+$$
$$\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{2}{3}c^2e\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+$$
$$\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{x}\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+$$
$$\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

$$\Big\downarrow \text{ 243}$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+$$
$$\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

$$\Big\downarrow \text{ 47}$$

3.1294. $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx$

$$\frac{2}{3}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

$$\Big\downarrow \text{5419}$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{2}{3}c^2e\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(2,\right.\right.$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

output
```
(2*c^2*e*(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*
c*(Log[x^2] - Log[1 + c^2*x^2]))/2))/3 - ((a + b*ArcTan[c*x])*(d + e*Log[1
 + c^2*x^2]))/(3*x^3) + (b*c^3*(e*Log[-(c^2*x^2)] - ((1 + c^2*x^2)*(d + e*
Log[1 + c^2*x^2]))/(c^2*x^2) - Log[1 - x^(-2)]*(d + e*Log[1 + c^2*x^2]) +
e*PolyLog[2, x^(-2)]))/6
```

3.1294. $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx$

3.1294.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.1294. $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^4}\,dx$

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

3.1294. $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx$

3.1294.4 Maple [F]

$$\int \frac{(a + b\arctan(cx))\,(d + e\ln(c^2x^2 + 1))}{x^4}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

3.1294.5 Fricas [F]

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^4}\,dx = \int \frac{(b\arctan(cx) + a)(e\log(c^2x^2 + 1) + d)}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^4, x)
```

3.1294.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**4,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.1294.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx$$
$$=\int \frac{(b\arctan(cx)+a)(e\log\left(c^2x^2+1\right)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="maxima"
)
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*d - 1/3*(2*(c*arctan(c*x) + 1/x)*c^2 + log(c^2*x^2 + 1)/x^3)*a*e + b*e*i
ntegrate(arctan(c*x)*log(c^2*x^2 + 1)/x^4, x) - 1/3*a*d/x^3
```

3.1294.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.1294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^4, x)
```

3.1295 $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^5}\,dx$

3.1295.1 Optimal result

Integrand size = 26, antiderivative size = 225

$$\begin{aligned}
\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx &= -\frac{ac^2e}{4x^2}-\frac{5bc^3e}{12x}-\frac{11}{12}bc^4e\arctan(cx)-\frac{bc^2e\arctan(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)\\
&\quad+\frac{1}{4}ac^4e\log\left(1+c^2x^2\right)-\frac{bc(d+e\log(1+c^2x^2))}{12x^3}+\frac{bc^3(d+e\log(1+c^2x^2))}{4x}\\
&\quad+\frac{1}{4}bc^4\arctan(cx)\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{4x^4}\\
&\quad-\frac{1}{4}ibc^4e\,\mathrm{PolyLog}(2,-icx)+\frac{1}{4}ibc^4e\,\mathrm{PolyLog}(2,icx)
\end{aligned}$$

output
```
-1/4*a*c^2*e/x^2-5/12*b*c^3*e/x-11/12*b*c^4*e*arctan(c*x)-1/4*b*c^2*e*arct
an(c*x)/x^2-1/2*a*c^4*e*ln(x)+1/4*a*c^4*e*ln(c^2*x^2+1)-1/12*b*c*(d+e*ln(c
^2*x^2+1))/x^3+1/4*b*c^3*(d+e*ln(c^2*x^2+1))/x+1/4*b*c^4*arctan(c*x)*(d+e*
ln(c^2*x^2+1))-1/4*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4-1/4*I*b*c^4*e
*polylog(2,-I*c*x)+1/4*I*b*c^4*e*polylog(2,I*c*x)
```

3.1295.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx =$$
$$-\frac{3ad+bcdx+3ac^2ex^2-3bc^3dx^3+5bc^3ex^3+3bd\arctan(cx)+3bc^2ex^2\arctan(cx)-3bc^4dx^4\arctan(cx}{}$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```

output
```
-1/12*(3*a*d + b*c*d*x + 3*a*c^2*e*x^2 - 3*b*c^3*d*x^3 + 5*b*c^3*e*x^3 + 3
*b*d*ArcTan[c*x] + 3*b*c^2*e*x^2*ArcTan[c*x] - 3*b*c^4*d*x^4*ArcTan[c*x] +
 11*b*c^4*e*x^4*ArcTan[c*x] + 6*a*c^4*e*x^4*Log[x] + 3*a*e*Log[1 + c^2*x^2
] + b*c*e*x*Log[1 + c^2*x^2] - 3*b*c^3*e*x^3*Log[1 + c^2*x^2] - 3*a*c^4*e*
x^4*Log[1 + c^2*x^2] + 3*b*e*ArcTan[c*x]*Log[1 + c^2*x^2] - 3*b*c^4*e*x^4*
ArcTan[c*x]*Log[1 + c^2*x^2] + (3*I)*b*c^4*e*x^4*PolyLog[2, (-I)*c*x] - (3
*I)*b*c^4*e*x^4*PolyLog[2, I*c*x])/x^4
```

3.1295.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^5}\,dx$$
$$\Big\downarrow \; 5556$$
$$-2c^2e\int \left(-\frac{-3bc^3x^3+bcx+3a}{12x^3\left(c^2x^2+1\right)}-\frac{b\left(1-c^2x^2\right)\arctan(cx)}{4x^3}\right)dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{4x^4}+\frac{1}{4}bc^4\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{12x^3}+\frac{bc^3\left(e\log\left(c^2x^2+1\right)+d\right)}{4x}$$
$$\Big\downarrow \; 2009$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{4x^4}-$$
$$2c^2e\left(-\frac{1}{8}ac^2\log\left(c^2x^2+1\right)+\frac{1}{4}ac^2\log(x)+\frac{a}{8x^2}+\frac{11}{24}bc^2\arctan(cx)+\frac{b\arctan(cx)}{8x^2}+\frac{1}{8}ibc^2\operatorname{PolyLog}(2,-icx)-\right.$$
$$\frac{1}{4}bc^4\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{12x^3}+\frac{bc^3\left(e\log\left(c^2x^2+1\right)+d\right)}{4x}$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```

output
```
-1/12*(b*c*(d + e*Log[1 + c^2*x^2]))/x^3 + (b*c^3*(d + e*Log[1 + c^2*x^2])
)/(4*x) + (b*c^4*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2]))/4 - ((a + b*ArcTan[
c*x])*(d + e*Log[1 + c^2*x^2]))/(4*x^4) - 2*c^2*e*(a/(8*x^2) + (5*b*c)/(24
*x) + (11*b*c^2*ArcTan[c*x])/24 + (b*ArcTan[c*x])/(8*x^2) + (a*c^2*Log[x])
/4 - (a*c^2*Log[1 + c^2*x^2])/8 + (I/8)*b*c^2*PolyLog[2, (-I)*c*x] - (I/8)
*b*c^2*PolyLog[2, I*c*x])
```

3.1295.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1295.4 Maple [F]

$$\int\frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^5}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

3.1295.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^5, x)
```

3.1295.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+e\log(c^2x^2+1))}{x^5}\,dx$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**5, x)
```

3.1295.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="maxima"
)
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
 + 1/4*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c^2 - log(c^2*x^2 +
1)/x^4)*a*e - 1/12*(72*c^6*x^4*integrate(1/12*x*arctan(c*x)/(c^2*x^2 + 1),
 x) + 8*c^4*x^4*arctan(c*x) - 72*c^2*x^4*integrate(1/12*arctan(c*x)/(c^2*x
^5 + x^3), x) + 2*c^3*x^3 - (3*c^3*x^3 - c*x + 3*(c^4*x^4 - 1)*arctan(c*x)
)*log(c^2*x^2 + 1))*b*e/x^4 - 1/4*a*d/x^4
```

3.1295.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="giac")
```

output
```
Timed out
```

3.1295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^5,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^5, x)
```

3.1296 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx$

3.1296.1 Optimal result

Integrand size = 26, antiderivative size = 248

$$\begin{aligned}
&\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx\\
&=-\frac{7bc^3e}{60x^2}-\frac{2c^2e(a+b\arctan(cx))}{15x^3}+\frac{2c^4e(a+b\arctan(cx))}{5x}+\frac{c^5e(a+b\arctan(cx))^2}{5b}\\
&\quad-\frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e\log\left(1+c^2x^2\right)-\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{20x^4}\\
&\quad+\frac{bc^3\left(1+c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{10x^2}-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{5x^5}\\
&\quad+\frac{1}{10}bc^5\left(d+e\log\left(1+c^2x^2\right)\right)\log\left(1-\frac{1}{1+c^2x^2}\right)-\frac{1}{10}bc^5e\,\mathrm{PolyLog}\left(2,\frac{1}{1+c^2x^2}\right)
\end{aligned}$$

output
```
-7/60*b*c^3*e/x^2-2/15*c^2*e*(a+b*arctan(c*x))/x^3+2/5*c^4*e*(a+b*arctan(c
*x))/x+1/5*c^5*e*(a+b*arctan(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*ln(
c^2*x^2+1)-1/20*b*c*(d+e*ln(c^2*x^2+1))/x^4+1/10*b*c^3*(c^2*x^2+1)*(d+e*ln
(c^2*x^2+1))/x^2-1/5*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5+1/10*b*c^5*
(d+e*ln(c^2*x^2+1))*ln(1-1/(c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1/(c^2*x^2+
1))
```

3.1296.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx$$
$$=\frac{1}{60}\left(-\frac{8c^2e(a+b\arctan(cx))}{x^3}-24c^4e\left(-\frac{a+b\arctan(cx)}{x}-\frac{c(a+b\arctan(cx))^2}{2b}+bc\left(\log(x)-\frac{1}{2}\log\left(1+c^2x^2\right)\right)\right)-6bc^5e\left(2\log(x)-\log\left(1+c^2x^2\right)\right)+7bc^3e\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log\left(1+c^2x^2\right)\right)-\frac{3bc\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}+\frac{6bc^3\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}-\frac{12(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}-\frac{3bc^5\left(d+e\log\left(1+c^2x^2\right)\right)^2}{e}+6bc^5\left(\log\left(-c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)+e\,\mathrm{PolyLog}\left(2,1+c^2x^2\right)\right)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

output
```
((-8*c^2*e*(a + b*ArcTan[c*x]))/x^3 - 24*c^4*e*(-((a + b*ArcTan[c*x])/x) -
 (c*(a + b*ArcTan[c*x])^2)/(2*b) + b*c*(Log[x] - Log[1 + c^2*x^2]/2)) - 6*
b*c^5*e*(2*Log[x] - Log[1 + c^2*x^2]) + 7*b*c^3*e*(-x^(-2) - 2*c^2*Log[x]
+ c^2*Log[1 + c^2*x^2]) - (3*b*c*(d + e*Log[1 + c^2*x^2]))/x^4 + (6*b*c^3*
(d + e*Log[1 + c^2*x^2]))/x^2 - (12*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2
*x^2]))/x^5 - (3*b*c^5*(d + e*Log[1 + c^2*x^2])^2)/e + 6*b*c^5*(Log[-(c^2*
x^2)]*(d + e*Log[1 + c^2*x^2]) + e*PolyLog[2, 1 + c^2*x^2]))/60
```

3.1296.3 Rubi [A] (warning: unable to verify)

Time = 2.24 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.17, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5552, 2925, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^6}dx$$

$$\Big\downarrow \text{5552}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx+\frac{1}{5}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^5\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2925}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx+\frac{1}{10}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^6\left(c^2x^2+1\right)}dx^2-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx+\frac{b\int \frac{d+e\log\left(c^2x^2+1\right)}{x^8}d\left(c^2x^2+1\right)}{10c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{b\int -\frac{d+e\log\left(c^2x^2+1\right)}{x^8}d\left(c^2x^2+1\right)}{10c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^6x^8}d\left(c^2x^2+1\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2789}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^6x^6}d\left(c^2x^2+1\right)+\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d\left(c^2x^2+1\right)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2756}$$

3.1296. $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}dx$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d(c^2x^2+1)-\frac{1}{2}e\int \frac{1}{c^4x^6}d(c^2x^2+1)+\frac{e\log\left(c^2x^2+1\right)+d}{2c^4x^4}\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \; 54$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d(c^2x^2+1)-\frac{1}{2}e\int \left(-\frac{1}{c^2x^2}+\frac{1}{x^2}+\frac{1}{c^4x^4}\right)d(c^2x^2+1)+\frac{e\log\left(c^2x^2+1\right)+d}{2c^4x^4}\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d(c^2x^2+1)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\log\left(-c^2x^2\right)+\log\left(c^2x^2+1\right)\right)+\frac{e\log\left(c^2x^2+1\right)+d}{2c^4x^4}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \; 2789$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)+\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^4}d(c^2x^2+1)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\log\left(-c^2x^2\right)+\right.\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \; 2751$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)-e\int -\frac{1}{c^2x^2}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\frac{1}{2}e\left(\right.\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \; 16$$

3.1296. $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^6}\,dx$

$$\frac{2}{5}c^2e\int\frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int-\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2779}$$

$$\frac{2}{5}c^2e\int\frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(e\int\frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2}{5}c^2e\int\frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$

$$\Big\downarrow \text{5453}$$

$$\frac{2}{5}c^2e\left(\int\frac{a+b\arctan(cx)}{x^4}dx-c^2\int\frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)+\frac{1}{3}bc\int\frac{1}{x^3\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{3x^3}\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$

$$\Big\downarrow \text{243}$$

3.1296. $\int\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx$

$$\frac{2}{5}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx\right)+\frac{1}{6}bc\int \frac{1}{x^4(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{(c^2x^2+1)(e\log(c^2x^2+1)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(c^2x^2+1)+d)+e\log(-c^2x^2)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \; 54$$

$$\frac{2}{5}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx\right)+\frac{1}{6}bc\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{(c^2x^2+1)(e\log(c^2x^2+1)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(c^2x^2+1)+d)+e\log(-c^2x^2)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2(-\log(x^2))+c^2\log(c^2x^2+1)-\frac{1}{x^2}\right)\right)-\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{(c^2x^2+1)(e\log(c^2x^2+1)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(c^2x^2+1)+d)+e\log(-c^2x^2)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \; 5453$$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2(-\log(x^2))+c^2\right.\right.$$
$$\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{(c^2x^2+1)(e\log(c^2x^2+1)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(c^2x^2+1)+d)+e\log(-c^2x^2)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \; 5361$$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}\right.$$
$$\frac{(a+b\arctan(cx))(e\log(c^2x^2+1)+d)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{(c^2x^2+1)(e\log(c^2x^2+1)+d)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)(e\log(c^2x^2+1)+d)+e\log(-c^2x^2)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \; 243$$

3.1296. $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^6}\,dx$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int\frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx)}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow 47$$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int\frac{1}{x^2}dx^2-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+}{}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$

$$\Big\downarrow 14$$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b}{}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$

$$\Big\downarrow 16$$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)\right)-\frac{a+b\arcta}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$

$$\Big\downarrow 5419$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}+$$
$$\frac{2}{5}c^2e\left(-\left(c^2\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)\right)-\frac{a+b\arctan(cx)}{3x^3}\right.$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\right.\right.$$

3.1296. $\int\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

output
```
-1/5*((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5 + (2*c^2*e*(-1/3*(
a + b*ArcTan[c*x])/x^3 - c^2*(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[
c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2) + (b*c*(-x^(-2) -
c^2*Log[x^2] + c^2*Log[1 + c^2*x^2]))/6))/5 - (b*c^5*(e*Log[-(c^2*x^2)] -
(e*(-(1/(c^2*x^2)) - Log[-(c^2*x^2)] + Log[1 + c^2*x^2]))/2 + (d + e*Log[1
 + c^2*x^2])/(2*c^4*x^4) - ((1 + c^2*x^2)*(d + e*Log[1 + c^2*x^2]))/(c^2*x
^2) - Log[1 - x^(-2)]*(d + e*Log[1 + c^2*x^2]) + e*PolyLog[2, x^(-2)]))/10
```

3.1296.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

3.1296. $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^6}\,dx$

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

3.1296.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(c^2x^2+1))}{x^6}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

3.1296.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^6}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^6, x)
```

3.1296.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**6,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.1296.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^6}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="maxima"
)
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d + 1/15*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3
)*c^2 - 3*log(c^2*x^2 + 1)/x^5)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^
2 + 1)/x^6, x) - 1/5*a*d/x^5
```

3.1296.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="giac")
```

output
```
Timed out
```

3.1296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^6,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^6, x)
```

3.1297 $\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$

3.1297.1 Optimal result

Integrand size = 22, antiderivative size = 562

$$
\begin{aligned}
&\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx\\
&=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\arctan(cx)}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))\\
&\quad-\frac{1}{2}ex^2(a+b\arctan(cx))-\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2}{1-icx}\right)}{c^2g}\\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-i\sqrt{g})(1-icx)}\right)}{2c^2g}\\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+i\sqrt{g})(1-icx)}\right)}{2c^2g}-\frac{bex\log\left(f+gx^2\right)}{2c}\\
&\quad-\frac{be(c^2f-g)\arctan(cx)\log\left(f+gx^2\right)}{2c^2g}+\frac{e\left(f+gx^2\right)(a+b\arctan(cx))\log\left(f+gx^2\right)}{2g}\\
&\quad+\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2c^2g}-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-i\sqrt{g})(1-icx)}\right)}{4c^2g}\\
&\quad-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+i\sqrt{g})(1-icx)}\right)}{4c^2g}
\end{aligned}
$$

output
```
-1/2*b*(d-e)*x/c+b*e*x/c+1/2*b*(d-e)*arctan(c*x)/c^2+1/2*d*x^2*(a+b*arctan
(c*x))-1/2*e*x^2*(a+b*arctan(c*x))-b*e*(c^2*f-g)*arctan(c*x)*ln(2/(1-I*c*x
))/c^2/g-1/2*b*e*x*ln(g*x^2+f)/c-1/2*b*e*(c^2*f-g)*arctan(c*x)*ln(g*x^2+f)
/c^2/g+1/2*e*(g*x^2+f)*(a+b*arctan(c*x))*ln(g*x^2+f)/g+1/2*b*e*(c^2*f-g)*a
rctan(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)-I*g^(1/2)
))/c^2/g+1/2*b*e*(c^2*f-g)*arctan(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(1-I*
c*x)/(c*(-f)^(1/2)+I*g^(1/2)))/c^2/g+1/2*I*b*e*(c^2*f-g)*polylog(2,1-2/(1-
I*c*x))/c^2/g-1/4*I*b*e*(c^2*f-g)*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(
1-I*c*x)/(c*(-f)^(1/2)-I*g^(1/2)))/c^2/g-1/4*I*b*e*(c^2*f-g)*polylog(2,1-2
*c*((-f)^(1/2)+x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)+I*g^(1/2)))/c^2/g-b*e*ar
ctan(x*g^(1/2)/f^(1/2))*f^(1/2)/c/g^(1/2)
```

3.1297.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1140 vs. $2(562) = 1124$.

Time = 6.75 (sec) , antiderivative size = 1140, normalized size of antiderivative = 2.03

$$\int x(a + b\arctan(cx)) \left(d + e\log\left(f + gx^2\right)\right) \, dx$$

$$= \frac{-2bcdgx + 6bcegx + 2ac^2dgx^2 - 2ac^2egx^2 + 2bdg\arctan(cx) - 2beg\arctan(cx) + 2bc^2dgx^2\arctan(cx) -}{}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(-2*b*c*d*g*x + 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 + 2*b*d*g*
ArcTan[c*x] - 2*b*e*g*ArcTan[c*x] + 2*b*c^2*d*g*x^2*ArcTan[c*x] - 2*b*c^2*
e*g*x^2*ArcTan[c*x] - 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]
+ (4*I)*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^
2*f*g]] - (4*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqr
t[c^2*f*g]] - 4*b*c^2*e*f*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*b
*e*g*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 2*b*c^2*e*f*ArcSin[Sqrt[
(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2
*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)]
- 2*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[
c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c
^2*f*g])/(c^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*Arc
Tan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sq
rt[c^2*f*g])/(c^2*f - g)] - 2*b*e*g*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*Arc
Tan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sq
rt[c^2*f*g])/(c^2*f - g)] - 2*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*
Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)]
 + 2*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I)*ArcTan[c*x]
)*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Lo
g[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)...
```

3.1297.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$

$$\Big\downarrow \text{5554}$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(c^2x^2+1\right)}+\frac{e\left(gx^2+f\right)\log\left(gx^2+f\right)}{2g\left(c^2x^2+1\right)}\right)dx+\frac{1}{2}dx^2(a+b\arctan(cx))+\frac{e\left(f+gx^2\right)\log\left(f+gx^2\right)(a+b\arctan(cx))}{2g}-\frac{1}{2}ex^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}dx^2(a+b\arctan(cx))+\frac{e(f+gx^2)\log(f+gx^2)(a+b\arctan(cx))}{2g}-\frac{1}{2}ex^2(a+b\arctan(cx))-$$
$$bc\left(-\frac{(d-e)\arctan(cx)}{2c^3}+\frac{e\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c^2\sqrt{g}}+\frac{e\arctan(cx)\left(c^2f-g\right)\log(f+gx^2)}{2c^3g}+\frac{e\arctan(cx)\left(c^2f-g\right)\text{lo}}{c^3g}\right.$$

input
```
Int[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]))/2 + (e*(f + g*
x^2)*(a + b*ArcTan[c*x])*Log[f + g*x^2])/(2*g) - b*c*(((d - e)*x)/(2*c^2)
- (e*x)/c^2 - ((d - e)*ArcTan[c*x])/(2*c^3) + (e*Sqrt[f]*ArcTan[(Sqrt[g]*x
)/Sqrt[f]])/(c^2*Sqrt[g]) + (e*(c^2*f - g)*ArcTan[c*x]*Log[2/(1 - I*c*x)])
/(c^3*g) - (e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c
*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(2*c^3*g) - (e*(c^2*f - g)*ArcTan[c*
x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))
])/(2*c^3*g) + (e*x*Log[f + g*x^2])/(2*c^2) + (e*(c^2*f - g)*ArcTan[c*x]*L
og[f + g*x^2])/(2*c^3*g) - ((I/2)*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*
x)])/(c^3*g) + ((I/4)*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g
]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(c^3*g) + ((I/4)*e*(c^2*f -
 g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*
(1 - I*c*x))])/(c^3*g))
```

3.1297.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

3.1297.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.54 (sec) , antiderivative size = 10102, normalized size of antiderivative = 17.98

method	result	size
default	Expression too large to display	10102
parts	Expression too large to display	10102

input
```
int(x*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.1297.5 Fricas [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int(b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*x*arctan(c*x) + a*d*x + (b*e*x*arctan(c*x) + a*e*x)*log(g*x^2
 + f), x)
```

3.1297.6 Sympy [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

3.1297.7 Maxima [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int (b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d - 1/
2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g - 1/2*(2*c*f*arctan(g*x/s
qrt(f*g)) + (4*c^4*g*integrate(1/2*x^3*arctan(c*x)/(c^2*g*x^2 + c^2*f), x)
 + 4*c^2*g*integrate(1/2*x*arctan(c*x)/(c^2*g*x^2 + c^2*f), x) - 2*c*x + (
c*x - (c^2*x^2 + 1)*arctan(c*x))*log(g*x^2 + f))*sqrt(f*g))*b*e/(sqrt(f*g)
*c^2)
```

3.1297.8 Giac [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int (b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1297.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*log(f + g*x^2)), x)
```

3.1298 $\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$

3.1298.1 Optimal result

Integrand size = 21, antiderivative size = 656

$$
\begin{aligned}
&\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx \\
&= -2aex-2bex\arctan(cx)+\frac{2ae\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}} \\
&\quad+\frac{ibe\sqrt{-f}\log(1+icx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe\sqrt{-f}\log(1-icx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}} \\
&\quad+\frac{ibe\sqrt{-f}\log(1-icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe\sqrt{-f}\log(1+icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}} \\
&\quad+\frac{be\log\left(1+c^2x^2\right)}{c}+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right) \\
&\quad-\frac{b\log\left(-\frac{g(1+c^2x^2)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c}-\frac{ibe\sqrt{-f}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(i-cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}} \\
&\quad+\frac{ibe\sqrt{-f}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(1+icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}} \\
&\quad-\frac{ibe\sqrt{-f}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(i+cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\operatorname{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right)}{2c}
\end{aligned}
$$

output
```
-2*a*e*x-2*b*e*x*arctan(c*x)+b*e*ln(c^2*x^2+1)/c+x*(a+b*arctan(c*x))*(d+e*
ln(g*x^2+f))-1/2*b*ln(-g*(c^2*x^2+1)/(c^2*f-g))*(d+e*ln(g*x^2+f))/c-1/2*b*
e*polylog(2,c^2*(g*x^2+f)/(c^2*f-g))/c+1/2*I*b*e*ln(1+I*c*x)*ln(c*((-f)^(1
/2)-x*g^(1/2))/(c*(-f)^(1/2)-I*g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*ln(1
-I*c*x)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g
^(1/2)+1/2*I*b*e*ln(1-I*c*x)*ln(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)-I*g
^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*ln(1+I*c*x)*ln(c*((-f)^(1/2)+x*g^(1/
2))/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*polylog(2,(I-c*
x)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g^(1/2)-1/2*I*b*e*polylog(
2,(c*x+I)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/2*I*b*e*p
olylog(2,(1-I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g^(1/2)+1/
2*I*b*e*polylog(2,(1+I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+g^(1/2)))*(-f)^(1/2)/g
^(1/2)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*f^(1/2)/g^(1/2)
```

3.1298.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1352 vs. $2(656) = 1312$.

Time = 3.26 (sec) , antiderivative size = 1352, normalized size of antiderivative = 2.06

$$\int (a + b\arctan(cx)) \left(d + e\log\left(f + gx^2\right)\right) \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + b*d*x*ArcTan[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]])/Sqrt[g] - (b*d*Log[1 + c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*
e*(x*ArcTan[c*x] - Log[1 + c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*g*(((-Log
[(-I)/c + x] - Log[I/c + x] + Log[1 + c^2*x^2])*Log[f + g*x^2])/(2*g) + (L
og[(-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/
c)] + PolyLog[2, (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/c)])/(
2*g) + (Log[(-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqr
t[g])/c)] + PolyLog[2, (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqrt[g])/c)]
)/(2*g) + (Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqr
t[g])/c)] + PolyLog[2, (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqrt[g])/c)]
)/(2*g) + (Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g
])/c)] + PolyLog[2, (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g])/c)])/(2*g
)))/c - (b*e*(4*c*x*ArcTan[c*x] + 4*Log[1/Sqrt[1 + c^2*x^2]] + (c^2*f*(4*A
rcTan[c*x]*ArcTanh[Sqrt[-(c^2*f*g)]/(c*g*x)] - 2*ArcCos[(c^2*f + g)/(-(c^2
*f) + g)]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - (ArcCos[(c^2*f + g)/(-(c^2*f
) + g)] - (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[(-2*c^2*f*(I*g + Sq
rt[-(c^2*f*g)])*(-I + c*x))/((c^2*f - g)*(c^2*f - c*Sqrt[-(c^2*f*g)]*x))]
- (ArcCos[(c^2*f + g)/(-(c^2*f) + g)] + (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f
*g)]])*Log[((2*I)*c^2*f*(g + I*Sqrt[-(c^2*f*g)])*(I + c*x))/((c^2*f - g)*(
c^2*f - c*Sqrt[-(c^2*f*g)]*x))] + (ArcCos[(c^2*f + g)/(-(c^2*f) + g)] -...
```

3.1298.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5544, 2925, 2841, 2840, 2838, 5451, 2009, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx)) \left(d+e\log\left(f+gx^2\right)\right) dx$$

$$\Big\downarrow \text{5544}$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-bc\int \frac{x\left(d+e\log\left(gx^2+f\right)\right)}{c^2x^2+1}dx+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2925}$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{c^2x^2+1}dx^2+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2841}$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)(d+e\log(f+gx^2))}{c^2}-\frac{eg\int \frac{\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)}{gx^2+f}dx^2}{c^2}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2840}$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)(d+e\log(f+gx^2))}{c^2}-\frac{e\int \frac{\log\left(1-\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)}{x^2}d(gx^2+f)}{c^2}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)(d+e\log(f+gx^2))}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{5451}$$

$$-2eg\left(\frac{\int (a+b\arctan(cx))dx}{g}-\frac{f\int \frac{a+b\arctan(cx)}{gx^2+f}dx}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)(d+e\log(f+gx^2))}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\int \frac{a+b\arctan(cx)}{gx^2+f}dx}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)(d+e\log(f+gx^2))}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{5445}$$

3.1298. $\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(a\int\frac{1}{gx^2+f}dx+b\int\frac{\arctan(cx)}{gx^2+f}dx\right)}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 218$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(b\int\frac{\arctan(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 5443$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int\frac{\log(1-icx)}{gx^2+f}dx-\frac{1}{2}i\int\frac{\log(icx+1)}{gx^2+f}dx\right)\right)}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 2856$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int\left(\frac{\sqrt{-f}\log(1-icx)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}\log(1-icx)}{2f\left(\sqrt{g}x+\sqrt{-f}\right)}\right)dx-\frac{1}{2}i\int\left(\right.\right.\right.}{g}\right.$$

$$x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 2009$$

3.1298. $\int(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\left(-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{i\sqrt{-f}c+\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(cx+i)}{\sqrt{-f}c+i\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}\right.\right.\right.}{}\right.$$

$$x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-$$
$$\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\operatorname{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

input
```
Int[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]) - 2*e*g*((a*x + b*x*ArcTan[c*
x] - (b*Log[1 + c^2*x^2])/(2*c))/g - (f*((a*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(
Sqrt[f]*Sqrt[g]) + b*((-1/2*I)*((Log[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]
*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1 + I*c*x]*Log
[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]
) - PolyLog[2, (Sqrt[g]*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])]/(2*Sqrt[-f]*S
qrt[g]) + PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])]/(2*Sq
rt[-f]*Sqrt[g])) + (I/2)*((Log[1 - I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(
c*Sqrt[-f] + I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1 - I*c*x]*Log[(c*(S
qrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - Po
lyLog[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])]/(2*Sqrt[-f]*Sqrt[
g]) + PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f] + I*Sqrt[g])]/(2*Sqrt[-f]
*Sqrt[g])))))/g) - (b*c*((Log[-((g*(1 + c^2*x^2))/(c^2*f - g))]*(d + e*Log
[f + g*x^2]))/c^2 + (e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/c^2))/2
```

3.1298.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.1298. $\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$

rule 5544
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2
*e*g   Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

3.1298.4 Maple [F]

$$\int (a+b\arctan (cx)) \left(d+e\ln \left(g\,x^{2}+f\right)\right) dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

3.1298.5 Fricas [F]

$$\int (a+b\arctan(cx)) \left(d+e\log\left(f+gx^2\right)\right) \, dx = \int (b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right) dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f), x
)
```

3.1298.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\arctan(cx)) \left(d+e\log\left(f+gx^2\right)\right) \, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

3.1298.7 Maxima [F]

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\int (b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
(2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 + f))*a*e
 + a*d*x + b*e*integrate(arctan(c*x)*log(g*x^2 + f), x) + 1/2*(2*c*x*arcta
n(c*x) - log(c^2*x^2 + 1))*b*d/c
```

3.1298.8 Giac [F]

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\int (b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1298.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\int (a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

input
```
int((a + b*atan(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int((a + b*atan(c*x))*(d + e*log(f + g*x^2)), x)
```

3.1299 $\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x}\,dx$

3.1299.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\,(d+e\log(f+gx^2))}{x}\,dx = {} & ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\\ & +\frac{1}{2}ibd\,\mathrm{PolyLog}(2,-icx)\\ & -\frac{1}{2}ibd\,\mathrm{PolyLog}(2,icx)\\ & +\frac{1}{2}ae\,\mathrm{PolyLog}\left(2,1+\frac{gx^2}{f}\right)\\ & +be\mathrm{Int}\left(\frac{\arctan(cx)\log\left(f+gx^2\right)}{x},x\right)\end{aligned}$$

output
```
b*e*CannotIntegrate(arctan(c*x)*ln(g*x^2+f)/x,x)+a*d*ln(x)+1/2*a*e*ln(-g*x
^2/f)*ln(g*x^2+f)+1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)+1
/2*a*e*polylog(2,1+g*x^2/f)
```

3.1299.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx=\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

3.1299.3 Rubi [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5550, 5355, 2838, 5548, 2904, 2841, 2752, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

$$\Big\downarrow \text{5550}$$

$$d\int \frac{a+b\arctan(cx)}{x}dx+e\int \frac{(a+b\arctan(cx))\log\left(gx^2+f\right)}{x}dx$$

$$\Big\downarrow \text{5355}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(gx^2+f\right)}{x}dx+ d\left(\frac{1}{2}ib\int \frac{\log(1-icx)}{x}dx-\frac{1}{2}ib\int \frac{\log(icx+1)}{x}dx+a\log(x)\right)$$

$$\Big\downarrow \text{2838}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(gx^2+f\right)}{x}dx+ d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{5548}$$

$$e\left(a\int \frac{\log(gx^2+f)}{x}dx+b\int \frac{\arctan(cx)\log(gx^2+f)}{x}dx\right)+ d\left(a\log(x)+\frac{1}{2}ib\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\,\mathrm{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{2904}$$

$$e\left(\frac{1}{2}a\int \frac{\log(gx^2+f)}{x^2}dx^2+b\int \frac{\arctan(cx)\log(gx^2+f)}{x}dx\right)+ d\left(a\log(x)+\frac{1}{2}ib\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\,\mathrm{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{2841}$$

$$e\left(\frac{1}{2}a\left(\log\left(-\frac{gx^2}{f}\right)\log(f+gx^2)-g\int \frac{\log\left(-\frac{gx^2}{f}\right)}{gx^2+f}dx^2\right)+b\int \frac{\arctan(cx)\log(gx^2+f)}{x}dx\right)+ d\left(a\log(x)+\frac{1}{2}ib\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\,\mathrm{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{2752}$$

$$e\left(b\int \frac{\arctan(cx)\log(gx^2+f)}{x}dx+\frac{1}{2}a\left(\mathrm{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log(f+gx^2)\right)\right)+ d\left(a\log(x)+\frac{1}{2}ib\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\,\mathrm{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{7299}$$

$$e\left(b\int \frac{\arctan(cx)\log(gx^2+f)}{x}dx+\frac{1}{2}a\left(\mathrm{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log(f+gx^2)\right)\right)+ d\left(a\log(x)+\frac{1}{2}ib\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\,\mathrm{PolyLog}(2,icx)\right)$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
$Aborted
```

3.1299. $\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x}\,dx$

3.1299.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5548
```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTan[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_S
ymbol] :> Simp[a   Int[Log[f + g*x^2]/x, x], x] + Simp[b   Int[Log[f + g*x^
2]*(ArcTan[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 5550
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
(d_)))/(x_), x_Symbol] :> Simp[d   Int[(a + b*ArcTan[c*x])/x, x], x] + Simp
[e   Int[Log[f + g*x^2]*((a + b*ArcTan[c*x])/x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.1299. $\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x}\,dx$

3.1299.4 Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(g\,x^2+f))}{x}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

3.1299.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(gx^2+f)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/
x, x)
```

3.1299.6 Sympy [N/A]

Not integrable

Time = 123.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(f+gx^2))}{x}\,dx = \int \frac{(a+b\operatorname{atan}(cx))\,(d+e\log(f+gx^2))}{x}\,dx$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*log(f + g*x**2))/x, x)
```

3.1299.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx=\int \frac{(b\arctan\left(cx\right)+a)(e\log\left(gx^2+f\right)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

output
```
a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*e*arctan(c*x) + a*e)*lo
g(g*x^2 + f))/x, x)
```

3.1299.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

output
```
Timed out
```

3.1299.9 Mupad [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

3.1300 $\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

3.1300.1 Optimal result

Integrand size = 24, antiderivative size = 672

$$
\begin{aligned}
\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x^2}\,dx
&= \frac{2ae\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\log(1+icx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}} \\
&+ \frac{ibe\sqrt{g}\log(1-icx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{ibe\sqrt{g}\log(1-icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}} \\
&+ \frac{ibe\sqrt{g}\log(1+icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x} \\
&+ \frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+gx^2)) - \frac{1}{2}bc\log\left(-\frac{g(1+c^2x^2)}{c^2f-g}\right)(d+e\log(f+gx^2)) \\
&+ \frac{ibe\sqrt{g}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(i-cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{ibe\sqrt{g}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} \\
&- \frac{ibe\sqrt{g}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(1+icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{ibe\sqrt{g}\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(i+cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} \\
&- \frac{1}{2}bce\operatorname{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right) + \frac{1}{2}bce\operatorname{PolyLog}\left(2,1+\frac{gx^2}{f}\right)
\end{aligned}
$$

output
```
-(a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+
f))-1/2*b*c*ln(-g*(c^2*x^2+1)/(c^2*f-g))*(d+e*ln(g*x^2+f))-1/2*b*c*e*polyl
og(2,c^2*(g*x^2+f)/(c^2*f-g))+1/2*b*c*e*polylog(2,1+g*x^2/f)-1/2*I*b*e*ln(
1+I*c*x)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)-I*g^(1/2)))*g^(1/2)/(-f
)^(1/2)+1/2*I*b*e*ln(1-I*c*x)*ln(c*((-f)^(1/2)-x*g^(1/2))/(c*(-f)^(1/2)+I*
g^(1/2)))*g^(1/2)/(-f)^(1/2)-1/2*I*b*e*ln(1-I*c*x)*ln(c*((-f)^(1/2)+x*g^(1
/2))/(c*(-f)^(1/2)-I*g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2*I*b*e*ln(1+I*c*x)*ln
(c*((-f)^(1/2)+x*g^(1/2))/(c*(-f)^(1/2)+I*g^(1/2)))*g^(1/2)/(-f)^(1/2)+1/2
*I*b*e*polylog(2,(I-c*x)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*g^(1/2)/(-f)^(1
/2)+1/2*I*b*e*polylog(2,(c*x+I)*g^(1/2)/(c*(-f)^(1/2)+I*g^(1/2)))*g^(1/2)/
(-f)^(1/2)-1/2*I*b*e*polylog(2,(1-I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+g^(1/2)))
*g^(1/2)/(-f)^(1/2)-1/2*I*b*e*polylog(2,(1+I*c*x)*g^(1/2)/(I*c*(-f)^(1/2)+
g^(1/2)))*g^(1/2)/(-f)^(1/2)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/
2)
```

3.1300.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 552, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx$$

$$= \frac{1}{2}\left(-\frac{2(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\right.$$

$$+\frac{e\sqrt{g}\left(4a\sqrt{-f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)+ib\sqrt{f}\left(\log(1+icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)+\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(i-cx)}{c\sqrt{-f}+i\sqrt{g}}\right)\right)-ib\sqrt{f}\cdots\right.}{\cdots}$$

$$+bc\left(\left(\log\left(-\frac{gx^2}{f}\right)-\log\left(-\frac{g(1+c^2x^2)}{c^2f-g}\right)\right)\left(d+e\log\left(f+gx^2\right)\right)\right.$$

$$\left.\left.-e\operatorname{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right)+e\operatorname{PolyLog}\left(2,1+\frac{gx^2}{f}\right)\right)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
((-2*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x + (e*Sqrt[g]*(4*a*Sqrt[
-f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt
[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I - c*
x))/(c*Sqrt[-f] + I*Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqrt[
-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 - I*c
*x))/(I*c*Sqrt[-f] + Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt
[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 + I*
c*x))/(I*c*Sqrt[-f] + Sqrt[g])]) + I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqr
t[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I + c
*x))/(c*Sqrt[-f] + I*Sqrt[g])])))/Sqrt[-f^2] + b*c*((Log[-((g*x^2)/f)] - L
og[-((g*(1 + c^2*x^2))/(c^2*f - g))])*(d + e*Log[f + g*x^2]) - e*PolyLog[2
, (c^2*(f + g*x^2))/(c^2*f - g)] + e*PolyLog[2, 1 + (g*x^2)/f]))/2
```

3.1300.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 649, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5552, 2925, 2863, 2009, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}dx$$

$$\Big\downarrow \text{5552}$$

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx+bc\int \frac{d+e\log\left(gx^2+f\right)}{x\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{x^2\left(c^2x^2+1\right)}dx^2-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2863}$$

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \left(\frac{d+e\log\left(gx^2+f\right)}{x^2}-\frac{c^2\left(d+e\log\left(gx^2+f\right)\right)}{c^2x^2+1}\right)dx^2-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+$$

$$\Big\downarrow \text{5445}$$

$$2eg\left(a\int \frac{1}{gx^2+f}dx+b\int \frac{\arctan(cx)}{gx^2+f}dx\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+$$

$$\Big\downarrow \text{218}$$

$$2eg\left(b\int \frac{\arctan(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+$$

$$\Big\downarrow \text{5443}$$

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int \frac{\log(1-icx)}{gx^2+f}dx-\frac{1}{2}i\int \frac{\log(icx+1)}{gx^2+f}dx\right)\right)-$$
$$\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+$$

$$\Big\downarrow \text{2856}$$

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int \left(\frac{\sqrt{-f}\log(1-icx)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}\log(1-icx)}{2f\left(\sqrt{g}x+\sqrt{-f}\right)}\right)dx-\frac{1}{2}i\int \left(\frac{\sqrt{-f}\log(icx+1)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-}}{2f}\right.\right.\right.$$
$$\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$\frac{1}{2}bc\left(-\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+$$

$$\Big\downarrow \text{2009}$$

3.1300. $\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

$$-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$
$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\left(-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{i\sqrt{-f}c+\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(cx+i)}{\sqrt{-f}c+i\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log(1-icx)\log\left(\frac{c(\sqrt{-f}-\sqrt{}}{c\sqrt{-f}+i}\right.}{2\sqrt{-f}\sqrt{g}}\right.\right.\right.$$
$$\frac{1}{2}bc\left(-\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\operatorname{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-(((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x) + 2*e*g*((a*ArcTan[(Sqrt
[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) + b*((-1/2*I)*((Log[1 + I*c*x]*Log[(c*(
Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (
Log[1 + I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/(
2*Sqrt[-f]*Sqrt[g]) - PolyLog[2, (Sqrt[g]*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[
g])]/(2*Sqrt[-f]*Sqrt[g]) + PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f]
 + Sqrt[g])]/(2*Sqrt[-f]*Sqrt[g])) + (I/2)*((Log[1 - I*c*x]*Log[(c*(Sqrt[-
f] - Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1
- I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt
[-f]*Sqrt[g]) - PolyLog[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])]
/(2*Sqrt[-f]*Sqrt[g]) + PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f] + I*Sqr
t[g])]/(2*Sqrt[-f]*Sqrt[g])))) + (b*c*(Log[-((g*x^2)/f)]*(d + e*Log[f + g*
x^2]) - Log[-((g*(1 + c^2*x^2))/(c^2*f - g))]*(d + e*Log[f + g*x^2]) - e*P
olyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)] + e*PolyLog[2, 1 + (g*x^2)/f]))/2
```

3.1300.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

3.1300.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(g\,x^{2}+f\right)\right)}{x^{2}}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

3.1300.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)(e\log(gx^2+f)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/
x^2, x)
```

3.1300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

output
```
Timed out
```

3.1300.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)(e\log(gx^2+f)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + (2*g*arctan
(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + b*e*integrate(arctan(c
*x)*log(g*x^2 + f)/x^2, x) - a*d/x
```

3.1300.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.1300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

3.1301 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx$

3.1301.1 Optimal result

Integrand size = 24, antiderivative size = 528

$$
\begin{aligned}
&\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx \\
&= \frac{bce\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}-\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2}{1-icx}\right)}{f} \\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{\left(c\sqrt{-f}-i\sqrt{g}\right)(1-icx)}\right)}{2f} \\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{\left(c\sqrt{-f}+i\sqrt{g}\right)(1-icx)}\right)}{2f}-\frac{aeg\log\left(f+gx^2\right)}{2f} \\
&\quad-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(f+gx^2\right)\right) \\
&\quad-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}+\frac{ibeg\operatorname{PolyLog}(2,-icx)}{2f} \\
&\quad-\frac{ibeg\operatorname{PolyLog}(2,icx)}{2f}+\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2f} \\
&\quad-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{\left(c\sqrt{-f}-i\sqrt{g}\right)(1-icx)}\right)}{4f} \\
&\quad-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}+\sqrt{g}x\right)}{\left(c\sqrt{-f}+i\sqrt{g}\right)(1-icx)}\right)}{4f}
\end{aligned}
$$

output
```
a*e*g*ln(x)/f-b*e*(c^2*f-g)*arctan(c*x)*ln(2/(1-I*c*x))/f-1/2*a*e*g*ln(g*x
^2+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x-1/2*b*c^2*arctan(c*x)*(d+e*ln(g*x^2+f)
)-1/2*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2+1/2*b*e*(c^2*f-g)*arctan(c*x
)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)-I*g^(1/2)))/f+1/2*
b*e*(c^2*f-g)*arctan(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(1-I*c*x)/(c*(-f)^
(1/2)+I*g^(1/2)))/f+1/2*I*b*e*g*polylog(2,-I*c*x)/f-1/2*I*b*e*g*polylog(2,
I*c*x)/f+1/2*I*b*e*(c^2*f-g)*polylog(2,1-2/(1-I*c*x))/f-1/4*I*b*e*(c^2*f-g
)*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(1-I*c*x)/(c*(-f)^(1/2)-I*g^(1/2)
))/f-1/4*I*b*e*(c^2*f-g)*polylog(2,1-2*c*((-f)^(1/2)+x*g^(1/2))/(1-I*c*x)/
(c*(-f)^(1/2)+I*g^(1/2)))/f+b*c*e*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2
)
```

3.1301.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1217 vs. $2(528) = 1056$.

Time = 5.20 (sec) , antiderivative size = 1217, normalized size of antiderivative = 2.30

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx =$$

$$-\frac{2adf+2bcdfx+2bdf\arctan(cx)+2bc^2dfx^2\arctan(cx)-4bce\sqrt{f}\sqrt{g}x^2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)-4ibc^2efx^2\operatorname{arcs}}{}$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
-1/4*(2*a*d*f + 2*b*c*d*f*x + 2*b*d*f*ArcTan[c*x] + 2*b*c^2*d*f*x^2*ArcTan
[c*x] - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - (4*I)*b*
c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]
] + (4*I)*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[
c^2*f*g]] - 4*b*e*g*x^2*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + 4*b*c
^2*e*f*x^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 2*b*c^2*e*f*x^2*Ar
cSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (
-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^
2*f - g)] + 2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^
((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTa
n[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] - 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[(c^2
*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*
I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[
(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^
((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*c^2*e*f*x^2*ArcSin[S
qrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sq
rt[c^2*f*g]))/(c^2*f - g)] - 2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]
*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)
] - 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g
+ 2*Sqrt[c^2*f*g]))/(c^2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[1 + (E^(...
```

3.1301.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}dx$$

$$\Big\downarrow \text{5556}$$

$$-2eg\int \left(-\frac{a+bcx}{2x\left(gx^2+f\right)}-\frac{b\left(c^2x^2+1\right)\arctan(cx)}{2x\left(gx^2+f\right)}\right)dx-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

$$\Big\downarrow \text{2009}$$

$$-2eg\left(\frac{a\log\left(f+gx^2\right)}{4f}-\frac{a\log(x)}{2f}+\frac{b\arctan(cx)\left(c^2f-g\right)\log\left(\frac{2}{1-icx}\right)}{2fg}-\frac{b\arctan(cx)\left(c^2f-g\right)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(1-icx)(c\sqrt{-f}-i\sqrt{g})}\right)}{4fg}\right.$$

$$\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[f + g*x^2]))/x - (b*c^2*ArcTan[c*x]*(d + e*Log[f + g*
x^2]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) - 2*e*g*(-
1/2*(b*c*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (a*Log[x])/(2*f)
 + (b*(c^2*f - g)*ArcTan[c*x]*Log[2/(1 - I*c*x)])/(2*f*g) - (b*(c^2*f - g)
*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1
 - I*c*x))])/(4*f*g) - (b*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqr
t[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/(4*f*g) + (a*Log[f + g*x
^2])/(4*f) - ((I/4)*b*PolyLog[2, (-I)*c*x])/f + ((I/4)*b*PolyLog[2, I*c*x]
)/f - ((I/4)*b*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x)])/(f*g) + ((I/8)*b
*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*
Sqrt[g])*(1 - I*c*x))])/(f*g) + ((I/8)*b*(c^2*f - g)*PolyLog[2, 1 - (2*c*(
Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/(f*g))
```

3.1301.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

3.1301.4 Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(g\,x^{2}+f\right)\right)}{x^{3}}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

3.1301.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^{2}\right)\right)}{x^{3}}\,dx=\int \frac{(b\arctan(cx)+a)(e\log\left(gx^{2}+f\right)+d)}{x^{3}}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/
x^3, x)
```

3.1301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^{2}\right)\right)}{x^{3}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

output
```
Timed out
```

3.1301.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx=\int \frac{(b\arctan\left(cx\right)+a)(e\log\left(gx^2+f\right)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - 1/2*(g*(log(g*x^2 +
 f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e + 1/2*(2*c*g*x^2*arctan(g*x/
sqrt(f*g)) + (4*c^2*g*x^2*integrate(1/2*x*arctan(c*x)/(g*x^2 + f), x) + 4*
g*x^2*integrate(1/2*arctan(c*x)/(g*x^3 + f*x), x) - (c*x + (c^2*x^2 + 1)*a
rctan(c*x))*log(g*x^2 + f))*sqrt(f*g))*b*e/(sqrt(f*g)*x^2) - 1/2*a*d/x^2
```

3.1301.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.1301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
            return 1
        elif isinstance(expn,list):
            return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
        elif  is_sqrt(expn):
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        elif isinstance(expn,Pow):    #type(expn,'`^`')
            if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
                return expnType(expn.args[0])    #ExpnType(op(1,expn))
            elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
                if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                    return 1
                else:
                    return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
            else:
                return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
        elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
            m1 = expnType(expn.args[0])
            m2 = expnType(list(expn.args[1:]))
            return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
        elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
            return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
        elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
        elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif is_appell_function(expn.func):
            m1 = max(map(expnType, list(expn.args)))
            return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif isinstance(expn,RootSum):
            m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
            return max(7,m1)
        elif str(expn).find("Integral") != -1:
            m1 = max(map(expnType, list(expn.args)))
            return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        else:
            return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```